Q1) Is it possible to draw a triangle with sides 4.9 cm, 10.2 cm and 16 cm?

Solution:

The sum of any two sides of a triangle is always greater than the measure of the third side of the triangle.

The given dimensions are 4.9 cm, 10.2 cm and 16 cm.

It is observed that 4.9 cm + 10.2 cm = 15.1 cm < 16 cm

Here, the sum of two sides is less than the measure of the third side.

Thus, the given dimensions cannot be the sides of a triangle.

Q2) A point lies on *y*-axis at a distance of 4 units from the origin. What are the coordinates of that point?

Solution:

We know that the *x*-coordinate of any point lying on the *y*-axis is 0.

Thus, the coordinates of the point are (0, 4).

Q3) Factorise the polynomial: $p(x) = (x^2 - 3x)^2 - 38\,(x^2 - 3x) - 80$

Solution:

$$p(x) = (x^2 - 3x)^2 - 38(x^2 - 3x) - 80$$

Let $x^2 - 3x = y$.

$$\begin{aligned}\therefore\ p(x) &= y^2 - 38y - 80\\ &= y^2 - 40y + 2y - 80\\ &= y(y-40) + 2(y-40)\\ &= (y-40)(y+2)\end{aligned}$$

Substituting the value of *y*:

$$p(x) = (x^2 - 3x - 40)(x^2 - 3x + 2)$$

$x^2 - 3x - 40$ and $x^2 - 3x + 2$ can further be factorised as:

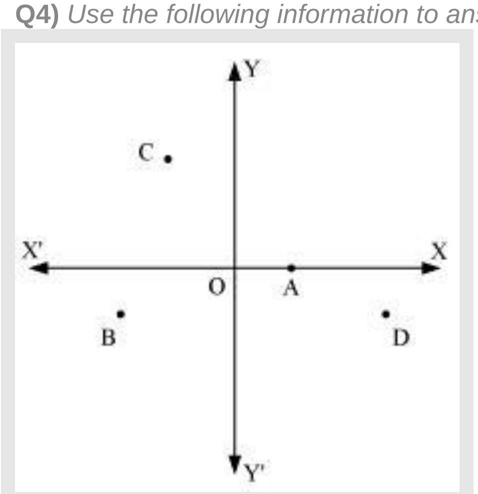
$$x^2-3x-40$$
$$=x^2-8x+5x-40$$
$$=x(x-8)+5(x-8)$$
$$=(x-8)(x+5)$$

$$x^2-3x+2$$
$$=x^2-2x-x+2$$
$$=x(x-2)-1(x-2)$$
$$=(x-2)(x-1)$$

$$\therefore p(x)=(x-1)(x-2)(x-8)(x+5)$$

Thus, $p(x)$ can be factorised as $(x-1)(x-2)(x-8)(x+5)$.

Q4) *Use the following information to answer the next question.*



With respect to the given figure, which points have a

(i) negative abscissa

(ii) positive ordinate

Solution:

(i) Points B and C have negative abscissa.

(ii) Point C has a positive ordinate.

Q5)

In which quadrant does the point (−3, 7) lie?

Solution:

We know that in the second quadrant, *x*-coordinate is negative and *y*-coordinate is positive.

Thus, the point (−3, 7) lies in the second quadrant.

Q6) Express $0.34\overline{89}$ in $\frac{p}{q}$ form.

Solution:

Let *x* = $0.34\overline{89}$ = 0.34898989…

⇒ 100*x* = 34.898989… … (1)

⇒ 10000*x* = 3489.8989… … (2)

Subtracting (1) from (2), we obtain

10000*x* − 100*x* = 3489.8989… − 34.898989…

9900*x* = 3455

$$\Rightarrow x = \frac{3455}{9900} = \frac{691}{1980}$$

Thus, $0.34\overline{89}$ can be written in $\frac{p}{q}$ form as $\frac{691}{1980}$.

