Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -2 \\ -10 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} -1 \\ 5 \end{pmatrix}$	CA
$\|\|A-B\|\|$	5.83	length of AB
$\|\|B-C\|\|$	10.20	length of BC
$\|\|C-A\|\|$	5.10	length of CA
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} 5 \\ -3 \end{pmatrix}$	AB
c_1	-5	
$\mathbf{n_2}$	$\begin{pmatrix} -10 \\ 2 \end{pmatrix}$	BC
c_2	-10	
$\mathbf{n_3}$	$\begin{pmatrix} 5 \\ 1 \end{pmatrix}$	CA
c_3	-5	
Area	10	Area of Triangle
$\angle A$	137.72°	Angles
$\angle B$	19.65°	
$\angle C$	22.6°	

TABLE 1: Vectors.

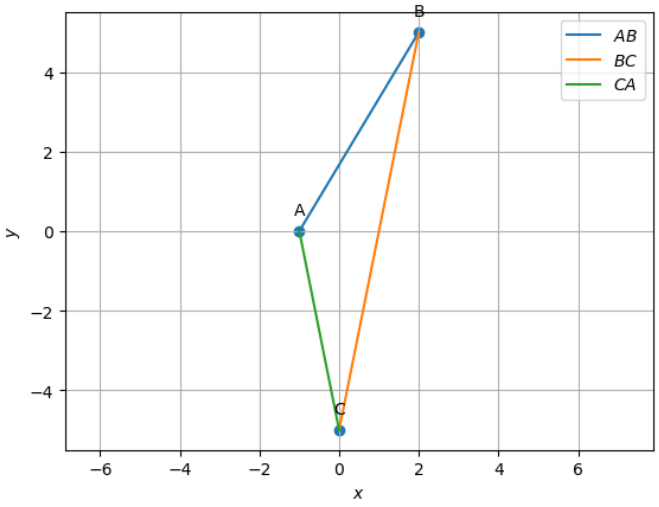

Fig. 1: triangle plotted using python

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} -0.5 \\ -2.5 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	
c_4	0	
$\mathbf{m_5}$	$\begin{pmatrix} -2.5 \\ -7.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} -7.5 \\ 2.5 \end{pmatrix}$	
c_5	-2.5	
$\mathbf{m_6}$	$\begin{pmatrix} 0.5 \\ 7.5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} 7.5 \\ -0.5 \end{pmatrix}$	
c_6	2.5	
$\mathbf{G}$	$\begin{pmatrix} 0.33 \\ 0 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
rank$\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

TABLE 2: Median.

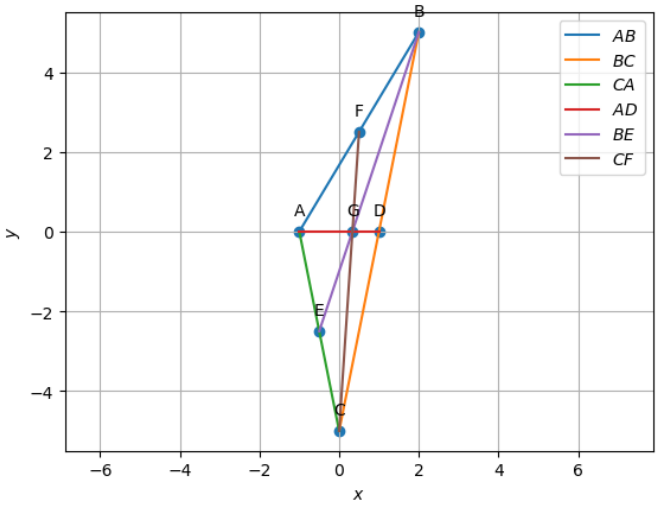

Fig. 2: medians plotted using python

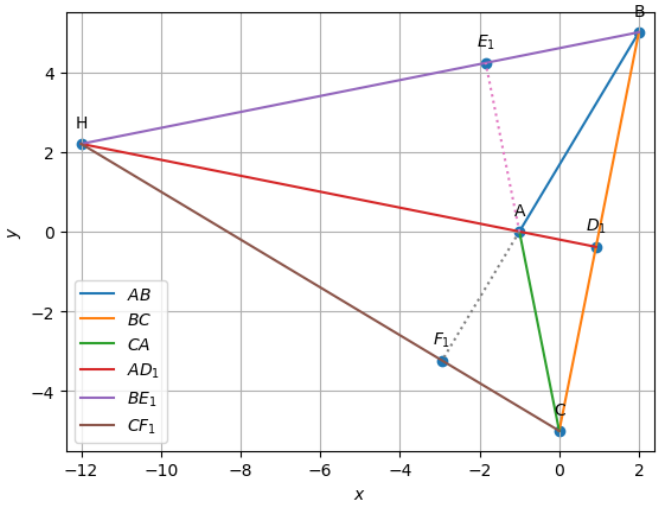

Fig. 3: altitudes plotted using python

3 Altitude

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}0.92\\-0.38\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-1.84\\4.23\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}-2.94\\-3.23\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}1.92\\-0.38\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}-0.38\\-1.92\end{pmatrix}$	
c_7	0.38	
$\mathbf{m_8}$	$\begin{pmatrix}-3.84\\-0.76\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}-0.76\\3.84\end{pmatrix}$	
c_8	17.69	
$\mathbf{m_9}$	$\begin{pmatrix}-2.94\\1.76\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}1.76\\2.94\end{pmatrix}$	
c_9	-14.7	
$\mathbf{H}$	$\begin{pmatrix}-12\\2.2\end{pmatrix}$	Orthocentre

TABLE 3: Altitude.

