Question 9.3.7

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There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?
Solution:

Parameter	Values	Description
n	10	Number of items
p	0.05	Probability of being defective
q	0.95	Probability of not being defective
$\mu = np$	0.5	Mean
$\sigma^2 = npq$	0.475	Variance

Table 1: Definition of parameters and their values

1. Binomial: The cdf using binomial is given by

$$F_Y(n) = \Pr(Y \leq n) \tag{1}$$
$$= \sum_{k=0}^{n} {}^{10}C_k p^k (1-p)^{10-k} \tag{2}$$

We require $\Pr(Y \leq 1)$. Since $n = 1$,

$$F_Y(1) = \Pr(Y \leq 1) \tag{3}$$
$$= \sum_{k=0}^{1} {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{4}$$
$$= 0.9138 \tag{5}$$

2. Gaussian: $Y \sim \mathcal{N}\left(\mu, \sigma^2\right)$

To obtain cdf,

$$
\begin{align}
\Pr(Y \leq 1) &= F_Y(1) \tag{6}\\
F_Y(x) &= \Pr(Y \leq x) \tag{7}\\
&= \Pr(Y-\mu \leq x-\mu) \tag{8}\\
&= \Pr(\frac{Y-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}) \tag{9}\\
&= 1-\Pr(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}) \tag{10}
\end{align}
$$

We know that,

$$
\begin{align}
\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{11}\\
\Pr(X > x) = Q(x) \tag{12}
\end{align}
$$

Hence,

$$
\begin{align}
F_Y(x) &= 1-Q\left(\frac{x-\mu}{\sigma}\right), \text{if } x > \mu \tag{13}\\
&= Q\left(\frac{\mu-x}{\sigma}\right), \text{if } x < \mu \tag{14}\\
\implies F_Y(1) &= 1-Q\left(\frac{0.5}{\sqrt{0.475}}\right) \tag{15}\\
&= 0.766 \tag{16}
\end{align}
$$

With correction of 0.5,

$$
\begin{align}
\Pr(Y \leq 1.5) &= F_Y(1.5) \tag{17}\\
F_Y(1.5) &= 1-Q\left(\frac{1}{\sqrt{0.475}}\right) \tag{18}\\
&= 0.927 \tag{19}
\end{align}
$$

From (5) and (19)

$$
\Pr(Y \leq 1) \approx F_Y(1.5) \tag{20}
$$

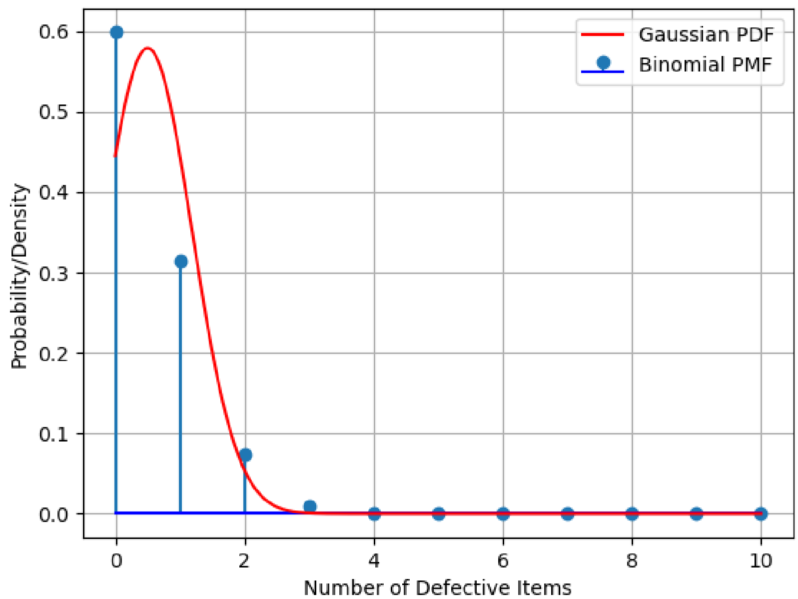

Figure 1: Binomial pmf vs Gaussian pdf