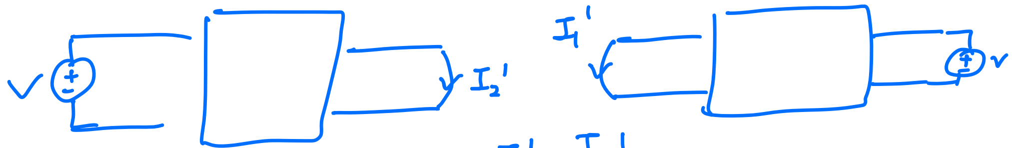
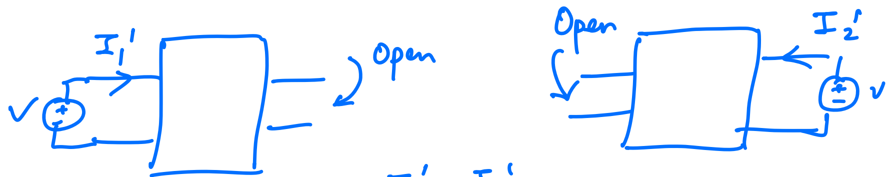
Reciprocity



$I_1' = I_2'$

Symmetry



$I_1' = I_2'$

Reciprocity of Z-parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

$V_2 = 0$

$0 = Z_{21} I_1 + Z_{22} I_2$

$I_1 = -\frac{Z_{22}}{Z_{21}} I_2$

$V = Z_{11} \; -\frac{Z_{22}}{Z_{21}} I_2 + Z_{12} I_2$

$V = -\frac{\Delta Z}{Z_{21}} I_2 \Rightarrow I_2' = V \cdot \frac{Z_{21}}{\Delta Z}$

$0 = Z_{11} I_1 + Z_{12} I_2$

$I_2 = -\frac{Z_{11}}{Z_{12}} I_1$

$V = Z_{21} I_1 + Z_{22} \; -\frac{Z_{11}}{Z_{12}} I_1 = -\frac{\Delta Z}{Z_{12}} I_1$

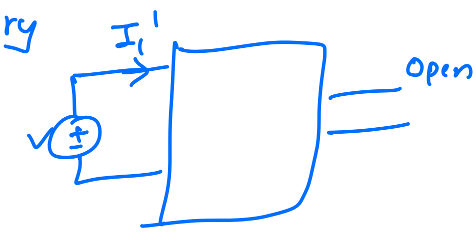
$I_1 = -\underline{V Z_{12}} \Rightarrow I_1' = V \cdot \underline{Z_{12}}$

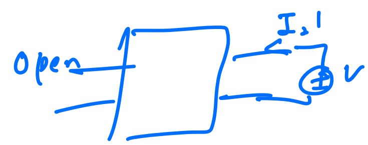
ΔZ ΔZ

$$I_1' = I_2' \Rightarrow Z_{12} = Z_{21}$$

"Reciprocity in Z-parameter"

Symmetry





$$V_1 = Z_{11} I_1 + Z_{12} \cancel{I_2}^{\,0}$$

$$V_2 = Z_{21} I_1 + Z_{22} \cancel{I_2}^{\,0}$$

$$I_1' = I_1 = \frac{V}{Z_{11}}$$

$$V_1 = Z_{11} \cancel{I_1}^{\,0} + Z_{12} I_2$$

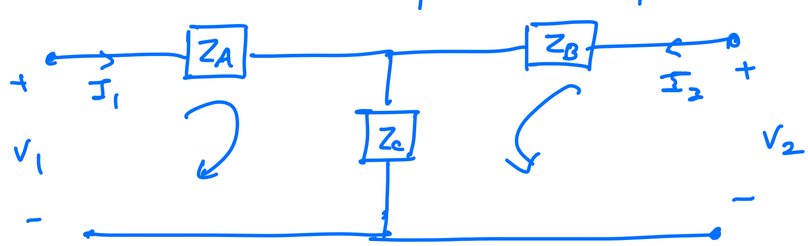
$$\underset{V}{V_2} = Z_{21} \cancel{I_1}^{\,0} + Z_{22} I_2$$

$$I_2' = I_2 = V/Z_{22}$$

$$Z_{11} = Z_{22}$$

"Symmetry in Z-parameter"

