AMS 512 PROJECT

Minimal Tracking Error Problem
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Submitted to:

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Abstract

We need to find optimal weights of a portfolio of 10 randomly selected stocks from the S&P 500 universe such that it minimizes the tracking error.

$$min_w \quad \sigma(r_p - r_b)$$

$$subject\ to\ w'e = 1$$

$$w'\mu - Er_b \geq R_*$$

$$w \geq 0$$

We will plot the inverse distribution function of the return of the initial equally-weighted portfolio and compare it with that of the benchmark (S&P 500 index) as the first figure of Figure 9.4 as given in the book *Advanced Staochastic Models, Risk Assessment, and Portfolio Optimization* by Rachev, Stoyanov and Fabozzi. Also, after minimizing the tracking error, we will plot the inverse distribution function of the return of this optimal portfolio and compare it again with the return of the benchmark as the second figure of Figure 9.4 as given in the book.

We use data from the time period December 31, 2002 to December 31, 2003. Using MATLAB (program and data values in Appendices A and B), we plot the two aforementioned plots and an additional plot which helps us compare how close the curves are before and after optimization.

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Chapter 1: Introduction

A **portfolio** is a collection of investments held by an institution or a private individual. Portfolios are constructed and held as a part of investment strategy and for the purpose of diversification. The concept of diversification is a key aspect of investment management as including a number of assets in a portfolio that may greatly reduce portfolio risk while not necessarily reducing performance.

1.1 Concepts

Optimal portfolio selection is a careful decision making about the portfolio composition. A portfolio is optimal portfolio if it is the most preferred, with respect to a set of measures, in a given set of feasible portfolios.

One such way to optimize a portfolio is via mean-variance analysis (M-V analysis) and popularly known as *modern portfolio theory* (MPT). This method was introduced by Harry Markowitz in the 1950. Markowitz suggested that the portfolio choice must be made with respect to the following two criteria:

(a) the expected portfolio return and

(b) the variance of the portfolio return (used as a proxy for risk).

1.1.1 Mean-Variance Analysis

A portfolio is preferred if it has a higher expected return and a lower variance. There are convenient methods to compute the resulting optimization problems and geometric interpretations of the trade-off between the expected return and variance.

In section 5.2.3 of the book *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization* by Rachev, Stoyanov and Fabozzi, it is noted that the key characteristics of investor's preferences determine the shape of the utility function. For example, all nonsatiable investors have nondecreasing utility functions and all risk-averse investors have concave utility functions. Therefore, different classes of investors can be characterized by the general unifying properties of their utility functions.

1.1.2 Stochastic Dominance Relation

To define stochastic dominance relations or a stochastic ordering, let us suppose two portfolios A and B. Let a class of investors not prefer A to B. Now, no matter what the utility functions for A and B are, the probability distributions of the two portfolios differ in such a way, that an investor from this given class will not prefer A. In this case, we say that portfolio B dominates portfolio A with respect to this class of investors. This relation is called *stochastic dominance relation* or a *stochastic ordering.*

Generally, the M-V analysis is not consistent with the *second-order stochastic dominance* (SSD) unless the joint distribution of investment returns is a multivariate normal, which is a very restrictive assumption. M-V analysis correctly describes the choices made by investors with a quadratic utility functions. Using variance as a proxy for risk is another well-known drawback of M-V analysis since variance is a measure of uncertainty and not risk measure. However, this deficiency was recognized and corrected by Markowitz by introducing downside semistandard deviation.

We discuss M-V analysis as it is necessary to understand the minimal tracking error problem which is the focus of this report. Here, the expected portfolio return is used as a measure for reward and the variance of portfolio return indicates the level of diversification.

The main principle behind M-V analysis can be summed as follows:

(a) Find a portfolio, with minimum variance, from a list of feasible portfolios with a given lower bound on the expected performance (maximum diversification).

(b) Find a portfolio, with maximum expected performance, from a list of feasible portfolios with a given upper bound on the variance of portfolio return (upper bound on level of diversification).

Also, here are a bunch of expressions that might be useful in our study and understanding of minimal tracking error problem.

1.1.3 A few useful identities

$$mean\ \mu = \frac{\sum_{i=1}^{N} x_i}{N}$$

$$standard\ deviation\ \sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

$$var(x) = \sigma^2(x)$$

$$cov(X, Y) = E(X - \mu_X)(Y - \mu_Y)$$

$$\sigma(X \pm Y) = \sqrt{var(X) + var(Y) \pm 2 * cov(X, Y)}$$

1.2 Active Portfolio Return

Active portfolio return is defined as the difference $r_p - r_b$ in which r_p stands for the return of the portfolio and r_b stands for return of the benchmark.. A measure of the performance of the portfolio relative to the benchmark is the average active return, also known as the portfolio alpha and denoted by α_p. Alpha is calculated as the difference between the average of the observed portfolio returns and the average of the observed benchmark returns,

$$\hat{\alpha}_p = \bar{r}_p - \bar{r}_b, \qquad (1)$$

where $\hat{\alpha}_p$ denoted estimated alpha.

$\bar{r}_p = \frac{1}{k}\sum_{i=1}^{k} r_{pi}$ denotes the average of the observed portfolio returns r_{p1}, r_{p2}, ..., r_{pk}.

$\bar{r}_b = \frac{1}{k}\sum_{i=1}^{k} r_{bi}$ denotes the average of the observed benchmark returns r_{b1}, r_{b2}, ..., r_{bk}.

1.3 Tracking Error

We can also use the standard deviation of the active portfolio return, $\sigma(r_p - r_b)$, to measure the closeness of the portfolio returns to the benchmark returns. This term is known as **tracking error**. There are two types of tracking errors: ex post or backward-looking tracking error and ex ante or forward-looking tracking error. When a tracking error is calculated using historical observations, it is called **backward-looking tracking error**. In the specified historical period, if portfolio returns and benchmark returns are equal, i.e., $r_{pi} = r_{bi}$ for all i, then the observed active return is zero and hence the tracking error is zero. Intuitively, the closer the tracking error is to zero, the risk profile of the portfolio is closer to the risk profile of the benchmark.

In the ex ante case, portfolio alpha is the mathematical expectation of the active return,

$$\alpha_p = E(r_p - r_b)$$

$$= w'\mu - Er_b$$

where $r_p = w'X$ in which w is the vector of portfolio weights, X is a random vector describing future asset returns, and $\mu = EX$ is a vector of the expected asset returns. The tracking error equals the standard deviation of the active return,

$$TE(w) = \sigma(r_p - r_b)$$

where $\sigma(Y)$ denotes the standard deviation of the random variable Y. Tracking error in this case is referred to as ex ante or **forward-looking tracking error**.

1.4 Active and Passive Strategies

In an active strategy, an investor uses financial and economic indicators along with various other tools to forecast the market and achieve higher gains whereas in a passive strategy, an investor invests in accordance with a pre-determined strategy that doesn't entail any forecasting, commonly mimicing the performance of an externally specified index .

