MATH693A Homework 1

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1. $f(\bar{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial}{\partial x_1}(f(x)) \\ \frac{\partial}{\partial x_2}(f(x_2)) \end{bmatrix} = \begin{bmatrix} 200(x_2 - x_1^2)(-2x_1) + 2(x_1 - 1) \\ 200(x_2 - x_1^2) \end{bmatrix}$$

$$= \begin{bmatrix} 400x_1^3 - 400x_1x_2 + 2x_1 - 2 \\ 200(x_2 - x_1^2) \end{bmatrix}$$

$$\nabla^2 f(x) = \begin{bmatrix} \frac{\partial^2}{\partial x_1^2} & \frac{\partial^2}{\partial x_1 x_2} \\ \frac{\partial^2}{\partial x_2 x_1} & \frac{\partial^2}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} 1200x_1^2 - 400x_2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}$$

We need Iterations k, $\bar{x}^T$, $f(\bar{x})$, $\bar{p}^{SD}$, α

Problem 1a(i)

Newton Method

$x_0 = [1.2 ; 1.2]^T$, **Stopping Criteria:** $\|f\| < 10^{-8}$

Present here the first 6 values from your iterations for the variables in the Table

Iteration # k	$\bar{x}_k^T$	$f(\bar{x}_k)$	$\bar{p}_k^{SD}$	α_k
0	1.200000000000000, 1.200000000000000	5.800000000000000	-0.00408163265306, 0.230204081632653	1.000000000000000
1	1.195918367346939, 1.430204081632653	0.038384034418534	-0.195267745952645, -0.467031908145293	0.500000000000000
2	1.098284494370616, 1.196688127560007	0.018762343235567	-0.033796335111792, -0.064695278624755	1.000000000000000
3	1.064488159258824, 1.131992848935252	0.004289183002068	-0.052496044128596, -0.110620642498651	1.000000000000000
4	1.011992115130228, 1.021372206436601	0.000903273286643	-0.007731028105105, -0.012891644319282	1.000000000000000
5	1.004261087025123, 1.008480562117319	0.000018514093528	-0.004210752701645, -0.008397621375135	1.000000000000000
6	1.000050334323477, 1.000082940742184	0.000000033970388	-0.000050156464258, -0.000082587539385	1.000000000000000

Problem 1a(ii)

Newton Method

$x_0 = [1.2 ; 1.2]^T$, **Stopping Criteria:** $\|f\| < 10^{-8}$

Present here the last 6 values from your iterations for the variables in the Table

Iteration # k	$\bar{\boldsymbol{x}}_k{}^T$	$f(\bar{\boldsymbol{x}}_k)$	$\bar{\boldsymbol{p}}_k{}^{SD}$	α_k
1	1.195918367346939, 1.430204081632653	0.038384034418534	-0.195267745952645, -0.467031908145293	0.500000000000000
2	1.098284494370616, 1.196688127560007	0.018762343235567	-0.033796335111792, -0.064695278624755	1.000000000000000
3	1.064488159258824, 1.131992848935252	0.004289183002068	-0.052496044128596, -0.110620642498651	1.000000000000000
4	1.011992115130228, 1.021372206436601	0.000903273286643	-0.007731028105105, -0.012891644319282	1.000000000000000
5	1.004261087025123, 1.008480562117319	0.000018514093528	-0.004210752701645, -0.008397621375135	1.000000000000000
6	1.000050334323477, 1.000082940742184	0.000000033970388	-0.000050156464258, -0.000082587539385	1.000000000000000

Problem 1a(iii)

Newton Method $\boldsymbol{x_0} = [-1.2\ ;\ 1]^T$, **Stopping Criteria:** $\|f\| < 10^{-8}$ Present here the first 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{\boldsymbol{x}}_k{}^T$	$f(\bar{\boldsymbol{x}}_k)$	$\bar{\boldsymbol{p}}_k{}^{SD}$	α_k
0	-1.200000000000000 1.000000000000000	24.199999999999996	0.024719101123596 0.380674157303370	1.000000000000000
1	-1.175280898876404 1.380674157303370	4.731884325266608	1.938395770052997 -4.555708012051856	0.125000000000000
2	-0.932981427619779 0.811210655796888	4.087398662072179	0.150441348648932 -0.221474279984497	1.000000000000000
3	-0.782540078970848 0.589736375812392	3.228672588621933	0.322542959901112 -0.482172987241705	1.000000000000000
4	-0.459997119069736 0.107563388570687	3.213898091447576	0.066951484928344 0.042438980612811	1.000000000000000
5	-0.393045634141392 0.150002369183498	1.942585420621423	0.734534900158846 -0.572928969929725	0.250000000000000
6	-0.209411909101680 0.006770126701067	1.600193693646818	0.143692887705857 -0.023098782905090	1.000000000000000

