iii. A departmental store has a single cashier. During the rush hours, customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Assuming Poisson distribution for arrival rate and Exponential distribution for service rate, find: **5**

(a) Probability that the cashier is idle.
(b) Average number of customers in the queuing system.
(c) Average time a customer spends in the system.
(d) Average number of customers in the queue.
(e) Average time a customer spends in the queue waiting for service.

Q. 6 i. (a) Write applications of Simulation. **4**
(b) Define: Pure and mixed strategy.

ii. Canteen of a college keeps stock of a popular brand of cold drink. Previous experience shows daily demand as: **6**

Demand	0	10	20	30	40	50
Probability	0.04	0.16	0.15	0.48	0.12	0.05

Using the following random numbers: 48, 78, 19, 51, 56, 77, 15, 14, 68, 09:
(a) Simulate the demand for next 10 days.
(b) Find average daily demand on the basis of simulated data.

OR iii. Find the optimal strategy for the player A and B and also find the value of game: **6**

Player B

$$\text{Player A}\quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 1 & 2 & 3 \\ 2 & 7 & 6 \\ 4 & 8 & 1 \end{bmatrix} \end{array}$$

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Management Studies
End Sem Examination May-2024
MS5CO25 Operations Research

Programme: MBA Branch/Specialisation: Management

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Operations Research attempts to find ____________ solution to a problem. **1**
(a) Perfect (b) Optimum
(c) Both (a) and (b) (d) None of these

ii. In which model of operations research, everything is defined and results are not uncertain? **1**
(a) Deterministic model (b) Probabilistic model
(c) Both (a) and (b) (d) None of these

iii. Every linear programming problem is associated with another linear programming problem is called- **1**
(a) Primal (b) Dual
(c) Non-linear programming (d) None of these

iv. Intersection of key column and key row in a linear programming problem is known as- **1**
(a) Incoming variable (b) Outgoing variable
(c) Key element (d) All of these

v. In a transportation problem a feasible solution is called basic feasible solution if the number of non-negative allocations is equal to- **1**
(a) m – n + 1 (b) m – n – 1
(c) m + n – 1 (d) All of these

vi. The method used for solving an assignment problem is called- **1**
(a) Hungarian method (b) MODI method
(c) Inverse method (d) None of these

vii. In Markov analysis, the sum of state probabilities must be- **1**
(a) Equal to 1 (b) Less than 1
(c) Greater than 1 (d) All of these

viii. The term 'Jockeying' in a queuing theory refers to- **1**
(a) Not entering the long queue
(b) Leaving the queue
(c) Shifting from one queue to another parallel queue
(d) None of these

ix. An outcome has a probability of 35% of occurring. To simulate this outcome, which random number on a scale of 00-99 should be associated- **1**
(a) 00 - 34 (b) 00 – 35
(c) 65 - 99 (d) None of these

x. A saddle point exists when- **1**
(a) Maximin = minimax (b) Maximin = minmin
(c) Maximin = maximax (d) None of these

Q.2 i. Write any two limitations of operations research. **2**
ii. Describe any 3 models of operations research. **3**
iii. Explain various scopes of operations research. **5**
OR iv. Describe importance of operations research in decision making. **5**

Q.3 i. Write dual of the following linear programming problem: **3**

Maximize $Z = 5x_1 + 10x_2 + 8x_3$

Subject to $3x_1 + 5x_2 + 2x_3 \leq 60$

$4x_1 + 4x_2 + 4x_3 \leq 72$

$2x_1 + 4x_2 + 5x_3 \leq 100$

and $x_1, x_2, x_3 \geq 0$

ii. Solve the following linear programming problem by using graphical method- **7**

Maximize $Z = 48x_1 + 40x_2$

Subject to $2x_1 + x_2 \leq 90$

$x_1 + 2x_2 \leq 80$

$x_1 + x_2 \leq 50$

and $x_1, x_2 \geq 0$

OR iii. Solve the following linear programming problem by using Simplex method: **7**

Maximize $Z = 4x_1 + 10x_2$

Subject to $2x_1 + x_2 \leq 10$

$2x_1 + 5x_2 \leq 20$

$2x_1 + 3x_2 \leq 18$

and $x_1, x_2 \geq 0$

Q.4 i. What are the 3 different methods for obtaining an initial basic feasible solution to a transportation problem? Write in brief. **3**

ii. Find the initial basic feasible solution for the following transportation problem by using Vogel's Approximation Method (VAM). **7**

