Syllabus for Quiz 2 (Nov 20): Graph Theory & Network Topology
[2^{nd} part of Module 3]

$s \to \infty$ " Highest power of $p(s)$ & $q(s)$ should differ by atmost one"

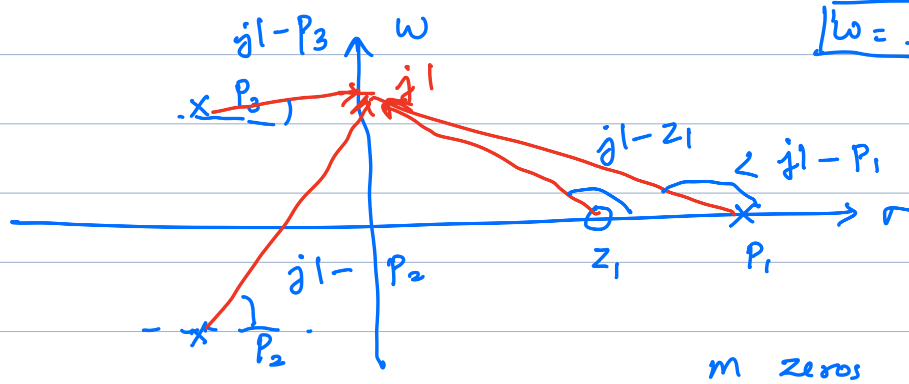
$s \to 0$ $s = 0.1$ $s^1 > s^2 > \ldots$
" Lowest power of $p(s)$ & $q(s)$ should differ by atmost one"

L.T & Freq. Response (Bode Plot)

$Z(s)$

$$Z(j\omega) = Z(s)\Big|_{\substack{\sigma = 0 \\ s = j\omega}}$$

$\boxed{\omega = 1}$



$$Z(s) = k \cdot \frac{(s - Z_1) \cdots (s - Z_m)}{(s - P_1) \cdots (s - P_n)}$$

m zeros

↳ n poles

$$Z(j \cdot 1) = Z(s)\Big|_{s = j \cdot 1}$$

$$= k \frac{(j1 - Z_1) \cdots (j1 - Z_m)}{(j1 - P_1) \cdots (j1 - P_n)}$$

$$|Z(j1)| = |k| \frac{|j1 - z_1| \cdots |j1 - z_m|}{|j1 - p_1| \cdots |j1 - p_n|}$$

$$|Z(j1)| = |k| \cdot \frac{\ln(j1 - z_1) \cdots \ln(j1 - z_m)}{\ln(j1 - P_1) \cdots \ln(j1 - P_n)}$$

$$\angle Z(j1) = \underbrace{\angle k}_{0} + \sum_i \angle j1 - z_i$$

$$- \sum_i \angle j1 - P_i$$

$|Z(j\omega)|$ "Magnitude response" $\angle Z(j\omega)$ "Phase response"

$$20 \log_{10} |Z(j\omega)| = 20 \log |k| + \sum_i 20 \log |j\omega - z_i|$$

$$- \sum_i 20 \log |j\omega - P_i|$$

Bode plots

↳ "First order", "asymptotic" approximations.

$$20 \log_{10} |Z(j\omega)| = 20 \log_{10} |k| + \sum_i 20 \log_{10} |j\omega - z_i|$$

$$- \sum_i 20 \log_{10} |j\omega - p_i|$$

Case 1 : $Z(s) = k$

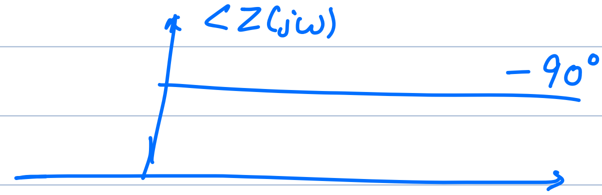
$$20 \log_{10} |Z(j\omega)| = 20 \log_{10} |k|$$

