HW 4

Zhaoyang Chen

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```
library(tidyverse)
library(glmnet)
library(pls)
library(leaps)
library(caret)
library(mltools)
library(DAAG)
library(splines)
library(margins)
library(iml)
```

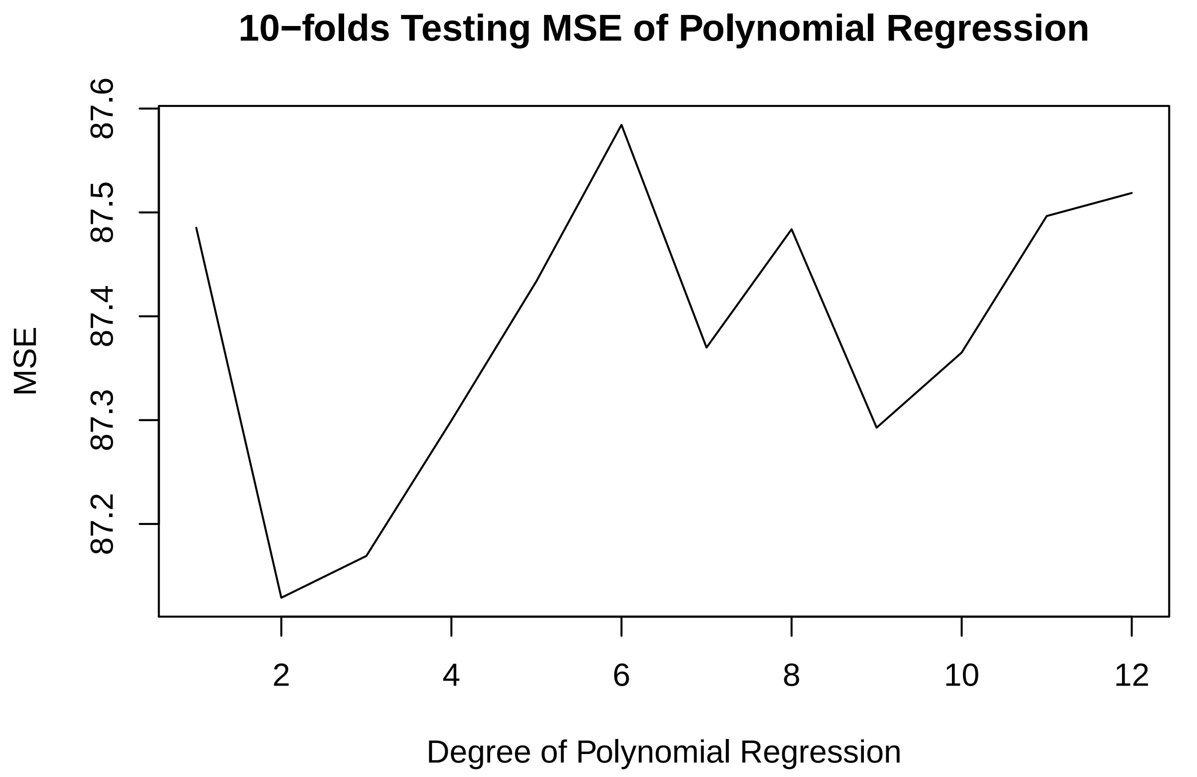
Load data

```
train = read.csv('gss_train.csv')
test = read.csv('gss_test.csv')

