Q1) The given table shows the salary of 60 workers at a small factory. Draw the frequency polygon of this data.

Salary	Number of workers
3000 − 4000	15
4000 − 5000	25
5000 − 6000	9
6000 − 7000	11

Solution:

The histogram of the given data is drawn by taking salary along horizontal axis and number of workers along vertical axis. Then, the bars of each class interval corresponding to its frequency are drawn.

The frequency polygon can be obtained by joining the mid-points of each bar of each class interval of this histogram.

The class-mark of the class interval, 2000 − 3000, (preceding the first class interval) of zero frequency,

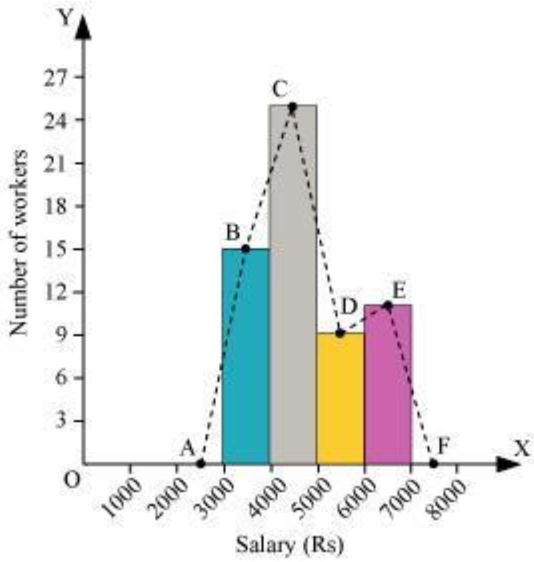
$$\frac{2000+3000}{2}=2500$$

The class-mark of the class interval, 7000 − 8000, (succeeding the last class interval) of zero frequency,

$$\frac{7000+8000}{2}=7500$$

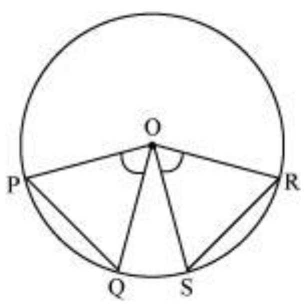
The required polygon can be obtained by joining the mid-points of each bar of the histogram by means of line segments along with the class marks of the two class intervals of zero frequency before the first class interval and after the last class interval.

Thus, the frequency polygon so formed is as follows.



Q2) Prove that equal chords of a circle subtend equal angles at the centre.

Solution:



Let PQ and RS be two equal chords of a circle.

We have to prove that ∠POQ = ∠ROS

In ΔPOQ and ΔROS,

PO = OR (Radius of circle)

PQ = RS (Given)

OQ = OS (Radius of circle)

∴ ΔPOQ ≅ROS (By SSS congruency criterion)

We know that corresponding parts of congruent triangles are equal.

∴ ∠POQ = ∠ROS

Thus, equal chords of a circle subtend equal angles at the centre.

Q3) The marks out of 100 obtained by 8 students in a class are as follows:

74, 46, 57, 33, 10, 29, 91, 92

If marks of a student are chosen at random, then find the probability that they are more than the mean marks.

Solution:

Mean marks of 8 students $= \frac{74+46+57+33+10+29+91+92}{8}$

$$= \frac{432}{8}$$
$$= 54$$

The marks more than mean marks (i.e., 54) are 74, 57, 91, and 92.

These are four observations.

∴ Probability that if the marks of a student are chosen at random, then they are more than the mean marks

$$= \frac{\text{Number of observations, which are greater than mean marks}}{\text{Total number of observations}}$$

$$= \frac{4}{8}$$
$$= \frac{1}{2}$$

Q4) After 10 years, Roshni's mother's age will be 2 times the age of Roshni. If her mother is 24 years old now, then determine the present age of Roshni graphically.

Solution:

Let the present ages of Roshni and her mother be *x* and *y* respectively.