4 Perpendicular Bisector

parameters	value	description
$\mathbf{m_{10}}$	$\begin{pmatrix}-10\\2\end{pmatrix}$	AD_1
$\mathbf{n_{10}}$	$\begin{pmatrix}-2\\-10\end{pmatrix}$	
c_{10}	-2	
$\mathbf{m_{11}}$	$\begin{pmatrix}-5\\-1\end{pmatrix}$	BE_1
$\mathbf{n_{11}}$	$\begin{pmatrix}1\\-5\end{pmatrix}$	
c_{11}	12	
$\mathbf{m_{12}}$	$\begin{pmatrix}-5\\3\end{pmatrix}$	CF_1
$\mathbf{n_{12}}$	$\begin{pmatrix}-3\\-5\end{pmatrix}$	
c_{12}	-14	
$\mathbf{O}$	$\begin{pmatrix}6.5\\-1.1\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	7.58	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	84.54°	$\angle BAC = 2\angle B0C$
$\angle BAC$	137.72°	
$\angle AOC$	39.3°	$\angle AOC = 2\angle ABC$
$\angle ABC$	19.6°	
$\angle AOB$	314.7°	$\angle AOB = 2\angle BCA$
$\angle BCA$	22.6°	

TABLE 4: Perpendicular Bisector.

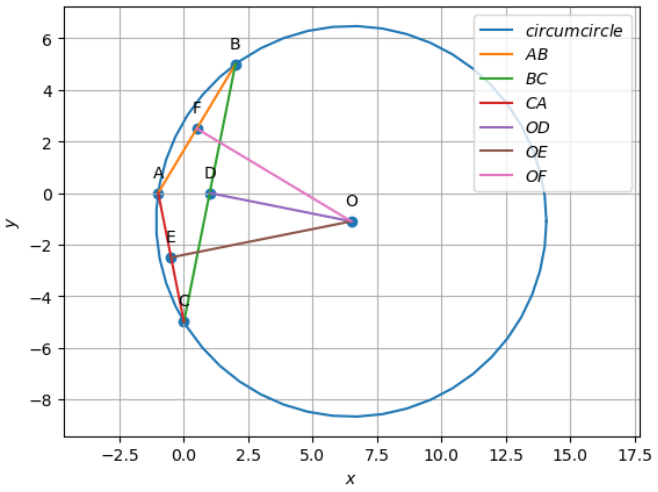

Fig. 4: perpendicular bisectors plotted using python

5 Angle Bisector

parameters	value	description
$\mathbf{m_{13}}$	$\begin{pmatrix}-0.71\\0.12\end{pmatrix}$	AI
$\mathbf{n_{13}}$	$\begin{pmatrix}0.123\\0.711\end{pmatrix}$	
c_{13}	-0.123	
$\mathbf{m_{14}}$	$\begin{pmatrix}-0.71\\-1.84\end{pmatrix}$	BI
$\mathbf{m_{14}}$	$\begin{pmatrix}1.84\\-0.71\end{pmatrix}$	
c_{14}	0.123	
$\mathbf{m_{15}}$	$\begin{pmatrix}0\\-1.96\end{pmatrix}$	CI
$\mathbf{n_{15}}$	$\begin{pmatrix}1.96\\0\end{pmatrix}$	
c_{15}	0	
$\mathbf{I}$	$\begin{pmatrix}0\\-0.17\end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix}0.93\\-0.36\end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix}-0.93\\-0.36\end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix}-0.81\\0.31\end{pmatrix}$	Point of contact with AB
$\|\mathbf{I}-\mathbf{D_3}\|$	0.946	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I}-\mathbf{E_3}\|$		
$\|\mathbf{I}-\mathbf{F_3}\|$		
r		
$\angle BAI$	68.86°	$\angle BAI = \angle CAI$
$\angle CAI$		
$\angle ABI$	9.826°	$\angle ABI = \angle CBI$
$\angle CBI$		
$\angle ACI$	11.309°	$\angle ACI = \angle BCI$
$\angle BCI$		

TABLE 5: Angle Bisectors.

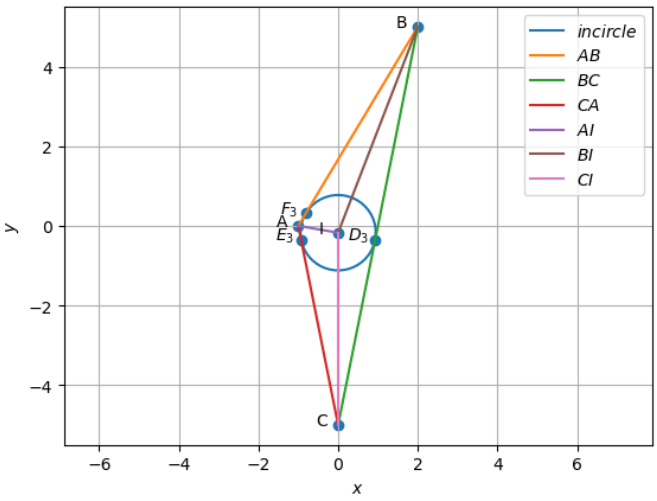

Fig. 5: Angle bisectors plotted using python