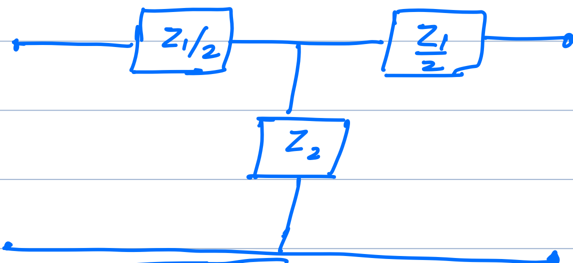
Announcements

1. No tutorial this week.
2. Tutorial time to be confirmed (Some difficulties associated with Friday slot)

   Wed 5-6pm → $\boxed{\text{Fri 3-4 pm}}$?

Review



$$Z_0 = \sqrt{Z_1 Z_2 + Z_2^2/4} \qquad Z_1 = j\omega L_1$$

$$\gamma = \alpha + j\beta \qquad Z_2 = j\omega L_2.$$

Case A

$$\sinh\left(\frac{\gamma}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

Z_1 & Z_2 same reactance.

$$\alpha = 2\sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}}, \quad \beta = 0$$

Case B:

Z_1 & Z_2 are of different reactance type.

$$Z_1(s) = sL \qquad Z_1(j\omega) = j\omega L$$

$$Z_2(s) = \frac{1}{sC} \qquad Z_2(j\omega) = \frac{1}{j\omega C}$$

$$\boxed{\frac{Z_1}{4Z_2}(j\omega)} = \frac{j\omega L}{4 \cdot \frac{1}{j\omega C}} = j^2 \frac{\omega^2 L C}{4} = \boxed{\frac{-\omega^2 L C}{4}}$$

$$\sqrt{\frac{Z_1}{4Z_2}} = j\left(\sqrt{\frac{\omega^2 LC}{L}}\right)$$

$$\sinh\left(\frac{\gamma}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}}$$

$\gamma = \alpha + j\beta$.

$$\sinh\left(\frac{\alpha + j\beta}{2}\right) = j\left|\sqrt{\frac{Z_1}{4Z_2}}\right|$$

$$\sinh\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) + j\cosh\left(\frac{\alpha}{2}\right)\underline{\sin}\left(\frac{\beta}{2}\right)$$

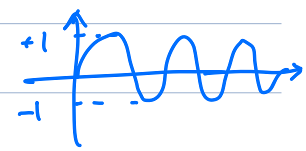
Case (i) $\qquad$ Case (ii) $\quad = j\left|\sqrt{\frac{Z_1}{4Z_2}}\right|$

$\sinh\left(\frac{\alpha}{2}\right) = 0 \qquad \cos\left(\frac{\beta}{2}\right) = 0$

Case (i) $\quad \sinh\left(\frac{\alpha}{2}\right) = 0$

$$\Rightarrow \alpha = 0$$

$$\underbrace{\cosh\left(\frac{\alpha}{2}\right)}_{1}\sin\left(\frac{\beta}{2}\right) = \left|\sqrt{\frac{Z_1}{4Z_2}}\right|$$



$$\sin\left(\frac{\beta}{2}\right) = \left|\sqrt{\frac{Z_1}{4Z_2}}\right|$$

$$\beta = 2\sin^{-1}\left(\left|\sqrt{\frac{Z_1}{4Z_2}}\right|\right)$$

$\alpha = 0$ (No attenuation)

$\beta = 2\sin^{-1}\left(\left|\underbrace{\sqrt{\frac{Z_1}{4Z_2}}}_{+ve}\right|\right)$ (depends on freq of operation)

$$-1 \le \underline{\frac{Z_1}{4Z_2}} < 0$$

Case (ii) $\quad \cos\left(\frac{\beta}{2}\right) = 0$

$$\frac{\beta}{2} = (2n-1)\,\pi/2 \qquad \beta = (2n-1)\pi$$

$$\beta = \pi,\ 3\pi,\ 5\pi \ldots$$

$$\beta = \pi$$

$$\cosh\left(\frac{\alpha}{2}\right)\ \underbrace{\sin\left(\frac{\beta}{2}\right)}_{\substack{\sin\left(\frac{\pi}{2}\right) \\ 1}} = \left|\sqrt{\frac{Z_1}{4Z_2}}\right|$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

$$\cosh\left(\frac{\alpha}{2}\right) = \left|\sqrt{\frac{Z_1}{4Z_2}}\right|$$

$$\alpha = 2\cosh^{-1}\left(\left|\sqrt{\frac{Z_1}{4Z_2}}\right|\right)$$

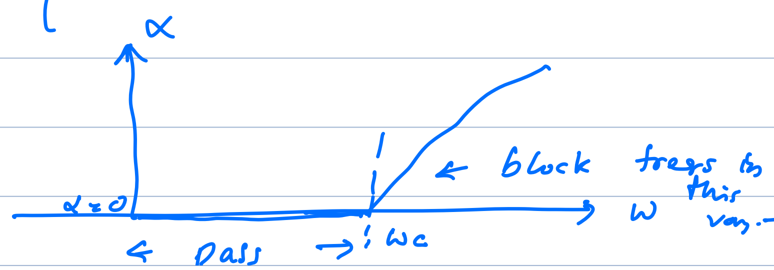
$$\frac{Z_1}{4Z_2} \leq -1$$

$$\frac{Z_1}{4Z_2} \leq -1 \qquad \alpha = 2\cosh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) \ \text{(depends on freq. ω)}$$

$$\beta = \pi$$

$-1 \leq \frac{Z_1}{4Z_2} < 0$	$\frac{Z_1}{4Z_2} \leq -1$
$\alpha = 0$	$\alpha = 2\cosh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right)$
$\beta = 2\sin^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right)$	$\beta = \pi$

LPF



frequencies in this range

At $w = w_c$: $\dfrac{Z_1(jw_c)}{4\, Z_2(jw_c)} = -1$

$Z_1(jw_c) + 4\, Z_2(jw_c) = 0$

Network Syntheses → Simple L & C

1) Constant k — $Z_1(jw_c) = jw_cL$

2) m - assumptions — $Z_2(jw_c) = \dfrac{1}{jw_cC}$

T-circuit