Problem Set 6

By Chu Zhuang

```
# import relevant packages
import random
import math
import numpy as np
import pandas as pd
import seaborn
import sklearn
from sklearn.svm import SVC
from sklearn.model_selection import train_test_split
import matplotlib as mpl
import matplotlib.pyplot as plt
```

1. Non Linear Separation

First of all, construct a clear non-linear two-class dataset with two features of 100 obersations:

```
#construct the dataset
X=[[random.uniform(-1,1),random.uniform(-1,1)] for i in range(100)]
Y0=[x1+x1*x1+x2+x2*x2 for (x1,x2) in X]
```

```
#transform into two classes
y_prob=[math.exp(y)/(1+math.exp(y)) for y in Y0]
Y=[prob>0.6 for prob in y_prob]
```

```
#convert into features and label dataset, as np format
np_X=np.array(X)  #features
np_Y=np.array(Y)  #labels
```

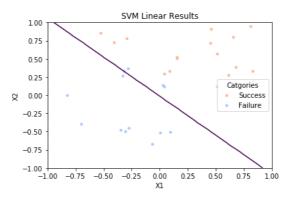
```
#split training and test dataset 70%/30%
from sklearn.model_selection import train_test_split
X_train,X_test, y_train, y_test =train_test_split(np_X,np_Y,test_size=0.3, random_state=4)
```

```
#function to plot ROC
import matplotlib.pyplot as plt
{\tt def\ plot\_ROC\_curve(model,data\_feature,true\_label,method\_des):}
   clf=model
    classes = clf.classes_
       probs = clf.predict_proba(data_feature)
    except AttributeError:
       print("The {} classifier does not apear to support prediction probabilties, so an ROC curve can't be
created. You can try adding `probability = True` to the model specification or use a different
model.".format(type(clf)))
       return
   predictions = clf.predict(data_feature)
    #setup axis for plotting
   fig, ax = plt.subplots(figsize = (5,5))
    #We can return the AUC values, in case they are useful
    aucvals = []
    for classIndex, className in enumerate(classes):
                                                            #Setup binary classes
       truths = [1 if c == className else 0 for c in true_label]
       predict = [1 if c == className else 0 for c in predictions]
        scores = probs[:, classIndex]
        #Get the ROC curve
       fpr, tpr, thresholds = sklearn.metrics.roc_curve(truths, scores)
        #fpr, tpr, thresholds = sklearn.metrics.roc_curve(truths, predictions)
       auc = sklearn.metrics.auc(fpr, tpr)
       aucVals.append(auc)
       #Plot the class's line
       ax.plot(fpr, tpr, label = "{} (AUC ${:.3f}$)".format(str(className).split(':')[0], auc))
    #Make the plot nice, then display it
    ax.set_title('Receiver Operating Characteristics')
    plt.plot([0,1], [0,1], color = 'k', linestyle='--')
    ax.set_xlabel('False Positive Rate')
    ax.set_ylabel('True Positive Rate')
    ax.legend(loc = 'lower right')
    plt.show()
    #plt.close()
```

```
#build the SVM linear model
svm_linear=SVC(kernel = 'linear', probability = True)
svm_linear.fit(X_train,y_train)
#predict Y based on SVM, training
svm pY=svm linear.predict(X train)
{\tt svm\_pY\_test=svm\_linear.predict}(X\_{test})
#calculate error rate, AUC score on Training Dataset
error_rate_linear=1 - sklearn.metrics.accuracy_score(y_train,svm_pY)
auc_score_linear=sklearn.metrics.roc_auc_score(y_train,svm_pY)
print('Results for Training:')
print('Error Rate of linear SVM model:',round(error_rate_linear,4))
print('AUC score of best linear SVM model:',round(auc_score_linear,4))
Results for Training:
Error Rate of linear SVM model: 0.1
AUC score of best linear SVM model: 0.9
#calculate error rate, AUC score on Test Dataset
\verb|error_rate_linear_test=| 1 - sklearn.metrics.accuracy_score(y_test,svm_py_test)| \\
auc\_score\_linear\_test=sklearn.metrics.roc\_auc\_score(y\_test,svm\_pY\_test)
print('Results for Test:')
print('Error Rate of linear SVM model:',round(error_rate_linear_test,4))
print('AUC score of best linear SVM model:',round(auc_score_linear_test,4))
Results for Test:
Error Rate of linear SVM model: 0.1
AUC score of best linear SVM model: 0.8846
#save the training and test results separately
method_label=[]
method_label.append('Linear SVM')
train_error_rate=[error_rate_linear]
train_auc=[auc_score_linear]
test_error_rate=[error_rate_linear_test]
test_auc=[auc_score_linear_test]
#plot prediction results based on Test dataset
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(np_Y)))
#plot points of success first
index_true=np.where(y_test==True)[0]
\verb"index_true="index_true.astype" (np.int16)"
x1=X_test[index_true,0]
x2=X test[index true.1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index_false=np.where(y_test==False)[0]
\verb|index_false=index_false.astype(np.int16)|\\
x1=X_test[index_false,0]
x2=X_test[index_false,1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
#predict the results of X1,X2, based on logistic regression bayes
Z = svm_linear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('SVM Linear Results')
```

plt.show()

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.



As we could see above clearly, there is a linear decision boundary in test data set, while some mis-classified datapoints (blue one above the decision boundary) since the datasets is non-linear separated.

Over all, for SVM linear model, error rate is around 0.1 for both Training and Test dataset. AUC score is little bit higher for training 0.9 than Testing 0.8846 for linear SVM model.

