ME609 – PHASE II CODING ASSIGNMENT

GROUP NO. 7

CAUCHY'S STEEPEST DESCENT METHOD

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BACKGROUND

Gradient descent is based on the observation that if the multi-variable function $F(\mathbf{x})$ is defined and differentiable in a neighborhood of a point $\mathbf{a}$, then $F(\mathbf{x})$ decreases *fastest* if one goes from $\mathbf{a}$ in the direction of the negative gradient of F at $\mathbf{a}$, $-\nabla F(\mathbf{a})$. It follows that, if

$$\mathbf{a}_{n+1} = \mathbf{a}_n - \gamma \nabla F(\mathbf{a}_n)$$

for $\gamma \in \mathbb{R}_+$ small enough, then $F(\mathbf{a_n}) \geq F(\mathbf{a_{n+1}})$. In other words, the term $\gamma \nabla F(\mathbf{a})$ is subtracted from $\mathbf{a}$ because we want to move against the gradient, toward the minimum. With this observation in mind, one starts with a guess $\mathbf{x}_0$ for a local minimum of F, and considers the sequence $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \ldots$ such that

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \gamma_n \nabla F(\mathbf{x}_n),\ n \geq 0.$$

IMPLEMENTATION DETAILS

α is found from unidirectional search using exhaustive search and bisection method

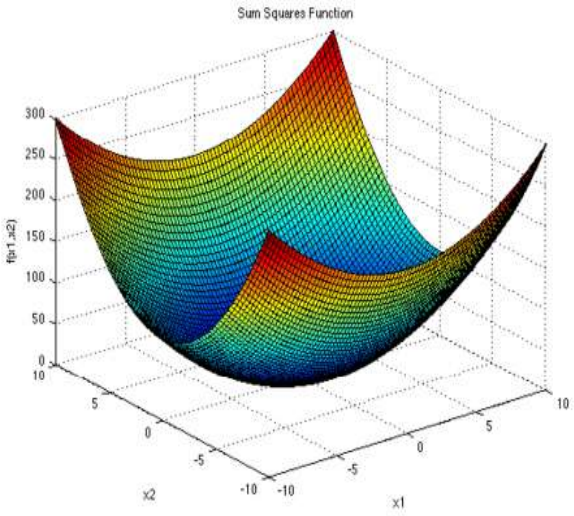
```
Z = Z - alpha_final*dX;                    % DESCENT UPDATE.

%Termination condition check.
dZ = partial_derivative(function_name,d,Z,delta_x);

if abs(dot(dX,dZ))<epsilon^2,
    break;
end
%Step 5
if norm(alpha_final*dX)/norm(Z) < epsilon,
    break;
end
```

RESULTS – QUESTION 1

SUM SQUARES FUNCTION



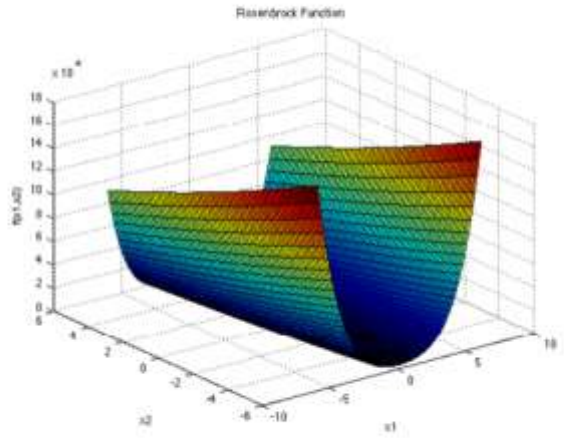
$$f(\mathbf{x}) = \sum_{i=1}^{d} i x_i^2$$

Optima = [0 0 0 … 0 0]

