Problem Set 2

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2020/2/2

```
Course: MACS30100 Perspectives on Computational Modeling (Winter 2020)
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```

The Bayes Classifier

1.

a. Set seed

```
# Set seed
set.seed(110)
```

b. Simulate a dataset

```
x_1 = runif(200, -1, 1)
x_2 = runif(200, -1, 1)
```

c. Calculate Y

```
ep = rnorm(200, 0, 0.25)
y = x_1 + x_1^2 + x_2 + x_2^2 + ep
```

d. Calculate the probability of success

The log-odds is calculated by the following equation.

$$\log\left(\frac{Pr(success)}{1 - Pr(success)}\right) = X_1 + X_1^2 + X_2 + X_2^2 + \epsilon$$

By transforming the equation, the probability of success is:

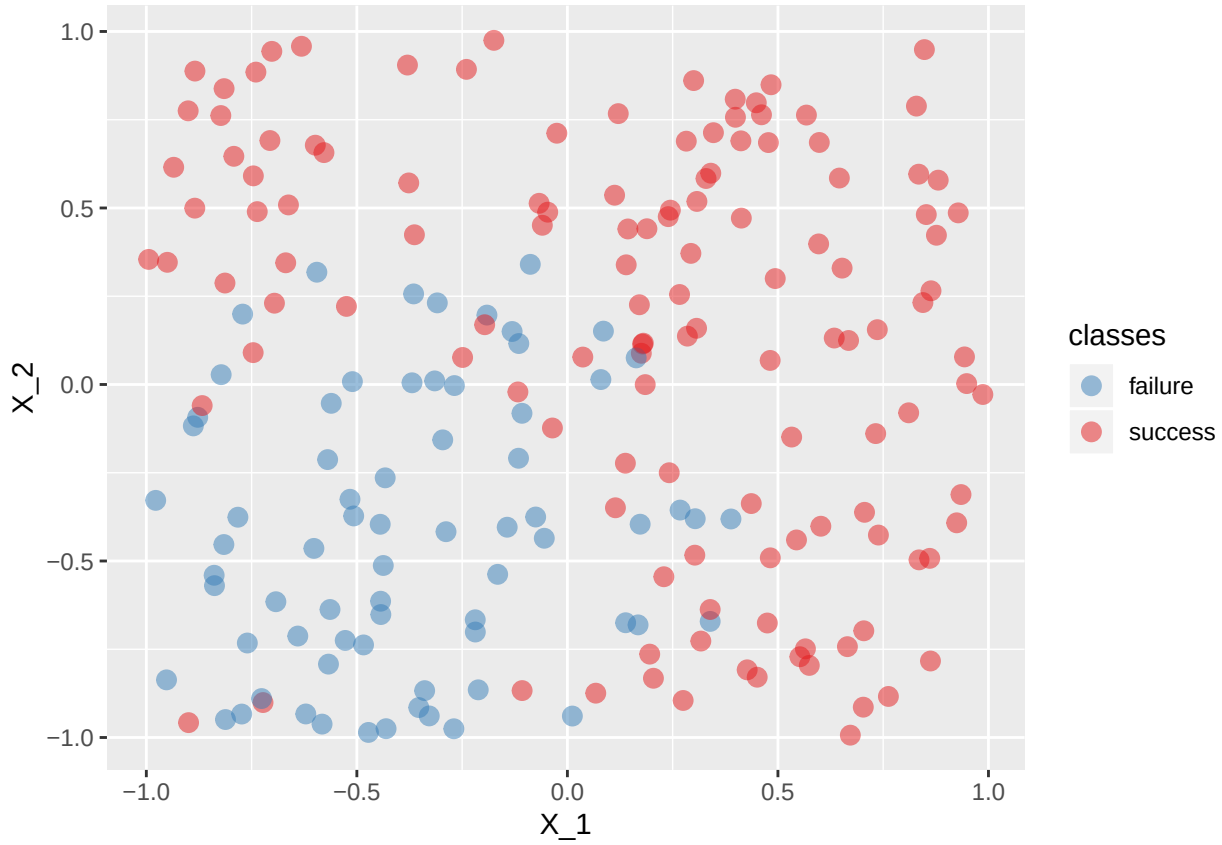
$$Pr(success) = \frac{e^{X_1 + X_1^2 + X_2 + X_2^2 + \epsilon}}{1 + e^{X_1 + X_1^2 + X_2 + X_2^2 + \epsilon}} = \frac{e^Y}{1 + e^Y}$$

