Question 12.13.3.1

Anupama Kulshreshtha
EE22BTECH11009

For a loaded die, the probabilities of outcomes are given as under: $\Pr(1) = \Pr(2) = 0.2, \Pr(3) = \Pr(5) = \Pr(6) = 0.1$ and $\Pr(4) = 0.3$

The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more' ,respectively. Determine whether or not A and B are independent.

Solution: Let X, Y and Z be random variables with definition given as under:

X	Number appearing on dice the first time
Y	Number appearing on dice the second time
Z	Sum of the numbers appearing on the dice

Table 1: Definition of Random variables.

Let A be the event $X = Y$
$A = ((1,1),(2,2),(3,3),(4,4),(5,5),(6,6))$
We know,

$$\Pr(A) = \sum_{k=0}^{6} \Pr(k)^2 \tag{1}$$

$$= 0.2X0.2 + 0.2X0.2 + 0.1X0.1 + 0.3X0.3 + 0.1X0.1 + 0.1X0.1 \tag{2}$$

$$= 0.2 \tag{3}$$

Let B be the event $Z \geq 10$
$B = ((4,6),(5,5),(6,4),(5,6),(6,5),(6,6))$
We know,

$$\Pr(Z = n) = \sum_{k=(n-6)}^{6} \Pr(k)\Pr(n-k) \tag{4}$$

$$\implies \Pr(Z = 10) = 0.07 \tag{5}$$

$$\implies \Pr(Z = 11) = 0.02 \tag{6}$$

$$\implies \Pr(Z = 12) = 0.01 \tag{7}$$

Hence,

$$\Pr(B) = \Pr(Z = 10) + \Pr(Z = 11) + \Pr(Z = 12) \tag{8}$$
$$= 0.1 \tag{9}$$

Now, A and B will be independent if,

$$\Pr(A \cap B) = \Pr(A)\Pr(B) \tag{10}$$
$$A \cap B = ((5,5),(6,6)) \tag{11}$$
$$\Pr(A \cap B) = 0.1X0.1 + 0.1X0.1 \tag{12}$$
$$= 0.02 \tag{13}$$
$$= \Pr(A)\Pr(B) \tag{14}$$

Hence, events A and B are independent.