ii. Let $a * H$ and $b * H$ be two cosets of H. Either $a * H$ and $b * H$ are disjoint or they are identical, where G is a group H is a subgroup of G and a, b, G. **5**

iii. State and prove Lagrange's theorem. **5**

Q.6 Attempt any two:

i. Determine the discrete numeric function corresponding to generating function: **5**

(a) $A(z) = \frac{7}{1-3z}$

(b) $A(z) = (1+z)^n + (1-z)^n$

ii. Solve the recurrence relation $a_r - 2a_{r-1} + a_{r-2} = 7$ **5**

iii. Solve $a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r,\ r \geq 2$ with boundary conditions $a_0 = 1$ and $a_1 = 1$ by the generation function method. **5**

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Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2022

CA5BS04 Mathematics of Computer Application

Programme: BCA+MCA (Integrated)/MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Let $P(n)$ be the statement that $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for $n > 0$. What is the statement for $P(1)$- **1**

(a) $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(b) $0^2 = \frac{0(0+1)(0+1)}{6}$

(c) $1^2 = \frac{1(1+1)(2+1)}{6}$

(d) None of these

ii. In how many ways 4 boys and 3 girls can be seated in a row so that they are alternate? **1**

(a) 144 (b) 288 (c) 12 (d) None of these

iii. Maximum number of edges is simple connected graph with n vertices is- **1**

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n^2(n-1)}{2}$ (d) $\frac{n^2(n+1)}{2}$

iv. A graph with all vertices having equal degree is known as a ____________. **1**

(a) Multi graph (b) Regular graph

(c) Simple graph (d) Bipartite graph

v. A graph is tree if and only if it is _______________. **1**

(a) Minimally connected (b) Circuit less

(c) Connected (d) None of these

vi. Number of pendant vertices in a binary tree is always _________. **1**

(a) Odd (b) Even (c) Prime (d) None of these

vii. The monoid is a ____________. **1**

(a) Groupoid (b) A group

(c) A commutative group (d) None of these

viii. Which sentence is true? **1**

(a) Set of all natural numbers a group under multiplication

(b) Set of all rational negative numbers forms a group under multiplication

(c) Set of all non-singular matrices of order 2 forms a group under multiplication

(d) None of these

ix. For the recurrence relation $a_r - 2a_{r-1} = 3.2^r$, the general form of the particular solution is- **1**

(a) $P2^r$ (b) $Pr2^r$ (c) P (d) 2^r

x. For the recurrence relation $a_r + 3a_{r-1} - 5a_{r-2} = 5$, order is- **1**

(a) 1 (b) 2 (c) 3 (d) None of these

Q.2 Attempt any two:

i. In how many ways 6 children can be arranged in a line, such that two particular children of them are always together **5**

ii. Prove that $n^3 + 2n$ is divisible by 3 for all $n \geq 2$ by method of induction **5**

iii. Define: **5**

(a) Direct Proof (b) Indirect Proof

Q.3 Attempt any two:

i. Prove that the sum of the degree of all vertices in G is twice the number of edges in G, where G is a graph. **5**

ii. A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G. **5**

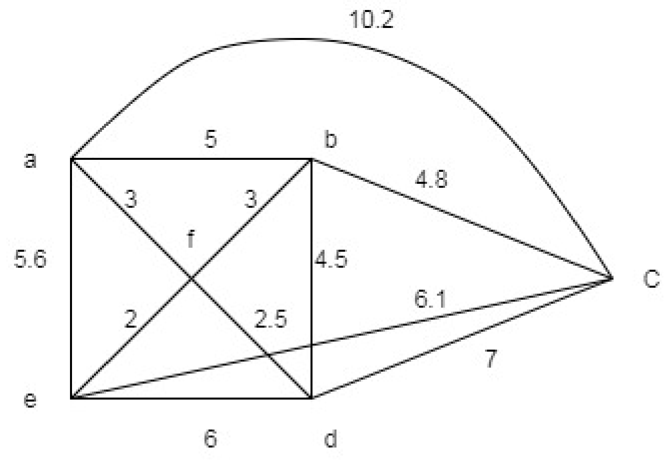
iii. Define following points: **5**

(a) Sub graph (b) Walk (c) Path (d) Circuit

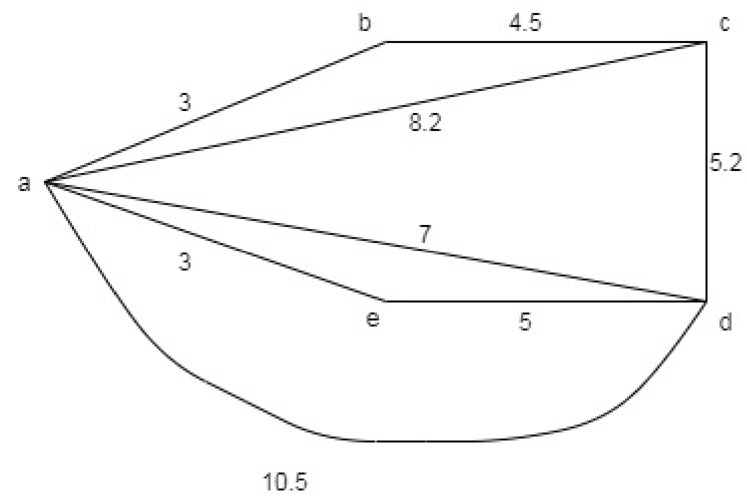
Q.4 Attempt any two:

i. Show that a Tree T with n vertices has exactly $(n-1)$ edges. **5**

ii. Find out the minimal spanning tree using Kruskal's algorithm in the following graph. **5**



iii. Find a minimal spanning tree of the following graph using Prims algorithms. **5**



