

Probability and Statistics (UCS410)

Experiment 6

(Joint probability mass and density functions)

1. The joint probability density of two random variables X and Y is

$$f(x, y) = \begin{cases} 2(2x + 3y)/5; & 0 \leq x, y \leq 1 \\ 0; & \text{elsewhere} \end{cases}$$

Then write a R-code to

- (i) check that it is a joint density function or not? (Use `integral2()`)



```
1 library('pracma')
2 install.packages('pracma')
3
4 f=function(x,y){2*(2*x+3*y)/5}
5 I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
6 print(I$Q)
```

6:11 (Top Level) R Script

Console Terminal Background Jobs

```
R 4.3.1 ~ /
> f=function(x,y){2*(2*x+3*y)/5}
> I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
> print(I$Q)
[1] 1
>
```

- (ii) find marginal distribution $g(x)$ at $x = 1$.



```
1 library('pracma')
2 install.packages('pracma')
3
4 f=function(x,y){2*(2*x+3*y)/5}
5 I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
6 print(I$Q)
7
8 gx_1= function(y){f(1,y)}
9 gx1= integral(gx_1,0,1)
10 print(gx1)
```

8:1 gx_1(y) R Script

Console Terminal Background Jobs

```
R 4.3.1 ~ /
> gx_1= function(y){f(1,y)}
> gx1= integral(gx_1,0,1)
> print(gx1)
[1] 1.4
>
```

(iii) find the marginal distribution $h(y)$ at $y = 0$.

```
1 library('pracma')
2 install.packages('pracma')
3
4 f=function(x,y){2*(2*x+3*y)/5}
5 I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
6 print(I$Q)
7
8 gx_1= function(y){f(1,y)}
9 gx1= integral(gx_1,0,1)
10 print(gx1)
11
12 hy_0= function(x){f(x,0)}
13 hy0= integral(hy_0,0,1)
14 print(hy0)
```

12:1 hy_0(x)

Console Terminal Background Jobs

R 4.3.1 ~/

```
> hy_0= function(x){f(x,0)}
> hy0= integral(hy_0,0,1)
> print(hy0)
[1] 0.4
>
```

(iv) find the expected value of $g(x, y) = xy$.

```
1 library('pracma')
2 install.packages('pracma')
3
4 f=function(x,y){2*(2*x+3*y)/5}
5 I=integral2(f,xmin=0,xmax=1,ymin=0,ymax=1)
6 print(I$Q)
7
8 gx_1= function(y){f(1,y)}
9 gx1= integral(gx_1,0,1)
10 print(gx1)
11
12 hy_0= function(x){f(x,0)}
13 hy0= integral(hy_0,0,1)
14 print(hy0)
15
16 f_xy=function(x,y){x*y*f(x,y)}
17 E_xy= integral2(f_xy,0,1,0,1)
18 print(E_xy$Q)
```

16:1 f_xy(x, y)

Console Terminal Background Jobs

R 4.3.1 ~/

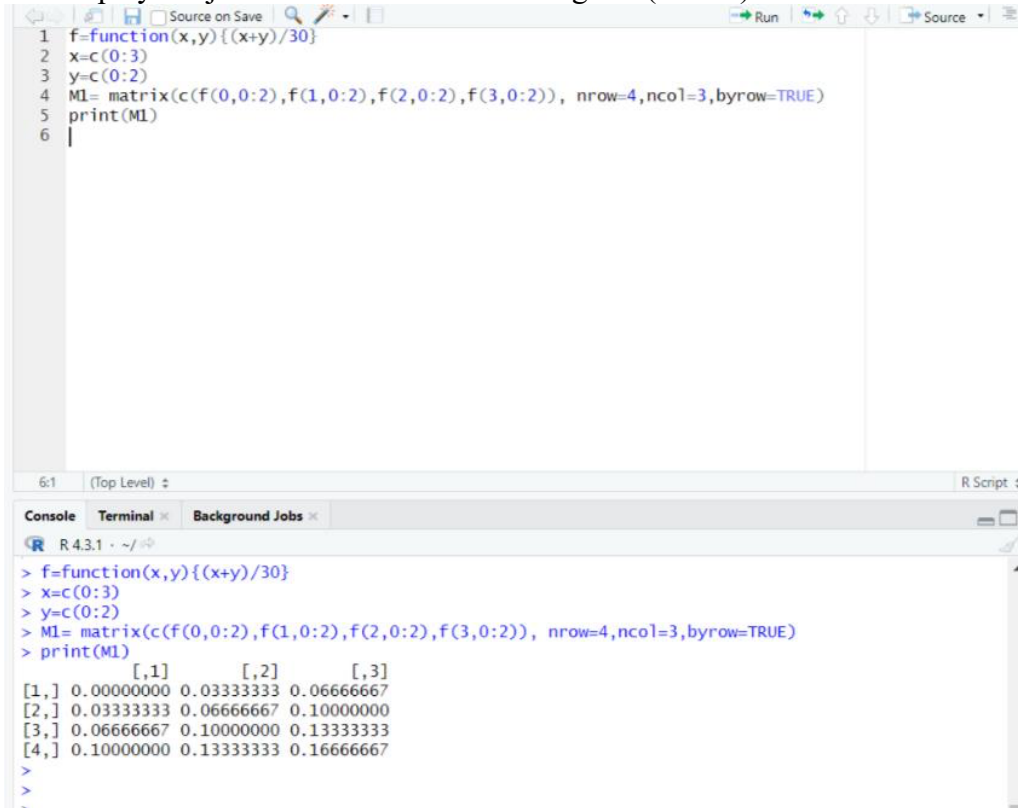
```
> f_xy=function(x,y){x*y*f(x,y)}
> E_xy= integral2(f_xy,0,1,0,1)
> print(E_xy$Q)
[1] 0.3333333
>
```

