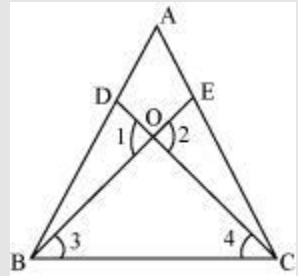
Q1) *Use the following information to answer the next question.*

The given figure shows an isosceles triangle ABC, where AB = AC. D and E are points on the side AB and AC such that $\angle 1 = \frac{5}{2}\angle 3 \text{ and } \angle 2 = \frac{5}{2}\angle 4$.	

If AE = 3 cm, OD = 2 cm, BD = 7 cm, and OC = 6 cm, then find the sum of the perimeters of ΔABE and ΔACD?

Solution:

From the given figure, it is observed that ∠1 = ∠2 [Vertically opposite angles]

$$\therefore \frac{5}{2}\angle 3 = \frac{5}{2}\angle 4 \quad \left[\angle 1 = \frac{5}{2}\angle 3 \text{ and } \angle 2 = \frac{5}{2}\angle 4\right]$$

⇒ ∠3 = ∠4 … (1)

∴In ΔOBC,

∠3 = ∠4

⇒ OB = OC = 6 cm [Sides opposite to equal angles of a triangle are equal]

It is given that ΔABC is isosceles with AB = AC.

∴ ∠ABC = ∠ACB … (2)

Subtracting (1) from (2), we obtain

∠ABC − ∠3 = ∠ACB − ∠4

⇒ ∠ABE = ∠ACD

Comparing ΔABE and ΔACD,

AB = AC [Given]

∠ABE = ∠ACD [Shown above]

∠A = ∠A [Common]

∴ ΔABE ≅ ΔACD [By ASA congruency criterion]

⇒ BE = CD [CPCT]

⇒ BE = CD = OC + OD = 6 cm + 2 cm = 8 cm

AE = AD = 3 cm [CPCT]

Then,

AC = AB = AD + BD = 3 cm + 7 cm = 10 cm

Thus,

Perimeter of ΔABE + Perimeter of ΔACD

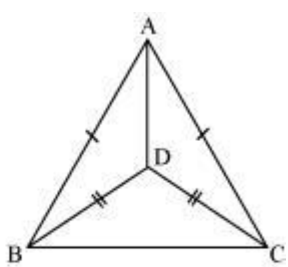
= 2 × Perimeter of ΔABE

= 2 × (AB + BE + AE)

= 2 × (10 + 8 + 3) cm

= 42 cm

Q2)



In the given figure, ΔABC and ΔDBC are isosceles, where AB = AC and DB = DC. Show that AD is the angle bisector of ∠BAC.

Solution:

Comparing ΔADB and ΔADC,

AB = AC (given)

BD = CD (given)

AD = AD (common)

$\therefore \Delta ADB \cong \Delta ADC$ [SSS congruence rule]

$\Rightarrow \angle BAD = \angle CAD$ (corresponding parts of congruent triangles are equal)

Thus, AD is the angle bisector of $\angle BAC$.

Q3)

What is the decimal expansion of $\frac{30}{14}$?

Solution:

The decimal expansion of $\frac{30}{14}$ can be obtained as:

$$\frac{30}{14} = \frac{30 \div 2}{14 \div 2} = \frac{15}{7}$$

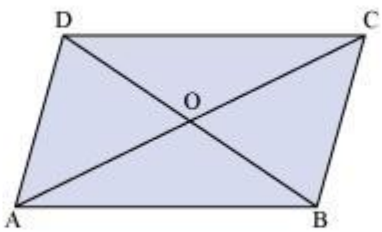
$$\begin{array}{r|l} & 2.142857 \\ \hline 7 & 15 \\ & \underline{14} \\ & 10 \\ & \underline{7} \\ & 30 \\ & \underline{28} \\ & 20 \\ & \underline{14} \\ & 60 \\ & \underline{56} \\ & 40 \\ & \underline{35} \\ & 50 \\ & \underline{49} \\ & 1 \end{array}$$

$$\therefore \frac{30}{14} = 2.\overline{142857}$$

Q4) If the diagonals of a parallelogram are of lengths 26 cmand 20 cm and one of its sides is 13 cm, then find the area of the parallelogram.

Solution:

Let ABCD be the parallelogram in which diagonal AC = 26 cm, diagonal BD = 20 cm, and BC = 13 cm. Let AC and BD intersect at O.



We know that diagonals of a parallelogram bisect each other.

