Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2022

EN3BS15 Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: AU/CE/FT /ME/RA

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. $E^{-1}(f(x)) =$ _____ where E is shift operator and h is interval of differencing. **1**

(a) $f(x)$ (b) $f(x+h)$ (c) $f(x-h)$ (d) None of these

ii. To find the value of y corresponding to x= 2 is done by using _____method for the given data: **1**

x	0	1	4	5	8
y	12	68	153	180	200

(a) Newtons Forward (b) Stirlings

(c) Lagrange's (d) None of these

iii. To apply _____ rule number of intervals n must be multiple of 3. **1**

(a) Trapezoidal (b) Simpsons 3/8

(c) Weedle's (d) None of these

iv. Runge Kutta method of first order is known as _____ methods. **1**

(a) Euler's (b) Modified Euler

(c) Improved Euler (d) None of these

v. If mean of the exponential distribution is ½ then variance is ____. **1**

(a) ½ (b) ¼ (c) 1 (d) None of these

vi. If variance of the binomial distribution is 3 and mean is 5 then probability of failure of the event is_____. **1**

(a) 3/5 (b) 2 (c) 5/3 (d) None of these

vii. If values of one of the variable x decreases correspondingly y also decreases, then correlation between them is _____. **1**

(a) Positive (b) Negative (c) Perfect (d) None of these

viii. The two regression lines x on y and y on x always intersect at ___. **1**

(a) Mean (b) Mode (c) Median (d) None of these

P.T.O.

ix. When we reject the null hypothesis though it is true then it is known as error of _____ kind. **1**

(a) First (b) Second (c) Third (d) None of these

x. The probability level below which we reject the null hypothesis is called _____. **1**

(a) Level of significance (b) Confidence level

(c) Critical level (d) None of these

Q.2 Attempt any two:

i. Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ in factorial notation and find its successive differences. **5**

ii. Use Newton's divided difference formula to find $f(x)$ at $x = 3$ from the following data: **5**

x	0	1	2	4	5	6
$f(x)$	11	14	15	5	6	19

iii. Obtain the value of $\frac{dy}{dx}$ at $x = 90$ using the following data: **5**

x	60	75	90	105	120
y	28.2	38.2	43.2	40.9	37.7

Q.3 Attempt any two:

i. Apply Simpson's 3/8 rule to evaluate $\int_0^2 \frac{1}{1+x^3} dx$ to two decimal places by dividing the range in to eight equal parts. **5**

ii. Find the second approximation using Picard's method for the solution of $\frac{dy}{dx} = 1 + xy$ given that $y(0) = 1$. **5**

iii. Using Runge Kutta method of order four finds $y(0.1), y(0.2)$ given that $\frac{dy}{dx} = y - x; y(0) = 2$, taking $h = 0.1$. **5**

Q.4 Attempt any two:

i. Prove that mean deviation about mean of the normal distribution is 4/5 times the standard deviation. **5**

ii. The probability that a bomb dropped from a plane will strike the target is 1/5. If 6 bombs are dropped, find the probability that- **5**

(a) Exactly two will strike the target

(b) At least two will strike the target

iii. Fit Poisson distribution to the following data: **5**

x	0	1	2	3	4
$f(x)$	122	60	15	2	1

Q.5 Attempt any two:

i. Fit a straight line to the following data: **5**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

ii. Find the two lines of regression from the following data: **5**

x	78	89	97	69	59	79	68	61
y	125	137	156	112	107	136	123	108

iii. Find the rank correlation coefficient from the following data: **5**

x	68	64	75	50	64	80	75	40	55	64
y	62	58	68	45	81	60	68	48	50	70

Q.6 Attempt any two:

i. Find the student's t-statistics for the following variable values in a sample: -4, -2, -2, 0, 2, 2, 3, 3 taking the mean of the universe to be zero. **5**

ii. A dice is tossed 120 times with the following results: **5**

No. Turned up	1	2	3	4	5	6	Total
Frequency	30	25	18	10	22	15	120

Test the hypothesis that the dice is unbiased ($\chi^2_{.05,5} = 11.07$).

iii. Test whether the two sets of observations: **5**

I:	17	27	18	25	27	29	27	23	17
II:	16	16	20	16	20	17	15	21	

Indicates sample drawn from the same universe. [The value of z at 5% level of significance and for 8 and 7 degree of freedom is .6575].