Dim	Optima found	Function value	# function evaluations	Epsilon
5	-0.0172 -0.0000 0.0000 0.0000 -0.0038	3.6909e-04	102	1e-03
5	-0.0076 -0.0000 -0.0000 0.0000 -0.0017	7.1817e-05	126	1e-04
5	-0.0015 -0.0000 -0.0000 0.0000 -0.0003	2.7275e-06	150	1e-05
5	1.0e-03 * [-0.2881 -0.0000 0.0000 0.0000 -0.0643]	1.0365e-07	168	1e-06
10	-0.0268 -0.0000 -0.0000 -0.0000 0.0000 0.0000 -0.0000 -0.0000 -0.0000 -0.0030	8.0763e-04	174	1e-03
10	-0.0045 -0.0000 -0.0000 -0.0000 0.0000 -0.0000 -0.0000 0.0000 0.0000 0.0004	2.1735e-05	228	1e-04
10	-0.0030 -0.0000 -0.0000 -0.0000 -0.0000 0.0000 -0.0000 0.0000 0.0000 0.0003	9.7091e-06	240	1e-05
10	1.0e-03 * -0.3970 -0.0000 -0.0000 0.0000 0.0000 -0.0000 -0.0000 -0.0000 0.0000 0.0363	1.7079e-07	300	1e-06

RESULTS – QUESTION 2

ROSENBROCK FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$$

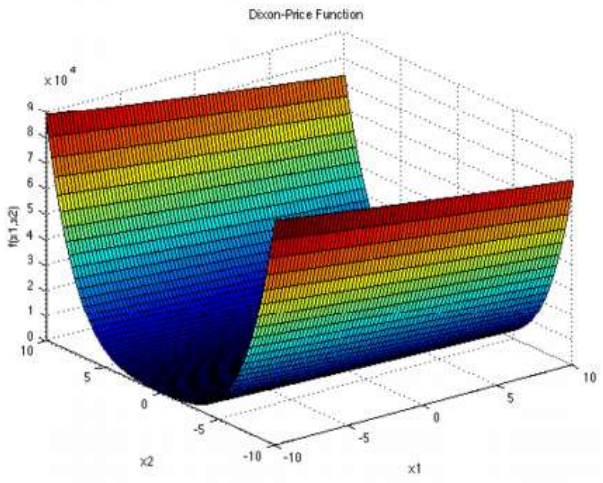
Global Minimum:

$$f(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (1, \ldots, 1)$$

Dim	Optima found						Function value	# function evaluations	Epsilon
3	1.2111	1.4695	2.1634				0.2673	1562	1e-03
3	1.0620	1.1281	1.2731				0.0203	2784	1e-04
3	0.9959	0.9917	0.9835				8.5441e-05	24654	1e-05
6	0.9568	0.9162	0.8385	0.7033	0.4936	0.2408	0.3805	2454	1e-03
6	0.9931	0.9864	0.9728	0.9463	0.8952	0.8009	0.0149	12150	1e-04
6	0.9991	0.9981	0.9962	0.9925	0.9850	0.9702	3.0036e-04	30000 (Max Iter Reached)	1e-05

RESULTS – QUESTION 3

DIXON-PRICE FUNCTION



$$f(\mathbf{x}) = (x_1 - 1)^2 + \sum_{i=2}^{d} i\,(2x_i^2 - x_{i-1})^2$$

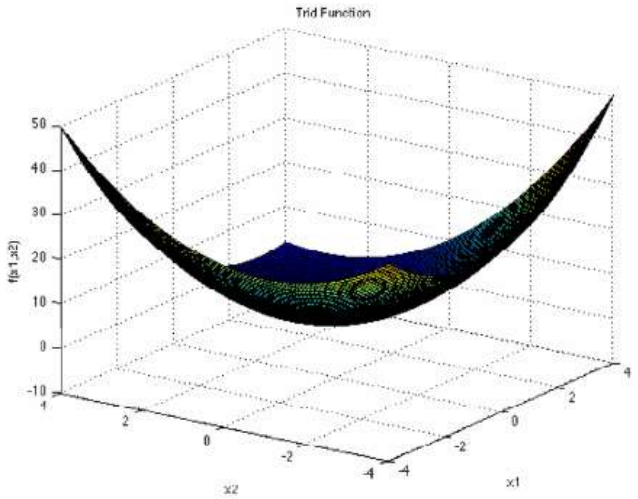
Global Minimum:

$$f(\mathbf{x}^*) = 0, \text{ at } x_i = 2^{-\frac{2^i - 2}{2^i}}, \text{ for } i = 1, \ldots, d$$

