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Gaussian Process
Human Subject Motion

Introduction:

Gaussian processes can be considered as probability distribution in the space of functions. This exercise involves implementing a Gaussian process for regression on 5 different traces of 12 different human subjects on three different axis, x, y and z. The motion of the human subject is captured by 50 different markers placed along the body of the human subject.

Gaussian Process is defined by its mean, μ and covariance ***K.*** Both the mean and covariance are in turn functions to define which *hyperparameters* are to be estimated. This exercise attempts to estimate the optimal hyperparameters which can define the motion of the human subject.

Method:

This exercise has been done using the scientific library of scikit-learn called *GaussianProcessRegressor*.

The process is based on the algorithm described by Rasmussen and Williams in "*Gaussian Processes for Machine Learning*".

input: X (inputs), $\mathbf{y}$ (targets), k (covariance function), σ_n^2 (noise level), $\mathbf{x}_*$ (test input)

$$L := \text{cholesky}(K + \sigma_n^2 I)$$

$$\left.\begin{aligned} \boldsymbol{\alpha} &:= L^\top \backslash (L \backslash \mathbf{y}) \\ \bar{f}_* &:= \mathbf{k}_*^\top \boldsymbol{\alpha} \end{aligned}\right\} \text{predictive mean eq. (2.25)}$$

$$\left.\begin{aligned} \mathbf{v} &:= L \backslash \mathbf{k}_* \\ \mathbb{V}[f_*] &:= k(\mathbf{x}_*, \mathbf{x}_*) - \mathbf{v}^\top \mathbf{v} \end{aligned}\right\} \text{predictive variance eq. (2.26)}$$

$$\log p(\mathbf{y}|X) := -\tfrac{1}{2}\mathbf{y}^\top \boldsymbol{\alpha} - \textstyle\sum_i \log L_{ii} - \tfrac{n}{2}\log 2\pi \qquad \text{eq. (2.30)}$$

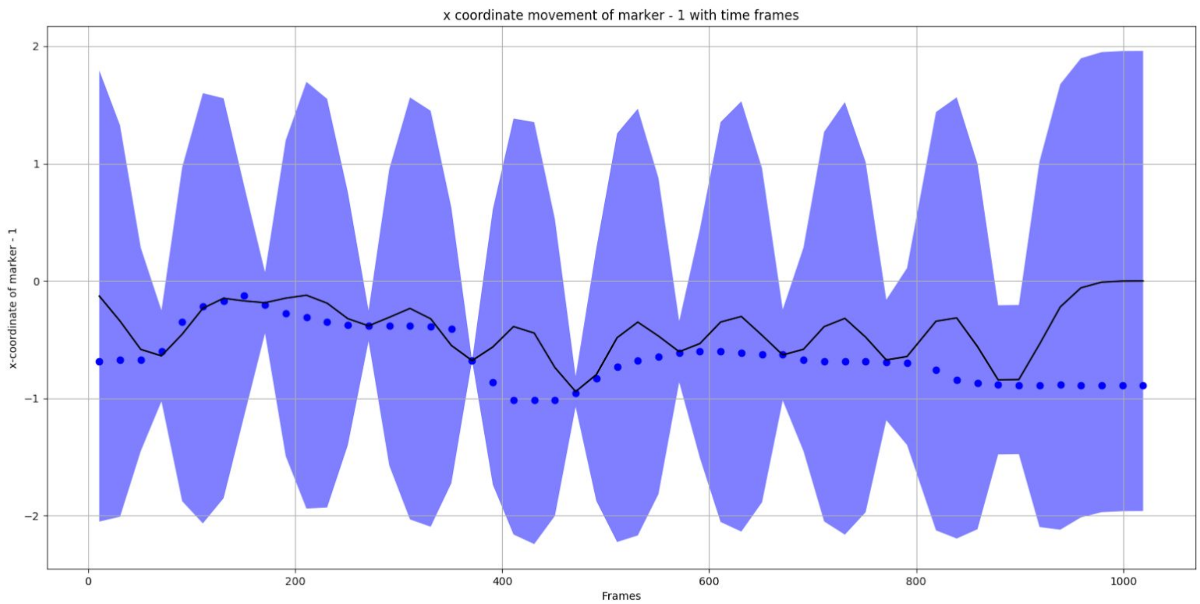
return: $\bar{f}_*$ (mean), $\mathbb{V}[f_*]$ (variance), $\log p(\mathbf{y}|X)$ (log marginal likelihood)

95% confidence interval signifies the probabilistic nature of the Gaussian process. It is calculated on the basis of the mean square error generated.

