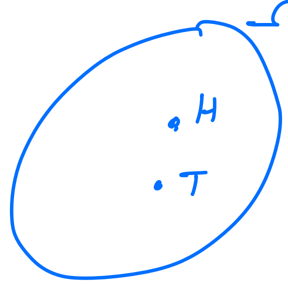
Announcements

1. Matlab Resources posted
2. Ph.D students can also form groups

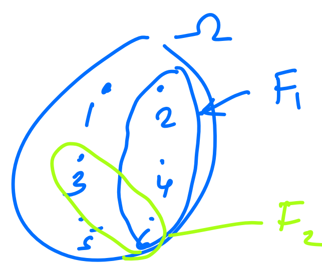
Random Experiment - Eg. Tossing a coin

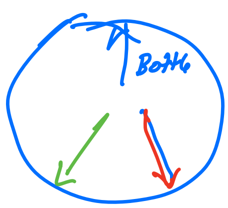
1. Sample Space : Ω

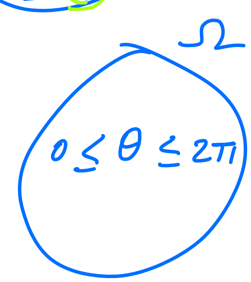
Ω : All outcomes of a random exp



Dice throw







2. Event Space : $\mathcal{F}$

Event : "The dice comes up with an even number"

$F_1 =$

F_2 : Divisible by 3

$F_1 \in \mathcal{F}$

$F_2 \in \mathcal{F}$

3. Probability Measure : P

$P(F) \rightarrow 0 - 1$

$$\downarrow$$
$$F \in \mathcal{F}$$

$(\Omega, \mathcal{F}, P)$

Axioms of probability.

Axioms of Event space

1. $\Omega \in \mathcal{F}$

   Eg: Dice throw

   $\Omega : \{1, 2, 3, 4, 5, 6\}$

   Event: F_1 = Dice comes up either 1, 2, 3, 4, 5, 6

   $F_1 \in \mathcal{F}$

   $\Omega \in \mathcal{F}$

2. $A \in \mathcal{F}$, $A^c \in \mathcal{F}$

   $(\bar{A})$

3. (A) $A_1, A_2, \dots, A_n \in \mathcal{F}$

   $\bigcup_{i=1}^{n} A_i \in \mathcal{F}$

   (B) $A_1, A_2, \dots \in \mathcal{F}$

   $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$

Axioms of Probability Measure.

1. $P(F) \geq 0$ $F \in \mathcal{F}$
2. $P(\Omega) = 1$
3. (A) $A_1, A_2, \dots, A_n \in \mathcal{F}$

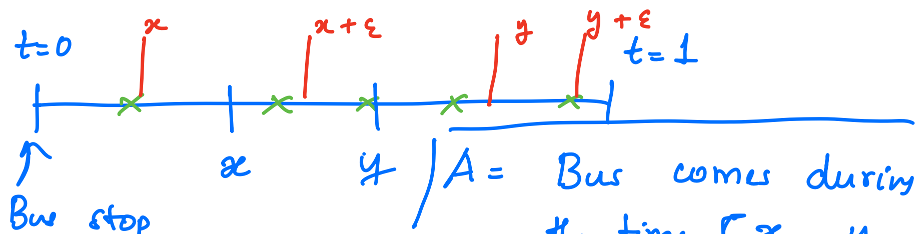
   (disjoint)

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$$

(B) $A_1, A_2, \ldots \in \mathcal{F}$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

Example: Suppose you are waiting for a bus. Bus can come in the next 1 hour, equally likely in the next 1 hour



A = Bus comes during the time $[x, y]$

Define this probability measure.

$$P(A) = y - x$$

Check if this P satisfy all the axioms?

1. $P(A) \geq 0$
2. $\Omega = [\overset{x}{0}, \overset{y}{1}]$

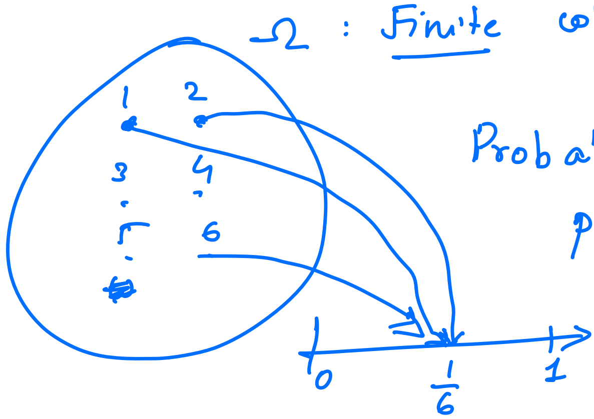
   $P(\Omega) = 1 - 0 = 1$
3. $A_1 = (x, x+\varepsilon)$

   $A_2 = (y, y+\varepsilon)$

   $P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Discrete probability spaces

Ω : Finite collection of outcomes.



Probability mass function (pmf)

p

$\omega \in \Omega$ $\qquad p(\omega) \geq 0$

$$\sum_{\omega \in \Omega} p(\omega) = 1$$

Relation between pmf & probability measure.

$$P(F) = \sum_{\omega \in F} p(\omega)$$

$F \in \mathcal{F}$

F : Dice come up even

Satisfy all axioms

Example 2-9 (PP) take turns randomly.

2 players A & B draw one balls from a box containing m white balls and n black balls.

A player wins the game when he picks a white ball.

Qn: what is the probability that the player who starts the game wins?

Suppose that A starts the game.

Events when A wins:

$X_0 \hookrightarrow$ A : white.

$X_1 \hookrightarrow$ A: black, B: black, A: white.

$X_2 \hookrightarrow$ A: B, B: B, A: B, B: B

A: white

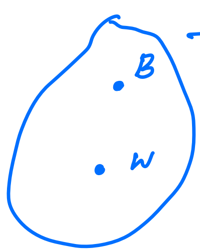
X_k : A & B draws k black balls each & then A draw a white ball.

A wins $= X_0 \cup X_1 \cup X_2 \cup \cdots$

$P(A \text{ wins}) = P(X_0 \cup X_1 \cup X_2 \cup \cdots)$

$X_0, X_1, X_2 \ldots$ are all disjoint

$= P(X_0) + P(X_1) + P(X_2) \cdots$



Ω: A starts

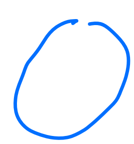
$p(B) = n/m+n$

$p(W) = m/m+n$

$$P(X_0) = \frac{m}{m+n}$$

A black

B black



# of B balls $= n-2$

# of W balls

Ω: A : B
B : B

$$p(W) = \frac{m}{m+(n-2)}$$

$= m$

$$P(X_1) = \left(\frac{n}{m+n}\right)\left(\frac{n-1}{m+(n-1)}\right)\left(\frac{m}{m+(n-2)}\right)$$

$$P(X_2) = \left(\overset{A:B}{\quad}\right)\left(\overset{B:B}{\quad}\right)\left(\overset{A:B}{\quad}\right)\left(\overset{B:B}{\quad}\right)$$

$$\left(\overset{A:W}{\quad}\right)$$

$\vdots$

$k = \frac{n}{2}$