Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2018

CA5BS03 Mathematics of computer Applications

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If $A = \{x: 1 < x < 4, x \in I\}, B = \{x: 2 < x < 7, x \in I\}$ then $A \cap B$ **1**

(a) $\{3\}$ (b) $\{2,3,4\}$ (c) $\{5,6\}$ (d) ϕ

ii. If $A = \{4,5,9\}$ and $R = \{(4,5), (4,9), (5,9)\}$ then relation R is **1**

(a) Reflexive (b) Symmetric

(c) Transitive (d) Anti − symmetric

iii. Every square matrix satisfies its **1**

(a) Characteristic polynomial (b) Characteristic equation

(c) Characteristic vectors (d) Characteristic roots

iv. The system of equations is said to be consistent if **1**

(I) $\rho(A) = \rho(A:B) = \text{n}$ (II) $\rho(A) = \rho(A:B) < \text{n}$

(a) Both (I) & (II) are true (b) Only (I) is true

(c) (I) is true but (II) is false (d) (I) is false but (II) is true

v. A vertex of degree zero is called **1**

(a) Pendant vertex (b) Isolated vertex

(c) Walk (d) Tree

vi. If there are two or more than two edges having the same pair of end vertices, then such edges are called **1**

(a) Adjacent edges (b) Parallel edges

(c) Self loop (d) Incident edges

vii. If A and B be two events of a sample space S then A and B are called independent event if **1**

(a) $P(A \cap B) = P(A).P(B)$ (b) $P(A \cap B) \neq P(A).P(B)$

(c) $P(A \cap B) = P(A) + P(B)$ (d) $P(A \cap B) = \emptyset$

viii. The mean of Binomial 's distribution is given by **1**

(a) npq (b) np (c) nq (d) $\sqrt{npq}$

ix. The order of the recurrence relation $y_{x+2} - 5y_{x+1} + 6y_x = 0$, is **1**

(a) 1 (b) 2 (c) 0 (d) 3

x. A recurrence relation of degree one is called **1**

(a) Characteristic (b) Homogeneous

(c) Linear (d) None of these

Q.2 i. If $f: R \to R$ defined by $f(x) = x^2 \, for \, all \, x \in R$ and $g: R \to R$ defined by $g(x) = sinx \, for \, all \, x \in R$ then show that: $(f \circ g)x \neq (g \circ f)x$. **4**

ii. Show that the relation "less than or equal" ($\leq$) on the set of positive integers is a partial order relation. **6**

OR iii. A class has 175 students**.** The following is the description showing the number of students studying one or more of the following subjects in this class: Mathematics 100; Physics 70; Chemistry 46; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; and none of the three subjects 22. Find: **6**

(a) How many students study all the three subjects?

(b) How many students study exactly one subjects out of three.

Q.3 i. Find the rank and nullity of matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ **4**

ii. Verify Cayley – Hamilton theorem for the matrix A and hence compute A^{-1}: A = $\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **6**

OR iii. Find the Eigen values and Eigen vectors of the matrix: **6**

A = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

Q.4 i. Define the following: simple graph, self-loop, parallel edge, incident edge. **4**

ii. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$**.** **6**

OR iii. Write Kruskal's and Prim's Algorithm for finding minimal spanning tree. **6**

Q.5 i. What is the chance that a leap year selected at random will contain 53 Wednesdays? **4**

ii. A coin is tossed 4 times, what is the probability to getting: **6**

(a) Two heads, (b) At least two heads.

OR iii. Show that in a Poisson distribution mean and variance are same. **6**

Q.6 i. Solve the following recurrence relation: **4**

$$a_r - 6a_{r-1} + 8a_{r-2} = 0 \; given \; a_0 = 3 \; and a_1 = 2.$$

ii. Solve the following recurrence relation: **6**

$$a_r - 5a_{r-1} + 6a_{r-2} = 2 + r, r \geq 2, given \; a_0 = 1 \; and a_1 = 1.$$

OR iii. Determine the generating function of the numeric function a_r, where $a_r = \begin{cases} 2^r, if \; r \; is \; even \\ -2^r, if \; r \; is \; odd. \end{cases}$ **6**

