Course Review

Course Review **Question 1 Page 479**

a) An even function is symmetric with respect to the y-axis. An odd function is symmetric with respect to the origin.

b) Substitute $-x$ for x in $f(x)$. If $f(-x) = f(x)$ for all x, the function is even. If $f(-x) = -f(x)$ for all x, the function is odd.

Course Review **Question 2 Page 479**

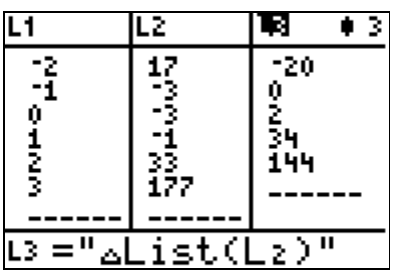
Answers may vary. A sample solution is shown.
A polynomial function has the form $f(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$. For a polynomial function of degree n, where n is a positive integer, the nth differences are equal (or constant).

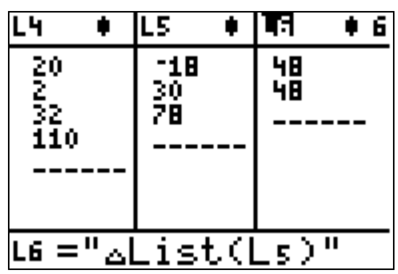
Course Review **Question 3 Page 479**

$f(x)$ extends from quadrant 3 to 4, since it has an even exponent and negative coefficient.
$g(x)$ extends from quadrant 2 to 1, since it has an even exponent and positive coefficient.
$h(x)$ extends from quadrant 3 to 1, since it has an odd exponent and positive coefficient.

Course Review **Question 4 Page 479**

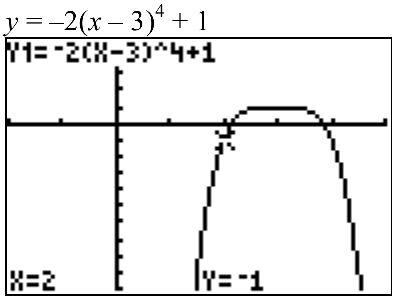
Enter the data into a graphing calculator.
Press STAT, EDIT. Enter the values from x into L1 and the values from y into L2. To find the first differences, type in as shown below for L3. Continue until the differences are constant. L6 is the fourth differences, so the degree of the polynomial is 4.



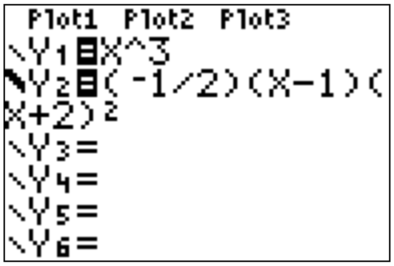


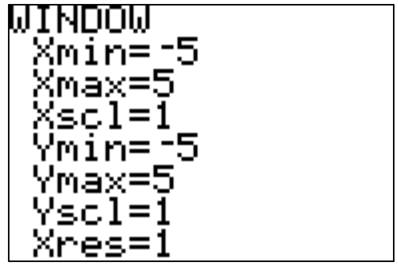
Course Review **Question 5 Page 479**

$y = -2(x - 3)^4 + 1$

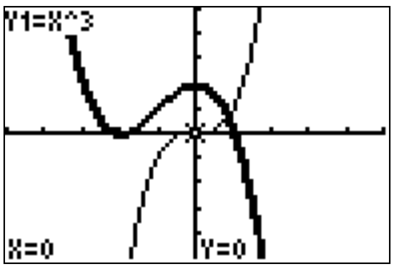


Course Review **Question 6 Page 479**

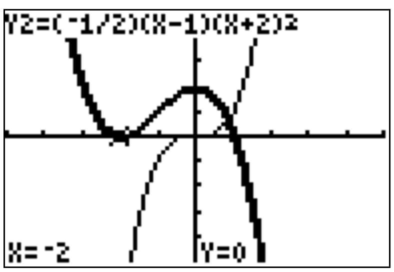


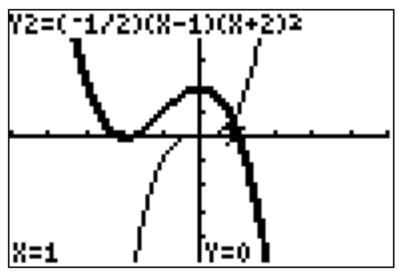


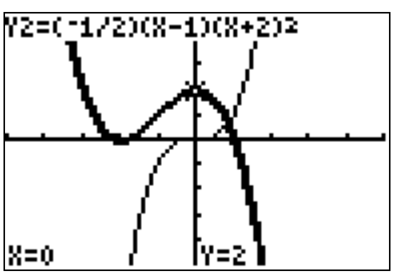
$f(x) = x^3$: x-intercept 0, y-intercept 0, $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$



$g(x) = -\frac{1}{2}(x-1)(x+2)^2$: x-intercept –2 and 1, y-intercept 2, $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$







Course Review **Question 7 Page 479**

a) $f(3) = 2(3)^4 + 5(3)^3 - (3)^2 - 3(3) + 1$
$= 280$
$f(1) = 2(1)^4 + 5(1)^3 - (1)^2 - 3(1) + 1$
$= 4$
$$\text{Average slope or Average rate of change} = \frac{280-4}{3-1} = 138$$

b) $f(0.999) = 2(0.999)^4 + 5(0.999)^3 - (0.999)^2 - 3(0.999) + 1$
$\doteq 3.982\ 03$
$f(2.999) = 2(2.999)^4 + 5(2.999)^3 - (2.999)^2 - 3(2.999) + 1$
$\doteq 279.658\ 15$
$$\begin{aligned}\text{Average rate of change} &= \frac{f(1) - f(0.999)}{1-0.999}\\ &\doteq \frac{4-3.982\ 03}{0.001}\\ &\doteq 17.97\end{aligned}$$
$$\begin{aligned}\text{Average rate of change} &= \frac{f(3) - f(2.999)}{1-0.999}\\ &\doteq \frac{280-279.658\ 15}{0.001}\\ &\doteq 342\end{aligned}$$
The instantaneous rate of change (instantaneous slope) at 1 is approximately 18.
The instantaneous rate of change (instantaneous slope) at 3 is approximately 342.

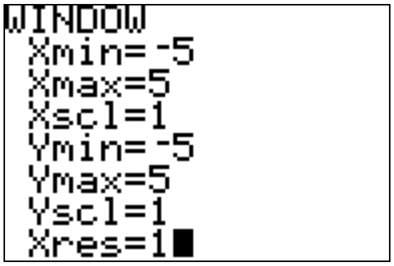
c) Answers may vary. A sample solution is shown.
The graph is increasing for $1 < x < 3$.

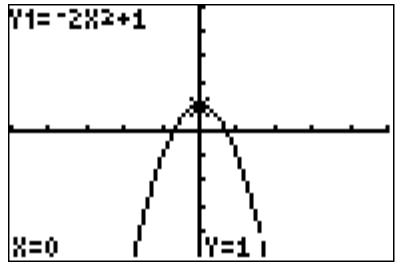
Course Review **Question 8 Page 479**

a) From the graph, the zeros occur at–3, –1, 2, and 5.
The graph extends from quadrant 2 to 1, so the function has even degree with positive leading coefficient.
$y = (x+3)(x+1)(x-2)(x-5)$

b) From the graph, the zeros occur at –5 and 1 (order 2).
The graph extends from quadrant 2 to 4, so the function has odd degree with negative leading coefficient.
$y = -(x+5)(x-1)^2$

Course Review **Question 9 Page 479**





For $x < 0$, the slope is positive and decreasing.
For $x > 0$, the slope is negative and decreasing.

Course Review **Question 10 Page 479**

a)
$$
\begin{array}{r l}
 & \;\; 2x^2 + 4x \\
2x-1 & \overline{)\,4x^3 + 6x^2 - 4x + 2} \\
 & \;\; \underline{4x^3 - 2x^2} \\
 & \qquad\quad 8x^2 - 4x \\
 & \qquad\quad \underline{8x^2 - 4x} \\
 & \qquad\qquad\quad 0 + 2
\end{array}
$$

$$\frac{4x^3 + 6x^2 - 4x + 2}{2x - 1} = 2x^2 + 4x + \frac{2}{2x - 1},\ x \neq \frac{1}{2}$$

b)
$$
\begin{array}{r l}
 & \;\; 2x^2 + 4x + 4 \\
x-2 & \overline{)\,2x^3 + 0x^2 - 4x + 8} \\
 & \;\; \underline{2x^3 - 4x^2} \\
 & \qquad\quad 4x^2 - 4x \\
 & \qquad\quad \underline{4x^2 - 8x} \\
 & \qquad\qquad\quad 4x + 8 \\
 & \qquad\qquad\quad \underline{4x - 8} \\
 & \qquad\qquad\qquad 16
\end{array}
$$

$$\frac{2x^3 - 4x + 8}{x - 2} = 2x^2 + 4x + 4 + \frac{16}{x - 2},\ x \neq 2$$

c)

$$\begin{array}{r|l} & x^2 - 5x + 15 \\ \hline x+2 & x^3 - 3x^2 + 5x - 4 \\ & \underline{x^3 + 2x^2} \\ & \quad -5x^2 + 5x \\ & \quad \underline{-5x^2 - 10x} \\ & \qquad\qquad 15x - 4 \\ & \qquad\qquad \underline{15x + 30} \\ & \qquad\qquad\quad -34 \end{array}$$

$$\frac{x^3 - 3x^2 + 5x - 4}{x+2} = x^2 - 5x + 15 - \frac{34}{x+2},\ x \neq -2$$

d)

$$\begin{array}{r|l} & 5x^3 - 8x^2 + 10x - 6 \\ \hline x+1 & 5x^4 - 3x^3 + 2x^2 + 4x - 6 \\ & \underline{5x^4 + 5x^3} \\ & \quad -8x^3 + 2x^2 \\ & \quad \underline{-8x^3 - 8x^2} \\ & \qquad\qquad 10x^2 + 4x \\ & \qquad\qquad \underline{10x^2 + 10x} \\ & \qquad\qquad\qquad -6x - 6 \\ & \qquad\qquad\qquad \underline{-6x - 6} \\ & \qquad\qquad\qquad\quad 0 \end{array}$$

$$\frac{5x^4 - 3x^3 + 2x^2 + 4x - 6}{x+1} = 5x^3 - 8x^2 + 10x - 6,\ x \neq -1$$

Course Review **Question 11 Page 479**

Note that different methods were used to factor.