Problem 1a(iv)

Newton Method $\boldsymbol{x_0} = [-1.2\ ;\ 1]^T$, **Stopping Criteria:** $\|f\| < 10^{-8}$ Present here the last 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{\boldsymbol{x}}_k{}^T$	$f(\bar{\boldsymbol{x}}_k)$	$\bar{\boldsymbol{p}}_k{}^{SD}$	α_k
15	0.802785534446063 0.633221011934572	0.051535404874557	0.060705273659376 0.108710233497953	1
16	0.863490808105439 0.741931245432525	0.019992777967696	0.078587878341622 0.139404951403058	1
17	0.942078686447061 0.881336196835582	0.007169243633384	0.025913131024650 0.055000471497104	1
18	0.967991817471711 0.936336668332686	0.001069613679054	0.028218493287811 0.055302031567464	1
19	0.996210310759522 0.991638699900150	0.000077768464028	0.003269068300201 0.007309642457909	1

20	0.999479379059724 0.998948342358059	0.000000282466949	0.000519510558405 0.001049166988210	1

Problem 1a(iv)

Newton Method $x_0 = [1.2\ ;\ 1.2]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the <u>first</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
0	1.200000000000000 1.200000000000000	5.800000000000000	-0.004081632653061 0.230204081632653	1.000000000000000
1	1.195918367346939 1.430204081632653	0.038384034418534	-0.195267745952645 -0.467031908145293	0.500000000000000
2	1.098284494370616 1.196688127560007	0.018762343235567	-0.033796335111792 -0.064695278624755	1.000000000000000
3	1.064488159258824 1.131992848935252	0.004289183002068	-0.052496044128596 -0.110620642498651	1.000000000000000
4	1.011992115130228 1.021372206436601	0.000903273286643	-0.007731028105105 -0.012891644319282	1.000000000000000
5	1.004261087025123 1.008480562117319	0.000018514093528	-0.004210752701645 -0.008397621375135	1.000000000000000
6	1.000050334323477 1.000082940742184	0.000000033970388	-0.000050156464258 -0.000082587539385	1.000000000000000

Problem 1a(vi)

Newton Method $x_0 = [1.2\ ;\ 1.2]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the <u>last</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
3	1.098284494370616 1.196688127560007	0.018762343235567	-0.033796335111792 -0.064695278624755	0.500000000000000
4	1.064488159258824 1.131992848935252	0.004289183002068	-0.052496044128596 -0.110620642498651	1.000000000000000
5	1.011992115130228 1.021372206436601	0.000903273286643	-0.007731028105105 -0.012891644319282	1.000000000000000
6	1.004261087025123 1.008480562117319	0.000018514093528	-0.004210752701645 -0.008397621375135	1.000000000000000
7	1.000050334323477 1.000082940742184	0.000000033970388	-0.000050156464258 -0.000082587539385	1.000000000000000
8	1.000000177859219 1.000000353202799	0.000000000000032	-0.000000177859130 -0.000000353202652	1.000000000000000

Problem 1a(vii)

Newton Method $x_0 = [-1.2\ ;\ 1]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the <u>first</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
0	-1.200000000000000 1.000000000000000	24.199999999999996	0.024719101123596 0.380674157303370	1.000000000000000

