SET Theory

Set: A set is a collection of well defined or distinct objects

eg. (i) N → set of natural numbers
(ii) W → set of whole numbers
(iii) Q → set of rational numbers
(iv) ~~R~~ Z → set of integers
(v) C → set of complex no.
(vi) Z^+ → set of +ve integers
(vii) Z^- → set of -ve integers.

* sets are usually denoted by capital letters.

Representation of sets

(1) Roaster or tabular form

* In this, the elements of a set are listed within a pair of brackets { } and are seperated by commas.

Eg. (i) let N denote the set of first three natural nos
$N = \{1, 2, 3\}$

(ii) set of vowels
$A = \{a, e, i, o, u\}$

(iii) set of vowels in a word MATHEMATICS
$A = \{A, E, I\}$

** The order in which elements are listed is immaterial but elements must not be repeated.

(2) Set Builder form

A rule or a formula or a statement is written with in a pair of brackets so that the set must passes a single property to become the member of that set.

Ex $P = \{1, 2, 3, 4, 5\}$ → roaster form

$P = \{x : x \in N \text{ and } x \leq 5\}$ (: such that; $\in$ belongs to)

p is a set of elements x such that ...

Eg. set of natural numbers

$A = \{x : x \in N\}$

set of integers less than 2 & greater than 4

$A = \{x : x \in Z \text{ and } 2 \not> x \not> 4\}$

(iii) write the set $A = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \frac{8}{9}\}$

$A = \{x : x = \frac{n}{n+1}, n \in N \text{ and } 1 \leq x \leq 8\}$

Types of sets

(i) Finite set

Ex $A = \{x : x \in N, 1 < x < 4\}$

(2) Infinite set

$A = \{x : x \in N, x > 5\}$

(3) Empty set

* No elements

$A = \{x : x \in N, 1 < x < 2\}$

Note $\phi \neq \{\phi\}$ (element)

* denoted by ϕ or $\{\}$; * cardinal no. = 0

(4) Equal set

* Cardinality same & elements also same

Eg $A = \{1, 2, 3\}$

$B = \{2, 1, 3\}$

$|A| = 3$ and $|B| = 3$

(5) Equivalent set

Eg $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$

** cardinality same
$|A| = |B| = 3$

(6) Singleton set

Eg.. $A = \{x : x \in N, 1 < x \leq 2\}$

* only one element is present in the set

** Cardinality of a set

* Cardinal number
* No. of distinct elements of ~~of~~ a set
  different
* denoted by $n(A)$ or $|A|$

Eg. $A = \{a, d, e, l, p\} \Rightarrow n(A) = |A| = 5$

$B = \{a, a, b, d, d\} \rightarrow n(B) = |B| = 3$

$C = \{a, \{c, d\}, k\} \rightarrow n(C) = |C| = 3$

(1) (2) (3) distinct element

(7) power set

let A be the set

$A = \{a, b, c\}$

$P(A) = 2^n$

$P(A) = 2^3 = 8$

* The family consisting of all the subsets of a set is called power set of that set. denoted by $P(A)$.
proper subset $= 2^n - 1$
where $n \rightarrow$ no. of elements in set A

$= \{\{\phi\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}$

* $P(A) = \{x : x \subseteq A\}$

* Cartesian product of sets

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$A \times B = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

SUBSET

* If all the elements of A is contained in B then

$A \subset B$

↓ subset, superset

Eg $A = \{1, 5, 6\}$

$B = \{1, 2, 3, 4, 5, 6\}$

so $A \subset B$

* Note: ϕ is a subset of every set

i.e. • $\phi \subset A$, $\phi \subset B$

• $A \subseteq A$

• $A \subset B \rightarrow$ all $x \in A \Rightarrow x \in B$

• $A \not\subset B$

at least one $x \in A$ such that $x \notin B$

* proper subset

$A = \{1, 2, 3\}$

$B = \{1, 2\}$

$B \subset A$ (proper subset)

* improper subset

$A \subseteq B$

$B \subseteq A$ ∵ Cardinal no. should be equal

Q Two finite sets have m and n elements the total no. of power subset of the first set is 56 more than the total no. of power set of the second set. Find values M & N