```
pr <- exp(y) / (1 + exp(y))
```

e. Plot each point from the dataset

```
# Dataframe
df <- data.frame(X_1 = x_1, X_2 = x_2, Pr = pr)
df <- df %>%
  mutate(cl = case_when(pr > 0.5 ~ 'success',
                        pr <=  0.5 ~ 'failure'),
         cl_n = case_when(pr > 0.5 ~ 1,
```

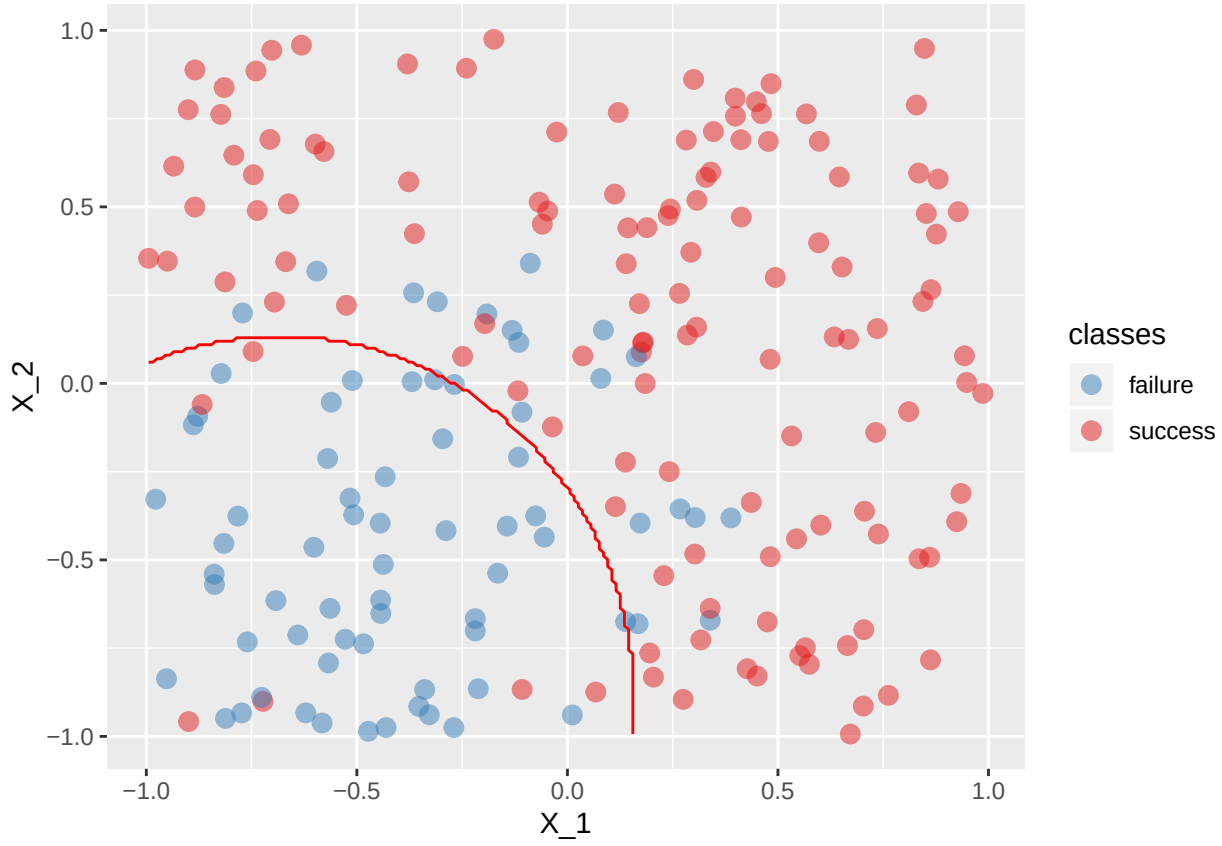
```
                            pr <= 0.5 ~0))
twoClassColor <- brewer.pal(3,'Set1')[1:2]
names(twoClassColor) <- c('success','failure')

df %>%
  ggplot(aes(x=X_1, y=X_2)) +
  geom_point(aes(color = cl), size = 3, alpha = .5) +
  scale_colour_manual(name = 'classes', values = twoClassColor)
```



f. Overlay with Bayes decision boundary

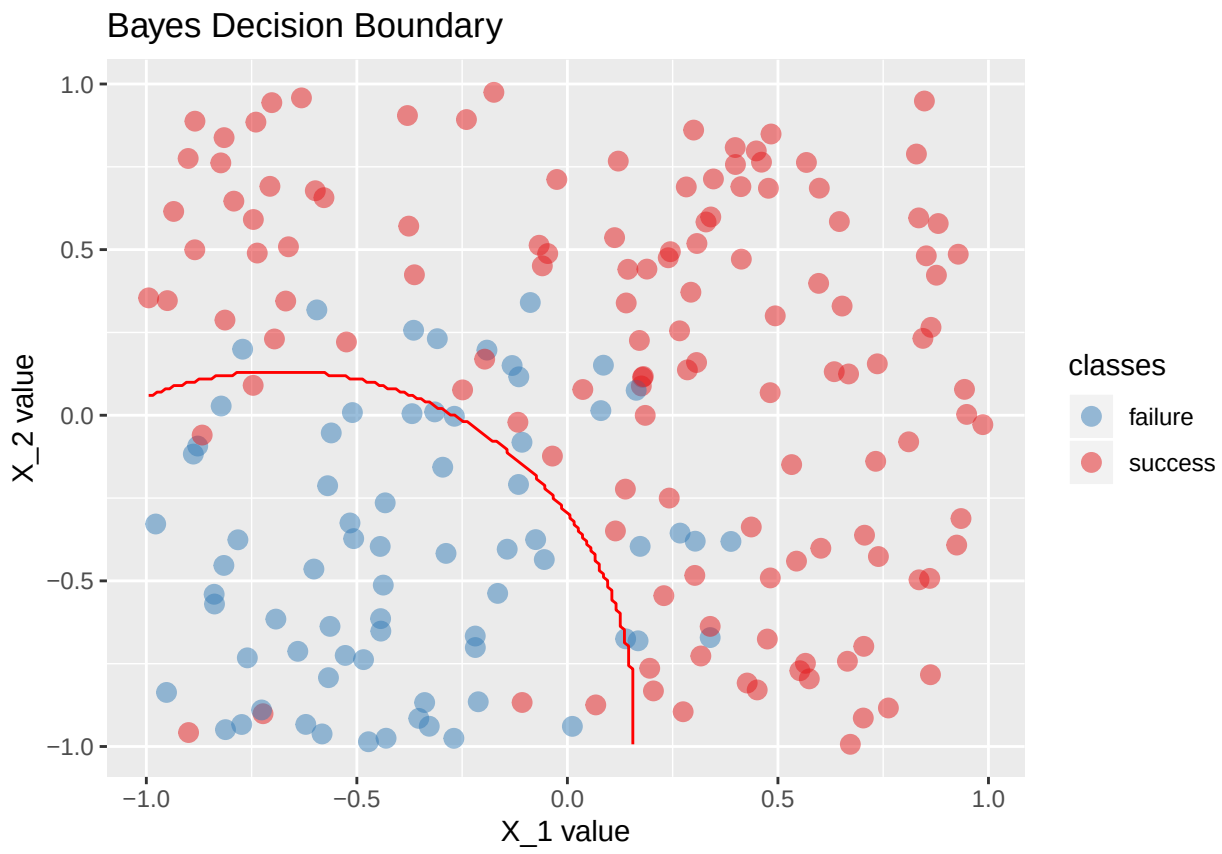
```
# Refering to  http://www.cmap.polytechnique.fr/~lepennec/R/Learning/Learning.html
V <- 10
T <- 4
TrControl <- trainControl(method = "repeatedcv",
                          number = V,
                          repeats = T)
df_model <- df %>%
  dplyr::select(X_1, X_2, cl)
nbp = 200
Pred1 <- seq(min(df_model$X_1), max(df_model$X_1), length = nbp)
Pred2 <- seq(min(df_model$X_2), max(df_model$X_2), length = nbp)
Grid <- expand.grid(X_1 = Pred1, X_2 = Pred2)

