PHASE-III

Penalty Function Method & Method of Multipliers

GROUP NO: G7

ANUPAM KHANDELWAL(160108008) & KONARK JAIN(160108022)

BACKGROUND

Constrained Optimization Problem formulation:

The problem of constrained optimization is formulated as follows:

Minimize: $f(x)$

Subject to: $g_j(x) \geq 0 \quad j = 1, 2, \ldots, J$

$h_k(x) = 0 \quad k = 1, 2, \ldots, K$

$x^{(L)} \leq x \leq x^{(U)}$ where x is a vector

- $g_j(x)$ are the inequality constraints
- $h_k(x)$ are the equality constraints
- $x^{(L)}$ and $x^{(U)}$ are the lower bounds and upper bounds for the vector x.

BACKGROUND

Penalty Function Method:

The penalty function P, is defined as:

$$\boxed{P(x,R) = f(x) + \Omega(R, g(x), h(x))}$$

where f(x) is the objective function, g(x) and h(x) are the inequality and equality constraints respectively, R is a set of penalty parameters, and Ω being the penalty term.

We are using bracket operator penalty for computing penalty terms.

$$\boxed{\Omega = R\langle g(x) \rangle^2}$$

where $\langle \alpha \rangle = \alpha$ when $\alpha < 0$ and zero otherwise.

In this method instead of minimizing the objective function directly, we instead minimize the penalty function so as to account for constraint violation.

BACKGROUND

Method of Multipliers:

The penalty function here is defined as:

$$\boxed{P\left(x,\sigma^{(t)},\tau^{(t)}\right) = f(x) + R\sum_{j=1}^{J}[\left(< g_j(x) + \sigma_j^{(t)} >\right)^2 - (\sigma_j^{(t)})^2] + \mathrm{R}\sum_{k=1}^{K}\left[(h_k(x)\ + \tau_k^{(t)}\right)^2 - \left(\tau_k(x)\right)^2\ ]}$$

where the σ_j and τ_k are updated iteratively as:

$$\boxed{\begin{array}{c} \sigma_j^{(t+1)} = < g_j\left(x^{(t)}\right)\ + \sigma_j^{(t)}\ > \\ \tau_k^{(t+1)} = h_k\left(x^{(t)}\right) + \tau_k^{(t)} \end{array}}$$

Similarly here also we minimize the penalty function instead.

RESULTS – PROBLEM 1

Problem 1:

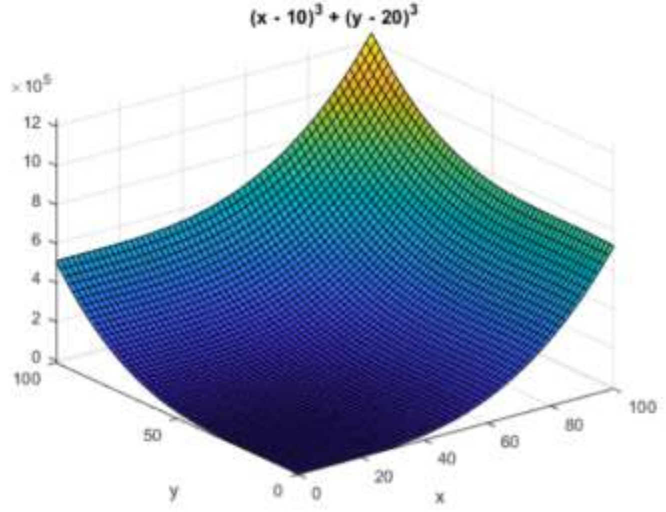
$$\min \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x_1} - \mathbf{10})^3 + (\boldsymbol{x_2} - \mathbf{20})^3,$$

$$\textit{subject to } \boldsymbol{g_1}(\boldsymbol{x}) = (\boldsymbol{x_1} - \mathbf{5})^2 + (\boldsymbol{x_2} - \mathbf{5})^2 - 100 \geq 0,$$

$$\boldsymbol{g_2}(\boldsymbol{x}) = 82.81 - (\boldsymbol{x_1} - \mathbf{5})^2 - (\boldsymbol{x_2} - \mathbf{5})^2 \leq 0,$$

$$\mathbf{13} \leq \boldsymbol{x_1} \leq \mathbf{100}, \qquad \mathbf{0} \leq \boldsymbol{x_2} \leq \mathbf{100}.$$

- Number of variables: 2 variables.
- The global minima: x* = (14.095,0.84296), f(x*) = -6961.81388.