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Que 1. (i) (a) $\{3\}$ +1

(ii) (c) Transitive +1

(iii) (b) Characteristic equation +1

(iv) (a) both (I) & (II) are true. +1

(v) (b) Isolated vertex +1

(vi) (b) Parallel edges +1

(vii) (a) $P(A \cap B) = P(A).P(B)$ +1

(viii) (b) np +1

(ix) (b) 2 +1

(x) (c) Linear. +1

Que. 2(i) Let $x \in R$, then $(g \circ f)(x) = g(f(x)) = g(x^2) = \sin x^2$ (+2)

and $(f \circ g)(x) = f(g(x)) = f(\sin x) = (\sin x)^2 = \sin^2 x$

Clearly $(g \circ f)(x) \neq (f \circ g)(x)$ (+2)

2 (ii) Let I^+ be the set of all positive integers.

(a) Reflexive - For each $n \in I^+$ we have

"$n \leq n$"

$\Rightarrow$ The relation "$\leq$" is reflexive (+2)

(b) Anti-symmetric - For each $m, n \in I^+$, we have

$m \leq n$ and $n \leq m \Rightarrow m = n$ (+2)

$\Rightarrow$ The relation "$\leq$" is anti-symmetric.

(c) Transitive - For each $m, n, p \in I^+$, we have

$m \leq n$; and $n \leq p \Rightarrow m \leq p$.

$\therefore$ The relation "$\leq$" is transitive.

Hence by definition of partial order relation, the relation "$\leq$" (less than or equal) is a partial order relation. (+2)

OR

(iii) Let X denote the set of all students in class.
Let M, P and C denote the set of students studying Mathematics, Physics and chemistry respectively.

Then we have $|X| = 175$, $|M| = 100$, $|P| = 70$, $|C| = 46$

$|M \cap P| = 30$, $|M \cap C| = 28$, $|P \cap C| = 23$, $|(M \cup P \cup C)'| = 22$ (+1)

$\therefore |M \cup P \cup C| = |X| - |(M \cup P \cup C)'| = 175 - 22 = 23$

By the Principle of inclusion and exclusion

$|M \cap P \cap C| = |M \cup P \cup C| - |M| - |P| - |C| + |M \cap P| + |M \cap C| + |P \cap C|$ (+1)

$\therefore |M \cap P \cap C| = 153 - 100 - 70 - 46 + 30 + 28 + 23 = 18$ (+1)

Hence the no. of students who study all the three subjects is 18.

Let M_1, P_1 and C_1 be the set of students who study only mathematics physics and chemistry respectively. Then

$M_1 = M - P - C$, $P_1 = P - M - C$, $C_1 = C - M - P$

$\therefore |M_1| = |M - P - C| = |M| - |M \cap P| - |M \cap C| + |M \cap P \cap C|$

$\therefore |M_1| = 100 - 30 - 28 + 18 = 60$ (+1)

similarly

$|P_1| = |P - M - C| = |P| - |P \cap M| - |P \cap C| + |P \cap M \cap C|$

$\therefore |P_1| = 70 - 30 - 23 + 18 = 35$ (+1)

and $|C_1| = |C - M - P| = |C| - |C \cap M| - |C \cap P| + |C \cap M \cap P|$

$|C_1| = 46 - 28 - 23 + 18 = 13$

Hence the no. of students who study exactly one subject out of three $= |M_1| + |P_1| + |C_1| = 60 + 35 + 13 = 108$ (+1)

ue. 3(i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix} = 1[(4\times5) - (6\times2)] - 2[5 - 4] + 3[6\times1 - 2\times4]$

$\therefore |A| = 8 - 2 - 6 = 0$

$\therefore \rho(A) \leq 3$ (+2)

$\therefore |A|_{2\times2} = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 4 - 2 = 2 \neq 0$

$\therefore \rho(A) = 2$. (rank of A)

Nullity = Order of A - rank of A $= 3 - 2 = 1$ (+2) ue.3(ii)

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The characteristic eqn. of matrix A is given by $|A-\lambda I|=0$ (+1/2)

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 0 \\ 2 & -1-\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix} = 0$$