Model <- train(data=df_model, cl ~ ., method = "nb", trControl = TrControl,
               tuneGrid = data.frame(usekernel = c(FALSE), fL = c(0), adjust = c(1)))
```

```
Pred <- predict(Model, newdata = Grid)

df_model %>%
  ggplot(aes(x=X_1, y=X_2)) +
  geom_point(aes(color = cl), size = 3, alpha = .5) +
  geom_contour(data = cbind(Grid, classes = Pred),
               aes(z = as.numeric(classes)),
               color = "red", breaks = c(1.5)) +
scale_colour_manual(name = 'classes', values = twoClassColor)
```



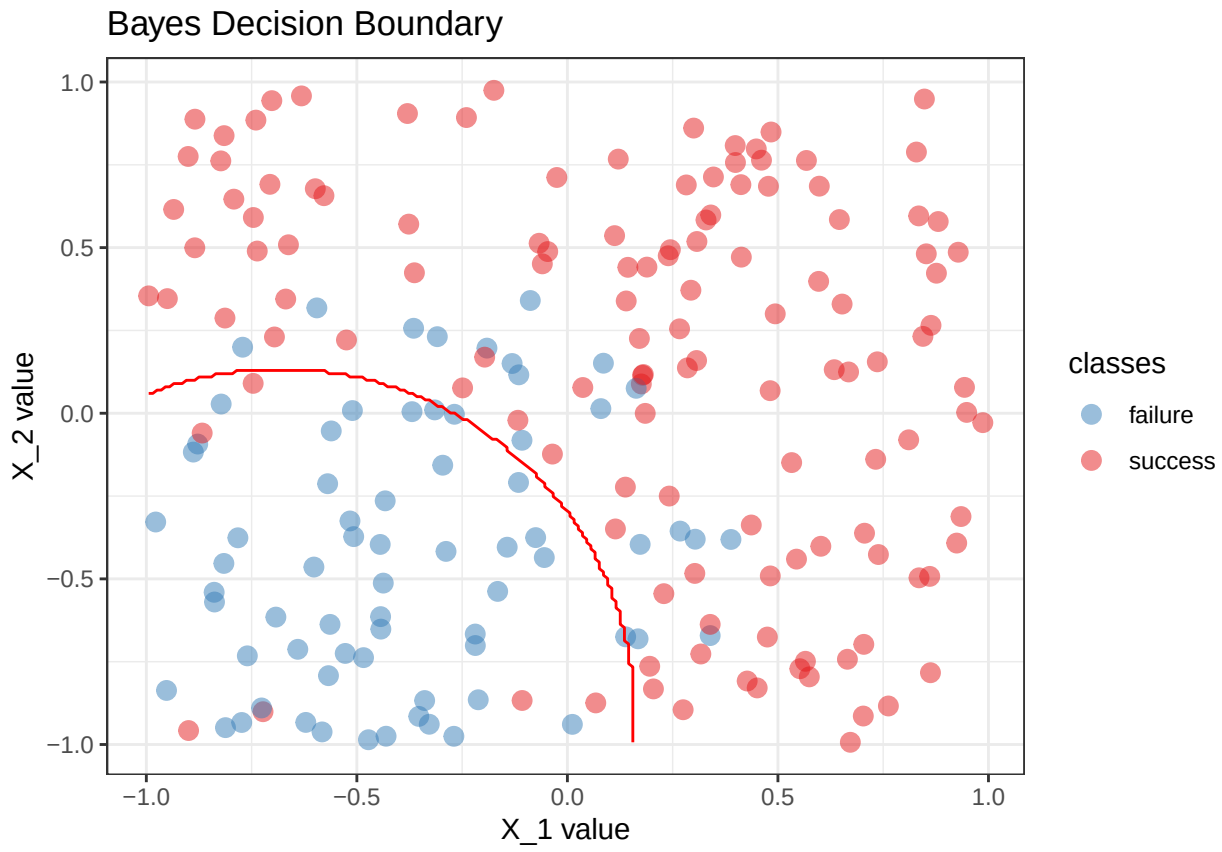
g. Title and axis labels

```
df_model %>%
  ggplot(aes(x=X_1, y=X_2)) +
  geom_point(aes(color = cl), size = 3, alpha = .5) +
  geom_contour(data = cbind(Grid, classes = Pred),
               aes(z = as.numeric(classes)),
               color = "red", breaks = c(1.5)) +
scale_colour_manual(name = 'classes', values = twoClassColor) +
  ggtitle("Bayes Decision Boundary") +
  xlab('X_1 value') +
  ylab('X_2 value')
```



h. Colored background

```
df_model %>%
  ggplot(aes(x=X_1, y=X_2)) +
  geom_point(aes(color = cl), size = 3, alpha = .5) +
  geom_contour(data = cbind(Grid, classes = Pred),
               aes(z = as.numeric(classes)),
               color = "red", breaks = c(1.5)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  ggtitle("Bayes Decision Boundary") +
  xlab('X_1 value') +
  ylab('X_2 value') +
  theme_bw()
```



Exploring Simulated Differences between LDA and QDA

2. In case Bayes decision boundary is linear

a. Repeat simulation 1,000 times

```
train_lda2 = c()
test_lda2 = c()
train_qda2 = c()
test_qda2 = c()

for (i in 1:1000) {
  # i data generation
  x_1 = runif(1000, -1, 1)
  x_2 = runif(1000, -1, 1)
  ep = rnorm(1000, 0, 0.25)
  y = case_when(x_1 + x_2 + ep >= 0 ~ TRUE,
                x_1 + x_2 + ep < 0 ~ FALSE)
  df2 <- data.frame(x_1, x_2, y)

  # ii split
  split <- initial_split(df2, prop = .7)
  train <- training(split)
  test <- testing(split)

  # iii training
  lda2 <- MASS::lda(y ~ x_1 + x_2, data = train)
```

```
  qda2 <- MASS::qda(y ~ x_1 + x_2, data = train)

    # iv error
  train_lda2 = c(train_lda2, sum(predict(lda2, train)$class != train$y) / 700)
  test_lda2 = c(test_lda2, sum(predict(lda2, test)$class != test$y) / 300)
  train_qda2 = c(train_qda2, sum(predict(qda2, train)$class != train$y) / 700)
  test_qda2 = c(test_qda2, sum(predict(qda2, test)$class != test$y) / 300)
}
```

b. Summarize the results