a)

$$\begin{aligned} P(1) &= 1^3 + 4(1)^2 + 1 - 6 \\ &= 0 \end{aligned}$$

$(x-1)$ is a factor

$$\begin{aligned} P(-2) &= (-2)^3 + 4(-2)^2 + (-2) - 6 \\ &= 0 \end{aligned}$$

$(x+2)$ is a factor

$$\begin{aligned} P(-3) &= (-3)^3 + 4(-3)^2 + (-3) - 6 \\ &= 0 \end{aligned}$$

$(x+3)$ is a factor

$$x^3 + 4x^2 + x - 6 = (x-1)(x+2)(x+3)$$

b) $P(-1) = 2(-1)^3 + (-1)^2 - 16(-1) - 15$
$= 0$
$(x+1)$ is a factor

$$
\begin{array}{r}
2x^2 - x - 15 \\
x+1 \overline{)\, 2x^3 + x^2 - 16x - 15} \\
\underline{2x^3 + 2x^2} \qquad\qquad\quad \\
-x^2 - 16x \qquad \\
\underline{-x^2 - x} \qquad\quad \\
-15x - 15 \\
\underline{-15x - 15} \\
0
\end{array}
$$

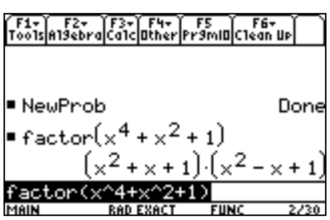
$$
\begin{aligned}
2x^3 + x^2 - 16x - 15 &= (x+1)(2x^2 - x - 15) \\
&= (x+1)(x-3)(2x+5) \\
&= (x-3)(x+1)(2x+5)
\end{aligned}
$$

c) $P(2) = 2^3 - 7(2)^2 + 11(2) - 2$
$= 0$
$(x-2)$ is a factor

–2	1	–7	11	–2
–		–2	10	–2
×	1	–5	1	0

$x^3 - 7x^2 + 11x - 2 = (x-2)(x^2 - 5x + 1)$

d)



$x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$

Course Review **Question 12 Page 480**

a) $P(5) = 4(5)^3 - 7(5)^2 + 3(5) + 5$
$= 345$

b) $P\left(-\frac{2}{3}\right) = 6\left(-\frac{2}{3}\right)^4 + 7\left(-\frac{2}{3}\right)^2 - 2\left(-\frac{2}{3}\right) - 4$
$= \frac{44}{27}$

Course Review **Question 13 Page 480**

a) $P(-5) = 3(-5)^5 - 4(-5)^3 - 4(-5)^2 + 15$
$= -8960$

Since $P(-5) \neq 0$, $(x+5)$ is not a factor

b) $P(-1) = 2(-1)^3 - 4(-1)^2 + 6(-1) + 5$
$= -7$

Since $P(-1) \neq 0$, $(x+1)$ is not a factor

Course Review **Question 14 Page 480**

Parts a) through d) have been solved using different methods.

a)
$$x^4 - 81 = 0$$
$$(x^2 - 9)(x^2 + 9) = 0 \quad \text{difference of squares}$$
$$(x-3)(x+3)(x^2+9) = 0 \quad \text{difference of squares}$$
$$x = 3 \text{ or } x = -3$$

Note: There are no real values of x for which $x^2 + 9 = 0$.

b) $P(-1) = (-1)^3 - (-1)^2 - 10(-1) - 8$
$= 0$

$P(-2) = (-2)^3 - (-2)^2 - 10(-2) - 8$
$= 0$

$P(4) = (4)^3 - (4)^2 - 10(4) - 8$
$= 0$

$x = -2$ or $x = -1$ or $x = 4$

c)

$$
\begin{aligned}
(8x^3+27)&=0\\
(2x+3)\left[(2x)^2-2x(3)+3^2\right]&=0\\
(2x+3)(4x^2-6x+9)&=0\\
x&=-\frac{3}{2}
\end{aligned}
$$

d) $12x^4+16x^3-7x^2-6x=0$

$x(12x^3+16x^2-7x-6)=0$

$$
\begin{aligned}
P\left(-\frac{1}{2}\right)&=12\left(-\frac{1}{2}\right)^3+16\left(-\frac{1}{2}\right)^2-7\left(-\frac{1}{2}\right)-6\\
&=0
\end{aligned}
$$

Divide $\left(12x^3+16x^2-7x-6\right)$ by $\left(2x+1\right)$

$$
\begin{array}{r|l}
 & 6x^2+5x-6 \\
\hline
2x+1 & 12x^3+16x^2-7x-6 \\
 & \underline{12x^3+6x^2} \\
 & \quad 10x^2-7x \\
 & \quad \underline{10x^2+5x} \\
 & \qquad -12x-6 \\
 & \qquad \underline{-12x-6} \\
 & \qquad\qquad 0
\end{array}
$$

$x(2x+1)(6x^2+5x-6)=0$

$x(2x+1)(2x+3)(3x-2)=0$

$$x=-\frac{3}{2} \text{ or } x=-\frac{1}{2} \text{ or } x=0 \text{ or } x=\frac{2}{3}$$

Course Review **Question 15 Page 480**

a) Answers may vary. A sample solution is shown.

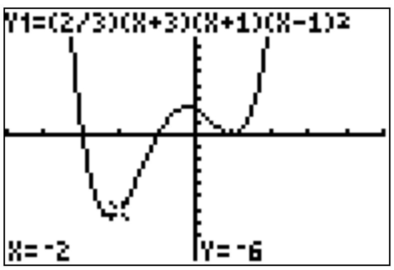
$y = k(x+3)(x+1)(x-1)^2$

$y = 2(x+3)(x+1)(x-1)^2$

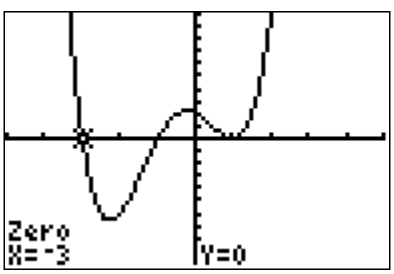
$y = -(x+3)(x+1)(x-1)^2$

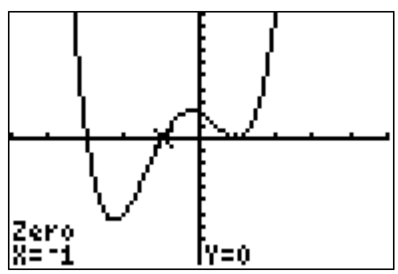
b)
$$
\begin{aligned}
y &= k(x+3)(x+1)(x-1)^2 \\
-6 &= k(-2+3)(-2+1)(-2-1)^2 \\
-6 &= k(1)(-1)(-3)^2 \\
k &= \frac{-6}{-9} \\
y &= \frac{2}{3}(x+3)(x+1)(x-1)^2
\end{aligned}
$$

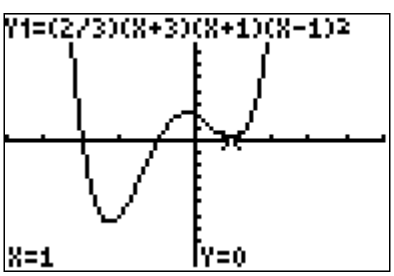
c)



d)







The function is positive for $x < -3$, $-1 < x < 1$, $x > 1$.

Course Review **Question 16 Page 480**

a) Case 1:

$x - 4 > 0$ and $x + 3 > 0$

$x > 4$ $x > -3$

$x > 4$ is a solution, since it includes $x > -3$

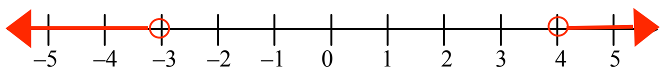
Case 2:

$x - 4 < 0$ and $x + 3 < 0$

$x < 4$ $x < -3$

$x < -3$ is a solution, since it includes $x < 4$

The solution is $x < -3$ or $x > 4$



b) $(2x - 3)(x + 2) < 0$

Case 1:

$2x - 3 < 0$ and $x + 2 > 0$

$2x < 3$ $x > -2$

$x < \frac{3}{2}$

$-2 < x < \frac{3}{2}$ is a solution

Case 2:

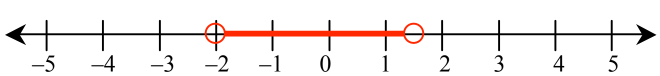
$2x - 3 > 0$ and $x + 2 < 0$

$2x > 3$ $x < -2$

$x > \frac{3}{2}$

No solution

The solution is $-2 < x < \frac{3}{2}$



c) $x^3 - 2x^2 - 13x - 10 \le 0$

Factor using the factor theorem

$$\begin{aligned} P(-1) &= (-1)^3 - 2(-1)^2 - 13(-1) - 10 \\ &= 0 \end{aligned}$$

$(x+1)$ is a factor

Divide

1	1	–2	–13	–10
–		1	–3	–10
×	1	–3	–10	0

$(x+1)(x^2 - 3x - 10) \le 0$

$(x+1)(x-5)(x+2) \le 0$

When $(x+1)(x-5)(x+2) = 0$, $x = -1$ or $x = 5$ or $x = -2$

When $(x+1)(x-5)(x+2) < 0$

Case 1:

$x+1<0$ and $x-5<0$ and $x+2<0$

$x<-1$ $\quad$ $x<5$ $\quad$ $x<-2$

$x<-2$ is a solution, since it includes $x<-1$, $x<5$

Case 2:

$x+1>0$ and $x-5>0$ and $x+2<0$

$x>-1$ $\quad$ $x>5$ $\quad$ $x<-2$

No solution

Case 3:

$x+1>0$ and $x-5<0$ and $x+2>0$

$x>-1$ $\quad$ $x<5$ $\quad$ $x>-2$

$-1<x<5$ is a solution, since it includes $x>-2$

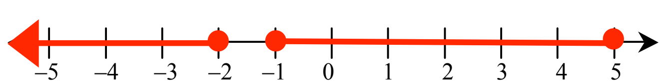
Case 4:

$x+1<0$ and $x-5>0$ and $x+2>0$

$x<-1$ $\quad$ $x>5$ $\quad$ $x>-2$

No solution

The solution is $x \le -2$ or $-1 \le x \le 5$



a) $x-2=0$

$x=2$

The vertical asymptote has equation $x = 2$.

As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $f(x)$ approaches 0. Thus, $f(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

b) $x+3=0$

$x=-3$

The vertical asymptote has equation $x = -3$.

As $x \to \pm\infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$\begin{aligned} g(x) &= \frac{\frac{x}{x}+\frac{5}{x}}{\frac{x}{x}+\frac{3}{x}} \\ &= \frac{1+\frac{5}{x}}{1+\frac{3}{x}} \end{aligned}$$

As $x \to \pm\infty$, $\frac{5}{x}$ and $\frac{3}{x}$ get very close to 0.

$$g(x) \to \frac{1+0}{1+0}$$

$$g(x) \to 1$$

The horizontal asymptote has equation $y = 1$.

c)
$$\begin{aligned} x^2-9&=0 \\ (x-3)(x+3)&=0 \\ x = 3 \text{ and } x &= -3 \end{aligned}$$

The vertical asymptotes have equations $x = -3$ and $x = 3$.

