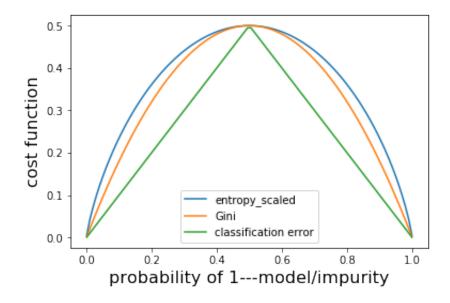
Question 1

```
p = np.linspace(0.001,0.999,100)
entropy_scaled = 0.5*(- p*np.log2(p) - (1 - p)*np.log2((1 - p)))
gini = (p)*(1 - (p)) + (1 - p)*(1 - (1-p))
classification_error = 1 - np.max([p, 1 - p], axis = 0)
plt.plot(p,entropy_scaled,label = 'entropy_scaled')
plt.plot(p,gini, label = 'Gini')
plt.plot(p,classification_error, label = 'classification error')
plt.legend()
plt.xlabel('probability of 1---model/impurity',size = 16)
plt.ylabel('cost function',size = 16)
plt.show()
```



When we are training a decision tree, we use the cost function to decide

- 1. whether to use this feature to split the node;
- 2. which value to use to split it
- 3. whether to stop splitting.

During the process of training, we need to decide whether to stop training, and we often would set a threshold of cost function for decision. I think Gini and Cross are more suitable as criterion than classification error. Gini and entropy are more sensitive to the impurity of data, so that a given threshold will naturally mean a lower impurity of data after classifying by the model which uses the Gini or entropy as cost function compared with that uses classification error.

When pruning the tree, we would cut the tree so that the new cost function(C_new) of the model will become: cost function(C_new) alpha * the complexity of model(C_new). For a given alpha, we would search around the subtree to decide an optimal tree that can minimize the C_new . Using the criterion of classification error can help us to search around the model space more efficiently as it naturally represent the impurity of data and it can adjust the impurity larger for a given decrease of cost function when the impurity is low(after the training).

- --- Have discussed it with Shengwenxin Ni whose suggestions are really helpful.
- ---- https://www.bogotobogo.com/python/scikit-learn/scikt_machine_learning_Decision_Tree_Learning_Information_Gain_IG_Impurity_Entropy_Gini_Classification_Error.php

Question 2

```
import pandas as pd
import numpy as np
import csv
from sklearn.model_selection import train_test_split
from sklearn.model selection import KFold
from sklearn.model selection import GridSearchCV
from sklearn.tree import DecisionTreeClassifier
from sklearn.linear model import ElasticNet
from sklearn.linear_model import LogisticRegression
from sklearn.naive_bayes import GaussianNB
from sklearn.ensemble import BaggingClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import AdaBoostClassifier
from sklearn.metrics import auc
from sklearn.metrics import roc curve
import copy
```

from sklearn import preprocessing

```
df_train = pd.read_csv('gss_train.csv')
df_test = pd.read_csv('gss_test.csv')
data_train = copy.copy(df_train)
data_test = copy.copy(df_test)
Y_train = df_train['colrac']
Y_test = df_test['colrac']
data_train.drop(['colrac'],axis = 1, inplace=True)
data_test.drop(['colrac'],axis = 1, inplace=True)
X_train = np.array(data_train)
X_scaled_train = preprocessing.scale(X_train)
X_test = np.array(data_test)
X_scaled_test = preprocessing.scale(X_test)
```

```
df_train.shape
```

```
(1476, 56)
```

?LogisticRegression

```
indice = np.random.permutation(np.array(list(range(len(X_train))))
# logistic regression; without hyperparameter
kf = KFold(n_splits=10)
error_rate = []
for train_indice, test_indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y_train = Y_train[train_indice]
    x_test = X_scaled_train[test_indice]
    y_test = Y_train[test_indice]
    clf = LogisticRegression(random_state=0).fit(x_train, y_train)
    error_rate.append(clf.score(x_test, y_test))
```

```
np.mean(error_rate)
```

