ii. Find the Karl Pearson's coefficient of correlation between x and y for the following data : **8**

x :	6	2	4	9	1	3	5	8
y :	13	8	12	15	9	10	11	16

OR iii. Find the two lines of regressions from the following data : **8**

x :	158	160	163	165	167	170	172	175	177	181
y :	163	158	167	170	160	180	170	175	172	175

Estimate y, when x = 164.

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2018

CA5BS01 Mathematical Foundation of Computer Science

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated.Answers of Q.1 (MCQs) should be written in full instead of only (i), (ii), (iii) or (iv).

Q.1 i. If S = {a, b, c} then power set of S equals to : **1**

(a) {a, b, c}
(b) {∅, {a}, {b}, {c}, {a, b}, {b, c}, {c, a}, {a, b, c}}
(c) {∅, {a}, {b}, {c}, {a, b, c}
(d) None of these.

ii. If A = {1, 2, 3} and B = {1, 3, 5, 7} then B – A = ... **1**

(a) {1, 2, 3, 5, 7} (b) {1, 3}
(c) {1, 3, 5} (d) {5, 7}

iii. If A = {1, 2, 3} then the identity relation on A is : **1**

(a) R = {(1, 2), (2, 3), (3, 1)} (b) R = {(1, 1), (2, 2), (3, 3)}
(c) R = {1, 2, 3} (d) None of these.

iv. The relation R in which "whenever (a, b) ∈ R and (b, a) ∈ R then a = b" is called: **1**

(a) Reflexive relation. (b) Transitive relation.
(c) Anti-symmetric relation. (d) Symmetric relation.

v. The rank of the unit matrix of order n is : **1**

(a) 0 (b) n (c) 1 (d) n-1.

vi. The eigen values of the matrix $A = \begin{bmatrix} -5 & -5 & 9 \\ 8 & 9 & 18 \\ -2 & -3 & -7 \end{bmatrix}$ are: **1**

(a) 1, 1, 1 (b) -1, -1, -1 (c) 1, -1, -1 (d) -1, 1, 1

P.T.O.

vii. Coefficient of range = ..., where L is the largest value and S is the smallest value. **1**

(a) $\frac{L-S}{L+S}$ (b) $\frac{L+S}{L-S}$ (c) $\frac{L}{S}$ (d) $\frac{S}{L}$

viii. Mode is the value of the variable which occurs .... in a distribution. **1**

(a) Most frequently. (b) Rarely.
(c) In the middle. (d) None of these.

ix. Correlation coefficient lies between : **1**

(a) -1 and +1 (b) -1/2 and +1/2
(c) 0 and 1 (d) -1 and 0.

x. The regression lines become identical if : **1**

(a) r = ±1 (b) r = 0 (c) r = 1 (d) r = ±2

Q.2 i. Define subset and proper subset with examples. **2**

ii. If $A \subseteq B$, then prove that $A \times C \subseteq B \times C$ for any set C. **3**

iii. For any three sets A, B and C, prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **5**

OR iv. State and prove De-Morgan's law for set theory. **5**

Q.3 i. Differentiate between relation and function. **2**

ii. Suppose A = {0, 1, 2, 3, 4}, B = {0, 1, 2, 3} and aRb if and only if a + b = 4, then find domain and range of R. **3**

iii. Prove that the relation R on the set Z of all integers defined by $(x, y) \in R \Leftrightarrow x - y$ is divisible by m is an equivalence relation on Z. **5**

OR iv. Prove that the function $f: Q \longrightarrow Q$ given by $f(x) = 2x - 3$ for all $x \in Q$ is a bijection. **5**

Q.4 i. Define rank and nullity of a matrix. **2**

ii. Reduce the following matrix to the normal form and find its rank and nullity: $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. **3**

iii. Determine the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. **5**

OR iv. Find for what value(s) of k the equations : **5**
$x + y + z = 1$, $x + 2y + 4z = k$ and $x + 4y + 10z = k^2$
have a solution and solve completely in each case.

