Shengwenxin_Ni_HW2

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0.1 Import Libraries

[1215]:
```
import random
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
import seaborn as sns
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
from sklearn.feature_selection import RFE
from sklearn import metrics
from sklearn.naive_bayes import GaussianNB
from sklearn.linear_model import LogisticRegression
from sklearn.neighbors import NearestNeighbors
from sklearn.metrics import roc_auc_score
from sklearn.neighbors import KNeighborsClassifier
import copy
```

1 Question 1

1.1 Helper functions

produc_list: Generate a random list of X values with the desired length. Will be used in the following questions as well.

produce_y1: Calculate the value of y (in terms of probability)

[879]:
```
def produce_list(N,seed):
    random.seed(seed)
    x = []
    for i in range(N):
        x.append(random.uniform(-1,1))
    return x

def produce_y1(x1,x2):
    error = random.normalvariate(0,0.25)
    l_odds = x1 + x1**2 + x2 + x2 ** 2 + error
    prob = np.exp(l_odds) / (1 + np.exp(l_odds))
    return prob
```

1.2 Main Function

[1138]:
```
x1 = produce_list(200,317)
x2 = produce_list(200,828)

random.seed(317)

s1 = []
s2 = []
f1 = []
f2 = []

for i in range(len(x1)):
    if produce_y1(x1[i],x2[i]) >= 0.5:
        s1.append(x1[i])
        s2.append(x2[i])
    else:
        f1.append(x1[i])
        f2.append(x2[i])
```

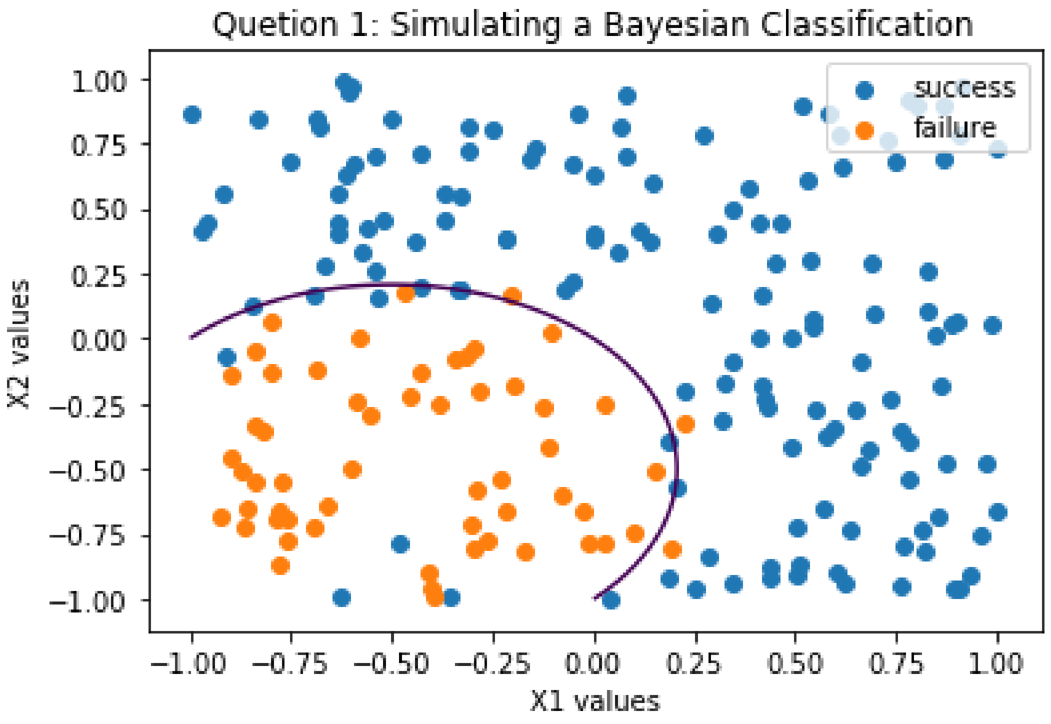
1.3 The Bayes Decision Boundary

Calculate the Bayes Decision Boundary based on the value of X1 and X2

[1139]:
```
x1 = np.linspace(-1,1,1000)
x2 = np.linspace(-1,1,1000)

Z = []
for i in x1:
    each = []
    for j in x2:
        each.append(i + i**2 + j + j ** 2)
    Z.append(each)
```

1.4 Generate the Graph

```
[1198]: plt.scatter(s1,s2)
        plt.scatter(f1,f2)
        plt.contour(x1,x2,Z,[0])
        plt.title('Quetion 1: Simulating a Bayesian Classification')
        plt.xlabel('X1 values')
        plt.ylabel('X2 values')
        plt.legend(['success','failure'],loc='upper right')
        plt.show()
```



Note: The solid purple line is the decision boundary.

2 Question 2

2.1 Helper Functions

produce_y2: Calculate the value of y (in terms of Boolean value)

err_rate: Calculate the error rate of a single round of simulation.