1	-1.175280898876404 1.380674157303370	4.731884325266608	1.938395770052997 -4.555708012051856	0.125000000000000
2	-0.932981427619779 0.811210655796888	4.087398662072179	0.150441348648932 -0.221474279984497	1.000000000000000
3	-0.782540078970848 0.589736375812392	3.228672588621933	0.322542959901112 -0.482172987241705	1.000000000000000
4	-0.459997119069736 0.107563388570687	3.213898091447576	0.066951484928344 0.042438980612811	1.000000000000000
5	-0.393045634141392 0.150002369183498	1.942585420621423	0.734534900158846 -0.572928969929725	0.250000000000000
6	-0.209411909101680 0.006770126701067	1.600193693646818	0.143692887705857 -0.023098782905090	1.000000000000000

Problem 1a(viii)

Newton Method $\boldsymbol{x_0} = [-1.2\ ;\ 1]^T$, **Stopping Criteria:** $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the <u>last</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
16	0.863490808105439 0.741931245432525	0.019992777967696	0.078587878341622 0.139404951403058	1
17	0.942078686447061 0.881336196835582	0.007169243633384	0.025913131024650 0.055000471497104	1
18	0.967991817471711 0.936336668332686	0.001069613679054	0.028218493287811 0.055302031567464	1
19	0.996210310759522 0.991638699900150	0.000077768464028	0.003269068300201 0.007309642457909	1
20	0.999479379059724 0.998948342358059	0.000000282466949	0.000519510558405 0.001049166988210	1
21	0.999998889618128 0.999997509346269	0.000000000008517	0.000001110321938 0.000002490532631	1

Problem 1a(ix)

Steepest Descent Method $\boldsymbol{x_0} = [1.2\ ;\ 1.2]^T$, **Stopping Criteria:** $\|f(\bar{x}_k)\| < 10^{-8}$ Present here the <u>first</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
0	1.200000000000000 1.200000000000000	5.800000000000000	-0.923548958248274, 0.383480536296861	0.125000000000000
1	1.084556380218966 1.247935067037108	0.520844867766157	0.907267511011767, -0.420553995894122	0.031250000000000
2	1.112908489938083 1.234792754665416	0.014171542193907	-0.929746279393009, 0.368200836439649	0.001953125000000
3	1.111092579236144 1.235511896924087	0.012438618590703	0.738272817499907, -0.674502221598083	0.000488281250000
4	1.111453064010314 1.235182550136198	0.012423898527980	-0.994927136179291, 0.100598179377539	0.000488281250000
5	1.110967259744601 1.235231670340972	0.012410443854420	0.737967725727800, -0.674836006585406	0.000488281250000
6	1.111327595548179 1.234902160572131	0.012395990435597	-0.994837249380813, 0.101483236272889	0.000488281250000

Problem 1a(x)

Steepest Descent Method $x_0 = [1.2\,;\ 1.2]^T$, **Stopping Criteria:** $\|f(\bar{x}_k)\| < 10^{-8}$ Present here the last 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$ (1.06-07)*	$\bar{p}_k{}^{SD}$	α_k(1.0e-04)*
7606	1.000081174691810 1.000179768326973	0.369083378513680	0.890153412020434 -0.455660951880210	0.152587890625000
7607	1.000094757354957 1.000172815492624	0.368953384990326	-0.899347775422549 0.437234009247351	0.152587890625000
7608	1.000081034396958 1.000179487154142	0.368836293106411	0.890160866132626 -0.455646389655414	0.152587890625000
7609	1.000094617173846 1.000172534541995	0.368706693310531	-0.899340579484607 0.437248810281162	0.152587890625000
7610	1.000080894325648 1.000179206429359	0.368589994581223	0.890168308133889 -0.455631850504385	0.152587890625000
7611	1.000094477216092 1.000172254039062	0.368460787267332	-0.899333394765663 0.437263587621092	0.076293945312500

Problem 1a(xi)

Steepest Descent Method $x_0 = [-1.2\,;\ 1]^T$, **Stopping Criteria:** $\|f(\bar{x}_k)\| < 10^{-8}$ Present here the first 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
0	-1.200000000000000 1.000000000000000	24.199999999999996	0.925847643695199 0.377896997426612	0.250000000000000
1	-0.968538089076200 1.094474249356653	6.321495316645379	-0.875432951101911 -0.483339578479767	0.125000000000000
2	-1.077967207963939 1.034056802046682	5.955234291215230	0.918220469788388 0.396069651528102	0.062500000000000
3	-1.020578428602165 1.058811155267188	4.112427323765298	-0.655722192375562 -0.755002255908012	0.007812500000000
4	-1.025701258230099 1.052912700142907	4.103537774540148	0.998948670078196 -0.045842715331914	0.007812500000000
5	-1.017896971745113 1.052554553929377	4.098936564058504	-0.628663418320573 -0.777677508010546	0.007812500000000
6	-1.022808404700743 1.046478948398044	4.091765532760610	0.999846763052762 -0.017505725146787	0.007812500000000

