

MACS30100 Homework 4

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```
knitr::opts_chunk$set(message = FALSE, warning = FALSE)
knitr::opts_chunk$set(fig.width = 7, fig.height = 3, fig.align = 'center')

# load necessary package
library(tidyverse)
library(broom)
library(rsample)
library(tibble)
library(rcfss)
library(knitr)
library(boot)
library(splines)
library(magrittr)
library(glmnet)
library(margins)
library(plotly)
library(pls)
library(caret)
library(iml)
library(elasticnet)
library(gridExtra)

# Set the theme as minimal
theme_set(theme_minimal())

# Import the data
gss_test <- read_csv("data/gss_test.csv")
gss_train <- read_csv("data/gss_train.csv")
```

Egalitarianism and income

Question 1

```
set.seed(0) # Ensure the reproducibility

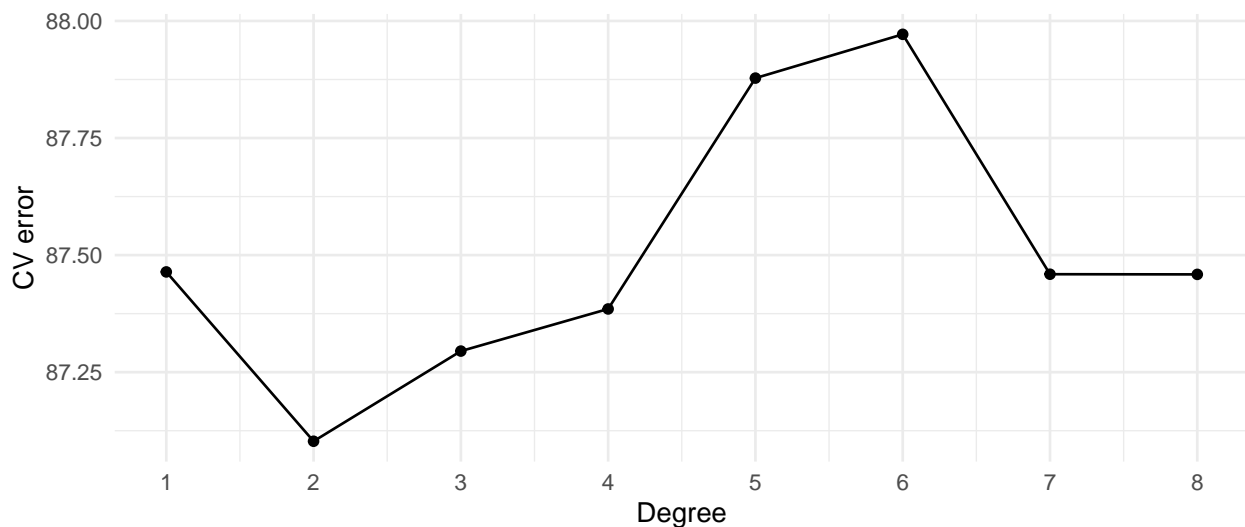
# Note: I use the cv.glm ("boot" package) to perform the 10-fold cross-validation
# which is different from the lab's way, but I found this is a simpler way (code)
```

```

# Fit the train data to polynomial functions and perform the CV by each degrees
degrees <- rep(NA, 8)
error_poly <- rep(NA, 8)
for (i in 1:8) {
  poly.fit <- glm(egalit_scale ~ poly(income06, i), data = gss_train)
  degrees[i] <- i
  error_poly[i] <- cv.glm(gss_train, poly.fit, K = 10)$delta[2]
  poly <- data.frame(degrees, error_poly)
}

# Plot the CV error by each degree
poly %>%
  ggplot(aes(x = degrees, y = error_poly)) +
  geom_point() +
  geom_line() +
  scale_x_continuous(breaks = seq(0, 10, 1)) +
  labs(x = "Degree", y = "CV error")

```



```

# Show the degree when the CV error is minimum
poly %>%
  filter(error_poly == min(error_poly))

```

```

##  degrees error_poly
## 1      2      87.1025

```

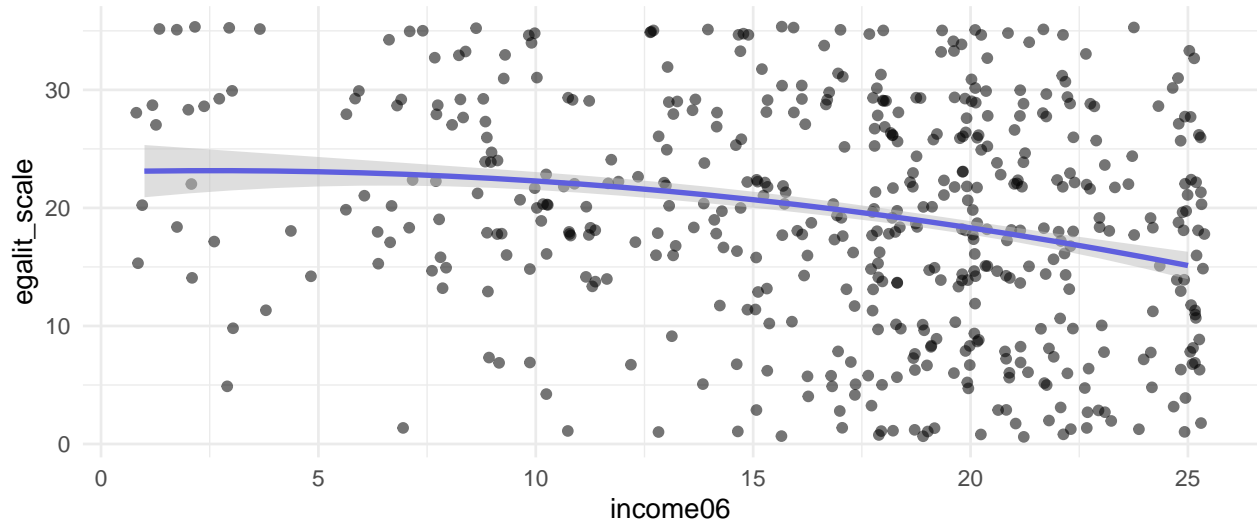
```

# Fit the test data and predict the egalit scale
poly_fit <- glm(egalit_scale ~ stats::poly(income06, 2), data = gss_train)
poly_pred <- augment(poly_fit, newdata = gss_test)

