1_Notebook_Project_LifeExpectancy_LinearRegression

December 20, 2020

1 Linear Regression

1. Convert Business Problem to Data Science Problem
2. Load Data
3. Understand the Data
4. Data Preprocessing
5. Exploratory Data Analysis
6. Model Building
7. Model Diagnostics
8. Predictions and Evaluations

[1]:
```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

1.1 Validate your System Libraries

Validate your System Libraries and if version is not updated, please update it.

[2]:
```
# python version
# python --version

# python version (method 2)
from platform import python_version

print('python: {}'.format(python_version()))

#numpy version
import numpy as np
print('numpy: {}'.format(np.version.version))

#pandas version
import pandas as pd
print('pandas: {}'.format(pd.__version__))

#seaborn version
import seaborn as sns
```

```python
print('seaborn: {}'.format(sns.__version__))

# matplotlib version
import matplotlib
print('matplotlib: {}'.format(matplotlib.__version__))

# sklearn version
import sklearn
print('The scikit-learn version is {}.'.format(sklearn.__version__))

# statsmodels version
import statsmodels
print('statsmodels: {}'.format(statsmodels.__version__))

# statsmodels version
import imblearn
print('imblearn : {}'.format(imblearn .__version__))

# Pandas also provides a utility function, pd.show_versions(), which reports␣
↪the version of its dependencies as well:
# pd.show_versions(as_json=False)
```

```
python: 3.6.9
numpy: 1.19.4
pandas: 1.1.5
seaborn: 0.11.0
matplotlib: 3.2.2
The scikit-learn version is 0.22.2.post1.
statsmodels: 0.10.2
imblearn : 0.4.3
```

```
/usr/local/lib/python3.6/dist-packages/sklearn/externals/six.py:31:
FutureWarning: The module is deprecated in version 0.21 and will be removed in
version 0.23 since we've dropped support for Python 2.7. Please rely on the
official version of six (https://pypi.org/project/six/).
  "(https://pypi.org/project/six/).", FutureWarning)
/usr/local/lib/python3.6/dist-packages/sklearn/utils/deprecation.py:144:
FutureWarning: The sklearn.neighbors.base module is  deprecated in version 0.22
and will be removed in version 0.24. The corresponding classes / functions
should instead be imported from sklearn.neighbors. Anything that cannot be
imported from sklearn.neighbors is now part of the private API.
  warnings.warn(message, FutureWarning)
```

1.2 1. Import Libraries

[3]:
```
pd.set_option('display.max_rows', 800)
pd.set_option('display.max_columns', 500)

import matplotlib.pyplot as plt
%matplotlib inline

# import all libraries and dependencies for machine learning
from sklearn import preprocessing
from sklearn.model_selection import train_test_split
import statsmodels.api as sm
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression
from statsmodels.stats.outliers_influence import variance_inflation_factor
from sklearn.metrics import mean_absolute_error, mean_squared_error,r2_score
import random
```

```
/usr/local/lib/python3.6/dist-packages/statsmodels/tools/_testing.py:19:
FutureWarning: pandas.util.testing is deprecated. Use the functions in the
public API at pandas.testing instead.
  import pandas.util.testing as tm
```

1.3 2. Load Data

[4]:
```
# Loading the dataset
df = pd.read_csv("/content/drive/MyDrive/Colab Notebooks/Life Expectancy Data.
 ↪csv")
```

1.4 3. Understanding the data

[5]:
```
#Pandas dataframe.info() function is used to get a concise summary of the␣
 ↪dataframe.
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2938 entries, 0 to 2937
Data columns (total 22 columns):
 #   Column                           Non-Null Count  Dtype
---  ------                           --------------  -----
 0   Country                          2938 non-null   object
 1   Year                             2938 non-null   int64
 2   Status                           2938 non-null   object
 3   Life expectancy                  2928 non-null   float64
 4   Adult Mortality                  2928 non-null   float64
 5   infant deaths                    2938 non-null   int64
 6   Alcohol                          2744 non-null   float64
```

```
 7   percentage expenditure           2938 non-null   float64
 8   Hepatitis B                      2385 non-null   float64
 9   Measles                          2938 non-null   int64
 10   BMI                             2904 non-null   float64
 11  under-five deaths                2938 non-null   int64
 12  Polio                            2919 non-null   float64
 13  Total expenditure                2712 non-null   float64
 14  Diphtheria                       2919 non-null   float64
 15   HIV/AIDS                        2938 non-null   float64
 16  GDP                              2490 non-null   float64
 17  Population                       2286 non-null   float64
 18   thinness  1-19 years            2904 non-null   float64
 19   thinness 5-9 years              2904 non-null   float64
 20  Income composition of resources  2771 non-null   float64
 21  Schooling                        2775 non-null   float64
dtypes: float64(16), int64(4), object(2)
memory usage: 505.1+ KB
```

[6]:
```python
# Exploring the dataset
df.shape
```

[6]:
```
(2938, 22)
```

2 Missing values

[7]:
```python
# Checking the null values in the dataset
df.isnull().sum()
```

[7]:
```
Country                              0
Year                                 0
Status                               0
Life expectancy                     10
Adult Mortality                     10
infant deaths                        0
Alcohol                            194
percentage expenditure               0
Hepatitis B                        553
Measles                              0
 BMI                                34
under-five deaths                    0
Polio                               19
Total expenditure                  226
Diphtheria                          19
 HIV/AIDS                            0
GDP                                448
Population                         652
 thinness  1-19 years               34
```

```
 thinness 5-9 years                34
Income composition of resources   167
Schooling                         163
dtype: int64
```

[8]:
```python
#The describe() function computes a summary of statistics pertaining to the␣
↪DataFrame columns.
df.describe()
```

[8]:
```
              Year  Life expectancy   Adult Mortality  infant deaths  \
count  2938.000000       2928.000000       2928.000000    2938.000000
mean   2007.518720         69.224932        164.796448      30.303948
std       4.613841          9.523867        124.292079     117.926501
min    2000.000000         36.300000          1.000000       0.000000
25%    2004.000000         63.100000         74.000000       0.000000
50%    2008.000000         72.100000        144.000000       3.000000
75%    2012.000000         75.700000        228.000000      22.000000
max    2015.000000         89.000000        723.000000    1800.000000

           Alcohol  percentage expenditure  Hepatitis B       Measles   \
count  2744.000000             2938.000000  2385.000000    2938.000000
mean      4.602861              738.251295    80.940461    2419.592240
std       4.052413             1987.914858    25.070016   11467.272489
min       0.010000                0.000000     1.000000       0.000000
25%       0.877500                4.685343    77.000000       0.000000
50%       3.755000               64.912906    92.000000      17.000000
75%       7.702500              441.534144    97.000000     360.250000
max      17.870000            19479.911610    99.000000  212183.000000

             BMI   under-five deaths         Polio  Total expenditure  \
count  2904.000000        2938.000000  2919.000000         2712.00000
mean     38.321247          42.035739    82.550188            5.93819
std      20.044034         160.445548    23.428046            2.49832
min       1.000000           0.000000     3.000000            0.37000
25%      19.300000           0.000000    78.000000            4.26000
50%      43.500000           4.000000    93.000000            5.75500
75%      56.200000          28.000000    97.000000            7.49250
max      87.300000        2500.000000    99.000000           17.60000

        Diphtheria     HIV/AIDS            GDP    Population  \
count  2919.000000  2938.000000    2490.000000  2.286000e+03
mean     82.324084     1.742103    7483.158469  1.275338e+07
std      23.716912     5.077785   14270.169342  6.101210e+07
min       2.000000     0.100000       1.681350  3.400000e+01
25%      78.000000     0.100000     463.935626  1.957932e+05
50%      93.000000     0.100000    1766.947595  1.386542e+06
75%      97.000000     0.800000    5910.806335  7.420359e+06
max      99.000000    50.600000  119172.741800  1.293859e+09
```

```
        thinness  1-19 years   thinness 5-9 years  \
count            2904.000000          2904.000000
mean                4.839704             4.870317
std                 4.420195             4.508882
min                 0.100000             0.100000
25%                 1.600000             1.500000
50%                 3.300000             3.300000
75%                 7.200000             7.200000
max                27.700000            28.600000

       Income composition of resources    Schooling
count                      2771.000000  2775.000000
mean                          0.627551    11.992793
std                           0.210904     3.358920
min                           0.000000     0.000000
25%                           0.493000    10.100000
50%                           0.677000    12.300000
75%                           0.779000    14.300000
max                           0.948000    20.700000
```

[9]:
```
# print the 5 records of the dataset by default. Pass number how many record␣
↪you want look
df.head()
```

[9]:
```
       Country  Year      Status  Life expectancy   Adult Mortality  \
0  Afghanistan  2015  Developing              65.0            263.0
1  Afghanistan  2014  Developing              59.9            271.0
2  Afghanistan  2013  Developing              59.9            268.0
3  Afghanistan  2012  Developing              59.5            272.0
4  Afghanistan  2011  Developing              59.2            275.0

   infant deaths  Alcohol  percentage expenditure  Hepatitis B  Measles   \
0             62     0.01               71.279624         65.0      1154
1             64     0.01               73.523582         62.0       492
2             66     0.01               73.219243         64.0       430
3             69     0.01               78.184215         67.0      2787
4             71     0.01                7.097109         68.0      3013

    BMI   under-five deaths   Polio  Total expenditure  Diphtheria   \
0   19.1                  83    6.0               8.16         65.0
1   18.6                  86   58.0               8.18         62.0
2   18.1                  89   62.0               8.13         64.0
3   17.6                  93   67.0               8.52         67.0
4   17.2                  97   68.0               7.87         68.0

