A Three Step Solution for a Homogeneous, Multiple Depot, Multiple Travelling Salesman Problem

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I. Introduction

In the Standard Travelling Salesman Problem, we have a single salesman and multiple targets and we have to find an optimal path that will visit all the targets exactly once and after that the salesman will return to its original location (depot). It is a single depot problem. In the problem assigned to us is the extended version of Standard Travelling Salesman Problem, here we have multiple depots and different target locations. So, homogenous unmanned aerial vehicles(UAVs) will start from the different depots, and each UAVs will be homogeneous in terms of their traveling speed, service time, flight endurance and payload capacities. Each target will be visited by exactly one UAVs. So in this problem we have to assign an optimal sequence of targets for each of the UAVs, such that the total distance of visiting all the targets is minimized. These UAVs can be used for both military and civil application for monitoring purposes.The paper defines a way to assign a set of locations to be visited in particular order to every vehicle and each vehicle has a distinct depot. The given problem is non polynomial (NP) hard problem. So in this paper we will provide the solution of the Homogenous Multiple Depot Multiple Travelling Salesman Problem (HMDMTSP) and for better understanding we will also make a pictorial representation of the tours for every salesman.

Definition of Certain terms:

1) Homogenous vehicles: All the vehicles will have similar capacity, power or ability. They will have same sensing or attacking capabilities if we talk in sense of UAV drones. They will have equal range of distance they can travel, amount of parcels they can carry etc. We will be given fixed number of vichles in parameters taken from the user.

2) Mutiple Depot: Depots meaning the starting point of the vehicles.So, Multiple depots means multiple travellars/UAV/vehicles may have different starting point. So, assiging a keeping the depot according to our targets may optimize our cost.But, it will increase the problem complexity. So, for this problem, we will take certain fixed depots. If you are thinking in case of parcel delivering, think that delivery hub have position fixed after some some calculation of optimal depot assigning.

3) Multiple travellar: It means there are multiple travellars/vehicles that will deliver the packets or will carry the attacks in case of UAV.

Problem Definition: In this paper, we are solving the Homogenous Multiple Depot Multiple Travelling Salesman problem. In which we have multiple distinct depots (starting points) and a set of target locations. According to the problem statement we have to find an optimal way of traversal such that every target location is visited by at least one of the unmanned aerial vehicles (UAVs) and the cost of the traversal is minimized. The cost, here taken is , total sum of distance covered by all the travelling salesman in the sequence of cities given by the solution.

Objectives:

- Minimize Distance:- One of the main objectives is minimize the total time and total distance traveled to complete the tour. It helps in reducing the operating cost and resources.

- Workload Management:- To balance the workload among the unmanned aerial vehicles, to ensure an equitable distribution of tasks.
- Scalability:- To provide a scalable solution, such that we can handle a large number of targets , UAVs and depots.
- Increase the success of the mission, by ensuring that all the target locations are visited efficiently.
- Make a pictorial representation of the solution for easy visualization and understanding.

Usecases:

- Vehicle Routing in Distribution Networks: The goods distributing companies often have similar problem, where they have multiple storehouse from where they have to transport or deliver the goods to different locations.In our indian scenario, we can see the companies like Zomato, Flipkart, Amazon, Myntra, BlueDart, Delhivery have to deliver a lot of parcel everyday. They can find this solution helpful in assigning the locations/routes to different delivery partners in best way. And hence they could minimize their total distance and hence the cost also.
- Telecommunications Network Planning:- From the point of view of telecommunications, where base stations from where data transmits , acts as depots , the HMDMTSP can be used to optimize the service routing problem for technicians visiting these locations.
- Emergency Response Planning: If we twick the problem and take time as cost factor, then we can use it in a situation where we have to assign ambulances to different hospitals (acting as depot) so that we can optimize the routes of these ambulances to provide faster service.
- Mail and Courier Services: In context of mail service companies, they could have multiple depots for rearranging and distribution of mails. The HMDMTSP can optimize the delivery routes of vehicles and hence reducing the distance and cost of the mail service company.