After 10 years:

Roshni's age = (x + 10)

Her mother's age = (y + 10)

According to the given situation,

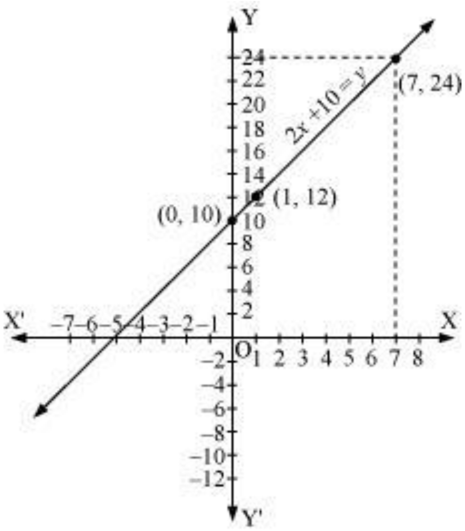
$2\ (x + 10) = y + 10$

$2x + 10 = y$ … (1)

The two solutions of equation (1) are as follows.

x	0	1
y	10	12

Plotting the points, (0, 10) and (1, 12), we obtain the following graph.



It can be seen from the graph that the value of x, corresponding to $y = 24$, is 7.

Thus, the present age of Roshni is 7 years.

Q5) The mean of *n*observations is 35. If 5 is subtracted from thrice of each observation, then find the mean of new observations.

Solution:

Let the*n*observations be $x_1, x_2, x_3, \ldots$ and x_n.

It is given that mean of these observations is 35.

$$\therefore \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = 35$$
$$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_n = 35n$$

When 5 is subtracted from thrice of each observation, the new observations will be

$3x_1 - 5, 3x_2 - 5, 3x_3 - 5, \ldots 3x_n - 5$

Now, mean of these new observations

$$= \frac{(3x_1 - 5) + (3x_2 - 5) + (3x_3 - 5) + \ldots + (3x_n - 5)}{n}$$
$$= \frac{3(x_1 + x_2 + x_3 + \ldots + x_n) - (5 + 5 + 5 + \ldots n \text{ times})}{n}$$
$$= \frac{3 \times 35n - 5 \times n}{n}$$
$$= \frac{100n}{n}$$
$$= 100$$

Q6) What is the sample space when three coins are tossed together?

Solution:

When three coins are tossed together, we can get eight possible outcomes. These are as follows:

1.Heads on all the coins.
2.Heads on the first two coins and tail on the last coin.
3. Head on the first and last coins and tail on the second coin.
4. Heads on the last two coins and tail on the first coin.
5. Head on the last coin and tails on the first two coins.
6.Head on the second coin and tails on the first and last coins.
7. Head on the first coin and tails on the last two coins.
8. Tails on all the coins.

Thus, the sample space when three coins are tossed together is {HHH, HHT, HTH, THH, TTH, THT, HTT, TTT}.

Q7) Check which of the following, (1, −1) or (2, 4), is the solution of the equation, 2 + 3*y* = 7*x*?

Solution:

The given linear equation is 2 + 3*y* = 7*x*

For (1, −1),

L.H.S. = 2 + 3 (−1) = 2 − 3 = −1

R.H.S. = 7*x* = 7 (1) = 7

L.H.S. ≠ R.H.S.

∴ (1, −1) is not a solution of the given equation.

For (2, 4),

L.H.S. = 2 + 3 (4) = 14

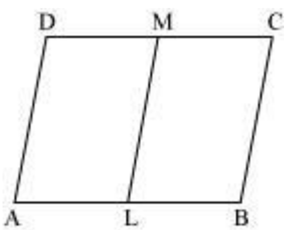
R.H.S. = 7 (2) = 14

L.H.S. = R.H.S.

∴ (2, 4) is a solution of the given equation.

Q8) Show that the line joining the mid points of two opposite sides of a parallelogram is parallel to the other pair of sides.

Solution:



Let ABCD be a parallelogram.

Let L and M be the mid points of sides AD and CD respectively.

∴ AL = BL = $\frac{AB}{2}$

DM = MC = $\frac{CD}{2}$

It is known that, the opposite sides of a parallelogram are equal and parallel.