Q6) The polynomial $p(x) = 2x^4 + x^3 - 14x^2 - ax - 6$ is exactly divisible by $q(x) = x^2 + 3x + 2$. By using the long division method, find the value of *a*?

Solution:

Using the long division method, $p(x)$ can be divided by $q(x)$.

This can be done as:

$$
\begin{array}{r|l}
 & 2x^2-5x-3 \\
\hline
x^2+3x+2 & 2x^4+x^3-14x^2-ax-6 \\
 & 2x^4+6x^3+4x^2 \\
 & - \quad - \quad - \\
\hline
 & -5x^3-18x^2-ax-6 \\
 & -5x^3-15x^2-10x \\
 & + \quad + \quad + \\
\hline
 & -3x^2+(10-a)x-6 \\
 & -3x^2-9x-6 \\
 & + \quad + \quad + \\
\hline
 & (10-a+9)x \\
\hline
\end{array}
$$

Since $p(x)$ is exactly divisible by $q(x)$, the remainder must be zero.

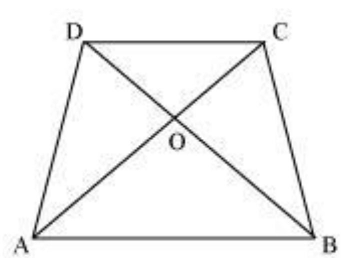
$\therefore 10 - a + 9 = 0$

$\Rightarrow a = 19$

Thus, the value of a is 19.

Q7) Prove that the sum of the lengths of diagonals of a convex quadrilateral is less than its perimeter.

Solution:



Let ABCD be a convex quadrilateral. Let the diagonals, AC and BD, intersect each other at point O.

We know that the sum of the lengths of two sides of a triangle is greater than the third side.

In ΔABC,

AC < AB + BC … (1)

In ΔBCD,

BD < BC + CD … (2)

In ΔACD,

AC < AD + CD … (3)

In ΔBAD,

BD < AD + AB … (4)

Adding equations (1), (2), (3), and (4), we obtain

2(AC + BD) < 2 (AB + BC + CD + AD)

(AC + BD) < (AB + BC + CD + AD)

∴ Sum of the lengths of diagonals < Perimeter

Thus, the sum of the lengths of diagonals of a convex quadrilateral is less than its perimeter.

Q8) What is the coefficient of x^2 in the polynomial $2\sqrt{2x^4}+5\sqrt{2x^2}-7\sqrt{2}$?

Solution:

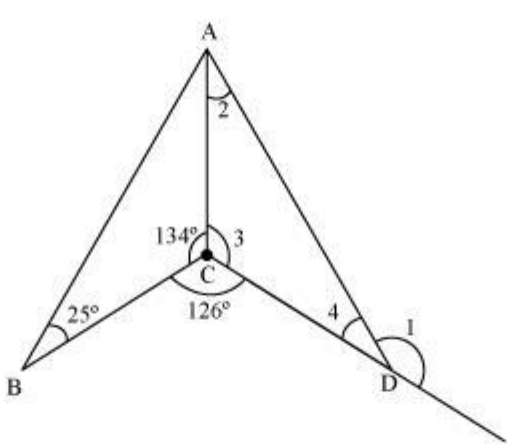
The given polynomial is firstly simplified as:

$$2\sqrt{2x^4}+5\sqrt{2x^2}-7\sqrt{2}$$
$$=2\sqrt{2}\,x^2+5\sqrt{2}\,x-7\sqrt{2}$$

Here, coefficient of x^2 is $2\sqrt{2}$.

Thus, the coefficient of x^2 in the given polynomial is $2\sqrt{2}$.

Q9)



In the given figure, AC is the bisector of ∠BAD.

Find the measures of ∠1, ∠2, ∠3, and ∠4.

Solution:

By applying angle sum property in ΔABC, we obtain

$\angle ABC + \angle BCA + \angle BAC = 180°$

$\Rightarrow 25° + 134° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 159° = 21°$

Now, AC is the bisector of ∠BAD.

