Probability of choosing a square:-

$1, 2, 3, \ldots\ldots n$

$\Rightarrow \frac{[\sqrt{n}]}{n} = P(A_n)$

$\Rightarrow P(A_n) = \frac{[\sqrt{n}]}{n} \leq \frac{1}{\sqrt{n}}$

$\therefore \lim_{n\to\infty} P(A_n) \leq \frac{1}{\sqrt{n}} \leq 0$ But probability is never less than 0

$\therefore \boxed{P(A_n) = 0} \rightarrow$ Improbable (Never use word "Impossible")

•) For prime numbers :- $\lim_{n\to\infty} \frac{\pi(n)}{n/\log n} = 1$

$\Rightarrow \boxed{\pi(n) \simeq \frac{n}{\log n}}$

•) $P(\text{choosing even}) = \frac{[\frac{n}{2}]}{n} = \begin{cases} \frac{1}{2} & \text{if } n \text{ is even} \\ \frac{1}{2} - \frac{1}{2n} & \text{if } n \text{ is odd.} \end{cases}$

$\therefore \boxed{\lim_{n\to\infty} P(\text{even}) = \frac{1}{2}}$

*) Sample space :- set of all possible outcomes of a random experiment.

No of elements in sample space = m

$|S| = m$

Repeat n times; $|S^n| = m^n \rightarrow$ outcomes

Event :- Subset of Sample space.

$A = \{(x,y); 1 \leq x \leq 6, 1 \leq y \leq 6; x+y \geq 10\}$

$= \{46, 64, 55, 56, 65, 66\}$

$$\boxed{P(A) = \frac{|A|}{|S|}}$$

only possible when sample space is finite.

Probability :- $P(A) = \frac{m(A)}{m(S)} = \frac{\text{measure of } A}{\text{measure of } S}$.

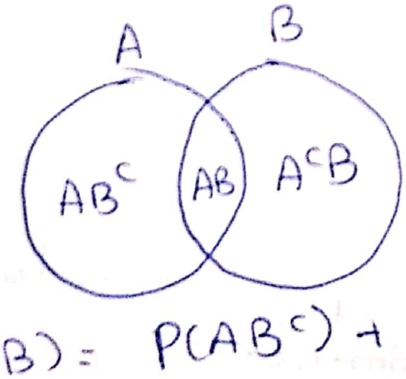
A, B, C.. are events.

A & B are mutually exclusive (M.E.) $(A \cap B = \phi)$

1.) If A & B are M.E. events then

$P(A \cup B) = P(A) + P(B)$

2.) $P(A \cup B) = P(A) + P(B) - P(AB)$



$\rightarrow P(A \cup B) = P(AB^c) + P(AB) + P(A^cB)$

$= P(AB^c) + P(B) \;\ldots\ldots (1)$

$\rightarrow P(A) = P(AB^c) + P(AB) \;\ldots\ldots (2)$

$\Rightarrow P(A \cup B) - P(B) + P(AB)$

$\therefore \boxed{P(A\cup B) = P(A) + P(B) - P(AB)}$

3.) $P(A\cup B\cup C) = P(A\cup B) + P(C) - P((A\cup B)\cap C)$

$= P(A\cup B) + P(C) - P(AC\cup BC)$

$= P(A) + P(B) - P(AB) + P(C) - [P(AC) + P(BC) - P(ABC)]$

$= P(A) + P(B) + P(C) - [P(AB) + P(BC) + P(AC)]$

$+ P(ABC)$

Note:- $P(A\cup B\cup C\cup D) \rightarrow$ Assignment :- 2

•) $P(\bigcup_{i=1}^{n} A_i) = P(A_1\cup A_2\cup A_3 \cdots \cup A_n)$ $\boxed{1\le i<j\le n}$ $\boxed{1\le i<j<k\le n}$

$= \underbrace{\sum_{i=1}^{n} P(A_i)}_{{}^nC_1} - \underbrace{\sum_{i<j} P(A_i\cap A_j)}_{{}^nC_2} + \underbrace{\sum_{i<j<k} P(A_i A_j A_k)}_{{}^nC_3}$

$\cdots\cdots + \underbrace{(-1)^{r-1}\sum_{i_1<i_2<\cdots<i_r} P(A_{i_1}A_{i_2}\cdots A_{i_r})}_{\text{General number (term)}} + \cdots + \underbrace{(-1)^{n-1} P(A_1 A_2 \cdots A_n)}_{{}^nC_n = 1,\ \text{Last term}}$

Total number of terms $= 2^n - 1$

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Eg:- There are n envelopes and n letters.

•) what is the probability that at least one letter is inserted in correct envelope:

•) what is the probability that none of letters is in correct envelope

•)

Let A_i be a event that is $l_i \in E_i$.

To find :) P{At least one letter in correct envelope}

$$= P(A_1 \cup A_2 \cup \dots \cup A_n)$$

Total no of ways to arrange n letters = $n!$

$$P(A_i) = \frac{(n-1)!}{n!} \qquad P(A_i A_r) = \frac{(n-2)!}{n!}$$

$$P(A_{i_1} A_{i_2} \dots A_r) = \frac{(n-r)!}{n!}$$

⋮

$$P(A_1 A_2 \dots A_n) = \frac{(n-n)!}{n!} = \frac{1}{n!}$$

$$\therefore P(A_1 \cup A_2 \cup \dots A_n) = {}^nC_1 \frac{(n-1)!}{n!} - {}^nC_2 \frac{(n-2)!}{n!}$$

$$+ {}^nC_3 \frac{(n-3)!}{n!} + \dots + (-1)^{r-1} {}^nC_r \frac{(n-r)!}{n!}$$

$$+ \dots (-1)^{n-1} \frac{(n-n)!}{n!}$$

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$$P(A_1 \cup A_2 \cup \dots A_n) = 1 - \frac{1}{2!} + \frac{1}{3!} + \dots + (-1)^{r-1}\frac{1}{r!} + \dots + (-1)^{n-1}\frac{1}{n!}$$

$\cdot)$ $1 -$ Pr (At least....)

$$= 1 - \left(1 - \frac{1}{2!} + \frac{1}{3!} + \dots (-1)^{n-1}\frac{1}{n!}\right)$$

$$\simeq \frac{1}{e}$$

Dearrangement :- 123 $\to$ 6 ways

321

$\boxed{312}$ $\to$ no-one is in correct position

Addition theorem of Probability

① Two dice are tossed. Find the probability of even no' on the first die or a total of 8.

Sol:) $P(A) = \frac{3 \times 6}{} \quad \underline{3}\ \underline{6} = 3 \times 6 = \frac{\boxed{18}}{36}$

$P(B) = \frac{5}{36}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{5}{36} - \frac{3}{36}$$

$$\Rightarrow \frac{23-3}{36} = \frac{\cancel{20}\ 5}{\cancel{36}\ 9} = \frac{5}{9}$$

A = Event that we get even no' on the first dice

B = Event that we get a total of 8.

$$A = \{ (2,1), (2,2) \cdots (2,6)$$
$$(4,1), (4,2) \cdots (4,6) \}$$
$$(6,1), (6,2) \cdots (6,6)$$

$$B = \{ (2,6), (3,5), (4,4), (5,3), (6,2) \}$$

$$P(A) = \frac{\text{No. of exhaustive events}}{\text{No. of Sample points}}$$

$$= \frac{18}{36}$$

$$P(B) = \frac{5}{36} \qquad P(A \cap B) = \frac{3}{36}$$

(2) A man is dealt four spade cards from a ordinary pack of 52 cards. If he is given three more cards. Find the probability that at least one of the additional cards is also a spade.

Sol:) Number of remaining cards: 48 cards

A = Probability that at least one of additional cards is space.

$\bar{A}$ = None of additional cards is spade.

$P(A) = 1 - P(\bar{A})$ By using complement theorem

$= 1 - \dfrac{{}^{39}C_3}{{}^{48}C_3}$

$\left({}^nC_r = \dfrac{n!}{(n-r)!\,r!}\right)$

$\left({}^nP_r = \dfrac{n!}{(n-r)!}\right)$

$48 - 9 = 39$

$\underline{\times} \; \underline{\times} \; \underline{\times}$

$\frac{39}{48} \times \frac{38}{47} \times \frac{37}{46}$

•) Cards (52)

Red → 26 (Diamond 13, hearts 13)

Black → 26 (Spades 13, clubs 13)

12 → 4 King, 4 Queen, 4 Joker.