data_cv = createFolds(as.integer(rownames(train)), k = 10, list = T, returnTrain = F) #create 10-folds
```

Egalitarianism and income

Question one

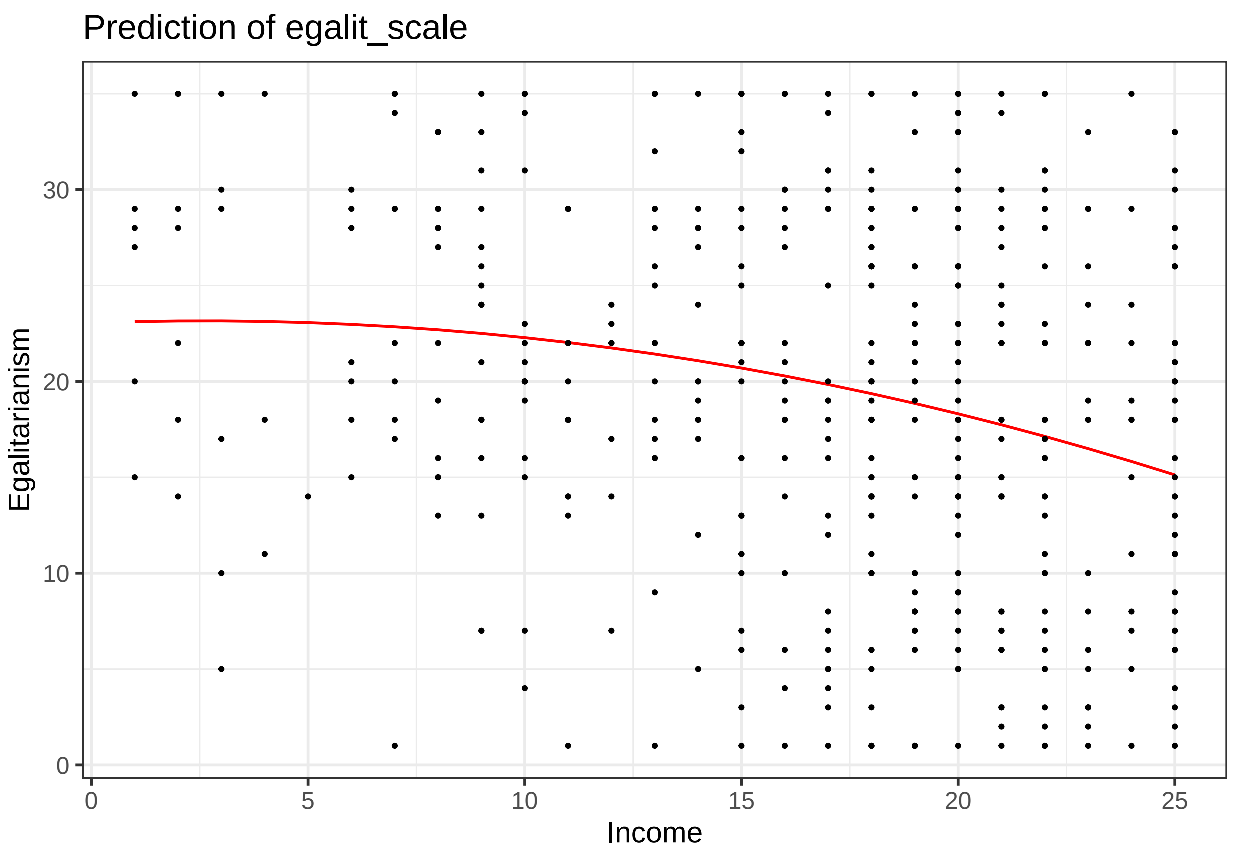
```
set.seed(1234)# set a random seed
train_poly = train[,c('egalit_scale','income06')] # select two target variables
test_poly_X = test[,'income06']
test_poly_y = test[,'egalit_scale']
models = list()
mses = c()
for(d in 1:12){
  mse = c()
  for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model1 = lm(egalit_scale ~ poly(income06, d, raw = T), data = tra)
    pred1 = predict(model1, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred1, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
  }
  mses = c(mses, mean(mse))
}
plot(mses, type = 'l', xlab = 'Degree of Polynomial Regression', ylab = 'MSE', main = '10-folds Testing
```



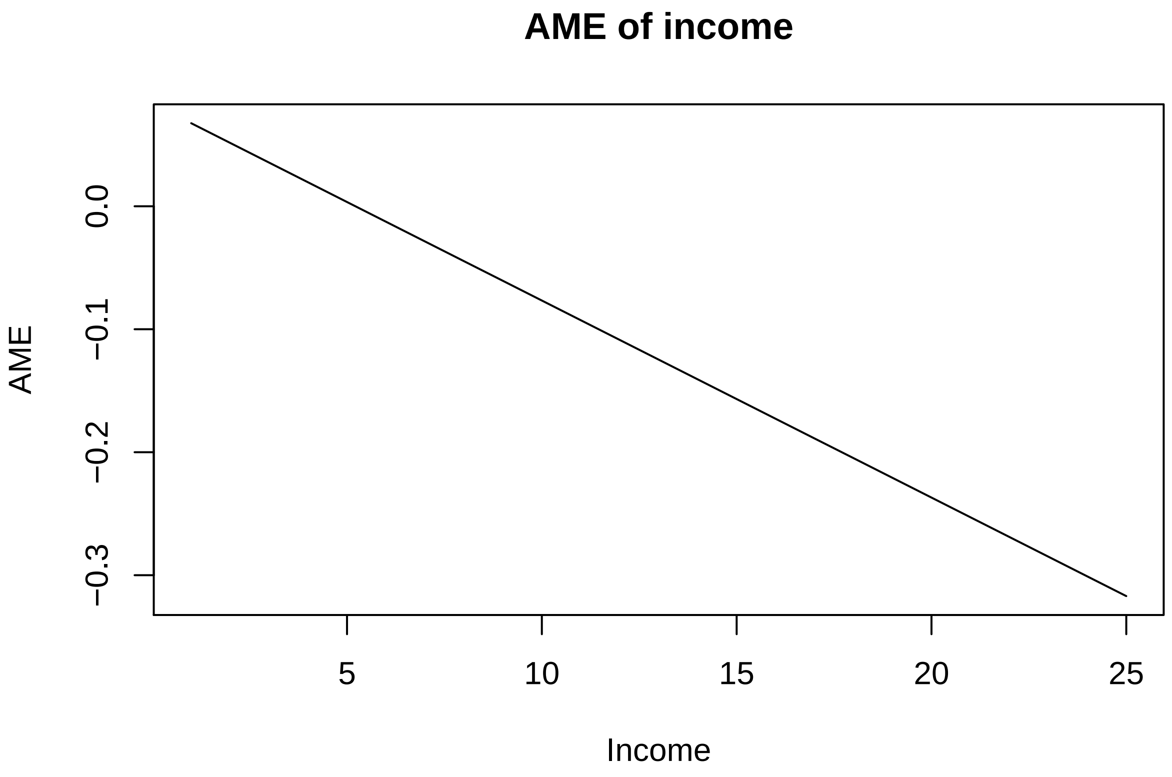
```
mses
```