$\Rightarrow -\lambda^3 - \lambda^2 + 5\lambda + 5 = 0$ or $\lambda^3 + \lambda^2 - 5\lambda - 5 = 0$ (+1/2)

To verify Cayley Hamilton theorem we will show that

$$A^3 + A^2 - 5A - 5I = 0 \longrightarrow (1)$$

$\therefore A^2 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 5 & 10 & 0 \\ 10 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (+2)

Thus

$$A^3 + A^2 - 5A - 5I = \begin{bmatrix} 5 & 10 & 0 \\ 10 & -5 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. $A^3 + A^2 - 5A - 5I = 0$ (+1)

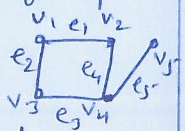
Hence Cayley Hamilton theorem verified.

To compute A^{-1}; multiplying eqn. (1) by A^{-1} we get

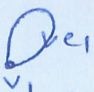
$A^2 + A - 5I - 5A^{-1} = 0$ or $A^{-1} = \frac{1}{5}(A^2 + A - 5I)$ (+1)

$\therefore A^{-1} = \frac{1}{5}\left(\begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix}\right)$ Answer. (+1)

ue.4(i) <u>Simple Graph</u> — A graph that has neither self loops nor parallel edges is called a simple graph. (+1)

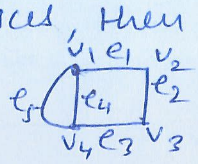


<u>Self loop</u> - An edge is said to be self loop or loop if its both end vertices are same. (+1)

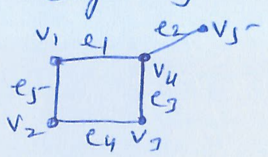


<u>Parallel Edge</u> - If there are two or more than two edges having the same pair of end vertices, then such edges are called parallel edges. (+1)

Here e_4, e_5 are parallel edges.



<u>Incident Edge</u>. Let v_i be an end vertex of some edge e_j, then we say that the edge e_j is incident on (or to or with) the vertex v_j. (+1)



Here edges e_1, e_2, e_3 are incident edges on vertex v_4.

Que. 4 (ii) Let G be a simple graph with n vertices $v_1, v_2, ---, v_n$ (say). The vertex v_1 can be joined to the remaining $(n-1)$ vertices $v_2, v_3, ---, v_n$ to obtain a maximum number of $(n-1)$ edges, namely $(v_1, v_2), (v_1, v_3), ---, (v_1, v_n)$. (+2)

The vertex v_2 can be jointed to the remaining $(n-2)$ vertices $v_3, v_4, ---, v_n$ to obtain a maximum number of $(n-2)$ edges, namely $(v_2, v_3), (v_2, v_4), ---, (v_2, v_n)$. Note that in this case we have not joined v_2 to v_1 since this edge (v_1, v_2) has already obtained. (+2)

Proceeding in this manner, the vertex v_{n-1} will give us only one new edge namely (v_{n-1}, v_n).

Hence maximum number of edges in the graph G is (+2)

$$= (n-1) + (n-2) + --- + 3 + 2 + 1$$

$$= \frac{1}{2} n(n-1).$$

Hence Proved.

R (iii) <u>Kruskal's algorithm.</u> The steps of the algorithm to construct a minimal spanning tree T for a connected weighted graph G with n vertices and m edges is as below:

<u>Step 1.</u> Arrange all edges of G in an increasing order to their weights. choose among them an edge e_i with least weight. Add this edge in T. (+1)

<u>Step 2</u> Choose another edge e_j with next higher weight in such a way that these edges do not form a cycle. If edge e_j does not form a cycle, then add e_j also in T. (+1)

<u>Step 3</u> Repeat step 2 untill all $(n-1)$ edges are added in T. and do not form a cycle. (+1)

<u>Prim's Algorithm.</u>

<u>Step 1.</u> Select an arbitrary vertex v_i and choose the edge with smallest weight in G. (+1)

<u>Step 2.</u> Select another edge e_j with smallest weight joining the vertex v_i. (+1)

<u>Step 3</u> Repeat step 2 untill all $(n-1)$ edges are added in the tree T. (+1)

Que. 5(i) A leap year consists of 366 days in which there are 52 complete weeks and 2 days are more. 52 weeks consists 52 wednesday and so we are to find the probability of being one wednesday out of 2 remaining days. (+1)
These two remaining days may make the following combination
(i) Tuesday and wednesday (ii) wednesday and Thursday
(iii) Thursday and Friday (iv) Friday and Saturday (+1)
(v) Saturday and Sunday (vi) Sunday and Monday (vii) Monday and Tuesday.

