

Announcements : - Tutorial Sheet on 2 port posted
 - Tutorial class on Wed

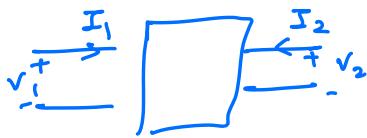
Last week - 2 port parameters (Z, Y, H, G, T)

Transmission Parameters ($ABCD$)

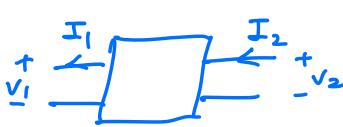
$$V_1 = A V_2 + B (-I_2)$$

$$I_1 = C V_2 + D (-I_2)$$

Inverse Transmission Parameters (T')

$$\begin{cases} V_2 = A' V_1 + B' (-I_1) \\ I_2 = C' V_1 + D' (-I_1) \end{cases}$$


$$A' = \frac{V_2}{V_1} \Big|_{I_1=0} \quad (\text{Forward voltage ratio})$$



$$C' = \frac{I_2}{V_1} \Big|_{I_1=0} \quad (\text{o/p ckt transfer admittance})$$

$$B' = \frac{V_2}{-I_1} \Big|_{V_1=0} \quad (\text{short ckt transfer impedance})$$

$$D' = \frac{I_2}{-I_1} \Big|_{V_1=0} \quad (\text{Forward current ratio})$$

$$\begin{cases} Z = Y^{-1} \\ H = G^{-1} \\ T = (T')^{-1} \end{cases}$$

Z -parameters in terms of transmission parameters.

$$\begin{aligned} V_1 &= A V_2 + B (-I_2) \\ I_1 &= C V_2 + D (-I_2) \end{aligned} \quad \left. \begin{aligned} V_1 &= Z_{11} I_1 + Z_{12} I_2 \\ V_2 &= Z_{21} I_1 + Z_{22} I_2 \end{aligned} \right.$$

$$C V_2 = I_1 + D I_2$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$$

$$Z_{21} = \frac{1}{C} \quad Z_{22} = \frac{D}{C}$$

$$V_1 = A \left(\frac{1}{C} I_1 + \frac{D}{C} I_2 \right) + B (-I_2)$$

$$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 - B I_2$$

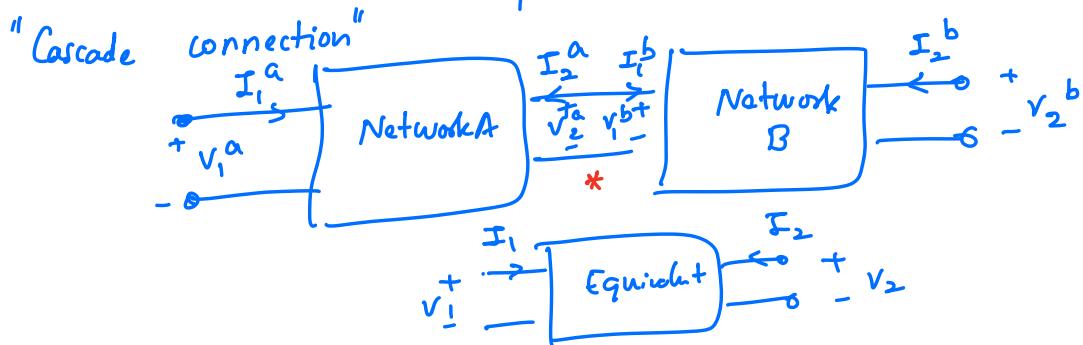
$$= \frac{A}{C} I_1 + \frac{(AD-BC)}{C} I_2$$

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \Delta T = AD - BC$$

$$V_1 = \frac{A}{C} I_1 + \frac{\Delta T}{C} I_2 : \quad Z_{11} = \frac{A}{C}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \rightarrow Z = \begin{bmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix} \quad Z_{12} = \frac{\Delta T}{C}$$

Interconnection of 2 port networks.