[903]:
```
def produce_y2(l):
    random.seed(426501)
    y = []
    for i in l:
        error = random.normalvariate(0,1)
        if i[0] + i[1] + error >= 0:
            y.append(True)
        else:
            y.append(False)
    return y


def err_rate(clf,x_train,y_train,x,y):
    clf.fit(x_train,y_train)
    df = pd.DataFrame({'X':x,'actual-Y':y})
    df['predicted-Y'] = clf.predict(x)
    df['error'] = (df['actual-Y'] != df['predicted-Y'])

    return (df['error']== True).sum()/df['error'].count()
```

2.2 Main Function

simulate-err: The function that produce a dataframe that keeps the resulte of the error rate of lda-test,lda-train,qda-test, qda-train, given the number of random data used in a single simulation. This function will be used in question 3&4 as well.

[1141]:
```
def simulate_err(n,seed,func,include_train = True):
    lda_train = []
    lda_test = []
    qda_train = []
    qda_test = []

    for i in range(1000):

        x1 = produce_list(n, seed)
        seed += 1
        x2 = produce_list(n, seed)
        X = list(zip(x1, x2))
        Y = func(X)

        seed += 1
        x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size = 0.
 ↪3, random_state = seed)

        lda_clf = LinearDiscriminantAnalysis()
        qda_clf = QuadraticDiscriminantAnalysis()

        lda_test.append(err_rate(lda_clf,x_train,y_train,x_test,y_test))
        qda_test.append(err_rate(qda_clf,x_train,y_train,x_test,y_test))

        if include_train:
            lda_train.append(err_rate(lda_clf,x_train,y_train,x_train,y_train))
            qda_train.append(err_rate(qda_clf,x_train,y_train,x_train,y_train))

    df = { 'lda_test': lda_test,'qda_test': qda_test}

    if include_train:
        df['lda_train'] = lda_train
        df['qda_train'] = qda_train
    df = pd.DataFrame(df)

    return df
```

