Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Science / Engineering

End Sem Examination May-2024

CA3CO19 Mathematics -II

Programme: BCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then A^{100}= ______. **1**

(a) $2^{100}A$ (b) $2^{99}A$ (c) $2^{0}A$ (d) None of these

ii. A matrix in which number of column is more than number of rows is called- **1**

(a) Horizontal matrix (b) Triangular matrix

(c) Vertical matrix (d) None of these

iii. If $f(x,y) = x^3 + y^3 - 3axy$, then the value of $\frac{\partial f}{\partial x}$ is- **1**

(a) $3x^2 + 3ay$ (b) $3x^2 - 3ay$

(c) $3y^2 - 3ay$ (d) None of these

iv. The degree of homogeneous function $u = f(\frac{y}{x})$ is- **1**

(a) 0 (b) 1 (c) 2 (d) 3

v. The degree of differential equation - $p^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^3$ - **1**

(a) 2 (b) 3 (c) 1 (d) None of these

vi. Integrating factor of $\frac{dy}{dx} - y = e^{3x}$ is - **1**

(a) e^{x} (b) e^{-x} (c) e^{2x} (d) None of these

vii. Roots of auxiliary equation of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$ are- **1**

(a) 1, 4 (b) -1, 4 (c) -1, -4 (d) None of these

viii. The particular integral of the ordinary differential equation $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=0$ is- **1**

(a) e^{2x} (b) 0 (c) e^{-2x} (d) None of these

ix. The arithmetic mean of 4,7, x and 9 is 7 then value of x is- **1**

(a) 8 (b) 7 (c) 9 (d) None of these

x. If number of terms in given statistical data is odd number then median = **1**

(a) $(\frac{n}{2}+1)^{th} term$ (b) $(\frac{n+1}{2}) th\ term$

(c) $\frac{n}{2}\ th\ term$ (d) None of these

Q.2 i. Define nullity of a matrix **2**

ii. If A= diag (1, -1, 2) and B= diag (2, 3, -1) then find 3A+4B **3**

iii. Solve $3x+3y+2z=1,\ \ x+2y=4,\ \ 10y+3z=-2,$ $2x-3y-z=5$ by matrix method. **5**

OR iv. Find the rank of the following matrix $\begin{bmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 4 & 1 & 3\end{bmatrix}$. **5**

Q.3 i. State Euler's theorem for homogeneous function. **3**

ii. If $u=f(r)\ where\ r^2=x^2+y^2$ then prove that **7**

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f"(r)+\frac{1}{r}f'(r)$$

OR iii. If $u=\frac{x^2y^2}{x^2+y^2}$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial x^2}=2u$ **7**

Q.4 i. Solve $(1+x^2)dy=(1+y^2)dx$ **3**

ii. Solve $(1+y^2)dx=(tan^{-1}y-x)dy$ **7**

OR iii. Solve $(ycosx+siny+y)dx+(sinx+xcosy+x)dy=0$ **7**

Q.5 i. Solve $\frac{d^3y}{dx^3}-8y=0$ **3**

ii. Solve $\frac{d^2y}{dx^2}+4y=e^x+sin2x$ **7**

OR iii. Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x sinx$ **7**

Q.6 i. Find Mode of the following series: **3**

15,20,16,15,14,21,15,17,16,17,15,18,20

ii. Calculate the coefficient of variation for following distribution: **7**

Expenditure (In Rs)	Less than 5	Less than 10	Less than 15	Less than 20	Less than 25
No. of hens	6	16	28	38	46

OR iii. Calculate Median for the following data: **7**

Class Interval	0-20	20-40	40-60	60-80	80-100
Frequency	2	7	10	3	3

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End Sem Examination May 2024 (1)
BCA / BCA+MCA
Mathematics - II (CA3 C019)

Q1 MCQ (+10)

i) b) $2^{99} A$
ii) a) Horizontal matrix
iii) b) $3x^2 - 3ay$
iv) a) 0
v) c) 1
vi) b) e^{-x}
vii) b) $-1, 4$
viii) b) None of these
ix) a) 8
x) b) $\left(\frac{n+1}{2}\right)^{th}$ term