```
##  [1] 87.48522 87.12897 87.16921 87.29956 87.43380 87.58433 87.36986
##  [8] 87.48378 87.29276 87.36508 87.49652 87.51871
```

```
best_model_1 = lm(egalit_scale ~ poly(income06, 2, raw = T), data = train_poly)
pred_1 =  predict(best_model_1, newdata = data.frame(income06=test_poly_X))
dt = data.frame(pred = as.data.frame(pred_1)[,1], income = test$income06)
ggplot(dt, aes(x = income, y = pred_1)) +
  theme_bw() +
  geom_line(color = 2) +
  geom_point(data =test, aes(x = income06, y = egalit_scale), size = 0.4) +
  labs(title = 'Prediction of egalit_scale', x = 'Income', y = 'Egalitarianism')
```



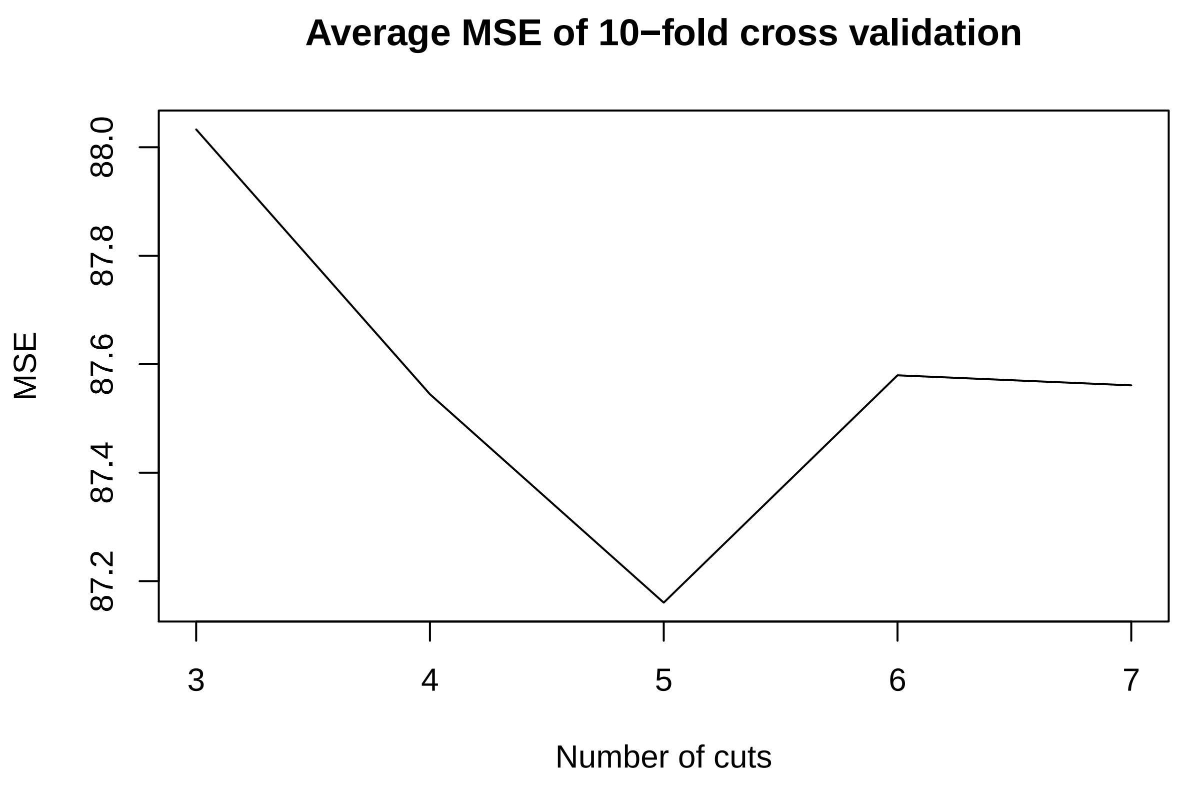
```
beta = best_model_1$coefficients
y = beta[2] + beta[3]*sort(train_poly$income06)
plot(sort(train_poly$income06), y, type = 'l', main = 'AME of income', xlab = 'Income', ylab = 'AME')
```



According to the result of 10-fold cross validation, we choose d as 2. However, the best model does not fit the test data well because there does not exist a clear trend on the scatter plot between income and Egalitarianism. Also, from the AME of income across its