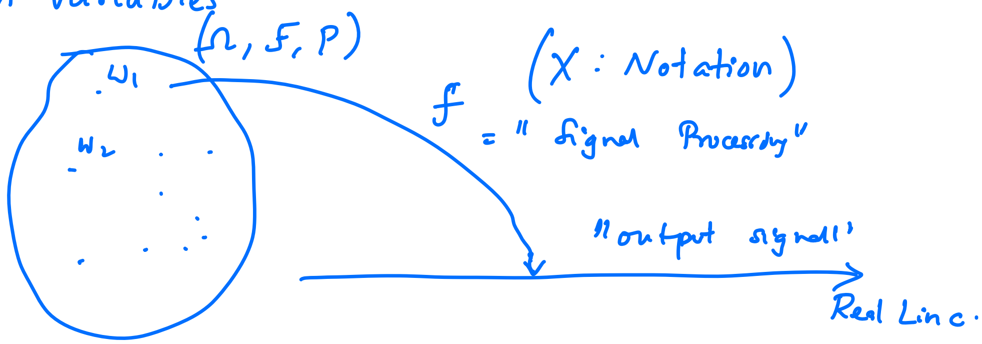
Last week - (i) Probability space - $(\Omega, \mathcal{F}, P)$

(ii) Discrete probability space

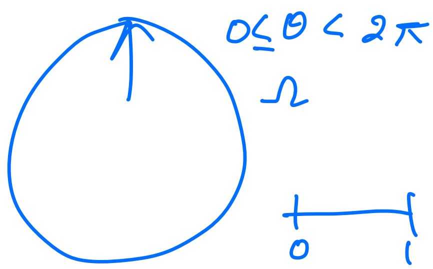
(iii) Conditional probabilities & Bayes Theorem

Quiz 1 - Oct 16 ? , Oct 14-15 ?

Random Variables



Example : Spin the bottle



$$X \quad f(\omega) = \begin{cases} +1 & 0 \leq \omega < \pi \\ -1 & \pi \leq \omega < 2\pi \end{cases}$$

" Binary Quantizer"

$$Y = \theta/2\pi$$

Distribution of random variable (X)

$$P_X(F) = P(X^{-1}(F)) = P(\{\omega : X(\omega) \in F\})$$

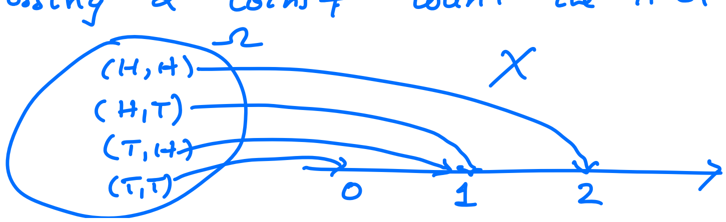
$F = \{1, 2\}$

Example : Spin the bottle + Binary Quantizer (X)

$$P_X(\{1\}) = P\{\omega : X(\omega) = 1\}$$

$$= \frac{1}{2}$$

Example : Tossing 2 coins + Count the # of heads (X)



$$P_X(\{1\}) = P(\{(H,T)\} \cup (T,H))$$
$$= P((H,T)) + P((T,H))$$
$$= 1/4 + 1/4$$
$$= 1/2$$

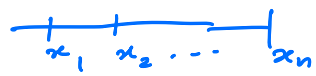
Special Case: Discrete Random Variable.

Probablity Mass Function

$$P_X(x) = P_X(\{x\})$$

$$F = \{x_1, x_2, \ldots x_n\}$$

$$P_X(F) = \sum_{x \in F} P_X(x)$$



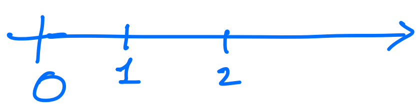
$$\sum_i P_X(x_i) = 1$$

Cumulative distribution of a random variable.

$$F_X(\alpha) = P_X((-\infty, \alpha])$$
$$= P(\{\omega : X(\omega) \leq \alpha\})$$

[Special Case of Discrete r.v $F_X(\alpha) = \sum_{k=-\infty}^{\alpha} P_X(k)$]

Eg: 2 Coin Throw + Count # of heads.