$$V_1 = V_1^a \quad V_2 = V_2^b$$

$$I_1 = I_1^a \quad I_2 = I_2^b$$

$$V_2^a = V_1^b \quad I_2^a = -I_1^b$$

$$\frac{V_1}{I_1} \begin{pmatrix} V_1^a \\ I_1^a \end{pmatrix} = \begin{pmatrix} A_a & B_a \\ C_a & D_a \end{pmatrix} \begin{pmatrix} V_2^a \\ -I_2^a \end{pmatrix} \begin{pmatrix} V_1^b \\ I_1^b \end{pmatrix}$$

$$\begin{pmatrix} V_1^b \\ I_1^b \end{pmatrix} = \begin{pmatrix} A_b & B_b \\ C_b & D_b \end{pmatrix} \begin{pmatrix} V_2^b \\ -I_2^b \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

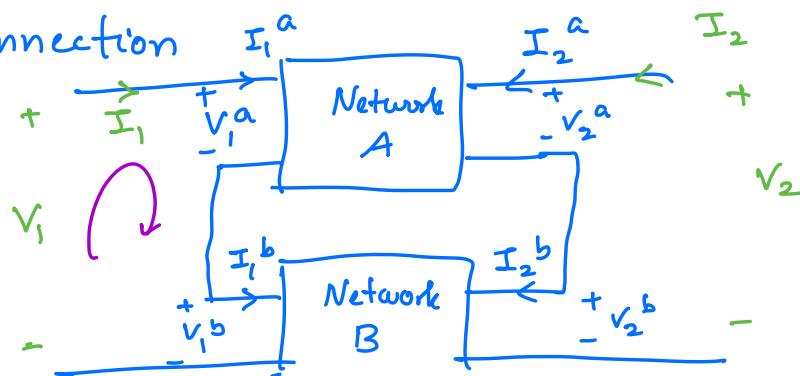
$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A_a & B_a \\ C_a & D_a \end{pmatrix} \begin{pmatrix} A_b & B_b \\ C_b & D_b \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$$[T] = [T_a] [T_b]$$

Inverse transmission

$$[T'] = [T'_b] [T'_a] \text{ [Exercise!]}$$

Series connection



$$I_1 = I_1^a = I_1^b$$

$$I_2 = I_2^a = I_2^b$$

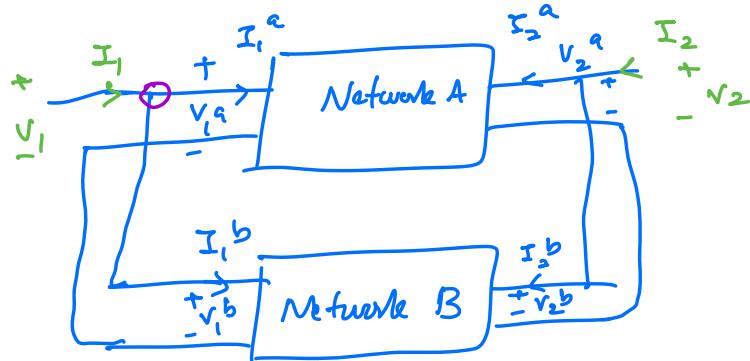
$$\left\{ \begin{array}{l} V_1 = V_1^a + V_1^b \\ V_2 = V_2^a + V_2^b \end{array} \right.$$

$$\begin{pmatrix} V_1^a \\ V_2^a \end{pmatrix} = \begin{pmatrix} Z_{11}^a & Z_{12}^a \\ Z_{21}^a & Z_{22}^a \end{pmatrix} \begin{pmatrix} I_1^a \\ I_2^a \end{pmatrix}$$

$$\begin{pmatrix} V_1^b \\ V_2^b \end{pmatrix} = \begin{pmatrix} Z_{11}^b & Z_{12}^b \\ Z_{21}^b & Z_{22}^b \end{pmatrix} \begin{pmatrix} I_1^b \\ I_2^b \end{pmatrix}$$

$$\begin{aligned}
 \left(\begin{array}{c} V_1 \\ V_2 \end{array} \right) &= \left(\begin{array}{c} V_1^a \\ V_2^a \end{array} \right) + \left(\begin{array}{c} V_1^b \\ V_2^b \end{array} \right) \\
 &= \left(\left(\begin{array}{cc} Z_{11}^a & Z_{12}^a \\ Z_{21}^a & Z_{22}^a \end{array} \right) + \left(\begin{array}{cc} Z_{11}^b & Z_{12}^b \\ Z_{21}^b & Z_{22}^b \end{array} \right) \right) \left(\begin{array}{c} I_1 \\ I_2 \end{array} \right) \\
 [Z] &= [Z_a] + [Z_b]
 \end{aligned}$$

Parallel Connection



$$V_1 = V_1^a = V_1^b$$

$$V_2 = V_2^a = V_2^b$$

$$\left\{
 \begin{array}{l}
 I_1 = I_1^a + I_1^b \\
 I_2 = I_2^a + I_2^b \\
 \left(\begin{array}{c} I_1^a \\ I_2^a \end{array} \right) = \left(\begin{array}{cc} Y_{11}^a & Y_{12}^a \\ Y_{21}^a & Y_{22}^a \end{array} \right) \left(\begin{array}{c} V_1^a \\ V_2^a \end{array} \right) \quad \begin{array}{c} V_1 \\ V_2 \end{array} \\
 \left(\begin{array}{c} I_1^b \\ I_2^b \end{array} \right) = \left(\begin{array}{cc} Y_{11}^b & Y_{12}^b \\ Y_{21}^b & Y_{22}^b \end{array} \right) \left(\begin{array}{c} V_1^b \\ V_2^b \end{array} \right) \quad \begin{array}{c} V_1 \\ V_2 \end{array}
 \end{array}
 \right.$$

$$\left. \left(\begin{array}{c} I_1 \\ I_2 \end{array} \right) = \left(\begin{array}{c} I_1^a \\ I_2^a \end{array} \right) + \left(\begin{array}{c} I_1^b \\ I_2^b \end{array} \right) \right)$$

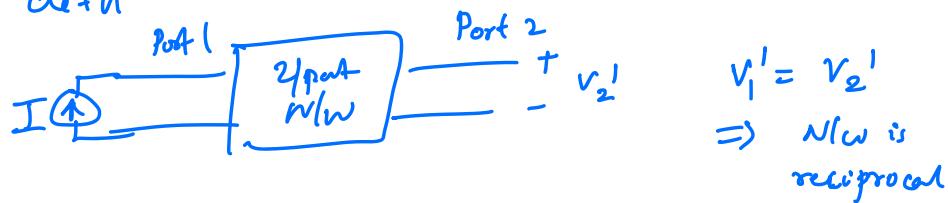
$$[Y] = [Y_a] + [Y_b]$$

Reciprocity



$$I_1' = I_2' \Rightarrow \text{Network is reciprocal}$$

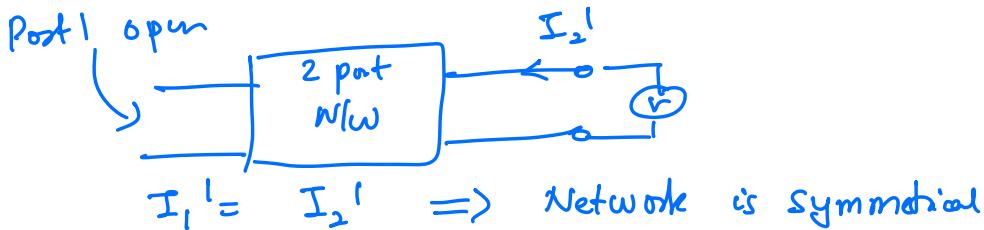
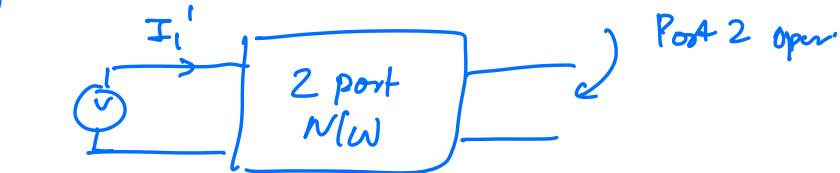
Equivalent defn



$$v_1' = v_2' \\ \Rightarrow \text{N/w is reciprocal}$$



Symmetry



$$I_1' = I_2' \Rightarrow \text{Network is symmetrical}$$

Network being symmetrical (reciprocal)

\Rightarrow Network parameters