potential values, we can see that the mean average effect is decreasing, which means that one unit change in income will lead to more reduction in egalitarianism as income becomes larger.

Question two

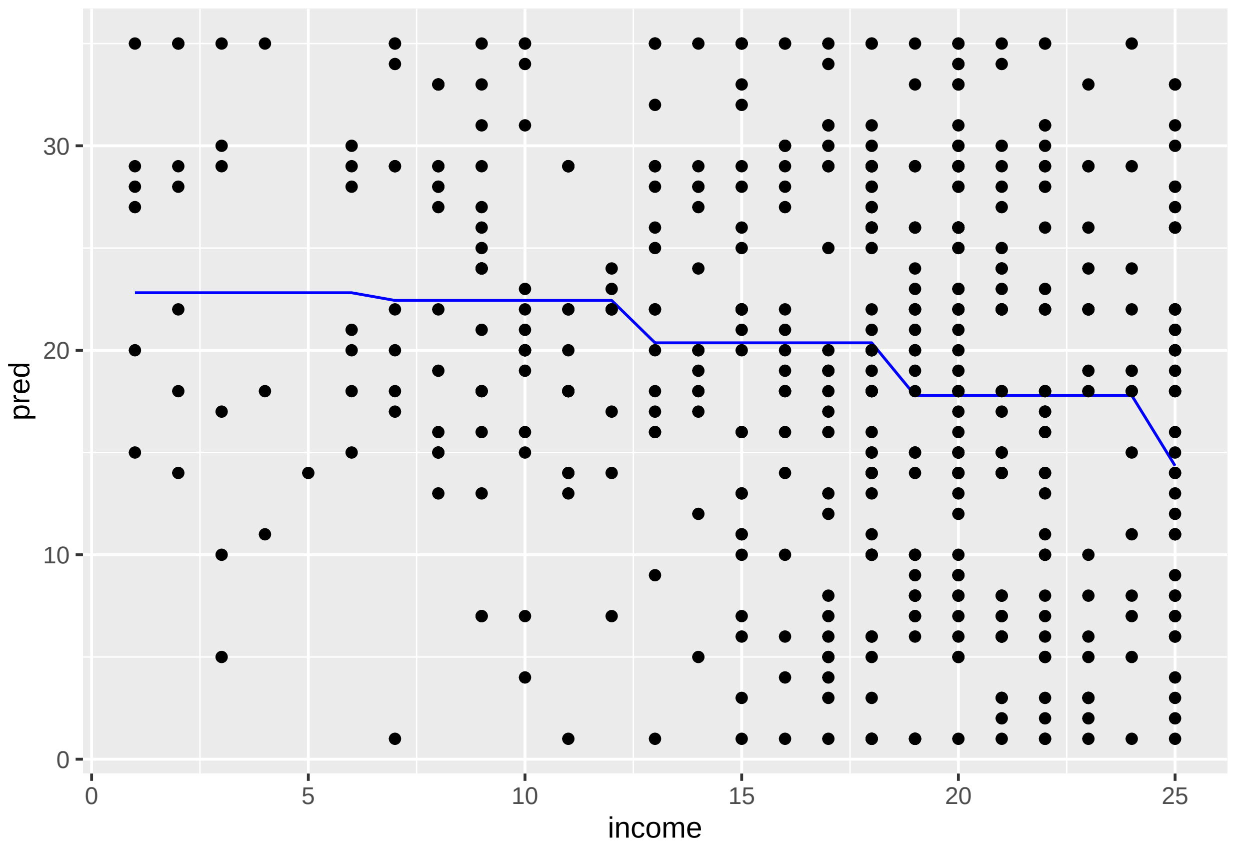
```
model_list = list()
mses = c()

mse = c()
n = 3
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06), data =
    pred = predict(model, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
}
mses = c(mses, mean(mse))

mse = c()
n = 4
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
y[4] = 0; y[5] = 1
sf_5 = stepfun(x = cut, y = y)
for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06) + sf_5(
    pred = predict(model, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
}
mses = c(mses, mean(mse))

mse = c()
n = 5
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
y[4] = 0; y[5] = 1
sf_5 = stepfun(x = cut, y = y)
y[5] = 0; y[6] = 1
sf_6 = stepfun(x = cut, y = y)
for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06) + sf_5(
    pred = predict(model, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
}
mses = c(mses, mean(mse))

