Last class : Basic terminology of graphs : Path, Loop
Incidence matrix, Tree

Tree

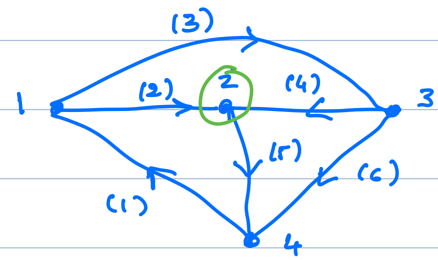
$$A = [\, A_t \,\vdots\, A_\ell \,]$$

$$\dim(A_t) = (n-1) \times (n-1)$$

$$\dim(A_\ell) = (n-1) \times (b-(n-1))$$

Property: $\text{Rank}(A_t) = n-1$ [Invertible]

KVL & KCL



$$A = \begin{array}{c} 1 \\ 2 \\ 3 \\ \\ \\ \end{array} \begin{array}{c} \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{bmatrix} \\ \begin{array}{cccccc} (1) & (2) & (3) & (4) & (5) & (6) \\ i_{(1)} & i_{(2)} & i_{(3)} & i_{(4)} & i_{(5)} & i_{(6)} \end{array} \end{array}$$

KCL : At any node, the algebraic sum of currents is 0.

KCL (Node 2) : $i_{(2)} + i_{(4)} - i_{(5)} = 0$

KCL equations in Graph Theory (GT)

$$A \cdot i = 0$$

i = Coloumn vector containing branch currents

$$A\, I(s) = 0$$

[KCL in Laplace domain in GT]

$$A = [A_t \quad A_\ell]$$

$$i = \begin{bmatrix} i_t \\ i_\ell \end{bmatrix}$$ i_t → Branch currents (tree); i_ℓ → Branch currents (links)

KCL:

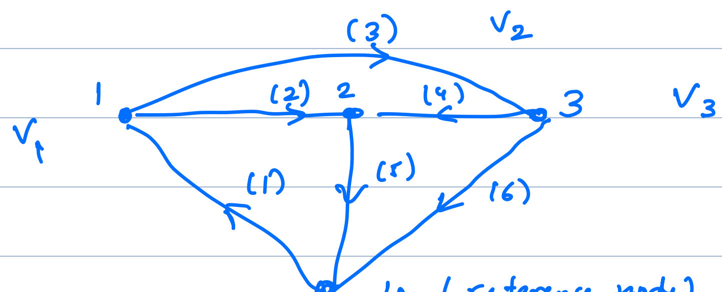
$$[A_t \quad A_\ell] \begin{bmatrix} i_t \\ i_\ell \end{bmatrix} = 0$$

$$A_t \cdot i_t + A_\ell\, i_\ell = 0$$

$$i_t = -A_t^{-1} A_\ell\, i_\ell \quad //$$

Nodal Analysis

[Figure out the node voltages]



4 (reference node) $v_4 = 0$

$$A = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & +1 & 0 & 1 \end{bmatrix} \quad \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$v_n = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

v_n ↗ node voltage

Voltage across branch (2) $= v_1 - v_2$

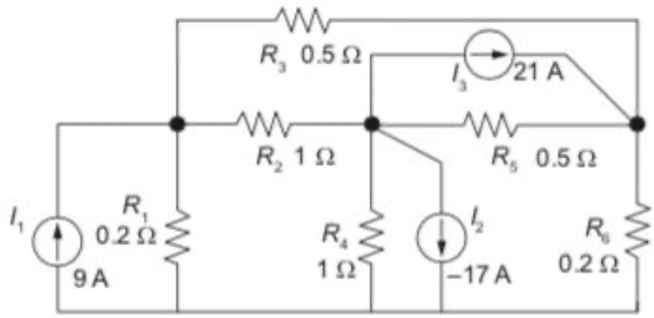
$$v = A^T v_n$$

v (branch voltages)

Example 17.4.1 in SK.

