$\Rightarrow (p \vee q) \wedge (p \vee q) \wedge (\sim p \vee \sim q) \wedge (\sim p \wedge \sim q)$

$\Rightarrow (p \vee q) \wedge (\sim p \vee \sim q)$

$\Rightarrow$ POS

Que. 3) Obtain PDNF & PCNF for the following formula :-

$(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

Soln.)

Using Truthtable :-

P	Q	R	A $P \wedge Q$	$\sim P$	B $\sim P \wedge R$	C $Q \wedge R$	D $A \vee B$	$C \vee D$
F	F	F	F	T	F	F	F	F
F	F	T	F	T	T	F	T	T
F	T	F	F	T	F	F	F	F
F	T	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	F
T	F	T	F	F	F	F	F	F
T	T	F	T	F	F	F	T	T
T	T	T	T	F	F	T	T	T

CVD mein True values se minterm count hoga aur false se maxterm

Minterm	Maxterm
—	$P \vee Q \vee R$
$\sim P \wedge \sim Q \wedge R$	—
—	$P \vee \sim Q \vee R$
$\sim P \wedge Q \wedge R$	—
—	$\sim P \vee Q \vee R$
—	$\sim P \vee Q \vee \sim R$
$P \wedge Q \wedge \sim R$	—
$P \wedge Q \wedge R$	—

$\Rightarrow$

SOP

PDNF :- $(\sim P \wedge \sim Q \wedge R) \vee (\sim P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge Q \wedge R)$

PCNF :- $(P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

POS

Que-8) Obtain PCNF & PDNF.

$\underbrace{(\sim P \rightarrow R)}_{A} \wedge \underbrace{(Q \leftrightarrow P)}_{B}$

Sol:

P	Q	R	$\sim P$	$\sim P \rightarrow R$ (A)	$Q \leftrightarrow P$ (B)	$A \wedge B$
F	F	F	T	F	T	F
F	F	T	T	T	T	T -
F	T	F	T	F	F	F
F	T	T	T	T	F	F
T	F	F	F	T	F	F
T	F	T	F	T	F	F
T	T	F	F	T	T	T -
T	T	T	F	T	T	T -

Minterm	Maxterm
-	$P \vee Q \vee R$
$\sim P \wedge \sim Q \wedge R$	
-	$P \vee \sim Q \vee R$
-	$P \vee \sim Q \vee \sim R$
-	$\sim P \vee Q \vee R$
-	$\sim P \vee Q \vee \sim R$
$P \wedge Q \wedge \sim R$	
$P \wedge Q \wedge R$	

SOP

PDNF :- $(\sim P \wedge \sim Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge Q \wedge R)$

PCNF :- POS - $(P \vee Q \vee R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

Without using truthtable :-

Q.1) Obtain the PCNF & PDNF of
$(P\wedge Q)\vee(\sim P\wedge Q)\vee(Q\wedge R)$

$\sim P\vee P\equiv T$
$\sim P\wedge P\equiv F$

Sol.) $(P\wedge Q\wedge T)\vee(\sim P\wedge Q\wedge T)\vee(Q\wedge R\wedge T)$

$\Rightarrow \{(P\wedge Q)\wedge(R\vee\sim R)\}\vee\{(\sim P\wedge Q)\wedge(\sim R\vee R)\}$
$\vee[(Q\wedge R)\wedge(\sim P\vee P)]$

$\Rightarrow [(P\wedge Q\wedge\sim R)\vee(P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge\sim R)\vee$
$(\sim P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge R)\vee(P\wedge Q\wedge R)$

Neglecting repeating ~~terms~~ terms.

$\Rightarrow (P\wedge Q\wedge\sim R)\vee(P\wedge Q\wedge R)\vee(\sim P\wedge Q\wedge\sim R)\vee$
$(P\wedge Q\wedge R) \Rightarrow$ PDNF

For PCNF,
Negation of minterm that are not in minterm of P, Q, R.