→ let A be the set having 'm' elements

B " " " " 'n' elements

$P(A) = 2^m$

$P(B) = 2^n$

$2^m = 56 + 2^n \Rightarrow 2^m - 2^n = 56$

$\Rightarrow 2^m - 2^n = 2^3 \times 7$

$2^n(2^{m-n} - 1) = 2^3 \times 7$

By comparing

$2^n = 2^3$

$\boxed{n = 3}$

$2^{m-n} - 1 = 7$

$2^{m-n} = 8$

$2^{m-n} = 2^3$ ∴ $m - n = 3$

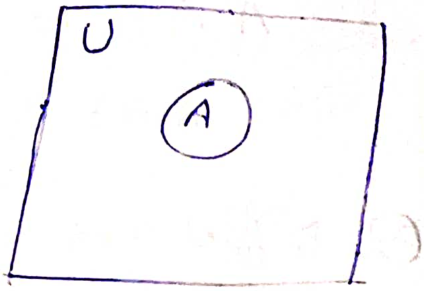
$\boxed{m = 6}$

Universal set

- All the objects under discussion

Eg. $U = \{x : x \text{ is an integer}, x \leq 8\}$

$A = \{1, 2, 4\}$



Disjoint set

Two sets A and B are said to be disjoint if $A \cap B = \phi$ {empty set or null element}

Eg. $A = \{1, 2, 3\}$, $B = \{a, b, c\}$

$A \cap B = \phi$ so A & B are disjoint

* Ordered Pair

An element of the form (a, b) is called an ordered pair

a -> 1^{st} element
b -> 2^{nd} "

* Equality of ordered pair

let (a, b) and (c, d) be any two ordered pair

$(a, b) = (c, d)$

when $a = c$
and $b = d$

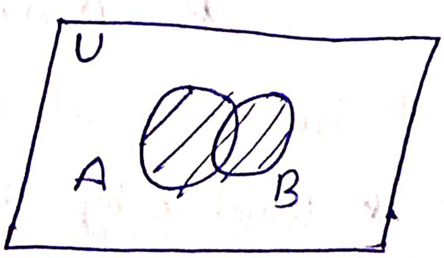
Operation of Sets

① Union:

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A = \{1, 2, 3, 4\}$, $B = \{2, 4, 5, 6\}$

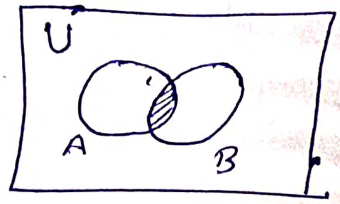
$A \cup B = \{1, 2, 3, 4, 5, 6\}$



2 Intersection:

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cap B = \{2, 4\}$



3 Difference of two sets

$A - B = \{x : x \in A \text{ and } x \notin B\}$

= Difference of B from A

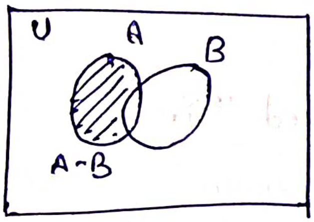
$A = \{1, 2, 3, 4\}$

$B = \{2, 4, 5, 6\}$

$A - B = \{1, 3\}$

* $A - B = A - (A \cap B)$

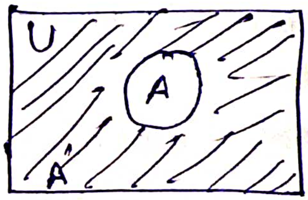
$A - B \neq B - A$



4 Complement of a set

$A = \{1, 2, 3, 4\}$ U → universal set

denoted by A^c, or $\bar{A}$ or A'



$A' = \{x : x \in U \text{ and } x \notin A\}$

$A' = U - A$

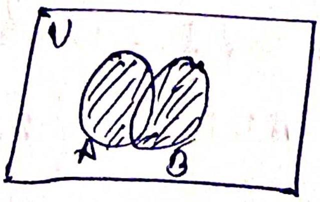
* $(A')' = A$
* $A' \cup A = U$
* $A' \cap A = \phi$
* $U' = \phi$
* $\phi' = U$