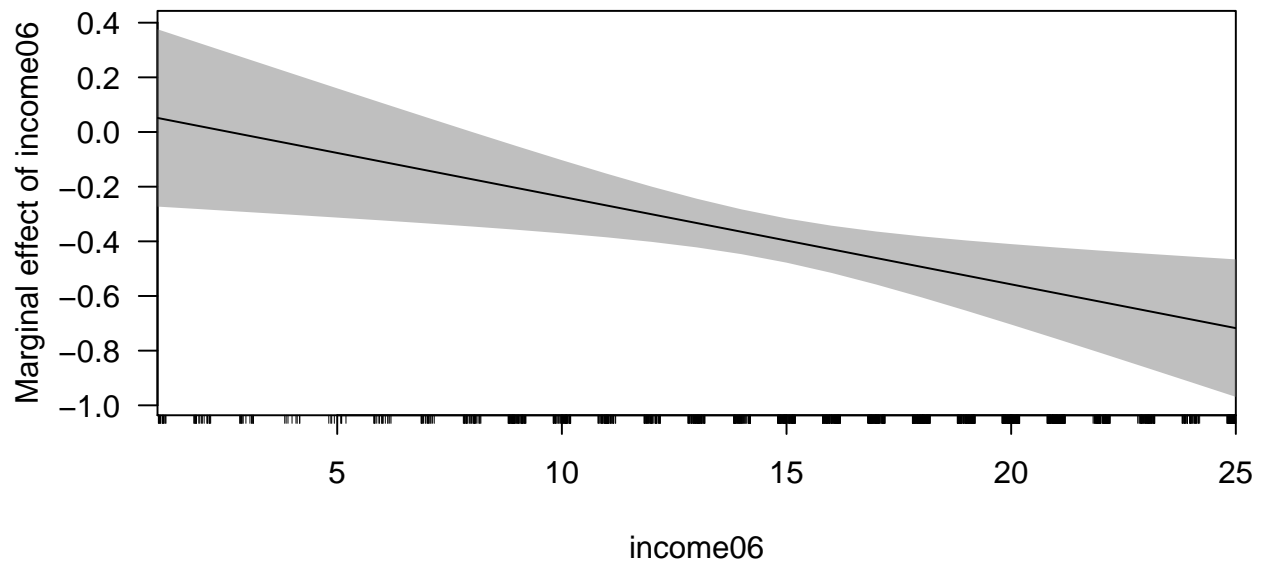
# Plot the income and egalit scale by jitter and prediction line by ggplot
poly_pred %>%
  ggplot(aes(x = income06)) +
  geom_jitter(aes(y = egalit_scale, alpha = 0.3)) +
  geom_line(aes(y = .fitted), data = poly_pred, color = "blue", size = 1) +

```

```
geom_ribbon(aes(ymin = .fitted - 2*.se.fit, ymax = .fitted + 2*.se.fit),
           fill = "gray", alpha = 0.5) +
theme(legend.position = 'none')
```



```
# Plot the marginal effect of income by cplot
cplot(poly_fit, "income06", what = "effect")
```



```
# Calculate the average marginal effect
summary(margins(poly_fit))
```

```
##   factor    AME    SE      z    p  lower  upper
## income06 -0.4507 0.0475 -9.4933 0.0000 -0.5438 -0.3577
```

- To predict `egalit_scale` as a function of `income06`, the quadratic function is the best fit among the polynomial regression.

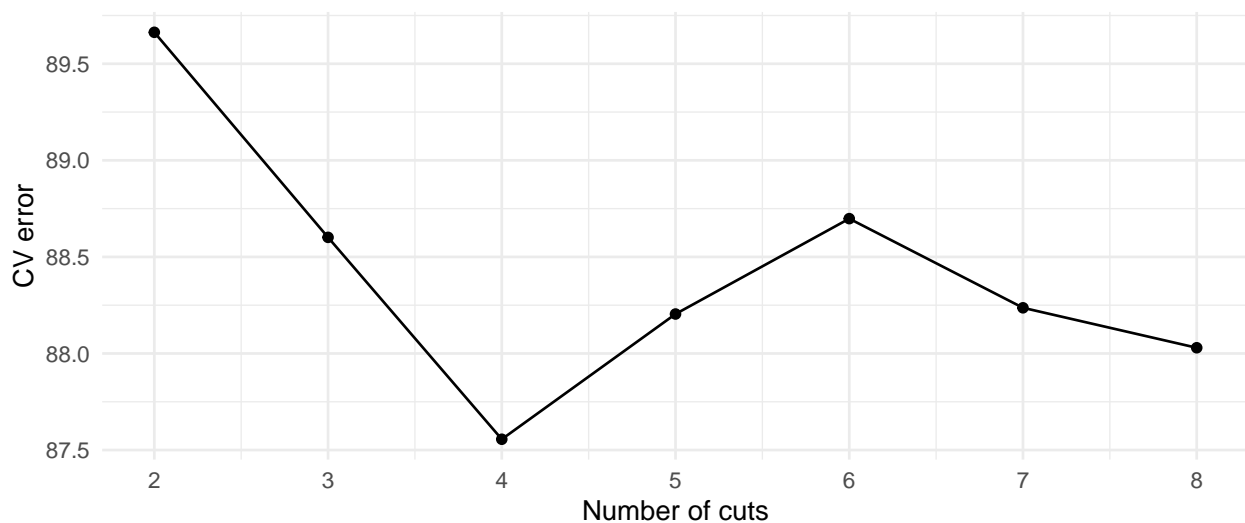
- The plot of the polynomial fit to the data shows the downward sloping. Thus, we can interpret that people with more income can be predicted to have lower feelings towards egalitarianism.
- The graph of the marginal effect of income also shows downward sloping. On average, the marginal effect is -0.4507 (SE: 0.0475). Thus, we can interpret that the change of the feeling towards egalitarianism could decrease when the income increases, but on average, we can predict one unit increase in income06 could decrease the feeling towards egalitarianism by 0.4507.

Question 2