Q2 i) Nullity: The number of zero's row in a matrix after elementary operations. (+1)
The difference of order of a matrix and rank of a matrix is called Nullity. It is denoted by N(A) or $\nu(A)$ (+1)
Nullity = order − rank
$N(A) = O(A) - r(A)$

ii) Given

$$A = diag(1, -1, 2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(+1)

$$B = diag(2, 3, -1) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$3A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \quad 4B = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$ (+1)

(2)

$$3A+4B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$ (1)

$$= \text{diag}(11, 9, 2)$$

1(i) The equations are

$$3x+3y+2z=1$$
$$x+2y=4$$
$$10y+3z=-2$$
$$2x-3y-z=5$$

The matrix equation $AX=B$ is given by (0.5)

$$\begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 0 \\ 0 & 10 & 3 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 5 \end{bmatrix}$$

Augmented matrix is given by (0.5)

$$[A:B] = \left[\begin{array}{ccc|c} 3 & 3 & 2 & 4 \\ 1 & 2 & 0 & +4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & 1 & 5 \end{array}\right]$$

Applying $R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 3 & 3 & 2 & 1 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & 1 & 5 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_4 \to R_4 - 2R_1$

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H

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \\ -4 \\ 0 \end{bmatrix}$$

$x + 2y = 4 \quad \text{— (i)}$

$-3y + 2z = -11 \quad \text{— (ii)}$

$z = -4 \quad \text{— (iii)}$

On solving we get

$\boxed{x = 2} \quad \boxed{y = 1} \quad \boxed{z = -4}$

(3)

$$[A:B] \approx \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 10 & 3 & -2 \\ 0 & -7 & -1 & -3 \end{array}\right]$$ (+1)

$R_3 \to R_3 + \frac{10}{3}R_2$, $R_4 \to R_4 - \frac{7}{3}R_2$

$$[A:B] \approx \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29/3 & -116/3 \\ 0 & 0 & -17/3 & 68/3 \end{array}\right]$$

$R_3 \to 3R_3$ $R_4 \to 3R_4$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & -17 & 68 \end{array}\right]$$ (+1)

$R_4 \to R_4 + \frac{17}{29}R_3$ we get

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 \to \frac{1}{29}R_3$ $$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ (+1)

which is echelon form $\rho(A:B) = \rho(A) = 3$
$=$ no of unknown

Given system of equation is consistent and has unique solution.

Reduced matrix form is given by

$R_2(-\frac{1}{3}) \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & -11/3 \\ 0 & 10 & 3 & -2 \end{array}\right]$ +0.5

$R_3 \to R_3 - 10R_2$

$\sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & -2/3 & -11/3 \\ 0 & 0 & \frac{29}{3} & \frac{104}{3} \end{array}\right]$ +0.5

rank$[A:B] = 3$

Clearly rank(A) = rank$[A:B] = 3$
= no of unknowns

Equation has unique solution and it is given by $AX = B$

$x + 2y = 4$

$y - \frac{2}{3}z = \frac{-11}{3}$

$\frac{29}{3}z = \frac{104}{3}$

$\boxed{z = \frac{104}{29}}$ +1

solving $x =$ $y =$

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OR ii)
$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

Applying operations on A
$R_2 \to R_2 - 3R_1 \quad R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & -7 & 7 \end{bmatrix}$$ +1

$R_3 \to R_3 - R_2$ +1

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & -7 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_2(-\frac{1}{7}) \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 2 \end{bmatrix}$ +1

$R_3(\frac{1}{2}) \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 1 \end{bmatrix}$ +1

This is the echelon form
Hence rank of Matrix $\boxed{\rho(A) = 3}$ +1

Q3 i) Euler's Theorem: If $u = f(x, y)$ is a homogeneous function of degree n in x and y i.e $u = x^n f(y/x)$ then +3

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu}$$

ii) Given $u = f(r), \quad r^2 = x^2 + y^2$
Partially Diff $\frac{\partial r}{\partial x} = \frac{x}{r}, \quad \frac{\partial r}{\partial y} = \frac{y}{r}$ +1

$$\frac{\partial u}{\partial x} = f'(r)\frac{\partial r}{\partial x} = f'(r)\frac{x}{r}$$