If an **active strategy** is followed, then the goal of the portfolio manager is to gain a higher alpha at the cost of deviating from the bechmark portfolio's risk profile, thus, obtaining a higher forward-looking error.

However, if a **passive strategy** is followed, then the general goal is to construct a portfolio such that the forward-looking tracking error is as small as possible in order to match the benchmark portfolio's risk profile. Therefore, the alpha gained is slightly different from zero. As a result, passive strategies are characterized by very small alphas and very small forward-tracking errors.

1.5 Enhanced Indexing

In order to avoid the two extremes, we have strategies that are in the middle between the active and the passive strategies called **enhanced indexing**. A portfolio constructed by such a strategy allows the risk profile of the benchmark portfolio to be close but not identical to the risk profile of the constructed portfolio. Hence, enhanced indexing strategies have small to medium-ized forward-looking tracking errors and small to medium-sized alphas.

Chapter 2: Minimal Tracking Error Problem

The classical **minimal tracking error problem** is as follows

$$\begin{aligned} &min_w \quad \sigma(r_p - r_b) \\ &subject\ to\ w'e = 1 \\ &\qquad w'\mu - Er_b \geq R_* \\ &\qquad w \geq 0 \end{aligned}$$

where r_p is return of the portfolio, r_b is return of the benchmark, $r_p - r_b$ is active portfolio return, and e is a column vector of ones. R_* denoted the lower bound of the expected alpha. Thus, the goal is to find a portfolio closest to the benchmark while setting a limit on the expected alpha. The specific problem for this project is solving the above optimization problem with $\hat{\sigma}(r_p - r_b)$ as the objective function and setting R_* to zero.

2.1 Optimization Problem

Thus we need to solve the following optimization problem:

$$\begin{aligned} &min_w \quad \hat{\sigma}(r_p - r_b) \\ &subject\ to\ w'e = 1 \\ &\qquad w'\mu - Er_b \geq 0 \\ &\qquad w \geq 0 \end{aligned}$$

2.2 Data Set and Data Cover

Our dataset includes 10 randomly selected stocks from the S&P 500 universe and the benchmark is the S&P 500 index. The data covers one-year period from December 31, 2002 to December 31, 2003.

The 10 random stocks chosen in this project are 3M Company(MMM), Akamai Technologies, Inc.(AKAM), Coach, Inc.(COH), The Dow Chemical Company(DOW), H. J. Heinz Company(HNZ), JPMorgan Chase & Co.(JPM), Pfizer Inc.(PFE), Schlumberger Limited(SLB), VeriSign, Inc.(VRSN), and Xerox Corp.(XRX). Also, we have used the values of the adjusted close for calculating the **daily rate of return**

$$r_{it} = (\frac{AC_{it} - AC_{i(t-1)}}{AC_{i(t-1)}})100\%$$

where r_t is the rate of return of the i^{th} stock for day t, AC_t is the adjusted close of the i^{th} stock for day t and AC_{t-1} is the adjusted close of the i^{th} stock for day $t-1$. We also calculate the daily rate of return for the benchmark (ˆ GSPC) in a similar fashion. This is r_b. We begin with an initial portfolio of 10 equally weighted random stocks. Hence, their weights $w_i = 0.1$ initially, for $i = 1$ to 10.

The portfolio's total rate of return for a day t is calculated as the weighted sum of the individual daily rate of returns.

$$r_{pt} = w_1 r_{p1(t)} + w_2 r_{p2(t)} + ... + w_{10} r_{p10(t)} \qquad (2)$$

Refer to Appendix B for the price values of the mentioned stocks.

2.3 Initial Portfolio vs. Benchmark

To plot the 1st figure of the Initial Portfolio vs. Benchmark of Figure 9.4 on page 302 of the book Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization by Rachev, Stoyanov and Fabozzi, we do the following steps.

Step 1: Calculate the daily rate of return of all the ten stocks $r_{p1}, r_{p2}, ..., r_{p10}$.

Step 2: Calculate the daily rate of return of the benchmark r_b.

Step 3: Using (2), calculate the total rate of return for the portfolio, r_p.

Step 4: Center data values r_p and r_b by subtracting the mean of r_p and r_b respectively from each of the corresponding data values so that the new mean is zero.

$$r_p = r_p - \bar{r}_p \text{ and } r_b = r_b - \bar{r}_b.$$

Step 5: Calculate the standard deviation of the rate of return of the portfolio, $\sigma(r_p)$, and of the rate of return of the benchmark, $\sigma(r_b)$.

Step 6: Plot the inverse distribution $F^{-1}(t)$ vs. time t for r_p and r_b.

For the data values specified in Appendix B, we get $\sigma(r_p) = 1.50022$ and $\sigma(r_b) = 1.07302$. The MATLAB code is given in Appendix A and the adjusted close prices of 10 random stocks and $\hat{\text{G}}$SPC for the time period December 31, 2002 to December 31, 2003 in appendix B.

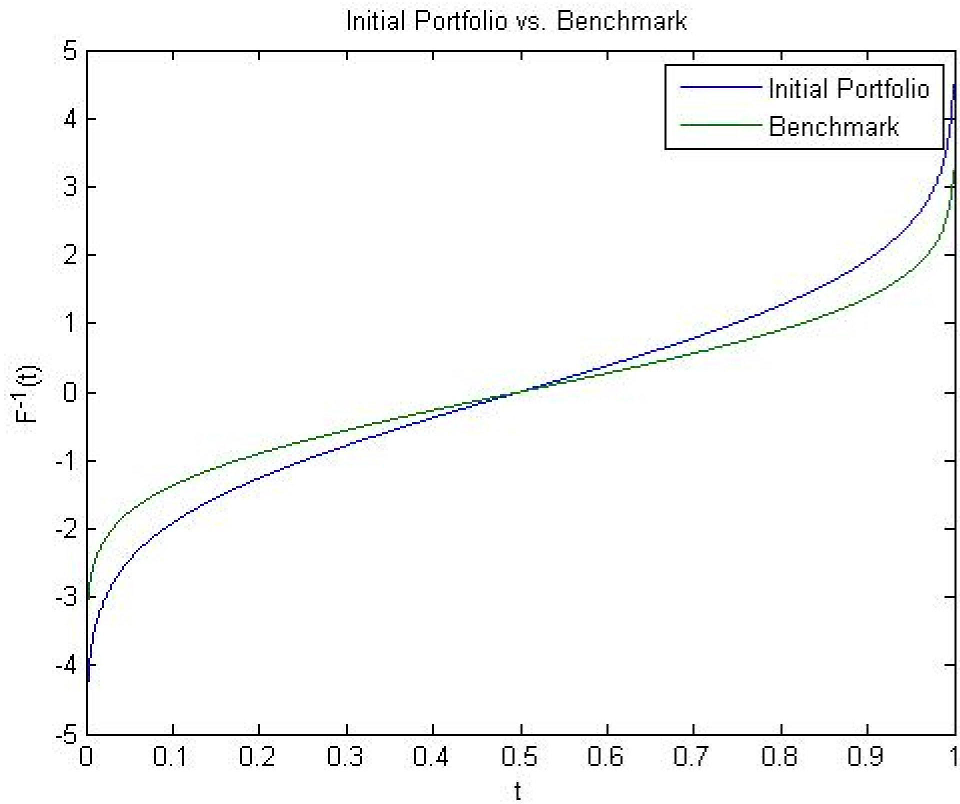

Figure 1 : *The inverse distribution function of the S&P* 500 *index* (*the benchmark*) *and the equally weighted portfolio* (*initial portfolio*)