Case: Independent current sources

Independent voltage source (cut-sets)

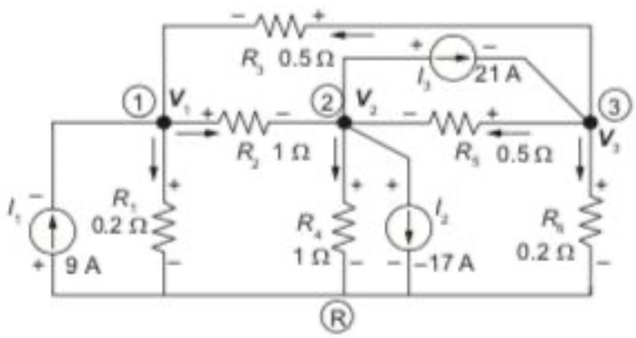


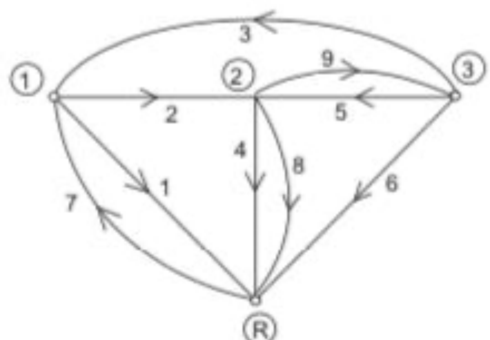
Step 1: Figure out the graph corresponding to this ckt

Independent current sources

$$A = \left[\quad \vdots \quad \right]$$

$Y = \text{diag}(5, 1 \ldots\ldots)$





$$\begin{array}{c} \\ \mathbf{A} = \end{array} \begin{array}{c} \begin{array}{ccccccccc} (1) & (2) & (3) & (4) & (5) & (6) & (7) & (8) & (9) \end{array} \\ \left[\begin{array}{cccccc:ccc} 1 & 1 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & -1 \end{array}\right] \end{array} \begin{array}{c} \\ ① \\ ② \\ ③ \end{array}$$

A_p $\qquad$ A_g

← Passive elements → ← Current source →

KCL equations: $A\, i = 0$

$$\begin{bmatrix} A_p & A_g \end{bmatrix} \begin{bmatrix} i_p \\ i_g \end{bmatrix} = 0$$

Is A_p invertible?

Known

$$i_g = \begin{bmatrix} 9 \\ 21 \\ -17 \end{bmatrix}$$

Node voltages: $V_n = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$

$$v = A^T v_n$$

branch voltages

$$\begin{bmatrix} v_p \\ v_g \end{bmatrix} = [A_p \quad A_g]^T v_n$$

$$v_p = A_p^T v_n$$

R (Branch current) = Branch voltage

ωL

$1/j\omega C$ For all passive elements:

Branch current = Branch admittance × Branch voltage.

$$i_{(1)} = Y_1 V_1$$
$$i_{(2)} = Y_2 \cdot V_2$$
$$\vdots$$
$$i_p = Y \, V_p$$

branch voltages (passive)

branch current (passive)

$$Y = \text{diag}(Y_1, Y_2, \ldots)$$

$$= \begin{bmatrix} Y_1 & & 0 \\ & Y_2 & \\ 0 & & \ddots \end{bmatrix}$$

$$[A_p \quad A_g] \begin{bmatrix} i_p \\ \vdots \end{bmatrix} = 0 \quad \text{(KCL)}$$

i_g

$$\begin{bmatrix} A_p & A_g \end{bmatrix} \begin{bmatrix} Y \cdot v_p \\ i_g \end{bmatrix} = 0$$

$$\begin{bmatrix} A_p & A_g \end{bmatrix} \begin{bmatrix} Y \cdot A_p^T v_n \\ i_g \end{bmatrix} = 0$$

$$A_p Y \cdot A_p^T v_n + A_g i_g = 0$$

$$v_n = -(A_p Y A_p^T)^{-1} A_g i_g$$

↳ Equation for node voltages.

Exercise: Please work out for Example 17.4.1 in SK

$(A_p Y \cdot A_p^T)$ - Invertible? [Formal proof textbook]

$(n-1) \times p \quad p \times p \quad p \times (n-1)$

$\text{rank}(A) = (n-1)$