0.79470950542379115

```
# naive bayesian model
kf = KFold(n_splits=10)
error_rate = []
for train_indice, test_indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y_train = Y_train[train_indice]
    x_test = X_scaled_train[test_indice]
    y_test = Y_train[test_indice]
    clf = GaussianNB().fit(x_train, y_train)
    error_rate.append(clf.score(x_test, y_test))
```

```
np.mean(error_rate)
```

```
#elastic net regression with grid search
elst = ElasticNet()
parameters = {'alpha':[0.1,0.5,1,10,100], 'l1_ratio':np.linspace(0.1,1,11)}
clf = GridSearchCV(elst, parameters, cv = 10)
clf.fit(X_scaled_train,Y_train)
```

```
clf.best_estimator_
```

```
elst = ElasticNet(alpha=0.1,l1_ratio=0.1)
clf.fit(X_scaled_train,Y_train)
y_predict = clf.predict(x_test)
y_predict[y_predict>0.5] = 1
y_predict[y_predict<=0.5] = 0
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)</pre>
```

```
error_rate
```

```
0.18367346938775511
```

```
# decision tree
Tree = DecisionTreeClassifier()
#n_components = list(range(1,X.shape[1]+1,1))
criterion = ['gini', 'entropy']
splitter = ["best", "random"]
max_depth = [4,6,8,12]
parameters = {'criterion':criterion, 'splitter':splitter,
    'max_depth':max_depth}
clf = GridSearchCV(Tree, parameters, cv = 10)
clf.fit(X_scaled_train,Y_train)
```

```
GridSearchCV(cv=10, error score=nan,
             estimator=DecisionTreeClassifier(ccp alpha=0.0, class weight=None,
                                              criterion='gini', max_depth=None,
                                              max_features=None,
                                              max leaf nodes=None,
                                              min impurity decrease=0.0,
                                              min_impurity_split=None,
                                              min_samples_leaf=1,
                                              min samples split=2,
                                              min_weight_fraction_leaf=0.0,
                                              presort='deprecated',
                                              random state=None,
                                               splitter='best'),
             iid='deprecated', n_jobs=None,
             param_grid={'criterion': ['gini', 'entropy'],
                         'max depth': [4, 6, 8, 12],
                         'splitter': ['best', 'random']},
             pre_dispatch='2*n_jobs', refit=True, return_train_score=False,
             scoring=None, verbose=0)
```

```
clf.best estimator
```

```
clf.best_score_
```

```
0.78656462585034015
```

```
clf.best_estimator_
```

```
clf.best_score_
```

```
0.770279463136606
```

```
GridSearchCV(cv=10, error_score=nan,
             estimator=RandomForestClassifier(bootstrap=True, ccp_alpha=0.0,
                                              class weight=None,
                                              criterion='gini', max depth=None,
                                              max features='auto',
                                              max leaf nodes=None,
                                              max samples=None,
                                              min impurity decrease=0.0,
                                              min impurity split=None,
                                              min samples leaf=1,
                                              min samples split=2,
                                              min_weight_fraction_leaf=0.0,
                                              n estimators=100, n jobs=None,
                                              oob score=False,
                                              random_state=None, verbose=0,
                                              warm start=False),
             iid='deprecated', n_jobs=None,
             param grid={'bootstrap': [True, False],
                         'criterion': ['gini', 'entropy'],
                         'max features': ['auto', 'sqrt', 'log2', 55],
                         'max samples': [4, 6, 8, 12],
                         'n_estimators': [10, 20, 50, 100, 150]},
             pre dispatch='2*n jobs', refit=True, return train score=False,
             scoring=None, verbose=0)
```

```
clf.best_estimator_
```

```
RandomForestClassifier(bootstrap=False, ccp_alpha=0.0, class_weight=None, criterion='gini', max_depth=None, max_features='auto', max_leaf_nodes=None, max_samples=6, min_impurity_decrease=0.0, min_impurity_split=None, min_samples_leaf=1, min_samples_split=2, min_weight_fraction_leaf=0.0, n_estimators=100, n_jobs=None, oob_score=False, random_state=None, verbose=0, warm_start=False)
```

```
clf.best score
```

```
# Boosting
Boosting = AdaBoostClassifier()
n_estimators = [10,20,50,70,80]
learning_rate = np.linspace(0.1,1,10)
algorithm = ['SAMME', 'SAMME.R']
parameters = {'n_estimators':n_estimators, 'learning_rate':learning_rate,
    'algorithm':algorithm}
clf = GridSearchCV(Boosting, parameters, cv = 10)
clf.fit(X_scaled_train,Y_train)
```

```
clf.best_estimator_
```

```
AdaBoostClassifier(algorithm='SAMME.R', base_estimator=None,
learning_rate=0.400000000000000, n_estimators=80,
random_state=None)
```

```
clf.best_score_
```

