Question 9.3.7

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There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?
Solution:

Parameter	Values	Description
n	10	Number of items
p	0.05	Probability of being defective
q	0.95	Probability of not being defective
$\mu = np$	0.5	Mean
$\sigma^2 = npq$	0.475	Variance

Table 1: Definition of parameters and their values

1. Binomial: The cdf using binomial is given by

$$F_Y(n) = \Pr(Y \leq n) \tag{1}$$

$$= \sum_{k=0}^{n} {}^{10}C_k p^k (1-p)^{10-k} \tag{2}$$

We require $\Pr(Y \leq 1)$. Since $n = 1$,

$$F_Y(1) = \Pr(Y \leq 1) \tag{3}$$

$$= \sum_{k=0}^{1} {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{4}$$

$$= 0.9138 \tag{5}$$

2. Gaussian: $Y \sim \mathcal{N}\left(\mu, \sigma^2\right)$

To obtain cdf,

$$\Pr(Y \leq 1) = F_Y(1) \tag{6}$$

$$F_Y(x) = \Pr(Y \leq x) \tag{7}$$

$$= \Pr(Y - \mu \leq x - \mu) \tag{8}$$

$$= \Pr(\frac{Y - \mu}{\sigma} \leq \frac{x - \mu}{\sigma}) \tag{9}$$

$$= 1 - \Pr(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}) \tag{10}$$

We know that,

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \tag{11}$$

$$\Pr(X > x) = Q(x) \tag{12}$$

Hence,

$$F_Y(x) = 1 - Q\left(\frac{x - \mu}{\sigma}\right), \text{if } x > \mu \tag{13}$$

$$= Q\left(\frac{\mu - x}{\sigma}\right), \text{if } x < \mu \tag{14}$$

$$\implies F_Y(1) = 1 - Q\left(\frac{0.5}{\sqrt{0.475}}\right) \tag{15}$$

$$= 0.766 \tag{16}$$

With correction of 0.5,

$$\Pr(Y \leq 1.5) = F_Y(1.5) \tag{17}$$

$$F_Y(1.5) = 1 - Q\left(\frac{1}{\sqrt{0.475}}\right) \tag{18}$$

$$= 0.927 \tag{19}$$

From (5) and (19)

$$\Pr(Y \leq 1) \approx F_Y(1.5) \tag{20}$$

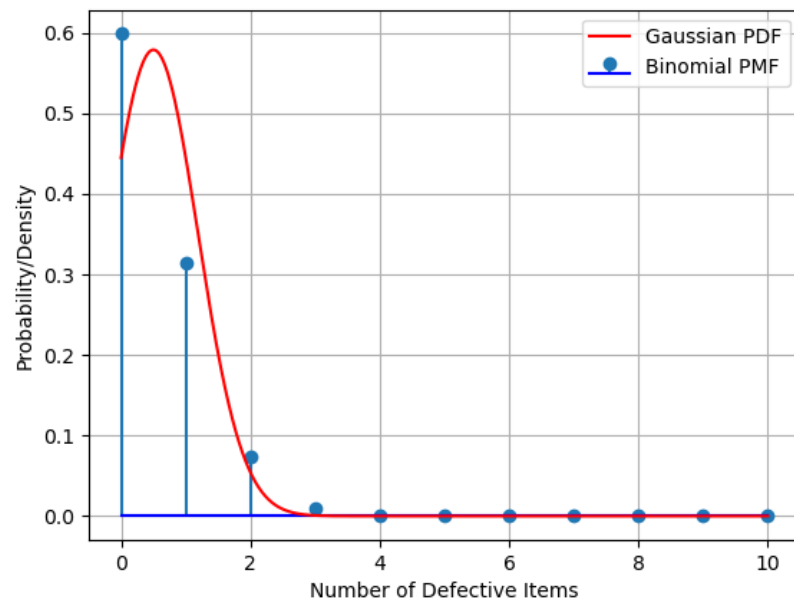| Binomial	Gaussian (without correction)	Gaussian (with correction)
0.9138	0.766	0.927

Table 2: Probability obtained using different methods

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Figure 1: Binomial pmf vs Gaussian pdf