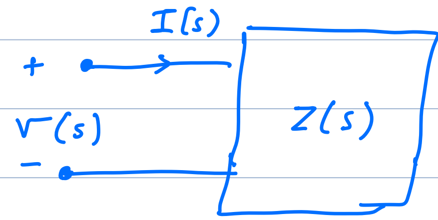
Announcements: Quiz 2 moved to Nov 20, 4-6pm (Sat)

Agenda: Stability of networks



$$Z(s) = \frac{V(s)}{I(s)}$$

$$V(s) = Z(s) \cdot I(s) \quad - (1)$$

$$I(s) = \frac{V(s)}{Z(s)} \quad - (2)$$

zeros

$$Z(s) = \frac{a_0}{b_0} \frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$$

poles

Poles of $Z(s)$

$\Rightarrow$ Zeros of $1/Z(s)$

Zeros of $Z(s)$ $\Rightarrow$ Poles of $1/Z(s)$

Stability conditions of zeros of $Z(s)$

= Stability conditions of poles of $Z(s)$

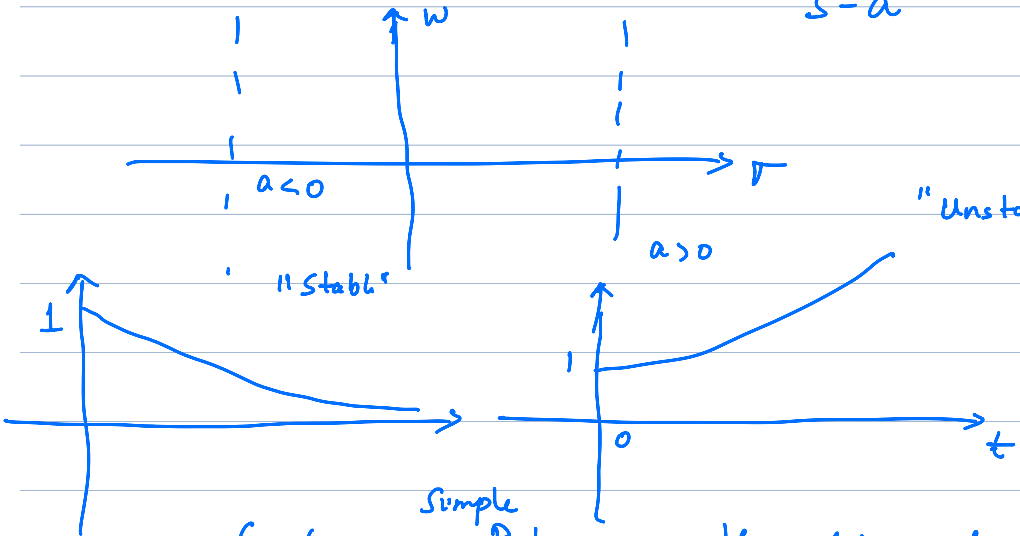
Assumptions (Network analysis)

$Z(s) = \frac{p(s)}{q(s)}$ $\Rightarrow$ Polynomials p and q has real coefficients

$\Rightarrow$ Roots are reals or they are complex conjugate of each other.

