Combination (selection)

$$^nC_r = \frac{\lfloor n}{\lfloor n-r \rfloor r}$$

Q $^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 3 : 4 : 5$ find n & r

$$^nC_{r-1} : {}^nC_r : {}^nC_{r+1} :: 3 : 4 : 5$$

$$\frac{\frac{n!}{(n-r-1)!\,(r-1)!}}{\frac{n!}{(n-(r+1))!\,(r+1)!}} = \frac{3}{4}$$

$r = 27$
$n = 62$

Combination

Q In howmany ways can a committe of 5 members be selected from 6 men and 5 ladies consisting of three men and 2 ladies.

Soln 5 members → 6 men and 5 ladies
→ 3 men & 2 ladies

$^6C_3 \times {}^5C_2$

Q A committe of 5 Ps to be formed out of 6 men and 4 ladies. In howmany ways this can be done when
(i) Atleast two ladies are included
(ii) Atmost two ladies are included.

Ans of 5 → 6 mens & 4 ladies

(i) Atleast two ladies are included.

$${}^6C_3 \times {}^4C_2 + {}^6C_2 \times {}^4C_3 + {}^6C_1 \times {}^4C_4 = 186$$

case 1 | case 2 | case 3

(ii) Atmost two ladies are included

$${}^6C_5 \times {}^4C_0 + {}^6C_4 \times {}^4C_1 + {}^6C_3 \times {}^4C_2$$

case 1 | case 2 | case 3

Q An examination paper containing 12 questions. Consist of two parts, Part A and Part B. Part A contains 7 questions and Part B contains 5 questions. A candidate is required to attempt 8 questions, selecting at least 3 from each part. In how many ways the candidate can select the question.

Soln

12

Atleast 3 should.

8

part A → 7

part B → 5

Case I $${}^7C_5 \times {}^5C_3 = \frac{7!}{2! \times 5!} \times \frac{5!}{2! \times 3!} = \frac{7 \times 6 \times 5 \times 4}{4} = 210$$

Case II $${}^7C_4 \times {}^5C_4 = \frac{7!}{3! \times 4!} \times \frac{5!}{1 \times 4!} = \frac{7 \times 6 \times 5 \times 5}{6} = 175$$

Case III $${}^7C_3 \times {}^5C_5 = \frac{7!}{4! \times 3!} \times \frac{5!}{(5-5)!\,5!} = \frac{7 \times 6 \times 5}{6} = 35$$

total = 210 + 175 + 35

total = 420

Q Everybody in a room shakes hands with everybody else. The total no. of handshakes is 66. How many people are there in the room

Ans $${}^nC_2 = 66$$

$$\frac{n!}{(n-2)!\,2!} = 66$$

$$n = 12$$

Q Determine the number of 5 cards combination out of a deck of 52 cards. If there is exactly one Ace in each combination.

Soln

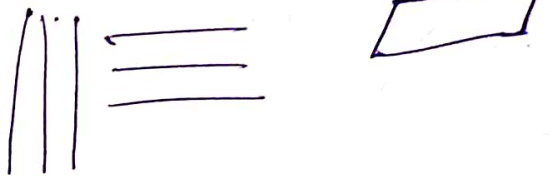
52 have 4 Ace

$${}^{4}C_1 \times {}^{48}C_4$$

Q If n parallel lines in a plane are intersected by a family of n parallel lines. Find the number of parallelograms forms

Soln

$${}^{m}C_2 \times {}^{n}C_2$$



Q A box contains 5 different red and 6 different white balls. In howmany ways can six ball be selected so that their are atleast two balls of each colour.

