Announcements

1. Lab Exp 2 due on Jan 2
2. Test 2

Recap:

White Noise (Discrete)

Flat p.s.d $\Longleftrightarrow$ WSS, uncorrelated, zero mean.

$$X = \text{White Noise} \longrightarrow \boxed{h[n]} \longrightarrow Y \qquad S_X(f) = \sigma^2$$

$$S_Y(f) = \sigma^2 \cdot |H(f)|^2$$

$$R_Y(k) = \sigma^2 \sum_{n=k}^{\infty} h[n]\, h[n-k]$$

h is causal

— (*)

Example: Suppose an uncorrelated WSS process with mean m, variance σ^2 is input to an LTI system with impulse response:

$$h[k] = r^k \quad ; \quad k \geq 0, \quad |r| < 1$$

Find mean and autocorrelation of output?

$$Y[n] = \sum_{k=0}^{\infty} h[k]\, X[n-k]$$

$$E[Y[n]] = \sum_{k=0}^{\infty} h[k]\, \underbrace{E[X[n-k]]}_{m}$$

$$= m \sum_{k=0}^{\infty} h[k]$$

$$= m \cdot \sum_{k=0}^{\infty} r^{k}$$

$$m_y = \frac{m}{1 - r} \quad //$$

$$C_y[k] = \sigma^2 \sum_{n=k}^{\infty} h[n]\, h[n-k]$$

Equation (*)

$$= \sigma^2 \sum_{n=k}^{\infty} r^{n}\, r^{n-k}$$

$$= \sigma^2 \cdot r^{-k} \sum_{n=k}^{\infty} r^{2n}$$

$$= \sigma^2\, r^{-k} \cdot r^{2k} \underbrace{\sum_{n=k}^{\infty} r^{2(n-k)}}_{\sum_{l=0}^{\infty} r^{2l}}$$

$[l = n-k]$

$$= \sigma^2\, r^{k} \cdot \sum_{l=0}^{\infty} r^{2}$$

$$= \sigma^2 \cdot r^{k} \cdot \frac{1}{1 - r^2} \quad //$$

$$C_y[k] = R_y[k] - m_y^2 \quad //$$

White Noise (Continous)

RP with the following auto-correlation function $K_x(\tau) = \sigma^2 \delta(\tau)$

$$\boxed{S_x(f) = \sigma^2 \quad \forall f}$$

$\Leftrightarrow$ Uncorrelated, WSS with zero mean.

$$R_X(0) = E[X^2(t)]$$

$$= \int_{-\infty}^{\infty} S_X(f)\, df$$

Good model for "thermal noise" of electronic components

$$E[X^2(t)] = \int_{-\infty}^{\infty} \sigma^2\, df$$

$\rightarrow$ Infinite power

Example: Suppose the transfer function of an LTI system is given by

$$H(\omega) = \mathrm{sgn}(\omega) \left(\frac{\omega}{2\pi}\right)^2 \exp(-j(\omega\, 8/\pi))\, W(\omega)$$

ω-domain (not in f-domain) $\omega = 2\pi f$

$$W(\omega) = \begin{cases} 1, & |\omega| \le 40\pi \\ 0, & \text{else.} \end{cases}$$

Let X (input) be a WSS process with autocorrelation function

$$R_X(\tau) = \frac{5}{2}\delta(\tau) + 2$$

Compute total average output power?

$$S_X(\omega) = \mathcal{F}[R_X]$$

$$= \frac{5}{2} + 4\pi\,\delta(\omega)$$

$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega)$$

$$= \left[\frac{5}{2} + 4\pi\,\delta(\omega)\right] |H(\omega)|^2$$

$$= \frac{5}{2} |H(\omega)|^2$$

$$= \frac{5}{2} \left(\frac{\omega}{2\pi}\right)^4 W(\omega)$$

$$R_Y(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_Y(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-40\pi}^{40\pi} 5/2 \, (\omega/2\pi)^4 \, d\omega$$

$$= 3.2 \times 10^6 //$$

Special Case: White Gaussian Noise
(White Gaussian Process)

White Noise (Continous time)

$$X(t) \sim N(0, \sigma^2)$$

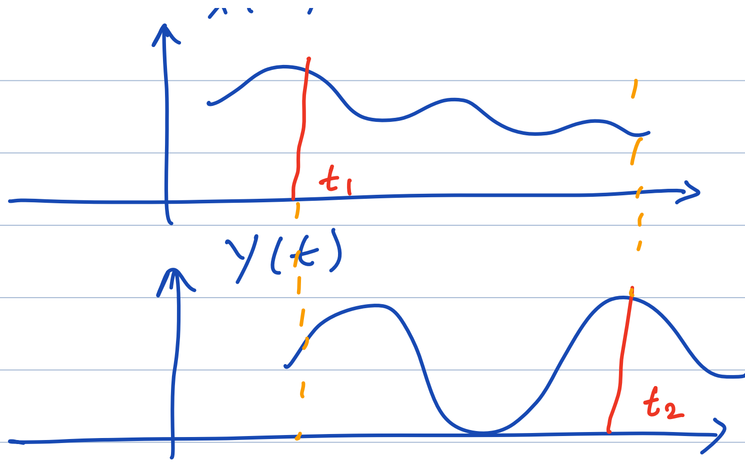
White Noise (Discrete time)

$$X[n] \sim N(0, \sigma^2)$$

Theorem: The derivative of Brownian motion process is white Gaussian noise.

More than one random process
"Vector random process"

$X(t)$



$$Z(t) = (X(t), Y(t))$$

SSS: $F_{Z(t_1) Z(t_2) \cdots Z(t_k)} = F_{Z(t_1+\tau) Z(t_2+\tau) \cdots Z(t_k+\tau)}$

Jointly Wide Sense Stationary (2 random process)

Cross-correlation function

$$R_{XY}(t_1, t_2) = E[X(t_1) Y(t_2)]$$

$$R_{YX}(t_1, t_2) = E[Y(t_1) X(t_2)]$$