The Study of Some Inverse Thermoelastic Problem on Different Solids

29 April 2019

Department of Mathematics

Rashtrasant Tukadoji Maharaj Nagpur University, Nagpur

Research Scholar

Ms. Ranjana S. Ghume

Supervisor

Dr. N. W. Khobragade

INTRODUCTION AND HISTORICAL REVIEW

Boundary Value Problem

The heat conduction equation along with boundary conditions prescribed on the surfaces of the body is called the **boundary condition problem.**

The Boundary Conditions

The differential equation of heat conduction will have numerous solutions unless a set of boundary conditions and an initial condition (for the time independent problem) are prescribed. The boundary conditions that prescribe the conditions at the boundary surfaces of the region may be linear or nonlinear. For convenience the linear boundary conditions will be separated into the following three groups:

Boundary conditions of the first kind

Temperature is prescribed along the boundary surface and for the general case it is a function of both time and position, i.e. $T = f_i(r_s, t) = 0$. If T = 0 then this special case is called as homogeneous boundary condition

Boundary condition of the second kind

The normal derivative of temperature is prescribed at the boundary surface and it may be a function of both time and position. For a boundary surface that fits the coordinate surface of an orthogonal coordinate system it is given in the form

$$\frac{\partial T}{\partial n_i} = f_i(\bar{r}_s, t) \quad \text{on the boundary surface } s_i$$

$$\frac{\partial T}{\partial n_i} = 0 \quad \text{on the boundary surface } s_i$$

This special case is called the homogeneous boundary condition of he second kind. An insulated boundary condition satisfies this condition.

Boundary condition of the third kind

A linear combination of the temperature and its normal derivative is prescribed at the boundary surface. For a boundary surface that fits the coordinate surface of an orthogonal coordinate system it is given as

$$k_i \frac{\partial T}{\partial n_i} + h_i T = f_i(\bar{r}_s, t) \qquad \text{on the boundary } s_i$$

Which is called the homogeneous boundary condition of the third kind

Homogeneous And Nonhomogeneous Boundary-Value Problems Of Heat Conduction

In this thesis we shall be concerned primarily with the solution of linear boundary value problems of heat conduction. For convenience in the analysis, the time- dependent boundary value problems of heat conduction will be considered in two different groups: homogeneous problems and nonhomogeneous problems.

The time-dependent boundary value problem of heat conduction will be referred to as a homogeneous problem when both the differential equation and the boundary conditions are homogeneous. The problem in the form:

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{in region R, } t > 0$$

$$k_i \frac{\partial T}{\partial n_i} + h_i T = 0 \qquad \text{on boundary } s_i,\ t > 0$$

$$T = F(r) \qquad \text{in region R, } t = 0$$

Will be referred to as the homogeneous problem because both the differential equation and the boundary condition are homogeneous. The boundary condition in above equation could be a homogeneous boundary condition of the first or second kind. The boundary value problem of heat conduction will be referred to as nonhomogeneous if the differential equation, or the boundary conditions or both are nonhomogeneous. For example, the boundary-value problem of heat conduction in the form

$$\nabla^2 T + \frac{g(\bar{r},t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{in region R, } t > 0$$

$$k_i \frac{\partial T}{\partial n_i} + h_i T = f_i(\bar{r},t) \qquad \text{on boundary } s_i,\ t > 0$$

$$T = F(\bar{r}) \qquad \text{in region R, } t = 0$$

is nonhomogeneous because the differential equation and the boundary condition are nonhomogeneous (i.e. functions g(r, t) and f_1(r, t) do not include T as product).

HISTORICAL PREVIEW

- **Duhamel** [**2**] first formulated the fundamental equations of thermoelasticity, taking in to consideration the deformation produced by the temperature changes. Duhamel's analysis showed that the stresses produced by temperature gradient can be calculated separately from those produced by mechanical forces and that the total stresses can be obtained by superposition. This seems to be the first use of superposition in stress analysis. He applied his famous fundamental equations to several specific cases, including the circular cylinder and the sphere subjected to radially varying temperature.
- **Mura** [**9**] and **Danilovskaya**[**6**] generalized previous solution to accommodate convective boundary conditions.
- **Sternberg**and **Chakravorthy**[**26**] obtained the missing displacements in the closed form as in [**6**] and showed that the dynamic effects induced by gradual heating or cooling of the boundary are substantially different from those encountered in even at extremely high, but finite, time gradients of the surface temperature.

- Inertia terms have been taken into account in several thermoelastic investigations since the appearance of Danilovskaya’s original paper [**6**].

- **Boley** [**28**] studied thermally induced beam and plate vibrations. A particularly important contribution to the subject under discussion is due to **Nowacki [32**] who obtained several closed exact solutions to the uncoupled three-dimensional thermoelastic equations of motion, corresponding to the time-dependent heat source in the interior of medium which occupies the entire space. Here the mention a purely formal dynamic treatment by **Ignaczak** [**18**] of the thermoelastic problem for the half- space, in the presence of time-dependent heat source at the boundary will not be out of context.

- Several authors have investigated the thermoelastic problems for various bodies subjected to various initial and boundary conditions and the extensive bibliographies can be found in [1 to 140].

- The inverse thermoelastic problems consist of determination of heating medium, the heat flux at the boundary surfaces of solids when the condition of the displacement and stresses are known at the same point of solids under consideration. The inverse problem is very important in view of its relevance to various industrial machine subjected to heating such as main shaft of lathe and turbine and the role of rolling machine.

- Therefore further studies on thick and thin bodies are needed with some heat generation hence there is ample opportunity for doing research in this area and we propose to study thermoelastic analysis of thick and thin bodies.

DEFINITIONS AND STANDARD RESULTS LAPLACE TRANSFORM

Let f(x) be a function of x defined for x >0, then the Laplace transform of f(x) is defined by

F(x) is called the inverse Laplace transform of f(p), where p is the Laplace transform parameter.

FINITE FOURIER TRANSFORM

(A). If f(x) satisfies Dirichlet's conditions in the interval (0, a) and if for that range its finite Fourier Sine transform is defined to be

$$\bar{f}_s(m) = \int_0^a f(x)\, Sin\left(\frac{m\pi x}{a}\right) dx$$

then at each point of $(0, a)$ at which f(x) is continuous

$$f(x) = \frac{2}{a} \sum_{m=1}^{\infty} \bar{f}_s(m)\, Sin\left(\frac{m\pi x}{a}\right)$$

(B). If f(x) satisfies Dirichlet's conditions in the interval (0, a) and if for that range its finite Fourier Cosine transform is defined to be

$$\bar{f}_c(m) = \int_0^a f(x)\, Cos\left(\frac{m\pi x}{a}\right) dx$$

then at each point of $(0, a)$ at which f(x) is continuous

$$f(x) = \frac{\bar{f}_c(0)}{a} + \frac{2}{a} \sum_{m=1}^{\infty} \bar{f}_c(m)\, Cos\left(\frac{m\pi x}{a}\right)$$

(C). Properties of Sine and Cosine transforms:

$$(1)\ \int_0^a \frac{\partial f}{\partial x} Sin\left(\frac{m\pi x}{a}\right)dx = -\left(\frac{m\pi}{a}\right)\bar{f}_c(m)$$

$$(2)\ \int_0^a \frac{\partial f}{\partial x} Cos\left(\frac{m\pi x}{a}\right)dx = (-1)^m f(a) - f(0) + \left(\frac{m\pi}{a}\right)\bar{f}_s(m)$$

$$(3)\ \int_0^a \frac{\partial^2 f}{\partial x^2} Sin\left(\frac{m\pi x}{a}\right)dx = \left(\frac{m\pi}{a}\right)\left[(-1)^{m+1} f(a) + f(0)\right] - \left(\frac{m^2\pi^2}{a^2}\right)\bar{f}_s(m)$$

(D). If f(x) satisfies Dirichlet's conditions in the interval (0, ∞) and if for that range, its Fourier transform is defined to be

$$\overline{f}_s(\alpha) = \int_0^\infty f(x)\sin(\alpha x)\,dx$$

$$f(x) = \frac{2\alpha}{\pi}\int_0^\infty \overline{f}_s(\alpha)\sin(\alpha x)\,d\alpha$$

(E). Properties of Sine transforms:

1. $$\int_0^\infty \frac{\partial f}{\partial x} Sin(\alpha x)dx = -\alpha\,\bar{f}_c(\alpha)$$

2. $$\int_0^\infty \frac{\partial^2 f}{\partial x^2} Sin(\alpha x)dx = [\alpha f(0)] - \left(\alpha^2\right)\bar{f}_s(\alpha)$$

where $f_s(\alpha)$ is the Fourier Sine transform respectively.

FINITE HANKEL TRANSFORM

(A). If f(x) satisfies Dirichlet's conditions in the interval $(0,a)$ then its finite Hankel transform in that range is defined to be

$$\bar{f}_\mu(\xi_i) = \int_0^a x\, f(x)\, J_\mu(x\xi_i)\, dx$$

where ξ_i is the root of the transcendental equation

$$J_\mu(a\xi_i) = 0.$$

then at any point of $(0,a)$ at which the function f(x) is continuous,

where the sum is taken over all the positive roots of the above equation.

(B). Properties of the Hankel Transform (1.1.32)

If f(x) satisfies Dirichlet's conditions in the closed interval $[0,a]$ then

(i) Finite Hankel transform of $\frac{\partial f}{\partial x}$, i.e.

$$H_\mu\left[\frac{\partial f}{\partial x}\right] = \int_0^a \frac{\partial f}{\partial x}\, x\, J_\mu(x\xi_i)\, dx$$

$$= \frac{\xi_i}{2\mu}\left[(\mu-1)H_{\mu+1}\{f(x)\} - (\mu+1)H_{\mu-1}\{f(x)\}\right]$$

(ii) $$H_\mu\left[\frac{\partial^2 f}{\partial x^2}+\frac{1}{x}\frac{\partial f}{\partial x}\right]=\frac{\xi_i}{2}\left[-H_{\mu-1}\left\{\frac{\partial f}{\partial x}\right\}+H_{\mu+1}\left\{\frac{\partial f}{\partial x}\right\}\right]$$

(C). If f(x) satisfies Dirichlet's conditions in the range $b \le x \le a$ and if its finite Hankel transform in that range is defined to be

$$H[f(x)]=\bar{f}_\mu(\xi_i)$$

$$=\int_b^a x f(x)\left[(J_\mu(x\xi_i)G_\mu(a\xi_i)-J_\mu(a\xi_i)G_\mu(x\xi_i)\right]dx$$

in which ξ_i is a root of the transcendental equation

$$\left[J_\mu(\xi_i b)G_\mu(\xi_i a)-J_\mu(\xi_i a)G_\mu(\xi_i b)\right]=0$$

then at which the function is continuous,

$$f(x)=\sum_i \frac{2\xi_i^2 J_\mu^2(\xi_i b)\,\bar{f}_\mu(\xi_i)}{J_\mu^2(a\xi_i)-J_\mu^2(b\xi_i)}[J_\mu(x\xi_i)G_\mu(a\xi_i)-J_\mu(a\xi_i)G_\mu(x\xi_i)]$$

(D) Property of the Hankel transform defined as

(i) $$\int_a^b \left[\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x}\right][J_\mu(x\xi_i)G_\mu(a\xi_i) - J_\mu(a\xi_i)G_\mu(x\xi_i)]dx$$

$$= -\xi_i^2 \bar{f}_\mu(\xi_i) + a[J_\mu(x\xi_i)G_\mu(a\xi_i) - J_\mu(a\xi_i)G_\mu(x\xi_i)]_{x=a}$$

$$+ b[J_\mu(x\xi_i)G_\mu(a\xi_i) - J_\mu(a\xi_i)G_\mu(x\xi_i)]_{x=b} = -\xi_i^2 \bar{f}_\mu(\xi_i)$$

FINITE MARCHI-FASULO INTEGRAL TRANSFORM

The finite Marchi-Fasulo integral transform of f(z), $-h < z < h$ is defined to be

$$\bar{F}(n) = \int_{-h}^{h} f(z) P_n(z) dz$$

then at each point of (-h, h) at which f(z) is continuous,

where

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

The eigen values a_n are the solutions of the equation

$$= \left[\alpha_2 a \cos(ah) - \beta_2 \sin(ah)\right] \times \left[\beta_1 \cos(ah) - \alpha_1 a \sin(ah)\right]$$

FINITE MARCHI-ZGRABLICH INTEGRAL TRANSFORM

The finite Marchi-Zgrablich integral transform of $f(r)$ is defined as

$$\bar{f}_p(m) = \int_a^b r\, f(r)\, S_p(\alpha, \beta, \mu_m r)\, dr$$

where α_1, α_2, β_1 and β_2 are the constants involved in the boundary conditions $\alpha_1 f(r) + \alpha_2 f'(r)_{r=a} = 0$ and $\beta_1 f(r) + \beta_2 f'(r)_{r=b} = 0$ for the differential equation $f'(r) + (1\ r) f'(r) - (p^2 / r^2) f(r) = 0$, $f\ (n)$ is the transform of $f(r)$ with respect to kernel $S_p(\alpha, \beta, \mu_m r)$ and weight function r .The inversion of equation of the above equation is given by where kernel function $S_p(\alpha, \beta, \mu_m r)$ can be defined as

$S_p(\alpha, \beta, \mu_m r) = J_p(\mu_m r)[Y_p(\alpha, \mu_m a) + Y_p(\beta, \mu_m b)] - Y_p(\mu_m r)[J_p(\alpha, \mu_m a) + J_p(\beta, \mu_m b)]$ and $J_p(\mu r)$ and $Y_p(\mu r)$ are

Bessel function of first and second kind respectively.

OPERATIONAL PROPERTY:

$$\int_a^b r^2 \left(f''(r) + (1/r) f'(r) - (p^2/r^2) f(r) \right) S_p(\alpha, \beta, \mu_m r)\, dr$$

$$= (b/\beta_2) S_p(\alpha, \beta, \mu_m b) \left[\beta_1 f(r) + \beta_2 f'(r) \right]_{r=b} - (a/\alpha_2) S_p(\alpha, \beta, \mu_m a) \left[\alpha_1 f(r) + \alpha_2 f'(r) \right]_{r=a} - \mu_m^2 \bar{f}_p(m)$$

2. Inverse Thermoelastic Problem of A Thin Rectangular Plate

The main results of this chapter have been published as detailed below:

1.Thermal stresses of a three dimensional thermoelastic problem of a thin rectangular plate, Scientific reviews and Chemical Communications, 2(3), pp. 446-457, 2012, **USA**

2.1 INTRODUCTION

Durge et al. [2003] has studied the inverse unsteady state thermoelastic problem of a thick rectangular plate. In the present chapter, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement and thermal stress at the edge y = b of a thin rectangular plate occupying the region $D: -a \le x \le a$; $0 \le y \le b$; $-h \le z \le h$ with known boundary conditions.

2.2 STATEMENTOF THE PROBLEM

$0 \le y \le b, -h \le z \le h$.

u_x, u_y and u_z in the X, Y, Z direction respectively are in the integral form as

(2.2.1)

(2.2.2)

(2.2.3)

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t) = -\alpha E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right) T(x,y,z,t) \quad (2.2.4)$$

Where $T\ (x,\ y,\ z,\ t)$ denotes the temperature of thin rectangular plate satisfying the following differential equation as

(2.2.5)

Where k is thermal diffusivity of the material subject to initial conditions

$$T(x, y, z, 0) = 0 \tag{2,2.6}$$

And the boundary conditions are

$$\left[T(x, y, z, t) + k_1 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=a} = F_1(y, z, t) \tag{2.2,7}$$

$$\left[T(x, y, z, t) + k_2 \frac{\partial T(x, y, z, t)}{\partial x}\right]_{x=-a} = F_2(y, z, t) \tag{2.2.8}$$

$$\left[T(x, y, z, t)\right]_{y=b} = G(x, z, t) \quad \text{(Unknown)} \tag{2.2.9}$$

$$\left[T(x, y, z, t) + C \frac{\partial T(x, y, z, t)}{\partial y}\right]_{y=0} = g(x, z, t) \tag{2.2.10}$$

$$\left[T(x, y, z, t) + k_3 \frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=h} = F_3(x, y, t) \tag{2.2.11}$$

$$\left[T(x, y, z, t) + k_4 \frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=-h} = F_4(x, y, t) \tag{2.2.12}$$

The interior condition is

(2.2.13)

The stresses components in terms of $U(x, y, z, t)$

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) \tag{2.2.14}$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} \right) \tag{2.2.15}$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \tag{2.2.16}$$

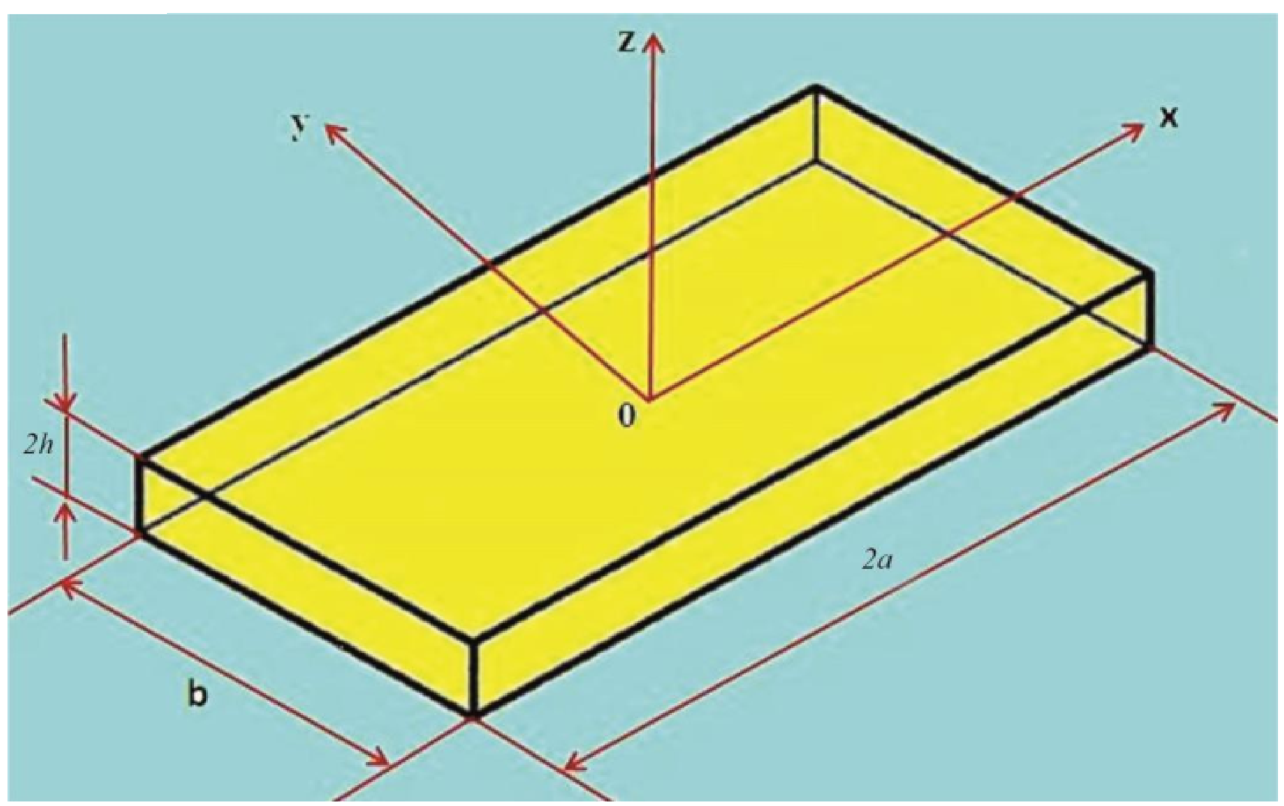


Figure 1: Geometry of the problem

2.3 SOLUTION OF THE PROBLEM

By applying finite Marchi – Fasulo transform and Laplace transform to the equations (2.2.5) to (2.2.13), and then taking their inversion, we obtain

$$T(x,y,z,t)=\frac{k}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)\right] \tag{2.3.1}$$

$$+\frac{2k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(y)\,\tau_3(t)-\psi_2(y)\,\tau_4(t)}{\left[1+(c\varsigma\pi/\xi)^2\right]}\right]-\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]A_3(m,n,y,t)$$

$$G(x,z,t)=\frac{k}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\phi_1(b)\,\tau_1(t)-\phi_2(b)\,\tau_2(t)\right]$$

$$+\frac{2k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(b)\,\tau_3(t)-\psi_2(b)\,\tau_4(t)}{\left[1+(c\varsigma\pi/\xi)^2\right]}\right] \tag{2.3.2}$$

where

$$\phi_1(y)=\frac{\sinh\,(y/c)-\cosh\,(y/c)}{\sinh(\xi/c)},\qquad \psi_1(y)=\sin\left(\frac{\varsigma\pi}{\xi}\right)y-\left(\frac{c\varsigma\pi}{\xi}\right)\cos\left(\frac{\varsigma\pi}{\xi}\right)y,$$

$$\phi_2(y)=\frac{\sinh\left(\frac{y-\xi}{c}\right)-\cosh\left(\frac{y-\xi}{c}\right)}{\sinh(\xi/c)}\qquad \psi_2(y)=\sin\left(\frac{\varsigma\pi}{\xi}\right)(y-\xi)-\left(\frac{c\varsigma\pi}{\xi}\right)\cos\left(\frac{\varsigma\pi}{\xi}\right)(y-\xi)$$

$$\tau_3(t) = \int_0^t [\bar{\bar{f}}(m,n,t-u) - A_1(m,n,t-u)]\, e^{-ku\left[q^2+\left(\frac{\varsigma\pi}{\xi}\right)^2\right]} du$$

$$\tau_4(t) = \int_0^t [\bar{\bar{g}}(m,n,t-u) - A_2(m,n,t-u) e^{-ku\left[q^2+\left(\frac{\varsigma\pi^2}{\xi}\right)^2\right]} du$$

$$A_1(m,n,t) = \left[\left(\chi + c\frac{d\chi}{dz}\right)_{z=\xi}\right],$$

$$A_2(m,n,t) = \left[\left(\chi + c\frac{d\chi}{dz}\right)_{z=0}\right],$$

$$A_3(m,n,z,t) = L^{-1}[\chi]$$

Here $\bar{\bar{f}}(m,n,t)$ and $\bar{\bar{g}}(m,n,t)$ denote the Marchi – Fasulo transforms of $\bar{f}(m,z,t)$ and $\bar{g}(m,z,t)$ respectively.