    HIV/AIDS         GDP  Population   thinness  1-19 years  \
0        0.1  584.259210  33736494.0                   17.2
```

```
1        0.1  612.696514    327582.0                   17.5
2        0.1  631.744976  31731688.0                   17.7
3        0.1  669.959000   3696958.0                   17.9
4        0.1   63.537231   2978599.0                   18.2

    thinness 5-9 years  Income composition of resources  Schooling
0                 17.3                            0.479       10.1
1                 17.5                            0.476       10.0
2                 17.7                            0.470        9.9
3                 18.0                            0.463        9.8
4                 18.2                            0.454        9.5
```

[10]:

```
# print the last 5 records of the dataset by default. Pass number how many
record you want look
df.tail()
```

[10]:

```
       Country  Year      Status  Life expectancy   Adult Mortality  \
2933  Zimbabwe  2004  Developing              44.3            723.0
2934  Zimbabwe  2003  Developing              44.5            715.0
2935  Zimbabwe  2002  Developing              44.8             73.0
2936  Zimbabwe  2001  Developing              45.3            686.0
2937  Zimbabwe  2000  Developing              46.0            665.0

      infant deaths  Alcohol  percentage expenditure  Hepatitis B  Measles   \
2933             27     4.36                     0.0         68.0       31
2934             26     4.06                     0.0          7.0      998
2935             25     4.43                     0.0         73.0      304
2936             25     1.72                     0.0         76.0      529
2937             24     1.68                     0.0         79.0     1483

       BMI   under-five deaths   Polio  Total expenditure  Diphtheria   \
2933   27.1                  42   67.0               7.13         65.0
2934   26.7                  41    7.0               6.52         68.0
2935   26.3                  40   73.0               6.53         71.0
2936   25.9                  39   76.0               6.16         75.0
2937   25.5                  39   78.0               7.10         78.0

       HIV/AIDS         GDP  Population   thinness  1-19 years  \
2933       33.6  454.366654  12777511.0                     9.4
2934       36.7  453.351155  12633897.0                     9.8
2935       39.8   57.348340    125525.0                     1.2
2936       42.1  548.587312  12366165.0                     1.6
2937       43.5  547.358879  12222251.0                    11.0

       thinness 5-9 years  Income composition of resources  Schooling
2933                  9.4                            0.407        9.2
2934                  9.9                            0.418        9.5
2935                  1.3                            0.427       10.0
```

```
2936                    1.7                              0.427        9.8
2937                   11.2                              0.434        9.8
```

[11]:
```python
num_col = df.select_dtypes(include=np.number).columns
print("Numerical columns: \n",num_col)

cat_col = df.select_dtypes(exclude=np.number).columns
print("Categorical columns: \n",cat_col)
```

```
Numerical columns:
 Index(['Year', 'Life expectancy ', 'Adult Mortality', 'infant deaths',
       'Alcohol', 'percentage expenditure', 'Hepatitis B', 'Measles ', ' BMI ',
       'under-five deaths ', 'Polio', 'Total expenditure', 'Diphtheria ',
       ' HIV/AIDS', 'GDP', 'Population', ' thinness  1-19 years',
       ' thinness 5-9 years', 'Income composition of resources', 'Schooling'],
      dtype='object')
Categorical columns:
 Index(['Country', 'Status'], dtype='object')
```

2.1 4. Data Pre-processing

[12]:
```python
# Remove the extra space from column names

df = df.rename(columns=lambda x: x.strip())
```

[13]:
```python
# Import label encoder
from sklearn import preprocessing

# label_encoder object knows how to understand word labels.
label_encoder = preprocessing.LabelEncoder()

# Encode labels in column 'Status'.
df['Status']= label_encoder.fit_transform(df['Status'])

df.head(10)
```

[13]:
```
       Country  Year  Status  Life expectancy  Adult Mortality  infant deaths  \
0  Afghanistan  2015       1             65.0            263.0             62
1  Afghanistan  2014       1             59.9            271.0             64
2  Afghanistan  2013       1             59.9            268.0             66
3  Afghanistan  2012       1             59.5            272.0             69
4  Afghanistan  2011       1             59.2            275.0             71
5  Afghanistan  2010       1             58.8            279.0             74
6  Afghanistan  2009       1             58.6            281.0             77
7  Afghanistan  2008       1             58.1            287.0             80
8  Afghanistan  2007       1             57.5            295.0             82
9  Afghanistan  2006       1             57.3            295.0             84
```

	Alcohol	percentage expenditure	Hepatitis B	Measles	BMI	\
0	0.01	71.279624	65.0	1154	19.1	
1	0.01	73.523582	62.0	492	18.6	
2	0.01	73.219243	64.0	430	18.1	
3	0.01	78.184215	67.0	2787	17.6	
4	0.01	7.097109	68.0	3013	17.2	
5	0.01	79.679367	66.0	1989	16.7	
6	0.01	56.762217	63.0	2861	16.2	
7	0.03	25.873925	64.0	1599	15.7	
8	0.02	10.910156	63.0	1141	15.2	
9	0.03	17.171518	64.0	1990	14.7	

	under-five deaths	Polio	Total expenditure	Diphtheria	HIV/AIDS	\
0	83	6.0	8.16	65.0	0.1	
1	86	58.0	8.18	62.0	0.1	
2	89	62.0	8.13	64.0	0.1	
3	93	67.0	8.52	67.0	0.1	
4	97	68.0	7.87	68.0	0.1	
5	102	66.0	9.20	66.0	0.1	
6	106	63.0	9.42	63.0	0.1	
7	110	64.0	8.33	64.0	0.1	
8	113	63.0	6.73	63.0	0.1	
9	116	58.0	7.43	58.0	0.1	

	GDP	Population	thinness 1-19 years	thinness 5-9 years	\
0	584.259210	33736494.0	17.2	17.3	
1	612.696514	327582.0	17.5	17.5	
2	631.744976	31731688.0	17.7	17.7	
3	669.959000	3696958.0	17.9	18.0	
4	63.537231	2978599.0	18.2	18.2	
5	553.328940	2883167.0	18.4	18.4	
6	445.893298	284331.0	18.6	18.7	
7	373.361116	2729431.0	18.8	18.9	
8	369.835796	26616792.0	19.0	19.1	
9	272.563770	2589345.0	19.2	19.3	

	Income composition of resources	Schooling
0	0.479	10.1
1	0.476	10.0
2	0.470	9.9
3	0.463	9.8
4	0.454	9.5
5	0.448	9.2
6	0.434	8.9
7	0.433	8.7
8	0.415	8.4
9	0.405	8.1

[14]:
```
print(df.isna().sum())
print(df.shape)
```

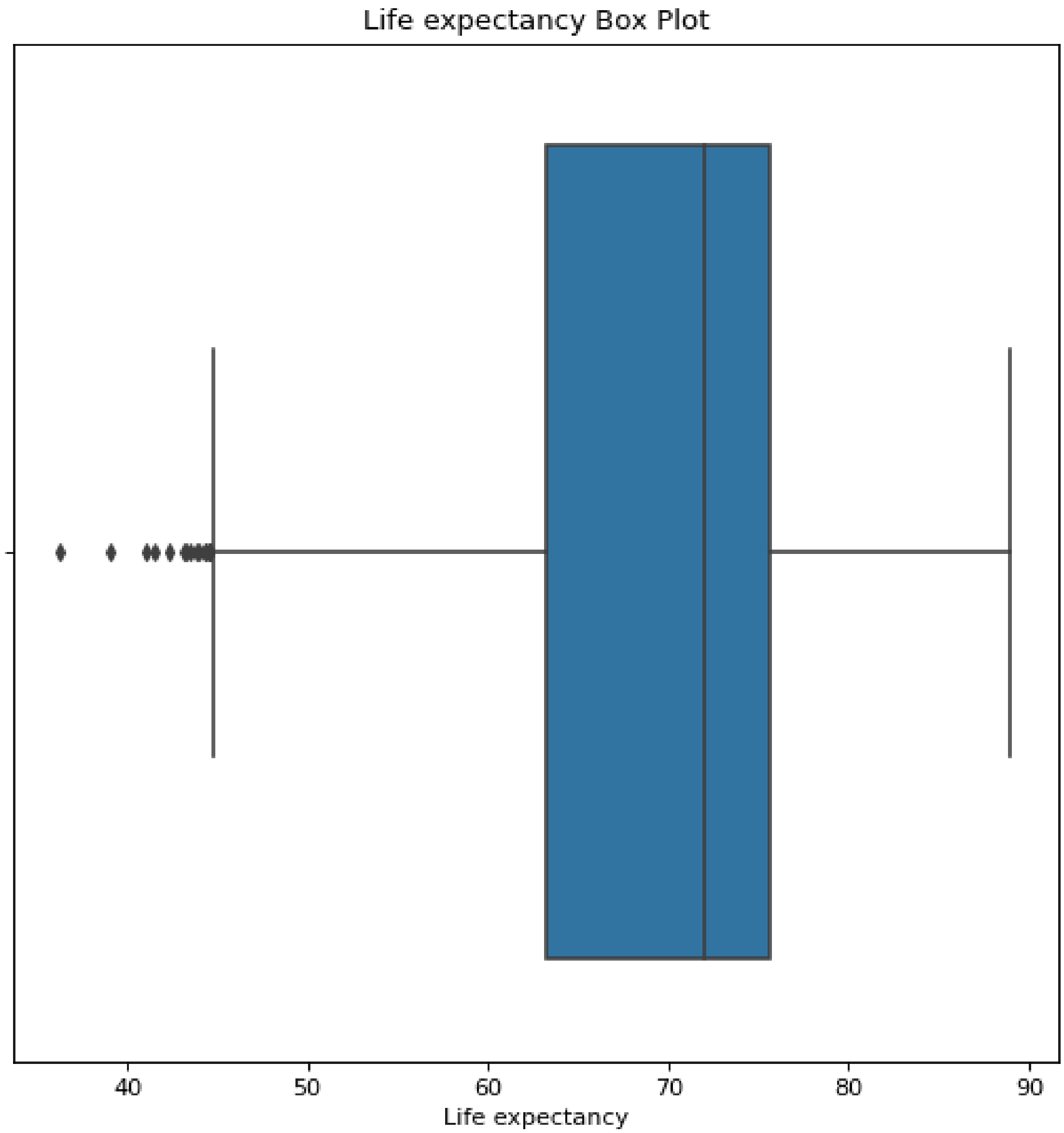
```
Country                             0
Year                                0
Status                              0
Life expectancy                    10
Adult Mortality                    10
infant deaths                       0
Alcohol                           194
percentage expenditure              0
Hepatitis B                       553
Measles                             0
BMI                                34
under-five deaths                   0
Polio                              19
Total expenditure                 226
Diphtheria                         19
HIV/AIDS                            0
GDP                               448
Population                        652
thinness  1-19 years               34
thinness 5-9 years                 34
Income composition of resources   167
Schooling                         163
dtype: int64
(2938, 22)
```

[15]:
```
# Replace using mean
for i in df.columns.drop('Country'):
    df[i].fillna(df[i].mean(), inplace = True)
```

2.2 5. Exploratory Data Analysis

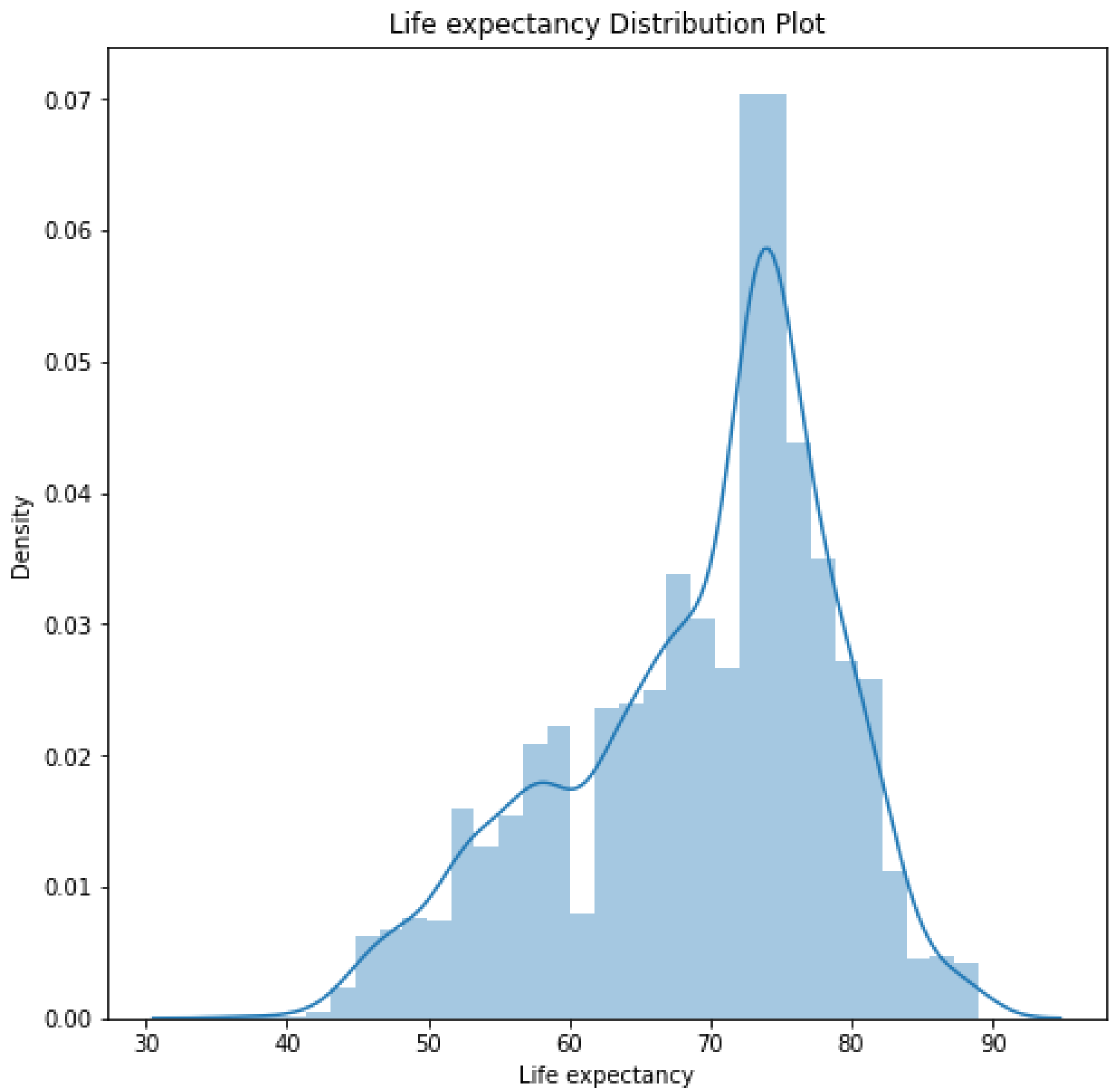
[16]:
```
# Let's check the distribution of y variable (Life Expectancy)
plt.figure(figsize=(8,8), dpi= 80)
sns.boxplot(df['Life expectancy'])
plt.title('Life expectancy Box Plot')
plt.show()
```