Q.5 Attempt any two:

i. Define abelian group. Prove that the set $G = \{1,3,7,9\}$ is an abelian group under multiplication modulo 10. **5**

Medi Caps University

Faculty of Engineering

End Sem (odd) Examination Dec-2022

CA5BS04

Mathematics of Computer Application

Q.1. (i) (c) $1^2 = \frac{1(1+1)(2+1)}{6}$ 1

(ii) (a) 144 1

(iii) (a) $\frac{n(n-1)}{2}$ 1

(iv) (b) Regular graph 1

(v) (a) Minimally Connected 1

(vi) (d) None of these 1

(vii) (a) Groupoid 1

(viii) (c) Set of all non-singular matrices of order 2 forms a group under 1

(ix) (b) $P r 2^{r}$ 1

(x) (b) 2 1

Q.2 (i) Given six children are to be arranged in a line
If two particular children are always together then they can be arranged in 2! ways. +1
~~Now~~ then it is a permutation of $6-2+1 = 5$ children which can be arranged in 5! ways +1
∴ the total ~~arran~~ ways of arrangement of 6 children with restriction $= 5!\,2!$ +2
$= 240$ +1
Ans.

Q.2 (ii) Prove that n^3+2n is divisible by 3, $n \geq 2$

$P(n) = n^3 + 2n$

for $n=2$ $P(2) = 2^3 + 2(2) = 12 = 3 \times 4$

$P(2)$ is divisible by 3 +1

let $P(m)$ is divisible by 3

$m^3 + 2m = 3K$ — (1) +1

where K is some positive integer

Now

$P(m+1) = (m+1)^3 + 2(m+1)$ +1

$= m^3 + 1^3 + 3m^2 + 3m + 2m + 2$

$= (m^3 + 2m) + 3(m^2 + m + 1)$ +1

$= 3K + 3(m^2 + m + 1)$ {from (1) +1

$\Rightarrow P(m+1)$ is divisible by 3

∴ By mathematical induction $P(n)$ is divisible by 3

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Q.2 (iii) Direct Proof: A direct proof of $p \Rightarrow q$ is a logically valid argument that begins with the assumptions that p is true and in one or more applications of the law of detachment, concludes that q must be true. +2

Eg. The product of two odd integers is odd. +½

Indirect Proof: Proof by Contra-positive.

In this we use the fact that the proposition $p \Rightarrow q$ is logically equivalent to its contrapositive $(\sim q \Rightarrow \sim p)$ +2

i.e. $(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

for eg.: If Ammu does not agree with Communalists, then she is not orthodox. is the same as 'If Ammu is orthodox, then she agrees with communalists. +½

Q.3(i) let $G = (V, E)$ be a graph and let no. of edges in G be e.
then prove that

$$\sum_{v \in V} deg(v) = 2e \text{ where } v \in V \text{ is any vertex}$$

we shall prove the theo by induction method +1

Step I. If $e = 0$ i.e. no. of edges in G is zero. Also in this case degree of each vertex $v \in V$ is zero. Then

$$\sum_{v \in V} deg(v) = 0 = 2 \times 0$$
$$= 2e$$ +1

∴ theorem is true in this case.

Step II If $e = 1$ i.e. If there is only one edge in G. In this case the graph G has only two vertices and the degree of each vertex is one

$$\sum deg(v) = 1 + 1 = 2 = 2 \times 1$$
$$= 2e$$ +1

∴ the theorem is true in this case

Step III: Now assume that the theorem is true for all graphs having $(e-1)$ edges.

let G be a graph having e edges.
Delete one edge, say $e' = (a, b)$ from G.
Thus a new graph G', say is obtained having $e-1$ edges where $G' = G - \{e'\}$.
Therefore by hypothesis we have in G'

$$\sum deg(v) = 2(e-1)$$ +1

Now if we replace edge $e'(a,b)$ to obtain the graph G, then the degree of each of vertice a and b will be increased by one

$\therefore$ adding the edge $e'=(a,b)$ to G' to obtain G.

$$\sum_{v \in V} deg(v) = 2(e-1) + 2$$ +1

$$\sum deg(v) = 2e$$

Q.3(ii) Let there be n vertices in G. Out of these n vertices 3 are of degree 4 and $(n-3)$ vertices are of degree 3 +1

So degree of $G = 3 \times 4 + 3 \times (n-3)$ +1

But from the theo.

