Announcements (i) Tutorial on Oct 13

(ii) Syllabus for Test 2

Part B: Lecture 1 – Lecture 6

[Basic understanding of Part A assumed]

Last class : Image parameters: Image impedance Z_{im1} Z_{im2}

Image transfer constant γ

Reciprocal N/w

$Z_{im1} = \sqrt{\frac{AB}{CD}}$ $Z_{im2} = \sqrt{\frac{DB}{CA}}$ $e^{\gamma} = \sqrt{AD} + \sqrt{BC}$

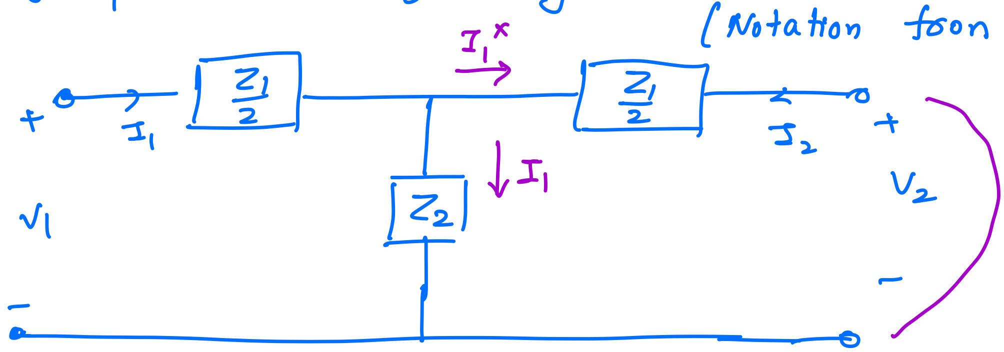
Reciprocal & Symmetrical N/w ($A = D$)

$Z_{im1} = \sqrt{\frac{B}{C}}$ $Z_{im2} = \sqrt{\frac{B}{C}}$ $e^{\gamma} = \underline{A + \frac{B}{Z_0}}$

$Z_{im1} = Z_{im2} = Z_0$ $\frac{V_1}{V_2} = \frac{I_1}{-I_2} = e^{\gamma}$

Image parameters of a symmetrical T- network.

(Notation from SK 16.7.1)



Transmission parameters

$V_1 = A\,V_2 + B\,(-I_2)$

$I_1 = C\,V_2 + D\,(-I_2)$

$$I_2 = 0 \qquad A = \frac{V_1}{V_2} \qquad V_2 = V_1 \cdot \frac{Z_2}{Z_2 + \frac{Z_1}{2}}$$

$$A = \frac{V_1}{V_2} = 1 + \frac{Z_1}{2Z_2}$$

$$C = \frac{I_1}{V_2}$$

$$V_2 = I_1 \cdot Z_2 \qquad C = \frac{I_1}{V_2} = \frac{1}{Z_2}$$

$$V_2 = 0$$

$$B = \frac{V_1}{-I_2} = \frac{V_1}{I_1} \cdot \frac{I_1}{-I_2}$$

$$\frac{V_1}{I_1} = \frac{Z_1}{2} + Z_2 \parallel \frac{Z_1}{2} \quad -(1)$$

$$-I_2 = I_1 \cdot \frac{Z_2}{Z_2 + \frac{Z_1}{2}} \quad -(2)$$

$$B = \frac{V_1}{-I_2} = \left(\frac{Z_1}{2} + \frac{Z_1/2 \cdot Z_2}{Z_2 + \frac{Z_1}{2}} \right) \left(\frac{Z_2 + \frac{Z_1}{2}}{Z_2} \right)$$

$$= \frac{Z_1}{2} + \frac{Z_1}{2} \left(Z_2 + \frac{Z_1}{2} \right) \frac{1}{Z_2}$$

