Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering / Science

End Sem (Odd) Examination Dec-2022

BC3BS05 / CS3BS04 / IT3BS01 Discrete Mathematics

Programme: B.Tech. / B.Sc. Branch/Specialisation: CSE / IT / Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The trivial subset of set $X=\{a, b, c\}$ is **1**

(a) X (b) $\{\emptyset, X\}$ (c) $\{\emptyset\}$ (d) None of these

ii. Let A and B be two disjoint sets then $|A \cup B|$ - **1**

(a) $|A \cup B| = |A| + |B|$ (b) $|A \cup B| = |A| - |B|$

(c) $|A \cup B| = |A||B|$ (d) None of these

iii. If $f : X \rightarrow Y$ and A , B are two subsets of Y then- **1**

(a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$

(b) $f^{1}(A \cup B) = f^{-1}(A) \cap f^{-1}(B)$

(c) $f^{-1}(A \cap B) = f^{-1}(A) \cup f^{-1}(B)$

(d) None of these

iv. The number of maximal elements in the set $\{1,2,3,4,5\}$ under relation divisibility is- **1**

(a) 2 (b) n (c) 3 (d) None of these

v. In group $G = \{1, -1, i, -i\}$ order of element i with respect to multiplication is- **1**

(a) 1 (b) 2 (c) 4 (d) None of these

vi. Let I be a set of integers under addition operation H is subgroup of even integers then elements in coset of H in G is- **1**

(a) $\{0, \pm 1, \pm 2 \ldots\ldots\}$

(b) $\{1,2,3 \ldots\ldots\}$

(c) $\{0, \pm 1, \pm 2 \ldots\ldots\}$ and $\{1,2,3 \ldots\ldots\}$

(d) None of these

vii. Which is planar graph? **1**

(a) K_4 (b) K_5 (c) K_6 (d) None of these

viii. The degree of pendant vertex is- **1**

(a) 1 (b) 0 (c) 3 (d) 2

ix. The homogeneous solution of $a_r + Aa_{r-1} + Ba_{r-2} = 0$, when roots of axillary equation are real and distinct- **1**

(a) $c_1 {m_1}^r + c_2 {m_2}^r$ (b) $(c_1 + rc_2)m^r$

(c) $c_1 e^{m1} + c_2 e^{m2}$ (d) None of these

x. In recurrence relation generating function of sequence $(y_n\}$ is given by- **1**

(a) $\sum_{h=0}^{n} y_h t^h$ (b) $\sum_{h=0}^{\infty} y_h t^h$

(c) $\sum_{h=0}^{n} y_{h+1} t^{h+1}$ (d) $\sum_{h=0}^{n-1} y_h t^h$

Q.2 Attempt any two:

i. Define reflexive, symmetric and transitive relation. With example. **5**

ii. How many solutions does equation $x_1+x_2+x_3+x_4=13$ have where x_1, x_2, x_3 are non-negative integers with $0 \le x_i \le 5$, i=1,2,3,4 **5**

iii. Show that if 5 points are selected in a square whose sides have length 1 inch, at least two of the points must be no more than $\sqrt{2}$ inches apart. **5**

Q.3 Attempt any two:

i. Let B be the set of all positive divisors of 30 i.e $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and the operations + and * on B are defined as $a+b$ $=L.C.M$ *of* a and b , $a*b= H.C.F.$ *of* a and b , $a'=30/a$. Prove that (B, +, *, ') is Boolean Algebra. **5**

ii. Show that the relation "divides" on N is a partial order relation. **5**

iii. Change the Boolean function into disjunctive normal form **5**

$f(x, y, z) = [x+ (x'+ y)'].[x + (y'. z')']$

Q.4 Attempt any two:

i. If H_1 and H_2 are two subgroups of a group (G, °), then $H_1 \cap H_2$ is also a subgroup of G but union of two subgroups is not necessarily a subgroup explain with an example. **5**

ii. Find all generators in the cyclic group $\{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7. **5**

iii. Prove that every cyclic group is abelian group. **5**

Q.5 Attempt any two:

i. Define following with example: **5**

(a) Graph colouring and chromatic number

(b) Vertex disjoint subgraph

ii. Prove that number of edges in a tree with n vertices is $n-1$. **5**

iii. If the number of vertices in a graph is 10 each of degree 3. Find number of edges and number of regions in the graph. **5**

Q.6 Attempt any two:

i. Solve the recurrence relation $a_r + 5a_{r-1} + 5a_{r-2} = 2 + r$ **5**

ii. Find numeric function of generating function: **5**

$A(z) = (1 + z)^n + (1 - z)^n$

iii. There are 10 students in the class, of which 8 are girls and 2 are boys. Find the number of ways to select: **5**

(a) 2 girls and 1 boy

(b) 1 girl and 2 boys

******

No. | Marks

Medi-Caps University
Faculty of Engg / Science
End Sem (odd) Examination Dec-2022
BC3BS05 / CS3BS04 / IT3BS01
Discrete Mathematics.

