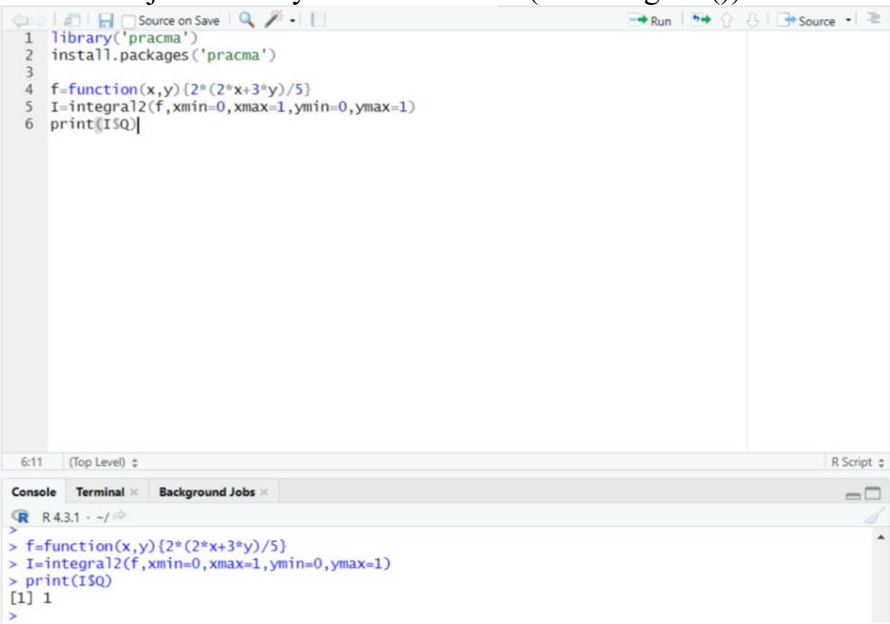
Probability and Statistics (UCS410)
Experiment 6
(Joint probability mass and density functions)

1. The joint probability density of two random variables X and Y is

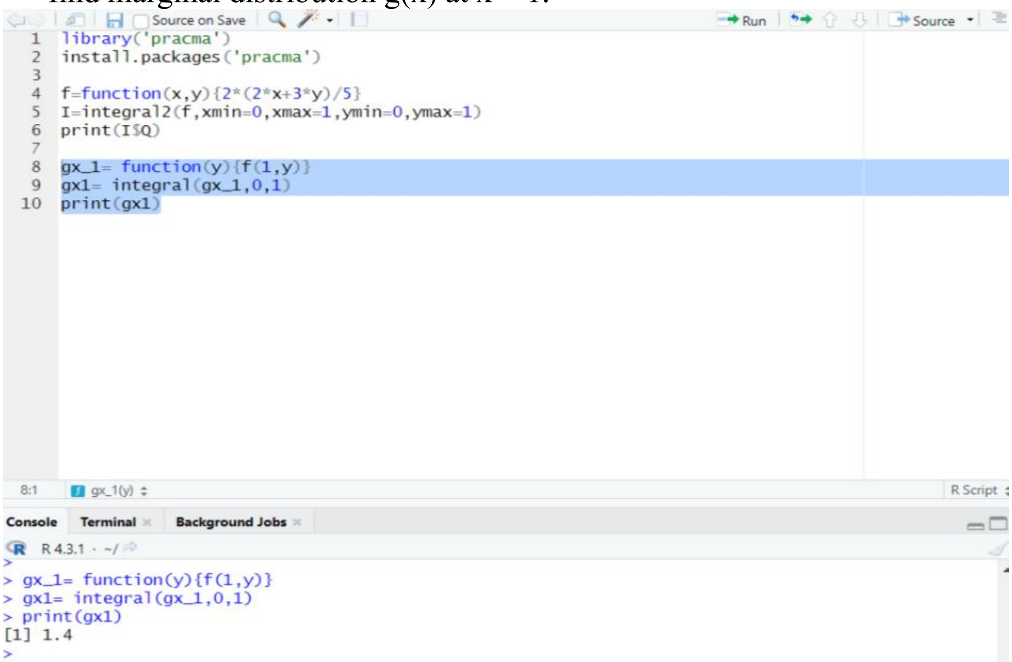
$$f(x,y) = \begin{cases} 2(2x+3y)/5; & 0 \le x, y \le 1 \\ 0; & elsewhere \end{cases}$$

Then write a R-code to

(i) check that it is a joint density function or not? (Use integral2())



(ii) find marginal distribution g(x) at x = 1.



(iii) find the marginal distribution h(y) at y = 0.

```r
library('pracma')
install.packages('pracma')

f=function(x,y){2*(2*x+3*y)/5}
I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
print(I$Q)

gx_1= function(y){f(1,y)}
gx1= integral(gx_1,0,1)
print(gx1)

hy_0= function(x){f(x,0)}
hy0= integral(hy_0,0,1)
print(hy0)
```

12:1 hy_0(x)

Console Terminal Background Jobs

R 4.3.1 · ~/

```
>
> hy_0= function(x){f(x,0)}
> hy0= integral(hy_0,0,1)
> print(hy0)
[1] 0.4
>
```

(iv) find the expected value of g(x, y) = xy.

```r
library('pracma')
install.packages('pracma')

f=function(x,y){2*(2*x+3*y)/5}
I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
print(I$Q)

gx_1= function(y){f(1,y)}
gx1= integral(gx_1,0,1)
print(gx1)

hy_0= function(x){f(x,0)}
hy0= integral(hy_0,0,1)
print(hy0)

f_xy=function(x,y){x*y*f(x,y)}
E_xy= integral2(f_xy,0,1,0,1)
print(E_xy$Q)
```