$\therefore \angle BAC = \angle CAD$

$\Rightarrow \angle 2 = 21°$ ($\because \angle BAC = 21°$)

We know that the measure of one complete angle is 360°.

$\therefore \angle BCD + \angle BCA + \angle ACD = 360°$

$\Rightarrow 126° + 134° + \angle 3 = 360°$

$\Rightarrow \angle 3 = 360° - 260° = 100°$

By exterior angle property of triangles, we have

$\angle 1 = \angle 2 + \angle 3$

$\Rightarrow \angle 1 = 21° + 100° = 121°$

By applying angle sum property in ΔACD, we obtain

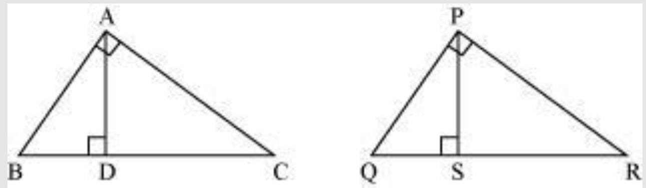
$\angle ACD + \angle CDA + \angle CAD = 180°$

$\Rightarrow 100° + \angle 4 + 21° = 180°$

$\Rightarrow \angle 4 = 180° - 121° = 59°$

Q10)

Use the following information to answer the next question.

The given figure shows two triangles ABC and PQR, where ∠BAC = ∠QPR = 90°. D and S are points on the side BC and QR respectively such that AD⊥BC and PS⊥QR.	

If AB = PQ = 6 cm, AC = 8 cm, AD = 4.8 cm, and QS = 3.6 cm, then what is the perimeter of ΔPQR?

Solution:

ΔABC is right-angled at A.

Applying Pythagoras Theorem in ΔABC, we obtain

$\therefore BC^2 = AB^2 + AC^2 = (6\text{ cm})^2 + (8\text{ cm})^2 = 100\text{ cm}^2$

⇒ BC = 10 cm

ΔABD is right-angled at D.

Applying Pythagoras Theorem in ΔABD, we obtain

$\therefore BD^2 = AB^2 - AD^2 = (6\text{ cm})^2 - (4.8\text{ cm})^2 = 12.96\text{ cm}^2$

⇒ BD = 3.6 cm

Comparing ΔABD and ΔPQS, we obtain

AB = PQ [Each is 6 cm]

∠ADB = ∠PSQ [Each is 90°]

BD = QS [Each is 3.6 cm]

∴ΔABD ≅ ΔPQS [By RHS congruency criterion]

⇒ ∠ABD = ∠PQS [CPCT]

Now, comparing ΔABC and ΔPQR, we obtain

∠BAC = ∠QPR [Each is 90°]

AB = PQ [Each is 6 cm]

∠ABC = ∠PQR [∠ABD = ∠PQS]

∴ΔABC ≅ ΔPQR [By ASA congruency criterion]

⇒ AC = PR = 8 cm and BC = QR = 10 cm [CPCT]

Thus, perimeter of ΔPQR = PQ + QR + PR = 6 cm + 10 cm + 8 cm = 24 cm

Q11) Express $0.43\overline{7}$ as a fraction in the simplest form.

Solution:

Let *y* = $0.43\overline{7}$

Then, *y* = 0.43777777…

On multiplying both sides by 100, we get

100*y*= 43.777777777… (1)

On multiplying both sides of equation (1) by 10, we get

1000*y*= 437.777777… (2)

On subtracting equation (1) from equation (2), we get

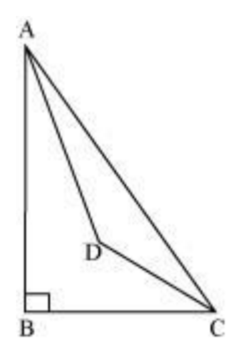
900*y*= 394

$$y = \frac{394}{900}$$

$$y = \frac{197}{450}$$

Thus, $0.43\overline{7}$ can be expressed as a fraction in the simplest form as $\frac{197}{450}$.