```
/usr/local/lib/python3.6/dist-packages/seaborn/_decorators.py:43: FutureWarning:
Pass the following variable as a keyword arg: x. From version 0.12, the only
valid positional argument will be `data`, and passing other arguments without an
explicit keyword will result in an error or misinterpretation.
  FutureWarning
```



[17]:
```
plt.figure(figsize=(8,8))
plt.title('Life expectancy Distribution Plot')
sns.distplot(df['Life expectancy']);
```

```
/usr/local/lib/python3.6/dist-packages/seaborn/distributions.py:2551:
FutureWarning: `distplot` is a deprecated function and will be removed in a
future version. Please adapt your code to use either `displot` (a figure-level
function with similar flexibility) or `histplot` (an axes-level function for
histograms).
  warnings.warn(msg, FutureWarning)
```



Summary : The y variable is having very few outliers and is almost linearly distributed. So the assumption for linear regression holds true

[18]:
```
num_col = df.select_dtypes(include=np.number).columns
print("Numerical columns: \n",num_col)

cat_col = df.select_dtypes(exclude=np.number).columns
print("Categorical columns: \n",cat_col)
```

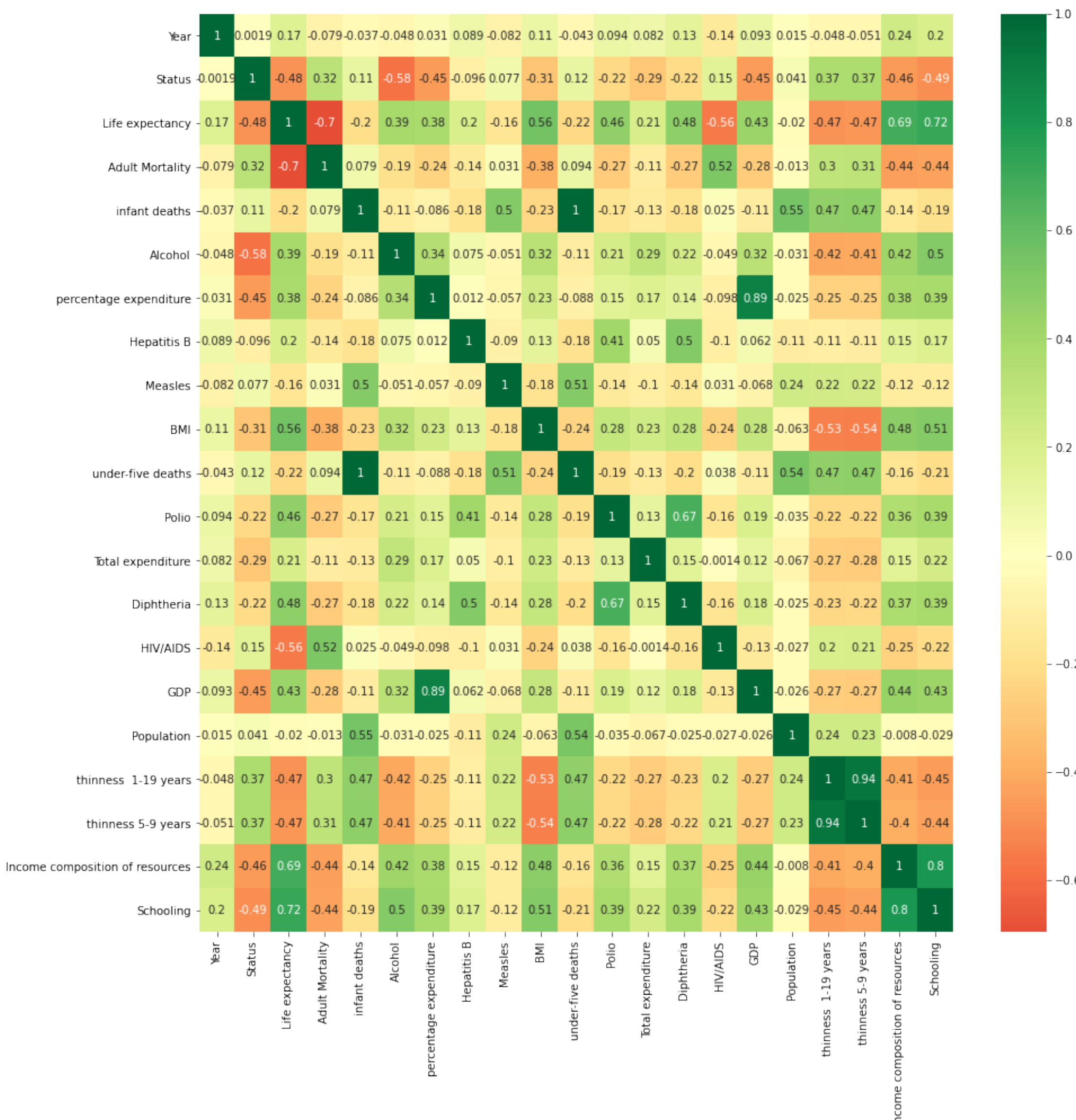
```
Numerical columns:
 Index(['Year', 'Status', 'Life expectancy', 'Adult Mortality', 'infant deaths',
       'Alcohol', 'percentage expenditure', 'Hepatitis B', 'Measles', 'BMI',
       'under-five deaths', 'Polio', 'Total expenditure', 'Diphtheria',
```