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	Marks
Q.1	
i) (c) $f(x-h)$	[1]
ii) (c) Lagrange's	[1]
iii) (b) Simpsons 3/8	[1]
iv) (a) Euler's	[1]
v) (b) 1/4	[1]
vi) (a) 3/5	[1]
vii) (a) Positive	[1]
viii) (a) Mean	[1]
ix) (a) First	[1]
x) (a) Level of significance	[1]

Q.2

i) $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$

$= A x^{[4]} + B x^{[3]} + C x^{[2]} + D x^{[1]} + E x^{[0]}$ [2]

1	1	-12	24	-30	$9 = E$
	0	1	-11	13	
2	1	-11	13		$-17 = D$
	0	2	-18		
3	1	-9			$-5 = C$
	0	3			
	$1 = A$	$-9 = B$			

[3]

Marks

$f(x) = x^{(4)} - 6x^{(3)} - 5x^{(2)} - 17x^{(1)} + 9$ [4]

$\therefore \Delta f(x) = 4x^{(3)} - 18x^{(2)} - 10x^{(1)} - 17x^{(0)}$

$\Delta^2 f(x) = 12x^{(2)} - 36x^{(1)} - 10x^{(0)}$

$\Delta^3 f(x) = 24x^{(1)} - 36x^{(0)}$

$\Delta^4 f(x) = 24x^{(0)}$

$\Delta^n f(x) = 0 \quad ; n \geq 5$ [5]

Q2 i)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
0	11					
		3				
1	14		-1			
		1		$-1/4$		
2	15		-2		$1/4$	
		-5		1		$-1/24$
4	5		2		0	
		1		1		
5	6		6			
		13				
6	19					

[2]

$$y = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + (x-x_0)(x-x_1)(x-x_2)\Delta^3 f(x_0) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 f(x_0) + (x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)\Delta^5 f(x_0)$$ [3]

$$= 11 + (x-0)3 + (x-0)(x-1)(-1) + (x-0)(x-1)(x-2)(-1/4) + (x-0)(x-1)(x-2)(x-4)(1/4) + (x-0)(x-1)(x-2)(x-4)(x-5) \times \left(\frac{-1}{24}\right)$$ [4]

$$y_{x=3} = 11 + 9 - 6 + 6\left(\frac{-1}{4}\right) - 6\left(\frac{1}{4}\right) + 12\left(\frac{-1}{24}\right)$$

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$= 14 - \frac{6}{2} - \frac{1}{2}$

$= \frac{21}{2} = 10.5$ [5]

iii)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$60 = x_{-2}$	28.2				
		10			
$75 = x_{-1}$	38.2		-5		
		$5 = \Delta y_{-1}$		$-2.3 = \Delta^3 y_{-2}$	
$90 = x_0$	43.2		-7.3		8.7
		$-2.3 = \Delta y_0$		$6.4 = \Delta^3 y_{-1}$	
$105 = x_1$	40.9		-.9		
		-3.2			
$120 = x_2$	37.7				

[2]

$$\left.\frac{dy}{dx}\right|_{x=x_0} = \frac{1}{h}\left[\left\{\frac{\Delta y_0 + \Delta y_{-1}}{2}\right\} - \frac{1}{6}\left\{\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right\} + \cdots\right]$$ [3]

$$= \frac{1}{15}\left[\frac{5 - 2.3}{2} - \frac{1}{6}\left[\frac{6.4 - 2.3}{2}\right]\right]$$ [4]

$$= \frac{1}{15}[1.35 - .341666]$$

$$= .0672222$$ [5]

Q3

i) $h = \frac{2-0}{8} = \frac{1}{4}$; $y = \frac{1}{1+x^3}$ [1]

x:	0	1/4	1/2	3/4	1	5/4	6/4	7/4	8/4 = 2
y:	1	.9846	.8889	.7033	.5	.3386	.2286	.1572	.1111
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

[3]

$$\int_0^2 \frac{1}{1+x^3}\,dx = \frac{3h}{8}\left[y_0 + y_8 + 3(y_1 + y_2 + y_4 + y_5 + y_7) + 2(y_3 + y_6)\right]$$

[4]

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$= \frac{3}{4\times 8}[11.5828]$

$= 1.0859$ [5]

To apply simpsons 3/8 rule interval must lee multiple of 3 if considered with this then it should lee considered.

