A Comparison of Canonical Discriminant Analysis and Principal Component Analysis for Spectral Transformation

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Abstract

A study was conducted in Michigan's Upper Peninsula to test the strength and weakness of canonical discriminant analysis (CDA) as a spectral transformation technique to separate ground scene classes which have close spectral signatures. Classification accuracies using CDA transformed images were compared to those using principal component analysis (PCA) transformed images. Results showed that Kappa *accuracies using CDA images were significantly higher than those derived using PCA at $\alpha = 0.05$. Comparison of CDA and PCA eigen structure matrices indicated that there is no distinct pattern in terms of source variable contributions and load signs between the canonical discriminant functions and the principal components.*

Introduction

One challenging task in satellite image processing is to find an effective technique to best discriminate ground information classes whose spectral signatures are similar. Remote sensing scientists may be interested in building a decision rule for classification with a minimum of error rate. Or, their emphasis may be on constructing and interpreting a discrimination space where maximum separation of classes is obtained (Tabachnick and Fidell, 1996). Canonical discriminant analysis (CDA) is a multivariate technique which can be used to determine the relationships among a categorical variable and a group of independent variables. One primary purpose of CDA is to separate classes (populations) in a lower dimensional discriminant space.

However, unlike the use of principal component analysis (PCA), which has been broadly accepted in remote sensing research (Richards, 1984; Singh and Harrison, 1985; Fung and LeDrew, 1987; Longhlin, 1991; Eklundh and Singh, 1993; Eastman and Fulk, 1993; Gong, 1993; Beaubien, 1994; Hirosawa *et al.*, 1996; Roger, 1994; Roger, 1996), CDA is only occasionally described as a transformation technique (Lillesand and Kiefer, 1994; Mather, 1987). Most commercial image processing software packages do not include it as a standard algorithm. The lower popularity of CDA may be due to its "hand-operated" nature and a lack of sufficient examination by remote sensing scientists.

Canonical discriminant analysis, however, has some unique features compared to PCA, especially with regard to its capability to separate classes. Principal component analysis essentially is a data reduction technique to reduce the number of feature variables while maintaining most of the information found in the original image data in the first several components. It is particularly efficient for analyzing multidimensional data that have some degree of correlation (redundancy). There is no guarantee that the first several PCs will necessarily provide the best discriminating power (Dillon *et al.*, 1989; Kshirsager *et al.*, 1990; Jackson, 1991). With CDA, data redundancy can be reduced while discriminating power is preserved in the first several canonical discriminant functions (McLachlan, 1992). Differing from PCA, spectral transformation by CDA uses coefficients obtained from class-related training samples. The transformation process is human "guided" and class dependent. Although CDA derives canonical variables that summarize between-class variation in much the same way that PCA summarizes total image band variation (SAS/STAT, 1990), a single principal component can not discriminate any better than the first canonical discriminant function (Jackson, 1991). The trade-off here is that CDA involves human effort and knowledge in selecting training samples, while PCA maintains a relatively automated and scene dependent process. The introduced human knowledge (extra information) in training may enhance class separation, thus improving classification performance.

This study, therefore, was designed to explore the strength as well as weakness of CDA as a spectral transformation technique and to compare it to PCA in improving classification accuracy. The null hypothesis is that spectral transformation based on canonical discriminant analysis does not improve classification accuracy compared to the use of principal component analysis.

Background

A preliminary analysis was conducted to evaluate the potential of using canonical discriminant transformation in image preprocessing of remotely sensed satellite data to improve classification performance in Pawnee National Grasslands in northeastern Colorado (Maxwell, 1976). The primary objective of the research was to obtain a reduced data set using the canonical

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discriminant transformation. Maxwell found that the final classification using the transformed variables resulted in only a slight improvement in classification accuracy compared to using raw data.

