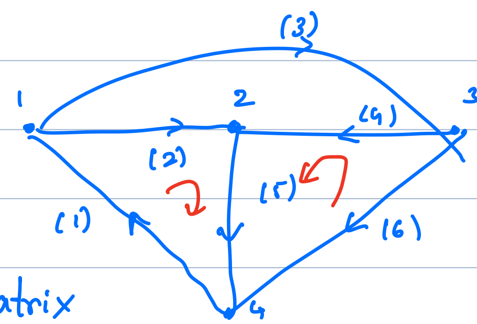
Last class: (i) Incidence matrix

(ii) Nodal Analysis

[Tutorial] – No tutorial this week.

Tutorial sessions are "back" to Wed 5–6 pm.

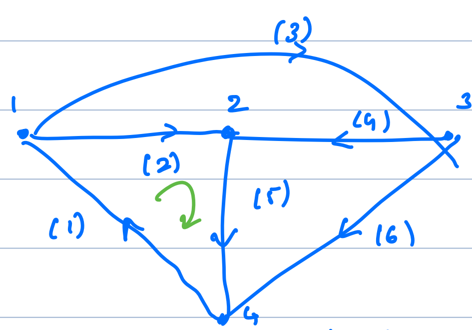


Circuit Matrix

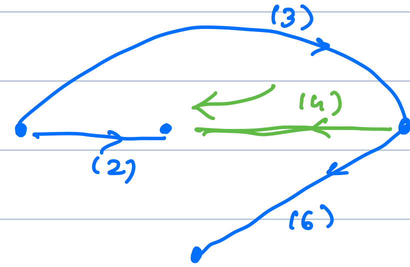
B_a ← all

$$B_a = \begin{array}{c|cccccc|c} & (1) & (2) & (3) & (4) & (5) & (6) & \\ \hline \text{Loop 1} & 1 & 1 & 0 & 0 & 1 & 0 & \text{Loop 1} \\ \text{Loop 2} & & & & & & & \\ \vdots & & & & & & & \\ \text{Loop } l & & & & & & & \end{array}$$

$$\underset{i^{th}\text{ loop} \quad j^{th}\text{ branch}}{b_{ij}} = \begin{cases} 0, & \text{if the loop } i \text{ doesn't contain branch } j \\ 1, & \text{branch same direction as loop} \\ -1, & \text{opposite direction.} \end{cases}$$



b branches



(n–1) branches

Fundamental circuit matrix (B_f) $\left(\begin{array}{c} b-(n-1) \\ \text{loops} \end{array} \right)$

$$B_f = \begin{array}{c} \\ f\text{-}c\text{-}1 \\ f\text{-}c\text{-}2 \\ f\text{-}c\text{-}3 \end{array} \begin{array}{c} \begin{array}{cccccc} (1) & (2) & (3) & (4) & (5) & (6) \end{array} \\ \left[\begin{array}{cccccc} 0 & -1 & 1 & 1 & 0 & 0 \\ & & & & & \\ & & & & & \end{array} \right] \end{array}$$

$\to$ Circuit obtained by putting one of the links in the tree (f - circuit)

$b-(n-1)$

Properties of B_f

$$B_f = \begin{array}{c} \begin{array}{ccc|ccc} (2) & (3) & (6) & (4) & (1) & (5) \end{array} \\ \left[\begin{array}{ccc|ccc} & & & 1 & 0 & 0 \\ & & & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array} \right] \\ \begin{array}{cc} \leftarrow \text{tree} \rightarrow & \leftarrow \text{links} \rightarrow \end{array} \end{array} \begin{array}{c} \\ f\text{-}c\text{-}1 \ (4) \\ f\text{-}c\text{-}2 \ (1) \\ f\text{-}c\text{-}3 \ (5) \\ \\ \end{array}$$

B_{ft} $\qquad$ $B_{fl} = I_{b-(n-1)}$

$\text{rank}\,(B_{fl}) = b-(n-1)$

$\text{rank}\,(B_f) \geq b-(n-1)$

Prop (Proof Skip) $\text{rank}\,(B_f) = b-(n-1)$

KVL equations:

$v(t) = [\,v_{(1)}(t) \quad v_{(2)}(t) \quad \cdots \quad v_{(b)}(t)\,]$

branch voltages

$B_f \cdot v(t) = 0$ [KVL in matrix form].