Q12)



In the given figure, AD and CD are the bisectors of ∠BAC and ∠ACB respectively and AB > BC.

Show that: AD > CD

Solution:

It is given that AD and CD are the bisectors of ∠BAC and ∠ACB respectively.

$\therefore \angle CAD = \dfrac{\angle BAC}{2} \qquad ...(1)$

$\angle ACD = \dfrac{\angle ACB}{2} \qquad ...(2)$

It is also given that AB > BC.

It is known that if two sides of a triangle are unequal, then the angle opposite to longer side is greater.

$\therefore \angle ACB > \angle BAC$

$$\Rightarrow \frac{\angle ACB}{2} > \frac{\angle BAC}{2}$$

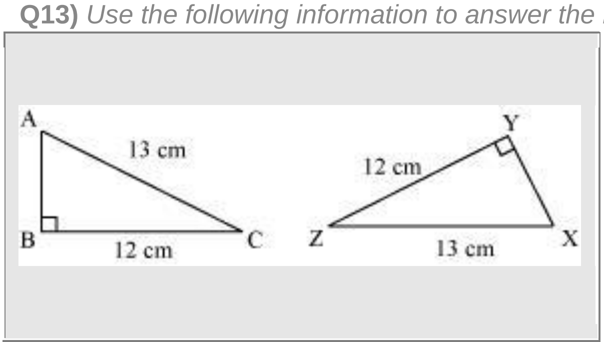
$\Rightarrow \angle ACD > \angle CAD$ [Using equations (1) and (2)]

It is also known that if two angles of a triangle are unequal, then the side opposite to greater angle is longer.

$\therefore$ AD > CD

Hence, the result is proved.

Q13) *Use the following information to answer the next question.*



Is ΔABC ≅ ΔXYZ? Justify your conclusion.

Solution:

In ΔABC and ΔXYZ,

∠ABC = ∠XYZ = 90°

AC = XZ = 13 cm

BC = YZ = 12 cm

Therefore, by RHS congruency rule, ΔABC ≅ ΔXYZ

Q14) The information in which alternative is correctly matched?

- A)

Polynomial	Degree
$3x^3 + 7x^2 + 8$	8

- B)

Polynomial	Degree
$9x^4 + 5x^3 + 2x + 8$	9

- C)

Polynomial	Degree
$17x^5 + 4$	5

- D)

Polynomial	Degree
$3x^2 + 1$	6

Answer:

C

Solution:

The highest power of the variable in a polynomial is called the degree of the polynomial.

The highest power of variable x in the polynomial $17x^5 + 4$ is 5. Hence, the degree of this polynomial is 5.

Thus, the information in alternative **C** is correctly matched.

The correct answer is C.

Q15) How can the value of $(19.9)^3$ be found using a suitable identity?

Solution:

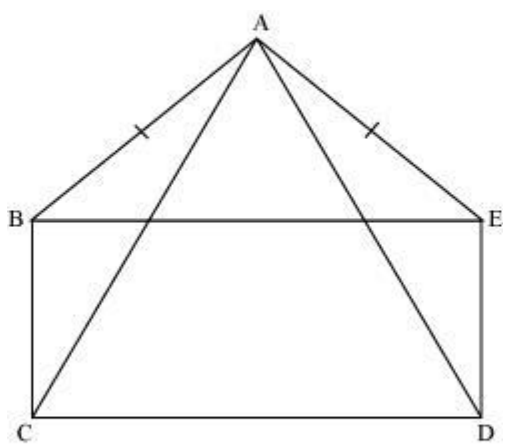
$(19.9)^3$ can also be written as $(20 - 0.1)^3$

Using identity $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$,

$$\begin{aligned}(20-0.1)^3 &= (20)^3 - (0.1)^3 - 3(20)(0.1)(20-0.1)\\ &= 8,000 - 0.001 - 6 \times 19.9\\ &= 8,000 - 0.001 - 119.4\\ &= 7,880.599\end{aligned}$$

Thus, the value of $(19.9)^3$ is 7,880.599.