```
set.seed(0) #Ensure the reproducibility

# Fit the train data to step functions and perform the CV by each number of cuts
cuts <- rep(NA, 7)
error_steps <- rep(NA, 7)
for (i in 2:8) {
  gss_train$income_cut <- cut(gss_train$income06, i)
  step.fit <- glm(egalit_scale ~ income_cut, data = gss_train)
  cuts[i] <- i
  error_steps[i] <- cv.glm(gss_train, step.fit, K = 10)$delta[2]
  steps <- data.frame(cuts, error_steps)
}

# Plot the CV error by each number of cuts
steps <- steps[-1, ]
steps %>%
  ggplot(aes(x = cuts, y = error_steps)) +
  geom_point() +
  geom_line() +
  scale_x_continuous(breaks = seq(2, 8, 1)) +
  labs(x = "Number of cuts", y = "CV error")
```



```
# Show the steps when the CV error is minimum
```

```
steps %>%
```

```
  filter(error_steps == min(error_steps))
```

```
##    cuts error_steps
```

```
## 1     4      87.55596
```

```
# Fit the test data and predict the egalit scale
```

```
gss_test$income_cut <- cut(gss_test$income06, 4)
```

```
steps_fit <- glm(egalit_scale ~ income_cut, data = gss_test)
```

```
steps_pred <- augment(steps_fit, newdata = gss_test)
```

```
# Plot the income and egalit scale by jitter and prediction line
```

```
steps_pred %>%
```

```
  ggplot(aes(x = income06)) +
```

```
  geom_vline(xintercept = c(7, 13, 19), linetype = "dotted") +
```

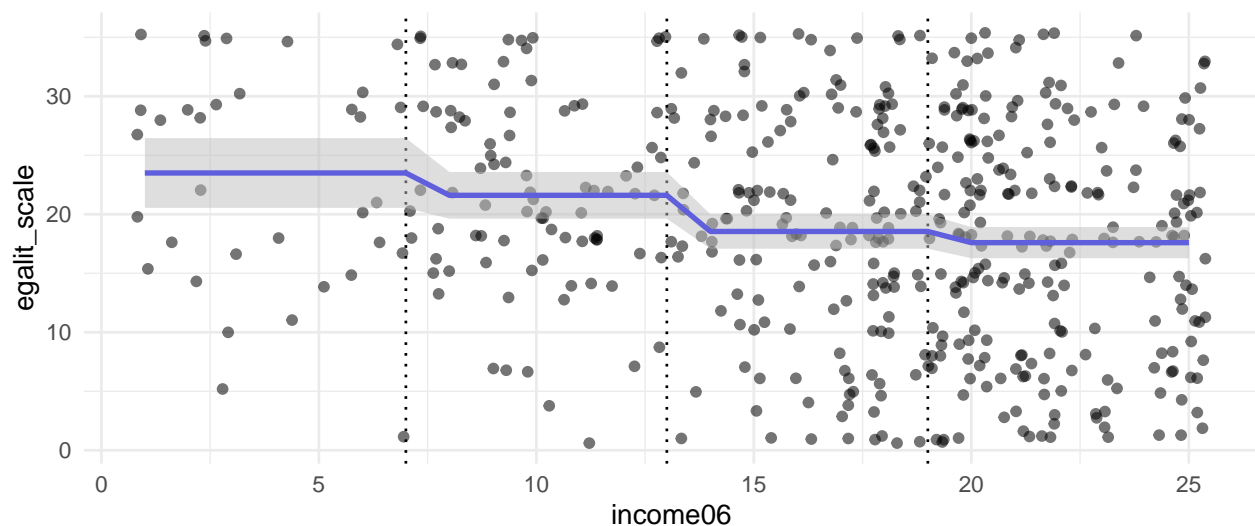
```
  geom_jitter(aes(y = egalit_scale, alpha = 0.3)) +
```

```
  geom_line(aes(y = .fitted), data = steps_pred, color = "blue", size = 1) +
```

```
  geom_ribbon(aes(ymin = .fitted - 2*.se.fit, ymax = .fitted + 2*.se.fit),
```