Factory	Ware House				Capacity
	W1	W2	W3	W4	
F1	21	16	25	13	**11**
F2	17	18	14	23	**13**
F3	32	27	18	41	**19**
Demand	**6**	**10**	**12**	**15**	

OR iii. Solve the following assignment problem for minimization of distance travelled: **7**

	P	Q	R	S	T
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Q.5 Attempt any two:

i. (a) Define transition probability matrix with example. **5**
(b) Define various queue disciplines.

ii. In a certain market, only two brands of cold drinks A and B are sold. Given that a man last purchased brand A, there is 80% chance that he would buy the same brand in the next purchase, while if a man purchased brand B, there is 90% chance that his next purchase would be brand B. Using this information: **5**
(a) Develop transition probability matrix.
(b) Interpret the state transition matrix in terms of:
I. Retention & loss II. Retention & gain.
(c) Draw state transition diagram.

Faculty of Management Studies
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MCQ.

Q1.

(i) (b) optimum +1

(ii) (a) Deterministic +1

(iii) (b) Dual +1

(iv) (c) Key element +1

(v) (c) m+n-1 +1

(vi) (a) Hungarian method +1

(vii) (a) Equal to 1 +1

(viii) (c) Shifting from one queue to another parallel queue. +1

(ix) (a) 00-34 +1

(x) (a) Maximin = Minmax +1

Q2 (i) Limitation of operations Research. (Any Two)

a) Magnitude of Computation : Modern problem involve large number of Variables and hence to find interrelationship among makes it difficult. Thus use of operations Research is limited to very large organizations. +1

(b) Analysis of Only Quantifiable factors: OR can evaluate only the effects of numeric and quantifiable factors. It does not consider the complexities involved with humans & their behaviours. +1

(c) A wide gap between the Managers and OR Researchers.

(d) Money & time Costs: when Basic Data are subjected to frequent changes, incorporating them in to OR. models is a costly affair.

(ii) Models of OR [Any three].

(a) Iconic : Pictorial representation of various aspect of a system
Eg. Toys, miniature of building, blueprints of Home, globes. +1

(b) Analogue : These models are small physical systems that has similar characteristics and work like a system it represents.
Eg: maps in diff colors represent water, desert and other geographical feature +1

(c) Mathematical Model; These Models employ a set of mathematical symbols to represent the decision variable of the system. The variables are related by mathematical system. +1

d) Static : does not take time in to account
Eg LPP, assignment, transportation.

e) Dynamic : considers time as one of the imp factor.
Eg. replacement, Dynamic Prog.

f) Deterministic : which doesnot take uncertainity in to account i.e. Sure. Eg. LPP, Transportation, assignment.

other models are Descriptive, predictive, Analytic, Simulation etc.

Q2. (iii) Various Scope of OR. [Any five with small descriptions]

a) Personnel Management
* Selection of suitable person on minimum salary.
* Skills & wages balancing
* to determine best age of retirement of Employees.
* Scheduling Training for Employees. +1

b) In finance & Accounting.
* Cash flow Planning
* Credit Policy Analysis, investment analysis
* claim and complaint procedures.
* to find Profit Plan for Company. +1

c) In Purchasing
* optimal buying
* quantities & timing of purchase
* optimal reordering

d) In Research and development

* Project Selection.
* Determination of area of Research and development +1
* Reliability and Alternative design.

e) In LIC +1
* what should be the premium rates for various modes of policies.
* How best the profit could be distributed in the case with profit Policies

others scopes are agriculture, industry, Marketing, production management. etc.

Q 2 (iv) (i) Better decision (1.5) (iii) Better coordination (1.5) +5
(ii) Better control (1.5) (iv) Better system (1.5)

Q3 (1) Dual of LPP.

$$\text{Max } Z = 5x_1 + 10x_2 + 8x_3$$

$$\text{St. } \quad 3x_1 + 5x_2 + 2x_3 \le 60$$
$$4x_1 + 4x_2 + 4x_3 \le 72$$
$$2x_1 + 4x_2 + 5x_3 \le 100$$
$$\& \quad x_1, x_2, x_3 \geq 0$$