$\bar{f}(m,z,t)$ $\bar{g}(m,z,t)$ denote the finite Marchi – Fasulo transform of $f(x,z,,t)$ and $g(x,z,t)$ respectively.

$$\bar{\bar{f}}(m,n,t) = \int_{-h}^{h} \bar{f}(m,z,t)\, P_n(z)\, dz ,$$

$$\bar{\bar{g}}(m,n,t) = \int_{-h}^{h} \bar{g}(m,z,t)\, P_n(z)\, dz ,$$

$$\lambda_n = \int_{-h}^{h} P_n^2(z)\, dz$$

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_3 + \alpha_4)\cos(a_n h) + (\beta_3 - \beta_4)\sin(a_n h)$$

$$W_n = (\beta_3 + \beta_4)\cos(a_n h) + (\alpha_4 - \alpha_3)\, a_n \sin(a_n h)$$

Equations (2.3.1) and (2.3.2) are the desired solution of the given problem with $\beta_3 = \beta_4 = 1$, $\alpha_3 = k_3$, $\alpha_4 = k_4$.

2.4 DETERMINATION OFAIRYSSTRESS FUNCTION

Substituting the values of $T(x, y, z, t)$ from equation (2.3.1) in equation (2.2.4) one

obtains

$$+\frac{2\alpha E k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{1}{1+\left(c\varsigma\pi/\xi\right)^2}\right]\times\left[\frac{\psi_1(z)\,\tau_3(t)-\psi_2(z)\,\tau_4(t)}{a_m^2+a_n^2+\left(\varsigma\pi/\xi\right)^2}\right]$$
$$-\alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{A_3\,(m,n,z,t)}{a_m^2+a_n^2-l_0}\right] \tag{2.4.1}$$

2.5 DETERMINATION OF DISPLACEMENTCOMPONENTS

Substituting the values (2.4.1) in the equation (2.2.1) to (2.2.3) one obtains

$$u_x = \frac{\alpha k}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{(1+\nu)a_m^2}{a_m^2+a_n^2-1/c^2}\right]$$

$$\times\left[\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)\right] + \frac{2\alpha k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{(1+\nu)a_m^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right] \times\left[\frac{\psi_1(y)\tau_3(t)-\psi_2(y)\,\tau_4(t)}{1+(c\varsigma\pi/\xi)^2}\right]$$

$$-\alpha\sum_{m,n=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{(1+\nu)\,a_m^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t)\,, \qquad (2.5.1)$$

$$u_y = \frac{\alpha k}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-(1+\nu)/c^2}{a_m^2+a_n^2-1/c^2}\right]\left[\phi'_1(b)\,\tau_1(t)-\phi_2'(b)\,\tau_2(t)\right] + \frac{2\alpha k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{(1+\nu)\,(\varsigma\pi/\xi)^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]\left[\frac{1}{(1+(c\varsigma\pi/\xi)^2}\right]$$

$$\times\left[\psi'_1(b)\,\tau_3(t)-\psi'_2(b)\,\tau_4(t)\right] - \alpha\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-(1+\nu)\,l_0}{a_m^2+a_n^2-l_0}\right]A'_3(m,n,b,t) \qquad (2.5.2)$$

$$u_z = \frac{\alpha k}{c^2}\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n b}{\lambda_n}\right]\left[\frac{(1+\nu)\,a_n^2}{a_m^2+a_n^2-1/c^2}\right]\times\left[\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n b}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{(1+\nu)a_n^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]\left[\frac{\psi_1(y)\tau_3(t)-\psi_2(y)\,\tau_4(t)}{1+(c\varsigma\pi/\xi)^2}\right]$$

$$-\alpha\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n b}{\lambda_n}\right]\left[\frac{(1+\nu)\,a_n^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t)\,, \qquad (2.5.3)$$

where

$$\phi'_1(b) = \frac{\cosh(b/c) - \sinh(b/c) - 1}{1/c \sinh(\xi/c)} \qquad \psi'_1(b) = \frac{-\cos(\varsigma\pi/\xi)b - (c\varsigma\pi/\xi)\sin(\varsigma\pi/\xi)b + 1}{(\varsigma\pi/\xi)}$$

$$\phi'_2(b) = \frac{\cosh((b-\xi)/c) - \sinh((b-\xi)/c) - \cosh(b/c) - \sinh(b/c)}{(1/c)\sinh(\xi/c)} \qquad \psi'_2(b) = \frac{-\cos(\varsigma\pi/\xi)(b-\xi) - (c\varsigma\pi/\xi)\sin(\varsigma\pi/\xi)(b-\xi) + \cos\varsigma\pi}{(\varsigma\pi/\xi)}$$

$$A'_3(m,n,h,t) = \int_0^h A_3(m,n,z,t)dz$$

2.6 DETERMINATION OF STRESSFUNCTION

Substituting values of (2.4.1) in equations (2.2.14) to (2.2.16) one

$$\sigma_{xx} = \frac{\alpha E k}{c^2}\sum_{m,n,=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_n^2 + 1/c^2}{a_m^2 + a_n^2 - 1/c^2}\right]\left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha E k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_n^2 - (\varsigma\pi/\xi)^2}{a_m^2 + a_n^2 + (\varsigma\pi/\xi)^2}\right] \times \frac{\left[\psi_1(y)\,\tau_3(t) - \psi_2(y)\,\tau_4(t)\right]}{\left[1 + (c\varsigma\pi/\xi)^2\right]} \qquad (2.6.1)$$

$$+\frac{2\alpha Ek\pi}{\varsigma^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_m^2-a_n^2}{a_m^2+a_n^2+(\varsigma\pi/\varsigma)^2}\right]$$

(2.6.2)

(2.6.3)

$$-\alpha E\sum_{m,n,=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{l_0-a_m^2}{a_m^2+a_n^2-l_0}\right]A_3(m,n,y,t)$$

2.7 SPECIAL CASE AND NUMERICALRESULTS

Set $f(x,z,t)=(1-e^{-t})(x+a)^2(x-a)^2(z+h)^2(z-h)^2\,e^{\xi}$,

$g(x,z,t)=(1-e^{-t})(x+a)^2(x-a)^2(z+h)^2(z-h)^2$ in the equations (2.3.1)- (2.6.3) to obtain

$$\frac{T(x,y,z,t)}{\delta}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{\eta=1}^{\infty}(-1)^{(\eta+1/2)}\left(\eta+\frac{1}{2}\right)\left(\frac{P_n(x)}{\mu_n}\right)\left(\frac{P_m(z)}{\lambda_m}\right)\left(\frac{1}{1-q^2}\right)\times\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n{}^2}\right]\times[\Phi(y)e-\Psi(y)]$$

$$\times\int_0^t(1-e^{-t'})\,e^{-0.86\left(q^2+\left(\eta+\frac{1}{2}\right)^2\pi^2\right)(t-t')}dt' \tag{2.7.1}$$

$$\frac{G(x,z,t)}{\delta}=\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right][\phi_1(b)\,\tau_1(t)-\phi_2(b)\,\tau_2(t)]$$

$$+\frac{2k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(b)\,\tau_3(t)-\psi_2(b)\,\tau_4(t)}{\left[1+(c\varsigma\pi/\xi)^2\right]}\right]-\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]A_3(m,n,b,t)$$

$$\tag{2.7.2}$$

$$U(x,y,z,t)=\alpha Ek\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{[\phi_1(z)\,\tau_1(t)-\phi_2(z)\,\tau_2(t)]}{a_m^2+a_n^2-1}\right]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{1}{1+(\varsigma\pi/\xi)^2}\right]\times\left[\frac{\psi_1(z)\,\tau_3(t)-\psi_2(z)\,\tau_4(t)}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]$$

$$-\alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{A_3(m,n,z,t)}{a_m^2+a_n^2-l_0}\right] \tag{2.7.3}$$

Set ,

$$\delta = \frac{8(k_1+k_2)k\pi}{h^2},\ a=1.5,\ k=0.86,\quad ,b=3, c=1,\ h=2, \xi=2\ t=1\,\mathrm{s} \tag{2.7.4}$$

$$\frac{T(x,y,z,t)}{\delta} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{\eta=1}^{\infty}(-1)^{(\eta+1/2)}\left(\eta+\frac{1}{2}\right)\left(\frac{P_n(x)}{\mu_n}\right)\left(\frac{P_m(z)}{\lambda_m}\right)\left(\frac{1}{1-q^2}\right)$$

$$\times\left[\frac{a_n\cos^2(a_n)-\cos(a_n)\sin(a_n)}{a_n{}^2}\right]\times\left[\Phi(y)e-\Psi(y)\right]\quad\times\int_0^t(1-e^{-t'})\,e^{-0.86\left(q^2+\left(\eta+\frac{1}{2}\right)^2\pi^2\right)(t-t')}dt'$$

$$\frac{G(x,z,t)}{\delta} = \sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\phi_1(b)\,\tau_1(t)-\phi_2(b)\,\tau_2(t)\right] \tag{2.7.5}$$

$$+\frac{2k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{\psi_1(b)\,\tau_3(t)-\psi_2(b)\,\tau_4(t)}{\left[1+(\varsigma\pi/\xi)^2\right]}\right] - \sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]A_3(m,n,b,t)$$

$$U(x,y,z,t)=\alpha Ek\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\left[\phi_1(z)\,\tau_1(t)-\phi_2(z)\,\tau_2(t)\right]}{a_m^2+a_n^2-1}\right]$$

$$+\frac{2\alpha Ek\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{1}{1+(\varsigma\pi/\xi)^2}\right]$$

$$\times\left[\frac{\psi_1(z)\,\tau_3(t)-\psi_2(z)\,\tau_4(t)}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right]-\alpha E\sum_{m,n=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{A_3\,(m,n,z,t)}{a_m^2+a_n^2-l_0}\right] \tag{2.7.6}$$

$$u_x=\alpha k\sum_{m,n=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{(1+\nu)a_m^2}{a_m^2+a_n^2-1}\right]\times\left[\phi_1(y)\,\tau_1(t)-\phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha k\pi}{\xi^2}\sum_{m,n,\varsigma=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m a}{\lambda_m}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left|\frac{(1+\nu)a_m^2}{a_m^2+a_n^2+(\varsigma\pi/\xi)^2}\right|\times\left[\frac{\psi_1(y)\tau_3(t)-\psi_2(y)\,\tau_4(t)}{1+(\varsigma\pi/\xi)^2}\right]$$

$$-\alpha\sum_{m,n=1}^{\infty}\left[\frac{(k_1+k_2)\sin 2a_m(1.5)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{(1+\nu)\,a_m^2}{a_m^2+a_n^2-l_0}\right]\times A_3\,(m,n,y,t)\,, \tag{2.7.7}$$

$$u_y = \alpha k \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-(1+v)}{a_m^2 + a_n^2 - 1}\right] \left[\phi'_1(b)\,\tau_1(t) - \phi_2'(b)\,\tau_2(t)\right]$$

$$+\frac{2\alpha k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{(1+v)\left(\varsigma\pi/\xi\right)^2}{a_m^2 + a_n^2 + \left(\varsigma\pi/\xi\right)^2}\right]\left[\frac{1}{\left(1+(c\varsigma\pi/\xi\right)^2}\right]$$

$$\times\left[\psi'_1(b)\,\tau_3(t) - \psi'_2(b)\,\tau_4(t)\right] \;-\alpha \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-(1+v)\,l_0}{a_m^2 + a_n^2 - l_0}\right] A'_3(m,n,b,t) \tag{2.7.8}$$

$$u_z = \alpha k \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n(3)}{\lambda_n}\right]\left[\frac{(1+v)\,a_n^2}{a_m^2 + a_n^2 - 1}\right] \quad \times\left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{(k_3+k_4)\sin 2a_n(3)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos\varsigma\pi}\right]$$

$$\times\left[\frac{(1+v)a_n^2}{a_m^2 + a_n^2 + \left(\varsigma\pi/\xi\right)^2}\right]\left[\frac{\psi_1(y)\tau_3(t) - \psi_2(y)\,\tau_4(t)}{1+\left(\varsigma\pi/\xi\right)^2}\right] \tag{2.7.9}$$

$$-\alpha \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{\left(k_3+k_4\right)\sin 2a_n(3)}{\lambda_n}\right]\left[\frac{(1+v)\,a_n^2}{a_m^2 + a_n^2 - l_0}\right] A_3(m,n,y,t),$$

$$\sigma_{xx} = \frac{\alpha E k}{c^2} \sum_{m,n,=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_n^2 + 1/c^2}{a_m^2 + a_n^2 - 1/c^2}\right] \times \left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha E k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_n^2 - (\varsigma\pi/\xi)^2}{a_m^2 + a_n^2 + (\varsigma\pi/\xi)^2}\right] \times \frac{\left[\psi_1(y)\,\tau_3(t) - \psi_2(y)\,\tau_4(t)\right]}{\left[1 + (c\varsigma\pi/\xi)^2\right]}$$

$$-\alpha E \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{a_m^2 - (k_0{}^2\pi^2/\xi^2)}{a_m^2 + a_n^2 - l_0}\right] A_3(m,n,y,t) \qquad (2.7.10)$$

$$\sigma_{yy} = \left(\frac{\alpha E k}{c^2}\right) \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_m^2 - a_n^2}{a_m^2 + a_n^2 - 1/c^2}\right] \left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right]$$

$$+\frac{2\alpha E k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-a_m^2 - a_n^2}{a_m^2 + a_n^2 + (\varsigma\pi/\xi)^2}\right]$$

$$\times\left[\frac{\psi_1(y)\,\tau_3(t) - \psi_2(y)\,\tau_4(t)}{\left[1 + (c\varsigma\pi/\xi)^2\right]}\right] - \alpha E \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{-a_m^2 - a_n^2}{a_m^2 + a_n^2 - l_0}\right] A_3(m,n,y,t) \qquad (2.7.11)$$

$$\sigma_{zz} = \left(\frac{\alpha E k}{c^2}\right) \sum_{m,n=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{1/c^2 - a_m^2}{a_m^2 + a_n^2 - 1/c^2}\right]\left[\phi_1(y)\,\tau_1(t) - \phi_2(y)\,\tau_2(t)\right] \tag{2.7.12}$$

$$+\frac{2\alpha E k\pi}{\xi^2} \sum_{m,n,\varsigma=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{\varsigma}{\cos(\varsigma\pi)}\right]\left[\frac{-(\varsigma\pi/\xi)^2 - a_m^2}{a_m^2 + a_n^2 + (\varsigma\pi/\xi)^2}\right] \times \frac{\left[\psi_1(y)\,\tau_3(t) - \psi_2(y)\,\tau_4(t)\right]}{\left[1 + (\varsigma\pi/\xi)^2\right]}$$

$$-\alpha E \sum_{m,n,=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{P_n(z)}{\lambda_n}\right]\left[\frac{l_0 - a_m^2}{a_m^2 + a_n^2 - l_0}\right] A_3(m,n,y,t)$$

2.9 CONCLUSION

- The temperature distribution, unknown temperature gradient, displacements, and thermal stresses on the edge y = b of a thin rectangular plate have been obtained, when the boundary conditions are known with the aid of finite Marchi-Fasulo transform and Laplace transform techniques.