Prog: B Tech / B Sc Branch: CSE / IT / Comp sci.

Qu. 1. MCQs (10)

i. b) $\{\phi, X\}$
ii. a) $|A \cup B| = |A| + |B|$
iii. a) $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$
iv. c) 3
v. c) 4
vi. d) None of these. None of these
vii. a) K_4
viii. a) One 1
ix. a) $c_1 m_1^r + c_2 m_2^r$
x. b) $\sum_{h=0}^{\infty} y_h t^h$

Let R be a relation in the set A.

Qu 2 i. Reflexive relation -

R is called reflexive relation if every ~~relation~~ element of A is R-related to itself.

$(a,a) \in R \quad \forall\ a \in A$

aRa

Ex. If $A = \{1,2,3\}$ then $R = \{(1,1),(2,2),(3,3),(1,3)\}$ +1

ii. Symmetric relation -

R is called symmetric relation if a is R-related to b then b is also R-related to a.

$(a,b) \in R \Rightarrow (b,a) \in R \quad \forall\ a,b \in A$

Marks.

10. Ex. If $A=\{1,2,3\}$ then $R=\{(1,2),(2,1)\}$ +2

iii. Transitive relation.

R is called transitive relation if a is R-related to b and b is R-related to c, then a is also R related to c.

$(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \quad \forall a,b,c \in A$

$aRb, bRc \Rightarrow aRc$

Ex. If $A=\{1,2,3\}$ then $R=\{(1,2),(2,3),(1,3)\}$ +2

ii) Let S denotes the set of all integers solution

A_i denote the set of integer solution with $x_i > 5$

Then nu. of solution with $0 \le x_i \le 5$ will be +1

$|A_1' \cap A_2' \cap A_3' \cap A_4'| = ?$

By Principle of inclusion and inclusion.

$$|A_1' \cap A_2' \cap A_3' \cap A_4'| = |S| - \sum_{i=1}^{4} |A_i| + \sum_{i,j=1}^{4} |A_i \cap A_j|$$

$$- \sum_{i,j,k=1}^{4} |A_i \cap A_j \cap A_k| + |A_1 \cap A_2 \cap A_3 \cap A_4| \quad \text{--- (1)}$$ +1

where $|S| = {}^{13+4-1}C_{13} = 560$

$|A_i| = {}^{(13-6)+4-1}C_{13-6} = 120$

$|A_i \cap A_j| = {}^{(13-6-6)+4-1}C_{13-6-6} = 4$ +1

$|A_i \cap A_j \cap A_k| = 0$ as sum of x_i's would exceed 13.

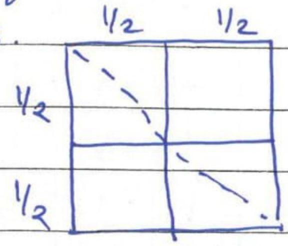
$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0$ +1

By eq (1)

$|A_1' \cap A_2' \cap A_3' \cap A_4'| = 560 - {}^4C_1 \times 120 + {}^4C_2 \times 4 - {}^4C_3 \times 0 + 0$

$= 104$ +1

No. (iii) First divide the square into four small squares of length $\frac{1}{2}$ inches. | Marks



+1

Now given 5 points (pigeons) will be placed in the 4 small square (pigeonholes).
Hence by Principle of pigeonhole some small square must contain atleast 2 points and its distance can not exceed (no more than) the length of the hypotaneous of length $= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{2}}{2}$ +2

$= \frac{1}{\sqrt{2}}$ +2

Qu. (3) (i). given $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$
$a + b = $ LCM of a & b
$a * b = $ HCF of a & b
$a' = 30/a$

Required composition tables —

+	1	2	3	5	6	10	15	30
1	1	2	3	5	6	10	15	30
2	2	2	6	10	6	10	30	30
3	3	6	3	15	6	30	15	30
5	5	10	15	5	30	10	15	30
6	6	6	6	30	6	30	30	30
10	10	10	30	10	30	10	30	30
15	15	30	15	15	30	30	15	30
30	30	30	30	30	30	30	30	30

For LCM
+

a	a'
1	30
2	15
3	10
5	6
6	5
10	3
15	2
30	1

For complement
a'

M T W T F S S
Page No.:
Date:
YOUVA

For HCF *

Marks.