```
    fill = "gray", alpha = 0.5) +
```

```
  theme(legend.position = 'none')
```



```
# Eliminate the income_cut
```

```
gss_train[,46] <- NULL
```

```
gss_test[,46] <- NULL
```

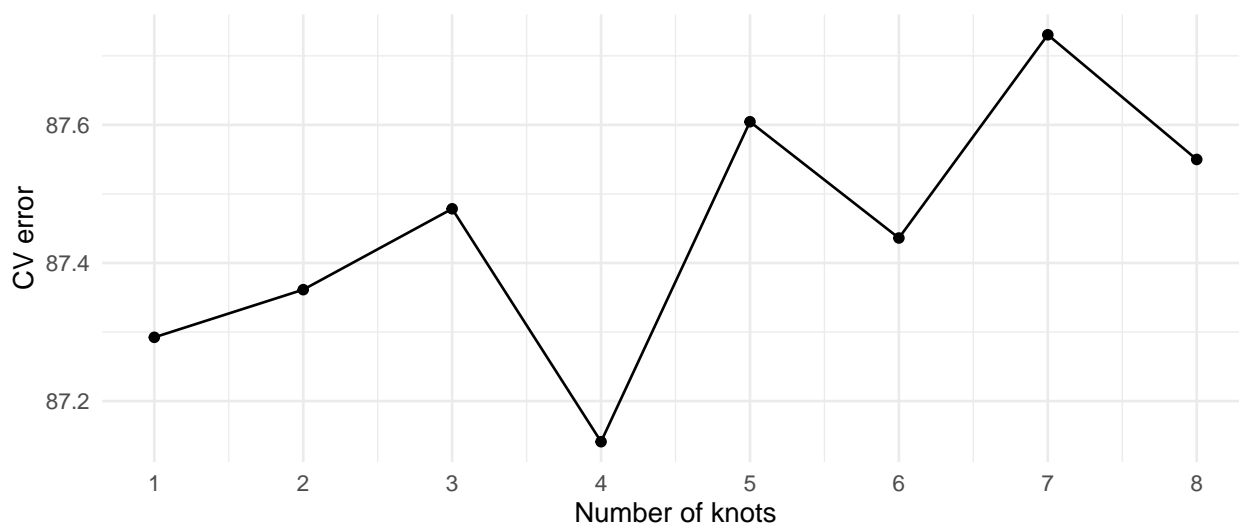
- Fit a step function to predict `egalit_scale` as a function of `income06`, and chose the optimal number of cuts: 4 cuts and plot the data as above.
- The interpretation is that the categories of the people with more income can be predicted to have lower feelings towards egalitarianism. The standard errors are larger in low income categories partly because of the fewer sample size.

Question 3

```
set.seed(1) #Ensure the reproducibility

# Fit the train data to spline functions and perform the CV by each number of knots
knots <- 1:8
error_spline <- rep(NA, 8)
for (i in 1:8) {
  spline_fit <- glm(egalit_scale ~ ns(income06, df = i + 3), data = gss_train)
  error_spline[i] <- cv.glm(gss_train, spline_fit, K = 10)$delta[2]
}
spline <- data.frame(knots, error_spline)

# Plot the CV error by each number of knots
spline %>%
  ggplot(aes(x = knots, y = error_spline)) +
  geom_point() +
  geom_line() +
  scale_x_continuous(breaks = seq(1, 8, 1)) +
  labs(x = "Number of knots", y = "CV error")
```



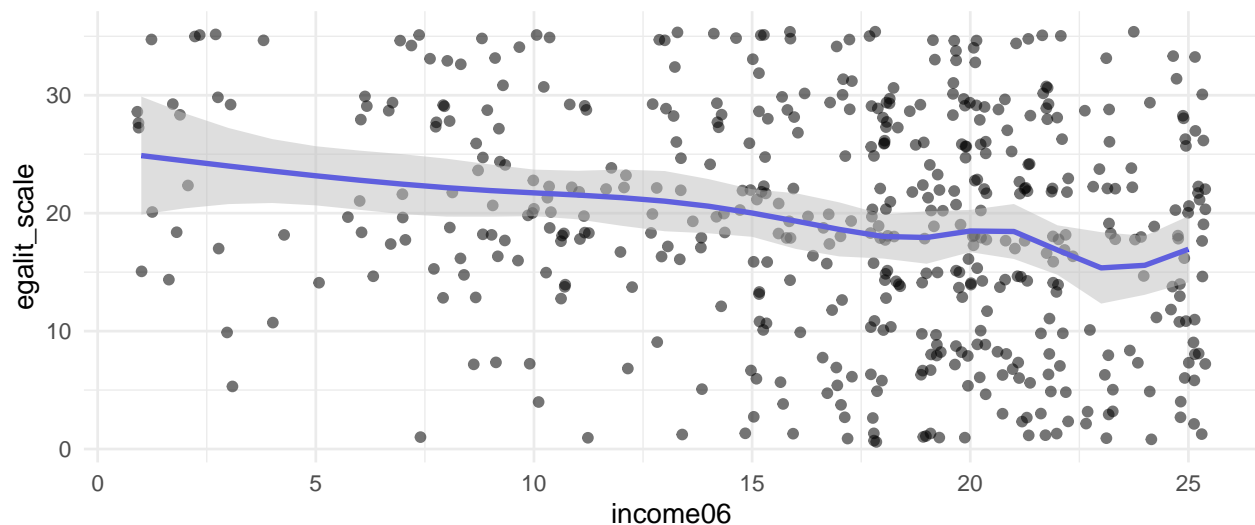
```
# Show the knots when the CV error is minimum
spline %>%
  filter(error_spline == min(error_spline))
```

```
##   knots error_spline
## 1     4      87.14114
```

```
# Fit the test data and predict the egalit scale
spline_fit <- glm(egalit_scale ~ splines::ns(income06, df = 4 + 3), data = gss_test)
spline_pred <- augment(spline_fit, newdata = gss_test)
```