Therefore,

$$OC = \frac{AC}{2} = \frac{26\,\text{cm}}{2} = 13\,\text{cm}$$
$$OB = \frac{BD}{2} = \frac{20\,\text{cm}}{2} = 10\,\text{cm}$$

In ΔOBC, OB = 10 cm (let *a*), OC = 13 cm (let *b*), and BC = 13 cm (let *c*)

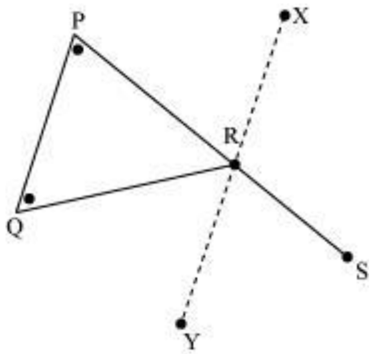
Now,

$$s = \frac{a+b+c}{2} = \frac{10\,\text{cm} + 13\,\text{cm} + 13\,\text{cm}}{2} = 18\,\text{cm}$$
$$\therefore \text{Area of } \Delta OBC = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18 \times (18-10) \times (18-13) \times (18-13)}\ \text{cm}^2$$
$$= \sqrt{18 \times 8 \times 5 \times 5}\ \text{cm}^2$$
$$= 60\ \text{cm}^2$$

We know that the diagonals of a parallelogram divide it into four triangles of equal areas.

∴ Area of parallelogram ABCD = 4 ×Area of ΔOBC = 4 ×60 cm^2= 240 cm^2

Q5)



ΔPQR is an isosceles triangle with PR = PQ. An exterior angle is drawn by extending the side PR of ΔPQR.

A line XY is drawn such that it passes through R and bisects the exterior angle QRS.

Prove that XY || PQ.

Solution:

It is given that ΔPQR is an isosceles triangle with PR = RQ.

We know that angles opposite to equal sides are equal.

$\therefore \angle PQR = \angle RPQ$ … (1)

Also, it is given that XY bisects $\angle QRS$.

$\therefore \angle QRY = \angle YRS$ … (2)

We know that the measure of an exterior angle of a triangle is equal to the sum of the measures of its two opposite interior angles.

$\therefore \angle QRS = \angle RPQ + \angle PQR$

$\Rightarrow \angle QRS = 2\angle PQR$ [Using (1)]

$\Rightarrow \angle QRY + \angle YRS = 2\angle PQR$ ($\because \angle QRS = \angle QRY + \angle YRS$)

$\Rightarrow 2\angle QRY = 2\angle PQR$

$\Rightarrow \angle QRY = \angle PQR$

Thus, the alternate interior angles made by the lines XY and PQ, when cut by the transversal RQ, are equal.

Therefore, XY||PQ

Hence, proved

Q6) Evaluate the following products using algebraic identities.

(a) 993^3 $\left(1\frac{1}{2}\text{ marks}\right)$

(b) 1002^3 $\left(1\frac{1}{2}\text{ marks}\right)$

Solution:

(a) $993^3 = (1000 - 7)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$\therefore 993^3 = (1000)^3 - (7)^3 - 3(1000)(7)(1000 - 7)$

$= 1000000000 - 343 - 21000(993)$

$= 1000000000 - 343 - 20853000$

$= 1000000000 - 20853343$

$= 979146657$

(b) $(1002)^3 = (1000 + 2)^3$

We know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

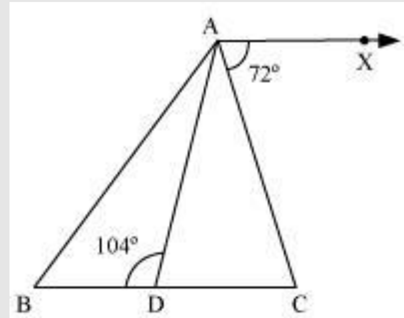
$\therefore 1002^3 = (1000)^3 + (2)^3 + 3(1000)(2)(1000 + 2)$

$= 1000000000 + 8 + 6000(1002)$

$= 1000000008 + 6012000$

$= 1006012008$

Q7 *Use the following information to answer the next question.*

In the given figure, AD is the bisector of ∠BAC and AX is parallel to BC.	

What is the measure of ∠ABC?

Solution:

In the given figure, AX is parallel to BC.

∴ ∠XAC = ∠ACD (Alternate interior angles)

⇒ ∠ACD = 72°

Similarly, ∠XAD = ∠ADB

⇒ ∠XAD = 104°

⇒ ∠XAC + ∠CAD = 104°

⇒ ∠CAD = 104° − 72° = 32°

∴ ∠BAC = 2 × 32° = 64°

⇒ ∠ABC = 180° − 72° − 64° = 44°

Thus, the measure of ∠ABC is 44°.