3 ii)

Here $f(x,y) = 1+xy$

$x_0 = 0$

$y_0 = 1$

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$= 1 + \int_0^x (1 + x\cdot 1)\,dx$$

$$= 1 + x + \frac{x^2}{2}$$ [2.5]

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$= 1 + \int_0^x \left[1 + x\left(1 + x + \frac{x^2}{2}\right)\right]dx$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}.$$ [5]

3 iii) Given $x_0 = 0$, $y_0 = 2$, $h = .1$

$k_1 = h f(x_0, y_0) = .1(2-0) = .2$

$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = .1[2.1 - .05] = .205$

$k_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = .1[2.1025 - .05]$

$= .20525$

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0. $K_4 = h f(x_0+h, y_0+k_3)$

$= .1\,[2.20525 - .1] = .210525$

$y_{(.1)} = y_1 = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= 2 + .20517$

$= 2.2052$ [2.5]

To find $y_{.2}$

$K_1 = h f(x_1, y_1) = .1\,[2.2052 - .1]$

$= .21052$

$K_2 = h f\left(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}\right) = .1\,[2.31046 - .15]$

$= .216046$

$K_3 = h f\left(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}\right) = .1\,[2.313223 - .15$

$= .2163223$

$K_4 = h f(x_1+h, y_1+k_3) = .1\,[2.4215223 - .2]$

$= .22215223$

$y_{(.2)} = y_2 = y_1 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= 2.4214$ [5]

Q4 (i) $M.D = \int_{-\infty}^{\infty} |x-\mu| f(x)\,dx$

$= \int_{-\infty}^{\infty} |x-\mu| \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} dx$

Put $t = \frac{x-\mu}{\sqrt{2}\sigma} \Rightarrow dx = \sqrt{2}\sigma\,dt$

$M.D = \int_{-\infty}^{\infty} |\sigma\sqrt{2}t| \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} \sqrt{2}\sigma\,dt$ [2]

Marks

$= \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \cdot 2 \int_0^\infty t e^{-t^2} dt$ [3]

put $u = t^2 \Rightarrow du = 2t\,dt$

$= \sqrt{\frac{2}{\pi}}\sigma \int_0^\infty e^{-u} du$ [4]

$= \sqrt{\frac{2}{\pi}}\sigma \left[-e^{-u}\right]_0^\infty$

$= \sqrt{\frac{2}{\pi}}\sigma = .8\sigma = \frac{4}{5}\sigma$ approx [5]

Q4 ii) $p = \frac{1}{5}$ & $n = 6$

$q = 1 - \frac{1}{5} = \frac{4}{5}$ [1]

Using Binomial Distribution

$P(r) = {}^nC_r\, p^r q^{n-r}$; $r = 0, 1, 2, \ldots n$

$= {}^6C_r \left(\frac{1}{5}\right)^r \left(\frac{4}{5}\right)^{6-r}$ [2]

(a) $P(r=2) = {}^6C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^4$

$= 15 \cdot \frac{4^4}{5^6}$

$= .24576$ [3.5]

(b) P (at least two will strike the target)

$= P(r \geq 2)$

$= P(r=2) + P(r=3) + P(r=4) + P(r=5) + P(r=6)$

$= 1 - P(r=0) - P(r=1)$

$= 1 - .2621 - .3932 = .3446$ [5]

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(iii) Q4 Mean $m = \frac{\Sigma fx}{\Sigma f} = \frac{100}{200} = .5$ Marks [1]

$f(x) = N \cdot \frac{e^{-m} m^x}{x!} \quad 200 \cdot \frac{e^{-.5} (.5)^x}{x!}; x = 0, 1, \ldots 4$ [2]

x	$f(x)$
0	121
1	61
2	15
3	3
4	0

[3] [4] [5]

(i) Q5 Equation of straight line for the given data is $y = a + bx$

Its equation of normal are

$\Sigma y = na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$ [1]

x	y	$u = x - 5$	$v = y - 12$	uv	u^2
1	9	-4	-3	12	16
2	8	-3	-4	12	9
3	10	-2	-2	4	4
4	12	-1	0	0	1
5	11	0	-1	0	0
6	13	1	1	1	1
7	14	2	2	4	4
8	16	3	3	12	9
9	15	4	3	12	16
		$\Sigma u = 0$	$\Sigma v = 0$	$\Sigma uv = 57$	$\Sigma u^2 = 60$

[3]

$0 = 9a + 0 \Rightarrow a = 0$; $57 = 0 + 60b \Rightarrow b = .95$

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Req. eq. of straight line

$y - 12 = .95(x - 5)$

$y = 7.25 + .95x$ [5]

(ii)

Q5

x	y	$u = x - 78$	$v = y - 125$	uv	u^2	v^2
78	125	0	0	0	0	0
89	137	11	12	132	121	144
97	156	19	31	589	361	961
69	112	-9	-13	117	81	169
59	107	-19	-18	342	361	324
79	136	1	11	11	1	121
68	123	-10	-2	20	100	4
61	108	-17	-17	289	289	289
$\Sigma x =$ 600	$\Sigma y =$ 1004	$\Sigma u =$ -24	$\Sigma v =$ 4	$\Sigma uv =$ 1506	$\Sigma u^2 =$ 1314	$\Sigma v^2 =$ 2012

