RELATIONS

$A = \{2, 8\}$

$B = \{1, 4, 7\}$

$A \times B = \{(2,1), (2,4), (2,7), (8,1), (8,4), (8,7)\}$

$ARB = A \rightarrow B$ $\quad$ $ARB = A > B = \{(2,1), (8,1), (8,4), (8,7)\}$

$BRA = B \rightarrow A$ $\quad$ $ARB = A < B = \{(2,4), (2,7)\}$

Domain of ARB ($A > B$) $= \{2, 8\}$ $\quad$ Domain $A < B = \{2\}$

Range of $A > B = \{1, 4, 7\}$ $\quad$ Range $A < B = \{4, 7\}$

let $R = ARB = A > B = \{(2,1), (8,1), (8,4), (8,7)\}$

R^{-1} = inverse = $\{(1,2), (1,8), (4,8), (7,8)\}$

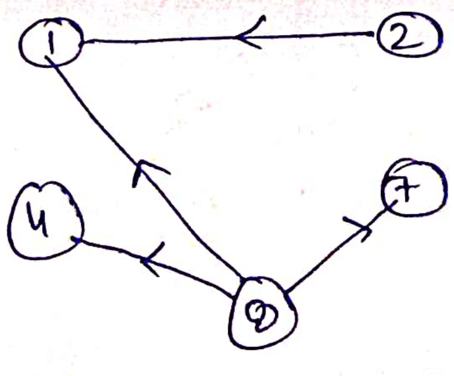
R' = complement = $\{(2,4), (2,7)\}$ $\quad$ ∵ Jo relation mai se bach gye Jo relation mei nhi hai ab wahi lenge

Representation of Relation

① Matrices

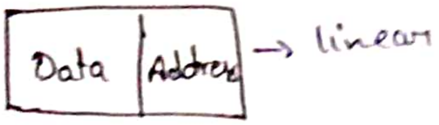
$$\begin{array}{cc} & \begin{array}{ccc} 1 & 4 & 7 \end{array} \leftarrow B \\ A \rightarrow \begin{array}{c} 2 \\ 8 \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

② Graph

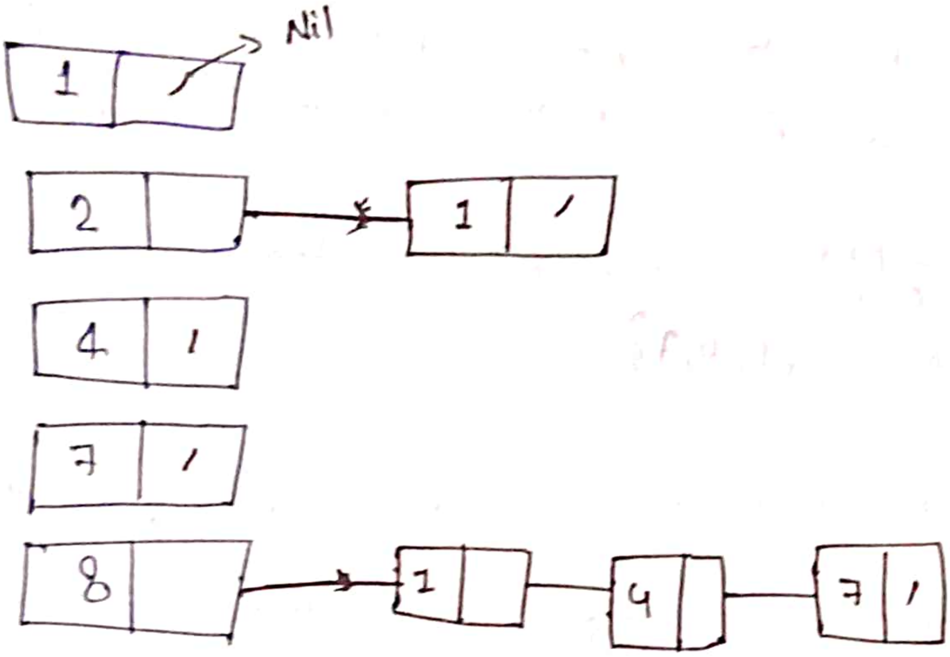


$ARB = A > B$

3) Adjacency list



ARB = A > B = {(2,1), (8,1), (8,4), (8,7)}



Properties of Relation or Types of Relation

① Reflexive

A relation R on set A is said to be a reflexive relation if $(a, a) \in R$ for every $a \in A$

eg $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$ → Reflexive

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$ → Not reflexive

(ii) Irreflexive

A relation R on set A is said to be irreflexive if $(a, a) \notin R$ for every $a \in A$

Eg. $A = \{1, 2, 3, 4\}$

$R = \{(1,2), (4,1), (3,1)\} \rightarrow$ irreflexive ✓

$R = \{(1,2), (2,2), (3,4)\} \rightarrow$ irreflexive ✗

③ Symmetric

A relation R on set A is said to be symmetric if and only if $(a,b) \in R \Leftrightarrow (b,a) \in R$

Eg. $A = \{1, 2, 3\}$

$R = \{(1,2), (2,1), (1,1), (1,3), (3,1)\}$ ✓

④ Asymmetric

A Relation R on set A is said to be asymmetric if

$(a,b) \in R \Leftrightarrow (b,a) \notin R$

ye kisi bhi trh ki symmetry allow nhi karta

Eg $A = \{1, 2, 3\}$

$R = \{(1,2), (3,1)\}$ ✓

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & (1,1) & (1,2) & (1,3) \\ 2 & (2,1) & (2,2) & (2,3) \\ 3 & (3,1) & (3,2) & (3,3) \end{array}$$

⑤ Anti Symmetric

A relation R on set A is said to be anti symmetric if $(a,b) \in R$ then $(b,a) \in R$ then $a = b$.

Eg. $A = \{1, 2, 3\}$

$R = \{(1,2), (1,1), (2,2), (3,1), (3,2)\}$ ✓

⑥ Transitive

A relation R on set A is said to be transitive if and only if $(a,b) \in R$ & $(b,c) \in R \Rightarrow (a,c) \in R$

Eg. $A = \{1, 2, 3\}$

$R = \{(1,2), (2,3), (1,3)\}$

Q Which of the following relations are symmetric and which are anti-symmetric

(a) $R_1 = \{(a,b) : a \le b\} \rightarrow$ Anti-symmetric

(b) $R_2 = \{(a,b) : a > b\} \rightarrow$ Anti-symmetric

(c) $R_3 = \{(a,b) : a = b \text{ or } a = -b\} \rightarrow$ symmetric

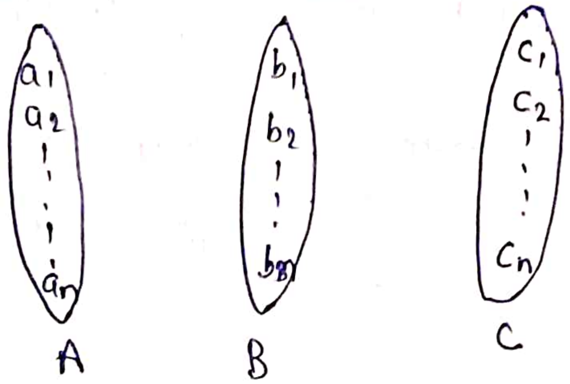
(d) $R_4 = \{(a,b) : a = b\} \rightarrow$ symmetric

(e) $R_5 = \{(a,b) : a < b+1\} \rightarrow$ Anti-symmetric

(f) $R_6 = \{(a,b) : a+b \le 3\} \rightarrow$ Symmetric

A

Composite Relation (composition of relation)



$R = A \times B$ $\quad S = B \times C$

$ROS = A \times C$ $\quad R \rightarrow S$ $\quad SoS \Rightarrow S \rightarrow S$

$SoR \rightarrow S \rightarrow R$ $\quad RoR \Rightarrow R \rightarrow R$

Q If there are two relations R & S on $A = \{1, 2, 3\}$ such that $R = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$

and $S = \{(1,2), (1,3), (2,1), (3,3)\}$

then find RoS & S^2.