```
# Plot the income and egalit scale by jitter and prediction line
```

```
spline_pred %>%
  ggplot(aes(x = income06)) +
  geom_jitter(aes(y = egalit_scale, alpha = 0.3)) +
  geom_line(aes(y = .fitted), data = spline_pred, color = "blue", size = 1) +
  geom_ribbon(aes(ymin = .fitted - 2*.se.fit, ymax = .fitted + 2*.se.fit),
            fill = "gray", alpha = 0.5) +
  theme(legend.position = 'none')
```



- I fit a natural regression spline to predict `egalit_scale` as a function of `income06`. I selected the optimal number of degrees of freedom: 7, and plot the results of the optimal model.
- The interpretation is that the people with more income can be predicted to have lower feelings towards egalitarianism. Looking at the standard errors (gray zone), this is a better fit to the data than the step function.

Egalitarianism and everything

Question 4

data pre-processing: feature standardization

I converted category variables of the data to numeric variables, and then standardized features (except `egalit_scale`). By doing this, all features' mean became 0 and ones' standard deviation became 1.

```
# Convert category variable to numeric variable
train_numeric <- data.matrix(data.frame(unclass(gss_train)))
test_numeric <- data.matrix(data.frame(unclass(gss_test)))

# Standardize the features except egalit_scale (mean:0, sd:1)
train_scale <- scale(data.frame(train_numeric) %>% select(!egalit_scale))
train_scale <- as.data.frame(train_scale)
train_scale$egalit_scale <- data.frame(train_numeric)$egalit_scale
```

```
test_scale <- scale(data.frame(test_numeric) %>% select(!egalit_scale))
test_scale <- as.data.frame(test_scale)
test_scale$egalit_scale <- data.frame(test_numeric)$egalit_scale
```

```
# Check the feature standardization only in test data just in case
test_scale %>% summarise_each(mean) # mean = 0 (except egalit_scale)
```

```
##          age          attend authoritarianism          black          born
## 1 2.009071e-16 -1.642366e-16  1.308494e-16 4.982492e-17 -1.790799e-16
##          childs          colath          colrac          colcom          colmil
## 1 6.401359e-18 -3.838538e-17 1.427727e-16 -2.198206e-16 8.909393e-17
##          colhomo          colmslm          con_govt          degree          evangelical
## 1 1.781258e-16 -7.823849e-18 -8.394435e-17 9.985348e-17 -6.562733e-17
##          grass          happy          hispanic_2          homosex          income06
## 1 -1.283555e-16 -9.681453e-18 8.04919e-17 -2.837702e-16 -3.883775e-18
##          marital          mode          news          owngun          partyid_3
## 1 7.119492e-17 6.112239e-17 -5.958318e-17 -1.201182e-16 -9.921093e-17
##          polviews          pornlaw2          pray          pres08          reborn_r
## 1 2.484182e-16 -1.169628e-16 -5.530443e-17 3.054107e-17 -1.549949e-16
##          relig          science_quiz          sex          sibs          social_connect
## 1 7.230397e-18 -2.274123e-16 2.180443e-16 5.410103e-17 -7.284431e-17
##          social_cons3          south          spend3          teensex          tolerance
## 1 1.678754e-16 -1.570639e-16 1.640343e-16 -3.341476e-17 -1.770606e-16
##          tvhours          vetyears          wordsum          zodiac          egalit_scale
## 1 5.409179e-17          0 -2.34579e-16 -8.67784e-17          19.11359
```

```
test_scale %>% summarise_each(sd) # sd = 1 (except egalit_scale)
```

```
## age attend authoritarianism black born childs colath colrac colcom colmil
## 1 1 1 1 1 1 1 1 1 1 1
## colhomo colmslm con_govt degree evangelical grass happy hispanic_2 homosex
## 1 1 1 1 1 1 1 1 1 1
## income06 marital mode news owngun partyid_3 polviews pornlaw2 pray pres08
## 1 1 1 1 1 1 1 1 1 1
## reborn_r relig science_quiz sex sibs social_connect social_cons3 south spend3
## 1 1 1 1 1 1 1 1 1 1
## teensex tolerance tvhours vetyears wordsum zodiac egalit_scale
## 1 1 1 1 1 1 1 9.515674
```

(a) Linear regression

```
# Fit the linear regression using 10 fold CV to minimize MSE
(glm <- train(egalit_scale~., data = train_scale,
             method = 'lm', metric = 'RMSE',
             trControl = trainControl(method = 'cv', number = 10)))
```

```
## Linear Regression
##
## 1481 samples
## 44 predictor
```



```
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 1334, 1333, 1332, 1332, 1334, 1333, ...
## Resampling results:
##
##      RMSE      Rsquared    MAE
##  8.004338  0.3137795  6.394753
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

```
# Calculate the MSE (= RMSE^2)
(8.002394)^2
```

```
## [1] 64.03831
```

In case of the linear regression, the MSE is 64.03831.