Maher (1987) found out that using canonical discriminant transformation of raw satellite data greatly facilitated visual interpretation in a false-color spectral space constructed by the first three canonical component images. He broadly discussed the definition, objectives, calculation procedures, hypothesis testing, and applications of canonical discriminant analysis in image pre-processing. Mather suggested that the use of multiple discriminant analysis in remote sensing be aimed at maximizing between-group spectral variance for visual interpretation and analysis. The use of discriminant function images for improving classification accuracy was not recommended.

Recently, Lillesand and Kiefer (1994) discussed the potential application of CDA in image pre-processing. They suggested that canonical discriminant analysis will not only improve classification efficiency, but will also improve classification accuracy due to the increased spectral separability of classes.

The Computational Procedure

The objective of CDA is to search for linear combinations l's ($l_1', l_2', \ldots, l_k'$) of independent variables x to achieve maximum separation of classes (populations) y. In a simple case of two classes with multivariate observations x, discriminant analysis can be carried out to transform x to univariate y ($y = l'x$) such that the y's of the two classes are separated as much as possible. The separation of the y is defined as the absolute difference of the means ($\bar{y}_1$ and $\bar{y}_2$) in standard deviation units (Johnson and Wichern, 1992): i.e.,

$$Separation = \frac{|\bar{y}_1 - \bar{y}_2|}{s_y} \quad (1)$$

where

$$s_y^2 = \frac{\sum_{j=1}^{n_1} (y_{1j} - \bar{y}_1)^2 + \sum_{j=1}^{n_2} (y_{2j} - \bar{y}_2)^2}{(n_1 + n_2 - 2)} \quad (2)$$

is the pooled estimate of the variance. The linear combination

$$y = \hat{l}'x = (\bar{x}_1 - \bar{x}_2)' S_{pooled}^{-1} x \quad (3)$$

maximizes the ratio

$$\frac{(\bar{y}_1 - \bar{y}_2)^2}{s_y^2} = \frac{(\hat{l}'\bar{x}_1 - \hat{l}'\bar{x}_2)^2}{\hat{l}' S_{pooled} \hat{l}}. \quad (4)$$

When the objective is to obtain linear combinations separating more than two populations, CDA can be carried out based on the same principle (Johnson and Wichern, 1992): i.e.,

$$\left(\frac{\textit{Sum of squared distances from populations to overall means}}{\textit{Variance of Y}}\right) \quad (5)$$

$$= \frac{l'\left[\sum_{i=1}^{g} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})'\right] l}{l'\left[\sum_{i=1}^{g} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)'\right] l} = \frac{l'Ml}{l'Nl} \quad (6)$$

where

$$\bar{x}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij} \; (\textit{Mean vector}), \; \bar{x} = \frac{\sum_{i=1}^{g} n_i \bar{x}_i}{\sum_{i=1}^{g} n_i} \; (\textit{Overall mean scalar})$$

and

$$M = \sum_{i=1}^{g} (\bar{x}_i - \bar{x})(\bar{x}_i - \bar{x})', \; N = \sum_{i=1}^{g} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)(x_{ij} - \bar{x}_i)'.$$

The vectors of coefficients l's that maximize ratio (6) are given by $l_1' = e_1, l_2' = e_2, \ldots, l_k' = e_k$, where $e_1, \ldots, e_s$ are the standardized eigenvectors of matrix $\mathbf{N}^{-1}\mathbf{M}$ (Johnson and Wichern, 1992). The linear combination $l_k'x = e_k x$ ($k \le s$) is the sample kth discriminant, also called the kth canonical discriminant function. A primary assumption is that the g classes should have a common covariance matrix of full rank; but it is not necessary that they have multivariate normal distributions (Johnson and Wichern, 1992).

The dimensionality of canonical discriminant space is usually equal to the number of nonzero eigenvalues of $\mathbf{N}^{-1}\mathbf{M}$, which is the smaller of the two numbers, $g - 1$ and p (x's dimension). However, it is possible that the number of significant canonical discriminant dimensions may be smaller (Tatsuoka, 1971).