1.2 Build Non-linear SVM model (Radial Kernel): calculate the error rate and plot the graph

```
#build the SVM linear model
from sklearn.svm import SVC
svm_nlinear=SVC(kernel = 'rbf', gamma='auto',probability = True)
svm_nlinear.fit(X_train,y_train)

#predict Y based on SVM, training
svm_pY=svm_nlinear.predict(X_train)
svm_pY_test=svm_nlinear.predict(X_test)
```

```
#calculate error rate, AUC score on Training Dataset
error_rate_nlinear=1 - sklearn.metrics.accuracy_score(y_train,svm_pY)
auc_score_nlinear=sklearn.metrics.roc_auc_score(y_train,svm_pY)
print('Results for Training:')
print('Error Rate of linear SVM model:',round(error_rate_nlinear,4))
print('AUC score of best linear SVM model:',round(auc_score_nlinear,4))
```

```
Results for Training:
Error Rate of linear SVM model: 0.0429
AUC score of best linear SVM model: 0.9542
```

```
#calculate error rate, AUC score on Test Dataset
error_rate_nlinear_test=1 - sklearn.metrics.accuracy_score(y_test,svm_pY_test)
auc_score_nlinear_test=sklearn.metrics.roc_auc_score(y_test,svm_pY_test)
print('Results for Test:')
print('Error Rate of linear SVM model:',round(error_rate_nlinear_test,4))
print('AUC score of best linear SVM model:',round(auc_score_nlinear_test,4))
```

```
Results for Test:
Error Rate of linear SVM model: 0.0
AUC score of best linear SVM model: 1.0
```

```
#save the training and test results separately
method_label=['Linear SVM', 'Non-linear SVM']
train_error_rate=[error_rate_linear,error_rate_nlinear]
train_auc=[auc_score_linear,auc_score_nlinear]
test_error_rate=[error_rate_linear_test,error_rate_nlinear_test]
test_auc=[auc_score_linear_test,auc_score_nlinear_test]

df_svm=pd.DataFrame({'Train_Error':train_error_rate,'Train_AUC':train_auc,'Test_Error':test_error_rate,'Test_AUC':test_auc},index=method_label)
```

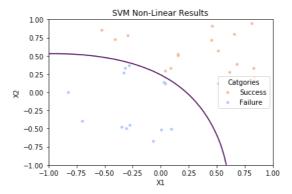
#plot prediction results based on Test dataset

```
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(np_Y)))
#plot points of success first
index\_true = np.where(y\_test == True)[0]
index\_true=index\_true.astype(np.int16)
x1=X_test[index_true,0]
x2=X_test[index_true,1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index_false=np.where(y_test==False)[0]
index false=index false.astvpe(np.int16)
x1=X test[index false.0]
x2=X_test[index_false,1]
\verb|plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])|\\
#np.mesharid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
#predict the results of X1,X2, based on logistic regression bayes
Z = svm_nlinear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:.1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('SVM Non-Linear Results')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

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For the non-linear SVM model, on the same test dataset, we could see that it better captures the non-linear separation in the dataset; with only few data point close to the boundary and non mis-classification.

Both the Training error and test error of non-linear SVM outperforms the linear SVM model (as shown in the table below). Also for AUC score, the non-linear SVM model is higher than linear one, and even equals to 1 for non-linear SVM model on this non-linear dataset.

Overal, non-linear SVM performs better than linear SVM on the non-linear dataset.

```
#show training results for both SVM model df_svm
```

```
.dataframe tbody tr th {
    vertical-align: top;
}
.dataframe thead th {
    text-align: right;
}
```

	Train_Error	Train_AUC	Test_Error	Test_AUC
Linear SVM	0.100000	0.900000	0.1	0.884615
Non-linear SVM	0.042857	0.954167	0.0	1.000000

SVM vs. Logistic Regression

1. Construct the dataset-overlapping and nolinear with observations:n=500, and features: p=2.

```
#construct the dataset
import math
X=[[random.uniform(-1,1),random.uniform(-1,1)] for i in range(500)]
Y0=[x1+x1*x1+x2+x2*x2+random.normalvariate(0,0.5) for (x1,x2) in X]
```

```
#transform into two classes
y_prob=[math.exp(y)/(1+math.exp(y)) for y in Y0]
Y=[prob>0.5 for prob in y_prob]
```

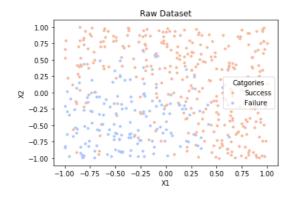
```
#convert into features and label dataset, as np format
np_X=np.array(X)  #features
np_Y=np.array(Y)  #labels
```

2. Plot the observations (Raw Dataset)

```
#plot the raw data
labels=['Success'.'Failure']
pallet = seaborn.color\_palette(palette='coolwarm', n\_colors = len(set(np\_Y)))
#plot points of success first
index_true=np.where(np_Y==True)[0]
index\_true=index\_true.astype(np.int16)
x1=np_X[index_true,0]
x2=np_X[index_true,1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index\_false=np.where(np\_Y==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np X[index false.1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('x2')
plt.title('Raw Dataset')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.



4. Fit a Logistic Regression

```
#build linear logistic regression model
from sklearn.linear_model import LogisticRegression
lg_linear=LogisticRegression(solver='liblinear')
lg_linear.fit(np_X,np_Y)
```

5. Plot prediction for linear Logistic Regression

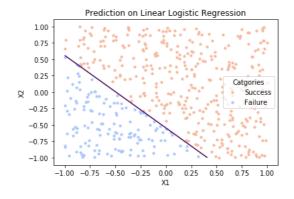
```
#predict Y based on train feature
lg_pY_linear=lg_linear.predict(np_X)
```

```
#plot the predicted results of linear Logistic Regression
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(lg_pY_linear)))
#plot points of success first
index_true=np.where(lg_pY_linear==True)[0]
index_true=index_true.astype(np.int16)
x1=np_X[index_true,0]
x2=np X[index true.1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index_false=np.where(lg_pY_linear==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np_X[index_false,1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
#predict the results of X1,X2, based on logistic regression bayes
Z = lg_linear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Prediction on Linear Logistic Regression')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

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6. Non linear Logistic Regression

First of all, transform the \boldsymbol{X} to derive non-linear predictors