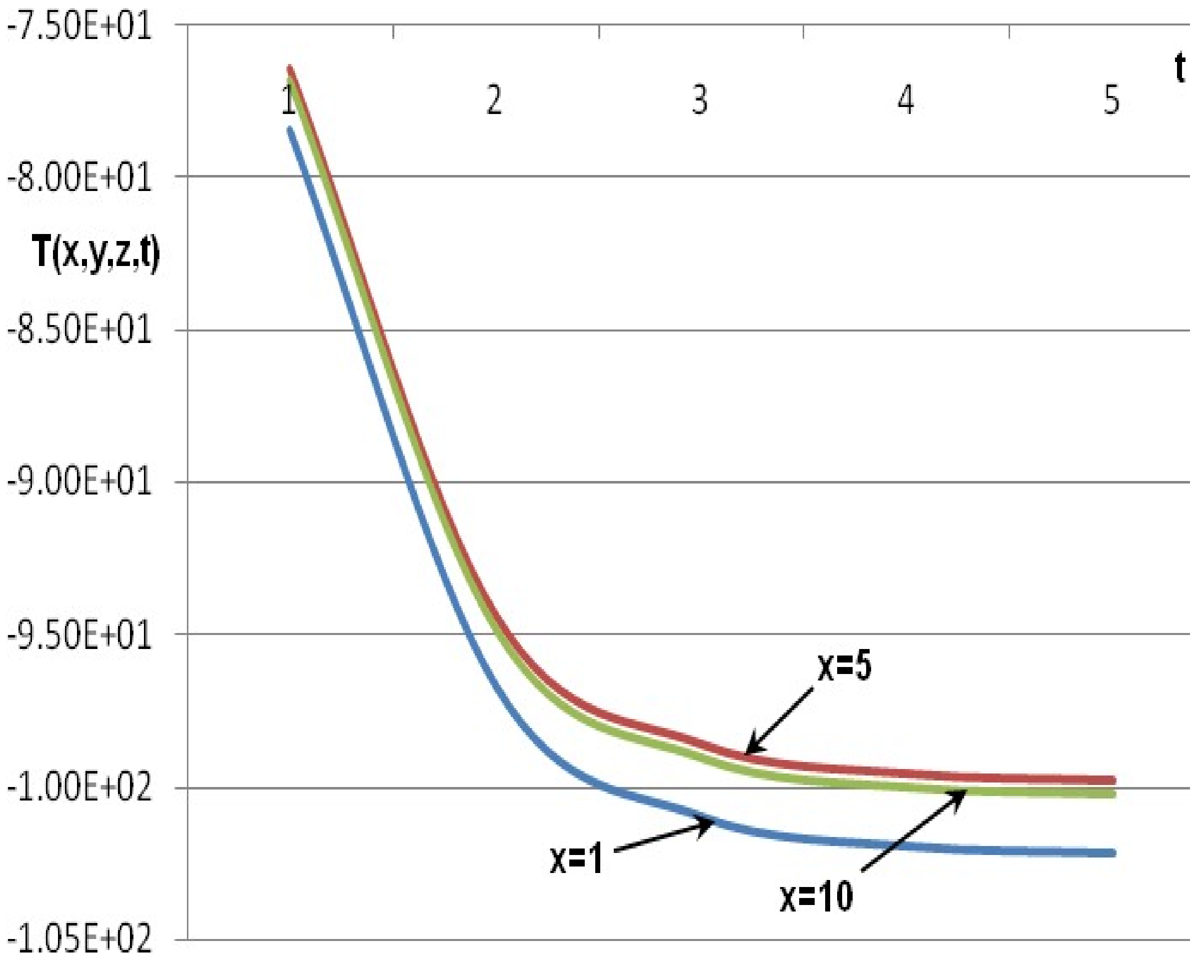


Fig (1). T(x,y,z,t) versus t for different values of x

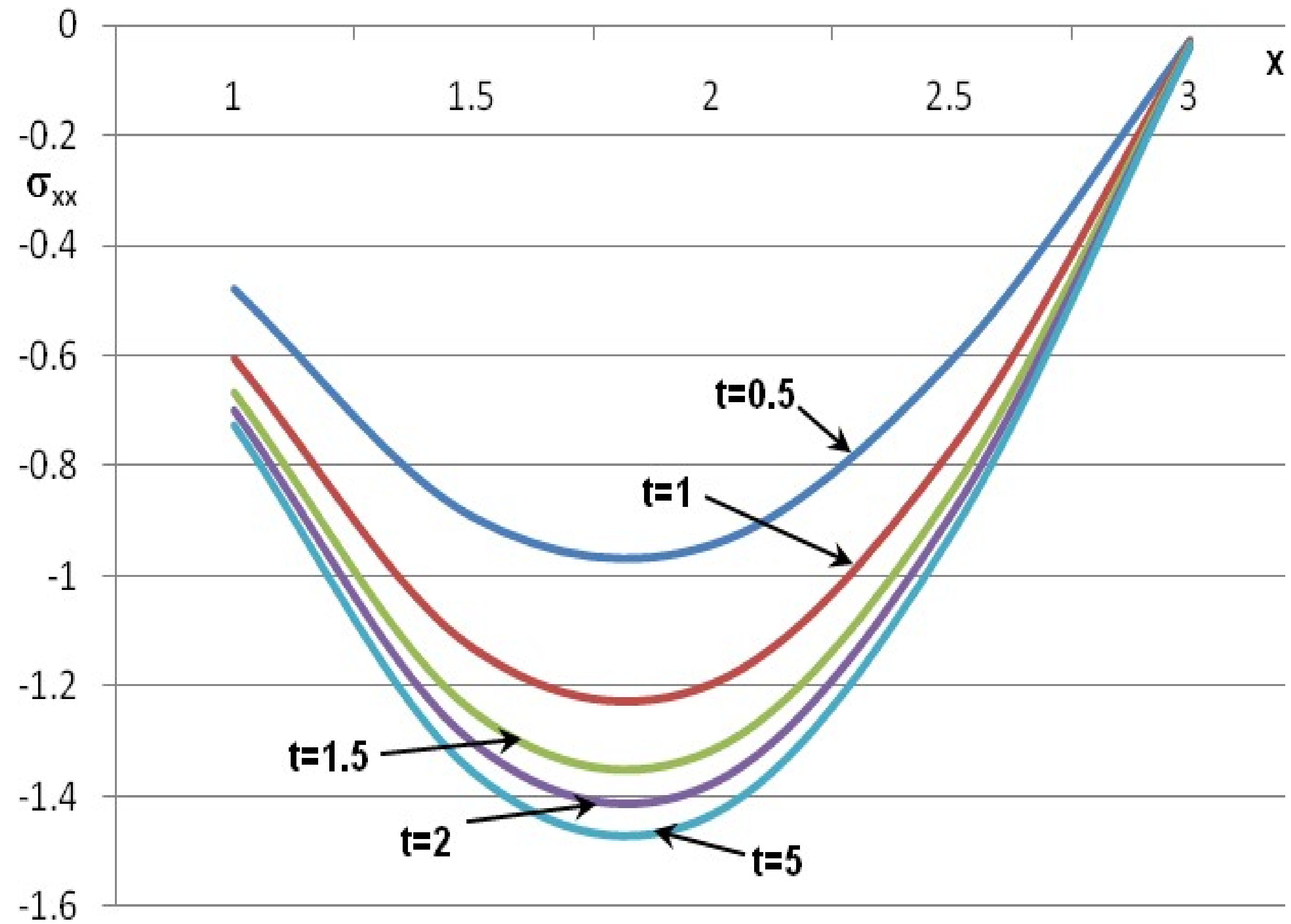


Fig (2). σ_{xx} versus x for different values of t

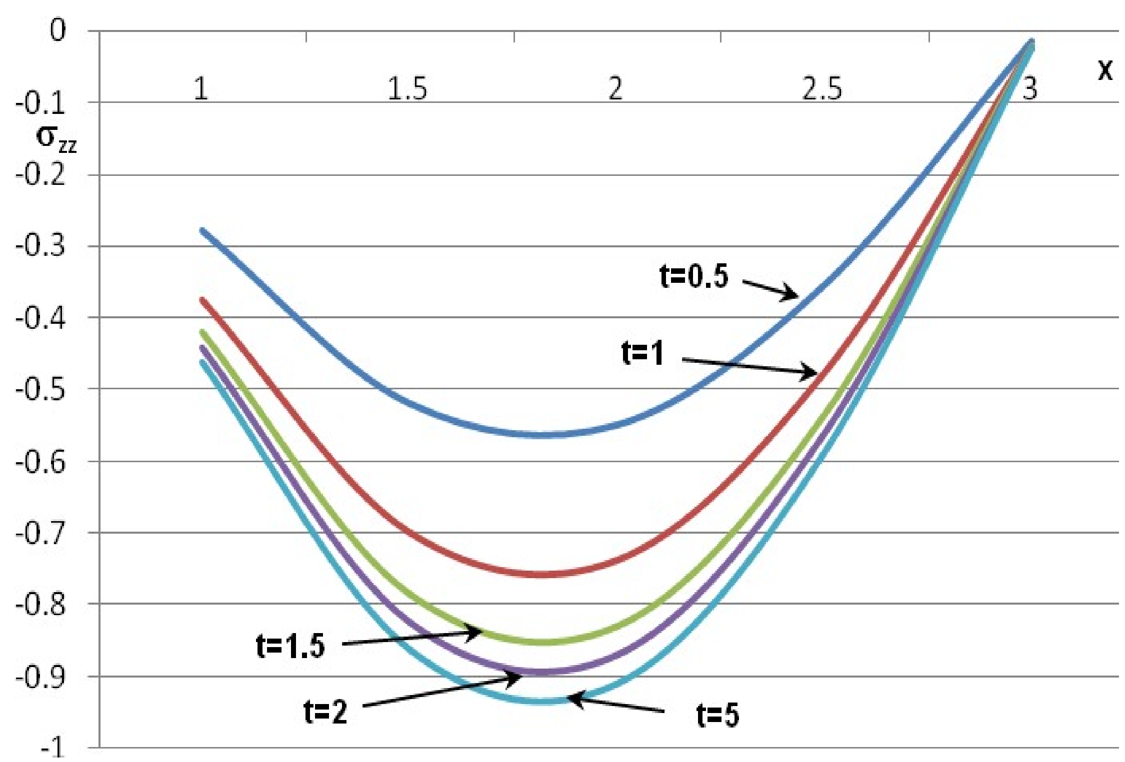


Fig (3). σ_{zz} versus x for different values of t

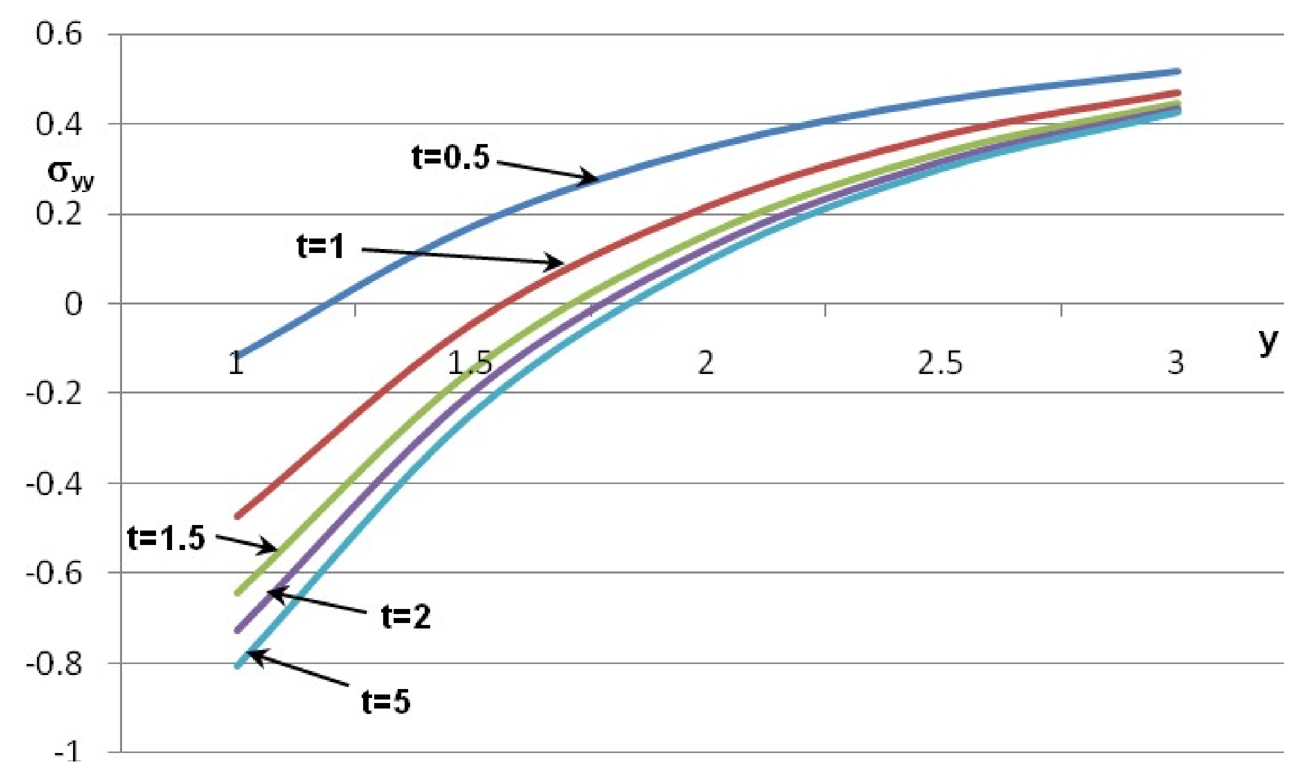


Fig (4). σ_{yy} versus y for different values of t

3. Deflection of A Thick Rectangular Plate

The main results of this chapter have been published as detailed below:

1. Deflection of a thick rectangular plate, Canadian Journal on Science and Engineering Mathematics, Volume 3 no. 2, pp. 61-64, Feb 2012, **Canada**

3.1 INTRODUCTION

In this chapter, an attempt has been made to study the transient thermoelastic problem to determine the temperature distribution and thermal deflection of the plate occupying the space D: $\{[x, y, z] \in R^3 : -a \le x \le a\ ;\ -b \le y \le b\ ,\ 0 \le z \le h\}$ with the stated boundary conditions. The heat conduction equation is solved with the help of finite Marchi fasulo transform and finite Fourier cosine transform techniques.

3.2 STATEMENTOF THE PROBLEM

Consider a thick isotropic rectangular plate occupying the space D. The differential equation satisfied by the deflection $\omega(x, y, z, t)$ as **Roy Choudhary** [**2003**] is

$$D\nabla^4\omega(x, y, z, t) = \frac{-\nabla^2 M_T(x, y, z, t)}{1-\vartheta} \tag{3.2.1}$$

where $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$

and the resultant thermal momentum M_T is defined as

$$M_T(x, y, z, t) = \propto E \int_0^h zT(x, y, z, t)dz \tag{3.2.2}$$

where α and E are the coefficient of liner expansion, Young's modulus respectively. Since the edge of the rectangular plate is fixed and clammed,

$$\frac{\partial \omega}{\partial z} = 0 \text{ at } z = 0, \quad \xi \tag{3.2.3}$$

The temperature of the plate at time t satisfying the differential equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \tag{3.2.4}$$

where k is the thermal diffusivity to the material of the plate, subject to the initial and boundary conditions:

$$T(x, y, z, o) = F(x, y, z) \tag{3.2.5}$$

$$\left[T + k_1 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a} = f_1(y,z,t) \tag{3.2.6}$$

$$\left[T + k_2 \frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=-a} = f_2(y,z,t) \tag{3.2.7}$$

$$\left[T + k_3 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=b} = f_3(x,z,t) \tag{3.2.8}$$

$$\left[T + k_4 \frac{\partial T(x,y,z,t)}{\partial y}\right]_{y=-b} = f_4(x,z,t) \tag{3.2.9}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=0} = g(x,y,t) \tag{3,2.10}$$

$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=\xi} = f(x,y,t) \text{ (known)} \tag{3.2.11}$$

$$\left[T(x,y,z,t)\right]_{z=h} = G(x,y,t) \text{ (unknown)} \tag{3.2.12}$$

Equations (3.2.1) to (3.2.12) constitute the mathematical formulation of the problem under consideration.

3.3 SOLUTION OF THE PROBLEM

Applying finite Marchi Fasulo transform and finite Fourier cosine transform to the equations (3.2.4) to (3.2.11) one obtains

$$\frac{\partial \overline{\overline{T}}}{\partial t} + (ka_p^2)\overline{\overline{T}}^* = k\left[(-1)^s \overline{\overline{f}}^*(m,n,t) - \overline{\overline{g}}^*(m,n,t) - \frac{s^2\pi^2}{h^2}\overline{\overline{T}}_c(s) + \phi\right] \tag{3.3.1}$$

where $a_p^2 = a_m^2 + b_n^2$

Equation (3.1) is a first order differential equation whose solution is given by

$$\overset{=}{T}^{*}(m,n,s,t) = k\left[(-1)^{s}\overset{=}{f}^{*}(m,n,t) - \overset{=}{g}^{*}(m,n,t) - \frac{s^{2}\pi^{2}}{h^{2}}\overset{=}{T}_{c}(s) + \phi\right] + \overset{=}{F}^{*}(m)e^{-(ka_{p}^{2})t} \tag{3.3.2}$$

where $\bar{T}$ denotes the Marchi Fasulo transform of T and $\bar{\bar{T}}$ denotes the Marchi Fasulo transform of $\bar{T}$, m & n are the Marchi Fasulo transform parameter. $\bar{\bar{T}}^{*}$ denotes the Fourier cosine transform of $\bar{\bar{T}}$ and s is a Fourier cosine transform parameter. By applying the inversion of Fourier cosine transform and Marchi Fasulo transform to the equation (3.3.2), one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \sum_{m,n,s=1}^{\infty}\left[\frac{P_{m}(x)}{\lambda_{m}}\right]\left[\frac{R_{n}(y)}{\mu_{n}}\right]\left\{\begin{array}{l}\left[k(c-1)^{s}\overset{=}{f}(m,n,t) - \overset{=}{g}(m,n,t) - \dfrac{s^{2}\pi^{2}}{\xi^{2}}\overset{=}{T}_{c}(s) + \phi\right] \\ + \left[\dfrac{2}{\xi}\overset{=}{F}_{c}(s) \times \cos\left(\dfrac{s\pi z}{\xi}\right).e^{-(ka_{p}^{2})t}\right]\end{array}\right\} \tag{3.3.3}$$

$$G(x,y,t) = \sum_{m,n,s=1}^{\infty}\left[\frac{P_{m}(x)}{\lambda_{m}}\right]\left[\frac{R_{n}(y)}{\mu_{n}}\right]\left\{\begin{array}{l}\left[k(c-1)^{s}\overset{=}{f}(m,n,t) - \overset{=}{g}(m,n,t) - \dfrac{s^{2}\pi^{2}}{\xi^{2}}\overset{=}{T}_{c}(s) + \phi\right] \\ + \left[\dfrac{2}{\xi}\overset{=}{F}_{c}(s) \times \cos\left(\dfrac{s\pi h}{\xi}\right).e^{-(ka_{p}^{2})t}\right]\end{array}\right\} \tag{3.3.4}$$

where

$$\overset{=}{f}(m,n,t) = \int_{-a}^{a}\bar{f}(m,z,t)\,P_{m}(x)\,dx\,,$$

$$\overset{=}{g}(m,n,t) = \int_{-a}^{a}\overline{g}(m,z,t)\,P_{m}(x)\,dx\,,\quad \lambda_{m} = \int_{-a}^{a}P_{m}^{2}(x)\,dx$$

$$P_m(x) = Q_m \cos(a_m x) - W_m \sin(a_m x)$$
$$R_n(y) = Q_n \cos(a_n y) - W_n \sin(a_n y)$$
$$Q_m = a_m(\alpha_3 + \alpha_4)\cos(a_m a) + (\beta_3 - \beta_4)\sin(a_m a)$$
$$W_m = (\beta_3 + \beta_4)\cos(a_m a) + (\alpha_4 - \alpha_3)\, a_m \sin(a_m a)$$

Equation (3.3.3) is the desired solution of the given problem with $\beta_3 = \beta_4 = 1$, $\alpha_3 = k_3$, $\alpha_4 = k_4$.

3.4 DETERMINATION OF THERMALDEFLECTION

Substituting the value of temperature the $T(x, y, z, t)$ from equation (3.3.3) to the equation (3.2.2) we obtain the expression for distribution

$$M_T = K\alpha E \sum_{m,n,s=1}^{\infty} \left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{R_n(y)}{\mu_n}\right]\left\{\begin{array}{l}(-1)^s\left[\frac{2}{\xi}\overline{\overline{F}}_c(s)e^{-(ka_p^2)t}\left(\frac{s\pi}{\xi}\right)^2\right]+\overline{\overline{f}}(m,n,t)\Big\} \\ -\overline{\overline{g}}(m,n,t)-\frac{s^2\pi^2}{\xi^2}\overline{\overline{T}}_c(s)+\phi\end{array}\right\} \tag{3.4.1}$$

We assume the solution of equation (3.2.1) satisfying condition (2.3) as

$$\omega(x, y, z, t) = \sum_{m=1}^{\infty} c_m(t)xy\sin\left(\frac{m\pi z}{\xi}\right).[z(z-\xi)] \tag{3.4.2}$$

It can be easily seen that

$$\frac{\partial\omega}{\partial z} = 0 \text{ at } z = 0, \xi$$

Hence solution (3.4.2) satisfies the condition (3.2.3).

Now,

$$\nabla^4\omega(x,y,z,t)=8\sum_{m=1}^{\infty}c_m(t)\sin\left(\frac{m\pi z}{\xi}\right).[z(z-\xi)] \tag{3.4.3}$$

and

$$\nabla^2 M_T=\frac{K\alpha E}{D(1-\upsilon)}\sum_{m,n,s=1}^{\infty}\left[\frac{P_m^{11}(x)}{\lambda_m}\right]\left[\frac{R_n^{11}(y)}{\mu_n}\right]\times\left\{\begin{array}{l}\left[(-1)^s\overline{\overline{f}}-\overline{\overline{g}}-\dfrac{s^2\pi^2}{\xi^2}\overline{\overline{T}}_c(s)+\phi\right]\\ +\left[\dfrac{2}{\xi}\overline{\overline{F}}_c\, e^{-(ka_p^2).t}\left(\dfrac{s\pi}{\xi}\right)^2(-1)^s\right]\end{array}\right\} \tag{3.4.4}$$

Comparing equations (3.4.2) and (3.4.3), one obtains

$$C_m(t)=\frac{K\alpha E}{8}\sum_{m,n,s=1}^{\infty}\left[\frac{P_m^{11}(x)}{\lambda_m}\right]\left[\frac{R_n^{11}(y)}{\mu_n}\right]\frac{1}{\sin\left(\frac{m\pi}{n}\right).z[z(z-\xi)]}$$
$$\times\left[\frac{1}{D(1-\upsilon)}\right]\left[(-1)^s\overline{\overline{f}}-\overline{\overline{g}}-\frac{s^2\pi^2}{\xi^2}\overline{\overline{T}}_c(s)+\frac{2}{\xi}\overline{\overline{F}}_c\,e^{-(ka_p^2).t}\times\left(\frac{s\pi}{\xi}\right)^2(-1)^s\right] \tag{3.4.5}$$

Substituting equation (3.4.4) in equation (3.4.2), we get

$$\omega(x,y,t)=\frac{K\alpha E}{8D(1-\upsilon)}\sum_{m,n,s=1}^{\infty}\left[\frac{P_m^{11}(x)}{\lambda_m}\right]\left[\frac{R_n^{11}(y)}{\mu_n}\right]a^2b^2$$
$$\times\left[(-1)^s\overline{\overline{f}}+\frac{2}{\xi}\overline{\overline{F}}_c\,e^{-(ka_p^2).t}\left(\frac{s\pi}{\xi}\right)^2-\overline{\overline{g}}-\left(\frac{s\pi}{\xi}\right)^2Tc^{(s)}\right] \tag{3.4.6}$$

where

$P_m^{11}(x)=\nabla^2P_m(x)$, $R_n^{11}(y)=\nabla^2R_n(y)$.

3.5 SPECIAL CASE AND NUMERICAL RESULTS

Setting $f(x,y,t)=(1-\bar{e}^{t})(x+a)^2(x-a)^2(y-b)^2(y+b)^2 e^h(1+c)$

and $g(x,y,t)=(1-\bar{e}^{t})(x+a)^2(x-a)^2(y+b)^2(y-b)^2(1+c)$,

$\beta=28.132k(k_1+k_2)(k_2+k_3)(1+C).$,

$$\delta=\frac{28.132k(k_1+k_2)(k_2+k_3)\alpha E(1+C)}{8D(1-\vartheta)}$$

$$a=4, b=2, h=1, t=1sec, k=0.86$$

in equation (3.3.3) and (3.4.6), one obtain the expressions for temperature distribution and thermal deflection as

$$\frac{T(x,y,z,t)}{\beta}=\sum_{m,m,s=1}^{\infty}\left[\frac{P_m(x)}{\lambda_m}\right]\left[\frac{R_n(y)}{\mu_n}\right]\left[(-1)^s-1\right]$$
$$\times\left[\frac{4a_n\cos^2(4a_n)-\cos(4a_n)\sin(4a_n)}{a_n{}^2}\right]$$
$$\times\left[\frac{2bm\cos^2(2bm)-\cos(2bm).\sin(2bm)}{b_m{}^2}\right] \tag{3.5.1}$$
$$+2\frac{1}{1+(9.8596)s^2}\left[e^1(-1)^s-1\right].\cos(3.145).ze^{-k(a_n^2+b_m^2)}$$

$$\frac{\omega(x,y,t)}{\delta} = \sum_{m,n,s=1}^{\infty}\left[\frac{P_m^{11}(x)}{\lambda_m}\right]\left[\frac{R_n^{11}(x)}{\mu_n}\right](a^2.b^2) \times\left[\frac{4a_n\cos^2(4a_n)-\cos(4a_n)\sin(4a_n)}{a_n^{\ 2}}\right]$$

$$\times\left[\frac{2bm\cos^2(2bm)-\cos(2bm).\sin(2bm)}{b_m^{\ 2}}\right].\left[(-1)^s-1\right]$$

$$+19.7192\left[\frac{e^1(-1)^s-1}{1+(9.8596).s^2}\right]s^2.e^{-k(a_n^2+b_m^2)}. \tag{3.5.2}$$

3.6 CONCLUSION

- The temperature distribution, unknown temperature gradient and thermal deflection on upper plane surface of a thick rectangular plate have been derived when the boundary conditions are known, with the aid of Marchi- Fasulo transform and finite Fourier cosine transform techniques. The series solution is convergent.
- The expressions (3.5.1) and (3.5.2) are represented graphically. The temperature distribution and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications.