Two types of significance tests are usually of interest in canonical discriminant analysis. The first is an overall test to determine whether there is any significant linear relationship between the canonical variable y's and the independent variable x's. In this application, the x's are band variables. If overall significance is found, a second test is performed to detect the significance of succeeding canonical correlations after the first, and to determine how many of the canonical discriminant dimensions contribute significantly to group differentiation (Rencher, 1995; SAS/STAT, 1990; Tatsuoka, 1971; Wherry, 1984).

Even though canonical discriminant variables are primarily artificial, they may often be associated with physical variables. However, interpretation of canonical discriminant functions should be undertaken with caution (Johnson and Wichern, 1992; Tabachnick and Fidell, 1996; Tatsuoka, 1971; Rencher, 1995; Wherry, 1984). Many papers indicate that the correlation (also referred to as "loading" or "structure") between the canonical discriminant functions and the original variables are more informative and are the best measure of source variable importance. Other analyses have shown that the correlation only provides information on the contribution of source variables in a univariate context, rather than in a multivariate setting (Johnson and Wichern, 1992; Rencher, 1995). The "canonical structure" does not indicate how the original variables contribute jointly to the separation of information classes in canonical discriminant space. For this reason, the canonical coefficients, the l's, are more informative and may be used to assess the contributions of the original variables (Johnson and Wichern, 1992; Rencher, 1995) by evaluating their signs and magnitude.

However, the coefficients reflect not only the difference in contribution of the source variables, but also differences in scaling of the variables. To remove the effect of scaling, the coefficients may be standardized by dividing the initial estimates by the standard deviations of the corresponding variables (Rencher, 1995). The canonical coefficients are also not very stable in regard to sampling effects; therefore, a large sample size is usually preferred in order to obtain reliable coefficients (Johnson and Wichern, 1992; Scheiner and Gurevitch, 1993).

Methods

Study Sites and Data Collection

This study was conducted at two sites in Michigan's Upper Peninsula: Thompson and Raco Plain (Figure 1). Major conifer

Figure 1. Map showing locations of the two study sites, Thompson and Raco Plain, Michigan.

and hardwood species, such as jack pine (*Pinus banksiana*), red pine (*P. resinosa*), black spruce (*Picea mariana*), white spruce (*P. glauca*), balsam fir (*Abies balsamea*), tamarack (*Larix laricina*), northern white cedar (*Thuja occidentalis*), sugar maple (*Acer saccharum*), big-tooth aspen (*Populus grandidentata*), yellow birch (*Betula alleghaniensis*), and paper birch (*B. papyrifera*) occur at both sites along with other species.

Thompson (10 km by 10 km) is located east of Escanaba along the southern shore of the central part of the Peninsula. The dominant species are jack pine, red pine, black spruce, northern white cedar, and sugar maple. Site conditions are characterized by limestone bedrock and sand lake plain. The elevation ranges from 185 to 200 meters. Raco Plain (11 km to 12 km) is located in the eastern upper part of the Peninsula. The dominant trees are jack pine, red pine, and their mixtures with various structure conditions. Northern hardwoods are also dominant on uplands in this area. Site conditions are characterized by sand lake plain. Elevation ranges from 280 to 310 meters.

A subscene covering each of the study areas was extracted from Landsat Thematic Mapper scenes from August and September, 1991. Two sets of aerial photos were purchased for accuracy assessment, and they were dated May, 1994 (Thompson; 1:40,000-scale; black-and-white infrared) and May, 1992 (Raco Plain; 1:15,840-scale; color infrared). During the assessment process, sampling locations were generated using ERDAS IMAGINE and identified on the rectified digital photos. Forest classes and structure conditions were manually interpreted using stereoscopic methods on the aerial photos. A total of five field visits were conducted to the two sites in May, 1994 and June and September, 1995. Forest classes and structure conditions were examined and large forest patches were identified on the ground for training purposes.