Thus the no. of favourable cases are 2 ((i) and (ii)) only.
and the exhaustive no. of cases are 7. (+2)
Hence the required probability is $= \frac{2}{7}$. Answer

5(ii) Given $n=4$, let $p =$ prob. of getting a head in a single throw of one coin i.e. $p=\frac{1}{2}$ $\therefore q=\frac{1}{2}$ (+1)
By Binomial Distribution. $P(X=r) = {}^nC_r p^r q^{n-r}$; $r=0,1,\ldots n$.
(i) P(two Heads) $= P(r=2) = {}^4C_2 \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = 6\times\frac{1}{16} = \frac{3}{8}$ $= 0.375$ (+2.5)

(ii) P(at least two Heads) $= P(r=2 \text{ or } r=3 \text{ or } r=4)$

$$= {}^4C_2 \left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2 + {}^4C_1\left(\tfrac{1}{2}\right)^1\left(\tfrac{1}{2}\right)^3 + {}^4C_4\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^4$$

$$= \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{11}{16} = 0.6875$$ Answer (+2.5)

OR (iii) For Poisson Distribution $P(X=r) = (e^{-m}.m^r)/\lfloor r$; $r=0,1,\ldots\infty$
Now we know that the k^{th} moment about origin
$\mu_k' = \Sigma r^k P(X=r)$ (+1)
$\therefore$ First moment about origin $\mu_1' = \sum_{r=0}^{\infty} r.P(X=r)$

$$\therefore \mu_1' = \sum_{r=0}^{\infty} r.\frac{e^{-m}.m^r}{\lfloor r} = e^{-m}\sum_{r=0}^{\infty}\frac{m^r}{(r-1)!} = e^{-m}\left[m+\frac{m^2}{\lfloor 1}+\frac{m^3}{\lfloor 2}+\ldots\right]$$

$$= me^{-m}\left[1+\frac{m}{\lfloor 1}+\frac{m^2}{\lfloor 2}+\ldots\right] = me^{-m}.e^m = m$$ (+2)

i.e. $\boxed{\mu_1' = m}$ Mean

Second moment about origin $\mu_2' = \sum_{r=0}^{\infty} r^2 P(X=r)$

$$\mu_2' = \sum_{r=0}^{\infty} r P(X=r) + \sum_{r=0}^{\infty} r(r-1) P(X=r)$$ (+1)

$$= m + e^{-m}\left[m^2+\frac{m^3}{\lfloor 1}+\frac{m^4}{\lfloor 2}+\ldots\right] = m+e^{-m}.m^2.e^m$$

$\mu_2' = m^2+m$
Now variance $\mu_2 = \mu_2' - (\mu_1')^2 = m^2+m-m^2$ (+2)
i.e. $\boxed{\mu_2 = m.}$ Variance

Que 6(i) The given recurrence relation is $a_r - 6a_{r-1} + 8a_{r-2} = 0$
given $a_0 = 3$ and $a_1 = 2$. $\longrightarrow (1)$

The characteristic equ. is $m^2 + 8 - 6m = 0 \Rightarrow m = 2, 4$. (+1)

The general sol. of (1) is given by $a_r = c_1 2^r + c_2 4^r \longrightarrow (2)$

putting $r=0$ in (2), we get $a_0 = c_1 + c_2 \Rightarrow 3 = c_1 + c_2 \longrightarrow (3)$ (+1)

putting $r=1$ in (2) we get $a_1 = 2c_1 + 4c_2 \Rightarrow 2 = 2c_1 + 4c_2 \longrightarrow (4)$ (+1)

solving equ. (3) and (4) we get $c_1 = 5$, $c_2 = -2$

putting the values of c_1 and c_2 in (2) we get required sol. of (1)

$$a_r = 5 \times 2^r - 2 \times 4^r.$$ Answer. (+1)