```
       'HIV/AIDS', 'GDP', 'Population', 'thinness  1-19 years',
       'thinness 5-9 years', 'Income composition of resources', 'Schooling'],
      dtype='object')
Categorical columns:
 Index(['Country'], dtype='object')
```

[19]:

```
# Let's check the multicollinearity of features by checking the correlation
↪matric

plt.figure(figsize=(15,15))
p=sns.heatmap(df[num_col].corr(), annot=True,cmap='RdYlGn',center=0)
```



[20]:
```
# Pair Plots to know the relation between different features
ax = sns.pairplot(df[num_col])
```

```
Output hidden; open in https://colab.research.google.com to view.
```

Few of the features are having the linear relationship with y variable. So linear regression would be good approach for the same

2.3 6. Model Building

[21]:
```
# Train test split
X=df.drop(columns=['Life expectancy','Country'])
y=df[['Life expectancy']]

X_train, X_test, y_train, y_test = train_test_split(X,y,test_size=0.
↪3,random_state=1234)
```

[22]:
```
X.columns
```

[22]:
```
Index(['Year', 'Status', 'Adult Mortality', 'infant deaths', 'Alcohol',
       'percentage expenditure', 'Hepatitis B', 'Measles', 'BMI',
       'under-five deaths', 'Polio', 'Total expenditure', 'Diphtheria',
       'HIV/AIDS', 'GDP', 'Population', 'thinness  1-19 years',
       'thinness 5-9 years', 'Income composition of resources', 'Schooling'],
      dtype='object')
```

[23]:
```
X.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2938 entries, 0 to 2937
Data columns (total 20 columns):
 #   Column                           Non-Null Count  Dtype
---  ------                           --------------  -----
 0   Year                             2938 non-null   int64
 1   Status                           2938 non-null   int64
 2   Adult Mortality                  2938 non-null   float64
 3   infant deaths                    2938 non-null   int64
 4   Alcohol                          2938 non-null   float64
 5   percentage expenditure           2938 non-null   float64
 6   Hepatitis B                      2938 non-null   float64
 7   Measles                          2938 non-null   int64
 8   BMI                              2938 non-null   float64
 9   under-five deaths                2938 non-null   int64
 10  Polio                            2938 non-null   float64
 11  Total expenditure                2938 non-null   float64
 12  Diphtheria                       2938 non-null   float64
 13  HIV/AIDS                         2938 non-null   float64
```

```
 14  GDP                               2938 non-null   float64
 15  Population                        2938 non-null   float64
 16  thinness  1-19 years              2938 non-null   float64
 17  thinness 5-9 years                2938 non-null   float64
 18  Income composition of resources   2938 non-null   float64
 19  Schooling                         2938 non-null   float64
dtypes: float64(15), int64(5)
memory usage: 459.2 KB
```

2.4 Approach 1 : Adding 1 varaible after 1

2.4.1 Building model with 1 variable

[24]:
```python
# Select only one feature in regression model
X_train1 = X_train['Income composition of resources']
```

[25]:
```python
# Add a constant
X_train1 = sm.add_constant(X_train1)

# Create a first ols model
model_1 = sm.OLS(y_train, X_train1).fit()
```

[26]:
```python
# Check parameters created
model_1.params
```

[26]:
```
const                              48.440947
Income composition of resources    33.059741
dtype: float64
```

[27]:
```python
# Summary of the model
print(model_1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.490
Model:                            OLS   Adj. R-squared:                  0.490
Method:                 Least Squares   F-statistic:                     1974.
Date:                Sun, 20 Dec 2020   Prob (F-statistic):         1.09e-302
Time:                        16:09:50   Log-Likelihood:                -6894.3
No. Observations:                2056   AIC:                         1.379e+04
Df Residuals:                    2054   BIC:                         1.380e+04
Df Model:                           1
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              48.4409      0.492     98.412      0.000
```

```
47.476      49.406
Income composition of resources    33.0597      0.744     44.427      0.000
31.600      34.519
==============================================================================
Omnibus:                      138.959   Durbin-Watson:                   2.047
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              617.560
Skew:                           0.121   Prob(JB):                    7.92e-135
Kurtosis:                       5.674   Cond. No.                         6.86
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

R-squared is very less for the model, so there's a need to add more features as R-square itself is not able to explain the expenses

2.4.2 Building model with 2 variable

[28]:
```python
# Add one more feature in regression model
X_train2 = X_train[['Income composition of resources','Schooling']]
```

[29]:
```python
# Add a constant
X_train2 = sm.add_constant(X_train2)

# Create second ols model
model_2 = sm.OLS(y_train, X_train2).fit()
```

[30]:
```python
# Check parameters created
model_2.params
```

[30]:
```
const                              43.145928
Income composition of resources    16.273079
Schooling                           1.320315
dtype: float64
```

[31]:
```python
# Summary of the model
print(model_2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.562
Model:                            OLS   Adj. R-squared:                  0.561
Method:                 Least Squares   F-statistic:                     1316.
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:50   Log-Likelihood:                -6738.3
No. Observations:                2056   AIC:                         1.348e+04
Df Residuals:                    2053   BIC:                         1.350e+04
Df Model:                           2
```

```
Covariance Type:            nonrobust
================================================================================
===================
                                   coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                           43.1459      0.540     79.895      0.000
42.087      44.205
Income composition of resources    16.2731      1.146     14.197      0.000
14.025      18.521
Schooling                        1.3203      0.072     18.340      0.000
1.179       1.461
==============================================================================
Omnibus:                      182.792   Durbin-Watson:                   2.037
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              596.381
Skew:                          -0.427   Prob(JB):                    3.14e-130
Kurtosis:                       5.497   Cond. No.                         101.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

R-squared and Adj. R squared has increased for the model, but we can still improvise over it so let's add more features

2.4.3 Building model with 3 variable

[32]:
```
# Adding one more feature in regression model
X_train3 = X_train[['Income composition of resources','Schooling', 'Adult␣
↪Mortality']]
```

[33]:
```
# Add a constant
X_train3 = sm.add_constant(X_train3)

# Create third fitted model
model_3 = sm.OLS(y_train, X_train3).fit()
```

[34]:
```
# Check parameters created
model_3.params
```

[34]:
```
const                              56.227689
Income composition of resources    10.637516
Schooling                           1.003654
Adult Mortality                    -0.034790
dtype: float64
```

[35]:
```
# Summary of the model
print(model_3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.721
Model:                            OLS   Adj. R-squared:                  0.720
Method:                 Least Squares   F-statistic:                     1765.
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:51   Log-Likelihood:                -6275.3
No. Observations:                2056   AIC:                         1.256e+04
Df Residuals:                    2052   BIC:                         1.258e+04
Df Model:                           3
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              56.2277      0.577     97.502      0.000
55.097      57.359
Income composition of resources    10.6375      0.930     11.438      0.000
8.814      12.461
Schooling                           1.0037      0.058     17.236      0.000
0.889       1.118
Adult Mortality                    -0.0348      0.001    -34.168      0.000
-0.037      -0.033
==============================================================================
Omnibus:                      379.309   Durbin-Watson:                   1.962
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1628.478
Skew:                          -0.829   Prob(JB):                         0.00
Kurtosis:                       7.032   Cond. No.                     1.72e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.72e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

We have achieved a R-squared of `0.72` by manually picking the highly correlated variables. Now lets use RFE to select the independent variables which accurately predicts the dependent variable `Life expectancy`.