T equivalent of a reciprocal 2 port network.



$$V_1 = I_1 Z_A + (I_1 + I_2) Z_C = (Z_A + Z_C) I_1 + Z_C I_2$$

$$V_2 = I_2 . Z_B + (I_1 + I_2) Z_C = Z_C I_1 + (Z_B + Z_C) I_2$$

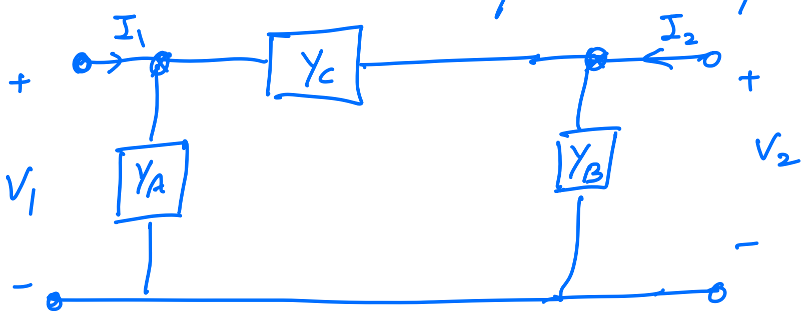
$$* \; Z = \begin{pmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{pmatrix} \qquad Z_{21} = Z_{12}$$

Y- parameters : Exercise

Reciprocity : $Y_{12} = Y_{21}$

Symmetry : $Y_{11} = Y_{22}$

π - equivalent of a reciprocal 2-port N/w



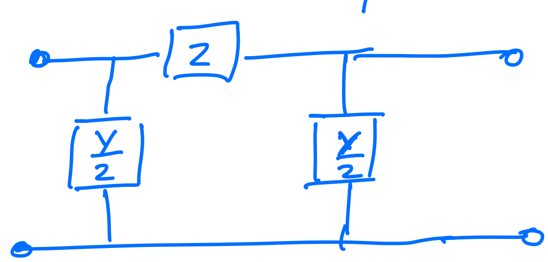
$$I_1 = V_1 \cdot Y_A + (V_1 - V_2) Y_C$$
$$= (Y_A + Y_C) V_1 + (-Y_C) V_2$$

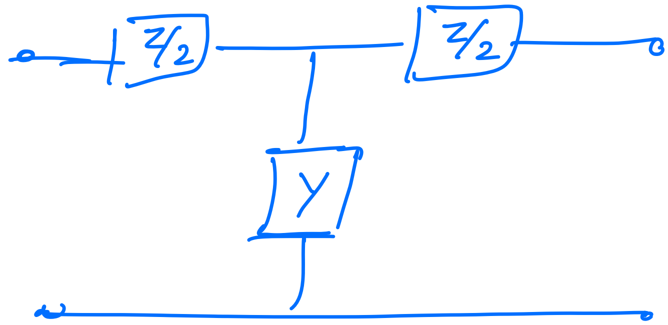
$$I_2 = V_2 \cdot Y_B + Y_C \cdot (V_2 - V_1)$$
$$= (-Y_C) V_1 + (Y_B + Y_C) V_2$$

$$* \; Y = \begin{pmatrix} Y_A + Y_C & -Y_C \\ -Y_C & Y_B + Y_C \end{pmatrix} \qquad Y_{12} = Y_{21}$$

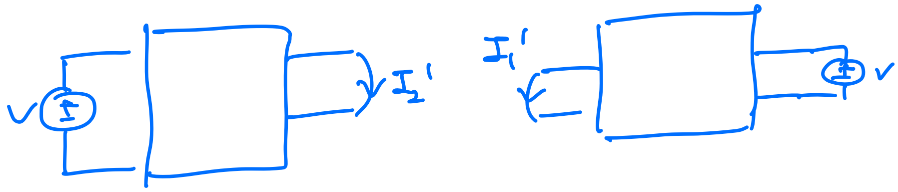
T & π equivalent for reciprocal & symmetric

