Monte Carlo Simulation: Expected Profit

Why Monte Carlo?

When dealing with many varying parameters, calculating the expected value directly can be very difficult. Calculating profits contains many varying parameters such as No. of leads per sale, cost of single lead etc.

Monte Carlo method approximates the expected value by simulating many random samples from the system and averaging the results. It handles high-dimensional problems by focusing on random samples rather than exhaustive calculation.

Problem statement:

Predicting the expected value of profit based on various parameters.

Parameter:		Range:
• Profit per sale(P)	-	47-53
• Number of leads per month(L)	-	1200-1800
• Conversion rate in percentage(R)	-	1%-5%
• Cost of single lead(C)	-	0.2-0.8
• Fixed overhead(F)	-	800

Simulating Using Excel

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1															
2															
3	Variables	Min	Max											Iteration	Average
4	P	47	53											10	604.274111
5	L	1200	1800											100	772.3819024
6	R	1%	5%											1000	656.6088141
7	C	0.2	0.8											10000	691.1763909
8					Simulation									100000	697.944385
9	Constant				S.No.	L	R	P	F	C		Y			
10	F	800			1	1307.546	4.1144964043314500%	51.70056	800	0.55386		1257.237			
11					2	1628.983	3.5972704078834800%	48.19918	800	0.402336		1369.021			
12					3	1735.402	4.7771580504088600%	49.0953	800	0.480473		2436.329		Legends	
13					4	1490.824	4.0009933909580800%	48.30994	800	0.439747		1425.994		P	Profit Per Sale
14					5	1274.411	1.1330998873347000%	49.92748	800	0.221503		-361.316		L	Number of leads per month
15					6	1425.645	1.2828480678261900%	51.43972	800	0.625923		-751.571		R	Conversion Rate in percentage
16					7	1772.58	2.2526716672333600%	47.46802	800	0.586814		55.24271		C	Cost of Single lead
17					8	1465.053	2.5317035457218000%	50.29006	800	0.573737		224.7423		F	Fixed Overhead
18					9	1257.356	2.2060758056366700%	49.78066	800	0.649716		-236.097		Y	Profits
19					10	1693.433	2.2583751510809500%	51.83875	800	0.330315		623.1587			
20					11	1255.613	1.1636993414197000%	49.07298	800	0.53168		-750.551			

Theoretical Method

$$
\begin{aligned}
\mathrm{E}\left(L \cdot \left(\tfrac{R}{100}\right) \cdot P - (F + L \cdot C)\right) &= E\left(L \cdot \left(\tfrac{R}{100}\right) \cdot P\right) - E\left(F + L \cdot C\right) \\
&= E(L) \cdot E\left(\tfrac{R}{100}\right) \cdot E(P) - E(F) - E(L) \cdot E(C) \\
&= 1500 \cdot \tfrac{3}{100} \cdot 50 - 800 - 1500 \cdot 0.5 \\
&= 700
\end{aligned}
$$

Note In above equation random variables L, R, P, C are independent that's why we can use E(X.Y) = E(X).E(Y) where X and Y are random variables. And also the expected value of a uniformly distributed random variable X is E(X) = (a+b)/2 where X ~ U(a,b).

Result

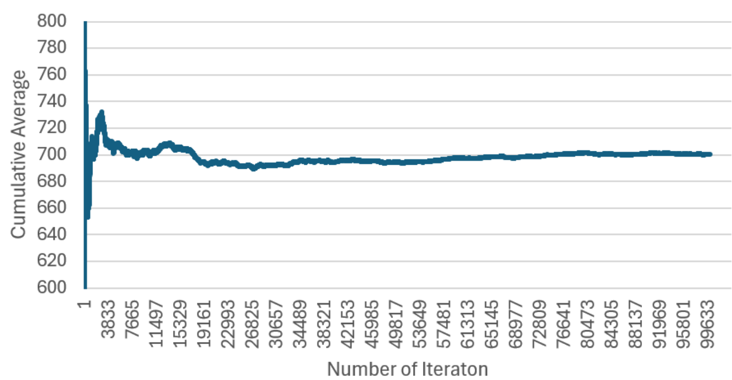
The monte carlo simulation yielded an expected profit of approximately **699.39**. This was calculated by averaging the profit outcomes from all **1,00,000** simulation runs.

The theoretical method yields an expected value of **700.**

Justification

Monte Carlo Simulation relies on this principle by generating a large number of random samples and using the average of these samples to estimate the expected value. The **Law of Large Numbers** ensures that this average becomes increasingly accurate as the number of simulations grows.

We considered the result from 1,00,000 simulations as the expected profit because when we conducted the simulations, we observed that the profit value did not change significantly between 1,000, 10,000 and 1,00,000 simulations.



$$\overline{X}_n \xrightarrow{\text{a.s.}} \mu \qquad \text{when } n \to \infty.$$

$$\Pr\left(\lim_{n\to\infty} \overline{X}_n = \mu\right) = 1.$$

The cumulative average graph stables after a particular number of iterations which supports the law of large numbers.

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