Defining Classification Schemes

Because the derivation of canonical coefficients depends entirely on training samples, reliable training statistics must be obtained to adequately represent the spectral characteristics of the defined classification schemes. The initial classification scheme was adapted from an existing system (Maclean, 1994, personal communication). Training samples for each class were later located on the TM images, and a group of more than 50 continuous pixels was taken from each class in each image for training.

Data and Image Processing

All data and image processing were performed on a SUN UNIX workstation and a PC Windows 95 microcomputer using ERDAS IMAGINE and SAS software. A CDA data and image processing model (Figure 2) was designed using ERDAS IMAGINE Spatial Modeler and the SAS CANDISC Procedure.

Prior to performing any of the four steps, non-target classes were masked out to facilitate data processing for the remaining areas where forests are located. That is, any large, clear, and homogeneous non-forest area (e.g., lakes, clear cutting area, and agriculture lands) were classified in the first pass and masked out. The same procedure was applied to both CDA and PCA transformations.

The canonical coefficients were calculated directly using the SAS CANDISC Procedure and sample data taken from the source TM subset. The source data were then standardized using the total sample means and standard deviations. The total-sample standardized canonical coefficients were selected to transform the source data, and a total of six CDA component images were generated. An unsupervised algorithm (ERDAS IMAGINE, 1995) was applied to a subset of the component images to segregate a number of spectral clusters. Only the first four CDA and PCA component images were used in the classifications (see the next section). The clusters were then recoded to appropriate information classes based on field investigation results and existing old maps, classifications, and forest inventory data. Accuracy assessments were carried out independently through stereoscopic aerial photo interpretation.

A discrete multivariate technique, called *Kappa* (K_{hat}) analysis, was applied to each error matrix (Congalton *et al.*, 1983; Congalton, 1991). The K_{hat} values from each error matrix were calculated and compared. The difference between the two K_{hat}

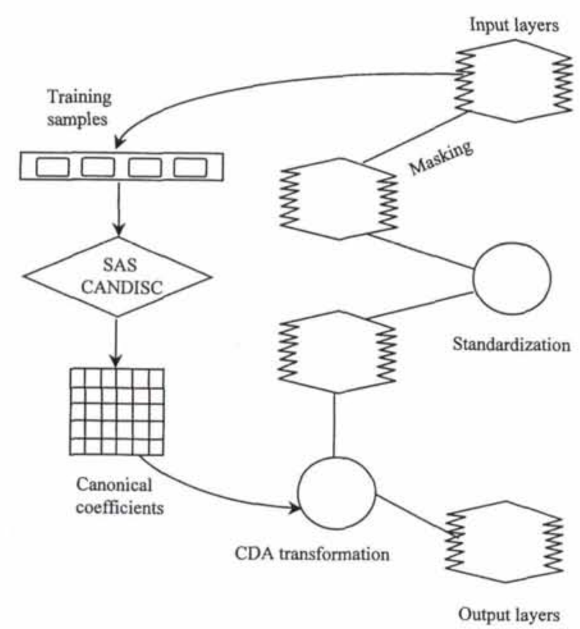

Figure 2. Canonical discriminant transformation model showing steps of the transformation process.

TABLE 1. EIGENVECTORS AND EIGENVALUES FOR THE STANDARDIZED CANONICAL DISCRIMINANT TRANSFORMATION FOR THE RACO PLANE SITE, MICHIGAN'S UPPER PENINSULA.

Standardized	CAN1	CAN2	CAN3	CAN4	CAN5	CAN6
BAND1	0.119	0.118	−1.343	−1.338	−0.439	0.974
BAND2	−0.189	−0.1878	−0.4263	−0.5945	1.9999	−0.9969
BAND3	−0.344	0.7226	−0.9885	1.9893	−1.7130	−0.9767
BAND4	5.314	0.0767	−0.7272	1.6386	−0.0606	0.8287
BAND5	−0.084	1.1947	1.7450	−2.0751	−1.4443	−1.5917
BAND7	−0.649	0.9855	0.6451	1.3740	1.5363	2.3235
Eigenvalue	28.58	5.79	2.46	0.26	0.07	0.03
Proportion	0.7683	0.1558	0.0662	0.0069	0.0019	0.0008
Cumulative	0.7683	0.9241	0.9904	0.9973	0.9992	1.0000

values was evaluated by a standard Z test at $\alpha = 0.05$ (Congalton, 1991). A FORTRAN program written by Congalton was used to perform the *Kappa* analysis. Traditional producer's and user's accuracies were also calculated and compared for individual class accuracy.