∴ AB = CD

$$\Rightarrow \frac{AB}{2} = \frac{CD}{2}$$
$$\Rightarrow AL = MD$$

If a pair of opposite sides of a quadrilateral is equal and parallel, then the quadrilateral is a parallelogram.

In quadrilateral ALMD,

AL = DM and AL || DM.

∴ ALMD is a parallelogram.

⇒ AD || LM

It is known that, the lines parallel to the same line are parallel to each other.

∴ AD || LM || BC

Thus, the line joining the mid points of two opposite sides of a parallelogram is parallel to the other pair of opposite sides.

Q9) A bicycle starts with a speed of 4 m/s and travels with an acceleration of

$\frac{1}{2}\mathrm{m/s^2}$. How can the velocity at different times be represented by a graph?

Solution:

The final velocity of bicycle can be represented by the following equation:

$v = u + at$

Where, v = Final velocity

u = Initial velocity = 4 m/s

a = Acceleration $= \frac{1}{2}\mathrm{m/s^2}$

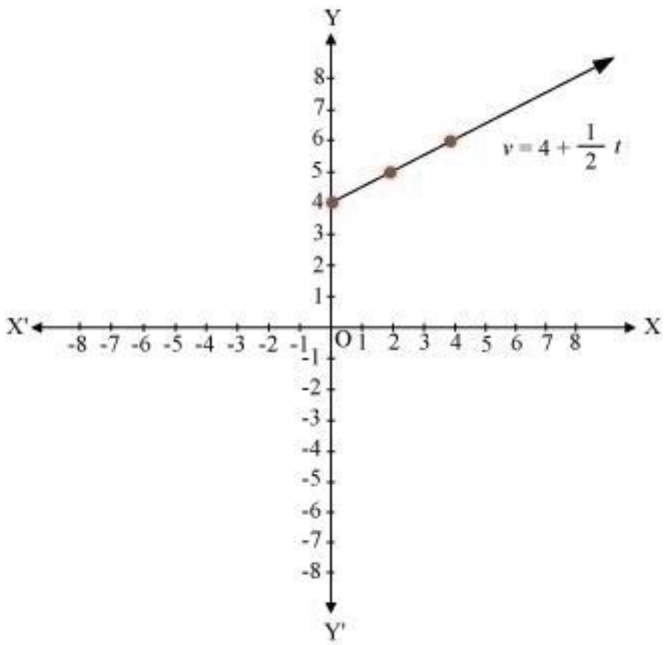
t = Time

$$v = 4 + \frac{1}{2}t$$

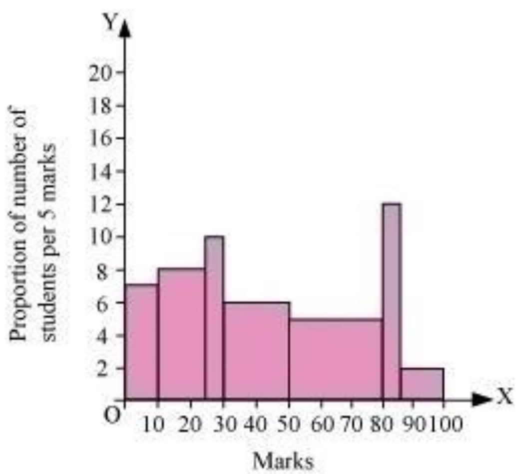
We can find solution to this equation as follows:

t	0	2	4
$v = u + \frac{1}{2}t$	4	5	6

Thus, the graph of bicycle's velocity at different times can be shown as:



Q9) Read the following histogram drawn from the data of the whole students of a class and answer the following questions.



(a) What information does this histogram depict? (1 mark)

(b) Convert the above histogram into frequency distribution table. (2 marks)

(c) How many students are there in the class? (1 mark)

(d) Find the class mark of each class interval. (1 mark)

(e) In which class interval is there the maximum number of students and in which class interval is there minimum number of students? (1 mark)

Solution:

(a) The given histogram depicts information regarding the number of students with respect to the marks secured out of 100 in a test or tests.