(b) Elastic net regression

```
set.seed(0) # Ensure the reproducibility

# Fit the elastic net regression using 10 fold CV to minimize MSE
(en <- train(egalit_scale~., data = train_scale,
  method = 'glmnet', metric = 'RMSE',
  trControl = trainControl(method = 'cv', number = 10),
  tuneLength = 10))
```

```
## glmnet
##
## 1481 samples
## 44 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 1332, 1333, 1332, 1334, 1333, 1333, ...
## Resampling results across tuning parameters:
##
##  alpha  lambda      RMSE      Rsquared    MAE
##  0.1     0.002118067  7.971155  0.3153620  6.354929
##  0.1     0.004893009  7.971155  0.3153620  6.354929
##  0.1     0.011303486  7.971155  0.3153620  6.354929
##  0.1     0.026112519  7.971155  0.3153620  6.354929
##  0.1     0.060323307  7.968860  0.3156240  6.353541
##  0.1     0.139354665  7.962791  0.3163555  6.349738
##  0.1     0.321927360  7.950305  0.3180480  6.343337
##  0.1     0.743693973  7.932957  0.3210822  6.346044
##  0.1     1.718029578  7.953600  0.3210112  6.405191
##  0.1     3.968871254  8.071438  0.3160035  6.573949
##  0.2     0.002118067  7.971198  0.3153667  6.354904
##  0.2     0.004893009  7.971198  0.3153667  6.354904
```

##	0.2	0.011303486	7.971198	0.3153667	6.354904
##	0.2	0.026112519	7.970364	0.3154650	6.354395
##	0.2	0.060323307	7.965828	0.3160170	6.351466
##	0.2	0.139354665	7.956189	0.3172800	6.345091
##	0.2	0.321927360	7.937803	0.3200559	6.339387
##	0.2	0.743693973	7.938018	0.3210721	6.371257
##	0.2	1.718029578	7.999136	0.3172993	6.473632
##	0.2	3.968871254	8.251110	0.2973389	6.773910
##	0.3	0.002118067	7.971220	0.3153703	6.354890
##	0.3	0.004893009	7.971220	0.3153703	6.354890
##	0.3	0.011303486	7.971220	0.3153703	6.354890
##	0.3	0.026112519	7.968981	0.3156404	6.353450
##	0.3	0.060323307	7.962938	0.3164007	6.349444
##	0.3	0.139354665	7.949279	0.3183070	6.341149
##	0.3	0.321927360	7.932004	0.3211066	6.343692
##	0.3	0.743693973	7.952675	0.3196806	6.397348
##	0.3	1.718029578	8.069018	0.3094130	6.564938
##	0.3	3.968871254	8.413989	0.2798254	6.937287
##	0.4	0.002118067	7.971275	0.3153641	6.354929
##	0.4	0.004893009	7.971275	0.3153641	6.354929
##	0.4	0.011303486	7.971189	0.3153752	6.354880
##	0.4	0.026112519	7.967591	0.3158199	6.352496
##	0.4	0.060323307	7.959959	0.3168086	6.347187
##	0.4	0.139354665	7.943281	0.3192203	6.338244
##	0.4	0.321927360	7.933329	0.3210813	6.354252
##	0.4	0.743693973	7.968756	0.3182112	6.424344
##	0.4	1.718029578	8.150972	0.2988712	6.661247
##	0.4	3.968871254	8.545275	0.2685114	7.069786
##	0.5	0.002118067	7.971311	0.3153624	6.354932
##	0.5	0.004893009	7.971311	0.3153624	6.354932
##	0.5	0.011303486	7.970624	0.3154467	6.354495
##	0.5	0.026112519	7.966267	0.3159932	6.351588
##	0.5	0.060323307	7.956945	0.3172355	6.345076
##	0.5	0.139354665	7.938223	0.3200186	6.337530
##	0.5	0.321927360	7.938103	0.3205741	6.365777
##	0.5	0.743693973	7.992129	0.3155617	6.457879
##	0.5	1.718029578	8.229123	0.2887109	6.743469
##	0.5	3.968871254	8.678108	0.2552305	7.204141
##	0.6	0.002118067	7.971314	0.3153636	6.354890
##	0.6	0.004893009	7.971314	0.3153636	6.354890
##	0.6	0.011303486	7.970007	0.3155249	6.354060
##	0.6	0.026112519	7.964968	0.3161630	6.350715
##	0.6	0.060323307	7.953927	0.3176716	6.343317
##	0.6	0.139354665	7.934492	0.3206236	6.338157
##	0.6	0.321927360	7.945162	0.3197017	6.378577
##	0.6	0.743693973	8.020505	0.3121024	6.496059
##	0.6	1.718029578	8.294651	0.2806336	6.813347
##	0.6	3.968871254	8.784309	0.2533486	7.309072
##	0.7	0.002118067	7.971300	0.3153665	6.354913
##	0.7	0.004893009	7.971300	0.3153665	6.354913
##	0.7	0.011303486	7.969391	0.3156038	6.353648
##	0.7	0.026112519	7.963747	0.3163235	6.349861
##	0.7	0.060323307	7.950781	0.3181357	6.341310
##	0.7	0.139354665	7.932875	0.3209039	6.341217

```
## 0.7 0.321927360 7.950959 0.3190766 6.389521
## 0.7 0.743693973 8.055092 0.3073967 6.538365
## 0.7 1.718029578 8.344313 0.2761476 6.865130
## 0.7 3.968871254 8.907687 0.2493906 7.426060
## 0.8 0.002118067 7.971288 0.3153699 6.354902
## 0.8 0.004893009 7.971288 0.3153699 6.354902
## 0.8 0.011303486 7.968779 0.3156824 6.353235
## 0.8 0.026112519 7.962483 0.3164923 6.348925
## 0.8 0.060323307 7.947884 0.3185664 6.339542
## 0.8 0.139354665 7.932719 0.3209636 6.345325
## 0.8 0.321927360 7.955653 0.3187149 6.398892
## 0.8 0.743693973 8.093841 0.3017719 6.584624
## 0.8 1.718029578 8.397186 0.2709978 6.917660
## 0.8 3.968871254 9.050237 0.2395210 7.554349
## 0.9 0.002118067 7.971308 0.3153682 6.354910
## 0.9 0.004893009 7.971308 0.3153682 6.354910
## 0.9 0.011303486 7.968172 0.3157607 6.352823
## 0.9 0.026112519 7.961180 0.3166710 6.347920
## 0.9 0.060323307 7.945302 0.3189515 6.338320
## 0.9 0.139354665 7.933090 0.3209596 6.349541
## 0.9 0.321927360 7.962066 0.3180807 6.409852
## 0.9 0.743693973 8.128323 0.2970259 6.625935
## 0.9 1.718029578 8.454165 0.2647376 6.973821
## 0.9 3.968871254 9.194544 0.2295790 7.674570
## 1.0 0.002118067 7.971324 0.3153667 6.354918
## 1.0 0.004893009 7.971225 0.3153787 6.354863
## 1.0 0.011303486 7.967567 0.3158393 6.352399
## 1.0 0.026112519 7.959860 0.3168538 6.346903
## 1.0 0.060323307 7.942848 0.3193218 6.337524
## 1.0 0.139354665 7.934275 0.3208405 6.354033
## 1.0 0.321927360 7.970328 0.3171393 6.422626
## 1.0 0.743693973 8.161933 0.2923939 6.664329
## 1.0 1.718029578 8.511690 0.2581813 7.030251
## 1.0 3.968871254 9.341309 0.2291685 7.812307
##
## RMSE was used to select the optimal model using the smallest value.
## The final values used for the model were alpha = 0.3 and lambda = 0.3219274.
```

```
# Calculate the MSE (= RMSE^2)
(7.965442)^2
```

```
## [1] 63.44827
```