As we can see from Figure 1, that the inverse distribution function of the initial portfolio is above the inverse distribution function of the benchmark returns close to $t = 1$ meaning the probability of a large positive return of the initial portfolio is larger than that of the benchmark. Also, notice that we have taken the rate of returns in terms of percentages as stated in section 2.2.1, thus, we obtain the range of Y-axis from -5 to 5. If we omit the multiplication by 100, and take rate of return to be just,

$$r_{it} = \frac{AC_{it} - AC_{i(t-1)}}{AC_{i(t-1)}},$$

we would get the range of Y-axis -0.05 to 0.05 as on page 302 in the book.

2.4 Optimal Portfolio vs. Benchmark

Now, we need to optimize our chosen portfolio such that the tracking error is minimized.

$$\sigma(r_p - r_b) = \sqrt{\sigma^2(r_p) + \sigma^2(r_b) - 2 * cov(r_p, r_b)} \tag{3}$$

where $cov(r_p, r_b)$ = covariance of r_p and r_b. We, once again, start from an equally weighted portfolio, i.e., $w_i = 0.1$ for $i = 1, 2, ..., 10$. We would need to introduce new steps to the above steps 1-6. However, the first four steps remain the same. We would need to introduce an optimization step after Step 4 that will minimize the expression (3). Figure 2 plots the inverse distribution for the optimal portfolio and the benchmark obtained by this method. In this case, the standard deviation of the optimal portfolio, $\sigma(r_p) = 1.2460$ whereas the standard deviation of the benchmark remains the same as before, i.e., $\sigma(r_b) = 1.07302$. Thus, we observe that the standard deviation has dropped from 1.50022 of the intial portfolio to 1.2460 of the optimal portfolio obtained by minimizing the tracking error subject to certain constraints.

We observe in Figure 2 below that the inverse distribution curve of the return of the benchmark is closer to the inverse distribution curve of the return on the optimal portfolio than it was to that of the initial portfolio. Also, as in Figure 1, the inverse distribution function of the optimal solution is above the inverse distribution function of the benchmark returns close to $t = 1$ meaning the probability of a large positive return of the optimal portfolio is larger than that of the benchmark.

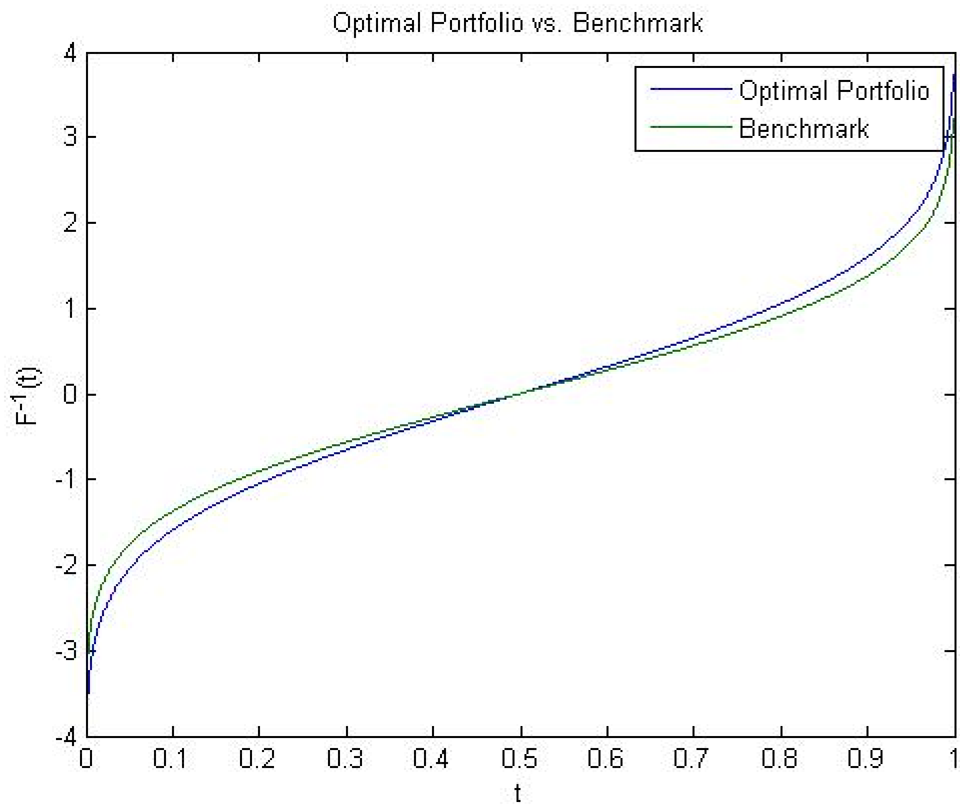

Figure 2: The inverse distribution function of the S&P 500 index (the benchmark) and the optimal portfolio obtained by minimizing the tracking error.

In the following we need to find x, the vector weights for the optimal portfolio stocks. Let us go through the MATLAB code to see what it does:

Step 1: Import the values of adjusted close prices for each day from December 30, 2002 to December 31, 2003 from an .xls file called rawdata.xslx and store it in a 254 x 11 '$adjcl$' matrix. We need the adjusted close price for December 30, 2002 to calculate the rate of return of the day December 31, 2002.

Step 2: Calculate the rate of return for each day for all the ten stocks and the S&P 500 index. Store it in a 253 x 11 'rr' matrix.

Step 3: Center the data by subtracting the mean of the corresponding column from each corresponding column entry. Store this in a 253 x 11 'cr' matrix.

Step 4: We initialize 10 x 1 matrix $x0$ with all 0.1s as our initial portfolio is equally-weighted.

Step 5: We assign $lb = 0$ and $ub = 1$ as the weights x_i satisfy the constraint

$$0 \leq x_i \leq 1 \text{ for } i = 1 \text{ to } 10.$$

Step 6: Aeq is a row vector with all 1's and $beq = 1$ as we need,

$$\sum_{i=1}^{10} x_i = 1$$

Step 7: We construct a 1 x 10 matrix A such that it's first row is the negative of the means of our centered data of the stocks, thus satisfying our constraint

$$Ax \geq b \equiv -Ax \leq -b$$

where $A = \mu$, row vector of the means of returns of the centered data and $b = \mu_b = E(r_b)$, mean of the centered benchmark return data. Therefore, we get the constraint

$$w'\mu - Er_b \geq 0$$

Step 8: Define our objective function as $\sigma(r_p - r_b)$ where r_p is the weighted sum of the columns of matrix cr corresponding to the stocks and r_b is the coulmn in cr corresponding to the S&P 500 index.