(b) We know that length of the rectangle of a class interval

$$= \frac{\text{Frequency of the class interval}}{\text{Width of the class}} \times \text{Least width}$$

⇒ Frequency of the class interval $= \frac{\text{Length of rectangle of the class interval} \times \text{Width of the class}}{\text{Least width}}$

It can beobserved from the given histogram that the least width of the class interval is 5.

Now, the frequency table of the given histogram can be drawn as follows:

Marks	Width of the class	Length of rectangle	Frequency of the class
0 − 10	10	7	$\frac{7\times10}{5}=14$
10 − 25	15	8	$\frac{8\times15}{5}=24$
25 − 30	5	10	$\frac{5\times10}{5}=10$
30 − 50	20	6	$\frac{6\times20}{5}=24$
50 − 80	30	5	$\frac{5\times30}{5}=30$
80 − 85	5	12	$\frac{12\times5}{5}=12$
85 − 100	15	2	$\frac{2\times15}{5}=6$

(c) Number of students in the class = 14 + 24 + 10 + 24 + 30 + 12 + 6 = 120

(d) Class mark of a class interval $= \frac{\text{Lower limit} + \text{Upper limit}}{2}$

Therefore, class mark of the interval 0 − 10 $= \frac{0+10}{2} = 5$

Class mark of the interval 10 − 25 $= \frac{10+25}{2} = 17.5$

Class mark of the interval 25 − 30 $= \frac{25+30}{2} = 27.5$

Class mark of the interval 30 − 50 $= \frac{30+50}{2} = 40$

Class mark of the interval 50 − 80 $= \frac{50+80}{2} = 65$

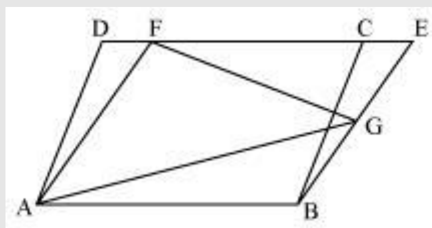
Class mark of the interval 80 − 85 $= \frac{80+85}{2} = 82.5$

Class mark of the interval 85 − 100 $= \frac{85+100}{2} = 92.5$

(e) From the frequency distribution table of the given histogram, we can observe that the class interval 50 − 80 has the maximum number of students (i.e., 30) and the class interval 85 − 100 has the minimum number of students (i.e., 6).

Q10) *Use the following information to answer the next question.*

In the given figure, ABCD and ABEF are parallelograms.



Prove that

(i) area (ABCD) = area (ABEF)

(ii) $\text{area}(AFG) = \frac{1}{2}\text{area}(ABCD)$

Solution:

(i)

We know that parallelograms on the same base and between the same parallels are equal in area.

Parallelograms ABCD and ABEF are on the same base AB and between the same parallels.

∴ area (ABCD) = area (ABEF) … (1)

(ii)

We know that if a triangle and a parallelogram are on the same base and between the same parallels, then the area of triangle is half the area of parallelogram.

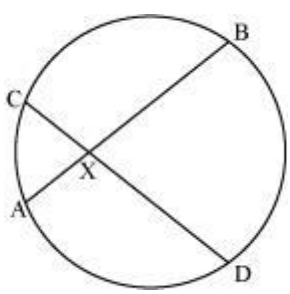
ΔAFG and parallelogram ABEF are on the same base AF and between the same parallels AF and BE.

$$\therefore \text{area}(\Delta AFG) = \frac{1}{2}\text{area}(ABEF)$$

Using result (1), we obtain

$$\therefore \text{area}(\Delta AFG) = \frac{1}{2}\text{area}(ABCD)$$

Q11)



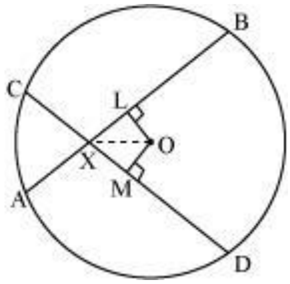
Two equal chords AB and CD of a circle intersect each other at X as shown in the given figure. Show that AX = CX

Solution:

Let O be the centre of the circle.