A $R \rightarrow S$ (piche vale element ko dekhenge agar S vale m aage hai

$RoS = \{(1,2)\ (1,3)\ (1,1)\ (2,3)\ (3,2)\ (3,3)\}$

$S^2 = S \circ S = \{(1,1)(1,3)(2,2)(2,3)(3,3)\}$

Q Prove that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$ if R and S are the true relations and R^{-1} and S^{-1} are the inverse of R and S

A Let A, B and C are the sets such that

$$R = A \times B$$

$$S = B \times C$$

$$R \circ S = A \times C \quad - (i)$$

$$S^{-1} = C \times B, \quad R^{-1} = B \times A$$

$$S^{-1} \circ R^{-1} = C \times A \quad - (ii)$$

By eq^n (i)

$$(R \circ S)^{-1} = C \times A \quad - (iii)$$

By eq^n (ii) & (iii)

$$(R \circ S)^{-1} = S^{-1} \circ R^{-1}$$

Reflexive Closure

If R is a relation on set A such that R is not reflexive then its closure can be find as:-

$$R' = R \cup \Delta$$

where Δ is the diagonal element.

eg $A = \{1, 2, 3\}$

$R = \{(1,2), (2,3)(2,1)\}$

$\Delta = \{(1,1)(2,2), (3,3)\}$

$R' = R \cup \Delta = \{(1,2)(2,3)(2,1)(1,1)(2,2)(3,3)\}$

* Symmetric closure

If R is a relation on set A such that R is not symmetric then its closure can be find as

$$R^* = R \cup R^{-1} \rightarrow \text{inverse}$$

eg. $A = \{1,2,3\}$

$R = \{(1,2)(2,3)(3,1)\}$

$R^{-1} = \{(2,1)(3,2)(1,3)\}$

$R^* = \{(1,2)(2,3)(3,1)(2,1)(3,2)(1,3)\}$

Transitive closure:

If R is a relation on set A such that R is not transitive then its closure

$$R^+ = R \cup R^2 \cup R^3 \cup R^4 \text{ - - - -}$$

eg $A = \{1,2,3,4\}$

$R = \{(1,2)(2,3)(3,4)\}$ $\quad$ $R = \{(1,2)(2,3)(3,4)\}$

$R^2 = \{(1,3)(2,4)\}$

$R^3 = \{(1,4)\}$

$R^4 = \times$

$R^+ = R \cup R^2 \cup R^3 = \{(1,2)(2,3)(3,4)(1,3)(2,4)(1,4)\}$

> Equivalence Relation

→ Fulfill these three conditions
① Reflexive, ② ⇒ Symmetric, ③ ⇒ Transitive

Q If $A = \{0,1,2,3 \text{ - - -}\}$ and $R = \{(x,y) : x-y = 8k$, k is an integer$\}$ then prove that R is an equivalence relation

Sol^n let x, y be the element that belongs to A such that $x, y \in A$

Reflexive:

if $x \in A$ then xRx

$x - x$ is divisible by 8

therefore it is reflexive.

Symmetric:

if $x, y \in A$ then $xRy \Rightarrow yRx \in A$

$\therefore$ $x - y$ is divisible by 8

$\Rightarrow$ $y - x$ is also divisible by 8

therefore it is symmetric

Transitive: If $x, y, z \in A$ then xRy & $yRz \in A$ then $xRz \in A$

$\therefore$ $x - y$ is divisible by 8

& $y - x$ is also " " 8

$x - z$ is also " " 8

therefore it is transitive

so it is equivalence relation.

Q If A is a set of positive integers and a relation R is defined on A as follows

$(a,b) R (c,d) \Leftrightarrow a+d = b+c \quad \forall\ a, b, c, d \in R$

then prove that R is an equivalence relation.

Soln Reflexive:

$(a,b) R (a,b)$

Acc. to condition $a + \cancel{d} = b + c$

$\Rightarrow$ $a + b = b + a$

so, it is reflexive

Symmetric:

$(a,b) R (c,d) \Leftrightarrow (c,d) R (a,b)$

$\because a + d = b + c$

$\Rightarrow$ $b + c = a + d$

$= c+b = d+a$

so, it is ~~transitive~~ symmetric

③ Transitive:

$(a,b)\,R\,(c,d)$ & $(c,d)\,R\,(e,f) \Leftrightarrow (a,b)\,R\,(e,f)$ - -

$a+f = b+e$

$$a+d = b+c$$
$$c+f = d+e$$

$$\Rightarrow \quad a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$$

$$a+f = b+e$$

so it is transitive

hence it is equivalence relation.

Q1 If A is set of real numbers and R is the relation of A

$R = \{(x,y) \in A \times A : x-y$ is an integer$\}$

OR xRy iff $x-y$ is an integer

Q2 let A denote the set of real numbers and a relation R is defined as

$(a,b)\,R\,(c,d)$ iff $a^2+b^2 = c^2+d^2$

Q3 If $N = \{1,2,3 \dots\}$ and relation is defined as

$(a,b)\,R\,(c,d)$ iff ~~ab =~~ $ad = bc$.

In all given question show that equivalence relation

Q1
Sol ① reflexive:
$x \in A$
so xRx
$x - x$ is an integer
therefore it is reflexive

② Symmetric
if $x, y \in A \Rightarrow xRy \Leftrightarrow yRx \in A$
$x-y$ is an integer
so $y-x$ is also an integer
therefore it is symmetric

③ Transitive
if $x, y, z \in A \Rightarrow xRy$ & $yRz \in A \Leftrightarrow xRz \in A$
$x-y$ is an integer
and $y-z$ is an integer
also $x-z$ is an integer
therefor it is transitive
~~so~~ hence it is equivalence relation.

Q2
Solve Reflexive
$(a,b) R (a,b)$
$a^2+b^2 = a^2+b^2$ (Acc. to condition $a^2+b^2 = c^2+d^2$)
so it is reflexive

Symmetric
$(a,b) R (c,d) \Leftrightarrow (c,d) R (a,b)$
$\therefore a^2+b^2 = c^2+d^2$
$\Rightarrow c^2+d^2 = a^2+b^2$
so it is symmetric

<u>transitive</u> : if $a, b, c, d, e, f \in A$

$(a,b) R (c,d)$ & $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$

$\because \quad a^2+b^2 = c^2+d^2$

$c^2+d^2 = e^2+f^2$

$a^2+b^2+\cancel{c^2}+\cancel{d^2} = \cancel{c^2}+\cancel{d^2}+e^2+f^2$

$a^2+b^2 = e^2+f^2$

$(a,b) R (e,f)$

so it is transitive

hence it have equivalene relation

<u>Q3</u> Relation is defined as

$(a,b) R (c,d)$ iff ~~ab = cd~~

$ad = bc$

(1) Reflexive

(a,b) and $(c,d) \in N$

$(a,b) R (a,b)$

$ab = ba$

so it is reflexive

(2) symmetric $(a,b) R (c,d) \Rightarrow (c,d) R (a,b)$

$\because \quad ad = bc$

$cb = da$

so it is symmetric

(3) <u>Transitive</u>.