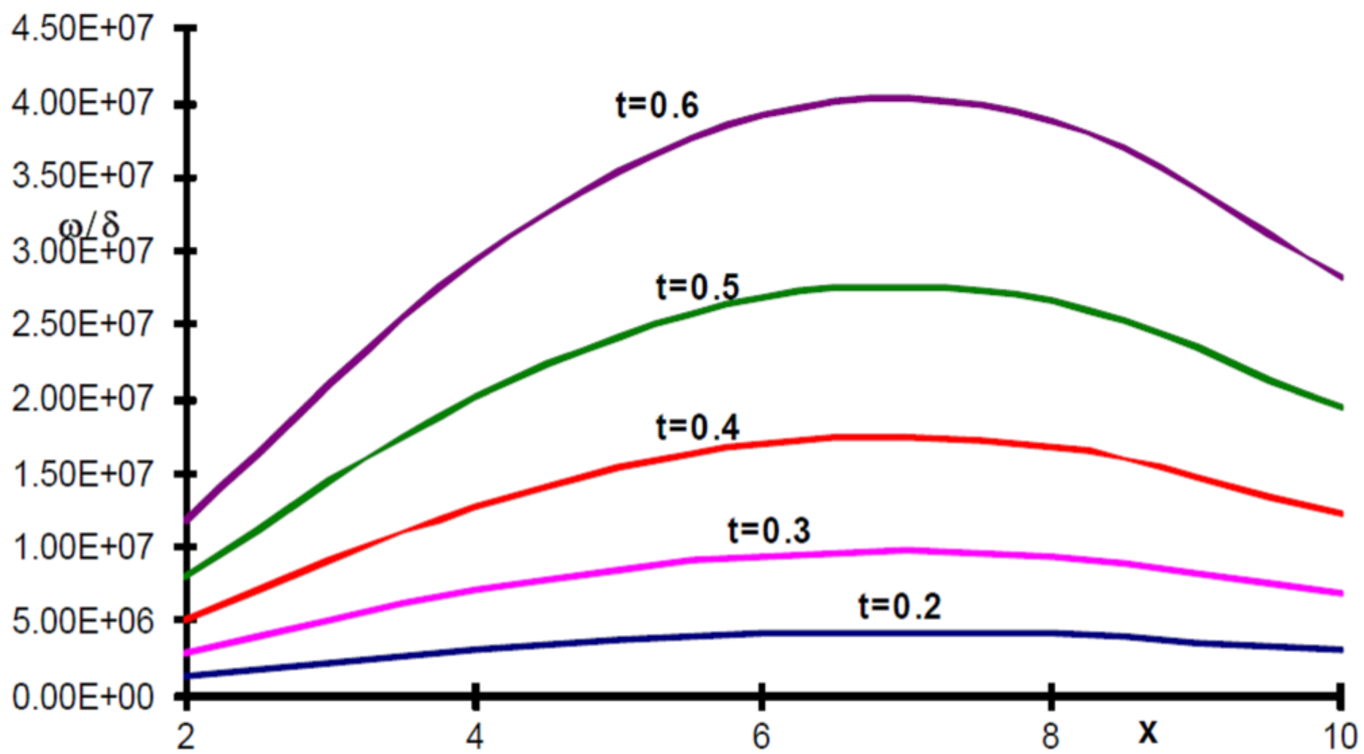


Fig (1). ω/δ versus x for different values of t

Fig (2). T/β versus y for different values of t

4. Inverse Thermoelastic Problem of Semi Infinite Rectangular Beam due to Heat Generation

The main results of this chapter have been published as detailed below:

1.Inverse thermoelastic problem of semi-infinite rectangular beam due to heat generation, Int. J. of Engineering and Innovative Technology (IJEIT) ,Volume 3 issue 4, pp. 429-434, Oct 2013, **USA.**

4.1 INTRODUCTION

This chapter is a generalization of chapter 2. In this chapter, an attempt has been made to determine the temperature distribution, unknown temperature gradient, displacement function and thermal stresses of a thin rectangular beam occupying the region D : $-a \le x \le a$; $0 \le y \le b$, $0 \le z \le \infty$ with known boundary conditions. Here Marchi- Fasulo transforms and Fourier cosine transform techniques have been used to find the solution.

4.2 STATEMENTOF THE PROBLEM

Consider a thin rectangular beam occupying the space D : $-a \le x \le a$; $0 \le y \le b$, $0 \le z \le \infty$. The displacement components u_x , u_y *and* u_z in the x and y and z directions respectively as **Durge et al**. **[2003]** are

$$u_x = \int_{-a}^{a} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - v \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \tag{4.2.1}$$

$$u_y = \int_{0}^{b} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2} - v \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \tag{4.2.2}$$

$$u_z = \int_{0}^{\infty} \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - v \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \tag{4.2.3}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)^2 U(x,y,z,t)=-\lambda E\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times T(x,y,z,t) \tag{4.2.4}$$

where T(x,y,z,t) denotes the temperature of a rectangular beam satisfy the following differential equation as

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{g(x,y,z,t)}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{4.2.5}$$

where k is the thermal conductivity and diffusivity of the material, subject to initial condition

$$T(x,y,z,0)=f(x,z,t) \tag{4.2.6}$$

The boundary conditions are

$$\left[T(x,y,z,t)+k_1\frac{\partial T(x,y,z,t)}{\partial x}\right]_{x=a}=f_1(y,z,t) \tag{4.2.7}$$

(4.2.8)

(4.2.9)

(4.2.10)

(4.2.11)

(4.2.12)

$$\left.\frac{\partial T(x,y,z,t)}{\partial z}\right|_{z=\infty} = h(x,y,t) \tag{4.2.13}$$

$$+\frac{\partial^2 U}{\partial z^2}\Big]$$

The stress components in terms of U(x, y, z, t) **Durge et al. [2003]** are given by

$$\sigma_{xx} = \left[\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right] \tag{4.2.14}$$

$$\sigma_{yy} = \left[\frac{\partial^2 U}{\partial z^2} + \frac{\partial^2 U}{\partial x^2}\right] \tag{4.2.15}$$

$$\sigma_{zz} = \left[\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right] \tag{4.2.16}$$

The equations (4.2.1) to (4.2.16) constitute the mathematical formulation of the problem under consideration.

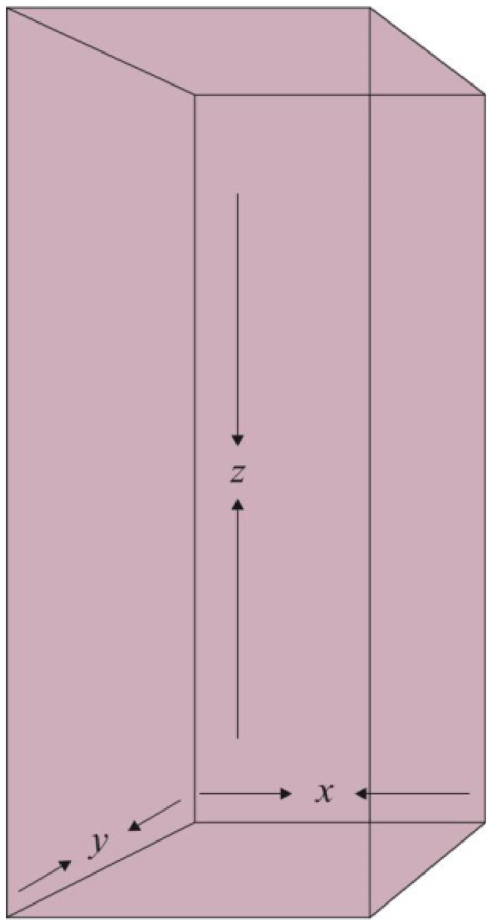


Figure 1: Geometry of the problem

4.3 SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo transform and finite Fourier sine and cosine transform to the equations, we get

$$\frac{d\overline{\overline{T}}^*}{dt} + \alpha q^2 \overline{\overline{T}}^* = \frac{\alpha \overline{\overline{g}}^*}{k} + \Psi$$

This is a linear equation whose solution is given by

$$\overline{\overline{T}}^{*}(m,n,\eta,t) = e^{-\alpha q^2 t}\left(\overline{\overline{f}}^{*} + \int_0^t \left[\frac{\alpha\,\overline{\overline{g}}^{*}}{k} + \Psi\right] e^{\alpha q^2 t'}\,dt'\right) \tag{4.3.1}$$

where,

$$\Psi = \frac{P_n(a)}{k_1} f_1 - \frac{P_n(-a)}{k_2} f_2 + \frac{m\pi}{b}\left[(-1)^{m+1} f_4 + f_3\right],$$

Now,applyinginversionofFourier Cosine transform,Fouriersinetransformandfinite Marchi-Fasulo transform to the equation (4.3.1), one obtains the expression for temperature distribution as

$$T(x,y,z,t) = \left(\frac{4\eta}{\xi\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\;\;\Lambda(z) \tag{4.3.2}$$

$$G(x,z,t) = \left(\frac{4\eta}{\xi\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin(pb)\;\;\Lambda(z) \tag{4.3.3}$$

where,

$$\Lambda(z) = \int_0^{\infty} B(t)\cos(\eta z)\,dz\;,\quad B(t) = e^{-\alpha q^2 t}\left(\overline{\overline{f}}^{*} + \int_0^t \left[\frac{\alpha\,\overline{\overline{g}}^{*}}{k} + \Psi\right] e^{\alpha q^2 t'}\,dt'\right)$$

Equations (4.3.2) and (4.3.3) are the required solutions.

4.4 AIRY'SSTRESS FUNCTIONS

Substituting value of temperature distribution T(x,y,z,t) from (4.3.2) in equation (4.2.4) equation one

$$U(x,y,z,t) = -\left(\frac{4\eta\pi E}{\xi\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \Lambda(z) \tag{4.4.1}$$

where $\Lambda(z) = \int_0^{\infty} B(t)\cos(\eta z)dz$

4.5 DISPLACEMENTCOMPONENTS

$$u_x = -\left(\frac{4\eta\lambda}{\xi\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(-p^2\sin py\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{array}\right\}dx \tag{4.5.1}$$

$$u_y = -\left(\frac{4\eta\lambda}{\xi\pi}\right)\Lambda(z)\int_0^b \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dy \tag{4.5.2}$$

$$u_z = -\left(\frac{4\eta\lambda}{\xi\pi}\right)\Lambda(z)\int_0^\infty \left\{ \begin{array}{l} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py) \\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py \end{array} \right\} dz \tag{4.5.3}$$

4.6 STRESS FUNCTION

$$\sigma_{xx} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}(-p^2\sin py) - \eta^2\sum_{m=1}^{\infty}\sin py\right] \tag{4.6.1}$$

$$\sigma_{yy} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\sin py\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right) - \eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right] \tag{4.6.2}$$

$$\sigma_{zz} = -\left(\frac{4\eta\lambda E}{\xi\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2 \sin py)\right] \tag{4.6.3}$$

4.7 SPECIALCASE AND NUMERICAL RESULT

Set $$f(x,y,z,t) = (x-a)^2(x+a)^2(z+e^{-z})(e^{y-t}) \tag{4.7.1}$$

$$g(x,y,t) = (x-a)^2(x+a)^2(e^{y-t})e^{-b}$$

$$\bar{f}(n,y,z,t) = (z+e^{-z})(e^{y-t}) \times \left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right] \tag{4.7.2}$$

$a = 2$, $k = 0.86$, , $b = 3$, $\xi = 2$, $t = 1$ sec in the above equations

$$T(x,y,z,t) = \left(\frac{4\eta}{\xi\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\ \ \Lambda(z) \times \left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t}) \tag{4.7.3}$$

$$G(x,z,t) = \left(\frac{4\eta}{\xi\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin(pb)\ \ \Lambda(z) \times \left[\frac{a_n\cos^2(a_n a) - \cos(a_n a)\sin(a_n a)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t}) \tag{4.7.4}$$

$$U(x,y,z,t) = -\left(\frac{2\eta\pi E}{\pi}\right)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\,\Lambda(z) \quad \times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right](z+e^{-z})(e^{y-t})$$

(4.7.5)

$$u_x = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{-a}^{a}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left(-p^2\sin py\right)-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}-\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\\ -\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py+\lambda\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{array}\right\}dx$$

(4.7.6)

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right](z+e^{-z})(e^{y-t})$$

$$u_y = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_{0}^{b}\left\{\begin{array}{l}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py-\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py\\ +\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{array}\right\}dy \qquad (4.7.7)$$

$$\times\left[\frac{a_n\cos^2(2a_n)-\cos(2a_n)\sin(2a_n)}{a_n{}^2}\right](z+e^{-z})(e^{y-t})$$

$$u_z = -\left(\frac{2\eta\lambda}{\pi}\right)\Lambda(z)\int_0^{\infty}\left\{\begin{aligned}&\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2\sin py)\\ &+\upsilon\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\eta^2\left(\frac{P_n(x)}{\lambda_n}\right)\sin py - \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\sin py\end{aligned}\right\}dz \tag{4.7.8}$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t})$$

$$\sigma_{xx} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\left[\sum_{m=1}^{\infty}(-p^2\sin py) - \eta^2\sum_{m=1}^{\infty}\sin py\right] \tag{4.7.9}$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t})$$

$$\sigma_{yy} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\sum_{m=1}^{\infty}\sin py\left[\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right) - \eta^2\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)\right] \tag{4.7.10}$$

$$\times\left[\frac{a_n\cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t})$$

$$\sigma_{zz} = -\left(\frac{2\eta\lambda E}{\pi}\right)\Lambda(z)\left[\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n''(x)}{\lambda_n^2}\right)\sin py + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{P_n(x)}{\lambda_n}\right)(-p^2 \sin py)\right] \tag{4.7.11}$$

$$\times\left[\frac{a_n \cos^2(2a_n) - \cos(2a_n)\sin(2a_n)}{a_n^{\ 2}}\right](z+e^{-z})(e^{y-t})$$

4.9 MATERIALPROPERTIES

The numerical calculations has been carried out for an Aluminum (pure) rectangular beam with the material properties as,

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 117 Btu/(hr. ftOF)

Thermal diffusivity α = 3.33 ft^2/hr. Poisson ratio ν = 0.35

Coefficient of linear thermal expansion α_t = 12.84 x 10^{-6}1/F

Lame constant μ = 26.67

Young's modulus of elasticity E = 70G Pa

4.10 DIMENSIONS

The constants associated with the numerical calculation are taken as

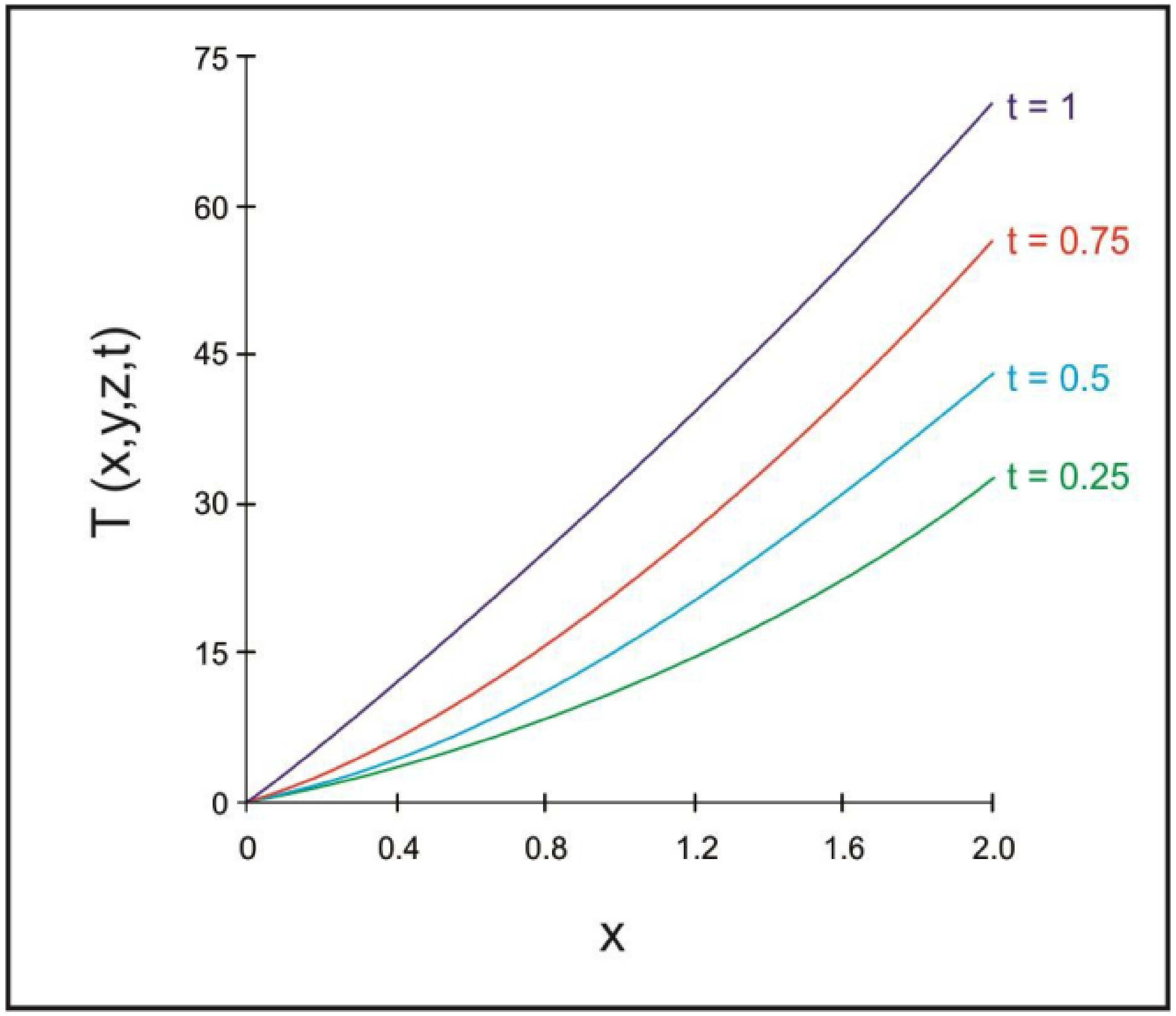
Length of rectangular beam x = 4ft

Breath of rectangular beam y = 3 ft

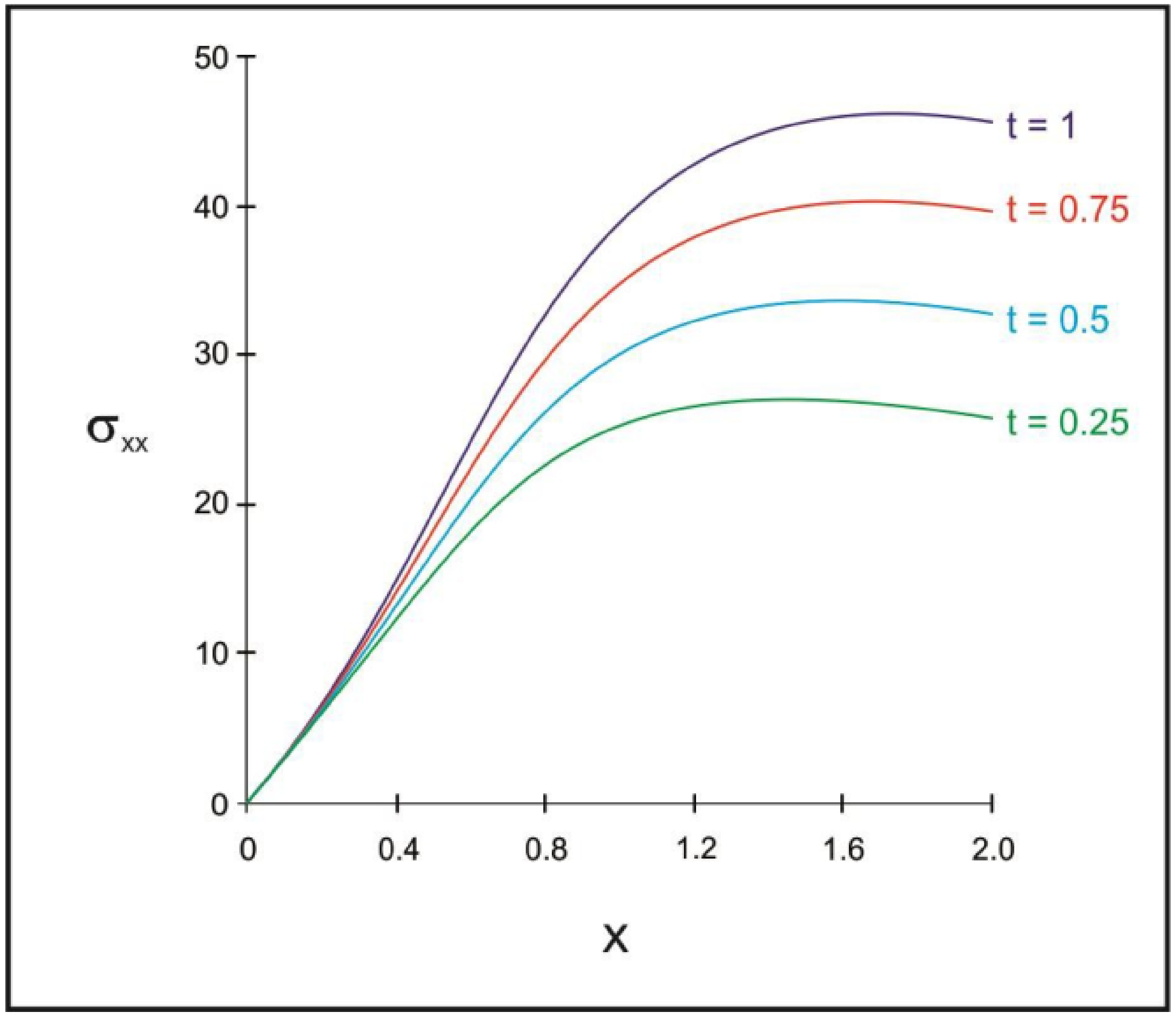
Height of rectangular beam z = 10^3ft

4.11 CONCLUSION

- The temperature distribution displacements and thermal stresses at any point of a thin rectangular object nave been obtained, when the boundary conditions are known
- The results are obtain in the form of infinite series.