Draw OL⊥AB and OM⊥CD.

Join OX.



It is known that equal chords are equidistant from the centre.

∴OL = OM … (1)

Comparing ΔOLX and ΔOMX,

∠OLX = ∠OMX (each is a right angle)

OX = OX (common)

OL = OM {from equation (1)}

∴ΔOLX ≅ ΔOMX [RHS congruency criterion]

It is known that corresponding parts of congruent triangles are equal.

∴LX = MX … (2)

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

∴AL = $\frac{AB}{2}$, and CM = $\frac{CD}{2}$.

But it is given that AB = CD.

∴AL = CM … (3)

Subtracting equation (2) from equation (3),

AL − LX = CM − MX

⇒ AX = CX

Hence, the result is proved.

Q12) What is the ratio of the radius to the height of the cylinder?

Solution:

(a) Let the base radius and the height of the cylinder be *r* and *h* respectively.

$$\frac{\text{Total surface area}}{\text{Curved surface area}} = \frac{3}{1}$$

$$\Rightarrow \frac{2\pi rh + \pi r^2}{2\pi rh} = \frac{3}{1}$$

$$\Rightarrow \frac{2h + r}{2h} = 3$$

$$\Rightarrow 1 + \frac{r}{2h} = 3$$

$$\Rightarrow \frac{r}{2h} = 2$$

$\Rightarrow r = 4h$

$\Rightarrow r:h = 4:1$

Thus, the required ratio is 4:1.

(b) If r = 28 cm, then $h = \frac{1}{4} \times 28 \text{ cm}$ = 7 cm

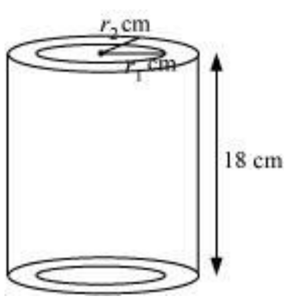
∴Volume of cylinder = $\pi r^2 h$ $= \frac{22}{7} \times 28 \times 28 \times 7 \text{ cm}^3 = 17248 \text{ cm}^3$

Thus, the volume of the cylinder is 17248 cm^3.

Q13) The volume and total surface area of a hollow metallic cylinder, whose height is 18 cm, are 3564 cm^3 and 2772 cm^2. Find the thickness of the metallic cylinder.

Solution:

Let r_2 and r_1 be the external and internal radii of the cylinder. Its height, h= 18 cm



Now, volume of the cylinder $= \left[\pi\left(r_2^2 - r_1^2\right)h\right] \text{ cm}^3$

$$\Rightarrow 3564 = \pi\left(r_2^2 - r_1^2\right)h$$
$$\Rightarrow \pi\left(r_2 - r_1\right)\left(r_2 + r_1\right)h = 3564 \qquad ...(1)$$

It is given that total surface area of the metallic cylinder is 2772 cm^2.

$$\therefore 2\pi r_1 h + 2\pi r_2 h + 2\pi\left(r_2^2 - r_1^2\right) = 2772$$
$$\Rightarrow 2\pi\left(r_1 + r_2\right)h + 2\pi\left(r_2 + r_1\right)\left(r_2 - r_1\right) = 2772$$
$$\Rightarrow 2\pi\left(r_1 + r_2\right)\left(h + r_2 - r_1\right) = 2772 \qquad ...(2)$$

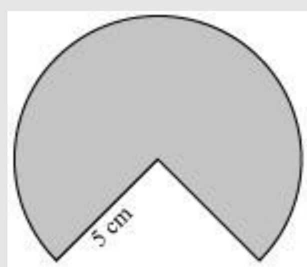
Dividing equation (1) by (2), we obtain

$$\frac{\pi\left(r_2 - r_1\right)\left(r_2 + r_1\right)h}{2\pi\left(r_1 + r_2\right)\left(h + r_2 - r_1\right)} = \frac{3564}{2772}$$
$$\Rightarrow \frac{\left(r_2 - r_1\right)\times 18}{2\left(18 + r_2 - r_1\right)} = \frac{9}{7}$$
$$\Rightarrow 7\left(r_2 - r_1\right) = 18 + \left(r_2 - r_1\right)$$
$$\Rightarrow 6\left(r_2 - r_1\right) = 18$$
$$\Rightarrow r_2 - r_1 = \frac{18}{6} = 3$$

Therefore, the thickness of the metallic cylinder is 3 cm.