Results and Discussions

Selection of the Components for Classifications

All of the derived canonical components (discriminant functions) contribute to class differentiation ($p < 0.05$). However, only a very small portion (less than 1 percent) of information (variance) was accounted for by the last two components when their eigenvalues (Table 1) were examined. Similar to principal component transformation (Figure 3), the first four canonical component images account for most of the variance and contain the "target" information (Figure 4). The last two primarily contained "unexplained," "noisy" elements, like PC5 and PC6. They appeared as "stripes" running diagonally from the upper left to the lower right corner on the images (Figure 4). These stripes were represented as spatially correlated, structured noise (Roger, 1996). The issue of noisy variance associated with the last two or three PCA components has been strongly argued and empirically verified for remotely sensed satellite data (Fung and LeDrew, 1987; Roger, 1994; Roger, 1996). However, this issue has not been considered with CDA components in the past. The possible impact of the noise elements on classification performance has not been examined.

Because the "noise" or "non-target" variance is minimal compared to the data variance in the first four components, it seems unlikely that excluding the last two CDA and PCA components from entering the classifications would drastically degrade the classification accuracy. In fact, for both CDA and PCA, such a noise-adjustment procedure may actually help improve overall classification performance. That is, CDA can be used as a transformation technique not only to increase class separations, but also to reduce data dimension and noise. Maxwell (1976) found that classification results using all of the CDA function images show only a slight improvement in accuracy,

