VISUALIZATION OF STREAMLINES FOR FLOW PAST A STOKESLET

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THEORY

A stokeslet is the solution to a *point force* embedded in Stokes flow. In a lot of the literature, spherical particles are approximated as Stokeslets. Therefore, understanding the Stokeslet is very fundamental to studying the interaction between particles at low Reynold's numbers.

For a 2D velocity flow field $\boldsymbol{u} = u\hat{\boldsymbol{i}} + v\hat{\boldsymbol{j}}$, define a stream function such that

$$v = -\frac{\partial \psi}{\partial x} \quad u = \frac{\partial \psi}{\partial y} \tag{1}$$

Governing equations of Stokes Flow are given as,

$$\nabla p - \mu \nabla^2 \boldsymbol{u} = \boldsymbol{f} \tag{2}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{3}$$

Taking curl of the Momentum equations, we get

$$\nabla^2(\mathbf{\nabla} \times \mathbf{u}) = \mathbf{\nabla} \times \mathbf{\nabla} p = 0 \tag{4}$$

$$\therefore \nabla^4 \psi = 0 \tag{5}$$

We see that the above equation is bi-harmonic. The streamlines for a Stokeslet in infinite domain, as obtained from the accompanying code, are as shown in the figure below.

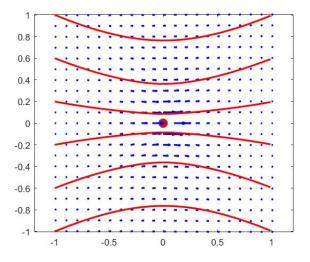


Figure 1: Flow past a Stokeslet