```
df2_result <- data.frame(train_lda2, test_lda2, train_qda2, test_qda2)
head(df2_result)
```

train_lda2	test_lda2	train_qda2	test_qda2
0.0957143	0.0766667	0.0957143	0.0766667
0.0828571	0.1266667	0.0857143	0.1200000
0.0800000	0.1033333	0.0771429	0.1000000
0.0971429	0.0900000	0.0957143	0.0900000
0.0828571	0.0933333	0.0842857	0.0900000
0.0928571	0.0900000	0.0957143	0.0933333

```
df2_result <- df2_result %>%
  gather(dset, error)
df2_result %>%
  group_by(dset) %>%
  summarise(mean = mean(error), sd = sd(error))
```

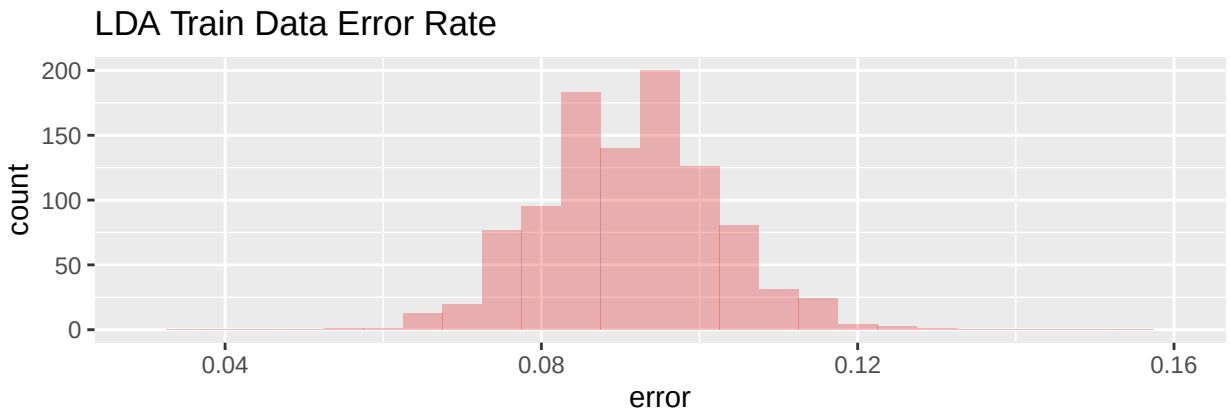
dset	mean	sd
test_lda2	0.0934167	0.0168609
test_qda2	0.0938367	0.0169069
train_lda2	0.0911400	0.0109330
train_qda2	0.0909057	0.0110357

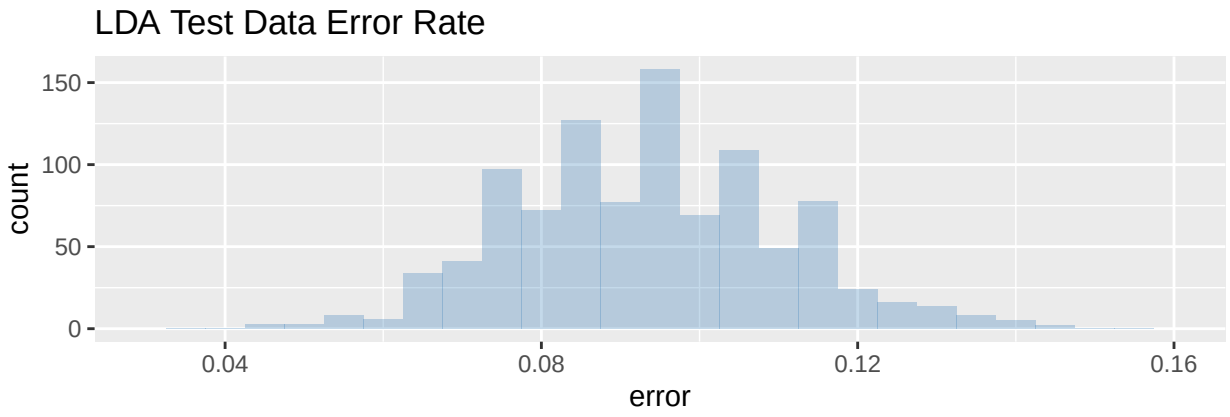
```
lda_p1 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'train_lda2'),
                 fill = brewer.pal(3,'Set1')[1],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.03,0.16)) +
  ggtitle("LDA Train Data Error Rate")

lda_p2 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'test_lda2'),
                 fill = brewer.pal(3,'Set1')[2],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.03,0.16)) +
```

```
  ggtitle("LDA Test Data Error Rate")