Q14) *Use the following information to answer the next question.*

A cone of height 3 cm is formed by folding the given figure.



What is the base radius of the cone so formed?

Solution:

It can be seen that when the given figure is folded, its radius becomes the slant height of the cone.

∴ Slant height, *l* = 5 cm

Height, h = 3 cm

We know that,

$$l^2 = h^2 + r^2$$
$$r^2 = l^2 - h^2$$
$$r^2 = (5\text{ cm})^2 - (3\text{ cm})^2$$
$$r^2 = (25 - 9)\text{ cm}^2$$
$$r^2 = 16\text{ cm}^2$$
$$r = 4\text{ cm}$$

Therefore, the base radius of the cone so formed is 4 cm.

Q15) If a solid sphere is cut into two hemispheres, then by what percent is the surface area increased?

Solution:

Let r be the radius of the sphere.

Surface area of sphere, $S_1 = 4\pi r^2$ … (1)

When the sphere is cut into two hemispheres,

Radius of each hemisphere = r

Surface area of each hemisphere = $3\pi r^2$

Surface area of both hemispheres, $S_2 = 2 \times 3\pi r^2 = 6\pi r^2$ … (2)

Increase in surface area = $S_2 - S_1 = 6\pi r^2 - 4\pi r^2 = 2\pi r^2$

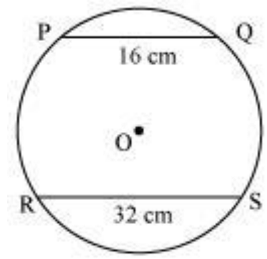
Percentage increase $= \frac{2\pi r^2}{4\pi r^2} \times 100 = \frac{200}{4}$ % = 50%

Thus, if a sphere is cut into two hemispheres, then the surface area is increased by 50%.

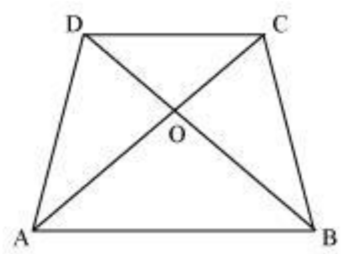
Q16) Prove that the sum of the lengths of diagonals of a convex quadrilateral is less than its perimeter.

OR

In the given figure, PQ and RS are two parallel chords of length 16 cm and 32 cm respectively. The distance between PQ and RS is 23 cm. Find the radius of the circle.



Solution:



Let ABCD be a convex quadrilateral. Let the diagonals, AC and BD, intersect each other at point O.

We know that the sum of the lengths of two sides of a triangle is greater than the third side.

In ΔABC,

AC < AB + BC … (1)

In ΔBCD,

BD < BC + CD … (2)

In ΔACD,

AC < AD + CD … (3)

In ΔBAD,

BD < AD + AB … (4)

Adding equations (1), (2), (3), and (4), we obtain

2(AC + BD) < 2 (AB + BC + CD + AD)

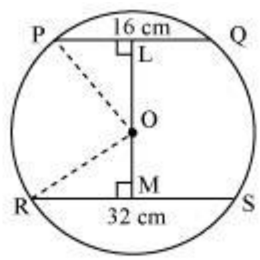
(AC + BD) < (AB + BC + CD + AD)

∴ Sum of the lengths of diagonals < Perimeter

Thus, the sum of the lengths of diagonals of a convex quadrilateral is less than its perimeter.

OR

PQ and RS are two parallel chords of lengths 16 cm and 32 cm respectively.



Let OL and OM be perpendiculars to the chords PQ and RS.

Joining OR and OP,

LM is the distance between chords PQ and RS.

∴ LM = 23 cm

Let OM = *x*

∴OL = (23 − *x*)

Let *r* be the radius of the circle.