mse = c()
n = 6
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
y[4] = 0; y[5] = 1
sf_5 = stepfun(x = cut, y = y)
y[5] = 0; y[6] = 1
sf_6 = stepfun(x = cut, y = y)
y[6] = 0; y[7] = 1
sf_7 = stepfun(x = cut, y = y)
for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06) + sf_5(
    pred = predict(model, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
}
mses = c(mses, mean(mse))

mse = c()
n = 7
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
y[4] = 0; y[5] = 1
sf_5 = stepfun(x = cut, y = y)
y[5] = 0; y[6] = 1
sf_6 = stepfun(x = cut, y = y)
y[6] = 0; y[7] = 1
sf_7 = stepfun(x = cut, y = y)
y[7] = 0; y[8] = 1
sf_8 = stepfun(x = cut, y = y)
for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06) + sf_5(
    pred = predict(model, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
}
mses = c(mses, mean(mse))

plot(x = 3:7, y = mses, type = 'l', main = 'Average MSE of 10-fold cross validation', xlab = 'Number of
```



```r
n = 5
cut = seq(min(train_poly$income06), max(train_poly$income06), length.out = n)
y = rep(0, n+1)
y[1] = 1
sf_1 = stepfun(x = cut, y = y)
y[1] = 0; y[2] = 1
sf_2 = stepfun(x = cut, y = y)
y[2] = 0; y[3] = 1
sf_3 = stepfun(x = cut, y = y)
y[3] = 0; y[4] = 1
sf_4 = stepfun(x = cut, y = y)
y[4] = 0; y[5] = 1
sf_5 = stepfun(x = cut, y = y)
y[5] = 0; y[6] = 1
sf_6 = stepfun(x = cut, y = y)

model = lm(egalit_scale ~ sf_1(income06) + sf_2(income06) + sf_3(income06) + sf_4(income06) +
             sf_5(income06) + sf_6(income06), data = train)
pred = predict(model, newdata = data.frame(income06=test$income06))
temp = data.frame(pre = pred, act = test$egalit_scale)
mean((temp$pre - temp$act)^2)
```