Let y_1, y_2, y_3 dual Variables Then. +1

dual is $\text{Min } Z' = 60y_1 + 72y_2 + 100y_3$

$$\text{St } \quad 3y_1 + 4y_2 + 2y_3 \geq 5 \quad +1$$
$$5y_1 + 4y_2 + 4y_3 \geq 10$$
$$2y_1 + 4y_2 + 5y_3 \geq 8 \quad y_1, y_2, y_3 \geq 0 \quad +1$$

Q3(ii) graphical method for.

$$\text{Max } Z = 48x_1 + 40x_2$$

s.t.

$$2x_1 + x_2 \leq 90$$
$$x_1 + 2x_2 \leq 80$$
$$x_1 + x_2 \leq 50 \quad \& \; x_1, x_2 \geq 0$$

Step I Convert inequalities in to equation.

$$2x_1 + x_2 = 90 \quad — (1)$$
$$x_1 + 2x_2 = 80 \quad — (2)$$
$$x_1 + x_2 = 50 \quad — (3).$$

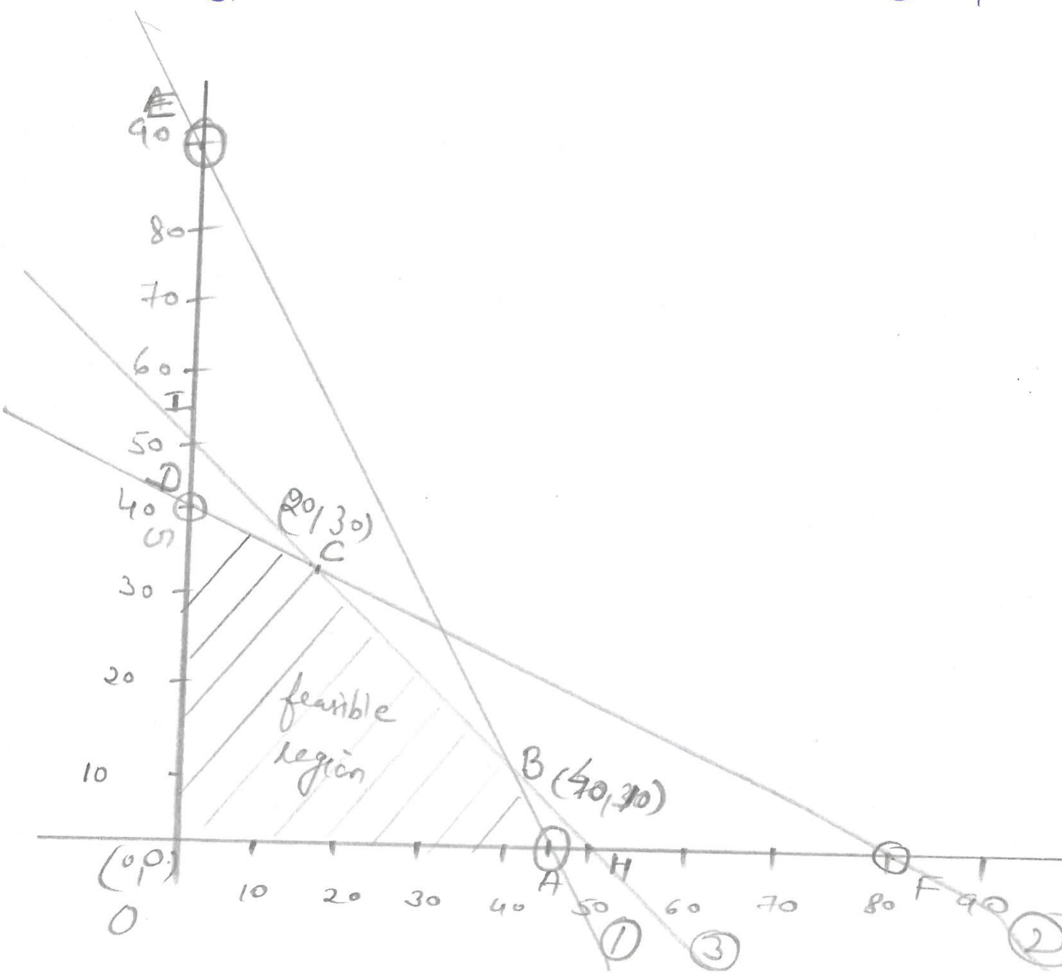
+2

Step II get points on axes of these lines

		A / F / H	E / G / I
pts. on	(1)	A (45,0)	E (0,90)
"	(2)	F (80,0)	G (0,40)
	(3)	H (50,0)	I (0,50)

+2

Step III Draw three lines using (1),(2),(3) points on axes.



