Announcements

1. Problem set posted
2. Tutorial this week (Wed 5pm)

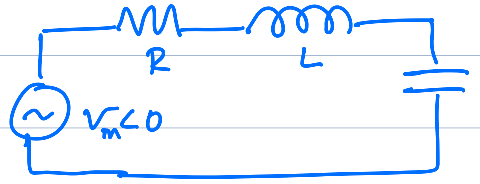
Resonance

Resonant freq:

$$Q\text{-factor }(Q) = 2\pi \cdot \frac{\text{Max Energy stored in a cycle}}{\text{Energy dissipated in a cycle}}$$

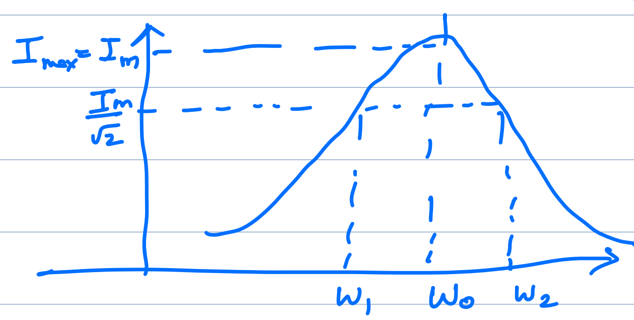
$$Q\,(\text{Capacitor}) = \omega R_p C$$

$$Q\,(\text{Inductor}) = \frac{\omega L}{R_s}$$



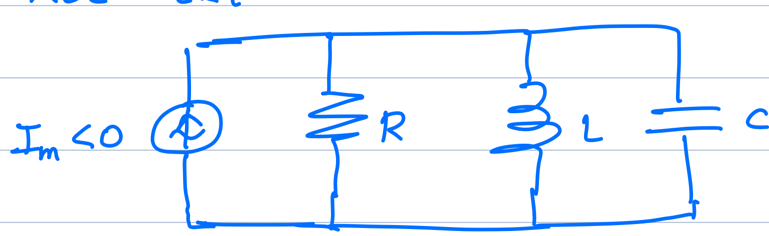
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$I_m = \frac{V_m}{R}$$



$$BW = \omega_2 - \omega_1$$

Parallel RLC ckt



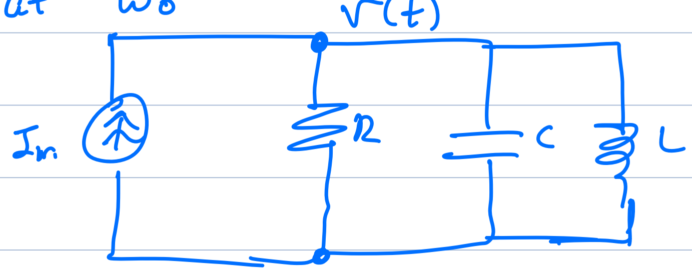
$$Y(j\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$\frac{1}{j\omega_0 L} + j\omega_0 C = 0$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Q-factor (Parallel RLC ckt) at ω_0 $= 2\pi \cdot \frac{\text{Max energy}}{\text{Energy dissipated}}$



Energy dissipated in a cycle $= \frac{1}{2} \cdot I_m^2 \cdot R \cdot T$

$= \frac{1}{2} I_m^2 \cdot \frac{R}{f}$

Max Energy in a cycle $= \max\limits_t \left(E_c(t) + E_L(t) \right)$

$E_c(t) = \frac{1}{2} \cdot C \cdot v^2(t)$

$v(t)$ at $\omega = \omega_0$ $= I_m \cdot R \angle 0$

$= I_m R \cos(\omega_0 t)$

$E_c(t) = \frac{1}{2} \cdot C \cdot I_m^2 R^2 \cos^2(\omega_0 t)$

$E_L(t) = \frac{1}{2} \cdot L \cdot I_L^2(t)$

$I_L(j\omega_0) = \frac{v(j\omega_0)}{j\omega_0 L} = \frac{I_m \cdot R}{j\omega_0 L}$

$I_L = I_m R \angle -90^0$

$$\overline{\omega_0 L}$$

$$I_L(t) = \frac{I_m R}{\omega_0 L} \cos(\omega_0 t - 90^\circ)$$

$$= \frac{I_m R}{\omega_0 L} \sin(\omega_0 t)$$

$$E_L(t) = \frac{1}{2} \cdot L \cdot \left(\frac{I_m R}{\omega_0 L} \sin(\omega_0 t) \right)^2$$