$$\angle Z(j\omega) = \angle k$$

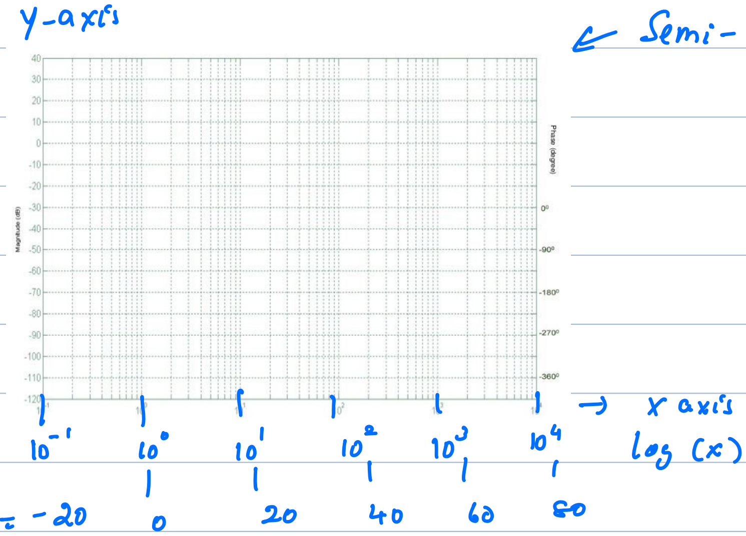
Case 2: $Z(s) = \frac{1}{s}$ $Z(j\omega) = \frac{1}{j\omega} = \boxed{-j}/\omega$

$\angle -90°$

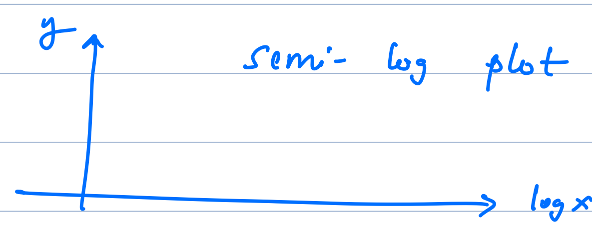
$$20\log_{10}|Z(j\omega)| = 20\log_{10}\frac{1}{\omega}$$
$$= -20\log(\omega)$$
$$\angle Z(j\omega) = -90°$$



linear axis

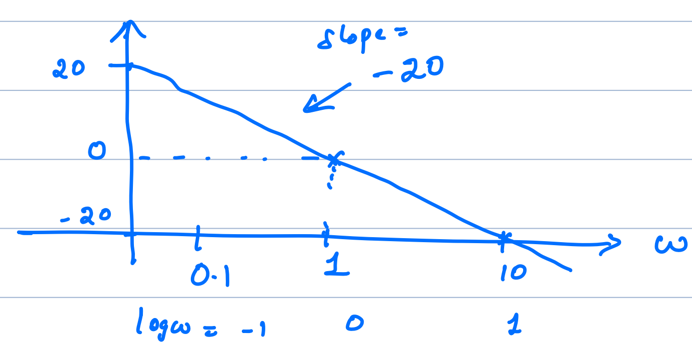


$20\log 10^{-1} = -20$



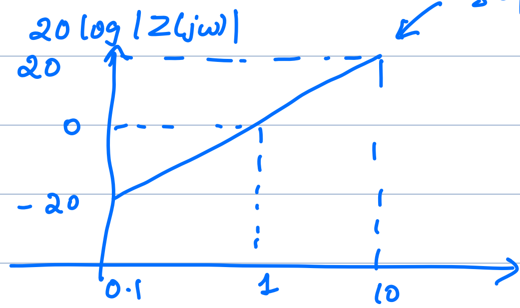
$20\log|Z(j\omega)|$

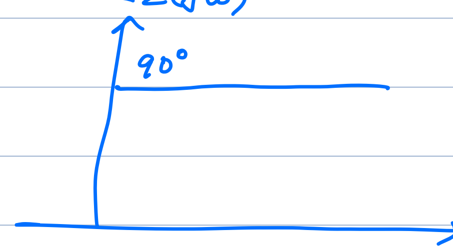
$= -20\log\omega$



Case 3: $Z(s) = s$

slope = +20





Case 4: $Z(s) = \dfrac{1}{\underline{\underline{1+sT}}} = \dfrac{1}{T(\frac{1}{T}+s)}$

constant

$$20\log|Z(j\omega)| = -20\log|1+j\omega T|$$
$$= -20\log\sqrt{\underbrace{1+\omega^2T^2}}$$

" Asymptoten "