Problem 1a(xii)

Steepest Descent Method $x_0 = [-1.2\,;\ 1]^T$, **Stopping Criteria:** $\|f(\bar{x}_k)\| < 10^{-8}$ Present here the last 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$ 1.0e-07	$\bar{p}_k{}^{SD}$	α_k
8294	0.999905081983656 0.999826878272635	0.369161209155554	0.899324924712471 -0.437281007809519	0.152587890625000
8295	0.999918804592981 0.999820205893976	0.369031025773857	-0.890120931130621 0.455724399130831	0.152587890625000
8296	0.999905222425453 0.999827159696453	0.368913736898251	0.899317757372434 -0.437295748063728	0.152587890625000

8297	0.999918944925413 0.999820487092875	0.368783948218213	-0.890128445423160 0.455709721915770	0.152587890625000
8298	0.999905362643225 0.999827440671396	0.368667053415349	0.899310601230380 -0.437310464675444	0.152587890625000
8299	0.999919085033991 0.999820767843261	0.368537658165347	-0.890135947481463 0.455695068001924	0.076293945312500

Problem 1a(xiii)

Steepest Descent Method $x_0 = [1.2\ ;\ 1.2]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the first 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$	$\bar{p}_k{}^{SD}$	α_k
0	1.200000000000000 1.200000000000000	5.800000000000000	-0.923548958248274 0.383480536296861	0.125000000000000
1	1.084556380218966 1.247935067037108	0.520844867766157	0.907267511011767 -0.420553995894122	0.031250000000000
2	1.112908489938083 1.234792754665416	0.014171542193907	-0.929746279393009 0.368200836439649	0.001953125000000
3	1.111092579236144 1.235511896924087	0.012438618590703	0.738272817499907 -0.674502221598083	0.000488281250000
4	1.111453064010314 1.235182550136198	0.012423898527980	-0.994927136179291 0.100598179377539	0.000488281250000
5	1.110967259744601 1.235231670340972	0.012410443854420	0.737967725727800 -0.674836006585406	0.000488281250000
6	1.111327595548179 1.234902160572131	0.012395990435597	-0.994837249380813 0.101483236272889	0.000488281250000

Problem 1a(xiv)

Steepest Descent Method $x_0 = [1.2\ ;\ 1.2]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the last 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{x}_k{}^T$	$f(\bar{x}_k)$(1.0e-17)*	$\bar{p}_k{}^{SD}$	α_k(1.0e-09)*
18056	1.000000001276563 1.000000002818952	0.869594976121798	0.890005127502390 -0.455950515976739	0.232830643653870
18057	1.000000001483784 1.000000002712793	0.869261495518173	-0.899459402743036 0.437004328144632	0.232830643653870
18058	1.000000001274362 1.000000002814541	0.868987389242565	0.890012828641190 -0.455935483214575	0.232830643653870
18059	1.000000001481584 1.000000002708385	0.868653367289376	-0.899452032306886 0.437019497939180	0.232830643653870
18060	1.000000001272164 1.000000002810137	0.868381676977442	0.890020516853457 -0.455920475061063	0.232830643653870
18061	1.000000001479388 1.000000002703985	0.868047259352685	-0.899444673243412 0.437034643677194	0.116415321826935

Problem 1a(xv)

Steepest Descent Method $x_0 = [-1.2\ ;\ 1]^T$, Stopping Criteria: $\|\nabla f(\bar{x}_k)\| < 10^{-8}$ Present here the first 6 values from your iterations for the variables in the Table