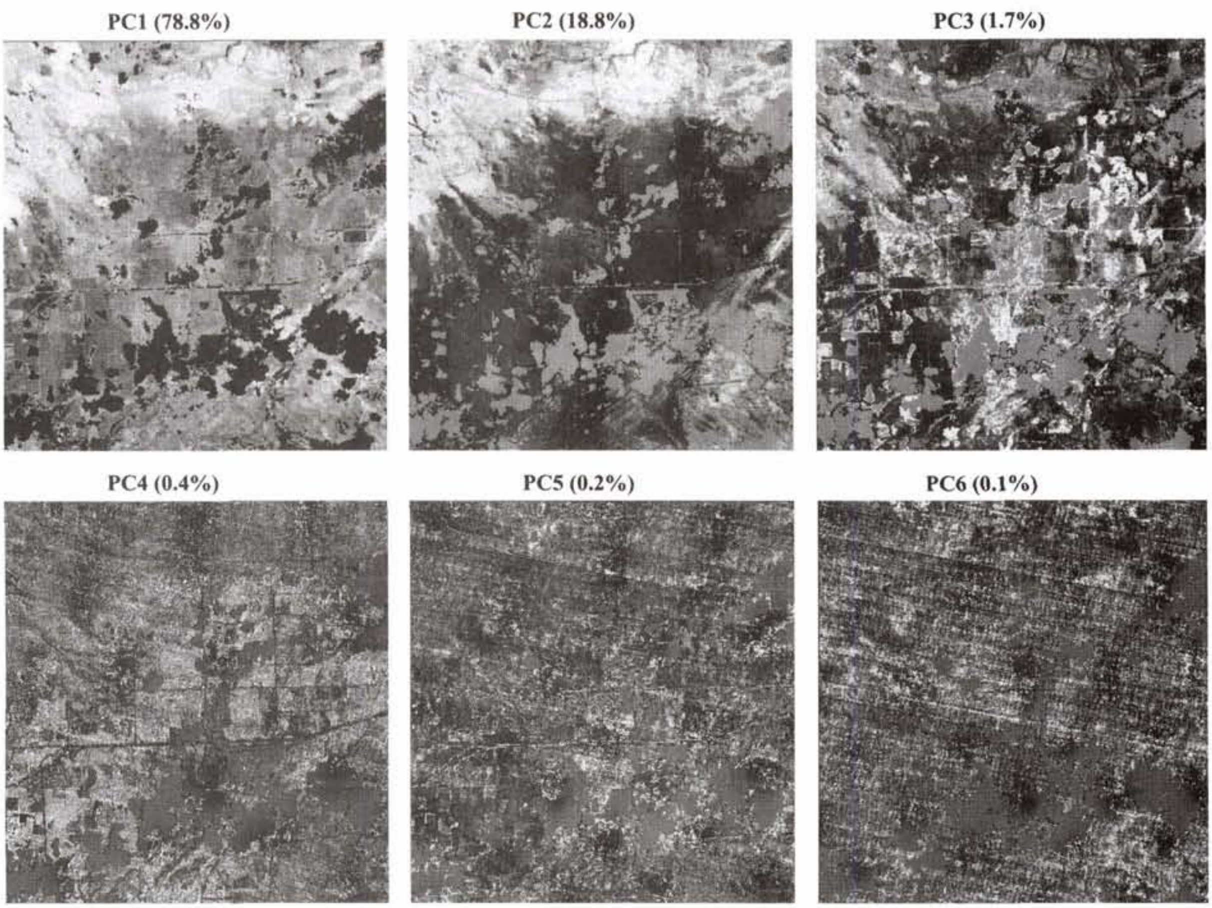

Figure 3. Map showing transformed component images using principal component analysis, Raco Plain, Michigan. The percentage of the source scene variance contained in each canonical component is indicated. The last two components show noise variances.

