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Analysis of Structures

Project Report

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July 2019

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1 Introduction to programming languages

Programming languages provides a means to communicate with the computers through instructions often referred to as a code or set of codes fed in syntactical order in an apprehensive manner as dictated by the guidelines of a Programming Language. These set of instructions/codes devised for carrying out a task or a multitudes of tasks are called Programs. The guidelines of a Programming Language are the syntax which are predefined by the developers and that could be easily formulated by the computer to be comprehended in it's own language, termed as the Machine Language. This method of formulation is famously known as Compilation.

In view of this subject, following are the two programming languages explained concisely in which the assigned tasks have been written.

1.1 Python

Originally created by Guido Van Rossum and released in 1991, Python is a multi-paradigm programming language. It fully supports Object-oriented programming and structured programming. In addition, it also features a dynamic type system and automatic memory management.Rather than having all of its functionality built into its core, Python was developed predominantly to be highly extensible. The compact modularity has made it particularly popular as a means of adding programmable interfaces to existing applications.

This leads to successful inclusion of Python as scripting language in many software products, including in finte-element-method softwares, such as Abaqus.

1.2 Abaqus

Abaqus FEA formerly known as ABAQUS is a software suite for finite element analysis, originally released in 1978. Abaqus is pre-dominantly used in the automotive, aerospace, and industrial products industries. The product is popular due to the wide material modeling capability, and the program's ability to be customized, for example, users can define their own material models so that new materials could also be simulated in Abaqus.

Abaqus was initially designed to address non-linear physical behavior; as a result, the package has an extensive range of material models such as elastomeric (rubberlike) and hyperelastic (soft tissue) material capabilities.[1]

2 Project 1 – Calculation of section values for the combined profile using thin walled approach in Python

A python program code is written to perform numerical calculations in order to determine – area, center of mass, static moments and moment of inertia of a given combined Profile. The combinations consists of two L-profiles and one O-profile and is symmetric about z-axis.

To achieve this task, various classes are built upon collecting attributes and methods with which objects (or) instances are generated.The combined profile is generated by creating classes like – base, node, element, profile and list. For simplification in programming, thin-walled approach is implemented for the profile.

The aim of the task is –

- To validate the results obtained from the program with analytical results.
- To discuss the errors in the evaluations.

2.1 Problem definition

With an overview of Object-Oriented Programming, it is required to develop a python program that calculates the section properties of a combination of several profiles placed juxtaposing each other in the below figure.

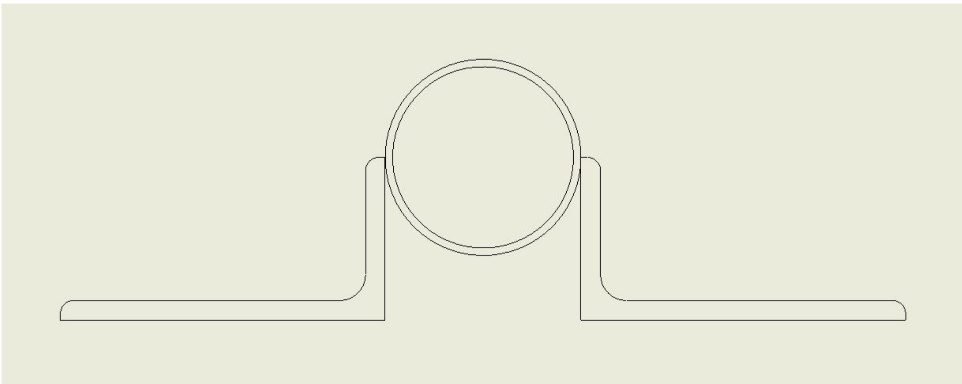


Figure 1: Actual view of the combined profile

The profile comprises of two L-Profiles of varying cross sections which is combined with one O-profile. The following sectional properties are needed to be evaluated with the help of this program –

- Area of combined profile.
- Static moments.
- Centre of mass.
- Moment of inertia.

The dimensional properties of the profiles as per the DIN standards are mentioned in the tables below. 1 and 2 –

Profile	k[mm]	bb[mm]	z[mm]
L 100x50x6	100	50	6
L 100x75x9	100	75	9
L 120x80x8	120	80	8

Table 1: L-profile dimensions

Profile	d[mm]	Tt[mm]
O 60.3x2.3	60.3	2.3

Table 2: O-profile dimensions

2.2 Thin-walled approximation

Thin-walled structures comprise an important and growing proportion of engineering construction with areas of application becoming increasingly diverse ranging from aircraft, bridges, ships and oil rigs to storage vessels, industrial buildings and warehouses.To analyze and calculate the section properties of combined profile, we have used the Thin Walled Approach.The Profile has been approximated as 2-Dimensional line elements having constant thickness throughout the section and fillets across the edges can be neglected. The simplified geometry of combined profile using Thin Walled Approach is shown in the below figure.

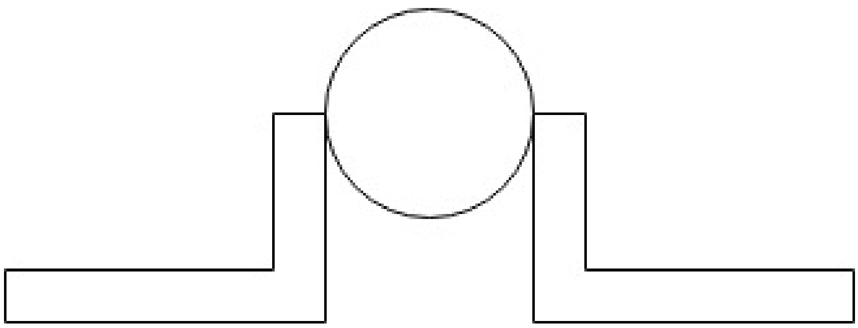


Figure 2: Simplified geometry of the combined profile

The sectional properties required to be approximated as listed above are,

- **Area:** Analytically the area of the profile is an integral function of all the infinitesimally small areas of the elements of the profile discretised for Finite element analysis. The corresponding approximation can be derived using Quadrature Rule,however, for a simplified geometry as depicted in the figure- 2, it is convenient to produce it by the expressions below.

$$A = \int_A e_\mu.dA \approx \sum_{i=1}^{nEle} A_i = \sum_{i=1}^{nEle} e_{\mu,i}.L_i.t_i \quad (1)$$

Where,

- L_i – Length of line i
- t_i – Thickness of line i
- $e_{\mu,i}$ – Relative elasticity of line i (1 for only one material)

- **First moment of an area:** Static or the first moments of an area is given below with its corresponding approximation for sum over individual elemental properties.

$$S_y = \int_A e_\mu.z.dA \approx \sum_{i=1}^{nEle} e_{\mu,i}.\overline{z}_i.A_i \tag{2}$$

$$S_z = \int_A e_\mu.y.dA \approx \sum_{i=1}^{nEle} e_{\mu,i}.\overline{y}_i.A_i \tag{3}$$

Where,

- A_i – Area of line i
- $\overline{y}_i$ – The y coordinate of the center of line i
- $\overline{z}_i$ – The z coordinate of the center of line i

- **Centre of mass:** The Centre of mass for a structure having uniformly distributed mass can be analytically integrated over the area and correspondingly approximated with the summation expression as:

$$y_c = \frac{\int_A y.dA}{\int_A dA} \approx \frac{\sum_{i=1}^{nEle} y_i A_i}{\sum_{i=1}^{nEle} A_i} \tag{4}$$

$$z_c = \frac{\int_A z.dA}{\int_A dA} \approx \frac{\sum_{i=1}^{nEle} z_i A_i}{\sum_{i=1}^{nEle} A_i} \tag{5}$$

- **Second moments of an area or Moment of inertia:** Moment of Inertia of the body is the tendency by virtue of which the body resists rotational motion or continue to maintain its rotational motion. The Area Moment of Inertia or the Second moment of Inertia about the Centre of Mass of a cylindrical body is expressed as:

$$I_y = \int_A e_\mu.z^2.dA \approx \sum_{i=1}^{nEle} e_{\mu,i}.\left(\frac{(z_{b,i} - z_{a,i})^2}{12} + \overline{z}_i^2\right).A_i \tag{6}$$

$$I_z = \int_A e_\mu.y^2.dA \approx \sum_{i=1}^{nEle} e_{\mu,i}.\left(\frac{(y_{b,i} - y_{a,i})^2}{12} + \overline{y}_i^2\right).A_i \tag{7}$$

where,

- A_i – The area of line i
- $\overline{y}_i$ – The y coordinate of the center of line i
- $\overline{z}_i$ – The z coordinate of the center of line i
- $y_{a,i}$ – The y coordinate of the first point of line i
- $z_{a,i}$ – The z coordinate of the first point of line i
- $y_{b,i}$ – The y coordinate of the second point of line i
- $z_{b,i}$ – The z coordinate of the second point of line i

On application of the Parallel axis theorem (Steiner theorem), the Moment of inertia of the body about any arbitrary point at a distance z_i from Y-axis and y_i from Z-Axis is given as,

$$I_y = I_{y,c} + \sum_{i=1}^{nEle} z_i^2.A_i \tag{8}$$

$$I_z = I_{z,c} + \sum_{i=1}^{nEle} y_i^2.A_i \tag{9}$$

2.3 Problem simplification

The actual profile contains complex geometry, like fillets at the edges and corners. The program has been coded to calculate the sectional properties of the rectangular elements and hence, it is required to be simplified. This simplification is undoubtedly sure to render the results imprecise however, accurate, which is further discussed in the section.

The simplified geometry discretised to nodes and elements is depicted in the figure below –

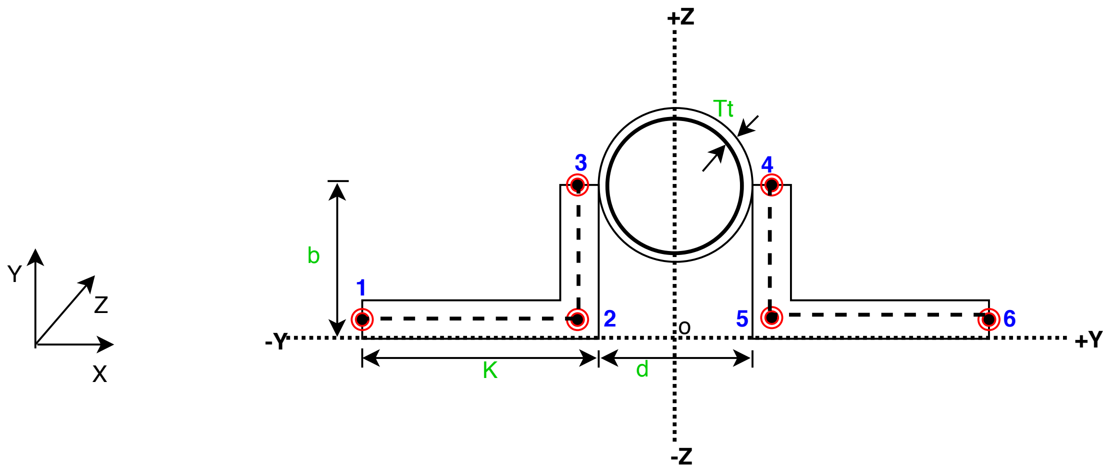


Figure 3: Discretised geometry of the combined profile

Node	Coordinates	
	Y	Z
1	-r - k	z/2
2	-r	z/2
3	-r	bb
4	r	bb
5	r	z/2
6	r + k	z/2

Table 3: Node chart with coordinates

Elements	Node-1	Node-2	Thickness
1	1	2	z
2	2	3	z
3	4	5	z
4	5	6	z

Table 4: Element chart

2.4 Programmer's Guide

After simplification of geometry, it is required to develop different classes and to generate inheritance between them. The basic introduction of the UML diagram is explained below.

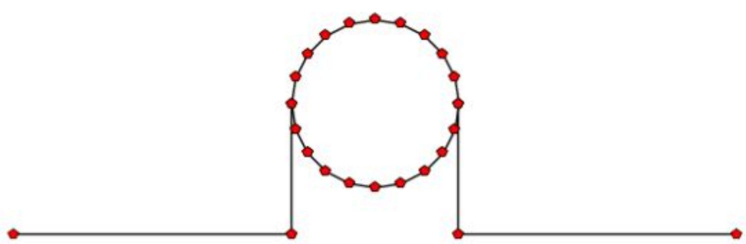


Figure 4: Combined profile using lines as elements and nodes as points

2.4.1 UML Diagram:

UML(Unified Modeling Language) diagram is used to visualize design of a system. UML structure diagrams emphasize the things that must be present in the system being modelled. Since structure diagrams represent the structure, they are used extensively in documenting the architecture of software systems. In our description of the examples we want to implement, we use the class diagram which describes the structure of a system by showing the system's classes, their attributes, and the relationship among the classes. The UML diagram consists of a rectangular box, which is divided into three sections.

- **First Section** – Contains name of class.
- **Second Section** – Contains name of attribute.
- **Third Section** – Contains name of method.

Some symbols are used to develop the UML diagram. Every symbols are used for specific purpose.

- The white filled arrowhead points from inheriting class to the inherited class.
- Black filled rhombus represents the composition. The parts which exists in profile are described as a composition.
- White filled rhombus represents the aggregation. The parts which do not exists in profile are described as an aggregation.
- + : It represents that the items are accessible from outside which means it represent public attributes.
- − : It represents that the items are not accessible from outside which means it represents private attributes.

The class hierarchy of the combined profile is shown in the diagram below –

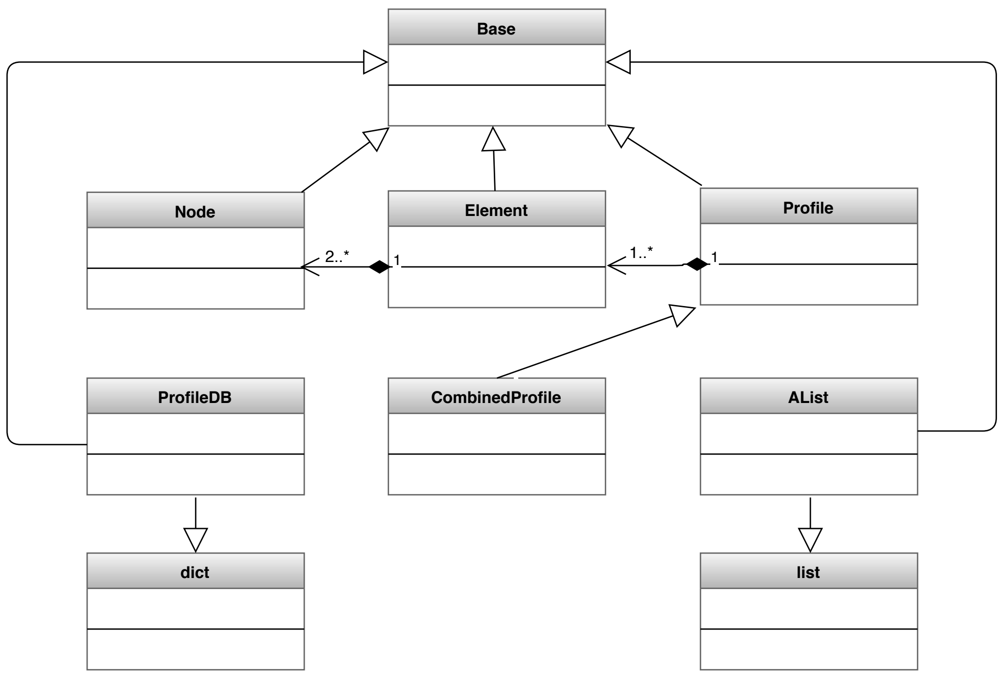


Figure 5: Class hierarchy of developed program

2.4.2 General base class:

The class `Base` can inherit more than one class. The class `Base` connects all derived classes viz. node, element, profile. It contains logging for all classes. It has three properties viz. common log file, instance counter and appendLog. The class `Base` composes the message substance support to the screen into a log file. When the output windows pops up, we can see an instance created and deleted. The actual time is also called and printed on our logging comment. The UML diagram of `Base` class with its attributes and methods are shown below.

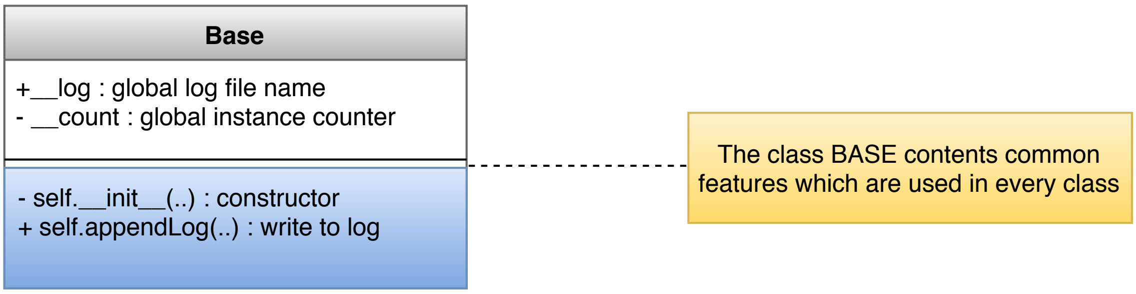


Figure 6: UML diagram of Base class

2.4.3 Node class:

The class `Node` contains coordinates which are later connected to create an element. It has two attributes viz. node number and List for points coordinates. A list method able to print the instance data to a given output stream. The values of coordinates are passed through constructor and assigned to the attributes. The UML diagram of the `Node` class is shown below.

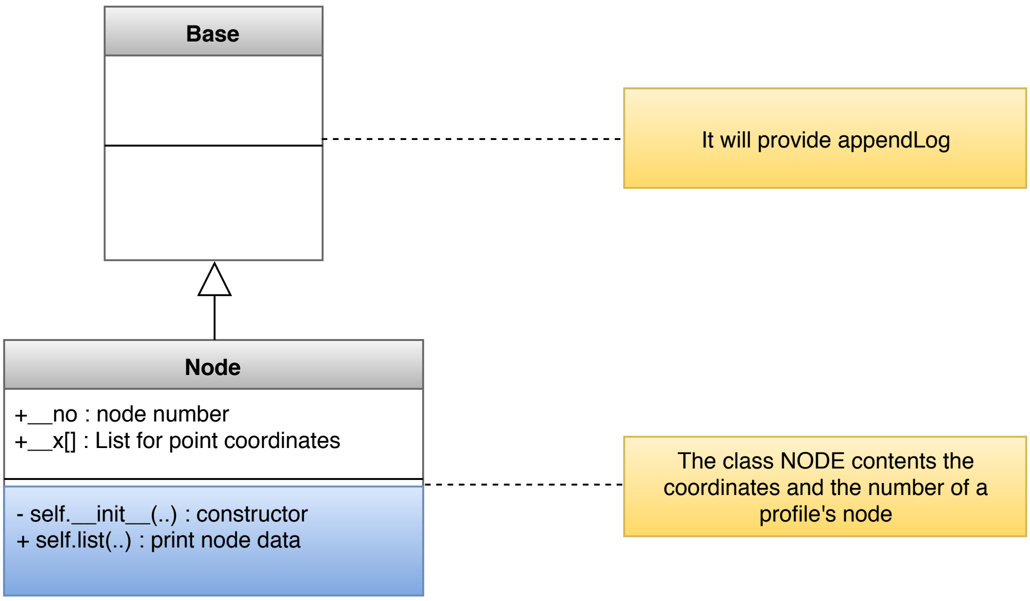


Figure 7: UML diagram of Node class

It is obvious to define node number as an integer. In node class `_no` is defined as an Integer. An error must occur while entering the floating value or string for the node number and it is declared using `raise` function in node class.

2.4.4 Element class:

The class `Element` will be able to connect two nodes. It is also able to calculate the element's section values viz. area, static moment, center coordinate and moment of inertia. The node number, thickness and coordinates are passed through the constructor. With the help of different method like `getL()`, `getA()`, `getC()`, `getS()` and `getI()`, section properties of the elements are calculated.

- `delX(..)`: Calculates the projected length from predefined point.
- `getL(..)`: Calculates the length of the element.
- `getA(..)`: Calculates the area of the element by multiplying the length of the element and its thickness.
- `getC(..)`: Calculates the center coordinates of the element.
- `getS(..)`: Calculates the static moment of the element.
- `getI(..)`: Calculates the moment of inertia of the element.
- `list(..)`: It is used to print the calculated data of the element in log file.

The class `Element` inheriting the `Node` class. The UML diagram of the Element class is shown below.

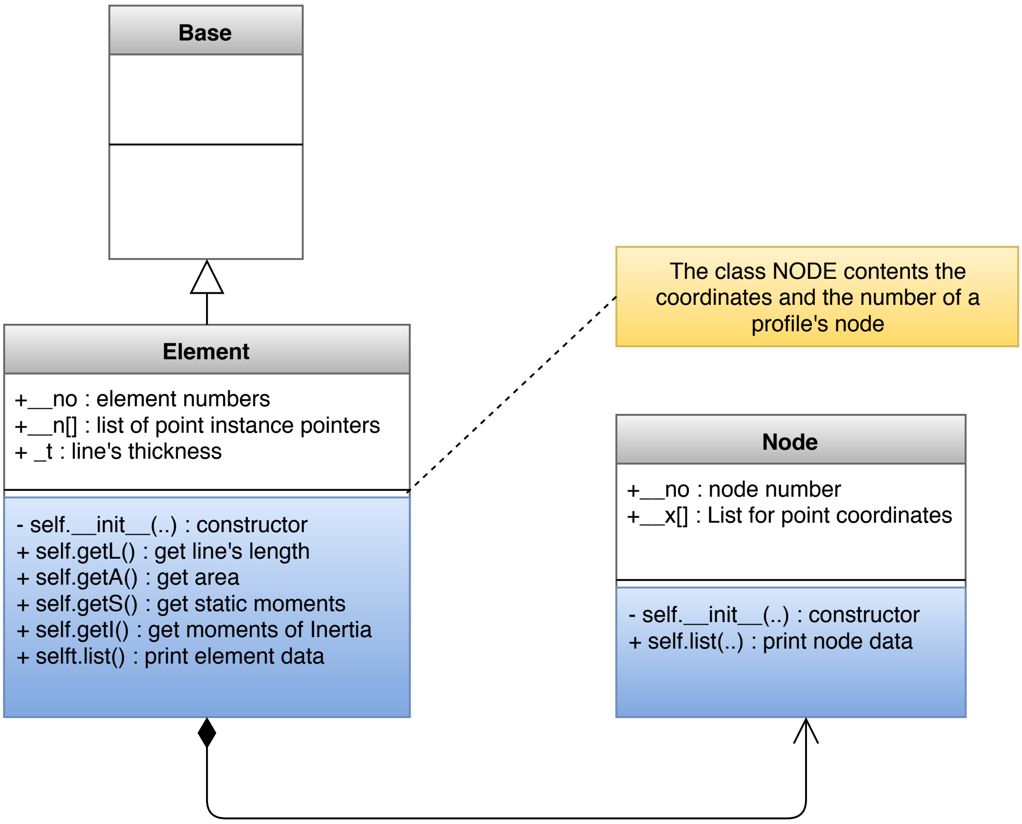


Figure 8: UML diagram of Element class

2.4.5 General profile class

Every profile contains the common features. This common features are implemented in General Profile class. The class `Profile` has two attributes viz profile's name and element container. The element container is introduced using `AList` class. With this method elements are added into the container. To calculate the global section values of the thin walled geometry, different methods viz. getPvalues and getResults are introduced in Profile class.

