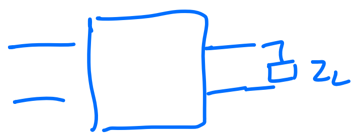
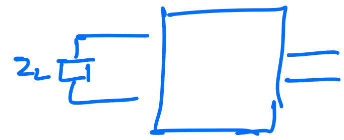
Last class





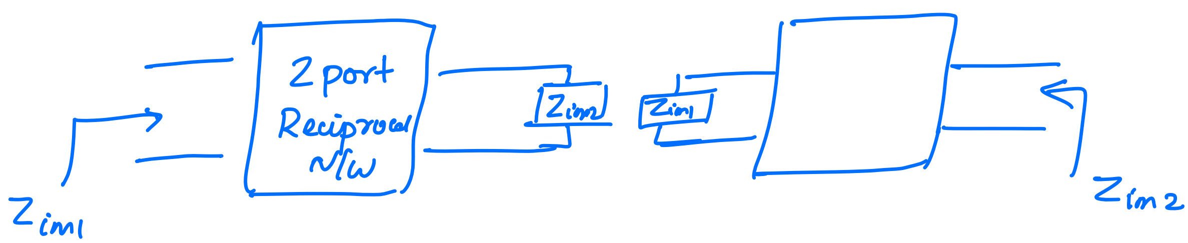
$$* \; Z_1 = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}} \qquad Z_2 = Z_{22} - \frac{Z_{12} Z_{21}}{Z_L + Z_{11}}$$

$$Z_{1o} = \frac{A}{C} \qquad Z_{2o} = \frac{D}{C}$$

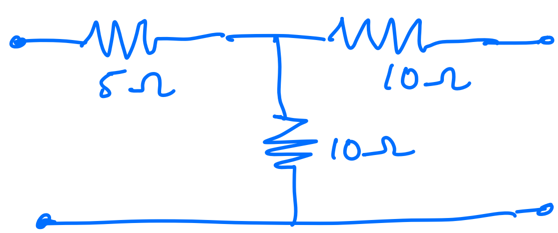
$$Z_{1s} = \frac{B}{D} \qquad Z_{2s} = \frac{B}{A}$$

Image Parameters (Reciprocal N/w)

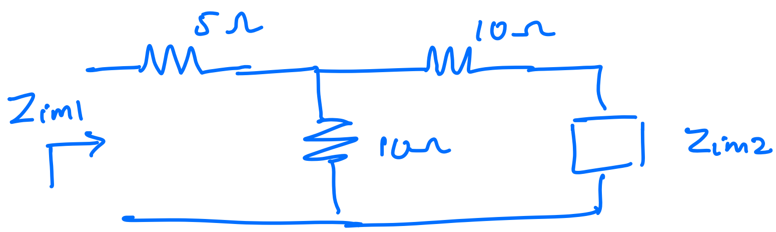
$$\underbrace{Z_{im1} \quad Z_{im2}}_{\text{Image impedance}} \qquad \underset{\sim}{\gamma} \quad \text{Image transfer constant}$$



Example:
(16.8.1 in SK)



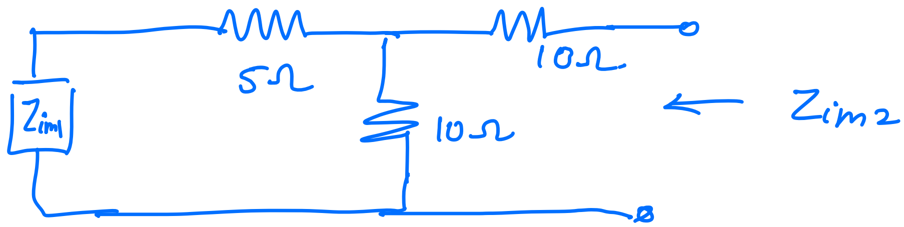
$$Z = \begin{pmatrix} 15 & 10 \\ 10 & 20 \end{pmatrix}$$