Question 3 & Question 4

```
import matplotlib.pyplot as plt
```

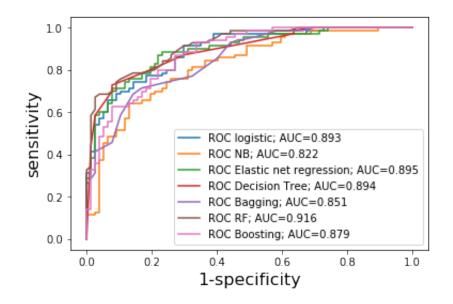
```
from sklearn.metrics import auc
from sklearn.metrics import roc_curve
from sklearn.metrics import roc_auc_score
```

```
model error rate = {}
# logistic error rate
kf = KFold(n splits=10)
error_rate = []
for train indice, test indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y train = Y train[train indice]
    x test = X scaled train[test indice]
    y_test = Y_train[test_indice]
    clf = LogisticRegression(random state=0).fit(x train, y train)
    y predict = clf.predict(x test)
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc auc score(y test, y predict prob)
plt.plot(fpr,tpr,label = 'ROC logistic; AUC={}'.format(round(auc,3)))
model_error_rate['logistic'] = np.mean(error_rate)
# Naive Bayes
kf = KFold(n splits=10)
error_rate = []
for train_indice, test_indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y_train = Y_train[train_indice]
    x test = X scaled train[test indice]
    y test = Y train[test indice]
    clf = GaussianNB().fit(x_train, y_train)
    y predict = clf.predict(x test)
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y predict prob = clf.predict proba(x test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc auc score(y test, y predict prob)
plt.plot(fpr,tpr,label = 'ROC NB; AUC={}'.format(round(auc,3)))
model_error_rate['Naive Bayes'] = np.mean(error_rate)
#Elastic net regression
kf = KFold(n splits=10)
```

```
error rate = []
for train indice, test indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y_train = Y_train[train_indice]
    x test = X scaled train[test indice]
    y_test = Y_train[test_indice]
    clf = ElasticNet(alpha=0.1, copy_X=True, fit_intercept=True,
           11_ratio=0.100000000000000001, max_iter=1000, normalize=False,
           positive=False, precompute=False, random_state=None,
           selection='cyclic', tol=0.0001, warm_start=False).fit(x_train,
y_train)
    y predict = clf.predict(x test)
    y_predict[y_predict>0.5] = 1
    y_predict[y_predict<=0.5] = 0</pre>
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y_predict_prob = clf.predict(x_test)
fpr, tpr, = roc curve(y test, y predict prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Elastic net regression; AUC=
{}'.format(round(auc,3)))
model_error_rate['Elastic net regression'] = np.mean(error_rate)
#Decision tree (CART)
kf = KFold(n_splits=10)
error_rate = []
for train_indice, test_indice in kf.split(indice):
    x train = X scaled train[train indice]
    y_train = Y_train[train_indice]
    x_test = X_scaled_train[test_indice]
    y test = Y train[test indice]
    clf = DecisionTreeClassifier(ccp_alpha=0.0, class_weight=None,
criterion='gini',
                       max_depth=4, max_features=None, max_leaf_nodes=None,
                       min impurity decrease=0.0, min impurity split=None,
                       min_samples_leaf=1, min_samples_split=2,
                       min_weight_fraction_leaf=0.0, presort='deprecated',
                       random_state=None, splitter='random').fit(x_train,
y_train)
    y_predict = clf.predict(x_test)