$\boxed{B_a \cdot v(t) = 0}$

$$\left[\leftarrow \text{loop } 1 \rightarrow \right] \left[\updownarrow \right] = 0$$

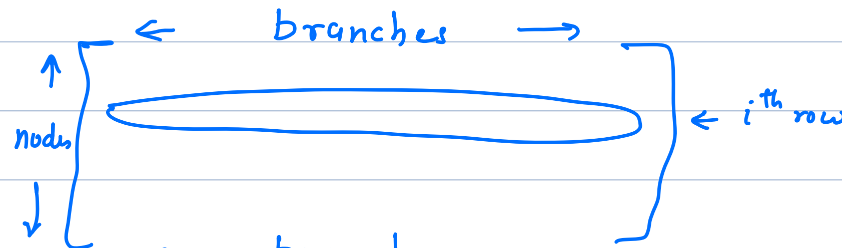
branch voltages

KCL equations : $A \, i(t) = 0$

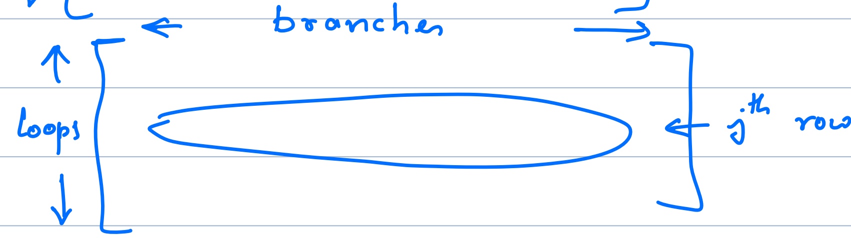
in terms of B matrix?

[Relation b/w A & B matrix]

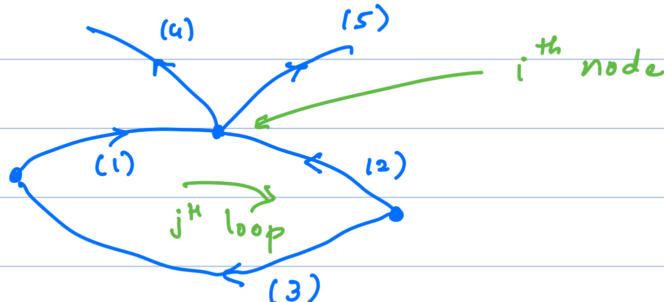
$A =$



$B =$



$A_i^T \; B_j$



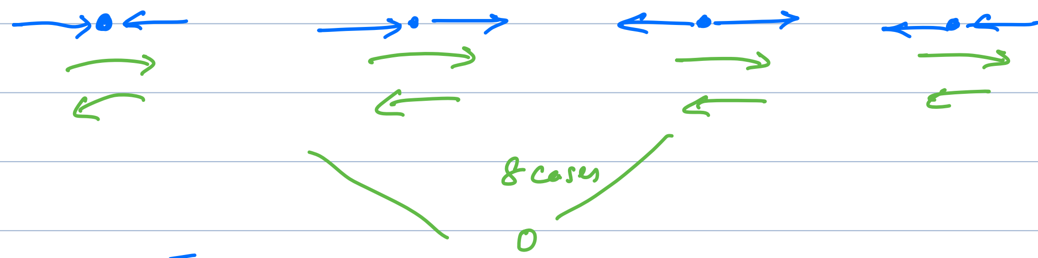
$$A_i = \begin{array}{c} \begin{matrix} (1) & (2) & (3) & (4) & (5) \end{matrix} \\ \begin{bmatrix} -1 & -1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$B_j = \begin{bmatrix} 1 & -1 & 1 & 0 & 0 \end{bmatrix}$$

$$A_i \; B_j^T = 0 \; //$$

Key idea: Any loop, for any node
there would be 2 branches
All other branches are either
not incident at that node. or

not in the loop



$$A_i \; B_j^T = 0$$

Property:

$$A B^T = 0$$

$$A_a \; B_a^T = 0$$

$$A \; B_f^T = 0 \quad \leftarrow$$ Orthogonality relation between A & B matrix

$$\dim(A) = (n-1) \times b$$

$$\dim(B_f) = b-(n-1) \times b$$

$$B_f = [\, B_{ft} \quad B_{fl} \,]$$

$$= [\, B_{ft} \quad I \,]^I$$

$$A = [\, A_t \quad A_l \,]$$

$$[\, A_t \quad A_l \,] \begin{bmatrix} B_{ft}^T \\ I \end{bmatrix} = 0$$

$$A_t \; B_{ft}^T + A_l = 0$$

$$B_{ft} = -(A_t^{-1} A_l)^T \; //$$

KCL using B matrix

$$A \quad i(t) = 0$$

$$\begin{bmatrix} A_t & A_l \end{bmatrix} \begin{bmatrix} i_t \\ i_l \end{bmatrix} = 0$$

$$A_t \, i_t + A_l \, i_l = 0$$

$$i_t = -\boxed{A_t^{-1} A_l}\, i_l$$

$$= B_{ft}^T \, i_l \quad //$$