NCERT Assignment

EE22BTECH11009-ANUPAMA KULSHRESHTHA

There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Solution:

We define random variable X as shown in table 1

$X = 0$	Sample contains no defective item
$X = 1$	Sample contains 1 defective item
n	Number of trials
p	Probability of single successful item
k	Number of successful outcomes

TABLE 1: Definition of X and parameters.

Then

$$p_X(1) = \frac{5}{100} \tag{1}$$
$$= 0.05 \tag{2}$$
$$p_X(0) = 1 - p_X(1) \tag{3}$$
$$= 0.95 \tag{4}$$

We use binomial distribution to solve this question, whose formula is given by

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Here, $p = 0.05$ and $q = 0.95$ The CDF of binomial is given by

$$\Pr(X \le n) = \begin{cases} 0, & n < 0 \\ \sum_{k=0}^{n} \binom{10}{k} p^k q^{10-k}, & 0 \le n \le 10 \\ 1, & \text{otherwise} \end{cases} \tag{5}$$

Here, the desired probability is $\Pr(X \le 1)$ Hence,

$$\Pr(X \le 1) = \sum_{k=0}^{1} \binom{10}{k} p^k q^{10-k} \tag{6}$$
$$= \binom{10}{0} p^0 q^{10-0} + \binom{10}{1} p^1 q^{10-1} \tag{7}$$
$$= 1(0.05)^0 (0.95)^{10} + 10(0.05)(0.95)^9 \tag{8}$$
$$= 0.9138 \tag{9}$$