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A PROBABILISTIC STUDY OF VARIATION IN NUMBER OF CHILD DEATHS

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Abstract

The level and trend of mortality indicate the standard of development of a nation, thus the study of mortality especially infant and child mortality is one of the important area of human demography. In this paper an attempt has been made to study the pattern of child deaths among females. The parameters involved in the model under consideration have been estimated. Various set of data from NFHS II for Uttar Pradesh has been used to discuss the applicability of the proposed model.

Key words: Child death, Probability distribution, Fertility

1. Introduction

The Infant and child mortality is known as a good and sensitive indicator of development of a nation and impact of government intervention programs and policies. Effective control in reduction of mortality has been one of the remarkable achievements across the world. However, the benefits of this reduction are not equally shared by all sections of population to an adequate level. In the recent past, India has experienced significant reduction in infant and child mortality but the present level of infant and child mortality is still fairly high as compared to the other developed countries. Collectively these two types of mortality is known as under five mortality (we will use child death instead of under five mortality here after). It is affected by several socio-demographic factors. For example, mother's age at birth, education, autonomy, occupation, family income, religion/castes and media exposures are the major factors that affect child death in India.

Child death has been major concern of the researchers and demographers because of its apparent relationship with the level of fertility (Rustein et al, 1975) and indirect relationship with the acceptance of modern contraceptive means (Kabir et. al; 1993). The connections between child death and fertility or vice versa are the root of

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many explanations of demographic transition and thus important for shaping population policy in less developed countries. As far as the child replacement hypothesis is concerned, high fertility is a necessary biological and behavioral response to high mortality i.e. the parents try to replace those children who die, and their aim to produce enough children to ensure the survival of some intended number up to adulthood (Chowdhury et. al., 1976). Thus, child death has positive impact on fertility (Bhuyan et.al 1996) and reduction in child death or enhance in the probability of survival of children is one of the known reasons of reduction in the fertility.

Studies on early age mortality in the last five decades are mostly confined to infant death, but, it has been realized that child death also need to be examined in addition to infant mortality. A number of parametric models for the study of the age patterns of mortality have been developed over the year. First parametric model for duration of mortality was developed by Gompertz (1825). The very first attempt to study infant mortality through parametric modeling was proposed by Keyfitz (1977) who used a hyperbolic function for the study infant mortality. After that many attempt have been made to study age at infant mortality through mathematical models. In various countries data on age of infant and child mortality are heavily distorted. It suffers from substantial errors that is occurs due to recall lapse and partly due to tendency of digit preference of the respondent which result in under reporting and misreporting of age of death.

To overcome such limitation and to remove variation due to this bias, mathematical models on number of child deaths to specified type of female are used in the previous papers. The discrete probability models were proposed the estimates of the parameters were obtained. The suitability of proposed discrete probability models were checked on the basis of goodness of fit test using observed data. Application of discrete probability models in study of human fertility has received higher attention of researchers because fertility outcomes are essentially of stochastic nature. Several models and corresponding class of statistical distribution function have been proposed to describe it; these include use of the modified form of Binomial and Poisson (Dandekar, 1955 and Singh, 1963, 68), the negative binomial and modified form of negative binomial (Brass, 1958) distribution to describe the number of births to a female but there is a less number of models have been developed for number of infant or child death to a female.

In this paper an attempt has been made to develop a probability model to explain the pattern of child death for all females in the society. The applications of this model are illustrated through the real data taken from Uttar Pradesh. For this purpose the National Family Health Survey –II (1998-1999) are used.

