Q1) Use the following information to answer the next question.

The polynomial $p(x) = x^2 + Sx + T$, where S and T are constants, leaves 2 as the remainder when it is divided by x.

If (x + 2) is a factor of the polynomial p(x), then what are the respective values of S and T?

- A) -3 and -4
- B) -3 and -2
- C) 3 and 2
- D) 3 and 4

Answer:

С

Solution:

The given polynomial is $p(x) = x^2 + Sx + T$.

According to the remainder theorem, when a polynomial p(x) is divided by a linear polynomial (x - a), the remainder obtained is p(a).

It is given that p(x) leaves 2 as the remainder when it is divided by x.

∴
$$p(0) = 2$$

$$\Rightarrow (0)^2 + S(0) + T = 2$$

$$\Rightarrow T = 2$$

$$\therefore p(x) = x^2 + Sx + 2$$

According to the factor theorem, if f(x) is a polynomial of degree $n \ge 1$ and g(x) is a linear polynomial, then g(x) is a factor of f(x) if f(x) = 0 at zero of g(x).

The zero of (x + 2) is -2.

Since (x + 2) is a factor of p(x), we must have p(-2) = 0.

$$p(-2) = 0$$

$$\Rightarrow (-2)^2 + S(-2) + 2 = 0$$

$$\Rightarrow$$
 4 - 2S + 2 = 0

$$\Rightarrow$$
 6 - 2S = 0

$$\Rightarrow 2S = 6$$

$$\Rightarrow$$
 S = 3

Thus, the respective values of *S* and *T* are 3 and 2.

The correct answer is C.

Q2) If (3x - 1) is a factor of the polynomial $27x^3 + 9x^2 - bx + 3$, then what is the value of *b*?

- A) 1
- B) 5
- C) 15
- D) 17

Answer:

С

Solution:

The given polynomial is $p(x) = 27x^3 + 9x^2 - bx + 3$.

According to the factor theorem, if p(x) is a polynomial of degree $n \ge 1$ and h(x) is a linear polynomial, then h(x) is a factor of p(x) if p(x) = 0 at zero of h(x).

The zero of (3x - 1) is $\frac{1}{3}$.

 $p\left(\frac{1}{3}\right) = 0$ Since (3x - 1) is a factor of p(x), we must have

$$\therefore p\left(\frac{1}{3}\right) = 0$$

$$\Rightarrow 27\left(\frac{1}{3}\right)^3 + 9\left(\frac{1}{3}\right)^2 - b\left(\frac{1}{3}\right) + 3 = 0$$

$$\Rightarrow 27 \times \frac{1}{27} + 9 \times \frac{1}{9} - \frac{b}{3} + 3 = 0$$

$$\Rightarrow 1 + 1 - \frac{b}{3} + 3 = 0$$

$$\Rightarrow 5 - \frac{b}{3} = 0$$

$$\Rightarrow \frac{b}{3} = 5$$

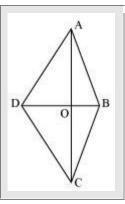
$$\Rightarrow b = 5 \times 3 = 15$$

Thus, the required value of p is 15.

The correct answer is C.

Q3) Use the following information to answer the next question.

The given figure shows a kite ABCD, where AB = BC = 26 cm, AD = CD = 28 cm, and BD = 30 cm.



What is the length of the diagonal AC?

- A) 36 cm
- B) 38.6 cm
- C) 44.8 cm
- D) 48 cm

Answer:

С

Solution:

In \triangle ABD, let a = 30 cm, b = 28 cm, and c = 26 cm

∴Semi-perimeter (s) =
$$\frac{a+b+c}{2} = \frac{30 \text{ cm} + 28 \text{ cm} + 26 \text{ cm}}{2} = 42 \text{ cm}$$

It is known that, area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the sides of the triangle and s is the semi-perimeter of the triangle.

Therefore, area of
$$\triangle ABD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-30)(42-28)(42-26)} \text{ cm}^2$$

$$= \sqrt{42\times12\times14\times16} \text{ cm}^2$$

$$= \sqrt{6\times7\times2\times6\times2\times7\times4\times4} \text{ cm}^2$$

$$= 336 \text{ cm}^2$$

Since ABCD is a kite, where AB = BC, AD = CD,

AC⊥ BD and AO = OC

 $\frac{1}{2} \times base \times height$ We also know that area of triangle = $\frac{1}{2}$

$$∴ Area of ΔABD = \frac{1}{2} × BD × AO = \frac{1}{2} × (30 cm) × AO$$

$$\therefore \frac{1}{2} \times (30 \text{ cm}) \times AO = 336 \text{ cm}^2$$

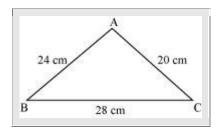
$$\Rightarrow (15 \text{ cm}) \times AO = 336 \text{ cm}^2$$

$$\Rightarrow AO = \frac{336}{15} \text{ cm} = 22.4 \text{ cm}$$

$$AC = 2AO = 2 \times 22.4 \text{ cm} = 44.8 \text{ cm}$$

The correct answer is C.

Q4) Use the following information to answer the next question.



What is the height of the given \triangle ABC corresponding to the side BC?

Assume
$$\sqrt{6} = 2.45$$

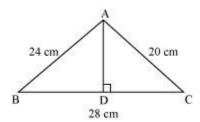
- A) 18.9 cm
- B) 17.34 cm
- C) 16.8 cm
- D) 15.75 cm

Answer:

С

Solution:

Let AD be the height of triangle ABC corresponding to the side BC.