Iteration # k	$\bar{\boldsymbol{x}}_k^T$	$f(\bar{\boldsymbol{x}}_k)$	$\bar{\boldsymbol{p}}_k^{SD}$	α_k
0	-1.200000000000000 1.000000000000000	24.199999999999996	0.925847643695199 0.377896997426612	0.250000000000000
1	-0.968538089076200 1.094474249356653	6.321495316645379	-0.875432951101911 -0.483339578479767	0.125000000000000
2	-1.077967207963939 1.034056802046682	5.955234291215230	0.918220469788388 0.396069651528102	0.062500000000000
3	-1.020578428602165 1.058811155267188	4.112427323765298	-0.655722192375562 -0.755002255908012	0.007812500000000
4	-1.025701258230099 1.052912700142907	4.103537774540148	0.998948670078196 -0.045842715331914	0.007812500000000
5	-1.017896971745113 1.052554553929377	4.098936564058504	-0.628663418320573 -0.777677508010546	0.007812500000000
6	-1.022808404700743 1.046478948398044	4.091765532760610	0.999846763052762 -0.017505725146787	0.007812500000000

Problem 1a(xvi)

Steepest Descent Method $\boldsymbol{x_0 = [-1.2\ ;\ 1]^T}$, **Stopping Criteria:** $\|\nabla \boldsymbol{f}(\bar{\boldsymbol{x}}_k)\| < \mathbf{10^{-8}}$ Present here the <u>last</u> 6 values from your iterations for the variables in the Table				
Iteration # k	$\bar{\boldsymbol{x}}_k^T$	$f(\bar{\boldsymbol{x}}_k)$(1.0e-16)*	$\bar{\boldsymbol{p}}_k^{SD}$	α_k 1.0e-09 *
17907	0.999999997541248 0.999999994551224	0.342704702412173	-0.890170080526777 0.455628387762387	0.465661287307739
17908	0.999999997126730 0.999999994763392	0.342587312615089	0.899301246426697 -0.437329701913087	0.465661287307739
17909	0.999999997545500 0.999999994559745	0.342477858246364	-0.890177511458841 0.455613869513369	0.465661287307739
17910	0.999999997130979 0.999999994771906	0.342360669666160	0.899294120468562 0.437344355046084	0.465661287307739
17911	0.999999997549745 0.999999994568252	0.342251708127245	-0.890184930294359 0.455599374315666	0.465661287307739
17912	0.999999997135221 0.999999994780407	0.342134776332514	0.899287005378874 -0.437358985224606	0.232830643653870

Problem 1b(i)

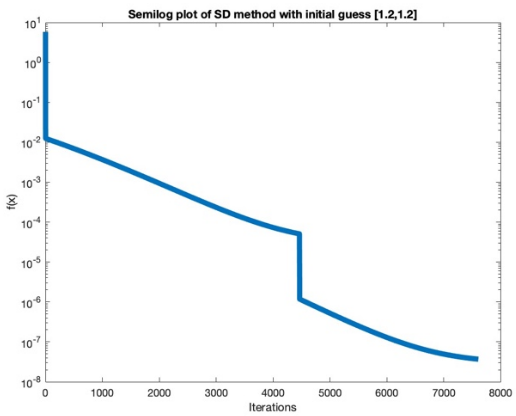
Present here the number of iterations number of iterations required in order to achieve convergence		
Line Search Method	**Stopping Criteria**	**Number of iterations required in order to achieve convergence**
Newton Method		
$x_0 = (1.2\,,1.2)^T$	$\|\nabla f(\bar{x}_k)\| < 10^{-8}$	8
$x_0 = (1.2\,,1.2)^T$	$|f(\bar{\boldsymbol{x}}_k)| < 10^{-8}$	7
$x_0 = (-1.2\ ;\ 1)^T$	$\|\nabla f(\bar{x}_k)\| < 10^{-8}$	21
$x_0 = (-1.2\ ;\ 1)^T$	$|f(\bar{\boldsymbol{x}}_k)| < 10^{-8}$	20
Steepest Descent Method		
$x_0 = (1.2\,,1.2)^T$	$\|\nabla f(\bar{x}_k)\| < 10^{-8}$	18061
$x_0 = (1.2\,,1.2)^T$	$|f(\bar{\boldsymbol{x}}_k)| < 10^{-8}$	7611
$x_0 = (-1.2\ ;\ 1)^T$	$\|\nabla f(\bar{x}_k)\| < 10^{-8}$	17912
$x_0 = (-1.2\ ;\ 1)^T$	$|f(\bar{\boldsymbol{x}}_k)| < 10^{-8}$	8299