grid.arrange(lda_p1, lda_p2, nrow = 2)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```






```
qda_p1 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'train_qda2'),
                 fill = brewer.pal(3,'Set1')[1],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.03,0.16)) +
  ggtitle("QDA Train Data Error Rate")

qda_p2 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'test_qda2'),
                 fill = brewer.pal(3,'Set1')[2],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.03,0.16)) +
  ggtitle("QDA Test Data Error Rate")
```

```r
grid.arrange(qda_p1, qda_p2, nrow = 2)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).

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```
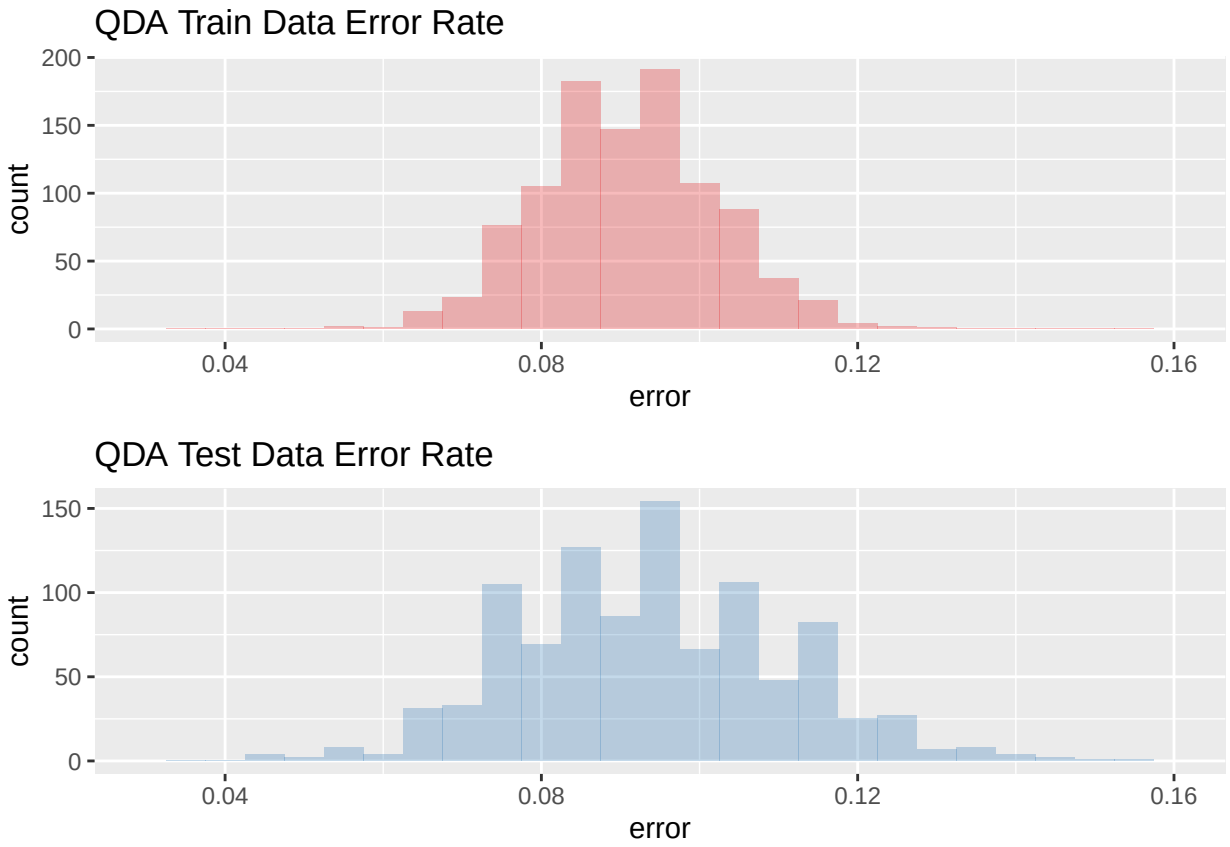



3. In case Bayes decision boundary is non-linear

```r
train_lda2 = c()
test_lda2 = c()
train_qda2 = c()
test_qda2 = c()

for (i in 1:1000) {
  # i data generation
  x_1 = runif(1000, -1, 1)
  x_2 = runif(1000, -1, 1)
  ep = rnorm(1000, 0, 0.25)
  y = case_when(x_1 + x_1^2 + x_2 + x_2^2 + ep >= 0 ~ TRUE,
                x_1 + x_1^2 + x_2 + x_2^2 + ep < 0 ~ FALSE)
  df2 <- data.frame(x_1, x_2, y)

  # ii split
  split <- initial_split(df2, prop = .7)
  train <- training(split)
  test <- testing(split)
```

```
  # iii training
  lda2 <- MASS::lda(y ~ x_1 + x_2, data = train)
  qda2 <- MASS::qda(y ~ x_1 + x_2, data = train)

    # iv error
  train_lda2 = c(train_lda2, sum(predict(lda2, train)$class != train$y) / 700)
  test_lda2 = c(test_lda2, sum(predict(lda2, test)$class != test$y) / 300)
  train_qda2 = c(train_qda2, sum(predict(qda2, train)$class != train$y) / 700)
  test_qda2 = c(test_qda2, sum(predict(qda2, test)$class != test$y) / 300)
}
```

b. Summarize the results

```
df2_result <- data.frame(train_lda2, test_lda2, train_qda2, test_qda2)
head(df2_result)
```

train_lda2	test_lda2	train_qda2	test_qda2
0.1514286	0.1566667	0.1057143	0.1133333
0.1385714	0.1166667	0.1042857	0.0966667
0.1514286	0.1333333	0.1071429	0.1133333
0.1385714	0.1400000	0.1100000	0.1133333
0.1271429	0.1533333	0.0871429	0.1000000
0.1314286	0.1300000	0.1128571	0.0966667

```
df2_result <- df2_result %>%
  gather(dset, error)
df2_result %>%
  group_by(dset) %>%
  summarise(mean = mean(error), sd = sd(error))
```

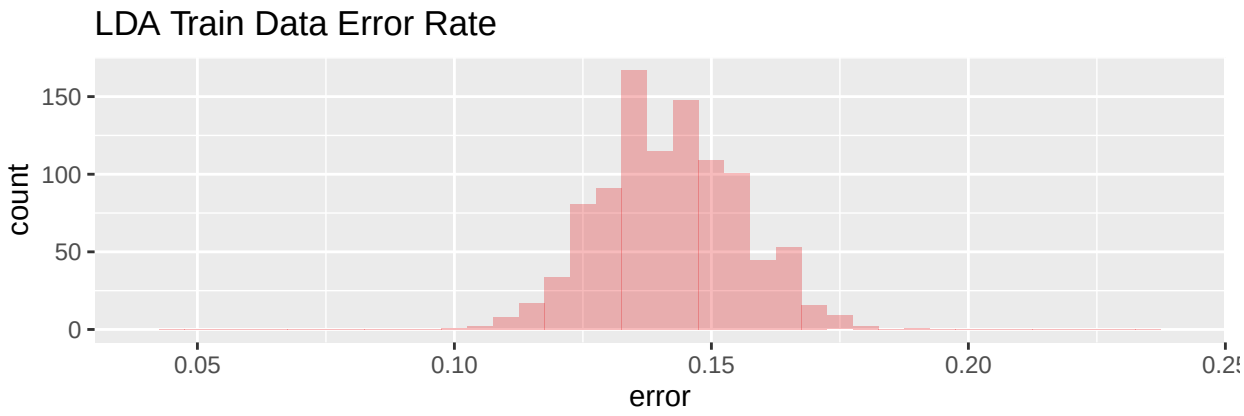
dset	mean	sd
test_lda2	0.1427233	0.0199135
test_qda2	0.1067600	0.0171807
train_lda2	0.1420243	0.0133372
train_qda2	0.1051986	0.0110349

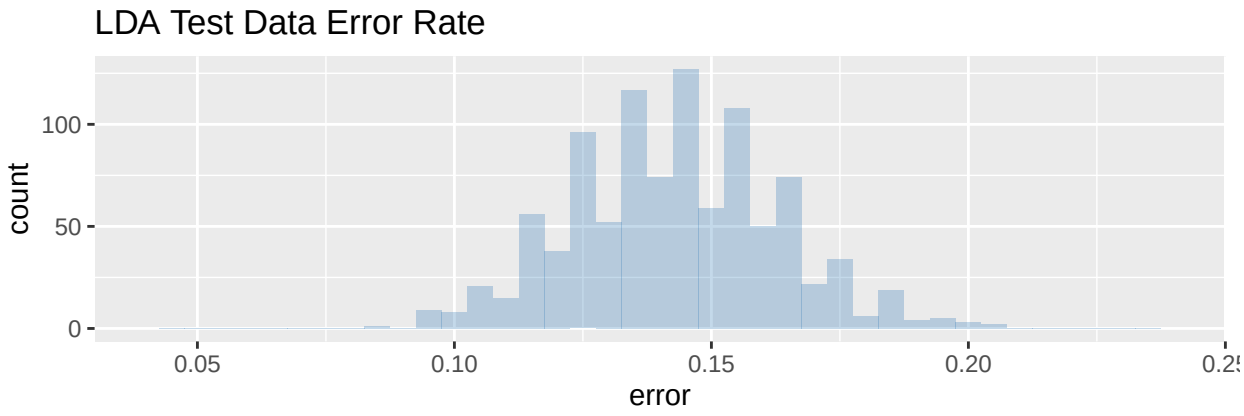
```
lda_p1 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'train_lda2'),
                 fill = brewer.pal(3,'Set1')[1],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.04,0.24)) +
  ggtitle("LDA Train Data Error Rate")