✪) Conditional Probability of an event given other event :-

•) $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$

•) $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$, $P(A) \neq 0$

Q) From a city population, the probability of selecting

(i) A male or a smoker is $7/10$

(ii) A male smoker (B) is $2/5$

(iii) A male if a smoker is already selected is $2/3$

Find the probability of (a) Non-smoker

(b) A male.

(c) A smoker if a male was selected.

A = Event of Choosing a male

B = Event of Choosing a smoker

Given: (i) $P(A \cup B) = 7/10$

(ii) $P(A \cap B) = 2/5$

(iii) $P(A|B) = 2/3$.

(By Conditional theorem-)

we know, $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \frac{\cancel{2}}{3} = \frac{\cancel{2}}{5} \times \frac{1}{P(B)}$

$\Rightarrow \frac{1}{P(B)} = \frac{5}{3}$. $\boxed{P(B) = \frac{3}{5}}$

(i) $P(\text{Non-Smoker}) = 1 - P(B) = 1 - \frac{3}{5} = \frac{2}{5}$

By Complement theorem

(ii) By addition theorem,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{7}{10} = P(A) + \frac{3}{5} - \frac{2}{5}$

$\Rightarrow P(A) = \frac{7}{10} - \frac{1}{5}$

$= \frac{7}{10} - \frac{2}{10} = \frac{1}{2}$

(iii) $P(B|A) = \frac{2/5}{1/2}$

$= \frac{4}{5}$.

Independent events :- Several events are said to be independent if the ~~operand~~ occurence (or non-occurence) of an event say A is not affected by the supplementary knowledge concerning the occurence of any number of remaining events.

Say :> A, B are two independent events

$P(A \cap B) = P(A) P(B)$

So; $P(A_1 \cap A_2 \cap A_3 \cdots \cap A_n) = P(A_1) P(A_2) \cdots P(A_n)$

Then conditional probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$(P(B) = 0)$ $= \frac{P(A) \cancel{P(B)}}{\cancel{P(B)}}$

$P(A|B) = P(A)$

$P(A|S) = P(A)$

$\boxed{P(A|B) = P(A)}$

$\boxed{P(B|A) = P(B)}$

Q) If A and B are independent events then prove that A^c and B^c are also independent.

Sol:) Given: A & B are independent events.

$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$

To prove: A^c and B^c are also independent;

i.e; $P(A^c \cap B^c) = P(A^c) P(B^c)$

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$P(A' \cap B') = P(A \cup B)'$ (By de-Morgan law)

$= 1 - P(A \cup B)$ (By complement theorem)

$= 1 - [P(A) + P(B) - P(A \cap B)]$ (By addition theorem)

$= 1 - P(A) - P(B) + P(A)P(B)$

$= (1 - P(A)) - P(B)(1 - P(A))$

$= (1 - (P(A))(1 - P(B))$

$= P(A^c) P(B^c)$

$\therefore P(A' \cap B') = P(A') P(B')$

Hence, A^c & B^c are independent to each other.

---

4) Multiplication theorem of probability

Let A and B be two events. Then

$$\boxed{P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)} \quad P(A), P(B) \neq 0$$

$P(A)P(B|A) = \cancel{P(A)} \dfrac{P(A \cap B)}{\cancel{P(A)}}$ (By conditional probability)

$= P(A \cap B)$

Given: $P(B|A)$ and $P(A|B)$ are Conditional probability.

To prove:- $P(A \cap B) = P(A)P(B|A)$

RHS :) $\cancel{P(A)} \dfrac{P(A \cap B)}{\cancel{P(A)}} = P(A \cap B)$

Proof:) Let S be the given Sample space

$P(A) = \frac{n(A)}{n(S)}$ (1) $P(B) = \frac{n(B)}{n(S)}$ (2) $P(A \cap B) = \frac{n(A \cap B)}{n(S)}$ (3)

$P(A|B) = \frac{n(A \cap B)}{n(B)}$

As B is the new sample space so the probability of A given B is

So, Rewriting (3)

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{n(B)}{n(S)} . P(A|B) = P(B)P(A|B)$

Pairwise independent events

Consider n events $A_1, A_2, \ldots, A_n$, defined on sample space S so that

$P(A_i) \geq 0, \quad i = 1, 2, \ldots, n$

These events are said to be pairwise independent if every pair of events ~~are~~ is independent.

$P(A_i \cap A_j) = P(A_i) P(A_j)$

$i \neq j = 1, 2, \ldots n$

Mutually independent events:-

Let S denotes the sample space of, n number of events then the events are said to be mutually independent if

$P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_n}) = P(A_{i_1}) P(A_{i_2}) \ldots P(A_{i_n})$

Q.) If A, B, C are random events in a sample space and if they are pairwise independent and A is independent of $B \cup C$. Show that A, B, C are mutually independent.

Given: A, B, C are pairwise independent

$\therefore$ $P(A \cap B) = P(A) P(B)$

$P(B \cap C) = P(B) P(C)$

$P(A \cap C) = P(A) P(C)$

Given A is independent of $B \cup C$

$\therefore$ $P(A \cap (B \cup C)) = P(A) P(B \cup C)$

To prove:- $P(A \cap B \cap C) = P(A) P(B) P(C)$

We know that;

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

(By addition theorem)

$$= P(A) + P(B) + P(C) - P(A)P(B) - P(A)P(C) - P(B)P(C) + P(A \cap B \cap C)$$

$$P(A \cap (B \cup C)) = P(A)[P(B) + P(C) - P(B \cap C)]$$

$$= P(A)P(B) + P(A)P(C) - P(A)P(B \cap C)$$

$$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$- P(A \cap (B \cup C)) - P(A)P(B)P(C)$$
$$+ P(A \cap B \cap C)$$

$$\Rightarrow P(A \cap B \cap C) = P(A)P(B)P(C) - (P(A) + P(B) + P(C))$$
$$+ P(A \cap (B \cup C)) + P(A \cup B \cup C) + P(B)P(C)$$

As $\boxed{P(A \cap (B \cup C)) + P(A \cup B \cup C) = P(A) + P(B) + P(C)}$

$\therefore \boxed{P(A \cap B \cap C) = P(A)P(B)P(C)}$ Prooved.

$$B \cup C = X$$

$$P(A \cap X) + P(A \cup X) = P(A) + P(X)$$
$$= P(A) + P(B \cup C)$$
$$= P(A) + P(B) + P(C) - P(B)P(C)$$

Baye's Theorem

If $E_1, E_2, \dots, E_n$ are mutually disjoint events such that $P(E_i) \neq 0$, $i = 1, 2, \dots, n$ then for any arbitrary event A which is a subset of $\bigcup_{i=1}^{n} E_i$ such that $P(A) > 0$, we have

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$$

<u>Proof</u> :- <u>LHS</u> : $P(E_i|A)$

<u>RHS</u> : $\dfrac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)}$

Given $A \subset \bigcup_{i=1}^{n} E_i$

$\Rightarrow A \cap \left(\bigcup_{i=1}^{n} E_i\right) = A$

$\Rightarrow \bigcup_{i=1}^{n} (A \cap E_i)$ (By distributive law)

Since $A \cap E_i \subset E_i$ are mutually disjoint events

So

$$P(A) = P\left(\bigcup_{i=1}^{n} (A \cap E_i)\right)$$

$$= \sum_{i=1}^{n} P(A \cap E_i) \quad \text{(By axiom of additivity)}$$

$$= \sum_{i=1}^{n} P(E_i)\, P(A|E_i)$$

$P(A \cap E_i) = P(A)\,P(E_i/A)$

$\Rightarrow P(E_i|A) = \dfrac{P(E_i)(P(A|E_i))}{P(A)}$

Hence; $\boxed{P(E_i|A) = \dfrac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)}}$

Prior Probabilities $\longrightarrow$ New information $\longrightarrow$ Apply Baye's theorem $\downarrow$ Posterior probability

$P(E_i)$ = Prior probability

$P(A)$ = Likelihoods

$P(E_i|A)$ = Posterior probability.