if $a, b, c, d, e, f \in N$

$(a,b) R (c,d)$ & $(c,d) R (e,f) \Rightarrow (a,b) R (e,f)$

$\because \quad ad = bc$

$cf = de$

$\Rightarrow$ ~~af = be~~

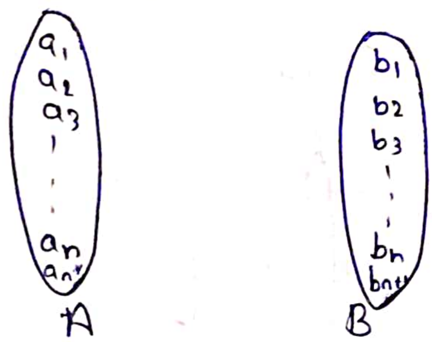
$ad \times cf = bc \times de$

$\Rightarrow \quad af = be$

so it is transitive
hence it has equivalene relation

Functions

$F: A \to B$



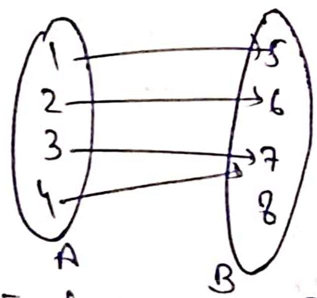
Domain: set of all possible output

Co-domain: " " " " "

Range : set of actual o/p or outcome

eg.

$f: A \to B$



Domain = $\{1, 2, 3, 4\}$

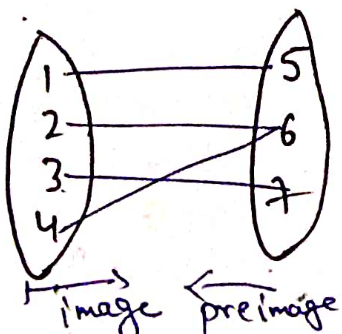
Codomain = $\{5, 6, 7, 8\}$

Range = $\{5, 6, 7\}$

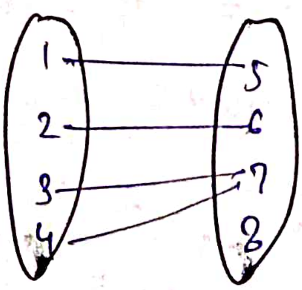
Types of Functions

① One to One (Injective)

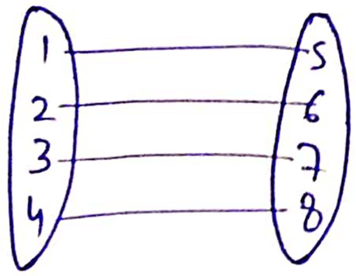
② Onto (surjective)



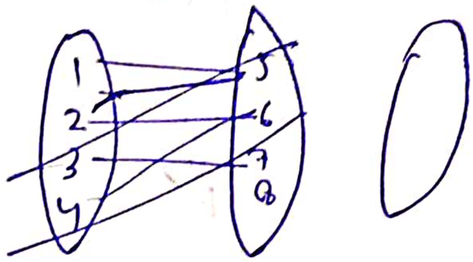
③ Into



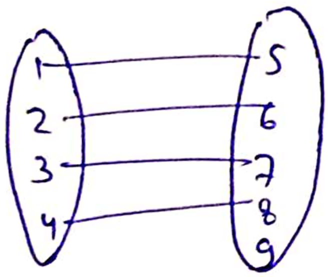
(4) One - One Onto (Bijective)



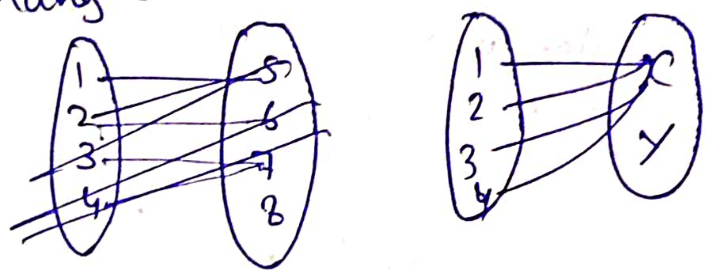
(6) Many - One



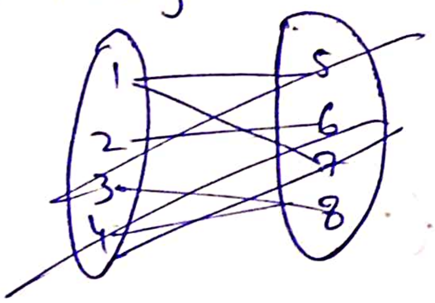
(5) one - one into



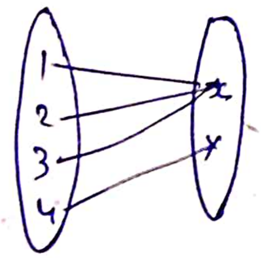
(7) Many - one ~~onto~~ into



(8) Many - one Onto



or



1 element ki kai sari preimage ho skti hai lekin ek ~~image~~ element ki ~~b~~ ek hi ~~pre~~ image bn skti hai

Inverse of a function

$f(x) = 4x + 5$

$\therefore \ f^{-1}(x) = ?$

sol $y = 4x + 5$

$\Rightarrow \dfrac{y-5}{4} = x$

$\Rightarrow f^{-1}(x) = \dfrac{x-5}{4}$

Composition of function

$f(x) = 4x+5$

$g(x) = x^2$

~~fofo~~ fog, gof

so $f(g(x)) = 4x^2+5$

$g(f(x)) = (4x+5)^2$

Q1 If $f(x) = \frac{1}{1-x}$, $g(x) = \frac{x-1}{x}$

find $g(f(x))$

Q2 $f(x) = x^3 - \frac{1}{x^3}$ find $f(x) + f\left(\frac{1}{x}\right)$

Q3 $f(x) = \log\left(\frac{1+x}{1-x}\right)$ show that $f(x) + f(y) = f\left(\frac{x+y}{1+xy}\right)$

Q4 $f(x) = x+2$, $h(x) = 3$ find $h \cdot f(g(f^{-1}(x))) \cdot h(f(x))$

$g(x) = \frac{1}{x^2+1}$

Q5 $x = \{1,2,3\}$ & f, g, h, & s mapped x to x

$f = \{(1,2)\ (2,3)\ (3,1)\}$

$g = \{(1,2)\ (2,1)\ (3,3)\}$

$h = \{(1,1)\ (2,2)\ (3,1)\}$

$s = \{(1,1)\ (2,2)\ (3,3)\}$

find fog, gof, sog, gos, sof, fos, & fohog.

Q1 Sol

$f(x) = \frac{1}{1-x}$, $g(x) = \frac{x-1}{x}$

$g(f(x)) = \dfrac{\frac{1}{1-x} - 1}{\frac{1}{1-x}} = \dfrac{\frac{1-(1-x)}{1-x}}{\frac{1}{1-x}} = x$ Ans

Q2
Sol^n

$$f(x) = x^3 - \frac{1}{x^3}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = \cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3} = 0$$

$$f(x) + f\left(\frac{1}{x}\right) = 0$$

Q3
Sol^n

$$f(x) + f(y) = \log\left(\frac{1+x}{1-x}\right) + \log\left(\frac{1+y}{1-y}\right)$$

$$= \log(1+x) - \log(1-x) + \log(1+y) - \log(1-y)$$

$$= \log[(1+x)(1+y)] - [\log(1-x)(1-y)]$$

RHS

$$\log\left(\frac{1+\frac{x+y}{1+xy}}{1-\frac{(x+y)}{(1+xy)}}\right) \Rightarrow \log\left(\frac{x+y+1+xy}{1+xy-x-y}\right)$$

put as $f(x)$ is value of x.