The methods used in this class are as follows –

- `addElement(..)`: Method is used to add elements in profile.
- `getResults(..)`: Calculates the section properties of the profile.
- `getPValues(..)`: Calculates the principle values of the profile.
- `list(..)`: It is used to print the calculated values of section properties on screen.
- `plot(..)`: It is used to draw lines and nodes of the profile and it will plot the graphical picture of our profile. It will also save the profile in `.png` format.

The UML diagram of the `Profile` class is shown below.

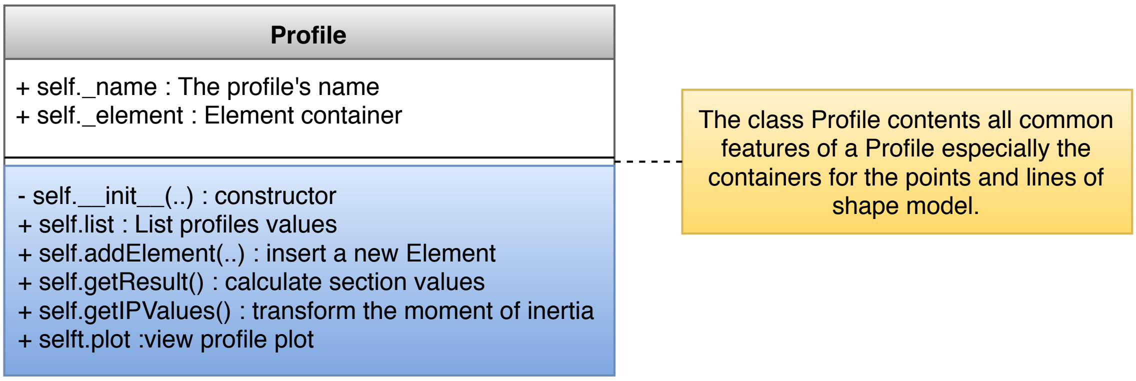


Figure 9: UML diagram of Profile class

2.4.6 AList class

The class `AList` inherited from `Base` and `list` class. The class Base is used to print and list is used to store the instance pointers. If the index is greater than the list's length, then it is required to increase the list's length, otherwise the item to be inserted will be appended. For this, add method is implemented in AList class.

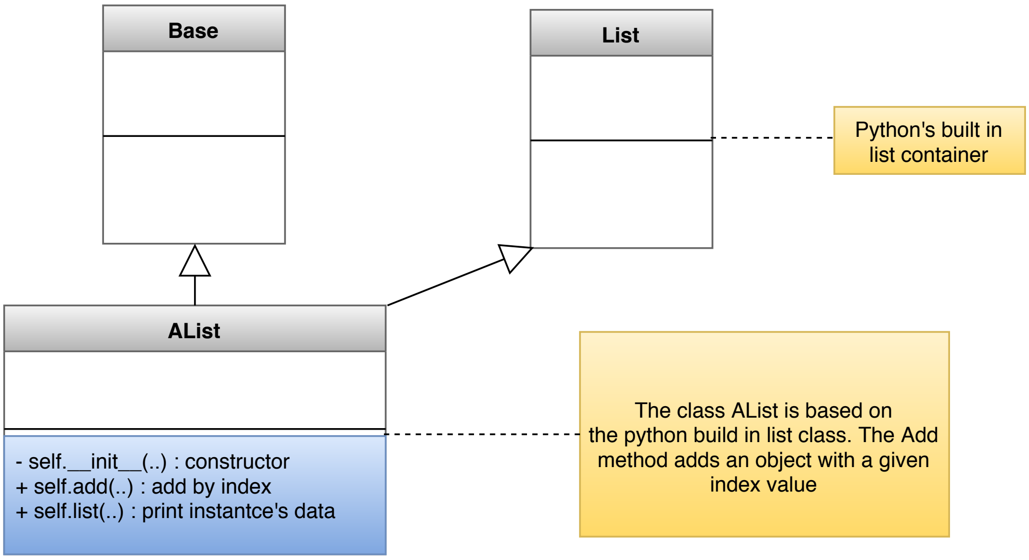


Figure 10: UML diagram of AList class

2.4.7 CombinedProfile class

The class `CombinedProfile` is user defined Class. Various dimensional parameters are defined according to the geometry of the combined profile. The user coordinates are selected as shown in Table- 3. The nodes and elements are created according to chosen dimensional parameter.

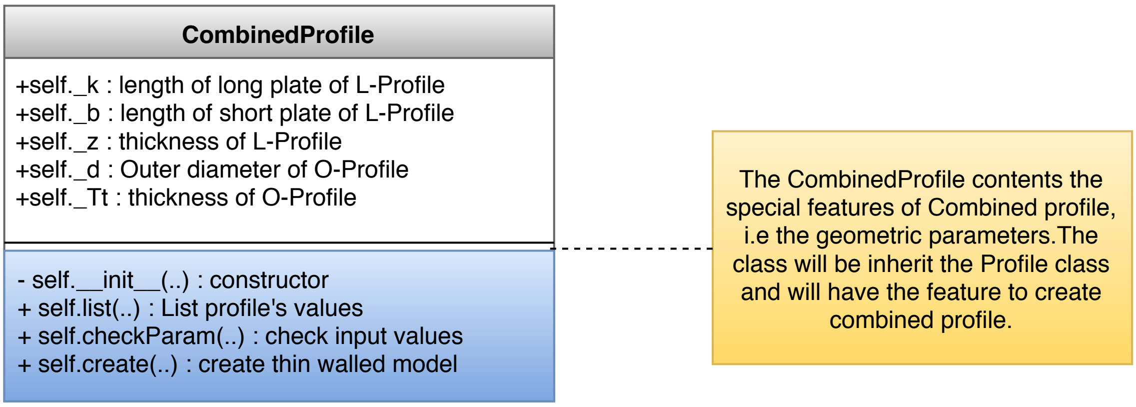


Figure 11: UML diagram of CombinedProfile class

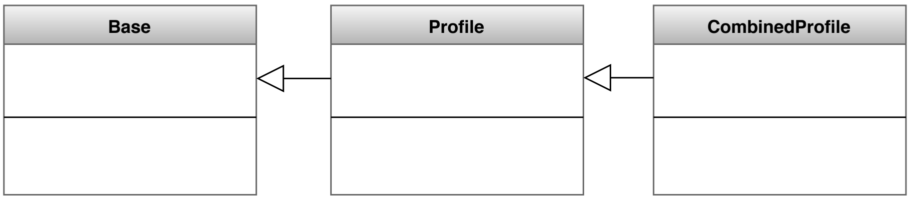


Figure 12: CombinedProfile class hierarchy

User must enter valid dimensional parameters as per requirement. It is essential to check the entered dimensions of the profile, hence the function for checking the same is implemented in `CombinedProfile` class as shown in figure-13. If user enter invalid data of the profile, then user will get the message in output window referring to invalid dimension and the execution of the program will be stopped.

```python
    # check paramaeter
    def checkParam(self):

        nDim = 0.
        if self._k < nDim:
            raise Exception ("> Invalid length of long plate of L-Profile : %f" % self._k)
        if self._b < nDim:
            raise Exception ("> Invalid length of short plate of L-Profile : %f" % self._b)
        if self._z < nDim:
            raise Exception ("> Thickness of L-Profile : %f" % self._z)
        if self._d < nDim:
            raise Exception ("> Outer diameter of O-Profile : %f" % self._d)
        if self._Tt < nDim:
            raise Exception ("> Thickness of O-Profile : %f" % self._Tt)
```

Figure 13: Checking of CombinedProfile Parameter

2.4.8 Profile Database class

For arbitrary profile instances we can implement profile data base class, which is inherited from dictionary. It can get the keys from the stored instances. We can get the instance pointer after giving a specific key. The class `ProfileDB` gives an access for the calculation and plotting of multiple profiles simultaneously.

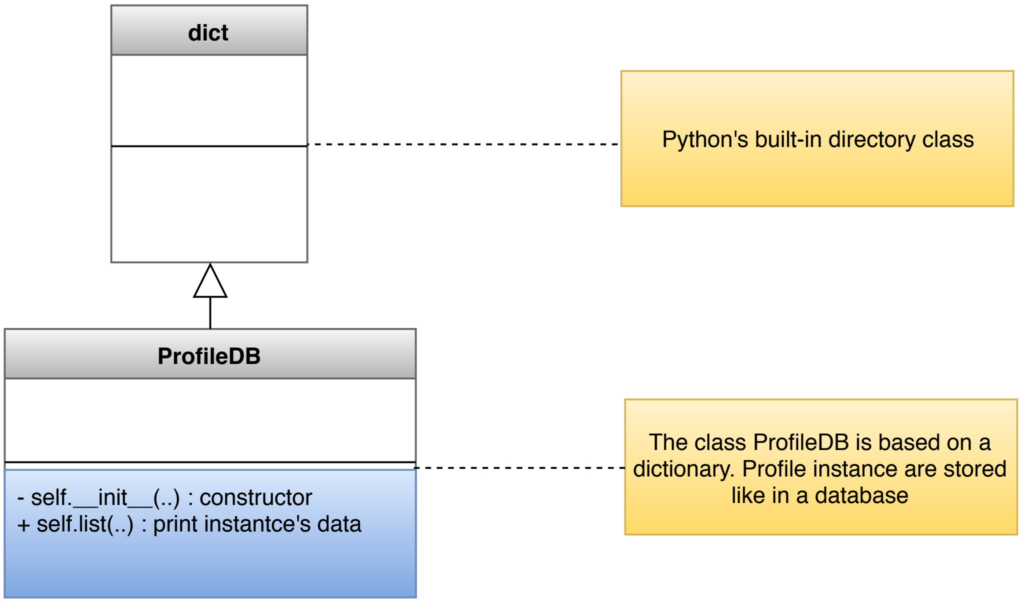


Figure 14: UML diagram of ProfileDB class

2.4.9 Developed Program

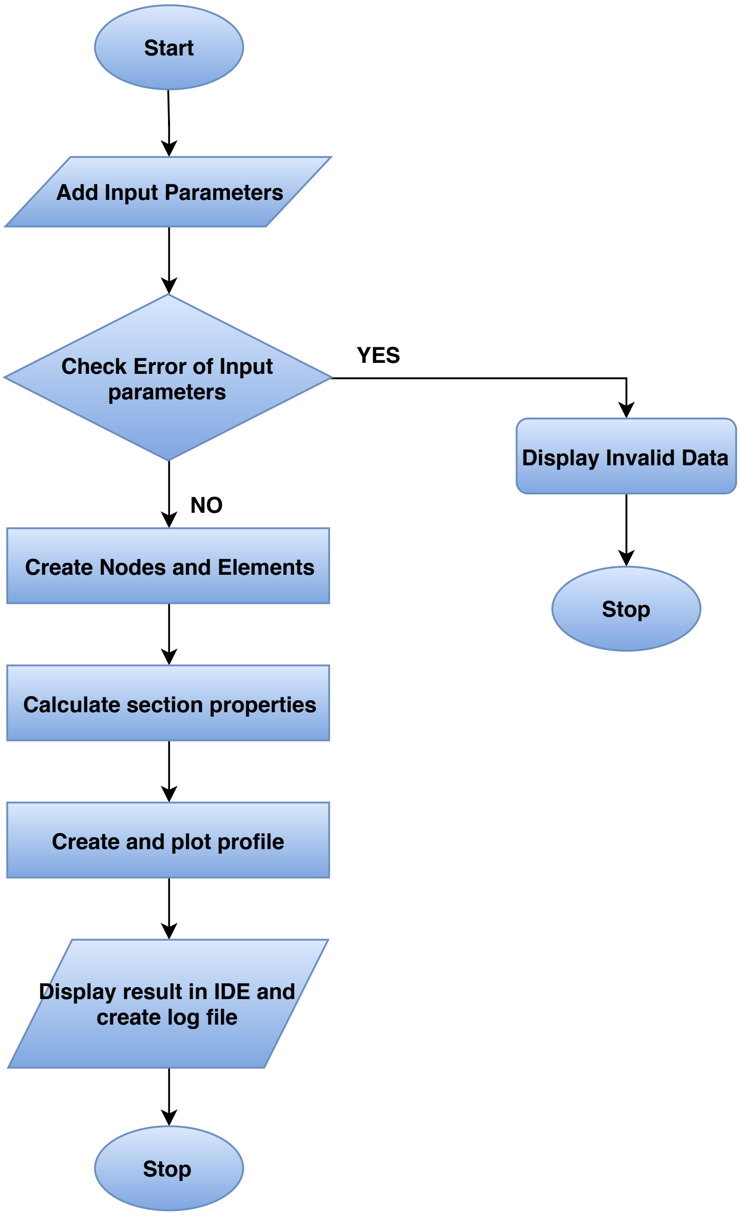


Figure 15: Flow diagram of developed program

2.5 User Manual

This section provides the instruction to use the developed python program to compute the section properties of given combined profile. A little example has been introduced step by step to use the developed program.

- **Open the developed program** – Open the **PythonWin** app or **jetBrains PyCharm Community Edition** from the task bar of the computer and open 'mainTWA.py' file from the project folder **Project-1**. User can see the window as shown in Figure-16 after opening in IDE.

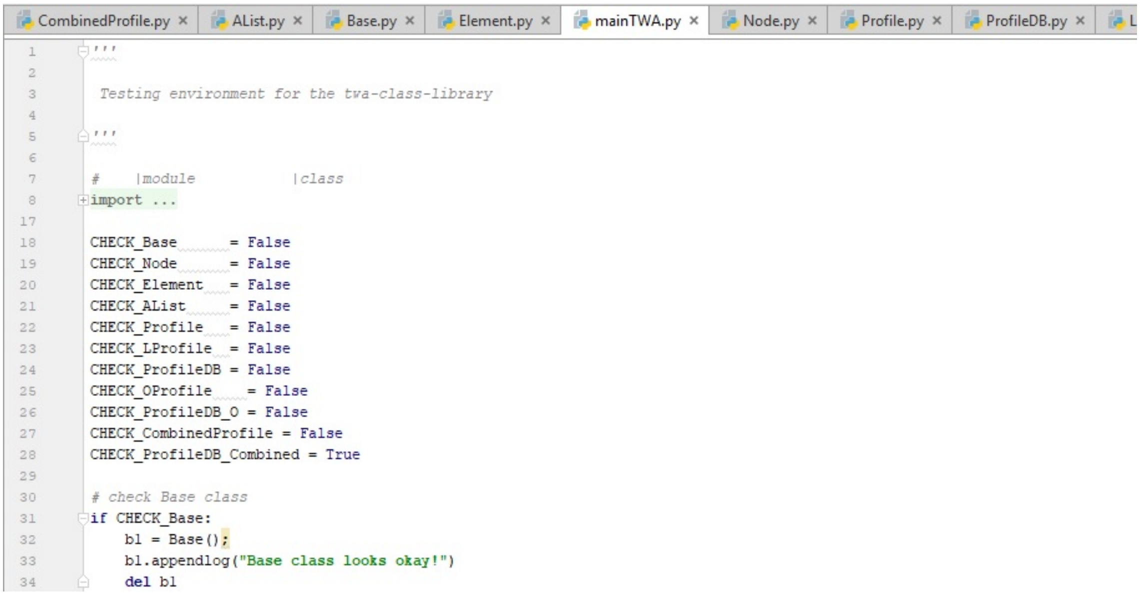


Figure 16: Window of mainTWA file

- **Input Data to the program and execute the program** – The developed program is capable to calculate section properties of three different combinations shown in Table-5. There are three instances for three combinations namely `profile_1`, `profile_2`, `profile_3` are created.

  The section properties of any of these combined profiles can be calculated by uncommenting the corresponding instance in file `mainTwa.py`. The comment can be done by using "#" in front of the corresponding instance. In this example, `profile_1` is uncommented. User can also create new instance as per requirement.

Profile	Name	$k[mm]$	$b[mm]$	$z[mm]$	$d[mm]$	$Tt[mm]$
profile_1	Combined-1	100	50	6	60.3	2.3
profile_2	Combined-2	100	75	10	60.3	2.3
profile_3	Combined-3	120	80	8	60.3	2.3

Table 5: Combined profile dimensions

- Click on the 'Run' option which can be found on the menubar which is located on the top of the window as shown in Figure-17.

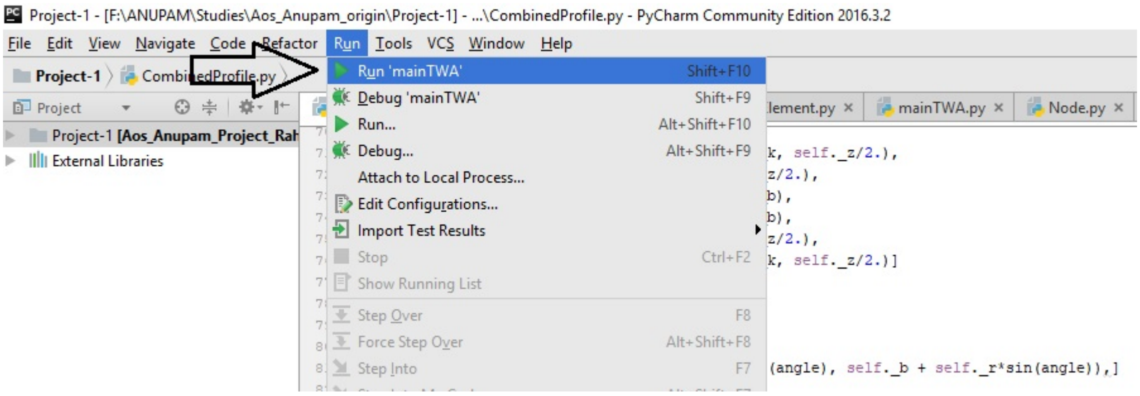


Figure 17: Run the Program

- **Visualization of the Output** After successful execution of the program, user can see the generated image of combined profile and the calculated section properties of the combined profile in output window of IDE as shown in Figure-18. The User can also access the generated log file `CombinedProfile.log` of the combined profiles in project folder.

