Total No. of Questions: 25
MU/926

Enrollment No : ____________________

MEDICAPS UNIVERSITY

Faculty of Engineering / Science
End Semester Examination May 2025
EN3BS12 / BC3BS03 Engineering Mathematics -II

Programme	:	B.Tech. / B.Sc.	**Branch/Specialisation**	:	ALL
Duration	:	3 hours	**Maximum Marks**	:	60

Note: *All questions are compulsory. Internal choices, if any, are indicated. Assume suitable data if necessary. Notations and symbols have their usual meaning.*

Section 1 (Answer all question(s))

Marks CO BL

Q1. The Laplace transform of $sin(4t)$ is- 1 1 2

(A) $4/(s^2+16)$ (B) $s/(s^2+16)$
(C) $1/(s^2+16)$ (D) None of these

Q2. If $L\{f(t)\}=F(s)$ then $L\left\{\int_0^t f(\tau)d\tau\right\}=---?$ 1 1 1

(A) $\int_0^s f(\omega)d\omega$ (B) $F'(s)$
(C) $F(s)/s$ (D) None of these

Q3. If $F(s)$ is complex Fourier Transform of $f(x)$, then Fourier transform of $f(ax)$ is- 1 1 1

(A) $F(as)$ (B) $F(s/a)$
(C) $(1/a)F(s/a)$ (D) None of these

Q4. If $f(x)$ is an odd function defined periodic in the range $-\pi \le x \le \pi$, then the Fourier coefficient of cosine terms i.e. $a_{n,} n=0...\infty$ is- 1 2 3

(A) 1 (B) 0
(C) $a_0 = 1 and a_{\mathrm{n}} = 0, n = 1...\infty$ (D) None of these

Q5. The solution of $(D^2-6DD'+9D'^2)z=0$ is $z=$ ________, where $D=\frac{\partial}{\partial \mathbf{x}}$ and $D'=\frac{\partial}{\partial \mathbf{y}}$. 1 1 2

(A) $f_1(y+3x)+xf_2(y+3x)$ (B) $f_1(y-3x)+xf_2(y-3x)$
(C) $f_1(y+3x)+\chi_2(y-3x)$ (D) None of these

Q6. The complete integral of $z=px+qy-p^2$ is _______, where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$. 1 2 3

(A) $z=ax+by-a^2b$ (B) $z=px+qy-a^2b$
(C) z=ax+by-p^2q' (D) None of these

Q7. Divergence and curl of a vector field are respectively- 1 1 2

(A) Vector and vector (B) Scalar and scalar
(C) Scalar and vector (D) None of these

Q8.	The $\bar{F} = y\hat{i} + z\hat{j} + x\hat{k}$ then the $\int_c \bar{F}.d\bar{r}$ -	1	1	1
	(A) $\int_{\mathbb{C}} xdx + ydy + zdz$ (B) $\int_c ydx + xdy + zdz$ (C) $\int_c ydx + zdy + xdz$ (D) None of these			
Q9.	If x and x_a are the exact and approximate values respectively of a quantity, then relative error is given by-	1	1	1
	(A) $\mid \frac{x-x_a}{x} \mid$ (B) $\mid \frac{x-x_a}{x} \mid *100$ (C) $\mid (x-x_a) \mid$ (D) None of these			
Q10.	In the Gauss elimination method, the coefficient matrix is reduced to-	1	1	2
	(A) Diagonal matrix (B) Lower triangular matrix (C) Upper triangular matrix (D) None of these			

Section 2 (Answer any 2 question(s))

		Marks	CO	BL
Q11.	Find the Laplace transform of $\left\{e^{-t}\,\frac{\sin t}{t}\right\}$.	5	3	4
Q12.	Apply convolution theorem to evaluate $\mathcal{L}^{-1}\left\{\frac{1}{(S^2+1)(S^2+9)}\right\}$.	5	2	3
Q13.	Solve the following differential equation by Laplace transform- $(D^2+9)y = cos2t$ given that $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$	5	2	3