Symmetric Difference

* Denoted by $A \oplus B$ or $A \Delta B$
* $A \oplus B = (A - B) \cup (B - A)$

eg $A = \{1, 2, 3, 4\}$

$B = \{2, 3, 5, 6\}$ $\Rightarrow A \oplus B = \{1, 4, 5, 6\}$



Q. $A = \{\phi, b\}$

Construct the following sets

(i) $A - \phi$ (ii) $\{\phi\} - A$ (iii) $A \cup P(A)$ (iv) $A \cap P(A)$

(v) $\phi - A$

A. (i) $\{\phi, b\} - \phi = A$

(ii) $\{\phi\} - \{\phi, b\} = \phi$ or $\{\}$

~~(iii)~~ $\{\phi, \{\phi\}, \{b\}, \{\phi, b\}\} = P(A)$

(iv) $A \cap P(A) = A$

(v) $\phi - A = \phi$

(iii) $\{(\phi, b), (\phi), \{b\}, \phi\}$

Algebra of set theory

(1) Commutative law

(i) $A \cup B = B \cup A$

(ii) $A \cap B = B \cap A$

(i) proof Let $x \in A \& B$

LHS $\therefore x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$ — (i)

RHS $\therefore x \in B \cup A$

$x \in B$ or $x \in A$

$\therefore x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$ — (ii)

(i) and (ii)

$A \cup B = B \cup A$

② Associative law

(ii) $A \cap (B \cap C) = (A \cap B) \cap C$

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

let $x \in A$ & B & C

LHS $\because x \in [A \cup (B \cup C)]$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in [(A \cup B) \cup C]$ — (i)

RHS $x \in [(A \cup B) \cup C]$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow x \in A$ or $x \in B$ or $x \in C$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in [A \cup (B \cup C)]$ — (ii)

③ Idempotent

(i) $A \cup A = A$ (ii) $A \cap A = A$

let $x \in A$

LHS $x \in (A \cup A)$

$\Rightarrow x \in A$ or $x \in A$

$x \in A$

RHS $\because x \in A$

$\Rightarrow x \in A$ or $x \in A$ $\Rightarrow x \in (A \cup A)$

(4) Identity

universal set

(i) $A \cup \phi = A$ (ii) $A \cap U = A$

let $x \in A$

LHS : $x \in (A \cup \phi)$

$x \in A$ or $x \in \phi$

$\Rightarrow x \in A$ — (i)

RHS : $x \in A$

$x \in A$ or $x \in \phi$

$\Rightarrow x \in (A \cup \phi)$ — (ii)

from (i) and (ii)

$A \cup \phi = A$

(5) De-Morgan's :

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

let $x \in A, B$

LHS : $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$x \in (A' \cap B')$ — (i)

RHS : $x \in (A' \cap B')$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A \cup B) \Rightarrow x \in (A \cup B)'$

⑥ Distributive law

(i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) proof. Let $x \in A, B, C$

LHS

$\therefore \quad x \in [A \cup (B \cap C)]$

$\Rightarrow \quad x \in A$ or $x \in (B \cap C)$

$\Rightarrow \quad x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow \quad x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow \quad x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow \quad x \in [(A \cup B) \cap (A \cup C)]$ — (i)

RHS $\quad x \in [(A \cup B) \cap (A \cup C)]$

$\therefore \quad x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow \quad x \in A$ or $x \in (B \cap C)$

$\Rightarrow \quad x \in [A \cup (B \cap C)]$ — (ii)

**

$x \in A - B$

$x \in A$ and $x \notin B$

Q1 $\quad y - x' = y \cap x$

Q2 $\quad (x - y) \cap y = \phi$

Q3 $\quad (A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Q4 $\quad A - (A \cap B) = A - B$

Q5 $\quad x \cup (y - z) = (x \cup y) - (z - x)$

(i) Let $z \in x, y$

LHS $\because z \in (y - x')$

$\because z \in y$ and $z \notin x'$

$\Rightarrow z \in y$ and $z \in x$

$\Rightarrow z \in (y \cap x)$

RHS $z \in (y \cap x)$

$\Rightarrow z \in y$ and $z \in x$

$z \in y$ and $z \notin x'$

$\Rightarrow z \in (y - x')$

(ii) let $z \in (x - y) \cap y$

$\Rightarrow z \in x$ and $\underbrace{z \notin y \text{ and } z \in y}_{z \in \phi}$