Let a = 28 cm, b = 20 cm, and c = 24 cm

Then, semi-perimeter (s) of
$$\triangle ABC = \frac{a+b+c}{2} = \frac{28+20+24}{2}$$
 cm = 36 cm

By using Heron's formula, we obtain

Area of
$$\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{36 \times (36-28) \times (36-20) \times (36-24)} \text{ cm}^2$$

$$= \sqrt{36 \times 8 \times 16 \times 12} \text{ cm}^2$$

$$= \sqrt{12 \times 3 \times 8 \times 8 \times 2 \times 12} \text{ cm}^2$$

$$= 12 \times 8\sqrt{3 \times 2} = 96\sqrt{6} \text{ cm}^2$$

It is also known that,

Area of
$$\triangle ABC = \frac{1}{2} \times Base \times Height = \frac{1}{2} \times BC \times AD$$

$$\therefore 96\sqrt{6} = \frac{1}{2} \times 28 \times AD$$

$$\Rightarrow 96\sqrt{6} = 14 \times AD$$

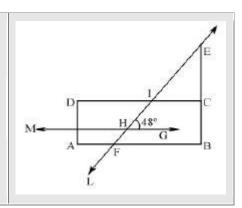
$$\Rightarrow AD = \frac{96\sqrt{6}}{14} = \frac{96}{14} \times 2.45 = 16.8 \text{ cm}$$

Thus, the height of \triangle ABC corresponding to the side BC is 16.8 cm.

The correct answer is C.

Q5) Use the following information to answer the next question.

The given figure shows rectangle ABCD, side BC of which is extended to point E. Line MG is parallel to sides AB and CD of the rectangle. Line LE intersects sides AB and CD at points F and I respectively and meets BE at point E. Also, ∠EHG = 48°.



The measure of∠BEF is

- A) 42°
- B) 48°
- C) 52°
- D) 58°

Answer:

Solution:

In the figure, line MGis parallel to side AB, and line LE acts as the transversal.

Therefore, ∠EHG and ∠BFE are corresponding angles and are thus, congruent.

$$\therefore \angle BFE = \angle EHG = 48^{\circ}$$

Since ∠FBE is an angle of the rectangle, ∠FBE = 90°

Now, in \triangle FBE, \angle FBE + \angle BFE + \angle BEF = 180°

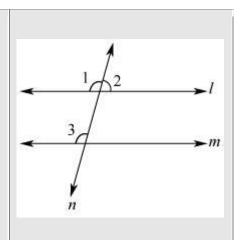
$$\angle BEF = 180^{\circ} - (90^{\circ} + 48^{\circ})$$

$$\angle BEF = 180^{\circ} - 138^{\circ}$$

The correct answer is A.

Q6) Use the following information to answer the next question.

The given figure shows a pair of parallel lines l and m, which are intersected by a transversal n.



If the measures of $\angle 1$ and $\angle 2$ are in the ratio 3:1, then the measure of $\angle 3$ is

- A) 70°
- B) 90°
- C) 105°
- D) 135°

Answer:

D

Solution:

Let the measure of $\angle 2$ be x°

Thus, measure of $\angle 1 = 3x^{\circ}$

∠1 and ∠2 form a linear pair.

$$\therefore \angle 1 + \angle 2 = 180^{\circ}$$

$$3x^{\circ} + x^{\circ} = 180^{\circ}$$

$$4x = 180^{\circ}$$

$$x = 45^{\circ}$$

Thus, measure of $\angle 1 = 3 \times 45^{\circ} = 135^{\circ}$

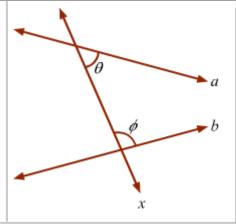
Since lines I and m are parallel and I is the transversal, $\angle 1$ and $\angle 3$ are corresponding angles and are thus, congruent.

Thus, $\angle 1 = \angle 3 = 135^{\circ}$

The correct answer is D.

Q7) Use the following information to answer the question.

In the given figure, the transversal *x* intersects the two lines *a* and *b*.



If
$$\theta = \frac{7z}{2}$$
 and $\phi = \frac{9z}{8}$, which of the following inequalities is correct?

A)

$$z < \left(29\frac{36}{37}\right)^{\circ}$$

$$z < \left(38\frac{34}{37}\right)^{\circ}$$

$$z > \left(40\frac{17}{37}\right)^{\circ}$$

$$z > \left(49\frac{11}{37}\right)^{\circ}$$

View Solution

Answer:

В

Solution:

It can be observed from the given figure that the distance between the lines a and b is decreasing when these lines are extended in the side of the marked angles (and $\sqrt{}$.

Hence, by Euclid's fifth postulate:

$$\theta + \phi < 180^{\circ}$$

$$\therefore \frac{7z}{2} + \frac{9z}{8} < 180^{\circ} \qquad \left[\text{Given, } \theta = \frac{7z}{2} \text{ and } \phi = \frac{9z}{8} \right]$$

$$\Rightarrow \frac{28z + 9z}{8} < 180^{\circ}$$

$$\Rightarrow \frac{37z}{8} < 180^{\circ}$$

$$\Rightarrow z < \left(\frac{1440}{37} \right)^{\circ}$$

$$\Rightarrow z < \left(38 \frac{34}{37} \right)^{\circ}$$

The correct answer is B.

Q8) How many lines can pass through two distinct points?

- A) 0
- B) 1
- C) 2
- D) 3

Answer:

В

Solution:

Euclid's first postulate states that a straight line can be drawn from any one point to any other point, i.e. there is a unique line that passes through two distinct points.

The correct answer is B.