Section 3 (Answer any 2 question(s))

		Marks	CO	BL
Q14.	Find the Fourier complex transform of $f(x) = e^{-\lvert x \rvert}$.	5	2	3
Q15.	Find the Fourier series expansion of $f(x) = x - x^2, -\pi < x < \pi$.	5	2	3
Q16.	Find half range sine series of the function$f(x) = x^2 - 2, 0 < x \leq 2$.	5	2	3

Section 4 (Answer any 2 question(s))

		Marks	CO	BL
Q17.	Solve by Lagrange's method $x^2\,(y-z)p + y^2(z-x)q = z^2(x-y)$.	5	2	3
Q18.	Solve the following- $\left(D^2 - DD' - 2D'^2\right)z = (y-1)e^x$.	5	1	2
Q19.	Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by the method of separation of variables where $u(x,0) = 6e^{-3x}$.	5	2	3

Section 5 (Answer any 2 question(s))

		Marks	CO	BL
Q20.	Find the directional derivatives of $\phi = x^2yz + 4xz^2$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$ at the point $(1,-2,-1)$.	5	4	5
Q21.	Find the circulation of $\overline{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$ along the curve C where C is rectangle whose vertices are $(0,0),(a,0),(a,b),(0,b)$.	5	2	3
Q22.	Give the statement of Gauss divergence theorem and apply it to show that $\int\int_S \nabla\left(x^2+y^2+z^2\right)ds = 6V$ where S is any closed surface enclosing volume V.	5	2	3

Section 6 (Answer any 2 question(s))

		Marks	CO	BL
Q23.	Apply Newton -Raphson method to solve $3x = cosx + 1$, correct upto three places of decimals.	**5**	**2**	**3**
Q24.	Using Gauss-Seidel method , solve the linear system of equations- $27x + 6y - z = 85$ $6x + 15y + 2z = 72$ $x + y + 54z = 110$	**5**	**2**	**3**
Q25.	Find the real root of the equation $x^3 - 9x + 1 = 0$, by Regula Falsi method, correct upto 2 places of decimals.	**5**	**2**	**3**

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Solution of EN3BS12 (Engg. Mathematics-II)

Date:- 09/05/2025

Q-1	(i)	a)	$\frac{4}{s^2+16}$	+1
Q-2	(ii)	c)	$\frac{F(s)}{s}$	+1
Q-3	(iii)	c)	$\frac{1}{a}F\left(\frac{s}{a}\right)$	+1
Q-4	(iv)	b)	0	+1
Q-5	(v)	a)	$f_1(y+3x)+x\,f_2(y+3x)$	+1
Q-6	(vi)	d)	None of these	+1
Q-7	(vii)	c)	Scalar and vector	+1
Q-8	(viii)	c)	$\int_c y\,dx + z\,dy + x\,dz$	+1
Q-9	(ix)	a)	$\left\lvert \frac{x-x_a}{x} \right\rvert$	+1
Q-10	(x)	c)	Upper Triangular Matrix	+1

Q12 Let $f(s) = \frac{1}{s^2+1}$ and $g(s) = \frac{1}{s^2+9}$

Then

$$F(t) = L^{-1}\{f(s)\} = L^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t$$

$$\& \quad G(t) = L^{-1}\{g(s)\} = L^{-1}\left\{\frac{1}{s^2+9}\right\} = \frac{1}{3}\sin 3t \qquad +1$$