```
## [1] 87.58953
```

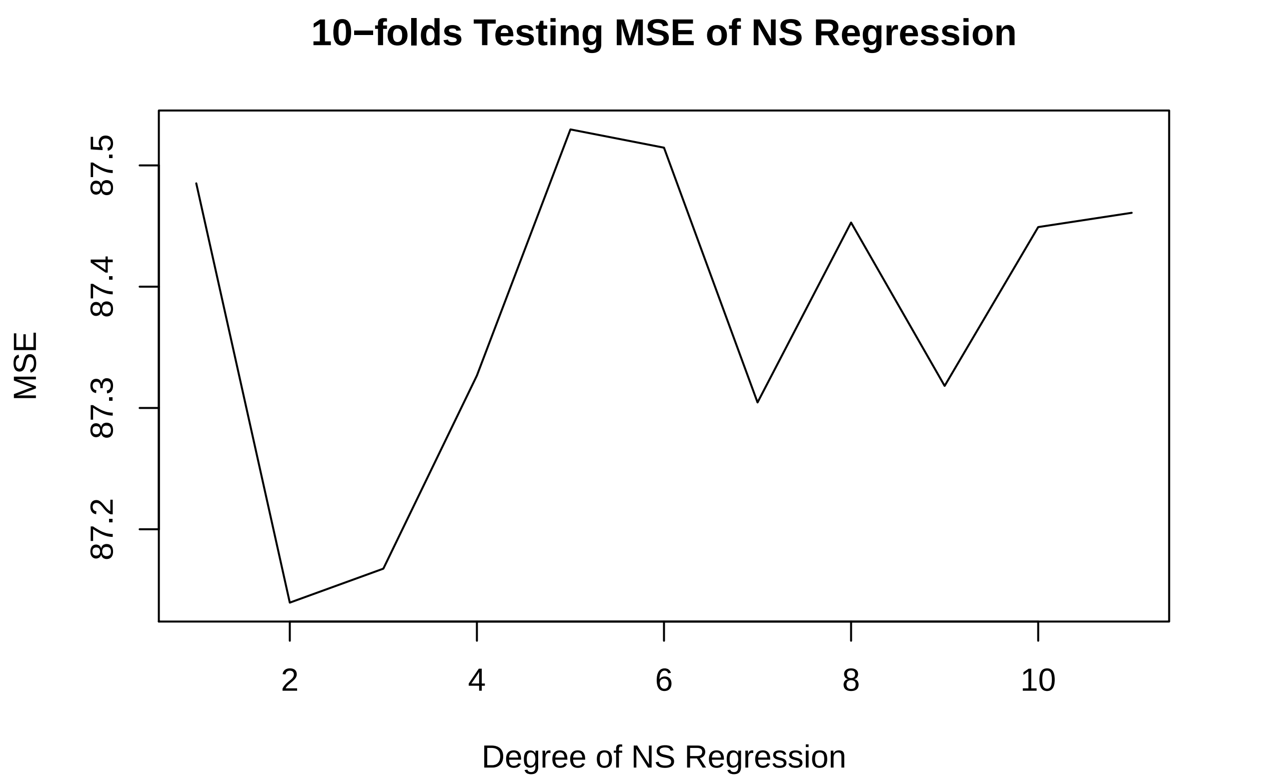
```r
df = data.frame('income' = test$income06, 'pred' = pred)
ggplot(data = df, aes(x = income, y = pred)) +
  geom_line(color = 'blue') +
  geom_point(data = test, aes(x = income06, y = egalit_scale))
```



Compared with the polynomial regression, we can see that step function has less variation but more bias. However, its trend is similar to polynomial regression, which means that egalitarianism will decline as income becomes larger.

Question three

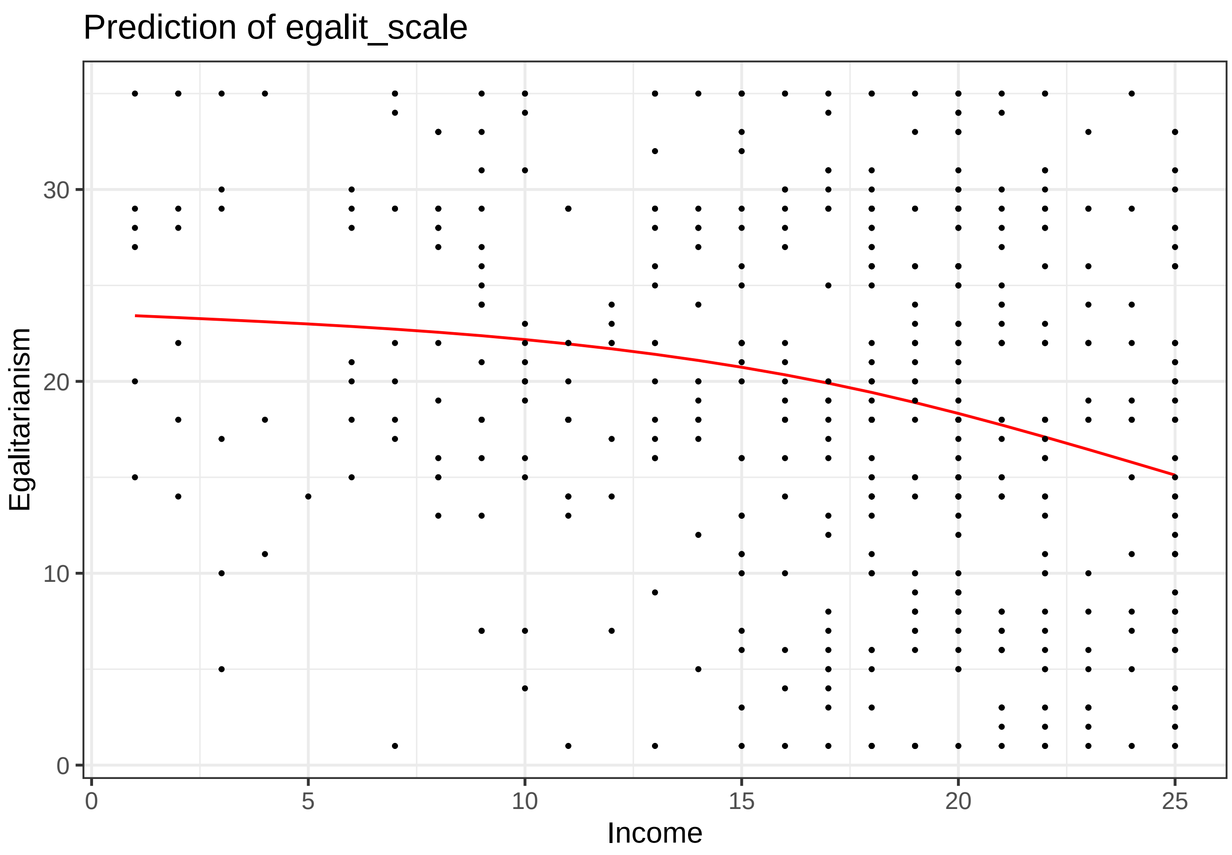
```
models = list()
mses = c()
for(k in 1:11){
  mse = c()
  for(i in 1:10){
    validation_id = data_cv[[i]]
    val = train_poly[validation_id,]
    tra = train_poly[-validation_id,]
    best_model_2 = glm(egalit_scale ~ ns(income06, df = k), data = tra)
    pred = predict(best_model_2, newdata = data.frame(income06=val$income06))
    temp = data.frame(pre = pred, act = val$egalit_scale)
    mse = c(mse, mean((temp$pre - temp$act)^2))
  }
  mses = c(mses, mean(mse))
}
plot(mses, type = 'l', xlab = 'Degree of NS Regression', ylab = 'MSE', main = '10-folds Testing MSE of 
```