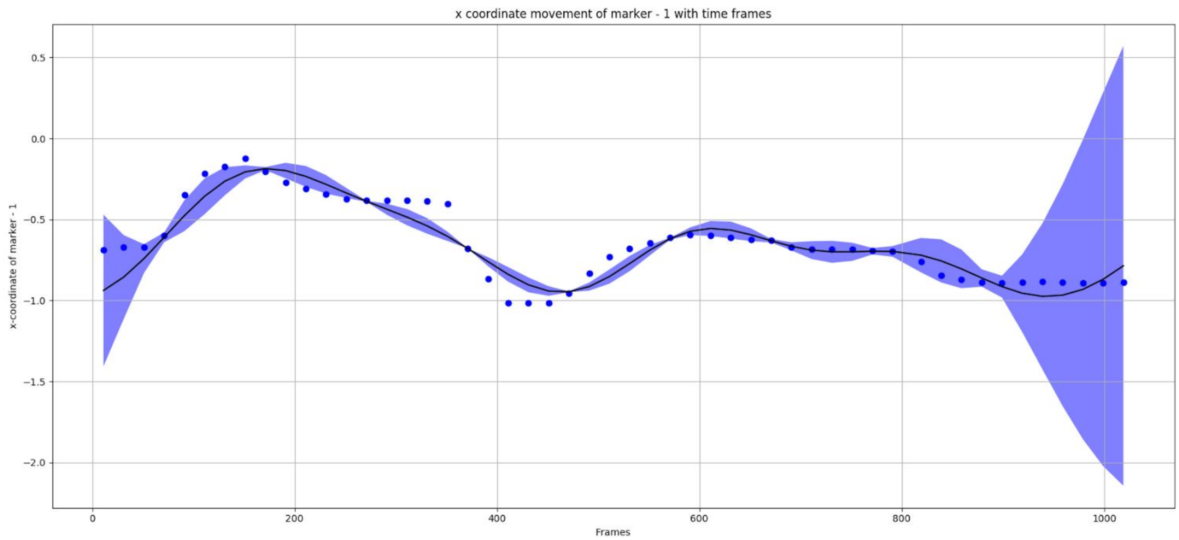
<u>Plots of the GP model fitting one marker of one trail</u>

The first step involved capturing various samples from a single trail. The below plot shows the movement of subject “AG” in the first trail along the 1st marker on X-axis (0_x from block1-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM).

Legend: Blue-dots : Actual movement from the csv file
Black-line: Predicted movement by the Gaussian process
Dark Region: pointwise 95% confidence interval
The same legend is used for all plots