+1

Step (IV) Feasible region :

The common region of all the given constraints is known as feasible region. Here which is polygon. OABCD

Step (V) optimum Solution

optimum Solution lies on any one corner point of feasible region

Corner pt.	Value of objective function. $Z = 48x_1 + 40x_2$
O (0,0)	$Z = 48 \times 0 + 40 \times 0 = 0$
A (45,0)	$Z = 45 \times 48 + 40 \times 0 = 2160$
B. (40,10)	$Z = 48 \times 40 + 40 \times 10 = 2320$ (Maximum)
C. (20,30)	$Z = 48 \times 20 + 40 \times 30 = 2160$
D (0,40)	$Z = 48 \times 0 + 40 \times 40 = 1600$

+1

The maximum value of objective function is 2320 at. $x_1 = 40$ and $x_2 = 10$.

+1

OR

(iii) Simplex method

Max $Z = 4x_1 + 10x_2$

St. $2x_1 + x_2 \le 10$

$2x_1 + 5x_2 \le 20$

$2x_1 + 3x_2 \le 18$ $x_1, x_2 \ge 0$

Step I

Set up standard form using slack variables s_1, s_2, s_3

revised form of LPP will be

Max $Z = 4x_1 + 10x_2 + 0.s_1 + 0.s_2 + 0.s_3$

St. $2x_1 + x_2 + s_1 + 0.s_2 + 0.s_3 = 10$

$2x_1 + 5x_2 + 0.s_1 + s_2 + 0.s_3 = 20$

$2x_1 + 3x_2 + 0.s_1 + 0.s_2 + s_3 = 18$

$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

+2

Step II

Find IBFS by putting all nonbasic variable to 0

i.e. $x_1 = 0$, $x_2 = 0$, $s_1 = 10$ $s_2 = 20$ $s_3 = 18$, $Z = 0$

+1

Step III Prepare initial Simplex table

I Simplex table

	C_j	4	10	0	0	0		
C_B	X_B	x_1	x_2	s_1	s_2	s_3	X_0	X_0/x_2
0	s_1	2	1	1	0	0	10	$10/1 = 10$
0	s_2	2	5	0	1	0	20	$\frac{20}{5} = 4 \leftarrow$
0	s_3	2	3	0	0	1	18	$18/3 = 6$
	$C_j - Z_j$	4	10 $\uparrow$	0	0	0		

+1

8

Incoming Vector x_2, departing S_2

Key (5)

II Simplex table

	C_j	4	10	0	0	0		
C_B	X_B	x_1	x_2	S_1	S_2	S_3	X_0	$X_0/$
0	S_1	8/5	0	1	$-1/5$	0	6	
10	x_2	2/5	1	0	1/5	0	4	
	S_3	4/5	0	0	$-3/5$	1	6	
$C_j - Z_j$		0	0	0	-2	0		

+2

All $C_j - Z_j \le 0$

current Sol^n optimal.

$x_1 = 0$, $x_2 = 4$, $S_1 \to 6$, $S_2 = 0$ $S_3 = 6$

$Z_{max} = 4 \times 0 + 4 \times 10 = 40$ +1

Q4. (1) (i) Three methods For obtaining an IBFS of a Transportation Problem.

(a) NWCR

North west Corner rule

The method attaches greater importance to the cell situated to the upper left corner of the table and makes as much as possible an allocation to the cell with both demand & supply restriction. +1

(b) Least cost method (LCM)

This method advocates that allocation should be based on minimum cost of transpotation. +1

First allocation must be made to the cell with the least cost of traspotation per unit satisfying demand supply constraints. Then next repeating process.

(c) Vogel's Approximation method. (VAM)

The Vogel's Approximation method takes into account not only the least cost C_{ij} but also the cost that just exceeds C_{ij}. The IBFS obtained by Vogel's method is either optimal or very close to the optimal solutions. +1

Q4 (ii)

	W_1	W_2	W_3	W_4	Supply.
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Demand	6	10	12	15	43

Total Supply = Total Demand = 43 +1

Prob. is balanced to find IBFS we use VAM as.