Graph 1: Temperature distribution vs. x



Graph 2: Stress distribution vs. x

5.Thermoelastic Problem of a Thin Circular Plate due to Partially Distributed Heat Supply

The main results of this chapter have been published as detailed below:

1.Thermal stresses of a three dimensional thermoelastic problem of a thin rectangular plate, International Journal of Engineering and Innovative Technology , Volume 3 Issue 6, pp. 446-457, December 2013, **USA**

5.1 INTRODUCTION

The direct and inverse problems of thermo-elasticity of thick circular plate are considered by **Nowacki [1957].**The quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature has determined **Wankhede[1992]**.**Noda et al [2003]** has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and fixed circular edge thermally insulated .

In all aforementioned ,investigation they have not considered any thermoelastic problem with radiation type boundary conditions .This paper is concerned with transient thermoelastic problems of a thin circular plate occupying the space $D : 0 \le r \le a, -h \le z \le h$ due to heat generation with radiation type boundary conditions..

5.2 STATEMENTOF THE PROBLEM

Consider a thin circular plate of thickness $2h$ occupying the space $D : 0 \le r \le a, -h \le z \le h$. The differential equation governing the displacement function $U(r, z, t)$ as **Nowacki[1957]** is,

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+\nu)a_t T \tag{5.2.1}$$

$$\text{with } \frac{\partial U}{\partial r} = 0 \text{ at } r = a \tag{5.2.2}$$

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial Z^2}+\frac{\theta_0}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.2.3}$$

Subject to the initial conditions

$$T(r,z,0)=F(r,z) \tag{5.2.4}$$

and the boundary conditions

(5.2.5)

(5.2.6)

(5.2.7)

(5.2.8)

Where k_1 and k_2 are the radiation constants on the two plane surfaces. The stress functions σ_{rr} and $\sigma_{\theta\theta}$ as **Nowacki[1957]** are given by

$$\sigma_{rr}=-2\mu\frac{1}{r}\frac{\partial U}{\partial r} \tag{5.2.9}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 U}{\partial r^2} \tag{5.2.10}$$

where μ is the Lame's constant, while each of the stress functions σ_{rz}, σ_{zz}, $\sigma_{\theta z}$ are zero within the plane in the plane state of stress.

The equation (5.2.1) to (5.2.10) constitute the mathematical formulation of the problem under consideration.

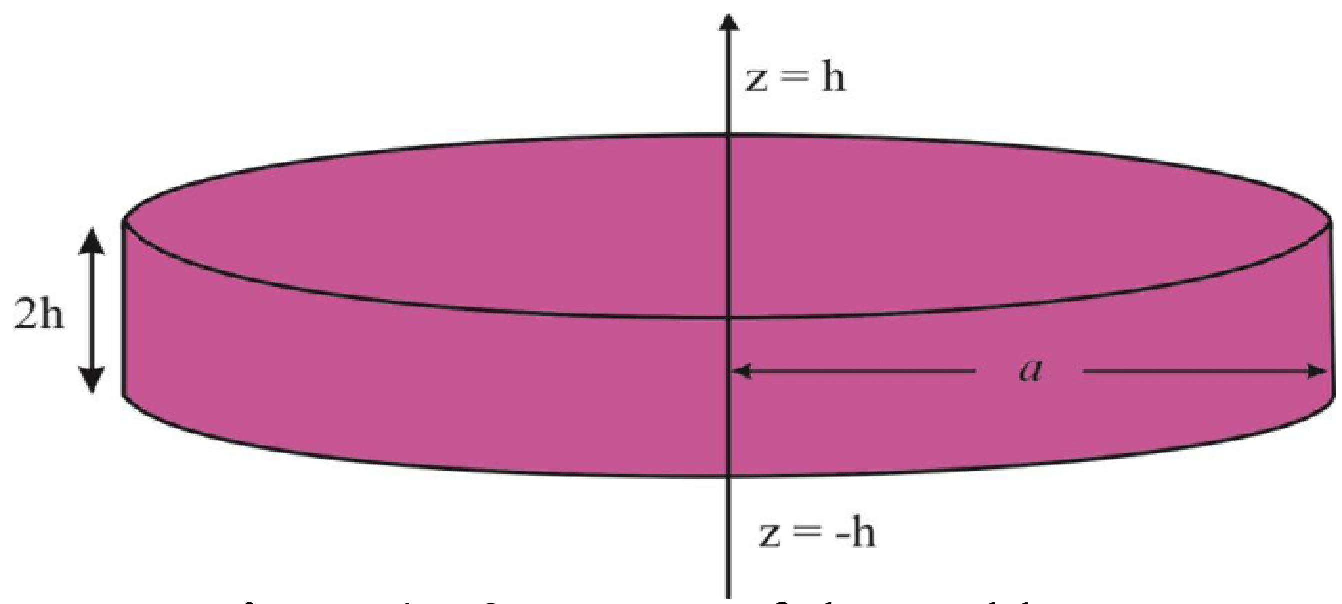


Figure 1 : Geometry of the problem

5.3 SOLUTION OF THE PROBLEM

Applying finite Marchi-Fasulo integral transform to the equation (5.2.3) to (5.2.5) and using (5.2.6), (5.2.7) one obtains

$$\frac{\partial^2 \overline{T}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{T}}{\partial r} - a_n^2 \overline{T} + \frac{\theta_0}{k} = \frac{1}{\alpha}\frac{\partial \overline{T}}{\partial t} - \left[\frac{P_n(h)}{k_1} f_1 + \frac{P_n(-h)}{k_2} f_2 (-\frac{Q_0}{k}) \right] \tag{5.3.1}$$

$$\frac{d^2\overline{T}^*}{dr^2}+\frac{1}{r}\frac{d\overline{T}^*}{dr}-q^2\overline{T}^* = X \tag{5.3.2}$$

Where $q^2 = a_n^2 - \frac{1}{k} + \frac{s}{\alpha}$, $\quad \overline{\theta}^*(r,z,t) = \overline{T}^*(r,z,t)$,

And $X = -\left[\overline{F}^*(r,z) + \frac{P_n(h)}{k_1} f_1 + \frac{P_n(-h)}{k_2} f_2(-\frac{Q_0}{k})\right]$

The equation (5.3.2) is a Bessel's equation whose solution is given by

$$\overline{T}^*(r,n,s) = AI_0(qr) + BK_0(qr) + P.I. \tag{5.3.3}$$

Where A, B are constants and are modified Bessel's function of first and second kind of order zero respectively. As $r \to 0$, $k_0 \to \infty$ but by physical consideration $T^*(r,n,s)$ remains finite. Therefore B must be zero. Using equation

$$\left[\overline{T}^*(r,n,s)\right]_{r=\xi} = \bar{f}^*(n,s) \tag{5.3.4}$$

And equation (5.3.3) one obtains the value of A and B.

Substituting the values of A and B in (5.3.3) and their inversion of Laplace transform and finite Marchi Fasulo integral transform leads to

$$T(r,z,t) = \frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\phi(r,z,t) \tag{5.3.5}$$

$$g(z,t) = \frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{z}\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m a)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\phi(r,z,t) \tag{5.3.6}$$

where λ_m are positive roots of transcendental equation

$J_0(\lambda_m a) = 0$ and

$\psi(a) = P.I$, $\phi(r,z,t) = L^{-1}(P.I)$

$P_n(z) = Q_n \cos(a_n z) - w_n \sin(a_n z)$

$$Q_n = a_n(\alpha_1+\alpha_2)\cos(a_n h) + (\beta_1-\beta_2)\sin(a_n h)$$

$$w_n = (\beta_1+\beta_2)\cos(a_n h) + (\alpha_2-\alpha_1)a_n \sin(a_n h)$$

Equation (5.3.5) is the desired solution of the given problem with $\beta_1 = \beta_2 = 1$ and $\alpha_1 = k_1$, $\alpha_2 = k_2$.

5.4 DISPLACEMENT FUNCTION

Substituting the value of $T(r, z, t)$ from (5.3.5) in (5.2.1) one obtains the thermoelastic displacement function $U(r, z, t)$ as

$$U(r,z,t) = -(1+\nu)a_t \frac{2k}{\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(\lambda_m r)}{\lambda_m J_1^{/}(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2+a_n^2)(t-t')} dt'$$

$$+\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\phi(r,z,t) \qquad (5.4.1)$$

5.5 STRESS FUNCTION

Using (5.4.1) in (5.2.9) and (5.2.10) the stress functions are obtained as

$$\sigma_{rr} = (1+\nu)a_t \frac{4\mu k}{r\xi}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0'(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2+a_n^2)(t-t')} dt' \qquad (5.5.1)$$

$$\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\phi^{/}(r,z,t)$$

$$\sigma_{\theta\theta} = (1+\nu)a_t \frac{4\mu k}{\xi} \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m J_0''(\lambda_m r)}{J_1(\lambda_m \xi)}$$

$$\times \int_0^t [\bar{f}(n,t') - \psi(a)]\, e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt' + \sum_{n=1}^{\infty} \frac{p_n(z)}{\lambda_n} \phi''(r,z,t) \tag{5.5.2}$$

5.6 SPECIALCASE AND NUMERICAL RESULT

Set $f(z,t)=e^{(t-\varepsilon)}z^2$ (5.6.1)

By applying Marchi-Fasulo transform we get

$$\bar{f}(z,t) = \frac{2e^{t+\xi}}{a_n^2}(a_n^2 z - 2z - 2)\sin(2a_n h) \tag{5.6.2}$$

Put $\frac{2k}{\xi} = \alpha, a = 2, h = 1, t = 1\sec$

$$\frac{T(r,z,t)}{\alpha} = \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi)} \times \int_0^t [\frac{2e^{t+\xi}}{a_n^2}(a_n^2 - 2z - 2)\sin(2a_n h) - \psi] e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt'$$

$$+ \frac{1}{\alpha} \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \phi(r,z,t) \tag{5.6.3}$$

$$\frac{g(z,t)}{\alpha} = \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m J_0(2\lambda_m)}{J_1(\lambda_m \xi)} \times \int_0^1 [\frac{2e^{1+\xi}}{a_n^2}(a_n^2 - 2z - 2)\sin(2a_n) - \psi] e^{-k(\lambda_m^2 + a_n^2)(t-t')} dt'$$

$$+\frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\,\phi(r,z,t) \tag{5.6.4}$$

$$\frac{U(r,z,t)}{\alpha}=-(1+\nu)a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(\lambda_m r)}{\lambda_m J_1^{/}(\lambda_m\xi)}\quad\times\int_0^t[\bar{f}(n,t')-\psi(a)]\,e^{-k(\lambda_m^2+a_n^2)(t-t')}dt'$$

$$+\frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\,\phi(r,z,t) \tag{5.4.1}$$

$$\frac{\sigma_{rr}}{\alpha}=(1+\nu)a_t\frac{2\mu}{r}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0'(\lambda_m r)}{J_1(\lambda_m\xi)}\times\int_0^t[\bar{f}(n,t')-\psi(a)]\,e^{-k(\lambda_m^2+a_n^2)(t-t')}dt' \tag{5.5.1}$$

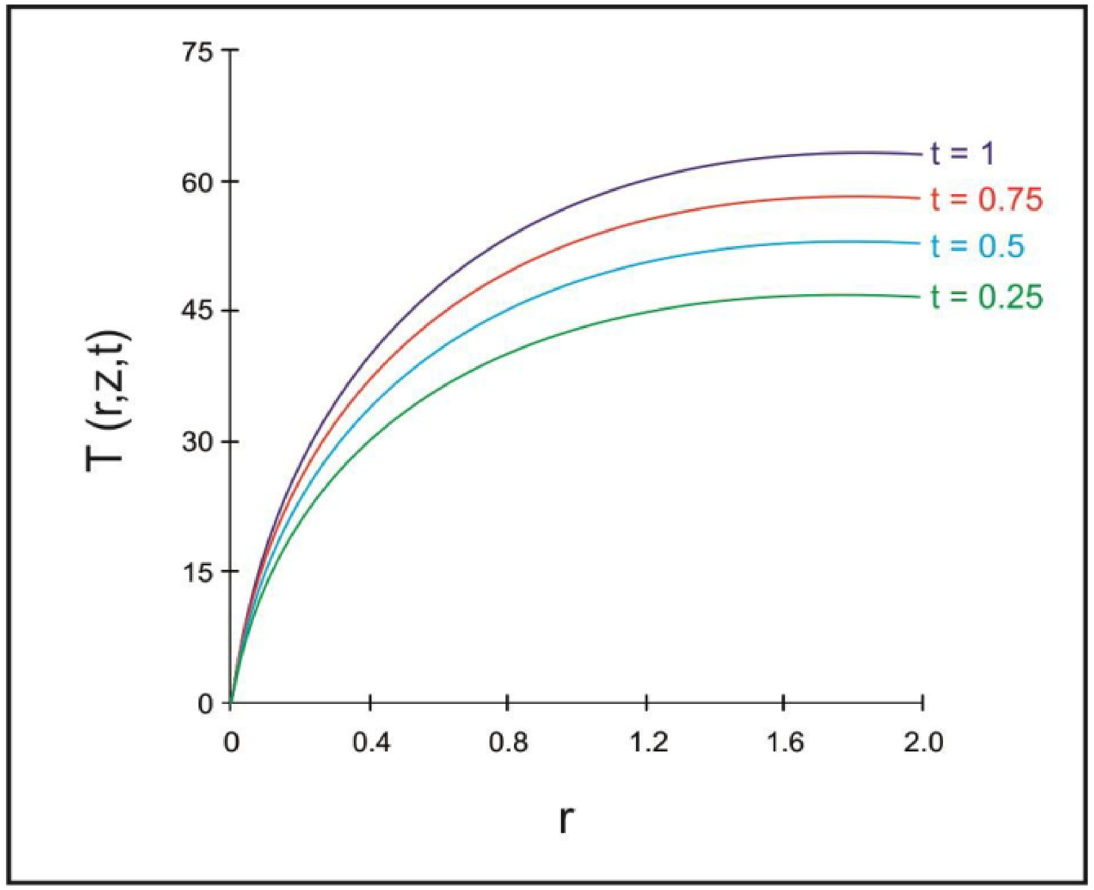
$$\frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\,\phi^{/}(r,z,t)$$

$$\frac{\sigma_{\theta\theta}}{\alpha}=(1+\nu)2\mu a_t\sum_{n=1}^{\infty}\frac{P_n(z)}{\lambda_n}\sum_{m=1}^{\infty}\frac{\lambda_m J_0''(\lambda_m r)}{J_1(\lambda_m\xi)} \tag{5.5.2}$$

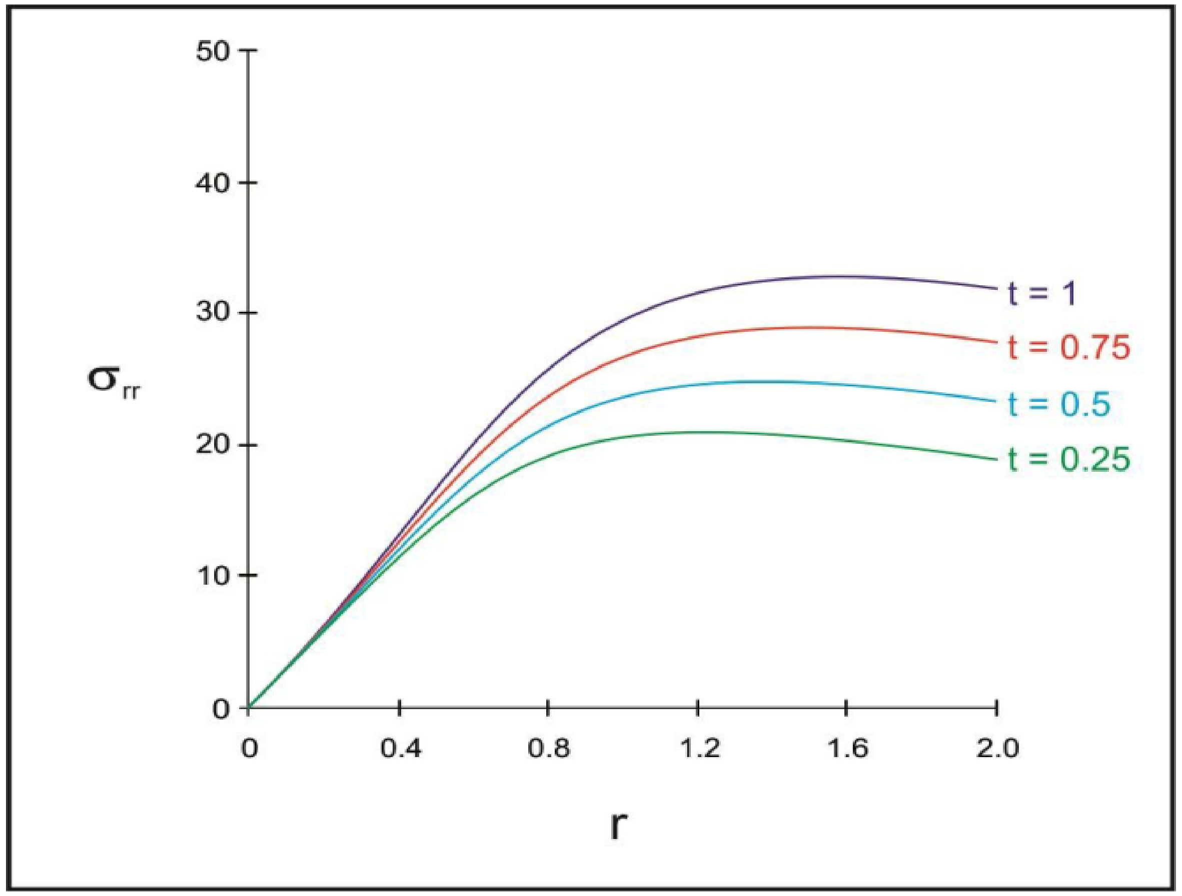
$$\times\int_0^t[\bar{f}(n,t')-\psi(a)]\,e^{-k(\lambda_m^2+a_n^2)(t-t')}dt'+\frac{1}{\alpha}\sum_{n=1}^{\infty}\frac{p_n(z)}{\lambda_n}\phi^{//}(r,z,t)$$

5.8 CONCLUSION

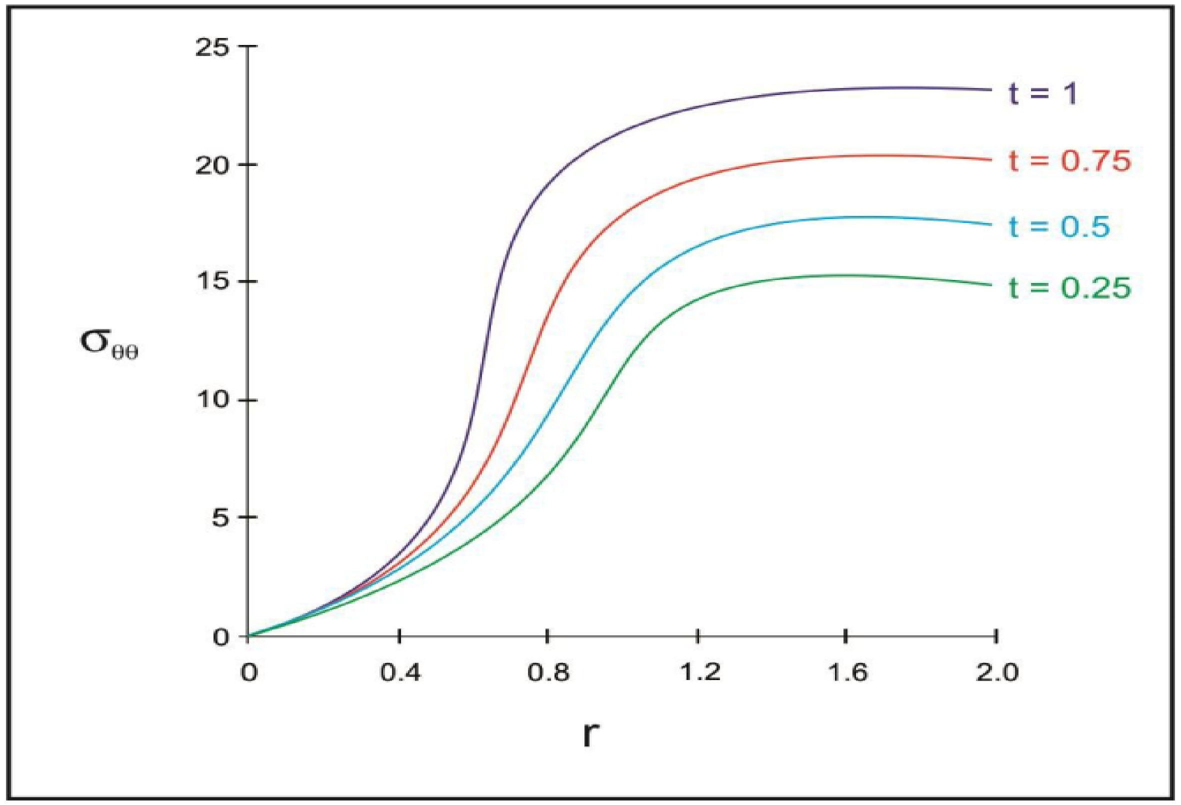
- The temperature distribution, displacement and thermal stresses of thick circular plate are investigated with known boundary conditions.
- Finite integral transform techniques are used to obtain numerical results. Any particular cases of special interest can be assigned to the parameters and functions in expressions.
- The temperature, displacement and thermal stresses that are obtained can be useful to the design of structure or machines in engineering applications.



