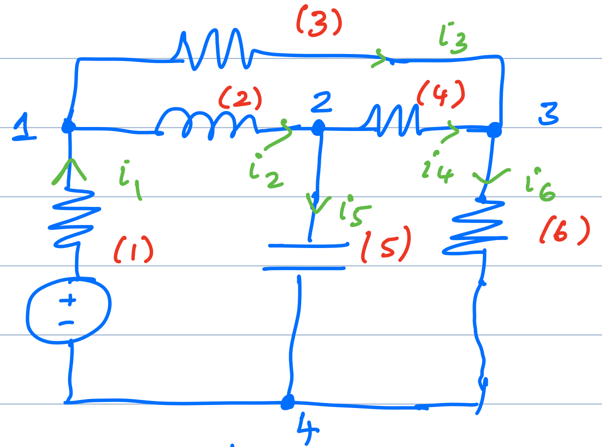
Graph Theory & Network Topology
[Notation from SK Textbook]

Example network

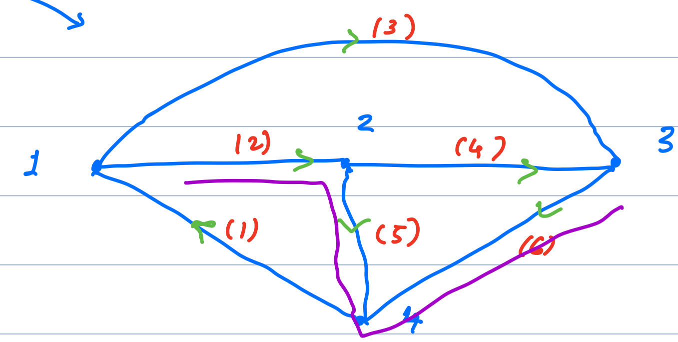


↓ Graph

(1) Nodes in ckt same as nodes in Graph

(2) Replace ckt components with lines

Undirected Graph representing the ckt



" Directed Graph "

Terminology & Properties

(1) Incident branches at a node

Branches whose one end falls at a particular node

Ex 2 : (2), (4), (5)

2. $\boxed{\text{Rank}}$ of a graph = # nodes − 1

nodes − n

branches − b

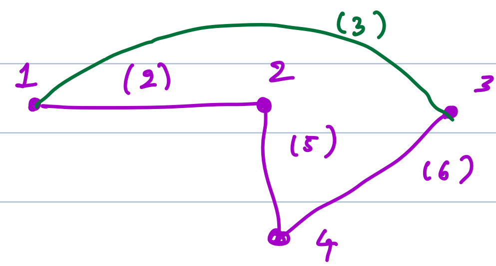
3. Sub graph : Subset of nodes & branches from the original graph

Proper subset : Strict subset "

4. Path : (1) 2 nodes have only one incident branch "Terminal nodes"

"A sequence of branches" (2) All other nodes has 2 incident branch

Eg: (2) – (5) – (6)



1 & 3 have only one incident branch

2 & 4 have exactly 2 incident branch

5. Loop : A loop is simply a path in which terminal nodes coincide

(2) – (5) – (6) – (3)

6. Connected graph : A connected graph is one where at least one "path" exists b/w any pair of nodes

All incidence matrix (A_a) "all"

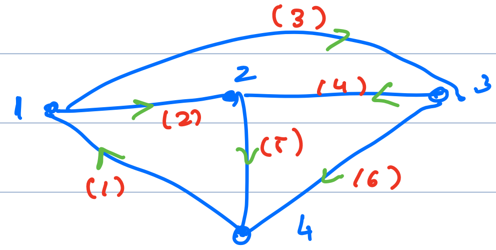
$\dim(A_a)$: $n \times b$

$$A_a = \begin{bmatrix} \leftarrow & r_1 & \rightarrow \\ & \vdots & \\ \leftarrow & r_n & \rightarrow \end{bmatrix}$$