Q.5 i. Define Standard Deviation. **2**

ii. Calculate the median from the following table : **3**

Marks	Frequency
0-10	22
10-20	38
20-30	46
30-40	34
40-50	20

iii. Define Mean. Suppose two classes of 40 and 50 strength have mean Biostatistics marks of 52 and 61 respectively. Find out the combined mean of the two groups. **5**

OR iv. The following table gives the wages of employees in two factories. In which factory, there is greater variation in the distribution of wages per employee? **5**

	Factory A	Factory B
Number of Employees	50	100
Average wage per month per worker	120	85
Variance of the wage per employee per month	9	16

Q.6 i. Define coefficient of correlation and for what value of r the coefficient of correlation is perfect negative and perfect positive. **2**

End Sem (odd) Exam . Dec - 2018

Solution: CA5BS01

Mathematical foundation of Computer Science

MCQ

Q1

(i) (b) $\{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$ 1

(ii) (d) $\{5,7\}$ 1

(iii) (b) $R = \{(1,1), (2,2), (3,3)\}$ 1

(iv) (d) Symmetric 1

(v) (b) n 1

(vi) (b) $-1, -1, -1$ 1

(vii) (a) $\frac{L-S}{L+S}$ 1

(viii) (a) Most frequently 1

(ix) (a) $-1, +1$ 1

(x) (a) $r = \pm 1$

Q2 (i) Subset $A \subseteq B$ +1

Proper subset $A \subset B$ +1

$A = \{1,2,3\}$ $B = \{1,2,3,4\}$.

$A \subset B$ If $B = \{1,2,3\}$ $A \subseteq B$

(ii) Let $(a,c) \in A \times C$ Then +1

$\Rightarrow a \in A$ and $c \in C$ +1

$\Rightarrow a \in B$ and $c \in C$ since $A \subseteq B$ +1

$\Rightarrow a \in B \times C$

To Prove LHS RHS (2)

(iii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let (x,y) be an arb. element of $A \times (B \cap C)$

Then $(x,y) \in A \times (B \cap C) \Leftrightarrow x \in A$ and $y \in (B \cap C)$ +1

$\Leftrightarrow x \in A$ and $y \in B$ and $y \in C$

$\Leftrightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$ +2

$\Leftrightarrow (x,y) \in (A \times B)$ and $(x,y) \in (A \times C)$

+2

$\Leftrightarrow (x,y) \in (A \times B) \cap (A \times C)$

OR

(iii) Demargon' Law of Set Theory

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

(a) Let $x \in (A \cup B)'$

$\Rightarrow x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$ $2\frac{1}{2}$

$(A \cup B)' \subseteq A' \cap B'$

Let $x \in A' \cap B' \Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)'$

$\therefore (A \cup B)' \subseteq A' \cap B'$ $2\frac{1}{2}$

(b) is ~~same~~ as. (1)'

(3)

Q3 (i) Relation

Let A and B be two sets. A relation from A to B is a subset of $A \times B$ and is denoted by R

R is relation from A to B $\iff R \subseteq A \times B$

$R = \{(x, y) : x R y$, where $x \in A$ and $y \in B\}$ +1

Domain of $R = \{x : x \in A$ & $(x, y) \in R$ for $y \in B\}$

Range of $R = \{y : y \in B$ & $(x, y) \in R$ for some $x \in A\}$.

(ii) Function : Let A and B be two non-empty sets if every element of the first set A is related to an unique element of B by some definite rule f, called mapping or function +1

$f : A \to B$. $\quad A \xrightarrow{f} B$

Domain of f = first set A

Co-domain of f = Second set B.

Range of a mapping $f(A) = \{f(x) : x \in A\}$

(ii)

$a = \{0, 1, 2, 3, 4\}$ +1

$b = \{0, 1, 2, 3\}$.

The values of a, b satisfying $a + b = 4$

a =	$\cancel{0}$	1	2	3	4	✓ Domain
b =	$\cancel{x}$	3	2	1	0	✓ Range

+2

4

(iii) (a) R is reflexive for each $a \in I$ $a-a \in I$ 1

(b) also $(a-b)/m$ here $-(a-b) = b-a$ is also divisible by m (+ve integer)

$\therefore$ $aRb \Rightarrow bRa$ symmetric +2

(c) $a, b, c \in I$ we have

aRb. $bRc \Rightarrow (a-b)$ & $(b-c)$ are both divisible by m.