```
#transform to non-linear features

X1=[(x1,x1*x1,x2,x2*x2,x1*x2) for (x1,x2) in X]

np_X1=np.array(X1)
```

```
#build non linear logistic regression model
lg_nlinear=LogisticRegression(solver='liblinear')
lg_nlinear.fit(np_X1,np_Y)
```

7. Plot prediction for non-Linear Logistic Regression

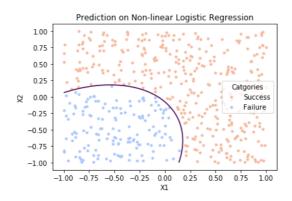
```
#predict Y based on train feature
lg_pY_nlinear=lg_nlinear.predict(np_X1)
```

```
#plot the predicted results, colored according to the new labeled class
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(lg_pY_linear)))
#plot points of success first
index_true=np.where(lg_pY_nlinear==True)[0]
index_true=index_true.astype(np.int16)
x1=np X[index true.0]
x2=np X[index true.1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index_false=np.where(lg_pY_nlinear==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np_X[index_false,1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
xy=zip(xx.ravel(),yy.ravel())
xy1=[(xx,xx*xx,yy,yy*yy,xx*yy) for (xx,yy) in xy]
np_xy1=np.array(xy1)
\#predict the results of X1,X2, based on logistic regression bayes
#Z = lg_nlinear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = lg_nlinear.predict_proba(np_xy1)
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('X2')
plt.title('Prediction on Non-linear Logistic Regression')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

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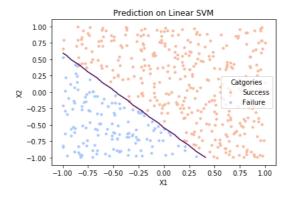
```
#build the SVM linear model
svm_linear=SVC(kernel = 'linear', probability = True)
svm_linear.fit(np_X,np_Y)

#predict Y based on SVM, training
svm_pY_linear=svm_linear.predict(np_X)
```

```
#plot the predicted results
labels=['Success','Failure']
pallet = seaborn.color\_palette(palette='coolwarm', n\_colors = len(set(svm\_py\_linear)))
#plot points of success first
index_true=np.where(svm_pY_linear==True)[0]
index_true=index_true.astype(np.int16)
x1=np_X[index_true,0]
x2=np_X[index_true,1]
\verb|plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])|\\
#plot points of failure
index_false=np.where(svm_pY_linear==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np_X[index_false,1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
\#predict the results of X1,X2, based on logistic regression bayes
Z = svm_linear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('x2')
plt.title('Prediction on Linear SVM')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

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9. Non-Linear SVM model

```
#build the SVM linear model
svm_nlinear=SVC(kernel = 'rbf', gamma='auto',probability = True)
svm_nlinear.fit(np_X,np_Y)

#predict Y based on SVM, training
svm_pY_nlinear=svm_nlinear.predict(np_X)
```

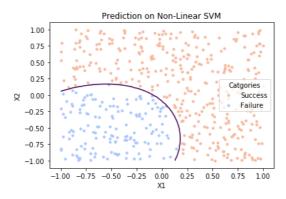
```
#plot the predicted results
labels=['success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(svm_pY_nlinear)))

#plot points of success first
index_true=np.where(svm_pY_nlinear==True)[0]
index_true=index_true.astype(np.int16)
x1=np_X[index_true,0]
```

```
x2=np_X[index_true,1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index\_false=np.where(svm\_pY\_nlinear==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np X[index false.1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
#predict the results of X1,X2, based on logistic regression bayes
Z = svm_nlinear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5], color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.vlabel('X2')
plt.title('Prediction on Non-Linear SVM')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

C:\Users\zhuangchu\anaconda3\lib\site-packages\ipykernel_launcher.py:27: UserWarning: The following kwargs were not used by contour: 'color'

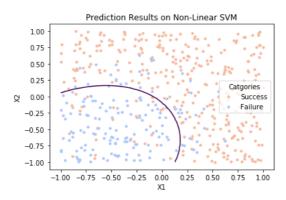


Also, for better visualization, I further plot the prediction results based on true label:

```
#plot the predicted results (Nonlinear SVM)
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(np_Y)))
#plot points of success first
index\_true=np.where(np\_Y==True)\,[0]
index_true=index_true.astype(np.int16)
x1=np_X[index_true,0]
x2=np X[index true.1]
\verb|plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])|\\
#plot points of failure
index_false=np.where(np_Y==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np_X[index_false,1]
\verb|plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])|\\
#np.mesharid. set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
#predict the results of X1,X2, based on logistic regression bayes
Z = svm_nlinear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = Z[:,1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('X1')
plt.ylabel('x2')
plt.title('Prediction Results on Non-Linear SVM')
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

C:\Users\zhuangchu\Anaconda3\lib\site-packages\ipykernel_launcher.py:27: UserWarning: The following kwargs were not used by contour: 'color'

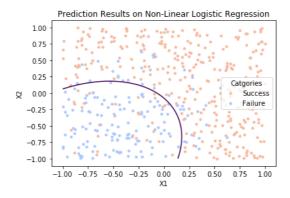