16:1 f_xy(x, y)

Console Terminal Background Jobs

R 4.3.1 · ~/

```
>
> f_xy=function(x,y){x*y*f(x,y)}
> E_xy= integral2(f_xy,0,1,0,1)
> print(E_xy$Q)
[1] 0.3333333
>
```

2. The joint probability mass function of two random variables X and Y is

$$f(x, y) = \{(x + y)/30; \;\; x = 0, 1, 2, 3; \;\; y = 0, 1, 2\}$$

Then write a R-code to

(i) display the joint mass function in rectangular (matrix) form.

```r
f=function(x,y){(x+y)/30}
x=c(0:3)
y=c(0:2)
M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
print(M1)
```

```
> f=function(x,y){(x+y)/30}
> x=c(0:3)
> y=c(0:2)
> M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
> print(M1)
           [,1]       [,2]       [,3]
[1,] 0.00000000 0.03333333 0.06666667
[2,] 0.03333333 0.06666667 0.10000000
[3,] 0.06666667 0.10000000 0.13333333
[4,] 0.10000000 0.13333333 0.16666667
>
>
```

(ii) check that it is joint mass function or not? (use: Sum())

```r
f=function(x,y){(x+y)/30}
x=c(0:3)
y=c(0:2)
M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
print(M1)
sum(M1)
```

```
> sum(M1)
[1] 1
>
>
```

(iii) find the marginal distribution g(x) for x = 0, 1, 2, 3. (Use:apply())

```r
f=function(x,y){(x+y)/30}
x=c(0:3)
y=c(0:2)
M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
print(M1)
sum(M1)
gx=apply(M1,1,sum)
cat("The marginal probabilities are")
print((gx))
print(sum(gx))
```

```
gx=apply(M1,1,sum)
cat("The marginal probabilities are")
he marginal probabilities are> print((gx))
1] 0.1 0.2 0.3 0.4
print(sum(gx))
1] 1
```

(iv) find the marginal distribution h(y) for y = 0, 1, 2. (Use:apply())

```r
f=function(x,y){(x+y)/30}
x=c(0:3)
y=c(0:2)
M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
print(M1)
sum(M1)
gx=apply(M1,1,sum)
cat("The marginal probabilities are")
print((gx))
print(sum(gx))
hy=apply(M1,2,sum)
cat("The marginal probabilities are")
print((hy))
print(sum(hy))
```

```
>
> hy=apply(M1,2,sum)
> cat("The marginal probabilities are")
The marginal probabilities are> print((hy))
[1] 0.2000000 0.3333333 0.4666667
> print(sum(hy))
[1] 1
```

(v) find the conditional probability at x = 0 given y = 1.

```r
f=function(x,y){(x+y)/30}
x=c(0:3)
y=c(0:2)
M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
print(M1)
sum(M1)
gx=apply(M1,1,sum)
cat("The marginal probabilities are")
print((gx))
print(sum(gx))
hy=apply(M1,2,sum)
cat("The marginal probabilities are")
print((hy))
print(sum(hy))
p_x0_y1=M1[1,2]/hy[2]
print(p_x0_y1)
```

```
>
> p_x0_y1=M1[1,2]/hy[2]
> print(p_x0_y1)
[1] 0.1
>
```

(vi) find E(x), E(y), E(xy), V ar(x), V ar(y), Cov(x, y) and its correlation coefficient.

```r
print(p_x0_y1)
E_x= sum(x*gx)
print(E_x)
E_y=sum(y*hy)
print(E_y)
E_x2=sum(x^2*gx)
E_y2= sum(y^2*hy)
print(E_x2)
print(E_y2)
Var_X= E_x2-(E_x)^2
print(Var_X)
Var_Y= E_y2-(E_y)^2
print(Var_Y)
x=c(0:3)
y=c(0:2)
f1=function(x,y){x*y*(x+y)/30}
M2= matrix(c(f1(0,0:2),f1(1,0:2),f1(2,0:2),f1(3,0:2)),nrow=4,ncol = 3, byrow=TRUE)
print(M2)
E_xy=(sum(M2))
print(sum(M2))
Cov_xy= E_xy - E_x*E_y
print(Cov_xy)
r_xy=Cov_xy/sqrt(Var_X*Var_Y)
print(r_xy)
```

```
>
>
> E_x= sum(x*gx)
> print(E_x)
[1] 2
> E_y=sum(y*hy)
> print(E_y)
[1] 1.266667
> E_x2=sum(x^2*gx)
> E_y2= sum(y^2*hy)
> print(E_x2)
[1] 5
> print(E_y2)
[1] 2.2
> Var_X= E_x2-(E_x)^2
> print(Var_X)
[1] 1
> Var_Y= E_y2-(E_y)^2
> print(Var_Y)
[1] 0.5955556
> x=c(0:3)
> y=c(0:2)
> f1=function(x,y){x*y*(x+y)/30}
> M2= matrix(c(f1(0,0:2),f1(1,0:2),f1(2,0:2),f1(3,0:2)),nrow=4,ncol = 3, byrow=TRUE)
> print(M2)
     [,1]       [,2]      [,3]
[1,]    0 0.00000000 0.0000000
[2,]    0 0.06666667 0.2000000
[3,]    0 0.20000000 0.5333333
[4,]    0 0.40000000 1.0000000
> E_xy=(sum(M2))
> print(sum(M2))
[1] 2.4
> Cov_xy= E_xy - E_x*E_y
> print(Cov_xy)
[1] -0.1333333
> r_xy=Cov_xy/sqrt(Var_X*Var_Y)
> print(r_xy)
[1] -0.1727737
>
```