Rough work

P	Q	R	Minterm
F	F	F	$\sim P\wedge\sim Q\wedge\sim R$
F	F	T	$\sim P\wedge\sim Q\wedge R$
F	T	F	✓ $\sim P\wedge Q\wedge\sim R$
F	T	T	✓ $\sim P\wedge Q\wedge R$
T	F	F	$P\wedge\sim Q\wedge\sim R$
T	F	T	$P\wedge\sim Q\wedge R$
T	T	F	✓ $P\wedge Q\wedge\sim R$
T	T	T	✓ $P\wedge Q\wedge R$

$S \Rightarrow (\sim P\wedge\sim Q\wedge\sim R)\vee(\sim P\wedge\sim Q\wedge R)$
$\vee(P\wedge\sim Q\wedge\sim R)\vee(P\wedge\sim Q\wedge R)$

$\sim S \Rightarrow (P\vee Q\vee R)\wedge(P\vee Q\vee\sim R)\wedge$
$(\sim P\vee\sim Q\vee R)\wedge(\sim P\vee Q\vee\sim R)$
$\Downarrow$
PCNF

Q.2) $[P\to(Q\wedge R)]\wedge[\sim P\to(\sim Q\wedge\sim R)]$

Soln.) Rule No-11

$[\sim P\vee(Q\wedge R)]\wedge[\sim(\sim P)\vee(\sim Q\wedge\sim R)]$

$[\sim P\vee(Q\wedge R)]\wedge[P\vee(\sim Q\wedge\sim R)]$

$\Rightarrow\{(\sim P\vee Q)\wedge(\sim P\vee R)\}\wedge[(P\vee\sim Q)\wedge(P\vee\sim R)]$

$\Rightarrow [(\sim P \vee Q \vee F) \wedge (\sim P \vee R \vee F)] \wedge [(P \vee \sim Q \vee F) \wedge (P \vee \sim R \vee F)]$

$\Rightarrow [[(\sim P \vee Q) \vee (\sim R \wedge R)] \wedge [\sim P \vee R \vee (\sim Q \wedge Q)] \wedge [(P \vee \sim Q) \vee (\sim R \wedge R)] \wedge [(P \vee \sim R) \vee (\sim Q \wedge Q)]$

$\Rightarrow (\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (\sim P \vee R \vee \sim Q) \wedge (\sim P \vee R \vee Q) \wedge (P \vee \sim Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim R \vee \sim Q) \wedge (P \vee \sim R \vee Q)$

Neglecting same terms.

$\Rightarrow (\sim P \vee Q \vee \sim R) \wedge (\sim P \vee R \vee \sim Q) \wedge (P \vee \sim Q \vee \sim R) \wedge (P \vee \sim Q \vee R) \wedge (P \vee \sim R \vee \sim Q) \wedge (P \vee \sim R \vee Q) \Rightarrow$ PCNF

For PDNF:-

~~Rough work~~

~~P Q R~~ ~~Maxterm~~

Maxterm

$\sim P \vee Q \vee \sim R$

$\sim P \vee Q \vee R$

$\sim P \vee Q \vee \sim R$

$\sim P \vee Q \vee R$

$P \vee \sim Q \vee \sim R$

$P \vee \sim Q \vee R$

$P \vee Q \vee \sim R$

$P \vee Q \vee R$

$S \Rightarrow (P \vee Q \vee R) \wedge (\sim P \vee Q \vee \sim R)$

Negation of S

$\Rightarrow \sim S \Rightarrow (P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge \sim R)$

$\Rightarrow$ PDNF

Date
Page

o) Predicate Logic:-

(I) Quantifiers :-

Quantifier is one which is used to quantify nature of variables.