Q16)



In the given figure, BCDE is a rectangle and ΔABE is isosceles.

Prove that ∠CAD = 2 (∠ACB).

Solution:

It is given that AB = AE.

The angles opposite to the equal sides of an isosceles triangle are equal.

∴∠ABE = ∠AEB

Also, ∠CBE = ∠BED (Each is 90°)

∠ABE + ∠CBE = ∠AEB + ∠BED

⇒ ∠ABC = ∠AED ... (1)

Comparing ΔABC and ΔAED:

AB = AE (Given)

∠ABC = ∠AED {From equation (1)}

BC = ED (Opposite sides of rectangle)

∴ΔABC ≅ ΔAED [SAS congruence rule]

Hence, AC = AD (Corresponding parts of congruent triangles are equal)

ΔACD is isosceles, where AC = AD.

∴∠ACD = ∠ADC

Now, applying angle sum property of triangle in ΔACD:

∠ACD + ∠ADC + ∠CAD = 180°

⇒ 2 ∠ACD + ∠CAD = 180°

⇒ 2 (∠BCD − ∠ACB) + ∠CAD = 180°

⇒ 2 (90° − ∠ACB) + ∠CAD = 180°

⇒ 180° − 2∠ACB +∠CAD = 180°

⇒ ∠CAD = 2 (∠ACB)

Hence, the result is proved.

Q17) Find three irrational numbers between $\frac{40}{9}$ and $2\sqrt{6}$.

Solution:

$$
\begin{array}{r|l}
 & 4.44 \\
9 &)40 \\
 & \underline{36} \\
 & 40 \\
 & \underline{36} \\
 & 40 \\
 & \underline{36} \\
 & 4
\end{array}
$$

$$\therefore \frac{40}{9} = 4.\overline{4}$$

	2.449...
2	6.000000
	4
44	200
	176
484	2400
	1936
4889	46400
	44001
	2399

$$\therefore 2\sqrt{6} = 2 \times 2.449 = 4.898$$

A number that has a non-terminating and non-recurring decimal expansion is an irrational number.

Therefore, to find three irrational number between $\frac{40}{9}$ and $2\sqrt{6}$, it is required to search for three numbers that are non-terminating and non−recurring and also lying between the numbers $4.\overline{4}$ and 4.898.

Thus, the three irrational numbers between $\frac{40}{9}$ and $2\sqrt{6}$ are

4.46101101110..., 4.67484884888...and 4.7214345414...

Q18) After how many places of decimal does the decimal expansion of the rational number $\frac{31}{2^3 \times 5^2}$ terminate?

Solution:

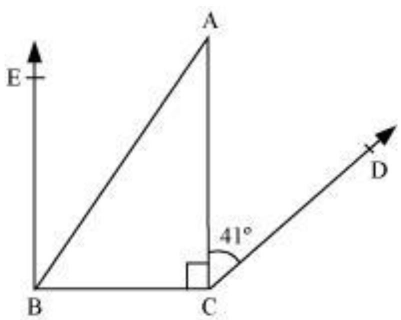
The number of places after which the decimal expansion of a rational number with denominator 2^m5^n, where m and n are non-negative integers, terminates is equal to the highest value among m and n.

Here, the highest between 3 and 2 is 3. Therefore, $\frac{31}{2^3 \times 5^2}$ will terminate after three digits of decimal.

This can be verified as:

$\frac{31}{2^3 \times 5^2} = \frac{31}{200} = 0.155$

Q19)



In the given figure, AB||CD and BE||AC

(a) Find ∠ABC.

(b) Find ∠EBA and hence show that EB⊥BC.