II. Literature Review

This literature review discusses about 5 papers implementing different methodology and algorithms.

A. HMDMTSP Survey

In this paper, HMDMTSP Survey released by Laporte and his team in 2017, they discussed about various questions that can arises related to Homogenous Multiple Depot, Multiple travelling salesman problem. They have also discussed all the regarding solution for that questions. They also categoriges the types of the question that can be generated over this topic. They have conducted a lot of reviews, discussions, surveys to find out what problem will arise if they changes time windows, make the vehicles homogenous, depots positions,vessels and how will it effect the upcoming result. After that they have discussed possible solutions for this variations over the standard HMDMTSP problem. And this is the main key findings of this research paper and hence this papers provides valuable information for researchers dealing with the optimization problem.

B. Vehicle Routing Problem

The paper ”Vehicle routing problem” published by Toth and Vigo in 2014, focuses on car routing. It is actually Car routing detail introduction problem. This VRP probelm is findout the complexity and explore solutions. Its main/ prime focus is on HMDMTSP problem and its variations. They have performed in-depth analysis of HMDMTSP and various other vehicle Routing Problems and founded their complexities. They revisited existing solution till the date to solve the problem.They have implemented the solutions and examined thier results. After that, they compared various results of the different solutions of other variants.

C. Multi-Depot Vehicle Routing

The paper ”Multi-Depot Vehicle Routing” authored by Duhamel and his team in 2020. The main aim of the Duhamel and his team members to research over controlling the multi-warehouse traffic field variants of the routing problem.They designed approach or solution to investigate cases/scenerios with multiple locations and how they affect the routing optimization. In methodology, they have used various aspects of vehicle routing problem , which has multiple sources, depots, heterogeniety etc. And it resulted into new unseen challanges and complexities.

D. An Algorithm for Homogenous VRP

This paper is published by Ma and Liu in 2018. They have started the research with aim to solve Homogenous Vehicle Routing Problem, a variant of HMDMTSP and develop a optimal solution that. They have presented a new algorithm to solve HVRP effectively. Their methodology is designed to accomodate Homogenous fleets where so many different kinds of vehicles are considered in the routing problem. Their alsorithm showed that quality of the proposed solution increases with the tools having different characterstics. This research contributes to the development of different algorithms for handling differnt vessels in HMDMTSP-related variants.

E. Metaheuristic for HMDMTSP

This paper is published by Bräysy and Dullaert in 2005.It is a old research over this topic. We have taken to increase our range of view over the topic, like what are the research is going in different time era. They have studied metaheuristic methods and solved HMDMTSP using that.After that , they evaluated its effectiveness. Thier methodology is do qualitative analysis of various meta-heuristic methods such as simulated annealing, tabu search and genetic algorithms while solving the HMDMTSP example.They studied the effectiviness of different metaheuritics over the HMDMTSP and compared qualitatively. This study concluded that meta-heuristics provide better solutions for HMDMTSP. These heuritics have been proven effective and have ability to solve the technical problem of delivery. And hence this paper, provides a valuable learning resource for fellow researchers over this topic.This detailed description provides a better understanding of the objectives, methods, and key findings of each study in the context of HMDMTSP and transportation issues.

III. Methodology And System Design

Before stating the methodology that we are proposing, lets recall the problem statement. We are trying to assign a certain sequence of targets to different homogenous vehicles such that in the last overall cost of delivering the packets get minimized. So, This become a resource allocation problem. The initial problem has homogenous, multiple depot, multiple travellars/vehicles, so combining them in optimal way is much complex and hard. Actually that is a NP-Hard problem. We will try to reduce the complexity of the problem to that level, that it can be easily be solvable with some alteration to the classical travelling salesman problem. Even if, classical travelling salesman problem, can not find the optimal answer due to computational constraint, we can use certain approximation and optimization technique to find a better solution in our constraint like less computational power and time or others .

A. ***Random Solution***

In this step, we will generate a random set of routes or sequence of cities for every traveler, starting from different depots. Then we will do a sufficient of iteration of randomness, like 100000 is computationally possible, to get a better random solution. we will save this route to the Solution.json and after that we will project the solution to the map of Turkey that we taken in dataset.

