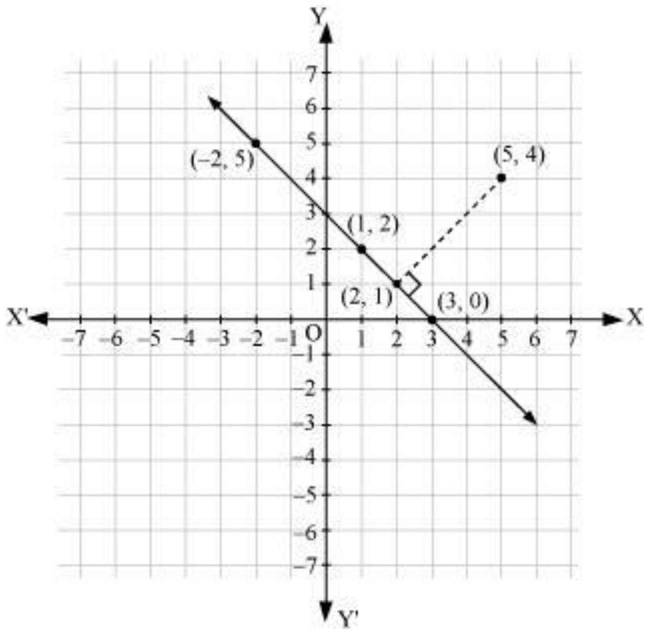
Q1) Among the four points (−2, 5), (1, 2), (3, 0) and (5, 4), which three points lie on a straight line?

What are the coordinates of the foot of the perpendicular drawn from the remaining point to the straight line?

Solution:

The given four points can be plotted on the Cartesian plane as:



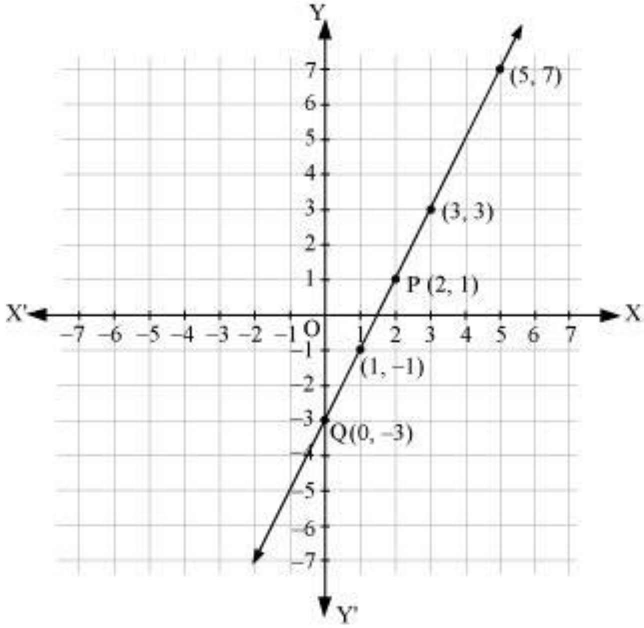
It can be seen that, (−2, 5), (1, 2) and (3, 0) lie on a straight line.

The coordinates of the foot of the perpendicular drawn from (5, 4) to the line are (2, 1).

Q2) The coordinates of the points P and Q are (2, 1) and (0, −3) respectively. Draw a line passing through the points P and Q on the Cartesian plane and determine the coordinates of three different points other than P and Q, which lie on the line PQ.

Solution:

The point P (2, 1) can be located by moving 2 units on the positive *x*-axis from the origin (0, 0), and then 1 unit along the positive *y*-axis. The point Q (0, −3) can be located by moving 3 units on the negative *y*-axis. The line passing through P and Q is obtained by joining P and Q and extending in both the directions. This can be done as:



It is seen that, (3, 3), (1, −1) and (5, 7) are three different points lying on the line $\overleftrightarrow{PQ}$ other than P and Q.

Q3) If the points ($2a + 3$, $b - 3$) and ($5a - 3$, $1 - 3b$) are coincident, then locate the points on the Cartesian plane.

Solution:

It is given that the points ($2a + 3$, $b - 3$) and ($5a - 3$, $1 - 3b$) are coincident.