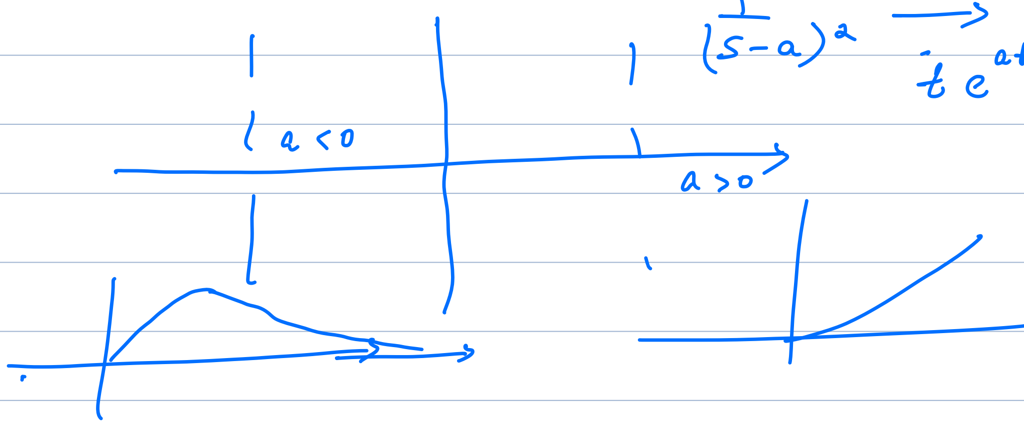
Simple Pole : Real

$$\frac{1}{s-a} \xrightarrow{\mathcal{L}^{-1}} e^{at}\,u(t)$$

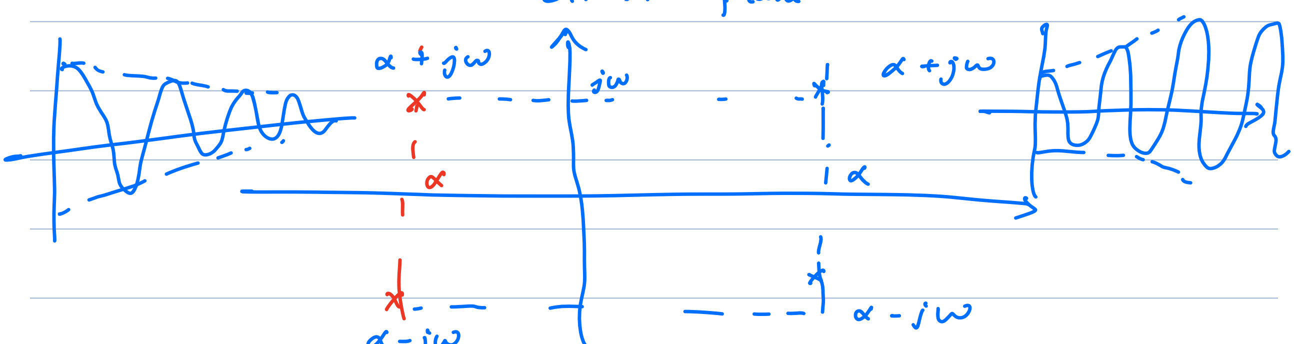


Conclusion : Simple Pole on the LH of s-plane for stability.

$$\frac{1}{(s-a)^2} \xrightarrow{\mathcal{L}^{-1}} t\,e^{at}\,u(t)$$



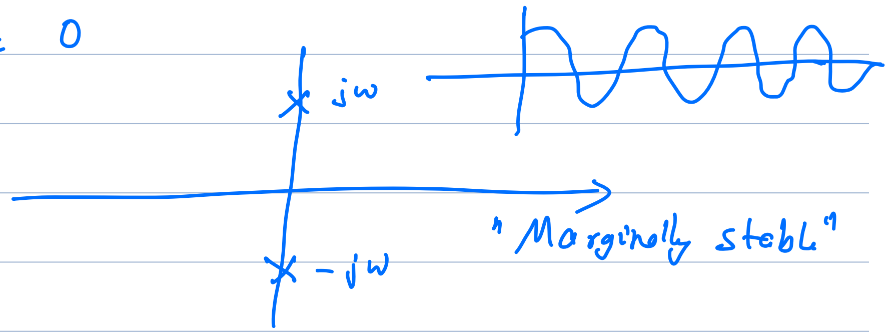
Conclusion : Poles with multiplicity should be LH of s-plane.



$$\mathcal{L}^{-1}\left[\frac{\omega}{(s-\alpha)^2+\omega^2}\right] \xrightarrow{\mathcal{L}^{-1}} e^{\alpha t}\cos(\omega t)$$

Complex conjugate poles should be on left half of s-plane

$\alpha = 0$



"Marginally stable"

Multiple jω poles

"Unstable"

$$\mathcal{L}^{-1}\left[\frac{s}{(s^2+\omega^2)^2}\right] = \frac{t}{2\omega}\sin(\omega t)$$

Conclusion: As long as poles on jω axis are not repeated, the network is marginally stable.

$$\mathcal{L}^{-1}\left[\frac{1}{(s^2+\omega_1^2)(s^2+\omega_2^2)}\right]$$

$$\longrightarrow a_0 \cos \omega_1 t + a_2 \cos \omega_2 t$$

"Marginally stable"

Necessary conditions for stability

1) Poles and zeros on the left half of s plane.