Q4
Sol^n

$$f(x) = x+2, \quad g(x) = \frac{1}{x^2+1}, \quad h(x) = 3$$

$$f^{-1}(x) = x-2$$

$$f^{-1}(g(x)) = \frac{1}{x^2+1} - 2$$

$$= h \cdot f(g(f^{-1}(x)) \cdot h f(x))$$

$$= 9 \cdot f(g(x-2))$$

$$= 9 \cdot f\left(\frac{1}{(x-2)^2+1}\right)$$

$$= 9\left\{\left[\frac{1}{(x-2)^2+1}\right] + 2\right\}$$

Q5

$f \circ g = f(g(x))$

$f(g(1)) = f(2) = 3$

$f(g(2)) = f(1) = 2$

$f(g(3)) = f(3) = 1$

$f \circ g = \{(2,3), (1,2), (3,1)\}$

similarly $g \circ f = g(f(x))$

$g(f(1)) = g(2) = 1$

$g(f(2)) = g(3) = 3$

$g(f(3)) = g(1) = 2$

so $g \circ f = \{(2,1)\ (3,3), (1,2)\}$

(iii). $s \circ g = s(g(x))$

$s(g(1)) = \cancel{s(g(2))}\ s(2) = 2$

$s(g(2)) = s(1) = 1$

$s(g(3)) = s(3) = 3$

$s \circ g = \{(2,2), (1,1), (3,3)\}$

(iv) - -

Mathematical induction

① Base step.

$n \in N \rightarrow n = 1$ (Base step

$\forall n \in$ positive integer $N \rightarrow n = 0$ (Base step)

$n \geq 2 \rightarrow n = 2$ Base

② Inductive-Hypothesis

let for $n = k$ it is true — (i)

③ Inductive step:

for $n = k+1$

from (i) LHS = RHS

Q. Prove that the sum of n natural number is $\frac{n(n+1)}{2}$

Sol let $p(n) = 1 + 2 + 3 + \cdots n = \frac{n(n+1)}{2}$

(i) Base step → for $n = 1$

LHS = 1

and RHS = $\frac{1(1+1)}{2} = 1$ so it is true

(ii) Inductive hypothesis

let for $n = k$ it is true

$P(k) = 1 + 2 + 3 + \cdots k = \frac{k(k+1)}{2}$ —— (i)

(iii) Inductive step:

for $n = k+1$

$P(k+1) = 1 + 2 + 3 + \cdots + k+1 = \frac{(k+1)(k+2)}{2}$

LHS $\underbrace{1 + 2 + 3 + \cdots + k}_{\text{from (i)}} + k+1$

$\frac{k(k+1)}{2} + k+1$

$\Rightarrow \frac{(k+1)(k+2)}{2} = \text{RHS}$

Q2 $\sum_{n=1}^{n} n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall\ n \in N$

Sol ① Base step

$P(n) = 1^2 + 2^2 + 3^2 + \cdots n^2 = \frac{n(n+1)(2n+1)}{6}$

for $n = 1$

LHS as $1^2 = 1$

$RHS = \frac{1(1+1)(2\times1+1)}{6} = 1$

② inductive hypothesis

let for $n = K$ it is true

$P(K) = 1^2 + 2^2 + 3^2 + \cdots K^2 = \frac{K(K+1)(2K+1)}{6}$ —— (i)

③ inductive step

for $n = K+1$

$P(K+1) = 1^2 + 2^2 + 3^2 + \cdots (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

LHS $\underbrace{1^2 + 2^2 + 3^2 + \cdots + K^2}_{\text{from (i)}} + (K+1)^2$

$\frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$= \left(\frac{K+1}{6}\right)(2K^2 + K + 6K + 6)$

$= \left(\frac{K+1}{6}\right)(2K^2 + 7K + 6) \Rightarrow \frac{(K+1)(2K+3)(K+2)}{6} = RHS$

Q3 $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots n} = \frac{2n}{n+1} \quad \forall\ n \in N$

Sol ① Base step for $n = 1$

$LHS = 1$

$RHS = \frac{2\times1}{1+1} = 1$

② Inductive Hypothesis :

let for $n = K$ .... it is true

$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots K} = \frac{2K}{K+1}$ —— (i)

3) Inductive step:

for $n = k+1$

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots(k+1)} = \frac{2(k+1)}{k+2}$$

LHS $\underbrace{1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+3+\cdots k}}_{\text{from eqn (1)}} + \frac{1}{1+2+3+\cdots(k+1)}$

$$= \frac{2K}{K+1} + \frac{1}{1+2+3+\cdots(k+1)}$$

$$= \frac{2K}{K+1} + \frac{1}{\frac{(k+1)(k+2)}{2}} \Rightarrow \frac{2K}{K+1} + \frac{2}{(k+1)(k+2)}$$

$$\therefore 1+2+3+\cdots+n+(n+1) = \frac{(n+1)(n+2)}{2}$$

$$= \frac{2(k+1)}{k+2}$$

Q $7 + 77 + 777 + \cdots \underbrace{77\cdots7}_{n \text{ times}} = \frac{7}{81}\left[10^{n+1} - 9n - 10\right] \forall n \in N$

Sol (1) Base step for $n = 1$

LHS = 7

and RHS $= \frac{7}{81}\left[10^{1+1} - 9 - 10\right] = \frac{7 \times 81}{81} = 7$

LHS=RHS

(2) Inductive hypothesis

let for $n = k$

$$P(K) = 7 + 77 + 777 + \cdots \underbrace{77\cdots7}_{K \text{ times}} = \frac{7}{81}\left[10^{K+1} - 9K - 10\right] \quad \text{—(i)}$$

(3) For $n = k+1$

$$P(K+1) = 7 + 77 + 777 + \cdots + \underbrace{77\cdots7}_{(K+1) \text{ times}} = \frac{7}{81}\left[10^{K+2} - 9(K+1) - 10\right]$$

LHS $\underbrace{7 + 77 + 777 + \cdots + \underbrace{77\cdots7}_{K \text{ times}}}_{\text{from eqn (i)}} + \underbrace{77\cdots7}_{(K+1) \text{ times}}$

$$\Rightarrow \frac{7}{81}\left[10^{K+1} - 9K - 10\right] + \underbrace{777\cdots7}_{(K+1) \text{ times}}$$

$$= \frac{7}{81}\left(10^{K+1} - 9K - 10\right) + \frac{7}{9}\left(\underbrace{99\ldots 9}_{(K+1)\text{ times}}\right)$$

$$= \frac{7}{9}\left[\frac{10^{K+1} - 9K - 10}{9} + \left(10^{K+1} - 1\right)\right]$$

$$= \frac{7}{9}\left[\frac{10^{K+1} - 9K - 10 + 9 \times \left(10^{K+1} - 1\right)}{9}\right]$$

$$= \frac{7}{81}\left[10 \cdot 10^{K+1} - 9(K+1) - 10\right]$$

Q Show that $n^3 + 2n$ is divisible by 3 for $n \geq 1$.