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y predict prob = clf.predict proba(x test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Decision Tree; AUC={}'.format(round(auc,3)))
model error rate['Decision tree'] = np.mean(error rate)
# Bagging
kf = KFold(n splits=10)
error rate = []
```

```
for train_indice, test_indice in kf.split(indice):
    x train = X scaled train[train indice]
    y_train = Y_train[train_indice]
    x_test = X_scaled_train[test_indice]
    y test = Y train[test indice]
    clf = BaggingClassifier(base estimator=None, bootstrap=True,
bootstrap_features=False,
                  max_features=1.0, max_samples=12, n_estimators=40,
                  n_jobs=None, oob_score=False, random_state=None, verbose=0,
                  warm_start=True).fit(x_train, y_train)
    y_predict = clf.predict(x_test)
    error rate.append(sum(np.ones(len(y test))[y predict!=y test])/len(y test))
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Bagging; AUC={}'.format(round(auc,3)))
model error rate['Bagging'] = np.mean(error rate)
# Random Forest
kf = KFold(n splits=10)
error_rate = []
for train indice, test indice in kf.split(indice):
    x_train = X_scaled_train[train_indice]
    y_train = Y_train[train_indice]
    x_test = X_scaled_train[test_indice]
    y_test = Y_train[test_indice]
    clf = RandomForestClassifier(bootstrap=False, ccp alpha=0.0,
class_weight=None,
                       criterion='gini', max_depth=None, max_features='auto',
                       max_leaf_nodes=None, max_samples=6,
                       min_impurity_decrease=0.0, min_impurity_split=None,
                       min_samples_leaf=1, min_samples_split=2,
                       min_weight_fraction_leaf=0.0, n_estimators=100,
                       n jobs=None, oob score=False, random state=None,
                       verbose=0, warm start=False).fit(x train, y train)
    y_predict = clf.predict(x_test)
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC RF; AUC={}'.format(round(auc,3)))
model_error_rate['RF'] = np.mean(error_rate)
#Boosting
kf = KFold(n splits=10)
error rate = []
for train_indice, test_indice in kf.split(indice):
    x train = X scaled train[train indice]
    y_train = Y_train[train_indice]
```

```
x_test = X_scaled_train[test_indice]
    y test = Y train[test indice]
    clf = AdaBoostClassifier(algorithm='SAMME.R', base_estimator=None,
                   learning_rate=0.4, n_estimators=80,
                   random state=None).fit(x train, y train)
   y_predict = clf.predict(x_test)
    error_rate.append(sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test))
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Boosting; AUC={}'.format(round(auc,3)))
model error rate['Boosting'] = np.mean(error rate)
plt.xlabel('1-specificity',size = 16)
plt.ylabel('sensitivity', size = 16)
plt.legend()
plt.show()
```



```
sorted(model_error_rate.items(), key = lambda x:x[1])
```

```
[('Boosting', 0.19446589446589446),
  ('RF', 0.19787185144328001),
  ('logistic', 0.20529049457620885),
  ('Elastic net regression', 0.20665563522706379),
  ('Decision tree', 0.22566648280933993),
  ('Bagging', 0.25138812281669426),
  ('Naive Bayes', 0.2649016363302078)]
```