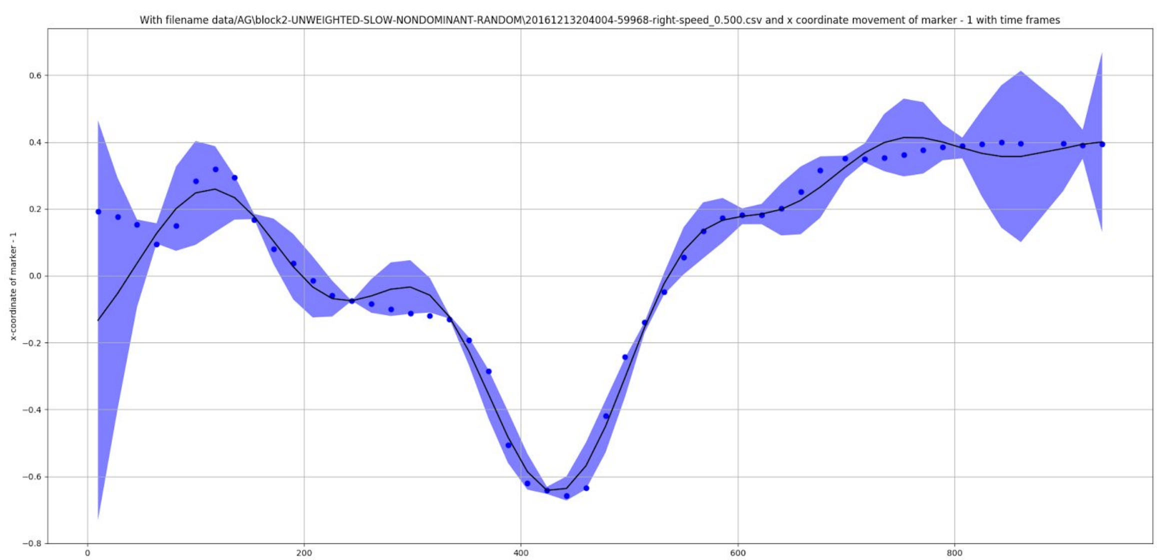
We see that the estimation of the hyperparameters is poor. We will now try to find an optimal hyperparameter to address this. After several trials, the below plot was obtained.
<u>Note:</u> An interesting thing to note here is that with the increase in samples, the prediction improves.



Variation of hyperparameters across data-traces

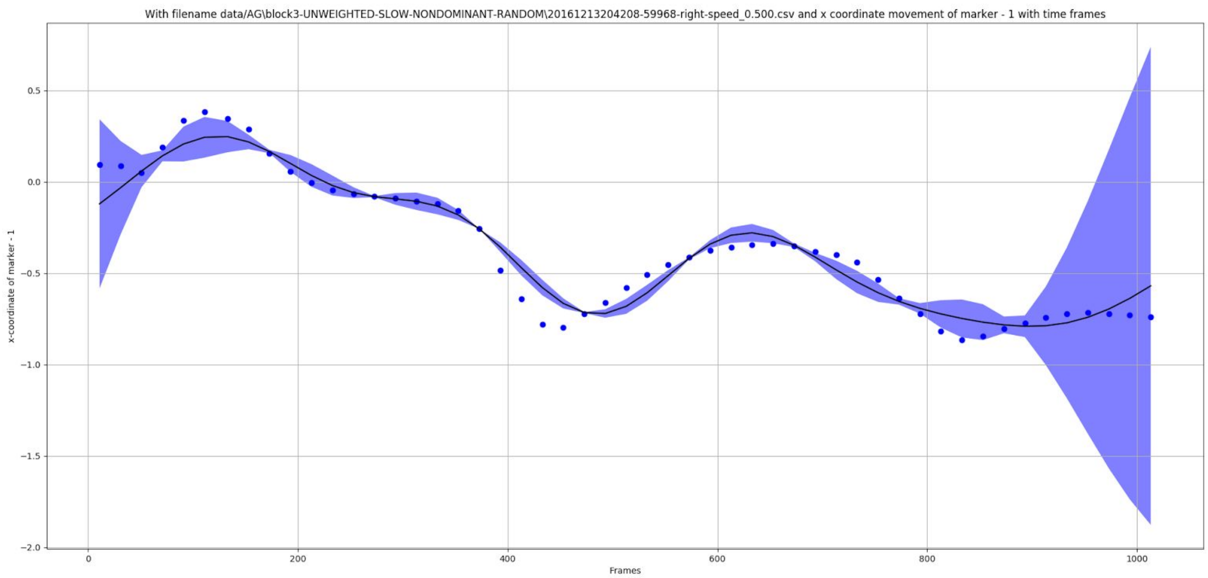
Let us consider the other data-traces of the subject, 'AG'. Using the same hyperparameters, we find the following traces. The figure in the previous section belonged to 'block1-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM'

Plot using data-trace 2: block2-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM

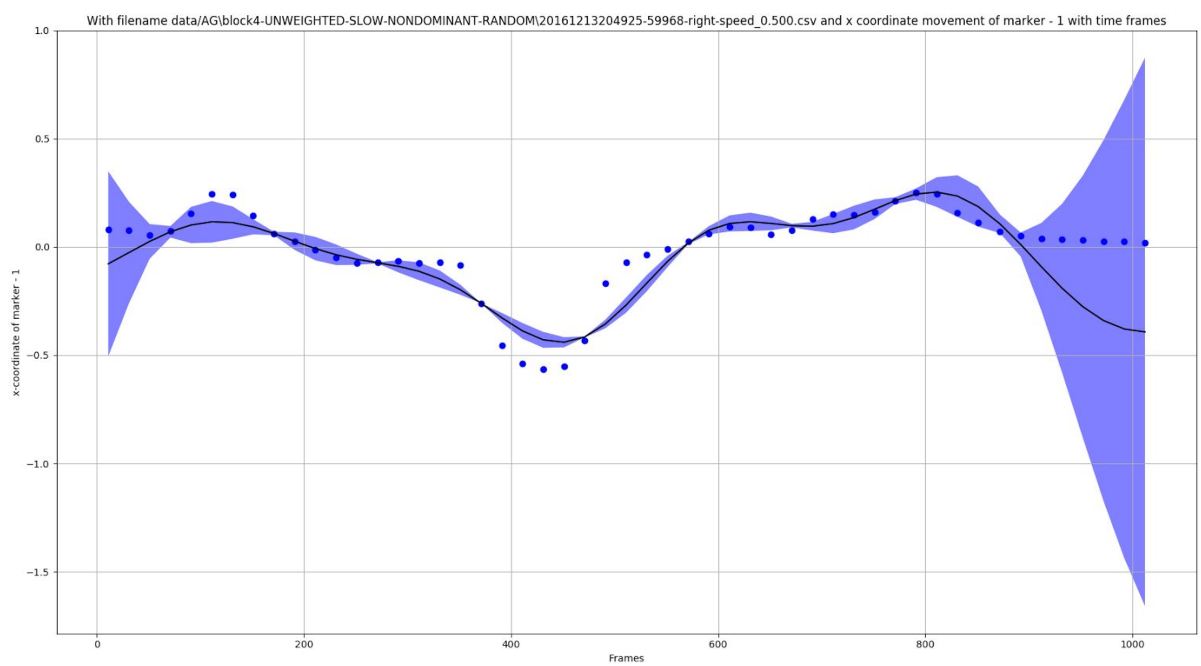


This shows good results with trace 2 using the same hyperparameters.

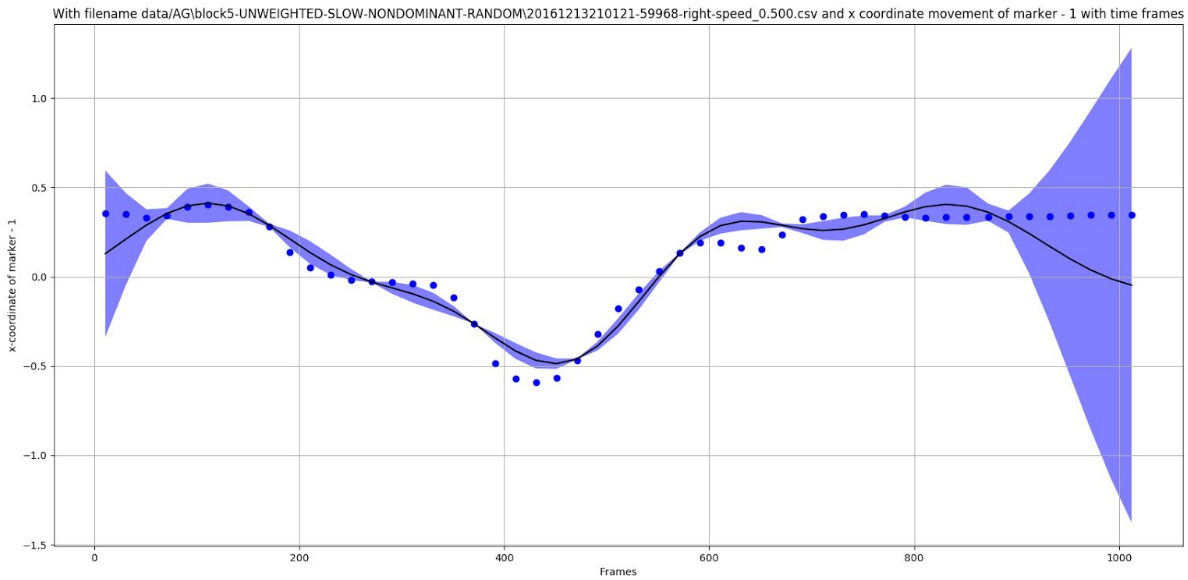
Plot using data-trace 3: block3-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM



Plot using data-trace 4: block4-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM



Plot using data-trace 5: block5-UNWEIGHTED-SLOW-NONDOMINANT-RANDOM



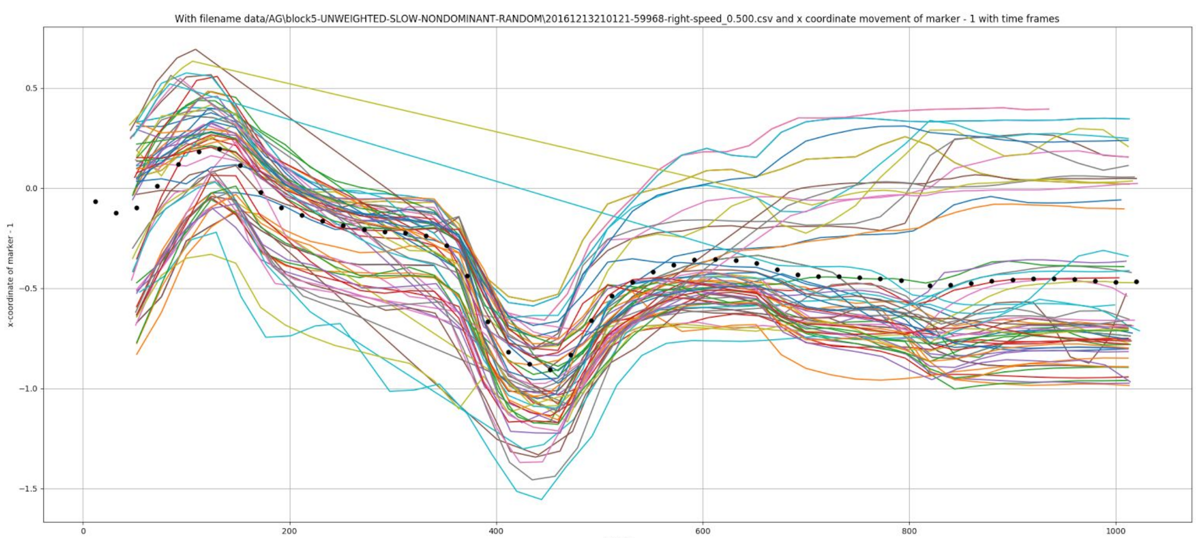
Surprisingly, we find that the motion of the human subject is more and less similar in all traces of the same subject. The hyperparameters remain the same and predict almost accurately.

<u>GP fitting trails over subjects</u>

Legend: black dotted line - prediction

All other curves are the 0_x positions of 40 different frames across all subjects

The black dotted line shows the average of all Gaussian process predictions.



Conclusion:

It seems that one set of hyperparameters can predict the motion of the human subjects. Although this exercise was done using the scikit-learn library, it would be interesting to attempt gradient descent method of estimation of hyperparameters which could be the next step to the experiment. The results of the this library can be compared to the manual implementation of the algorithm.

References:

- *Gaussian Processes for Machine Learning"* by Rasmussen and Williams.
- Gaussian Process for dummies