In case of the elastic net regression, the MSE is 63.44827 when $\alpha = 0.3$ and $\lambda = 0.3219274$.

(c) Principal component regression

```
set.seed(0) # Ensure the reproducibility

# Fit the elastic net regression using 10 fold CV to minimize MSE
(pcr <- train(egalit_scale~, data = train_scale,
```

```
method = 'pcr', metric = 'RMSE',
trControl = trainControl(method = 'cv', number = 10),
tuneLength = 25))
```

```
## Principal Component Analysis
##
## 1481 samples
## 44 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 1332, 1333, 1332, 1334, 1333, 1333, ...
## Resampling results across tuning parameters:
##
##  ncomp  RMSE      Rsquared    MAE
##  1      9.606186  0.006361916  8.040998
##  2      8.328120  0.252193770  6.751873
##  3      8.186049  0.277047321  6.566935
##  4      8.156189  0.282728286  6.541748
##  5      8.156475  0.282689107  6.541171
##  6      8.109372  0.290257530  6.503810
##  7      8.118816  0.288664486  6.511752
##  8      8.119589  0.288645500  6.512461
##  9      8.119517  0.288646874  6.512472
## 10      8.116325  0.289363604  6.511273
## 11      8.110378  0.290473587  6.501737
## 12      8.115127  0.289607235  6.508659
## 13      8.118796  0.288895620  6.512840
## 14      8.100607  0.291851875  6.484557
## 15      8.098285  0.292251624  6.484410
## 16      8.100139  0.292014483  6.477554
## 17      8.098603  0.292486398  6.477943
## 18      8.058409  0.299536327  6.444167
## 19      8.063595  0.298488470  6.443831
## 20      8.026826  0.305005690  6.412105
## 21      8.034788  0.303371277  6.411667
## 22      8.040898  0.302350809  6.420260
## 23      8.047291  0.301420489  6.426302
## 24      8.053313  0.300474965  6.433934
## 25      8.055754  0.300030545  6.441273
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was ncomp = 20.
```

```
# Calculate the MSE (= RMSE^2)
(8.052263)^2
```

```
## [1] 64.83894
```

In case of the principal component regression, the MSE is 64.83894 when the number of component is 20.

(d) Partial least squares regression

```
set.seed(0) # Ensure the reproducibility

# Fit the elastic net regression using 10 fold CV to minimize MSE
(pls <- train(egalit_scale~., data = train_scale,
              method = 'pls', metric = 'RMSE',
              trControl = trainControl(method = 'cv', number = 10),
              tuneLength = 20))

## Partial Least Squares
##
## 1481 samples
## 44 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 1332, 1333, 1332, 1334, 1333, 1333, ...
## Resampling results across tuning parameters:
##
##  ncomp  RMSE      Rsquared  MAE
##  1      8.066524  0.2977267  6.434851
##  2      8.001162  0.3096623  6.395425
##  3      7.980793  0.3132387  6.363744
##  4      7.982828  0.3134459  6.360445
##  5      7.977556  0.3144310  6.356143
##  6      7.975842  0.3147450  6.358435
##  7      7.975199  0.3148393  6.358046
##  8      7.974292  0.3150010  6.356897
##  9      7.973964  0.3150458  6.356680
## 10      7.974097  0.3150265  6.356763
## 11      7.974065  0.3150328  6.356692
## 12      7.974038  0.3150353  6.356649
## 13      7.974047  0.3150344  6.356649
## 14      7.974043  0.3150348  6.356648
## 15      7.974047  0.3150343  6.356650
## 16      7.974045  0.3150344  6.356649
## 17      7.974045  0.3150345  6.356649
## 18      7.974045  0.3150345  6.356649
## 19      7.974045  0.3150345  6.356649
## 20      7.974045  0.3150345  6.356649
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was ncomp = 9.

# Calculate the MSE (= RMSE^2)
(7.973964)^2

## [1] 63.5841
```

In case of the principal component regression, the MSE is 63.5841 when the number of component is 9.