lda_p2 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'test_lda2'),
                 fill = brewer.pal(3,'Set1')[2],
```

```
                alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.04,0.24)) +
  ggtitle("LDA Test Data Error Rate")

grid.arrange(lda_p1, lda_p2, nrow = 2)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```

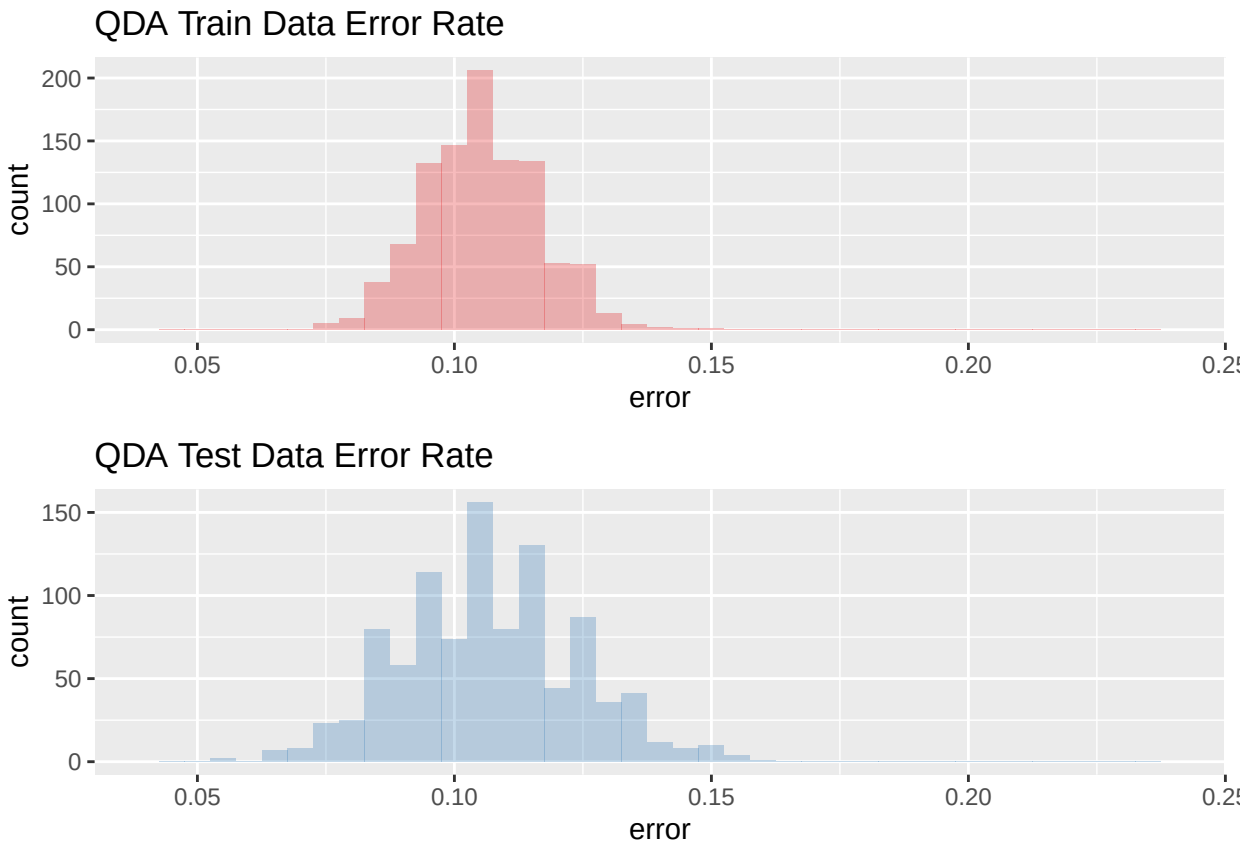





```
qda_p1 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'train_qda2'),
                 fill = brewer.pal(3,'Set1')[1],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.04,0.24)) +
  ggtitle("QDA Train Data Error Rate")

qda_p2 <- df2_result %>%
  ggplot(aes(x=error)) +
  scale_colour_manual(name = 'classes', values = twoClassColor) +
  geom_histogram(data=subset(df2_result, dset == 'test_qda2'),
                 fill = brewer.pal(3,'Set1')[2],
                 alpha = 0.3, binwidth = 0.005) +
  scale_x_continuous(limits = c(0.04,0.24)) +
```

```
  ggtitle("QDA Test Data Error Rate")

grid.arrange(qda_p1, qda_p2, nrow = 2)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).

## Warning: Removed 2 rows containing missing values (geom_bar).
```



