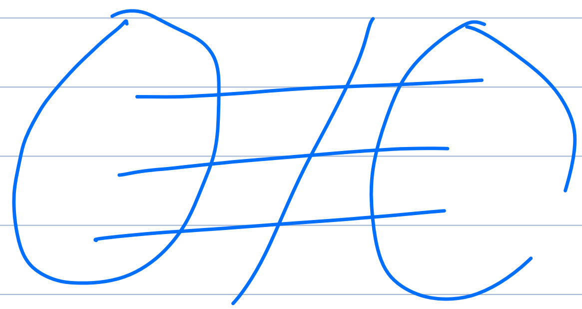
Announcements: Lecture on Nov 15 (Following Thu timetable)

: Syllabus : Nov 21 @ 2pm

(Network Topology)



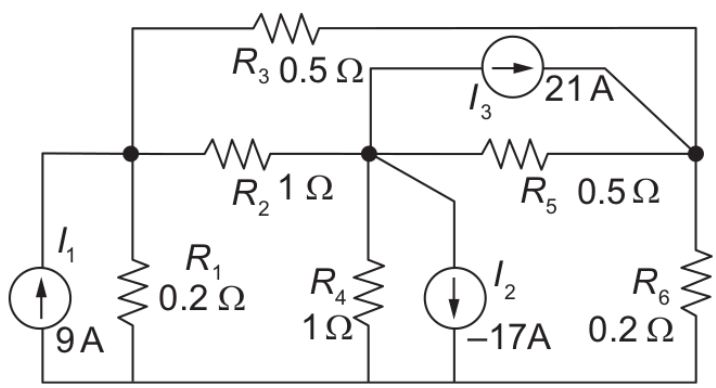
Q_a

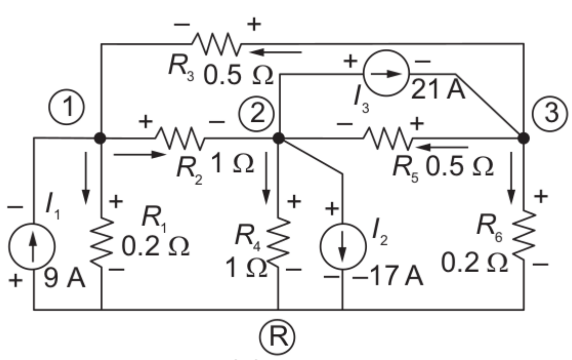
Q_f

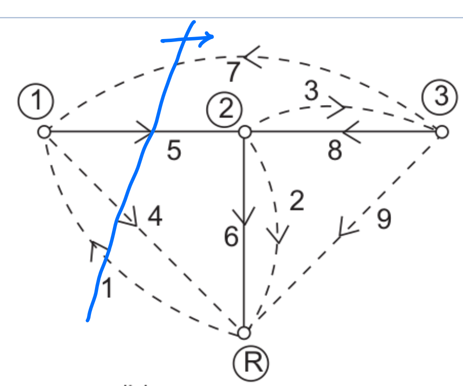
$Q_f \cdot i = 0$: $A i = 0$

KVL: $v = Q_f^T v_t$

Example: Independent current sources







9

1 2 3 14 5 6 7 8 9 7

$$Q_f = \begin{array}{c} \leftarrow 5 \rightarrow \\ \leftarrow 6 \rightarrow \\ \leftarrow 8 \rightarrow \end{array} \left[\begin{array}{ccc|cccccc} -1 & 0 & 0 & 1 & 1 & 0 & -1 & 0 & 0 \\ & & & & & & & & \\ & & & & & & & & \end{array} \right]$$

5, 7, 4, 1 $\qquad Q_{fg} \qquad\qquad Q_{fp}$

$$Q_f \, i = 0 \qquad\qquad i_g = \begin{bmatrix} 9 \\ -17 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} Q_{fg} & Q_{fp} \end{bmatrix} \begin{bmatrix} i_g \\ i_p \end{bmatrix} = 0$$

$$Q_{fg} \, i_g + Q_{fp} \, i_p = 0$$

$$i_p = y_p \cdot v_p$$

admittance matrix

$$y_p = \text{diag}(\qquad)$$

$$\begin{bmatrix} 5 & & 0 \\ & & \\ 0 & & \end{bmatrix}$$

$$Q_{fg} \, i_g + Q_{fp} \cdot y_p \cdot v_p = 0$$

KVL in terms of Q

$$v = Q_f^T \, v_t$$

$$\begin{bmatrix} v_g \\ v_p \end{bmatrix} = \begin{bmatrix} Q_{fg}^T \\ Q_{fp}^T \end{bmatrix} v_t$$

$$v_p = Q_{fp}^T \, v_t$$

$$Q_{fg}\, i_g + Q_{fp}\, y_p\, Q_{fp}^T\, v_t = 0$$

$$\underline{\underline{v_t}} = -\left[Q_{fp}\, y_p\, Q_{fp}^T \right]^{-1} Q_{fg}\, i_g$$