```
#plot the predicted results (Nonlinear Logistic Regression)
labels=['Success','Failure']
pallet = seaborn.color_palette(palette='coolwarm', n_colors = len(set(np_Y)))
#plot points of success first
index_true=np.where(np_Y==True)[0]
index_true=index_true.astype(np.int16)
x1=np_X[index_true,0]
x2=np_X[index_true,1]
plt.scatter(x1,x2,s=10,c=pallet[1],label=labels[0])
#plot points of failure
index_false=np.where(np_Y==False)[0]
index_false=index_false.astype(np.int16)
x1=np_X[index_false,0]
x2=np_X[index_false,1]
plt.scatter(x1,x2,s=10,c=pallet[0],label=labels[1])
#np.meshgrid, set regions
xx, yy = np.meshgrid(np.linspace(-1,1,100), np.linspace(-1,1,100))
xy=zip(xx.ravel(),yy.ravel())
xy1=[(xx,xx*xx,yy,yy*yy,xx*yy) for (xx,yy) in xy]
np xv1=np.arrav(xv1)
\#predict the results of X1,X2, based on logistic regression bayes
#Z = lg_nlinear.predict_proba(np.c_[xx.ravel(), yy.ravel()])
Z = lg_nlinear.predict_proba(np_xy1)
Z = Z[:.1].reshape(xx.shape)
#plot the contour and regions of success (in blue) or failure(in red)
plt.contour(xx, yy, Z, [0.5],color='r')
plt.legend(loc = 'center right', title = 'Catgories')
plt.xlabel('x1')
plt.ylabel('X2')
plt.title('Prediction Results on Non-Linear Logistic Regression')
plt.show()
```

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

'c' argument looks like a single numeric RGB or RGBA sequence, which should be avoided as value-mapping will have precedence in case its length matches with 'x' & 'y'. Please use a 2-D array with a single row if you really want to specify the same RGB or RGBA value for all points.

 $\verb|C:\Users\zhuangchu\Anaconda3\lib\site-packages\ipykernel_launcher.py: 31: UserWarning: The following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contour: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by contours: 'color' and the following kwargs were not used by color and the following kwargs were not used by contours: 'color and the following kwargs were not used by contours: 'color and the following kwargs were not used by contours: 'color and the following kwargs were not used by color and the following kwargs were not used by contours: 'color and the following kwargs were not used by contours: 'color and the following kwargs were not used by contours: 'color and the following kwargs were not used by contours: 'color and the following kwargs were not used by color and the following kwargs were not used by color and the following kwargs were not used by the f$



10. As we could see from the visualizations above, for this dataset, both non-linear logistic regression and non-linear SVM performs close to each other and very well (capture the 2-degree polynomial relationship between Y and X).

However it really depends on the dataset constructed. Since in this dataset, the relationship is simple (2 degree polynomial) and the non-linear logistic regression also built upon this function, so the non-linear logistic regression model yields good performance. However, if the non-linear relationship is complex or unknown, the non-linear logistic regression model which needs to be built on pre-defined fixed models might fail. In this case, non-linear SVM is more flexible and could perform better, while it might not strictly find and follow the polynomial relationship in the dataset.

The performance of Non-linear logistic regression based on the right guess of the relationshop in the dataset; while SVM is more flexible and welcomes more complex non-linear relationship. However, if there is not a clear hyperplane to separate the predicted classes (no matter linear or non-linear), the SVM could fail as well.

Tuning Cost

11. Generate Data which are barely linearly separable:

```
#construct the dataset
x=[[random.uniform(-1,1),random.uniform(-1,1)] for i in range(500)]
y0=[x1+x2+random.normalvariate(0,0.5) for (x1,x2) in X]

#transform into two classes
y_prob=[math.exp(y)/(1+math.exp(y)) for y in Y0]
Y=[prob>0.5 for prob in y_prob]

#convert into features and label dataset, as np format
np_X=np.array(X)  #features
np_Y=np.array(Y)  #labels

#split training and test dataset 70%/30%
X_train,X_test, y_train, y_test =train_test_split(np_X,np_Y,test_size=0.3, random_state=4)
```

12/13. Cross validation of Cost in linear-SVM:

```
#10-fold cross validation
from sklearn.model_selection import StratifiedKFold
skf = StratifiedKFold(n_splits=10,random_state=4)
#kf = StratifiedKFold(n_splits=10, random_state=4)
#define variable to save error rates
train_error_allc=[]
cv_error_allc=[]
train_error_allc2=[]
test_error_allc=[]
#tune cost parameters
#np.linspace(0.1,1,10)
for i in range(len(cost)):
    svm_linear=SVC(C=cost[i], kernel = 'linear', probability = True) #build the model for each cost
    train error=[]
    cv_error=[]
    for train_index, test_index in skf.split(X_train,y_train):
       X_train10, X_test10 = X_train[train_index], X_train[test_index]
       y_train10, y_test10 = y_train[train_index], y_train[test_index]
       svm_linear.fit(X_train10,y_train10)
        #calculate accuracy based on CV training samples
        \verb|er_score| = 1-sklearn.metrics.accuracy_score(y\_train10, svm_linear.predict(X\_train10))|
        \verb|train_error.append(er_score)|
        #calculate accuracy based on test(validation) samples
        \verb|er_score| = 1-sklearn.metrics.accuracy_score(y_test10, svm_linear.predict(X_test10))|
        cv_error.append(er_score)
    train_error_allc.append(np.mean(train_error))
    cv_error_allc.append(np.mean(cv_error))
    #calculate test error
    svm_linear.fit(X_train,y_train)
```

```
er_score=1-sklearn.metrics.accuracy_score(y_test, svm_linear.predict(X_test))
test_error_allc.append(er_score)
er_score=1-sklearn.metrics.accuracy_score(y_train, svm_linear.predict(X_train))
train_error_allc2.append(er_score)
```

```
df_cost=pd.DataFrame({'CV_Train_Error':train_error_allc,'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Train_Error_nocV':train_error_allc,'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Train_Error_nocV':train_error_allc,'Index=cost)
df_cost
```

