

NCERT Assignment

EE22BTECH11009-ANUPAMA KULSHRESHTHA

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution:

We define random variable X as shown in table 1

| | |
|---------|-----------------------------------|
| $X = 0$ | Sample contains no defective item |
| $X = 1$ | Sample contains 1 defective item |

TABLE 1: Definition of X .

Then

$$p_X(1) = \frac{5}{100} \quad (1)$$

$$= 0.05 \quad (2)$$

$$p_X(0) = 1 - p_X(1) \quad (3)$$

$$= 0.95 \quad (4)$$

We use binomial distribution to solve this question, whose formula is given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where n is the number of trials, k is the number of successful outcomes, and p is the probability of single successful outcome

Here, $p = 0.05$ and $q = 0.95$ The CDF of binomial is given by

$$\Pr(X \leq n) = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^n \binom{10}{k} p^k q^{10-k}, & 0 \leq n \leq 10 \\ 1, & \text{otherwise} \end{cases} \quad (5)$$

Here, the desired probability is $\Pr(X \leq 1)$ Hence,

$$\Pr(X \leq 1) = \sum_{k=0}^1 \binom{10}{k} p^k q^{10-k} \quad (6)$$

$$= \binom{10}{0} p^0 q^{10-0} + \binom{10}{1} p^1 q^{10-1} \quad (7)$$

$$= 1(0.05)^0 (0.95)^{10} + 10(0.05)(0.95)^9 \quad (8)$$

$$= 0.5987 + 0.3151 \quad (9)$$

$$= 0.9138 \quad (10)$$