The following graph Figure 3 shows how the distance between the initial portfolio and the benchmark decreases when tracking error is minimized.

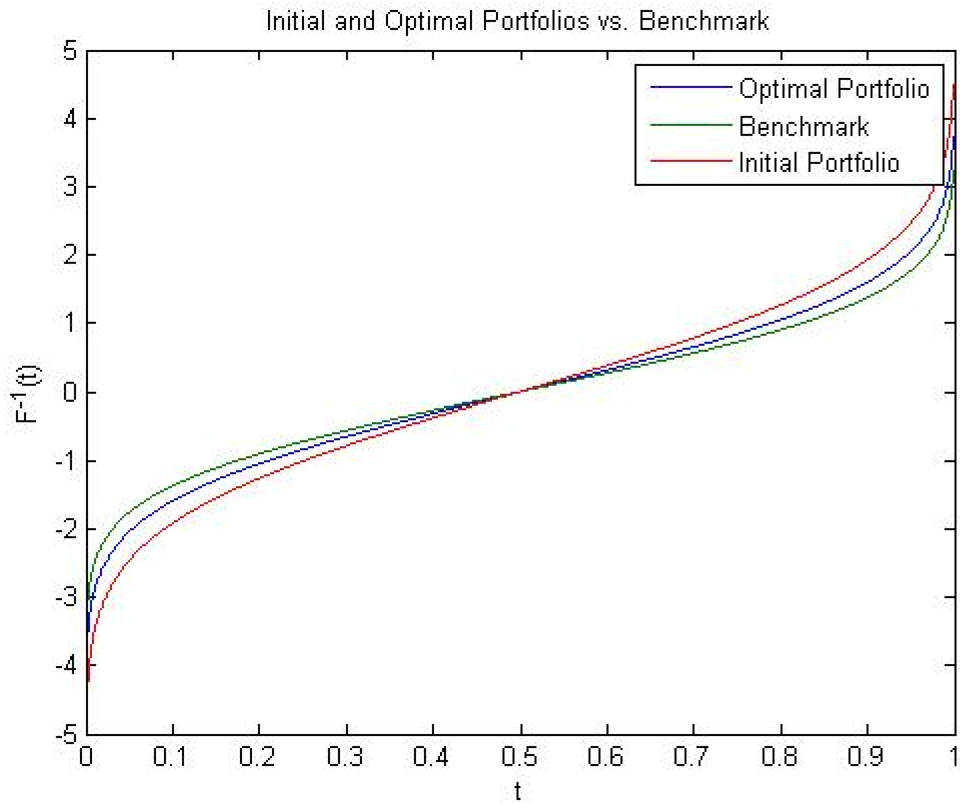

Figure 3 : *The inverse distribution function of the S&P* 500 *index* (*the benchmark*), *the initial portfolio and the optimal portfolio obtained by minimizing the tracking error.*

Chapter 3: Future Work

We need to optimize the portfolio further by using the following expressions as objective function in the classical minimal tracking error problem:

$$\hat{\theta}_p^*(r_{p0}, r_{b0}) \qquad \text{and}$$

$$\hat{l}_p^*(r_{p0}, r_{b0})$$

where functional $\theta_p^*(X, Y)$ is positively homogenous of degree $\frac{1}{p}$, $l_p^*(X, Y)$ is positively homegenous of degree 1 irrespective of the value of p and the index 0 signifies that the corresponding returns are centered. The above functionals are defined as:

$$\theta_p^*(X, Y) = [\int_{-\infty}^{\infty} (max(F_X(t) - F_Y(t), 0))^p dt]^{\frac{1}{p}}, \qquad p \geq 1$$

$$l_p^*(X, Y) = [\int_0^1 (max(F_Y^{-1}(t) - F_X^{-1}(t), 0))^p dt]^{\frac{1}{p}}, \qquad p \geq 1$$

where X and Y are zero-mean random variables, $F_X(t) = P(X < t)$ is the distribution function of X and $F_X^{-1}(t) = inf\{x : F_X(x) \geq t\}$ is the generalized inverse of the distribution function.

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APPENDIX A

AMS512.m

```
adjcl = xlsread('rawdata.xlsx','A2:K255');
for j=1:11
   for k=1:253
        rr(k,j)=(adjcl(k,j)-adjcl(k+1,j))/adjcl(k+1,j);
   end
end
for m = 1:11
    cr(:,m)=rr(:,m)-mean(rr(:,m));
end


x0=[0.1*ones(10,1)];
lb=0;
ub=1;
Aeq=[ones(1,10)];
beq=1;
A=[];
for n=2:11
    A=[A mean(cr(:,n))];
end
%A=[mean(cr(:,2)) mean(cr(:,3)) mean(data(:,4)) mean(data(:,5))
mean(data(:,6)) mean(data(:,7)) mean(data(:,8)) mean(data(:,9))
mean(data(:,10)) mean(data(:,11))];
A=[(-1)*A;eye(10);(-1)*eye(10)];
b=[(-1)*mean(cr(:,1)); ones(10,1); zeros(10,1)];
objfun=@(x) std(cr(:,2:11)*x-cr(:,1));
options = optimset('Algorithm','active-set');
```

```
options=optimset(options,'MaxFunEvals',1000);
[x,fval,exitflag,output]=fmincon(objfun,x0,A,b,Aeq,beq,lb,ub,[],o
ptions)
for i=1:10
    if x(i)<0.0001
       x(i)=0;
    end
end
initport=cr(:,2:11)*x0
mui=0;
sdi=std(initport);
iyi = -3*sdi:1e-3:3*sdi; %covers more than 99% of the curve
ixi = cdf('normal', iyi, mui, sdi);
mub=0;                   %mean of benchmark is zero as data is
centered
sdb=std(cr(:,1));      %standard deviation of benchmark
iyb = -3*sdb:1e-3:3*sdb; %covers more than 99% of the curve
ixb = cdf('normal', iyb, mub, sdb);
figure(1);
plot(ixi, iyi, ixb, iyb);
xlabel('t');
ylabel('F^{-1}(t)');
title('Initial Portfolio vs. Benchmark');
legend('Initial Portfolio','Benchmark');
optport=cr(:,2:11)*x;
mup=0;                   %mean of portfolio is zero as data is
centered
```

```
sdp=std(optport);        %standard deviation of optimal portfolio
iyp = -3*sdp:1e-3:3*sdp; %covers more than 99% of the curve
ixp = cdf('normal', iyp, mup, sdp);
figure(2);
plot(ixp, iyp, ixb, iyb);
xlabel('t');
ylabel('F^{-1}(t)');
title('Optimal Portfolio vs. Benchmark');
legend('Optimal Portfolio','Benchmark');
figure(3);
plot(ixp, iyp, ixb, iyb, ixi, iyi);
xlabel('t');
ylabel('F^{-1}(t)');
title('Initial and Optimal Portfolios vs. Benchmark');
legend('Optimal Portfolio','Benchmark','Initial Portfolio');
```