$\Rightarrow z \in \phi$

(iii) let $z \in A, B$

$\Rightarrow z \in A$ and $z \notin B$ or $z \in B$ and $z \notin A$

$\Rightarrow z \in A$ or $z \in B$ and $z \in B$ and $z \notin A$

$\Rightarrow z \in (A \cup B)$ and $z \in (B \cap A)$

$z \in (A \cup B) \cap z \in (A \cap B)$

$z \in (A \cup B) \cap (A \cap B)$

* $[x \notin B$ or $x \in B] = U$

* $x \notin B$ and $x \in B = \phi$

(iv) $A - (A \cap B) = A - B$

let $z \in A, B$

LHS $z \in A$ and $z \notin (A \cap B)$

$z \in A$ and $z \notin (A \cap B)$

$z \in A$ and $z \notin A$ or $z \notin B$

$(z \in A$ and $z \notin A)$ or $(z \in A$ and $z \notin B)$

ϕ or $(z \in A$ and $z \notin B)$

$\Rightarrow z \in [A - B]$

(iii) let $x \in A, B$

LHS

$x \in [(A-B) \cup (B-A)]$

$\Rightarrow x \in (A-B)$ or $x \in (B-A)$

$\Rightarrow x \in A$ and $x \notin B$ or $x \in B$ and $x \notin A$

$\Rightarrow [x \in A$ and $x \notin B)$ or $x \in B]$ and $[x \in A$ and $x \notin B)$ or $x \notin A]$ Distribu

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \notin B$ or $x \in B)]$ and $[x \in A$ or $x \notin A$ and $x \notin B$ or $x \notin A]$

$\Rightarrow [x \in (A \cup B)$ and $U]$ and $[U$ and $(x \notin B$ or $x \notin A]$

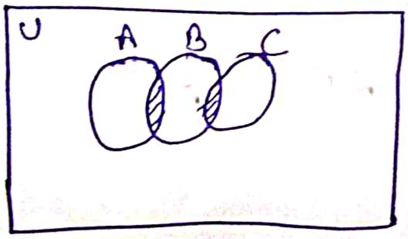
$\Rightarrow x \in (A \cup B)$ and $x \notin (B \cap A)$

$\Rightarrow x \in [(A \cup B) - (B \cap A)]$

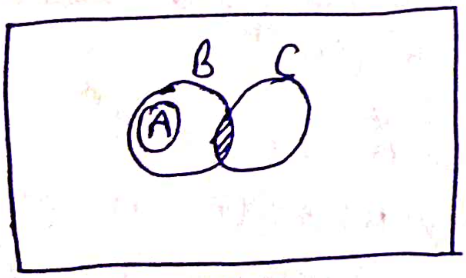
Q. Draw a venn diagram of sets A, B and C

(i) A and B have elements in common

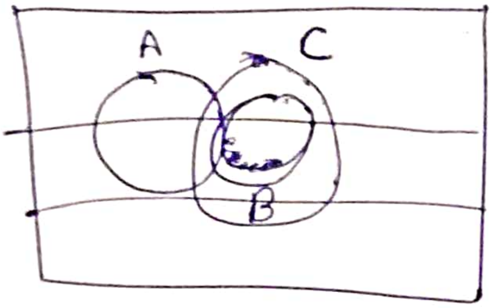
(ii) B and C " " " " but A and C are disjoint

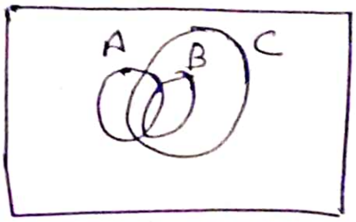


Q. A is subset of B, set A and C are disjoint but B and C have elements in common.



Q. $A \cup B \subseteq A \cup C$ but $B \not\subset C$. Draw VENN diagram.





Inclusion & Exclusion Principle

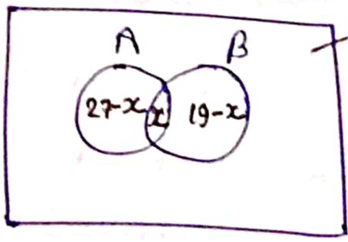
(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

~~(ii) $n(A \cap B) = n(A) + n(B)$.~~

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Q. If $n(U) = 60$, $n(A) = 27$, $n(B) = 19$ and $n(A \cup B)' = 31$ use venn diagram to find

(i) $n(A \cap B)$ (ii) $n(A - B)$



$n(U) = 60$

sol (i)