2.5 Approach 2 : RFE and eleminating by using p-value and VIF

[36]:
```python
# Running RFE with important column count to be 15
lm = LinearRegression()
lm.fit(X_train, y_train)
```

```
rfe = RFE(lm, 15)
rfe = rfe.fit(X_train, y_train)
```

```
/usr/local/lib/python3.6/dist-packages/sklearn/utils/validation.py:760:
DataConversionWarning: A column-vector y was passed when a 1d array was
expected. Please change the shape of y to (n_samples, ), for example using
ravel().
  y = column_or_1d(y, warn=True)
```

[37]:
```
list(zip(X_train.columns,rfe.support_,rfe.ranking_))
```

[37]:
```
[('Year', False, 2),
 ('Status', True, 1),
 ('Adult Mortality', True, 1),
 ('infant deaths', True, 1),
 ('Alcohol', True, 1),
 ('percentage expenditure', False, 3),
 ('Hepatitis B', True, 1),
 ('Measles', False, 5),
 ('BMI', True, 1),
 ('under-five deaths', True, 1),
 ('Polio', True, 1),
 ('Total expenditure', True, 1),
 ('Diphtheria', True, 1),
 ('HIV/AIDS', True, 1),
 ('GDP', False, 4),
 ('Population', False, 6),
 ('thinness  1-19 years', True, 1),
 ('thinness 5-9 years', True, 1),
 ('Income composition of resources', True, 1),
 ('Schooling', True, 1)]
```

[38]:
```
# Selecting the important features (in the support)
imp_columns = X_train.columns[rfe.support_]
imp_columns
```

[38]:
```
Index(['Status', 'Adult Mortality', 'infant deaths', 'Alcohol', 'Hepatitis B',
       'BMI', 'under-five deaths', 'Polio', 'Total expenditure', 'Diphtheria',
       'HIV/AIDS', 'thinness  1-19 years', 'thinness 5-9 years',
       'Income composition of resources', 'Schooling'],
      dtype='object')
```

[39]:
```
# Creating X_train dataframe with RFE selected variables
X_train_rfe = X_train[imp_columns]
```

After passing the arbitary selected columns by RFE we will manually evaluate each models p-value and VIF value. Unless we find the acceptable range for p-values and VIF we keep dropping the variables one at a time based on below criteria. - High p-value High VIF : Drop the variable - High p-value Low VIF : Drop the variable with high p-value first - Low p-value Low VIF : accept the variable

2.5.1 Checking VIF

Variance Inflation Factor or VIF, gives a basic quantitative idea about how much the feature variables are correlated with each other. It is an extremely important parameter to test our linear model. The formula for calculating VIF is:

[40]:
```
random.seed(0)

# Add a constant
X_train_rfec = sm.add_constant(X_train_rfe)

# Build the model with RFE features
lm_rfe = sm.OLS(y_train,X_train_rfec).fit()

#Summary of linear model
print(lm_rfe.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.820
Model:                            OLS   Adj. R-squared:                  0.819
Method:                 Least Squares   F-statistic:                     620.1
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:51   Log-Likelihood:                -5823.0
No. Observations:                2056   AIC:                         1.168e+04
Df Residuals:                    2040   BIC:                         1.177e+04
Df Model:                          15
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              56.3507      0.815     69.162      0.000
54.753      57.949
Status                             -2.0823      0.313     -6.663      0.000
-2.695      -1.469
Adult Mortality                    -0.0200      0.001    -20.474      0.000
-0.022      -0.018
infant deaths                       0.0944      0.010      9.677      0.000
0.075       0.114
Alcohol                             0.0403      0.031      1.289      0.198
-0.021       0.102
Hepatitis B                        -0.0209      0.005     -4.303      0.000
-0.030      -0.011
BMI                                 0.0488      0.006      7.963      0.000
0.037       0.061
under-five deaths                  -0.0714      0.007     -9.983      0.000
```

```
-0.085      -0.057
Polio                                  0.0272      0.005      4.962      0.000
0.016       0.038
Total expenditure                      0.0768      0.042      1.835      0.067
-0.005       0.159
Diphtheria                             0.0456      0.006      7.807      0.000
0.034       0.057
HIV/AIDS                              -0.4970      0.024    -20.592      0.000
-0.544      -0.450
thinness  1-19 years                  -0.0739      0.061     -1.212      0.226
-0.193       0.046
thinness 5-9 years                     0.0032      0.060      0.054      0.957
-0.114       0.120
Income composition of resources        6.4836      0.769      8.435      0.000
4.976       7.991
Schooling                              0.6908      0.051     13.544      0.000
0.591       0.791
==============================================================================
Omnibus:                      110.684   Durbin-Watson:                   1.979
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              321.018
Skew:                          -0.238   Prob(JB):                     1.96e-70
Kurtosis:                       4.876   Cond. No.                     2.45e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.45e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Since the p value for few of the features is not siginificant, we need to drop it but before that let's check the VIF score as well

[41]:
```python
# Create a dataframe that will contain the names of all the feature variables
 ↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe.values, i) for i in
 ↪range(X_train_rfe.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

[41]:

	Features	VIF
6	under-five deaths	178.16
2	infant deaths	177.70
14	Schooling	44.59
13	Income composition of resources	30.42
9	Diphtheria	30.31

```
7                            Polio   26.28
11              thinness  1-19 years   19.47
12                thinness 5-9 years   19.31
4                        Hepatitis B   19.00
5                                BMI    8.28
8                  Total expenditure    7.74
0                             Status    7.13
1                    Adult Mortality    4.42
3                            Alcohol    4.35
10                          HIV/AIDS    1.70
```

Since the variable **thinness 5-9 years** is having a very high p value , we would remove the feature from training dataset

[42]:
```python
# Dropping insignificant variables

X_train_rfe1 = X_train_rfe.drop(['thinness 5-9 years'], 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe1c = sm.add_constant(X_train_rfe1)
lm_rfe1 = sm.OLS(y_train, X_train_rfe1c).fit()

#Summary of linear model
print(lm_rfe1.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.820
Model:                            OLS   Adj. R-squared:                  0.819
Method:                 Least Squares   F-statistic:                     664.7
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:51   Log-Likelihood:                -5823.0
No. Observations:                2056   AIC:                         1.168e+04
Df Residuals:                    2041   BIC:                         1.176e+04
Df Model:                          14
Covariance Type:            nonrobust
================================================================================
===================
                                  coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                          56.3537      0.813     69.336      0.000
54.760      57.948
Status                         -2.0819      0.312     -6.665      0.000
-2.695      -1.469
Adult Mortality                -0.0200      0.001    -20.484      0.000
-0.022      -0.018
```

```
infant deaths                          0.0945      0.010      9.703      0.000
0.075       0.114
Alcohol                                0.0403      0.031      1.288      0.198
-0.021       0.102
Hepatitis B                           -0.0209      0.005     -4.304      0.000
-0.030      -0.011
BMI                                    0.0487      0.006      8.014      0.000
0.037       0.061
under-five deaths                     -0.0715      0.007    -10.003      0.000
-0.085      -0.057
Polio                                  0.0272      0.005      4.963      0.000
0.016       0.038
Total expenditure                      0.0767      0.042      1.835      0.067
-0.005       0.159
Diphtheria                             0.0456      0.006      7.811      0.000
0.034       0.057
HIV/AIDS                              -0.4969      0.024    -20.599      0.000
-0.544      -0.450
thinness  1-19 years                  -0.0710      0.029     -2.427      0.015
-0.128      -0.014
Income composition of resources        6.4837      0.768      8.437      0.000
4.977       7.991
Schooling                              0.6909      0.051     13.548      0.000
0.591       0.791
==============================================================================
Omnibus:                      110.689   Durbin-Watson:                   1.979
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              320.883
Skew:                          -0.238   Prob(JB):                     2.09e-70
Kurtosis:                       4.876   Cond. No.                     2.45e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.45e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[43]:

```
# Create a dataframe that will contain the names of all the feature variables␣
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe1.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe1.values, i) for i in␣
↪range(X_train_rfe1.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

[43]:
```
                              Features     VIF
6                    under-five deaths  177.82
2                        infant deaths  177.16
13                           Schooling   44.55
12  Income composition of resources   30.42
9                           Diphtheria   30.30
7                                Polio   26.28
4                          Hepatitis B   18.99
5                                  BMI    8.19
8                    Total expenditure    7.74
0                               Status    7.10
1                      Adult Mortality    4.41
3                              Alcohol    4.35
11                thinness  1-19 years    4.07
10                            HIV/AIDS    1.70
```

Since the variable `under-five deaths` is having a very high VIF score, we would remove the feature from training dataset

[44]:
```python
# Dropping insignificant variables

X_train_rfe2 = X_train_rfe1.drop('under-five deaths', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe2c = sm.add_constant(X_train_rfe2)
lm_rfe2 = sm.OLS(y_train, X_train_rfe2c).fit()

#Summary of linear model
print(lm_rfe2.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.811
Model:                            OLS   Adj. R-squared:                  0.810
Method:                 Least Squares   F-statistic:                     675.4
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:51   Log-Likelihood:                -5872.2
No. Observations:                2056   AIC:                         1.177e+04
Df Residuals:                    2042   BIC:                         1.185e+04
Df Model:                          13
Covariance Type:            nonrobust
================================================================================
===================
                                     coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                             55.1520      0.823     67.005      0.000
```

```
53.538      56.766
Status                                  -2.0413      0.320     -6.382      0.000
-2.668      -1.414
Adult Mortality                         -0.0204      0.001    -20.368      0.000
-0.022      -0.018
infant deaths                           -0.0025      0.001     -2.797      0.005
-0.004      -0.001
Alcohol                                 -0.0038      0.032     -0.121      0.904
-0.066       0.058
Hepatitis B                             -0.0243      0.005     -4.887      0.000
-0.034      -0.015
BMI                                      0.0502      0.006      8.071      0.000
0.038       0.062
Polio                                    0.0307      0.006      5.484      0.000
0.020       0.042
Total expenditure                        0.0825      0.043      1.928      0.054
-0.001       0.166
Diphtheria                               0.0536      0.006      9.039      0.000
0.042       0.065
HIV/AIDS                                -0.5147      0.025    -20.893      0.000
-0.563      -0.466
thinness  1-19 years                    -0.0578      0.030     -1.932      0.053
-0.116       0.001
Income composition of resources          7.1638      0.784      9.140      0.000
5.627       8.701
Schooling                                0.7020      0.052     13.448      0.000
0.600       0.804
==============================================================================
Omnibus:                      110.170   Durbin-Watson:                   1.988
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              309.575
Skew:                          -0.250   Prob(JB):                     5.98e-68
Kurtosis:                       4.834   Cond. No.                     2.30e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.3e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[45]:
```python
# Create a dataframe that will contain the names of all the feature variables
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe2.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe2.values, i) for i in
↪range(X_train_rfe2.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
```

