Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Commerce

End Sem (Even) Examination May-2019

CM3CO05 Business Mathematics

Programme: B.Com.(Hons) Branch/Specialisation: Commerce

Duration: 3 Hrs. **Maximum Marks: 60**

Note: (a) All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.
(b) Use of simple (non-programmable) calculator is allowed.

Q.1	i.	If $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then AB is (a) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 5 \end{bmatrix}$	**1**
	ii.	The value of $\begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 4 & 4 \end{vmatrix}$ is (a) 1 (b) 4 (c) 0 (d) -1	**1**
	iii.	If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,4\}$, $B = \{2,3,6,8\}$, then $(A \cup B)'$ is (a) $\{5,6,7,8,9\}$ (b) $\{1,4,5,7,9\}$ (c) $\{5,7,9\}$ (d) $\{1,2,3,4,6,8\}$	**1**
	iv.	Which of the following functions is a polynomial function? (a) $7x+5$ (b) 3 (c) $4x^2-3x+6$ (d) All of these	**1**
	v.	The value of $\lim_{x \to 0} \frac{\sin x}{x}$ is (a) 0 (b) 1 (c) -1 (d) e	**1**
	vi.	The derivative of xe^x is (a) $e^x(x+1)$ (b) $x+e^x$ (c) xe^x+1 (d) $x(e^x+1)$	**1**

P.T.O.

vii. The integral of $\sec x.\tan x$ is **1**

(a) $\cos x + c$ (b) $\sin x + c$ (c) $\tan x + c$ (d) $\sec x + c$

viii. The value of $\int_0^1 (2x+1)dx$ is **1**

(a) 1 (b) 0 (c) 2 (d) None of these

ix. The common ratio of the series $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ...$ is **1**

(a) 2 (b) 1/2 (c) 1 (d) 1/4

x. The first, second and fourth terms of a proportion are 16, 24 and 54 respectively. Then the third term is **1**

(a) 36 (b) 28 (c) 48 (d) 32

Q.2 Attempt any two:

i. If $2x - y = \begin{bmatrix} 0 & 2 & 11 \\ 2 & 8 & 15 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 15 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$, then find the values of x and y. **5**

ii. Solve the following system of equations by Cramer's rule: **5**
$2x + 5y = 9, x + y = 3$.

iii. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{bmatrix}$. **5**

Q.3 Attempt any two:

i. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many like both coffee and tea? **5**

ii. If $U = \{1,2,3,4,5,6,7,8,9\}, A = \{2,4,6,8\}, B = \{2,3,5,7\}$, then verify that: (a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$. **5**

iii. A publisher find that the production cost directly attributed to each book is Rs. 60 and that the fixed cost is Rs. 90000. If each book can be sold for Rs. 90, determine the cost function, revenue function, break-even point and profit function. **5**

Q.4 Attempt any two:

i. Differentiate the following functions with respect to x: **5**

(a) $\frac{e^x}{1+x}$ (b) $x^{5/2} \log x$.

ii. The total cost of a daily output of x tons of coal is Rs. $\frac{x^3}{10} - 3x^2 + 50x$. What is the value of x, when average cost is minimum? **5**

iii. The cost function of a firm is given by $C(x) = 2x^2 - 4x + 5$. Find **5**

(a) The average cost (b) The marginal cost at $x = 10$.

Q.5 Attempt any two:

i. Evaluate the following: **5**

(a) $\int \log x\, dx$ (b) $\int \frac{(1-x)^3}{x} dx$.

ii. Find the consumer's surplus and producer's surplus defined by the demand function $D(x) = 20 - 5x$ and supply curve $S(x) = 4x + 8$. **5**

iii. Determine the cost of producing 200 cars, if the marginal cost (in rupees per unit) is given by $MC(x) = \frac{15}{2}x^2 - 4x + 8000$. **5**

Q.6 Attempt any two:

i. The first and the last term of an A.P. are - 4 and146 and sum of the A.P. is 7171. Find the number of terms in the A.P. and common difference. **5**

ii. (a) Find the value of $P(12,4) \div P(10,3)$. **5**

(b) If $P(4,2) = n.C(4,2)$,then find the value of n.