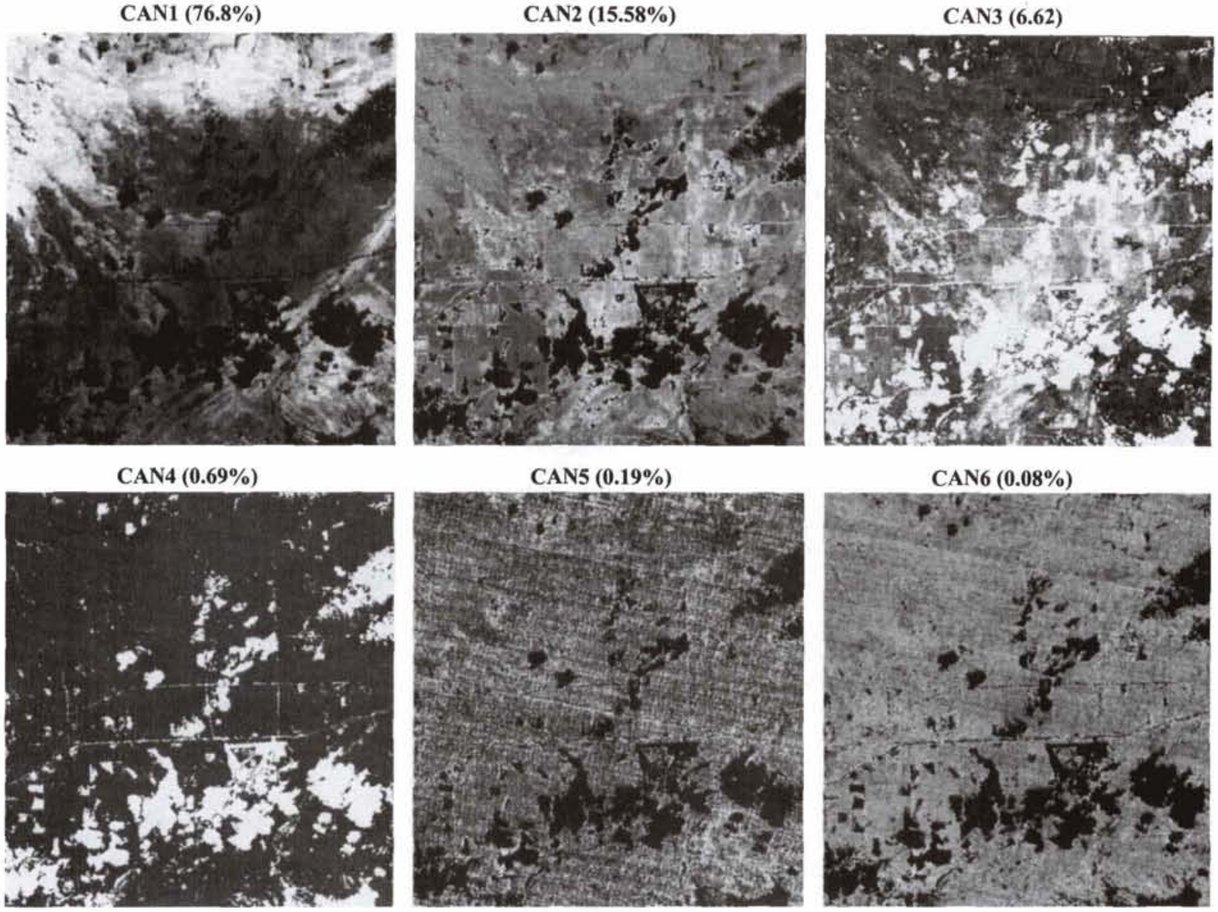

Figure 4. Map showing transformed component images using canonical discriminant analysis, Raco Plain, Michigan. The percentage of the source scene variance contained in each canonical component is indicated. The last two components show similar noise variances compared to PC images in Figure 3.

and the insignificant results can probably be attributed to the use of all variables for classification and the high F values for the source variables. The use of all canonical functions for classification virtually extracted all of the source information, regardless of the useful or "noise" variance. It turns out that CDA does increase the separation among classes by enlarging the between-group variance for the first several components. When only the first two CDA images were used for classification, significant improvements were obtained because of the maximization of between-groups variance. Visual comparisons in this study show that classifications derived from the first four CDA components were better than those from the whole data set. When selecting the component images for final classification, therefore, only the first four components were selected and the last two CDA components were removed from the input classification data. The last two PCA images were also dropped prior to classifications.

Interpretation of the Source Variable Contributions

Both the first canonical discriminant function and the first principal component are heavily weighted to the infrared bands (Tables 1 and 2). This may be attributed to the predominantly forested landscapes of the study areas and the use of masked images with minimum non-forest information. The second components, compared to the first components, are weighted relatively lightly in the infrared bands. The first two canonical and principal components represent between 92 and 97 percent of the total variances, respectively. The third and fourth components are weighted equally in the visible and infrared bands and comprise between 3 and 8 percent of the total variance. The loading "signs" for both the canonical and principal components tend to follow a similar pattern, but seems to be physical-content (source data) related. Both the first components derived from the Thompson image have positive coefficients for all the visible and infrared bands. In the Raco Plain image, they are a mixture of positive and negative coefficients. This could be linked possibly to the different vegetation content of the two areas. At Thompson, conifer stands occupy more than 80 percent of the total area. At Raco Plain, however, conifer stands occupy approximately 60 percent of the total area. The comparisons show that the canonical and principal components are not significantly distinct in terms of source variables' contributions and load signs.

Comparison of Classification Accuracy

At the Thompson site, the canonical discriminant analysis approach yields a higher overall classification accuracy of 75.6 percent ($K_{hat} = 0.728$), which is a more than 6 percent increase compared to that produced by the PCA method (69.1 percent, $K_{hat} = 0.654$). At the Raco Plain site, similarly, the CDA approach yields a higher overall accuracy of 70.8 percent ($K_{hat} = 0.681$), while the PCA yields an overall accuracy of 63.8 percent ($K_{hat} = 0.604$). The net increase in accuracy is 7 percent. The improved accuracies indicate that the maximization of class

