Question 12.13.3.1

Anupama Kulshreshtha
EE22BTECH11009

For a loaded die, the probabilities of outcomes are given as under: $\Pr(1) = \Pr(2) = 0.2, \Pr(3) = \Pr(5) = \Pr(6) = 0.1$ and $\Pr(4) = 0.3$

The die is thrown two times. Let A and B be the events, 'same number each time', and 'a total score is 10 or more' ,respectively. Determine whether or not A and B are independent.

Solution: Let X, Y and Z be random variables with definition given as under:

X	Number appearing on dice the first time	
Y	Number appearing on dice the second time	
Z	Sum of the numbers appearing on the dice	X + Y
W	Difference of the numbers appearing on the dice	X - Y

Table 1: Definition of Random variables.

$$p_X(k) = \begin{cases} 0.2, & k = 1, 2 \\ 0.1, & k = 3, 5, 6 \\ 0.3, & k = 4 \end{cases} \tag{1}$$

$$p_X(k) = p_Y(k) \tag{2}$$

PMF of W using z-transform: applying the z-transform on both the sides

$$M_W(z) = M_{X-Y}(z) \tag{3}$$

Using the expectation operator:

$$E\left[z^{-W}\right] = E\left[z^{-X+Y}\right] \tag{4}$$

$$= M_X(z) \cdot M_Y(z^{-1}) \tag{5}$$

Extracting the PMF by considering the definition of z-transform

$$M_W(z) = p_W(0) + p_W(1)z^{-1} + p_W(1)z^{-2} + ... + p_W(k)z^k + ... \tag{6}$$

$$= 0.01(2z^{-5} + 4z^{-4} + 9z^{-3} + 12z^{-2} + 13z^{-1} + 20$$
$$+ 13z^1 + 12z^2 + 9z^3 + 4z^4 + 2z^5) \tag{7}$$

defined for all the values of $-5 \leq k \leq 5$

Now, Z can take values ranging from $\{2 \text{ to } 12\}$.

PMF of Z using z-transform: applying the z-transform on both the sides

$$M_Z(z) = M_{X+Y}(z) \tag{8}$$
$$E\left[z^{-Z}\right] = M_X(z) \cdot M_Y(z) \tag{9}$$

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = (0.1z^{-6} + 0.1z^{-5} + 0.3z^{-4} + 0.1z^{-3} + 0.2z^{-2} + 0.2z^{-1})^2 \tag{10}$$

defined for all the values of $2 \leq k \leq 12$

For event A, Finding the probability for $W = 0$

$$p_W(0) = 0.2 \tag{11}$$

For event B, Finding the probability for $Z \geq 10$

$$p_Z(10) = 0.07 \tag{12}$$
$$p_Z(11) = 0.02 \tag{13}$$
$$p_Z(12) = 0.01 \tag{14}$$

Hence,

$$\Pr(B) = \Pr(Z = 10) + \Pr(Z = 11) + \Pr(Z = 12) \tag{15}$$
$$= 0.1 \tag{16}$$

Now, A and B will be independent if,

$$\Pr(AB) = \Pr(A)\Pr(B) \tag{17}$$
$$AB = ((5,5),(6,6)) \tag{18}$$
$$\Pr(AB) = 0.1 \times 0.1 + 0.1 \times 0.1 \tag{19}$$
$$= 0.02 \tag{20}$$
$$= \Pr(A)\Pr(B) \tag{21}$$

Hence, events A and B are independent.

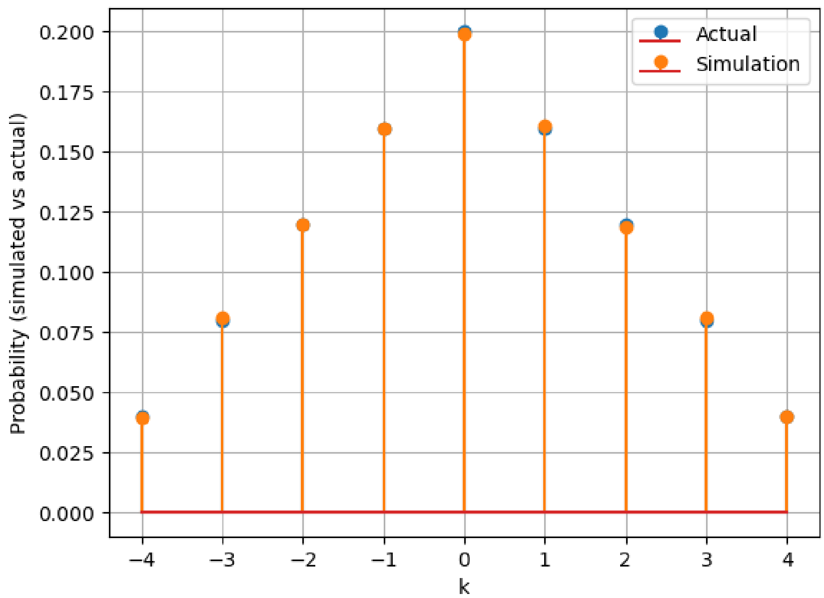

Figure 1: Stem plot for P(W)