2. Probability Model for the Number Child Death

Let x denote the number of child death to a female. Let, the distribution of number of child death to females of given parity n and risk of child death p follows a binomial distribution.

$$P[X = x \mid n, p] = \binom{n}{x} p^x (1-p)^{n-x} \quad ; \quad 0 \le p \le 1, \; n > 0 \tag{1}$$

where $x = 0, 1, 2 \ldots\ldots\ldots\ldots\ldots n$

As discussed in the previous papers, it is assumed that probability of death of child varies from female to female but remains constant at each birth for a given female. We assume that probability of death of child 'p' follows beta distribution with parameter a and b which is given by

$$g(p) = \frac{1}{\beta(a,b)} p^{(a-1)} (1-p)^{(b-1)} \; ; \quad 0 \le p \le 1; \quad a, b > 0 \tag{2}$$

Since p varies from 0 to 1 use of beta distribution as a probability distribution of 'p' is justified due to its flexibility. Therefore, the joint distribution of x and p for given n is given by

$$P[X = x \cap P = p \mid n] = P[X = x/n, p] \times g(p)$$

$$= \binom{n}{x} p^x (1-p)^{n-x} \frac{1}{\beta(a,b)} p^{(a-1)} (1-p)^{b-1} \tag{3}$$

Therefore marginal distribution of X at fixed n, is written as

$$P[X = x \mid n] = \int_0^1 \binom{n}{x} p^x (1-p)^{n-x} \frac{1}{\beta(a,b)} p^{(a-1)} (1-p)^{(b-1)} dp \tag{4}$$

Further in this model, we assume that the number of parity (Number of children ever born to a female) is also a random variable and follows a Poisson distribution.

$$P[n = k] = \frac{e^{-\lambda} \lambda^k}{k!} \tag{5}$$

where k=0,1,2,…………..n and λ is the average parity

The joint distribution of X and n is, therefore, written as

$$P[X = x \cap n = k] = P[X = x \mid n] \times P[n = k] \tag{6}$$

Hence, Marginal distribution of X is given as

$$P[X = x] = \sum_{k=x}^{\infty} \int_0^1 \binom{k}{x} p^x (1-p)^{k-x} \frac{1}{\beta(a,b)} p^{a-1} (1-p)^{b-1} dp \times \frac{e^{-\lambda} \lambda^k}{k!} \tag{7}$$

After simplification, it reduced to

$$P[X = x] = \frac{\lambda^x}{\beta(a,b) x!} \int_0^1 e^{-\lambda p} p^{a+x-1} (1-p)^{b-1} dp \tag{8}$$

It is easy to verify that $\sum_{x=0}^{\infty} P[X = x] = 1$, Thus, $P[X = x]$ is a probability mass function for child death to females.

3. Estimation Procedure

In this paper method of moment has been used as estimation procedure to estimate the parameters λ, a and b of model considered for number of child death to females of all parity. The method of moments generally provides estimate which are consistent but not as efficient as the method maximum likelihood. But method of moment is often used because it lead to very simple in computations than maximum likelihood method. Also the complexities in mathematical derivation involved in method of moment are less than that involved in maximum likelihood method. The first three moments of the probability model considered here are as follows

$$E(X) = \frac{\lambda\beta\,(a+1,b)}{\beta(a,b)} = \frac{\lambda a}{a+b} \tag{9}$$

$$E(X^2) = \frac{\lambda^2\beta(a+2,b)}{\beta(a,b)} + \frac{\lambda\beta(a+1,b)}{\beta(a,b)} = \frac{\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)} \tag{10}$$

$$\begin{aligned} E(X^3) &= \frac{\lambda^3\beta(a+3,b)}{\beta(a,b)} + \frac{3\lambda^2\beta(a+2,b)}{\beta(a,b)} + \frac{\lambda\beta(a+1,b)}{\beta(a,b)} \\ &= \frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)} + \frac{3\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)} \end{aligned} \tag{11}$$

Let $\mu_1^{'}$, $\mu_2^{'}$ and $\mu_3^{'}$ denotes the three raw moments about zero for the data, thus replacing $E(X)$, $E(X^2)$ and $E(X^3)$ by $\mu_1^{'}$, $\mu_2^{'}$ and $\mu_3^{'}$ in the above equation we can obtained three equations with three unknown parameters λ, a and b, as given below,

$$\mu_1^{'} = \frac{\lambda a}{a+b} \tag{12}$$

$$\mu_2^{'} = \frac{\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)} \tag{13}$$

$$\mu_3^{'} = \frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)} + \frac{3\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)} \tag{14}$$