$$= \frac{Z_1}{2} + \frac{Z_1}{2} + \frac{Z_1^2}{4Z_2}$$

$$= Z_1 + \frac{Z_1^2}{4Z_2} = Z_1 \left(1 + \frac{Z_1}{4Z_2} \right)$$

$$D = A = 1 + Z_1/2Z_2$$

$$T = \begin{bmatrix} 1 + \frac{Z_1}{2Z_2} & Z_1\left(1 + \frac{Z_1}{4Z_2}\right) \\ \frac{1}{Z_2} & 1 + \frac{Z_1}{2Z_2} \end{bmatrix}$$

$$Z_{im1} = Z_{im2} = Z_0 = \sqrt{B/C} \qquad B = C \cdot Z_0^2$$

$$= \sqrt{\frac{Z_1\left(1 + Z_1/4Z_2\right)}{1/Z_2}}$$

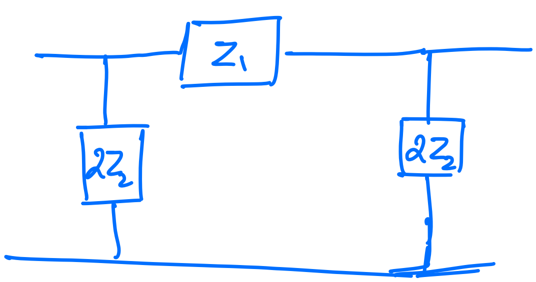
$$= \sqrt{Z_1\left(Z_2 + \frac{Z_1}{4}\right)}$$

$$e^{\gamma} = A + \frac{B}{Z_0}$$

$$= 1 + \frac{Z_1}{2Z_2} + C \cdot Z_0$$

$$= 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2} \quad //$$

Symmetrical π-network. (Exercise)



$$(Z_0)_\pi = \frac{Z_1 Z_2}{\sqrt{Z_1\left(Z_2 + \frac{Z_1}{4}\right)}}$$

$$(Z_0)_T = \sqrt{Z_1\left(Z_2 + \frac{Z_1}{4}\right)}$$

$$(Z_0)_\pi \, (Z_0)_T = Z_1 Z_2$$

$$(e^{\gamma})_\pi = (e^{\gamma})_T$$

Getting back to T- network.

$$e^r = 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

$$Z_2 e^r = Z_2 + \frac{Z_1}{2} + Z_0$$

$$Z_0 = Z_2 (e^r - 1) - \frac{Z_1}{2}$$

$$Z_0 = \sqrt{Z_1 \left(Z_2 + \frac{Z_1}{4} \right)}$$

$$Z_0^2 = Z_1 Z_2 + Z_1^2/4$$

$$\left(Z_2 (e^r - 1) - \frac{Z_1}{2} \right)^2 = Z_1 Z_2 + Z_1^2/4$$

$$Z_2^2 (e^r - 1)^2 + \cancel{\frac{Z_1^2}{4}} - Z_1 Z_2 (e^r \cancel{-1}) = \cancel{Z_1 Z_2} + \cancel{\frac{Z_1^2}{4}}$$

$$Z_2^{\cancel{2}} (e^r - 1)^2 - Z_1 \cancel{Z_2} e^r = 0$$

$$(e^r - 1)^2 = \frac{Z_1}{Z_2} \cdot e^r$$

$$e^{2r} - 2e^r + 1 = \frac{Z_1}{Z_2} \cdot e^r$$

$$e^r - 2 + e^{-r} = Z_1$$

$$\overline{Z_2}$$

$$e^{r} + e^{-r} = 2 + \frac{Z_1}{Z_2}$$

$$\boxed{\frac{e^{r} + e^{-r}}{2}} = 1 + \frac{Z_1}{2Z_2}$$

$$\boxed{\cosh(r) = 1 + 2\sinh^2\left(\frac{r}{2}\right)} = 1 + \frac{Z_1}{2Z_2}$$

