NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA

End Semester Examination, Spring 2022

Subject: Formal Languages and Automata Theory   Subject Code: CS2004   Full Marks: 50

Time: 3 hours   Number of pages: 2

Questions 1 – 10 are mandatory. Any marks fetched in Question 11 will be added to the secured marks and scaled to 50. Mere answer without justification will not fetch any mark.

A Turing machine M can be either defined by a 7 tuple $(Q, \sum, \tau, \delta, q_0, \Box, F)$ or $(Q, \sum, \tau, \delta, q_0, q_{accept}, q_{reject})$, where Q is the set of states, $\sum$ is the set of input alphabet, τ is the set of tape symbols, blank symbol $\Box(or \sqcup) \in \tau$, $\sum \subset \tau$, $\delta : Q \times \tau \to Q \times \tau \times \{L, R\}$ is the transition function, q_0 is the start state, F denotes the set of final states in the first model, q_{accept} denotes the single final state and q_{reject} denotes the single reject state in the second model.

1. Construct an LBA (Linear Bounded Automata) that accepts the following language $L_1 = \{a^n b^n c^n : n \geq 1\}$ (Recall that a Linear bounded automata is a special type of Turing machine). Define the LBA as a Turing machine and draw the state transition diagram for LBA recognizing L_1. 5 marks

2. Construct a Turing machine for the following language.
$L_2 = \{w\#w : w \in \{0, 1\}^*, \#$ is a special tape symbol.$\}$. 5 marks

3. Define a Nondeterministic Turing machine. Prove that any Nondeterministic Turing machine has the same computation power as a Deterministic Turing machine (you may assume that any Deterministic Turing machine has the same computational power as a Multidimensional Turing machine). 5 marks

4. Describe the procedure of encoding done by the Universal Turing machine to encode any arbitrary Turing machine. 5 marks

5. Consider the following computational problem. 5 marks
Input: A Turing machine M, a string w.
Question: Decide whether M halts while processing w.
The corresponding language is given by
$H_{TM} = \{\langle M, w\rangle : M$ is a Turing machine that halts on string $w\}$.
Prove that H_{TM} is undecidable (Use the fact that there exists a language that is Turing acceptable whose complement is not Turing acceptable).

6. Construct a PDA for the following language. 5 marks
$L_6 = \{vv^R : v \in \{a, b\}^*, v^R$ denotes the reverse of the string $v\}$.
Can a Deterministic PDA accept L_6? Construct a Deterministic PDA that accepts $RL_6 = \{vcv^R : v \in \{a, b\}^*, v^R$ denotes the reverse of the string $v, c \neq a, c \neq b\}$.

7. Are the class of Determisitic Context-free Languages closed under union? Justify your answer with a suitable example. (You may either give a formal proof that DCFL's are closed under union or give a counter example showing that they are not closed. In the latter case, you must prove that the two input languages are DCFL and prove that their union is not a DCFL. You may assume that $a^n b^n c^n, n \geq 1$, is not context free). 5 marks

8. Differentiate between Regular, Context-free, Context-sensitive and Unrestricted grammar, with suitable examples. Design a DFA D for the following language. 5 marks
$L_8 = \{w \in \{a, b\}^* : |w| \geq 2$, and the second to last symbol from right of w is $b\}$.
For instance, D accepts $aabb$, but rejects $abab$.

9. Consider the following grammar $G = (V, T, R, S)$: 5 marks
$V = \{S, A, B\}$ is the set of variables;
$T = \{a, b\}$ is the set of terminals;
S is the start variable;
R is the set of production rules as given below.
$S \to ASA|aB$
$A \to B|S$
$B \to b|\epsilon$.
Convert G to an equivalent grammar H in Chomsky Normal Form.

10. (a) Prove that $L_{10a} = \{ww : w \in \{a, b\}^*\}$ is not regular. 2.5 marks

(b) Write the context free grammar that generates the following language.
$L_{10b} = \{a^i b^j c^k : i, j, k \geq 0, i = k \text{ or } i = j\}$. 2.5 marks

11. Consider the computational problem of deciding whether 5 marks
$L(M)$ is regular given that M is a Turing machine. The corresponding language be $Regular_{TM}$.
$Regular_{TM} = \{\langle M \rangle : M \text{ is a Turing machine that accepts a regular language}\}$.
Prove that $Regular_{TM}$ is undecidable.
(Hint. Use techniques similar to one used to prove that $Empty_{TM}$ is undecidable.)

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