```
.dataframe tbody tr th {
    vertical-align: top;
}
.dataframe thead th {
    text-align: right;
}
```

	CV_Train_Error	CV_Error	Test_Error	Train_Error_noCV
0.001	0.488571	0.488571	0.513333	0.488571
0.010	0.164122	0.170971	0.160000	0.162857
0.050	0.160945	0.162400	0.153333	0.160000
0.100	0.159358	0.170971	0.153333	0.162857
0.500	0.158093	0.165257	0.173333	0.162857
1.000	0.156821	0.165257	0.173333	0.162857
10.000	0.156822	0.165257	0.166667	0.157143
100.000	0.156504	0.165257	0.166667	0.154286
1000.000	0.156504	0.165257	0.166667	0.154286
10000.000	0.156504	0.165257	0.166667	0.157143
100000.000	0.158409	0.165257	0.173333	0.160000

- 12. As we could see above, with the increasing of cost, the training errors decreases and at the same time CV error decreases as well (while a slightly larger than training error), which indicates that the bias decreases along with the increase of cost. However, the decrease of CV error is not inline with the decrease of cost; after c=0.5, the cv error stablizes at 0.165257, while the error rate of training still goes down, indicating the potential of 'over-fitting' with larger cost value.
 - If considering both CV training error and CV error, the smallest best cost value is 100 (while 0.5 and 1 also perform quite close)
- 13. When expanding to test dataset, we could see that however, when c=0.05/0.1, the model has the fewest test error; while the CV error and training error is not optimized then (slightly larger then the fewest errors for training and cv, c=0.5/1). This discrepency between results of SVM classification on test dataset and training dataset further validates the advantage of training but poorer at generalization of high cost value. When training SVM model, it is important to keep in mind to loose the cost value a little bit which might permit better results for testing and further prediction.

Application: Predicting attitudes towards racist college professors

```
#load the data
df_gss_train=pd.read_csv('data/gss_train.csv')
df_gss_test=pd.read_csv('data/gss_test.csv')
df_gss_train.head()
```

```
.dataframe tbody tr th {
   vertical-align: top;
}
.dataframe thead th {
   text-align: right;
}
```

	age	attend	authoritarianism	black	born	childs	colath	colrac	colcom	colmil	 partyid_3_Ind	partyid_3_Rep	ı
0	21	0	4	0	0	0	1	1	0	1	 1	0	1
1	42	0	4	0	0	2	0	1	1	0	 1	0	C
2	70	1	1	1	0	3	0	1	1	0	 0	0	C
3	35	3	2	0	0	2	0	1	0	1	 1	0	(
4	24	3	6	0	1	3	1	1	0	0	 1	0	1

Organize the data:

```
#organize the data to fit in model, for feature and predict value
#drop the predict value from the feature set
df_gss_trainO=df_gss_train.drop('colrac',axis=1)
df_gss_testO=df_gss_test.drop('colrac',axis=1)

np_gss_train_feature=df_gss_trainO.values
np_gss_train_y=df_gss_train['colrac'].values

np_gss_test_feature=df_gss_testO.values
np_gss_test_y=df_gss_test['colrac'].values
```

15. Fit in SVM model and tune Cost by 10-fold cross validation

```
#10-fold cross validation
skf = StratifiedKFold(n_splits=10,random_state=4)
#define variable to save error rates
#train_error_allc=[]
cv error allc=[]
train error allc2=[]
test_error_allc=[]
auc_allc=[]
#tune cost parameters
cost=[0.001,0.01,0.05,0.1,0.5,1,10]
for i in range(len(cost)):
   svm_linear=SVC(C=cost[i], kernel = 'linear', probability = True) #build the model for each cost
    test_error=[]
    cv_error=[]
    for train_index, test_index in skf.split(np_gss_train_feature,np_gss_train_y):
       X_train10, X_test10 = np_gss_train_feature[train_index], np_gss_train_feature[test_index]
       y_train10, y_test10 = np_gss_train_y[train_index], np_gss_train_y[test_index]
        {\tt svm\_linear.fit}(X\_{\tt train10}, y\_{\tt train10})
        #calculate accuracy based on test(validation) samples
        er_score=1-sklearn.metrics.accuracy_score(y_test10, svm_linear.predict(X_test10))
        cv_error.append(er_score)
    #train_error_allc.append(np.mean(train_error))
    cv_error_allc.append(np.mean(cv_error))
    #calculate test error
    svm_linear.fit(np_gss_train_feature,np_gss_train_y)
    er\_score = 1-sklearn.metrics.accuracy\_score(np\_gss\_test\_y, svm\_linear.predict(np\_gss\_test\_feature))
    test_error_allc.append(er_score)
    #calculate auc for test dataset
    auc_score=sklearn.metrics.roc_auc_score(np_gss_test_y, svm_linear.predict(np_gss_test_feature))
    auc_allc.append(auc_score)
    #er_score=1-sklearn.metrics.accuracy_score(np_gss_train_y, svm_linear.predict(np_gss_train_feature))
    #train_error_allc2.append(er_score)
```

```
df_cost=pd.DataFrame({'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Test_AUC':auc_allc},index=cost)
df_cost
```

```
.dataframe tbody tr th {
    vertical-align: top;
}
.dataframe thead th {
    text-align: right;
}
```

	CV_Error	Test_Error	Test_AUC
0.001	0.239674	0.219067	0.779386
0.010	0.205949	0.196755	0.800141
0.050	0.201224	0.206897	0.789176
0.100	0.203945	0.219067	0.776018
0.500	0.203251	0.215010	0.779791
1.000	0.205958	0.217039	0.777905
10.000	0.207314	0.219067	0.776018

From the table above we could see that when cost value equals to 0.05, the cv error is smallest (0.201224), while generally the averaged cv error is very close around 0.20 when the cost value changes from 0.01,0.05,0.5,1,10. However, when the cost value equals to 0.01, it yields the smallest **test error** which is even less than 0.20 (AUC score around 0.80),and for cost=0.05, its test error is the second smallest. Generally, the linear SVM performs well on this dataset with the **best tuned cost value** as 0.01/0.05.

16-1. Radial SVM

Fit in Radial SVM and Tune Cost and Gamma parameters by 10-fold cross validation.