0.

*	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

+1

Closure law – Clear that all elements of composition tables belong in set B. i.e. closure law hold. +0.5

Commutative law. $\forall\ a, b \in B$

$a+b = b+a$ & $a*b = b*a$ +0.5

ex. $3+5 = 5+3$ $\quad$ $10*15 = 15*10$

$15 = 15$ $\quad$ $5 = 5$

As each row is identical to its corresponding column in both the tables (+, *) commutative law holds so.

Distributive law – $\forall\ a, b, c \in B$

$a + (b*c) = (a+b) * (a+c)$ +1

ex $3 + (5*10) = (3+5) * (3+10)$

$3 + 5 = 15 * 30$

$15 = 15$

Similar for $a*(b+c) = (a*b) + (a*c)$

Distributive law hold.

Identity law – As per composition table +1

30 is identity element for HCF (*) &

1 is identity element for LCM (+)

Identity law hold.

Complement law – For each $a \in B$, there exists +1

~~For a table~~ complementary elements such that

For LCM + $\quad a + a' = 30$

HCM * $\quad a * a' = 1$

ex $3+10 = 30$ & $3*10 = 1$

No. | Marks

As all laws satisfied $(B, +, *, ')$ is called Boolean algebra.

(ii) To show the "divides" relation is partial order relation, relation should be —

Reflexive — $\forall\ a \in N$ +1

$a | a$ or $a R a$

Hence relation is reflexive.

Anti symmetric — $\forall\ a, b \in N$ +2

If $a|b$ then there exist no natural number such that $b|a$ if a, b are two distinct natural no.'s & is possible only when they are identical so.

$a | b,\ b | a \Rightarrow a = b$

or $a R b,\ b R a \Rightarrow a = b$

Hence relation is anti-symmetric.

Transitive — $\forall\ a, b, c \in N$

$a | b,\ b | c \Rightarrow a | c$ +2

or $a R b,\ b R c \Rightarrow a R c$

Hence relation is transitive.

Thus the relation "divides" on N is partial order relation.

(iii) $f(x, y, z) = [x + (x' + y)'] . [x + (y' . z')']$

$= [x + (x . y')] . [x + y + z]$ +1

$= (x + x.y').x + (x + x.y').y + (x + x.y').z$

$= x.x + x.y'.x + x.y + x.y'.y + x.z + x.y'.z$ +1

$= x + x.y' + x.y + 0 + x.z + x.y'.z$

$= x.1.1. + x.y'.1 + x.y.1 + x.1.z + x.y'.z$ +1

$= x.(y+y').(z+z') + x.y'(z+z') + x.y.(z+z') + x.(y+y').z + x.y'.z$

$= x.y.z + x.y.z' + x.y'.z + x.y'.z' + x.y'z + x.y'.z' +$ +1
$x.y.z + x.y.z' + x.y.z + x.y'.z + x.y'.z$

$= x.y.z + x.y.z' + x.y'.z + x.y'.z'$ +1

Marks.

Qu. 4. (i). H_1 & H_2 are two subgroup of $(G, \circ)$
Then $H_1 \cap H_2 \neq \phi$
To prove $H_1 \cap H_2$ is a subgroup, we have to show $a \in H_1 \cap H_2$, $b \in H_1 \cap H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$ +1
~~Now~~ $a \in H_1 \cap H_2 \Rightarrow a \in H_1$ and $a \in H_2$
$b \in H_1 \cap H_2 \Rightarrow b \in H_1$ and $b \in H_2$
But given that H_1 & H_2 are subgroups
Therefore $a \in H_1$, $b \in H_1 \Rightarrow ab^{-1} \in H_1$ +1
$a \in H_2$, $b \in H_2 \Rightarrow ab^{-1} \in H_2$
Hence $ab^{-1} \in H_1$, $ab^{-1} \in H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$
Thus $a \in H_1 \cap H_2$, $b \in H_1 \cap H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$ +1
Hence proved.
But union of two subgroups is not necessarily a subgroup. For ex. Let G be a additive group of I and If $H_1 = \{0, \pm 2, \pm 4, \pm 6 \dots\}$ & $H_2 = \{0, \pm 3, \pm 6, \pm 9 \dots\}$ +1
Then $H_1 \cup H_2 = \{0, \pm 2, \pm 3, \pm 4, \pm 6 \dots\}$
Obviously $H_1 \cup H_2$ does not satisfy closure property
Ex. $3, 4 \in H_1 \cup H_2$ but $7 \notin H_1 \cup H_2$
Hence $H_1 \cup H_2$ is not a subgroup of G. +1
However $H_1 \cap H_2 = \{0, \pm 2, \pm 6, \dots\}$ is a subgroup of G.