APPENDIX B

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
1111.92	69.25	10.76	18.19	31.18	27.43	29.26	25.09	24.88	15.06	12.69
1109.64	68.64	11.08	17.86	30.8	27.42	29.16	24.91	25.25	15.15	12.55
1109.48	68.82	11.27	17.99	31.51	27.43	29.1	24.86	25.31	15.35	12.56
1095.89	68.58	10.95	18.08	30.99	27.18	28.86	24.68	25.13	14.88	12.5
1094.04	68.61	10.92	17.79	30.87	27.25	28.78	24.66	24.99	15.04	12.69
1096.02	68.91	11.14	17.93	30.71	27.3	28.82	24.52	24.55	14.88	12.61
1092.94	69.43	10.97	17.56	30.67	27.13	28.76	24.36	24.85	14.54	12.12
1088.66	68.47	11.2	17.83	30.18	27.13	28.5	24.34	24.16	14.69	11.95
1089.18	67.95	11.35	17.57	30.32	26.96	28.4	24.46	23.98	14.65	11.98
1076.48	67.97	10.95	17.03	29.8	26.83	28.41	24.4	22.92	14.16	11.42
1075.13	67.58	10.82	16.66	29.99	26.86	28.46	24.58	22.9	14.36	11.12
1068.04	66.78	11.16	17.08	29.68	27.07	28.09	24.46	22.82	14.33	10.95
1074.14	66.35	11.42	17.49	29.66	26.94	27.95	24.43	23.07	14.65	11.02
1071.21	66.16	11.37	17.28	29.54	26.92	27.87	24.63	22.68	14.64	11.06
1059.05	65.92	10.54	17.04	29.39	26.86	27.62	24.22	22.64	14.11	10.6
1060.18	66.8	10.35	17.35	29.65	27.05	27.75	24.43	22.56	14.3	10.84
1069.3	66.74	10.98	17.35	29.13	27.36	28.25	24.46	22.52	14.57	11.08
1061.5	65.76	11.5	17.69	28.76	27.24	27.85	24.29	22.31	14.66	11.15
1069.72	66.21	11.71	17.88	29.24	27.31	28.19	24.41	21.82	14.99	11.48
1064.73	65.96	13.29	18.51	29.04	27.24	28.52	24.11	21.4	15.03	11.23
1066.62	65.93	13.79	18.79	28.62	27.16	28.53	24.17	21.68	15.24	11.03
1070.12	66.47	13.6	19.24	28.65	27.01	28.41	24.27	21.17	15.18	11.21
1058.2	64.37	13.4	19.2	27.94	26.98	28.2	23.84	21.25	14.98	11.2
1058.45	64.57	13.03	18.83	28.17	26.98	28.29	24.03	21.25	14.74	10.99
1053.89	64.48	13.11	18.5	27.75	27.04	28.31	23.99	21.12	14.89	10.52
1052.08	64.79	12.83	18.46	27.61	26.65	28.1	24.07	20.97	15.05	9.76
1035.28	63.13	11.37	17.46	27.09	26.74	27.88	23.57	20.96	14.24	9.69
1033.65	62.55	11.24	16.73	27.26	26.8	27.59	23.9	21.05	14.26	9.59
1042.44	62.59	11.2	17.17	27.39	26.98	27.69	24.45	21.08	14.25	9.51
1034.15	62.61	11.04	17.59	27.31	26.86	27.8	24.28	20.89	13.81	9.35
1043.63	63.41	11.37	17.7	27.7	26.98	28.21	24.2	21.28	14.46	9.56
1050.35	63.68	10.97	18.17	27.79	26.82	28.25	24.21	21.56	14.91	9.7
1058.41	64.59	11.2	18.24	27.55	26.62	28.57	23.76	21.57	15.57	9.67
1058.53	63.98	11.01	18.26	27.68	26.49	28.59	23.01	21.15	15.83	9.79
1046.57	63.66	10.44	17.55	27.45	26.15	28.71	22.48	20.97	15.25	9.46
1047.11	63.93	11.04	17.41	27.67	26.18	28.59	22.25	20.88	15.4	9.45
1053.21	64.1	10.3	17.57	28.01	26.62	28.71	22.46	21.12	15.77	9.65
1058.05	64.55	9.08	17.59	28.27	26.56	28.93	22.52	21.13	15.63	9.7
1051.81	64.2	9.1	17.61	28.06	26.36	29.1	22.37	21.15	15.94	9.8
1053.25	64.47	9.42	17.55	27.99	26.42	29.16	22.23	21.07	15.49	9.67

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
1059.02	63.89	9.31	17.8	28.19	26.61	29.25	22.35	20.91	15.76	9.77
1050.71	63.96	7.9	17.09	28.05	26.4	28.6	22.34	21.28	14.69	9.65
1046.94	63.3	8	17.23	28.2	26.28	28.55	21.83	20.95	14.69	9.73
1048.11	62.9	5.96	17.43	27.68	26.05	28.62	22.11	21.06	14.66	9.84
1046.79	62.64	5.99	17.02	27.41	26.1	28.32	22.27	21.4	14.84	9.88
1031.13	61.42	5.87	16.63	27.5	26.14	27.84	21.89	21.3	14.12	9.74
1028.91	61.37	5.76	16.22	27.23	26.22	27.7	21.8	21.34	13.33	9.83
1033.77	61.57	5.9	16.26	26.95	26.26	27.88	21.92	21.74	13.52	10.11
1030.36	61.48	5.88	16.1	25.93	26.3	27.87	21.65	21.72	13.8	10.6
1046.03	61.23	6.35	16.32	26.69	26.32	29.21	22.27	23.27	14.47	10.57
1044.68	61.21	5.79	15.15	26.45	26.42	29.16	21.92	22.58	13.95	10.56
1039.32	60.05	5.45	15.15	26.19	26.47	28.95	21.6	22.42	14.02	10.21
1050.07	59.68	5.79	15.04	26.16	26.59	29.35	21.83	22.67	14.29	10.33
1046.76	59.77	5.56	15.19	26.03	26.59	29.16	21.63	22.31	13.68	10.23
1049.48	60.55	5.75	14.81	25.77	26.65	29.3	21.83	22.72	13.83	10.13
1045.35	60.43	5.49	14.36	25.7	26.64	29.03	21.68	22.86	13.72	10.37
1038.06	60.04	5.07	14.26	25.41	26.49	28.3	21.74	23.01	13.61	10.22
1038.73	60.12	5.18	14.4	25.46	26.56	28.44	21.89	22.79	13.48	9.93
1033.78	58.95	5.2	13.87	25.4	26.5	28.37	21.68	22.88	13.27	9.82
1039.25	59.44	4.7	14.02	25.26	26.58	28.75	21.67	22.86	13.1	10.1
1034.35	59.15	4.6	14.29	25.56	26.45	28.32	21.74	22.81	13.06	9.88
1029.85	59.22	4.63	14.12	25.64	26.38	28.02	21.76	22.69	12.92	9.81
1020.24	57.8	4.47	13.7	24.94	26.31	27.9	21.95	22.5	12.61	9.57
1018.22	57.5	4.27	13.69	24.78	25.85	27.92	21.77	22.41	12.54	9.54
995.97	56.01	4.29	13.16	24.21	25.62	27.09	21.48	21.92	12.44	9.43
1006.58	56.98	4.49	13.05	24.6	25.48	27.34	21.84	22.28	12.43	9.62
996.85	58.16	4.6	13.08	24.25	25.39	26.98	21.6	22.07	11.9	9.37
1003.27	57.42	4.71	13.5	24.04	25.38	27.21	21.58	22.37	12.2	9.5
1009.38	57.23	5.05	13.61	24.08	25.57	27.51	21.84	23.07	12.68	9.67
1029.03	57.88	5.22	13.92	24.31	25.59	27.97	22.34	22.97	13.57	9.9
1022.82	57.62	5.12	13.98	24.22	25.26	27.61	22.01	23.13	13.58	9.74
1036.3	57.57	5.12	14.14	24.71	25.59	28	22.23	23.1	14.17	9.79
1039.58	57.52	5.1	14.13	24.78	25.5	28.2	22.66	22.31	14.64	9.9
1025.97	56.87	5.14	14.22	24.61	25.35	27.35	22.7	21.86	14.51	9.33
1029.32	57.09	4.79	14.2	24.89	25.26	27.3	22.62	21.92	14.61	9.56
1014.81	56.13	4.54	13.95	24.71	25.19	26.8	22.52	21.79	14.08	9.38
1018.63	56.05	4.65	13.81	24.88	25.21	26.83	22.55	21.89	13.93	9.53
1016.42	56.32	4.59	13.72	24.38	25.19	26.57	22.51	21.79	14.69	9.52
1010.92	55.39	4.32	13.52	24.17	25.11	26.34	22.46	21.8	14.18	9.68
1023.17	55.72	4.51	13.48	24.78	24.99	27.03	22.4	22.18	14.59	9.7