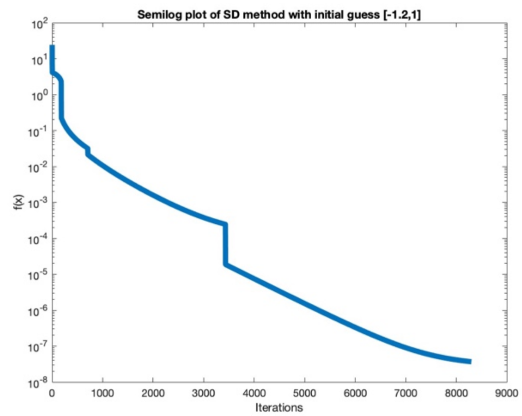
Problem 1b(ii)
Discuss your observation with regards to number of iterations required in order to achieve convergence:

We can see that Newton’s method converges a lot faster than Steepest descent. We also see that when we are calculating the minimum using am initial guess closer to the actual minimum the number of iterations is less for Newtons method. Interesting we see that steepest descent takes a fewer iterations for the farther point compared to the closer.

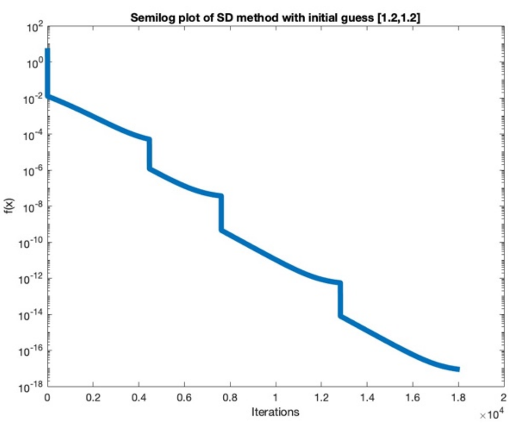
Problem 2(i) and (ii)

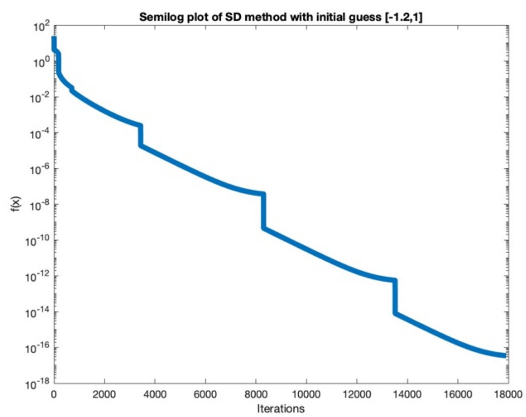
1. Steepest Descent
   - Function(x) > 10^-8