[If the real part is zero, then pole ore zero should be simple]

2)

Denominator: $(s + \underset{a>0}{a})[(s + \underset{\alpha>0}{\alpha})^2 + \underset{\beta^2>0}{\beta^2}]$ $\quad a > 0$, $\alpha > 0$

Poles on the left half of s plane.

All the coeffients of a stable network should be positive.

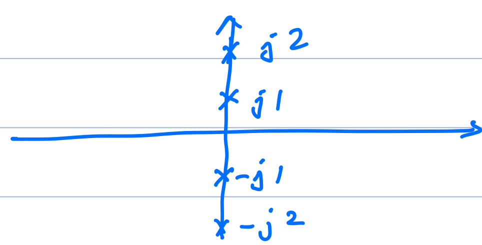
$a>0$, $b>0$, $\alpha>0$

$$(s+a)(s+b)[(s+\alpha)^2 + \beta^2]$$
$$= s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

Can we expect $a_3 = 0$?
$a_2 = 0$?
$a_1 = 0$?
$a_0 = 0$?

"No" : There is no cancellation with negative terms

3. All of the coeffients should be non-zero



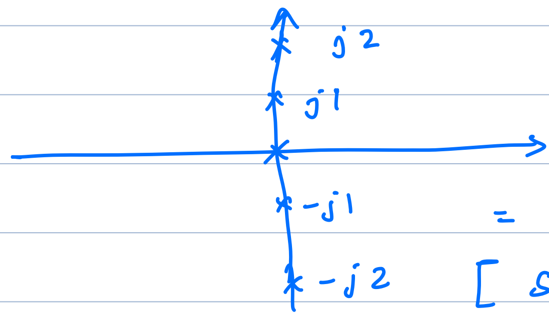
$$Z(s) = \frac{1}{(s^2+1)(s^2+4)}$$

$$(s^2+1)(s^2+4) = s^4 + 5s^2 + 4$$
$$(s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)$$

$a_3 = 0$, $a_1 = 0$

" Odd coeffionts are missing"

Exception: All odd coefficients are missing.



$$Z(s) = \frac{1}{s(s^2+1)(s^2+4)}$$

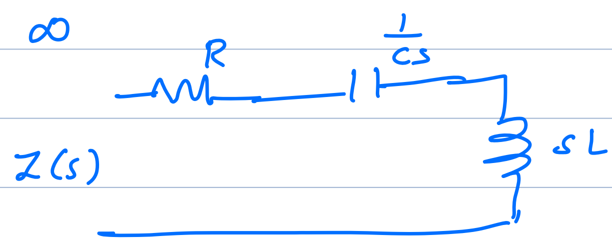
$$= s^5 + 5s^3 + 4s$$

$$[\, s^5 + a_0 s^4 + a_1 s^3 + a_2 s^2 + a_1 s + a_0 \,]$$

" Even coefficients are missing"

4. [Passive network]

$s \to \infty$



" Inductor dominates"

Any passive networks behaves like an inductor

$s \to 0$ " Capacitor dominates"

$$Z(s) = \frac{p(s)}{q(s)} \quad \boxed{\approx sL, R}$$

$s \to \infty$ $\quad Z(s) = sL, R$

$s \to 0$ $\quad Z(s) = \frac{1}{sC}, R$

$\Rightarrow$ Highest degree of $p(s)$ and $q(s)$ should differ by at most one.

$$Z(s) = \frac{p(s)}{q(s)} = \frac{s^m + a_1 s^{m-1} + \cdots}{s^n + b_1 s^{n-1} + \cdots}$$

$$s \to \infty \qquad Z(s) \approx \frac{s^m}{s^n} = s^{m-n}$$

sL, R

$m - n = 1$ or 0

Example: $$Z(s) = \frac{4s^4 + s^2 - 3s + 1}{s^3 + 2s^2 + 2s + 40}$$

"Unstable"

Violates two necessary condition: All coefficients should be positive

→ Coefficient of s^3 is missing