Graph 1 : Temperature distribution vs. r



Graph 2 : Radial stresses vs. r



Graph 3 : Tangential stresses vs. r

6. Inverse Thermoelastic Problem of a Semi Infinite Circular Beam due to Heat Generation

The main results of this chapter have been published as detailed below:

1. Interior thermoelastic problem of a semi-infinite circular beam due to heat generation, Int. J. of Engineering and Innovative Technology (IJEIT) , Volume 3(4), pp. 433-436, October 2013, **USA.**

6.1 INTRODUCTION

Khobragade and **Roy H.S[2013]** have investigated transient thermoelastic problems of circular plate with heat generation. In this article we analyse inverse thermoelastic problems of temperature and thermal stresses of thick semi-infinite circular beam due to heat generation. The results presented here will be more useful in engineering applications.In this chapter, we analyzed inverse thermo elastic problem of temperature and thermal stresses of thick, semi-infinite circular beam due to heat generation. The governing heat conduction equation has been solved by using Marchi-Zgrablich and Fourier Cosine transform techniques. The result presented here will be more useful in engineering applications.

6.2 STATEMENTOF THE PROBLEM

Consider a thick circular beam occupying the space D : $a \le r \le b$, $0 \le z < \infty$. The material is homogeneous and isotropic. The differential equation governing the displacement potential function $\phi(r, z, t)$ as **Noda et al. [2003]** is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left[\frac{1+\upsilon}{1-\upsilon}\right]\alpha_t T \qquad (6.2.1)$$

where, ν and αt are the Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate and T is temperature of the plate satisfying the differential equation as **Noda et al. [2003]** is

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\chi(r,z,t)=\frac{1}{k}\frac{\partial T}{\partial t} \tag{6.2.2}$$

Subject to initial condition

$$T(r,z,0)=f(r,z) \tag{6.2.3}$$

and boundary conditions are

$$\left[T(r,z,t)+k_1\frac{\partial T(r,z,t)}{\partial r}\right]_{r=a}=g_1(z,t) \tag{6.2.4}$$

$$\left[T(r,z,t)+k_2\frac{\partial T(r,z,t)}{\partial r}\right]_{r=\xi}=g_2(z,t) \quad \text{(known)} \tag{6.2.5}$$

$$\left[T(r,z,t)\right]_{r=b}=G(z,t) \quad \text{(unknown)} \tag{6.2.6}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=0}=f_1(r,t) \tag{6.2.7}$$

$$\left[\frac{\partial T(r,z,t)}{\partial z}\right]_{z=\infty}=f_2(r,t) \quad ,0\le r\le a, \quad t>0 \tag{6.2.8}$$

where k is the thermal diffusivity of the material of the plate.

The displacement function in the cylindrical co-ordinate system are represented by the Goodier thermoelastic function ϕ and Love's function L as **Noda etal. [1957]** are

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 L}{\partial r \partial z} \tag{6.2.9}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2} \tag{6.2.10}$$

in which Goodier thermoelastic potential must satisfy the equation as **Noda et al.[2003]** is

$$\nabla^2 \phi = \left(\frac{1+\nu}{1-\nu}\right) a_t T \tag{6.2.11}$$

The Love's function must satisfy

$$\nabla^2 (\nabla^2 L) = 0 \tag{6.2.12}$$

where,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

The component of stresses are represented by the use of the potential ϕ and Love's function L as **Noda et al. [2003]** are

$$\sigma_{rr} = 2G\left\{\left[\frac{\partial^2 \phi}{\partial r^2} - \nabla^2 \phi\right] + \frac{\partial}{\partial z}\left[\upsilon \nabla^2 L - \frac{\partial^2 L}{\partial r^2}\right]\right\} \tag{6.2.13}$$

$$\sigma_{\theta\theta} = 2G\left\{\left[\frac{1}{r}\frac{\partial \phi}{\partial r} - \nabla^2 \phi\right] + \frac{\partial}{\partial z}\left[\upsilon \nabla^2 L - \frac{1}{r}\frac{\partial^2 L}{\partial r^2}\right]\right\} \tag{6.2.14}$$

$$\sigma_{zz} = 2G\left\{\left[\frac{\partial^2 \phi}{\partial z^2} - \nabla^2 \phi\right] + \frac{\partial}{\partial z}\left[(z-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right]\right\} \tag{6.2.15}$$

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \phi}{\partial r \partial z} + \frac{\partial}{\partial r}\left[(1-\upsilon)\nabla^2 L - \frac{\partial^2 L}{\partial z^2}\right]\right\} \tag{6.2.16}$$

The equations (6.2.1) to (6.2.16) constitute the mathematical formulation of the problem under consideration

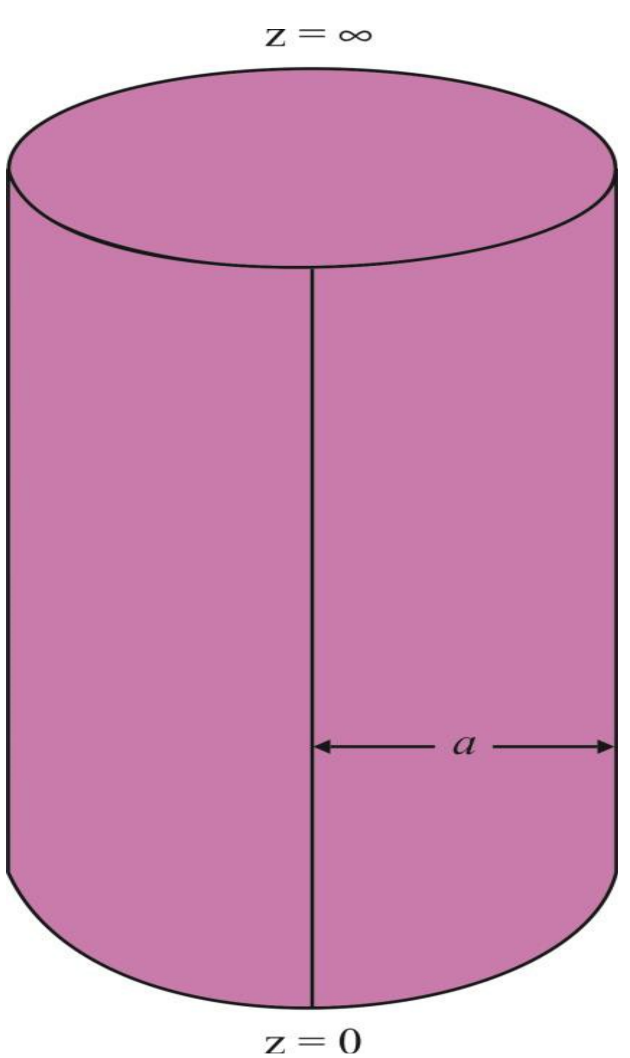


Figure 1: Geometry of the problem

6.3 SOLUTIONOF THE PROBLEM

Applying finite **Marchi-Zgrablichtransform**to the equations (6.2.2) and using equations (6.2.4), (6.2.5) one obtains

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\chi(r,z,t)=\frac{1}{k}\frac{\partial T}{\partial t} \tag{6.3.1}$$

By using the operational property of finite Marchi-Zgrablich transform, we get

$$\frac{\partial^2 \overline{T}}{\partial z^2}-\mu_n^2\overline{T}+\overline{\chi}=\frac{1}{k}\frac{\partial \overline{T}}{\partial t}+g(z,t) \tag{6.3.2}$$

Again, applying Fourier cosine transform to the equation (6.3.2), we get

$$\frac{d\overline{T}_c^{*}}{dt}+kp^2\overline{T}_c^{*}=\phi_1^{*}+\overline{\chi}_1^{*} \tag{6.3.3}$$

where

$$\overline{\chi}_1^{*}=k\overline{\chi}_c^{*} \quad and \ \phi_1^{*}=k\mu-k\mu_n^2\overline{T}_c^{*}-kg_c^{*}$$

Equation (6.3.3) is a linear equation whose solution is given by

$$\overline{T}^{*}(n,z,t)=e^{-kp^2t}\int_0^t\left(\phi_1^{*}+\overline{\chi}_1^{*}\right)e^{-kp^2t'}dt'+Ce^{-kp^2t}$$

Using (6.2.3) , we get

$C = F^{*}(m, n)$

Thus, we have,

$$\overline{T}^{*}(n,z,t) = e^{-kp^2 t}\left[\int_0^t \left(\phi_1^{*} + \overline{\chi}_1^{*}\right) e^{-kp^2 t'} dt' + \overline{F}^{*}(m,n)\right] \tag{6.3.4}$$

Applying inversion of Fourier cosine transform and Marchi-Zgrablich transform to the equation (6.3.4) one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n}\left\{e^{-kp^2 t}\left[\int_0^t \left(\phi_1^{*} + \overline{\chi}_1^{*}\right) e^{-kp^2 t'} dt' + \overline{F}^{*}(m,n)\right]\right\} \times S_0(k_1, k_2, \mu_n r) \tag{6.3.5}$$

$$G(z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n}\left\{e^{-kp^2 t}\left[\int_0^t \left(\phi_1^{*} + \overline{\chi}_1^{*}\right) e^{-kp^2 t'} dt' + \overline{F}^{*}(m,n)\right]\right\} \times S_0(k_1, k_2, \mu_n b) \tag{6.3.6}$$

These are the desired solutions of the given problem.

Let us assume Love's function L , which satisfy condition (6.2.11) as

$$L(r,z) = \sum_{n=1}^{\infty} \frac{1}{C_n} \psi S_0(k_1, k_2, \mu_n r) \tag{6.3.7}$$

where,

$$\psi = e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n)\right]$$

The displacement potential is given by

$$\phi = A\sum_{n=1}^{\infty} \frac{1}{C_n} \psi S_0(k_1, k_2, \mu_n r)\left[\psi + B(t)\right] \tag{6.3.8}$$

where, $A = \left(\dfrac{1+\upsilon}{1-\upsilon}\right)\alpha_t$

$$B(t) = e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n)\right] dt$$

6.4 DISPLACEMENTFUNCTION

Substituting the equations (6.3.7) and (6.3.8) in the equation (6.2.8) one obtains

$$u_r = A\sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0{}'(k_1, k_2, \mu_n r)\left[\psi + B(t)\right] - \sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi S_0{}'(k_1, k_2, \mu_n r) \tag{6.4.1}$$

$$u_z = 2(1-\upsilon)\left[\sum_{n=1}^{\infty} \frac{\mu_n{}^2}{C_n} \psi S_0{}'(k_1, k_2, \mu_n r) + \frac{1}{r}\sum_{n=1}^{\infty} \frac{\mu_n}{C_n} \psi S_0{}'(k_1, k_2, \mu_n r)\right] \tag{6.4.2}$$

6.5 STRESSFUNCTIONS

Substituting the values from the equation (6.3.7) and (6.3.8) in the equation (6.2.10) to (6.2.13) we get,

(6.5.1)

$$\sigma_{\theta\theta} = 2G\left\{\begin{array}{l} \dfrac{A}{r}\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] \\ \left[\begin{array}{l} A\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] + \\ \dfrac{A}{r}\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)\left[\psi + B(t)\right] \end{array}\right] + \dfrac{\partial}{\partial z}\left[\begin{array}{l} \upsilon\left[\begin{array}{l} A\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \\ +\dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) \end{array}\right] \\ -\dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty}\dfrac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \end{array}\right] \end{array}\right\} \tag{6.5.2}$$

(6.5.3)

$$\sigma_{rz} = 2G\left[(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n^{\,3}}{C_n}\psi S_0'''(k_1,k_2,\mu_n r) - \frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\ \sum_{n=1}^{\infty}\frac{\mu_n^{\,2}}{C_n}\psi S_0''(k_1,k_2,\mu_n r)\right] \tag{6.5.4}$$

where,

$$A = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \ and\ \psi$$

$$= e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right)e^{kp^2 t'}dt' + \overline{F}^*(m,n)\right]$$

$$B(t) = \int \psi dt$$

6.6 SPECIALCASE AND NUMERICAL RESULT

Set $F(r, z) = \delta(r - r_0)(z - e^{-z})$ (6.6.1)

Applying Marchi- Zgrablich to the equation (6.6.1) one obtains

$$\overline{F}(n, z) = r_0\,(z - e^{-z})S_0(k_1, k_2, \mu_n r_0) \tag{6.6.2}$$

Put $a = 2, \xi = 2.3, b = 2.5, t = 1\sec$ in equations

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n}\left\{ e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n r) \tag{6.6.3}$$

$$G(z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n}\left\{ e^{-kp^2 t}\left[\int_0^t \left(\phi_1^* + \overline{\chi}_1^*\right) e^{-kp^2 t'} dt' + \overline{F}^*(m,n)\right]\right\} \times S_0(k_1,k_2,\mu_n b) \tag{6.6.4}$$

$$\phi = A\sum_{n=1}^{\infty} \frac{1}{C_n}\psi S_0(k_1,k_2,\mu_n r)\left[\psi + B(1)\right] \tag{6.6.5}$$

$$u_r = A\sum_{n=1}^{\infty} \frac{\mu_n}{C_n} S_0{}'(k_1,k_2,\mu_n r)\left[\psi + B(1)\right] - \sum_{n=1}^{\infty} \frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) \tag{6.6.6}$$

$$u_z = 2(1-\upsilon)\left[\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) + \frac{1}{r}\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r)\right] \tag{6.6.7}$$

$$\tag{6.6.8}$$

$$\sigma_{\theta\theta} = 2G\left\{\begin{array}{l} \dfrac{A}{r}\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)\big[\psi + B(1)\big] \\ \\ \left[\begin{array}{l} A\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}S_0{}''(k_1,k_2,\mu_n r)\big[\psi + B(1)\big] + \\ \dfrac{A}{r}\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}S_0{}'(k_1,k_2,\mu_n r)\big[\psi + B(1)\big] \end{array}\right] + \dfrac{\partial}{\partial z}\left[\begin{array}{l} \upsilon\left[\begin{array}{l} A\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \\ +\dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n}{C_n}\psi S_0{}'(k_1,k_2,\mu_n r) \end{array}\right] \\ -\dfrac{1}{r}\displaystyle\sum_{n=1}^{\infty}\frac{\mu_n{}^2}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r) \end{array}\right] \\ \end{array}\right\} \tag{6.6.9}$$

(6.6.10)

$$\sigma_{rz} = 2G\left[(1-\upsilon)\sum_{n=1}^{\infty}\frac{\mu_n^{\;3}}{C_n}\psi S_0'''(k_1,k_2,\mu_n r) - \frac{(1-\upsilon)}{r^2}\sum_{n=1}^{\infty}\ \sum_{n=1}^{\infty}\frac{\mu_n^{\;2}}{C_n}\psi S_0{}''(k_1,k_2,\mu_n r)\right] \tag{6.6.11}$$

6.8 MATERIAL PROPERTIES

The numerical calculation has been carried out for an Aluminum (pure) circular plate with the material properties as

Density ρ =169 lb/ft ^{3}

Specific heat = 0.208 Btu/lbOF

Thermal conductivity K = 15.9 x 10^6Btu/(hr. ftOF) Thermal diffusivity α = 3.33 ft^2/hr.

Poisson ratio ν = 0.35

Coefficient of linear thermal expansion

$\alpha_t = 12.84 \times 10^{-6} 1/F$

Lame constant $\mu = 26.67$

Young's modulus of elasticity E = 70G Pa

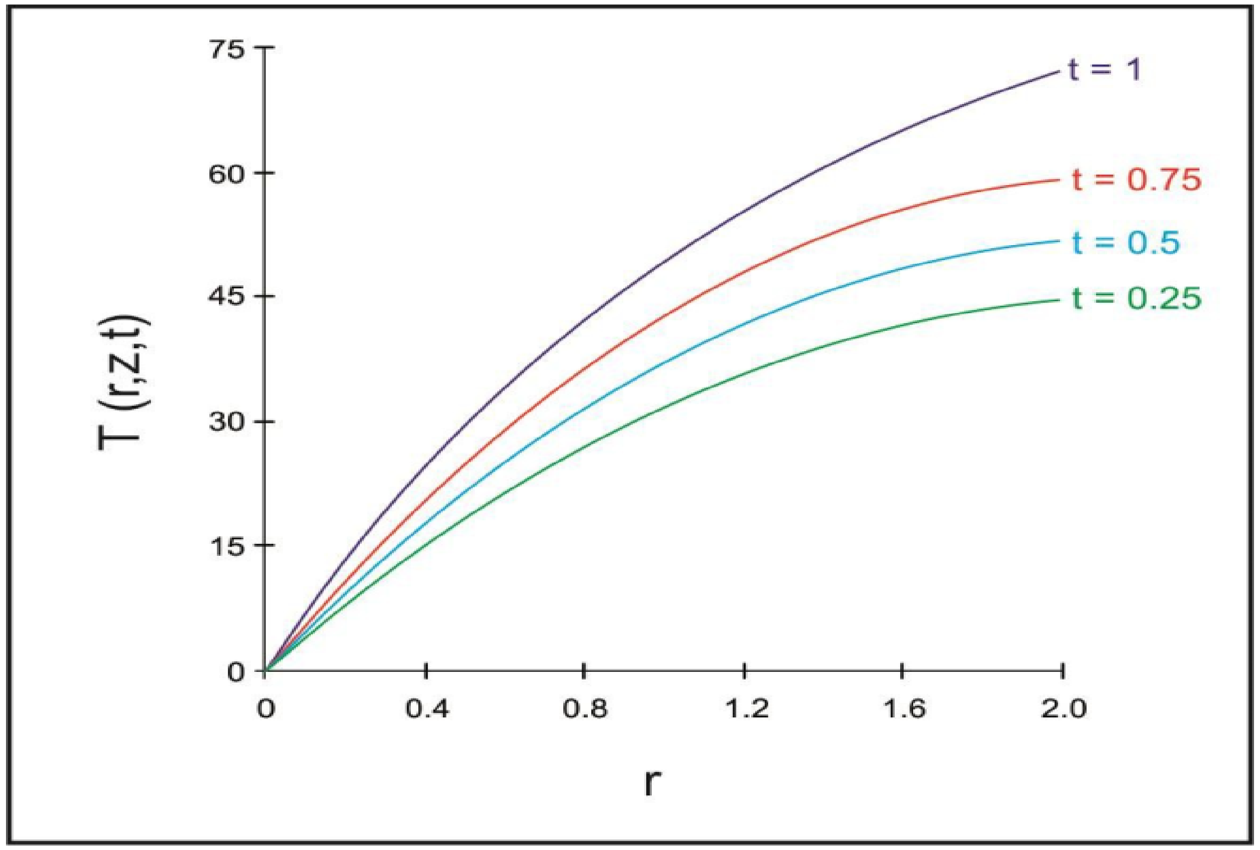
6.9 DIMENSIONS

The constants associated with the numerical calculation are taken as

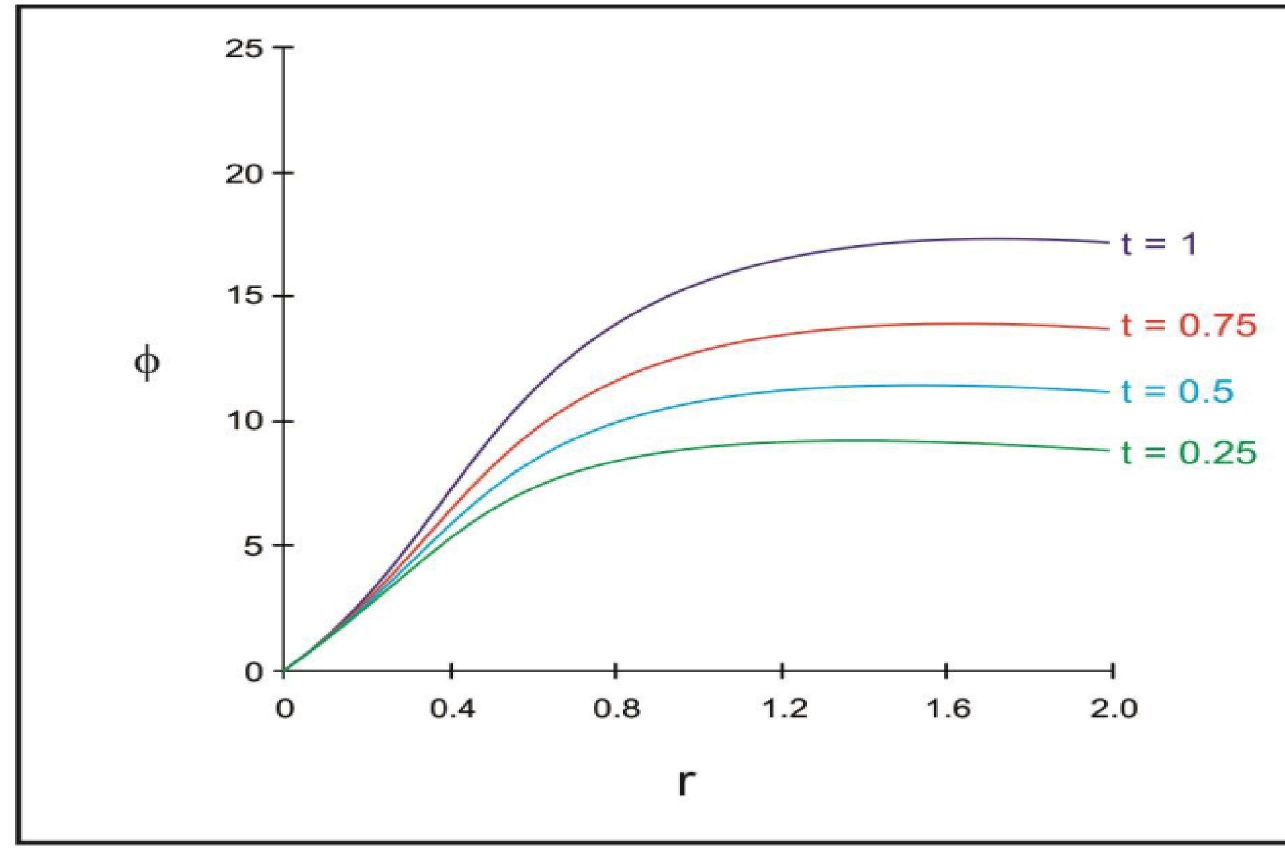
Radius of the disk a = 2ft Radius of the disk b = 2.5 ft

