Analysis of iterative time:-

1] for (i=0; i<n; i=i+c) { }

iteration	1	2	3	4	...	$K \le n-1$
iterator_value (i)	0	c	2c	3c	...	$(K-1)c \le n-1$

$$\to (K-1)c \le n-1 \Rightarrow K-1 \le \frac{n-1}{c}$$

$$\Rightarrow K \le \frac{n-1}{c}+1 \Rightarrow K \le \frac{n}{c} - \frac{1}{c} + 1$$

$$\Rightarrow T(n) = \underline{\underline{\frac{n}{c}}} - \frac{1}{c} + 1 \quad \left(\because T(n) = K \text{ in terms of } n\right)$$

$\Rightarrow$ Time complexity $= \underline{\underline{O(n)}}$.

2] for (i=n; i>0; i=i/c) { }

iteration	1	2	3	4	...	$K \ge 1$
i	n	$\frac{n}{c}$	$\frac{n}{c^2}$	$\frac{n}{c^3}$	...	$\frac{n}{c^{K-1}} \ge 1$

$$\frac{n}{c^{K-1}} \ge 1 \Rightarrow c^{K-1} \le n. \Rightarrow (K-1)\cdot \log c \le \log n.$$

$$\Rightarrow K-1 \le \frac{\log n}{\log c} \Rightarrow K-1 \le \log_c n \Rightarrow K \le \log_c n + 1$$

$$\Rightarrow T(n) = \underline{\underline{\log_c n}} + 1 \Rightarrow \underline{\underline{O(\log n)}}.$$

3] for (i=2 ; i<n ; i=pow(i,c)) { }

iteration	1	2	3	4	5	...	$K \le n-1$
i	2	2^{c}	2^{c^2}	2^{c^3}	2^{c^4}	...	$2^{c^{K-1}} \le n-1$

$$\Rightarrow 2^{c^{K-1}} \le n-1 \Rightarrow c^{K-1} \cdot \log_2 2 \le \log(n-1)$$

$$\Rightarrow c^{K-1} \le \log(n-1) \Rightarrow (K-1)\log c \le \log\log(n-1)$$

$$\Rightarrow K \le \frac{\log\log(n-1)}{\log c} = T(n) \Rightarrow O(\log\log n)$$

4] for (i=0; i*i < n ; i++) { }   Imp. (*)

iteration.	1	2	3	4	...	$K \le n-1$
iterator_value. (i*i) or (i^2)	0	1	2^2	3^2	...	$(K-1)^2 \le n-1$
i .	0	1	2	3	...	K−1

$$\Rightarrow (K-1)^2 \le n-1 \Rightarrow K \le (n-1)^{1/2} + 1 = T(n) \Rightarrow O(\sqrt{n})$$

or $O(n^{1/2})$.

5] for (i=1, j=1; j<n; i++) { j = j + i } **Imp** (*)

iteration	1	2	3	4	...	$K \le n-1$
iter-val (j)	1	1+1	1+1+2	1+1+2+3	...	$1+1+2+3+\dots+(K-1) \le n-1$
i	1	2	3	4	...	K

Same for loop using while loop:-

```
i=1, j=1;
while (j<n){
    j=j+i;
    i++;
}
```

$$\Rightarrow 1+1+2+3+\dots+(K-1) \le n-1$$

$$\cancel{1}+1+2+3+\dots+K\cancel{-1} \le n-1$$

$$1+2+3+\dots+K \le n-1$$

$$\frac{K\cdot(K+1)}{2} \le n-1 \Rightarrow K^2+K \le 2n-2.$$

$$\Rightarrow K^2 \le 2n-2-K. \Rightarrow K \le (2n-2-K)^{1/2} = T(n)$$

$$\Rightarrow O(n^{1/2}) \text{ or } O(\sqrt{n})$$

6] $p = 0$

for ($i=1$; $i<n$; $i=i*2$) { p++; }

for ($j=1$; $j<p$; $j=j*2$) { }.

Imp ⊛

First loop:-

iteration	1	2	3	4	...	$k \le n-1$
iter_val (i)	1	2	2^2	2^3	...	$2^{k-1} \le n-1$
p	0	1	2	3	...	$k-1$

Here at start of iteration k, p is k−1. But when k^{th} iteration completes, $p=k$. It means when the first loop execution completed, $p=k$.

Second loop:-

iteration	1	2	3	4	...	$k \le p-1$
iter_val (j)	1	2	2^2	2^3	...	$2^{k-1} \le p-1$

For first loop:-

$k \le \log(n-1)+1$

$\Rightarrow k = p$

For second loop:-

$k \le \log(p-1)+1$

T(n) = First loop time + second loop time.

$$= [\log(n-1)+1] + [\log(p-1)+1]$$

replace p.

$$= [\log(n-1)+1] + [\log(\underbrace{\log(n-1)+1}_{p}-1)+1]$$

$$= [\underbrace{\log(n-1)+1}_{\text{loop 1 time}}] + [\underbrace{\log\log(n-1)+1}_{\text{loop 2 time}}]$$

$$T(n) = \log(n-1) + \log\log(n-1) + 2.$$

$\Rightarrow O(\log n)$

7] for (i=1; i<=n; i++)
{ for (j=1; j≤n; j++){ }
}

First loop:-

iteration.	1	2	3	...	$K \le n$
iter-val (i)	1	2	3	...	$K \le n$

second loop:-

iteration	1	2	3	...	$K \le n$
iter-val(j)	1	2	3	...	$K \le n$.

$$T(n) = n * n \Rightarrow O(n^2)$$

8] for (i=0; i<n; i++)
{ for (j=0; j<i; j++){ }
}

Imp *

First loop:-

iteration	1	2	3	...	$K \le n-1$
iter-val(i)	0	1	2	...	$K-1 \le n-1$

$$\Rightarrow K-1 \le n-1$$
$$K \le n$$

Second loop:-

it	1	2	...	$K \le i-1$
it.val. (j)	0	1	...	$K-1 \le i-1$

$$\Rightarrow K-1 \le i-1.$$
$$K \le i$$

Here for every iteration i of outer loop, the inner loop runs i times.

outer loop i = 1, 2, 3, 4, ..., n

$$\Rightarrow 1+2+3+4+\ldots+n = T(n).$$

Total inner loop executions for corresponding i.

$$\Rightarrow T(n) = \frac{n\cdot(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2} \Rightarrow O(n^2).$$

9] p=0;

for (i=1; p<=n; i++) { p=p+i; }   Imp ⊛

iteration	1	2	3	4	...	$K \le n$
iter_val (p)	0	0+1	0+1+2	0+1+2+3	...	$0+1+2+3+\dots+(K-1) \le n$
i	1	2	3	4	...	K

$$\Rightarrow 0+\underbrace{1+2+3+\dots+(K-1)} \le n.$$

$$\frac{(K-1)\cdot K}{2} \le n \Rightarrow K^2 \le 2n+K.$$

$$\Rightarrow K \le (2n+K)^{1/2} = T(n)$$

$$\Rightarrow O(\sqrt{n}) \text{ or } O(n^{1/2})$$