

Algorithms (IT 301)

Assignment I

Last date of submission: 21st February, 2017.

1. An inversion in an array $A[1..n]$ is a pair of indices (i, j) such that $i < j$ and $A[i] > A[j]$. The number of inversions in an n -element array is between 0 (if the array is sorted) and $\binom{n}{2}$ (if the array is sorted backward).

Design and analyze a divide-and-conquer algorithm to count the number of inversions in an n -element array in $O(n \log n)$ time. Assume all the elements of the input array are distinct.

2. Suppose you are given two sets of n points, one set $\{p_1, p_2, \dots, p_n\}$ on the line $y = 0$ and the other set $\{q_1, q_2, \dots, q_n\}$ on the line $y = 1$. Create a set of n line segments by connect each point p_i to the corresponding point q_i . Design and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect, in $O(n \log n)$ time. [Hint: Use your solution to problem 1.] Assume a reasonable representation for the input points, and assume the x -coordinates of the input points are distinct.
3. Now suppose you are given two sets $\{p_1, p_2, \dots, p_n\}$ and $\{q_1, q_2, \dots, q_n\}$ of n points on the unit circle. Connect each point p_i to the corresponding point q_i . Design and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect in $O(n \log^2 n)$ time. [Hint: Use your solution to problem 2.] Assume a reasonable representation for the input points, and assume all input points are distinct.