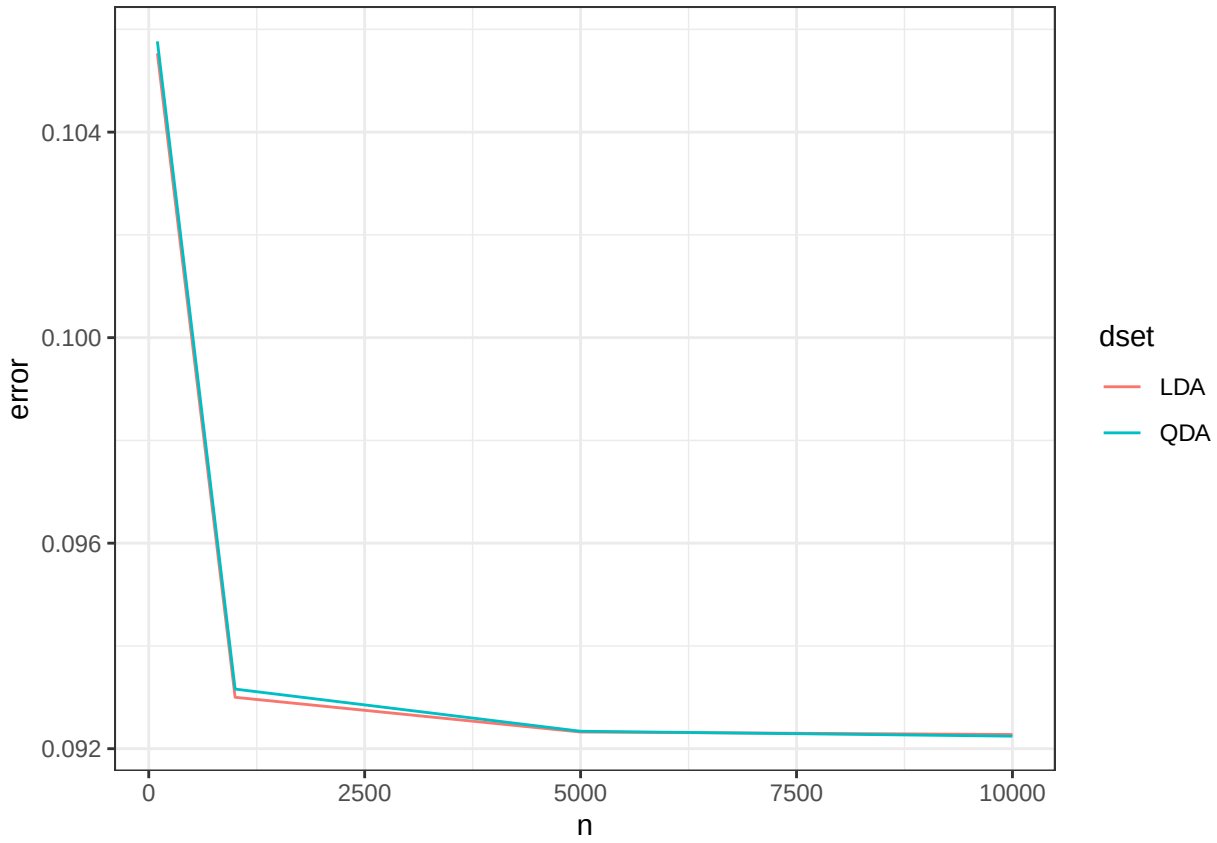
4. Increasing the sample size

```
train_lda4_mean = c()
test_lda4_mean = c()
train_qda4_mean = c()
test_qda4_mean = c()
for (n in c(100, 1000, 5000, 10000)) {
  train_lda4 = c()
  test_lda4 = c()
  train_qda4 = c()
  test_qda4 = c()
  for (i in 1:1000) {
    # i data generation
    x_1 = runif(n, -1, 1)
    x_2 = runif(n, -1, 1)
    ep = rnorm(n, 0, 0.25)
    y = case_when(x_1 + x_2 + ep >= 0 ~ TRUE,
                  x_1 + x_2 + ep < 0 ~ FALSE)
```

```
    df <- data.frame(x_1, x_2, y)

    # ii split
    split <- initial_split(df, prop = .7)
    train <- training(split)
    test <- testing(split)

    # iii training
    lda4 <- MASS::lda(y ~ x_1 + x_2, data = train)
    qda4 <- MASS::qda(y ~ x_1 + x_2, data = train)

    # iv error
    train_lda4 = c(train_lda4, sum(predict(lda4, train)$class != train$y) / (0.7*n))
    test_lda4 = c(test_lda4, sum(predict(lda4, test)$class != test$y) / (0.3*n))
    train_qda4 = c(train_qda4, sum(predict(qda4, train)$class != train$y) / (0.7*n))
    test_qda4 = c(test_qda4, sum(predict(qda4, test)$class != test$y) / (0.3*n))
  }
  train_lda4_mean = c(train_lda4_mean, mean(train_lda4))
  test_lda4_mean = c(test_lda4_mean, mean(test_lda4))
  train_qda4_mean = c(train_qda4_mean, mean(train_qda4))
  test_qda4_mean = c(test_qda4_mean, mean(test_qda4))
}
```

b. Summarize the results

```
df4 <- data.frame(n=c(100, 1000, 5000, 10000), LDA=test_lda4_mean, QDA=test_qda4_mean)
df4 <- df4 %>%
  gather(dset, error, -n)
df4 %>%
  ggplot(aes(x=n, y=error, color=dset)) +
  geom_line() +
  theme_bw()
```