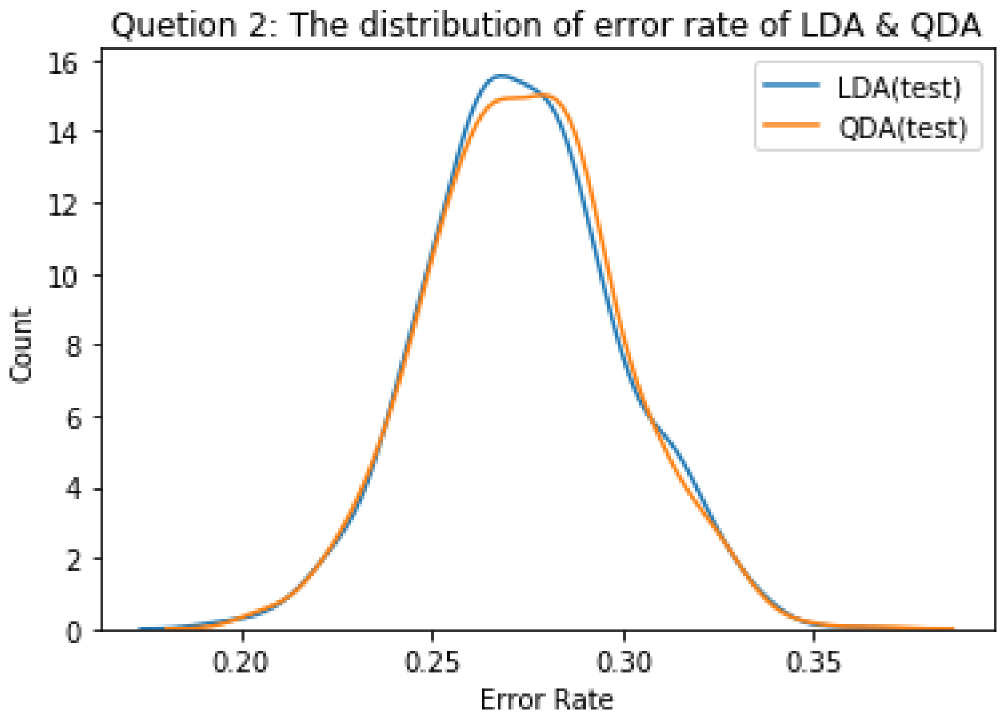
2.3 Statistical Table

[1142]:
```
df2 = simulate_err(1000,828,produce_y2)
df2.describe()
```

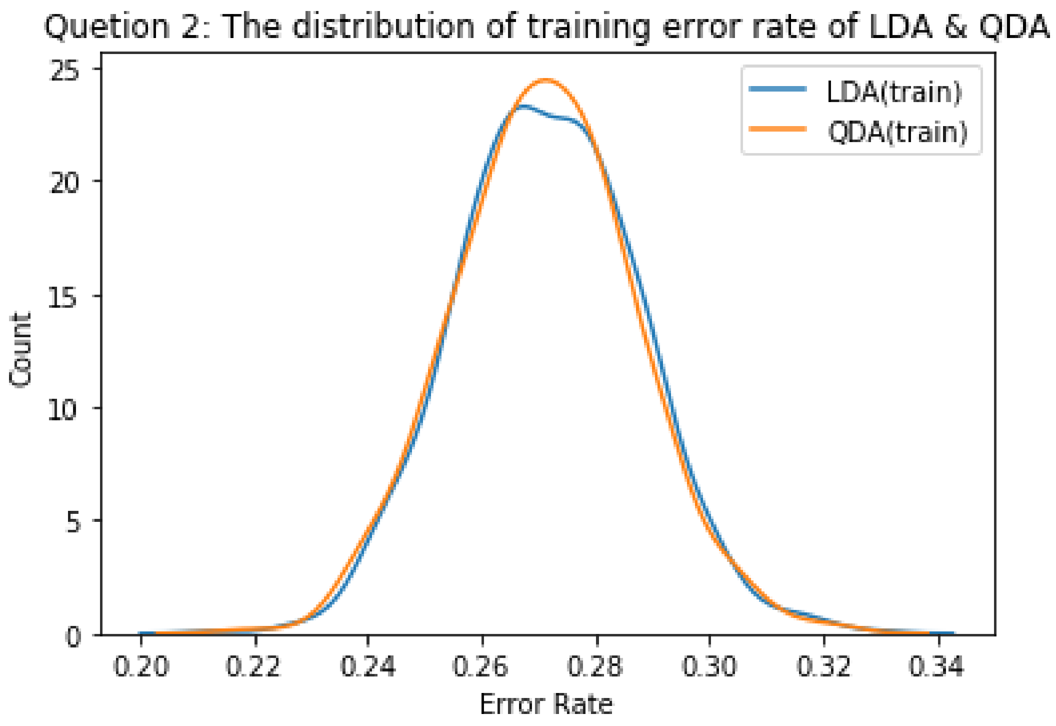
[1142]:
```
          lda_test     qda_test    lda_train    qda_train
count  1000.000000  1000.000000  1000.000000  1000.000000
mean      0.273490     0.273750     0.271691     0.271017
std       0.025289     0.025219     0.015979     0.015986
min       0.193333     0.200000     0.212857     0.215714
25%       0.256667     0.256667     0.261071     0.260000
50%       0.273333     0.273333     0.271429     0.271429
75%       0.290000     0.290000     0.282857     0.281429
max       0.356667     0.366667     0.330000     0.325714
```

2.4 Graph (Distribution of error rates)

[1146]:
```
sns.distplot(df2['lda_test'],hist= False,label = 'LDA(test)')
sns.distplot(df2['qda_test'],hist= False,label = 'QDA(test)')

plt.title('Quetion 2: The distribution of testing error rate of LDA & QDA')
plt.xlabel('Error Rate')
plt.ylabel('Count')

plt.legend()
plt.show()
```



[1147]:
```
sns.distplot(df2['lda_train'],hist= False,label = 'LDA(train)')
sns.distplot(df2['qda_train'],hist= False,label = 'QDA(train)')

plt.title('Quetion 2: The distribution of training error rate of LDA & QDA')
plt.xlabel('Error Rate')
plt.ylabel('Count')

plt.legend()
plt.show()
```



2.5 Comments/ Conclusion

Given the results above, I believe that LDA and QDA perform equally on both the training set and the testing set. My conclusion can be supported by two illustrations (the distribution of training error/rate of LDA & QDA). We can see that in both graphs, two distribution lines (LDA and QDA) almost coincide.

However, theoretically, we expect LDA to perform better because the decision boundary is linear ($f(X) = X_1 + X_2$). The results generated above do not reflect such expected pattern. Perhaps, this is because of the relative large irreducible error compared to the value of X_1, X_2

3 Question 3

3.1 Helper Functions

produce_y3: Calculate the value of y (in terms of Boolean value)

[906]:
```
def produce_y3(l):
    random.seed(426501)
    y = []
    for i in l:
        error = random.normalvariate(0,1)
        if i[0] + i[0]**2 + i[1] + i[1]**2 + error >= 0:
            y.append(True)
        else:
            y.append(False)
    return y
```