As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $h(x)$ approaches 0. Thus, $h(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

d) Since $x^2 + 4 = 0$ has no real roots, there are no vertical asymptotes.

As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $k(x)$ approaches 0. Thus, $k(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

a) i) The vertical asymptote has equation $x = -4$.
As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $f(x)$ approaches 0.
Thus, $f(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

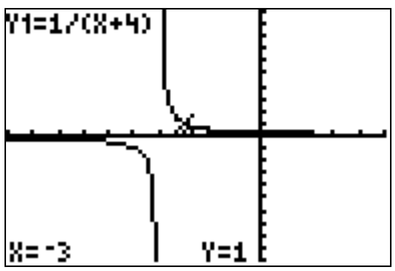
ii) No x-intercept
Let $x = 0$

$$y = \frac{1}{0+4}$$

$$y = \frac{1}{4}$$

y-intercept is $\frac{1}{4}$

iii)



iv) The function is decreasing for $x < -4$ and $x > -4$.

v) $\{x \in \mathbb{R}, x \neq -4\}$, $\{y \in \mathbb{R}, y \neq 0\}$

b) i) The vertical asymptote has equation $x = 2$.
As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $f(x)$ approaches 0.
Thus, $f(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

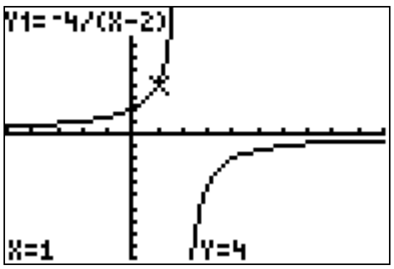
ii) No x-intercept
Let $x = 0$

$$y = \frac{-4}{0-2}$$

$$y = 2$$

y-intercept is 2

iii)



iv) The function is increasing for $x < 2$ and $x > 2$.

v) $\{x \in \mathbb{R}, x \neq 2\}$, $\{y \in \mathbb{R}, y \neq 0\}$

c) i) The vertical asymptote has equation $x = -3$.

As $x \to \pm\infty$, the numerator and denominator both approach infinity.

Divide each term by x.

$$g(x) = \frac{\frac{x}{x} - \frac{1}{x}}{\frac{x}{x} + \frac{3}{x}}$$

$$= \frac{1 - \frac{1}{x}}{1 + \frac{3}{x}}$$

As $x \to \pm\infty$, $\frac{1}{x}$ and $\frac{3}{x}$ get very close to 0.

$$g(x) \to \frac{1-0}{1+0}$$

$$g(x) \to 1$$

The horizontal asymptote has equation $y = 1$.

ii) Let $y = 0$

$$0 = \frac{x-1}{x+3}$$

$$0 = x - 1$$

$$x = 1$$

The x-intercept is 1

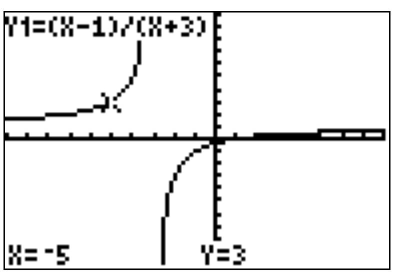
Let $x = 0$

$$y = \frac{0-1}{0+3}$$

$$y = -\frac{1}{3}$$

The y-intercept is $-\frac{1}{3}$

iii)



iv) The function is increasing for $x < -3$ and $x > -3$.

v) $\{x \in \mathbb{R}, x \neq -3\}$, $\{y \in \mathbb{R}, y \neq 1\}$

d) i) The vertical asymptote has equation $x = -\frac{1}{5}$

As $x \to \pm\infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$g(x) = \frac{\frac{2x}{x} + \frac{3}{x}}{\frac{5x}{x} + \frac{1}{x}}$$

$$= \frac{2 + \frac{3}{x}}{5 + \frac{1}{x}}$$

As $x \to \pm\infty$, $\frac{1}{x}$ and $\frac{3}{x}$ get very close to 0.

$$g(x) \to \frac{2+0}{5+0}$$

$$g(x) \to \frac{2}{5}$$

The horizontal asymptote has equation $y = \frac{2}{5}$

ii) Let $y = 0$

$$0 = \frac{2x+3}{5x+1}$$

$$0 = 2x + 3$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

The x-intercept is $-\frac{3}{2}$

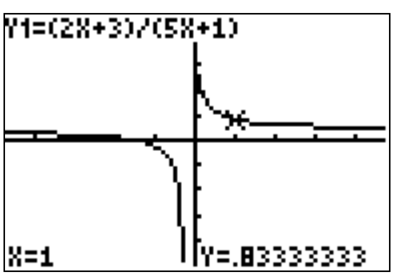
Let $x = 0$

$$y = \frac{2(0)+3}{5(0)+1}$$

$$y = 3$$

The y-intercept is 3

iii)



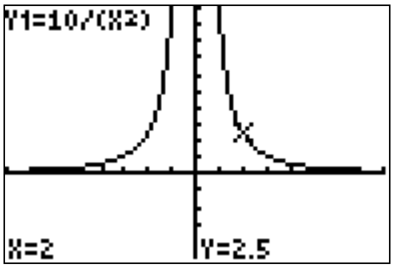
iv) The function is decreasing for $x < -\frac{1}{5}$ and $x > -\frac{1}{5}$

v) $\{x \in \mathbb{R}, x \neq -\frac{1}{5}\}$, $\{y \in \mathbb{R}, y \neq \frac{2}{5}\}$

e) i) The vertical asymptote has equation $x = 0$.
As $x \to \pm\infty$, the denominator approaches ∞, so $f(x)$ approaches 0.
Thus, $f(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

ii) There are no intercepts.

iii)



iv) The function is increasing for $x < 0$ and decreasing for $x > 0$.

v) $\{x \in \mathbb{R}, x \neq 0\}$, $\{y \in \mathbb{R}, y > 0\}$

f) i) $k(x)=\dfrac{3}{(x-9)(x+3)}$, $x \neq -3$, $x \neq 9$

The vertical asymptotes have equations $x = -3$ and $x = 9$.
As $x \to \pm\infty$, the denominator approaches $\pm\infty$, so $f(x)$ approaches 0.
Thus, $f(x)$ approaches a horizontal line at $y = 0$, but does not cross it.
The horizontal asymptote has equation $y = 0$.

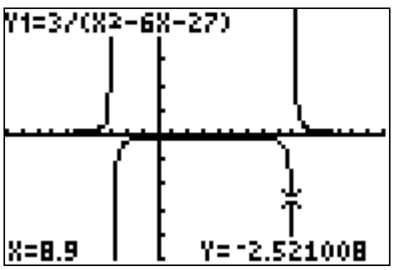
ii) No x-intercept
Let $x = 0$

$$y=\frac{3}{0^2-6(0)-27}$$

$$y=-\frac{1}{9}$$

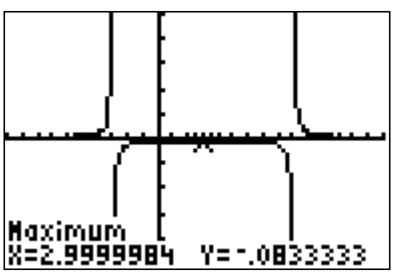
The y-intercept is $-\dfrac{1}{9}$

iii)



iv) The function is increasing for $x < -3$ and $-3 < x < 3$ and decreasing for $3 < x < 9$ and $x > 9$.

v)



$\{x \in \mathbb{R}, x \neq -3, x \neq 9\}$, $\{y \in \mathbb{R}, y \leq -\dfrac{1}{12}, y > 0\}$

$f(x)=\dfrac{1}{(x-7)(x+3)}, x\neq -3, x\neq 7$

The vertical asymptotes have equations $x = –3$ and $x = 7$.
There will be an maximum or minimum point exactly halfway between the asymptotes.

$x=\dfrac{7-3}{2}$

$x=2$

The intervals to be analyzed are $x<-3,\ -3<x<2,\ 2<x<7,\ x>7$

Consider the interval $x < –3$.
At $x = –5, f(x) \doteq 0.041\,667$.

$$\begin{aligned}\frac{f(-5)-f(-4.999)}{-5-(-4.999)}&=\frac{0.041667-0.041691}{-0.001}\\&\doteq 0.024\end{aligned}$$

The slope is approximately 0.024 at the point (–5, 0.041 667).

At $x = –4, f(x) \doteq 0.090\,909$.

$$\frac{f(-4)-f(-3.999)}{-4-(-3.999)}\doteq 0.099$$

The slope is approximately 0.099 at the point (–4, 0.090 909).
The slope is positive and increasing on the interval $x < –3$.

Consider the interval $–3 < x < 2$
At $x = –1, f(x) = –0.0625$.

$$\frac{f(-1)-f(-0.999)}{-1-(-0.999)}\doteq 0.023$$

The slope is approximately 0.023 at the point (–1, –0.0625).

At $x = 0, f(x) \doteq –0.047\,619$.

$$\frac{f(0)-f(-0.001)}{0-(-0.001)}\doteq 0.009$$

The slope is approximately 0.009 at the point (0, –0.047 619).
The slope is positive and decreasing on the interval $–3 < x < 2$.

Consider the interval $2 < x < 7$
At $x = 3, f(x) \doteq –0.041\,667$.

$$\frac{f(3)-f(2.999)}{3-(2.999)}\doteq -0.003$$

The slope is approximately –0.003 at the point (3, –0.041 667).

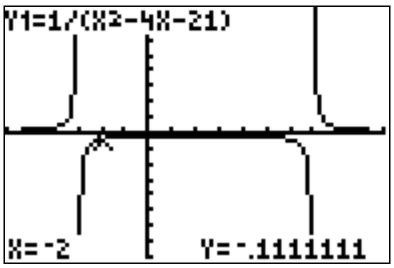
At $x = 4, f(x) \doteq –0.047\,619$.

$$\frac{f(4)-f(3.999)}{4-(3.999)}\doteq -0.009$$

The slope is approximately –0.009 at the point (4, –0.047 619).
The slope is negative and decreasing on the interval $2 < x < 7$.

Consider the interval $x > 7$.
At $x = 8, f(x) \doteq 0.090\ 909$.
$\frac{f(8)-f(7.999)}{8-(7.999)} \doteq -0.099$
The slope is approximately –0.099 at the point (8, 0.090 909).