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
1031.64	55.95	4.71	13.76	24.97	25.08	27.34	22.25	22.42	14.85	9.82
1021.39	56.23	4.65	13.83	24.9	24.98	26.98	21.57	21.95	14.2	9.68
1027.97	56.77	4.88	14.21	25.19	24.99	27.36	21.63	21.64	14.05	9.95
1026.27	57.82	4.89	14.28	25.67	24.81	27.5	21.83	22.12	13.75	10.07
1021.99	56.83	4.47	14	25.65	24.46	27.38	21.53	22.34	14.17	9.98
1008.01	57.77	4.16	13.99	25.43	23.99	27	21.15	22.43	13.84	9.91
1002.84	57.68	4.12	13.82	25.39	23.81	26.76	21.07	22.33	13.84	9.68
996.79	57.96	4.09	13.67	25.17	23.71	26.24	21.14	21.46	13.44	9.59
996.73	58.44	4.1	13.26	24.97	23.8	26.34	21.22	21.62	13.08	9.55
993.71	58.13	3.94	13.24	25.11	24.21	26.62	21.12	21.5	13.01	8.77
993.06	58.19	4.01	13.37	25.26	23.81	26.76	20.89	21.86	13.14	8.93
1003.27	58.67	4.24	13.62	25.68	24.06	27.3	21.06	22.23	13.15	9.14
1000.3	58.31	4.07	13.45	25.4	24.16	27.48	21.73	22.06	13.39	9.24
1002.35	58.3	4.06	13.2	25.36	24.15	26.98	22.11	21.74	13.09	9.27
999.74	58.26	3.95	13.2	25.12	24.29	26.87	22.25	21.64	12.79	9.28
990.67	57.58	3.99	13.23	24.89	24.44	26.56	22.21	21.32	12.47	9.32
990.51	57.37	4.13	12.99	24.95	24.55	26.75	21.99	21.44	12.34	9.38
984.03	58.32	4	13.19	24.8	24.51	26.38	22.31	21.38	12.8	9.25
990.35	58.11	3.91	12.98	25	24.61	26.54	23.13	21.26	12.4	9.3
980.59	57.27	3.9	12.74	25	24.6	26.21	23.16	21.17	11.9	9.3
977.59	56.49	3.7	12.93	25.15	24.75	26.03	23.2	21	11.54	9.38
974.12	56.04	3.75	12.51	25.04	24.62	25.92	23.18	21.01	11.47	9.4
967.08	55.78	3.25	12.26	25.04	24.75	26.03	22.84	20.53	11.56	9.42
965.46	56.08	3.41	12.59	25.23	24.9	25.85	22.87	20.14	11.99	9.32
982.82	56.55	3.88	12.72	25.69	25.22	26.61	23.27	20.36	11.65	9.32
980.15	56.11	3.97	12.79	25.74	25.21	26.32	23	20.54	11.96	9.79
990.31	56.59	4.58	12.77	26	25.25	27.65	23.48	20.34	12.35	9.93
987.49	56.26	5.1	12.85	25.72	25.35	27.66	23.5	20.26	11.94	10.08
989.28	56.32	5.13	13.06	25.81	25.31	27.8	22.92	20.23	12.21	10.1
996.52	56.9	5.24	13.34	25.85	25.4	28	23.07	20.83	11.9	10.27
998.68	57.07	4.99	12.95	25.3	25.29	27.99	23.25	20.64	11.51	9.87
981.6	56.24	4.95	12.8	24.86	25.15	27.28	22.91	20.32	11.62	9.77
988.61	56	4.72	12.92	23.53	24.87	27.5	23.17	20.54	11.98	9.84
988.11	55.56	4.95	12.97	23.5	24.84	27.57	22.82	20.94	11.83	10.01
978.8	55.03	4.92	12.9	22.84	24.62	27.17	22.9	21.27	12.01	9.61
993.32	52.54	5.16	12.69	23.02	24.91	27.88	23.5	21.42	12.47	9.9
981.73	51.39	5.05	12.71	22.54	24.58	27.65	23.05	20.73	12.42	9.85
994.09	51.51	5.44	13.03	22.48	24.69	28.6	23.47	20.62	13.07	10.07
1000.42	51.7	5.59	13.24	22.77	24.68	29.42	23.83	20.83	13.97	10.56
1003.86	52.05	5.64	13.31	23.03	24.57	29.43	23.81	20.92	14.13	10.34