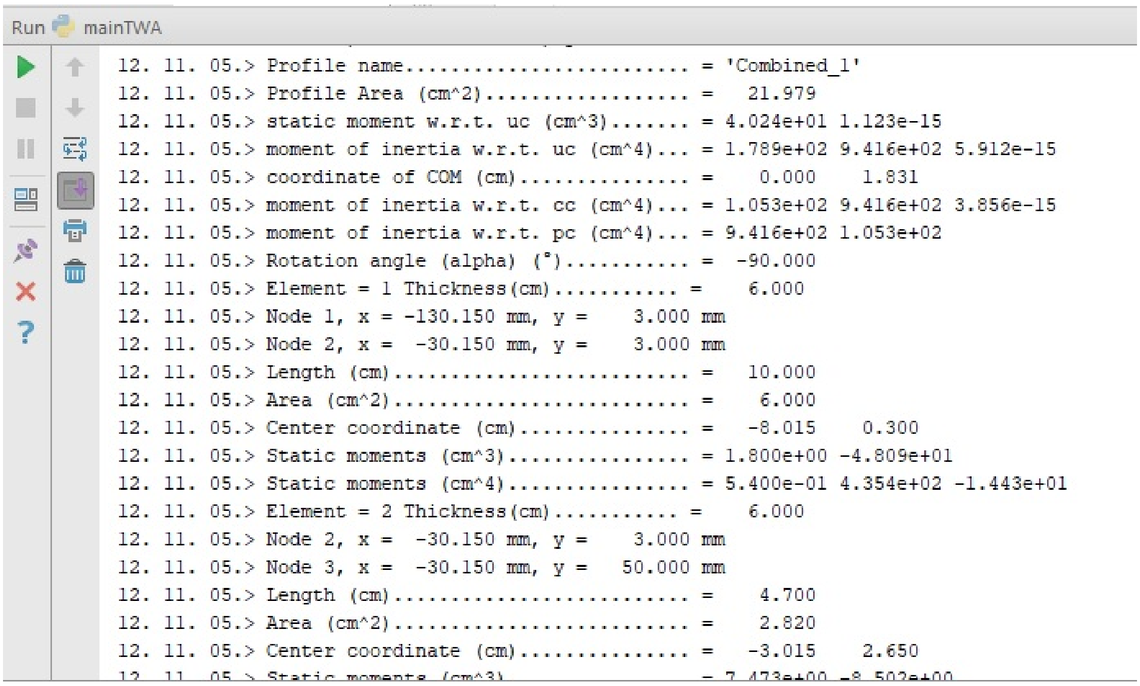


Figure 18: Output Window of executed program in IDE

2.5.0.1 Output log file of Combination-1

```
12. 11. 05.> Node 1, x = -130.150 mm, y =   3.000 mm
12. 11. 05.> Node 2, x =  -30.150 mm, y =   3.000 mm
12. 11. 05.> Node 3, x =  -30.150 mm, y =  50.000 mm
12. 11. 05.> Node 4, x =   30.150 mm, y =  50.000 mm
12. 11. 05.> Node 5, x =   30.150 mm, y =   3.000 mm
12. 11. 05.> Node 6, x =  130.150 mm, y =   3.000 mm
12. 11. 05.> Node 7, x =    0.000 mm, y =  80.150 mm
12. 11. 05.> Node 8, x =   -9.317 mm, y =  78.674 mm
12. 11. 05.> Node 9, x =  -17.722 mm, y =  74.392 mm
12. 11. 05.> Node 10, x =  -24.392 mm, y =  67.722 mm
12. 11. 05.> Node 11, x =  -28.674 mm, y =  59.317 mm
12. 11. 05.> Node 12, x =  -30.150 mm, y =  50.000 mm
12. 11. 05.> Node 13, x =  -28.674 mm, y =  40.683 mm
12. 11. 05.> Node 14, x =  -24.392 mm, y =  32.278 mm
12. 11. 05.> Node 15, x =  -17.722 mm, y =  25.608 mm
12. 11. 05.> Node 16, x =   -9.317 mm, y =  21.326 mm
12. 11. 05.> Node 17, x =   -0.000 mm, y =  19.850 mm
12. 11. 05.> Node 18, x =    9.317 mm, y =  21.326 mm
12. 11. 05.> Node 19, x =   17.722 mm, y =  25.608 mm
12. 11. 05.> Node 20, x =   24.392 mm, y =  32.278 mm
12. 11. 05.> Node 21, x =   28.674 mm, y =  40.683 mm
12. 11. 05.> Node 22, x =   30.150 mm, y =  50.000 mm
12. 11. 05.> Node 23, x =   28.674 mm, y =  59.317 mm
12. 11. 05.> Node 24, x =   24.392 mm, y =  67.722 mm
12. 11. 05.> Node 25, x =   17.722 mm, y =  74.392 mm
12. 11. 05.> Node 26, x =    9.317 mm, y =  78.674 mm
12. 11. 05.> Profile name......................... = 'Combined_1'
12. 11. 05.> Profile Area (cm^2).................. =   21.979
```

```
12. 11. 05.> static moment w.r.t. uc (cm^3)....... = 4.024e+01 1.123e-15
12. 11. 05.> moment of inertia w.r.t. uc (cm^4)... = 1.789e+02 9.416e+02 5.912e-15
12. 11. 05.> coordinate of COM (cm).............. =    0.000    1.831
12. 11. 05.> moment of inertia w.r.t. cc (cm^4)... = 1.053e+02 9.416e+02 3.856e-15
12. 11. 05.> moment of inertia w.r.t. pc (cm^4)... = 9.416e+02 1.053e+02
12. 11. 05.> Rotation angle (alpha) (Â°).......... =  -90.000
12. 11. 05.> Element = 1 Thickness(cm).......... =    6.000
12. 11. 05.> Node 1, x = -130.150 mm, y =    3.000 mm
12. 11. 05.> Node 2, x =  -30.150 mm, y =    3.000 mm
12. 11. 05.> Length (cm)......................... =   10.000
12. 11. 05.> Area (cm^2)......................... =    6.000
12. 11. 05.> Center coordinate (cm).............. =   -8.015    0.300
12. 11. 05.> Static moments (cm^3)............... = 1.800e+00 -4.809e+01
12. 11. 05.> Static moments (cm^4)............... = 5.400e-01 4.354e+02 -1.443e+01
12. 11. 05.> Element = 2 Thickness(cm).......... =    6.000
12. 11. 05.> Node 2, x =  -30.150 mm, y =    3.000 mm
12. 11. 05.> Node 3, x =  -30.150 mm, y =   50.000 mm
12. 11. 05.> Length (cm)......................... =    4.700
12. 11. 05.> Area (cm^2)......................... =    2.820
12. 11. 05.> Center coordinate (cm).............. =   -3.015    2.650
12. 11. 05.> Static moments (cm^3)............... = 7.473e+00 -8.502e+00
12. 11. 05.> Static moments (cm^4)............... = 2.499e+01 2.563e+01 -2.253e+01
12. 11. 05.> Element = 3 Thickness(cm).......... =    6.000
12. 11. 05.> Node 4, x =   30.150 mm, y =   50.000 mm
12. 11. 05.> Node 5, x =   30.150 mm, y =    3.000 mm
12. 11. 05.> Length (cm)......................... =    4.700
12. 11. 05.> Area (cm^2)......................... =    2.820
12. 11. 05.> Center coordinate (cm).............. =    3.015    2.650
12. 11. 05.> Static moments (cm^3)............... = 7.473e+00 8.502e+00
12. 11. 05.> Static moments (cm^4)............... = 2.499e+01 2.563e+01 2.253e+01
12. 11. 05.> Element = 4 Thickness(cm).......... =    6.000
12. 11. 05.> Node 5, x =   30.150 mm, y =    3.000 mm
12. 11. 05.> Node 6, x =  130.150 mm, y =    3.000 mm
12. 11. 05.> Length (cm)......................... =   10.000
12. 11. 05.> Area (cm^2)......................... =    6.000
12. 11. 05.> Center coordinate (cm).............. =    8.015    0.300
12. 11. 05.> Static moments (cm^3)............... = 1.800e+00 4.809e+01
12. 11. 05.> Static moments (cm^4)............... = 5.400e-01 4.354e+02 1.443e+01
12. 11. 05.> Object 4 is unprintable!
12. 11. 05.> Object 5 is unprintable!
12. 11. 05.> Element = 7 Thickness(cm).......... =    2.300
12. 11. 05.> Node 7, x =    0.000 mm, y =   80.150 mm
12. 11. 05.> Node 8, x =   -9.317 mm, y =   78.674 mm
12. 11. 05.> Length (cm)......................... =    0.943
12. 11. 05.> Area (cm^2)......................... =    0.217
12. 11. 05.> Center coordinate (cm).............. =   -0.466    7.941
12. 11. 05.> Static moments (cm^3)............... = 1.723e+00 -1.011e-01
12. 11. 05.> Static moments (cm^4)............... = 1.368e+01 6.278e-02 -8.001e-01
12. 11. 05.> Element = 8 Thickness(cm).......... =    2.300
12. 11. 05.> Node 8, x =   -9.317 mm, y =   78.674 mm
12. 11. 05.> Node 9, x =  -17.722 mm, y =   74.392 mm
12. 11. 05.> Length (cm)......................... =    0.943
12. 11. 05.> Area (cm^2)......................... =    0.217
12. 11. 05.> Center coordinate (cm).............. =   -1.352    7.653
```

```
12. 11. 05.> Static moments (cm^3)............... = 1.660e+00 -2.933e-01
12. 11. 05.> Static moments (cm^4)............... = 1.271e+01 4.093e-01 -2.238e+00
12. 11. 05.> Element = 9 Thickness(cm).......... =    2.300
12. 11. 05.> Node 9, x =  -17.722 mm, y =   74.392 mm
12. 11. 05.> Node 10, x =  -24.392 mm, y =   67.722 mm
12. 11. 05.> Length (cm)........................ =    0.943
12. 11. 05.> Area (cm^2)........................ =    0.217
12. 11. 05.> Center coordinate (cm)............. =   -2.106    7.106
12. 11. 05.> Static moments (cm^3)............... = 1.542e+00 -4.568e-01
12. 11. 05.> Static moments (cm^4)............... = 1.096e+01 9.700e-01 -3.238e+00
12. 11. 05.> Element = 10 Thickness(cm).......... =    2.300
12. 11. 05.> Node 10, x =  -24.392 mm, y =   67.722 mm
12. 11. 05.> Node 11, x =  -28.674 mm, y =   59.317 mm
12. 11. 05.> Length (cm)........................ =    0.943
12. 11. 05.> Area (cm^2)........................ =    0.217
12. 11. 05.> Center coordinate (cm)............. =   -2.653    6.352
12. 11. 05.> Static moments (cm^3)............... = 1.378e+00 -5.757e-01
12. 11. 06.> Static moments (cm^4)............... = 8.766e+00 1.531e+00 -3.650e+00
12. 11. 06.> Element = 11 Thickness(cm).......... =    2.300
12. 11. 06.> Node 11, x =  -28.674 mm, y =   59.317 mm
12. 11. 06.> Node 12, x =  -30.150 mm, y =   50.000 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm)............. =   -2.941    5.466
12. 11. 06.> Static moments (cm^3)............... = 1.186e+00 -6.381e-01
12. 11. 06.> Static moments (cm^4)............... = 6.497e+00 1.877e+00 -3.485e+00
12. 11. 06.> Element = 12 Thickness(cm).......... =    2.300
12. 11. 06.> Node 12, x =  -30.150 mm, y =   50.000 mm
12. 11. 06.> Node 13, x =  -28.674 mm, y =   40.683 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm)............. =   -2.941    4.534
12. 11. 06.> Static moments (cm^3)............... = 9.837e-01 -6.381e-01
12. 11. 06.> Static moments (cm^4)............... = 4.476e+00 1.877e+00 -2.896e+00
12. 11. 06.> Element = 13 Thickness(cm).......... =    2.300
12. 11. 06.> Node 13, x =  -28.674 mm, y =   40.683 mm
12. 11. 06.> Node 14, x =  -24.392 mm, y =   32.278 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm)............. =   -2.653    3.648
12. 11. 06.> Static moments (cm^3)............... = 7.915e-01 -5.757e-01
12. 11. 06.> Static moments (cm^4)............... = 2.900e+00 1.531e+00 -2.107e+00
12. 11. 06.> Element = 14 Thickness(cm).......... =    2.300
12. 11. 06.> Node 14, x =  -24.392 mm, y =   32.278 mm
12. 11. 06.> Node 15, x =  -17.722 mm, y =   25.608 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm)............. =   -2.106    2.894
12. 11. 06.> Static moments (cm^3)............... = 6.279e-01 -4.568e-01
12. 11. 06.> Static moments (cm^4)............... = 1.826e+00 9.700e-01 -1.330e+00
12. 11. 06.> Element = 15 Thickness(cm).......... =    2.300
12. 11. 06.> Node 15, x =  -17.722 mm, y =   25.608 mm
12. 11. 06.> Node 16, x =   -9.317 mm, y =   21.326 mm
12. 11. 06.> Length (cm)........................ =    0.943
```

```
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =   -1.352    2.347
12. 11. 06.> Static moments (cm^3)............... = 5.091e-01 -2.933e-01
12. 11. 06.> Static moments (cm^4)............... = 1.198e+00 4.093e-01 -6.948e-01
12. 11. 06.> Element = 16 Thickness(cm).......... =    2.300
12. 11. 06.> Node 16, x =   -9.317 mm, y =   21.326 mm
12. 11. 06.> Node 17, x =   -0.000 mm, y =   19.850 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =   -0.466    2.059
12. 11. 06.> Static moments (cm^3)............... = 4.467e-01 -1.011e-01
12. 11. 06.> Static moments (cm^4)............... = 9.200e-01 6.278e-02 -2.106e-01
12. 11. 06.> Element = 17 Thickness(cm).......... =    2.300
12. 11. 06.> Node 17, x =   -0.000 mm, y =   19.850 mm
12. 11. 06.> Node 18, x =    9.317 mm, y =   21.326 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =    0.466    2.059
12. 11. 06.> Static moments (cm^3)............... = 4.467e-01 1.011e-01
12. 11. 06.> Static moments (cm^4)............... = 9.200e-01 6.278e-02 2.106e-01
12. 11. 06.> Element = 18 Thickness(cm).......... =    2.300
12. 11. 06.> Node 18, x =    9.317 mm, y =   21.326 mm
12. 11. 06.> Node 19, x =   17.722 mm, y =   25.608 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =    1.352    2.347
12. 11. 06.> Static moments (cm^3)............... = 5.091e-01 2.933e-01
12. 11. 06.> Static moments (cm^4)............... = 1.198e+00 4.093e-01 6.948e-01
12. 11. 06.> Element = 19 Thickness(cm).......... =    2.300
12. 11. 06.> Node 19, x =   17.722 mm, y =   25.608 mm
12. 11. 06.> Node 20, x =   24.392 mm, y =   32.278 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.106    2.894
12. 11. 06.> Static moments (cm^3)............... = 6.279e-01 4.568e-01
12. 11. 06.> Static moments (cm^4)............... = 1.826e+00 9.700e-01 1.330e+00
12. 11. 06.> Element = 20 Thickness(cm).......... =    2.300
12. 11. 06.> Node 20, x =   24.392 mm, y =   32.278 mm
12. 11. 06.> Node 21, x =   28.674 mm, y =   40.683 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.653    3.648
12. 11. 06.> Static moments (cm^3)............... = 7.915e-01 5.757e-01
12. 11. 06.> Static moments (cm^4)............... = 2.900e+00 1.531e+00 2.107e+00
12. 11. 06.> Element = 21 Thickness(cm).......... =    2.300
12. 11. 06.> Node 21, x =   28.674 mm, y =   40.683 mm
12. 11. 06.> Node 22, x =   30.150 mm, y =   50.000 mm
12. 11. 06.> Length (cm)......................... =    0.943
12. 11. 06.> Area (cm^2)......................... =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.941    4.534
12. 11. 06.> Static moments (cm^3)............... = 9.837e-01 6.381e-01
12. 11. 06.> Static moments (cm^4)............... = 4.476e+00 1.877e+00 2.896e+00
12. 11. 06.> Element = 22 Thickness(cm).......... =    2.300
12. 11. 06.> Node 22, x =   30.150 mm, y =   50.000 mm
```

```
12. 11. 06.> Node 23, x =   28.674 mm, y =   59.317 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.941    5.466
12. 11. 06.> Static moments (cm^3)............... = 1.186e+00 6.381e-01
12. 11. 06.> Static moments (cm^4)............... = 6.497e+00 1.877e+00 3.485e+00
12. 11. 06.> Element = 23 Thickness(cm).......... =    2.300
12. 11. 06.> Node 23, x =   28.674 mm, y =   59.317 mm
12. 11. 06.> Node 24, x =   24.392 mm, y =   67.722 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.653    6.352
12. 11. 06.> Static moments (cm^3)............... = 1.378e+00 5.757e-01
12. 11. 06.> Static moments (cm^4)............... = 8.766e+00 1.531e+00 3.650e+00
12. 11. 06.> Element = 24 Thickness(cm).......... =    2.300
12. 11. 06.> Node 24, x =   24.392 mm, y =   67.722 mm
12. 11. 06.> Node 25, x =   17.722 mm, y =   74.392 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm).............. =    2.106    7.106
12. 11. 06.> Static moments (cm^3)............... = 1.542e+00 4.568e-01
12. 11. 06.> Static moments (cm^4)............... = 1.096e+01 9.700e-01 3.238e+00
12. 11. 06.> Element = 25 Thickness(cm).......... =    2.300
12. 11. 06.> Node 25, x =   17.722 mm, y =   74.392 mm
12. 11. 06.> Node 26, x =    9.317 mm, y =   78.674 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm).............. =    1.352    7.653
12. 11. 06.> Static moments (cm^3)............... = 1.660e+00 2.933e-01
12. 11. 06.> Static moments (cm^4)............... = 1.271e+01 4.093e-01 2.238e+00
12. 11. 06.> Element = 26 Thickness(cm).......... =    2.300
12. 11. 06.> Node 26, x =    9.317 mm, y =   78.674 mm
12. 11. 06.> Node 7, x =    0.000 mm, y =   80.150 mm
12. 11. 06.> Length (cm)........................ =    0.943
12. 11. 06.> Area (cm^2)........................ =    0.217
12. 11. 06.> Center coordinate (cm).............. =    0.466    7.941
12. 11. 06.> Static moments (cm^3)............... = 1.723e+00 1.011e-01
12. 11. 06.> Static moments (cm^4)............... = 1.368e+01 6.278e-02 8.001e-01
```

The generated image of the combined profile can be found in project folder as shown in Figure-19. Similarly user can also execute the program for other combinations.

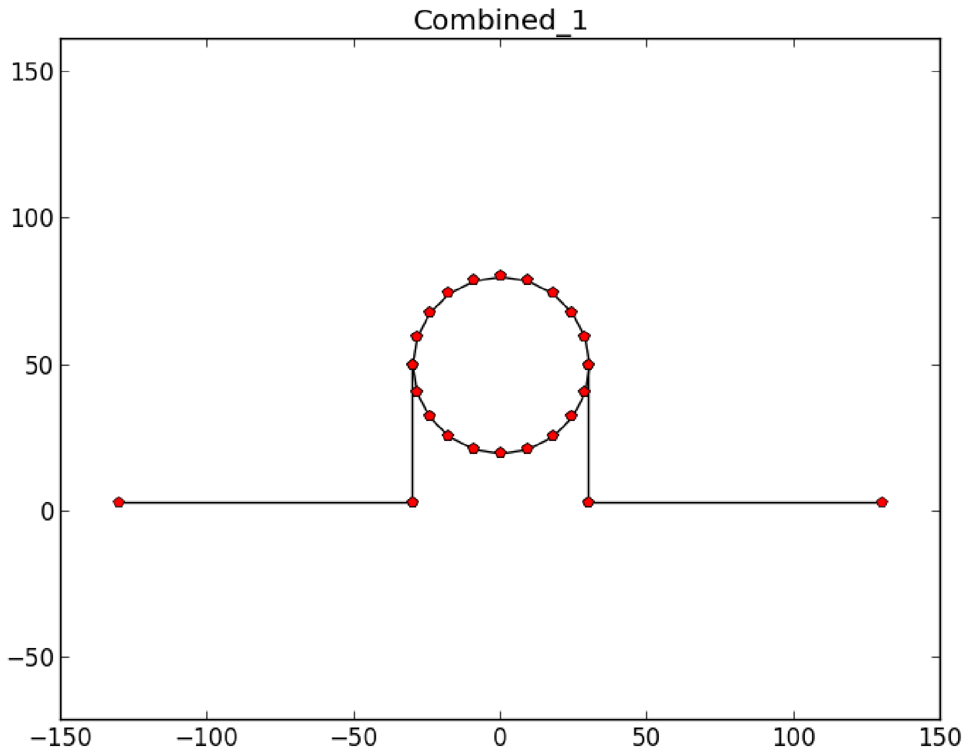


Figure 19: Combination-1

2.6 Result Analysis

To check the reliability of the developed python program, it is essential to compare the results with analytical results.

2.6.1 Analytical section properties

Analytically calculated physical section properties of the profiles, viz., area, Moment of inertia and Centre of mass are represented in DIN Standards. DIN EN 10056-1 for L-Profile and DIN 2448 for O-profile can be referred for values pertaining to the required parameter. The table below is an excerpt of these values for reference from the standards.

Point to be noted, the newest version of DIN Standard (DIN EN 10056-1) does not contain the details for L 100x75x9 (as given in the problem statement). Hence, L 100x75x10 is considered.

Profile (L `kxbbxz`)	$A_L[cm^2]$	$I_y[cm^4]$	$I_z[cm^4]$	$e_y[cm]$	$e_z[cm]$
L 100x50x6	8.71	89.9	15.4	3.51	1.05
L 100x75x10	16.6	162	77.6	3.19	1.95
L 120x80x8	15.5	226	80.8	3.83	1.87

Table 6: Sectional properties of L-profile

Profile(dxTt)	A[cm^2]	I[cm^4]
O 60.3x2.3	4.19	17.7

Table 7: Sectional properties of O-profile

Comparison of results drawn from the program using approximation method and that calculated using the analytical values from above table 6 and table 7 is discussed in the section.

2.6.2 Analytical calculation for combination-1 (L100x50x6 + O60.3x2.3)

Some abbreviations are used as follows –
L100x50x6 $= L_{100}$
L100x75x10 $= L_{75}$
L120x80x8 $= L_{120}$
O60.3x2.3 $= O_{60.3}$

- **Total Area:**

$$\begin{aligned} A &= (2 \times A_{L_{100}}) + (A_{O_{60.3}}) \\ &= (2 \times 8.71) + (4.19) \\ &= 21.61cm^2 \end{aligned}$$

- **Center of mass:** (y_c, z_c)

$$\begin{aligned} z_c &= \frac{\int_A z.dA}{\int_A dA} = \frac{S_y}{A} \\ &= \frac{(2 \times 8.71 \times 1.05) + (4.19 \times 5)}{21.61} \\ &= 1.81cm \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\int_A y.dA}{\int_A dA} = \frac{S_z}{A} \\ &= \frac{(8.71 \times -6.5215) + (8.71 \times 6.5215) + (0 \times 4.19)}{21.61} \\ &= 0cm \end{aligned}$$

- **Static Moment:**

$$\begin{aligned} S_y &= z_c \times A \\ &= 1.81 \times 21.61 \\ &= 39.241cm^3 \end{aligned}$$

$$\begin{aligned} S_z &= y_c \times A \\ &= 0 \times 21.61 \\ &= 0cm^3 \end{aligned}$$

- **Moment of inertia w.r.t User coordinates:**

$$\begin{aligned}
I_{yy} &= I_y + e_z^2 \times A \\
&= [2 \times (15.4 + (8.71 \times 1.05^2))] + [17.7 + (4.19 \times 5^2)] \\
&= 172.455 cm^4
\end{aligned}$$

$$\begin{aligned}
I_{zz} &= I_z + e_y^2 \times A \\
&= [2 \times (89.9 + (8.71 \times +6.525^2))] + [17.7 + (4.19 \times 0)] \\
&= 939.167 cm^4
\end{aligned}$$

$$\begin{aligned}
I_{yzu} &= I_{yz} + (e_y \times e_z) \times A \\
&= 0 cm^4
\end{aligned}$$

- **Moment of inertia w.r.t Center of mass:**

$$\begin{aligned}
I_{yc} &= I_y + (A \times d_z^2) \\
&= [2 \times (15.4 + (8.71 \times 0.76^2))] + [17.7 + (4.19 \times 3.19^2)] \\
&= 101.1996 cm^4
\end{aligned}$$

$$\begin{aligned}
I_{zc} &= I_z + (A \times d_y^2) \\
&= [2 \times (89.9 + (8.71 \times +6.525^2))] + [17.7 + (4.19 \times 0)] \\
&= 939.167 cm^4
\end{aligned}$$

$$\begin{aligned}
I_{yzc} &= I_{yz} + (d_y \times d_z) \times A \\
&= 0 cm^4
\end{aligned}$$

- **Percentage Error:**

$$e_A = \frac{Area_{analytical} - Area_{python}}{Area_{analytical}} \times 100 \qquad = \frac{21.61 - 21.98}{21.61} \times 100 \approx -1.71\%$$

$$e_{S_y} = \frac{S_{y_{analytical}} - S_{y_{python}}}{S_{y_{analytical}}} \times 100 \qquad = \frac{39.24 - 40.24}{39.24} \times 100 \approx -2.54\%$$

$$e_{I_{yy}} = \frac{I_{yy_{analytical}} - I_{yy_{python}}}{I_{yy_{analytical}}} \times 100 \qquad = \frac{172.45 - 178.95}{172.45} \times 100 \approx -3.76\%$$

$$e_{I_{zz}} = \frac{I_{zz_{analytical}} - I_{zz_{python}}}{I_{zz_{analytical}}} \times 100 \qquad = \frac{939.167 - 941.55}{939.167} \times 100 \approx -0.25\%$$

$$e_{I_{yc}} = \frac{I_{yc_{analytical}} - I_{yc_{python}}}{I_{yc_{analytical}}} \times 100 \qquad = \frac{101.20 - 105.27}{101.20} \times 100 \approx -4.02\%$$

$$e_{I_{zc}} = \frac{I_{zc_{analytical}} - I_{zc_{python}}}{I_{zc_{analytical}}} \times 100 \qquad = \frac{939.167 - 941.55}{939.167} \times 100 \approx -0.25\%$$

2.6.3 Analytical calculation for combination-2 (L100x75x10 + O60.3x2.3)

- **Total Area:**

$$\begin{aligned} A &= (2 \times A_{L75}) + (A_{O_{60.3}}) \\ &= (2 \times 16.6) + (4.19) \\ &= 37.39 cm^2 \end{aligned}$$

- **Center of mass:** (y_c, z_c)

$$\begin{aligned} z_c &= \frac{\int_A z.dA}{\int_A dA} = \frac{S_y}{A} \\ &= \frac{(2 \times 16.6 \times 1.95) + (4.19 \times 7.5)}{37.39} \\ &= 2.57 cm \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\int_A y.dA}{\int_A dA} = \frac{S_z}{A} \\ &= \frac{(16.6 \times -6.205) + (16.6 \times 6.205) + (0 \times 4.19)}{37.39} \\ &= 0 cm \end{aligned}$$

- **Static Moment:**

$$\begin{aligned} S_y &= z_c \times A \\ &= 2.29 \times 37.39 \\ &= 96.09 cm^3 \end{aligned}$$

$$\begin{aligned} S_z &= y_c \times A \\ &= 0 \times 37.39 \\ &= 0 cm^3 \end{aligned}$$

- **Moment of inertia w.r.t User coordinates:**

$$\begin{aligned} I_{yy} &= I_y + e_z^2 \times A \\ &= [2 \times (77.6 + (16.6 \times 1.95^2))] + [17.7 + (4.19 \times 7.5^2)] \\ &= 534.83 cm^4 \end{aligned}$$

$$\begin{aligned} I_{zz} &= I_z + e_y^2 \times A \\ &= [2 \times (162 + (16.6 \times 6.205^2))] + [17.7 + (4.19 \times 0)] \\ &= 1619.967 cm^4 \end{aligned}$$

$$\begin{aligned} I_{yzu} &= I_{yz} + (e_y \times e_z) \times A \\ &= 0 cm^4 \end{aligned}$$

- **Moment of inertia w.r.t Center of mass:**

$$\begin{aligned} I_{yc} &= I_y + (A \times d_z^2) \\ &= [2 \times (77.6 + (16.6 \times 0.62^2))] + [17.7 + (4.19 \times 4.93^2)] \\ &= 287.49 cm^4 \end{aligned}$$

$$
\begin{aligned}
I_{zc} &= I_z + e_y^2 \times A \\
&= [2 \times (162 + (16.6 \times 6.205^2))] + [17.7 + (4.19 \times 0)] \\
&= 1619.967cm^4
\end{aligned}
$$

$$
\begin{aligned}
I_{yzc} &= I_{yz} + (d_y \times d_z) \times A \\
&= 0cm^4
\end{aligned}
$$