At $x = 9, f(x) \doteq 0.041\ 667$.
$\frac{f(9)-f(8.999)}{9-(8.999)} \doteq -0.024$
The slope is approximately –0.024 at the point (9, 0.041 667).
The slope is negative and increasing on the interval $x > 7$.



Course Review **Question 20 Page 480**

a)
$$\begin{aligned}\frac{5}{x-3} &= 4\\ 5 &= 4(x-3)\\ 5 &= 4x-12\\ 4x &= 5+12\\ 4x &= 17\\ x &= \frac{17}{4}\end{aligned}$$

b)
$$\begin{aligned}\frac{2}{x-1} &= \frac{4}{x+5}\\ 2(x+5) &= 4(x-1)\\ 2x+10 &= 4x-4\\ 4x-2x &= 10+4\\ 2x &= 14\\ x &= 7\end{aligned}$$

c)
$$\begin{aligned}\frac{6}{x^2+4x+7} &= 2\\ 6 &= 2(x^2+4x+7)\\ 6 &= 2x^2+8x+14\\ 2x^2+8x+14-6 &= 0\\ 2x^2+8x+8 &= 0\\ 2(x^2+4x+4) &= 0\\ 2(x+2)^2 &= 0\\ x &= -2\end{aligned}$$

a) $\dfrac{3}{x-4} < 5$

Because $x-4 \neq 0,$ either $x < 4$ or $x > 4$

Case 1:$x > 4$

$3 < 5(x-4)$ Multiply both sides by $(x-4),$ which is positive if $x > 4$

$3 < 5x - 20$

$3 + 20 < 5x$

$5x > 23$

$x > \dfrac{23}{5}$

$x > \dfrac{23}{5}$ is within the inequality $x > 4,$ so the solution is $x > \dfrac{23}{5}$

Case 2:$x < 4$

$3 > 5(x-4)$ Multiply both sides by $(x-4),$ which is negative if $x < 4$

$3 > 5x - 20$

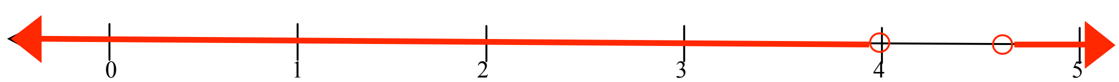
$3 + 20 > 5x$

$5x < 23$

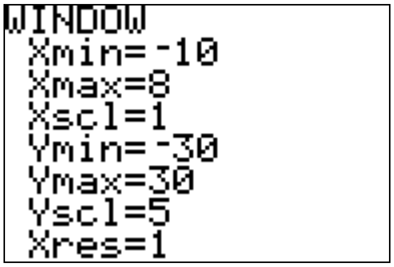
$x < \dfrac{23}{5}$

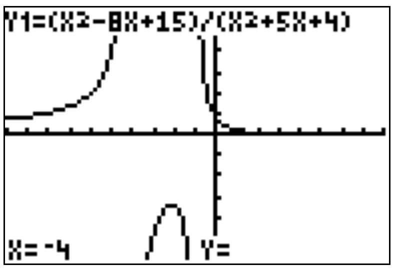
$x < 4$ is within the inequality $x < \dfrac{23}{5},$ so the solution is $x < 4$

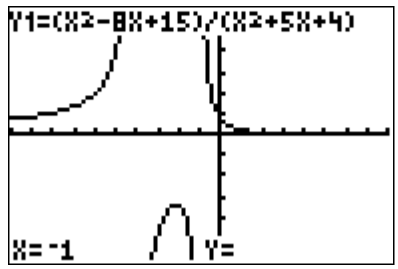
Combining the two cases, the solution to the inequality is $x < 4$ or $x > \dfrac{23}{5}$

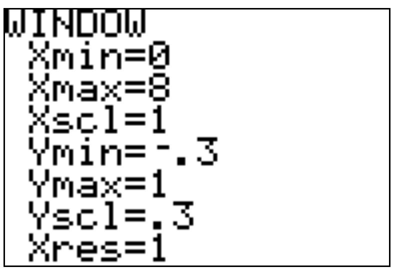


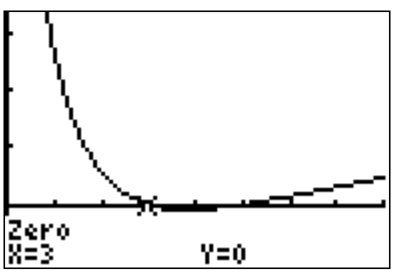
b) Enter the function $f(x)=\dfrac{x^2-8x+15}{x^2+5x+4}$ and view its graph.

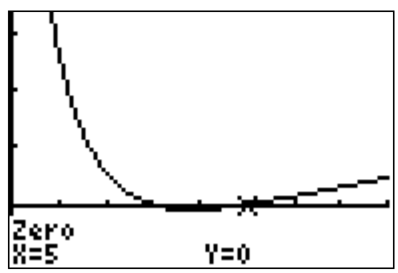






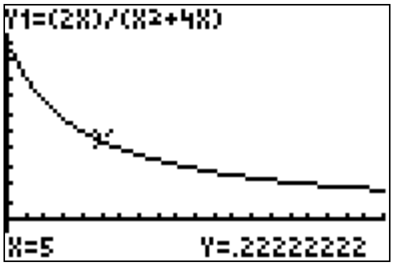






There are asymptotes at $x = -4$ and $x = -1$, so there are no solutions there.
Using the zero operation, the zeros are $x = 3$ and $x = 5$.
Using the graph and the zeros, you can see that $f(x) \geq 0$ for $x < -4$, $-1 < x < 3$, and $x \geq 5$.

a)



b) As $x \to \pm\infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$R(t) = \frac{\frac{2t}{t^2}}{\frac{t^2}{t^2} + \frac{4t}{t^2}}$$

$$= \frac{\frac{2}{t}}{1 + \frac{4}{t}}$$

As $x \to \pm\infty$, $\frac{2}{t}$ and $\frac{4}{t}$ get very close to 0.

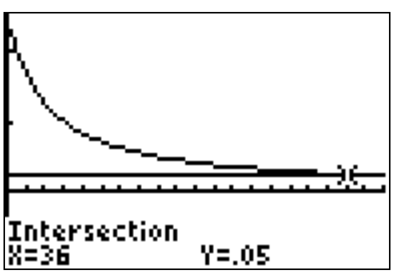
$$R(t) \to \frac{0}{1+0}$$

$$R(t) \to 0$$

The horizontal asymptote has equation $R(t) = 0$.

The chemical will not completely dissolve.

c)



$\{t \in \mathbb{R}, 0 \le x \le 36\}$

Course Review **Question 23 Page 481**

a) $135° \times \frac{\pi}{180°} = \frac{135\pi}{180}$
$= \frac{3\pi}{4}$

b) $-60° \times \frac{\pi}{180°} = -\frac{60\pi}{180}$
$= -\frac{\pi}{3}$

Course Review **Question 24 Page 481**

a) $\frac{\pi}{6} \times \frac{180°}{\pi} = 30°$

b) $\frac{9\pi}{8} \times \frac{180°}{\pi} = 202.5°$

Course Review **Question 25 Page 481**

Find the arc length

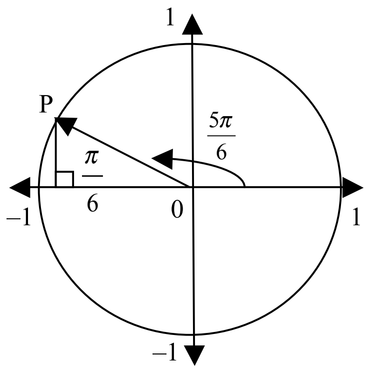
$$\begin{aligned}\theta &= \frac{a}{r}\\ a &= r\theta\\ &= 9\left(\frac{5\pi}{12}\right)\\ &= \frac{15\pi}{4}\end{aligned}$$

The perimeter of the sector is the sum of the arc length and 2 radii

$$\begin{aligned}P &= \frac{15\pi}{4} + 2(9)\\ &= \frac{15\pi}{4} + \frac{72}{4}\\ &= \frac{15\pi + 72}{4}\end{aligned}$$

Course Review **Question 26 Page 481**

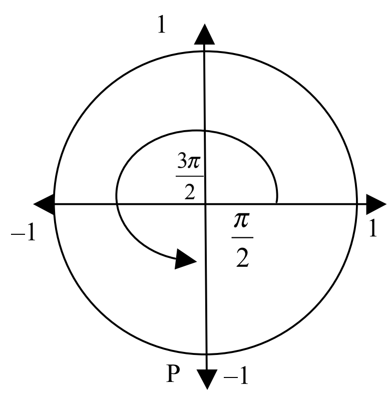
a)



The terminal arm of an angle of $\frac{\pi}{6}$ intersects the unit circle at a point with coordinates $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. Since the terminal arm of an angle of $\frac{5\pi}{6}$ is in the second quadrant, the coordinates of the point of intersection are $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

$$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$$

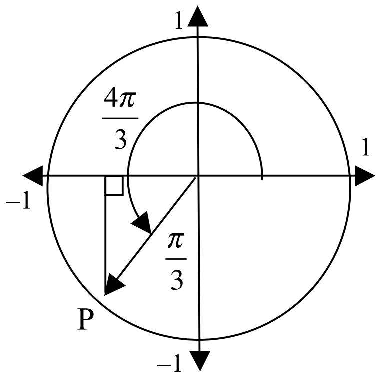
b)



The terminal arm of an angle of $\frac{\pi}{2}$ intersects the unit circle at a point with coordinates (0, 1). Since the terminal arm of an angle of $\frac{3\pi}{2}$ is in the second quadrant, the coordinates of the point of intersection are P(0, –1).

$$\sin\frac{3\pi}{2} = -1$$

c)



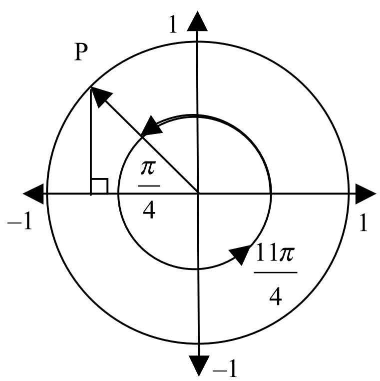
The terminal arm of an angle of $\frac{\pi}{3}$ intersects the unit circle at a point with

coordinates $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Since the terminal arm of an angle of $\frac{4\pi}{3}$ is in the second quadrant,

the coordinates of the point of intersection are $\mathrm{P}\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$.

$$\tan\frac{4\pi}{3} = \sqrt{3}$$

d)



The terminal arm of an angle of $\frac{\pi}{4}$ intersects the unit circle at a point with

coordinates $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Since the terminal arm of an angle of $\frac{11\pi}{4}$ is in the second quadrant,

the coordinates of the point of intersection are $\mathrm{P}\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$$\cot\frac{11\pi}{4} = -1$$