From equation (12) we have

$$a+b = \frac{\lambda a}{\mu_1^{'}} \tag{15}$$

This implies that $b = \frac{\lambda a}{\mu_1^{'}} - a$ (16)

Using equation (12) and (13) we have

$$\mu_2^{'} = \frac{\lambda^2\beta(a+2,b)}{\beta(a,b)} + \mu_1^{'} \quad \text{or} \quad \mu_2^{'} - \mu_1^{'} = \frac{\lambda^2 a(a+1)}{(a+b+1)(a+b)} \tag{17}$$

Now by using equation (12) we have

$$\frac{\mu_2^{'} - \mu_1^{'}}{\mu_1^{'}} = \frac{\lambda(a+1)}{(a+b+1)} \tag{18}$$

Putting the value of $(a+b)$ from equation (15) we have

$$\frac{\mu_2' - \mu_1'}{\mu_1'} = \frac{\lambda(a+1)}{(\frac{\lambda a}{\mu_1'}+1)} = \frac{\lambda(a+1)}{(\lambda a + \mu_1')} = \delta(say) \tag{19}$$

Solving for a the above equation will become

$$a = \frac{\lambda - \delta\mu_1'}{\lambda(\delta - 1)} \tag{20}$$

Now from equation (14), we have

$$\mu_3' = \frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)} + \frac{3\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)}$$

or

$$\mu_3' = \frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)} + 3\left[\frac{\lambda^2(a+1)a}{(a+b+1)(a+b)} + \frac{\lambda a}{(a+b)}\right] - \frac{3\lambda a}{(a+b)} + \frac{\lambda a}{(a+b)}$$

By using equation (12) and (13), we get

$$\mu_3' = \frac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)} + 3\mu_2' - 2\mu_1'$$

or, $\mu_3' - 3\mu_2' + 2\mu_1' = \dfrac{\lambda^3(a+2)(a+1)a}{(a+b+2)(a+b+1)(a+b)}$

Using equation (15), we have

$$\frac{\mu_3' - 3\mu_2' + 2\mu_1'}{\mu_1'} = \frac{\lambda^2(a+2)(a+1)}{(a+b+2)(a+b+1)}$$

Putting the value of $(a+b)$ from equation (15) we get

$$\frac{\mu_3' - 3\mu_2' + 2\mu_1'}{\mu_1'} = \frac{\lambda^2(a+2)(a+1)}{(\frac{\lambda a}{\mu_1'}+2)(\frac{\lambda a}{\mu_1'}+1)}$$

or, $$\frac{\mu_3' - 3\mu_2' + 2\mu_1'}{\mu_1'^3} = \frac{\lambda^2(a+2)(a+1)}{(\lambda a + 2\mu_1')(\lambda a + \mu_1')} = \eta(say) \tag{21}$$

Simplifying equation (21) we get,

$$\eta(\lambda a + 2\mu_1')(\lambda a + \mu_1') = \lambda^2(a+2)(a+1)$$

Or, $a^2\lambda^2(\eta - 1) + 3a(\lambda\eta\mu_1' - \lambda^2) + 2(\mu_1'^2\eta - \lambda^2) = 0$

Putting the value of a from equation (20) in the above equation and after simplifying the equation, we have

$$\left(\frac{\lambda-\delta\mu_1'}{(\delta-1)}\right)^2(\eta-1)+3\left(\frac{\lambda-\delta\mu_1'}{(\delta-1)}\right)(\eta\mu_1'-\lambda)+2(\mu_1'^2\eta-\lambda^2)=0 \qquad (22)$$

