

Question 12.13.3.1

Anupama Kulshreshtha
EE22BTECH11009

For a loaded die, the probabilities of outcomes are given as under: $\Pr(1) = \Pr(2) = 0.2, \Pr(3) = \Pr(5) = \Pr(6) = 0.1$ and $\Pr(4) = 0.3$

The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more', respectively. Determine whether or not A and B are independent.

Solution: Let X, Y and Z be random variables with definition given as under:

X	Number appearing on dice the first time
Y	Number appearing on dice the second time
Z	Sum of the numbers appearing on the dice
W	Difference of the numbers appearing on the dice

Table 1: Definition of Random variables.

$$p_X(k) = \begin{cases} 0.2, & k = 1, 2 \\ 0.1, & k = 3, 5, 6 \\ 0.3, & k = 4 \end{cases} \quad (1)$$

$$p_X(k) = p_Y(k) \quad (2)$$

z transform using expectation:

$$X(z) = E[X^2] = \sum_{k=0}^{\infty} x(k) \cdot z^{-k} \quad (3)$$

PMF of W using z-transform: applying the z-transform on both the sides

$$W = X - Y \quad (4)$$

$$M_W(z) = M_{X-Y}(z) \quad (5)$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X+Y}] \quad (6)$$

$$= \left(\sum_{i=1}^6 p_X(i) \cdot z^{-i} \right) \cdot \left(\sum_{j=1}^6 p_Y(j) \cdot z^j \right) \quad (7)$$

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = p_W(0) + p_W(1)z + \dots + p_W(k)z^k + \dots \quad (8)$$

$$= 0.01(2z^{-5} + 4z^{-4} + 9z^{-3} + 12z^{-2} + 13z^{-1} + 20 \\ + 13z^1 + 12z^2 + 9z^3 + 4z^4 + 2z^5) \quad (9)$$

defined for all the values of $-5 \leq k \leq 5$

Now,

$$Z = X + Y \quad (10)$$

Z can take values ranging from $\{2 \text{ to } 12\}$.

PMF of Z using z -transform: applying the z -transform on both the sides

$$z\{Z\} = z\{X + Y\} \quad (11)$$

$$M_Z(z) = M_{X+Y}(z) \quad (12)$$

Using the expectation operator:

$$E[z^{-Z}] = E[z^{-X-Y}] \quad (13)$$

$$= \left(\sum_{i=1}^6 p_X(i) \cdot z^{-i} \right)^2 \quad (14)$$

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = (0.1z^{-6} + 0.1z^{-5} + 0.3z^{-4} + 0.1z^{-3} + 0.2z^{-2} + 0.2z^{-1})^2 \quad (15)$$

defined for all the values of $2 \leq k \leq 12$

For event A, Finding the probability for $W = 0$

$$p_W(0) = 0.2 \quad (16)$$

For event B, Finding the probability for $Z \geq 10$

$$p_Z(10) = 0.07 \quad (17)$$

$$p_Z(11) = 0.02 \quad (18)$$

$$p_Z(12) = 0.01 \quad (19)$$

Hence,

$$\Pr(B) = \Pr(Z = 10) + \Pr(Z = 11) + \Pr(Z = 12) \quad (20)$$

$$= 0.1 \quad (21)$$

Now, A and B will be independent if,

$$\Pr(AB) = \Pr(A) \Pr(B) \quad (22)$$

$$AB = ((5, 5), (6, 6)) \quad (23)$$

$$\Pr(AB) = 0.1 \times 0.1 + 0.1 \times 0.1 \quad (24)$$

$$= 0.02 \quad (25)$$

$$= \Pr(A) \Pr(B) \quad (26)$$

Hence, events A and B are independent.

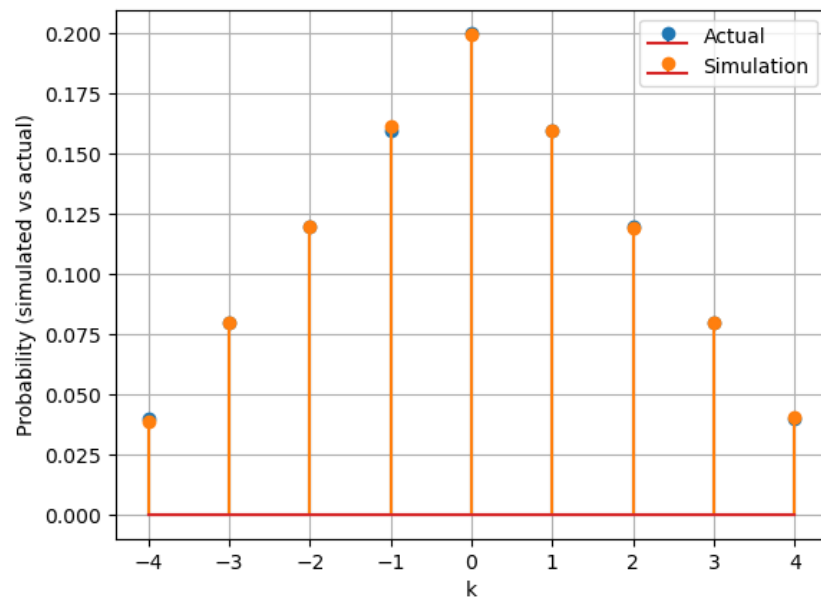


Figure 1: Stem plot for $P(Z)$