Sol ① Base step

for $n = 1$

$1^3 + 2 \times 1 = 3$ it is divisible by 3

② for $n = k$

$k^3 + 2k$ is divisible by 3 ---- (i)

③ For $n = k+1$

$\Rightarrow (k+1)^3 + 2(k+1)$

$\Rightarrow k^3 + 1 + 3k^2 + 3k + 2k + 2$

$\Rightarrow \underbrace{k^3 + 2k}_{\text{from (i)}} + \underbrace{3(k^2 + k + 1)}_{\text{also divisible by 3}}$

it is divisible by 3

Q1 $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ ∀ non-negative integer n

Solu ① Base step

for $n = 0$

LHS $= 1$

RHS $= 2^{0+1} - 1 = 1$

② Inductive hypothesis

for $n = k$

$1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1$ (i) .. it is true

③ Inductive step

for $n = k+1$

$1+2+2^2+ \cdots + 2^{K+1} = 2^{K+2}-1$

LHS

$$\underbrace{1+2+2^2+\cdots+2^K}_{\text{from (i)}\ 2^{K+1}-1} + 2^{K+1}$$

$$2^{K+1}-1+2^{K+1}$$

$$\Rightarrow 2^{K+1}-1+2^{K+1}$$

$$= 2^{K+2}-1 = \underline{RHS}$$

Q2 $1\cdot2 + 2\cdot3 + \cdots + n(n+1) = \dfrac{n(n+1)(n+2)}{3}, \quad n \in N$

Sol Base step: for $n=1$

$LHS = 2$

$RHS = \dfrac{1(1+1)(1+2)}{3} = 2$

(2) inductive hypothesis:

let for $n=K$ it is true

$P(K)$ $\quad 1\cdot2 + 2\cdot3 + \cdots + K(K+1) = \dfrac{K(K+1)(K+2)}{3}$ --- (i)

(3) inductive step:

for $n = K+1$

$P(K+1) = 1\cdot2 + 2\cdot3 + \cdots (K+1)(K+2) = \dfrac{(K+1)(K+2)(K+3)}{3}$

LHS $\quad \underbrace{1\cdot2 + 2\cdot3 + \cdots + K(K+1)}_{\text{from i}} + (K+1)(K+2)$

$$\frac{K(K+1)(K+2)}{3} + (K+1)(K+2)$$

$$\frac{(K+1)(K+2)}{3}\left[K+3\right] = \frac{(K+1)(K+2)(K+3)}{3} = RHS$$

Q3 $\dfrac{1}{1\cdot2} + \dfrac{1}{2\cdot3} + \dfrac{1}{3\cdot4} + \cdots + \dfrac{1}{n(n+1)} = \dfrac{n}{n+1}, \quad n \in N$

Sol Base step: let for $n=1$

$LHS = \dfrac{1}{2}$

$RHS = \dfrac{1}{1+1} = \dfrac{1}{2}$

(2) inductive hypothesis:

for $n=K$ it is true

$$P(K) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{K(K+1)} = \frac{K}{K+1} \quad \text{—— (i)}$$

(3) inductive step:

for $n = K+1$

$$P(K+1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{(K+1)(K+2)} = \frac{(K+1)}{(K+2)}$$

LHS

$$\Rightarrow \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)}$$

from (i)

$$\Rightarrow \frac{K}{K+1} + \frac{1}{(K+1)(K+2)}$$

$$\Rightarrow \frac{1}{K+1}\left[K + \frac{1}{K+2}\right] = \frac{1}{K+1}\left[\frac{K^2+2K+1}{(K+2)}\right]$$

$$= \frac{(K+1)\cancel{(K+1)}}{\cancel{(K+1)}(K+2)} = \frac{K+1}{K+2} = \text{RHS.}$$

Q4 $1^2 - 2^2 + 3^2 - 4^2 + \cdots (-1)^{n-1} n^2 = (-1)^{n-1} \frac{n(n+1)}{2} \quad \forall n \in N$

Sol: Base step for $n=1$

LHS $= 1$

$$\text{RHS} = (-1)^{1-1} \frac{1(1+1)}{2} = 1$$

inductive hypothesis:

Let for $n=K$ — it is true

$$P(K) = 1^2 - 2^2 + 3^2 - 4^2 + \cdots (-1)^{K-1} K^2 = (-1)^{K-1} \frac{K(K+1)}{2} \quad \text{—— (i)}$$

inductive step

for $n = K+1$

$$P(K+1) = 1^2 - 2^2 + 3^2 - 4^2 + \cdots (-1)^K (K+1)^2 = (-1)^K \frac{(K+1)(K+2)}{2}$$

$$\text{LHS} = 1^2 - 2^2 + 3^2 - 4^2 + \cdots (-1)^{K-1} K^2 + (-1)^K (K+1)^2$$

from eqn (i)

$$(-1)^{K-1} \frac{K(K+1)}{2} + (-1)^K (K+1)^2$$

$$(K+1)\left[\frac{(-1)^{K-1}K}{2} + (-1)^K(K+1)\right]$$

$$(-1)^K (K+1)\left[\frac{K}{-2} + K+1\right]$$

$$= (-1)^K (K+1)\left[\frac{-K+2K+2}{2}\right] = \frac{(-1)^K (K+1)(K+2)}{2} = RH.$$

Q5 Show that $n^4 - 4n^2$ is divisible by 3 for $n \geq 2$

A) Base step for $n = 2$

LHS $2^4 - 4(2)^2 = 0$ so it is divisible by 3

$\because$ Zero is divisible by every integer, with one exception: nothing is divisible by zero

(2) for $n = k$

$K^4 - 4K^2$ is divisible by 3 — (i)

(3) for $n = k+1$

$$(K+1)^4 + 4(K+1)^2$$

$$K^4 + 4K^3 + 6K^2 + 4K + 1 + [4K^2 + 4 + 8K]$$

$\because (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^2$

$$= K^4 - 4K^2 + 4K^3 + 6K^2 + 4K + 1 - 4 - 8K$$

$$= \underbrace{K^4 - 4K^2}_{(i)} + 4K^3 + 6K^2 - 4K - 3$$

$$+ 3[2K-3] + 4K^3 - 4K + 4K[K^2-1]$$

$$+ 4(K-1)K(K+1)$$

$\because$ we know that three consecutive numbers ka product always divisible by 3.

Q6 Prove that $n^5 - n$ is divisible by 5 $\forall$ non-negative integer!

Solve Base step for $n=1$

$1-1 = 0$ so zero is divisible by 5

inductive hypothesis:

for $n=k$

$k^5 - k$ is divisible by 5 —— (i)

inductive step

for $n=k+1$

$(k+1)^5 - (k+1)$

$(k+1)\left((k+1)^4 - 1\right)$

Q Prove that $(2n+7) < (n+3)^2 \; \forall n \in N$

Sol^n

(1) $n = 1$

$\therefore (2 \times 1 + 7) < (1+3)^2$

$\Rightarrow 9 < 16$

(2) $n = k$ (let)

$(2k+7) < (k+3)^2$ — (i)

(3) $n = k+1$

$\Rightarrow (2(k+1)+7) < (k+4)^2$

$(2k+9) < k^2 + 16 + 8k$

By eq^n (i)

$\Rightarrow (2k+7) < k^2 + 9 + 6k$

$2k+7+2 < k^2+9+6k+2$

$\Rightarrow (2k+9) < k^2 + 6k + 11$

$\Rightarrow (2k+9) < k^2 + 6k + 11 + 2k - 2k + 5 - 5$

$\Rightarrow$ $(2k+9) < (k+4)^2 - (2k+5)$

As the equality holds for natural no.