$\omega << \dfrac{1}{T}$: $20\log|Z(j\omega)|$

[Ex: $\omega = \dfrac{1}{100}\cdot\dfrac{1}{T}$]

$= -20\log 1$

$= 0$

$1+\left(\frac{1}{100}\right)^2\cdot\frac{1}{\cancel{T^2}}\cdot\cancel{T^2}$

$1+10^{-4}$

≈ 1

$\omega >> \dfrac{1}{T}$ $\quad 20\log|Z(j\omega)|$

$\omega T >> 1$

[Example: $\omega = 100\cdot\dfrac{1}{T}$]

$= -20\log(\omega T)$

$= -20\log(\omega)$

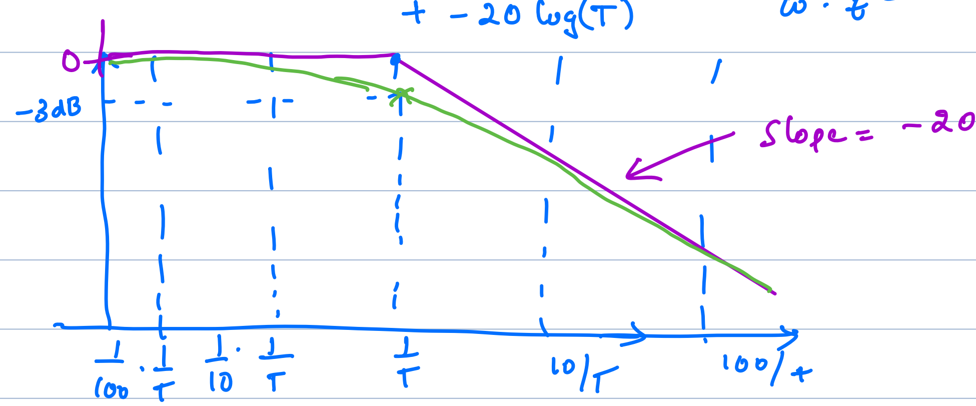
$+ -20\log(T)$

$\sqrt{1+100^2\cdot\frac{1}{\cancel{T^2}}\cdot\cancel{T^2}}$

$1+100^2$

$\approx 100^2$

$\omega^2\cdot t^2$



$-20 \log \sqrt{1+\omega^2 T^2} \qquad \omega = 1/T$

$-20 \log \sqrt{1+1}$

$= -3\,dB$ //

Phase response:

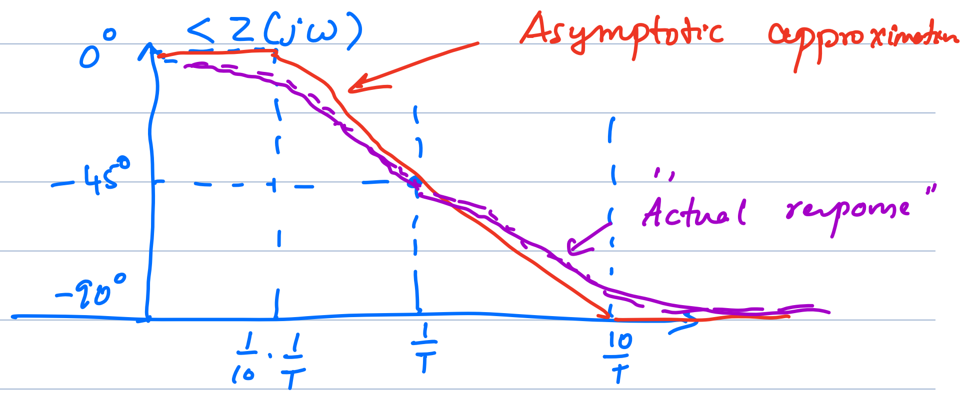
$$\angle Z(j\omega) = \angle \frac{1}{1+j\omega T} = -\tan^{-1}(\omega T)$$

$\omega << \frac{1}{T} \qquad \angle Z(j\omega) = -\tan^{-1}(\approx 0) = 0$

$\omega >> \frac{1}{T} \qquad \angle Z(j\omega) = -\tan^{-1}(\text{high number})$

$= -90°$

$\omega = \frac{1}{T} \qquad \angle Z(j\omega) = -\tan^{-1}(1) = -45°$



Case 5: $Z(s) = 1 + sT$