iii. If the fourth and seventh term of a series in G.P. are 54 and 1458, then find the series. **5**

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Q 1
Ans.
i) a) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$
ii) c) 0
iii) c) {5, 7, 9}
iv) d) All of these
v) b) 1
vi) a) $e^x(x+1)$
vii) d) $\sec x + c$
viii) c) 2
ix) b) 1/2
x) a) 36

Q 2
i) Given

$$2x - y = \begin{bmatrix} 0 & 2 & 11 \\ 2 & 8 & 15 \end{bmatrix} \quad \text{— (1)}$$

(+1)

$$x + 2y = \begin{bmatrix} 15 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix} \quad \text{— (2)}$$

Adding (1) & (2) by multiplying 2 in (1)

$$4x - \cancel{2y} = \begin{bmatrix} 0 & 4 & 22 \\ 4 & 16 & 30 \end{bmatrix}$$

$$x + \cancel{2y} = \begin{bmatrix} 15 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$

$$5x = \begin{bmatrix} 15 & 10 & 25 \\ 5 & 20 & 35 \end{bmatrix}$$

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$$x = \frac{1}{5}\begin{bmatrix} 15 & 10 & 25 \\ 5 & 20 & 35 \end{bmatrix}$$ (+2)

$$\boxed{x = \begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 7 \end{bmatrix}}$$

Putting value of x in equation 1.

$$2\begin{bmatrix} 3 & 2 & 5 \\ 1 & 4 & 7 \end{bmatrix} - y = \begin{bmatrix} 0 & 2 & 11 \\ 2 & 8 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 & 10 \\ 2 & 8 & 14 \end{bmatrix} - y = \begin{bmatrix} 0 & 2 & 11 \\ 2 & 8 & 15 \end{bmatrix}$$

$$y = \begin{bmatrix} 6 & 4 & 10 \\ 2 & 8 & 14 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 11 \\ 0 & 8 & 15 \end{bmatrix}$$

$$\boxed{y = \begin{bmatrix} 6 & 2 & -1 \\ 2 & 0 & -1 \end{bmatrix}}$$ (+2)

ii) Given System of Equation are

$$2x + 5y = 9$$
$$x + y = 3$$

Matrix form of above equations is

$$AX = B$$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$ (+2)

Determinant of $A = D = \begin{bmatrix} 2 & 5 \\ 1 & 1 \end{bmatrix} = 2 - 5 = -3$

$D_1 = \begin{bmatrix} 9 & 5 \\ 3 & 1 \end{bmatrix} = 9 - 15 = -6$

$D_2 = \begin{bmatrix} 2 & 9 \\ 1 & 3 \end{bmatrix} = 6 - 9 = -3$ (+2)

So the given equations has an unique solution

Now $x = \frac{D_1}{D} = \frac{-6}{-3} = 2$ (+1)

$y = \frac{D_2}{D} = \frac{-3}{-3} = 1$

Hence $x = 2$, $y = 1$ is the solution to given equations

iii) Determinant of the given Matrix

$$|A| = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \end{vmatrix}$$ (+1)

$$= 1\begin{vmatrix} -1 & 2 \\ -2 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} - 1\begin{vmatrix} 3 & -1 \\ 2 & -2 \end{vmatrix}$$

$$= 1(-3+4) - 2(9-4) - 1(-6+2)$$

$$= 1 - 10 + 4 = -5$$

or $|A| = -5 \neq 0$, thus inverse exist

Now $A^{-1} = \frac{Adj\,A}{|A|}$

To find Adj A, we have to find all Cofactors of $|A|$

C_{11} = Cofactor of 1 $= (-1)^{1+1} M_{11} = 1\begin{vmatrix} -1 & 2 \\ -2 & 3 \end{vmatrix}$

$= -3 + 4 = 1$

C_{12} = Cofactor of 2 $= (-1)^{1+2}\begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix}$