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
998.14	52.16	5.27	13.33	22.74	24.72	28.21	24.07	21.34	13.28	9.93
988.7	52.03	5.28	13.11	22.49	24.58	27.62	23.69	21.05	13.31	9.94
1002.21	52.2	5.3	13.31	22.96	24.67	28.14	24.08	21.69	14.42	10.11
1007.84	52.39	5.29	13.53	22.98	24.69	27.93	24.14	21.47	14.64	10.1
1004.42	52.52	5.05	12.73	23.02	24.59	27.67	24.21	21.44	13.75	10.05
985.7	51.82	4.71	12.27	22.54	24.42	26.93	24.09	21.68	13.21	9.79
993.75	52.43	4.79	12.33	22.93	24.65	27.12	24.38	21.67	13.14	10.06
982.32	52.02	4.47	12.27	22.77	24.56	26.99	24.3	21.59	13.15	9.81
974.5	52.06	4.78	11.99	22.8	24.45	26.7	24.03	21.47	12.74	9.74
976.22	51.91	4.94	12.03	22.75	24.32	26.48	24.63	21.52	12.89	9.79
985.82	52.59	4.91	12.24	23.35	24.42	26.87	25.33	21.55	13.14	9.78
975.32	52.19	4.77	12.11	22.93	24.52	26.55	24.86	21.73	12.5	9.65
983.45	52.44	4.73	11.77	23.06	24.95	26.79	24.91	21.44	11.98	9.81
981.64	52.3	5.04	11.81	22.84	24.69	26.44	24.9	21.32	12.38	9.56
995.69	52.78	5.58	12.24	23.17	25.06	27.18	25.03	21.33	12.91	9.52
994.7	53.04	5.75	12.28	23.14	24.64	26.91	25.05	21.6	13.32	9.56
1010.09	53.17	5.52	12.24	23.19	25.07	27.4	25.37	21.32	14.02	10.2
1011.66	52.5	5.27	12.01	23.17	25.13	28.19	25.46	21.44	14.09	10.45
1010.74	52.66	4.83	12.49	23.27	25	28.08	24.35	21.68	13.8	10.56
988.61	51.58	5.1	12.45	23.06	24.64	27.11	23.28	21.75	12.99	10.16
998.51	51.81	5.12	12.44	23.35	24.87	27.11	23.59	22.27	13.28	10.64
997.48	51.65	5.03	12.33	23.03	25.3	26.98	23.42	22.46	13.14	9.84
984.84	50.94	4.56	12.25	23.19	24.67	26.62	23.44	21.71	13.16	9.81
975.93	50.94	4.14	12.38	23.06	24.75	26.37	23.34	21.5	13.21	9.97
987.76	51.02	4.42	12.68	23.22	25	27.44	23.29	21.38	13.79	10.16
990.14	50.72	4.47	12.74	23.32	24.76	27.42	23.05	21.37	14.12	10.53
986.24	51.26	3.9	12.32	23.39	24.83	27.1	22.8	21.28	14.54	10.51
971.56	50.93	3.6	12.16	23.17	24.42	26.42	22.3	21.62	14.6	10.08
967	50.45	3.69	12.25	23.1	24.39	26.5	21.82	21.82	13.67	10.22
963.59	51.04	3.66	11.84	23.17	24.32	25.67	21.83	21.86	13.84	10.05
949.64	49.77	3.65	11.37	22.85	23.96	25.21	22.01	21.27	13.56	10.09
953.22	50.46	3.67	11.41	23	23.92	25.43	22.34	21.48	13.25	9.99
951.48	50.9	3.75	10.99	23.06	23.67	25.04	22.5	21.77	13.31	9.93
933.22	49.85	3.64	10.77	22.63	23.31	24.14	22.43	21.21	12.66	9.69
931.87	50.36	3.64	10.84	22.55	23.29	23.82	22.63	21.19	12.2	9.86
923.42	49.75	3.4	10.78	22.73	23.11	23.86	22.17	21.35	12.09	9.53
919.73	50.09	3.33	10.41	22.93	22.94	23.77	21.89	20.95	11.49	9.7
920.77	49.78	3	10.39	22.48	22.94	23.76	22.38	21.11	11.8	9.56
944.3	50.37	3.37	10.9	23.28	23.03	24.41	23.65	21.31	12.47	9.88
946.67	50.53	3.71	11.02	23.3	23.09	24.25	23.6	21.07	12.25	9.84

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
939.28	49.45	3.83	11.3	23.23	22.97	23.82	23.37	21.06	11.8	9.53
942.3	49.57	3.66	11.26	23.31	22.94	24	23.32	20.66	11.64	9.55
945.11	49.96	3.49	11.25	23.7	23	24.17	23.29	20.46	11.09	9.52
933.41	49.31	3.32	10.96	23.3	22.67	23.5	22.87	20.22	10.88	9.4
920.27	49.41	3.48	10.76	22.75	22.17	23.4	22.35	19.97	10.56	9.27
929.62	48.78	3.5	10.89	23.11	22.44	24.08	22.31	20.26	10.81	9.45
934.39	49.31	3.27	10.9	23.39	22.49	23.99	22.45	20.1	11.35	9.56
926.55	50.11	3.26	10.7	23.48	22.14	23.72	22.35	19.63	11.18	9.34
930.08	50.65	2.56	10.63	23.53	22.1	23.33	22.1	19.5	11.05	9.19
916.3	50.14	2.39	10.37	23.34	21.85	22.87	21.71	18.94	10.72	9.2
916.92	50.6	2.42	10.48	23.78	21.97	22.93	21.54	18.85	11.47	9.07
917.84	50.98	2.42	10.46	23.54	22.12	22.76	21.58	18.57	11	9.04
914.84	50.91	2.45	10.36	23.36	22.03	22.51	21.51	18.77	10.41	8.95
898.81	49.34	2.3	9.92	22.49	21.78	21.84	21.28	18.66	10.04	8.71
911.43	50.47	2.17	10.17	23.19	21.94	22.31	21.71	18.82	9.36	8.84
919.02	51.31	2.74	10.29	22.94	22.2	23	21.82	18.73	9.92	9.15
911.37	52.09	2.02	10.25	22.63	22.16	22.4	22.29	17.82	9.83	8.42
892.01	52.12	1.66	9.28	22.12	21.81	21.44	22.14	17.65	9.43	8.6
893.58	52.18	1.45	9.19	22.08	21.98	21.33	21.97	17.48	9.55	8.28
879.91	51.79	1.4	8.93	21.64	21.69	20.7	21.97	16.82	9.43	8.24
890.81	53.65	1.4	9.22	21.91	22.13	20.99	22.56	16.97	9.08	8.27
885.23	53.49	1.42	9.26	21.02	22.14	20.73	22.5	17.24	8.83	8.25
868.3	53.36	1.4	9.28	20.47	21.89	19.99	22.07	17.11	8.51	7.96
871.58	53.31	1.41	9.39	20.65	21.92	20.09	22.24	17.41	8.48	8.01
865.99	52.84	1.45	9.23	20.33	21.87	20.1	22.21	16.95	8.48	8.05
878.29	53.59	1.49	9.4	20.88	21.89	20.3	22.66	16.88	8.71	8.06
879.93	53.22	1.49	9.54	21.09	21.77	20.03	22.77	16.94	8.63	8.31
878.85	53.79	1.43	9.29	21.01	21.71	19.98	22.98	16.96	8.7	8.19
876.45	53.74	1.45	9.36	20.48	21.69	19.48	22.63	17.01	8.91	8.21
880.9	53.66	1.5	9.28	20.77	21.78	19.61	22.75	17.23	8.7	8.2
858.48	52.53	1.41	9.11	20.25	21.36	18.66	22.34	17.42	7.97	8.15
848.18	52.2	1.41	9.24	20.12	21.47	18.26	21.83	17.09	8.08	8
863.5	52.4	1.49	9.23	20.61	21.77	18.83	22.28	17.54	8.25	8.05
868.52	52.86	1.45	9.28	20.59	22.02	18.58	22.27	17.27	8.33	8.21
869.95	52.89	1.36	9.28	20.91	21.78	18.62	22.21	17.13	8.42	8.28
874.74	53.12	1.41	9.57	20.52	21.88	18.59	22.42	17.17	8.05	8.16
864.23	52.42	1.36	9.12	20.58	21.8	18.2	21.71	17.1	7.85	8.06
895.79	53.95	1.45	9.62	21.62	22.17	18.99	22.39	17.5	8.31	8.28
875.67	52.44	1.5	9.4	20.52	21.98	18.12	21.58	17.95	8.18	8.28
874.02	52.08	1.5	9.31	20.52	22.33	17.97	21.57	17.66	8.06	8.28