```
#10-fold cross validation
skf = StratifiedKFold(n_splits=10,random_state=4)
#define variable to save error rates
#train_error_allc=[]
cv_error_allc=[]
train_error_allc2=[]
test_error_allc=[]
auc allc=[]
#tune cost parameters
cost=[0.1,1,10,100,1000,100000]
gamma = [0.00001, 0.0001, 0.001, 0.01, 0.1, 1]
cg=[(c,g) for c in cost for g in gamma]
for i in range(len(cg)):
    svm_rbf=SVC(C=cg[i][0], kernel = 'rbf', gamma=cg[i][1], probability = True) #build the model for each cost
    test error=[]
    cv_error=[]
    for train_index, test_index in skf.split(np\_gss\_train\_feature,np\_gss\_train\_y):
        X_train10, X_test10 = np_gss_train_feature[train_index], np_gss_train_feature[test_index]
        y_train10, y_test10 = np_gss_train_y[train_index], np_gss_train_y[test_index]
        {\tt svm\_rbf.fit}(X\_{\tt train10}, y\_{\tt train10})
        #calculate accuracy based on CV training samples
        #er_score=1-sklearn.metrics.accuracy_score(y_train10, svm_linear.predict(X_train10))
        #train_error.append(er_score)
        #calculate accuracy based on test(validation) samples
        er\_score=1-sklearn.metrics.accuracy\_score(y\_test10, \ svm\_rbf.predict(X\_test10))
        {\sf cv\_error.append(er\_score)}
    #train_error_allc.append(np.mean(train_error))
    cv_error_allc.append(np.mean(cv_error))
    #calculate test error
    svm\_rbf.fit(np\_gss\_train\_feature, np\_gss\_train\_y)
    \verb|er_score| = 1-sklearn.metrics.accuracy_score(np_gss_test\_y, svm_rbf.predict(np_gss_test\_feature))|
    test error allc.append(er score)
    #calculate auc for test dataset
    auc\_score = sklearn.metrics.roc\_auc\_score (np\_gss\_test\_y, svm\_linear.predict (np\_gss\_test\_feature))
    auc\_allc.append(auc\_score)
    #er_score=1-sklearn.metrics.accuracy_score(np_gss_train_y, svm_rbf.predict(np_gss_train_feature))
    #train_error_allc2.append(er_score)
```

```
#aggregate the training and testing results
c=[c for (c,g) in cg]
g=[g for (c,g) in cg]
df_cost=pd.DataFrame({'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Test_AUC':auc_allc,'Cost':c,'Gamma':g},index=cg)
df_cost
```

```
.dataframe tbody tr th {
    vertical-align: top;
}
.dataframe thead th {
    text-align: right;
}
```

	CV_Error	Test_Error	Test_AUC	Cost	Gamma
(0.1, 1e-05)	0.474678	0.462475	0.776018	0.1	0.00001
(0.1, 0.0001)	0.474678	0.462475	0.776018	0.1	0.00010
(0.1, 0.001)	0.293730	0.286004	0.776018	0.1	0.00100
(0.1, 0.01)	0.345790	0.310345	0.776018	0.1	0.01000
(0.1, 0.1)	0.474678	0.462475	0.776018	0.1	0.10000
(0.1, 1)	0.474678	0.462475	0.776018	0.1	1.00000
(1, 1e-05)	0.474678	0.462475	0.776018	1.0	0.00001
(1, 0.0001)	0.281549	0.255578	0.776018	1.0	0.00010
(1, 0.001)	0.235707	0.215010	0.776018	1.0	0.00100
(1, 0.01)	0.246490	0.261663	0.776018	1.0	0.01000
(1, 0.1)	0.469943	0.458418	0.776018	1.0	0.10000
(1, 1)	0.474678	0.462475	0.776018	1.0	1.00000
(10, 1e-05)	0.285553	0.247465	0.776018	10.0	0.00001
(10, 0.0001)	0.228270	0.208925	0.776018	10.0	0.00010
(10, 0.001)	0.214089	0.200811	0.776018	10.0	0.00100
(10, 0.01)	0.259356	0.271805	0.776018	10.0	0.01000
(10, 0.1)	0.461871	0.448276	0.776018	10.0	0.10000
(10, 1)	0.474678	0.462475	0.776018	10.0	1.00000
(100, 1e-05)	0.222832	0.204868	0.776018	100.0	0.00001
(100, 0.0001)	0.208643	0.198783	0.776018	100.0	0.00010
(100, 0.001)	0.235698	0.221095	0.776018	100.0	0.00100
(100, 0.01)	0.259356	0.271805	0.776018	100.0	0.01000
(100, 0.1)	0.461871	0.448276	0.776018	100.0	0.10000
(100, 1)	0.474678	0.462475	0.776018	100.0	1.00000
(1000, 1e-05)	0.205260	0.200811	0.776018	1000.0	0.00001
(1000, 0.0001)	0.205967	0.192698	0.776018	1000.0	0.00010
(1000, 0.001)	0.272189	0.249493	0.776018	1000.0	0.00100
(1000, 0.01)	0.259356	0.271805	0.776018	1000.0	0.01000
(1000, 0.1)	0.461871	0.448276	0.776018	1000.0	0.10000
(1000, 1)	0.474678	0.462475	0.776018	1000.0	1.00000
(100000, 1e-05)	0.212720	0.198783	0.776018	100000.0	0.00001
(100000, 0.0001)	0.254662	0.237323	0.776018	100000.0	0.00010
(100000, 0.001)	0.274212	0.239351	0.776018	100000.0	0.00100
(100000, 0.01)	0.259356	0.271805	0.776018	100000.0	0.01000
(100000, 0.1)	0.461871	0.448276	0.776018	100000.0	0.10000
(100000, 1)	0.474678	0.462475	0.776018	100000.0	1.00000

```
#plot cross validation error for cost
import matplotlib.pyplot as plt
import math
cost0=[math.log(g,10) for g in cost]
y=df_cost.groupby('Cost').mean()['CV_Error']
y=y.values

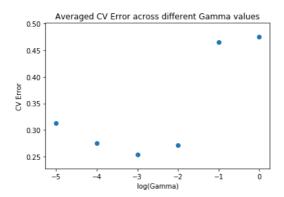
plt.scatter(cost0,y);
plt.xlabel('log(cost)');
plt.ylabel('CV_Error');
plt.title('Averaged CV_Error across different Cost values');
```