Q8)

Check whether $x = \frac{-1}{3}$ is the zero of the polynomial $p(x) = 3x + 1$.

Solution:

The point, $x = \frac{-1}{3}$, will be the zero of the polynomial $p(x) = 3x + 1$, if $p\left(\frac{-1}{3}\right) = 0$

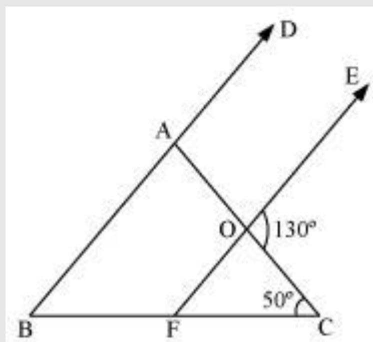
$p(x) = 3x + 1$

For $x = \frac{-1}{3}$,

$$p\left(\frac{-1}{3}\right) = 3\left(\frac{-1}{3}\right) + 1 = 0$$

Thus, $x = \frac{-1}{3}$ is a zero of polynomial $p(x)$.

Q9) *Use the following information to answer the next question.*

In the given figure, BD||EF.	

Determine the measure of ∠ABC.

Solution:

In ΔOFC, ∠EOC = ∠OCF + ∠OFC

∴ ∠OFC = 130° − 50° = 80°

BC is parallel to EF.

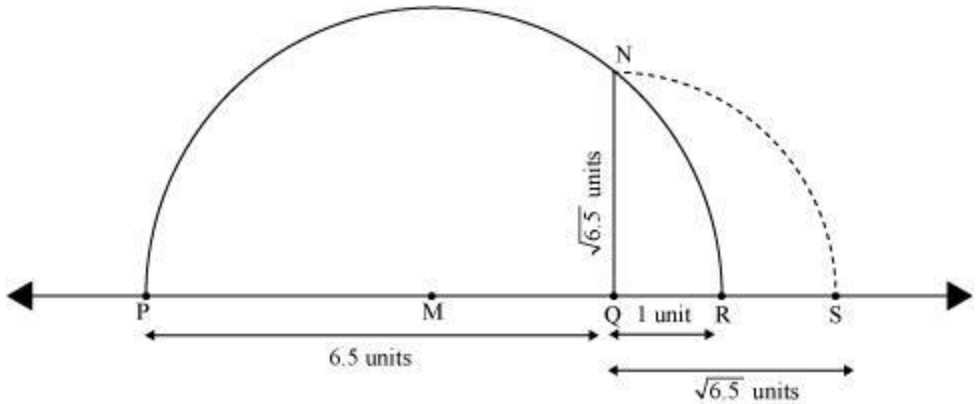
∴ ∠ABC = ∠OFC (Corresponding angles)

⇒ ∠ABC = 80°

Thus, the measure of ∠ABC is 80°.

Q10) Represent $\sqrt{6.5}$ on the number line.

Solution:



Draw a line segment PQ = 6.5 units and extend it to R such that QR = 1 unit.

M is the midpoint of PR.

With M as the centre and MR as the radius, draw a semicircle.

Draw NQ ⊥PR, intersecting the semicircle at N.

Then, QN = $\sqrt{6.5}$ units

Now, with Q as the centre and QN as the radius, draw an arc, meeting PR at S (when PR is extended).

Thus, QS = QN = $\sqrt{6.5}$ units

Q11) The polynomials $P(t) = 4t^3 - st^2 + 7$ and $Q(t) = t^2 + st + 8$ leave the same remainder when divided by $(t - 1)$. Find the value of s.

Solution:

When the polynomials $P(t)$ and $Q(t)$ are divided by $(t - 1)$, then the remainders are given by $P(1)$ and $Q(1)$ respectively (from the remainder theorem).

$\therefore P(1) = Q(1)$

$4(1)^3 - s(1)^2 + 7 = (1)^2 + s(1) + 8$

$4 - s + 7 = 1 + s + 8$

$-s + 11 = s + 9$

$2s = 2$

$\therefore s = 1$

Thus, the value of s is 1.

Q12) Find the value of *k* for which the cubic polynomial $3y^3-\frac{3}{2}y^2+ky+5$ is exactly divisible by $\left(y-\frac{1}{2}\right)$.

