NCERT Assignment

EE22BTECH11009-ANUPAMA KULSHRESHTHA

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution:

Let

$$X = \begin{cases} 1, & \text{sample contains 1 defective item} \\ 0, & \text{sample contains no defective item} \end{cases} \tag{1}$$

Then

$$p_X(1) = \frac{5}{100} \tag{2}$$
$$= 0.05 \tag{3}$$
$$p_X(0) = 1 - p_X(1) \tag{4}$$
$$= 1 - \frac{5}{100} \tag{5}$$
$$= \frac{95}{100} \tag{6}$$
$$= 0.95 \tag{7}$$

We use binomial distribution to solve this question, whose formula is given by

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the number of trials, k is the number of successful outcomes, and p is the probability of single successful outcome

As defective rate is 5%, $p = 0.05$ We need to find the probability of having 0 or 1 defective items in a sample of 10.

For k = 0;

$$\Pr(X = 0) = \binom{10}{0} * 0.05^0 * (1 - 0.05)^{10-0} \tag{8}$$
$$= 1 * 1 * 0.95^{10} \tag{9}$$
$$= 0.5987 \tag{10}$$

For k = 1;

$$\Pr(X = 1) = \binom{10}{1} * 0.05^1 * (1 - 0.05)^{10-1} \tag{11}$$
$$= 10 * 0.05 * 0.95^9 \tag{12}$$
$$= 0.3151 \tag{13}$$

To find the probability of event E which is not having more than 1 defective item, we sum the probabilities of having 0 or 1 defective items.

$$\Pr(E) = \Pr(X = 0) + \Pr(X = 1) \tag{14}$$
$$= 0.5987 + 0.3151 \tag{15}$$
$$= 0.9138 \tag{16}$$