Or, $\{\eta-2\delta^2+\delta\}\lambda^2+\{\eta\mu_1'(\delta-3)+\delta\mu_1'(3\delta-1)\}\lambda+\mu_1'^2(2\eta-\delta\eta-\delta^2)=0 \qquad (23)$

The value of δ and η can be calculated from the data directly by using the equation 19 and 21 respectively. After obtaining and substituting these values in the equation (23) we get a quadratic equation in λ, which is unknown and analytical solution of λ can be obtained. Once we get the value of λ the value of other parameters *a* and *b* can be easily obtained by using equation (20) and (16) respectively. After obtaining the estimate of the parameters the expected frequencies can be obtained by using the proposed probability model.

The estimate of λ i.e. the mean number of children ever born to a female obtained by above mentioned model gives the mean number of children born to the females having at least one child ever born. In order to obtain the mean number of children ever born to all females ($\tilde{\lambda}$) we have used a simple transformation which is discussed below:

We know that the mean number of children ever born is

$$\lambda=\frac{B}{N-n_0} \qquad \text{or,} \qquad B=\lambda(N-n_0) \qquad (24)$$

where, (N - n_0) i.e the number of females having at least one child ever born, is given for all categories and n_0 is the total number of childless females in a particular category which can be easily obtained from the data. N is the total number of females considered for this study and B is total number of births. It is clear that

$$\tilde{\lambda}=\frac{B}{N} \qquad (25)$$

By substituting the value of B from equation (24) in the above equation we get

$$\tilde{\lambda}=\frac{\lambda(N-n_0)}{N} \qquad (26)$$

This is mean number of children ever born to females in their whole reproductive periods. The estimates of $\tilde{\lambda}$ obtained by above mentioned formula is shown in Table number 11.

4. Application of the Model

To illustrate the application of the model discussed above the data collected in NFHS –II (1998-99) for the state Uttar Pradesh has been considered. In this data information on all live births to a woman and their survival status at the time of survey has been collected. In case of death of child age at death for each woman have been recorded separately. Women, who have no births in the preceding 5 years from the reference date of the survey, have been considered with the assumption that these females have already completed their family size. By this we are able to avoid censored observations from the data. Childless women have not been considered keeping the point in mind that the childless females cannot experience any child death.

The data set of the state Uttar Pradesh contains information of 3267 females. The whole state is divided into five regions namely hill, western, central, eastern and Bundelkhand region having distribution of females are 441, 996, 487, 1024 & 319 respectively. Also to check the suitability of the proposed model according to age groups the data of Uttar Pradesh is divided according to the present age group 30-39 and 40-49 years respectively and also according to place of residence i.e. rural and urban. The estimates of the model have also been obtained by considering the state of Uttar Pradesh whole as a one unit.

5. Results

The observed and expected number of frequencies of females according to number of child death for different regions of Uttar Pradesh is shown for in Tables 1 to 6. It can be observed from the tables that the calculated value of chi-square is smaller than corresponding tabular value at 5 percent level of significance for each data set considered. Although chi-square test for goodness of fit cannot be used on the data of Hill region due to fact that after estimating three parameters no degree of freedom has been left for test statistic but one can observe from the Table 1 that the expected frequencies are very close to observed frequencies in this case. Thus this refers the suitability of above proposed model for distribution of child death to females in all five regions of Uttar Pradesh. The estimated values of parameters along with risk of child death are also shown in these tables. It is clear that the risk of child death is highest in central region and lowest in hill region. The estimated value of the risk of infant death to female is almost similar to the observed values reported in the NFHS- II report except for the central region.