We know that a perpendicular from the centre bisects the chord.

∴ $\text{RM} = \frac{\text{RS}}{2} = \frac{32}{2} = 16 \text{ cm}$

$$\text{PL} = \frac{\text{PQ}}{2} = \frac{16}{2} = 8 \text{ cm}$$

Using Pythagoras theorem in ΔOMR, we obtain

$$(\text{OR})^2 = (\text{OM})^2 + (\text{MR})^2$$
$$r^2 = x^2 + (16 \text{ cm})^2$$
$$r^2 = x^2 + 256 \text{ cm}^2 \qquad ...(1)$$

Again using Pythagoras theorem in ΔOLP, we obtain

$$(\mathrm{OP})^2 = (\mathrm{OL})^2 + (\mathrm{PL})^2$$
$$r^2 = (23 \text{ cm} - x)^2 + (8 \text{ cm})^2$$
$$r^2 = 529 \text{ cm}^2 + x^2 - 46x \text{ cm} + 64 \text{ cm}^2$$
$$r^2 = 593 \text{ cm}^2 + x^2 - 46x \text{ cm} \qquad \ldots(2)$$

From (1) and (2), we obtain

$x^2 + 256 \text{ cm}^2 = 593 \text{ cm}^2 + x^2 - 46x \text{ cm}$

$46x \text{ cm} = (593 - 256) \text{ cm}^2$

$46x \text{ cm} = 337 \text{ cm}^2$

$$x = \frac{337}{46} \text{ cm}$$

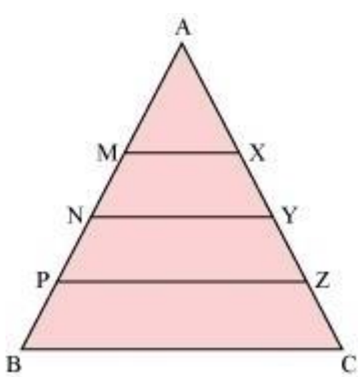
$x = 7.32$ cm

Putting $x = 7.32$ cm in equation (1), we obtain

$$r^2 = x^2 + 256 \text{ cm}^2$$
$$r^2 = (7.32 \text{ cm})^2 + 256 \text{ cm}^2$$
$$r^2 = 53.5824 \text{ cm}^2 + 256 \text{ cm}^2$$
$$r^2 = 309.5824 \text{ cm}^2$$
$$r = 17.59 \text{ cm} \approx 17.6 \text{ cm}$$

Thus, the radius of the circle is 17.56 cm.

Q17) The points M, N, and P divide the side AB of ΔABC into four equal parts. The lines MX, NY, and PZ are drawn parallel to side BC.



Prove that: $AX = \frac{1}{2}(YZ + ZC)$

Solution:

In ΔANY,

M is the mid-point of side AN and MX||NY

∴ By converse of mid-point theorem,

X is the mid-point of side AY.

∴ AX = XY … (1)

In ΔABC,

N is the mid-point of AB and NY||BC

∴ By converse of mid-point theorem,

Y is the mid-point of side AC.

∴ AY = YC

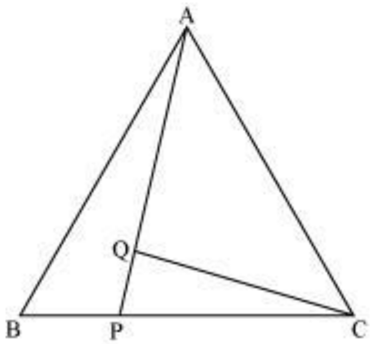
⇒ AX + XY = YZ + ZC

⇒ 2AX = YZ + ZC [From (1)]

$$AX = \frac{1}{2}(YZ + ZC)$$

Hence, proved

Q18)



ΔABC is shown in the given figure. P is a point on the side BC such that PC = 2 BP. Q is a point on AP such that QA = 5 PQ. Find the ratio of area of ΔAQC to that of area of ΔABC.

Solution:

It is given that P is a point on BC such that PC = 2BP

⇒ P divides BC in the ratio 1:2.