Eg) Suppose that a product is produced in three factories X, Y and Z. It is known that factory X produces thrice as many items as factory Y and factories Y and Z produces same number of items. Assume that it is known that 3 ~~person~~ cent of the items produced by each of the factories X and Z are defective. while 5 % of those manufactured by Y are defective. All items produced in three factories are stocked and an item or product in random is selected.

(i) what is the probability that this item is defective?

(ii) what is the probability that the selected item found to be defective was produced from factory X.?

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Sol:) Let E_1, E_2, E_3 be events

$P(E_1)$ = Probability of item drawn from X

$P(E_2)$ = Probability of item drawn from Y

$P(E_3)$ = Probability of item drawn from Z.

$P(A)$ = Probability of item being defective.

Let no. of goods produced Y be n

No. of goods produced in Z = n.

No. of goods produced in X = 3n.

Total defective goods:

Defective goods in X + defective goods in Y +

defective goods in Z

$$= \frac{3}{100} \times 3n + \frac{5}{100} \times n + \frac{3}{100} \times n$$

$$\Rightarrow \frac{9n}{100} + \frac{5n}{100} + \frac{3n}{100} \Rightarrow \frac{17n}{100}$$

(i) $P(A) = \frac{\text{No. of defective}}{\text{Total no. of goods}}$

$$= \frac{\frac{17\cancel{n}}{100}}{5\cancel{n}} = \boxed{\frac{17}{500}}$$

(ii)

$P(E_i \mid A) = \frac{9}{17}$

Note:) ✱) $P(AB) = P(A)P(B)$

if $P(B) = 0$, $AB \subset B$

$P(AB) \leq P(B) = 0$

✱) A & B are independent and m.e.

$P(A \cap B) = P(A)P(B) = 0$

Either $P(A)$ or $P(B)$ is zero.

✱) If $P(A), P(B) > 0$ then A & B can be either independent or m.e.

✱) For m.i of A, B, C , (1) → (4) must be true

$P(AB) = P(A)P(B)$ $P(BC) = P(B)P(C)$; $P(AC) = P(A)P(C)$

$P(ABC) = P(A)P(B)P(C)$

Q.)

$\boxed{112, 121, 211, 222}$

Four tickets are present. A ticket is drawn at random.

A → 1st digit is one

B → 2nd digit is one

C → 3rd digit is one.

$A = \{112, 121\}$

$B = \{112, 211\}$

$C = \{121, 211\}$

$AB = \{112\}$ $BC = \{211\}$ $AC = \{121\}$ $ABC = \phi$

$S = \{112, 121, 211, 222\}$

$P(A) = P(B) = P(C) = \frac{2}{4} = \frac{1}{2}$

$P(AB) = P(AC) = P(BC) = \frac{1}{4}$.

$\therefore P(AB) = P(A)P(B)$, $P(BC) = P(B)P(C)$, $P(AC) = P(A)P(C)$

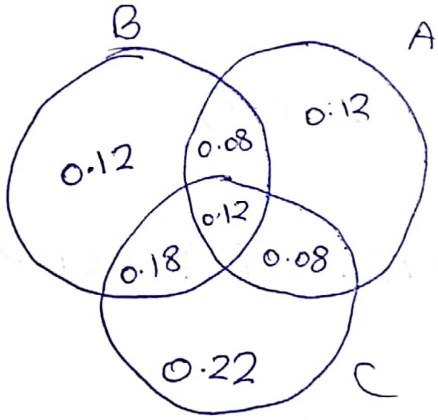
$\Rightarrow$ A, B, C are pairwise independent.

$\therefore P(ABC) = 0 \neq P(A)P(B)P(C) \neq \frac{1}{8}$

$\Rightarrow$ A, B, C are not mutually independent.

☆) Pairwise indepence does not imply mutually independent.

Q.)



$P(A) = 0.4$

$P(B) = 0.5$

$P(C) = 0.6$

$P(AB) = 0.2$

$P(AC) = 0.2$

$P(BC) = 0.3$

$P(ABC) = 0.12$

A, B $\rightarrow$ independent

B, C $\rightarrow$ independent

A, C $\rightarrow$ not independent

A, B, C $\rightarrow$ independent.

Mutally independence does not imply pairwise independence.

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Q.) If A and B are independent then A and B^c.

Given:

$P(AB) = P(A)P(B)$

To prove: $P(AB^c) = P(A)P(B^c)$

$A = AB^c \cup AB$

$\Rightarrow P(A) = P(AB^c) + P(AB)$

$\Rightarrow P(A) = P(AB^c) + P(A)P(B)$

$\Rightarrow \boxed{P(AB^c) = P(A)P(B^c)}$

Q.) If A & B are independent, A^c and B^c are independent.

Q.) If A, B and C are m.i., then A is independent of $B \cup C$.

Q.1) Consider a box with some balls; 25 white balls and 75 Black balls. Balls are taken one by one with replacement. what is the probability of drawing the 3rd white ball before the second black ball?

Sol:) WWW WBWWB

WWBW BWWW

Possible outcomes: WWW, BWWW, WBWW, WWBW

$$\text{Probability:-} \left(\frac{1}{4}\right)^3 + \frac{3}{4}\left(\frac{1}{4}\right)^3 \times 3$$

$$\Rightarrow \frac{1}{4^3} + \frac{9}{4^4}$$

WWBW

$$\Rightarrow \frac{4+9}{4^4} = \boxed{\frac{13}{4^4}}$$

$$\text{Probability:-} \left(\frac{25}{100}\times\frac{24}{99}\times\frac{23}{98}\right) + \left(\frac{75}{100}\times\frac{25}{99}\times\frac{24}{98}\times\frac{23}{97}\right)$$

$$+ \left(\frac{25}{100}\times\frac{75}{99}\times\frac{24}{98}\times\frac{23}{97}\right) + \left(\frac{25}{100}\times\frac{24}{99}\times\frac{75}{98}\times\frac{23}{97}\right)$$

$$\Rightarrow \frac{(25\times24\times23\times97) + 3(75\times25\times24\times23)}{100\times99\times98\times97}$$

Note: $P(A|B) = \frac{P(AB)}{P(B)} \Rightarrow P(AB) = P(A)P(B/A)$

$$P(ABC) = P(ABC)$$
$$= P(AB)P(C/AB)$$
$$= P(A)P(B/A)P(C/AB)$$

$\therefore P(A_1A_2 \dots A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1A_2) \dots P(A_n|A_1A_2 \dots A_{n-1})$

$W_1W_2W_3$ $\quad P(W_1W_2W_3) = P(W_1)P(W_2/W_1)P(W_3/W_1W_2)$

$$= \frac{25}{100} \times \frac{24}{99} \times \frac{23}{98}$$

$B_1W_2W_3W_4$ $\quad P(B_1W_2W_3W_4) = P(B_1)P(W_2/B_1)P(W_3/B_1W_2) \cdot P(W_4/B_1W_2W_3)$

$$= \frac{75}{100} \times \frac{25}{99} \times \frac{24}{98} \times \frac{23}{97}.$$

$W_1B_2W_3W_4$ $\quad P(W_1B_2W_3W_4) = P(W_1)P(B_2/W_1)P(W_3/W_1B_2) P(W_4/W_1B_2W_3)$

$W_1W_2B_3W_4$ $\quad P(W_1W_2B_3W_4) = P(W_1)P(W_2/W_1)P(B_3/W_1W_2) P(W_4/W_1W_2B_3)$

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Q.) Consider the quadratic equation:

$ax^2+bx+c=0$

a, b, c are determined by throwing a fair die.

what is the probability that the eqn has real roots?