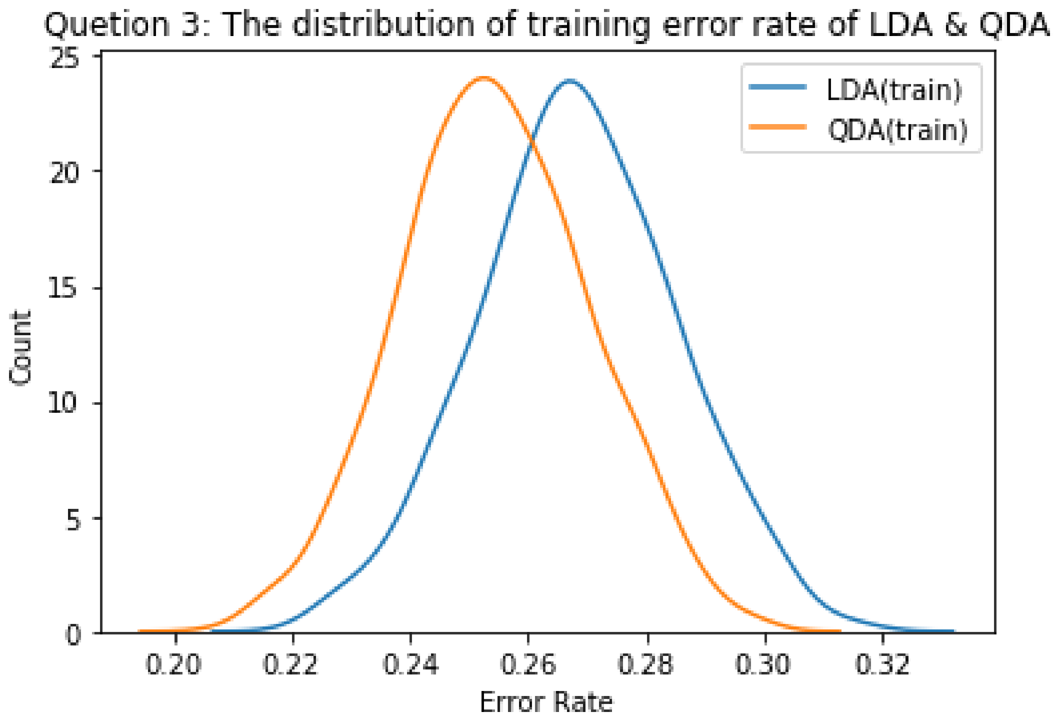
3.2 Main Function + Statistical Table

[907]:
```
df3 = simulate_err(1000,426501,produce_y3)
df3.describe()
```

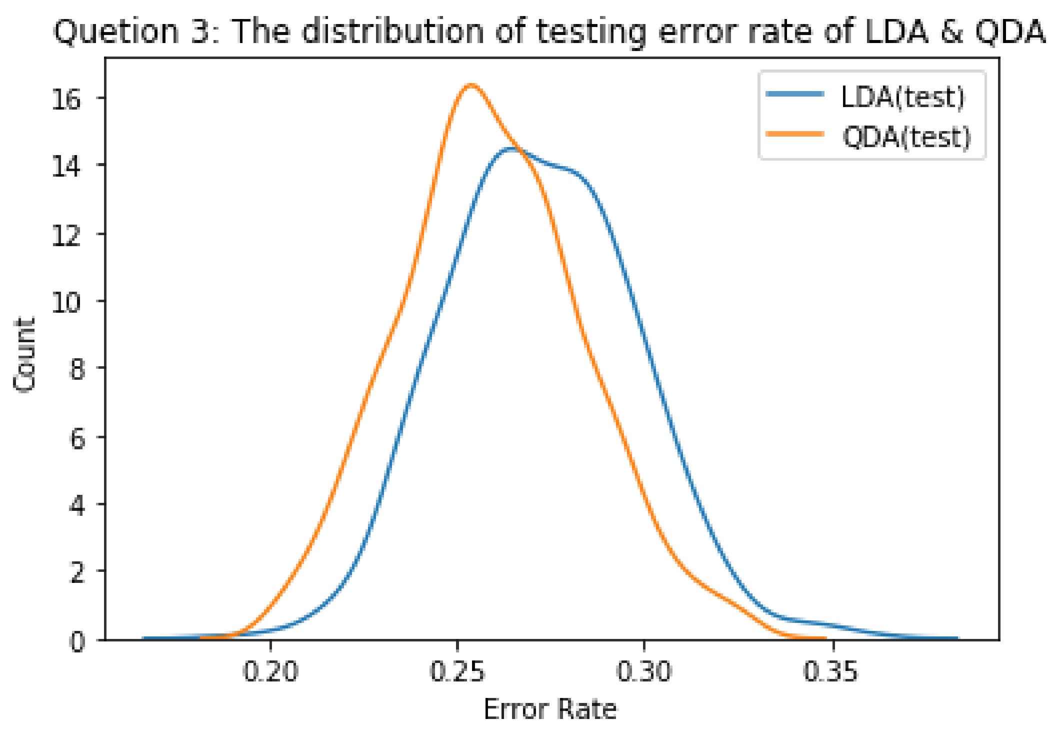
[907]:
```
          lda_test     qda_test    lda_train    qda_train
count  1000.000000  1000.000000  1000.000000  1000.000000
mean      0.272270     0.259200     0.268303     0.254401
std       0.025291     0.024572     0.016778     0.016023
min       0.186667     0.200000     0.220000     0.207143
25%       0.253333     0.243333     0.257143     0.242857
50%       0.270000     0.256667     0.268571     0.254286
75%       0.290000     0.274167     0.280000     0.265714
max       0.363333     0.330000     0.318571     0.300000
```

3.3 Graph (Distribution of error rates)

[1150]:
```
sns.distplot(df3['lda_train'],hist= False,label = 'LDA(train)')
sns.distplot(df3['qda_train'],hist= False,label = 'QDA(train)')

plt.title('Quetion 3: The distribution of training error rate of LDA & QDA')
plt.xlabel('Error Rate')
plt.ylabel('Count')

plt.legend()
plt.show()
```



[1149]:
```
sns.distplot(df3['lda_test'],hist= False,label = 'LDA(test)')
sns.distplot(df3['qda_test'],hist= False,label = 'QDA(test)')

plt.title('Quetion 3: The distribution of testing error rate of LDA & QDA')
plt.xlabel('Error Rate')
plt.ylabel('Count')

plt.legend()
plt.show()
```