Hence by convolution theorem,

$$L^{-1}\{f(s) \cdot g(s)\} = (F * G)(t)$$

$$= \int_0^t F(x)\, G(t-x)\, dx \qquad +1$$

$$= \int_0^t \sin x \times \frac{1}{3} \sin 3(t-x)\, dx$$

$$= \frac{1}{2} \times \frac{1}{3} \int_0^t 2 \sin x \sin(3t-3x)\, dx$$

$$= \frac{1}{6} \int_0^t [\cos(x-3t+3x) - \cos(x+3t-3x)]\, dx$$

$$[\because 2\sin a \sin b = \cos(a-b) - \cos(a+b)]$$

$$= \frac{1}{6} \int_0^t [\cos(4x-3t) - \cos(3t-2x)]\, dx$$

$$= \frac{1}{6}\left[\frac{\sin(4x-3t)}{4} - \frac{\sin(3t-2x)}{-2}\right]_0^t$$

$$= \frac{1}{6}\left[\left(\frac{\sin t}{4} - \frac{\sin(-3t)}{4}\right) + \left(\frac{\sin t}{2} - \frac{\sin 3t}{2}\right)\right]$$

$$= \frac{1}{6}\left[\frac{\sin t}{4} + \frac{\sin 3t}{4} + \frac{\sin t}{2} - \frac{\sin 3t}{2}\right]$$

$$= \frac{1}{6}\left(\frac{3}{4}\sin t - \frac{1}{4}\sin 3t\right)$$

$$= \frac{1}{24}(3\sin t - \sin 3t)$$ +3

Q.13 Solve $(D^2+9)y = \cos 2t$
with $y(0)=1$, $y(\frac{\pi}{2}) = -1$

The given differential equation can be written as

$$y'' + 9y = \cos 2t$$

Taking laplace transform of both sides

$$L\{y''\} + 9L\{y\} = L\{\cos 2t\}$$ +1

$$\Rightarrow \left[s^2 L\{y\} - s\,y(0) - y'(0)\right] + 9L\{y\} = \frac{s}{s^2+4}$$

$$\Rightarrow s^2 L\{y\} - s\times 1 - A + 9L\{y\} = \frac{s}{s^2+4}$$

where $y'(0) = A$

$$\Rightarrow (s^2+9)L\{y\} = s + A + \frac{s}{s^2+4}$$

$$\Rightarrow L\{y\} = \frac{s+A}{s^2+9} + \frac{s}{(s^2+4)(s^2+9)}$$

$$= \frac{s}{s^2+9} + \frac{A}{s^2+9} + \frac{s}{5(s^2+4)} - \frac{s}{5(s^2+9)}$$ +1

(using partial fractions)

$$\Rightarrow y = L^{-1}\left\{\frac{s}{s^2+9}\right\} + A\,L^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$+ \frac{1}{5}L^{-1}\left\{\frac{s}{s^2+4}\right\} - \frac{1}{5}L^{-1}\left\{\frac{s}{s^2+9}\right\}$$

$$= \cos 3t + \frac{A}{3}\sin 3t + \frac{1}{5}\cos 2t - \frac{1}{5}\cos 3t$$

$$y = \frac{4}{5}\cos 3t + \frac{A}{3}\sin 3t + \frac{1}{5}\cos 2t$$ +2

Since $y\left(\frac{\pi}{2}\right) = -1$

$\therefore -1 = y\left(\frac{\pi}{2}\right)$

$$\Rightarrow -1 = \frac{4}{5}\cos\frac{3\pi}{2} + \frac{A}{3}\sin\frac{3\pi}{2} + \frac{1}{5}\cos\frac{2\pi}{2}$$

$$\Rightarrow -1 = -\frac{A}{3} - \frac{1}{5}$$

$$\Rightarrow A = \frac{12}{5}$$

$\therefore$ Required solution is

$$y = \frac{4}{5}\cos 3t + \frac{4}{5}\sin 3t + \frac{1}{5}\cos 2t$$ +1

Section 3

Q.14 $f(x) = e^{-|x|}$

By definition of Fourier transform

$$F\{f(x)\} = \int_{-\infty}^{\infty} e^{-isx} f(x)\,dx$$ +1

$$= \int_{-\infty}^{0} e^{-isx} e^{-(-x)}\,dx + \int_{0}^{\infty} e^{-isx} e^{-(x)}\,dx$$