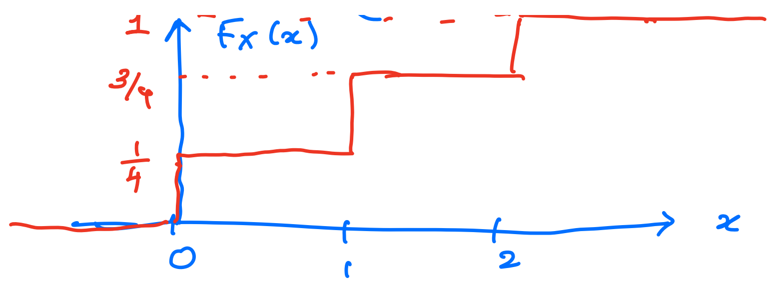
$$F_X(0) = P_X((-\infty, 0]) = \frac{1}{4}$$

$$F_X(\varepsilon) = P_X([-\infty, \varepsilon]) = 0 \qquad \varepsilon < 0$$

$$F_X(1) = P_X([-\infty, 1])$$
$$= P_X(\{0, 1\}) = 1/4 + \frac{1}{2} = 3/4$$

$$F_X(2) = P_X((-\infty, 2]) = 1.$$

$$F_X(3) = P_X([-\infty, 3]) = 1$$



Properties of $F_X(x)$

1A $F_X(+\infty) = 1$

$= P(X \in (-\infty, +\infty))$

$= 1$ //

1B. $F_X(-\infty) = 0$

$Pr(X \in (-\infty, -\infty)) = 0$

2. $F_X(x)$ is a non-decreasing function of x

$F_X(x_2) \geq F_X(x_1) \quad \forall \; x_2 \geq x_1$

$F_X(x_2) = Pr(X \in (-\infty, x_2])$

$= Pr(X \in (-\infty, x_1] \cup X \in [x_1, x_2])$

$= \underbrace{Pr(X \in (-\infty, x_1])}_{F_X(x_1)} + \underbrace{Pr(X \in [x_1, x_2])}_{\geq 0}$

$F_X(x_2) \geq F_X(x_1)$

3. $P_X(\alpha_1 < X \leq \alpha_2) = F_X(\alpha_2) - F_X(\alpha_1)$

Exercise! $(\alpha_2 \geq \alpha_1)$

(Pg 78)

"Standard" Discrete Random Variables

1. Bernoulli Random Variable $X \sim Ber(p)$

Range space is $\{1, 0\}$

$P_X(1) = p$ "Success"

$P_X(0) = 1-p$ "Failure"

$0 \leq p \leq 1$

Eg: Coin Toss $X \sim Ber(0.5)$

2. Binomial Random Variable $X \sim Bin(n, p)$

Range space is $\{0, 1, \ldots, n\}$ ↓ ~~+ve~~ integer

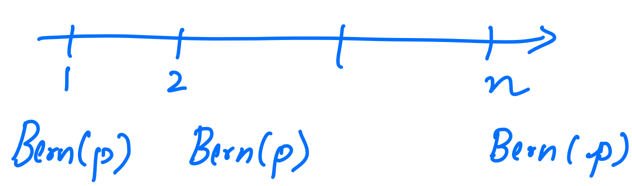
$P_X(i) = \binom{n}{i} p^i (1-p)^{n-i}$ $i = 0, 1, \ldots, n$

$n = 1$ $P_X(1) = \binom{1}{1} p^1 (1-p)^0 = p$

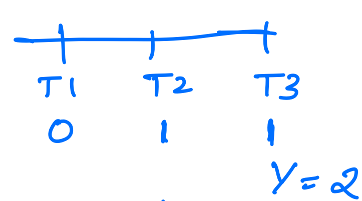
$P_X(0) = \binom{1}{0} p^0 (1-p)^1 = 1-p$

$Bin(1, p) = Bern(p)$

Connection between Bernoulli random variable & Binomial random variable



$n = 3$



Y = # of success from these n trials

$Y \sim Binomial(n, p)$

$n = 3$ $Y = 2$

$\binom{n}{i} p^i (1-p)^{n-i}$