$x^{(0)} = (13, 1)^T$
$\Delta = 0.00001$
R = 0.1(for both)
c = 10(for penalty)

Method name	Epsilon value	Optima Value	Function value	No. of function evaluations
Penalty	10^{-2}	(13.274, 2.542)	-5287.74	116
Penalty	10^{-3}	(13.966, 0.981)	-6818.58	137
Penalty	10^{-4}	(14.098, 0.844)	-6960.53	163
Multiplier	10^{-2}	(13.865, 1.124)	-6667.84	97
Multiplier	10^{-3}	(14.022, 0.923)	-6877.66	128
Multiplier	10^{-4}	(14.096, 0.868)	-6934.23	154

RESULTS – PROBLEM 2

Problem 2:

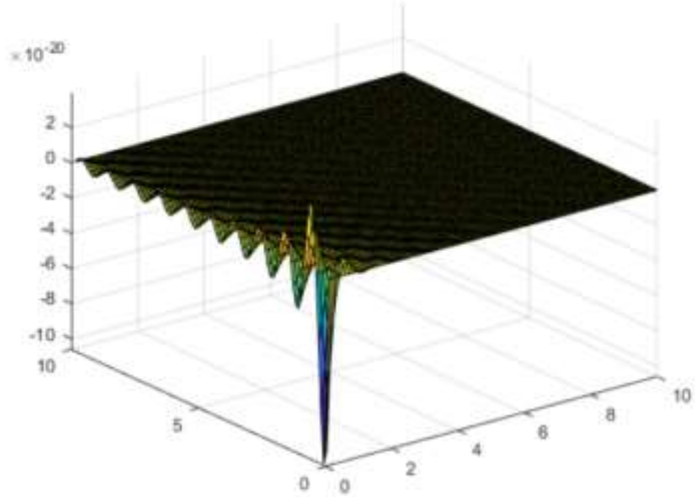
$$\max f(\boldsymbol{x}) = \frac{\sin^3(2\pi x_1)\sin(2\pi x_2)}{x_1^3(x_1+x_2)},$$

$$\text{subject to } g_1(\boldsymbol{x}) = x_1^2 - x_2 + 1 \le 0,$$

$$g_2(\boldsymbol{x}) = 1 - x_1 + (x_2-4)^2 \le 0,$$

$$0 \le x_1 \le 10, \qquad 0 \le x_2 \le 10$$

- Number of variables: 2 variables.
- The global minima: x∗ = (1.2279713, 4.2453733) , f (x∗) = 0.095825.



$x^{(0)} = (1,1)^T$
$\Delta = 0.00001$
R = 0.1(for both)
c = 10(for penalty)

Method name	Epsilon value	Optima Value	Function value	No. of iterations
Penalty	10^{-2}	(1.2293, 4.2424)	0.0958	4
Penalty	10^{-3}	(1.2280, 4.2456)	0.0958	7
Penalty	10^{-4}	(1.2280, 4.2454)	0.0958	11
Multiplier	10^{-2}	(1.2285, 4.2477)	0.0958	4
Multiplier	10^{-3}	(1.2284, 4.2472)	0.0958	5
Multiplier	10^{-4}	(1.2280, 4.2460)	0.0958	7

