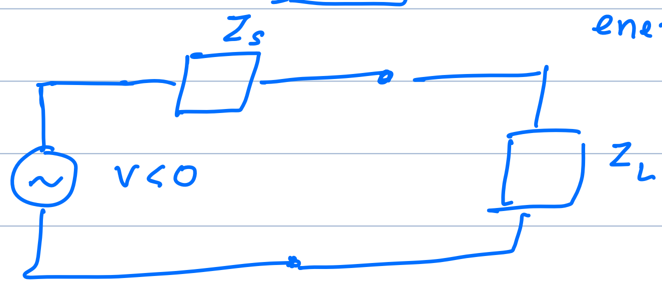
Announcement: Quiz 2 on $\boxed{\text{Nov 20, 4pm (4pm - 5.30 pm)}}$ + Seq

Resonance

Informally
" Circuit absorbs max energy"

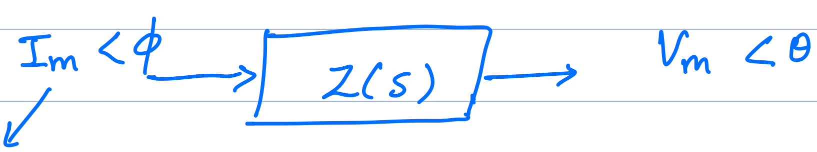


$Z_L = Z_s^*$
Conjugate.

Resonant freq of a ckt is defined as the freq where the ckt behaves purely resistive.

" Q - factor"

Review: Power & Energy in Sinusoidal steady state



$I_m e^{j(\omega t+\phi)}$

$$Z(j\omega) = |Z(j\omega)| \angle Z(j\omega)$$

$$V_m = I_m |Z(j\omega)|$$

$$\theta = \phi + \angle Z(j\omega)$$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

Instanteneous power $p(t) = v(t)\, i(t)$

$$p(t) = V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi)$$

$$= \frac{V_m I_m}{2} \left[\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi) \right]$$

Constant

time-varying

Average power $P = \frac{1}{T} \int_0^T p(t) \cdot dt$

$$= \frac{1}{T} \int_0^T \frac{V_m I_m}{2} \left[\underline{\cos(\theta - \phi)} + \underline{\cos(2\omega t + \theta + \phi)} \right] dt$$

$$= \frac{V_m \cdot I_m}{2} \cdot \cos(\theta - \phi)$$

Special Cases : 1. Resistor

$I_m \angle \phi$

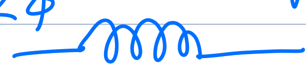
R

$V = R I_m \angle \phi$

Avg Power (R) $= \frac{I_m^2 R}{2}$

2. Capacitor / Inductor

$I_m \angle \phi$



$j\omega L$

$V = \omega L \cdot I_m \angle \phi + 90^\circ$

Avg. Power (L) $= 0$

Avg. Power (C) $= 0$

Energy "stored" by a capacitor.

$$E(t) = \int_0^t p(t)\, dt$$

$$= \int_0^t v(t) \cdot i(t)\, dt$$

$i(t) = C \cdot dv(t)$

$$\overline{dt}$$

$$= \int_0^t C \cdot v(t) \cdot \frac{dv(t)}{dt} \cdot dt$$

$$= \frac{1}{2} C \cdot [v^2(t) - v^2(0)]$$

Ass: Initial voltage of capacitor = 0 $\Rightarrow$ $v(0) = 0$

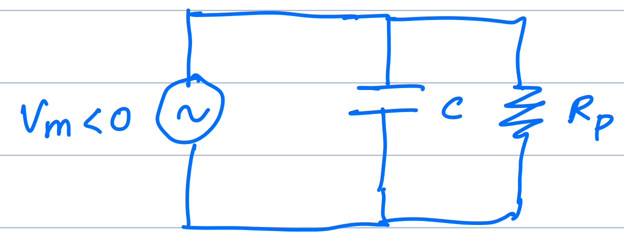
$$E(t) = \frac{1}{2} \cdot C \cdot v^2(t)$$

Inductor case: $E(t) = \frac{1}{2} \cdot L \cdot i^2(t)$

Q-factor = $2\pi \cdot \dfrac{\text{Max energy stored per cycle}}{\text{Energy dissipated per cycle}}$

[Sinusoidal Steady state]

Example: (Leaky Capacitor)



Max energy stored per cycle:

$$E(t) = \frac{1}{2} \cdot C \cdot v^2(t)$$

$$E_{max} = \frac{1}{2} C \cdot V_m^2$$

Power dissipated by R_p = $\frac{1}{2} \cdot \frac{V_m^2}{R_p}$

Energy dissipated by $R_p = \frac{1}{2} \cdot \frac{V_m^2}{R_p} \cdot T$

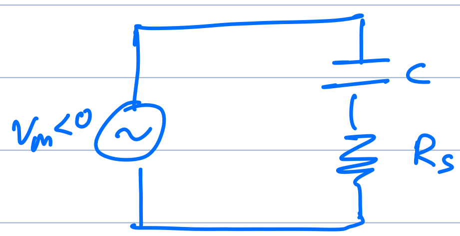
$$= \frac{1}{2} \cdot \frac{V_m^2}{R_p} \cdot \frac{1}{f}$$

Q-factor (Leaky Cap)

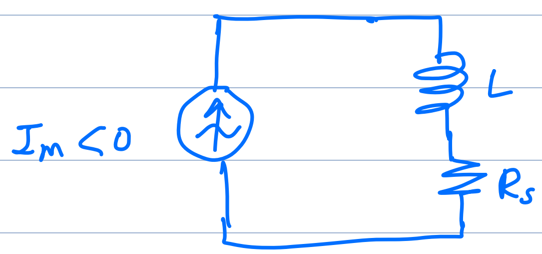
$$= 2\pi \cdot \frac{\cancel{\frac{1}{2}} \cdot C \cdot \cancel{V_m^2}}{\cancel{\frac{1}{2}} \cdot \frac{\cancel{V_m^2}}{R_p} \cdot \frac{1}{f}}$$

$$= \omega \cdot R_p \cdot C \; //$$

Exercise:



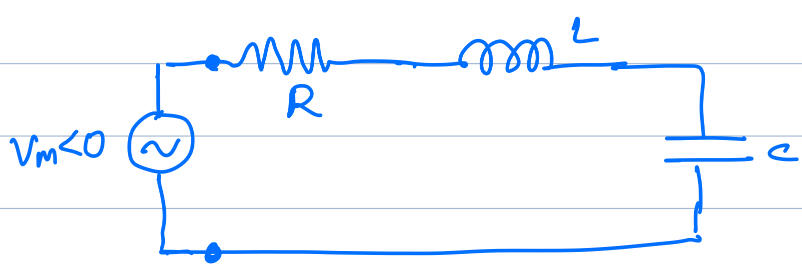
Inductor Case: Lossy Inductor:



Q-factor (Lossy Inductor)

$$= \omega \frac{L}{R_s} \; //$$

Series RLC circuit



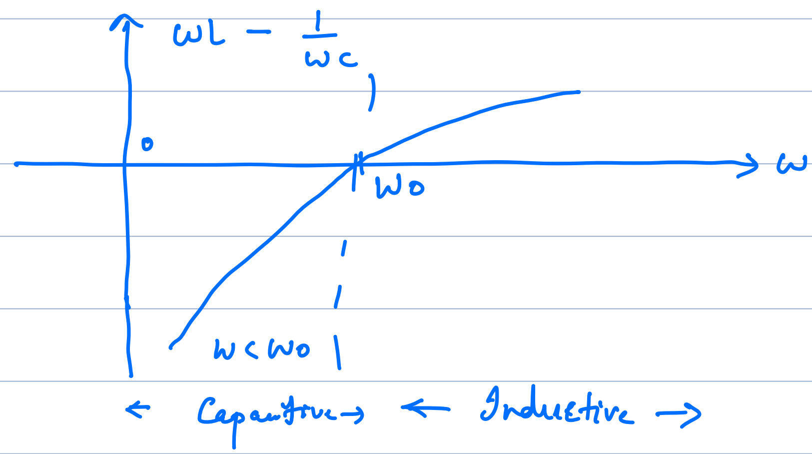
$$Z(j\omega) = R + j\omega L + \frac{1}{j\omega C}$$

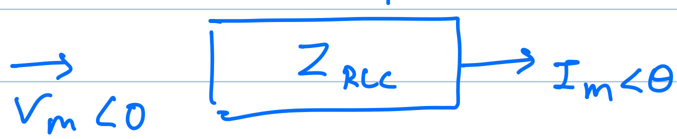
$$= R + j\omega L - \frac{j}{\omega C}$$

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow Z \text{ is purely resistive.}$$

Resonant frequency

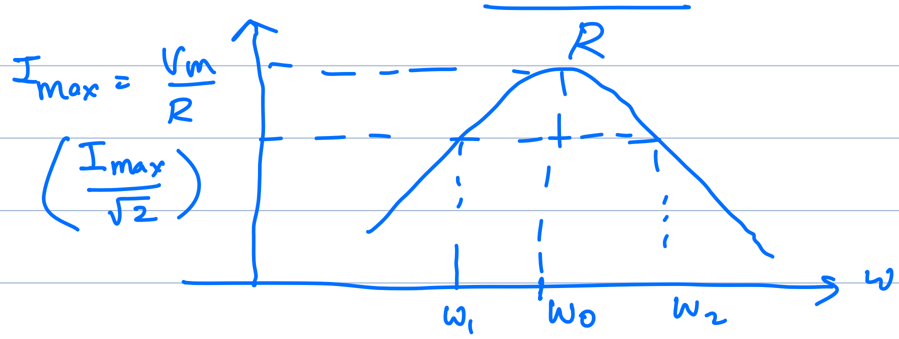
$$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$





$$I_m = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\theta = \tan^{-1} \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$



ω_1, ω_2 : Half power frequency (3 dB)