Q 1.1.5. The normal form of the equation of AB is

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{A}\right) = 0 \tag{1}$$

where

$$\mathbf{n}^\top \mathbf{m} = \mathbf{n}^\top \left(\mathbf{B} - \mathbf{A}\right) = 0 \tag{2}$$

$$\text{or, } \mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{3}$$

Find the normal form of the equations of AB, BC and CA.
Solution: The direction vector for CA vector is given by

$$\mathbf{m} = \mathbf{A} - \mathbf{C} \tag{4}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{6}$$

Now, normal vector is given by

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m} \tag{7}$$

$$= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{9}$$

$$\implies \mathbf{n}^\top = \begin{pmatrix} 4 & -4 \end{pmatrix} \tag{10}$$

Therefore, normal form of equation of line CA is

$$\mathbf{n}^\top(\mathbf{x} - \mathbf{C}) = 0 \tag{11}$$

$$\implies \mathbf{n}^\top \mathbf{x} - \mathbf{n}^\top \mathbf{C} = 0 \tag{12}$$

$$\implies \mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{C} \tag{13}$$

$$\implies \begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 & -4 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{14}$$

$$= -12 + 20 \tag{15}$$

$$= 8 \tag{16}$$

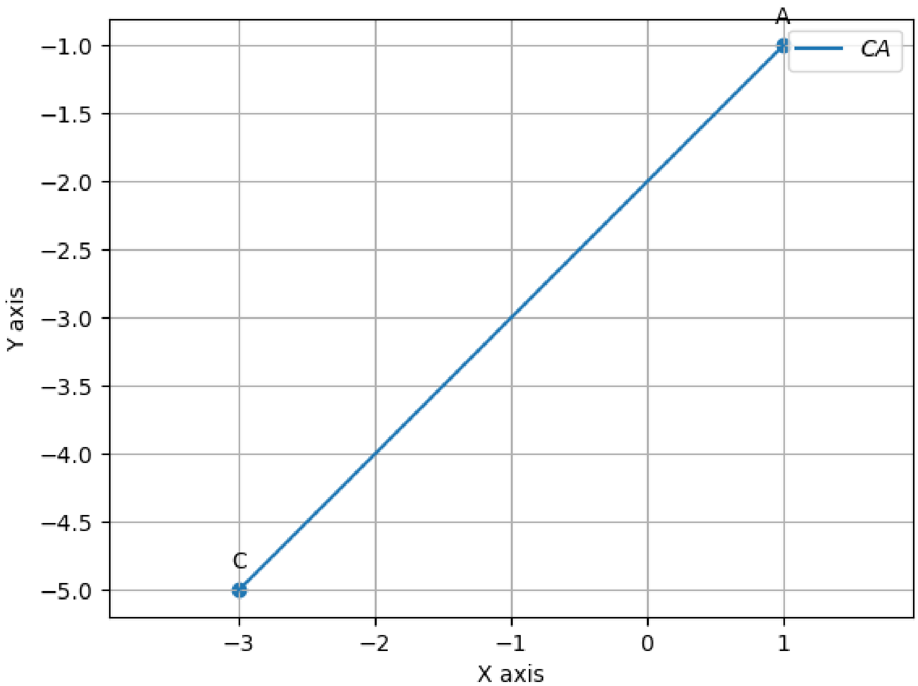

Figure 1: Line CA

Hence, the required equation of CA is

$$\begin{pmatrix} 4 & -4 \end{pmatrix} \mathbf{x} = 8 \tag{17}$$