```
Averaged CV Error across different Cost values

0.44 - 0.42 - 0.40 - 0.38 - 0.36 - 0.34 - 0.32 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.30 - 0.3
```

```
#plot cross validation error for Gamma
gamma0=[math.log(c,10) for c in gamma]
y=df_cost.groupby('Gamma').mean()['CV_Error']
y=y.values

plt.scatter(gamma0,y);
plt.xlabel('log(Gamma)');
plt.ylabel('CV_Error');
plt.title('Averaged CV_Error across different Gamma values');
```



```
#sor the model by smallest CV_Error
df_cost1=df_cost
df_cost1.sort_values(by = "CV_Error")[0:5]
```

```
.dataframe tbody tr th {
   vertical-align: top;
}
.dataframe thead th {
   text-align: right;
}
```

	CV_Error	Test_Error	Test_AUC	Cost	Gamma
(1000, 1e-05)	0.205260	0.200811	0.776018	1000.0	0.00001
(1000, 0.0001)	0.205967	0.192698	0.776018	1000.0	0.00010
(100, 0.0001)	0.208643	0.198783	0.776018	100.0	0.00010
(100000, 1e-05)	0.212720	0.198783	0.776018	100000.0	0.00001
(10, 0.001)	0.214089	0.200811	0.776018	10.0	0.00100

From the table and figure above, we could see that for the model has smallest cv error-0.20526 and test error-0.2008, the corresponding gamma value is 0.00001 and cost value is 1000; while then gamma=0.0001 and cost value=1000, the test error is smallest,less than 0.2, 0.192698. Therefore **the best parameters for radial kernel**SVM is gamma 0.0001/0.00001 and cost value=1000. and the best performance of the radial model is very close to the linear SVM, not outperforms, which might indicate the 'linear' relationship of the high dimension features between classes.

16-2. Polynomial SVM

Tuning Incomplete due to very slow tuning process in my laptop

 $\label{thm:polynomial} \mbox{Fit in Polynomial SVM and Tune Cost, Degree and Gamma parameters by 10-fold cross validation. }$

```
#10-fold cross validation
from sklearn.model_selection import StratifiedKFold
skf = StratifiedKFold(n_splits=10, random_state=4)
#define variable to save error rates
#train_error_allc=[]
cv_error_allc=[]
train error allc2=[]
test error allc=[]
auc_allc=[]
#tune cost parameters
cost=[0.01.0.1.1.10.100]
\#gamma = [1, 10, 100, 1000]
degree=[1,3,5,7,9] #np.linspace(1,10,11)
\#cgd=[(c,d,g) \text{ for c in cost for d in degree for g in gamma }]
cd=[(c,d) for c in cost for d in degree]
for i in range(len(cd)):
    svm_poly=SVC(C=cd[i][0], kernel = 'poly', degree=cd[i][1], gamma='auto',probability = True) #build the model for each cost
    cv_error=[]
    for train_index, test_index in skf.split(np_gss_train_feature,np_gss_train_y):
        X_train10, X_test10 = np_gss_train_feature[train_index], np_gss_train_feature[test_index]
        y_train10, y_test10 = np_gss_train_y[train_index], np_gss_train_y[test_index]
        {\tt svm\_poly.fit}(X\_{\tt train10,y\_train10})
        #calculate accuracy based on CV training samples
        \label{lem:score} \texttt{\#er\_score=1-sklearn.metrics.accuracy\_score(y\_train10, svm\_linear.predict(X\_train10))}
        #train_error.append(er_score)
        #calculate accuracy based on test(validation) samples
        er_score=1-sklearn.metrics.accuracy_score(y_test10, svm_poly.predict(X_test10))
        cv_error.append(er_score)
    #train_error_allc.append(np.mean(train_error))
    cv_error_allc.append(np.mean(cv_error))
    #calculate test error
    \verb| #svm_poly.fit(np_gss_train_feature,np_gss_train_y)|\\
    \verb|#er_score=1-sklearn.metrics.accuracy_score(np\_gss\_test\_y, svm\_poly.predict(np\_gss\_test\_feature))|
    #test_error_allc.append(er_score)
    #calculate auc for test dataset
    #auc_score=sklearn.metrics.roc_auc_score(np_qss_test_y, svm_poly.predict(np_qss_test_feature))
    #auc_allc.append(auc_score)
    \verb|#er_score=1-sklearn.metrics.accuracy_score(np_gss_train_y, svm_poly.predict(np_gss_train_feature))|
    #train_error_allc2.append(er_score)
```

```
#aggregate the training and testing results
c=[c for (c,d,g) in cgd[0:2]]
d=[d for (c,d,g) in cgd[0:2]]
g=[g for (c,d,g) in cgd[0:2]]
df_cost=pd.DataFrame({'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Cost':c,'Degree':d,'Gamma':g},index=cgd[0:2])
df_cost
```

```
.dataframe tbody tr th {
   vertical-align: top;
}
.dataframe thead th {
   text-align: right;
}
```

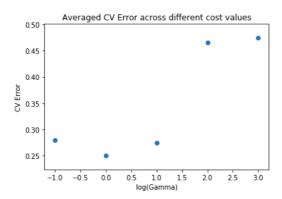
	CV_Error	Test_Error	Cost	Degree	Gamma
(0.01, 1, 1)	0.205949	0.196755	0.01	1	1
(0.01, 1, 10)	0.203945	0.219067	0.01	1	10

```
#aggregate the training and testing results
c=[c for (c,d,g) in cgd]
d=[d for (c,d,g) in cgd]
g=[g for (c,d,g) in cgd]
df_cost=pd.DataFrame({'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Test_AUC':auc_allc,'Cost':c,'Degree':d,'Gamma':g},index=cgd)
df_cost
```