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Again on Partially Differentiating we get

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$ +2

$$= 2(2-1)u$$

Note $= 2u$ +1

Direct Partial derivative Can also apply

Q.4 i) Given

$$(1+x^2)dy = (1+y^2)dx$$

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$ +1

This is V.S. form then integrating both the sides

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$ +1

$$\tan^{-1}y = \tan^{-1}x + \tan^{-1}c$$

$$\tan^{-1}y - \tan^{-1}x = \tan^{-1}c$$ +0.5

$$\tan^{-1}\frac{y-x}{1+xy} = \tan^{-1}c$$

$$\frac{y-x}{1+xy} = c$$ +0.5

$$\boxed{y = x = c(1+xy)}$$

ii) $$(1+y^2)dx = (\tan^{-1}y - x)\,dy$$

$$(1+y^2)\frac{dx}{dy} = \tan^{-1}y - x$$ +0.5

$$(1+y^2)\frac{dx}{dy} + x = \tan^{-1}y$$

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$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial x}\left(\frac{x f'(r)}{r}\right)$$

$$= \frac{r\left(x f''(r)\,\partial r/\partial x + f'(r)\right) - x f'(r)\,\partial r/\partial x}{r^2} \quad (+2)$$

$$= \frac{r\left(x f''(r)\frac{x}{r} + f'(r)\right) - x f'(r)\cdot x/r}{r^2}$$

$$= \frac{f''(r)x^2 + r f'(r) - \frac{x^2}{r} f'(r)}{r^2} \quad (+2)$$

Similarly

$$\frac{\partial^2 u}{\partial y^2} = \frac{f''(r)y^2 + r f'(r) - \frac{y^2}{r} f'(r)}{r^2}$$

On Adding we get

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{r^2}\left[f''(r)(x^2+y^2) + 2r f'(r) - \frac{f'(r)}{r}(x^2+y^2)\right]$$

$$= \frac{1}{r^2}\left[f''(r) r^2 + 2r f'(r) - \frac{f'(r)}{r} r^2\right] \quad (+1)$$

$$= f''(r) + \frac{1}{r} f'(r) \quad (+1)$$

OR iii) Given

$$u = \frac{x^2 y^2}{x^2+y^2}$$

$$u = \frac{x^4 (y/x)^2}{x^2(1+(y/x)^2)} \quad (+2)$$

$$u = x^2 F(y/x)$$

This is homogeneous function with degree $n = 2$ then by Euler's theorem we know that (+1)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$

(+1)

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$$

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$$\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$$ +0.5

This is L.D.E $\frac{dx}{dy} + Px = Q$
Where

$$P = \frac{1}{1+y^2} \quad Q = \frac{\tan^{-1}y}{1+y^2}$$

Integrating factor

$$IF = e^{\int P\,d\cancel{x}}$$

$$= e^{\int \frac{dy}{1+y^2}}$$ 1.5

$$= e^{\tan^{-1}y}$$

Solution of the equation

$$x\,I.F = \int Q\,IF\,dy + C$$ +0.5

$$x\,e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2}\,e^{\tan^{-1}y}\,dy + C$$

$$\tan^{-1}y = t \quad \frac{1}{1+y^2}dy = dt$$

$$= \int t\,e^t\,dt + C$$

$$= t e^t - e^t + C$$ +3

$$= e^t(t-1) + C$$

$$x\,e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + C$$

$$\boxed{x = (\tan^{-1}y - 1) + C e^{-\tan^{-1}y}}$$ +1

OR(iii) $(y\cos x + \sin y + y)dx + (\sin x + x\cos y + x)dy = 0$

Form of the equation $Mdx + Ndy = 0$ +1

$M = y\cos x + \sin y + y$, $N = \sin x + x\cos y + x$

$\frac{\partial M}{\partial y} = \cos x + \cos y + 1$, $\frac{\partial N}{\partial x} = \cos x + \cos y + 1$ +2