<u>Sol:)</u> $\left(b^2-4ac \geq 0 \right.$

$\left. \Rightarrow b^2 \geq 4ac \right)$

$S = \{(a,b,c) : 1 \leq a,b,c \leq 6\} = 216$

Fav outcomes: $\{(a,b,c) : b^2-4ac \geq 0\}$

ac	a	c	$b : b^2 \geq 4ac$	No. of cases
1	1	1	(2, 3, 4, 5, 6)	5
2	{1, 2}	{2, 1}	(3, 4, 5, 6)	$2 \times 4 = 8$
3	{1, 3}	{3, 1}	(4, 5, 6)	6
4	{1, 2, 4}	{4, 2, 1}	(4, 5, 6)	9
5	{1, 5}	{5, 1}	(5, 6)	4
6	{1, 2, 3, 6}	{6, 3, 2, 1}	(5, 6)	8
8	{2, 4}	{4, 2}	6	2

9 $\{3,3\}$ 6 1

Favourable: 43

Sample space: 216

Probababibility: $\frac{43}{216}$

Q.) The sum of two non-negative numbers is equal to $2n$.

$x+y=2n,\ x\geq 0, y\geq 0,\ \underline{n>0}$

what is the probabity that product is not less than $\frac{3}{4}\max(xy)$?

$\Pr\{xy \nless \frac{3}{4}\max(xy)\}$

$2n\ x(2n-x)$

•) $\max(xy) = \max x(2n-x) : 0<x<2n$ $\quad 2nx - x^2$

$2n-2x$

$n-x$

$f(x) = 2nx - x^2$

max at $x=n$ and hence $y=n$.

$\Rightarrow \boxed{\max(xy) = n^2}$

•) $\Pr\{xy \nless \frac{3}{4}n^2\}$

$= \Pr\{x(2n-x) \geq \frac{3}{4}n^2\}$ $\quad \Pr\{2x(2x-3n) - n(2x-3n) \leq 0\}$

$\Rightarrow \Pr\{(2x-n)(2x-3n) \leq 0\}$

$= \Pr\{2nx - x^2 \geq \frac{3}{4}n^2\}$

$= \Pr\{8nx - 4x^2 \geq 3n^2\} = \Pr\{4x^2 - 8nx + 3n^2 \leq 0\}$

$= \Pr\{4x^2 - 6nx - 2nx + 3n^2 \leq 0\}$

$\therefore Pr\{(2x-n)(2x-3n) \le 0\}$

$\left(\begin{array}{l} ab \le 0 \\ \Leftrightarrow a \ge 0,\ b \le 0 \\ (or)\ a \le 0,\ b \ge 0 \end{array} \right)$

Either: $Pr\{2x-n \ge 0$ and $2x-3n \le 0 \quad \cdots (1)$

$2x-n \le 0$ and $2x-3n \ge 0 \cdots (2)\}$

$\Rightarrow Pr\left\{\left(x \ge \frac{n}{2} \text{ and } x \le \frac{3n}{2}\right) \cup \left(x \le \frac{n}{2} \text{ and } x \ge \frac{3n}{2}\right)\right\}$

$\Rightarrow Pr\left\{x \ge \frac{n}{2},\ x \le \frac{3n}{2}\right\}$

$\Rightarrow Pr\left\{\frac{n}{2} \le x \le \frac{3n}{2}\right\}$

$$= \frac{\frac{3n}{2} - \frac{n}{2}}{2n - 0} = \frac{\frac{2n}{2}}{2n} = \frac{1}{2}.$$

Random variables:-

Functions:- •) If $f: A \to B$, then every element of A should have a corresponding image in B and no element in A can have more than 1 images in B

•) An expression like exponential, logarithmic and polynominal of variables & constants.

Eg:) $F(x) = y$

•) If $f: A \to B$, $\forall a \in A, \exists\ b \in B$.

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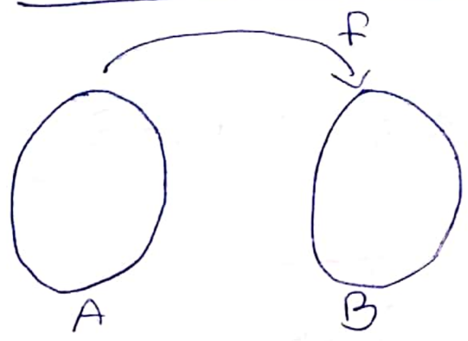
Assignment-3 :- Definition of functions:

A) Random Variable :- (Function)



1.) $f: A \to B$ is a random variable

2.) $A = S$

3.) $B = R$

Random variable is a real valued function whose domain is the Sample space.

Examples:-

Let P be the probability of no break up of circuit within one hour.

$q = 1-p \to$ Probability of breaking cup.

Let $X \to$ No of working circuits ~~break cups~~ after 1 hour

Possible values of X are 1, 0

(For one circuit) So either work or no work

X	Probability
0	P
1	q

Let Ω be Sample Space

$\Omega = \{S, F\}$ $\quad X(S) = 1 \quad X(F) = 0$

Now

Out of n circuits :-

Let X be no. of working circuits after 1 hr.

Possible values of X are 0, 1, 2, ... n.

$\Omega = \{S, F\}^n$

$= \{(e_1, e_2, \dots, e_n) : e_i = S \text{ or } F\}$

$Pr\{X=K\}$ $(0 \le K \le n)$

No of working circuits = K.

A Sample point.

$(\underbrace{S S S \ldots}_{K} \underbrace{F, F, \ldots, F}_{n-K})$ or $(F \underbrace{S, S \ldots S}_{K} F, F, F, \ldots F\}$ etc.

$n-K$

$\therefore Pr(X=K) = \binom{n}{K}$

Probability of a sample point: $p^K q^{n-K}$

$Pr(X=K) = \binom{n}{K} p^K q^{n-K}$; $K = 0, 1, 2, \ldots n$

Note:- $Pr\{x \in A\} = P\{\omega \in \Omega : X(\omega) \in A\}$

$$\boxed{Pr\{X=r\} = \binom{n}{r} p^r q^{n-r}, \quad r = 0, 1, 2, \ldots n}$$

X: Random Variable

x = Arbitrary variable in X.

$f(x) = Pr\{X=x\}$: f: Prob (mass) Function.

$f: R(X) \longrightarrow [0,1]$

& $$\boxed{f(x) = {}^nC_x \, p^x q^{n-x} \quad ; \; x = 0, 1, \ldots n}$$

$$\boxed{f(x) = 0 \quad \text{otherwise}}$$

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•) $\sum_{x=0}^{n} f(x) = \sum_{x=0}^{\infty} \binom{n}{x} q^{n-x} p^{x} = (q+p)^{n} = 1.$

Eg:) A & B play a game. For each head, B pays a dollar to A & A pays a dollar to B if tail occurs.

Let $P(H)=p$ and $P(T)=q$.

X = Gain of A after n tosses

Let $n=2$.

$X(TT) = -2$

$X(HT) = 0$

$X(TH) = 0$

$X(HH) = 2$

~~Bcoed~~	x	$f(x)$
~~qq2~~	-2	q^2
	0	$2pq$
	2	p^2

(If n tosses ~~throws~~ are done;

$X(\underbrace{TT\ldots T}_{n}) = -n$

$X(\underbrace{TT\ldots T}_{n-1}H) = -n+2$

$\vdots$

$X(\underbrace{HH\ldots H}_{n-1}T) = n-2$

$X(\underbrace{HH\ldots H}_{n}) = n$

x	$f(x)$
$-n$	q^{n}
$-(n-2)$	${}^{n}C_{n-1}\, p\, q^{n-1}$
$\vdots$	$\vdots$
$(n-2)$	${}^{n}C_{n-1}\, p^{n-1} q$
n	p^{n})

$\therefore X \sim B(n,p)$ $n =$ Bernouli trials.

$\Rightarrow \boxed{\frac{x+n}{2} \sim B(n,p)}$

If we add n to each x and divide by 2, then also Binomial expression is same.

$\therefore B\left\{\frac{x+n}{2} = K\right\}$ $B\left\{\frac{n+x}{2} = K\right\} = \binom{n}{K} p^K q^{n-K}$

0

1

2

3

4

⋮

n

According to definition; $P(X=K) = \binom{n}{K} p^K q^{n-K}$

$\boxed{\text{For } x = 0, 1, \cdots n}$

So we need to convert the x to this format.

In above example: we had $-n \leq x \leq n$

$\therefore$ we did $\frac{x+n}{2} = X$ So that $0 \leq X \leq n$ we can apply binomial distribution law.