Solution:

In the given figure, AB||CD

∴ ∠BAC = ∠ACD (Alternate interior angles)

∴ ∠BAC = 41°

In ΔABC,

∠ABC + ∠BCA + ∠BAC = 180° (By angle sum property)

⇒ ∠ABC + 90°+ 41°= 180° (∵ ∠BAC = 41°)

⇒ ∠ABC = 180°− 131°

⇒ ∠ABC = 49°

Now, BE||AC

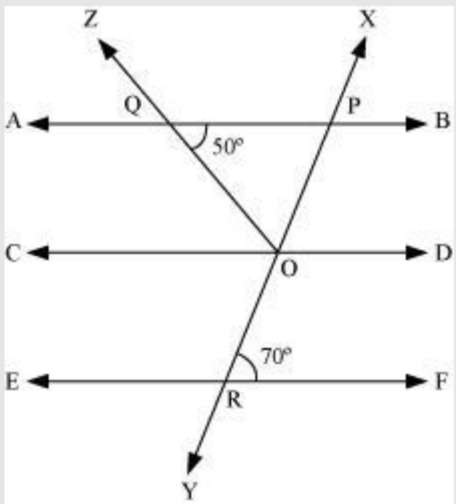
∴ ∠EBA = ∠BAC (Alternate interior angles)

⇒ ∠EBA = 41°

Now, ∠EBA + ∠ABC = 41°+ 49°= 90°

Thus, EB is perpendicular to BC.

Q20) *Use the following information to answer the next question.*

In the given figure, AB||CD||EF.	

What is the measure of ∠POQ?

A.60°

B.40°

C.50°

D.30°

Solution:

As AB||EF,

∴ ∠OPQ = ∠ORF = 70° (Alternate interior angles)

By angle sum property in ΔOPQ, ∠OPQ + ∠POQ + ∠PQO = 180°

⇒ 70° + ∠POQ + 50° = 180°

⇒ ∠POQ= 180° − (70° + 50°)

⇒ ∠POQ = 60°

The correct answer is A.

Q21) Find eight rational numbers between 7 and 8.

Solution:

Eight rational numbers are required to be calculated.

Thus, the two given numbers (7 and 8) can be rewritten with denominator as 8 + 1 = 9 as 7 = $\frac{63}{9}$ and 8 = $\frac{72}{9}$.

Now, eight rational numbers between $\frac{63}{9}$ and $\frac{72}{9}$ are $\frac{64}{9}, \frac{65}{9}, \frac{66}{9}, \frac{67}{9}, \frac{68}{9}, \frac{69}{9}, \frac{70}{9}$ and $\frac{71}{9}$

Thus, eight rational numbers between 7 and 8 are $\frac{64}{9}, \frac{65}{9}, \frac{66}{9}, \frac{67}{9}, \frac{68}{9}, \frac{69}{9}, \frac{70}{9}$ and $\frac{71}{9}$.

Q22) Factorise the following polynomials.

(i) $512x^3 - \frac{1}{27} - 64x^2 + \frac{8x}{3}$

(ii) $x^2 - 7x + 12$

OR

The polynomial, $p(x) = x^4 - 2x^3 - 13x^2 + kx + 24$, gives remainder 24 when divided by (x − 1). What is the value of k and with this value of k, factorise the polynomial p(x).

Solution:

(i) Let $p(x) = 512x^3 - \frac{1}{27} - 64x^2 + \frac{8x}{3}$

$$\Rightarrow p(x) = (8x)^3 - \left(\frac{1}{3}\right)^3 - 8x\left(8x - \frac{1}{3}\right)$$

$$\Rightarrow p(x) = (8x)^3 - \left(\frac{1}{3}\right)^3 - 3 \cdot 8x \cdot \frac{1}{3}\left(8x - \frac{1}{3}\right)$$

$$\Rightarrow p(x) = \left(8x - \frac{1}{3}\right)^3 \qquad \left[\text{Using } (a-b)^3 = a^3 - b^3 - 3ab(a-b)\right]$$

$$\Rightarrow p(x) = \left(8x - \frac{1}{3}\right)\left(8x - \frac{1}{3}\right)\left(8x - \frac{1}{3}\right)$$