(ii). Composition table for X_7

X_7	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

+2

No. Marks

Given cyclic group $G = \{1, 2, 3, 4, 5, 6\}$

If there exists an element $a \in G$ then

~~o(a) = (G)~~ $o(a) = o(G) = 6$

Clear that for $3, 5 \in G$ with multiplication modulo we can find all remaining elements of G.

$3^1 = 3$ & $5^1 = 5$

$3^2 = 2$ $5^2 = 4$

$3^3 = 6$ $5^3 = 6$

$3^4 = 4$ $5^4 = 2$

$3^5 = 5$ $5^5 = 3$

$3^6 = 1$ $5^6 = 1$ +2

i.e. 3 & 5 are generators of G.

While for $1, 2, 4, 6 \in G$ we can not find all elements of G under multiplication modulo 7. +1

Means 1, 2, 4 and 6 are not generators of G.

iii) Let G be a cyclic group, generated by a. +1

i.e $G = \{a\}$

Let $\forall x, y \in G$ then $x = a^r, y = a^s$ +1

where $r, s \in I$

Therefore $xy = a^r a^s$

$= a^{r+s}$ (by law of indices)

$= a^{s+r}$ (as integers obeys comm. law)

$= a^s a^r$

$= yx$ +2

As $xy = yx$, commutative law hold. +1

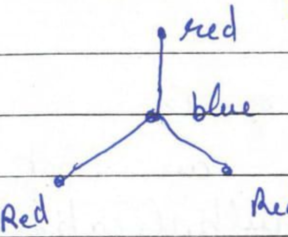
Hence Cyclic group G is abelian group

Marks.

Qu. (5) (i). a). By colouring of a graph, we mean to colour all the vertices of the graph with colours such that no two adjacent vertices have the same colour. After proper colouring, graph is called properly coloured graph. +1

The chromatic number of a ~~gra~~ graph is the minimum number of colours required for proper colouring the vertices of graph and denoted by $\chi(G)$. +1

Ex.



2 - colours are used +0.5

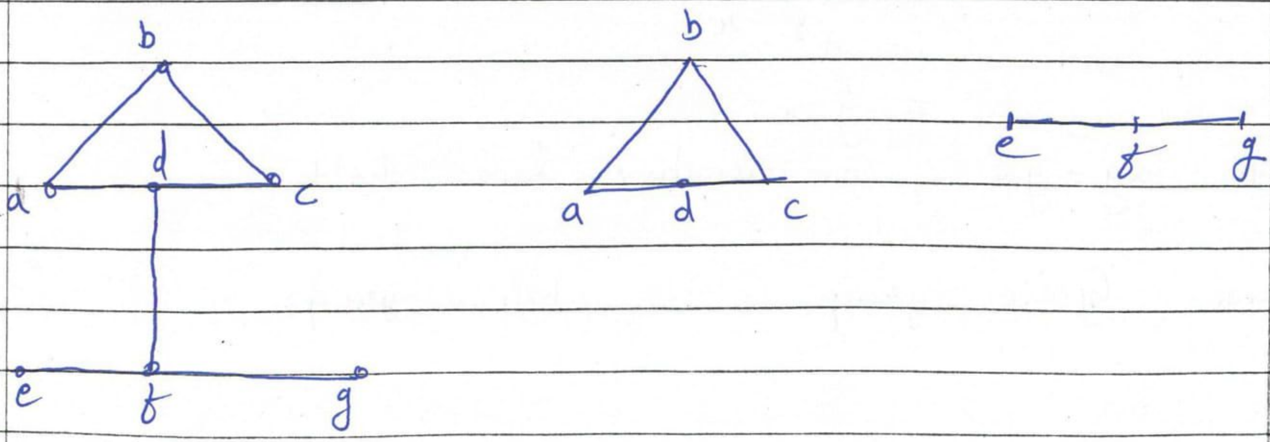
$\chi(G) = 2$.

b). Let $G=(V,E)$ be a graph, then two subgraphs H & K of G are called vertex disjoint subgraphs if H & K have no vertex in common +1

If $H = (V', E')$
$K = (V'', E'')$

then $V' \cap V'' = \phi$ +0.5

clearly $E' \cap E'' = \phi$



+1

G H K

No. (ii). We shall prove by induction method. Marks

If $n=1$, then tree T has no edges

If $n=2$, then tree T has one edge.