$$\left[\because\ |x| = \begin{cases} -x, & -\infty < x < 0 \\ x, & 0 < x < \infty \end{cases}\right.$$ +1

$$= \int_{-\infty}^{0} e^{x} e^{-isx}\, dx + \int_{0}^{\infty} e^{-isx} e^{-x}\, dx$$

$$= \int_{-\infty}^{0} e^{(1-is)x}\, dx + \int_{0}^{\infty} e^{-(is+1)x}\, dx$$

$$= \left[\frac{e^{(1-is)x}}{1-is} \right]_{-\infty}^{0} + \left[\frac{e^{-(1+is)x}}{1+is} \right]_{0}^{\infty}$$

$$= \left[\frac{1}{1-is} - \frac{0}{1-is} \right] + \left[\frac{0}{1+is} - \frac{1}{1+is} \right]$$

$(\because e^{-\infty} = 0)$

$$= \frac{1}{1-is} - \frac{1}{1+is}$$

$$= \frac{(1+is) - (1-is)}{1+s^2}$$

$$= \frac{2is}{1+s^2}$$

+3

Q.15 $f(x) = x - x^2 \quad , \quad -\pi < x < \pi$

Fourier Series of $f(x)$ in the interval $(-\pi, \pi)$ is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \to (1) \qquad +1$$

where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right) - \left(\frac{\pi^2}{2} + \frac{\pi^3}{3}\right)\right]$$

$$= -\frac{2}{3}\pi^2$$

$$\Rightarrow a_0 = -\frac{2}{3}\pi^2 \qquad +1$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{\mathrm{I}}{(x - x^2)}\, \underset{\mathrm{II}}{\cos nx}\,dx$$

$$= \frac{1}{\pi}\left[(x - x^2)\frac{\sin nx}{n} - (1 - 2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

(using successive integration)

$$= \frac{1}{\pi}\left[(1 - 2\pi)\frac{\cos n\pi}{n^2} - (1 + 2\pi)\frac{\cos n\pi}{n^2}\right]$$

$(\because \sin n\pi = 0)$

$$a_n = \frac{1}{\pi}\left(-4\pi \frac{(-1)^n}{n^2}\right) \quad [\because \cos n\pi = (-1)^n]$$

$$\Rightarrow a_n = \frac{-4(-1)^n}{n^2}$$ +1

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\sin nx\, dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{-\cos nx}{n}\right) - (1-2x)\left(-\frac{\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\cos n\pi}{n}\right]$$

$$b_n = \frac{-2(-1)^n}{n}$$ +1

$\therefore$ Equation (1) Becomes

$$x-x^2 = \frac{-\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx$$

$$\Rightarrow x-x^2 = \frac{-\pi^2}{3} + 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots\right]$$
$$+ 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right]$$ +1

Q.16 $f(x) = x^2 - 2, \quad 0 < x \le 2$

Half Range Sine series of $f(x)$ in $(0,2)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) \quad (\because l = 2) \quad +1$$

where

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{2}\int_0^2 (x^2-2)\sin\left(\frac{n\pi x}{2}\right)dx \quad +1$$

$$= \left[(x^2-2)\left(\frac{-2}{n\pi}\cos\frac{n\pi x}{2}\right) - (2x)\left(-\frac{4}{n^2\pi^2}\sin\frac{n\pi x}{2}\right) + (2)\left(\frac{8}{n^3\pi^3}\cos\frac{n\pi x}{2}\right)\right]_0^2$$

$$= \left(\frac{-4}{n\pi}\cos n\pi + \frac{16}{n^3\pi^3}\cos n\pi\right) - \left(\frac{4}{n\pi} + \frac{16}{n^3\pi^3}\right)$$

$$= \frac{-4}{n\pi}\left[(-1)^n + 1\right] + \frac{16}{n^3\pi^3}\left[(-1)^n - 1\right] \quad +2$$

$$\therefore f(x) = \sum_{n=1}^{\infty}\left[\frac{16}{n^3\pi^3}\{(-1)^n - 1\} - \frac{4}{n\pi}\{(-1)^n + 1\}\right]\sin\frac{n\pi x}{2} \quad +1$$