- **Percentage Error:**

$$e_A = \frac{Area_{analytical} - Area_{python}}{Area_{analytical}} \times 100 \qquad = \frac{37.39 - 38.34}{37.39} \times 100 \approx -2.54\%$$

$$e_{S_y} = \frac{S_{y_{analytical}} - S_{y_{python}}}{S_{y_{analytical}}} \times 100 \qquad = \frac{96.09 - 98.54}{96.09} \times 100 \approx -2.54\%$$

$$e_{I_{yy}} = \frac{I_{yy_{analytical}} - I_{yy_{python}}}{I_{yy_{analytical}}} \times 100 \qquad = \frac{534.83 - 549.65}{534.83} \times 100 \approx -2.77\%$$

$$e_{I_{zz}} = \frac{I_{zz_{analytical}} - I_{zz_{python}}}{I_{zz_{analytical}}} \times 100 \qquad = \frac{1619.96 - 1598.13}{1619.96} \times 100 \approx 1.34\%$$

$$e_{I_{yc}} = \frac{I_{yc_{analytical}} - I_{yc_{python}}}{I_{yc_{analytical}}} \times 100 \qquad = \frac{287.49 - 296.36}{287.49} \times 100 \approx -3.08\%$$

$$e_{I_{zc}} = \frac{I_{zc_{analytical}} - I_{zc_{python}}}{I_{zc_{analytical}}} \times 100 \qquad = \frac{1619.96 - 1598.13}{1619.96} \times 100 \approx 1.34\%$$

2.6.4 Analytical calculation for combination-3 (L120x80x8 + O60.3x2.3)

- **Total Area:**

$$\begin{aligned} A &= (2 \times A_{L_{120}}) + (A_{O_{60.3}}) \\ &= (2 \times 15.5) + (4.19) \\ &= 35.19cm^2 \end{aligned}$$

- **Center of mass:** (y_c, z_c)

$$\begin{aligned} z_c &= \frac{\int_A z.dA}{\int_A dA} = \frac{S_y}{A} \\ &= \frac{(2 \times 16.6 \times 1.87) + (4.19 \times 8)}{35.19} \\ &= 2.71cm \end{aligned}$$

$$\begin{aligned} y_c &= \frac{\int_A y.dA}{\int_A dA} = \frac{S_z}{A} \\ &= \frac{(16.6 \times -6.845) + (16.6 \times 6.845) + (0 \times 4.19)}{35.19} \\ &= 0cm \end{aligned}$$

- **Static Moment:**

$$\begin{aligned} S_y &= z_c \times A \\ &= 2.71 \times 35.19 \\ &= 95.36cm^3 \end{aligned}$$

$$\begin{aligned} S_z &= y_c \times A \\ &= 0 \times 35.19 \\ &= 0cm^3 \end{aligned}$$

- **Moment of inertia w.r.t User coordinates:**

$$\begin{aligned} I_{yy} &= I_y + e_z^2 \times A \\ &= [2 \times (80.8 + (15.5 \times 1.87^2))] + [17.7 + (4.19 \times 8^2)] \\ &= 555.86cm^4 \end{aligned}$$

$$\begin{aligned} I_{zz} &= I_z + e_y^2 \times A \\ &= [2 \times (226 + (15.5 \times 6.845^2))] + [17.7 + (4.19 \times 0)] \\ &= 1922.17cm^4 \end{aligned}$$

$$\begin{aligned} I_{yzu} &= I_{yz} + (e_y \times e_z) \times A \\ &= 0cm^4 \end{aligned}$$

- **Moment of inertia w.r.t Center of mass:**

$$\begin{aligned} I_{yc} &= I_y + (A \times d_z^2) \\ &= [2 \times (77.6 + (16.6 \times 1.12^2))] + [17.7 + (4.19 \times 5.29^2)] \\ &= 331.79cm^4 \end{aligned}$$

$$
\begin{aligned}
I_{zc} &= I_z + e_y^2 \times A \\
&= [2 \times (226 + (15.5 \times 6.845^2))] + [17.7 + (4.19 \times 0)] \\
&= 1922.17cm^4
\end{aligned}
$$

$$
\begin{aligned}
I_{yzc} &= I_{yz} + (d_y \times d_z) \times A \\
&= 0cm^4
\end{aligned}
$$

- **Percentage Error:**

$$e_A = \frac{Area_{analytical} - Area_{python}}{Area_{analytical}} \times 100 \qquad = \frac{35.19 - 35.70}{35.19} \times 100 \approx -1.44\%$$

$$e_{S_y} = \frac{S_{y_{analytical}} - S_{y_{python}}}{S_{y_{analytical}}} \times 100 \qquad = \frac{95.36 - 93.47}{95.36} \times 100 \approx 1.98\%$$

$$e_{I_{yy}} = \frac{I_{yy_{analytical}} - I_{yy_{python}}}{I_{yy_{analytical}}} \times 100 \qquad = \frac{555.86 - 573.21}{555.86} \times 100 \approx -3.12\%$$

$$e_{I_{zz}} = \frac{I_{zz_{analytical}} - I_{zz_{python}}}{I_{zz_{analytical}}} \times 100 \qquad = \frac{1922.17 - 1920.73}{1922.17} \times 100 \approx 0.074\%$$

$$e_{I_{yc}} = \frac{I_{yc_{analytical}} - I_{yc_{python}}}{I_{yc_{analytical}}} \times 100 \qquad = \frac{331.79 - 328.51}{331.79} \times 100 \approx 0.98\%$$

$$e_{I_{zc}} = \frac{I_{zc_{analytical}} - I_{zc_{python}}}{I_{zc_{analytical}}} \times 100 \qquad = \frac{1922.17 - 1920.73}{1922.17} \times 100 \approx 0.074\%$$

2.7 Result and discussion

The tabular representation of the results for comparison is shown as follows -

Section Properties	Combined profile			Remarks
	L_{100}	L_{75}	L_{120}	
Area(A)[cm^2]	21.98	38.34	35.70	Computed
	21.61	37.39	35.19	Analytical
	-1.71%	-2.54%	-1.44%	Error
Static Moment(S_y)[cm^3]	40.24	98.54	93.47	Computed
	39.24	96.09	95.36	Analytical
	-2.54%	-2.54%	1.98%	Error
Moment of inertia w.r.t uc(I_{yy})[cm^4]	178.95	549.65	573.21	Computed
	172.45	534.83	555.86	Analytical
	-3.76%	-2.77%	-3.12%	Error
Moment of inertia w.r.t uc(I_{zz})[cm^4]	941.55	1598.13	1920.73	Computed
	939.16	1619.96	1922.17	Analytical
	-0.25%	1.34%	0.074%	Error
Moment of inertia w.r.t COM(I_{yc})[cm^4]	105.27	296.36	328.51	Computed
	101.20	287.49	331.79	Analytical
	-4.02%	-3.08%	0.98%	Error
Moment of inertia w.r.t COM(I_{zc})[cm^4]	941.55	1598.13	1920.73	Computed
	939.16	1619.96	1922.17	Analytical
	-0.25%	1.34%	0.074%	Error

Table 8: Result Analysis Chart

From the Table-8 it can be seen that we are getting different values of moment of inertia in user coordinates than center coordinates because we have considered asymmetric origin for user coordinate and due to eccentricity we are getting the different values. Due to symmetry with respect to z-axis we are getting 0 static moment S_z in all combinations.

It can be observed that, percentage of variation in moment of inertia w.r.t center coordinates I_{yc} are on negative side, whereas I_{zc} are on positive side for two out of three combinations. Such a variation is due to the approximation of the profile.

The negative sign indicates that the calculated values are higher than the actual values and positive sign indicates the calculated values are smaller than actual values. However, the variation lies below 5% for all combinations. This variation can be considered as a factor of safety.

The reason for such variation is due to the negligence of fillets and assuming overlap of profiles in Thin Walled Approach leads to variation in the area and center of mass resulting the variation in moment of inertia. This leads to change in exural rigidity (EI) of the combined profile in which young's modulus remains same for all combinations. The higher moment of inertia makes more stiffer profile and hence, it is more safer to use.

Since % of variation in area and moment of inertia lies around or below 2.50%, it can be examined that the negligible variation in exural rigidity will not affect the maximum allowable load carrying capacity of the profile considering linear static analysis, therefore the developed program is on safer side.

It can be concluded that the developed python program for computation of the section properties of the combined profile is validated by comparing with analytical values. Hence, the developed code is reliable and on safer side.

3 Project 2 – Development and implementation of python script in Abaqus to create FE-model of Combined profile

3.1 Problem definition

It is required to develop python program to automate the Finite Element Model of combined profile in abaqus and to perform the linear static calculation. To achieve the task, it is required to develop different classes using python to automate FE-Model in abaqus and to perform the analysis of the combined profile. The python program should check the equilibrium condition of forces. It should be able to find the fiber nodes and calculate displacements along those nodes.

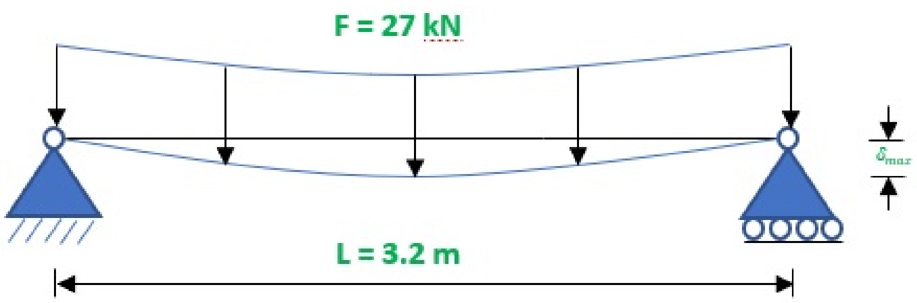


Figure 20: Simply supported beam at ends with uniformly distributed load

The beam has a length of 3.2 m. The beam is made of structural steel with Young's modulus of `210000 [MPa]`. The uniform distributed load of `27 kN` is applied on the two long plates. The vertical support has been provided on the edge at both ends (L=0m and L=3.2m). The rigid support has been provided on the vertex at L=3.2m. The Load and Boundary Conditions are shown in Figure - 21.

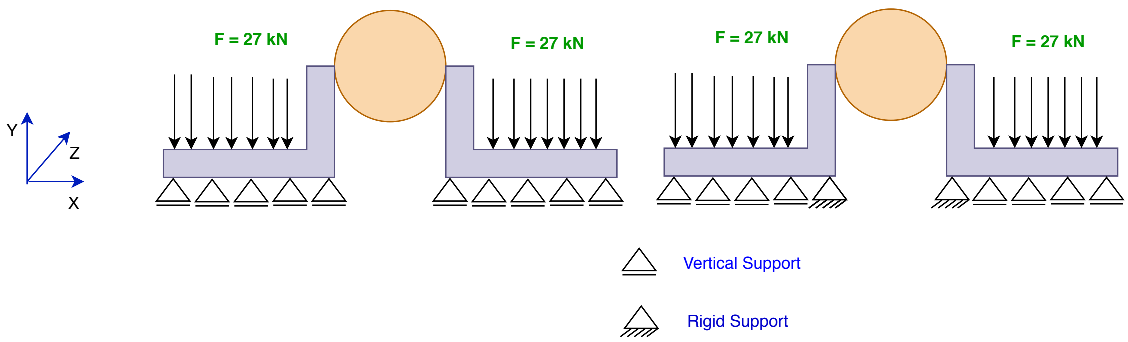


Figure 21: Load and Boundary Conditions at L=0m and L=3.2m

3.2 Programmer's guide

It is required to develop different classes in python to generate automatize FE-Model of the combined profile. The concept will remain same as we used in **TWA** to develop Base class and it will provide all common features of other classes. The class hierarchy of combined profile to generate FE-Model is as shown in figure below.

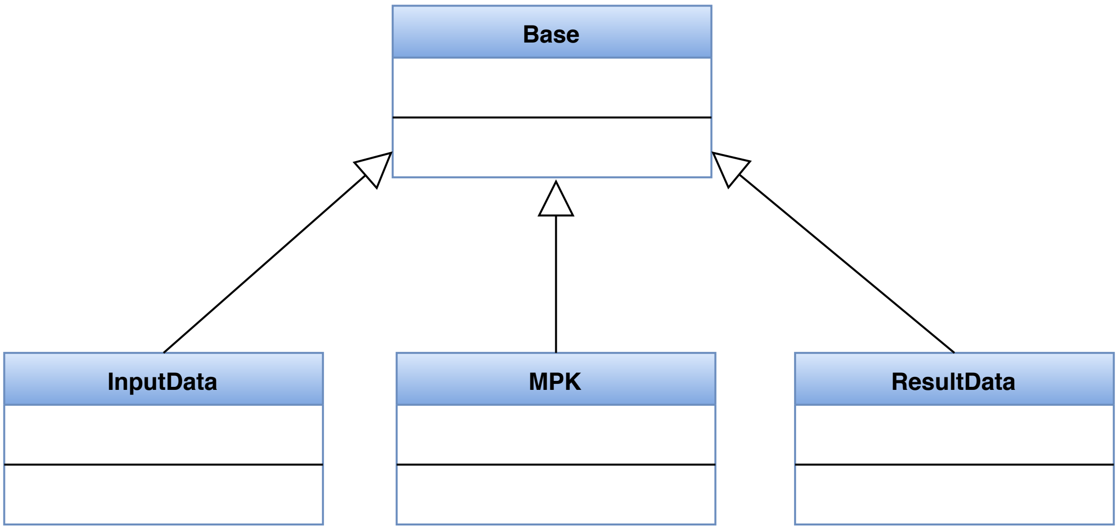


Figure 22: Class hierarchy to generate FE-Model of Combined Profile

3.2.1 Base

The class Base has the same function as we discussed in Thin Walled Approach. All classes will be derived from the `Base` class. It will provide all common features.

3.2.2 InputData

There are various input parameters of the combined profile implemented in `InputData` class to generate FE-Model. The important parameters are illustrated in UML diagram.

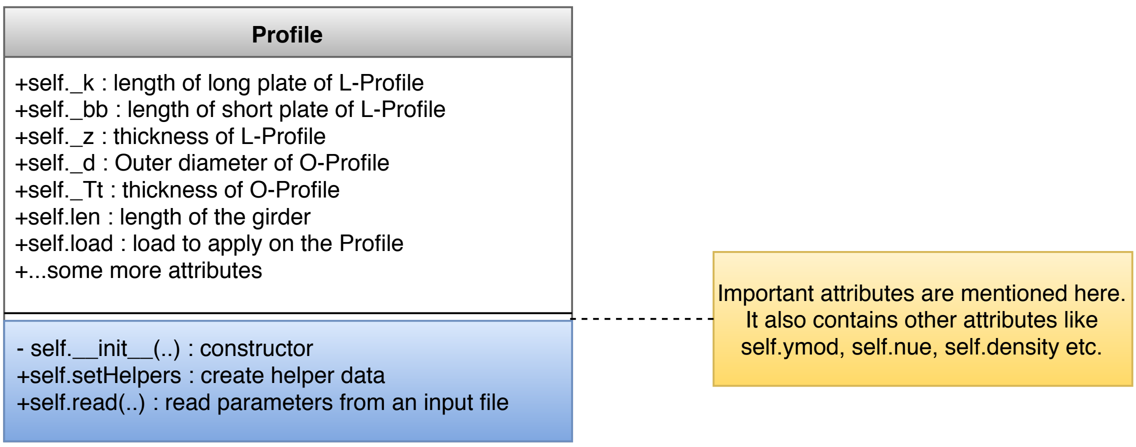


Figure 23: UML diagram of class InputData

Since we are using student version of abaqus, it facilitates only 1000 nodes. Some other parameters like young's modulus, poisson's ratio, density of the profile are defined as well in `InputData` class.

self.maxNumber = 1000, Maximum Node number

The number of seed for profile are set as below –
self.longplateSeed = 8 , Number of longplateSeed
self.shortplateSeed = 4 , Number of shortplateSeed
self.tubeSeed = 6 , Number of tubeSeed

The number of length seed are calculated as follows –

$$\begin{aligned}\textbf{self.lengthSeed} &= \frac{self.maxnumber}{(2 \times self.longplateSeed) + (4 \times self.shortplateSeed) + (1 \times self.tubeSeed) + 1} - 1 \\ &= \frac{1000}{(2 \times 8) + (4 \times 4) + (1 \times 6) + 1} - 1 = 24.64\end{aligned}$$

We have considered integer value for the length seed, so there are 24 elements means 25 nodes are available along the fiber. It is also essential to check the entered dimensions of the profile, hence the function for checking is implemented in `InputData` class same as explained in CombinedProfile class of `TWA` approach.

3.2.3 ResultData

To find out maximum displacements along the fiber nodes, it is required to select the nodes along the fiber. The node data is stored in dictionary **nodePos**. A **nodeDis** dictionary is used to store nodal displacements. A **nodeRFo** dictionary is used to store the reaction forces. The sum vector of the reaction forces is also defined in `ResultData` class using list **sumRFo**.

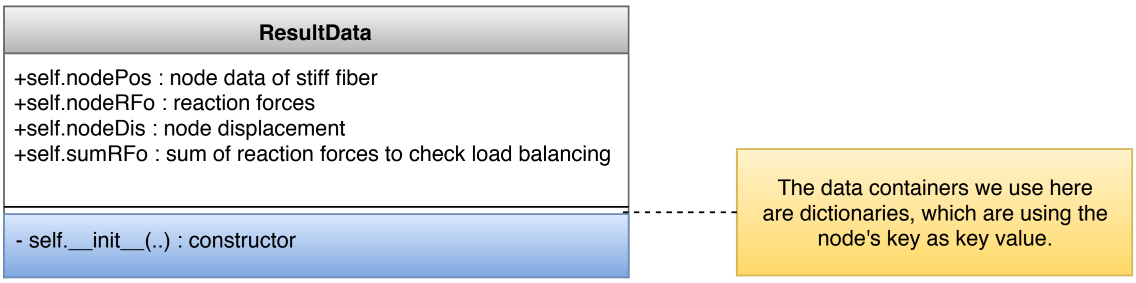


Figure 24: UML diagram of class ResultData

3.2.4 MPK class

The class `MPK` will implement to organize, calculate and analyze the data. The class constructor creates an InputData and ResultData object. It is essential to compile all modules before execution so we need to reload this modules.

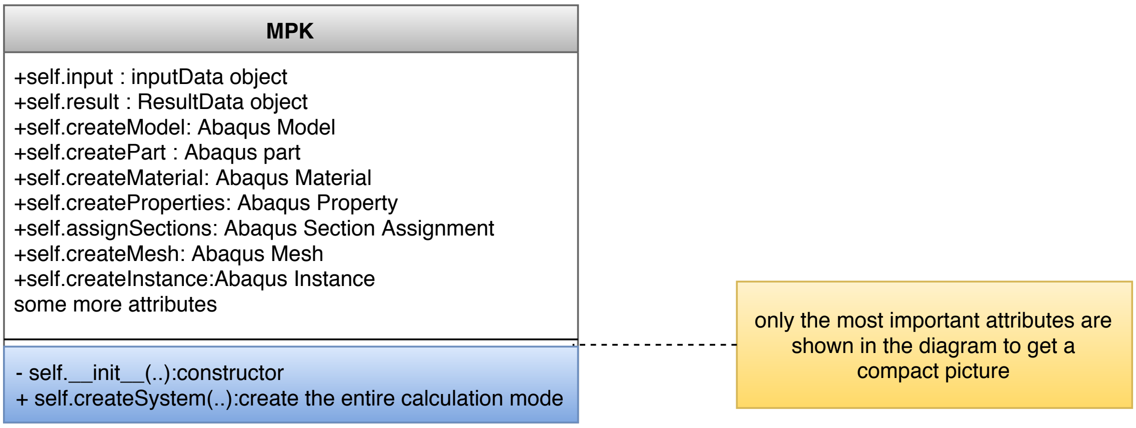


Figure 25: UML diagram of class MPK

The FE Model is generated step by step using the object tree of abaqus. All attributes can be stored as python file using the abaqus macro manager. The data of this attributes are implemented in MPK class and thereafter data are modified and optimized according to the requirement. The modeling chain of the MPK class is as shown in Figure - 27

- **createSystem:** This method is used to implement attributes of the MPK class.
- **createModel():** This method is used to generate model and if the model is available then it will print in log file with its given name.
- **createPart():** In this method part is created using the nodes which we used in Thin Walled approach. The lines are drawn through the nodes in sketch module as shown in Figure - 29 and this sketch has been extruded using shellextrude equal to the length of the girder defined in InputData class. The generated Part can be seen in Figure - 30.
- **createMaterial():** In this method, various parameters like young's modulus, poisson's ratio and density of the material are implemented.
- **createProperties():** In this method homogeneous shell section is implemented to create shell model of the combined profile.
- **assignSections():** In this method section properties are assigned to the longplate, shortplate and tube. For the selection of the longplate, short plate and tube, center coordinates of them are defined.
- **createMesh():** To generate Mesh, the edge has been defined as center line of the longplate, shortplate and tube. To generate mesh along the length, mid point coordinates of outer edges are defined. The mesh is generated by using the input value of the longplateSeed, shortplateSeed and tubeSeed, which are discussed in InputData class. The Mesh has been generated using S4R element type. The Mesh can bee seen in Figure - 34.
- **getFiberNodes():** To find out the maximum nodal displacement, it is required to select the nodes. For the precise selection of the nodes, the distance from center coordinates are calculated. It will also print node number with its coordinates in log file.
- **createInstance():** It is used to generate assembly of all parts.

- **createStep():** To perform the linear static analysis, it is required to create linear step, which is created by using createStep() method.
- **createBCs():** For boundary condition, the center edges of the lower short plates of the combinedprofile are selected. The applied boundary conditions can be seen in Figures - 32 and 33.
- **createLoads():** This method is used to apply load on the total two faces of the combined profile considering the two longplates . For the selection of the faces, the center coordinates of the all faces are implemented in the program. The input value of the load is taken from the InputData() class. The applied load can be seen in Figure - 31.
- **runJob():** After successfully implementing all methods, it is required to submit Job for the further calculations, which is done by using runJob() method.
- **analyseLinearStep():** This method is used to perform the post processing viz. sum of the reaction forces, maximum vertical displacement along the fiber and to plot the generated results in viewport.