$= (-1)(9-4) = -5$

C_{13} = Cofactor of -1 $= (-1)^{1+3}\begin{vmatrix} 3 & -1 \end{vmatrix} = -4$

C_{21} = Cofactor of 3 $=(-1)^{2+1}\begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} = (-1)8 = -8$

C_{22} = Cofactor of -1 $=(-1)^{2+2}\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5$

C_{23} = Cofactor of 2 $=(-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 2 & -2 \end{vmatrix} = (-1)(-6) = 6$

C_{31} = Cofactor of 2 $=(-1)^{3+1}\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$

C_{32} = Cofactor of -2 $=(-1)^{3+2}\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = (-1)5 = -5$

C_{33} = Cofactor of 3 $=(-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -7$

Adj A = transpose of Cofactor Matrix

$$C = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix}$$

(+3)

$$\text{Adj } A = \begin{bmatrix} 1 & -8 & 3 \\ -5 & 5 & -5 \\ -4 & 6 & -7 \end{bmatrix}$$

Hence $A^{-1} = \dfrac{\text{Adj } A}{|A|}$

$$A^{-1} = \frac{-1}{5}\begin{bmatrix} 1 & -8 & 3 \\ -5 & 5 & -5 \\ -4 & 6 & -7 \end{bmatrix}$$

(+1)

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Q3

i) Let $n(C) = 37$, $n(T) = 52$ (+2)

Number of People who like at least one of the two drinks $n(C \cup T) = 70$

Number of People who like both Coffee and tea $= n(C \cap T)$

We know that

$$n(C \cup T) = n(C) + n(T) - n(C \cap T) \quad (+1)$$

$$70 = 37 + 52 - n(C \cap T)$$

$$70 = 89 - n(C \cap T)$$

$$n(C \cap T) = 89 - 70$$

$$\boxed{n(C \cap T) = 19} \quad (+2)$$

Therefore 19 People like both Coffee & tea

ii) Given

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{2, 4, 6, 8\}$ $B = \{2, 3, 5, 7\}$

$A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B)' = \{1, 9\}$ — (1) (+1)

$A' = \{1, 3, 5, 7, 9\}$, $B' = \{1, 4, 6, 8, 9\}$

$A' \cap B' = \{1, 9\}$ — (2)

By (1) & (2) $(A \cup B)' = A' \cap B'$ (+1)

Now $A \cap B = \{2\}$ (+1)

$(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8, 9\}$ — (3) (+1)

By (3) & (4)

$$(A \cap B)' = A' \cup B' \quad (+1)$$

Q.4

i) a) Let $y = \dfrac{e^x}{1+x}$

$$\frac{dy}{dx} = \frac{D^r \frac{d}{dx} N^r - N^r \frac{d}{dx} D^r}{(D^r)^2} \quad (+1/2)$$

$$\frac{dy}{dx} = \frac{(1+x)\frac{d}{dx}e^x - e^x \frac{d}{dx}(1+x)}{(1+x)^2} \quad (+1/2)$$

$$= \frac{(1+x)e^x - e^x \cdot 1}{(1+x)^2} \quad (+1/2)$$

$$= \frac{\cancel{e^x} + xe^x - \cancel{e^x}}{(1+x)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{xe^x}{(1+x)^2}} \quad (+1)$$

b) Let $y = \underset{\mathrm{I}}{x^{5/2}} \underset{\mathrm{II}}{\log x}$

$$\frac{dy}{dx} = \mathrm{I}\frac{d}{dx}\mathrm{II} + \mathrm{II}\frac{d}{dx}\mathrm{I} \quad (+1/2)$$

$$\frac{dy}{dx} = x^{5/2}\frac{d}{dx}\log x + \log x \frac{d}{dx}x^{5/2} \quad (+1/2)$$

$$= x^{5/2} \cdot \frac{1}{x} + \log x \cdot \frac{5}{2}x^{5/2-1} \quad (+\tfrac{1}{2})$$

$$= x^{5/2}x^{-1} + \log x \frac{5}{2}x^{3/2}$$

$$= x^{3/2} + \frac{5}{2}\log x \, x^{3/2} = x^{3/2}\left(1 + \frac{5}{2}\log x\right)$$