Section 4

Q.17 $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

On comparing with $Pp+Qq=R$ we get
$P = x^2(y-z)$, $Q = y^2(z-x)$, $R = z^2(x-y)$

The auxiliary equation is
$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\Rightarrow \frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)} \quad (1) \qquad +1$$

Using multipliers $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$ +1

we get

$$\text{each fraction} = \frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{(y-z)+(z-x)+(x-y)}$$

$$= \frac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$$

$$\Rightarrow \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$$

On integration

$$\Rightarrow -\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = c_1$$

$$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = a \quad \text{where } a = -c_1 \qquad +1$$

Again, using multipliers $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ we get +1 each fraction $= \dfrac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{x(y-z)+y(z-x)+z(x-y)}$

$= \dfrac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$

$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$

On integration

$\Rightarrow \log x + \log y + \log z = \log b$

$\Rightarrow \log(xyz) = \log b$

$\Rightarrow xyz = b$

Hence general solution is

$$\phi\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z},\ xyz\right) = 0 \qquad +1$$

(other multipliers can also satisfy the solution)

Q.18 $(D^2 - DD' - 2D'^2)z = (y-1)e^x$

Auxiliary equation is

$m^2 - m - 2 = 0$

$\Rightarrow m = -1, 2$

$\therefore$ C.F. $= \phi_1(y-x) + \phi_2(y+2x)$ $\qquad +2$

$$P.I. = \frac{1}{D^2 - DD' - 2D'^2}(y-1)e^x$$

$$= \frac{1}{(D-2D')(D+D')}(y-1)e^x$$

$$= \frac{1}{(D+D')}\left[\frac{1}{(D-2D')}(y-1)e^x\right]$$

[Put $y+mx=c \Rightarrow y+2x=c \Rightarrow y=c-2x$]
as $m=2$

$$\therefore P.I. = \frac{1}{(D+D')}\int (c-2x-1)e^x dx$$

$$= \frac{1}{(D+D')}\left[(c-2x-1)e^x - (-2)e^x\right]$$

$$= \frac{1}{(D+D')}\left[(y-1)e^x + 2e^x\right] \quad (\because c = y+2x)$$

$$= \frac{1}{(D+D')}(y+1)e^x$$

[Put $y+mx=c \Rightarrow y-x=c$ as $m=-1$
$\Rightarrow y=c+x$]

$$\therefore P.I. = \int (c+x+1)e^x dx$$

$$= (c+x+1)e^x - (1)e^x$$

$$= (y+1)e^x - e^x \quad (\because c = y-x)$$

$$P.I. = ye^x$$

Hence complete solution is

$$z = C.F. + P.I.$$

$$= \phi_1(y-x) + \phi_2(y+2x) + ye^x$$

+3

Q.19 $\dfrac{\partial u}{\partial x} = 2\dfrac{\partial u}{\partial t} + u$, $u(x,0) = 6e^{-3x}$ $\longrightarrow$ (1)

Let the solution of eqn (1) be

$u(x,t) = X(x)\,T(t)$ $\longrightarrow$ (2)

or $u = XT$

$\therefore \dfrac{\partial u}{\partial x} = X'T$, $\dfrac{\partial u}{\partial t} = XT'$

Substituting these values in eqn (1)

$$X'T = 2XT' + XT$$

$$\Rightarrow (X' - X)T = 2XT'$$

$$\Rightarrow \frac{X' - X}{2X} = \frac{T'}{T} \longrightarrow (3)$$

(Separation of variables) +1

Since L.H.S. of eqn (3) is a function of x alone & R.H.S. is a function of t alone, therefore it will be true if each side is equal to the some constant (say) k