```
ValueError
                                                                                      Traceback (most recent call last)
<ipython-input-78-15e2e2816e72> in <module>
           3 d=[d for (c,d,g) in cgd[0:2]]
           4 g=[g for (c,d,g) in cgd[0:2]]
df_cost=pd.DataFrame({'CV_Error':cv_error_allc,'Test_Error':test_error_allc,'Test_AUC':auc_allc,'Cost':c,'Degree':d,'Gamma':g},index=cgd[0:2
1)
           6 df cost
~\Anaconda3\lib\site-packages\pandas\core\frame.py in __init_(self, data, index, columns, dtype, copy)
       409
                                elif isinstance(data, dict):
       410
--> 411
                                      mgr = init_dict(data, index, columns, dtype=dtype)
       412
                                 elif isinstance(data, ma.MaskedArray):
                                    import numpy.ma.mrecords as mrecords
       413
~\Anaconda3\lib\site-packages\pandas\core\internals\construction.py in init_dict(data, index, columns, dtype)
        255
                                        arr if not is_datetime64tz_dtype(arr) else arr.copy() for arr in arrays
        256
--> 257
                         return arrays_to_mgr(arrays, data_names, index, columns, dtype=dtype)
       258
~\Anaconda3\lib\site-packages\pandas\core\internals\construction.py in arrays_to_mgr(arrays, arr_names, index, columns, dtype)
          85
                        axes = [ensure_index(columns), index]
          86
---> 87
                         return create block manager from arrays(arrays, arr names, axes)
          88
          89
$$ \sim \Lambda anaconda $$ \ ib\ site-packages \ pandas\ core\ internals\ managers.py in create\_block\_manager\_from\_arrays (arrays, names, axes) $$
      1697
                               return mgr
      1698
                         except ValueError as e:
-> 1699
                              construction_error(len(arrays), arrays[0].shape, axes, e)
     1700
      1701
~\Anaconda3\lib\site-packages\pandas\core\internals\managers.py in construction_error(tot_items, block_shape, axes, e)
      1713
      1714
                         if passed == implied and e is not None:
 -> 1715
                                raise e
      1716
                        if block_shape[0] == 0:
                          raise ValueError("Empty data passed with indices specified.")
      1717
~\Anaconda3\lib\site-packages\pandas\core\internals\managers.py in create_block_manager_from_arrays(arrays, names, axes)
      1692
      1693
-> 1694
                               blocks = form_blocks(arrays, names, axes)
      1695
                                 mgr = BlockManager(blocks, axes)
      1696
                               mgr._consolidate_inplace()
~\Anaconda3\lib\site-packages\pandas\core\internals\managers.py in form_blocks(arrays, names, axes)
      1750
                   blocks = []
      1751
                         if len(items_dict["FloatBlock"]):
                                float_blocks = _multi_blockify(items_dict["FloatBlock"])
-> 1752
      1753
                               blocks.extend(float_blocks)
      1754
1844
                     for dtype, tup_block in grouper:
      1845
-> 1846
                                values. placement = stack arrays(list(tup block), dtvpe)
      1847
      1848
                                block = make_block(values, placement=placement)
\sim \Lambda (3) = -\alpha (3) = 
      1874
                   stacked = np.empty(shape, dtype=dtype)
      1875
                         for i, arr in enumerate(arrays):
-> 1876
                             stacked[i] = _asarray_compat(arr)
     1877
      1878
                       return stacked, placement
ValueError: could not broadcast input array from shape (39) into shape (2)
```

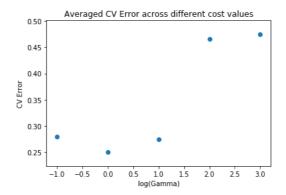
```
#plot cross validation error for cost
import matplotlib.pyplot as plt
import math
#cost=[0.1,1,10,100]
cost0=[math.log(c,10) for c in cost]
y=df_cost.groupby('Cost').mean()['CV_Error']
y=y.values

plt.scatter(cost0,y);
plt.xlabel('log(cost)');
plt.ylabel('CV_Error');
plt.title('Averaged CV_Error across different Cost values');
```



```
#plot cross validation error for Degree
#cost=[0.1,1,10,100]
d0=[math.log(d,10) for d in degree]
y=df_cost.groupby('Degree').mean()['CV_Error']
y=y.values

plt.scatter(d0,y);
plt.xlabel('log(Degree)');
plt.ylabel('CV_Error');
plt.title('Averaged CV_Error across different Degree values');
```



```
#plot cross validation error for gamma
gamma0=[math.log(g,10) for g in gamma]
y=df_cost.groupby('Gamma').mean()['CV_Error']
y=y.values

plt.scatter(gamma0,y);
plt.xlabel('log(Gamma)');
plt.ylabel('CV Error');
plt.title('Averaged CV Error across different Gamma values');
```

0.42 - 0.40 - 0.38 - 0.34 - 0.32 - 0.32 - 0.32 - 0.32 - 0.32 - 0.35 - 3.0 - 2.5 - 2.0 - 1.5 - 1.0 - 0.5 0.0

```
#sor the model by smallest CV_Error
df_cost1=df_cost
df_cost1.sort_values(by = "CV_Error")[0:5]
```

```
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    vertical-align: top;
}
.dataframe thead th {
    text-align: right;
}
```

	CV_Error	Test_Error	Test_AUC	Cost	Gamma
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(1000, 0.0001)	0.205967	0.192698	0.776018	1000.0	0.00010
(100, 0.0001)	0.208643	0.198783	0.776018	100.0	0.00010
(100000, 1e-05)	0.212720	0.198783	0.776018	100000.0	0.00001
(10, 0.001)	0.214089	0.200811	0.776018	10.0	0.00100

From the table and figure above, we could see that for the model has smallest cv error-0.20526 and test error-0.2008, the corresponding gamma value is 0.00001 and cost value is 1000; while then gamma=0.0001 and cost value=1000, the test error is smallest,less than 0.2, 0.192698. Therefore **the best parameters for radial kernel**SVM is gamma 0.0001/0.00001 and cost value=1000. and the best performance of the radial model is very close to the linear SVM, not outperforms, which might indicate the 'linear' relationship of the high dimension features between classes.