$r_1 + r_2 + \cdots + r_n = 0$

$\uparrow$
0 vector

$$
A_a = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccccc} (1) & (2) & (3) & (4) & (5) & (6) \end{array} \\
\left[\begin{array}{cccccc}
-1 & 1 & +1 & 0 & 0 & 0 \\
0 & -1 & 0 & -1 & 1 & 0 \\
0 & 0 & -1 & 1 & 0 & 1 \\
+1 & 0 & 0 & 0 & -1 & -1
\end{array}\right]
\end{array}
$$



$$
a_{ij} = \begin{cases}
0 & \text{if branch } j \text{ is not incident at node } i \\
-1 & \text{if branch } j \text{ is incident at node } i \text{ \& directed towards node } i \text{ "oriented"} \\
1 & \text{if branch } j \text{ is incident at node } i \text{ \& directed away from "oriented" node } i
\end{cases}
$$

i: node, j: branch

Properties

(1) Sum of the rows of $A_a = 0$

$\Rightarrow$ Rows of A_a are not linearly independent

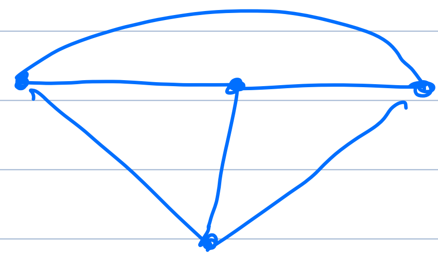
$\Rightarrow$ Rank $(A_a) = n-1$

[Skipped proof]

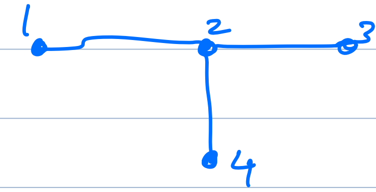
$\#$nodes $= n$
$\#$branches $= b$

Tree Subgraph (consisting of all nodes)

- A: Every node is connected to every other node
- B: Removal of any branch destroys Prop A

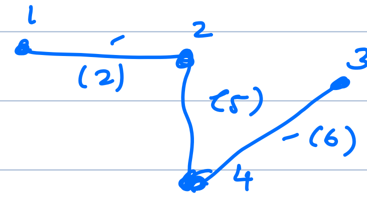


Tree - 1



Terminology

Tree - 2



Branches of tree: twigs

Complement of tree: co-tree

Branches of co-tree: links

Property of tree: (Proof skipped)

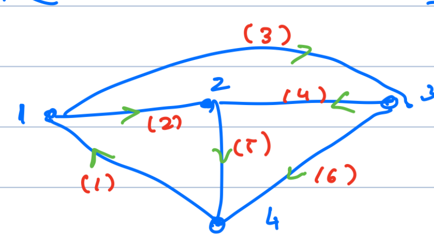
If a graph has n nodes then the tree has $(n-1)$ branches

Twigs: (2), (5), (6)

Links: (1), (3), (4)

Properties: Every connected graph has at least one tree.

$$A_a = \begin{array}{c|cccccc} & (1) & (2) & (3) & (4) & (5) & (6) \\ \hline 1 & -1 & 1 & +1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & -1 & 1 & 0 \\ 3 & 0 & 0 & -1 & 1 & 0 & 1 \\ 4 & +1 & 0 & 0 & 0 & -1 & -1 \end{array}$$



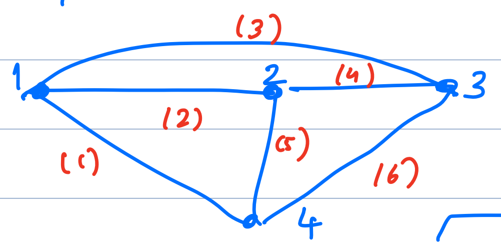
Incidence matrix (A) [Reduced incidence matrix]

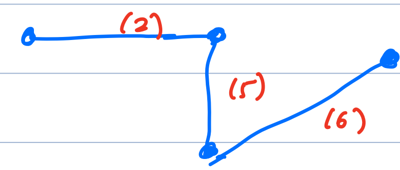
→ Get rid of the row corresponding to reference node

$$A = \begin{array}{cc} & \begin{matrix}(1) & (2) & (3) & (4) & (5) & (6)\end{matrix} \\ \begin{matrix}1\\2\\3\end{matrix} & \begin{bmatrix} -1 & 1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$\text{rank}(A) = n-1$$

Graph $\longrightarrow$ Tree





$$A = (n-1)\begin{array}{cc} & \overbrace{\begin{matrix}(2) & (5) & (6)\end{matrix}}^{n-1} \; \begin{matrix}(1) & (3) & (4)\end{matrix} \\ \begin{matrix}1\\2\\3\end{matrix} & \left[\underbrace{\begin{matrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\substack{A_t \\ \uparrow \\ \text{tree}}} \vdots \underbrace{\begin{matrix} -1 & +1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & +1 \end{matrix}}_{\substack{b-(n-1) \\ A_l \\ \uparrow \\ \text{link}}}\right] \end{array}$$

$$A = [A_t \vdots A_l]$$

$$\dim(A_t) = (n-1) \times (n-1)$$

$$\dim(A_l) = (n-1) \times (b-(n-1))$$