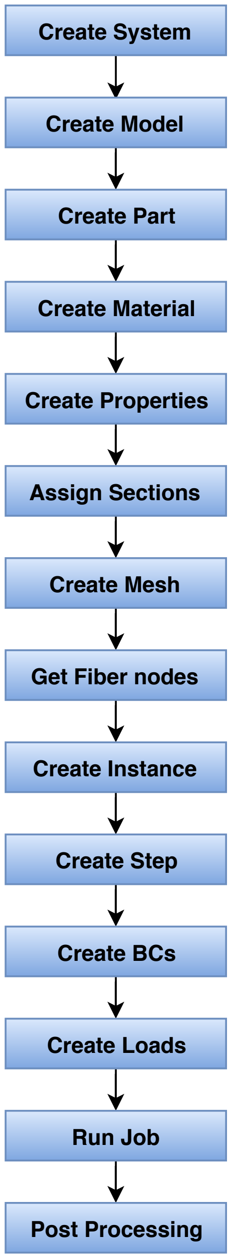


Figure 26: Modeling chain of MPK class

3.2.5 Flow diagram of the developed program

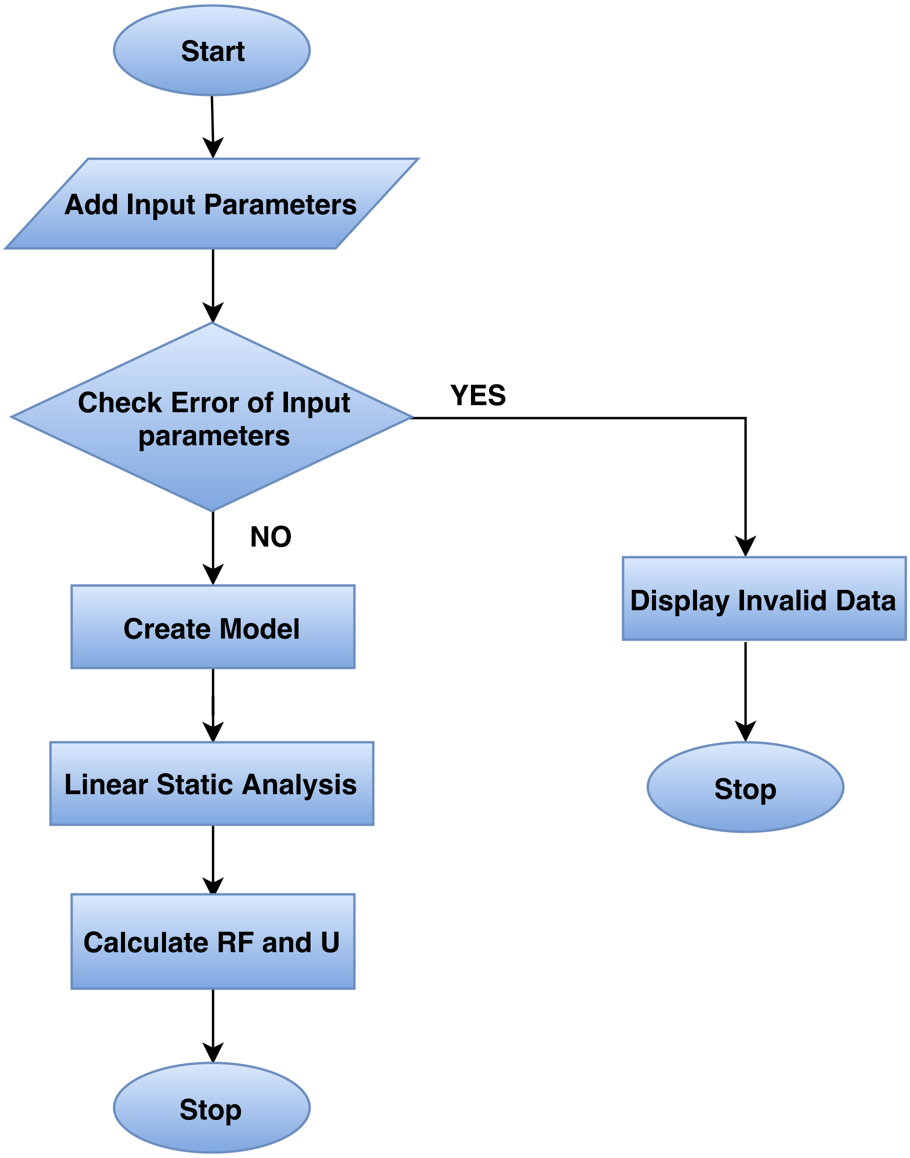


Figure 27: Flow diagram of developed Program

3.3 Aspects of the Abaqus graphical user interface

Abaqus/CAE kernel provides various facilities to generate FE-Model. Abaqus object tree provides various section to buildup FE-Model step by step. The scripting interface offers the possibility to run python script.

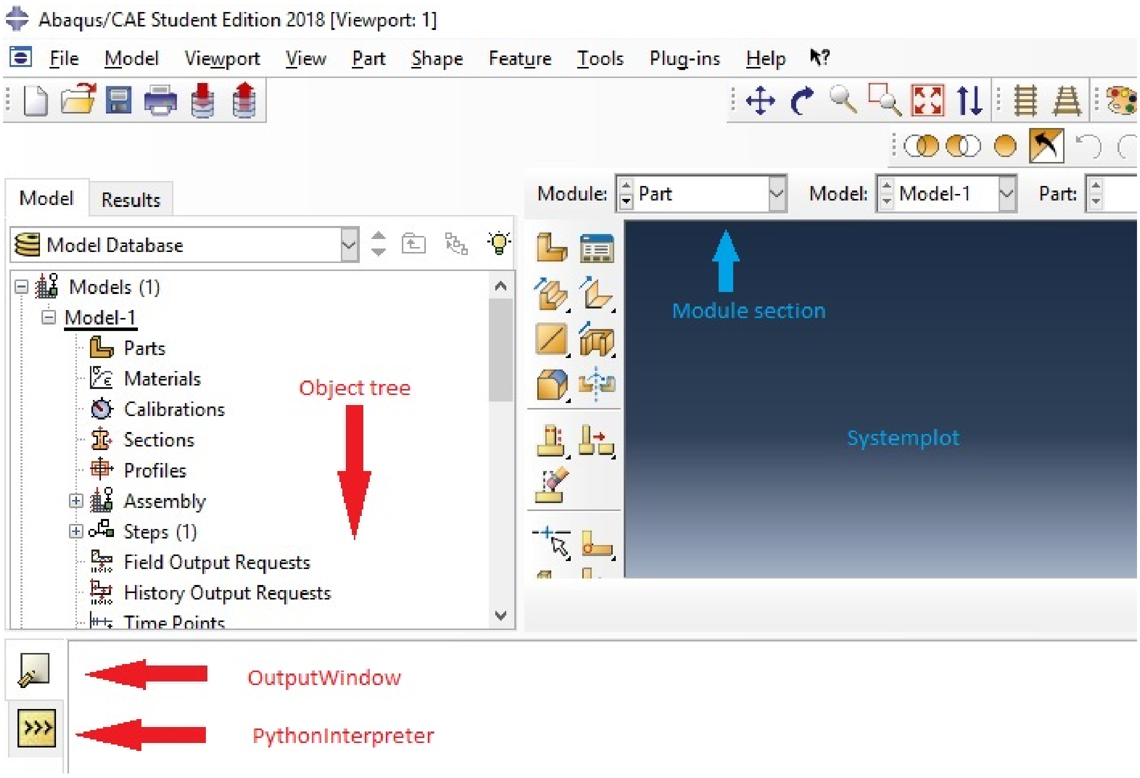


Figure 28: Abaqus graphical user interface

Abaqus offers the possibility to record interactive actions in a python by using abaqus macro manager. It is not mandatory to record macros, but it will make python programming easier. The Figure - 29 shows the facility of abaqus macro manager. An example to create sketch of the profile and its recording using abaqus macro manager is illustrated below.

File → Macro Manager → create → Name:createSketch → select work directory →Module:sketch → create → Name → continue → add:points → add lines → Done → stop recording.

User can record other modules following the same procedure and data will be stored in python file.

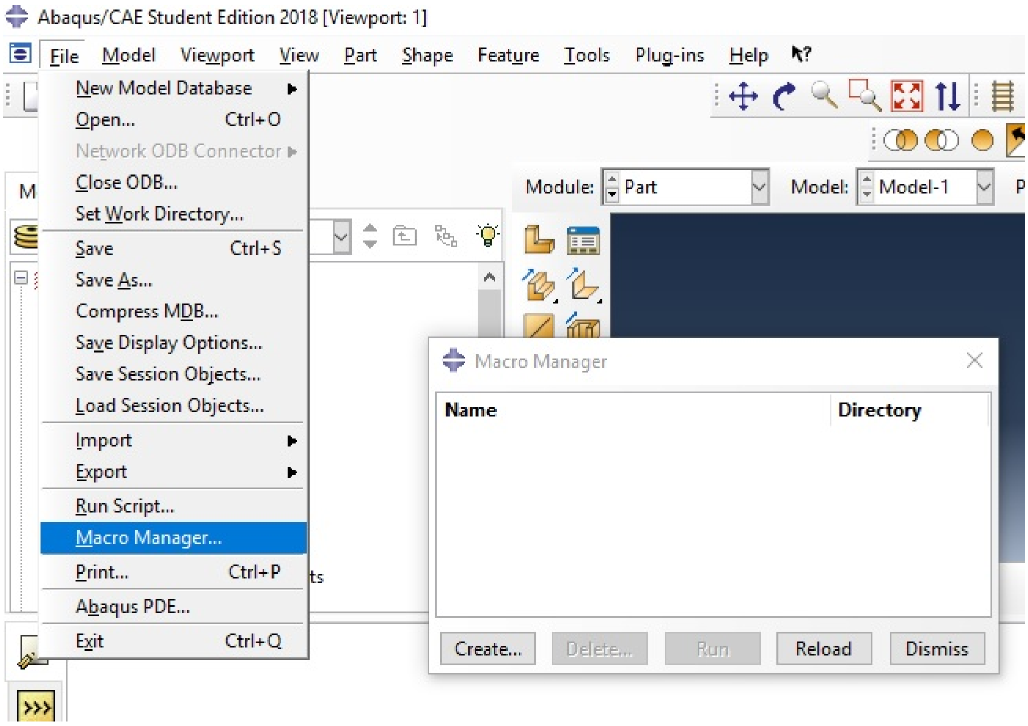


Figure 29: Abaqus Macro Manager

3.4 Visualization of various modules in Abaqus GUI

Sketch: The sketch has been created by drawing the lines through the nodes as shown in Figure-30.

Module → Sketch, Model → MPK, Sketch → MPK

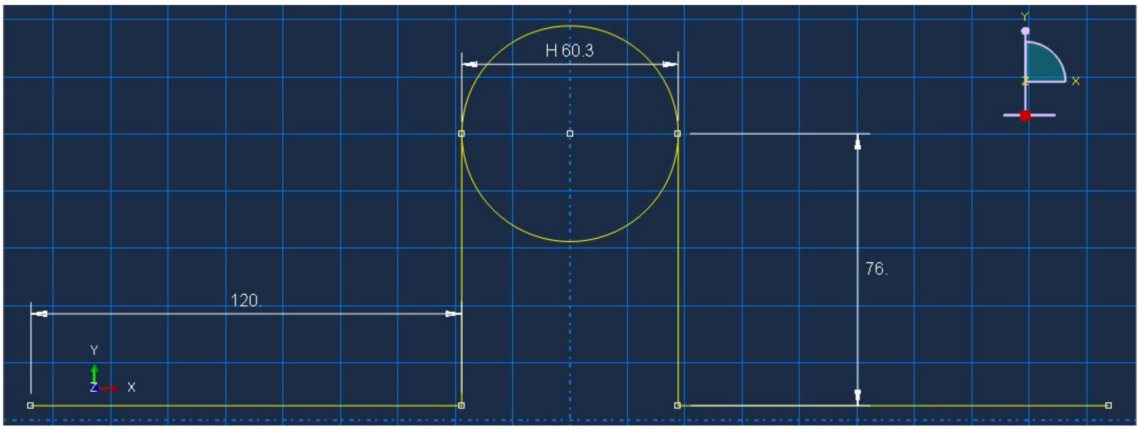


Figure 30: Combined Profile's sketch

Part: The part has been created by using extrusion in `Z` direction giving length of 3.2m as shown in Figure -31.

Module → Part, Model → MPK, Part → MPK

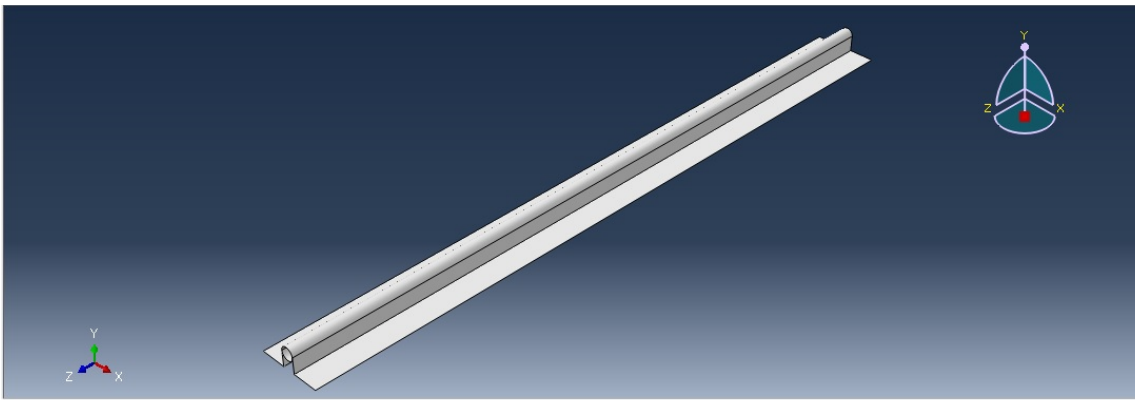


Figure 31: Combined Profile's Part

Load distribution: The load of 27 kN is distributed uniformly on the four faces of the girder as shown in Figure -32.

Module → Load, Model → MPK, Step → Linear

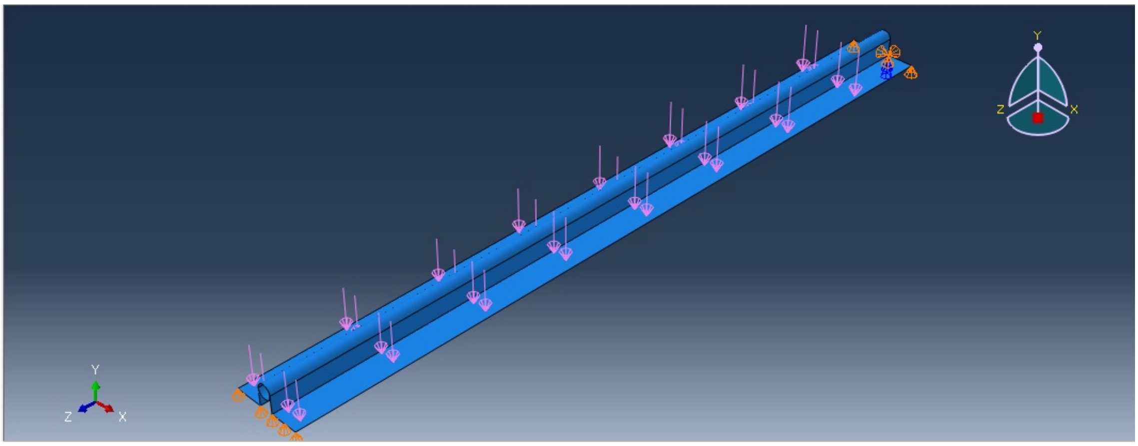


Figure 32: Load distribution

Boundary conditions: The boundary conditions are set on both end of the girder i.e at length L=0 m and at length L=3.2m. The vertical support has been provided on both end of the girder by selecting the edges and one point is fixed to restrict rigid body modes at length L=3.2m.

Vertical Support: U2 = 0
It means the displacement is fixed in Y direction.

Rigid body nodes: U1 = U3 = UR2 = 0
It means the displacement is fixed in X and Z direction and rotation is fixed in Y direction.

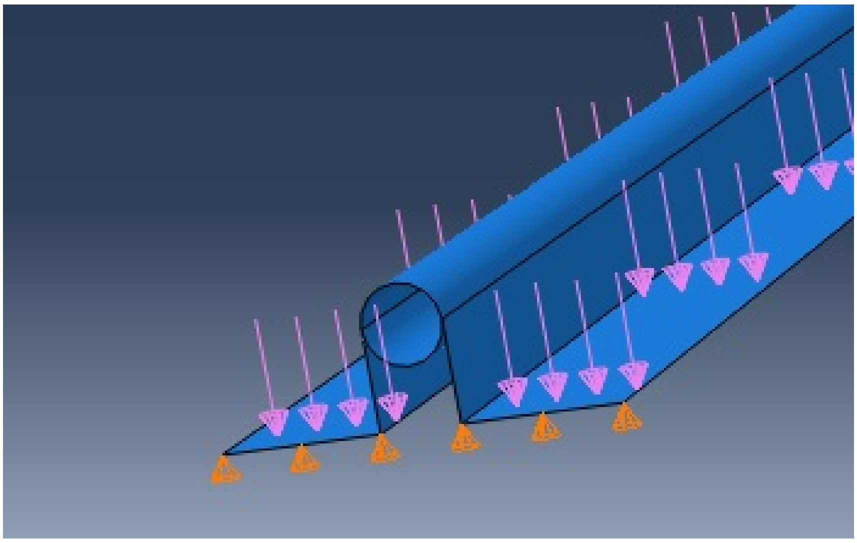


Figure 33: Boundary Condition at L=0, Vertical Support

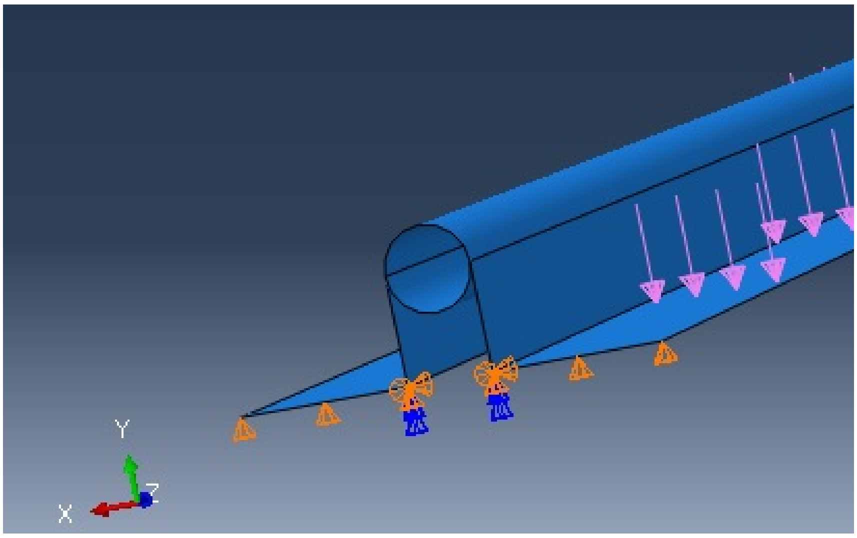


Figure 34: Boundary Condition at L=3.2m, Rigid body modes

Mesh The maximum allowable number of elements are 1000 in abaqus student version. So the number of seeds along the faces of the combined profile are so adjusted that, it can not exceed more than the limit of 1000. The calculation for the same is explained in InputData class represented in section 3.2.2.

Module → Mesh, Model → MPK, Object → assembly

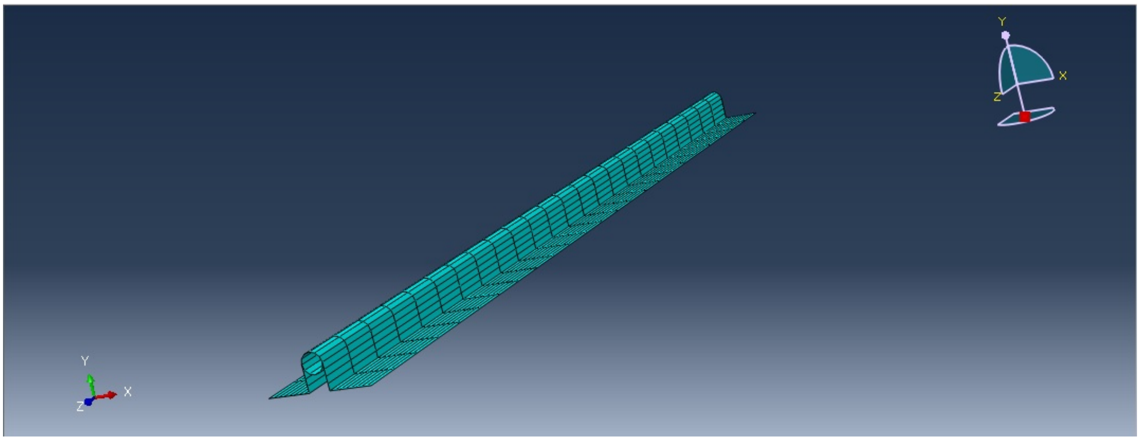


Figure 35: Combined profile's mesh

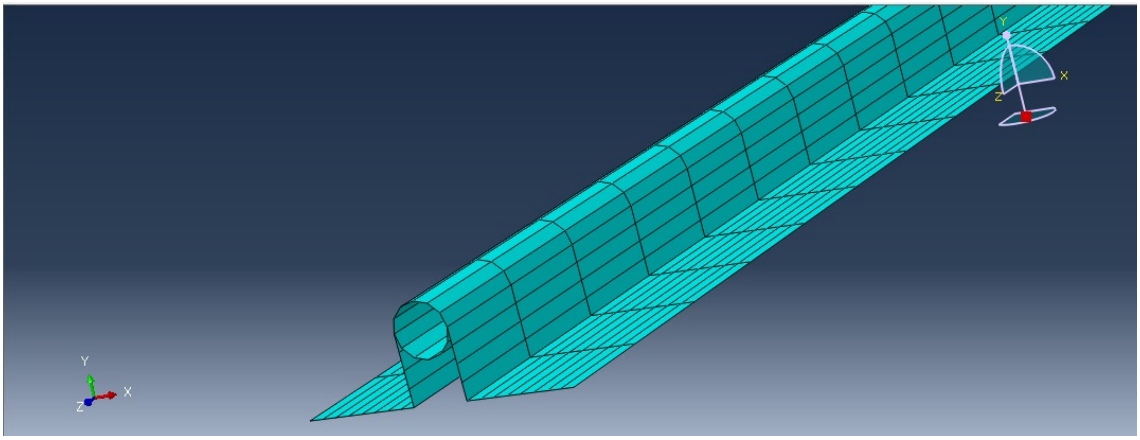


Figure 36: Closer view of the mesh

3.5 User Manual

This section provides the instruction to use the developed python program and its implementation in Abaqus to automate the FE-Model of the combined profile. The example of combination-1 has been explained here.

- **Open the developed program** – Open the `jetBrains PyCharm Community Edition` from the start menu of task bar of your computer and open `InputData.py` file from the project folder. User can see the window as shown in Figure -37 after opening in IDE.

Base.py × | InputData.py × | MPK.py × | ResultData.py × | runMPK.py ×

```
'''
Input data to control the calculation
- geometry
- loads
- meshing
- analysis type
'''

from Base import Base
profile_1 = True     # Combination-1  L100x50x6 + O60.3x2.3
profile_2 = False    # Combination-2  L100x75x10 + O60.3x2.3
profile_3 = False    # Combination-3  L120x80x8 + O60.3x2.3

class InputData(Base):
    # constructor
    def __init__(self):
        Base.__init__(self)

        self.prjName = "MPK"
        # profile parameters
        self.load = 27.0          # [kN]

        # mesh parameters
        self.maxNumber = 1000      # student edition limited 1000 nodes
        self.longplateSeed = 8     # number of long plate seeds
        self.shortplateSeed = 4    # number of short plate seeds
        self.tubeSeed = 6          # number of tube seeds
        # number of length seeds
```

Figure 37: Window of InputData file

- **Input data to the program** – There are three instances created namely profile_1, profile_2 and profile_3 respectively for three different combinations as shown in table-9.

  Set **True** key in front of the corresponding combined profile. In this case **True** key has been set for first combination. User can also insert the dimension of the profile as per requirement.

Profile	$k[mm]$	$bb[mm]$	$z[mm]$	$d[mm]$	$Tt[mm]$	$len[mm]$
profile_1	100	50	6	60.3	2.3	3200
profile_2	100	75	10	60.3	2.3	3200
profile_3	120	80	8	60.3	2.3	3200

Table 9: Combined profile dimensions

- **Execute the program** – Open the python file `runMPK.py` from project folder. Set the path of the working directory and path of the `runMPK.py` file as shown in Figure- 38.