(i) Universal Quantifier ($\forall x / x$) :-

(a) for all x
(b) for every x
(c) for each x
(d) Everything is x such that
(e) Eachthing is x such that

(ii) Existential Quantifier ($\exists x$) :-

(a) for some x
(b) there exists a x such that
(c) Some x such that
(d) There is atleast one x such that

Ques-1) Write the following statements in the symbolic form :-

(1) Something is Good
(2) Everything is Good
(3) Something is not Good
(4) Nothing is Good

Soln-1) $G(x) = x$ is Good

(1) $\exists x \; G(x)$
(2) $\forall x \; G(x)$
(3) $\exists x \sim G(x)$

Date
Page

0) Predicate Logic :-

(1) Quantifiers :-

Quantifier is one which is used to quantify nature of variables.

(i) Universal Quantifier ($\forall x / x$) :-

(a) for all x
(b) for every x
(c) for each x
(d) Everything is x such that
(e) Eachthing is x such that

(ii) Existential Quantifier ($\exists x$) :-

(a) for some x
(b) there exists a x such that
(c) Some x such that
(d) There is atleast one x such that

Ques-1) Write the following statements in the symbolic form :-
(1) Something is Good
(2) Everything is Good
(3) Something is not Good
(4) Nothing is Good

Soln 1)
$G(x) = x$ is Good

(1) $\exists x \, G(x)$
(2) $\forall x \, G(x)$
(3) $\exists x \sim G(x)$

Date
Page

(4) $\forall x \sim G(x)$

Que 2) Write in symbolic form -
(1) All men are good.
(2) Some men are good.
(3) Some men are not good.
(4) No men is good.

Sol^n 2) $M(x)$ = x is men.
$G(x)$ = x is good.

(1) $\forall x$ if x is a men then x is good
$\Rightarrow$ $\forall x(M(x) \longrightarrow G(x))$

(2) There is atleast one x, x is a men and x is good.
$\exists(x) \cdot (M(x) \wedge G(x))$

(3) $\exists(x)\ (M(x) \wedge \sim G(x))$
(4) $\forall(x)\ (M(x) \longrightarrow \sim G(x))$

Ques 3) Write in symbolic form -
(1) Every apple is red.
(2) Some people to trust other are rewarded.

Sol^n 1) $A(x)$ = x is apple
$G(x)$ = x is red

$\forall x$ if x is apple then x is red.

$\Rightarrow$ $\forall(x)\ (A(x) \rightarrow G(x))$

Soln(3) $P(x)$ = x is people
$T(x)$ = x trust
$R(x)$ = x is rewarded

$\exists x \, (P(x) \wedge T(x) \wedge R(x))$

Ques(4) (i) Every computer science student must take mathematics course.
(ii) Everybody must take a mathematics course for be a computer science student.

Soln(4) $S(x)$ = x is CS student
$M(x)$ = x takes mathematics course

(i) $\forall(x) \, (S(x) \longrightarrow M(x))$
(ii) $\forall(x) \, (M(x) \vee C(x))$

Ques(5) Every men in this locality is either an academician or a sportsmen.
Soln(5) $M(x)$ = x is men in the locality
$A(x)$ = x is academician
$S(x)$ = x is sportsman

$\forall(x) \, (M(x) \longrightarrow (A(x) \vee S(x)))$

For two Variables :-

Ques) (i) Everybody respects somebody.

(ii) There is someone respecting by everybody.

Sol's (i) ~~Varleg~~ R = respects

$\forall(x) \exists(y) R(x,y)$

or

$\forall(x) \exists(y)$ respects (x,y) //

Sol's (ii) $\exists y \forall x$ respects (y,x)

or

$\exists y \forall(x) R(y,x)$ //

Ques 7) (i) Some dogs are white but all cats are white.

(ii) Some people are not admired by everyone.

Sol's) (i)

$D(x) = x$ is dog

$C(y) = y$ is cat

$W(x) = x$ is white.

$W(y) = y$ is white

$\exists x \forall y [D(x) \land W(x) \land (C(y) \rightarrow W(y))]$

(ii) $\exists x \forall y \sim$ admired(x,y)

Ques 8) If water is hot, then shyam will in pool.

Sol's)

$W(x)$ = Water is hot

$S(x)$ = Shyam will swim in pool

$\forall x [W(x) \longrightarrow S(x)]$