∴PC = $\frac{2}{3}$ BC … (1)

Similarly, it is given that Q is a point on AP such that QA = 5 PQ

∴ QA = $\frac{5}{6}$ AP … (2)

Now, consider ΔAPC and ΔABC.

The height of both these triangles is same.

Also, area of ΔAPC = $\frac{1}{2}$ × PC × Height

Area of ΔABC = $\frac{1}{2}$ × BC × Height

∴ Area of ΔAPC = $\frac{1}{2}\times\left(\frac{2}{3}BC\right)$ × Height [Using (1)]

$$= \frac{2}{3}\left[\frac{1}{2} \times BC \times \text{Height}\right]$$
$$= \frac{2}{3} \times \text{area of} \Delta ABC$$

Thus, ar (ΔAPC) = $\frac{2}{3}$ × ar (ΔABC) … (3)

Now, consider ΔAQC and ΔAPC.

The height of both these triangles is same.

Also, ar (ΔAQC) = $\frac{1}{2}$ × AQ × Height

ar(ΔAPC) = $\frac{1}{2}$ × AP × Height

∴ar (ΔAQC) = $\frac{1}{2} \times \left(\frac{5}{6} AP\right) \times \text{Height} \quad [\text{Using (2)}]$

$$= \frac{5}{6}\left[\frac{1}{2} \times AP \times \text{Height}\right]$$
$$= \frac{5}{6} \times ar(\Delta APC)$$

Thus, area (ΔAQC) = $\frac{5}{6}$ × ar (ΔAPC) … (4)

$$\therefore \frac{ar(\Delta AQC)}{ar(\Delta ABC)} = \frac{\frac{5}{6} \times ar(\Delta APC)}{\frac{3}{2} \times ar(\Delta APC)}$$
$$= \frac{5}{6} \times \frac{2}{3}$$
$$= \frac{5}{9}$$

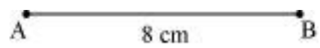
Thus, the ratio of area of ΔAQC to the area of ΔABC is 5:9.

Q19) Construct a perpendicular bisector of the line segment AB = 8 cm. Also, verify the validity of construction.

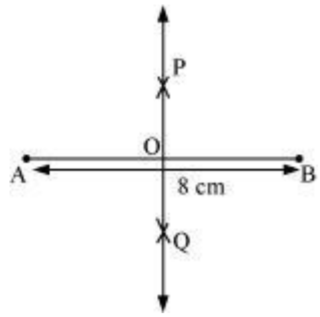
Solution:

The steps of construction are as follows:

(i) Draw a line segment AB = 8 cm using a ruler.



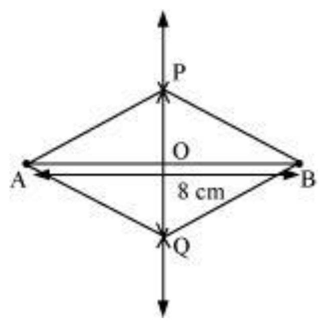
(ii) Draw two arcs taking A and B as centres and radius more than 4 cm on both sides of AB. Let the arcs intersect at points P and Q. Join PQ as:



We thus obtain PQ as the perpendicular bisector of line segment AB.

Validity of construction:

Firstly, join PA, PB, AQ, and BQ.



In ΔAPQ and ΔBPQ,

AP = BP

AQ = BQ

PQ = PQ

∴ΔAPQ ≅ ΔBPQ

∴By CPCT,

$\angle APQ = \angle BPQ$

Or, $\angle APO = \angle BPO$

In ΔAPO and ΔBPO,

$AP = BP$

$\angle APO = \angle BPO$

$PO = PO$

$\Delta APO \cong \Delta BPO$

$\therefore$By CPCT,

$AO = OB$

$\angle POA = \angle POB$

Also, $\angle POA + \angle POB = 180°$ (Linear pair)

$\therefore \angle POA = \angle POB = 90°$

Thus, PQ is the perpendicular bisector of the line segment AB.

Hence, the construction is verified.