$\therefore$ $2k+9 < (k+4)^2$

so it is true.

Q1 $(n+3)^2 \le 2^{n+3} \ \forall n \in N$

Q2 $n! \ge 2^{n-1} \ \forall n \in Z^+$

Q3 $11^{n+2} + 12^{2n+1}$ is divisible by 133 $\forall n \in N$

Q1 sol^n (1) $n = 1$

$(1+3)^2 \le 2^{1+3}$

$16 \le 16$

(2) Inductive hypothesis

let for $n = k$ it is true

$(k+3)^2 \le 2^{k+3}$ ——— (i)

(3) inductive step:

$n = k+1$

$(k+1+3)^2 \le 2^{k+4}$

$(k+4)^2 \le 2^{k+4}$

$k^2 + 16 + 8k \le 2^{k+4}$

from eq^n (i)

$k^2 + 9 + 6k \le 2^{k+3}$

$\cancel{k^2 + 9 + 6k + 2k - 2k + 7 \le 2^{k+3}}$

$\cancel{k^2 + 16 + 8k \le 2^{k+3} + 2k + 7}$

$k^2 + 9 + 6k + 7 + 2k \le \frac{2^{k+3} \cdot 2}{2} + 2k + 7$

$(k+4)^2 \le \frac{2^{k+4}}{2} + (2k+7)$

as for all the natural number inequality holds.

$$(k+4)^2 \le 2^{k+4}$$

Q2

Sol^n ① $n = 1$

$$1 \ge 2^{1-1} = 1 \ge 1.$$

② inductive hypothesis

for $n = k$

$k! \ge 2^{k-1}$ --- it is true

③ inductive step.

for $n = k+1$

$$(k+1)! \ge 2^{k+1-1}$$

$$(k+1)! \ge 2^{k}$$

$$\cancel{(k+1)\, k! \ge 2^{k}}$$

$\cancel{k! \ge}$

from eq^n ⓘ

$$k! \ge 2^{k-1}$$

$$(k+1)\, k! \ge 2^{k-1}(k+1)$$

$$(k+1)! \ge \frac{2^{k-1} \cdot 2\,(k+1)}{2}$$

$$(k+1)! \ge \frac{2^{k}}{2}(k+1)$$

as inequality holds for all natural number

$$(k+1)! \ge 2^{k}$$

Q3

Sol^n $11^{n+2} + 12^{2n+1}$

① Base step for $n = 1$

$\Rightarrow 11^{1+2} + 12^{2\times1+1}$

$\Rightarrow 11^3 + 12^3 \Rightarrow 1331 + 1728$

$= 3059$ so it is divisible by 133

② Inductive hypothesis

~~for~~ let $n = k$

$\Rightarrow 11^{k+2} + 12^{2k+1}$ - it is true

it is divisible by 133

(iii) for $n = k+1$

$$11^{(k+1)+2} + (12)^{2(k+1)+1}$$

$$11^{k+3} + 12^{2k+3}$$

$$\cancel{11^{k+2} \cdot 11 + 12^{2k+1} \cdot 2^{k}}$$

$$11^{k+2} \cdot 11 + 12^{2k+1} \cdot 12 \times 12$$

$$11^{k+2} \cdot 11 + 12^{2k+1} \cdot 144 \Rightarrow 11^{k+2} \cdot 11 + 12^{2k+1}(133+11)$$

$$\Rightarrow 11^{k+2} \cdot 11 + 12^{2k+1} \cdot 11 + 12^{2k+1} \cdot 133$$

$$11\big(\underbrace{11^{k+2} + 12^{2k+1}}_{\text{it is divisible by 133 from (i)}}\big) + \underbrace{133 \cdot 12^{2k+1}}_{\text{it is also divisible by 133}}$$

so $11^{k+3} + 12^{2k+3}$ is divisible by 133 it is true.

Generating Function

let $\{A\}$ be any sequence with terms $a_0, a_1, a_2 \cdots \infty$ is infinite series then $GF(A,Z)$ of a sequence A

$$G(A,Z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= a_0 + a_1 Z + a_2 Z^2 + \cdots \infty$$

Method to solve Recurrence relation using generating function

Step-I Multiply both sides by Z^n and sum upto $\boxed{k \text{ to } \infty}$

Step-II Write each term in terms of $G(A,Z)$

Step-III Solve $G(A,Z)$ then using standard generating function sequence cab be found as

(i) $a_n = c$

$$G(A,Z) = \frac{c}{1-Z}$$

(ii) $a_n = b^n$

$$G(A,Z) = \frac{1}{1-bZ}$$

(iii) $a_n = c \cdot b^n$

$$G(A,Z) = \frac{c}{1-bZ}$$

(iv) $a_n = n$

$$G(A,Z) = \frac{Z}{(1-Z)^2}$$

(i) $a_n = c \quad , n \geq 0$

$$G(A,Z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} c \cdot z^n \Rightarrow c\left[1 + Z + Z^2 + \cdots \infty\right]$$

$$\Rightarrow c\left[\frac{1}{1-Z}\right] = \frac{c}{1-Z} \qquad \because \left\{ \begin{array}{l} G.P \\ a = 1 \\ r = Z \\ S_\infty = \frac{a}{1-r} \end{array} \right\}$$

(ii) $a_n = b^n, \quad n \geq 0$

$$G(A,Z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} b^n z^n$$

$$\Rightarrow \quad [1 + bz + (bz)^2 + (bz)^3 + \cdots \infty]$$

$$\Rightarrow \quad \frac{1}{1-bz}$$

$\left\{ \begin{array}{l} G.P \\ a = 1 \\ r = bz \\ S_\infty = \frac{a}{1-r} \end{array} \right\}$

(iii) $a_n = c \cdot b^n, \; n \geq 0$

$$G(A,z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} c b^n z^n$$

$$= c\,[1 + (bz)^1 + (bz)^2 + (bz)^3 + \cdots \infty]$$

$$= \frac{c}{1-bz}$$

G.P. $\left\{ \begin{array}{l} a = 1 \\ r = bz \\ S_\infty = \frac{a}{1-r} \end{array} \right\}$

(iv) $a_n = n, \; n \geq 0$

$$G(A,z) = \sum_{n=0}^{\infty} a_n z^n$$

$$= \sum_{n=0}^{\infty} n z^n \Rightarrow [0 + z + 2z^2 + 3z^3 + \cdots \infty]$$

$$= z\,[1 + 2z + 3z^2 + 4z^3 + \cdots \infty]$$ {Binomial theorem}

$$= z\left[\frac{1}{(1-z)^2}\right] = \frac{z}{(1-z)^2}$$

Q Solve the Recurrence relation using generating function

Q $a_n + 2a_{n-1} = 0, \quad a_0 = 5$

Sol^n multiply by z^n

$$a_n z^n + 2a_{n-1} z^n = 0$$

$$\Rightarrow \sum_{n=1}^{\infty} a_n z^n + \sum_{n=1}^{\infty} 2a_{n-1} z^n = 0$$

$$\Rightarrow \sum_{n=1}^{\infty} a_n z^n + 2\sum_{n=1}^{\infty} a_{n-1} z^{n-1} z = 0$$