$Pr\{X=7\} = Pr\left\{\frac{11+X}{2} = \frac{11+7}{2}\right\}$

$= Pr\{Y=9\}$

$= \binom{11}{9}\left(\frac{1}{2}\right)^{11}$

$Pr\{X=8\} = Pr\left\{\frac{11+X}{2} = \frac{11+8}{2}\right\}$

$= Pr\{Y=9.5\} = 0$

Q) A coin is tossed until a head appears.

Let X = No of tosses required.

$S = \{H, TH, TTH, \ldots, \underbrace{TT\ldots T}_{K-1}H, \ldots\}$

Find the probability distribution.

$X(H) = 1$, $X(TH) = 2$, ...

$P(H) = P$, $P(T) = q$.

x	$f(x) = Pr\{X=x\}$
1	p
2	qp
3	q^2p
4	q^3p.
$\vdots$	

$Pr\{X=x\} = q^{x-1}p$

$\boxed{f(x) = q^{x-1}p \; ; x = 1, 2, \ldots}$

Geometric Distribution

X = No of failures till the 1st success.

$f(x) = Pr\{X=x\}$

$= q^x p$

(Substitute $X = x+1$)

$$\sum_{x=1}^{\infty} f(x) = p(1+q+q^2+\cdots)$$

$$= \boxed{\frac{p}{1-q} = 1}$$

$f(x)$ is a probability:-

i) $f(x) \geq 0$ $\quad f(x) \leq 1$

ii) $\sum_{x} f(x) = 1$

If a function f satisfies all these conditions, it is called a pmf. (probability mass function).

$$\boxed{f(x) = \begin{cases} 0 & \text{otherwise} \\ [0,1] & \forall \cancel{X} \in x \end{cases}}$$

Eg:) $F: R \to [0,1]$

$F(x) = Pr\{X \leq x\}$

$= \sum_{u \leq x} f(u)$

A coin is tossed twice.

X = No of heads

X	f(x)	F(X)
0	1/4	$Pr\{X \leq 0\} = \frac{1}{4}$
1	1/2	$Pr\{X \leq 1\} = \frac{3}{4}$
2	1/4	$Pr\{X \leq 2\} = 1$

Here F(x) is the cumulative function.

It will be cumulative distributive after we do the following distribution:-

If $x<0$, $F(x)=0$ $\qquad -\infty < X < \infty$

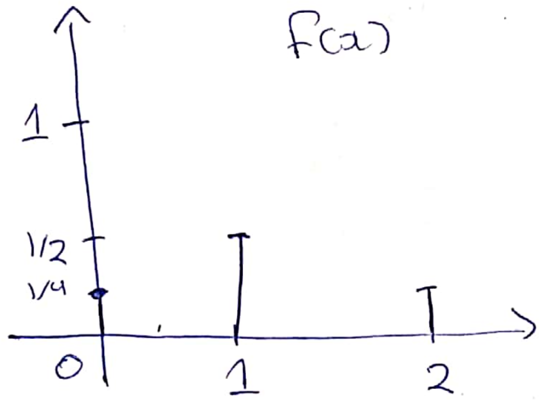
If $0 \le x < 1$, $F(x) = \frac{1}{4}$

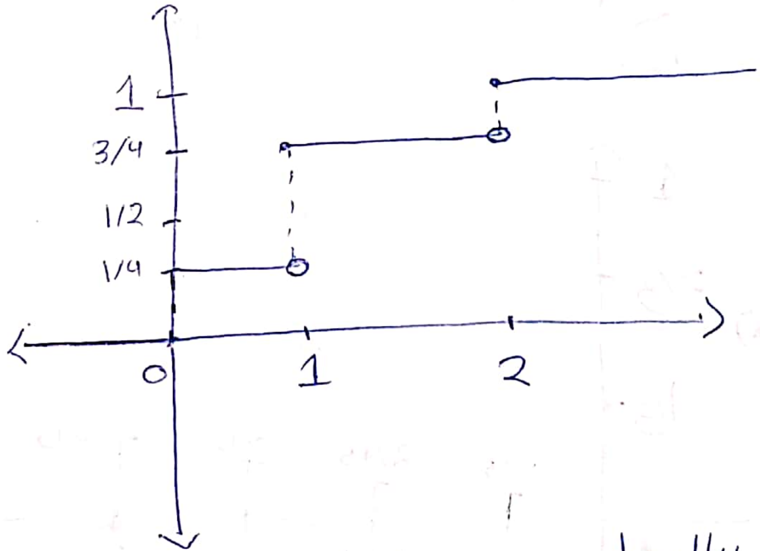
If $1 \le x < 2$, $F(x) = \frac{3}{4}$

If $x \geq 2$, $F(x) = 1$

Now $F(x)$ is Cumulative distribution func:-

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ 1/4 & \text{if } 0 \le x < 1 \\ 3/4 & \text{if } 1 \le x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$





Note: X is the random Variable which is actually a set.

$x \in X$ is a variable which takes the values.

$\therefore$ In function, we write $f(x)$ not $f(X)$.

Eg:- X = Number a die shows up. $\quad x$ is any element like 1 or 2. etc.,

$x = 1,2,3,4,5,6$ $\quad$ But $X = \{1,2,3,4,5,6\}$

Note 2: But always we write, $P(X)$ not $P(x)$.

Q.) X is a discrete random variable which takes the Values 1,2,3,4,5 with probabilities proportional to these values. Find $f(x)$, $F(x)$ & sketch

$Pr\{X>2\}: Pr\{X=3,4,5\}$

$= f(3)+f(4)+f(5)$

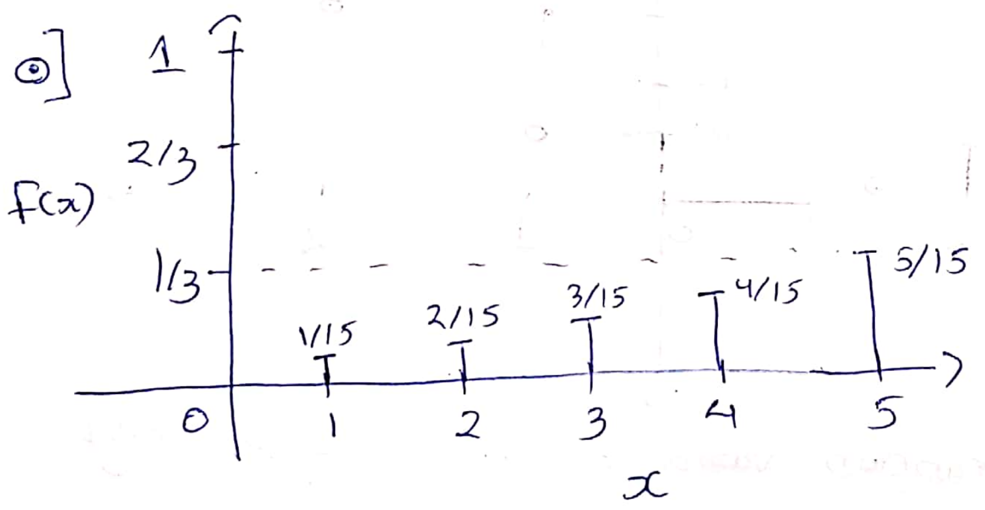
$= \frac{12}{15}$

$f(x) = Pr\{X=x\}$

$f(x) \propto x$

$\Rightarrow f(x) = Kx, \; x=1,2,\dots,5$

$\sum_{x=1}^{5} f(x) = 1 \Rightarrow K = \frac{1}{15}$, $f(x) = \frac{x}{15}$; $x=1,2,\dots 5$:



x	$f(x)$	$F(x)$
0		
1	1/15	1/15
2	2/15	3/15
3	3/15	6/15
4	4/15	10/15
5	5/15	1

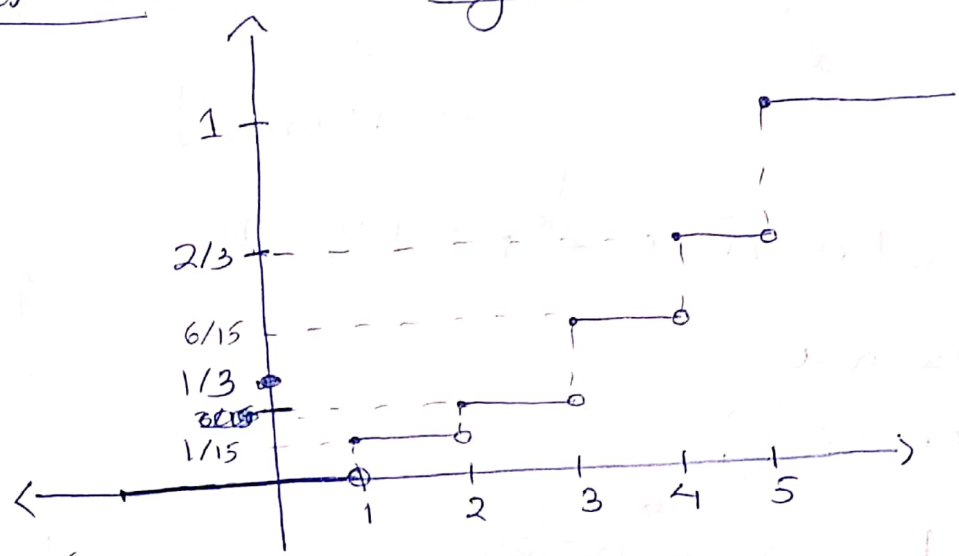
This is not the actual distribution fnc.

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1/15 & \text{if } 1 \le x < 2 \\ 3/15 & \text{if } 2 \le x < 3 \\ 6/15 & \text{if } 3 \le x < 4 \\ 10/15 & \text{if } 4 \le x < 5 \\ 1 & \text{if } x \geq 5 \end{cases}$$

This is the cumulative distribution func.

<u>F(x) vs x:-</u>

<u>Right continuous</u>



$$\lim_{x \to 2^-} F(x) = \frac{1}{15} \qquad \lim_{x \to 2^+} F(x) = \frac{3}{15}$$

$$\boxed{f(x) = F(x^+) - F(x^-)}$$ only at <u>$x = 1, 2, 3, 4, 5$</u>.

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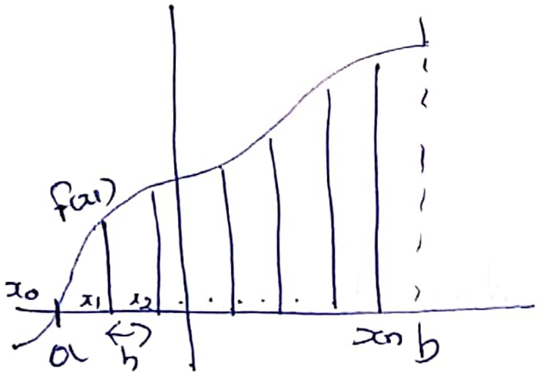
A) $\int_a^b f(x)\,dx$

Continuous Random Variable



$h = \frac{b-a}{n}$

$$\int_a^b f(x)\,dx \simeq h\left[\frac{f(x_0)+f(x_n)}{2} + f(x_1) + \cdots f(x_{n-1})\right]$$

Possible values of x: $\frac{1}{2^K}$; $K = 1, 2, \dots 10$.

Let ; $f(x) \propto x$

$\boxed{f(x) = Cx}$

$$\sum_{K=1}^{10} f(x) = 1 \Rightarrow \sum_{K=1}^{10} f\left(\frac{1}{2^K}\right) = 1$$

$$\Rightarrow C \sum_{K=1}^{10} \frac{1}{2^K} = 1$$

$$\Rightarrow C\left(\frac{\frac{1}{2}\left(1-\frac{1}{(2)^{10}}\right)}{1-\frac{1}{2}}\right) = 1$$

$$\Rightarrow C\left(\frac{1023}{1024}\right) = 1$$

$$\Rightarrow \boxed{C = \frac{1024}{1023}}$$

$$\boxed{f(x) = \frac{1024}{1023}\, x} \qquad x : \frac{1}{2^K} ; \; K = 1, 2, \ldots, 10$$

$$= \frac{1024}{1023} \cdot \frac{1}{2^K}$$

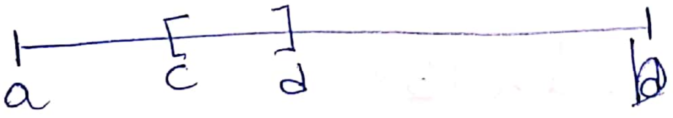
$$f(x) = \boxed{\frac{2^{10-K}}{1023} \qquad K = 1, 2, \ldots, 10}$$

·) A random variable X is called Continuous R.V., if it takes all the values in the interval.

Eg:- If X is continuous in $[a, b]$ with probability density function $f(x)$

then; 1) $f(x) \geq 0$

2) $\int_a^b f(x)\,dx = 1$.



$$Pr\{c \leq X \leq d\} = \int_c^d f(x)\,dx$$

A) Difference between discrete & Continuous R.V. :-

Discrete: $Pr\{X = a\} \geq 0$

Continuous: $Pr\{X = a\}$

$$= \lim_{\epsilon \to 0} Pr\{a - \tfrac{\epsilon}{2} \leq X \leq a + \tfrac{\epsilon}{2}\}$$

$$= \lim_{\epsilon \to 0} \int_{a-\epsilon/2}^{a+\epsilon/2} f(x)\,dx = 0 \qquad \boxed{\text{As } \epsilon \to 0,\ Pr\{X = a\} = 0}$$

*) If A is countable set of R.

Discrete: $P\{X \in R\} \geq 0$

Continuous: $P\{x \in R\} = 0$

$f(x) = K$ if $0 < x < \frac{1}{2^{10}}$

0 otherwise.

Solution:- $K: \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{0}^{1/2^{10}} K\,dx = 1$

$\Rightarrow \boxed{K = 2^{10}}$

$\therefore f(x) = \begin{cases} 2^{10} & 0 < x < 1/2^{10} \\ 0 & \text{otherwise} \end{cases}$

Note:) ☆☆ •) In Continuous Random variable, the probability density func $f(x)$ can be greater than one, less than one or 0.

•) In discrete RV, the probability density func $f(x)$ can never be greater than one as it indicates probability.

$\boxed{0 \leq f(x) \leq 1}$

Q.) $f(x) = \begin{cases} Kx & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$

Find K. Find F(x) and sketch F(x).

·) $F(x) = \Pr\{X \leq x\} = \int_{-\infty}^{x} f(u)\,du$

·) K: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow K \int_{0}^{1} x\,dx = 1$

$\Rightarrow \boxed{K = 2}$

·) $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$

can be written
$0 \leq x < 1$
$0 < x \leq 1$
$0 \leq x \leq 1$

Does not make any diff as they are single points in infinite points.

A) $F(x) = \begin{cases} 0 & x \leq 0 \\ \int_{-\infty}^{x} f(u)\,du & 0 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$

If $x \leq 0$, $F(x) = 0$

If $0 \leq x \leq 1$, $F(x) = \int_{-\infty}^{x} f(u)\,du = \int_{0}^{x} 2u\,du$

$= x^2$

If $x \geq 1$, $F(x) = 1$.

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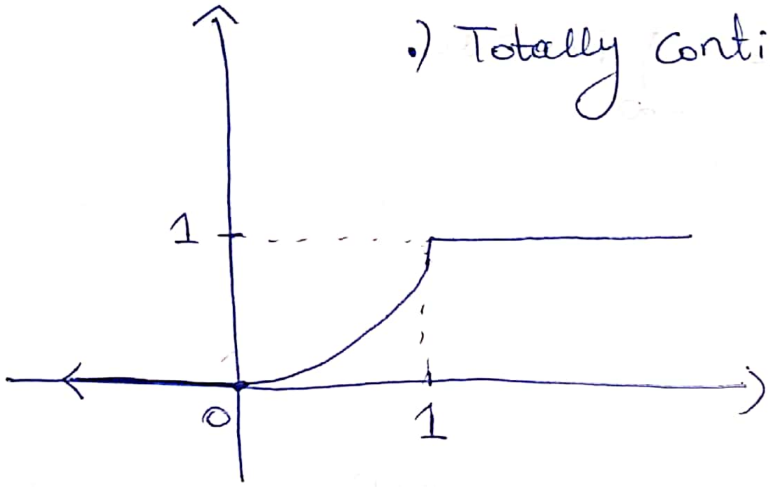
$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^2 & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$



•) Totally Continuous

A) $\boxed{Pr\{c \le x \le d\} = F(d) - F(c)}$

Q.) $f(x) = \begin{cases} Ke^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$ λ is Known Constant.