Solution:

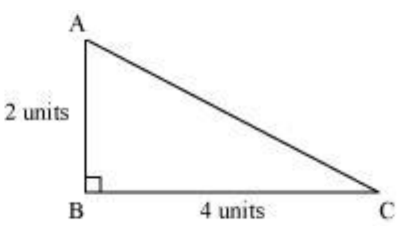
According to factor theorem, since $f(y)=3y^3-\frac{3}{2}y^2+ky+5$ is exactly divisible by $\left(y-\frac{1}{2}\right)$, $f\left(\frac{1}{2}\right)$ = 0.

$$\therefore 3\left(\frac{1}{2}\right)^3-\frac{3}{2}\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)+5=0$$
$$\Rightarrow 3\times\frac{1}{8}-\frac{3}{2}\times\frac{1}{4}+\frac{k}{2}+5=0$$
$$\Rightarrow \frac{3}{8}-\frac{3}{8}+\frac{k}{2}+5=0$$
$$\Rightarrow k=-10$$

Thus, the required value of *k* is −10.

Q13) If the sides containing the right angle of a right triangle are 2 units and 4 units long, then is the length of the hypotenuse a rational or an irrational number?

Solution:



On applying Pythagoras theorem, we obtain

$AC^2 = AB^2 + BC^2$

⇒ $AC^2 = (2 \text{ units})^2 + (4 \text{ units})^2$

⇒ AC^2 = 4 units + 16 units

⇒ AC^2 = 20 units

⇒ AC = $\sqrt{20}$ units

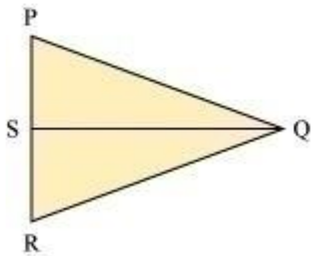
⇒ AC = $2\sqrt{5}$ units

Since 2 is a rational number and $\sqrt{5}$ is an irrational number, $2\sqrt{5}$ is an irrational number.

Thus, the length of the hypotenuse is an irrational number.

Q14)

In the given figure, an isosceles triangle PQR with PQ = QR is shown. S is a point on side PR. Prove that QR > QS.



Solution:

It is given that in ΔPQR, PQ = QR

⇒∠RPQ = ∠QRP … (1)

Now, ∠QSR is an exterior angle of ΔPQS.

∴ ∠QSR = ∠SPQ + ∠PQS

⇒∠QSR > ∠SPQ

i.e., ∠QSR > ∠RPQ

⇒ ∠QSR > ∠QRP [Using (1)]

We know that in a triangle, the side opposite to the larger angle is longer.

∴QR > QS

Hence, proved

Q15) Are real numbers closed under multiplication and division? Justify your answer with examples.

Solution:

Real numbers comprise rational numbers and irrational numbers.

Multiplication of two rational numbers results in a rational number, which is a real number.

For example: $\frac{1}{2}\times\frac{3}{5}=\frac{3}{10}$

Multiplication of two irrational numbers can result in either a rational or an irrational number, but in both the cases, it will be a real number.

For example: $\sqrt{3}\times\sqrt{3}=3$ and $\sqrt{2}\times\sqrt{3}=\sqrt{6}$

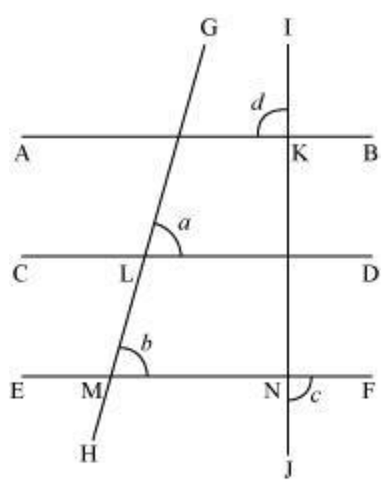
Multiplication of a rational number and an irrational number results in either a rational or an irrational number, but in both the cases it will be a real number.

For example: $0\times\sqrt{5}=0$ and $\sqrt{2}\times\sqrt{5}=\sqrt{10}$

Thus, real numbers are closed under multiplication.

Since division of any number by 0 is not defined, real numbers are not closed under division.

Q16)



In the given figure, *a* = *b* and *c* = *d*.

Show that AB ||CD.

Solution:

It is given that $a = b$ i.e., ∠GLD = ∠LMN

Here, ∠GLD are ∠LMN are the corresponding angles with respect to lines CD and EF.

If the corresponding angles formed by a transversal with two lines are equal, then the lines are parallel.

∴ CD || EF ... (1)

∠MNK = ∠FNJ (Vertically opposite angles)

⇒ ∠MNK = c

It is given that $c = d$.

∴ ∠IKA = ∠MNK

Here, ∠IKA and ∠MNK are corresponding angles with respect to lines AB and EF.

∴ AB || EF ... (2)

It is known that the lines parallel to the same line are parallel to each other.

Therefore, from (1) and (2), AB || CD.

Thus, AB and CD are parallel lines.