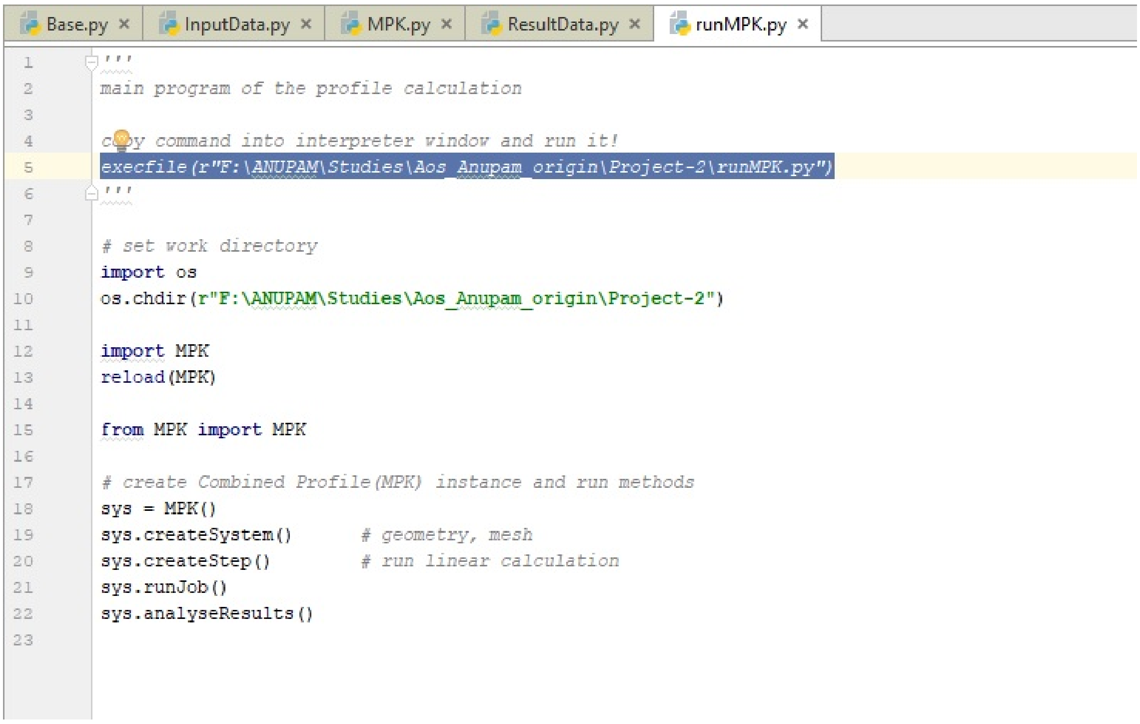


Figure 38: Copy path from runMPK python file

- Open the **Abaqus CAE** from the start menu of taskbar of your computer.
- Copy the path of **runMPK.py** from `runMPK.py` file as shown in Figure-38 and paste the path in python interpreter of the Abaqus CAE as shown in Figure-39.

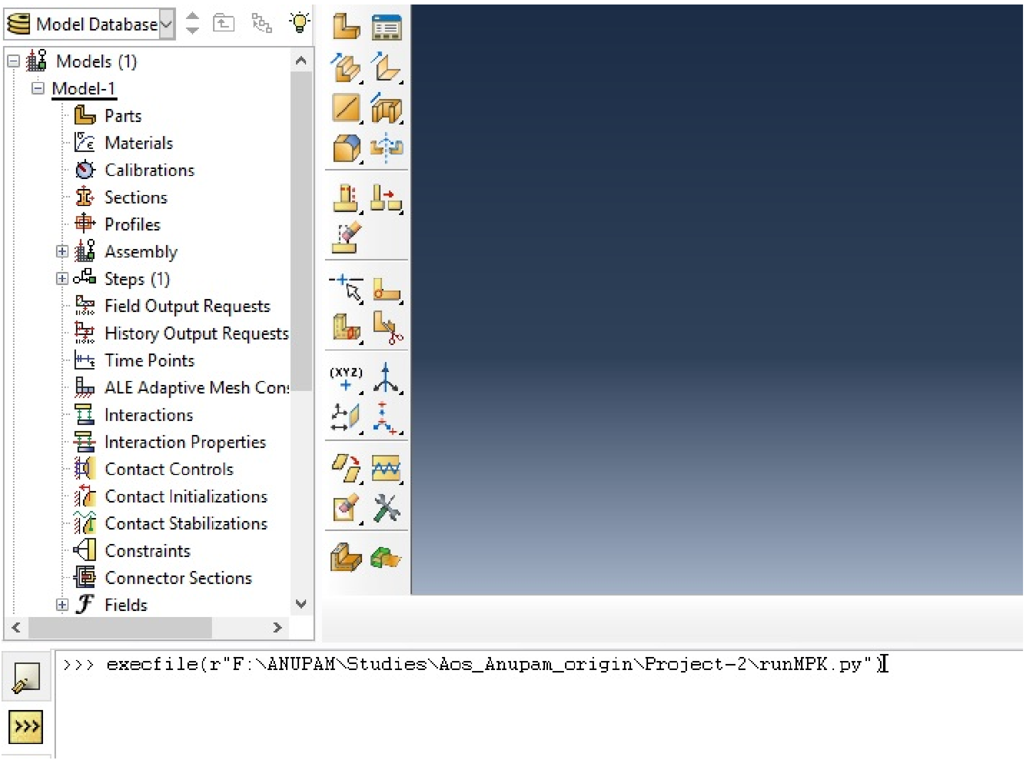


Figure 39: Implementation of path in Abaqus

- **Visualization of the output** – Visualize the maximum vertical displacement in abaqus graphic window in Visualization module as shown in Figure-40. It shows the displacement in **Y** direction.
- Check the results in Abaqus output window as shown in Figure-41 and generated `MPK.log` file in project folder. Similarly, user can implement the program for other combinations and can visualize the results.

Module → Visualization

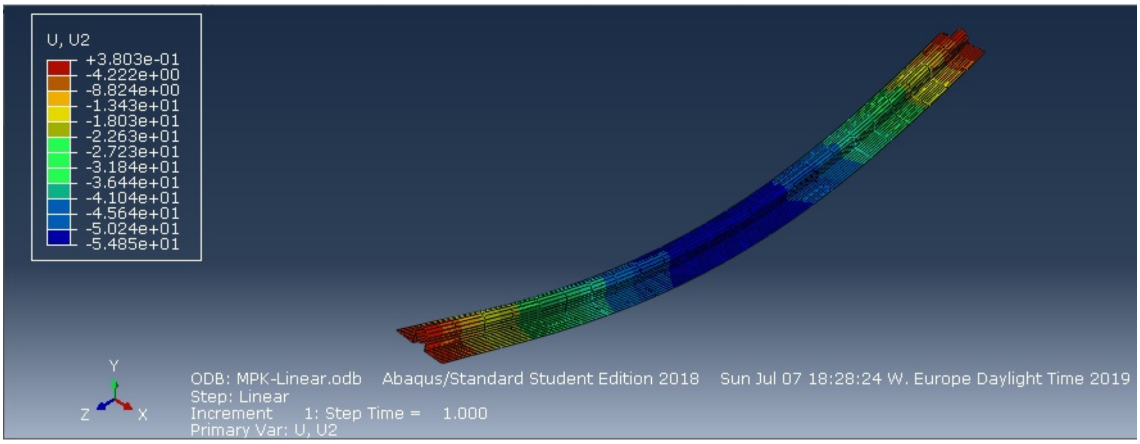


Figure 40: Displacement in Y direction

```
Job MPK-Linear: Abaqus/Standard completed successfully.
Job MPK-Linear completed successfully.

16.47.19 > open result database 'MPK-Linear.odb'...
16.47.19     5  0.000e+00  9.898e+03  0.000e+00
16.47.19     6  4.656e+03  5.841e+03 -6.294e-06
16.47.19     7  0.000e+00 -7.349e+01  0.000e+00
16.47.19     8  0.000e+00  2.636e+02  0.000e+00
16.47.19     9  0.000e+00 -7.349e+01  0.000e+00
16.47.19    10  0.000e+00  9.898e+03  0.000e+00
16.47.19    11 -4.656e+03  5.841e+03  6.237e-06
16.47.19    12  0.000e+00  2.636e+02  0.000e+00
16.47.19   112  0.000e+00 -2.994e+03  0.000e+00
16.47.19   113  0.000e+00  3.272e+02  0.000e+00
16.47.19   114  0.000e+00 -2.240e+02  0.000e+00
16.47.19   115  0.000e+00 -9.926e+01  0.000e+00
16.47.19   116  0.000e+00 -4.777e+01  0.000e+00
16.47.19   117  0.000e+00 -2.868e+01  0.000e+00
16.47.19   118  0.000e+00 -8.673e+00  0.000e+00
16.47.19   142  0.000e+00  1.180e+01  0.000e+00
16.47.19   143  0.000e+00  3.088e+01  0.000e+00
16.47.19   144  0.000e+00  2.503e+01  0.000e+00
16.47.19   145  0.000e+00  2.959e+01  0.000e+00
16.47.19   146  0.000e+00  1.102e+00  0.000e+00
16.47.19   147  0.000e+00  1.148e+02  0.000e+00
16.47.19   148  0.000e+00  4.321e+02  0.000e+00
16.47.19   149  0.000e+00 -8.673e+00  0.000e+00
16.47.19   150  0.000e+00 -2.868e+01  0.000e+00
16.47.19   151  0.000e+00 -4.777e+01  0.000e+00
16.47.19   152  0.000e+00 -9.926e+01  0.000e+00
16.47.19   153  0.000e+00 -2.240e+02  0.000e+00
16.47.19   154  0.000e+00  3.272e+02  0.000e+00
16.47.19   155  0.000e+00 -2.994e+03  0.000e+00
16.47.19   179  0.000e+00  4.321e+02  0.000e+00
16.47.19   180  0.000e+00  1.148e+02  0.000e+00
16.47.19   181  0.000e+00  1.102e+00  0.000e+00
16.47.19   182  0.000e+00  2.959e+01  0.000e+00
16.47.19   183  0.000e+00  2.503e+01  0.000e+00
16.47.19   184  0.000e+00  3.088e+01  0.000e+00
16.47.19   185  0.000e+00  1.180e+01  0.000e+00
16.47.19 > reaction force:       0.000      27.000      -0.000 kN
16.47.19 > max. vertical displacement: 53.51 mm
```

Figure 41: Displacements along the fiber node

3.5.0.1 Output log file of Combination-1

```
18.28.08 1 instances available.
18.28.08 2 instances available.
18.28.08 3 instances available.
18.28.08 delete object 3...
18.28.08 delete object 2...
18.28.08 delete object 1...
18.28.08 > create model 'MPK'...
18.28.08 > create part...
18.28.08 > create material...
18.28.08 > create properties...
18.28.08 > assign sections...
18.28.08 > create mesh...
18.28.08 > select nodes...
18.28.08 --no ---------x ---------y ---------z
18.28.08    5     -30.15       3.00    3200.00
18.28.08    6     -30.15       3.00       0.00
18.28.08   86     -30.15       3.00    3066.67
18.28.08   87     -30.15       3.00    2933.33
18.28.08   88     -30.15       3.00    2800.00
18.28.08   89     -30.15       3.00    2666.67
18.28.08   90     -30.15       3.00    2533.33
18.28.08   91     -30.15       3.00    2400.00
18.28.08   92     -30.15       3.00    2266.67
18.28.08   93     -30.15       3.00    2133.33
18.28.08   94     -30.15       3.00    2000.00
18.28.08   95     -30.15       3.00    1866.67
18.28.08   96     -30.15       3.00    1733.33
18.28.08   97     -30.15       3.00    1600.00
18.28.08   98     -30.15       3.00    1466.67
18.28.08   99     -30.15       3.00    1333.33
18.28.08  100     -30.15       3.00    1200.00
18.28.08  101     -30.15       3.00    1066.67
18.28.08  102     -30.15       3.00     933.33
18.28.08  103     -30.15       3.00     800.00
18.28.08  104     -30.15       3.00     666.67
18.28.08  105     -30.15       3.00     533.33
18.28.08  106     -30.15       3.00     400.00
18.28.08  107     -30.15       3.00     266.67
18.28.08  108     -30.15       3.00     133.33
18.28.08 > 950 nodes created.
18.28.08 > 912 elements created.
18.28.08 > 25 nodes along fiber.
18.28.08 create instance...
18.28.08 > create a 'Linear' step...
18.28.08 > create BCs...
18.28.08 > create load...
18.28.08 > run job 'MPK-Linear'...
18.28.31 > open result database 'MPK-Linear.odb'...
18.28.31     5  0.000e+00  9.898e+03  0.000e+00
18.28.31     6  4.656e+03  5.841e+03 -6.294e-06
18.28.31     7  0.000e+00 -7.349e+01  0.000e+00
18.28.31     8  0.000e+00  2.636e+02  0.000e+00
18.28.31     9  0.000e+00 -7.349e+01  0.000e+00
```

```
18.28.31    10  0.000e+00  9.898e+03  0.000e+00
18.28.31    11 -4.656e+03  5.841e+03  6.237e-06
18.28.31    12  0.000e+00  2.636e+02  0.000e+00
18.28.31   112  0.000e+00 -2.994e+03  0.000e+00
18.28.31   113  0.000e+00  3.272e+02  0.000e+00
18.28.31   114  0.000e+00 -2.240e+02  0.000e+00
18.28.31   115  0.000e+00 -9.926e+01  0.000e+00
18.28.31   116  0.000e+00 -4.777e+01  0.000e+00
18.28.31   117  0.000e+00 -2.868e+01  0.000e+00
18.28.31   118  0.000e+00 -8.673e+00  0.000e+00
18.28.31   142  0.000e+00  1.180e+01  0.000e+00
18.28.31   143  0.000e+00  3.088e+01  0.000e+00
18.28.31   144  0.000e+00  2.503e+01  0.000e+00
18.28.31   145  0.000e+00  2.959e+01  0.000e+00
18.28.31   146  0.000e+00  1.102e+00  0.000e+00
18.28.31   147  0.000e+00  1.148e+02  0.000e+00
18.28.31   148  0.000e+00  4.321e+02  0.000e+00
18.28.31   149  0.000e+00 -8.673e+00  0.000e+00
18.28.31   150  0.000e+00 -2.868e+01  0.000e+00
18.28.31   151  0.000e+00 -4.777e+01  0.000e+00
18.28.31   152  0.000e+00 -9.926e+01  0.000e+00
18.28.31   153  0.000e+00 -2.240e+02  0.000e+00
18.28.31   154  0.000e+00  3.272e+02  0.000e+00
18.28.31   155  0.000e+00 -2.994e+03  0.000e+00
18.28.31   179  0.000e+00  4.321e+02  0.000e+00
18.28.31   180  0.000e+00  1.148e+02  0.000e+00
18.28.31   181  0.000e+00  1.102e+00  0.000e+00
18.28.31   182  0.000e+00  2.959e+01  0.000e+00
18.28.31   183  0.000e+00  2.503e+01  0.000e+00
18.28.31   184  0.000e+00  3.088e+01  0.000e+00
18.28.31   185  0.000e+00  1.180e+01  0.000e+00
18.28.32 > reaction force:      0.000     27.000     -0.000 kN
18.28.32 > max. vertical displacement: 53.51 mm
```

3.6 Result Analysis

3.6.1 Simulation Result for Combination-1(L100x50x6 + O60.3x2.3)

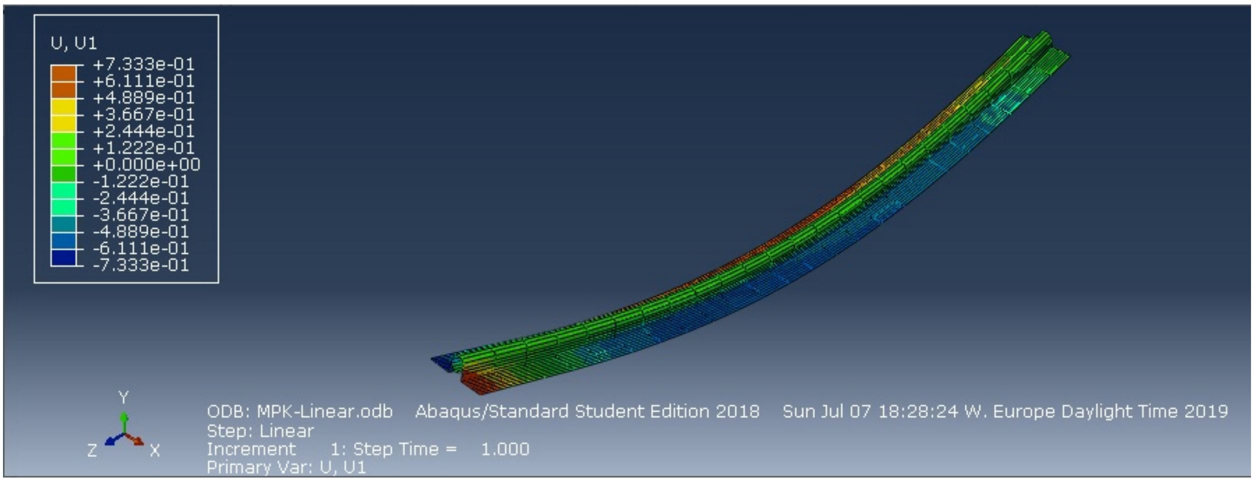


Figure 42: Displacement of the combined profile for Combination-1 in X direction

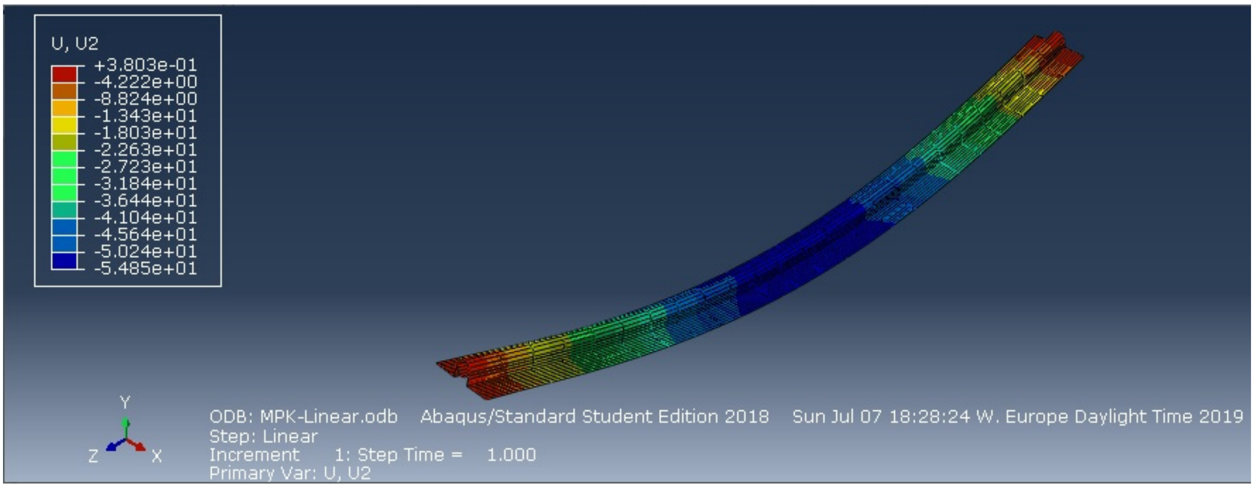


Figure 43: Displacement of the combined profile for Combination-1 in Y direction

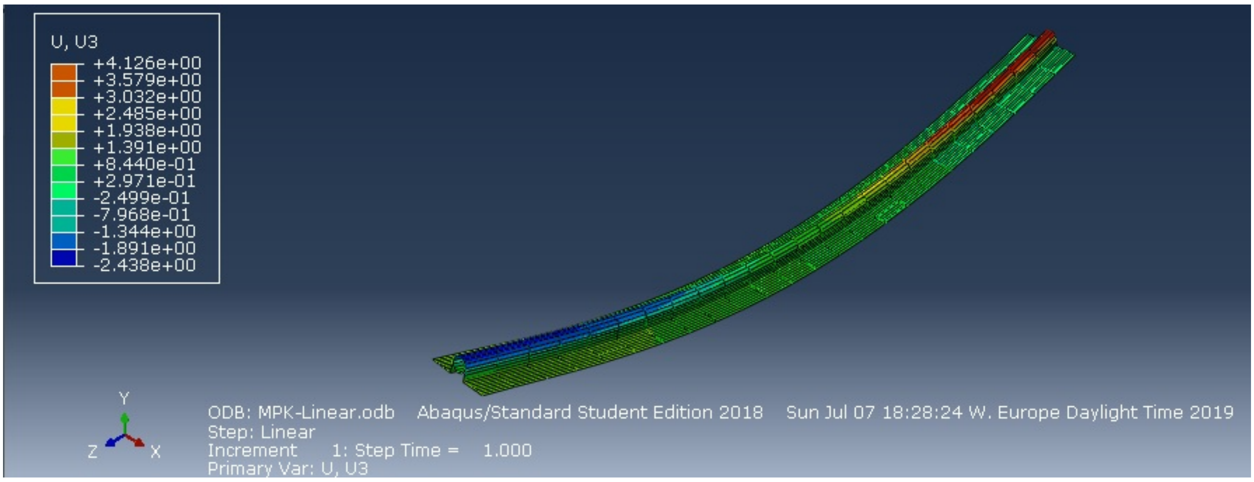
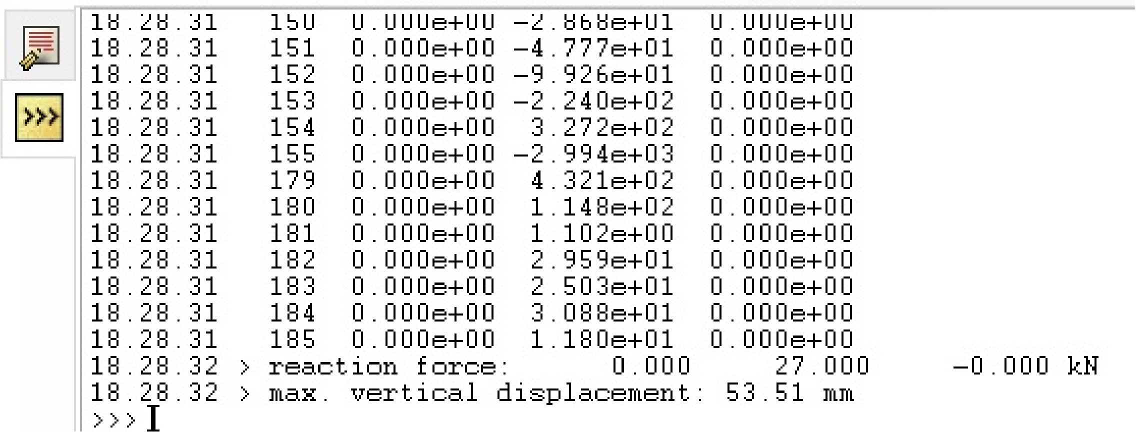


Figure 44: Displacement of the combined profile for Combination-1 in Z direction



```
18.28.31   150  0.000e+00 -2.868e+01  0.000e+00
18.28.31   151  0.000e+00 -4.777e+01  0.000e+00
18.28.31   152  0.000e+00 -9.926e+01  0.000e+00
18.28.31   153  0.000e+00 -2.240e+02  0.000e+00
18.28.31   154  0.000e+00  3.272e+02  0.000e+00
18.28.31   155  0.000e+00 -2.994e+03  0.000e+00
18.28.31   179  0.000e+00  4.321e+02  0.000e+00
18.28.31   180  0.000e+00  1.148e+02  0.000e+00
18.28.31   181  0.000e+00  1.102e+00  0.000e+00
18.28.31   182  0.000e+00  2.959e+01  0.000e+00
18.28.31   183  0.000e+00  2.503e+01  0.000e+00
18.28.31   184  0.000e+00  3.088e+01  0.000e+00
18.28.31   185  0.000e+00  1.180e+01  0.000e+00
18.28.32 > reaction force:      0.000      27.000      -0.000 kN
18.28.32 > max. vertical displacement: 53.51 mm
>>>
```

Figure 45: Displacements along the fiber node of Combination-1

3.6.2 Analytical Result for Combination-1(L100x50x6 + O60.3x2.3)

The moment of inertia I_{yc} of the beam has been taken from the analytical calculation done in thin wall approximation project.