3.4 Comments/ Conclusion

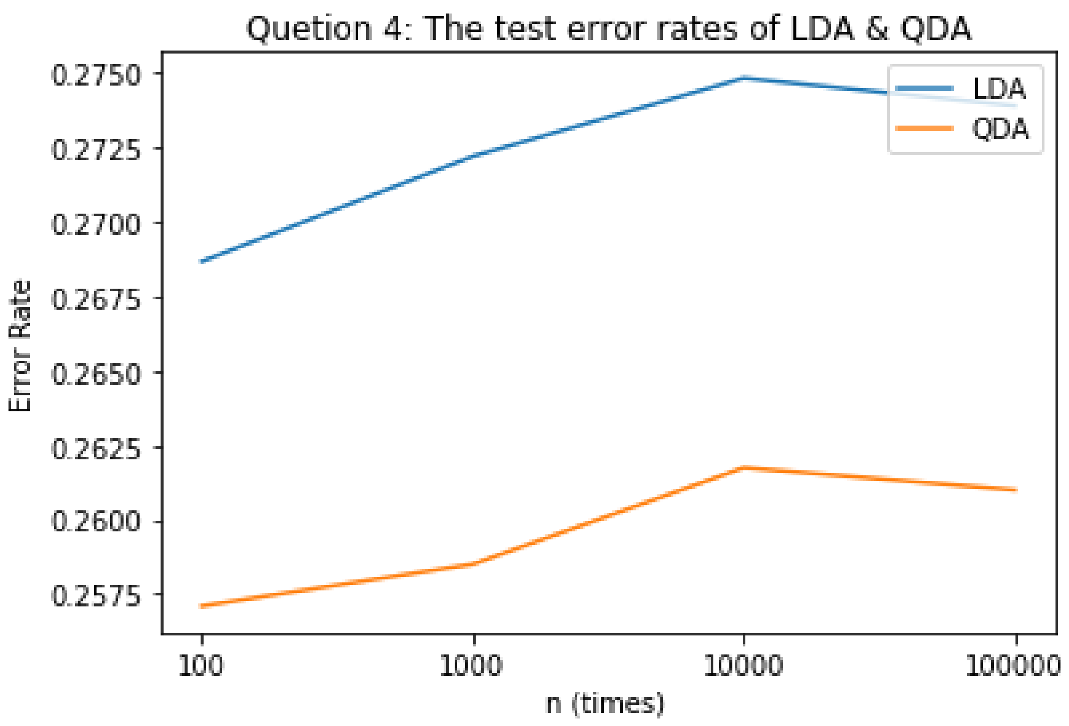
Given the results above, I believe that QDA slightly better than LDA on both the training set and the testing set. My conclusion can be supported by two illustrations (the distribution of training error/rate of LDA & QDA). We can see that in both graphs, the distribution line QDA is slightly left of that of LDA, which indicates a relatively lower error rate. Further, the above result conforms to the prediction that QDA would perform better with a non-linear Bayes decision boundary $f(X) = X_1 + X_1^2 + X_2 + X_2^2$

4 Question 4

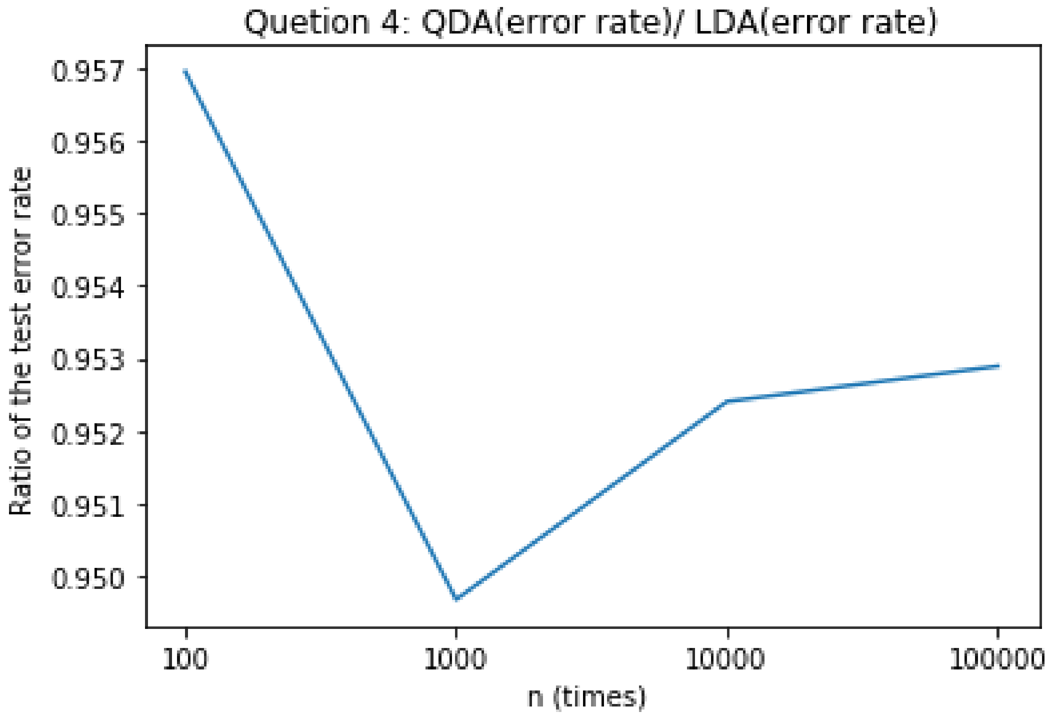
4.1 Main Function + Graph

[886]:
```
n_list = [100,1000,10000,100000]

lda_list = []
qda_list = []

for n in n_list:
    df = simulate_err(n,317,produce_y3,False)
    lda_list.append(df['lda_test'].mean())
    qda_list.append(df['qda_test'].mean())
```