```
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

[45]:

	Features	VIF
12	Schooling	44.53
11	Income composition of resources	30.29
8	Diphtheria	29.80
6	Polio	26.22
4	Hepatitis B	18.76
5	BMI	8.19
7	Total expenditure	7.73
0	Status	7.06
1	Adult Mortality	4.38
3	Alcohol	4.23
10	thinness 1-19 years	4.07
9	HIV/AIDS	1.69
2	infant deaths	1.47

Since the variable `Alcohol` is having a very high p value, we would remove the feature from training dataset

[46]:
```
# Dropping insignificant variables

X_train_rfe3 = X_train_rfe2.drop('Alcohol', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe3c = sm.add_constant(X_train_rfe3)
lm_rfe3 = sm.OLS(y_train, X_train_rfe3c).fit()

#Summary of linear model
print(lm_rfe3.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.811
Model:                            OLS   Adj. R-squared:                  0.810
Method:                 Least Squares   F-statistic:                     732.0
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:51   Log-Likelihood:                -5872.2
No. Observations:                2056   AIC:                         1.177e+04
Df Residuals:                    2043   BIC:                         1.184e+04
Df Model:                          12
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
```

```
--------------------
const                                55.1373      0.814     67.751      0.000
53.541      56.733
Status                               -2.0259      0.293     -6.904      0.000
-2.601      -1.450
Adult Mortality                      -0.0204      0.001    -20.439      0.000
-0.022      -0.018
infant deaths                        -0.0025      0.001     -2.826      0.005
-0.004      -0.001
Hepatitis B                          -0.0243      0.005     -4.887      0.000
-0.034      -0.015
BMI                                   0.0502      0.006      8.073      0.000
0.038       0.062
Polio                                 0.0307      0.006      5.484      0.000
0.020       0.042
Total expenditure                     0.0819      0.042      1.928      0.054
-0.001       0.165
Diphtheria                            0.0536      0.006      9.043      0.000
0.042       0.065
HIV/AIDS                             -0.5149      0.025    -20.942      0.000
-0.563      -0.467
thinness  1-19 years                 -0.0571      0.029     -1.949      0.051
-0.114       0.000
Income composition of resources       7.1646      0.784      9.144      0.000
5.628       8.701
Schooling                             0.7009      0.051     13.647      0.000
0.600       0.802
==============================================================================
Omnibus:                      110.297   Durbin-Watson:                   1.988
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              309.319
Skew:                          -0.251   Prob(JB):                     6.80e-68
Kurtosis:                       4.833   Cond. No.                     2.28e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.28e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[47]:

```
# Create a dataframe that will contain the names of all the feature variables␣
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe3.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe3.values, i) for i in␣
↪range(X_train_rfe3.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
```

```
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

[47]:

```
                          Features    VIF
11                       Schooling  42.11
10  Income composition of resources  30.28
7                       Diphtheria  29.80
5                            Polio  26.16
3                      Hepatitis B  18.73
4                              BMI   8.18
6                Total expenditure   7.49
0                           Status   6.05
1                  Adult Mortality   4.30
9              thinness  1-19 years   3.96
8                         HIV/AIDS   1.69
2                    infant deaths   1.45
```

Since the variable **Schooling** is having a very high VIF score, we would remove the feature from training dataset

[48]:

```
# Dropping insignificant variables

X_train_rfe4 = X_train_rfe3.drop('Schooling', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe4c = sm.add_constant(X_train_rfe4)
lm_rfe4 = sm.OLS(y_train, X_train_rfe4c).fit()

#Summary of linear model
print(lm_rfe4.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.794
Model:                            OLS   Adj. R-squared:                  0.793
Method:                 Least Squares   F-statistic:                     716.7
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:52   Log-Likelihood:                -5961.9
No. Observations:                2056   AIC:                         1.195e+04
Df Residuals:                    2044   BIC:                         1.202e+04
Df Model:                          11
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
```

```
const                               58.7499      0.804     73.101      0.000
57.174      60.326
Status                              -2.6868      0.302     -8.889      0.000
-3.280      -2.094
Adult Mortality                     -0.0217      0.001    -20.948      0.000
-0.024      -0.020
infant deaths                       -0.0029      0.001     -3.092      0.002
-0.005      -0.001
Hepatitis B                         -0.0249      0.005     -4.801      0.000
-0.035      -0.015
BMI                                  0.0626      0.006      9.740      0.000
0.050       0.075
Polio                                0.0360      0.006      6.188      0.000
0.025       0.047
Total expenditure                    0.1155      0.044      2.608      0.009
0.029       0.202
Diphtheria                           0.0572      0.006      9.249      0.000
0.045       0.069
HIV/AIDS                            -0.4918      0.026    -19.200      0.000
-0.542      -0.442
thinness  1-19 years                -0.0830      0.031     -2.719      0.007
-0.143      -0.023
Income composition of resources    13.9929      0.630     22.220      0.000
12.758      15.228
==============================================================================
Omnibus:                       88.663   Durbin-Watson:                   2.009
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              262.468
Skew:                          -0.121   Prob(JB):                     1.01e-57
Kurtosis:                       4.734   Cond. No.                     2.27e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.27e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[49]:
```python
# Create a dataframe that will contain the names of all the feature variables
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe4.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe4.values, i) for i in
↪range(X_train_rfe4.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

[49]:
```
                          Features    VIF
7                       Diphtheria  29.69
5                            Polio  25.80
3                      Hepatitis B  18.59
10  Income composition of resources  13.90
4                              BMI   7.83
6                Total expenditure   7.22
0                           Status   6.05
1                  Adult Mortality   4.30
9              thinness  1-19 years   3.95
8                         HIV/AIDS   1.69
2                    infant deaths   1.45
```

Since the variable `Diphtheria` is having a very high VIF score, we would remove the feature from training dataset

[50]:
```python
# Dropping insignificant variables

X_train_rfe5 = X_train_rfe4.drop('Diphtheria', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe5c = sm.add_constant(X_train_rfe5)
lm_rfe5 = sm.OLS(y_train, X_train_rfe5c).fit()

#Summary of linear model
print(lm_rfe5.summary())

# Create a dataframe that will contain the names of all the feature variables␣
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe5.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe5.values, i) for i in␣
↪range(X_train_rfe5.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.785
Model:                            OLS   Adj. R-squared:                  0.784
Method:                 Least Squares   F-statistic:                     748.8
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:52   Log-Likelihood:                -6004.1
No. Observations:                2056   AIC:                         1.203e+04
Df Residuals:                    2045   BIC:                         1.209e+04
Df Model:                          10
Covariance Type:            nonrobust
```

```
==============================================================================
===================
                                    coef    std err          t      P>|t|
[0.025      0.975]
------------------------------------------------------------------------------
-------------------
const                            59.1919      0.819     72.302      0.000
57.586      60.797
Status                           -2.6778      0.308     -8.682      0.000
-3.283      -2.073
Adult Mortality                  -0.0221      0.001    -20.928      0.000
-0.024      -0.020
infant deaths                    -0.0031      0.001     -3.256      0.001
-0.005      -0.001
Hepatitis B                      -0.0102      0.005     -2.018      0.044
-0.020      -0.000
BMI                               0.0645      0.007      9.838      0.000
0.052       0.077
Polio                             0.0649      0.005     12.954      0.000
0.055       0.075
Total expenditure                 0.1491      0.045      3.310      0.001
0.061       0.237
HIV/AIDS                         -0.4937      0.026    -18.889      0.000
-0.545      -0.442
thinness  1-19 years             -0.0823      0.031     -2.644      0.008
-0.143      -0.021
Income composition of resources  14.7697      0.637     23.190      0.000
13.521      16.019
==============================================================================
Omnibus:                       92.191   Durbin-Watson:                   2.014
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              273.609
Skew:                          -0.141   Prob(JB):                     3.86e-60
Kurtosis:                       4.765   Cond. No.                     2.17e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.17e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[50]:

	Features	VIF
5	Polio	17.90
3	Hepatitis B	16.46
9	Income composition of resources	13.42
4	BMI	7.82
6	Total expenditure	7.12

```
0                         Status   6.04
1                Adult Mortality   4.29
8           thinness  1-19 years   3.95
7                       HIV/AIDS   1.69
2                  infant deaths   1.45
```

[51]:
```
# Dropping insignificant variables

X_train_rfe6 = X_train_rfe5.drop('Polio', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe6c = sm.add_constant(X_train_rfe6)
lm_rfe6 = sm.OLS(y_train, X_train_rfe6c).fit()

#Summary of linear model
print(lm_rfe6.summary())

# Create a dataframe that will contain the names of all the feature variables␣
 ↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe6.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe6.values, i) for i in␣
 ↪range(X_train_rfe6.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.768
Model:                            OLS   Adj. R-squared:                  0.767
Method:                 Least Squares   F-statistic:                     752.1
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:52   Log-Likelihood:                -6085.2
No. Observations:                2056   AIC:                         1.219e+04
Df Residuals:                    2046   BIC:                         1.225e+04
Df Model:                           9
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              61.5258      0.831     74.081      0.000
59.897      63.155
Status                             -2.8026      0.321     -8.742      0.000
```

