

Machine Learning for Rendering

Practice 2

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Introduction

The goal of the practice 2 is to put in practice the Monte Carlo-related concepts seen in the theory class. This lab is divided in two different parts, and will run for two weeks. In Part I, you will work in a controlled testing environment named appWorkbench (more details are given below). In Part II, once your understanding of Monte Carlo methods is consolidated, you will apply the concepts to solve the illumination integral using the appRenderer of Practice 1.

Part I – Monte Carlo in the appWorkbench

The appWorkbench

The *appWorkbench* is a script for you to prototype, evaluate and analyze any particular Monte Carlo method for hemispherical integration. Throughout this course, you will resort to the appWorkbench to develop, validate and compare different methods such as, classic Monte Carlo (MC), Bayesian Monte Carlo (BMC), and variance reduction techniques such as Importance Sampling (IS).

Let us assume that we already have a MC integration method implemented in the appWorkbench, and that our goal now is to evaluate its performance for estimating the hemispherical integral of a given spherical function. The **first step** to use the *appWorkbench* for this purpose is to **specify the function we wish to integrate**. To this end, you must first focus on the (spherical) *Function* abstract base class [defined in the file *PyRT_Common.py*]. The *Function* abstract class has two abstract methods, which must be implemented by concrete child classes:

- `eval(ω_i)` : given a spherical direction ω_i , the eval method returns the value of the spherical function for that same direction
- `get_integral()` : return the “true” value of the integral of the function over a hemisphere centered at $\omega_i = (\theta_i, \phi_i) = (0,0)$, that is:

$$\text{get_integral}() = \int_{\Omega_{2\pi}} \text{eval}(\omega_i) d\omega_i$$

The concrete classes implementing the template defined by the abstract class *Function* can take several forms: a constant value over the hemisphere; a cosine lobe to a given power ($\cos^n \theta_i$); or a function given by the Arch environment map (i.e., with real-world radiance values). These functions are defined through the concrete classes *Constant* and *CosineLobe*. To use them, you just have to instantiate an object of the concrete class you wish to use.

The **second step** to analyze a MC integration method in the *appWorkbench* consists of **setting up the probability density function** (pdf) used by the Monte Carlo integration method. Recall that the role of the pdf is to determine the statistical distribution of the random directions used to sample the integrand. To set-up the pdf, you must first focus on the *PDF* abstract base class [see *PyRT_Common.py*], which has two main abstract methods:

- `generate_dir(u_1, u_2)` : given two random numbers u_1 and u_2 uniformly distributed in $[0,1]$, returns a spherical direction
- `get_val(ω_i)` : returns the probability of generating the direction ω_i (i.e., the value of the pdf for the direction ω_i)

This template is implemented by the child classes *UniformPDF* and *CosinePDF*. In this practice you will only use the *UniformPDF*, the *CosinePDF* being useful for later practices where importance sampling techniques will be explored.

Finally, the **third step** for analyzing a particular MC integration in the *appWorkBench* is to **define the experimental set-up**. The core idea is to experimentally measure the “error” of the MC integration method with a varying number of samples, ranging from *ns_min* to *ns_max*, with a step of *ns_step*. However, recall that the MC estimator is a random variable, meaning that for a fixed number of samples *ns*, the MC estimator can provide different results. We thus need to perform several estimates using *ns* samples and average the estimation “error” so as to precisely capture the performance of the MC estimator with *ns* samples. The number of estimates collected for a fixed number of samples is determined by the variable *n_estimates*. Thus, the experimental set-up is thus captured by a set of variables:

- *ns_min* (integer), which defines the minimum number of samples to be used in the monte carlo estimate
- *ns_max* (integer), which defines the maximum number of samples to be used in the monte carlo estimate
- *ns_step* (integer), which defines the step with which the number of samples *n_s* is increased
- *nEstimates* (integer), which defines the number of estimates to be performed for each particular number of samples

Based on the above-defined variables, a 2D matrix called *results* is then initialized with zeros. The role of this matrix is to store the (average) estimate error (absolute value) for each method and for each number of used samples. During the main loop of the script, the values of this matrix must be filled appropriately. Finally, once the error associated with each method and each number of samples is computed, the content of the *results* matrix is displayed. Fig. 1 shows an example of the final result produced by the *appWorkbench* when evaluating the performance of 4 different estimation methods (MC, MC IS, BMC and BMC IS) for estimating the value of particular integral. Note that, in this practice, you will only be able to visualize results for the MC method.

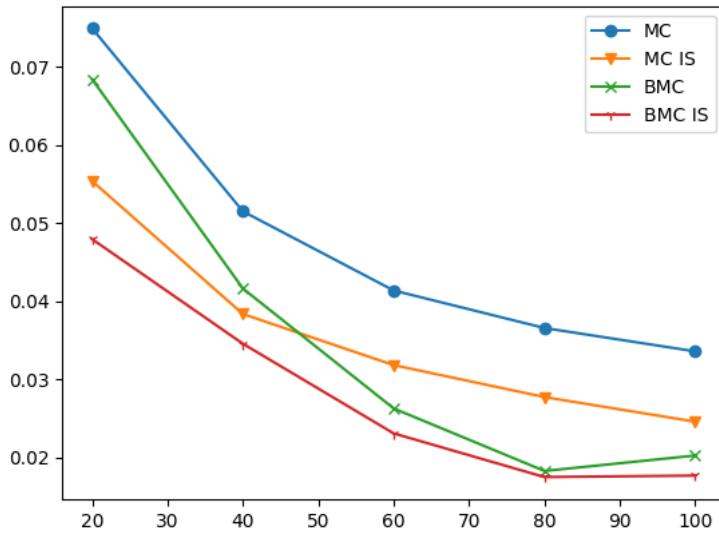


Figure 1. Average error of estimates of the value of the integral $I = \int_{\Omega_{2\pi}} L_i(\omega_i) \cos^4 d\omega_i$, using different Monte Carlo-based techniques as a function of the number of samples. For each sample count (ranging from 20 to 100), a total of 2500 estimates have been performed and averaged. Note the significant error reduction obtained when using the machine learning-based approach of Bayesian Monte Carlo (BMC).