Tables 6 and 7 present the females according to number of child deaths for the age group 30-39 and 40-49 respectively for Uttar Pradesh. Here also the observed and expected value of the female according to number of child deaths are quite close to each other and value of chi-square is insignificant at 5 percent level of significance. From the tables it is clear that the risk of child deaths is much higher for the females who belong to age group 30-49 than the females who belong to age-group 30-39. This may be due to the fact that there is more chance of infant as well as child death if a female produce children at higher ages. Table 8 and 9 shows the observed and expected frequencies of females experiencing child death with respect place of their residence in Uttar Pradesh i.e. urban and rural respectively. It is observed from the tables that he estimate of risk of child deaths in rural Uttar Pradesh is about 128 per thousand which is much more higher than the estimate of risk of child deaths of urban females (89 per thousand) in Uttar Pradesh. Table 10 reveals the suitability of the proposed model for the data set of Uttar Pradesh considered as a whole. By looking the observed and expected frequencies one can conclude that the model graduates the data superbly. It is found that the risk of child death is 133 per thousand for the females of Uttar Pradesh which is almost similar to what is observed from the data and reported in NFHS-II (1998-99)

The last Table 11 shows a collection of estimates of different parameters involved in the model according to the different regions/categories of Uttar Pradesh. It clearly shows that the mean number of children ever born to female is highest in eastern region and lowest in central region of Uttar Pradesh. Also the mean number of children ever born to females belonging to rural areas is higher than the females who belong to urban areas in the state Uttar Pradesh. The estimate of mean number of children ever born to a female in Uttar Pradesh is found out to be 4.38.

From the results discussed above we may conclude that the model proposed here may be considered to be suitable to describe the distribution of number of child death to females. The remarkable utility of this model is that it provides a clear visualization about unobservable risk of child deaths and also provides the mean number of children ever born to female through the distribution of female according to number of child deaths. Thus, this model provides a new dimension for comparing the risk of child death to females in different regions and categories. The estimates of the risk of child deaths and mean number of children ever born to a female for different region and categories of Uttar Pradesh are well supported by the estimates given in NFHS-II report except for the central region of Uttar Pradesh which may be due to selection bias or wrong choice of probability distribution for n i.e. the number of children ever born to the females. One may modify the proposed model by taking some shifted or truncated probability distribution for the number of children ever born to the females.

Table 1
Observed and Expected distribution of females according to the number of child deaths in Hill Region of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	350	350.401
1	60	59.151
2	20	19.899
3+	11	11.549
Total	441	441
Parameters		$\lambda = 4.5099$ $a = 0.3067$ $b = 4.0507$
$\chi^2_{0.5}$		-
d.f.		-
Risk of child death (p)		$p = 0.0704$

Table 2
Observed and Expected distribution of females according to the number of child deaths in Western Region of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	610	605.181
1	196	209.953
2	112	98.192
3	39	46.617
4	26	21.240
5+	13	14.817
Total	996	996
Parameters		$\lambda = 4.1954$ $a = 0.5224$ $b = 2.5349$
$\chi^2_{0.5}$		5.4418
d.f.		2
Risk of child death (p)		$p = 0.1709$

Table 3
Observed and Expected distribution of females according to the number of child deaths in Central Region of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	261	262.897
1	110	107.421
2	61	57.929
3	27	31.036
4	14	15.601
5+	16	12.116
Total	487	487
Parameters		$\lambda = 3.8271$, $a = 0.5289$, $b = 1.6764$
$\chi^2_{0.5}$		2.1722
d.f.		2
Risk of child death (p)		$p = 0.2398$

Table 4
Observed and Expected distribution of females according to the number of child deaths in Eastern Region of Uttar Pradesh.