2. The joint probability mass function of two random variables X and Y is

$$f(x, y) = \{(x + y)/30; \ x = 0, 1, 2, 3; \ y = 0, 1, 2\}$$

Then write a R-code to

- (i) display the joint mass function in rectangular (matrix) form.

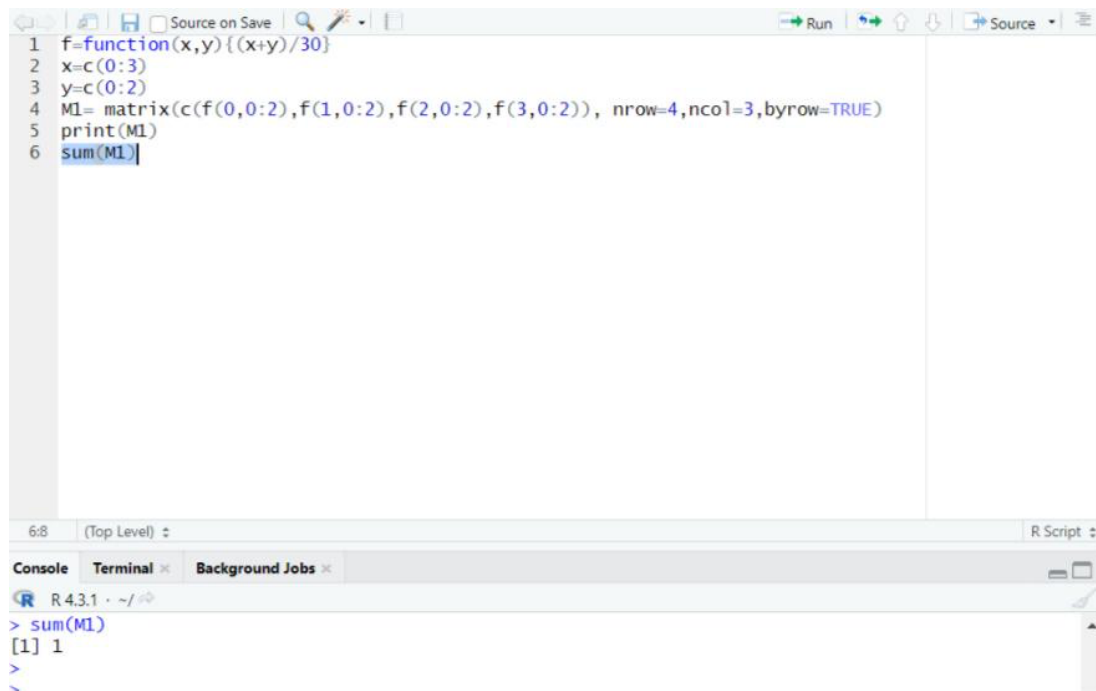


```
1 f=function(x,y){(x+y)/30}
2 x=c(0:3)
3 y=c(0:2)
4 M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
5 print(M1)
6
```

Console output:

```
> f=function(x,y){(x+y)/30}
> x=c(0:3)
> y=c(0:2)
> M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
> print(M1)
      [,1]      [,2]      [,3]
[1,] 0.00000000 0.03333333 0.06666667
[2,] 0.03333333 0.06666667 0.10000000
[3,] 0.06666667 0.10000000 0.13333333
[4,] 0.10000000 0.13333333 0.16666667
>
>
```

- (ii) check that it is joint mass function or not? (use: Sum())



```
1 f=function(x,y){(x+y)/30}
2 x=c(0:3)
3 y=c(0:2)
4 M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
5 print(M1)
6 sum(M1)
```

Console output:

```
> sum(M1)
[1] 1
>
>
```

- (iii) find the marginal distribution $g(x)$ for $x = 0, 1, 2, 3$. (Use: `apply()`)

```
1 f=function(x,y){(x+y)/30}
2 x=c(0:3)
3 y=c(0:2)
4 M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
5 print(M1)
6 sum(M1)
7 gx=apply(M1,1,sum)
8 cat("The marginal probabilities are")
9 print(gx)
10 print(sum(gx))
```