Course Review **Question 27 Page 481**

a) $$\cos\frac{\pi}{12} = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$$
$$= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

b) $$\sin\frac{11\pi}{12} = \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$$
$$= \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$$
$$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \left(-\frac{1}{2}\right) \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Course Review **Question 28 Page 481**

a) **L.S.** $= \sec x - \tan x$
$$= \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$ reciprocal identity and quotient identity
$$= \frac{1-\sin x}{\cos x}$$
R.S. $= \dfrac{1-\sin x}{\cos x}$

Since **L.S.** = **R.S.**, $\sec x - \tan x = \dfrac{1-\sin x}{\cos x}$ is an identity.

b) **L.S.** $= (\csc x - \cot x)^2$

$= \csc^2 x - 2\csc x \cot x + \cot^2 x$

$= \dfrac{1}{\sin^2 x} - 2\left(\dfrac{1}{\sin x}\right)\left(\dfrac{\cos x}{\sin x}\right) + \dfrac{\cos^2 x}{\sin^2 x}$ reciprocal and quotient identities

$= \dfrac{1 - 2\cos x + \cos^2 x}{\sin^2 x}$

$= \dfrac{(1-\cos x)^2}{1-\cos^2 x}$ factor the numerator
Pythagorean identity in the denominator

$= \dfrac{(1-\cos x)^2}{(1-\cos x)(1+\cos x)}$ factor the denominator

$= \dfrac{1-\cos x}{1+\cos x}$

R.S. $= \dfrac{1-\cos x}{1+\cos x}$

Since **L.S.** = **R.S.**, $(\csc x - \cot x)^2 = \dfrac{1-\cos x}{1+\cos x}$ is an identity.

c) **L.S.** $= \sin 2A$

$= \sin(A + A)$

$= \sin A \cos A + \cos A \sin A$ Compound angle formula

$= 2\sin A \cos A$ This is the double angle formula

R.S. $= \dfrac{2\tan A}{\sec^2 A}$

$= \dfrac{2\left(\dfrac{\sin A}{\cos A}\right)}{\left(\dfrac{1}{\cos^2 A}\right)}$ Reciprocal and Quotient identities

$= \dfrac{2\sin A}{\cos A} \times \cos^2 A$

$= 2\sin A \cos A$

Since **L.S.** = **R.S.**, $\sin 2A = \dfrac{2\tan A}{\sec^2 A}$ is an identity.

d) **L.S.** $= \cos(x+y)\cos(x-y)$

$= [\cos x\cos y - \sin x\sin y][\cos x\cos y + \sin x\sin y]$ compound angle formula

$= \cos^2 x\cos^2 y - \sin^2 x\sin^2 y$

$= \cos^2 x(1-\sin^2 y) - (1-\cos^2 x)\sin^2 y$ Pythagorean identity

$= \cos^2 x - \cos^2 x\sin^2 y - \sin^2 y + \cos^2 x\sin^2 y$

$= \cos^2 x - \sin^2 y$

$= \cos^2 x - (1-\cos^2 y)$ Pythagorean identity

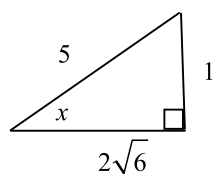
$= \cos^2 x + \cos^2 y - 1$

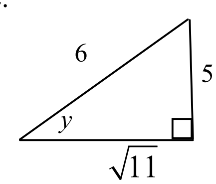
R.S. $= \cos^2 x + \cos^2 y - 1$

Since **L.S.** = **R.S.**, $\cos(x+y)\cos(x-y) = \cos^2 x + \cos^2 y - 1$ is an identity.

Course Review **Question 29 Page 481**

Use the Pythagorean Theorem to find the other side.





Using the CAST rule we know that cos x will be positive and cos y will be positive.

$a^2 = 5^2 - 1^2$
$a^2 = 25 - 1$
$a^2 = 24$
$a = \sqrt{24}$
$a = \sqrt{4\times 6}$
$a = 2\sqrt{6}$

$b^2 = 6^2 - 5^2$
$b^2 = 36 - 25$
$b^2 = 11$
$b = \sqrt{11}$

$\cos x = \dfrac{2\sqrt{6}}{5}$ and $\cos y = \dfrac{\sqrt{11}}{6}$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$= \dfrac{1}{5}\left(\dfrac{\sqrt{11}}{6}\right) + \dfrac{2\sqrt{6}}{5}\left(\dfrac{5}{6}\right)$

$= \dfrac{\sqrt{11}+10\sqrt{6}}{30}$

Course Review **Question 30 Page 481**

Since an angle of $\frac{5\pi}{8}$ lies in the second quadrant, it can be expressed as a sum of $\frac{\pi}{2}$ and an angle a. Find the measure of angle a.

$$\frac{5\pi}{8} = \frac{\pi}{2} + a$$
$$a = \frac{5\pi}{8} - \frac{\pi}{2}$$
$$a = \frac{5\pi}{8} - \frac{4\pi}{8}$$
$$a = \frac{\pi}{8}$$

Now apply the cofunction identity.

$$\begin{aligned}\cos\frac{5\pi}{8} &= \cos\left(\frac{\pi}{2} + \frac{\pi}{8}\right)\\ &= -\sin\frac{\pi}{8}\end{aligned}$$

$$y = \frac{\pi}{8}$$

Course Review **Question 31 Page 481**

a) $$\begin{aligned}\text{period} &= \frac{2\pi}{k}\\ &= \frac{2\pi}{2}\\ &= \pi\end{aligned}$$

The amplitude is 3.

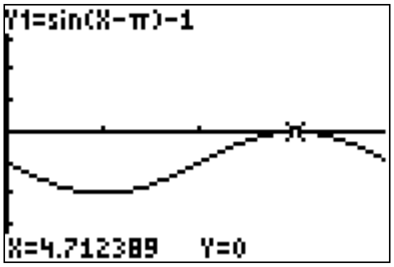
The phase shift is $\frac{\pi}{2}$ rad to the right.

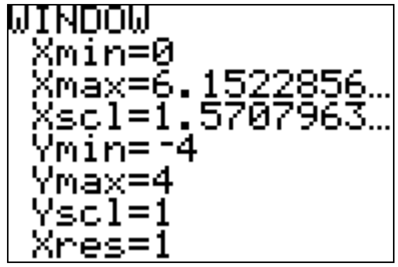
The vertical translation is 4 units upwards.

b) The range can be found from the amplitude and vertical shift.
The maximum value for y is $4 + 3 = 7$.
The minimum value for y is $4 - 3 = 1$.
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, 1 \le y \le 7\}$

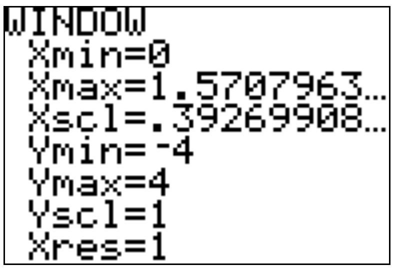
Course Review **Question 32 Page 481**

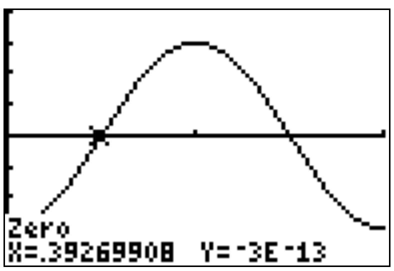
a) x-intercept: $\frac{3\pi}{2}$

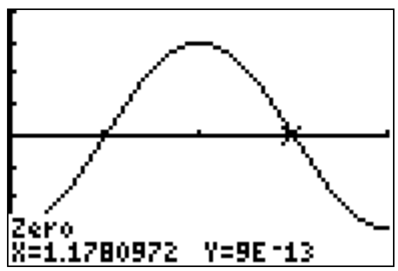




b)

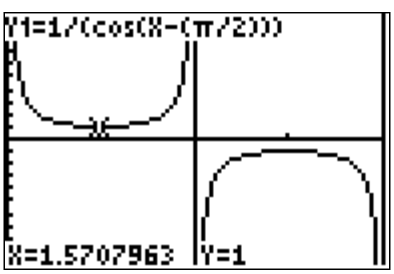






The x-intercepts are $\frac{\pi}{8}$ and $\frac{3\pi}{8}$.

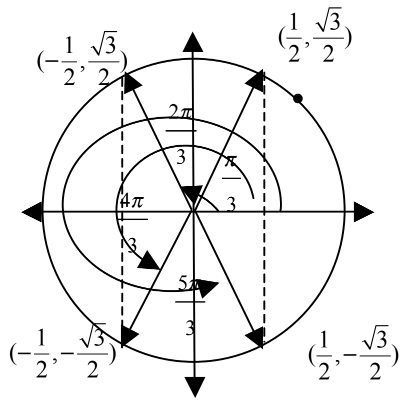
c)



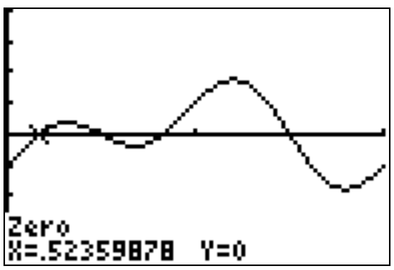
The asymptotes have equations $x = 0$, $x = \pi$, and $x = 2\pi$

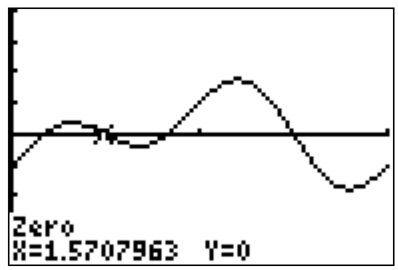
a) $\sin\theta = -\frac{\sqrt{3}}{2}$

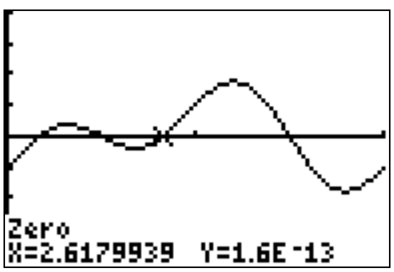
$\theta = \frac{4\pi}{3}$ or $\theta = \frac{5\pi}{3}$

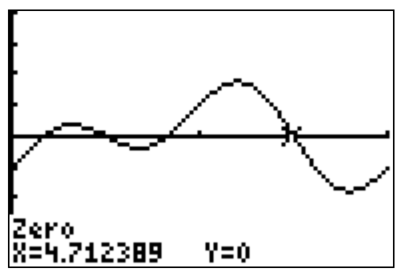


b)