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ii) Given total cost of output x

$$C(x) = \frac{x^3}{10} - 3x^2 + 50x$$

Average cost

$$AC = \frac{C(x)}{x} = \frac{\frac{x^3}{10} - 3x^2 + 50x}{x}$$

$$AC = \frac{x^2}{10} - 3x + 50 \quad (+1)$$

For minimum of AC Diff w.r.to x

$$\frac{d(AC)}{dx} = \frac{d}{dx} \frac{x^2}{10} - 3x + 50$$

$$= \frac{2x}{10} - 3 = \frac{x}{5} - 3 \quad (+3)$$

Put $\frac{d(AC)}{dx} = 0$

$$\frac{x}{5} - 3 = 0$$

$$\boxed{x = 15}$$

Now $\frac{d^2(AC)}{dx^2} = \frac{1}{5} > 0$

Hence average cost is minimum when output is $x = 15$

$$(AC)_{Min} = \frac{(15)^2}{10} - 3 \times 15 + 50 \quad (+1)$$

$$= \frac{225}{10} - 45 + 50$$

$$(AC)_{min} = 22.5 + 5 = 27.5$$

— x —

iii) Cost function

$$C(x) = 2x^2 - 4x + 5$$

Average Cost $A.C(x) = \frac{C(x)}{x}$

$$AC = \frac{2x^2 - 4x + 5}{x} \quad (+2)$$

$$\boxed{A.C = 2x - 4 + \frac{5}{x}}$$

Marginal Cost :

$$MC(x) = \frac{d}{dx} C(x)$$

$$= \frac{d}{dx} 2x^2 - 4x + 5 \quad (+2)$$

$$MC(x) = 4x - 4$$

When $x = 10$ $\quad MC(10) = 4 \times 10 - 4$

$$= 40 - 4 \quad (+1)$$

$$\boxed{MC(10) = 36 \text{ Rs}}$$

Q.5. i) a) $I = \int \underset{I}{\log x} . \underset{II}{1}\, dx$

By Parts Rule $(+\tfrac{1}{2})$

$$\int \underset{I}{u}\, \underset{II}{v}\, dx = u \int v\, dx - \int \left(\frac{d}{dx} u \int v\, dx \right) dx$$

$$I = \log x \int 1\, dx - \int \left(\frac{d}{dx} \log x \int 1\, dx \right) dx \quad (+1)$$

$$= x \log x - \int \frac{1}{\cancel{x}} . \cancel{x}\, dx$$

$\int \log x\, dx = x \ldots$ (+1)

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b) $\int \frac{(1-x)^3}{x} dx = \int \frac{1-x^3+3x^2-3x}{x} dx$

$= \int \frac{1}{x} - x^2 + 3x - 3 \, dx$ $(+1\frac{1}{2})$

$= \log x - \frac{x^3}{3} + \frac{3x^2}{2} - 3x + C$ $(+1)$

—x—

ii) Given

demand function

$D(x) = 20 - 5x$ —(1)

Supply curve

$S(x) = 4x + 8$ —(2)

Consumer's Surplus is given by

$CS = \int_0^a f(x)dx - k_0 a$ $(+\frac{1}{2})$

To find k_0 and a solving ① & ②

$20 - 5x = 4x + 8$

$\boxed{x = \frac{12}{9} = \frac{4}{3}}$ $(+\frac{1}{2})$

by ② $S = 4 \times \frac{4}{3} + 8 = \frac{40}{3} = k_0$ $(+\frac{1}{2})$

Now

$CS = \int_0^{4/3} (20 - 5x) dx - \frac{40}{3} \times \frac{4}{3}$

$= \left[20x - \frac{5x^2}{2}\right]_0^{4/3} - \frac{160}{9}$

$= 20 \times \frac{4}{3} - \frac{5}{2} \times \frac{16}{9} - \frac{160}{9}$ $(+1)$