Load applied P	27000 [N]
Length of the beam	3200 [mm]
Material	Structural Steel
Young's modulus E	210000 $[\frac{N}{mm^2}]$
Moment of Inertia I_{yc}	1011996 $[mm^4]$

Table 10: Section Properties

The maximum vertical displacement is calculated using the formula of uniform distributed load with given span length.

Maximum Displacement:

$$\begin{aligned}\delta_{max} &= \frac{(5 \times P \times L^3)}{384 \times E \times I} \\ &= \frac{(5 \times 27000 \times 3200^3)}{384 \times 210000 \times 1011996} \\ &= 54.2[mm]\end{aligned}$$

% Variation in displacement:

$$\begin{aligned}\%Variation &= \frac{\delta_{max_{analytical}} - \delta_{max_{abaqus}}}{\delta_{max_{analytical}}} \times 100\% \\ &= \frac{54.2 - 53.51}{54.2} \times 100\% \\ &= 1.27\%\end{aligned}$$

3.6.3 Simulation Result for Combination-2(L100x75x10 + O60.3x2.3)

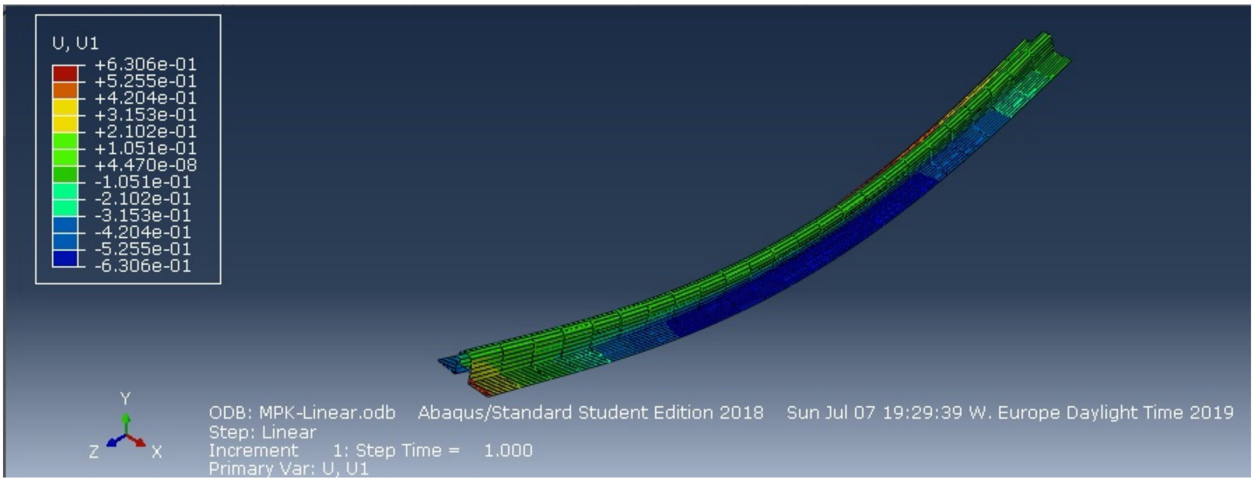


Figure 46: Displacement of the combined profile for Combination-2 in X direction

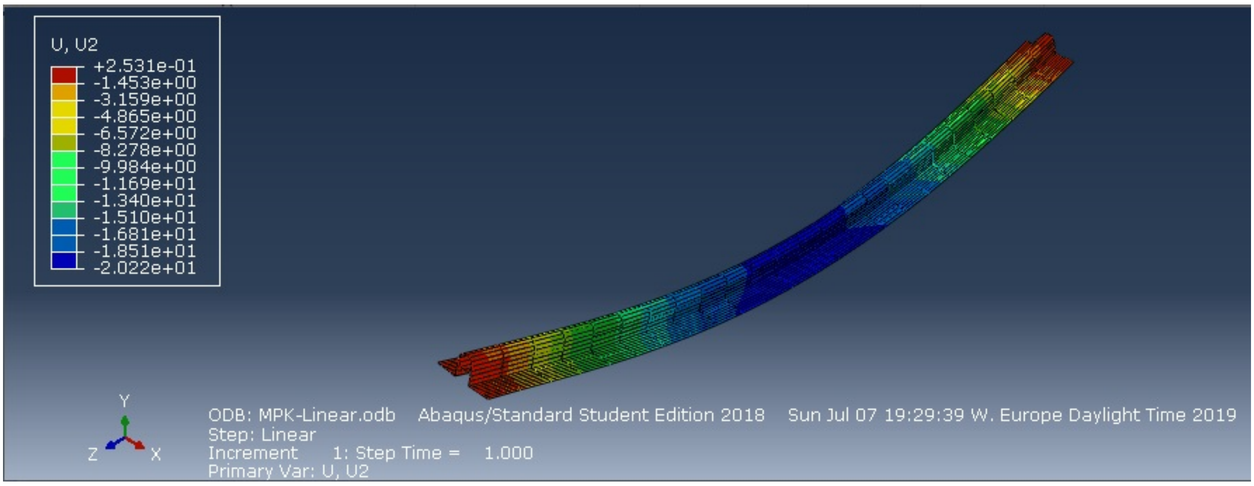


Figure 47: Displacement of the combined profile for Combination-2 in Y direction

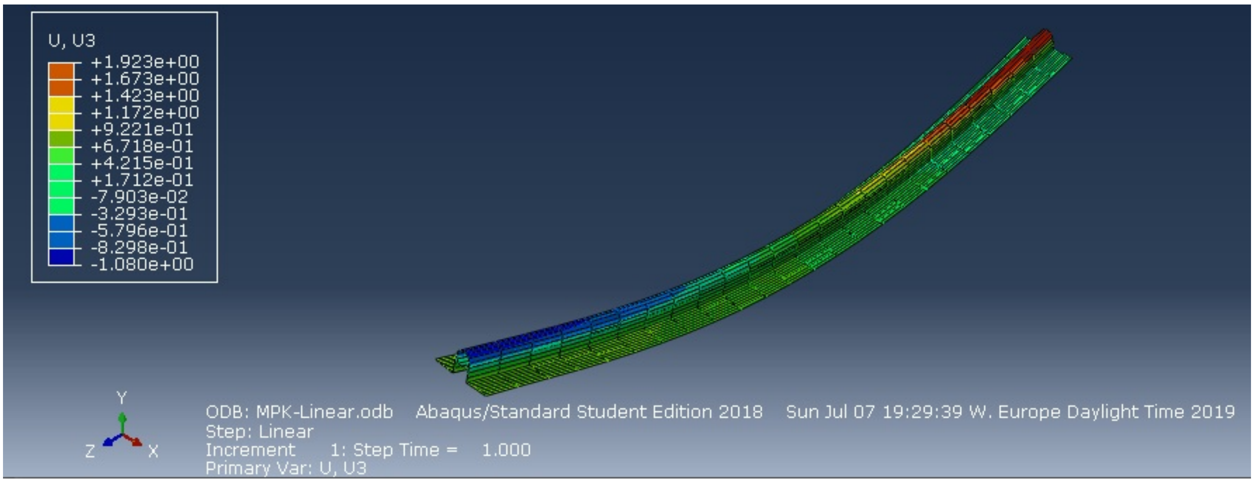


Figure 48: Displacement of the combined profile for Combination-2 in Z direction

```
19.29.47   146   0.000e+00 -5.220e+01   0.000e+00
19.29.47   147   0.000e+00 -8.910e+01   0.000e+00
19.29.47   148   0.000e+00 -1.408e+02   0.000e+00
19.29.47   149   0.000e+00 -1.519e+02   0.000e+00
19.29.47   150   0.000e+00 -2.445e+02   0.000e+00
19.29.47   151   0.000e+00 -2.756e+03   0.000e+00
19.29.47   175   0.000e+00  3.936e+02   0.000e+00
19.29.47   176   0.000e+00  1.940e+02   0.000e+00
19.29.47   177   0.000e+00  5.933e+01   0.000e+00
19.29.47   178   0.000e+00  3.628e+01   0.000e+00
19.29.47   179   0.000e+00  3.739e+01   0.000e+00
19.29.47   180   0.000e+00  4.219e+01   0.000e+00
19.29.47   181   0.000e+00  8.558e+00   0.000e+00
19.29.47 > reaction force:      0.000      27.000       0.000 kN
19.29.47 > max. vertical displacement: 19.26 mm
>>>
```

Figure 49: Displacement along the fiber node of Combination-2

3.6.4 Analytical Result for Combination-2(L100x75x10 + O60.3x2.3)

The moment of inertia I_{yc} of the beam has been taken from the analytical calculation done in thin wall approximation project.

Load applied P	27000 [N]
Length of the beam	3200 [mm]
Material	Structural Steel
Young's modulus E	210000 $[\frac{N}{mm^2}]$
Moment of Inertia I_{yc}	2874900 $[mm^4]$

Table 11: Section Properties

The maximum vertical displacement is calculated using the formula of uniform distributed load with given span length.

Maximum Displacement:

$$
\begin{aligned}
\delta_{max} &= \frac{(5 \times P \times L^3)}{384 \times E \times I} \\
&= \frac{(5 \times 27000 \times 3200^3)}{384 \times 210000 \times 2874900} \\
&= 19.08[mm]
\end{aligned}
$$

% Variation in displacement:

$$
\begin{aligned}
\%Variation &= \frac{\delta_{max_{analytical}} - \delta_{max_{abaqus}}}{\delta_{max_{analytical}}} \times 100\% \\
&= \frac{19.08 - 19.26}{19.08} \times 100\% \\
&= -0.94\%
\end{aligned}
$$

3.6.5 Simulation Result for Combination-3(L120x80x8 + O60.3x2.3)

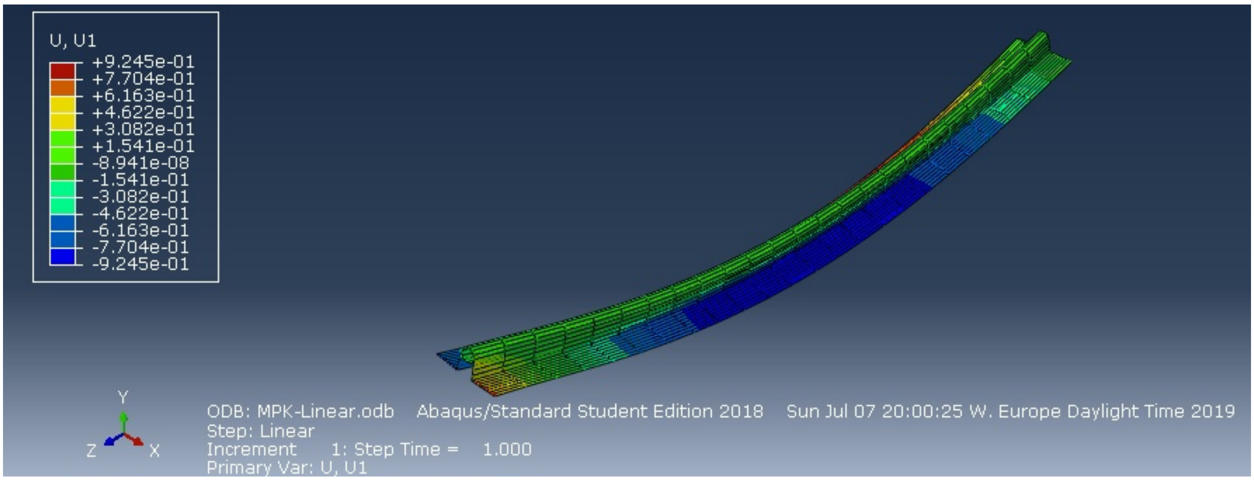


Figure 50: Displacement of the combined profile for Combination-3 in X direction

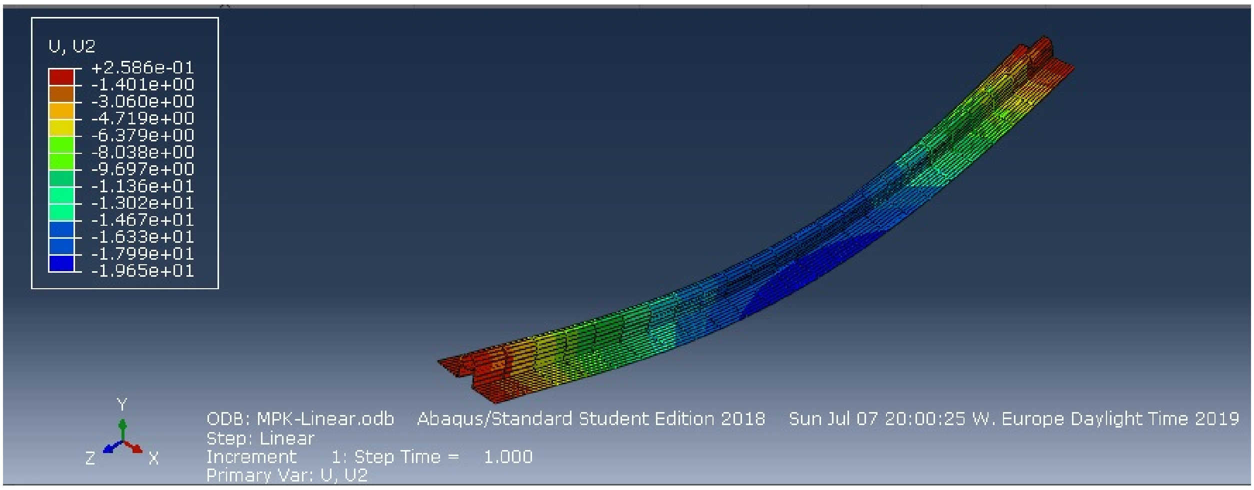


Figure 51: Displacement of the combined profile for Combination-3 in Y direction

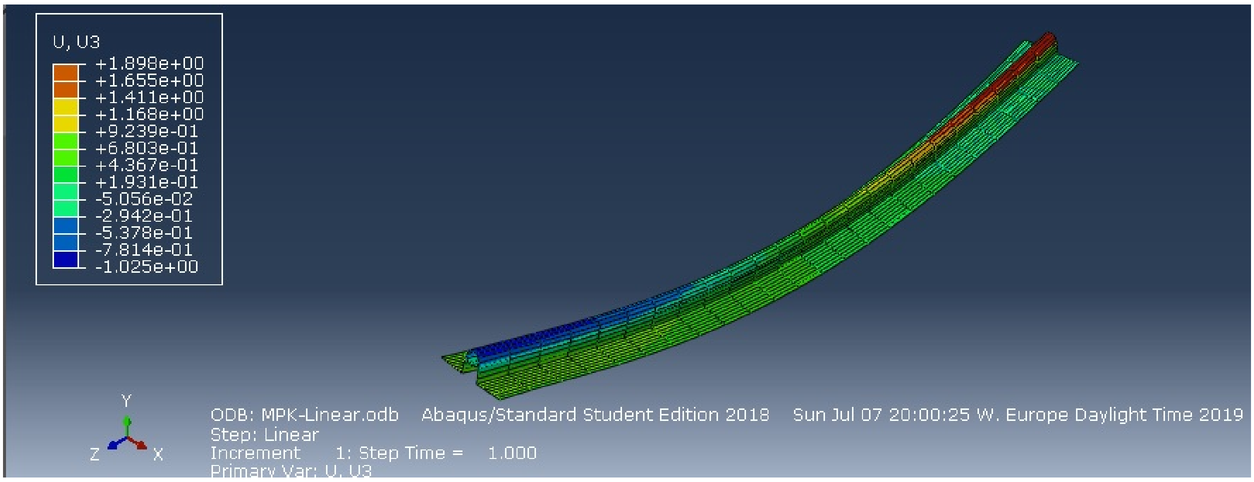
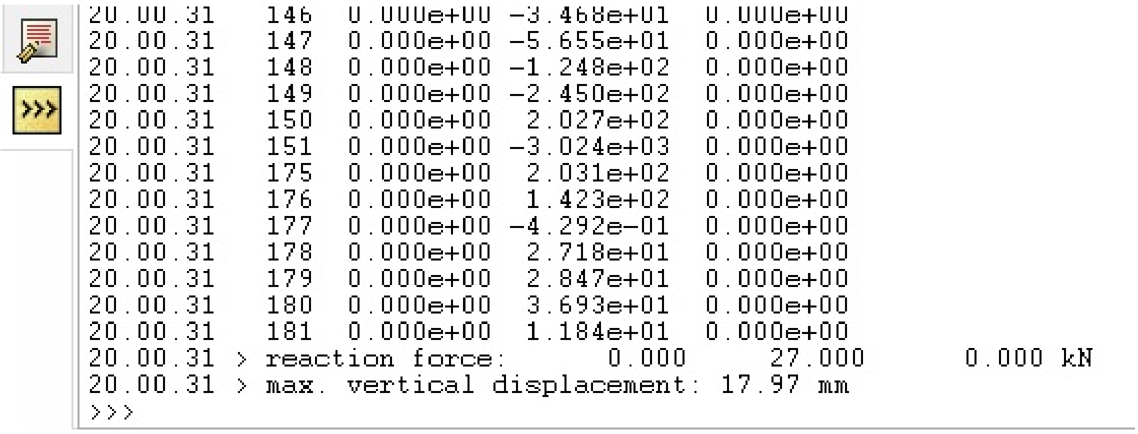


Figure 52: Displacement of the combined profile for Combination-3 in Z direction



```
20.00.31   146  0.000e+00 -3.468e+01  0.000e+00
20.00.31   147  0.000e+00 -5.655e+01  0.000e+00
20.00.31   148  0.000e+00 -1.248e+02  0.000e+00
20.00.31   149  0.000e+00 -2.450e+02  0.000e+00
20.00.31   150  0.000e+00  2.027e+02  0.000e+00
20.00.31   151  0.000e+00 -3.024e+03  0.000e+00
20.00.31   175  0.000e+00  2.031e+02  0.000e+00
20.00.31   176  0.000e+00  1.423e+02  0.000e+00
20.00.31   177  0.000e+00 -4.292e-01  0.000e+00
20.00.31   178  0.000e+00  2.718e+01  0.000e+00
20.00.31   179  0.000e+00  2.847e+01  0.000e+00
20.00.31   180  0.000e+00  3.693e+01  0.000e+00
20.00.31   181  0.000e+00  1.184e+01  0.000e+00
20.00.31 > reaction force:      0.000     27.000      0.000 kN
20.00.31 > max. vertical displacement: 17.97 mm
>>>
```

Figure 53: Displacement along the fiber node of Combination-3

3.6.6 Analytical Result for Combination-3(L120x80x8 + O60.3x2.3)

The moment of inertia I_{yc} of the beam has been taken from the analytical calculation done in thin wall approximation project.

Load applied P	27000 [N]
Length of the beam	3200 [mm]
Material	Structural Steel
Young's modulus E	210000 [$\frac{N}{mm^2}$]
Moment of Inertia I_{yc}	3009168.56 [mm^4]

Table 12: Section Properties

The maximum vertical displacement is calculated using the formula of uniform distributed load with given span length.

Maximum Displacement:

$$\begin{aligned}\delta_{max} &= \frac{(5 \times P \times L^3)}{384 \times E \times I} \\ &= \frac{(5 \times 27000 \times 3200^3)}{384 \times 210000 \times 3009168.56} \\ &= 18.23[mm]\end{aligned}$$

% Variation in displacement:

$$\begin{aligned}\%Variation &= \frac{\delta_{max_{analytical}} - \delta_{max_{abaqus}}}{\delta_{max_{analytical}}} \times 100\% \\ &= \frac{18.23 - 17.97}{18.23} \times 100\% \\ &= 1.42\%\end{aligned}$$

3.7 Result and discussion

For structural design, the response characteristics generally of interest are: support reactions, stresses or stress resultants (i.e., axial forces, shears and bending moments) and deflections. Linear structural analysis is based on two fundamental assumptions, namely,
(a) material linearity i.e., the structures are composed of linear elastic material, and
(b) geometric linearity implying that the structural deformations are so small that the equations of equilibrium can be expressed in the un-deformed geometry of the structure.

(a) Static Equilibrium

In a supported structure system when the external forces are in balance with the internal forces, or stresses, which exactly counteract the loads (Newton's Second Law), the structure is said to be in equilibrium. Since there is no translation motion, the vector sum of the external forces must be zero ($\sum F = 0$). Since there is no rotation, the sum of the moments of the external forces about any point must be zero ($\sum M = 0$), Which is exactly satisfied in our case for all three combinations as shown in Tables -13 and 14. User can check the static equilibrium condition in Visualization Module of abaqus using `CM`-Point moment at nodes and `RF`-Reaction force at nodes.

Applied Force [kN]	Reaction Force [kN]	Summation [kN]
$F_x = 0$	$F_x = 0$	$\sum F_x = 0$
$F_y = -27$	$F_y = 27$	$\sum F_y = 0$
$F_z = 0$	$F_z = 0$	$\sum F_z = 0$

Table 13: Summation of forces in all direction

Moment in x [kN.m]	Moment in y [kN.m]	Moment in z [kN.m]
$\sum M_x = 0$	$\sum M_y = 0$	$\sum M_z = 0$

Table 14: Summation of moments in all direction

(b) Deflection

The beam with given combined profile was subjected to uniform distributed load which was implemented in Abaqus using python script. Comparing analytical result and coding result we can observe that the finite element model returns the close vertical displacements result with maximum variation of just 1.02% as shown in Table -15 .

Combined profile	$\delta_{max_{abaqus}}$ [mm]	$\delta_{max_{analytical}}$ [mm]	% Variation
Combination-1	53.51	54.2	1.27%
Combination-2	19.26	19.08	-0.94%
Combination-3	17.97	18.23	1.42%

Table 15: Variation in displacements for all combinations

From Table-15 it can be seen that we are getting smaller variations for all three combinations. As explained earlier in TWA approach, we are getting more moment of inertia I_{yc} and hence the beam is more stiffer. It means the values obtained from developed program are on safer side. The girder can withstand the applied load because the computed maximum deflection is lesser than the actual deflection. The percentage of variation can be considered as factor of safety.

The reason for such variation is due to Thin Walled Approximation of the profile neglecting the fillets and curves and overlapping part of the elements. It can be also seen that our system satisfies the static equilibrium condition and the maximum vertical displacements are close enough with analytical result even considering shell structure instead of the solid structure.

Moreover, the shell elements are successfully utilized for effective results and it leads to huge computational time savings since they allow modeling of thin features with fewer mesh elements.

Since the percentage of variation lies below 1.50%, which is a good approximation and hence our program is on safer side. The program script has been successfully developed to create FE model of the combined profile and to calculate the vertical displacements.

Bibliography

[1] Dr. E. Baeck. Analysis of structures. 2016.