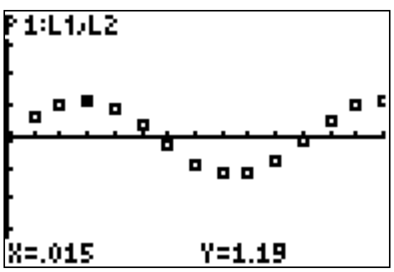


$\theta = \frac{\pi}{6}$ or $\theta = \frac{\pi}{2}$ or $\theta = \frac{5\pi}{6}$ or $\theta = \frac{3\pi}{2}$

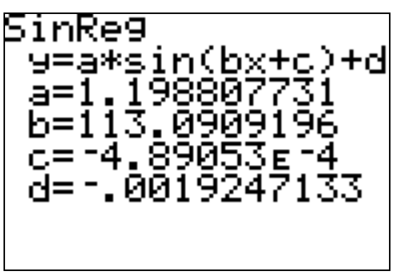
c)
$$\csc^2\theta - \csc\theta - 2 = 0$$
$$(\csc\theta - 2)(\csc\theta + 1) = 0$$
$$\csc\theta = 2 \text{ or } \csc\theta = -1$$
Case 1: $\csc\theta = 2$
$$\frac{1}{\sin\theta} = 2$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6} \text{ or } \theta = \frac{5\pi}{6}$$
Case 2: $\csc\theta = -1$
$$\frac{1}{\sin\theta} = -1$$
$$\sin\theta = -1$$
$$\theta = \frac{3\pi}{2}$$
$$\theta = \frac{\pi}{6} \text{ or } \theta = \frac{5\pi}{6} \text{ or } \theta = \frac{3\pi}{2}$$

Course Review **Question 34 Page 481**

a)

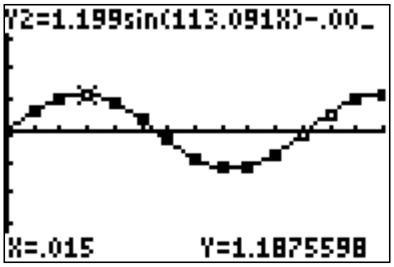


b)



$$y \doteq 1.199\sin(113.091x) - 0.002$$

c)



d) Average rate of change $= \dfrac{f(0) - f(0.001)}{0 - 0.001}$

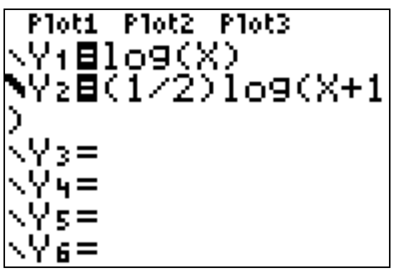
$\doteq 135.6$

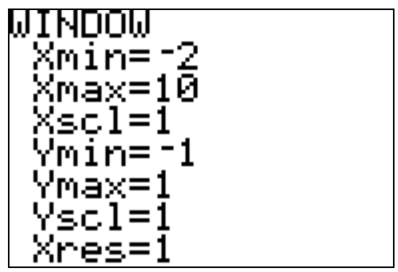
Course Review **Question 35 Page 481**

a) $\log_7 49 = 2$ **b)** $\log_a c = b$ **c)** $\log_8 512 = 3$ **d)** $\log_{11} y = x$

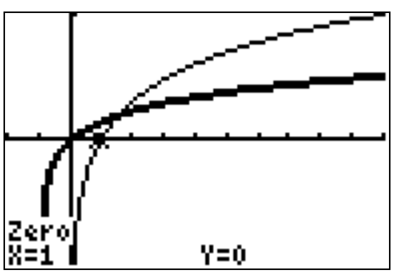
Course Review **Question 36 Page 482**

a)



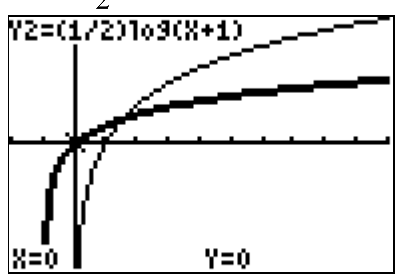


$f(x) = \log x$



The vertical asymptote occurs at $x = 0$.
There is no y-intercept.
The x-intercept is 1.

$g(x) = \dfrac{1}{2}\log(x+1)$



The vertical asymptote occurs at $x = -1$.
The y-intercept is 0.
The x-intercept is 0.

b) $f(x)$; $\{x \in \mathbb{R}, x > 0\}$, $\{y \in \mathbb{R}\}$
$g(x)$; $\{x \in \mathbb{R}, x > -1\}$, $\{y \in \mathbb{R}\}$

Course Review **Question 37 Page 482**

a) $3^8 = 6561$ **b)** $a^b = 75$ **c)** $7^4 = 2401$ **d)** $a^b = 19$

Course Review **Question 38 Page 482**

a) $$\begin{aligned} 2^x &= 256 \\ 2^x &= 2^8 \\ x &= 8 \end{aligned}$$

b) $$\begin{aligned} 15^x &= 15^1 \\ x &= 1 \end{aligned}$$

c) $$\begin{aligned} \log_6 6^{\frac{1}{2}} &= \frac{1}{2}\log_6 6 \\ &= \frac{1}{2} \end{aligned}$$

d) $$\begin{aligned} \log_3 3^5 &= 5\log_3 3 \\ &= 5 \end{aligned}$$

e) $\log_{12} 12 = 1$

f) $$\begin{aligned} \log_{11}\frac{1}{11} &= \log_{11} 11^{-1} \\ &= (-1)\log 11 \\ &= -1 \end{aligned}$$

Course Review **Question 39 Page 482**

a) $$\begin{aligned} \log_3 x &= 4 \\ 3^4 &= x \\ x &= 81 \end{aligned}$$

b) $$\begin{aligned} \log_x 125 &= 3 \\ x^3 &= 125 \\ x^3 &= 5^3 \\ x &= 5 \end{aligned}$$

c) $$\begin{aligned} \log_7 x &= 5 \\ 7^5 &= x \\ x &= 16\ 807 \end{aligned}$$

d) $$\begin{aligned} \log_x 729 &= 6 \\ x^6 &= 729 \\ x^6 &= 3^6 \\ x &= 3 \end{aligned}$$

e) $$\begin{aligned} \log_{\frac{1}{2}} 128 &= x \\ \left(\frac{1}{2}\right)^x &= 128 \\ 2^{-x} &= 2^7 \\ -x &= 7 \\ x &= -7 \end{aligned}$$

f) $$\begin{aligned} \log_{\frac{1}{4}} 64 &= x \\ \left(\frac{1}{4}\right)^x &= 64 \\ 4^{-x} &= 4^3 \\ -x &= 3 \\ x &= -3 \end{aligned}$$

Course Review **Question 40 Page 482**

$$\begin{aligned}125\ 000 &= 100\ 000(2)^{\frac{20}{h}}\\ 1.25 &= 2^{\frac{20}{h}}\\ \log 1.25 &= \frac{20}{h}\log 2\\ \frac{20}{h} &= \frac{\log 1.25}{\log 2}\\ h &= \frac{20\log 2}{\log 1.25}\\ h &\doteq 62\end{aligned}$$

The doubling period is approximately 62 min.

Course Review **Question 41 Page 482**

a)

$$\begin{aligned}7.8 &= -\log\left[H^+\right]\\ -7.8 &= \log\left[H^+\right]\\ 10^{-7.8} &= \left[H^+\right]\\ \left[H^+\right] &= 1.585\times 10^{-8}\end{aligned}$$

The concentration of hydronium ions in eggs is approximately 1.585×10^{-8} mol/L. Eggs are alkaline.

b)

$$\begin{aligned}\text{pH} &= -\log\left[7.9\times 10^{-4}\right]\\ &\doteq 3.1\end{aligned}$$

The pH of the vinegar solution is approximately 3.1.

a) $(3^x)^2 + 3^x - 21 = 0$

$$3^x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-21)}}{2(1)}$$

$$3^x = \frac{-1 \pm \sqrt{85}}{2}$$

$$3^x = \frac{-1 + \sqrt{85}}{2}$$

$$\log 3^x = \log\left(\frac{-1 + \sqrt{85}}{2}\right)$$

$$x \log 3 = \log\left(\frac{-1 + \sqrt{85}}{2}\right)$$

$$x = \frac{\log\left(\frac{-1 + \sqrt{85}}{2}\right)}{\log 3}$$

$$x \doteq 1.29$$

or

$$3^x = \frac{-1 - \sqrt{85}}{2}$$

A power of 3 is always positive, so it cannot be negative. This root is extraneous.

b) $4^x + 15(4)^{-x} - 8 = 0$

$4^{2x} + 15 - 8(4^x) = 0$ multiply each term by 4^x

$(4^x)^2 - 8(4^x) + 15 = 0$

$(4^x - 5)(4^x - 3) = 0$

$4^x = 5$ or $4^x = 3$

$x \log 4 = \log 5$ $\quad$ $x \log 4 = \log 3$

$x = \frac{\log 5}{\log 4}$ $\quad$ $x = \frac{\log 3}{\log 4}$

$x \doteq 1.16$ $\quad$ $x \doteq 0.79$

Course Review **Question 43 Page 482**

a) $$\begin{aligned}\log_8 4+\log_8 128&=\log_8(4\times128)\\&=\log_8 512\\&=3\end{aligned}$$

b) $$\begin{aligned}\log_7 7\sqrt{7}&=\log_7 7+\log_7\sqrt{7}\\&=1+\log_7 7^{\frac{1}{2}}\\&=1+\frac{1}{2}\log_7 7\\&=1+\frac{1}{2}\\&=\frac{3}{2}\end{aligned}$$

c) $$\begin{aligned}\log_5 10-\log_5 250&=\log_5\left(\frac{10}{250}\right)\\&=\log_5\left(\frac{1}{25}\right)\\&=\log_5 5^{-2}\\&=-2\log_5 5\\&=-2\end{aligned}$$

d) $$\begin{aligned}\log_6\sqrt[3]{6}&=\log_6 6^{\frac{1}{3}}\\&=\frac{1}{3}\log_6 6\\&=\frac{1}{3}\end{aligned}$$

