

Sardar Patel Institute of technology

Design and Analysis of Algorithms

Problem Statement:

4.5-4

Can the master method be applied to the recurrence $T(n) = 4T(n/2) + n^2 \lg n$?
Why or why not? Give an asymptotic upper bound for this recurrence.

Solution

$$T(n) = 4T(n/2) + n^2 \lg n$$

In the given recurrence, $a = 4$ and $b = 2$.

Hence, $n^{\log_b a} = n^{\log_2 4} = n^2$ and $f(n) = \Theta(n^2 \lg n)$.

Now, asymptotically $f(n) = n^2 \lg n$ is definitely larger than n^2 , but it is not polynomially larger than n^2 . So, we cannot apply master method to this recurrence.

Solution

$$T(n) = 4T(n/2) + n^2 \lg n$$

So, solving by expansion:

The image shows a handwritten solution on a lined notebook page. The recurrence relation $T(n) = 4T(n/2) + n^2 \lg n$ is written at the top. Below it, the expansion is shown step by step: $= 16T(n/2^2) + n^2 \lg n + n^2 \lg(n/2)$, followed by vertical dots indicating further expansion. The final result is $T(n) = 4^k T(n/2^k) + n^2 [\lg n + \dots + \lg \frac{n}{2^k}]$. The page also features a header with 'PAGE No.' and 'DATE' fields.

$$\begin{aligned} T(n) &= 4T\left(\frac{n}{2}\right) + n^2 \lg n \\ &= 16T\left(\frac{n}{2^2}\right) + n^2 \lg n + n^2 \lg\left(\frac{n}{2}\right) \\ &\vdots \\ T(n) &= 4^k T\left(\frac{n}{2^k}\right) + n^2 \left[\lg n + \dots + \lg \frac{n}{2^k} \right] \end{aligned}$$

Solution

$$T(n) = 4T(n/2) + n^2 \lg n$$

$$\frac{n}{2^k} = 1 \quad k = \log_2 n$$

$$T(n) = n^2 + n^2 \left[\log \frac{n}{2^{k-1}} + \log \frac{n}{2^{k-2}} + \dots + \log n \right]$$

$$\frac{n}{2^k} = 1 \quad \therefore \frac{n}{2^{k-1}} = 2, \quad \frac{n}{2^{k-2}} = 4$$

$$T(n) = n^2 + n^2 \left[\log 2 + \log 4 + \dots + \log n \right]$$

$$= n^2 + n^2 \left[\log_2 2 + \log_2 2^2 + \dots + \log_2 n \right]$$

Solution

$$T(n) = 4T(n/2) + n^2 \lg n$$

$$= n^2 + n^2 \left[\log_2 2 + \log_2 2^2 + \dots + \log_2 n \right]$$

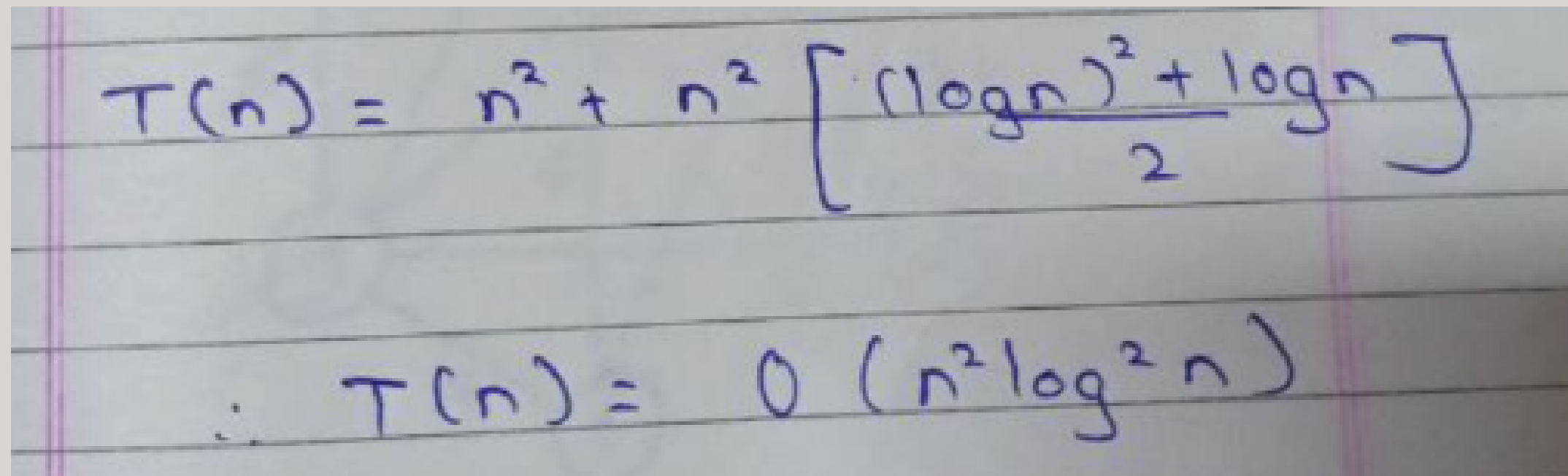
$$= n^2 + n^2 \left[1 + 2 + 3 + \dots + \log_2 \frac{n}{2} + \log_2 n \right]$$

$$= n^2 + n^2 \left[1 + 2 + 3 + \dots + \log_2 n - 1 + \log_2 n \right]$$

$$\therefore 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Solution

$$T(n) = 4T(n/2) + n^2 \lg n$$



The image shows a handwritten derivation on lined paper. The first line is $T(n) = n^2 + n^2 \left[\frac{(\lg n)^2 + \lg n}{2} \right]$. The second line is $\therefore T(n) = O(n^2 \log^2 n)$.

$$T(n) = n^2 + n^2 \left[\frac{(\lg n)^2 + \lg n}{2} \right]$$
$$\therefore T(n) = O(n^2 \log^2 n)$$

$$T(n) = \Theta(n^2 \log^2 n)$$