(Exercise)





Transmission Parameters



$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$V_2 = 0$

$$V = V_1 = B(-I_2)$$

$$I_2 = -\frac{V}{B}$$

$$I_2' = \frac{V}{B}$$

$V_1 = 0$

$$0 = A \cdot V - B I_2$$

$$I_2 = \frac{A \cdot V}{B}$$

$$I_1 = C \cdot V - D I_2$$

$$= C \cdot V - \frac{D A V}{B}$$

$$= -\frac{\Delta T}{B} \cdot V$$

$$I_1' = \frac{\Delta T}{B} \cdot V$$

$$\frac{\cancel{V}}{\cancel{B}} = \frac{\Delta T}{\cancel{B}} \cdot \cancel{V}$$

$$\Delta T = 1$$

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = 1$$

Symmetry: $A = D$

Inverse Transmission Parameters

R: $\det \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} = 1$

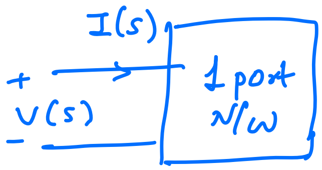
S: $A' = D'$

Hybrid Parameters

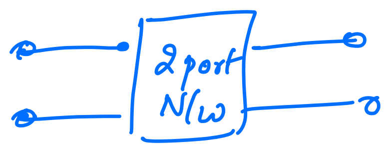
Reciprocity : $h_{12} = -h_{21}$

Symmetry : $\Delta H = 1$

Net parametrization: Image Impedance

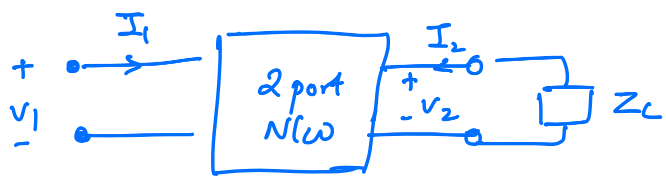


Input Impedance / Impedance $Z(s) = \dfrac{V(s)}{I(s)}$



Driving point impedance at port p

$$= \frac{V_p(s)}{I_p(s)}$$



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$V_2 = -I_2 \cdot Z_L$$

$$-I_2 \cdot Z_L = Z_{21} I_1 + Z_{22} I_2$$

$$I_2 = -\frac{Z_{21} I_1}{Z_L + Z_{22}}$$

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21} I_1}{Z_L + Z_{22}}$$

$$\frac{V_1}{I_1} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}}$$

Z_{10} = Driving point impedance on $Z_L \to \infty$

↑ open

$$= Z_{11}$$

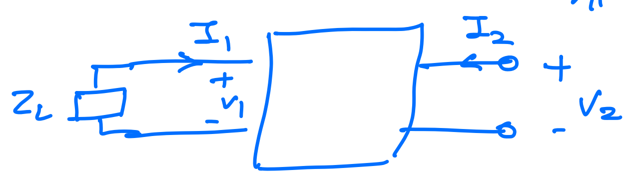
$Z_L = 0$ $\quad Z_{1s} = Z_{11} - \dfrac{Z_{12} Z_{21}}{Z_{22}}$

↑ short

$$= \frac{\Delta Z}{Z_{22}}$$

$$= \frac{1}{Y_{11}}$$

Exercise



$$\frac{V_2}{I_2} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_L}$$

$$Z_{20} = Z_{22}$$

↑ Port 1 is open

$Z_{2s} = Z_{22} - \dfrac{Z_{12} Z_{21}}{Z_{11}}$

$(Z_L = 0)$

$$= \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{11}}$$

$$= \Delta Z / Z_{11} = \frac{1}{Y_{22}}$$

$$\left.\begin{array}{lllll}
Z_{1o} & = & Z_{11} & = & \dfrac{A}{C} \\
Z_{1s} & = & 1/Y_{11} & = & B/D \\
Z_{2o} & = & Z_{22} & = & D/C
\end{array}\right\} \text{Use the lookup}$$

$$Z_{2s} = 1/Y_{22} = B/A$$