$n(A \cup B)' = n(U) - n(A \cup B)$

$31 = 60 - n(A \cup B)$

$\Rightarrow n(A \cup B) = 29$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$29 = 27 + 19 - n(A \cap B)$

$\boxed{n(A \cap B) = 17}$

OR

(ii) $n(A \cup B) = 27 - x + x + 19 - x$

$n(U) - n(A \cup B)' = 27 + 19 - x$

$60 - 31 = 27 + 19 - x$

$\boxed{x = 17}$

(ii)

$n(A-B) = n(A) - n(A \cap B)$

$= 27 - 17$

$= 10$

or

$n(A-B) = 27 - x$

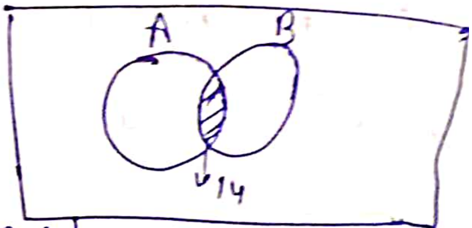
$= 27 - 17 = 10$

Q1 In a class of ~~6~~50 students, 28 playing cricket and 36 play hockey. Use Venn diagram to find

(i) Howmany play both the games

(ii) Howmany play only cricket. Consider each of the students take part ~~part~~ in atleast one game.

Q2 In a class of 50 students, 30 ~~studies~~ ~~History~~ History 25 study economics and 11 study bothe the languege. Draw a venn diagram to p represent it and find out howmany ~~of these~~ students do not study any of these subjects



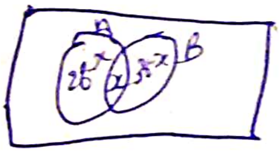
(i) let A and B are two sets

given $n(A \cup B) = 50 = n(U)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$50 = 28 + 36 - n(A \cap B)$

$n(A \cap B) = 64 - 50 = 14$



(ii) $28 - 14 = 14$ $\therefore$ only A $= n(A) - n(A \cap B)$

Q2

$n(A) = 30$

$n(B) = 25$

$n(A \cap B) = 11$

$n(U) = 50$

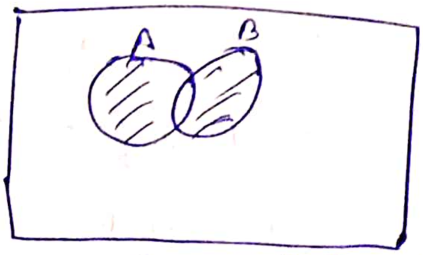
$\therefore$ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 30 + 25 - 11$

$= 44$

$n(A \cup B)' = n(U) - n(A \cup B)$

$= 50 - 44 = 6$



OR

$n(A-B) = n(A) - n(A \cap B)$

only A $= n(A) - n(A \cap B)$

$= 30 - 11 = 19$

$n(B-A) = n(B) - n(A \cap B)$

$= 25 - 11 = 14$

$n(A \cup B) = 19 + 11 + 14 = 44$

$n(A \cup B)' = 50 - 44 = 6$

Q. It is found that out of 100 students, 18 can drive neither a scooter nor a car while 25 can drive both these and 55 of them can drive a scooter. Howmany can drive a car

A

$n(U) = 100$

let $A \to$ for scooter

$B \to$ for Car

and $n(A) = 55$

$n(A \cap B) = 25$

$n(A \cup B)' = 18$ so $n(A \cup B) = 82$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$82 = 55 + n(B) - 25$

$\boxed{n(B) = 52}$

Q In an examination 70% of the students passed in hindi, 60% in english, 25% failed in both the subjects and 220 students passed in both the subjects find the total no. of students.