(10)

	W1	W2	W3	W4	Supply	Penalties P_1	P_2	P_3	P_4
F1	~~21~~	~~16~~	~~25~~	13 (11)	~~11~~	(3)	~~P_2~~	~~P_3~~	—
F2	17 (6)	18 (3)	14	23 (4)	~~13~~ ~~7~~ ~~3~~	(3)	(3)	(3)	(4)
F3	~~32~~	27 (7)	18 (12)	~~41~~	~~19~~ ~~12~~	(9)	(9)	(9)	(9)
Demand	~~6~~	~~16~~ 7	12	~~18~~ ~~4~~	43				
Penalties P_1	(4)	(2)	(4)	(10) ↑					
P_2	(15)	(9)	(4)	(18) ↑					
P_3	(15)	(9)	(4)	—					
P_4	↑ —	(9) ↑	(4)	—					

+2

+2

using VAM. IBFS $13\times11 + 17\times6 + 18\times3 + 23\times4$ +2

$+ 27\times7 + 18\times12 = 796.$

Q4. OR (iii)

Assignment Problem.

	P	Q	R	S	T
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Now, Select min of Each row and subtract this element from each row.

row reduction.

	P	Q	R	S	T
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

+2

Column reduction.

	P	Q	R	S	T
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

draw min number of lines covering all zeros.

+2

(Min is 15 here)

	P	Q	R	S	T
A	15	0	20	15	0
B	15	15	0	10	0
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

(v) do three operations

Simv(I) Select min of all uncovered element.

(ii) subtract this min element from All uncovered element

(iii) Add this min element to intersection pt of lines

(iv) covered element will be as it is

+2

Again repeat same process.

	P	Q	R	S	T
A	15	0	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	10	~~0~~
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	~~0~~	5
E	5	0	10	$\boxed{0}$	~~0~~

+1

optimal assignment is

	Cost
A → T	200
B → R	130
C → Q	110
D → P.	50
E → S	80
	570

+1

Q5 (I) (Any two.)

(a) Transition Prob. Matrix

Let us take a set of states $S_1, S_2, \ldots S_m$.

For the markov chain with set S.

the TPM is given by

$$\begin{array}{c|c} & \text{State } S_j \text{ (next state) } n=1 \\ \begin{array}{c}\text{current state}\\ (S_i)\\ n=0\end{array} & \begin{bmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1m} \\ P_{21} & P_{22} & P_{23} & \cdots & P_{2m} \\ P_{31} & P_{32} & P_{33} & \cdots & P_{3m} \\ P_{m1} & P_{m2} & P_{m3} & \cdots & P_{mn} \end{bmatrix} \end{array}$$

and have two important Properties +1

* $\sum_{j=1}^{n} P_{ij} = 1$ (sum of all elements in each row must be one)

* $0 \le P_{ij}$ ~~$\ne 0$~~ or $0 \le P_{ij} \le 1$
being probability

Eg.

$$\begin{array}{cc|cc} & & A & B\ (n=1) \\ (n=0) & \begin{array}{c}A\\B\end{array} & \begin{bmatrix} 0.70 & 0.30 \\ 0.80 & 0.20 \end{bmatrix} \end{array}$$

+1.5

(b) Various queue disciplines

FCFS (First Come first service)
Eg. window at railway station, bank etc +1

LCFS (Last Come first Service)
Eg godown, dinner lunch Plates in an event

SIRO (Service in random order) ,where the customers are selected for service at random irrespective of their arrival time. +1.5
Service on Priority Procedure, where the customer is chosen for service ahead of some other customers already in the queue.

(ii) (a) TPM.

$$\begin{array}{cc} & \text{next state} \\ & \begin{array}{cc} A & B \end{array} \\ \rightarrow \text{current State } \begin{array}{c} A \\ B \end{array} & \begin{pmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{pmatrix} \end{array}$$

+1

(b) Retention and Loss [Row interpretation]

P_{11} = Brand A retains 80% & losses 20% due to Brand B

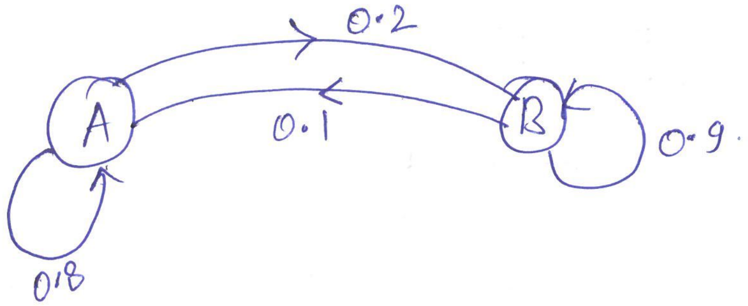
P_{12} = Brand B retains 90% & loses 10% due to A. +2

Retention and gain [Colum interpretation]

P_{21} = Brand A retains 80% of its customer & gain ~~20%~~ 10% of B.