RESULTS – PROBLEM 3

Problem 3:

$$\boldsymbol{min\ f(x) = x_1 + x_2 + x_3}$$

$$\textit{subject to } \boldsymbol{g_1(x) = -1 + 0.0025(x_4 + x_6) \le 0,}$$

$$\boldsymbol{g_2(x) = -1 + 0.0025(-x_4 + x_5 + x_7) \le 0,}$$

$$\boldsymbol{g_3(x) = -1 + 0.01(-x_6 + x_8) \le 0,}$$

$$\boldsymbol{g_4(x) = 100x_1 - x_1x_6 + 833.33252x_4 - 83333.333 \le 0,}$$

$$\boldsymbol{g_5(x) = x_2x_4 - x_2x_7 - 1250x_4 + 1250x_5 \le 0,}$$

$$\boldsymbol{g_6(x) = x_3x_5 - x_3x_8 - 2500x_5 + 1250000 \le 0,}$$

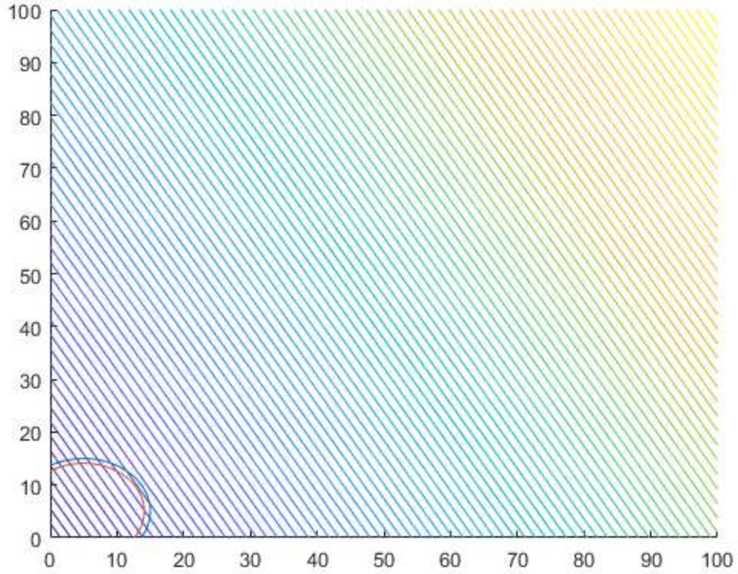
$$\boldsymbol{100 \le x_1 \le 10000}$$

$$\boldsymbol{1000 \le x_i \le 10000, i = 2, 3}$$

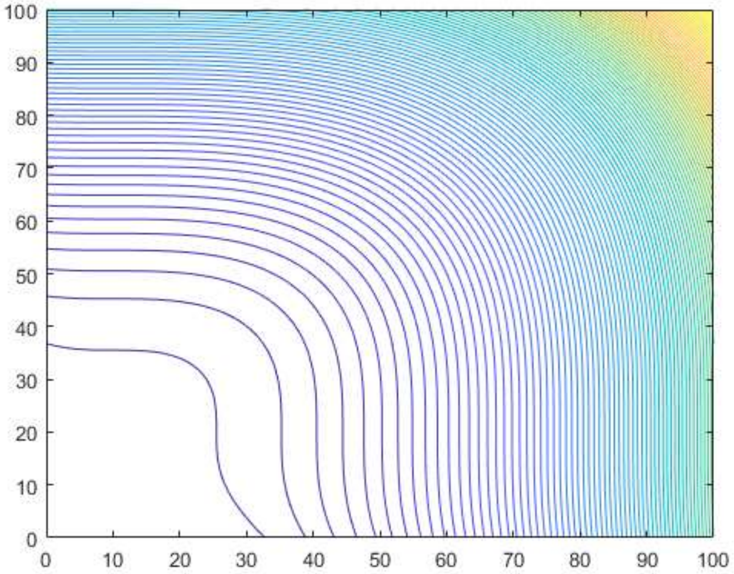
$$\boldsymbol{10 \le x_i \le 1000, i = 4, 5, \dots, 8}$$

- Number of variables: 8 variables.
- The global minima: x∗ = (579.3167, 1359.943, 5110.071, 182.0174, 295.5985, 217.9799, 286.4162,395.5979), f (x∗) = 7049.3307.

OBSERVATIONS



P2: Objective function contour



P2: Distorted penalty function contour

Heavy distortion in contour plot for penalty function was observed in penalty function method.