$\therefore \dfrac{X' - X}{2X} = k$ & $\dfrac{T'}{T} = k$

$$\Rightarrow X' - X = 2kX$$

$$\Rightarrow X' = (2k+1)X$$

$$\Rightarrow \frac{X'}{X} = 1 + 2k$$

$$\Rightarrow \frac{dX}{X} = 1 + 2k$$

Integrating

$$\log X = (1+2k)x + \log c_1$$

$$\Rightarrow X = c_1 e^{(1+2k)x}$$

Also, $\frac{T'}{T} = k \Rightarrow \frac{dT}{T} = k$

on integration

$\log T = kt + \log c_2$

$\Rightarrow T = c_2 e^{kt}$ +2

Putting values of X & T in eqn (2)

$u(x,t) = c_1 c_2 e^{(1+2k)x} e^{kt} \longrightarrow (4)$

Given, $u(x,0) = 6e^{-3x}$

Putting $t=0$ in eqn (4) we get

$u(x,0) = c_1 c_2 e^{(1+2k)x}$

$\Rightarrow 6e^{-3x} = c_1 c_2 e^{(1+2k)x}$

comparing both sides

$c_1 c_2 = 6$ & $(1+2k) = -3$

$\Rightarrow c_1 c_2 = 6$ & $k = -2$ +1

Putting these values in eqn (4), the required general soln is

$u(x,t) = 6e^{-3x} e^{-2t}$

$= 6e^{-(3x+2t)}$ +1

Section 5

Q. 20

$$\phi = x^2yz + 4xz^2$$

$$\therefore \text{grad}\,\phi = \nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$= \hat{i}(2xyz + 4z^2) + \hat{j}(x^2z) + \hat{k}(x^2y + 8xz)$$

$$= 8\hat{i} - \hat{j} - 10\hat{k}$$ at the point $(1, -2, -1)$ +2

Let $\hat{a}$ be the unit vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} - 2\hat{k}$.
Then

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{i} - \hat{j} - 2\hat{k}}{|2\hat{i} - \hat{j} - 2\hat{k}|}$$

$$= \frac{2\hat{i} - \hat{j} - 2\hat{k}}{\sqrt{4+1+4}}$$

$$= \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k})$$ +2

$\therefore$ Required directional derivative at $(1, -2, -1)$

$$= \hat{a} \cdot \text{grad}\,\phi$$

$$= \frac{1}{3}(2\hat{i} - \hat{j} - 2\hat{k}) \cdot (8\hat{i} - \hat{j} - 10\hat{k})$$

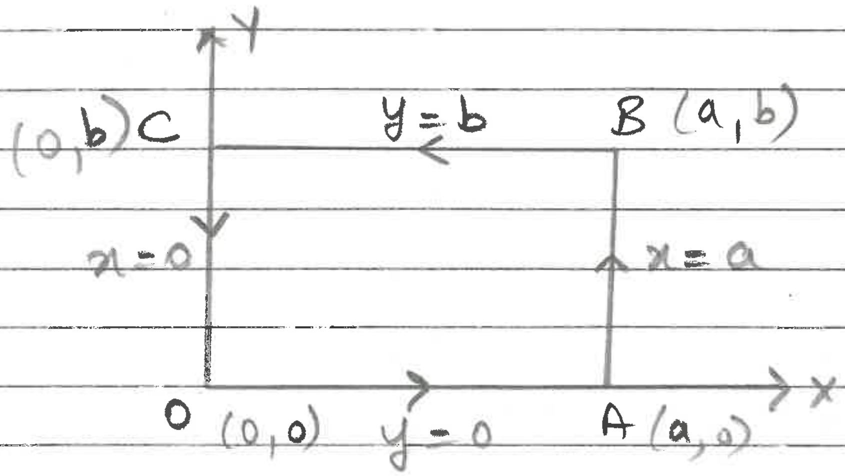
$$= \frac{1}{3}(16 + 1 + 20)$$

$$= \frac{37}{3}$$ +1

Q.21 $\vec{F} = (x^2+y^2)\hat{i} - 2xy\hat{j}$

$$\int_C \vec{F}\cdot d\vec{r} = \int_C \left[(x^2+y^2)\hat{i} - 2xy\hat{j}\right]\cdot(\hat{i}dx+\hat{j}dy)$$

$$= \int_C \left[(x^2+y^2)dx - 2xy\,dy\right] \rightarrow (1)$$ +1



+1

On line OA, $y = 0 \Rightarrow dy = 0$
& x varies from 0 to a.