TABLE 2. EIGENVECTORS AND EIGENVALUES FOR THE PCA TRANSFORMATION FOR THE RACO PLANE SITE, MICHIGAN'S UPPER PENINSULA

Non-Standardized	PC1	PC2	PC3	PC4	PC5	PC6
BAND1	0.0173	0.1971	−0.6763	−0.4122	−0.5735	−0.0675
BAND2	−0.0128	0.1336	−0.3538	0.0309	0.3860	0.8407
BAND3	0.0371	0.2317	−0.4844	−0.0963	0.6456	−0.5330
BAND4	−0.9649	−0.2079	−0.1306	−0.0882	−0.0062	−0.0304
BAND5	−0.259	0.8324	0.3834	0.3016	0.0437	−0.0061
BAND7	0.0119	0.3916	−0.1364	−0.8491	−0.3214	0.0593
Eignvalue	645.35	154.07	13.567	3.57	1.90	0.767
Proportion	0.788	0.188	0.017	0.004	0.002	0.001
Cumulative	0.788	0.976	0.993	0.997	0.999	1

separation, the removal of noise elements, and the introduction of human knowledge in training by the CDA procedure possibly helped improve the overall classification accuracy.

Accuracies for individual classes vary from 50 to near 100 percent. Generally, most user's and producer's accuracies of the classifications using the CAN images are higher than those using the PC images. The producer's accuracies suggested that jack pine, northern hardwoods, and lowland hardwoods were the easiest classes to map using the CAN images at the Thompson site. The relatively low overall accuracy and some individual accuracies suggest that the source TMdata may not be capable of separating similar information classes, such as red pine and black spruce, from each other. Additional ancillary information may be necessary to improve the classification results.

Examination of the error matrices showed that a majority of the misclassifications came from those transitional, structure-based classes or spectrally similar classes. These classes were previously identified through visual examination in the three-dimensional canonical discriminant spaces. The stand structure-related classes at Raco Plain proved very difficult to discriminate using the canonical component images. The most difficult pairs of classes to separate are young jack pine and red pine plantations and mature jack pine and mixed jack and red pine stands. The spectrally similar classes were also difficult to separate on the canonical component images. Although the CDA approach likely helped eliminate misclassifications for certain classes such as black spruce and white cedar, it did not for others like red pine, white spruce, and northern hardwood. Results of the *Kappa* analysis are used to compare the error matrices from the CDA and PCA approaches. Classification accuracy using CDA is better than that using PCA ($Z = 1.96$, $p < 0.05$). This holds for both of the independent transformation experiments conducted at the two sites. The results clearly reject the null hypothesis and suggest that the CDA transformation is of good quality and a better choice, even though it is still not perfect.

Conclusions

The use of canonical discriminant analysis as a spectral transformation technique and its application to forest type delineation using Landsat TM data in the Thompson and Raco Plain areas of Michigan's Upper Peninsula were evaluated. The CDAs overall classification accuracy proved to be greater than that of PCA ($p < 0.05$).

The last two canonical components usually contain largely noise variance, which accounts for less than 1 percent of the total variance found in the source variables. A sub-dimension (the first four components) is preferable to the whole derived canonical component data sets for final classifications, because the noise variance associated with the last two components is removed. Comparison of CDA and PCA eigen structure matrices revealed that there is no distinct pattern in terms of source variable contribution or load signs between the canonical and the principal components. The first two canonical components are heavily weighted to the infrared bands; the third and fourth components, in contrast, are relatively equally loaded on the visible and infrared bands. Similar loading patterns also are found in the principal components.

Results suggest that canonical discriminant analysis transformation is superior to principal component analysis transformation in improving the overall classification accuracy and that canonical discriminant analysis should be included as a standard spectral transformation algorithm in commercial image processing software. The relatively low classification accuracies for both studied sites indicate that the source TM data may not be capable of separating such classes as red pine and black spruce, or classes of various structure conditions. The use of ancillary data may be necessary to improve the classification results.

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