Question 5

```
set.seed(0) #Ensure the reproducibility

# Prepare features and response of test data
features <- test_scale %>% select(-egalit_scale)
response <- test_scale %>% select(egalit_scale)

# Create predictor object to pass to explainer functions
predictor.glm <- Predictor$new(
  model = glm, data = features, y = response, predict.fun = predict)

predictor.en <- Predictor$new(
  model = en, data = features, y = response, predict.fun = predict)

predictor.pcr <- Predictor$new(
  model = pcr, data = features, y = response, predict.fun = predict)

predictor.pls <- Predictor$new(
  model = pls, data = features, y = response, predict.fun = predict)

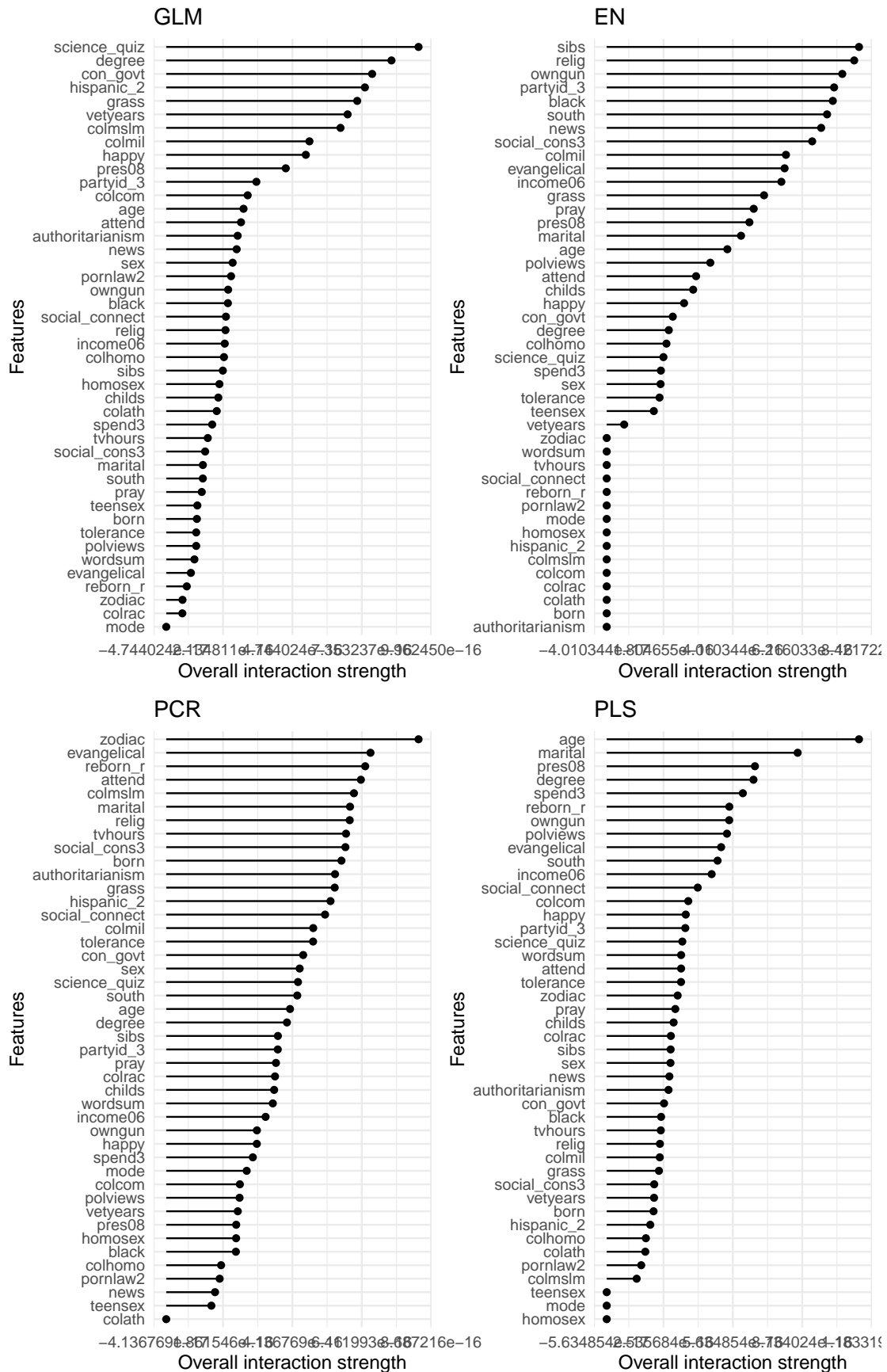
# Measure INXNs: identify features with large interaction effects in each model
interact.glm <- Interaction$new(predictor.glm) %>%
  plot() + ggtitle("GLM")

interact.en <- Interaction$new(predictor.en) %>%
  plot() + ggtitle("EN")

interact.pcr <- Interaction$new(predictor.pcr) %>%
  plot() + ggtitle("PCR")

interact.pls <- Interaction$new(predictor.pls) %>%
  plot() + ggtitle("PLS")

# Plot INXNs in each model
grid.arrange(interact.glm, interact.en, interact.pcr, interact.pls, nrow = 2, ncol = 2)
```



- The value of feature importance is how much the predicted outcome changes when the term is omitted. Looking at “the overall interaction strength”, they are almost zero in every model (e-16 or e-17). Especially in elastic net regression, many features (about 15) have low interaction strengths.
- Basically, the feature importances are ranked differently across different models. For example, “reborn_r” is higher rank in PCR and PLS while it is lower rank in GLM and EN. On the other hand, some features are in similar ranks. For example, “attend” are a little above the middle. However, since there seems to be no feature in the top rank of every model, we cannot surely say “this feature is important”.