Course Review **Question 44 Page 482**

a) $$\begin{aligned}2^x&=13\\\log 2^x&=\log 13\\x\log 2&=\log 13\\x&=\frac{\log 13}{\log 2}\\x&\doteq 3.7004\end{aligned}$$

b) $$\begin{aligned}5^{2x+1}&=97\\\log 5^{2x+1}&=\log 97\\(2x+1)\log 5&=\log 97\\2x\log 5+\log 5&=\log 97\\2x\log 5&=\log 97-\log 5\\x&=\frac{\log 97-\log 5}{2\log 5}\\x&\doteq 0.9212\end{aligned}$$

c) $$\begin{aligned}3^x&=19\\\log 3^x&=\log 19\\x\log 3&=\log 19\\x&=\frac{\log 19}{\log 3}\\x&\doteq 2.6801\end{aligned}$$

d) $$\begin{aligned}4^{3x+2}&=18\\\log 4^{3x+2}&=\log 18\\(3x+2)\log 4&=\log 18\\3x\log 4+2\log 4&=\log 18\\3x\log 4&=\log 18-2\log 4\\x&=\frac{\log 18-2\log 4}{3\log 4}\\x&\doteq 0.0283\end{aligned}$$

a) $\log_5(x+2)+\log_5(2x-1)=2$

$$\begin{aligned}
\log_5\left[(x+2)(2x-1)\right]&=2\\
5^2&=(x+2)(2x-1)\\
25&=2x^2+3x-2\\
2x^2+3x-2-25&=0\\
2x^2+3x-27&=0\\
(2x+9)(x-3)&=0\\
x&=-\frac{9}{2}\text{ or }x=3
\end{aligned}$$

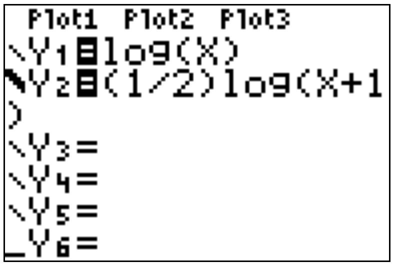
Both $\log_5(x+2)$ and $\log_5(2x-1)$ are undefined for $x=-\frac{9}{2}$ since the logarithm of a negative number is undefined. $x=-\frac{9}{2}$ is an extraneous root.
The solution is $x = 3$.

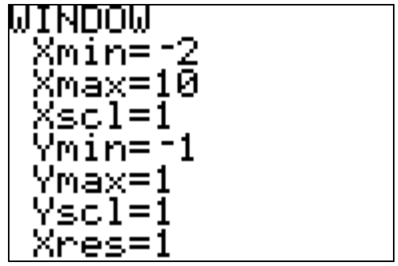
b) $\log_4(x+3)+\log_4(x+4)=\frac{1}{2}$

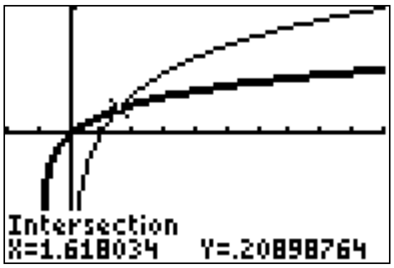
$$\begin{aligned}
\log_4\left[(x+3)(x+4)\right]&=\frac{1}{2}\\
4^{\frac{1}{2}}&=(x+3)(x+4)\\
2&=x^2+7x+12\\
x^2+7x+10&=0\\
(x+5)(x+2)&=0\\
x&=-5\text{ or }x=-2
\end{aligned}$$

Both $\log_4(x+3)$ and $\log_4(x+4)$ are undefined for $x = -5$ since the logarithm of a negative number is undefined. $x = -5$ is an extraneous root.
The solution is $x = -2$.

Course Review **Question 46 Page 482**







The point of intersection of $f(x)$ and $g(x)$ is approximately (1.62, 0.21).

Course Review **Question 47 Page 482**

a)

$$
\begin{aligned}
9 &= 10\left(\frac{1}{2}\right)^{\frac{3}{h}} \\
0.9 &= \frac{1}{2}^{\frac{3}{h}} \\
\log 0.9 &= \log \frac{1}{2}^{\frac{3}{h}} \\
\log 0.9 &= \frac{3}{h}\log\frac{1}{2} \\
\frac{3}{h} &= \frac{\log 0.9}{\log\frac{1}{2}} \\
h &= \frac{3\log\frac{1}{2}}{\log 0.9} \\
h &\doteq 19.7
\end{aligned}
$$

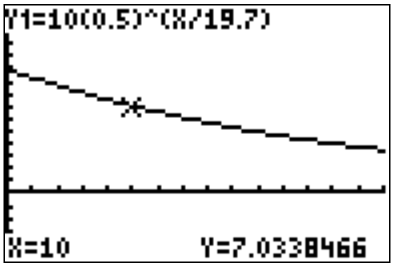
The half-life of bismuth-214 is approximately 19.7 min.

b)

$$
\begin{aligned}
A(t) &= 10\left(\frac{1}{2}\right)^{\frac{10}{19.7}} \\
&\doteq 7.03
\end{aligned}
$$

Approximately 7.03 mg of bismuth-214 remains after 10 min.

c)



d) Answers may vary. A sample solution is shown.
The graph would decrease faster because the sample would be decreasing at a faster rate.

Course Review **Question 48 Page 482**

a)

$$
\begin{aligned}
347\ 000 &= 124\ 000(2)^{\frac{6}{d}} \\
\frac{347}{124} &= 2^{\frac{6}{d}} \\
\log\left(\frac{347}{124}\right) &= \log 2^{\frac{6}{d}} \\
\log\left(\frac{347}{124}\right) &= \frac{6}{d}\log 2 \\
\frac{6}{d} &= \frac{\log\left(\frac{347}{124}\right)}{\log 2} \\
d &= \frac{6\log 2}{\log\left(\frac{347}{124}\right)} \\
d &\doteq 4.04
\end{aligned}
$$

The doubling time is approximately 4.04 years

b)

$$
\begin{aligned}
A(t) &= 124\ 000(2)^{\frac{19}{4.04}} \\
&\doteq 3\ 229\ 660
\end{aligned}
$$

The expected volume of computer parts in 2020 is approximately 3 229 660.

Course Review **Question 49 Page 482**

a) $y = 35\ 000(0.82)^t$

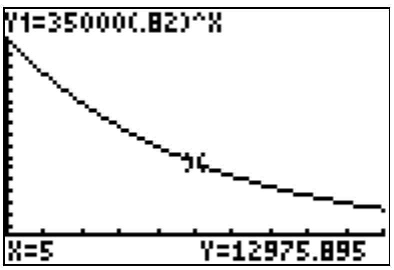
b)

$$
\begin{aligned}
y &= 35\ 000(0.82)^5 \\
&\doteq 12\ 975.89
\end{aligned}
$$

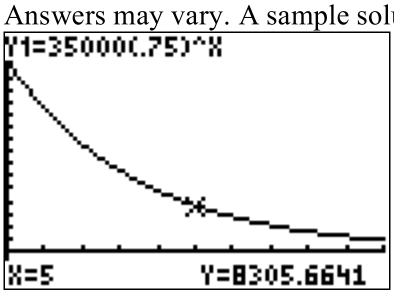
The car is valued at approximately \$12 975.89 after 5 years.

c)
$$\frac{1}{2} = (0.82)^t$$
$$\log\frac{1}{2} = \log 0.82^t$$
$$\log\frac{1}{2} = t\log 0.82$$
$$t = \frac{\log\frac{1}{2}}{\log 0.82}$$
$$t \doteq 3.5$$
It will take approximately 3.5 years for the car to depreciate to half its original value.

d)

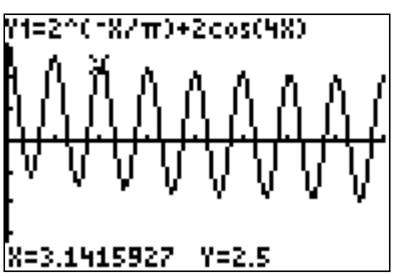


e) Answers may vary. A sample solution is shown. The graph would decrease faster.

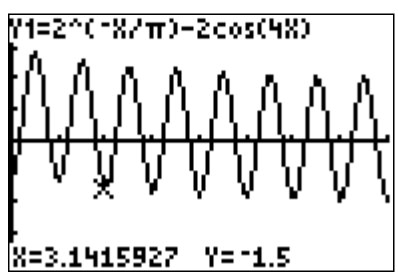


Course Review **Question 50 Page 483**

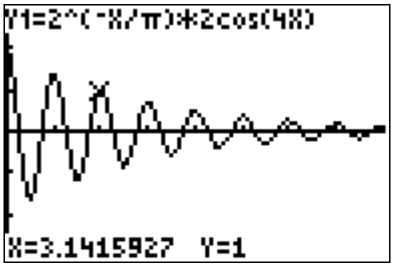
a)



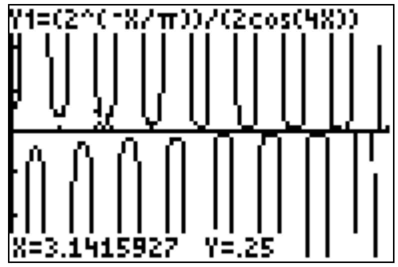
b)



c)



d)



Course Review **Question 51 Page 483**

a) $f(g(x)) = 2(x+3)^2 + 3(x+3) - 5$
$= 2(x^2 + 6x + 9) + 3x + 9 - 5$
$= 2x^2 + 12x + 18 + 3x + 4$
$= 2x^2 + 15x + 22$

b) $g(f(x)) = (2x^2 + 3x - 5) + 3$
$= 2x^2 + 3x - 2$

c) $f(g(x)) = 2x^2 + 15x + 22$ from a)
$f(g(-3)) = 2(-3)^2 + 15(-3) + 22$
$= 2(9) - 45 + 22$
$= -5$

d) $g(f(x)) = 2x^2 + 3x - 2$ from b)
$g(f(7)) = 2(7)^2 + 3(7) - 2$
$= 2(49) + 21 - 2$
$= 117$

Course Review **Question 52 Page 483**

a) $f(g(x)) = \dfrac{1}{4-x}$
$f(g(3)) = \dfrac{1}{4-3}$
$= 1$

b) $f(g(x)) = \dfrac{1}{4-x}$
$f(g(0)) = \dfrac{1}{4-0}$
$= \dfrac{1}{4}$

c) $f(g(x)) = \dfrac{1}{4-x}$
$f(g(4)) = \dfrac{1}{4-4}$
= undefined, does not exist

d) $g(f(x)) = 4 - \dfrac{1}{x}$
$g(f(4)) = 4 - \dfrac{1}{4}$
$= \dfrac{16}{4} - \dfrac{1}{4}$
$= \dfrac{15}{4}$

a) $f(g(x)) = \sqrt{x+1}$, $\{x \in \mathbb{R}, x \geq -1\}$

$x+1 \geq 0$ real if the square root of a positive number

$x \geq -1$

$g(f(x)) = \sqrt{x}+1$, $\{x \in \mathbb{R}, x \geq 0\}$

b) $f(g(x)) = \sin x^2$, $\{x \in \mathbb{R}\}$

$g(f(x)) = \sin^2 x$, $\{x \in \mathbb{R}\}$

c) $f(g(x)) = |x^2 - 6|$, $\{x \in \mathbb{R}\}$

$g(f(x)) = |x|^2 - 6$, $\{x \in \mathbb{R}\}$

d) $f(g(x)) = 2^{(3x+2)+1}$

$= 2^{(3x+3)}$

$\{x \in \mathbb{R}\}$

$g(f(x)) = 3(2^{x+1}) + 2$

$\{x \in \mathbb{R}\}$

e)
$$
\begin{aligned}
f(g(x)) &= (\sqrt{x-3}+3)^2 \\
&= x-3+6\sqrt{x-3}+9 \\
&= x+6\sqrt{x-3}+6
\end{aligned}
$$