Now, suppose that the theorem is true for all trees having less than n vertices. +1

Let T be a tree with n vertices. and there exists only one path between every pair of vertices. If we delete one edge from tree T then graph will be disconnected and will have two components

$$T-e = T_1 + T_2$$ +1

Let n_1 & n_2 be the number of vertices of T_1 & T_2 respectively where $n_1<n$ & $n_2<n$ such that $n_1+n_2=n$.

As no. of edges in $T_1 = n_1 - 1$

-"- $T_2 = n_2 - 1$ +1

Number of edges in $T-e = T_1 + T_2$

$$T-e = (n_1-1)+(n_2-1)$$
$$= (n_1+n_2) - 2$$
$$= n-2$$ +1

If we replace the edge, then

Number of edges in $T = n-2+1$
$$= n-1$$

Hence the theorem holds for all values of n and hence tree has $n-1$ edges with n vertices. +1

10. (iii). Given number of vertices $= n = 10$ | Marks.
degree of each $= 3$

Total degrees $= 3 \times 10 = 30$ +1

By Handshaking theorem

$$\Sigma d(v_i) = 2e$$
$$30 = 2e$$
$$e = 15$$ +2

By Euler's formula of planar graph

$$n - e + r = 2$$
$$r = 2 - n + e$$
$$= 2 - 10 + 15$$
$$= 7$$ +2

Ans number of edges $= 15$
number of regions $= 7$

Qu. 6 (i). Total sol $= a_r^h + a_r^p$ — (1)

Homogeneous sol — $m^2 + 5m + 5 = 0$, $m = \frac{-5 \pm \sqrt{5}}{2}$ +1

$$a_r^h = C_1\left(\frac{-5+\sqrt{5}}{2}\right)^r + C_2\left(\frac{-5-\sqrt{5}}{2}\right)^r \quad \text{— (2)}$$ +1

Particular sol

Trial sol corresponding to $2+r = A_0 + A_1 r$ +1

Put in given eq

$$(A_0 + A_1 r) + 5[A_0 + A_1(r-1)] + 5[A_0 + A_1(r-2)] = 2 + r$$
$$11 A_0 + 11 A_1 r - 15 A_1 = 2 + r$$

Comparing both sides $11 A_0 - 15 A_1 = 2$
$11 A_1 r = r$ or $11 A_1 = 1$ +1

Hence $A_1 = 1/11$, $A_0 = \frac{37}{121}$

Now we get $a_r^p = \left(\frac{37}{121}\right) + \left(\frac{1}{11}\right) r$ — (3) +1

Required sol $a_r = C_1\left(\frac{-5+\sqrt{5}}{2}\right)^r + C_2\left(\frac{-5+\sqrt{5}}{2}\right)^r + \left(\frac{37}{121}\right) + \left(\frac{1}{11}\right)^r$

No. ii $A(z) = (1+z)^n + (1-z)^n$ | Marks

By Binomial expansion

$(1+z)^n = \sum_{r=0}^{n} \binom{n}{r} z^r = {}^nC_0 z^0 + {}^nC_1 z + {}^nC_2 z^2 + \dots {}^nC_n z^n$

$= 1 + {}^nC_1 z + {}^nC_2 z^2 + \dots {}^nC_n z^n$ +1.5

$(1-z)^n = \sum_{r=0}^{n} \binom{n}{r} (-z)^r = {}^nC_0 (z)^0 + {}^nC_1 (-z)^1 + {}^nC_2 (-z)^2 + \dots {}^nC_n (-z)^n$

$= 1 - {}^nC_1 z + {}^nC_2 z^2 + \dots + (-1)^n \, {}^nC_n z^n$ +1.5

By adding $A(z) = 2 + 2\,{}^nC_2 z^2 + \dots + 2\,{}^nC_{2r} z^{2r} + \dots$ +1

$a_r = \begin{cases} 0 & \text{if } r \text{ is odd} \\ 2\,{}^nC_r & \text{if } r \text{ is even} \end{cases}$ +1

iii. Total students = 10
Number of girls = 8
Number of boys = 2

Number of ways to select

a) 2 girls and 1 boy
$^8C_2 \times {}^2C_1$ +1.5

$28 \times 2 = 56$ ways +1

b) 1 girl and 2 boys

$^8C_1 \times {}^2C_2$ +1.5

$8 \times 1 = 8$ way +1