```
mses
```

```
##  [1] 87.48522 87.13952 87.16752 87.32674 87.52964 87.51461 87.30455
##  [8] 87.45290 87.31819 87.44912 87.46089
```

```
best_model_2 = glm(egalit_scale ~ ns(income06, df = 2), data = train_poly)
pred =  predict(best_model_2, newdata = data.frame(income06=test_poly_X))
dt = data.frame(pred = as.data.frame(pred)[,1], income = test$income06)
ggplot(dt, aes(x = income, y = pred)) +
  theme_bw() +
  geom_line(color = 2) +
  geom_point(data =test, aes(x = income06, y = egalit_scale), size = 0.4) +
  labs(title = 'Prediction of egalit_scale', x = 'Income', y = 'Egalitarianism')
```



Egalitarianism and everything

Question four

Linear regression

```
train_cv = createFolds(1:1481, 10)
train_num = train %>% select_if(., is.numeric)
test_num = test %>% select_if(., is.numeric)
mse = c()
for (i in 1:10) {
  val_id = train_cv[[i]]
  val = train_num[val_id,]
  tra = train_num[-val_id,]
  model = lm(egalit_scale~., data = tra)
  pred = predict(model, newdata = val[,-5])
  mse = c(mse, mse(pred, val[,5]))
}
id = which(mse == min(mse))
best_model_1 = lm(egalit_scale~., data = train_num[-train_cv[[id]],])
pred = predict(best_model_1, newdata = test_num[,-5])
mse(pred, test_num[,5])
```

```
## [1] 85.04611
```

Elastic net regression

```r
alpha = seq(0, 1, length.out = 15)
cv_mse = c()
for(a in alpha) {
  model = cv.glmnet(x = as.matrix(train_num[,-5]), y = train_num[,5], alpha = a)
  cv_mse = c(cv_mse, min(model$cvm))
}
id = which(cv_mse == min(cv_mse))
best_model_2 = cv.glmnet(x = as.matrix(train_num[,-5]), y = train_num[,5], alpha = alpha[id])
best_lambda = best_model_2$lambda.min
best_alpha = alpha[id]
best_model_2 = glmnet(x = as.matrix(train_num[,-5]), y = train_num[,5], alpha = alpha[id], lambda = bes
pred = predict(best_model_2, newx = as.matrix(test_num[,-5]))
mse(pred, test[,15])
```

```
## [1] 84.57942
```

PCR

```r
model <- pcr(egalit_scale ~ .,
                data = select_if(train, is.numeric),
                center = TRUE,
                scale = TRUE,
                validation = "CV",
                segments = 10)

mse = as.vector(MSEP(model, estimate = "CV", intercept = FALSE)$val)
id = which(mse == min(mse))
best_model_3 = pcr(egalit_scale ~ ., data = train_num, comps = id)
pred = predict(best_model_3, newdata = test_num[,-5], comps = id)
mse(pred, test[,15])
```

```
## [1] 89.23381
```

PLS

```r
model <- plsr(egalit_scale ~ .,
                data = select_if(train, is.numeric),
                center = TRUE,
                scale = TRUE,
                validation = "CV",
                segments = 10)