(ii) The given equ. is $a_r - 5a_{r-1} + 6a_{r-2} = 2 + r$; $r \geq 2$
Given $a_0 = 1$ and $a_1 = 1$. $\longrightarrow (1)$

The characteristic equ. of (1) is $m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$

$\therefore a_r^{(h)} = c_1 2^r + c_2 3^r \longrightarrow (2)$ (+1)

The particular sol. (trial sol.) corresponding to the term $2+r$ on R.H.S. of (1) is $A_0 + A_1 r$

i.e. $a_r^{(p)} = A_0 + A_1 r \longrightarrow (3)$

putting equ. (3) in (1) we get

$(A_0 + A_1 r) - 5(A_0 + A_1(r-1)) + 6(A_0 + A_1(r-2)) = 2 + r$

or $(2A_0 - 7A_1) + 2A_1 r = 2 + r \longrightarrow (4)$

Comparing two sides of equ. (4) we get

$2A_0 - 7A_1 = 2$, $2A_1 = 1$ (+3)

on solving we get $A_0 = \frac{11}{4}$, $A_1 = \frac{1}{2}$.

putting A_0, A_1 in (3) we get $a_r^{(p)} = \frac{11}{4} + \frac{1}{2} r$

Thus total sol. of (1) is $a_r = a_r^{(h)} + a_r^{(p)}$

$\therefore a_r = c_1 2^r + c_2 3^r + \frac{11}{4} + \frac{1}{2} r$ Answer. (+2)

or

Q.6(iii) Let the generating function for a_r be

$A(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots \longrightarrow (1)$ (+1)

Given $a_r = \begin{cases} 2^r & ; \text{if } r \text{ is even} \\ 2^{-r} & ; \text{if } r \text{ is odd} \end{cases} \longrightarrow (2)$

putting $r = 0, 1, 2, 3, 4, \cdots$ in (2) we get

$a_0 = 1$ $a_1 = -2$ $a_2 = 2^2 = 4$ $a_3 = -2^3 = -8$, $a_4 = 2^4 = 16$. (+2)

$a_5 = -2^5 = -32, \cdots$

putting values in (1) we get

$A(z) = 1 - 2z + 4z^2 - 8z^3 + 16z^4 - 32z^5 + \cdots$ (+1)

$= 1 - 2z + (2z)^2 - (2z)^3 + (2z)^4 - (2z)^5 + \cdots$

i.e. $A(z) = (1+2z)^{-1}$ ($\because$ By Binomial theorem (+2)

or $A(z) = \frac{1}{(1+2z)}$ Answer

Q.3(iii) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$

The characteristic equ. of matrix A is given by $|A-\lambda I|=0$ +1/2

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\left[(3-\lambda)^2-1\right]=0 \Rightarrow (1-\lambda)(2-\lambda)(4-\lambda)=0$$

$\Rightarrow \lambda = 1, 2, 4$ are the eigen values of the matrix A. +(1/2)

Let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be eigen vector corresponding to eigen value

~~vector~~ $\lambda=1$. Then $(A-1.I)X_1=0$

$$\begin{bmatrix} 1-1 & 0 & 0 \\ 0 & 3-1 & -1 \\ 0 & -1 & 3-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore \Rightarrow \begin{aligned} 0x_1 + 2x_2 + x_3 &= 0 \\ 0x_1 - x_2 + 2x_3 &= 0 \end{aligned}$$

$$\therefore \frac{x_1}{4+1} = \frac{x_2}{0-0} = \frac{x_3}{0-0} = k \text{ (say)}$$

or $$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = 5k$$

$\therefore$ Eigen vector $X_1 = 5k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (+1)

Simi. Eigen vector $X_2 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ corresponding to eigen value

$\lambda = 2$. Then $(A-2I)X_2 = 0$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ (+1)

$\therefore$ Eigen vector $X_2 = k.\begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$.

and Eigen vector $X_3 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ corresponding to eigen value

$\lambda = 4$. Then $(A-4I)X_3 = 0$

$$\Rightarrow \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ (+2)

$\therefore$ Eigen vector $X_3 = k\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$