Thus, the given polynomial can be factorised as

$$512x^3 - \frac{1}{27} - 64x^2 + \frac{8x}{3} = \left(8x - \frac{1}{3}\right)\left(8x - \frac{1}{3}\right)\left(8x - \frac{1}{3}\right)$$

(ii) Let $p(x) = x^2 - 7x + 12$

For $x = 3$, $p(3) = (3)^2 - 7(3) + 12 = 9 - 21 + 12 = 0$

According to factor theorem, $(x - a)$ will be a factor of $p(x)$, if $p(a) = 0$

Therefore, $(x - 3)$ is a factor of $p(x)$.

$$\begin{array}{r|l} & \quad x - 4 \\ \hline (x-3) & x^2 - 7x + 12 \\ & x^2 - 3x \\ & \underline{- \quad +} \\ & \quad -4x + 12 \\ & \quad \underline{-4x + 12} \\ & \quad \underline{\quad \times \quad} \end{array}$$

Thus, $x^2 - 7x + 12$ can be factorised as $(x - 3)(x - 4)$.

OR

The given polynomial is, $p(x) = x^4 - 2x^3 - 13x^2 + kx + 24$

According to remainder theorem, when a polynomial $p(x)$ is divided by $(x - a)$, then the remainder is $p(a)$.

Here, $p(x)$ when divided by $(x - 1)$, gives the remainder as 24.

∴ $p(1) = 24$

$$(1)^4 - 2(1)^3 - 13(1)^2 + k(1) + 24 = 24$$
$$1 - 2 - 13 + k + 24 = 24$$
$$k = 14$$

$$p(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$$

$p(x)$ is factorised using factor theorem.

By hit and trial method, we obtain

$$p(-3) = (-3)^4 - 2(-3)^3 - 13(-3)^2 + 14(-3) + 24$$

= 81 + 54 − 117 − 42 + 24

= 0

∴ $(x + 3)$ is a factor of $p(x)$.

Again by hit and trial method, we obtain

$$p(4) = (4)^4 - 2(4)^3 - 13(4)^2 + 14(4) + 24$$

= 256 − 128 − 208 + 56 + 24

= 0

∴ $(x - 4)$ is also a factor of $p(x)$.

$(x + 3)(x - 4) = x^2 - x - 12$

$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
x^2 - x - 12 & x^4 - 2x^3 - 13x^2 + 14x + 24 \\
 & x^4 - x^3 \quad - 12x^2 \\
 & - \quad + \quad + \\
\hline
 & \quad - x^3 - x^2 + 14x + 24 \\
 & \quad - x^3 + x^2 + 12x \\
 & \quad + \quad - \quad - \\
\hline
 & \qquad - 2x^2 + 2x + 24 \\
 & \qquad - 2x^2 + 2x + 24 \\
 & \qquad + \quad - \quad - \\
\hline
 & \qquad\quad \times \\
\hline
\end{array}
$$

$\therefore$ $p(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

$= (x+3)(x-4)(x^2 - x - 2)$

By splitting the middle term, we obtain

$$
\begin{aligned}
p(x) &= (x+3)(x-4)\left[x^2 - 2x + x - 2\right] \\
&= (x+3)(x-4)\left[x(x-2) + 1(x-2)\right] \\
&= (x+3)(x-4)(x-2)(x+1)
\end{aligned}
$$

Thus, $p(x)$ can be factorised as

$p(x) = (x+3)(x-4)(x-2)(x+1)$

Q23) If $(x - 1)$ is a factor of $p(x) = kx^2 + (2k + 1)x - 13$, then find the value of k. Factorise $p(x)$.