$\therefore$ $(a-b)+(b-c)$ is divisible by m +2

$(a-c)$ " or aRc

$\therefore$ (a) (b), (c) $\Rightarrow$ equivalence relation.

OR

(iv) $f: Q \to Q$. $f(x) = 2x-3$ $\forall x \in Q$

(a) One-one

Let $f(x_1) = f(x_2)$

$2x_1 - 3 = 2x_2 - \cancel{x}.3$ +2

$2x_1 = 2x_2$

$x_1 = x_2$

$\therefore$ f is one one +2

$2x-3=y$

$x = \frac{y+3}{2}$

(b) for onto $\exists$ $\frac{y+3}{2} \in Q$

st. $f(x) = f\left(\frac{y+3}{2}\right) = \cancel{2}\left[\frac{y+3}{\cancel{2}}\right] - 3 = y$ onto

5

Q4. (i) Rank – order of highest order non vanaishing minor +1

Nullity →

$N(r) = n - r$ $\quad n =$ order, $r =$ rank +1

$A =$ must be square for nullity

(ii) $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_2 - R_1$ $\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$R_3 \leftarrow R_3 - 3R_1$; $R_4 \leftarrow R_4 - R_1$

$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$ +2

$\rho(A) = 2$ $\quad$ Nullity $= 2$ +1

(iii) for Eigen values.

$|A - \lambda I| = 0$ $\quad \begin{vmatrix} 6-\lambda & 2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 0$

$(\lambda - 2)(\; -\lambda^2 - 10\lambda - 16) = 0$ $2\frac{1}{2}$

$\lambda = 2, 2, 8$

eigen vector

$\lambda = 2$ (repeated)

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$4x - 2y + 2z = 0 \qquad z = K_2 \quad y = K_1$

$x = 2K_1/4$

$$X = \begin{bmatrix} \frac{K_1 - K_2}{2} \\ K_1 \\ K_2 \end{bmatrix}$$

$K_2 = 0 \quad K_1 = 2 \qquad \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \quad K_1 = 0, \; K_2 = 2$ $+ 2$

for $\lambda = 8 \qquad X = \begin{bmatrix} 2K \\ -K \\ K \end{bmatrix} = K \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ $+ 2\frac{1}{2}$

OR.
(iv)

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 4 & : & K \\ 1 & 4 & 10 & : & K^2 \end{bmatrix}$$ $+1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 3 & : & K-1 \\ 0 & 0 & 0 & : & K^2 - 3K + 2 \end{bmatrix}$$ $+2$

for solution $\rho(A:B) = \rho(A)$

$\therefore$ $k^2 - 3k + 2 = 0$ $\quad$ $k = 1$ or $k = 2$

Case (i) $k = 1$ infinite sol^n

$x + y + z = 1$ $\quad$ $y + 3z = 0$ $\quad$ $z = \lambda$ (say) +1

$x = 2\lambda + 1$, $y = -3\lambda$, $z = \lambda$.

+1

Case (ii) $k = 2$. $x + y + z = 1$ $\quad$ $y + 3z = 1$ & $z = \lambda$

$x = 2\lambda + 1$; $y = -3\lambda + 1$ $\quad$ $z = \lambda$.

Q5 (i) Standard deviation.

= square root of variance

variance $\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$

OR $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$

(ii) for median

Mark less than	C.f.
	22
" 10	60
" 20	106
" 30	140
" 40	160
" 50	

Median class $\frac{N}{2}$

$\frac{160}{2} = 80$ +1

Md class = 20 − 30

Median = $l + \frac{\frac{N}{2} - C.f}{f} \times i$

l = lower limit of median class.

f = freq. of median class.

i = magnitude of median class

C.f. = C.f of class preceding the median class.

$N = \Sigma f$

$$\text{Median} = 20 + \frac{\frac{160}{2} - 60}{46} \times 10$$ +2

$$= 24.34782$$

(vi) $n_1 = 40$ $n_2 = 50$ Mean = $\frac{\Sigma x}{n}$. formula +1 +1

$\bar{x}_1 = 52$ $\bar{x}_2 = 61$ $CM = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$ +1

$$\text{Combined mean} = \frac{40 \times 52 + 50 \times 61}{40 + 50}$$ +2

$$= 57$$

(iv) $n_1 = 50$ $n_2 = 100$.