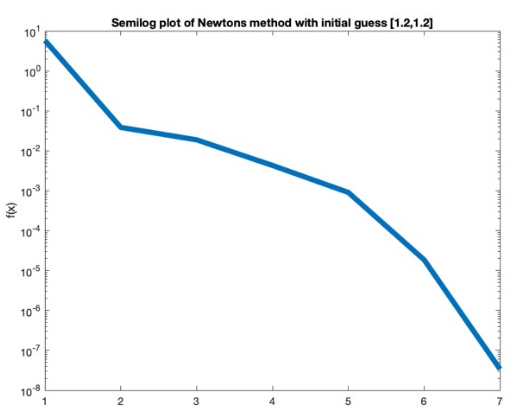


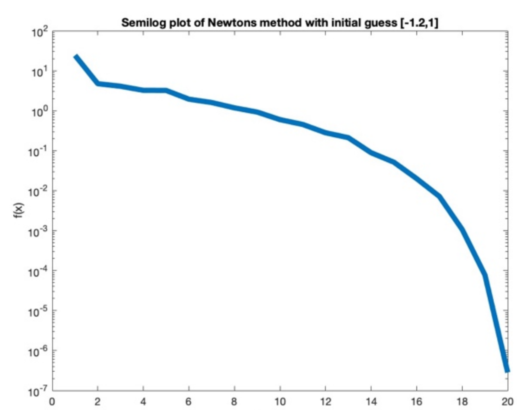
   - Gradient(f(x)) > 10^-8



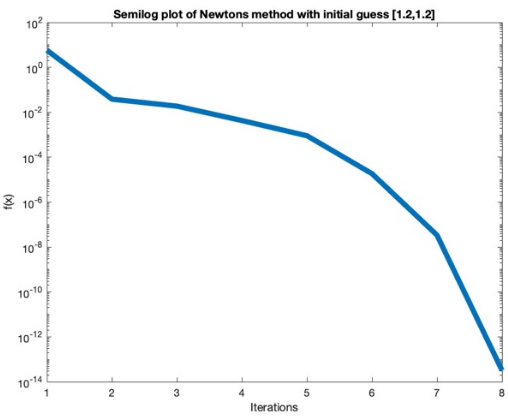


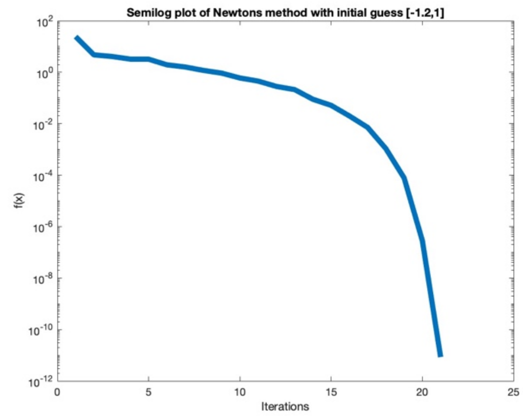
2. Newtons method
   - F(x) > 10^-8





   - Gradient(f(x)) > 10^-8





Problem 2(iii)

Compare the graph obtained in (i) with the one obtained in (ii). What can you infer about the convergence of the steepest descent and Newton algorithm.

We see that the graphs for steepest descent the graph is in a linear fashion with some bumps. Whereas for the newtons methods we clearly see that the method is quadratic. Which aligns with the theorem that steepest descent has a linear convergence and newtons method has a quadratic convergence.

Problem 3(a)

3) $f(x,y) = 5 - 5x - 2y + 2x^2 + 5xy + 6y^2$

Is this function convex?

→ Find hessian

→ Check if the eigenvalues are positive

$$f_x(x,y) = -5+4x+5y$$
$$f_y(x,y) = -2+5x+12y$$

$$f_{xx}(x,y) = \frac{\partial}{\partial x}(-5+4x+5y) = 4$$
$$f_{xy}(x,y) = \frac{\partial}{\partial x}(-5+4x+5y) = 5$$
$$f_{yy}(x,y) = \frac{\partial}{\partial y}(-2+5x+12y) = 12$$
$$f_{yx}(x,y) = \frac{\partial}{\partial y}(-2+5x+12y) = 5$$

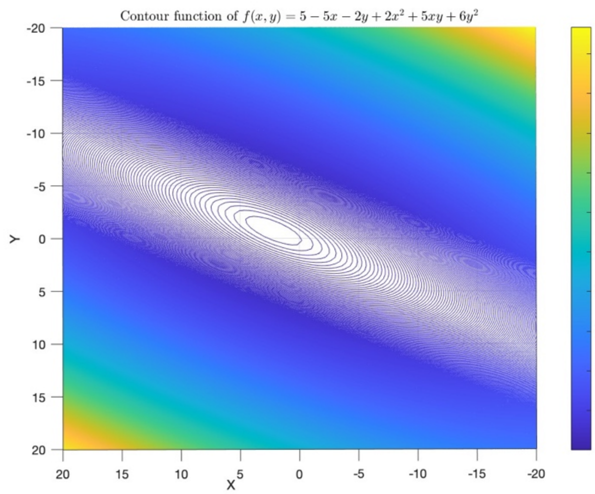
$$\Rightarrow \begin{bmatrix} 4 & 5 \\ 5 & 12 \end{bmatrix} \rightarrow \begin{bmatrix} 4-\lambda & 5 \\ 5 & 12-\lambda \end{bmatrix}$$