$$\sum_{n=1} a_n z^n + 2z \sum_{n=1}^{\infty} a_{n-1} z^{n-1} = 0$$

we know

$$\sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + \cdots \infty$$

$$\sum_{n=0}^{\infty} a_n z^n - a_0 = a_1 z + a_2 z^2 + \cdots \infty$$

$$\sum_{n=0}^{\infty} a_n z^n - a_0 = \sum_{n=1}^{\infty} a_n z^n$$

$$\sum_{n=0}^{\infty} a_n z^n - a_0 + 2z \sum_{n=1}^{\infty} a_{n-1} z^{n-1} = 0.$$

$$\sum_{n=1}^{\infty} a_{n-1} z^{n-1} = a_0 + a_1 z + a_2 z^2 + \cdots \infty$$

$$= \sum_{n=0}^{\infty} a_n z_n$$

$$\Rightarrow \sum_{n=0}^{\infty} a_n z^n - a_0 + 2z \sum_{n=0}^{\infty} a_n z_n = 0$$

$$G(A,z) - 5 + 2z\,G(A,z) = 0$$

$$G(A,z) = \frac{5}{1+2z} = \frac{5}{1-(-2z)}$$

$$\boxed{G(A,z) = 5\cdot(-2)^n}$$

$$G(A,z) = \frac{c}{1-bz}$$

$$G(A,z) = c\cdot b^n$$

Q $U_n = U_{n-1} + 2U_{n-2}$, $n \geq 2$, $u_0 = 3$, $u_1 = 7$

$\Rightarrow$ ~~multipu~~ $U_n - U_{n-1} - 2U_{n-2} = 0$

multiplying by z^n

$$U_n z^n - U_{n-1} z^n - 2U_{n-2} z^n = 0$$

$$\sum_{n=2}^{\infty} U_n z^n - \sum_{n=2}^{\infty} U_{n-1} z^n - 2\sum_{n=2}^{\infty} U_{n-2} z^n = 0$$

$$\sum_{n=0}^{\infty} U_n z^n - U_0 - U_1 z - z\sum_{n=2}^{\infty} U_{n-1} z^{n-1} - 2z^2 \sum_{n=2}^{\infty} U_{n-2} z^{n-2} = 0$$

$$\Rightarrow \sum_{n=0}^{\infty} U_n z^n - U_0 - U_1 z - z\sum_{n=0}^{\infty} U_n z^n - U_0 - 2z^2 \sum_{n=0}^{\infty} U_n z^n = 0$$

$\therefore \sum_{n=0}^{\infty} U_n z^n - U_0 - U_1 z - z\left[\sum_{n=0}^{\infty} U_n z^n - U_0\right] - 2z^2 \sum_{n=0}^{\infty} U_n \cdot z^n = 0$

$$G(U,z) - U_0 - U_1 z - z\left[G(U,z) - U_0\right] - 2z^2 G(U,z) = 0$$

$$G(U,z) - 3 - 7z - z\,G(U,z) + 3z - 2z^2 G(U,z) = 0$$

$$G(U,z) - z\,G(U,z) - 2z^2 G(U,z) = 3 + 4z$$

$$G(U,z)(1 - z - 2z^2) = 3 + 4z$$

$$G(U,z) = \frac{3+4z}{-[2z^2+z-1]}$$

$$\Rightarrow \quad G(U,z) = \frac{3+4z}{-(z+1)(2z-1)} = \frac{3+4z}{(1+z)(1-2z)}$$

By partial fraction

$$\frac{3+4z}{(1+z)(1-2z)} = \frac{A}{(1+z)} + \frac{B}{(1-2z)}$$

$$\Rightarrow \frac{3+4z}{\cancel{(1+z)(1-2z)}} = \frac{A(1-2z) + B(1+z)}{\cancel{(1+z)(1-2z)}}$$

$$\Rightarrow \quad 3 + 4z = A - 2Az + B + Bz$$

by comparing coeff.

$$A + B = 3$$
$$-2A + B = 4$$
$$3A = -1$$
$$A = \frac{-1}{3}$$

$$\therefore \quad A + B = 3 \Rightarrow B = \frac{10}{3}$$

$$G(U,z) = \frac{10}{3}\left[\frac{1}{1-2z}\right] - \frac{1}{3}\left[\frac{1}{1+z}\right]$$
$$= \frac{10}{3}\left[\frac{1}{1-2z}\right] - \frac{1}{3}\left[\frac{1}{1-(-z)}\right]$$

$$\boxed{G(U,z) = \frac{10}{3}2^n - \frac{1}{3}(-1)^n}$$ Ans

Q $U_n - 6U_{n-1} = 2^{n-1}$, $U_0 = 1$

sol multiply by z^n on both side

$$U_n z^n - 6U_{n-1} z^n = 2^{n-1} z^n$$

$$\sum_{n=1}^{\infty} U_n z^n - 6\sum_{n=1}^{\infty} U_{n-1} z^n = \sum_{n=1}^{\infty} 2^{n-1} z^n$$

$$\sum_{n=0}^{\infty} U_n z^n - U_0 - 6z\sum_{n=0}^{\infty} U_n z^n = \sum_{n=1}^{\infty} 2^{n-1} z^n$$

$$\therefore \sum_{n=1}^{\infty} U_{n-1} z^n = U_0 z + U_1 z^2 + U_2 z^2 + \dots$$

$$\sum_{n=0}^{\infty} U_n z^n = U_0 + U_0 + U_0 z^2$$

$$U_{-1} + z[U_0 + U_1 z$$

$$\sum_{n=1}^{\infty} U_n z^n = U_1 z + U_2 z^2$$

$$\sum_{n=0}^{\infty} U_n z^n = U_0 + U_1 z + \dots$$

$$\sum_{n=0}^{\infty} U_n z^n - U_0 - 6z\sum_{n=0}^{\infty} U_n z^n = \sum_{n=1}^{\infty} 2^{n-1} z^n$$

$$G(U,z) - U_0 - 6zG(U,z) = z + 2z^2 + 2^2 z^3 + 2^3 z^4 + \dots \infty$$

$$G(U,z) - U_0 - 6zG(U,z) = z[1 + 2z + (2z)^2 + (2z)^3 + \dots \infty]$$

G.P.

$$G(U,z) - U_0 - 6zG(U,z) = z\left[\frac{1}{1-2z}\right]$$

$$G(U,z)\left[1 - 6z\right] - U_0 = z\left[\frac{1}{1-2z}\right]$$

$$-5G(U,z) - 1 = \frac{z}{1-2z}$$

$$-5G(U,z) = \frac{z}{1-2z} + 1$$

$$\therefore G(U,z) = \frac{1-z}{(1-2z)(1-6z)}$$

$$G(U,z) = \frac{1-z}{(1-2z)(1-6z)}$$

$$G(U,z) = \frac{5}{4}\left(\frac{1}{1-6z}\right) - \frac{1}{4}\left(\frac{1}{1-2z}\right)$$

$$\boxed{G(U,z) = \frac{5}{4}6^n - \frac{1}{4}2^n}$$

A

Q (i) 1, 1, 1, 1, 1, 1, 1 find gen

Soln $\sum_{n=0}^{\infty} a_n z^n = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots \infty$

$a_0 = 1, a_1 = 1, a_2 = 1, \dots a_6 = 1$

$\Rightarrow \sum_{n=0}^{6} a_n z^n = 1 + z + z^2 + z^3 + \dots z^6$

acc. to sum of GP $= \frac{a(1+r^n)}{1-r}$

n → no. of terms

here $a = 1$

$r = z$

so $= \frac{1(1-z^7)}{1-z}$

$$\boxed{G(A,z) = \frac{1-z^7}{1-z}}$$ Ans

Q (ii) $\{a, a, a \dots\}$

(iii) $a_n = ar^n$

(iv) $\{2, 4, 8, 16, 32, \dots\}$

(v) $\{2, 3, 5, 9, 17, 33 \dots\}$

Permutation [Arrangement]

→ Arrangement of something.