P_{22} = Brand B retain 90% of its custom & gain 20% of A.

(c) State transition diagram. +2



Q5 (ii) (a) M/M/1 : ∞/FCFS case.

Arrival rate $\lambda = 20$ cust. / hour

Service rate $\mu = 24$ cust / hour.

$$\rho = \lambda/\mu = 20/24 = 0.83$$

(a) $P_0 = 1 - \rho = 1 - 0.83 = 0.17$ +1

(b) $L_s = \dfrac{\lambda}{\mu - \lambda} = \dfrac{20}{24-20} = 5$ customers +1

(c) $W_s = \dfrac{1}{\mu - \lambda} = \dfrac{1}{24-20} = \dfrac{1}{4} = 0.25$ hr. +1

(d) $L_q = \dfrac{\lambda^2}{\mu(\mu - \lambda)} = 4.17$ +1

(e) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)} = 0.21$ hr. +1

(Note: $L_s - L_q = \dfrac{\lambda}{\mu}$, $W_s - W_q = \dfrac{1}{\mu}$ may be used by students)

Q6 (i) (a) Application of Simulation.

Due to advantages like reduction in Complexity, costs time by using computer simulation the application areas are

* Business Process, investment & budgeting
* Social Problem, * Traffic control, * Transport design
* Basic scheme for estimation, * Inventory Problem

+2

Q.6 (ii) Allocation of random number to demand of cold drink (14)

Demand	Prob.	Cumulative Prob.	Random-Number interval
0	.04	.04	00 – 03
10	0.16	0.20	04–19
20	0.15	0.35	20 – 34
30	0.48	0.83	35–82
40	0.12	0.95	83–94
50	0.05	1.00	95–99

+3

Simulated demand for next 10 days.

Days	1	2	3	4	5	6	7	8	9	10
Random number	48	78	19	51	56	77	15	14	68	9
Simulated demand.	30	30	10	30	30	30	10	10	30	10

+ 220 +2

Average demand $\frac{220}{10}$ = 22 colddrink / day +1

OR

Q6 (iii)

$$\text{Player A} \quad \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 1 & 2 & 3 \\ A_2 & 2 & 7 & \boxed{6} \\ A_3 & \boxed{4} & \boxed{8} & 1 \end{array}$$

Player B

+1

First we try for saddle point

clearly the game has no saddle pt. So we will try for next Process.

Now we use principle of dominance to reduce the Payoff matrix

row $A_1 \leq$ row A_2. So. By row dominance rule.

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_2 & 2 & 7 & 6 \\ A_3 & 4 & 8 & 1 \end{array}$$

+2

again by colms dominance

Col. $B_2 \geq$ Col. B_3 or col. Col B_1 (any one)

by colm dominance rule

the reduced matrix is , Now using oddoment method.

15

$$\begin{array}{ll|cc|cc} & & \text{Player B} & & & \\ & & B_1 & B_3 & \text{odds} & \text{Prob} \\ \text{Player A} & A_2 & 2 & 6 & |3| & 3/7 \\ & A_3 & 4 & 1 & |-4| & 4/7 \\ & \text{odds} & |5| & |-2| & & \\ & \text{Prob} & 5/7 & 2/7 & & \end{array}$$

+2

Value of game If Player A opts A_2 (Expected gain)

$$V = 2 \times 5/7 + 6 \times \frac{2}{7} = \frac{10+12}{7} = \frac{22}{7}$$

If A opts A_3 then.

$$V = 4 \times \frac{5}{7} + 1 \times \frac{2}{7} = \frac{22}{7}$$

+1

The optimal strategy of Player A $\left[0, \frac{3}{7}, \frac{4}{7}, 0\right]$

The optimal strategy of Player B $[5/7, 0, 2/7, 0]$

Value of game = $22/7$

+1

*Q.6 (i) (b) Pure Strategy +1

Mixed strategy +1