Question 9.3.7

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There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?
Solution: The cdf using binomial is given by

Parameter	Values	Description
n	10	Number of items
p	0.05	Probability of being defective
X	1 if defective 0 if not defective	Bernoulli Random Variable
Y	$\sum_{i=1}^{n} X_i$	Binomial Random Variable

Table 1: Definition of parameters.

$$F_Y(n) = \Pr(Y \leq n) \tag{1}$$
$$= \sum_{k=0}^{n} {}^{10}C_k p^k (1-p)^{10-k} \tag{2}$$

We require $\Pr(Y \leq 1)$. Since $n = 1$,

$$F_Y(1) = \Pr(Y \leq 1) \tag{3}$$
$$= \sum_{k=0}^{1} {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{4}$$
$$= 0.9138 \tag{5}$$

Using Gaussian, Mean is given by

$$\mu = np \tag{6}$$
$$= 0.5 \tag{7}$$

Standard Deviation is given by

$$\sigma = \sqrt{np(1-p)} \tag{8}$$
$$= \sqrt{10 \times 0.05 \times (1 - 0.05)} \tag{9}$$
$$= 0.689 \tag{10}$$

We need to find

$$\Pr(X \leq 1) = \Pr(X < 1.5) \tag{11}$$

We have,

$$Z = \frac{X - \mu}{\sigma} \tag{12}$$
$$= 1.451 \tag{13}$$

$$\Pr(Z \leq 1.451) = 1 - \Pr(Z > 1.451) \tag{14}$$
$$= Q(1.4450) \tag{15}$$

From the plot, we have

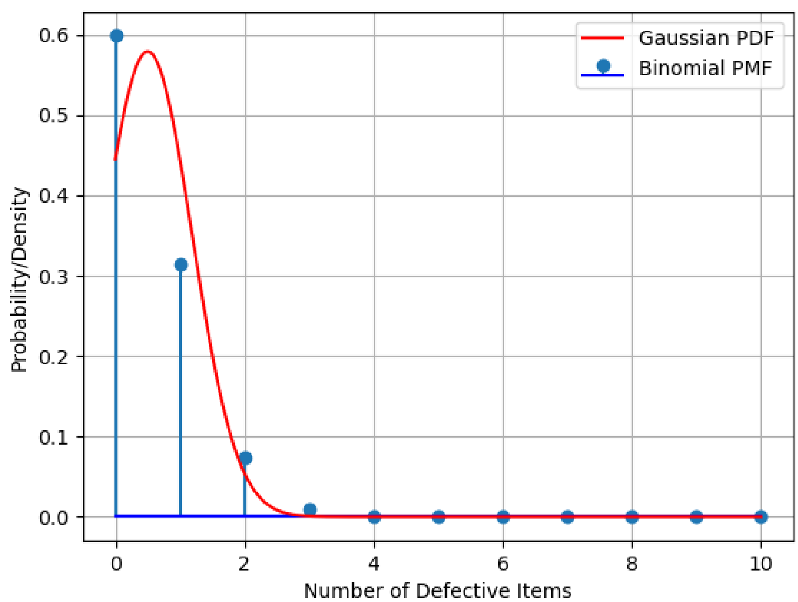

Figure 1: Binomial pmf vs Gaussian pdf

$$\Pr(Z \leq 1.451) = 0.9165 \tag{16}$$

From (5) and (16),

$$F_Y(1) \approx \Pr(Z \leq 1.451) \tag{17}$$