Soln

$${}^{5}C_2 \times {}^{6}C_4 + {}^{5}C_3 \times {}^{6}C_3 + {}^{5}C_4 \times {}^{6}C_2$$

Q Howmany three letter words can be made from the letter in Success.

permutation

Soln

$${}^{7}P_3 = \frac{7!}{4!} = 7 \times 6 \times 5$$

$$= 420$$

Case i with repetation

$${}^{7}P_3 = 420$$

Case ii without repetation

$$\frac{{}^{7}P_3}{2!\,3!} = \frac{7 \times 6 \times 5}{2 \times 6} = \frac{35}{2}$$

Case ① No repitition

S U C E

$^4P_3$

Case ② Two repetition

C → 2 CC [SUE] SS [CUE]

~~SUCC~~

$\frac{^4C_3}{2!} \times \frac{^4C_3}{2!}$

$\frac{^3P_2}{2!} \times 3! \times 2$

Case 3 Three repitition

SSS

$\frac{^3P_3}{3!}$

Generalized Pigeon Hole Principle

If n pigeon holes are occupied by k_{n+1} or more pigeons where k is positive Integer than atleast 1 hole is occupied by k+1 or more pigions

n → pigeon holes
kn+1 → pigeons
k+1 → Repetitions

Generalized Pigeon hole Principle

If n pigions are placed into k pigeon holes than there is atleast 1 pigion hole containing at least n-k pigeons

n → pigeons
k → pigeon holes
$\frac{n}{k} + 1$ → Repeatation

Q find the number of students in a class show that 3 of them are ~~worned~~ borned in the same ~~moth~~ months

* Every question have 3 objects. one of them is hidden

Sol^n
n = 12 (pigeon holes) (we have 12 months)
k+1 = 3 (repeatation) (months are rep
k = 2
so Pigeons, kn+1 = 2 × 12 + 1 = 25 Ans

Q What shell be the minimum ~~of~~ number of words that must begin with the same alphabet at 27 english words

A
n = 26
kn+1 = 27
k = 1
now for k+1 → 1+1 = 2 Repitition s.

Q Howmany students must there be in a class to guarintee that atleast ~~two~~ 2 students recieve the same score in the final examination if the Examination is graded on a scale form 0 to 100 points.

Sol^n

$$K+1 = 2 \Rightarrow K = 1$$

$$Kn+1 = ?$$

$$n = 101$$

So $Kn+1 = 1 \times 101 + 1 = 102$ Ans

Q If a bag contain many white, red and green balls than find the no. of balls to be choosen, to be sure of getting a pair of balls with same colour.

Sol^n

~~$Kn+1 = 3$~~

~~$K+1 = 2$~~

$$n = 3$$

$$K+1 = 2$$

$$K = 1$$

$$Kn+1 = 1 \times 3 + 1 = 4$$ Ans

Q Show that if we choose 9 single shoes outof 8 distinct pair of shoes than we are sure to have a pair.

Sol^n

$$K+1 = ?$$

$$n = 8$$

$$Kn+1 = 9$$

$$\Rightarrow \boxed{K+1 = 2}$$ Ans

* Jo chij fix hai vo pigeon holes hote hai. (in)

Q Show that if ~~20 are selected~~ 20 persons are selected for presenting a cultural program then 1 men select a subset of 3. Show that all the three persons would be ables to present their programs on the same day of the week.

Q Find the minimum number of boys borned in the same minuted out of 1500 boys borned on a day.

Soln 1

By generalized pigeon hole

$n = 20$ → pigeons

~~$k+1 = 3 \Rightarrow k = 2$~~

~~$kn+1 = 2 \times 20 + 1 = 41$~~

$\left[\frac{n}{k} + 1\right] = 3 \Rightarrow \frac{20}{7} + 1 = (3.)$ A

$K = 7$ (given in a week there is 7 days)

or

By pigeon hole

$n = 7$

$kn+1 = 20$

$k+1 = 3 \Rightarrow k = 2$

$2 \times 7 + 1$

22 Soln

$K = 60 \times 24$

$K = 1440$

$n = 1500$

$\left[\frac{n}{k} + 1\right] = \left[\frac{1500}{1400} + 1\right] = 2$ B