Modeling voter turnout

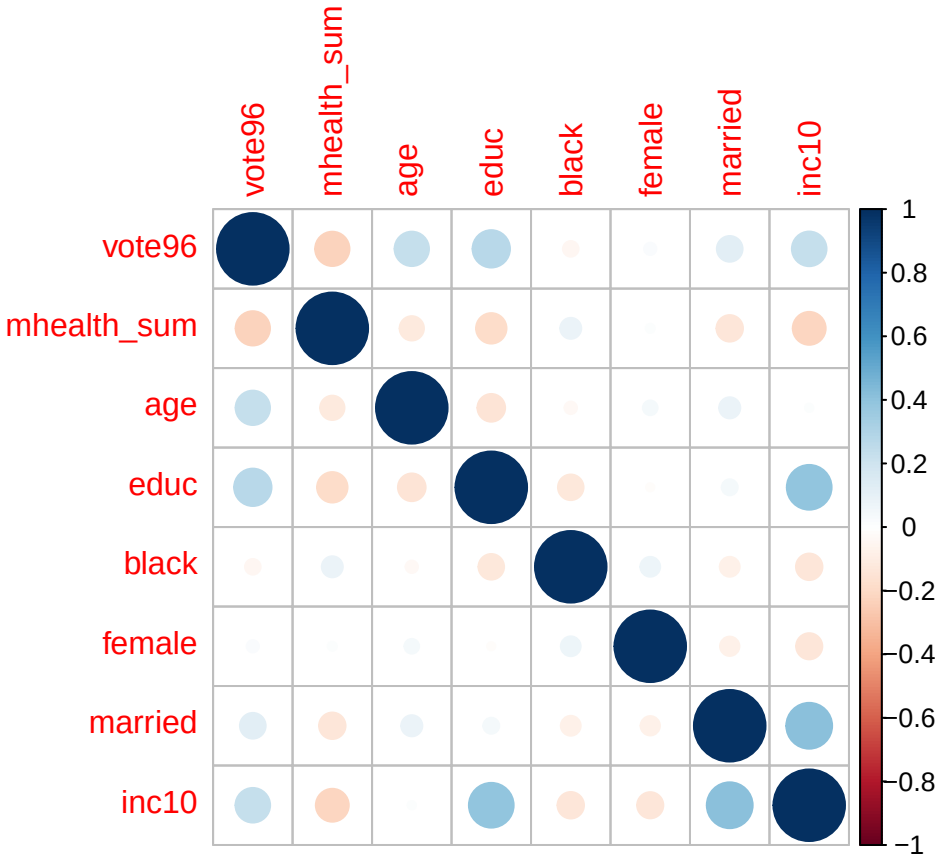
5. Building classifiers

a. Split data

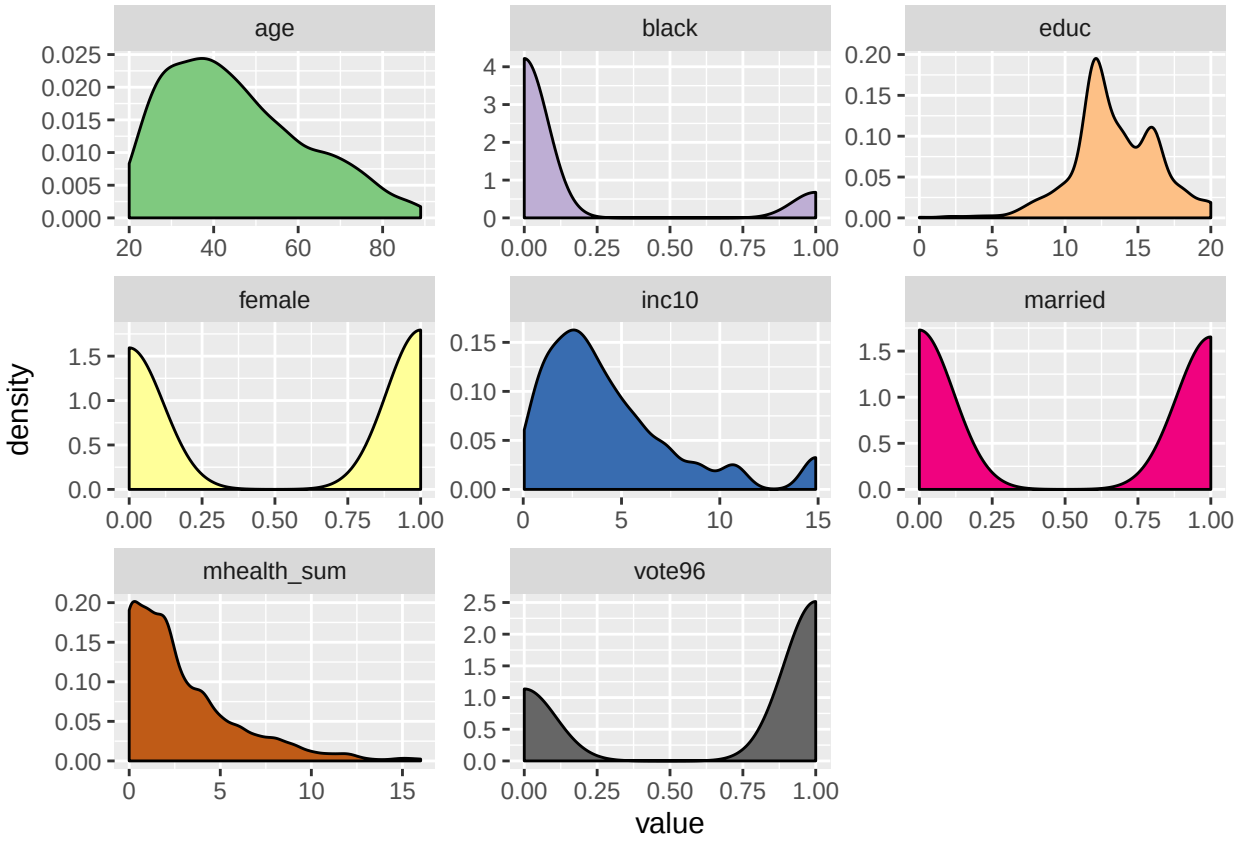
```
df5 <- read.csv('mental_health.csv')
split <- initial_split(df5, prop = .7)
train <- training(split)
test <- testing(split)
```

From the Lab notebook, below is some simple EDA.

```
# checking correlation across several features
train %>%
  drop_na() %>%
  select_if(is.numeric) %>%
  cor() %>%
  corrplot::corrplot()
```




```
# checking normality assumption for continuous features
train %>%
  drop_na() %>%
  gather(metric, value) %>% #https://www.rdocumentation.org/packages/tidyr/versions/0.8.3/topics/gather
  ggplot(aes(value, fill = metric)) +
  geom_density(show.legend = FALSE) +
  scale_fill_brewer(type = "qual") +
  facet_wrap(~ metric, scales = "free")
```



b. Estimation

c. Performance metrics

d. Results