[1210]:
```
q4_xticks = ['100','1000','10000','100000']
plt.xticks([0,1,2,3] ,q4_xticks)
plt.plot(lda_list)
plt.plot(qda_list)

plt.title('Quetion 4: The test error rates of LDA & QDA')
plt.xlabel('n (times) ')
plt.ylabel('Error Rate')
plt.legend(['LDA','QDA'],loc='upper right')
plt.show()
```



[1211]:
```
ratio = []
for i in range(len(qda_list)):
    ratio.append(qda_list[i]/lda_list[i])
ratio

plt.title('Quetion 4: QDA(error rate)/ LDA(error rate)')
plt.xlabel('n (times) ')
plt.xticks([0,1,2,3],q4_xticks)
plt.plot(ratio)
plt.ylabel('Ratio of the test error rate')
plt.show()
```



4.2 Comments and Conclusions

The result above shows that the ratio test error of QDA over LDA does not necessarily increase as n becomes larger. Instead, they fluctuate and converge to 0.953.

Analysis Since the decision boundary is non-linear, we can expect QDA performs better than LDA with a lower error rate. My result reflects such expectation, with the ratios of error rates staying below 1. Besides, LDA preferred to QDA for small training n. Thus, reversely, we can expect the ratio of the error rate of QDA over LDA to decrease. My result does not reflect such prediction. Here are several possible reasons:

1. the random state of python does not perform well
2. The differences between the calculated ratio of the error rates are too small to reflect the trend.
3. I'm a horrible programmer.

5 Question 5

5.1 Split Data (into training vs testing)

```
[1173]: df5 = pd.read_csv('mental_health.csv').dropna()
        y = copy.copy(df5['vote96'])
        x = copy.copy(df5[X_col])
        x_train, x_test, y_train, y_test = train_test_split(np.array(x),np.
         ↪array(y),test_size = 0.3,shuffle = True)
```

5.2 Helper Functions:

q5: Input a model and get a dictionary that contains the error rate and the information related to ROC.

plot_roc: Plot the graph of ROC

plot_area: Plot the graph of AUC.

plot_error: Plot the graph of the error rate

```
[1245]: def q5(clf, X_train,Y_train,X_test, Y_test,name):

            clf.fit(X_train,Y_train)
            Y_predicted = clf.predict(X_test)
            df= pd.DataFrame({'Y_actual':Y_test, 'Y_predicted': Y_predicted})
            df['error'] = (df['Y_actual'] != df['Y_predicted'])

            y_predict_prob = clf.predict_proba(x_test)[:,1]
            df['score'] = y_predict_prob
            fpr, tpr, thresholds = metrics.roc_curve(df['Y_actual'], df['score'])

            error_rate = (df['error']== True).sum()/df['error'].count()
            area = roc_auc_score(df['Y_actual'], df['score'])

            dic = {'fpr':fpr,'tpr':tpr,'error rate':error_rate,'area':area,'name':name}

            return dic

        def plot_roc(dic_list):
            legend = []
            plt.figure(figsize=(15,8))
            for dic in dic_list:
                fpr = dic['fpr']
                tpr = dic['tpr']
                plt.plot(fpr,tpr)
                legend.append(dic['name'])
```

```
    plt.legend(legend,loc='lower right')
    plt.plot([0, 1], [0, 1], color='navy', linestyle='--')
    plt.xlim([0.0, 1.0])
    plt.ylim([0.0, 1.05])
    plt.xlabel('False Positive Rate')
    plt.ylabel('True Positive Rate')
    plt.title('Question 5: ROC for each model')
    plt.show()

def plot_error(dic_list):
    name = []
    err = []

    plt.figure(figsize=(15,8))
    for dic in dic_list:
        name.append(dic['name'])
        err.append(dic['error rate'])
    plt.bar(name,err)
    plt.xlabel('Model')
    plt.ylabel('Error Rate')
    plt.title('Question 5: Error Rate for each model')


def plot_area(dic_list):

    name = []
    area = []

    plt.figure(figsize=(15,8))
    for dic in dic_list:
        name.append(dic['name'])
        area.append(dic['area'])
    plt.bar(name,area)
    plt.xlabel('Model')
    plt.ylabel('AUC')
    plt.title('Question 5: AUC for each model')
```

5.3 Logistic Regression Model

[1235]:
```
logi = LogisticRegression()
logi_dic = q5(logi,x_train,y_train,x_test,y_test,'LOGI')
```

```
/Users/nishengwenxin/opt/anaconda3/lib/python3.7/site-
packages/sklearn/linear_model/logistic.py:432: FutureWarning: Default solver
will be changed to 'lbfgs' in 0.22. Specify a solver to silence this warning.
  FutureWarning)
```

5.4 Linear Discriminant Model

[1236]:
```
lda = LinearDiscriminantAnalysis()
lda_dic = q5(lda,x_train,y_train,x_test,y_test,'LDA')
```