$$= \boxed{\frac{I_m^2 R^2}{2\omega_0^2 L}} \sin^2(\omega_0 t)$$

$$\downarrow$$

$$\frac{1}{LC}$$

$$= \frac{I_m^2 \cdot R^2 C}{2} \sin^2(\omega_0 t)$$

$$\max_t \left(E_c(t) + E_L(t) \right) = \frac{I_m^2 R^2 C}{2} \cos^2(\omega_0 t) + \frac{I_m^2 R^2 C}{2} \sin^2(\omega_0 t)$$

$$= \underline{\frac{I_m^2 R^2 C}{2}}$$

$$Q_0 = 2\pi \cdot \frac{\frac{I_m^2 R^2 \cdot C}{2}}{\frac{1}{2} \cdot I_m^2 \cdot R / f} = \underline{\underline{\omega_0 \cdot RC}}$$

$$= \frac{R \cdot}{\sqrt{\frac{L}{C}}} \quad //$$

$$Y(\omega) = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\omega = \omega_0 \qquad Y(\omega_0) = \frac{1}{R}$$

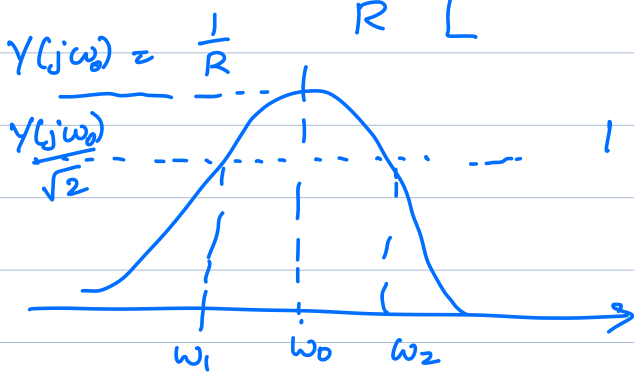
$$Y(j\omega) = \frac{1}{R}\left[1 + j\left(\omega \underbrace{\boxed{RC}}_{Q_0/\omega_0} - \frac{R}{\omega L}\right)\right]$$

$$Q_0 = \omega_0 RC$$

$$\frac{R}{\omega L} = \frac{RC}{\omega LC} = \frac{Q_0/\omega_0}{\omega \cdot \frac{1}{\omega_0^2}}$$

$$= Q_0 \frac{\omega_0}{\omega}$$

$$Y(j\omega) = \frac{1}{R}\left[1 + j\underbrace{\left(\omega \frac{Q_0}{\omega_0} - Q_0 \frac{\omega_0}{\omega}\right)}_{1}\right]$$



$$1 \pm j \cdot 1 \qquad (\sqrt{2})$$

$$\omega_1 \cdot \frac{Q_0}{\omega_0} - Q_0 \cdot \frac{\omega_0}{\omega_1} = -1$$

$$\omega_2 \cdot \frac{Q_0}{\omega_0} - Q_0 \cdot \frac{\omega_0}{\omega_2} = +1$$

$$\Downarrow$$

$$\omega_1 = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0}\right]$$

$$\omega_2 = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0}\right]$$

$$BW = \omega_2 - \omega_1 = \frac{\omega_0}{}$$

Q_0 //

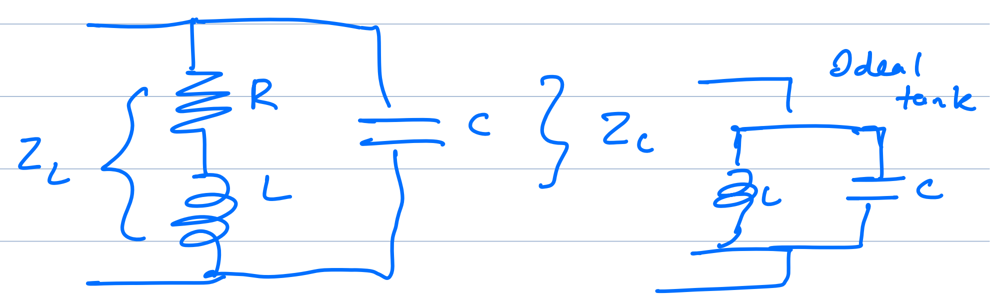
Series RLC ckt (Skip the derivation)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \omega_0 \cdot \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$BW = \omega_0 / Q_0$$

Example: Practical "Tank ckt"



$$Y(j\omega) = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$Z_L = R + j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

$$Y(j\omega) = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

At $\omega = \omega_0$

$$\cancel{\omega_0} C - \frac{\cancel{\omega_0} L}{R^2 + \omega_0^2 L^2} = 0$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Admittance (Impedance) at resonant freq

$$Y(j\omega_0) = \frac{R}{R^2 + \omega_0^2 L^2} = \frac{R}{R^2 + \left(\frac{1}{LC} - \frac{R^2}{L^2}\right)L^2}$$

$$= \frac{C \cdot R}{L} \quad //$$

Idea :