On line AB, $x = a \Rightarrow dx = 0$
& y varies from 0 to b.

On line BA, $y = b \Rightarrow dy = 0$
& x varies from a to 0

On line CO, $x = 0 \Rightarrow dx = 0$
& y varies from b to 0. +1

Substituting these values in eq^n (1)

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^a x^2 dx + \int_0^b (-2ay)\,dy$$

$$+ \int_a^0 (x^2+b^2)\,dx + 0$$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = \left[\frac{x^3}{3}\right]_0^a - 2a\left[\frac{y^2}{2}\right]_0^b + \left[\frac{x^3}{3} + b^2x\right]_a^0$$

$$= \frac{a^3}{3} - ab^2 - \frac{a^3}{3} - ab^2$$

$$= -2ab^2$$ +2

Q. 22 Gauss Divergence Theorem :-

If $\vec{F}$ be a continuously differentiable vector point function in a region V and S is a closed surface enclosing the region V then

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \operatorname{div}\vec{F}\,dV$$ +2

where $\hat{n}$ is the unit outward drawn normal vector of the surface.

To show :- $\iint_S \nabla(x^2+y^2+z^2)\,ds = 6V$

Now,

$$\nabla(x^2+y^2+z^2) = \hat{i}\frac{\partial}{\partial x}(x^2+y^2+z^2) + \hat{j}\frac{\partial}{\partial y}(x^2+y^2+z^2) + \hat{k}\frac{\partial}{\partial z}(x^2+y^2+z^2)$$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$ +1

Let $\vec{F} = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

By Gauss Divergence theorem

$$\iint_S \nabla(x^2+y^2+z^2)\,ds = \iiint_V \operatorname{div}\vec{F}\,dV$$

$$= \iiint_V \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)(2x\hat{i}+2y\hat{j}+2z\hat{k})\,dV$$

$$= \iiint_V \frac{\partial}{\partial x}(2x)+\frac{\partial}{\partial y}(2y)+\frac{\partial}{\partial z}(2z)\,dV$$

$$= (2+2+2)\iiint_V dV$$

$$= 6V$$ +2

Section 6

Q.23 $3x = \cos x + 1$

Let $f(x) = 3x - \cos x - 1 = 0$

$f'(x) = 3 + \sin x$

Here

$f(0.60) = -0.02534 = -ve$

$f(0.61) = 0.01035 = +ve$

$\therefore$ One root lies between 0.60 & 0.61

Taking $x_0 = 0.6$ +1

By Newton Raphson method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ +1

$$= x_n - \frac{3x_n - \cos x_n - 1}{3 + \sin x_n}$$

Taking $n=0$, the first approximation is

$$x_1 = x_0 - \frac{3x_0 - \cos x_0 - 1}{3 + \sin x_0}$$

$$= 0.6 - \frac{3(0.6) - \cos(0.6) - 1}{3 + \sin(0.6)}$$

$$= 0.6071$$

For $n=1$, second approximation is

$$x_2 = x_1 - \frac{3x_1 - \cos x_1 - 1}{3 + \sin x_1}$$

$$= (0.6071) - \frac{3(0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)}$$

$$= 0.6071$$

Since $x_1 = x_2$

$\therefore$ Real root of given eq^n correct to 3 places of decimal

$= 0.607$ +3

(Note:- Nearest Range of the root may be considered)