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

Q1 $^nP_4 = 20 \times {^nP_2}$ find n?

Q2 $^{(n+5)}P_{(n+1)} = \frac{11(n-1)}{2} \times {^{(n+3)}P_n}$. find n.

Q1 sol

$$^nP_4 = 20 \times {^nP_2}$$

$$\frac{\cancel{n!}}{(n-4)!} = 20 \times \frac{\cancel{n!}}{(n-2)!}$$

$$\cancel{\frac{(n-2)!}{(n-4)(n-3)(n-2)!} = 20 \Rightarrow 1 = 20(n-4)(n-3)}$$

$$\Rightarrow \cancel{1 = 20[n^2 - 3n - 4n + 12]}$$

$$\cancel{\Rightarrow 1 = 20[n^2 - 7n + 12]}$$

$$\Rightarrow (n-2)(n-3)\cancel{(n-4)!} = 20 \times \cancel{(n-4)!}$$

$$n^2 - 5n + 6 = 20$$

$$\Rightarrow n = 7, -2$$

Q2 Solⁿ

$$^{(n+5)}P_{(n+1)} = \frac{11(n-1)}{2} \times {^{(n+3)}P_n}$$

$$\Rightarrow \frac{(n+5)!}{(\cancel{n}+5-\cancel{n}-1)!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{(\cancel{n}+3-\cancel{n})}$$

$$\Rightarrow \frac{(n+5)!}{4!} = \frac{11(n-1)}{2} \times \frac{(n+3)!}{3!}$$

$$\frac{(n+5)(n+4)\cancel{(n+3)!}}{2\ \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = \frac{11(n-1)}{\cancel{2}} \times \frac{\cancel{(n+3)!}}{\cancel{3} \times \cancel{2}}$$

$$n^2 + 4n + 5n + 20 = 22n - 22$$

$$\Rightarrow \quad n^2 + 9n - 22n + 42 = 0$$

$$\Rightarrow \quad n^2 - 13n + 42 = 0$$

$$n^2 - 7n - 6n + 42 = 0$$

$$\boxed{n = 7, 6}$$

Permutation with Repeatation

Q Howmany ways are there to arrange the letters in the word (i) ALLAHABAD.

Sol^n $\quad n = 9$

$A \rightarrow 4$
$L \rightarrow 2$

Sol^n $\quad \Rightarrow \dfrac{9!}{4!\,2!}$

Q Find the no. of different messages that can be represented by sequences of 4 dashes and 3 dots

Ans $\quad \dfrac{7!}{4!\,3!} = \dfrac{7 \times \cancel{6}^{2} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2}} = 35$

Q The letters of the word TUESDAY are arranged in a line. Each arrangement ending with letter ~~as~~ S
1. Howmany different arrangements are possible?
Howmany of them start with letter D.

Ans $\quad$ TUESDAY

$\Rightarrow n = 7$

(i) $\underbrace{\square\square\square\square\square\square}_{6!}\boxed{S} \Rightarrow 6!$

(ii) $\boxed{D}\underbrace{\square\square\square\square\square}_{5!}\boxed{S} \Rightarrow 5!$

Q Howmany signals can be need by using 6 flags of different Colours when any number of them may be hosted at a time.

Sol^n

$$1 \text{ flag} = {}^6P_1$$
$$2 \text{ flag} = {}^6P_2$$
$$3 \text{ flag} = {}^6P_3$$
$$4 \text{ flag} = {}^6P_4$$
$$5 \text{ flag} = {}^6P_5$$
$$6 \text{ flag} = {}^6P_6$$

$\Rightarrow$ $6+6+6+6+6+6$

$= 36.$

Q Find the number of 4 digit numbers that can be formed using the digits 1, 2, 3, 4, and 5 if no digit is used more than 1's in a number. Howmany of these number will be even.

Sol^n (i) $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ } \rightarrow {}^5P_4 = 120$

(ii) $\boxed{\ }\boxed{\ }\boxed{\ }\boxed{2} \rightarrow {}^4P_3$

$\boxed{\ }\boxed{\ }\boxed{\ }\boxed{4} \rightarrow {}^4P_3$

$\Rightarrow 48$ Ans

Q It is required to sheet 5 men and 4 women in a row show that the women occupy the even places. Howmany such arrangements are possible

Sol^n

$\boxed{\ }\boxed{2}\boxed{\ }\boxed{4}\boxed{\ }\boxed{6}\boxed{\ }\boxed{8}\boxed{\ }$

$${}^4P_4 \times {}^5P_5 = 2880$$

Q In an examination hall there are 4 rows of chairs. Each row has 8 chairs, 1 behind the other. There are two classes appearing for the examination with 16 students in each class. It is desired that in each row all students belong to the same class and no two adjacent rows are alloted to the same Class. In howmany way can these 32 students be seated.

Sol^n Rows = 4

no. of chairs in each row = 8

no. of students in each class = 16

total students = 32

I A | B

II B | A

III A | B

IV B | A

$\Rightarrow 2(\overset{A}{16!} + \overset{B}{16!}) = 2 \times 16!$

Q. Howmany odd numbers can be formed greater than 80,000 using the digits 2, 3, 4, 5, and 8 if each digit is used only ones.

Sol^n

8				3

$3!$

$+$

8				5

$3!$

$= 2 \times 3!$

$= 12$ Ans

Q. There are 6 english, 4 sanskrit and 5 hindi books. In how many way can ~~be~~ they be arranged on a sell so as to keep all the books of same language together

$P_6 \quad P_4 \quad P_5$

E S H

6 4 5

$\underbrace{6}\ \underbrace{4}\ \underbrace{5}$

arrangement m multiply hota hai

3 packet.

(i) $^3P_3 = 3!$

(ii) $^6P_6 = 6!$

(iii) $^4P_4 = 4!$

(iv) $^5P_5 = 5!$

$\Rightarrow 3! \times 6! \times 4! \times 5!$

Q. A tea party is arranged for 16 people along the two sides of a long table with 8 chairs on each side 4 men wish to set on one particular side and 2 on the other side. In howmany ways can they be seated.

Soln

$^8P_4 \times {}^8P_2 \times {}^{10}P_{10}$

Q. (i) Find howmany arrangements can be made with the letters of word mathematics.

(ii) Howmany of them are the ~~words~~ vowels together

Soln

Mathematics

(i) $\frac{11!}{\lfloor 2 \lfloor 2 \lfloor 2}$

(ii)

$\underbrace{\text{MTHMTCS}}$ $\underbrace{\text{AEAI}}$

$\frac{8!}{\lfloor 2 \lfloor 2} \times \frac{\lfloor 4}{\lfloor 2}$

Q. In howmany ways can 10 books be arranged on a shelf show that a particular pair of books shell be

(i) Always together

(ii) Never together

Sol^n Always together

8 ① pair = 9! × 2! arr of 1.

(ii) all 10 books = $\lfloor 10$

always together = $2 \times \lfloor 9$

Never together = $\lfloor 10 - 2 \times \lfloor 9$