1 Annex-I

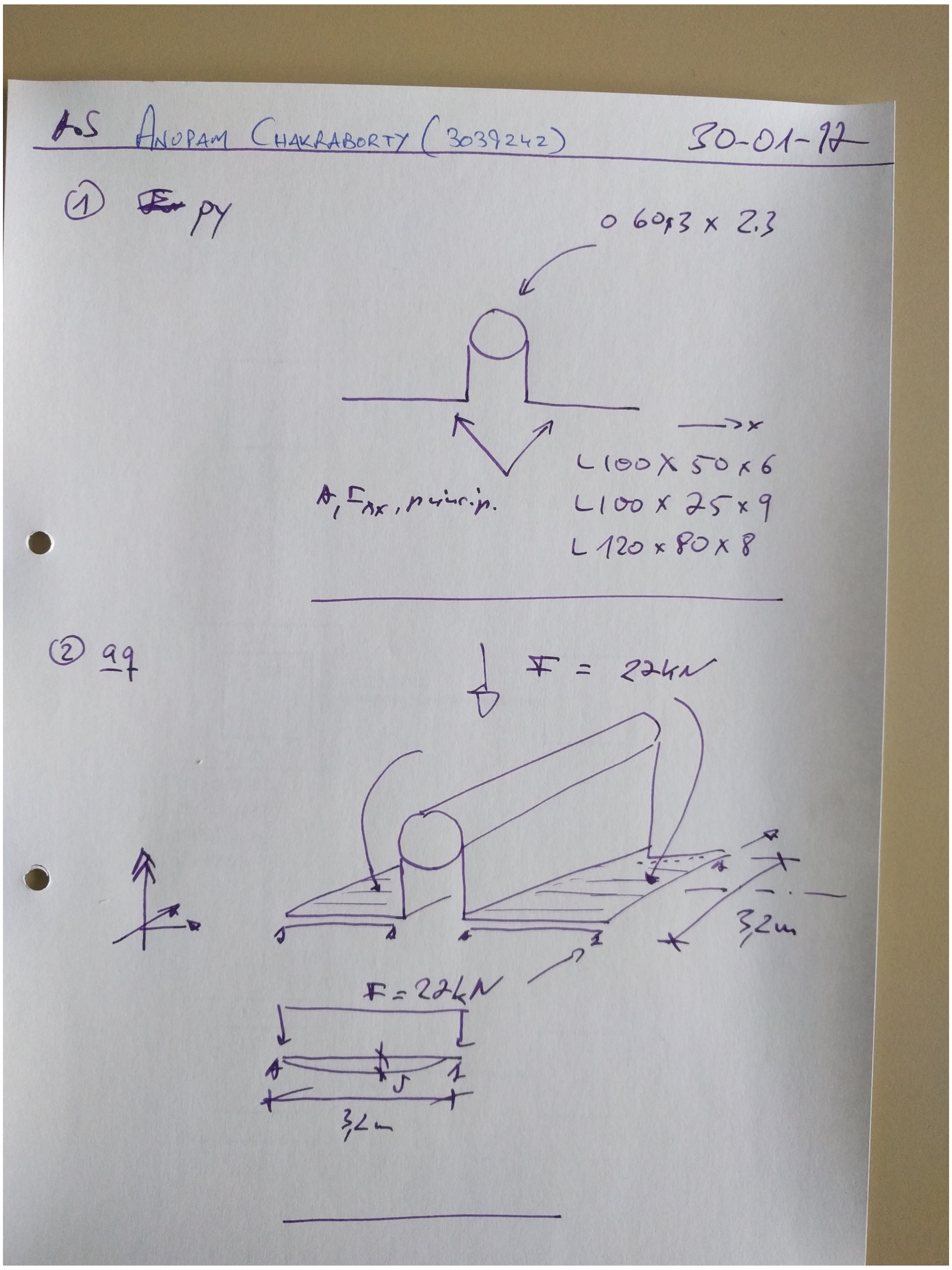


Figure 54: Task Sheet

2 Annex-II

DIN EN 10056-1:2017-06
EN 10056-1:2017 (D)

Tabelle 2 — Maße und statische Werte von warmgewalzten ungleichschenkligen Winkeln

Bezeichnung	Masse	Querschnitt	Maße				Abstände der Achsen				Statische Werte für die Biegeachse										Neigung der Achse V-V
											Achse y-y			Achse z-z			Achse u-u		Achse v-v		
			a	b	t	$r_{Übergang}$	c_y	c_z	c_u	c_v	I_y	i_y	$W_{el,y}$	I_z	i_z	$W_{el,z}$	I_u	i_u	I_v	i_v	tan α
	kg/m	cm²	mm	mm	mm	mm	cm	cm	cm	cm	cm⁴	cm	cm³	cm⁴	cm	cm³	cm⁴	cm	cm⁴	cm	
30 x 20 x 3	1,12	1,43	30	20	3	4	0,99	0,50	2,05	1,04	1,25	0,94	0,62	0,44	0,55	0,29	1,43	1,00	0,26	0,42	0,43
30 x 20 x 4	1,46	1,86	30	20	4	4	1,03	0,54	2,02	1,04	1,59	0,93	0,81	0,53	0,55	0,38	1,81	0,99	0,33	0,42	0,42
40 x 20 x 4	1,77	2,26	40	20	4	4	1,47	0,48	2,58	1,17	3,59	1,26	1,42	0,60	0,51	0,39	3,80	1,30	0,39	0,42	0,25
40 x 25 x 4	1,93	2,46	40	25	4	4	1,36	0,62	2,69	1,35	3,89	1,26	1,47	1,16	0,69	0,62	4,35	1,33	0,70	0,53	0,38
45 x 30 x 4	2,25	2,87	45	30	4	4,5	1,48	0,74	3,07	1,58	5,78	1,42	1,91	2,05	0,85	0,91	6,65	1,52	1,18	0,64	0,44
50 x 30 x 5	2,96	3,78	50	30	5	5	1,73	0,74	3,33	1,65	9,36	1,57	2,86	2,51	0,82	1,11	10,3	1,65	1,54	0,64	0,35
60 x 30 x 5	3,36	4,28	60	30	5	5	2,17	0,68	3,88	1,77	15,6	1,91	4,07	2,63	0,78	1,14	16,5	1,97	1,71	0,63	0,26
60 x 40 x 5	3,76	4,79	60	40	5	6	1,96	0,97	4,10	2,11	17,2	1,89	4,25	6,11	1,13	2,02	19,7	2,03	3,54	0,86	0,43
60 x 40 x 6	4,46	5,68	60	40	6	6	2,00	1,01	4,08	2,10	20,1	1,88	5,03	7,12	1,12	2,38	23,1	2,02	4,16	0,86	0,43
65 x 50 x 5	4,35	5,54	65	50	5	6	1,99	1,25	4,53	2,39	23,2	2,05	5,14	11,9	1,47	3,19	28,8	2,28	6,32	1,07	0,58
70 x 50 x 6	5,41	6,89	70	50	6	7	2,23	1,25	4,83	2,52	33,4	2,20	7,01	14,2	1,43	3,78	39,7	2,40	7,92	1,07	0,50
75 x 50 x 6	5,65	7,19	75	50	6	7	2,44	1,21	5,12	2,64	40,5	2,37	8,01	14,4	1,42	3,81	46,6	2,55	8,36	1,08	0,44
75 x 50 x 8	7,39	9,41	75	50	8	7	2,52	1,29	5,08	2,62	52,0	2,35	10,4	18,4	1,40	4,95	59,6	2,52	10,8	1,07	0,43
80 x 40 x 6	5,41	6,89	80	40	6	7	2,85	0,88	5,20	2,38	44,9	2,55	8,73	7,59	1,05	2,44	47,6	2,63	4,93	0,85	0,26
80 x 40 x 8	7,07	9,01	80	40	8	7	2,94	0,96	5,14	2,34	57,6	2,53	11,4	9,61	1,03	3,16	60,9	2,60	6,34	0,84	0,25
80 x 60 x 7	7,36	9,38	80	60	7	8	2,51	1,52	5,55	2,92	59,0	2,51	10,7	28,4	1,74	6,34	72,0	2,77	15,4	1,28	0,55
100 x 50 x 6	6,84	8,71	100	50	6	8	3,51	1,05	6,55	3,00	89,9	3,21	13,8	15,4	1,33	3,89	95,4	3,31	9,92	1,07	0,26
100 x 50 x 8	8,97	11,40	100	50	8	8	3,60	1,13	6,48	2,96	116	3,19	18,2	19,7	1,31	5,08	123	3,28	12,8	1,06	0,26
100 x 65 x 7	8,77	11,2	100	65	7	10	3,23	1,51	6,83	3,49	113	3,17	16,6	37,6	1,83	7,53	128	3,39	22,0	1,40	0,42
100 x 65 x 8	9,94	12,7	100	65	8	10	3,27	1,55	6,81	3,47	127	3,16	18,9	42,2	1,83	8,54	144	3,37	24,8	1,40	0,41
100 x 65 x 9	11,1	14,1	100	65	9	10	3,32	1,59	6,78	3,42	141	3,20	21,1	46,7	1,82	9,52	160	3,36	27,4	1,39	0,41
100 x 65 x 10	12,3	15,6	100	65	10	10	3,36	1,63	6,76	3,45	154	3,14	23,2	51,0	1,81	10,5	175	3,35	30,1	1,39	0,41
100 x 65 x 11	13,4	17,1	100	65	11	10	3,40	1,67	6,74	3,41	167	3,10	25,3	55,1	1,80	11,4	189	3,33	32,6	1,38	0,41
100 x 65 x 12	14,5	18,5	100	65	12	10	3,44	1,71	6,72	3,40	180	3,10	27,4	59,1	1,80	12,3	203	3,32	35,2	1,38	0,41

Figure 55: Dimensional Details and Sectional Properties of L-Profile - I

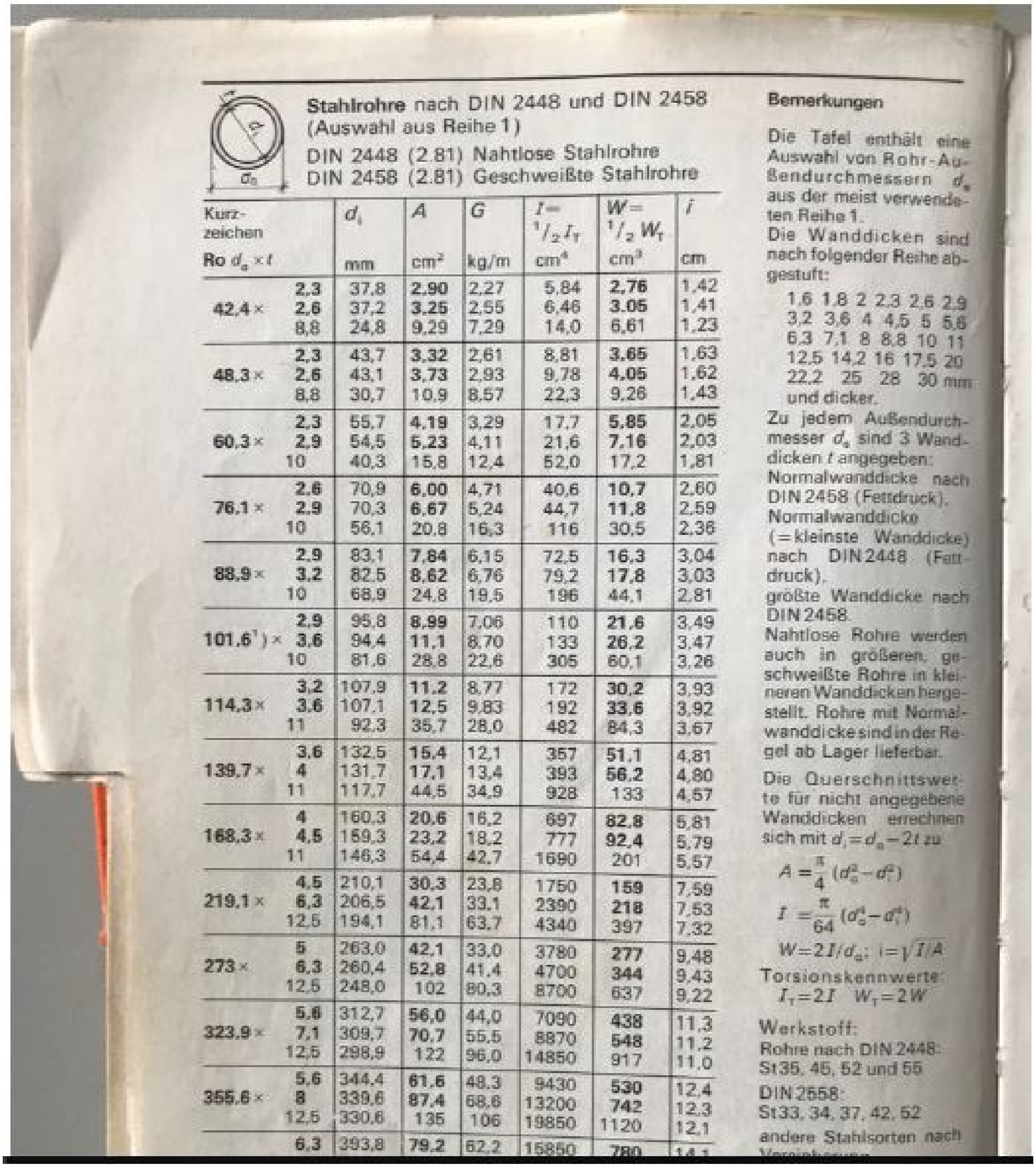
3 Annex-III

DIN EN 10056-1:2017-06
EN 10056-1:2017 (D)

Bezeichnung	Masse	Querschnitt	Maße				Abstände der Achsen				Statische Werte für die Biegeachse										Neigung der Achse V-V tan α
											Achse y-y			Achse z-z			Achse u-u		Achse v-v		
			a	b	t	$r_{Übergang}$	c_y	c_z	c_u	c_v	I_y	i_y	$W_{el,y}$	I_z	i_z	$W_{el,z}$	I_u	i_u	I_v	i_v	
	kg/m	cm²	mm	mm	mm	mm	cm	cm	cm	cm	cm⁴	cm	cm³	cm⁴	cm	cm³	cm⁴	cm	cm⁴	cm	
100 x 75 x 8	10,6	13,5	100	75	8	10	3,10	1,87	6,95	3,65	133	3,14	19,3	64,1	2,18	11,4	162	3,47	34,6	1,60	0,55
100 x 75 x 10	13,0	16,6	100	75	10	10	3,19	1,95	6,92	3,65	162	3,12	23,8	77,6	2,16	14,0	197	3,45	42,2	1,59	0,54
100 x 75 x 12	15,4	19,7	100	75	12	10	3,27	2,03	6,89	3,65	189	3,10	28,0	90,2	2,14	16,5	230	3,42	49,5	1,59	0,54
110 x 70 x 10	13,4	17,1	110	70	10	10	3,69	1,72	7,43	3,73	207	3,50	28,3	65,1	2,00	12,3	233	3,69	38,5	1,50	0,40
110 x 70 x 12	15,9	20,3	110	70	12	10	3,77	1,79	7,38	3,72	242	3,50	33,4	75,5	1,90	14,5	272	3,66	45,2	1,49	0,39
120 x 80 x 8	12,2	15,5	120	80	8	11	3,83	1,87	8,23	4,23	226	3,82	27,6	80,8	2,28	13,2	260	4,10	46,6	1,74	0,44
120 x 80 x 10	15,0	19,1	120	80	10	11	3,92	1,95	8,19	4,21	276	3,80	34,1	98,1	2,26	16,2	317	4,07	56,8	1,72	0,44
120 x 80 x 12	17,8	22,7	120	80	12	11	4,00	2,03	8,15	4,20	323	3,77	40,4	114	2,24	19,1	371	4,04	66,7	1,71	0,43
125 x 75 x 8	12,2	15,5	125	75	8	11	4,14	1,68	8,44	4,14	247	4,00	29,6	67,6	2,09	11,6	274	4,21	40,9	1,63	0,36
125 x 75 x 10	15,0	19,1	125	75	10	11	4,23	1,76	8,39	4,17	302	3,97	36,5	82,1	2,07	14,3	334	4,18	49,9	1,61	0,36
125 x 75 x 12	17,8	22,7	125	75	12	11	4,31	1,84	8,33	4,15	354	3,95	43,2	95,5	2,05	16,9	391	4,15	58,5	1,61	0,35
130 x 90 x 10	16,6	21,2	130	90	10	11	4,16	2,19	8,93	4,62	360	4,10	40,7	142	2,60	20,8	422	4,46	79,9	1,94	0,47
130 x 90 x 12	19,7	25,1	130	90	12	11	4,24	2,26	8,90	4,59	420	4,10	48,0	165	2,60	24,4	492	4,42	93,3	1,93	0,47
130 x 90 x 14	22,8	29,0	130	90	14	11	4,33	2,34	8,85	4,61	481	4,10	55,5	188	2,60	28,2	562	4,40	107	1,93	0,46
135 x 65 x 8	12,2	15,5	135	65	8	11	4,78	1,34	8,79	3,95	291	4,34	33,4	45,2	1,71	8,75	307	4,45	29,4	1,38	0,25
135 x 65 x 10	15,0	19,1	135	65	10	11	4,88	1,42	8,72	3,91	356	4,31	41,3	54,7	1,69	10,8	375	4,43	35,9	1,37	0,24
140 x 90 x 8	14,0	17,9	140	90	8	11	4,49	2,03	9,56	4,83	360	4,50	37,9	118	2,60	17,0	409	4,78	68,9	1,96	0,41
140 x 90 x 10	17,4	22,1	140	90	10	11	4,58	2,11	9,52	4,81	441	4,50	46,8	144	2,60	20,9	501	4,76	84,2	1,95	0,41
140 x 90 x 12	20,6	26,3	140	90	12	11	4,66	2,19	9,47	4,79	518	4,40	55,5	168	2,50	24,7	588	4,73	98,9	1,94	0,41
140 x 90 x 14	23,8	30,4	140	90	14	11	4,74	2,27	9,43	4,78	592	4,40	64,0	191	2,50	28,4	670	4,70	113	1,93	0,40
150 x 75 x 9	15,4	19,6	150	75	9	12	5,26	1,57	9,82	4,50	455	4,82	46,7	77,9	1,99	13,1	483	4,96	50,2	1,60	0,26
150 x 75 x 10	17,0	21,7	150	75	10	12	5,31	1,61	9,79	4,48	501	4,81	51,6	85,6	1,99	14,5	531	4,95	55,1	1,60	0,26
150 x 75 x 12	20,2	25,7	150	75	12	12	5,40	1,69	9,72	4,44	588	4,78	61,3	99,6	1,97	17,1	623	4,92	64,7	1,59	0,26
150 x 75 x 15	24,8	31,7	150	75	15	12	5,52	1,81	9,63	4,40	713	4,75	75,2	119	1,94	21,0	753	4,88	78,6	1,58	0,25

Figure 56: Dimensional Details and Sectional Properties of L-Profile - II

4 Annex-IV



Stahlrohre nach DIN 2448 und DIN 2458
(Auswahl aus Reihe 1)
DIN 2448 (2.81) Nahtlose Stahlrohre
DIN 2458 (2.81) Geschweißte Stahlrohre

Kurzzeichen Ro $d_a \times t$		d_i mm	A cm²	G kg/m	$I = {}^1/_2 I_T$ cm⁴	$W = {}^1/_2 W_T$ cm³	i cm
42,4 ×	**2,3**	37,8	**2,90**	2,27	5,84	**2,76**	1,42
	2,6	37,2	**3,25**	2,55	6,46	**3,05**	1,41
	8,8	24,8	9,29	7,29	14,0	6,61	1,23
48,3 ×	**2,3**	43,7	**3,32**	2,61	8,81	**3,65**	1,63
	2,6	43,1	**3,73**	2,93	9,78	**4,05**	1,62
	8,8	30,7	10,9	8,57	22,3	9,26	1,43
60,3 ×	**2,3**	55,7	**4,19**	3,29	17,7	**5,85**	2,05
	2,9	54,5	**5,23**	4,11	21,6	**7,16**	2,03
	10	40,3	15,8	12,4	52,0	17,2	1,81
76,1 ×	**2,6**	70,9	**6,00**	4,71	40,6	**10,7**	2,60
	2,9	70,3	**6,67**	5,24	44,7	**11,8**	2,59
	10	56,1	20,8	16,3	116	30,5	2,36
88,9 ×	**2,9**	83,1	**7,84**	6,15	72,5	**16,3**	3,04
	3,2	82,5	**8,62**	6,76	79,2	**17,8**	3,03
	10	68,9	24,8	19,5	196	44,1	2,81
101,6[1]) ×	**2,9**	95,8	**8,99**	7,06	110	**21,6**	3,49
	3,6	94,4	**11,1**	8,70	133	**26,2**	3,47
	10	81,6	28,8	22,6	305	60,1	3,26
114,3 ×	**3,2**	107,9	**11,2**	8,77	172	**30,2**	3,93
	3,6	107,1	**12,5**	9,83	192	**33,6**	3,92
	11	92,3	35,7	28,0	482	84,3	3,67
139,7 ×	**3,6**	132,5	**15,4**	12,1	357	**51,1**	4,81
	4	131,7	**17,1**	13,4	393	**56,2**	4,80
	11	117,7	44,5	34,9	928	133	4,57
168,3 ×	**4**	160,3	**20,6**	16,2	697	**82,8**	5,81
	4,5	159,3	**23,2**	18,2	777	**92,4**	5,79
	11	146,3	54,4	42,7	1690	201	5,57
219,1 ×	**4,5**	210,1	**30,3**	23,8	1750	**159**	7,59
	6,3	206,5	**42,1**	33,1	2390	**218**	7,53
	12,5	194,1	81,1	63,7	4340	397	7,32
273 ×	**5**	263,0	**42,1**	33,0	3780	**277**	9,48
	6,3	260,4	**52,8**	41,4	4700	**344**	9,43
	12,5	248,0	102	80,3	8700	637	9,22
323,9 ×	**5,6**	312,7	**56,0**	44,0	7090	**438**	11,3
	7,1	309,7	**70,7**	55,5	8870	**548**	11,2
	12,5	298,9	122	96,0	14850	917	11,0
355,6 ×	**5,6**	344,4	**61,6**	48,3	9430	**530**	12,4
	8	339,6	**87,4**	68,6	13200	**742**	12,3
	12,5	330,6	135	106	19850	1120	12,1
	6,3	393,8	**79,2**	62,2	15850	**780**	14,1

Bemerkungen

Die Tafel enthält eine Auswahl von Rohr-Außendurchmessern d_a aus der meist verwendeten Reihe 1.

Die Wanddicken sind nach folgender Reihe abgestuft:

1,6 1,8 2 2,3 2,6 2,9
3,2 3,6 4 4,5 5 5,6
6,3 7,1 8 8,8 10 11
12,5 14,2 16 17,5 20
22,2 25 28 30 mm
und dicker.

Zu jedem Außendurchmesser d_a sind 3 Wanddicken t angegeben:
Normalwanddicke nach DIN 2458 (Fettdruck).
Normalwanddicke (=kleinste Wanddicke) nach DIN 2448 (Fettdruck).
größte Wanddicke nach DIN 2458.
Nahtlose Rohre werden auch in größeren, geschweißte Rohre in kleineren Wanddicken hergestellt. Rohre mit Normalwanddicke sind in der Regel ab Lager lieferbar.

Die Querschnittswerte für nicht angegebene Wanddicken errechnen sich mit $d_i = d_a - 2t$ zu

$$A = \frac{\pi}{4}(d_a^2 - d_i^2)$$

$$I = \frac{\pi}{64}(d_a^4 - d_i^4)$$

$$W = 2I/d_a; \quad i = \sqrt{I/A}$$

Torsionskennwerte:
$I_T = 2I \quad W_T = 2W$

Werkstoff:
Rohre nach DIN 2448:
St35, 45, 52 und 55
DIN 2558:
St33, 34, 37, 42, 52
andere Stahlsorten nach

Figure 57: Dimensional Details and Sectional Properties of O-Profile