Q.24 $27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

Solving each eq^n of the given system for the unknown with the largest coefficient in terms of remaining unknowns

$$x = \frac{1}{27}(85 - 6y + z)$$

$$y = \frac{1}{15}(72 - 6x - 2z)$$

$$z = \frac{1}{54}(110 - x - y)$$ +1

we begin with $y = z = 0$ and using the most recent values of x, y, z we get

First Iteration:-

$$x^{(1)} = \frac{1}{27}(85 - 6\times0 + 0) = 3.148$$

$$y^{(1)} = \frac{1}{15}(72 - 6x^{(1)} - 2z^{(0)})$$

$$= 3.541$$

$$z^{(1)} = \frac{1}{54}(110 - x^{(1)} - y^{(1)})$$

$$= \frac{1}{54}(110 - 3.148 - 3.541)$$

$$= 1.913$$ +1

Second Iteration:-

$$x^{(2)} = \frac{1}{27}(85 - 6y^{(1)} + z^{(1)})$$

$$= \frac{1}{27}(85 - 6\times3.541 + 1.913)$$

$$= 2.432$$

$$y^{(2)} = \frac{1}{15}(72 - 6x^{(2)} - 2z^{(1)})$$
$$= \frac{1}{15}(72 - 6\times 2.432 - 2\times 1.913)$$
$$= 3.572$$

$$z^{(2)} = \frac{1}{54}(110 - x^{(2)} - y^{(2)})$$
$$= \frac{1}{54}(110 - 2.432 - 3.572)$$
$$= 1.926$$ +1

Third Iteration:-

$$x^{(3)} = \frac{1}{27}(85 - 6y^{(2)} + z^{(2)})$$
$$= \frac{1}{27}(85 - 6\times 3.572 + 1.926)$$
$$= 2.426$$

$$y^{(3)} = \frac{1}{15}(72 - 6x^{(3)} - 2z^{(2)})$$
$$= \frac{1}{15}(72 - 6\times 2.426 - 2\times 1.926)$$
$$= 3.572$$

$$z^{(3)} = \frac{1}{54}(110 - x^{(3)} - y^{(3)})$$
$$= \frac{1}{54}(110 - 2.426 - 3.572)$$
$$= 1.926$$ +1

Since second & third iterations give practically the same values, so we stop the process
Hence the solution is
$x = 2.426, \quad y = 3.572, \quad z = 1.926$ +1

Q. 25 Let $f(x) = x^3 - 9x + 1$
Here $f(2) = -9$, $f(3) = 1$
$\therefore$ One root lies between 2 and 3.

Taking $x_1 = 2$, $x_2 = 3$
$f(x_1) = -9$, $f(x_2) = 1$ +1

By regula Falsi method

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_1)$$ +1

$$= 2 - \frac{3-2}{1+9}(-9)$$

$$= 2.9$$

$$f(x_3) = f(2.9) = (2.9)^3 - 9(2.9) + 1$$
$$= -0.711$$ +1

Clearly $f(x_2) \cdot f(x_3) < 0$ therefore the root lies between 2.9 and 3.

Taking $x_1 = 2.9$, $x_2 = 3$
$f(x_1) = -0.711$, $f(x_2) = 1$

$$x_4 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} \cdot f(x_1)$$

$$x_4 = 2.9 - \frac{3-2.9}{1+0.711}(-0.711)$$
$$= 2.94$$

$f(x_4) = f(2.94) = -0.0478$ +1

Clearly $f(x_2).f(x_4) < 0$ therefore the root lies between 2.94 and 3

Taking $x_1 = 2.94$, $x_2 = 3$
$f(x_1) = -0.0478$, $f(x_2) = 1$.

$$x_5 = 2.94 - \frac{3-2.94}{1+0.0478}(-0.0478)$$
$$= 2.94$$

Since $x_4 = x_5 = 2.94$

$\therefore$ Required real root of the given eq^n correct to 2 decimal places
$= 2.94$ +1