```
-3.431      -2.174
Adult Mortality                    -0.0232      0.001    -21.164      0.000
-0.025      -0.021
infant deaths                      -0.0038      0.001     -3.849      0.000
-0.006      -0.002
Hepatitis B                         0.0135      0.005      2.766      0.006
0.004       0.023
BMI                                 0.0703      0.007     10.336      0.000
0.057       0.084
Total expenditure                   0.1839      0.047      3.932      0.000
0.092       0.276
HIV/AIDS                           -0.4923      0.027    -18.113      0.000
-0.546      -0.439
thinness  1-19 years               -0.0744      0.032     -2.297      0.022
-0.138      -0.011
Income composition of resources    16.2560      0.652     24.951      0.000
14.978      17.534
==============================================================================
Omnibus:                      124.779   Durbin-Watson:                   1.992
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              401.157
Skew:                          -0.247   Prob(JB):                     7.76e-88
Kurtosis:                       5.107   Cond. No.                     2.06e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 2.06e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[51]:

	Features	VIF
3	Hepatitis B	12.86
8	Income composition of resources	11.81
4	BMI	7.70
5	Total expenditure	6.94
0	Status	6.00
1	Adult Mortality	4.29
7	thinness 1-19 years	3.92
6	HIV/AIDS	1.69
2	infant deaths	1.45

[52]:
```
# Dropping insignificant variables

X_train_rfe7 = X_train_rfe6.drop('Hepatitis B', 1,)

# Adding a constant variable and Build a second fitted model

X_train_rfe7c = sm.add_constant(X_train_rfe7)
```

```python
lm_rfe7 = sm.OLS(y_train, X_train_rfe7c).fit()

#Summary of linear model
print(lm_rfe7.summary())

# Create a dataframe that will contain the names of all the feature variables␣
↪and their respective VIFs
vif = pd.DataFrame()
vif['Features'] = X_train_rfe7.columns
vif['VIF'] = [variance_inflation_factor(X_train_rfe7.values, i) for i in␣
↪range(X_train_rfe7.shape[1])]
vif['VIF'] = round(vif['VIF'], 2)
vif = vif.sort_values(by = "VIF", ascending = False)
vif
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.767
Model:                            OLS   Adj. R-squared:                  0.766
Method:                 Least Squares   F-statistic:                     842.4
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:09:52   Log-Likelihood:                -6089.0
No. Observations:                2056   AIC:                         1.220e+04
Df Residuals:                    2047   BIC:                         1.225e+04
Df Model:                           8
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              62.5312      0.748     83.601      0.000
61.064      63.998
Status                             -2.8044      0.321     -8.733      0.000
-3.434      -2.175
Adult Mortality                    -0.0233      0.001    -21.330      0.000
-0.025      -0.021
infant deaths                      -0.0043      0.001     -4.378      0.000
-0.006      -0.002
BMI                                 0.0712      0.007     10.456      0.000
0.058       0.085
Total expenditure                   0.1859      0.047      3.971      0.000
0.094       0.278
HIV/AIDS                           -0.4957      0.027    -18.225      0.000
-0.549      -0.442
thinness  1-19 years               -0.0701      0.032     -2.164      0.031
```

```
-0.134      -0.007
Income composition of resources    16.3814      0.651     25.164      0.000
15.105      17.658
==============================================================================
Omnibus:                      123.339   Durbin-Watson:                   1.995
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              400.688
Skew:                          -0.237   Prob(JB):                     9.81e-88
Kurtosis:                       5.110   Cond. No.                     1.90e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.9e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

[52]:

	Features	VIF
7	Income composition of resources	9.62
3	BMI	7.49
4	Total expenditure	6.53
0	Status	5.56
1	Adult Mortality	4.25
6	thinness 1-19 years	3.75
5	HIV/AIDS	1.68
2	infant deaths	1.41

2.6 Approach 3 : Stepwise Regression

[53]:
```
X_train.columns
```

[53]:
```
Index(['Year', 'Status', 'Adult Mortality', 'infant deaths', 'Alcohol',
       'percentage expenditure', 'Hepatitis B', 'Measles', 'BMI',
       'under-five deaths', 'Polio', 'Total expenditure', 'Diphtheria',
       'HIV/AIDS', 'GDP', 'Population', 'thinness  1-19 years',
       'thinness 5-9 years', 'Income composition of resources', 'Schooling'],
      dtype='object')
```

[54]:
```
X_train.head()
```

[54]:
```
      Year  Status  Adult Mortality  infant deaths  Alcohol  \
270   2001       1             21.0              0     4.90
1687  2011       1            124.0             34     5.30
822   2011       1            197.0              2     2.37
2030  2008       1            217.0             60     4.21
363   2004       1             17.0             81     6.85

      percentage expenditure  Hepatitis B  Measles   BMI  under-five deaths  \
270               251.658693         96.0        0  41.4                  0
```

```
1687             1117.196097         98.0        3   6.8                  39
822               549.278308         89.0        0  53.4                   2
2030              155.476762         88.0      341  21.6                  78
363               186.609049         96.0        0  46.9                  93

      Polio  Total expenditure  Diphtheria  HIV/AIDS          GDP  \
270    96.0               4.50        96.0       0.4  3419.275719
1687   97.0               6.40        97.0       0.1  9834.472689
822     9.0               6.81        89.0       0.3  3736.587130
2030   91.0               4.50        91.0       0.1  1919.466195
363    99.0               7.70        99.0       0.1  3623.476670

       Population  thinness  1-19 years  thinness 5-9 years  \
270      254984.0                   3.7                 3.7
1687     119917.0                   1.6                 1.6
822      619256.0                   1.7                 1.6
2030    9751864.0                   1.0                 9.7
363   184738458.0                   3.2                 3.2

      Income composition of resources  Schooling
270                             0.677       11.8
1687                            0.745       12.6
822                             0.666       13.0
2030                            0.655       11.5
363                             0.695       14.0
```

[55]:
```
X_train.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 2056 entries, 270 to 2863
Data columns (total 20 columns):
 #   Column                            Non-Null Count  Dtype
---  ------                            --------------  -----
 0   Year                              2056 non-null   int64
 1   Status                            2056 non-null   int64
 2   Adult Mortality                   2056 non-null   float64
 3   infant deaths                     2056 non-null   int64
 4   Alcohol                           2056 non-null   float64
 5   percentage expenditure            2056 non-null   float64
 6   Hepatitis B                       2056 non-null   float64
 7   Measles                           2056 non-null   int64
 8   BMI                               2056 non-null   float64
 9   under-five deaths                 2056 non-null   int64
 10  Polio                             2056 non-null   float64
 11  Total expenditure                 2056 non-null   float64
 12  Diphtheria                        2056 non-null   float64
 13  HIV/AIDS                          2056 non-null   float64
 14  GDP                               2056 non-null   float64
```

```
 15  Population                       2056 non-null   float64
 16  thinness  1-19 years             2056 non-null   float64
 17  thinness 5-9 years               2056 non-null   float64
 18  Income composition of resources  2056 non-null   float64
 19  Schooling                        2056 non-null   float64
dtypes: float64(15), int64(5)
memory usage: 337.3 KB
```

[56]:
```
y_train.head()
```

[56]:
```
      Life expectancy
270              68.2
1687             76.1
822              72.0
2030             67.5
363              72.0
```

[57]:
```
y_train.info()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 2056 entries, 270 to 2863
Data columns (total 1 columns):
 #   Column           Non-Null Count  Dtype
---  ------           --------------  -----
 0   Life expectancy  2056 non-null   float64
dtypes: float64(1)
memory usage: 32.1 KB
```

[ ]:
```
## By David Dale https://datascience.stackexchange.com/users/24162/david-dale

def stepwise_selection(X, y,
                       initial_list=[],
                       threshold_in=0.01,
                       threshold_out = 0.05,
                       verbose=True):
    """ Perform a forward-backward feature selection
    based on p-value from statsmodels.api.OLS
    Arguments:
        X - pandas.DataFrame with candidate features
        y - list-like with the target
        initial_list - list of features to start with (column names of X)
        threshold_in - include a feature if its p-value < threshold_in
        threshold_out - exclude a feature if its p-value > threshold_out
        verbose - whether to print the sequence of inclusions and exclusions
    Returns: list of selected features
    Always set threshold_in < threshold_out to avoid infinite looping.
    See https://en.wikipedia.org/wiki/Stepwise_regression for the details
    """
```

```
    included = list(initial_list)
    while True:
        changed=False
        # forward step
        excluded = list(set(X.columns)-set(included))
        new_pval = pd.Series(index=excluded)
        for new_column in excluded:
            model = sm.OLS(y, sm.add_constant(pd.
 ↪DataFrame(X[included+[new_column]]))).fit()
            new_pval[new_column] = model.pvalues[new_column]
        best_pval = new_pval.min()
        if best_pval < threshold_in:
            best_feature = new_pval.argmin()
            included.append(best_feature)
            changed=True
            if verbose:
                print('Add  {:30} with p-value {:.6}'.format(best_feature,␣
 ↪best_pval))

        # backward step
        # print(included)
        model = sm.OLS(y, sm.add_constant(pd.DataFrame(X[included]))).fit()
        # use all coefs except intercept
        pvalues = model.pvalues.iloc[1:]
        worst_pval = pvalues.max() # null if pvalues is empty
        if worst_pval > threshold_out:
            changed=True
            worst_feature = pvalues.argmax()
            included.remove(worst_feature)
            if verbose:
                print('Drop {:30} with p-value {:.6}'.format(worst_feature,␣
 ↪worst_pval))
        if not changed:
            break
    return included

result = stepwise_selection(X_train, y_train)

print('resulting features:')
print(result)
```

[59]:
```
X_train_stepwise = X_train[['Schooling', 'Adult Mortality', 'HIV/AIDS',␣
 ↪'Diphtheria', 'BMI', 'Income composition of resources', 'Status',␣
 ↪'percentage expenditure', 'Polio', 'Measles', 'Hepatitis B', 'under-five␣
 ↪deaths', 'infant deaths', 'thinness  1-19 years']]

# Adding a constant variable and Build a second fitted model
```

```
X_train_stepwise = sm.add_constant(X_train_stepwise)
lm_stepwise = sm.OLS(y_train, X_train_stepwise).fit()