Find K. Find F and sketch it

•) K: $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \int_{0}^{\infty} Ke^{-\lambda x}\,dx = 1$

$\Rightarrow K\left(\frac{e^{-\lambda x}}{-\lambda}\right)_0^\infty = 1$

$\Rightarrow \frac{K}{-\lambda}\left[0 - 1\right] = 1 \quad \Rightarrow \boxed{K = \lambda}$

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•) $F(x) = 0$ if $x \leq 0$

•) If $x > 0$, $F(x) = \int_{-\infty}^{x} f(u)\,du$

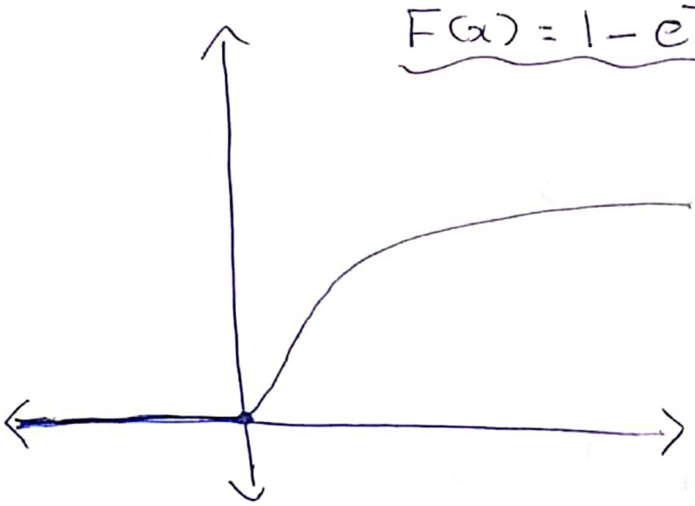
$$= \lambda \int_{0}^{x} e^{-\lambda u}\,du$$

$$= \lambda \left[\frac{e^{-\lambda u}}{-\lambda} \right]_0^x$$

$$\Rightarrow \frac{\lambda}{-\lambda} \left[e^{-\lambda x} - 1 \right]$$

$$\Rightarrow \boxed{1 - e^{-\lambda x}}$$

$F(x) = 1 - e^{-\lambda x}$



Note :) •) $\boxed{F'(x) = f(x)}$

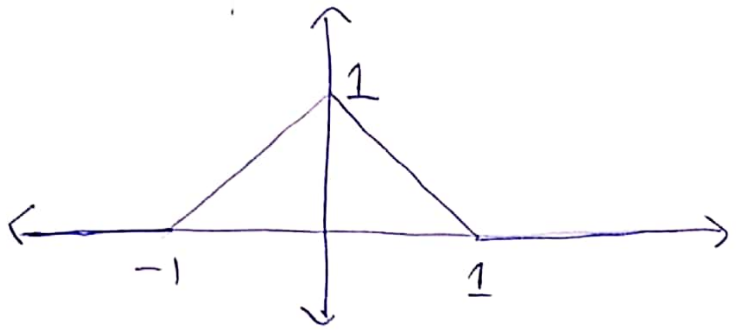
•) $\boxed{Pr(c \leq X \leq d) = F(d) - F(c)}$

Q.) $f(x) = \begin{cases} 1-|x| & \text{if } |x|<1 \\ 0 & \text{elsewhere} \end{cases}$

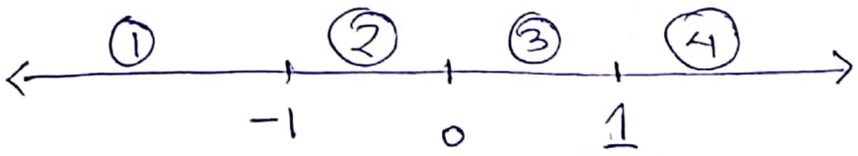
$\therefore \quad f(x) = \begin{cases} 1+x & -1<x\leq 0 \\ 1-x & 0\leq x<1 \\ 0 & \text{otherwise} \end{cases}$

Find probability distribution function.

•)



•)



$\Rightarrow$ •) If $x \leq -1$, $F(x) = 0$

•) If $-1 \leq x \leq 0$, $F(x) = \int_{-1}^{x} (1+u)\,du$

$$= \left(u + \frac{u^2}{2}\right)_{-1}^{x}$$

$$= \left(x + \frac{x^2}{2}\right) - \left(\frac{1}{2} - 1\right)$$

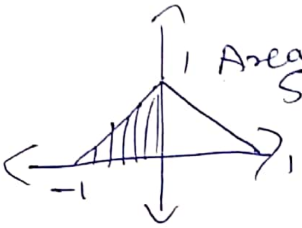
$$= x + \frac{x^2}{2} + \frac{1}{2} \Rightarrow \frac{x^2+2x+1}{2}$$

$$= \frac{(x+1)^2}{2}$$

$$\therefore F(0) = \frac{1}{2}.$$

.) If $0 \le x \le 1$ then $F(x) = \int_{-\infty}^{x} f(u)\,du$

$$= \left[\int_{-\infty}^{0} + \int_{0}^{x} f(u)\,du\right]$$



Area of Shaded

$$= F(0) + \int_0^x (1-u)\,du$$

$$= \frac{1}{2} + x - \frac{x^2}{2}$$

.) If $x \geq 1$, $F(x) = 1$.

Note: $F(x_1)$ gives area upto to x_1.

Eg: $F(0)$ gives $\frac{1}{2}$ i.e area upto $x=0$.

$$F(x) = \begin{cases} 0 & \text{if } x \le -1 \\ \frac{1}{2} + x + \frac{x^2}{2} & \text{if } -1 \le x \le 0 \\ \frac{1}{2} + x - \frac{x^2}{2} & \text{if } 0 \le x \le 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

$$Pr\{-0.2 < X < 0.7) = F(0.7) - F(-0.2)$$

$$= \left[\frac{1}{2} + (0.7) - \frac{(0.7)^2}{2}\right] - \left[\frac{1}{2} + (-0.2) + \frac{(-0.2)^2}{2}\right]$$

$\frac{1}{2} - 1)$

$\frac{{}^2 + 2x + 1}{2}$

Q.) Let X be the ~~no of years~~ thickness of washers in millimetres. Assume that X has the density $\boxed{f(x) = Kx, \text{ If } 0.9 < x < 1.1}$

$\boxed{= 0, \text{ otherwise}}$

Find K. What is the probability that a washer will have any thickness between $0.95 < x < 1.05$ mm.

Solution:- K: $\int_{-\infty}^{\infty} f(x) = 1$

$\Rightarrow \int_{0.9}^{1.1} Kx = 1$

$\Rightarrow \frac{K}{2} (x^2)_{0.9}^{1.1} = 1$

$\Rightarrow \frac{K}{2} (1.1-0.9)(1.1+0.9) = 1$

$\Rightarrow K = \cancel{2} \times \frac{\cancel{10}^{5}}{\cancel{2}} \times \frac{1}{\cancel{2}}$

$= 5$

(ii) $Pr(0.95 < X < 1.05) = F(1.05) - F(0.95)$

$F(x) = \int_{-\infty}^{x} 5x = 5\int_{0.9}^{x} x\,dx = \frac{5}{2}(x^2)_{0.9}^{x}$

$= \frac{5}{2}(x^2 - 0.81)$

thers

n

5)

a
2)
0.9

- 0.81)

$\therefore F(1.05) - F(0.95)$

$$= \frac{5}{2}\left[(1.05^2 - 0.81) - ((0.95)^2 - 0.81\right]$$

$$= \frac{5}{2}\left[(1.05 + 0.95)(1.05 - 0.95)\right]$$

$$= \frac{\cancel{5}}{\cancel{2}} \times \cancel{2} \times \frac{1}{10_2} = \underline{0.5}$$

Q.) Find the probability that none of the 3 bulbs in a traffic signal will have to be replaced during the 1500 hours of operation if the lifetime X of a bulb is a random variable

$$f(x) = \begin{cases} 6\left[0.25 - (x-1.5)^2\right] & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

where x is measured in multiples of thousand hours

Solution: If $p = Pr\{$ No failure of a bulb in 1st 1500 hours$\}$

Ans: p^3

$$Pr\{X > 1.5\}$$

$$= 1 - Pr\{X \leq 1.5\}$$

$$= 1 - 6\int_1^{1.5}\left[0.25 - (x-1.5)^2\right]dx$$

$\therefore \Pr\{X > 1.5\}$

$$= 6\int_{1.5}^{2} [0.25 - (x-1.5)^2]\,dx \qquad y = x - 1.5$$

$$= 6\int_{0}^{0.5} [0.25 - y^2]\,dy.$$

$$= 6\left[(0.25)y - \left(\frac{y^3}{3}\right)\right]_0^{0.5}$$

$$= \overset{2}{\cancel{6}}\left[\frac{0.25 \times 2}{\cancel{3}}\right]$$

$$\Rightarrow \underline{0.5} \qquad \underline{p^3 = (0.5)^3}$$

Q) Suppose that in an automatic process of filling oil into cans, the content of a can (in gallons) ~~is a random variable~~ $Y = 100 + X$, where X is a random variable with density

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $F(x)$.

•) In a lot of 1000 cans about how many will contain 100 gallons or more.

•) what is the probability that a can will contain

→ less than 99.5 gallons.

→ less than 99 gallons

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Solution :) $Pr\{Y \geq 100\}$

$= Pr\{X \geq 0\} = \frac{1}{2}$

Aliter :) OR put y in place of x, change range.

(c) $Pr\{\cancel{y} < 99.5\}$

$\Rightarrow Pr\{X < -0.5\}$

$$= \int_{-1}^{-0.5} (1+x)\,dx = \left(x + \frac{x^2}{2}\right)_{-1}^{-0.5}$$

$= \underline{0.125}$

(c) $Pr\{\cancel{y} < 99\}$

$\Rightarrow Pr\{X < -1\}$

$= 0$

$|Y - 100| < 1$

$Y - 100$

$1 - Y + 100$

Mean, Variance and expectation :-

X: Mark

x_1

x_2

$\vdots$

x_n

$\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

X: Random variable representing marks

x: An arbitrary mark.

•)

x	$f(x)$
x_1	$1/n$
x_2	$1/n$
$\vdots$	$\vdots$
x_n	$1/n$

$$\sum_{i=1}^{n} x_i f(x_i) = \bar{x}$$

•)

x	f	$f(x)$
x_1	f_1	f_1/N
x_2	f_2	f_2/N
$\vdots$	$\vdots$	$\vdots$
x_n	f_n	f_n/N

N = No of students

Discrete Frequency distribution.

$$\rightarrow \sum_{x} x f(x) = \frac{\sum_{i=1}^{N} x_i f_i}{N} = \bar{x}$$

$$\rightarrow \frac{1}{N} \sum_{i=1}^{n} f_i = 1.$$

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•)

x^2	$f(x)$
x_1^2	f_1/N
x_2^2	f_2/N
$\vdots$	$\vdots$
x_n^2	f_n/N

$$\sum_x x^2 f(x)$$

$$= \sum_{i=1}^{n} x_i^2 f(x_i)$$

$$= \frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 = \text{Mean of } x^2$$

$$= \overline{x^2}$$

$$\boxed{\begin{aligned}\sigma^2 &= \frac{1}{N}\sum_{i=1}^{N} f_i(x_i-\bar{x})^2 \\ &= \sum_{i=1}^{n}(x_i-\bar{x})^2 f(x_i) \\ &= \text{Mean of } (x-\bar{x})^2\end{aligned}}$$

σ = Standard deviation

σ^2 = Variance

General: $\boxed{\sum_{i=1}^{n} x_i^n f(x_i) \quad \text{Mean of } x^n}$

•) E(X) (Expectation value of X):-

↳ like Arithmetic mean.

P. mass fnc

If X is discrete, we define $\boxed{E(X) = \sum_x x f(x)}$

↳ Total range of x.

If X is continuous, we define $\boxed{E(X) = \int_{-\infty}^{\infty} x f(x)\,dx}$

P. density fnc

Mean of a ~~pro~~ Random Variable is denoted by 'μ' where E(x) = μ.

•) Mean of $(X-\mu)^2$ is called the Variance denoted by σ^2

Thus; $\sigma^2 = \sum_x (x-\mu)^2 f(x)$ ← <u>Discrete</u>

If continuous; $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$

σ is standard deviation.

•) Notation: $E(x^3)$ is denoted by μ_3' or m_3.

$E(x^r)$ is denoted by μ_r' or m_r.

where $E(x^r)$ or m_r or μ_r' is called the r^{th} moment about origin.

$$\boxed{m_r = E(x-a)^r \rightarrow r^{th} \text{ moment about } a.}$$

$\mu_r = E[(X-\mu)^r]$ is called the r^{th} central moment.

<u>Note:-</u> $m_1 = \mu$

$\mu_2 = \sigma^2$

•) more σ^2, more scattered.

•) less σ^2, less scattered.

Q.) $f(x) = Kx^3$, $x = 0,1,2,3,4,5$

$= 0$ otherwise

Find μ, σ^2

Soln :) $K : \sum_x f(x) = 1$

$\Rightarrow K\left(\frac{5 \cdot 6}{2}\right)^2 = 1 \quad \Rightarrow K = \frac{4}{25 \cdot 36 \cdot 9} = \frac{1}{225}$

1
16
81
256
625

•) $\mu = \sum_x x f(x)$

$= \frac{1}{225} \sum_{x=0}^{5} x(x^3)$

$= \frac{1}{225} \sum_{x=0}^{5} x^4$

$= \frac{1}{225} [1^4 + 2^4 + 3^4 + 4^4 + 5^4] = \underline{4.3}$

•) $\sigma^2 = \sum_x (x-\mu)^2 f(x)$

$= (0-4.3)^2 \times 0 + (1-4.3)^2 \times \frac{1}{225}$

$+ (2-4.3)^2 \cdot \frac{8}{225} + (3-4.3)^2 \frac{27}{225}$

$+ (4-4.3)^2 \frac{64}{225} + (5-4.3)^2 \frac{125}{225}$

$= 0.082$

$$E(2^x) = \sum_x 2^x f(x)$$

$$= \sum_x 2^x \cdot \frac{x^3}{225}$$

$$= \frac{2}{225} + 4 \times \frac{8}{225} + \frac{8 \times 27}{225} + 16 \times \frac{64}{225} + 32 \times \frac{125}{225}$$

$$= \underline{23.44}$$

Q.) $F(x) = Kx^2$ if $1 \le x \le 2$

0 elsewhere

Find K, μ & σ^2.

Soln:) $K: \int_{-\infty}^{\infty} f(x)dx = 1$

$$\Rightarrow \int_1^2 Kx^2 dx = 1 \quad \Rightarrow K\left(\frac{x^3}{3}\right) = 1$$

$$\Rightarrow K\left[\frac{8-1}{3}\right] = 1$$

$$\Rightarrow \boxed{K = \frac{3}{7}}$$

$F(x) = \frac{3x^2}{7}$ $1 \le x \le 2$

0 elsewhere.

$$\cdot) \mu = \frac{3}{7}\int_{-\infty}^{\infty} x^3 dx = \frac{3}{7}\int_1^2 x^3 dx = \frac{3}{7}\cdot\left(\frac{x^4}{4}\right)_1^2$$

$$= \frac{3}{7}\left[\frac{16}{4} - \frac{1}{4}\right]$$

$$= \frac{3}{7}\times\frac{15}{4} = \frac{45}{28} = \underline{1.607}$$

$$\cdot) E(x^2) = \frac{3}{7}\int_{-\infty}^{\infty} x^4 dx = \frac{3}{7}\left(\frac{x^5}{5}\right)_1^2 = \underline{2.657}$$