Solution:

$(x - 1)$ is a factor of $p(x) = kx^2 + (2k + 1)x - 13$. Therefore, by factor theorem, we obtain

$p(1) = 0$

$k(1)^2 + (2k + 1)(1) - 13 = 0$

$k + (2k + 1) - 13 = 0$

$k = 4$

$\therefore p(x) = 4x^2 + 9x - 13$

Splitting the middle term, we obtain

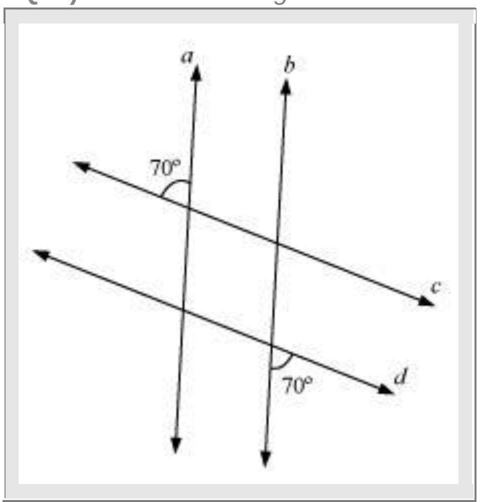
$p(x) = 4x^2 + 13x - 4x - 13$

$= x\,(4x + 13) - 1\,(4x + 13)$

$= (x - 1)\,(4x + 13)$

Thus, $p(x)$ can be factorised as $(x - 1)\,(4x + 13)$.

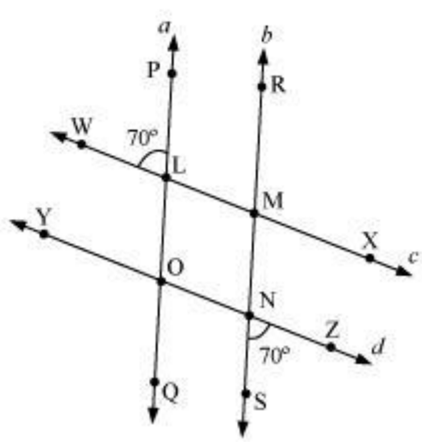
Q24) *Use the following information to answer the next question.*



In the given figure, $a \parallel b$. Check whether $c$is parallel to d. Give reasons to support your answer.

View Solution

Solution:



It is given that *a* || *b*.

It is known that if two parallel lines are cut by a transversal, then the pair of corresponding angles is equal.

Here, ∠WMR and ∠WLP are corresponding angles.

∴∠WMR = ∠WLP

⇒ ∠WMR = 70° (∠WLP = 70°)

∠YNR = ∠SNZ (Vertically opposite angles)

⇒ ∠YNR = 70° (∠SNZ = 70°)

Therefore, ∠WMR = ∠YNR = 70°

i.e., corresponding angles formed by transversal *b* on the lines *c* and *d* are equal.

It is also known that when a transversal cuts two lines, such that the pair of corresponding angles is equal, then the lines are parallel.

Hence, line *c*is parallel to line *d*.

Q25) For three variables *a*,*b*, and *c*, if $a+b+c=10$, $a^2+b^2+c^2=38$, and *abc* = 30, then find the value of $a^3+b^3+c^3$.

Solution:

$$a+b+c=10$$
$$\Rightarrow (a+b+c)^2=(10)^2$$
$$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=100$$
$$\Rightarrow 38+2(ab+bc+ca)=100 \qquad \left[a^2+b^2+c^2=38\right]$$
$$\Rightarrow ab+bc+ca=\frac{100-38}{2}=31$$

It is known that $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$\Rightarrow a^3+b^3+c^3-3\times 30=(a+b+c)\left[\left(a^2+b^2+c^2\right)-(ab+bc+ca)\right]$$
$$\Rightarrow a^3+b^3+c^3-90=10(38-31)=10\times 7=70$$
$$\Rightarrow a^3+b^3+c^3=70+90=160$$