1) Algorithm to generate the Random Solution::

- Take a dataset in the form of distance matrix and each entries say dij, will denote the distance between the ith and jth city.
- Generate a random set of Depots from the list of cities given in the turkish network/ dataset. And all the depots will be unique among themselves.
- For each depots, do the iteration given number of salesman times and generate the unique set of cities and for them . Doing this fir all depots will generate the complete routes which is saved in routes.
- Now, calculate the total distance covered for this solution.
- Now , repeat all the steps 100000 times and take the best one out of it to have a better random solution.
- Save the best random solution into solution.json file.
- Now project/draw the solution on tr2map.jpg given in resource file to have a image representation of the solution.

B. *Transformation of HMDMTSP to Assymetric single TSP*

As our base paper "A Transformation for a Homogeneous, Multiple Depot, Multiple Traveling Salesman Problem" discusses a method to reduce the Homogenous, multiple depot, multiple travellar salesman problem into multiple single Assymetric TSP and then multiple single TSP to a Single TSP. We will also use this transformation to reduce the complexity of the problem.To do this transformation, we will apply LKH heruistic and Noon-Bean Transformation. Since,

C. *Application of ATSP Solvers*

With the HMDMTSP transformed into an ATSP, the next step is to apply established algorithms for solving ATSP instances. In this methodology, we employ the Lin-KernighanHelsgaun (LKH) heuristic, a well-known and efficient algorithm for ATSP. The LKH heuristic is particularly suitable for solving large-scale instances of ATSP and can deliver highquality solutions within reasonable computational time.

1) Brute-Force:: At the first , we will implement a very naive and basic solution for this. We will generate 100000 random solution and find the best tour out of them on the basis of cost. The number of travellars/vehicle and number of depots will be our parameters that we can change.The cost metric will sum of total distances that all vehicles have travelled (in kilometers).

2) Optimization Using Hill Climbing: Hill Climbing is generally a local solution searching algorithm, in each iteration we make small modification and find the cost and based on the cost choose the path which is improving the current solution. In this approach we are using the sequence obtained from the brute force technique as the initial solution. After that we have generated the neighboring solution by doing the above 5 operation which are swap nodes in route, in this operation we select a random route among all possible routes and then swap any two nodes, swap hub with nodes in route, in this operation we select random routes and then replace then replace a hub with some random node, swap nodes between routes, in this operation we select two random routes and from both the routes we select random nodes and swap them, insert node in route, in this operation we delete a node and insert in to the desired position, insert nodes between routes, in this operation we remove the chosen node and inert it to the right of the desired node.After that we are calculating the objective function (cost) and based on this cost we are going to the solution which is minimizing the objective function. The termination point is when we have reached the maximum number of iteration or when there is no further improvement in the solution.There are many draw backs of this algorithm link it is initialization dependent, depends on the search space structure.

One of the major problems with this algorithm is that it may get stuck in local-minima, since hill-climbing never backtracks and only moves uphill, this may lead to a sub-optimal solution. Using Random walk was one another option we considered but it may never converge. Finally, we concluded on using simulated annealing for the optimization which is combination of hill-climbing and random-walk, which is basically a algorithm in which we accept worse solution initially but we get closer to the solution, the probability of choosing worse solution and decreases. This also helps in evading local-minima.

3) Optimization Using Simulated annealing: Simulated annealing is an optimization technique used to find the optimal solution (global minimum) of the cost function using some approximation. One of advantages with using simulated annealing was that it helped in avoiding premature convergence, it does that by allowing the algorithm to explore diverse solutions early in the optimization process. Simulated Annealing also does not require derivate information, making it applicable to problems where obtaining derivatives is challenging or computationally expensive. In the given problem we will first convert HMDMTSP to a Standard asymmetric TSP and then apply the optimization technique.Here we will modify the parameter in such a way that it minimizes the cost function, here the parameters are the cities in the list, and modifying the parameters is same as changing the order of exploring the cities. For generating the neighboring solution we have used the same five heuristics methods as in the intial solution, as using heuristic with simulated annealing solution can give a more optimal solution as compared to only randomly generating the neighboring solution. Now applying the simulated annealing technique to ATSP. Firstly create the initial order of cities to be visited (choose a random order). At each iteration swaps the cities in the list and then calculate the cost function if the new distance is greater than the previously calculated cost keep the list with certain probability otherwise if it is smaller keep the changes and update the list. After a certain amount of iterations there will be no significant change in the cost function and we will reach the optimal result.