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ +1

∴ Equation is exact

Solution of the equation is given by

$$\int M\,dx + \int N\,dy = C$$ +1

y = const (terms not containing x)
(or x = const)

$$\int (y\cos x + \sin y + y)\,dx + \int 0\,dy = C$$ +2

$$\boxed{y\sin x + x\sin y + xy = C}$$

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Q5

i) $\frac{d^3y}{dx^3} - 8y = 0$

$$\frac{d^3y}{dx^3} - 8y = 0$$ (+1)

$$(D^3 - 8)\,y = 0.$$

Auxilary eq.

$$m^3 - 8 = 0.$$

$$m^3 = 8$$ (+1)

$$m = 2, 2, 2$$

$CF = (c_1 + c_2x + c_3x^2)\,e^{2x}.$

$PI = 0$

$y = CF + PI$ (+1)

$y = (c_1 + c_2x + c_3x^2)\,e^{2x}.$

ii $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x.$

$(D^2 + 4)\,y = e^x + \sin 2x.$

Auxilary eq.

$$m^2 + 4 = 0$$ (+2)

$$m^2 = -4$$

$$m = \pm 2i$$

$CF = e^{0x}(c_1\cos 2x + c_2\sin 2x)$ (+1)

$CF = c_1\cos 2x + c_2\sin 2x.$

$PI = \frac{1}{D^2+4}\,e^x + \frac{1}{D^2+4}\sin 2x.$ (+1)

put $D = 1.$ put $D^2 = -(2)^2.$ ∵ it is '0'

So,

$\frac{1}{5}e^x. + \frac{x\sin 2x}{2D}.$

(+1)

$\frac{e^x}{5} + \frac{x}{2}\left(\frac{-\cos 2x}{2}\right)$

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$PI = \frac{e^x}{5} - \frac{x\cos 2x}{4}$ (+1)

$y = CF + PI$

$y = c_1\cos 2x + c_2\sin 2x + \frac{e^x}{5} - \frac{x\cos 2x}{4}.$ (+1)

OR iii) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x\sin x$

Auxilary eq:

$m^2 - 2m + 1 = 0$ (+2)

$m^2 - m - m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$ (+1)

$m = 1, 1$

$CF = (c_1 + c_2x)e^x.$

$PI = \frac{1}{D^2 - 2D + 1} xe^x\sin x.$

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Q6 i) 15, 20, 16, 15, 14, 21, 15, 17, 16, 17, 15, 18, 20

~~increasing order~~

~~14~~, ~~15~~.

+3

Here 15 occurs maximum times i.e (4)

So, mode of the following series is 15

ii) coficient of variation

Expenditure	0-5	5-10	10-15	15-20	20-25
freq.	6	10	12	10	8

$$\text{Coffiicient of Var.} = \frac{S.D.}{\text{mean}} \times 100$$

Exp.	f	x	fx
0-5	6	2.5	15
5-10	10	7.5	75
10-15	12	12.5	150
15-20	10	17.5	175
20-25	8	22.5	180

+2

$\Sigma f = 46$ $\Sigma fx = 595$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{595}{46}$$

$\boxed{\bar{x} = 12.93}$ +1

x	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
2.5	-10.43	108.78	652.70
7.5	-5.43	29.48	294.8
12.5	-0.43	0.18	2.21
17.5	4.57	20.88	208.8
22.5	9.57	91.58	732.67
	Total		1892.08

+2

+1

$$S.D.(\sigma) = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\frac{1892.08}{46}} = \sqrt{41.1324}$$

$= 6.413$

Coff. of Var. $= \frac{\sigma}{\bar{x}} \times 100$

$= \frac{6.413}{12.93} \times 100$ (+1)

$\boxed{C.V. = 49.597}$

iii) Median

Class interval	Frequency	cf
0-20	2	2
20-40	7	9
40-60	10	19
60-80	3	22
80-100	3	25
Total	$N = \Sigma f = 25$	

(+2)

median is $\frac{1}{2}(N+1)^{th}$ term $= \frac{1}{2}(25+1) = \frac{26}{2} = 13^{th}$ term

(+1)

which lies b/w 40-60

Median $= l + \frac{\frac{1}{2}N - F}{f} \times i$ (+1)

$= 40 + \frac{\frac{25}{2} - 9}{10} \times 20$ (+2)

$= 40 + \left(\frac{25}{2} - 9\right) \times 2 = 47$

(+1)

median 47.