

A MUST KNOW
USE OF **GCD**
IF YOU ARE
LEARNING
DSA



Mayank

01

SLOPE OF POINTS

You are given an array `points[]` of n points on cartesian plane

Each `points[i]` is represented by a pair<int, int> of it's x and y co-ordinate

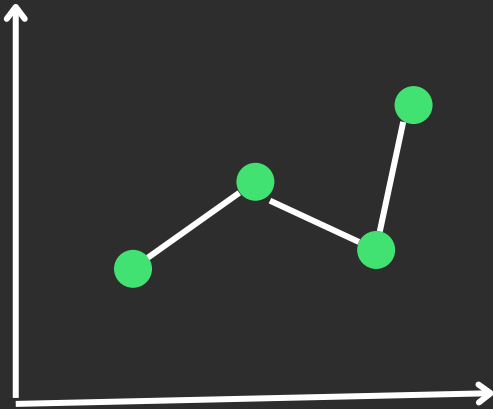
These points are also sorted in order of their x co-ordinate

You need to give minimum count of lines required to represent all these points in form of a histogram

Think! How to approach !!!

02

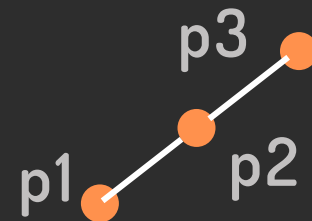
$$\text{SLOPE} = (Y_2 - Y_1) / (X_2 - X_1)$$



Suppose you are given co-ordinates like in the diagram & we can see we need atleast 3 lines to connect all these points

Suppose there are 3 adjacent points p_1 , p_2 , p_3 , when can we tell if they lie on a same line or different ??

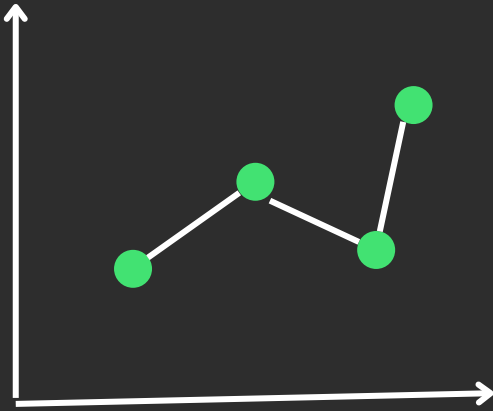
A line is made by connecting 2 points



So if slope of lines connecting p_1 - p_2 and p_2 - p_3 is same then we can say that these 3 points lie on same line

03

$$\text{SLOPE} = (Y_2 - Y_1) / (X_2 - X_1)$$



Suppose you are given co-ordinates like in the diagram & we can see we need atleast 3 lines to connect all these points

Suppose out of 3 points p_1, p_2, p_3 slope of line p_1-p_2 comes out to be $15/17$

Now $15/17 = 0.8823529.....$

Can you see there would be loss of precision even if we use double data type (c++) to store the slope as double can be precise upto 6 digits (by default)

So calculating slope with this formula is not feasible

04

$$4/5 == 16/20$$

So our problem is with division as after division if we store the resultant slope we may lose precision thus we may not get exact slopes

Again consider those 3 points p_1, p_2, p_3

$$\text{Slope of } p_1-p_2 = 4/5 \quad [(y_2-y_1)/(x_2-x_1)]$$

$$\text{Slope of } p_2-p_3 = 16/20 \quad [(y_2-y_1)/(x_2-x_1)]$$

But still you are using division operator... right ??

Let's reduce these slopes in their fraction representation (ignore division)

$$\text{Slope}(p_1, p_2) = 4/5$$

$$\text{Slope}(p_2, p_3) = 16/20 = 4 \cdot 4/5 \cdot 4 = 4/5$$

$$\text{Slope}(p_1, p_2) = 4/5$$

$$\text{Slope}(p_2, p_3) = 4/5$$

05

$$4/5 == 16/20$$

$$\text{Slope}(p1,p2) = 4/5$$

$$\text{Slope}(p2,p3) = 4/5$$

After reducing fractions of both slopes they come out to be $4/5$ and we can say their slopes are same so lie on a same line

In our previous solution what we did ...

- Maintain a variable 'slope'
- Iterate over all the points and calculate slope of line connecting current point (p2) to previous one (p1)
- let's call it 'currSlope'
- If $\text{slope} \neq \text{currSlope}$ which means we need a different line than previous ones

Approach is nice but the problem was with the way we calculated slope i.e.
 $\text{slope} = (y2-y1)/(x2-x1)$

06

$$4/5 == 16/20$$

$$\text{Slope}(p1,p2) = 4/5$$

$$\text{Slope}(p2,p3) = 16/20 = 4/5$$

A better solution would be store dx and dy values separately

$$dx = x2-x1, dy = y2-y1$$

$$\text{slope}(p1,p2) = 4/5 \text{ i.e. } dx1 = 4 \ \& \ dy1 = 5$$

$$\text{slope}(p2,p3) = 16/20 = 4/5 \text{ i.e. } dx2 = 4 \ \& \ dy2 = 5$$

Now if we consider dx & dy values of 2 lines (p1-p2, p2-p3) separately then

$dx1 == dx2 \ \&\& \ dy1 == dy2$, thus their slopes would be equal

So the final approach would be

- Maintain 2 variables 'dx' & 'dy'
- Iterate over all the points and calculate curr_dx & curr_dy wrt to line connecting current point & previous one (points[i] with points[i-1])
- if(dx == curr_dx && dy == curr_dy)
- we can say slope of curr line is same as previous, so we do not need a different line (just extend previous one as slopes are same)

Where is use of GCD ???

When you calculate dx & dy as we saw in previous slide, we would need to reduce their fractions, so 16/20 should be 4/5

So here comes GCD, $\text{GCD}(16, 20) = 4$, so divide 16 & 20 by 4 reducing them to 4 & 5, now store these reduced 4 & 5

08

A SMALL NOTE-

See, there are few other ways as well

Just tried to keep things simple

Main idea was to show you a practical use-case of GCD in DSA problems



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CODE

SMARTER

I hope you found it useful

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