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
866.45	52.18	1.55	9.28	20.78	22.08	17.57	21.03	17.76	8.22	8.28
862.79	51.99	1.48	9.35	20.43	22.41	17.38	20.81	17.12	7.9	8.3
833.27	50.41	1.36	8.91	19.64	22.05	16.37	20.28	16.95	7.45	8.05
831.9	50.36	1.39	9	19.36	21.77	16.73	20.51	17.07	7.31	8.16
804.19	48.76	1.32	8.88	18.33	21.5	15.98	20.43	16.87	6.67	7.93
800.73	48.58	1.33	8.71	18.12	21.65	15.98	20.32	17.09	6.7	7.86
807.48	48.85	1.32	8.63	18.26	21.46	16.51	20.53	17.55	6.87	7.9
828.89	49.98	1.36	8.81	18.74	21.82	17.17	21	18.04	7	8.04
822.1	49.57	1.33	8.65	18.47	21.74	16.94	20.59	18.41	7.1	8.09
829.85	50.22	1.4	8.52	19.03	21.84	17.57	20.45	18.06	6.97	7.89
821.99	49.66	1.4	8.39	18.69	21.89	17.25	20.06	18.21	6.87	7.94
834.81	50.09	1.5	8.54	19.38	22.3	17.6	20.58	18.59	7.07	7.99
841.15	50.33	1.53	8.61	19.66	22.38	17.47	20.89	18.7	7.12	8.28
837.28	50.66	1.58	8.31	19.43	22.52	17.45	20.6	18.37	6.87	8
827.55	50.1	1.6	8.18	19.31	22.44	17.21	20.25	18.68	6.55	7.96
838.57	50.47	1.54	8.19	19.08	22.65	17.37	20.1	18.34	6.76	8
832.58	50.19	1.4	7.64	19.42	22.63	17.18	20.09	19.01	6.74	7.9
848.17	50.93	1.46	8.01	20.28	22.92	17.62	20.27	18.86	7.16	8.18
837.1	50.12	1.38	7.84	19.81	22.71	17.48	20.06	18.21	6.9	8.22
845.13	50.53	1.44	7.81	20.23	23.06	17.33	20.38	17.91	6.98	8.51
851.17	50.57	1.43	7.9	20.52	23.07	17.33	20.6	17.81	6.82	8.39
834.89	49.96	1.33	7.66	19.93	22.8	16.83	20.01	17.39	6.61	7.9
817.37	48.89	1.23	7.32	19.69	22.44	16.4	20.01	16.99	6.75	7.76
818.68	48.82	1.23	7.42	19.84	22.57	16.57	20.14	17.16	6.67	7.72
829.2	49.16	1.27	7.55	20.13	22.62	16.94	20.45	17.45	7.07	7.9
835.97	49.39	1.35	7.57	20.47	23.14	17.02	20.72	17.47	7.13	7.95
829.69	48.92	1.42	7.51	20.33	23.08	16.93	20.53	16.78	7.12	7.89
838.15	49.13	1.27	7.78	20.51	23.36	17.18	20.86	17.1	7.31	8.04
843.59	49.18	1.26	7.86	20.73	23.54	17.67	20.79	17.05	7.34	8.05
848.2	49.16	1.23	7.68	20.61	23.45	17.75	20.9	17.14	7.49	8.06
860.32	49.92	1.31	7.76	20.65	23.52	18.16	21.32	16.86	7.75	8.18
855.7	49.74	1.33	7.7	20.93	23.55	17.98	21.27	16.87	7.62	8.14
844.61	49.49	1.39	7.76	20.16	22.89	17.75	20.81	16.6	7.64	8.28
864.36	50.56	1.43	7.7	20.95	23.21	18.1	21.4	16.85	8.08	8.56
858.54	50.25	1.36	7.64	20.89	23.67	18.11	21.02	16.39	8.2	8.69
847.48	49.96	1.31	7.56	20.75	23.57	17.94	20.88	16.14	7.89	7.4
861.4	50.46	1.4	7.73	21.24	23.86	18.34	21.19	16.71	8.26	7.59
887.34	51.51	1.5	7.87	21.93	24.38	19.36	21.72	17.2	9.7	7.97
878.36	50.92	1.48	7.83	21.78	24.62	19.02	21.51	17.03	9.32	8.07
887.62	50.18	1.49	7.03	22.01	24.97	19.58	21.37	17.33	8.96	7.87

^GSPC	MMM	AKAM	COH	DOW	HNZ	JPM	PFE	SLB	VRSN	XRX
901.78	50.45	1.54	7.29	22.15	25.02	20.17	21.04	18.21	8.65	7.97
914.6	50.58	1.7	7.5	22.43	24.94	20.63	20.98	18.73	9.31	8.53
918.22	50.73	1.76	7.41	22.14	24.64	20.86	21.33	18.48	9.76	8.32
931.66	51.11	1.81	7.31	22.37	24.84	21.25	21.63	18.28	9.74	8.19
926.26	50.54	1.77	7.7	22.18	24.89	21.01	21.54	18.45	9.51	8.05
927.57	50.57	1.77	8.05	22.1	24.51	20.86	21.47	18.69	9.67	7.95
927.57	51	1.78	8.04	22.04	24.88	21.03	21.71	18.72	9.79	7.71
909.93	50.2	1.66	8.11	21.83	24.45	20.62	21.54	18.39	8.92	7.6
922.93	50.75	1.74	8.33	22.21	24.52	21.44	22.05	18.21	8.88	7.64
929.01	50.8	1.86	8.14	22.24	24.72	21.55	22.31	18.97	8.1	7.76
908.59	50.43	1.9	8.01	21.86	24.2	19.98	22.16	19.15	7.9	7.61
909.03	50.62	1.88	8.13	21.93	24.13	19.59	21.98	19.28	7.84	7.5
879.82	49.24	1.73	7.93	21.39	23.96	18.22	21.31	18.83	7.41	7.4
879.39	49.1	1.77	7.94	21.21	23.99	18.22	21.27	18.66	7.4	7.19