5.5 Quadratic Regression Model

[1237]:
```
qda = QuadraticDiscriminantAnalysis()
qda_dic = q5(qda,x_train,y_train,x_test,y_test,'QDA')
```

5.6 Gaussian Naive Bayes Model

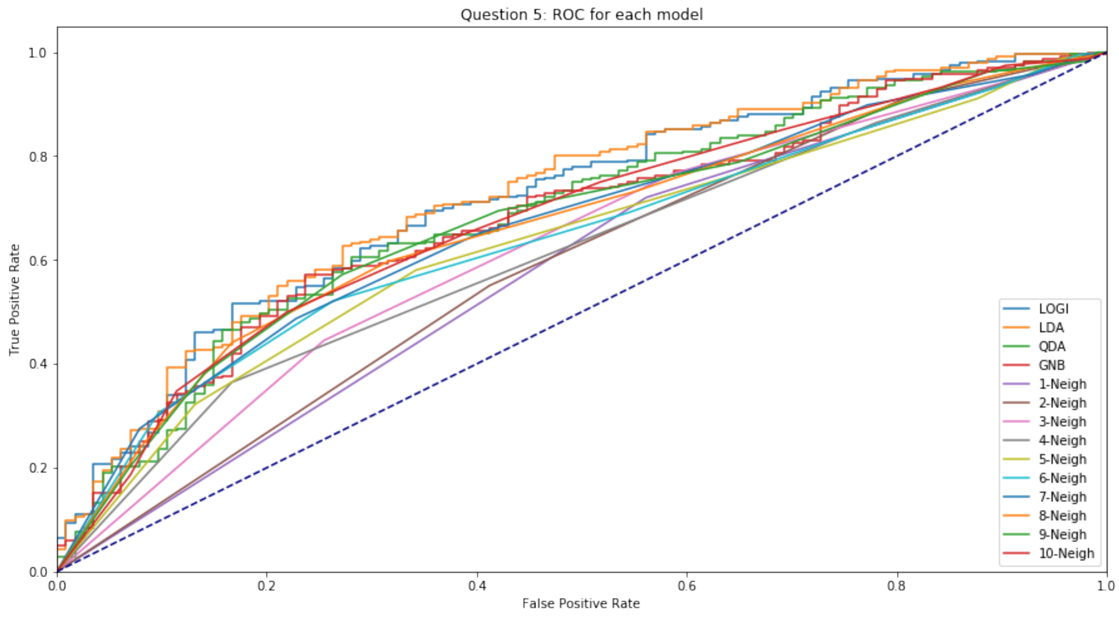
[1238]:
```
gnb = GaussianNB()
gnb_dic = q5(gnb,x_train,y_train,x_test,y_test,'GNB')
```

5.7 Nearest k-neighbors

[1181]:
```
num = 1
dic_neigh = []
while num <= 10:
    name = str(num) + '-Neigh'
    clf = KNeighborsClassifier(n_neighbors = num)
    dic = q5(clf,x_train,y_train,x_test,y_test,name)
    dic_neigh.append(dic)
    num += 1
```

5.8 Graph - ROC & AUC

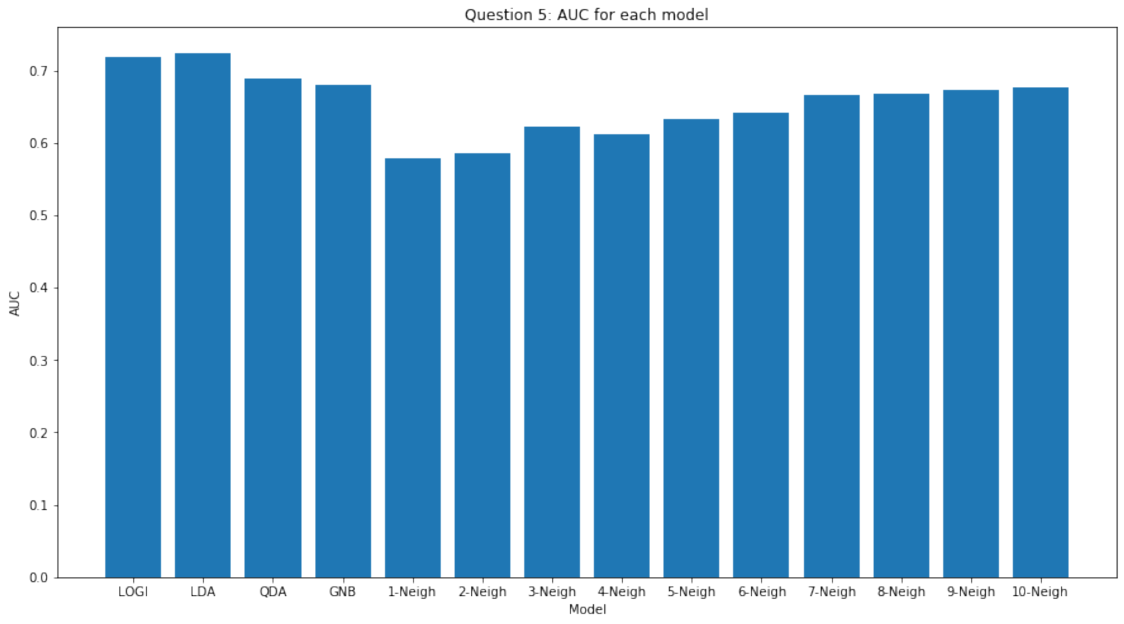
```
[1239]: dic_list = [logi_dic,lda_dic,qda_dic,gnb_dic] + dic_neigh
        plot_roc(dic_list)
```



```
[1247]: for dic in dic_list:
            print(dic['name'],'area:',dic['area'])
```

```
LOGI area: 0.7179601546238479
LDA area: 0.7245391019922689
QDA area: 0.6891911983348201
GNB area: 0.6798245614035089
1-Neigh area: 0.5794677371394588
2-Neigh area: 0.5864741302408564
3-Neigh area: 0.6222494796312815
4-Neigh area: 0.6125111507582516
5-Neigh area: 0.6325825156110617
6-Neigh area: 0.6420792447219744
7-Neigh area: 0.6656816830211121
8-Neigh area: 0.6675958965209634
9-Neigh area: 0.6725951531370801
10-Neigh area: 0.6774643175735949
```

```
[1248]: plot_area(dic_list)
```