$$Z_{im1} = 5 + 10 \,\|\, (10 + Z_{im2})$$

$$Z_{im1} = 5 + \frac{10\,(10 + Z_{im2})}{20 + Z_{im2}}$$

$$Z_{im1}(20 + Z_{im2}) = 200 + 15\, Z_{im2} \quad -(1)$$



$$Z_{im2} = 10 + 10 \,//\, (5 + Z_{im1})$$

$$= 10 + \frac{10(5 + Z_{im1})}{15 + Z_{im1}}$$

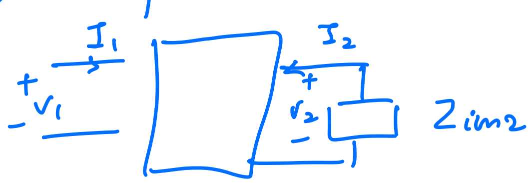
$$Z_{im2}(15 + Z_{im1}) = 200 + 20\, Z_{im1} \quad -(2)$$

(1) & (2)

$$Z_{im1} = 12.25\,\Omega$$

$$Z_{im2} = 16.33\,\Omega \quad //$$

Image impedance in terms of Transmission parameters

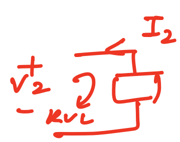


$$Z_{im1} = \frac{V_1}{I_1}$$

$$V_1 = A\, V_2 + B\,(-I_2)$$

$$I_1 = C\, V_2 + D\,(-I_2)$$

$$V_2 = -I_2\, Z_{im2} \; *$$



$$I_1 = C\, Z_{im2}\,(-I_2) + D\,(-I_2)$$

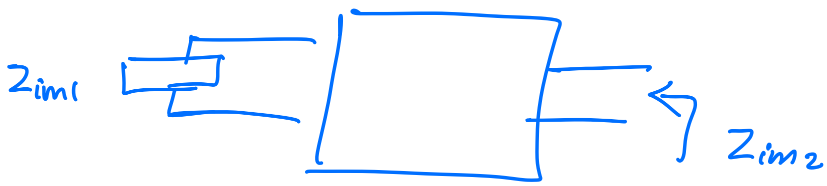
$$-I_2 = \frac{I_1}{C\, Z_{im2} + D}$$

$$V_1 = A\,(-I_2)\, Z_{im2} + B\,(-I_2)$$

$$V_1 = (A\, Z_{im2} + B)\,(-I_2)$$

$$V_1 = (A Z_{im2} + B) \frac{I_1}{C Z_{im2} + D}$$

$$Z_{im1} = \frac{V_1}{I_1} = \frac{A Z_{im2} + B}{C Z_{im2} + D} \quad (1)$$



$$Z_{im2} = \frac{D Z_{im1} + B}{C Z_{im1} + A} \quad (2)$$

} Skip the step of solving (1) & (2)

$$Z_{im1} = \sqrt{\frac{AB}{CD}} \qquad Z_{im2} = \sqrt{\frac{DB}{CA}}$$

*

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 1 \quad \text{(Reciprocity)}$$