#Summary of linear model
print(lm_stepwise.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:        Life expectancy   R-squared:                       0.823
Model:                            OLS   Adj. R-squared:                  0.822
Method:                 Least Squares   F-statistic:                     677.3
Date:                Sun, 20 Dec 2020   Prob (F-statistic):               0.00
Time:                        16:11:00   Log-Likelihood:                -5807.2
No. Observations:                2056   AIC:                         1.164e+04
Df Residuals:                    2041   BIC:                         1.173e+04
Df Model:                          14
Covariance Type:            nonrobust
================================================================================
===================
                                      coef    std err          t      P>|t|
[0.025      0.975]
--------------------------------------------------------------------------------
-------------------
const                              56.8039      0.756     75.122      0.000
55.321      58.287
Schooling                           0.6985      0.050     14.021      0.000
0.601       0.796
Adult Mortality                    -0.0197      0.001    -20.365      0.000
-0.022      -0.018
HIV/AIDS                           -0.4943      0.024    -20.731      0.000
-0.541      -0.448
Diphtheria                          0.0457      0.006      7.901      0.000
0.034       0.057
BMI                                 0.0491      0.006      8.134      0.000
0.037       0.061
Income composition of resources     5.8765      0.763      7.704      0.000
4.380       7.372
Status                             -1.8727      0.291     -6.433      0.000
-2.444      -1.302
percentage expenditure              0.0003   5.07e-05      5.555      0.000
0.000       0.000
Polio                               0.0272      0.005      5.004      0.000
0.017       0.038
Measles                         -2.367e-05   9.35e-06     -2.530      0.011
-4.2e-05   -5.32e-06
Hepatitis B                        -0.0194      0.005     -4.010      0.000
```

```
-0.029      -0.010
under-five deaths                    -0.0701      0.007     -9.950      0.000
-0.084      -0.056
infant deaths                         0.0940      0.010      9.797      0.000
0.075       0.113
thinness  1-19 years                 -0.0868      0.028     -3.056      0.002
-0.143      -0.031
==============================================================================
Omnibus:                      106.745   Durbin-Watson:                   1.971
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              308.003
Skew:                          -0.225   Prob(JB):                     1.31e-67
Kurtosis:                       4.842   Cond. No.                     1.05e+05
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 1.05e+05. This might indicate that there are
strong multicollinearity or other numerical problems.
```

2.7 Model Prediction and Evaluation

[60]:
```
# Predicting the price of training set.
X_test_stepwise = X_test[['Schooling', 'Adult Mortality', 'HIV/AIDS',␣
 ↪'Diphtheria', 'BMI', 'Income composition of resources', 'Status',␣
 ↪'percentage expenditure', 'Polio', 'Measles', 'Hepatitis B', 'under-five␣
 ↪deaths', 'infant deaths', 'thinness  1-19 years']]
X_test_stepwise = sm.add_constant(X_test_stepwise)
actual     = y_test["Life expectancy"]
prediction = lm_stepwise.predict(X_test_stepwise)
```

[61]:
```
#Evaluuvation: MSE
model_mse = mean_squared_error(prediction, actual)
print(model_mse)
```

```
15.972714682412724
```

[62]:
```
def mean_absolute_percentage_error(y_true, y_pred):
    y_true, y_pred = np.array(y_true), np.array(y_pred)
    return np.mean(np.abs((y_true - y_pred) / y_true)) * 100
```

[63]:
```
mean_absolute_percentage_error(actual, prediction)
```

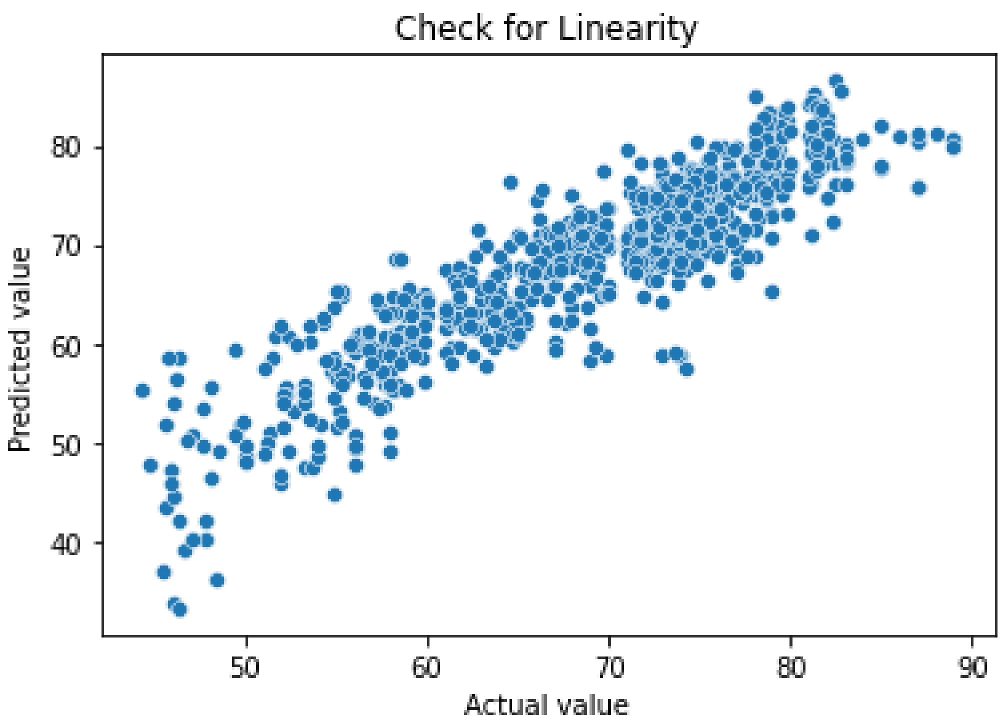
[63]:
```
4.558248666207725
```

[64]:
```
# Check for Linearity
sns.scatterplot(y_test['Life expectancy'],prediction)
plt.title('Check for Linearity')
plt.xlabel('Actual value')
```

```
plt.ylabel('Predicted value')
```

```
/usr/local/lib/python3.6/dist-packages/seaborn/_decorators.py:43: FutureWarning:
Pass the following variables as keyword args: x, y. From version 0.12, the only
valid positional argument will be `data`, and passing other arguments without an
explicit keyword will result in an error or misinterpretation.
  FutureWarning
```

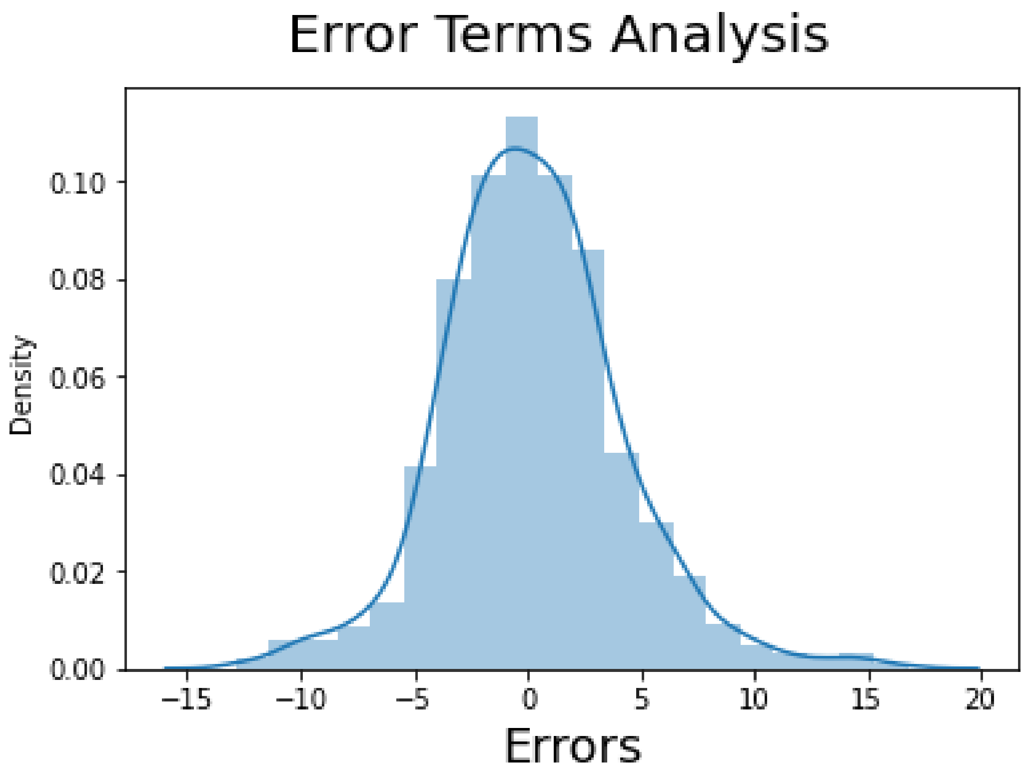
[64]: Text(0, 0.5, 'Predicted value')



[65]:
```
# Plot the histogram of the error terms
fig = plt.figure()
sns.distplot((y_test['Life expectancy'] - prediction), bins = 20)
fig.suptitle('Error Terms Analysis', fontsize = 20)
plt.xlabel('Errors', fontsize = 18)
```

```
/usr/local/lib/python3.6/dist-packages/seaborn/distributions.py:2551:
FutureWarning: `distplot` is a deprecated function and will be removed in a
future version. Please adapt your code to use either `displot` (a figure-level
function with similar flexibility) or `histplot` (an axes-level function for
histograms).
  warnings.warn(msg, FutureWarning)
```

[65]: Text(0.5, 0, 'Errors')



[ ]: