CS5800 Algorithms *Out: November 13, 2020*

Problem Set 5

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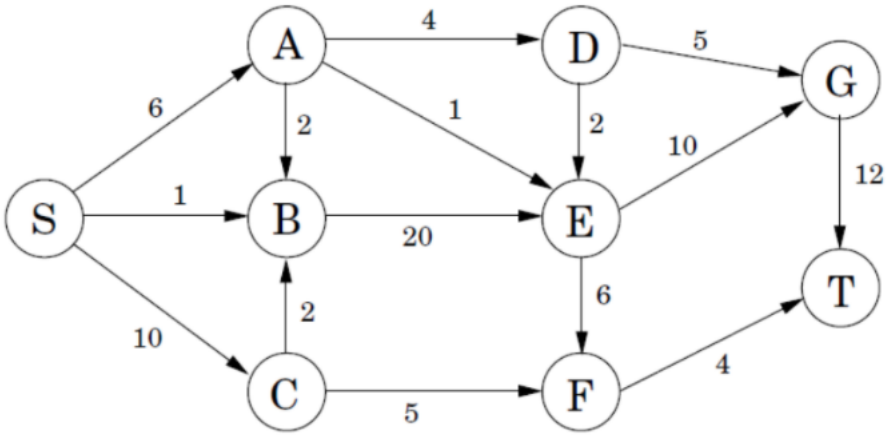
Problem 1 (Maximum Flow) 15

For the following network, with edge capacities as shown, find the maximum flow from s to t, along with a matching cut.



Problem 2 (Changing the capacities) 20

Let G be a network flow graph with source s and sink t. Suppose we increase the capacity of every edge starting in s by one unit. Indicate which of the following three statements is/are true. Justify your answer.

1. It is always the case that (i.e., for every G) the maximum flow from s to t increases by one.
2. It may be the case that (i.e., there exist G such that) the maximum flow from s to t does not increase.
3. It may be the case that (i.e., there exist G such that) the maximum flow from s to t increases by more than one unit.

Problem 3 (Updating the Maximum Flow) 10+10

You are given a directed graph G with n nodes and m edges, a source s, a sink t and a maximum flow f from s to t. Assume that the capacity of every edge is a positive integer. Describe an $O(n+m)$ time algorithm for updating the flow f in each of the following two cases.

1. The capacity of an edge e increases by 1.

2. The capacity of an edge e decreases by 1. (*Hint: decrease the flow from s to t through e by* 1*, then run one iteration of augmenting path.*)

Problem 4 (Dispensable Edge) 15

An edge of a directed graph G with source s and sink t is called *dispensable* if decreasing the capacity of this edge by 1 does not change the maximum flow from s to t in G. For every dispensable edge prove that there is no minimum s–t cut passing through it.

Problem 5 (Programming) 30

Coming soon