6.10 CONCLUSION

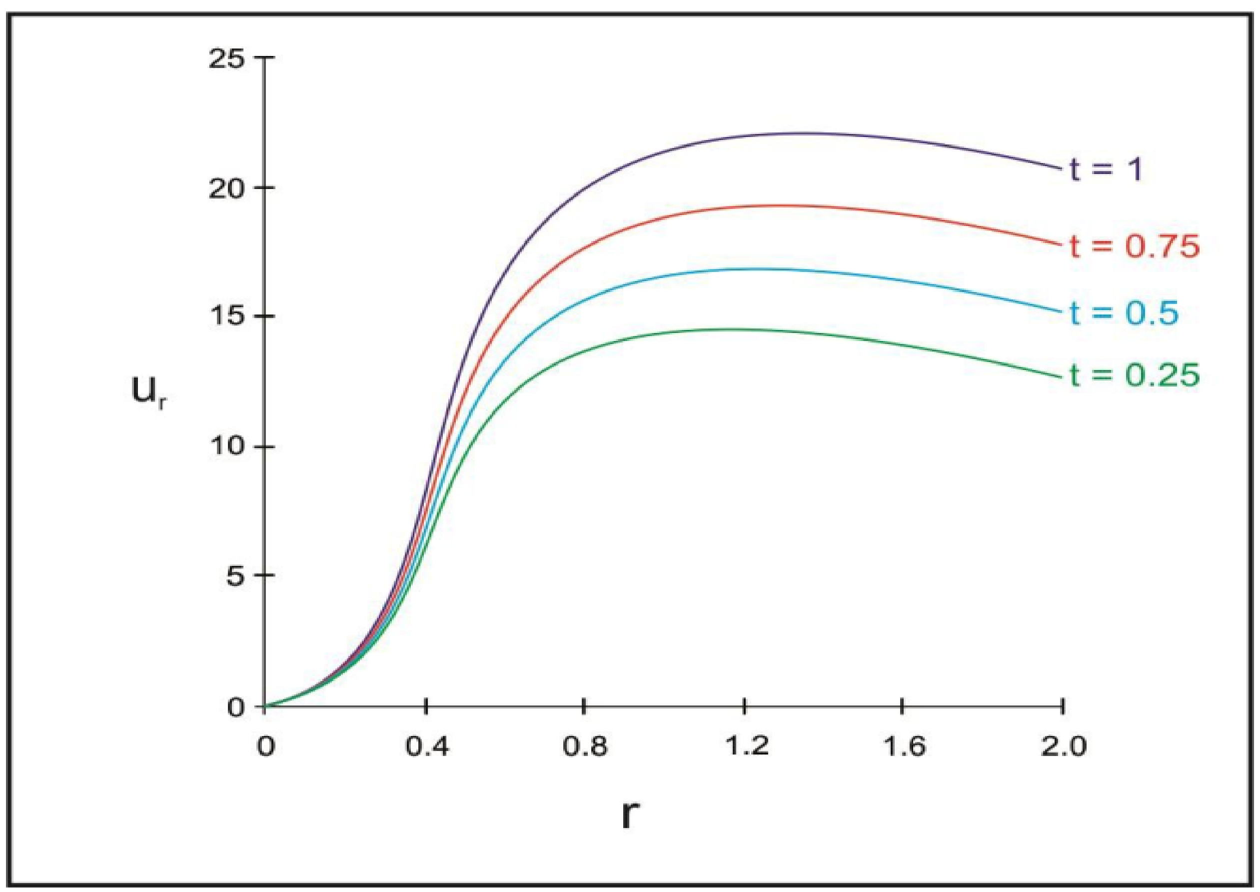
- In this study, we develop the analysis for the temperature field by introducing the methods of the Marchi- Zgrablich and Fourier cosine transform techniques.
- Determined the expression for temperature, displacement and thermal stresses of a semi-infinite, thick circular beam with known boundary conditions which is useful to design of structure or machines in engineering applications.



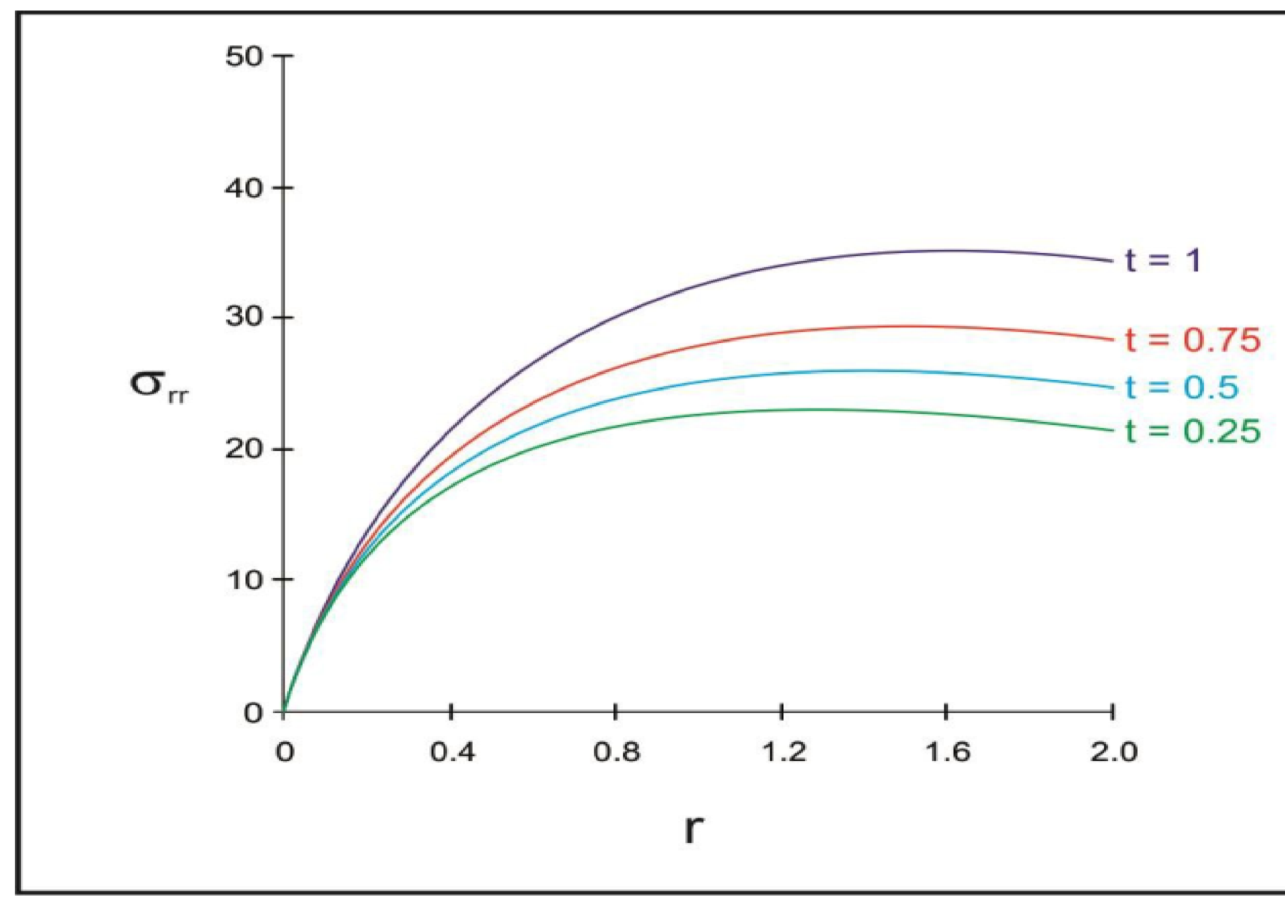
Graph 1: Temperature distribution vs. r



Graph 2: Displacement function vs. r



Graph 3: Stress Component vs. r



Graph 4: Radial stresses vs. r

7. Interior Thermoelastic Solution of a Hollow Cylinder

The main results of this chapter have been communicated for publication as detailed below:

1.Interior thermoelastic solution of a hollow cylinder, Int. J. of Engineering and Innovative Technology (IJEIT), Volume 4 Issue 1,July 2014, **USA.**

7.1 INTRODUCTION

In this chapter we modify the conceptual idea proposed by **Khobragadeet al [2002]** for circular plate and determine the temperature distributions, displacement and stress functions of a hollow cylinder with boundary conditions occupying the space $D = \{(x, y, z) \in R^3 : a \le (x^2 + y^2)^{1/2} \le b, 0 \le z \le h\}$, where $r = (x^2 + y^2)^{1\,2}$.

7.2 FORMULATIONOF THE PROBLEM

Consider a hollow cylinder as shown in the figure 1. The material of the cylinder is isotropic, homogenous and all properties are assumed to be constant. We assume that the cylinder is of a small thickness and its boundary surfaces remain traction free. The initial temperature of the cylinder is the same as the temperature of the surrounding medium, which is kept constant.

The displacement function $\phi(r, z, t)$ satisfying the differential equation as **Nowacki**[**1957**] is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{1+\nu}{1-\nu}\right) a_t T \tag{7.2.1}$$

with $\phi = 0$ at $r = a$ and $r = b$ (7.2.2)

Sierakowski and Sun [1966] is

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]=\frac{\partial T}{\partial t} \qquad (7.2.3)$$

where $\kappa = K/\rho c$ is the thermal diffusivity of the material of the cylinder, K is the conductivity of the medium, c is its specific heat and ρ is its calorific capacity (which is assumed to be constant respectively, subject to the initial and boundary condition

$$M_t(T,1,0,0)=0 \quad \text{for all} \quad a \le r \le b,\ 0 \le z \le h \qquad (7.2.4)$$

$$M_r(T,1,k_1,a)=F_1(z,t), \quad \text{for all} \quad 0 \le z \le h,\ t>0 \qquad (7.2.5)$$

$$M_r(T,1,k_2,b)=F_2(z,t) \quad \text{for all} \quad 0 \le z \le h,\ t>0 \qquad (7.2.6)$$

$$M_z(T,1,0,0)=F_3(r,t) \quad \text{for all} \quad a \le r \le b,\ t>0 \qquad (7.2.7)$$

$$M_z(T,1,0,\xi)=f(r,t) \quad \text{for all} \quad a \le r \le b,\ t>0 \text{ (known)} \qquad (7.2.8)$$

$$M_z(T,1,0,h)=G(r,t) \quad \text{for all} \quad a \le r \le b,\ t>0 \text{ (unknown)} \qquad (7.2.9)$$

being:

$$M_{\vartheta}(f,\bar{k},\bar{\bar{k}},\$) = (\bar{k}\, f + \bar{\bar{k}}\, \hat{f})_{\vartheta=\$}$$

where the prime (^) denotes differentiation with respect to ϑ, radiation constants are k and k on the curved surfaces of the plate respectively.

$$\nabla^2 U - \frac{U}{r^2} + (1-2\nu)^{-1} \frac{\partial e}{\partial r} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial r} \tag{7.2.10}$$

$$\nabla^2 W + (1-2\nu)^{-1} \frac{\partial e}{\partial z} = 2\left(\frac{1+\nu}{1-2\nu}\right) a_t \frac{\partial T}{\partial z} \tag{7.2.11}$$

The radial and axial displacement U and W satisfy the uncoupled thermoelastic equation as

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial r} \quad , \tag{7.2.12}$$

$$U = \frac{\partial \phi}{\partial r} \, , \tag{7.2.13}$$

$$W = \frac{\partial \phi}{\partial z} \tag{7.2.14}$$

where

The stress functions are given by

$$\tau_{rz}(a,z,t)=0\ ,\ \tau_{rz}(b,z,t)=0\ ,\ \tau_{rz}(r,0,t)=0 \tag{7.2.15}$$

$$\sigma_r(a,z,t)=p_i\ ,\ \sigma_r(b,z,t)=-p_o\ ,\ \sigma_z(r,0,t)=0 \tag{7.2.16}$$

where p_i and p_o are the surface pressure assumed to be uniform over the boundaries of the cylinder. The stress functions are expressed in terms of the displacement components by the following relations as **Sierakowski and Sun[1996]** are

$$\sigma_r=(\lambda+2G)\frac{\partial U}{\partial r}+\lambda\left(\frac{U}{r}+\frac{\partial W}{\partial z}\right) \tag{7.2.17}$$

$$\sigma_z=(\lambda+2G)\frac{\partial W}{\partial z}+\lambda\left(\frac{\partial U}{\partial r}+\frac{U}{r}\right) \tag{7.2.18}$$

$$\sigma_\theta=(\lambda+2G)\frac{U}{r}+\lambda\left(\frac{\partial U}{\partial r}+\frac{\partial W}{\partial z}\right) \tag{7.2.20}$$

$$\tau_{rz}=G\left(\frac{\partial W}{\partial r}+\frac{\partial U}{\partial z}\right) \tag{7.2.19}$$

where $\lambda = 2G\nu/(1-2\nu)$ is the Lame's constant, G is the shear modulus and U , W are the displacement

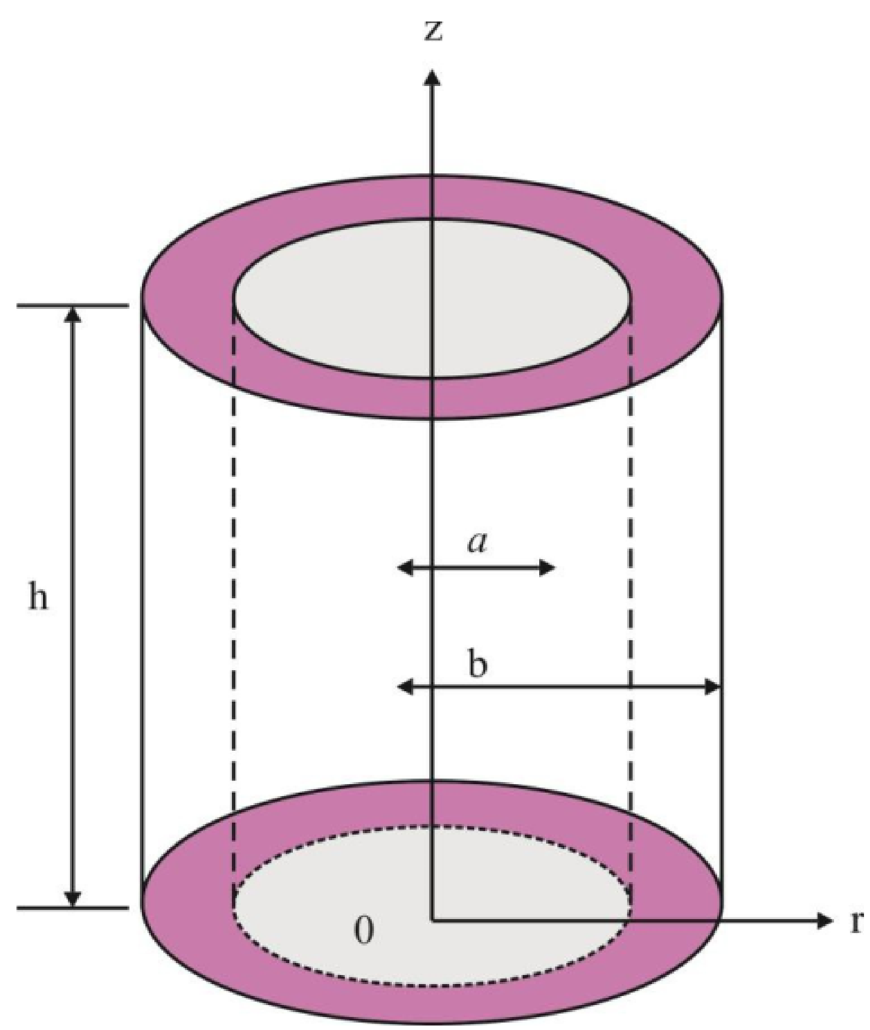


Figure 1: Geometry of the problem

7.3 SOLUTION OF THE OF THE PROBLEM

Applying transform technique and its inversion one obtains

$$\bar{T}^*(n,z,s) = \bar{f}(n,s)\cosh\{[(\mu_n^2 + (s/\kappa))^{1/2} z]/\sinh[(\mu_n^2 + (s/\kappa))^{1/2}\xi]\} \tag{7.3.1}$$

$$T(r,z,t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \tag{7.3.2}$$

$$G(r,t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm}{}' S_0(\bar{k}_1, \bar{k}_2, \mu_n r) \tag{7.3.3}$$

where

$$\varphi_{nm} = \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \int_0^t \bar{f}(n,t') e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt' ,$$

$$\varphi_{nm}{}' = \frac{\lambda_m \sin(\lambda_m h)}{\cos(\lambda_m \xi)} \int_0^t \bar{f}(n,t') e^{-\kappa(\mu_n^2 + \lambda_m^2)(t-t')} dt'$$

where n is the transformation parameter as defined in appendix, m is the Fourier sine transform parameter.