$\bar{x}_1 = 120$ $\bar{x}_2 = 85$

$\sigma_x^2 = 9$ $\sigma_y^2 = 16$.

$\sigma_x = 3$ $\sigma_y = 4$

$$C.V.(A) = \frac{\sigma_x}{\bar{x}} \times 100$$ +2

$$= \frac{3}{120} \times 100 = 2.5$$ +2

$$C.V(B) = \frac{\sigma_y}{\bar{y}} \times 100 = 4.71$$

Co.V. (B) > Co.V. (A)

Seri B (Variable) Than A.

(i) Coeff of Correlation.

As a measure of intensity or degree of linear relationship or degree of linear relationship between two variable Karl pearson developed a formula. Correlation Coeff.

It is a numerical measure of linear relationship

$$r_{xy} = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

$r = +1$ (Perfect +ve)

$r = -1$ (perfect -ve)

(ii)

x	y	x^2	y^2	xy
6	13	36	169	78
2	8	4	64	16
4	12	16	144	48
9	15	81	225	135
1	9	1	81	9
3	10	9	100	30
5	11	25	121	55
8	16	64	256	128
38	94	236	1160	499

$$r = \frac{\frac{1}{n}\Sigma xy - \frac{\Sigma x}{n}\frac{\Sigma y}{n}}{\sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}\sqrt{\frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2}}$$

$\bar{x} = 4.75$
$\bar{y} = 11.75$

$$= \frac{\frac{499}{8} - \frac{94}{8} \times \frac{38}{8}}{\sqrt{\frac{236}{8} - \left(\frac{38}{8}\right)^2}\sqrt{\frac{1160}{8} - \left(\frac{94}{8}\right)^2}}$$ +2

$$= \frac{62.375 - 11.75 \times 4.75}{\sqrt{29.5 - 22.5625}\sqrt{145 - (11.75)^2}}$$

$$= \frac{62.375 - 55.8125}{2.63 \times 2.63}$$

+3

$$= \frac{6.5625}{6.9325} = 0.948886$$

$$\approx 0.95$$

OR Lines of regression

$X - \bar{Y} = b_{yx}(X - \bar{X})$

$X - \bar{X} = b_{xy}(Y - \bar{Y})$ +2

$b_{yx} = \frac{r\sigma_y}{\sigma_x}$ $\quad b_{xy} = r\frac{\sigma_x}{\sigma_y}$

x	y	U = x-170	V = x-169	uv	u^2	v^2
158	163	-12	-6	72	144	36
160	158	-10	-11	110	100	121
163	167	-7	-2	14	49	4
165	170	-5	1	-5	25	1
167	160	-3	-9	27	9	81
170	180	0	11	0	0	121
172	170	2	1	2	4	1
175	175	5	6	30	25	36
177	172	7	3	21	49	9
181	175	11	6	66	121	36
1688	1690	5	0	337	526	446

+3

$$b_{vu} = \frac{k}{h} b_{yx} = \frac{1}{1} = b_{yx} = b_{vu} = r_{uv} \cdot \frac{\sigma_v}{\sigma_u}$$

$$r_{uv} = \frac{\frac{1}{n}\Sigma uv - \bar{u}\bar{v}}{\sqrt{\frac{\Sigma u^2}{n} - \bar{u}^2}\sqrt{\frac{\Sigma v^2}{n} - \bar{v}^2}}$$

$$\therefore b_{vu} = \frac{\frac{1}{n}\Sigma uv - \frac{\Sigma u}{n}\frac{\Sigma v}{n}}{\sqrt{\frac{\Sigma u^2}{n} - \left(\frac{\Sigma u}{n}\right)^2}} = \frac{33.7 - 0.5 \times 0}{\sqrt{52.6 - 0.25}} = 4.66$$

$$b_{uv} = \frac{33.7}{\sqrt{44.6}} = 5.05$$

$$V - \bar{v} = 4.66(u - \bar{u})$$

$$Y - 169 = 4.66(x - 170 - 5)$$

$$X - \bar{x} = 5.05(Y - 169)$$

when
$X = 164$
$Y = 117.74$

+3

[END] of Solution.