[2]

$$\bar{x} = \frac{\Sigma x}{n} = \frac{600}{8} = 75$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{1004}{8} = 125.5$$

$$b_{yx} = \frac{\Sigma uv - \frac{\Sigma u \Sigma v}{n}}{\Sigma u^2 - \frac{(\Sigma u)^2}{n}} = 1.217$$ [3]

$$b_{xy} = \frac{\Sigma uv - \frac{\Sigma u \Sigma v}{n}}{\Sigma v^2 - \frac{(\Sigma v)^2}{n}} = 0.752$$ [4]

Line of regression of y on x: $y - \bar{y} = b_{yx}(x - \bar{x})$

$y = 1.217x + 34.225$

Marks

Line of regression of x on y:

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x = .752y - 19.376 \quad [5]$$

(iii) Q5

x	y	R_x	R_y	$d = R_x - R_y$	d^2
68	62	7	6	1	1
64	58	5	4	1	1
75	68	8.5	7.5	1	1
50	45	2	1	1	1
64	81	5	10	-5	25
80	60	10	5	5	25
75	68	8.5	7.5	1	1
40	48	1	2	-1	1
55	50	3	3	0	0
64	70	5	9	-4	16

$\Sigma d^2 = 72$ [3]

$$\rho = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}m_1(m_1^2-1) + \frac{1}{12}m_2(m_2^2-1) + \cdots\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[72 + \frac{1}{12}\cdot 3\cdot(9-1) + \frac{1}{12}\cdot 2(4-1) + \frac{1}{12}\cdot 2(4-1)\right]}{10(100-1)} \quad [4]$$

$$= 0.545 \quad [5]$$

(vi) Q6

$$t = \frac{(\bar{x} - \mu)\sqrt{n}}{s}$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} \quad [1]$$

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x	$x-\bar{x}$	$(x-\bar{x})^2$
-4	-4.25	18.0625
-2	-2.25	5.0625
-2	-2.25	5.0625
0	-0.25	0.0625
2	1.75	3.0625
2	1.75	3.0625
3	2.75	7.5625
3	2.75	7.5625
		49.5000

Marks

[3]

$$\bar{x} = \frac{2}{8} = 0.25$$

$$S = \sqrt{\frac{49.5}{7}} = 2.659$$ [4]

$$t = \frac{(.25-0)\sqrt{8}}{2.659} = 0.27$$ [5]

(ii) Q6 Null Hypothesis H_0: The dice is unbiased one.

$$f_e = N \cdot P(x) = 120 \cdot \frac{1}{6} = 20$$ [1]

x	f_o	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2 / f_e$
1	30	10	100	5
2	25	5	25	1.25
3	18	-2	4	0.20
4	10	-10	100	5
5	22	2	4	0.21
6	15	-5	25	1.25
				$\chi^2 = 12.90$

[3]

degree of freedom $\nu = 6-1 = 5$ [4]

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Calculated value of χ^2 & tabulated value of χ^2
So reject the null hypothesis
$\Rightarrow$ the dice is biased one. [5]

6(i) Given $n_1 = 9$, $n_2 = 8$
$v_1 = 9-1 = 8$ $v_2 = 8-1 = 7$ [1]

$\bar{x} = \frac{\Sigma x}{n_1} = \frac{210}{9} = 23.33$ $\bar{y} = \frac{\Sigma y}{n_2} = \frac{141}{8} = 17.625$

H_0 : Two population variance are same.

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
17	-6.33	40.0689	16	-1.625	2.6406
27	3.67	13.4689	16	-1.625	2.6406
18	-5.33	28.4089	20	2.375	5.6406
25	1.67	2.7889	16	-1.625	2.6406
27	3.67	13.4689	20	2.375	5.6406
29	5.67	32.1489	17	-.625	0.3906
27	3.67	13.4689	15	-2.625	6.8906
23	-.33	0.1089	21	3.375	11.3906
17	-6.33	40.0689			
210		184.0001	141		37.8748

[3]

$$s_1^2 = \frac{\Sigma (x-\bar{x})^2}{n_1 - 1} = 23$$

$$s_2^2 = \frac{\Sigma (y-\bar{y})^2}{n_2 - 1} = 5.4107 \quad ; s_1^2 > s_2^2$$

$$z = \frac{1}{2} \log_e \frac{s_1^2}{s_2^2} = .7236$$ [4]

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Calculated value of z ≠ tabulated value of z

So reject the Null hypothesis

So the two variance are not same. [5]