No. of Child Deaths	Observed Frequency	Expected Frequency
0	523	522.745
1	252	253.794
2	130	128.467
3	65	63.788
4	33	30.598
5	8	14.092
6+	13	10.516
Total	1024	1024
Parameters		$\lambda = 6.5813$, $a = 0.8474$, $b = 5.0704$
$\chi^2_{0.5}$		3.4628
d.f.		3
Risk of child death (p)		$p = 0.1432$

Table 5

Observed and Expected distribution of females according to the number of child deaths in Bundelkhand Region of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	176	175.402
1	67	69.319
2	38	35.894
3	20	19.148
4	9	10.001
5+	9	9.236
Total	319	319
Parameters		$\lambda = 5.6415$ $a = 0.5677$ $b = 2.9316$
$\chi^2_{0.5}$		0.3473
d.f.		2
Risk of child death (p)		$p = 0.1623$

Table 6

Observed and Expected distribution of females according to the number of child deaths in age group 30-39 of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	911	907.687
1	287	300.749
2	130	115.583
3	41	43.942
4	14	15.990
5+	8	8.050
Total	1391	1392
Parameters		$\lambda = 4.0522$ $a = 0.6675$ $b = 4.2125$
$\chi^2_{0.5}$		2.6976
d.f.		2
Risk of child death (p)		$p = 0.1368$

Table 7
Observed and Expected distribution of females according to the number of child deaths in age group 40-49 of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	767	761.689
1	353	366.048
2	217	209.032
3	115	117.972
4	72	63.087
5	26	31.476
6	12	14.571
7+	12	10.126
Total	1574	1574
Parameters		$\lambda = 4.5405, a = 0.6444, b = 2.0457$
$\chi^2_{0.5}$		3.8931
d.f.		4
Risk of child death (p)		$p = 0.2396$

Table 8
Observed and Expected distribution of females according to the number of child deaths in urban females of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	562	565.221
1	157	150.255
2	60	58.689
3	14	24.300
4	15	10.031
5+	7	6.504
Total	815	815
Parameters		$\lambda = 5.7355, a = 0.4779, b = 4.8796$
$\chi^2_{0.5}$		7.2154
d.f.		2
Risk of child death (p)		$p = 0.0892$

Table 9
Observed and Expected distribution of females according to the number of child deaths in rural females of Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	1358	1343.394
1	528	565.618
2	301	278.288
3	143	138.798
4	71	67.959
5	25	32.252
6	14	14.743
7+	12	10.948
Total	2452	2452
Parameters		$\lambda = 6.8574$, $a = 0.6955$, $b = 4.7592$
$\chi^2_{0.5}$		6.5469
d.f.		4
Risk of child death (p)		$p = 0.1275$

Table 10
Observed and Expected distribution of females according to the number of child deaths in Uttar Pradesh

No. of Child Deaths	Observed Frequency	Expected Frequency
0	1920	1911.293
1	685	710.855
2	361	337.416
3	157	164.651
4	86	78.915
5	30	36.506
6	15	16.163
7+	13	11.201
Total	3267	3267
Parameters		$\lambda = 5.8817$, $a = 0.5937$, $b = 3.8625$
$\chi^2_{0.5}$		5.1525
d.f.		4
Risk of child death (p)		$p = 0.1332$

Table 11
Summary of parameters involved in the probability model for Uttar Pradesh data

Regions/categories	Total number of females (N)	Total number of childless females (n_0)	Total number of females having at least one child (N- n_0)	Estimated number of child deaths per	Estimated Mean number of births to females having at least one child ($\hat{\lambda}$)	Estimated Mean number of births to females ($\hat{\lambda}_1$)
Hill	557	116	441	70.4	4.5099	3.5707
Western	1320	324	996	170.9	4.1954	3.1656
Central	681	194	487	239.8	3.8271	2.7369
Eastern	1412	388	1024	143.2	6.5813	4.7728
Bundelkhand	415	96	319	162.3	5.6415	4.3365
Current Age Group (30-39)	1449	58	1391	136.8	4.0522	3.8901
Current Age Group (40-49)	1608	34	1574	239.6	4.5405	4.4445
Uttar Pradesh (Rural)	3382	930	2452	127.5	6.8574	4.9717
Uttar Pradesh (Urban)	1003	188	815	89.2	5.7355	4.6605
Uttar Pradesh	4385	1118	3267	133.2	5.8817	4.3821

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