Best model and defend my choice

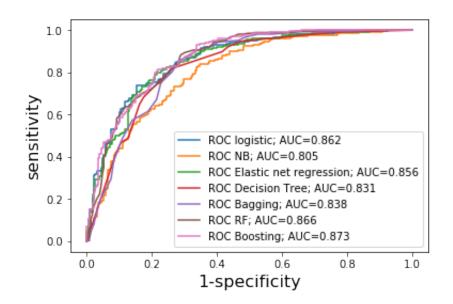
I think the Random Forest is the best model because it has the highest AUC, suggesting when the ture positive rate goes up, the false positive rate will go up relatively slower than other models, and produce a higher accuracy. Moreover, it has a low error rate compared with other models. Although the error rate is not the lowest, but the difference between boosting and random forest is not distinctive.

Question 5: model performance on test dataset

```
model error rate = {}
x_train = X_scaled_train
y train = Y train
x test = X scaled test
y_test = Y_test
# logistic error rate
clf = LogisticRegression(random_state=0).fit(x_train, y_train)
y_predict = clf.predict(x_test)
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)
y predict prob = clf.predict proba(x test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC logistic; AUC={}'.format(round(auc,3)))
model_error_rate['logistic'] = error_rate
# Naive Bayes
clf = GaussianNB().fit(x_train, y_train)
y_predict = clf.predict(x_test)
error rate = sum(np.ones(len(y test))[y predict!=y test])/len(y test)
y predict prob = clf.predict proba(x test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC NB; AUC={}'.format(round(auc,3)))
model_error_rate['Naive Bayes'] = error_rate
#Elastic net regression
clf = ElasticNet(alpha=0.1, copy_X=True, fit_intercept=True,
       l1_ratio=0.10000000000000001, max_iter=1000, normalize=False,
       positive=False, precompute=False, random state=None,
       selection='cyclic', tol=0.0001, warm_start=False).fit(x_train, y_train)
```

```
y_predict = clf.predict(x_test)
y predict[y predict>0.5] = 1
y_predict[y_predict<=0.5] = 0</pre>
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)
y predict prob = clf.predict(x test)
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Elastic net regression; AUC=
{}'.format(round(auc,3)))
model_error_rate['Elastic net regression'] = error_rate
#Decision tree (CART)
clf = DecisionTreeClassifier(ccp_alpha=0.0, class_weight=None,
criterion='gini',
                   max_depth=4, max_features=None, max_leaf_nodes=None,
                   min impurity decrease=0.0, min impurity split=None,
                   min_samples_leaf=1, min_samples_split=2,
                   min_weight_fraction_leaf=0.0, presort='deprecated',
                   random_state=None, splitter='random').fit(x_train, y_train)
y_predict = clf.predict(x_test)
error rate = sum(np.ones(len(y test))[y predict!=y test])/len(y test)
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Decision Tree; AUC={}'.format(round(auc,3)))
model error rate['Decision tree'] = error rate
# Bagging
clf = BaggingClassifier(base estimator=None, bootstrap=True,
bootstrap_features=False,
                  max_features=1.0, max_samples=12, n_estimators=40,
                  n_jobs=None, oob_score=False, random_state=None, verbose=0,
                  warm start=True).fit(x train, y train)
y_predict = clf.predict(x_test)
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC Bagging; AUC={}'.format(round(auc,3)))
model_error_rate['Bagging'] = error_rate
# Random Forest
clf = RandomForestClassifier(bootstrap=False, ccp_alpha=0.0, class_weight=None,
                       criterion='gini', max_depth=None, max_features='auto',
                       max leaf nodes=None, max samples=6,
                       min impurity decrease=0.0, min impurity split=None,
```

```
min_samples_leaf=1, min_samples_split=2,
                       min weight fraction leaf=0.0, n estimators=100,
                       n_jobs=None, oob_score=False, random_state=None,
                       verbose=0, warm_start=False).fit(x_train, y_train)
y predict = clf.predict(x test)
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)
y_predict_prob = clf.predict_proba(x_test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc_auc_score(y_test, y_predict_prob)
plt.plot(fpr,tpr,label = 'ROC RF; AUC={}'.format(round(auc,3)))
model_error_rate['RF'] = error_rate
#Boosting
clf = AdaBoostClassifier(algorithm='SAMME.R', base estimator=None,
               learning_rate=0.4, n_estimators=80,
               random state=None).fit(x train, y train)
y_predict = clf.predict(x_test)
error_rate = sum(np.ones(len(y_test))[y_predict!=y_test])/len(y_test)
y predict prob = clf.predict proba(x test)[:,1]
fpr, tpr,_ = roc_curve(y_test, y_predict_prob)
auc = roc auc score(y test, y predict prob)
plt.plot(fpr,tpr,label = 'ROC Boosting; AUC={}'.format(round(auc,3)))
model_error_rate['Boosting'] = error_rate
plt.xlabel('1-specificity', size = 16)
plt.ylabel('sensitivity', size = 16)
plt.legend()
plt.show()
```



```
[('RF', 0.19675456389452334),
  ('Elastic net regression', 0.20892494929006086),
  ('Boosting', 0.20892494929006086),
  ('Bagging', 0.21095334685598377),
  ('Decision tree', 0.21906693711967545),
  ('logistic', 0.2231237322515213),
  ('Naive Bayes', 0.26977687626774849)]
```

comments:

our best model: random forest in question 4 has a second highest AUC among the other models and the lowest error rate, suggesting the random forest can accomplish a good classification task with a good generalization ability.