$\therefore (2a + 3, b - 3) = (5a - 3, 1 - 3b)$

$\Rightarrow 2a + 3 = 5a - 3$ and $b - 3 = 1 - 3b$

$2a + 3 = 5a - 3$

$\Rightarrow 5a - 2a = 3 + 3$

$\Rightarrow 3a = 6$

$\Rightarrow a = 2$

$b - 3 = 1 - 3b$

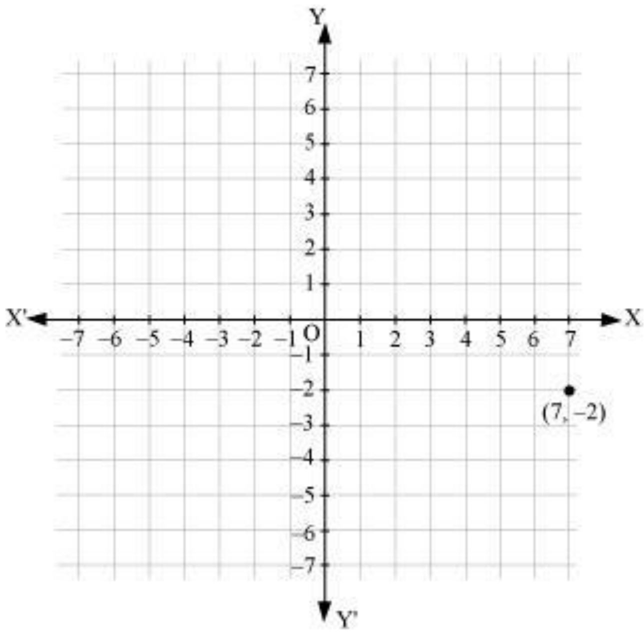
$\Rightarrow 3b + b = 1 + 3$

$\Rightarrow 4b = 4$

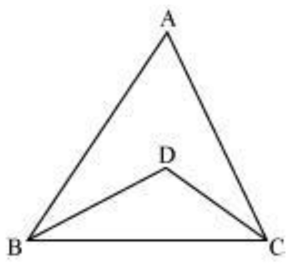
$\Rightarrow b = 1$

∴ The point is $(2a + 3, b - 3) = (2 \times 2 + 3, 1 - 3) = (7, -2)$

Now (7, −2) can be plotted on Cartesian plane by moving 7 units on the positive *x*-axis from the origin (0, 0) and then 2 units along negative *y*-axis. The point is shown below.



Q4)



In the given figure, BD and CD are bisectors of ∠ABC and ∠ACB respectively and CD < BD. Show that ∠ABC < ∠ACB.

Solution:

It is given that CD < BD.

It is known that if two sides of a triangle are unequal, then the angle opposite to the longer side is greater.

∴∠DBC < ∠DCB

$\Rightarrow 2\angle DBC < 2\angle DCB$ ...(1)

BD and CD are bisectors of $\angle ABC$ and $\angle ACB$ respectively.

$\therefore \angle ABC = 2\angle DBC$ and $\angle ACB = 2\angle DCB$ ...(2)

From (1) and (2), we have

$\angle ABC < \angle ACB$

Q5) Without actual division, state whether each of the following fractions is a terminating decimal or not. Give reasons to justify your answer.

(a) $\frac{3}{40}$

(b) $\frac{13}{42}$

Solution:

(a) For the fraction $\frac{3}{40}$, the numerator (3) and the denominator (40) are co-prime.

Now, the denominator of $\frac{3}{40}$ is 40, which can be prime factorised as $40 = 2^3 \times 5$.

As seen in the prime factorisation of 40, it has no prime factors other than 2 and 5.

Therefore,the fraction $\frac{3}{40}$ is a terminating decimal.

(b) For the fraction $\frac{13}{42}$, the numerator (13) and the denominator (42) are co-prime.

Now, the denominator of $\frac{13}{42}$ is 42, which can be prime factorised as $42 = 2 \times 3 \times 7$.

As seen in the prime factorisation of 42, it has prime factors (3 and 7) other than 2 and 5 as well.

Therefore, the fraction $\frac{13}{42}$ is a non-terminating decimal.

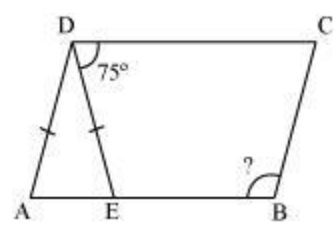
Q6) What is the value of the polynomial $p(x) = 5x^2 - 4$ at $x = \frac{5}{2}$?