A

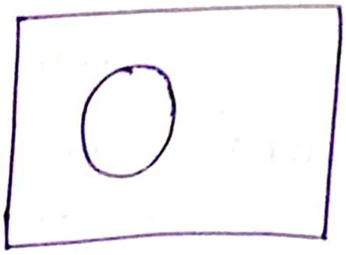
$n(A) = \frac{70}{100}$

$n(B) = \frac{60}{100}$

$n(A \cup B) = 220$

$n(A \cap B) = \frac{25}{100}$

$n(U) = ?$



As

~~$n(U) - n(A$~~

A

~~$n(A) = 70$~~

~~$n(B) = 60$~~

So let the total no. of students $= x$

$n(A) = \frac{70}{100}x$

$n(B) = \frac{60}{100}x$, $n(A \cap B) = 220$

$n(A \cup B)' = \frac{25}{100}x$, $n(U) = x$

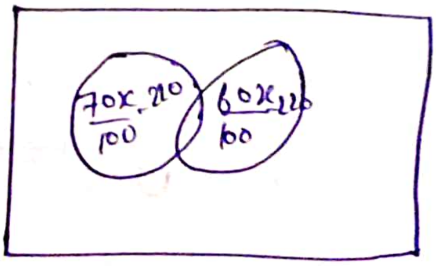
$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow x - \frac{25}{100}x = \frac{70}{100}x + \frac{60}{100}x - \cancel{n(A \cap B)}\ 220$

$\Rightarrow \frac{75x}{100} = \frac{130x}{100} - 220$

$220 = \frac{130x - 75x}{100}$

$\boxed{x = 400}$



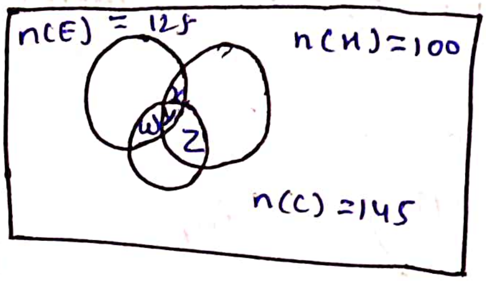
Q 125, 100 and 145 students appeared at the examination in the subjects Economics, hindi, and commerce respective 25 students appeared in economics only, 30 in economics and hindi, 80 in economics and commerce and 60 appeared in hindi & commerce. Use venn diagram to find the number of students who appeared.

(i) In all the three subjects

(ii) In hindi only

(iii) in commerce only

Sol



$n(\cancel{E}) = 125$

$n(\cancel{A}) = 100$

$n(C) = 145$

$n(E \text{ only}) = 25$

Economics & commerce $= 80 = w + y$

" & Hindi $= 30$

Hindi & commerce $= 60$

$\therefore \quad 125 = 25 + x + y + w$

$\Rightarrow \quad 125 = 25 + \underbrace{30}_{\text{Eco \& Hindi}} + w$

$\Rightarrow \boxed{w = 70}$

(i) $\therefore \; w + y = 80$

$y = 80 - 70 = 10$

$\boxed{y = 10}$

$\because \; y + z =$ hindi & Commerce

$10 + z = 60 \quad \Rightarrow \boxed{z = 50}$

$x + y + z + w = 150$ Ay

(ii) $100 =$ hindi only $+ x + y + z$

$\Rightarrow \; 100 - 20 - 10 - 50 =$ Hindi only

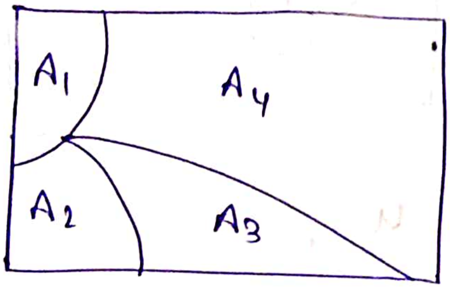
$\boxed{\text{Hindi only} = 20}$

(iii) $145 =$ comerce only $+ w + y + z$

$145 - 70 - 10 - 50 =$ commerceace only

$\boxed{\text{commerce only} = 15}$

Partition or Quotient set



let $A = A_1, A_2, A_3, A_4 \}$ {subsets}

partition set: $\rightarrow A_1 \cap A_2 = \phi$, $A_2 \cap A_3 = \phi$, $A_3 \cap A_4 = \phi$ . . . {disjoint}

$\rightarrow \{A_1 \cup A_2 \cup A_3 \cup A_4\} = A$

Ex $A = \{1, 2, 3, 4, 5, 6, 7\}$

$A_1 = \{1, 2, 3\}$

$A_2 = \{4, 5\}$ $\Rightarrow$ $A_1 \cap A_2 = \phi$

$A_3 = \{6, 7\}$ $A_2 \cap A_3 = \phi$

$A_1 \cap A_3 = \phi$

so A is a partition set.