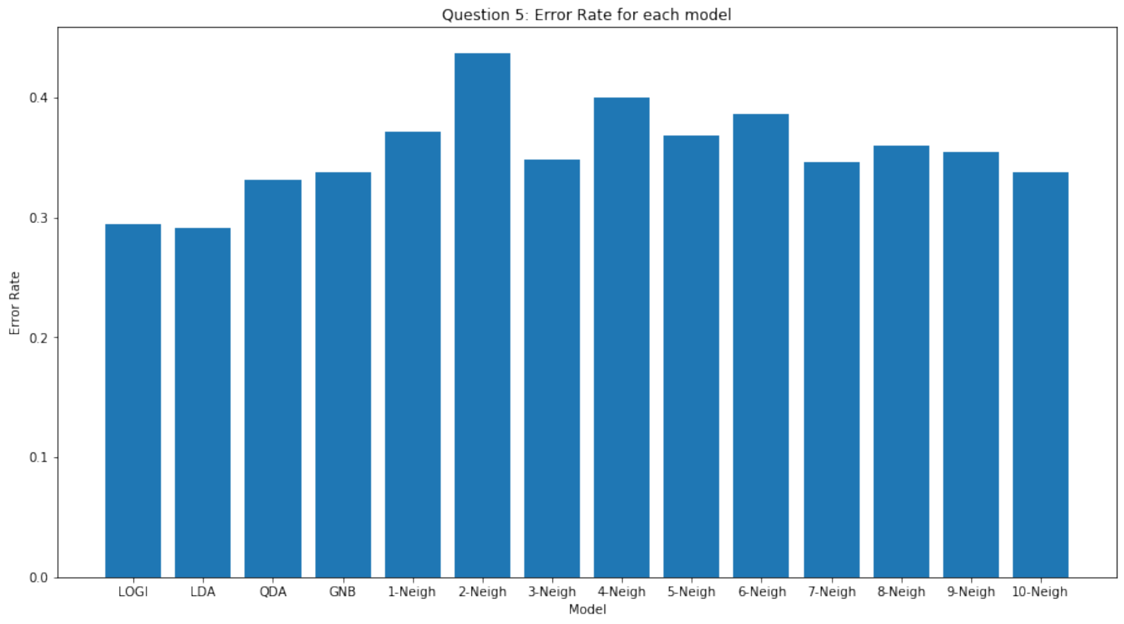


5.9 Graph - Error Rate

[1243]:
```
for dic in dic_list:
    print(dic['name'],'error_rate:',dic['error rate'])
```

```
LOGI error_rate: 0.29428571428571426
LDA error_rate: 0.2914285714285714
QDA error_rate: 0.3314285714285714
GNB error_rate: 0.33714285714285713
1-Neigh error_rate: 0.37142857142857144
2-Neigh error_rate: 0.43714285714285717
3-Neigh error_rate: 0.3485714285714286
4-Neigh error_rate: 0.4
5-Neigh error_rate: 0.36857142857142855
6-Neigh error_rate: 0.38571428571428573
7-Neigh error_rate: 0.3457142857142857
8-Neigh error_rate: 0.36
9-Neigh error_rate: 0.35428571428571426
10-Neigh error_rate: 0.33714285714285713
```

[1246]:
```
plot_error(dic_list)
```



5.10 Comments and Conlusions

LDA performs the best if the best is defined as giving the most accurate predictions. This statement can be supported by the following facts that:

1. LDA model has the lowest error rate.
2. LDA model has the greatest area under the curve.

The error rate refers to the frequency of errors. Therefore, it's clear that the lower the frequency of errors in prediction, the more accurate the prediction is. Thus, LDA wit the error rate 0.29142857142857144 is the best model in this sense. Besides, AUC, area under the curve denotes the overall performance of the classifier across all potential thresholds. Thus, higher AUC, better the performance. Again, LDA reaches an AUC of 0.7245391019922689, which is the highest among all models. Therefore, it's the best.

```
[ ]:
```