$\{x \in \mathbb{R}, x \geq 3\}$

$$
\begin{aligned}
g(f(x)) &= \sqrt{(x+3)^2 - 3} \\
&= \sqrt{x^2+6x+9-3} \\
&= \sqrt{x^2+6x+6}
\end{aligned}
$$

$x^2 + 6x + 6 \geq 0$

$$
\begin{aligned}
x &\geq \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)} \\
x &\geq \frac{-6 \pm \sqrt{12}}{2} \\
x &\geq \frac{-6 \pm \sqrt{4 \times 3}}{2} \\
x &\geq \frac{-6 \pm 2\sqrt{3}}{2} \\
x &\geq -3 \pm \sqrt{3} \\
x &\leq -3-\sqrt{3} \text{ or } x \geq -3+\sqrt{3}
\end{aligned}
$$

$\{x \in \mathbb{R}, x \leq -3-\sqrt{3}, x \geq -3+\sqrt{3}\}$

f) $f(g(x)) = \log 3^{x+1}$
$= (x+1)\log 3$
$= x\log 3 + \log 3$

$\{x \in \mathbb{R}\}$

$g(f(x)) = 3^{\log x+1}$

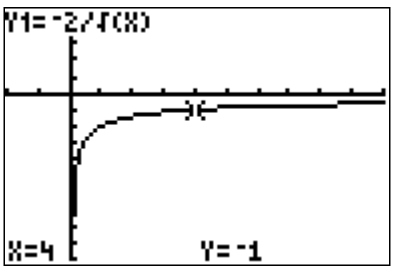
$\{x \in \mathbb{R}, x \ge 0\}$

Course Review **Question 54 Page 483**

a) $f(g(x)) = -\dfrac{2}{\sqrt{x}}$

b) $\{x \in \mathbb{R}, x > 0\}$

c)



$f(g(x))$ is neither even nor odd since it is not symmetric about the y-axis or the origin.

Course Review **Question 55 Page 483**

a)
$$\begin{aligned}
f(x) &= x^2 - 4\\
x &= y^2 - 4\\
y^2 &= x + 4\\
y &= \pm\sqrt{x+4}\\
f^{-1}(x) &= \pm\sqrt{x+4}\\
f(f^{-1}(x)) &= \left(\pm\sqrt{x+4}\right)^2 - 4\\
&= x + 4 - 4\\
&= x
\end{aligned}$$

b)
$$\begin{aligned}
f(x) &= \sin x\\
x &= \sin y\\
y &= \sin^{-1} x\\
f^{-1}(x) &= \sin^{-1} x\\
f(f^{-1}(x)) &= \sin(\sin^{-1} x)\\
&= x
\end{aligned}$$

c)

$$\begin{aligned} f(x) &= 3x \\ x &= 3y \\ y &= \frac{1}{3}x \\ f^{-1}(x) &= \frac{1}{3}x \\ f(f^{-1}(x)) &= 3\left(\frac{1}{3}x\right) \\ &= x \end{aligned}$$

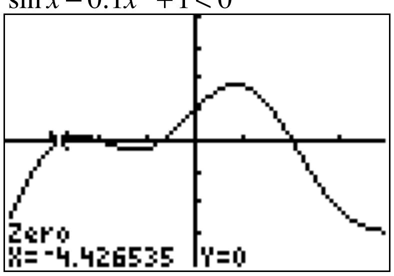
d)

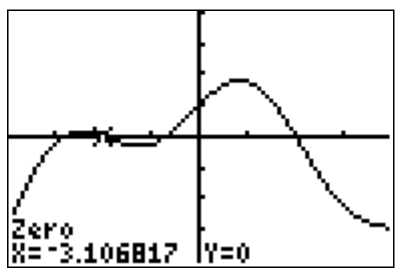
$$\begin{aligned} f(x) &= \frac{1}{x-2} \\ x &= \frac{1}{y-2} \\ y-2 &= \frac{1}{x} \\ y &= \frac{1}{x}+2 \\ f^{-1}(x) &= \frac{1}{x}+2 \\ f(f^{-1}(x)) &= \frac{1}{\left(\frac{1}{x}+2\right)-2} \\ &= \frac{1}{\frac{1}{x}} \\ &= x \end{aligned}$$

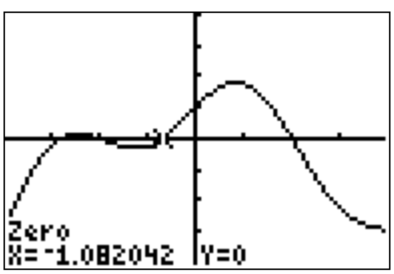
Course Review **Question 56 Page 483**

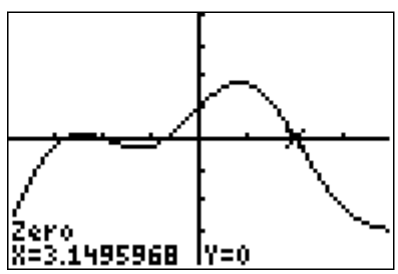
a) approximately $x < -4.4265$ or $-3.1068 < x < -1.0820$ or $x > 3.1496$

$\sin x - 0.1x^2 + 1 < 0$



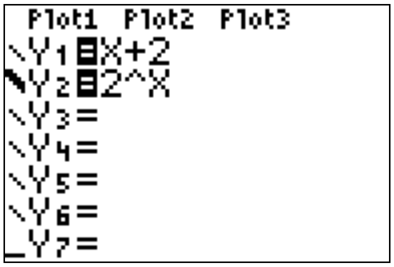


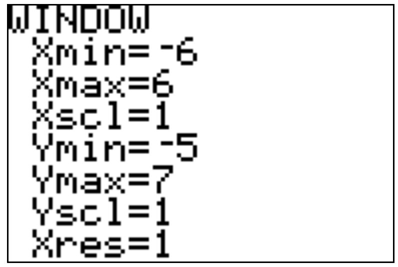


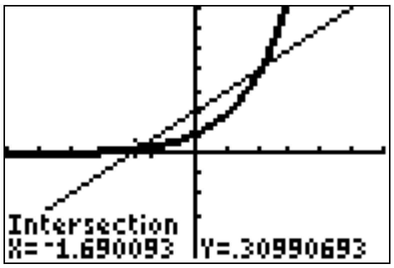


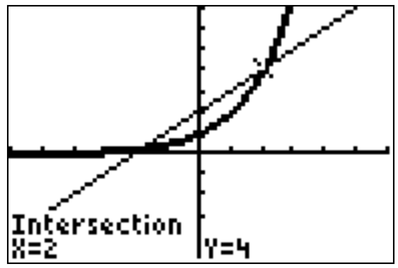
From the graph and the zeros find when $\sin x - 0.1x^2 + 1 < 0$.
$\sin x < 0.1x^2 - 1$ for the intervals: $(-\infty,-4.43)\cup(-3.11,-1.08)\cup(3.15,\infty)$

b)







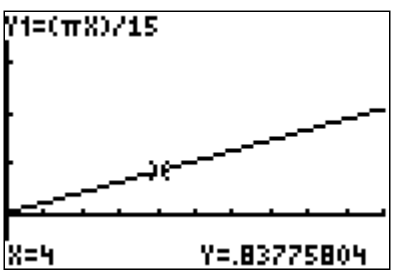


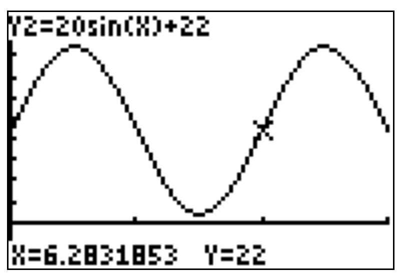
From the graph and the points of intersection:
$x + 2 \geq 2^x$ for the interval [–1.69, 2]

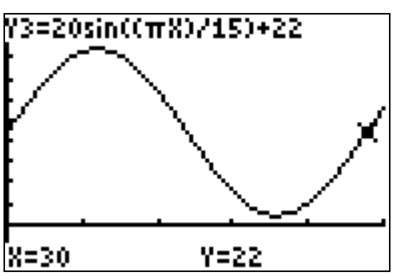
Course Review **Question 57 Page 483**

a) $h(t) = 20\sin\dfrac{\pi t}{15} + 22$

b)





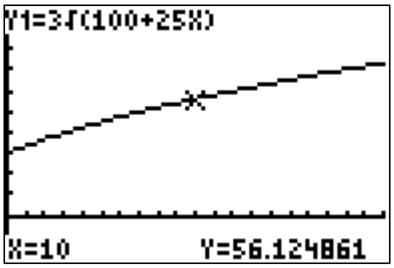


c) See the graphs above.
The period of $h(\theta)$ is 2π rad.
The period of $h(t)$ is 30 s.

Course Review **Question 58 Page 483**

a) $W(t) = 3\sqrt{100 + 25t}$ or $15\sqrt{4 + t}$

b) $\{t \in \mathbb{R}, t \geq 0\}$, $\{W \in \mathbb{Z}, W \in \geq 30\}$
Note that the workforce are people so W is an integer.

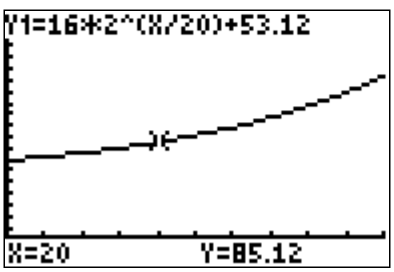


Course Review **Question 59 Page 483**

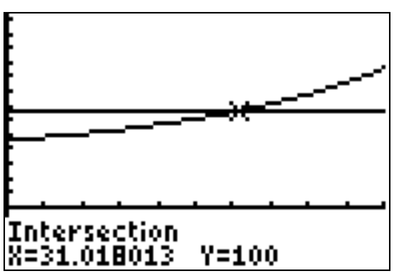
a) $C(t) = 1.28(12.5 \times 2^{\frac{t}{20}}) + 53.12$

$= 16 \times 2^{\frac{t}{20}} + 53.12$

b)



c)



It will take approximately 31 years for the concentration to reach 100 ppm.