10:15 (Top Level) 2

Console Terminal Background Jobs

R 4.3.1 ~ /

```
gx=apply(M1,1,sum)
cat("The marginal probabilities are")
The marginal probabilities are> print(gx)
[1] 0.1 0.2 0.3 0.4
print(sum(gx))
[1] 1
```

- (iv) find the marginal distribution $h(y)$ for $y = 0, 1, 2$. (Use: `apply()`)

```
1 f=function(x,y){(x+y)/30}
2 x=c(0:3)
3 y=c(0:2)
4 M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
5 print(M1)
6 sum(M1)
7 gx=apply(M1,1,sum)
8 cat("The marginal probabilities are")
9 print(gx)
10 print(sum(gx))
11 hy=apply(M1,2,sum)
12 cat("The marginal probabilities are")
13 print(hy)
14 print(sum(hy))
15
16
```

11:1 (Top Level) 2

Console Terminal Background Jobs

R 4.3.1 ~ /

```
>
> hy=apply(M1,2,sum)
> cat("The marginal probabilities are")
The marginal probabilities are> print(hy)
[1] 0.2000000 0.3333333 0.4666667
> print(sum(hy))
[1] 1
```

- (v) find the conditional probability at $x = 0$ given $y = 1$.

```
1 f=function(x,y){(x+y)/30}
2 x=c(0:3)
3 y=c(0:2)
4 M1= matrix(c(f(0,0:2),f(1,0:2),f(2,0:2),f(3,0:2)), nrow=4,ncol=3,byrow=TRUE)
5 print(M1)
6 sum(M1)
7 gx=apply(M1,1,sum)
8 cat("The marginal probabilities are")
9 print(gx)
10 print(sum(gx))
11 hy=apply(M1,2,sum)
12 cat("The marginal probabilities are")
13 print(hy)
14 print(sum(hy))
15 p_x0_y1=M1[1,2]/hy[2]
16 print(p_x0_y1)
17
18
```

15:1 (Top Level) 2

Console Terminal Background Jobs

R 4.3.1 ~ /

```
>
> p_x0_y1=M1[1,2]/hy[2]
> print(p_x0_y1)
[1] 0.1
```

(vi) find $E(x)$, $E(y)$, $E(xy)$, $\text{Var}(x)$, $\text{Var}(y)$, $\text{Cov}(x, y)$ and its correlation coefficient.

```

16 print(p_x0_y1)
17 E_x= sum(x*gx)
18 print(E_x)
19 E_y=sum(y*hy)
20 print(E_y)
21 E_x2=sum(x^2*gx)
22 E_y2= sum(y^2*hy)
23 print(E_x2)
24 print(E_y2)
25 Var_X= E_x2-(E_x)^2
26 print(Var_X)
27 Var_Y= E_y2-(E_y)^2
28 print(Var_Y)
29 x=c(0:3)
30 y=c(0:2)
31 f1=function(x,y){x*y*(x+y)/30}
32 M2= matrix(c(f1(0,0:2),f1(1,0:2),f1(2,0:2),f1(3,0:2)),nrow=4,ncol = 3, byrow=TRUE)
33 print(M2)
34 E_xy=(sum(M2))
35 print(sum(M2))
36 Cov_xy= E_xy - E_x*E_y
37 print(Cov_xy)
38 r_xy=Cov_xy/sqrt(Var_X*Var_Y)
39 print(r_xy)
40

```

kt12 (Top Level) R Script

```

R 4.3.1 ~ /
>
>
> E_x= sum(x*gx)
> print(E_x)
[1] 2
> E_y=sum(y*hy)
> print(E_y)
[1] 1.266667
> E_x2=sum(x^2*gx)
> E_y2= sum(y^2*hy)
> print(E_x2)
[1] 5
> print(E_y2)
[1] 2.2
> Var_X= E_x2-(E_x)^2
> print(Var_X)
[1] 1
> Var_Y= E_y2-(E_y)^2
> print(Var_Y)
[1] 0.5955556
> x=c(0:3)
> y=c(0:2)
> f1=function(x,y){x*y*(x+y)/30}
> M2= matrix(c(f1(0,0:2),f1(1,0:2),f1(2,0:2),f1(3,0:2)),nrow=4,ncol = 3, byrow=TRUE)
> print(M2)
      [,1] [,2] [,3]
[1,]  0 0.0000000 0.0000000
[2,]  0 0.0666667 0.2000000
[3,]  0 0.2000000 0.5333333
[4,]  0 0.4000000 1.0000000
> E_xy=(sum(M2))
> print(sum(M2))
[1] 2.4
> Cov_xy= E_xy - E_x*E_y
> print(Cov_xy)
[1] -0.1333333
> r_xy=Cov_xy/sqrt(Var_X*Var_Y)
> print(r_xy)
[1] -0.1727737
>

```