$$\cancel{1} + 2\sinh^2\left(\frac{r}{2}\right) = \cancel{1+} \frac{Z_1}{2Z_2}$$

$$\sinh(r/2) = \sqrt{\frac{Z_1}{4Z_2}} \quad //$$

$$\boxed{\text{Symmetrical T-network}}$$

$$\frac{V_1}{V_2} = \frac{I_1}{-I_2} = e^{r}$$

$$r(j\omega) = \alpha(j\omega) + j\,\beta(j\omega)$$

$$V_2 = V_1 \cdot e^{-r}$$

$$= V_1\, e^{-(\alpha(j\omega) + j\,\beta(j\omega))}$$

$$= V_1 \cdot e^{-\alpha(j\omega)}\, e^{-j\beta(j\omega)}$$

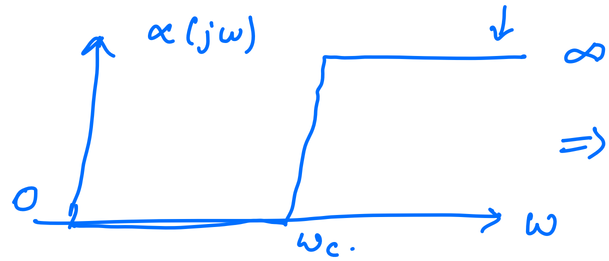
$V_1 = V \angle 0$ "Phasor notation"

$V_2 = v\, e^{-\alpha(j\omega)} \angle -\beta(j\omega)$

attenuation constant — phase constant.

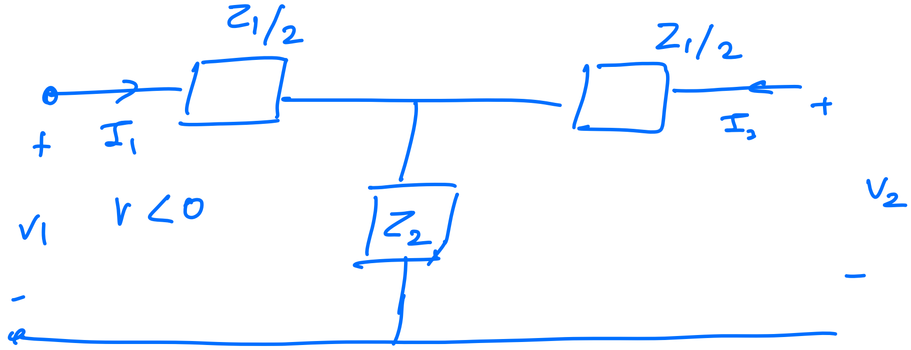
At ω: $\alpha(j\omega) > 0 \Rightarrow$ Output gets attenuated

$\alpha(j\omega) < 0 \Rightarrow$ Output gets amplified



⇒ Low pass filter

Application: Analog filters



$v e^{-\alpha} \angle 0$

Case 1: Z_1 & Z_2 are of the same "type" reactance

↳ Z_1 & Z_2 capacitors

Z_1 & Z_2 inductors

$$\sinh\left(\frac{\gamma}{2}\right) = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{sL_1}{4\,sL_2}}$$

$$= \sqrt{\frac{L_1}{4L_2}} \quad \text{(real \& positive term)}$$

$$\sinh(\gamma/2) = \sinh\left(\frac{\alpha + j\beta}{2}\right)$$

$$= \sinh\left(\frac{\alpha}{2}\right)\cos\left(\frac{\beta}{2}\right) + j\,\underbrace{\cosh\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)}_{0}$$

= real & positive number

$\cosh(\cdot)$ always positive.

$$\sin\left(\frac{\beta}{2}\right) = 0$$

$$\frac{\beta}{2} = 0,\ n\pi \qquad \angle \beta(j\omega) = 0$$

$$\beta = 0,\ 2n\pi$$

$$\sinh\left(\frac{\alpha}{2}\right)\cancelto{1}{\cos\left(\frac{\beta}{2}\right)} = \sqrt{\frac{Z_1}{4Z_2}}$$

$$\frac{\alpha}{2} = \sinh^{-1}\left(\sqrt{Z_1/4Z_2}\right)$$

$$\alpha = 2\sinh^{-1}\left(\sqrt{\frac{Z_1}{4Z_2}}\right) //$$