# NCERT Assignment

EE22BTECH11009-ANUPAMA KULSHRESHTHA

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

**Solution:**

Let

$$X = \begin{cases} 1, & \text{sample contains 1 defective item} \\ 0, & \text{sample contains no defective item} \end{cases} \tag{1}$$

Then

$$p_X(1) = \frac{5}{100} \tag{2}$$
$$= 0.05 \tag{3}$$
$$p_X(0) = 1 - p_X(1) \tag{4}$$
$$= 0.95 \tag{5}$$

We use binomial distribution to solve this question, whose formula is given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $n$ is the number of trials, $k$ is the number of successful outcomes, and $p$ is the probability of single successful outcome

Here, $p = 0.05$ and $q = 0.95$ The CDF of binomial is given by

$$\Pr(X \le n) = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} \binom{10}{k} p^k q^{10-k}, & 0 \le n \le 10 \\ 1, & \text{otherwise} \end{cases} \tag{6}$$

Here, the desired probability is $\Pr(X \le 1)$ Hence,

$$\Pr(X \le 1) = \sum_{k=0}^{1} \binom{10}{k} p^k q^{10-k} \tag{7}$$
$$= \binom{10}{0} p^0 q^{10-0} + \binom{10}{1} p^1 q^{10-1} \tag{8}$$
$$= 1(0.05)^0 (0.95)^{10} + 10(0.05)(0.95)^9 \tag{9}$$
$$= 0.5987 + 0.3151 \tag{10}$$
$$= 0.9138 \tag{11}$$