Tellegen's Theorem

Node voltage $\Rightarrow$ branch voltage.

$$v = A^T v_n$$

branch voltage   nod voltage

$$v^T i = (A^T v_n)^T i$$

$$= v_n^T\, \underbrace{A\, i}_{0}$$

$$= 0$$

inner product b/w branch voltage & branch currents

$$\sum_{j=1}^{b} \underbrace{v_j\, i_j}_{\text{instantaneous power}} = 0$$

" Total power delivered to all branches at every instant is 0"

$\times$ ——— Module 3 ⟶

Module 2: Laplace transform

$$\underbrace{\mathcal{L}[f(t)]}_{F(s)} = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt.$$

Region of convergence (ROC)

→ Region in the s-plane where F(s) exists

1. Linearity

2. $L\left[\frac{d}{dt} f(t)\right]$

3. $L\left[\int_0^t f(t)\right]$

4. $L[t\, f(t)]$

5. Initial & Final value theorem

$$F(s) \xrightarrow{\mathcal{L}^{-1}} F(t)$$

Partial Fraction expansion

$$F(s) = \frac{P(s)}{Q(s)} \qquad \begin{aligned} \text{order } (P) &= m \\ \text{order } (Q) &= n \end{aligned}$$

$m < n$

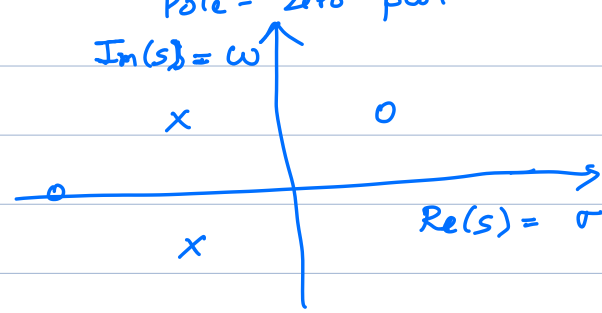
$$= \frac{a_0 s^m + a_1 s^{m-1} + \cdots + a_m}{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}$$

$$= \frac{a_0}{b_0} \frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

z_i's : zeros of F(s)

p_i's : poles of F(s)

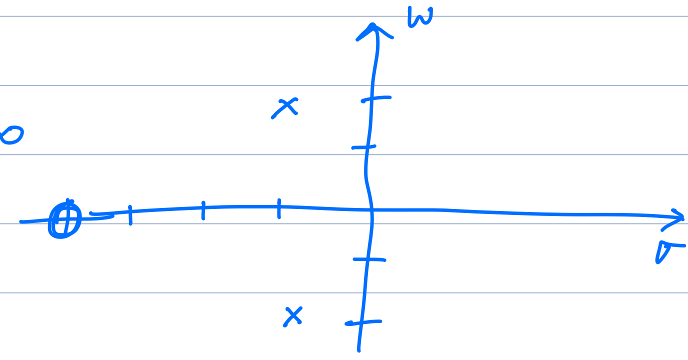
Pole - zero plot



Example: $F(s) = \dfrac{3(s+4)}{s^2+2s+5}$

Poles : $-1 \pm 2j$

Zeros : -4, ∞



$s \to \infty$

$$F(s) = \frac{3 \cdot s}{s^2} = 3/s = 0$$

$m < n$ $n-m$ zeros at infinity

$m > n$ $m-n$ poles at infinity

Multiplicity of Poles & Zeros $F(s) = \dfrac{1}{(s+a)^2}$

$F(s)$ has a pole at $s = -a$ with a multiplicity of 2.

Frequency response $F(j\omega)$ from $F(s)$

$$F(j\omega) = F(s) \Big|_{s = j\omega / \sigma = 0}$$