Solution:

$$p(x) = 5x^2 - 4$$
$$\therefore p\left(\frac{5}{2}\right) = 5\left(\frac{5}{2}\right)^2 - 4 = 5\left(\frac{25}{4}\right) - 4 = \frac{125}{4} - 4 = \frac{125-16}{4} = \frac{109}{4}$$

Thus, the value of the given polynomial at $x = \frac{5}{2}$ is $\frac{109}{4}$.

Q7)



In the given figure, ABCD is a parallelogram. E is a point on AB such that DA = DE.

What is the measure of ∠ABC?

Solution:

Since ABCD is a parallelogram, AB || CD.

Here, DE can be regarded as the traversal.

∴∠AED = ∠EDC [Alternate interior angles]

⇒ ∠AED = 75°

Since DA = DE, ΔDAE is isosceles.

∴ ∠EAD = ∠AED

⇒ ∠EAD = 75°

It is known that the interior angles on the same side of the transversal are supplementary.

∴∠EAD + ∠ABC = 180° [AD || BC]

⇒ 75° + ∠ABC = 180°

⇒ ∠ABC = 105°

Thus, the measure of ∠ABC is 105°.

Q8) What are the zeroes of the polynomial $p(u) = 3u^2 + 2u - 8$?

Solution:

The given polynomial can be factorised as:

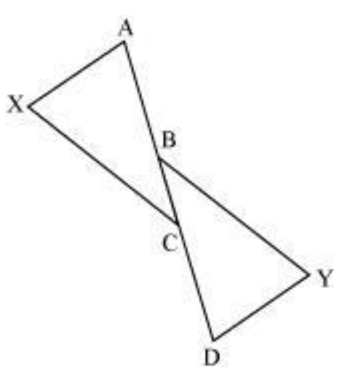
$$\begin{aligned} p(u) &= 3u^2 + 2u - 8 \\ &= 3u^2 + 6u - 4u - 8 \\ &= 3u(u+2) - 4(u+2) \\ &= (3u-4)(u+2) \end{aligned}$$

It can be observed that:

$$p(-2) = (3u-4)(0) = 0$$
$$p\left(\frac{4}{3}\right) = \left\{3\left(\frac{4}{3}\right) - 4\right\}(u+2) = \{4-4\}(u+2) = 0$$

Thus, the zeroes of the given polynomial are −2 and $\frac{4}{3}$.

Q9)



In the given figure, AX || DY, XC || BY and AB = CD.

Prove that ∠AXC = ∠BYD.

Solution:

It is given that AB = CD

⇒ AB + BC = CD + BC

⇒ AC = BD ... (1)

Comparing ΔAXC and ΔBYD:

∠XAC = ∠BDY (Alternate interior angles and AX || DY)

AC = BD {From equation (1)}

∠ACX = DBY (Alternate interior angles and CX || BY)

∴ΔAXC ≅ ΔDYB [ASA congruence rule]

The corresponding parts of congruent triangles are equal.

∴∠AXC = ∠BYD

Q10) What is the degree of the polynomial *p*(*x*)?

Solution:

(a) The given polynomial is $p(x)=\dfrac{x^{\frac{3}{2}}+\sqrt{2x}}{\sqrt{x}\left(\sqrt{3}-\sqrt{2}\right)}$

This polynomial can be simplified as:

$$p(x) = \frac{x^{\frac{3}{2}} + \sqrt{2}x^{\frac{1}{2}}}{x^{\frac{1}{2}}\left(\sqrt{3} - \sqrt{2}\right)}$$

$$= \frac{x^{\frac{3}{2}-\frac{1}{2}} + \sqrt{2}x^{\frac{1}{2}-\frac{1}{2}}}{\left(\sqrt{3} - \sqrt{2}\right)}$$

$$= \frac{x + \sqrt{2}x^0}{\left(\sqrt{3} - \sqrt{2}\right)}$$

$$= \frac{x}{\sqrt{3} - \sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

Thus, the degree of polynomial *p* (*x*) is 1

(b) Constant term $= \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}}$

This term can be rationalised as:

$$\frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{\sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)}{\left(\sqrt{3}\right)^2 - \left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)}{3 - 2}$$

$$= \sqrt{2}\left(\sqrt{3} + \sqrt{2}\right)$$

$$= \sqrt{6} + 2$$

On comparing $\sqrt{6} + 2$ with $a + \sqrt{b}$, it is obtained that *a* = 2 and *b* = 6.

Thus, the values of *a* and *b* are 2 and 6 respectively.