$$Z_{10} = \frac{A}{C} \qquad Z_{20} = \frac{D}{C}$$

$$Z_{1s} = \frac{B}{D} \qquad Z_{2s} = \frac{B}{A}$$

$$Z_{im1} = \sqrt{Z_{10} Z_{1s}} \qquad Z_{im2} = \sqrt{Z_{20} Z_{2s}}$$

Special case: Network is reciprocal and symmetric

$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 1$ $\qquad$ $A = D$

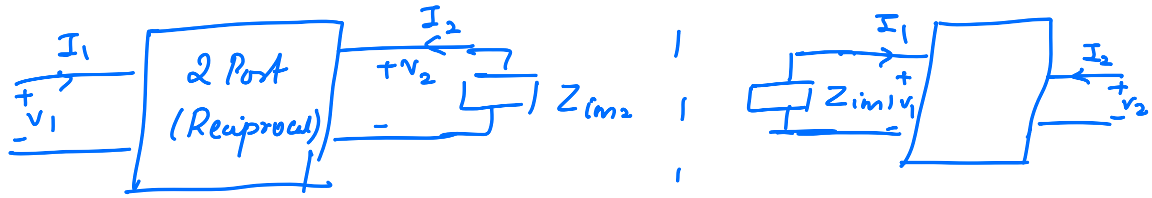
$$Z_{10} = Z_{20} = A/C \rightarrow Z_{oc}$$

$$Z_{1s} = Z_{2s} = \frac{B}{A} \rightarrow Z_{sc}$$

$$Z_{im1} = Z_{im2} = Z_0 \quad (Z_c) = \sqrt{\frac{B}{C}}$$

$\uparrow$ Characteristic impedance.

Image transfer constant (γ)



$$e^{\gamma} = \sqrt{\frac{V_1}{V_2} \cdot \frac{I_1}{(-I_2)}} \qquad ; \qquad e^{\gamma} = \sqrt{\frac{V_2}{V_1} \cdot \frac{I_2}{(-I_1)}}$$

In terms of transmission parameters.

$$V_1 = A\,V_2 + B\,(-I_2)$$

$$I_1 = C\,V_2 + D\,(-I_2)$$

$$V_2 = -Z_{im2}\,I_2$$

$$I_1 = C\,(-Z_{im2}\,I_2) + D\,(-I_2)$$

$$\frac{I_1}{-I_2} = D + C \cdot Z_{im2}$$

Z_{im2} (*)

$$= D + C \cdot \sqrt{\frac{DB}{CA}}$$

$$= D + \sqrt{\frac{BCD}{A}} = D + \frac{\sqrt{ABCD}}{A}$$

$$V_1 = A \cdot V_2 + B\,\frac{V_2}{Z_{im2}}$$

$$\frac{V_1}{V_2} = A + \frac{B}{Z_{im2}} = A + \frac{B}{\sqrt{\frac{DB}{CA}}}$$

$$= A + \sqrt{\frac{ABC}{D}} = A + \frac{\sqrt{ABCD}}{D}$$

$$e^{r} = \sqrt{\left(\frac{V_1}{V_2}\right)\left(\frac{I_1}{-I_2}\right)}$$

$$(e^{r})^2 = \left(A + \frac{\sqrt{ABCD}}{D}\right)\left(D + \frac{\sqrt{ABCD}}{A}\right)$$

$$= AD + \frac{ABCD}{AD} + 2\sqrt{ABCD}$$

$$= AD + BC + 2\sqrt{ABCD}$$

$$= (\sqrt{AD} + \sqrt{BC})^2$$

$$e^{r} = \sqrt{AD} + \sqrt{BC}$$

$$\det\begin{pmatrix} A & B \\ C & D \end{pmatrix} = 1 \qquad AD - BC = 1$$

$$BC = AD - 1$$

$$e^{r} = \sqrt{AD} + \sqrt{AD - 1}$$

Special Case: Reciprocal & Symmetric $(A = D)$

$$e^{r} = \sqrt{AD} + \sqrt{BC}$$

$$= A + \sqrt{BC}$$

$$= A + \frac{B}{\sqrt{\frac{B}{C}}} = A + \frac{B}{Z_0} \; //$$

$$\frac{V_1}{V_2} = A + \frac{\sqrt{ABCD}}{D} = A + \sqrt{BC}$$

$$(A = D) \qquad = e^{r} \quad //$$

$$\frac{I_1}{-I_2} = D + \frac{\sqrt{ABCD}}{A} = A + \sqrt{BC}$$

$$(A = D) \qquad = e^{r} \quad //$$

$$\frac{V_1}{V_2} = \frac{I_1}{-I_2} = e^{r} \quad //$$