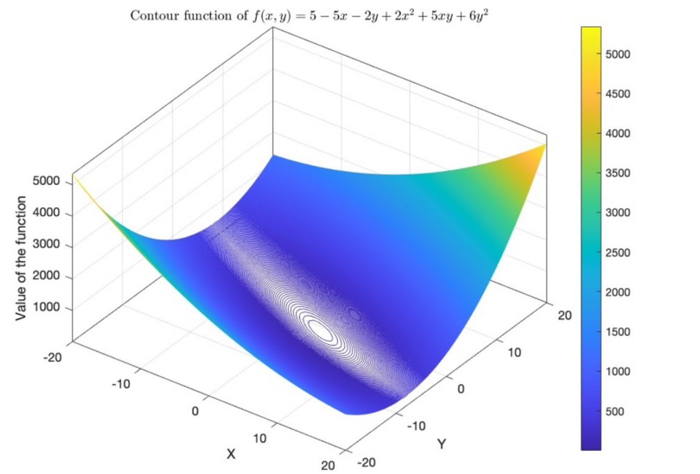
$$= (4-\lambda)(12-\lambda) - 25$$
$$= 48 - 12\lambda - 4\lambda + \lambda^2 - 25$$
$$= \lambda^2 - 16\lambda + 23$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{16 \pm \sqrt{16^2-(4)(23)}}{2} = 8 \pm \sqrt{41}$$

$\sqrt{41}$ is approx 6.4 so both eigenvalues are >0. Therefore the function is convex.

Problem 3(b)
Create a contour plot of the function using a programming language of your choice.





Problem 4(i) and (ii)
Present your solution below

4a) ST $x_K = 1+(0.5)^{2^k}$ is Q-quadratically convergent to 1

We can see that $x_k \to 1$ as $k \to \infty$

$$\frac{|x_{k+1}-1|}{|x_k-1|^2} = \frac{(0.5^{2^{k+1}})}{(0.5^{2^k})^2} = \frac{0.5^{2^{k+1}}}{(0.5)^{2^k}\cdot(0.5)^{2^k}} = \frac{0.5^{2^{k+1}-2^k}}{0.5^{2^k}}$$

$$= \frac{0.5^{2^k(2-1)}}{0.5^{2^k}} = 1$$

We see that we have an upper pound.

Therefore x_k is quadratically convergent

Checking for Q-Superlinearity:

$$\lim_{k \to 0} \frac{|x_{k+1}-1|}{|x_k-1|} = \frac{(0.5)^{2^{k+1}}}{0.5^{2^k}} = 0.5^{2^{k+1}-2^k} = 0.5^{2^k(2-1)} = 0.5^{2^k}$$

as $k \to \infty$ $0.5^{2^k} \to 0$. Therefore x_k is convergent super linearly

We can also say that it converges linearly as well since for a large k the value of the function is between 0 & 1 → range (0,1)

We can conclude that x_k is Q-Quadratically convergent!

4b) $x_k = \frac{1}{k!}$

We know that $\frac{1}{k!}$ tends to 0 as k tends to ∞

$$\frac{\frac{1}{(k+1)!}}{\frac{1}{k!}} = \frac{1}{k+1} \text{ which tends to 0 as k tends to } \infty$$

Since $\lim_{k \to \infty} \frac{\|\bar{x}_{k+1} - \bar{x}^*\|}{\|\bar{x}_k - \bar{x}^*\|} = 0$ we can say that $\frac{1}{k!}$ is Q superlinear

Since $\frac{\|\bar{x}_{k+1} - \bar{x}_*\|}{\|\bar{x}_k - \bar{x}^*\|} \in (0,1)$ we can say that $\frac{1}{k!}$ is ~~not~~ Q linear for a large enough k

Checking for q-quadratic

$$\frac{\frac{1}{(k+1)!}}{\left(\frac{1}{k!}\right)^2} = \frac{k!}{k+1} \text{ which tends to } \infty \text{ as k tends to } \infty$$

We can say that $\frac{1}{k!}$ is not Q-quadratic since we can't find an upper bound.

We can conclude that x_k is Q-superlinearly convergent!