This gives the probability of accepting the new solution:

$$p = \begin{cases} 1, & if \ f(y) \geq f(x) \\ e^{-(f(y)-f(x))/T}, & otherwise \end{cases}$$

The formula is basically a reflection that we accept the solution if it is better otherwise we check the probability. And it can be seen from the function that with decreasing temperature the chances of selecting a worse solution also decreases.

D. Reverse Transformation

Just after , we have solved our ATSP problem, we will do a reverse tranformation to get our desired solution for the original HMDMTSP solution. This involves mapping the targets created during ATSP transformation back to their respective vehicles and targets. The resulting solution provides the optimal sequence/order of targets for each vehicle type while satisfying the constraints of visiting each target at least once.

E. Cost Calculation and Evaluation

Finally, the cost of the solution for the HMDMTSP is calculated by summing up the costs of each vehicle's tour. This cost represents the total travel cost incurred by the UAV fleet while ensuring that all targets are observed at least once.

F. Performance Evaluation and Fine-Tuning Parameters

The methodology's performance is assessed through computational experiments using benchmark instances. Key performance metrics include solution quality, computational time, and scalability. The methodology may also involve fine-tuning parameters of the ATSP solver or considering additional constraints specific to the problem domain.

IV. Solution And Discussion

After running the program, we can see that hill climbing is giving a better solution than random method. And Simulated annealing is improving the hill climbing solution further to minimize the total distance covered. In the random solution method, we are visiting every branch of the solution to our limit and hence here we are not even trying to optimize the solution. Then we will try to optimize the solution through Hill climbing method. Let visualize the current solution as a node and upcoming solution as child nodes of this node. Then we are visiting that child nodes which is making our solution better. Since we are only accepting the better solution than the previous solution and doing this step for 100000 times, we are going towards best solution. In hill climbing, we are just only going toward up, one by one, and not accepting a worse solution than previous, so we can get stuck at local minima/maxima. Both Hill climbing and simulated annealing are optimization problems, but they differ in their approaches in searching for optimal solutions. Simulated Annealing is often considered more versatile than Hill Climbing, especially in solving complex optimization problems like the Heterogeneous Multiple Depot Multiple Traveling Salesman Problem (H-MDMTSP). Simulated Annealing introduces a controlled level of randomness, enabling global exploration. Simulated Annealing can be parallelized, allowing for the exploration of multiple solution paths simultaneously. This parallelization is advantageous in solving large-scale problems like H-MDMTSP, where efficient exploration of the solution space is critical. In the next step that is simulated annealing, we are also accepting the worse solution with a probability and from their we are try to have a jump for better solution. And hence , we could possibly get a better solution than Hill climbing method. In this method, we will always update our probability of accepting the worse solution. The conclusion is that, simulated annealing will provide the best solution here.

V. Conclusion

In conclusion, this methodology offers a systematic and effective approach to solving the challenging Homogenous, Multiple Depot, Multiple Travelling Salesman Problem (HMDMTSP) encountered in various real life events.It have application in surveillance missions involving UAVs.IT also have application in delivering any parcel like zomato/swiggy food, flipkart/Amazon parcels from different hub.By using Hill climbing and Simulated annealing, we have solved the problem.And we have found that best result are provided by simulated annealing.In future , we can solve the problem by by transforming the problem into an Asymmetric Travelling

Salesman Problem (ATSP) and applying efficient ATSP solvers like the Lin-Kernighan-Helsgaun heuristic, high-quality solutions can be obtained within reasonable computational time. Recommendations for future work may include further algorithm enhancements or extensions to address specific real-world constraints and scenarios.

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