$\boxed{CS = \frac{40}{9}}$

—x—

Now

$S(x) = 4x + 8$

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$$PS = K_0 a - \int_0^a s(x)\,dx \quad (+1/2)$$

$$= \frac{40}{3} \times \frac{4}{3} - \int_0^{4/3} (4x+8)\,dx$$

$$= \frac{160}{9} - \left(\frac{4x^2}{2} + 8x\right)_0^{4/3} \quad (+1)$$

$$= \frac{160}{9} - \left(2 \times \frac{16}{9} + \frac{32}{3}\right)$$

$$= \frac{160}{9} - \frac{32}{9} - \frac{32}{3}$$

$$= \frac{160 - 32 - 96}{9} = \frac{32}{9} \quad (+1)$$

$$\boxed{PS = \frac{32}{9}}$$

iii) Given

$$MC(x) = \frac{15}{2}x^2 - 4x + 8000$$

$$\frac{dc}{dx} = C'(x) = \frac{15}{2}x^2 - 4x + 8000 \quad (+1)$$

$\therefore$ Cost of Producing 200 units of cars is given by

$$C(200) = \int_0^{200} \left(\frac{15}{2}x^2 - 4x + 8000\right) dx \quad (+1)$$

$$= \left[\frac{15}{2}\frac{x^3}{3} - 4\frac{x^2}{2} + 8000x\right]_0^{200} \quad (+1)$$

$$= \left(\frac{5}{2}x^3 - 2x^2 + 8000x\right]_0^{200}$$

$$= \frac{5}{2} \times (200)^3 - 2 \times (200)^2 + 8000 \times 200 \quad (+1)$$

$$= \frac{5}{2} \times 8,000000 - 2 \times 40,000 + 1,600000$$

$$= 20,000000 - 80000 + 1,600000 \quad (+1)$$

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Q 6

i) Given ~~first term~~ (+1)

$a = -4$,

~~last term~~ $l = 146 = a_n$

Sum $S_n = 7171$

Now $l = a_n = a + (n-1)d$ —(1) (+1)

$S_n = \frac{n}{2}(a + l)$ —(2)

$7171 = \frac{n}{2}(-4 + 146)$

$142n = 7171 \times 2$

$n = \frac{7171 \times 2}{142}$, $\boxed{n = 101}$ (+2)

Number of terms $\boxed{n = 101}$

by (1) $146 = -4 + (101 - 1)d$

$150 = 100\, d$ (+1)

$d = \frac{150}{100}$ $\boxed{d = \frac{3}{2}}$

Common Difference $\boxed{d = 3/2}$

ii) a) $P(12, 4) = {}^{n}P_4 = \frac{12!}{(12-4)!}$ (+1)

$= \frac{12!}{8!}$

$P(10, 3) = {}^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!}$

$\frac{P(12, 4)}{P(10, 3)} = \frac{12!}{8!} \times \frac{7!}{10!}$ (+1 $\frac{1}{2}$)

$= \frac{{}^{3}\cancel{12} \times 11 \times \cancel{10!}}{\cancel{8}_{2} \times \cancel{7!}} \times \frac{\cancel{7!}}{\cancel{10!}} = \frac{33}{2}$

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b) $P(4, 2) = n\,C(4, 2)$ (+2)

$^4P_2 = n\ ^4C_2$

$$\frac{\cancel{4!}}{(4-2)!} = n\frac{\cancel{4!}}{2!(4-2)!} \quad (+1/2)$$

$1 = \frac{n}{2} \Rightarrow \boxed{n=2}$

iii) Given

4th term $a_4 = 54$ (+1)

7th term $a_7 = 1458$

Now $a_n = ar^{n-1}$ where (+1)

$a \rightarrow$ first term

$r \rightarrow$ common ratio

$a_4 = ar^3 = 54$

$a_7 = ar^6 = 1458$ (+1)

$$\frac{ar^6}{ar^3} = \frac{1458}{54}$$

$r^3 = 27$, $\boxed{r=3}$

$\therefore\ a\times(3)^3 = 54$, $27a = 54$ (+1)

$\boxed{a=2}$

Series a, ar, ar^2, ar^3 — —

$\therefore\ 2, 6, 18, 54$ — —