Dim	Optima found						Function value	# function evaluations	Epsilon
4	0.9995	0.7069	0.5945	0.5452			1.1486e-05	678	1e-05
4	0.9999	0.7071	0.5946	-0.5452			2.7706e-07	798	1e-06
4	1.0033	0.7083	0.5952	0.5455			7.5562e-09	966	1e-07
6	0.9784	0.6988	0.5910	0.5431	0.5217	-0.5102	4.8942e-04	762	1e-05
6	1.0009	0.7075	0.5948	0.5453	0.5222	0.5110	9.0025e-07	1086	1e-06
6	1.0000	0.7071	0.5946	0.5452	0.5221	-0.5109	2.4782e-09	2178	1e-07

RESULTS – QUESTION 4

TRID FUNCTION



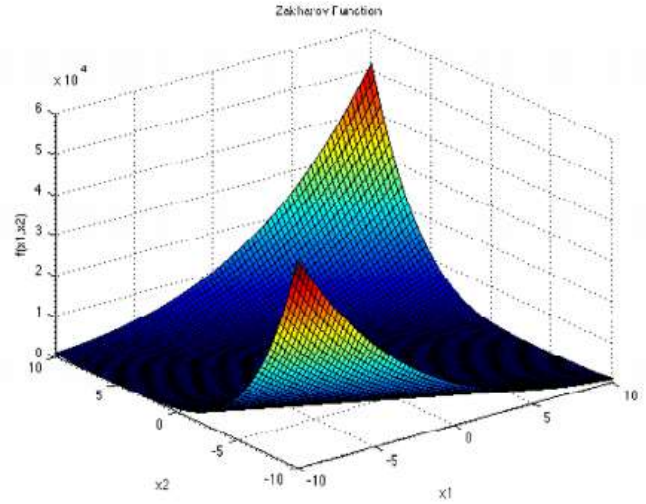
$$f(\mathbf{x}) = \sum_{i=1}^{d} (x_i - 1)^2 - \sum_{i=2}^{d} x_i x_{i-1}$$

$$f(\mathbf{x}^*) = -d(d+4)(d-1)/6, \text{ at } x_i = i(d+1-i), \text{ for all } i = 1, 2, \ldots, d$$

Dim	Optima found	Function value	# function evaluations	Epsilon
6	5.7624 9.5134 11.4661 11.3933 9.5719 5.7300	-49.8721	186	1e-05
6	5.9710 9.9522 11.9347 11.9404 9.9477 5.9735	-49.9985	330	1e-06
6	5.9991 9.9982 11.9979 11.9978 9.9984 5.9990	-50.0000	480	1e-07
10	9.7372 17.5171 23.2950 27.1876 29.0766 29.1159 27.1514 23.3250 17.4956 9.7483	-209.8100	498	1e-05
10	9.9679 17.9349 23.9138 27.8905 29.8870 29.8808 27.8962 23.9090 17.9383 9.9661	-209.9968	546	1e-06
10	10.0000 18.0001 24.0001 28.0002 30.0002 30.0002 28.0002 24.0001 18.0001 10.0001	-210.0000	1212	1e-07

RESULTS – QUESTION 5

ZAKHAROV FUNCTION



$$f(\mathbf{x}) = \sum_{i=1}^{d} x_i^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^4$$

Global Minimum:

$$f(\mathbf{x}^*) = 0, \text{ at } \mathbf{x}^* = (0, \ldots, 0)$$

Dim	Optima found					Function value	# function evaluations	Epsilon
2	1.0e-03 *	0.8197	0.2220			1.1205e-06	8	1e-05
2	1.0e-06 *	0.0570	0.1099			3.4455e-14	18	1e-06
2	1.0e-07 *	0.3731	-0.1888			1.7488e-15	24	1e-07
4	1.0e-04 *	-0.0306	-0.0507	-0.0808	-0.1248	2.1633e-09	8	1e-05
4	1.0e-03 *	0.0612	-0.5175	0.8159	-0.1436	1.1601e-06	18	1e-06
4	1.0e-05 *	-0.1306	0.1558	-0.0679	0.0053	4.5982e-12	24	1e-07