mse = as.vector(MSEP(model, estimate = "CV", intercept = FALSE)$val)
id = which(mse == min(mse))
best_model_4 = plsr(egalit_scale ~ ., data = train_num, comps = id)
pred = predict(best_model_4, newdata = test_num[,-5], comps = id)
mse(pred, test[,15])
```

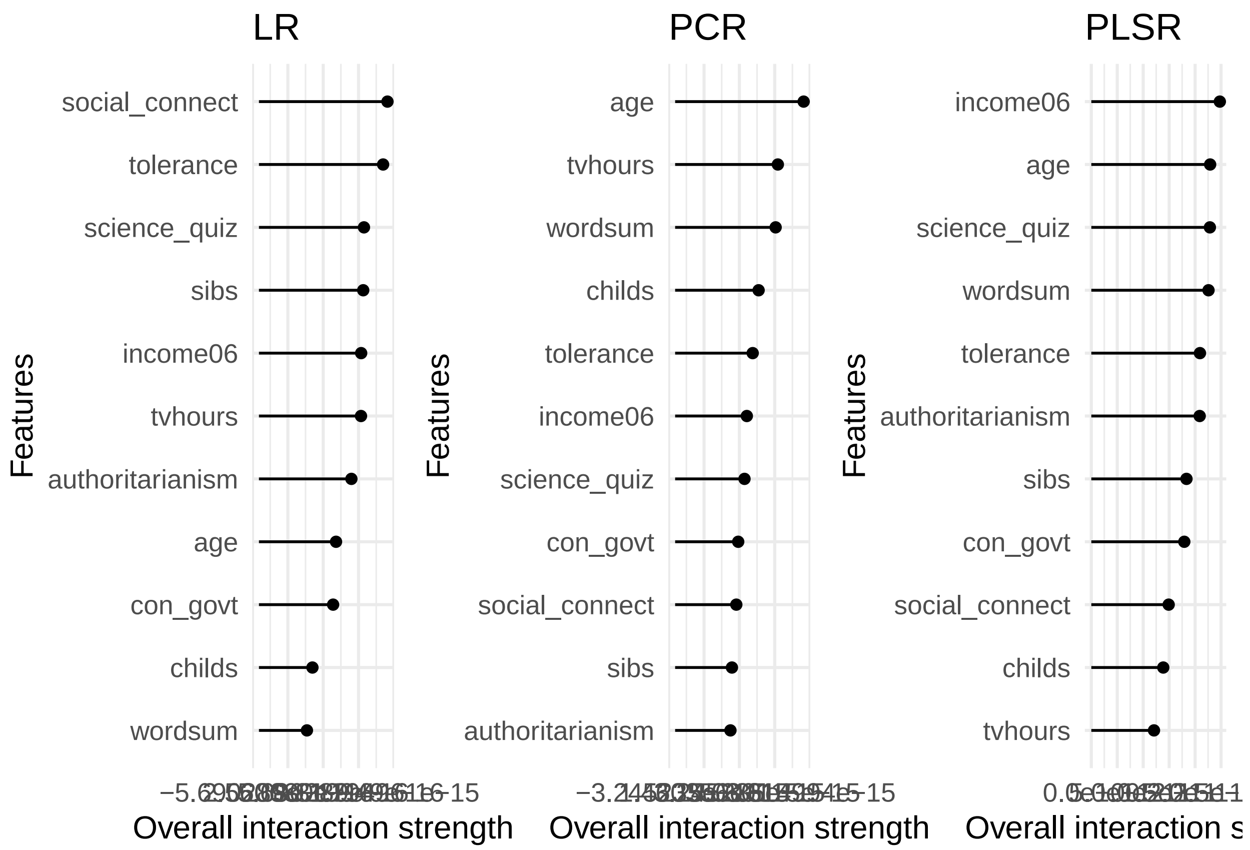
```
## [1] 90.26552
```

Question five

```r
features = train_num[,-5]
response = as.numeric(train$egalit_scale)

predictor.lr <- Predictor$new(
  model = best_model_1,
  data = features,
  y = response,
  predict.fun = function(model, newdata) {return(predict(model, newdata = newdata))}
)

predictor.ela <- Predictor$new(
  model = best_model_2,
  data = features,
  y = response,
  predict.fun = function(model, newdata) {return(predict(model, newx = as.matrix(newdata)))}
)

predictor.pcr <- Predictor$new(
  model = best_model_3,
  data = features,
  y = response,
  predict.fun = function(model, newdata) {return(predict(model, newdata = newdata, comps = 11))}
)

predictor.plsr <- Predictor$new(
  model = best_model_4,
  data = features,
  y = response,
  predict.fun = function(model, newdata) {return(predict(model, newdata = newdata, comps = 11))}
)

interact.lr <- Interaction$new(predictor.lr) %>%
  plot() +
  ggtitle("LR") +
  theme_minimal(base_size = 12)

# interact.ela <- Interaction$new(predictor.ela) %>%
#   plot() +
#   ggtitle("Elastic Net") +
#   theme_minimal(base_size = 12)

interact.pcr <- Interaction$new(predictor.pcr) %>%
  plot() +
  ggtitle("PCR") +
  theme_minimal(base_size = 12)

interact.plsr <- Interaction$new(predictor.plsr) %>%
  plot() +
  ggtitle("PLSR") +
  theme_minimal(base_size = 12)
```

I encountered some unknown problems when drawing the interaction plot of elastic net model so I will only show the other three plots.

```r
library(gridExtra)
grid.arrange(interact.lr, interact.pcr, interact.plsr, ncol = 3)
```



Feature Importance

For LR model, wordsum, authoritarianism, sibs and age are top four variables. For PCR model, age, childs, tolerance and wordsum are top four variables. For PLSR, tolerance, wordsum, con_govt and science_quiz are top four variables.

In the three models, we can find that wordsum is important in PCr, LR and PLSR, age is important in PCR and LR, tolerance is important in PCR and PLSR.

Feature Rank

LR regards authoritarianism as as important feature but PCR and PLSR list that feature at the tail of their rank, which might result from the different algorithm behind the model.

For PCR and PLSR, except childs and con_govt, we can find their rank are similar. This is because their algorithm both originate from dimension reduction. However, linear regression shows more differences since the original thought behind the model is to find the expectation of y conditioning on X.