Assignment 2.1: Estimate the value of a hemispherical integral

Your task in this assignment is to develop a Monte Carlo estimator with uniform sampling of the unit hemisphere. Then, use it to estimate the value of the integral over the hemisphere defined as:

$$I = \int_{\Omega_{2\pi}} \cos \theta_i d\omega_i$$

Once you have your Monte Carlo estimator ready, use it to compute a plot with estimate the error as a function of the number of samples. You will probably notice that the error plot is highly irregular and changes at each run (see two left-most examples in Figure 2). This is because of the variance inherent to the MC estimator. To alleviate the problem, you should average the estimate error over a number of different estimates for each sample count, by adapting the main loop of the script. An example of the final result is shown in Fig. 2 (right). In case your implementation is correct, you should observe the error converging to zero as the number of samples used in the MC estimate increases. This experiment should be repeated using different integrands that you might want to test, as well as varying number of minimum and maximum samples.

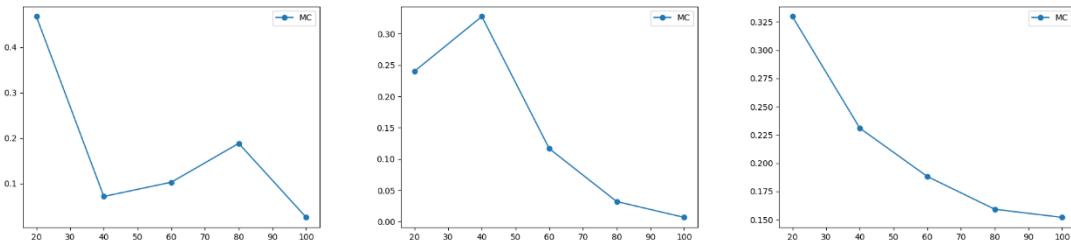


Figure 2. Left and center: Examples of different executions when measuring the estimate error as a function of the number of samples, using Monte Carlo. Right: Result when averaging over 1000 estimates.

[Optional] Assignment 2.2: Estimate the value of the hemispherical integral of the Arch Environment Map

Repeat Assignment 2.1 using the Arch Environment map, and assuming that the hemispherical integration is performed over the hemisphere defined around $[0, 1, 0]$. This implies that you create a new concrete class inheriting from the Abstract Base Class *Function*. How would you compute the reference value in this case?

Part II – Monte Carlo for rendering (appRenderer)

Assignment 2.3: Estimate the value of the illumination integral

With your MC estimator validated, you should work on its application to the computation of real images, in the context of the *appRenderer* script. To this end, you should develop the *compute_color()* method of the *CMCIntegrator* class so that it uses a Monte Carlo estimator to approximate the value of the illumination integral. In general terms, this can be achieved by implementing the *compute_color()* method such that, among other things, it contains the following steps:

```

(...)

Generate a sample set  $S$  of samples over the hemisphere

For each sample  $\omega_j \in S$ :
    Center the sample around the surface normal, yielding  $\omega'_j$ 
    Create a secondary ray  $r$  with direction  $\omega'_j$ 
    Shoot  $r$  by calling the method scene.closest_hit()

    If  $r$  hits the scene geometry, then:
         $L_i(\omega_j) = \text{object\_hit.emission};$ 
    Else:
        If the scene has an environment map
             $L_i(\omega_j) = \text{scene.env\_map.getValue}(\omega_j);$ 
        End If
    End If
    (...)

End For
(...)

Return result;

```

The first step is to generate a set of samples distributed over the hemisphere according to some probability density function. To this end, you must resort to the function `sample_set_hemisphere()`, which can be found in `PyRT_Common.py`. However, the resulting sample set is generated around the world normal vector $(0, 1, 0)$. Therefore, you must use the function `center_around_normal()` to rotate it, using the surface normal of the current scene point where you want to compute the illumination integral. This operation yields ω_j' , i.e., the direction for which we want to sample the incident radiance value $L_i(\omega_j')$. Follows the construction and shooting of a ray r with direction ω_j' . Then, depending on whether the ray r hits or not the scene geometry, we compute the value of the sample $L_i(\omega_j)$. Finally, based on the information collected by sampling, you should compute a Monte Carlo estimate of the illumination integral. Fig. 3 shows an example of the final result.



Figure 3: Image rendered using the Monte Carlo integrator. The result was generated using 40 samples per each estimate of the illumination integral, and using the `sphere_test_scene` with `areaLS=False` and `use_env_map=True`. The rendering time was approximately 175s.

[Optional] Assignment 2.4: Plot the Estimation Error

Plot the estimation error of the MC estimator for rendering using different numbers of samples. How would you compute the reference image? For the error metric you can use the root mean squared error. Besides the RMSE error plot, you can also create an image displaying the approximation error for each pixel given a rendered image.