7.4 DISPLACEMENT AND STRESS FUNCTION

Substituting the value of (7.3.2) in (7.2.1) one obtains the thermoelastic displacement fı

$$\phi(r,z,t) = \frac{r^2 k\, a_t (1+\nu)}{2\xi(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} S_0(k_1, k_2, \mu_n r) \tag{7.4.1}$$

Using (7.4.1) in the equations (7.2.11) and (7.2.12) one obtains

$$U = \frac{k\, a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \varphi_{nm} \left[2S_0\left(k_1, k_2, \mu_n r\right) + rS'_0\left(k_1, k_2, \mu_n r\right)\right] \tag{7.4.2}$$

$$W = \frac{r^2\, k\, a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \lambda_m\, \varphi_{nm} \cot(\lambda_m z)\, S_0(k_1, k_2, \mu_n r) \tag{7.4.3}$$

$$\sigma_r = \frac{k\, a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\varphi_{nm}}{C_n} \left[(\lambda + 2G)\left(r^2 S''_0(k_1, k_2, \mu_n r)\right) + 4rS'_0(k_1, k_2, \mu_n r) + 2S_0(k_1, k_2, \mu_n r)\right]$$

$$\times \lambda\left[2\, S_0(k_1, k_2, \mu_n r) + r\, S'_0(k_1, k_2, \mu_n r)\right] - r^2 \lambda_m\, S_0(k_1, k_2, \mu_n r) \tag{7.4.4}$$

$$\sigma_z = -\frac{k\, a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n}\right) \left[(\lambda + 2G) r^2 \lambda_m^{\ 2} S_0(k_1, k_2, \mu_n r)\right. \tag{7.4.5}$$

$$\left. - \lambda\left(r^2 S''_0(k_1, k_2, \mu_n r) + 5rS'_0(k_1, k_2, \mu_n r) + 4S_0(k_1, k_2, \mu_n r)\right)\right]$$

$$\sigma_{\theta} = \frac{k\, a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\varphi_{nm}}{C_n} \right) (\lambda + 2G)\left[r S_0'(k_1, k_2, \mu_n r) + 2 S_0(k_1, k_2, \mu_n r) \right] \tag{7.4.6}$$

$$+\lambda\left[r^2 S_0''(k_1, k_2, \mu_n r) + 4 r S_0'(k_1, k_2, \mu_n r) + \left(2 - r^2 \lambda_m^2\right) S_0(k_1, k_2, \mu_n r) \right]$$

$$\tau_{rz} = \frac{k\, a_t\, G\,(1+\nu)}{\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m\, \varphi_{nm} \cot(\lambda_m z)}{C_n} \right) \left[r^2 S_0'(k_1, k_2, \mu_n r) + 2 r S_0(k_1, k_2, \mu_n r) \right] \tag{7.4.7}$$

7.5 SPECIALCASE

Set $f(r,t) = (1 - e^{-t})\, \delta(r - r_0)$ (7.5.1)

Applying finite transform defined in Marchi Zgrablich [35] to the equation (5.1) one obtains

$$\bar{f}(n,t) = (1 - e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0) \tag{7.5.2}$$

Substituting the value of (7.5.2) in the equations (7.3.2) to (7.4.7) one obtains

$$T(r,z,t) = \frac{2\kappa}{\xi} \sum_{n=1}^{\infty} \frac{1}{C_n} \sum_{m=0}^{\infty} \frac{\lambda_m \sin(\lambda_m z)}{\cos(\lambda_m \xi)} \tag{7.5.3}$$

$$\times \int_0^t (1 - e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0)\, e^{-\kappa(\mu_n^2 + \lambda_m^2)(t - t')} S_0(k_1, k_2, \mu_n r)\, dt'$$

$$G(r,t)=\frac{2\kappa}{\xi}\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\lambda_m\sin(\lambda_m h)}{\cos(\lambda_m \xi)}$$

$$\times\int_0^t(1-e^{-t})\,r_0\,S_0(k_1,k_2,\mu_n r_0)\,e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}S_0(\bar{k}_1,\bar{k}_2,\mu_n r)dt' \tag{7.5.4}$$

$$\phi(r,z,t)=\frac{r^2 k\,a_t\,(1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\lambda_m\sin(\lambda_m z)}{\cos(\lambda_m \xi)}$$

$$\times\int_0^t(1-e^{-t})\,r_0\,S_0(k_1,k_2,\mu_n r_0)\,e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}\,S_0(k_1,k_2,\mu_n r)dt' \tag{7.5.5}$$

$$U=\frac{k\,a_t\,(1+\nu)}{2\xi(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{C_n}\sum_{m=0}^{\infty}\frac{\lambda_m\sin(\lambda_m z)}{\cos(\lambda_m \xi)}$$

$$\times\int_0^t(1-e^{-t})\,r_0\,S_0(k_1,k_2,\mu_n r_0)\,e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}dt'\left[2S_0\left(k_1,k_2,\mu_n r\right)+rS'_0\left(k_1,k_2,\mu_n r\right)\right] \tag{7.5.6}$$

$$\sigma_r = \frac{k\,a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \frac{\lambda_m \sin(\lambda_m\, z)}{C_n \cos(\lambda_m\, \xi)} \int_0^t (1-e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0\, e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}\, dt' \tag{7.5.7}$$

$$\times\left[(\lambda+2G)\left(r^2 S''_0(k_1,k_2,\mu_n r)\right)+4rS'_0(k_1,k_2,\mu_n r)+2S_0(k_1,k_2,\mu_n r)\right]$$

$$\times\lambda\left[2\,S_0(k_1,k_2,\mu_n r)+r\,S'_0(k_1,k_2,\mu_n r)\right]-r^2\lambda_m\;S_0(k_1,k_2,\mu_n r)$$

$$\sigma_z = -\frac{k\,a_t\,(1+\nu)}{2\xi\,(1-\nu)} \sum_{m,n=1}^{\infty} \left(\frac{\lambda_m \sin(\lambda_m\, z)}{C_n \cos(\lambda_m\, \xi)} \int_0^t (1-e^{-t})\, r_0\, S_0(k_1, k_2, \mu_n r_0)\, e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}\, dt' \right) \tag{7.5.8}$$

$$\times[(\lambda+2G)r^2\lambda_m{}^2 S_0(k_1,k_2,\mu_n r)\;-\lambda\left(r^2 S''_0(k_1,k_2,\mu_n r)+5rS'_0(k_1,k_2,\mu_n r)+4S_0(k_1,k_2,\mu_n r)\right)]$$

7.6 CONVERGENCE OF THE SERIES SOLUTION

In order for the solution to be meaningful the series expressed in equations (7.4.1) and (7.4.2) should converge for all $a \le r \le b$ and $0 \le z \le h$, and we should further investigate the conditions which has to be imposed on the functions f (r, t) so that the convergence of the series expansion for T(r, z, t) is valid. The temperature equations (7.4.1) and (7.4.2) can be expressed as

$$T(r,z,t)=\frac{2\kappa}{\xi}\sum_{n=1}^{M'}\frac{1}{C_n}\sum_{m=0}^{M'}\varphi_{nm}\,S_0(\bar{k}_1,\bar{k}_2,\mu_n r) \tag{7.6.1}$$

We impose conditions so that *T(z,r,t)* converge in some generalized *g(r,s)* sense to as $t\to 0$ in the transform domain. Taking into account of the asymptotic behaviors of $\mu_n, s_0(k_1,k_2,\mu_n r), c_n$ and as given in Marchi-Zgrablich, it is observed that the series expansion for *T(z,r,t)* will be convergent by one term approximation as)

$$\{\varphi'_{nm}\}=\int_0^t \bar{f}(n,t')\left\{e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}\right\}dt'=O\left\{1/\left(\mu_n^2+\lambda_m^2\right)^{\kappa}\right\},\kappa>0 \tag{7.6.2}$$

$$\{\varphi'_{nm}\}=\int_0^t \bar{f}(n,t')\left\{e^{-\kappa(\mu_n^2+\lambda_m^2)(t-t')}\right\}dt'=O\left\{1/\left(\mu_n^2+\lambda_m^2\right)^{\kappa}\right\},\kappa>0 \tag{7.6.3}$$

$$J_p(\alpha,\mu a)\,Y_p(\beta,\mu b)-J_p(\alpha,\mu a)\,Y_p(\beta,\mu b)=0 \tag{7.6.4}$$

and for other infinite series are used. The effects of truncating of numbers are brought out by the comparison table for solutions of different functions for 2 and 5 terms. It is evident from the table that the convergence is rapid for the temperature distribution during cooling process, radial and axial stress and somewhat slower in the case of other stresses as well as temperature distribution during heating process, while it is estimated from table that the possible error from the table is less that 2 percent.

Functions	**2 terms**	**5 terms**
$T(r, z, t)$	-4.35298	-6.7712
σ_r	-285.18	-180.967
σ_z	285.66	170.496
σ_θ	69.3182	60.9464
τ_{rz}	-5.90491	-6.35717

Table 1. Convergence of solution as the number of terms used in the equation are increased from 2 to 5.

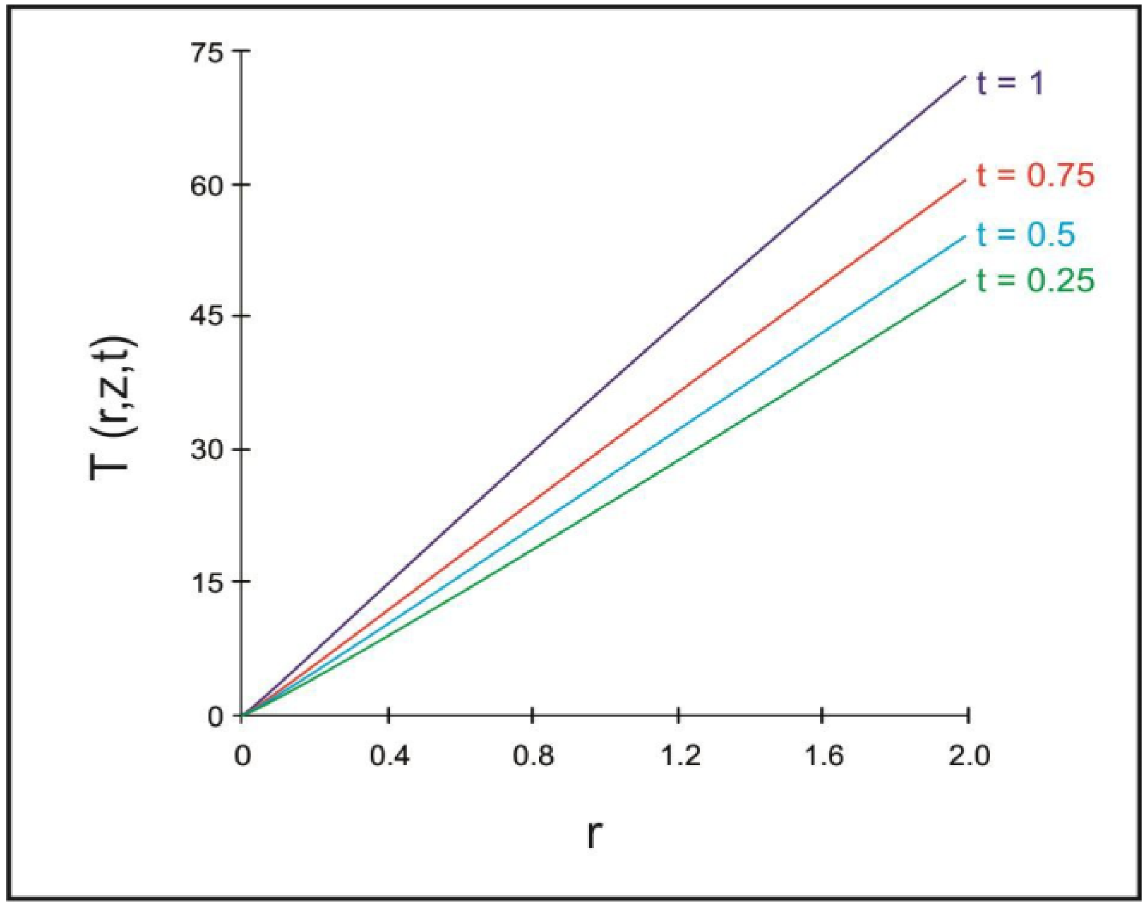
7.7 NUMERICAL RESULTS, DISCUSSIONAND REMARKS

To interpret the numerical computation we consider material properties of low carbon steel (AISI 1119), which can be used for medium duty shafts, studs, pins, distributor cams, cam shafts, and universal joints having mechanical and thermal properties $\kappa = 13.97[\mu m / s^2]$ $\upsilon = 0.29$, $\lambda = 51.9\ [W/(m-K)]$ and $a_t = 14.7\ \mu m / m -^0 C$.

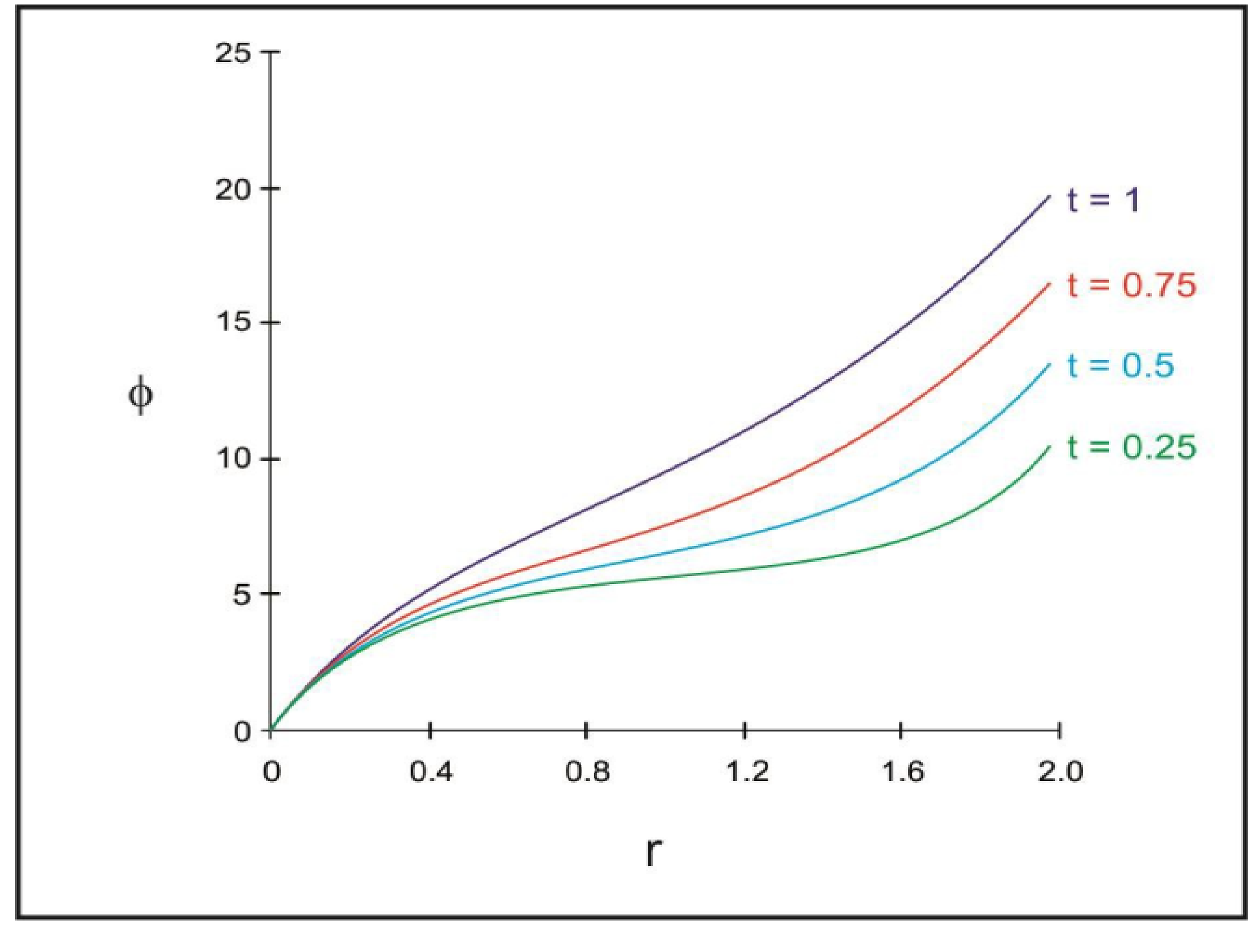
Setting the physical parameter with $a = 0.5$, $b = 1$ and $h = 3$.

7.8 CONCLUSION

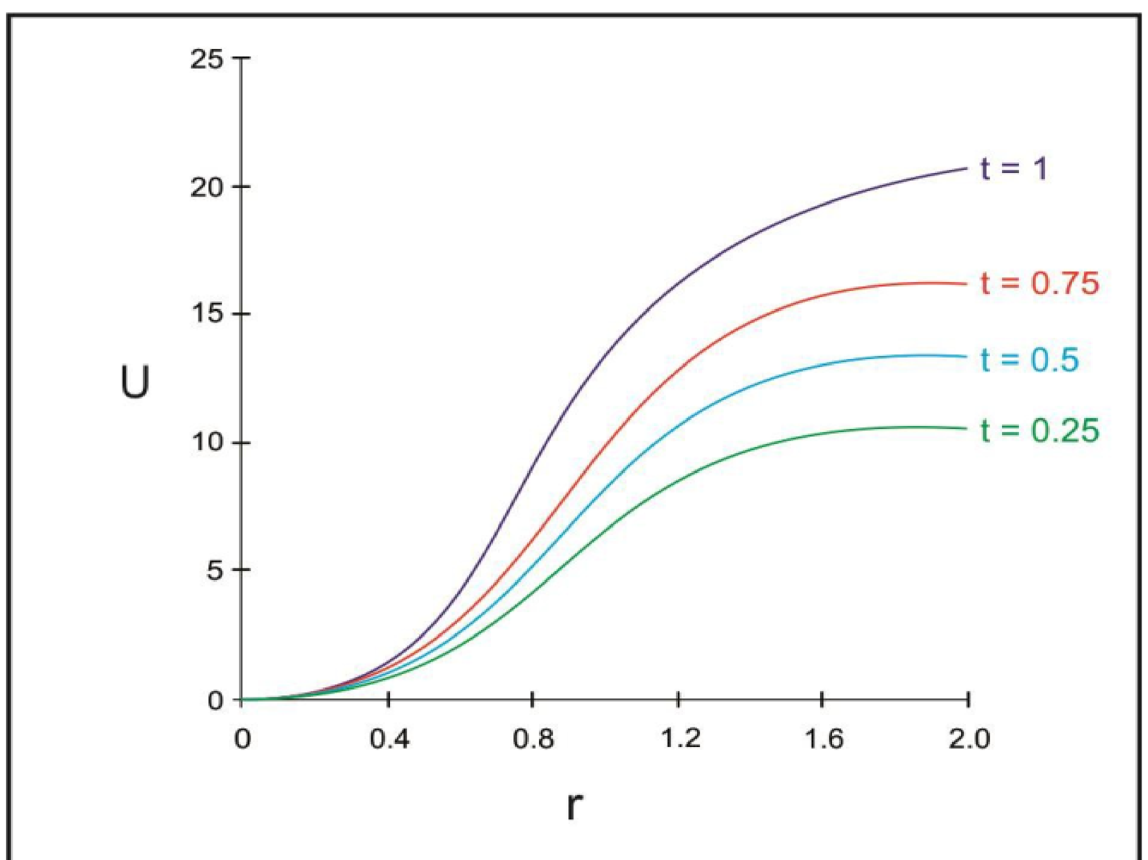
- Temperature distribution, displacement and stress function at the age $z=h$ occupying the region of the cylinder have been obtained with the known boundary conditions .
- We develop the analysis for the temperature field by introducing the transformation defined by Zgrablich and ,finite Fourier transform and Laplace transform technique with boundary conditions of radiation type .
- The series solutions converge provided we take sufficient number of terms in the series. Since the thickness of cylinder is very small, the series solution given here will be definitely convergent. Assigning suitable values to the parameters and functions in the series expressions can derive any particular case.
- The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in engineering applications.



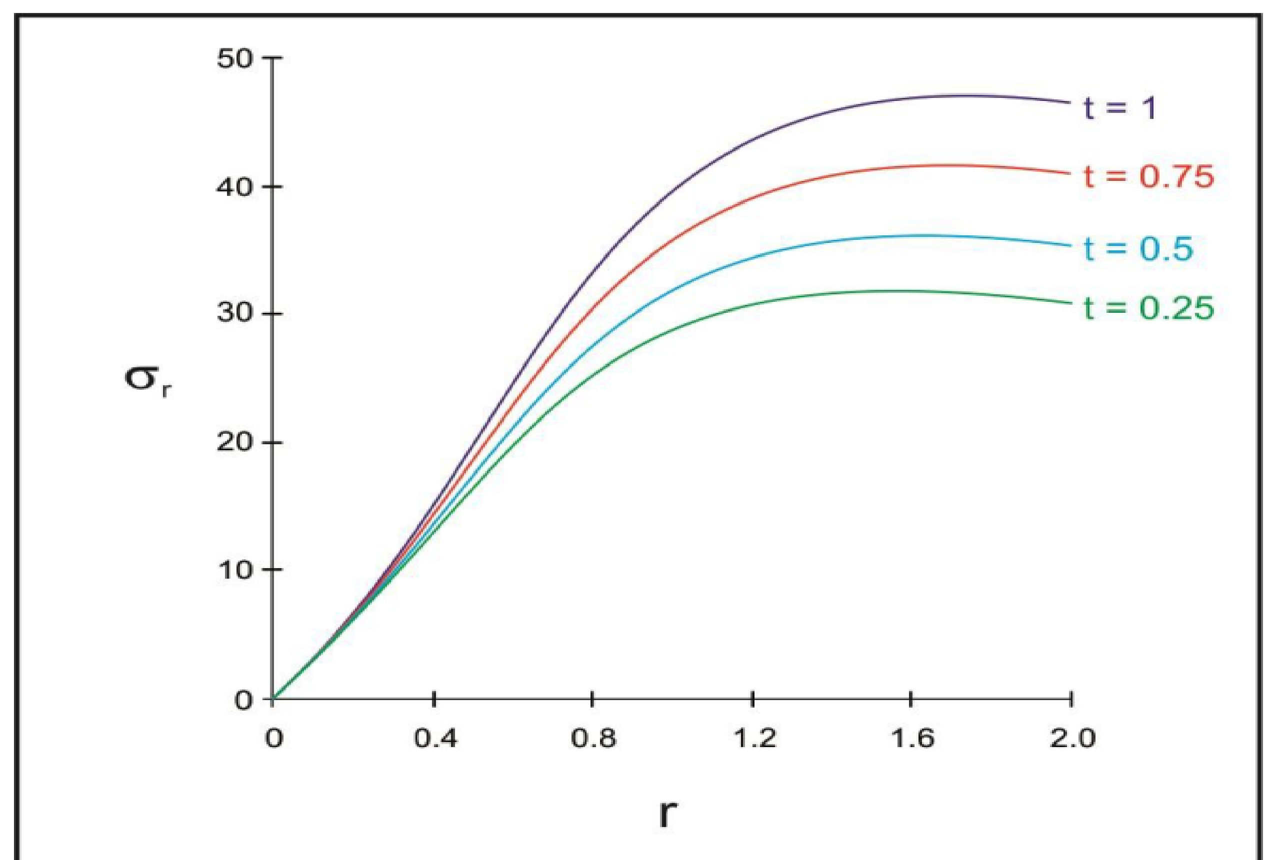
Graph 1: Temperature distribution vs r.



Graph 2: Distribution function vs r.



Graph 3: Thermal stress components vs r.



Graph 4: Radial stresses vsr

Importance of the study:

- The indirect thermoelastic problems have applications in the field of Aeronautics. The high velocity of modern aircraft give rise to aeronautic heating.
- In turn, this produces intense thermal stresses by lowering the elastic limit and reduces the strength of aircraft structure and spacecrafts.
- While the inverse thermoelastic problems are very important in view of its relevance to various industrial machines subjected to heating such as main shaft of lathe, turbine and roll of rolling mill.
- It is also useful to analyze the experimental data and measurement of aerodynamic heating. Thus thermoelastic problems play an important role in the development of science and technology especially aerospace engineering. The study of such problems will tremendously enhance the speed of the development in the industry.

Future scope

- By generalizing the results of the work done in this thesis one can derive the results for semi-infinite hollow cylinder, solid cylinder and semi-infinite circular beam.

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Thank You…