i.e. sum of degrees of all vertices in G is twice the no. of edges in G.

we have

degree of $G = 2 \times e = 2 \times 21$ +1

Henec

$$12 + 3(n-3) = 42$$

$$n-3 = \frac{42-12}{3} = 10$$

$$n = 13$$ +2

i.e. the total no. of vertices in G are 13

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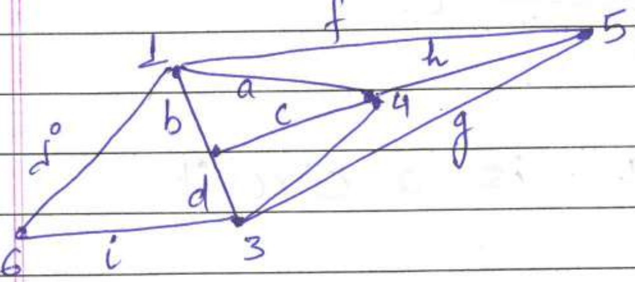
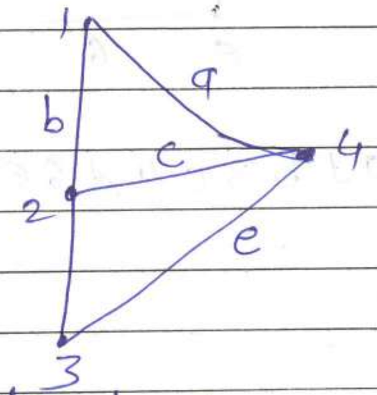
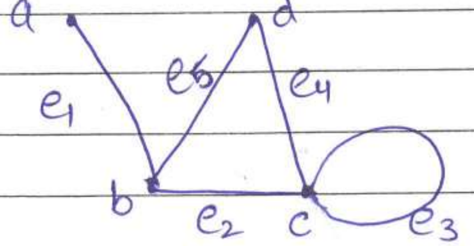
Q.3 (iii) a) Subgraph : A graph 'g' is said to be a subgraph of a graph G if all the vertices and all the edges of g in G, and each edge of g has the same end vertices in g as in G. +1





+1/4

b) Walk : A walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertice. No edge appears more than once in a walk. A vertex, however, may appear more than once. +1

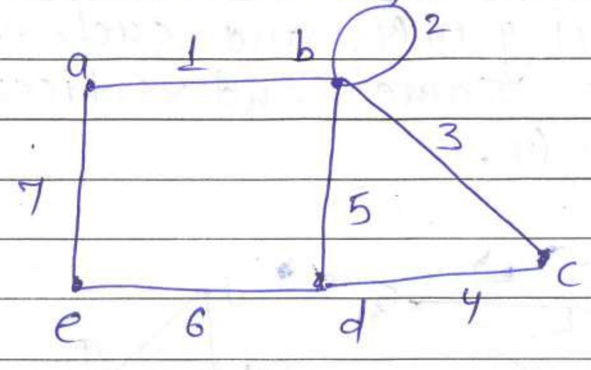


walk
$a e_1 b e_2 c e_3 c$ +1/4

c) Path : An open walk in which no vertex appears more than once is called a path +1

$a e_1 b e_2 c e_3 c$ is a path from above graph +1/4

Circuit : A Circuit is a closed walk in which all vertices are distinct except terminal vertices +1

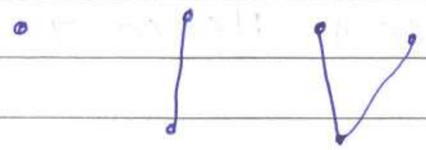


+1/4

a 1 b 3 c 4 d 6 e 7 a is a circuit

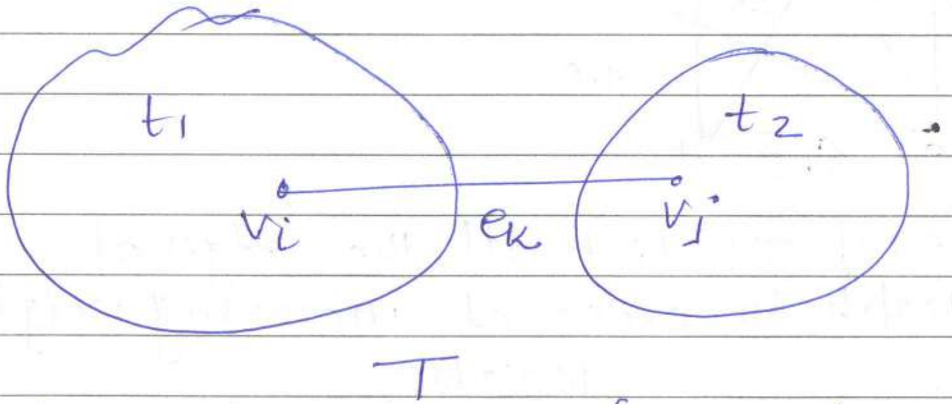
Q

Q.4. (i) The theorem will be proved by induction on the no. of vertices.
It is easy to see that the theo. is true for $n=1, 2, 3$



+1

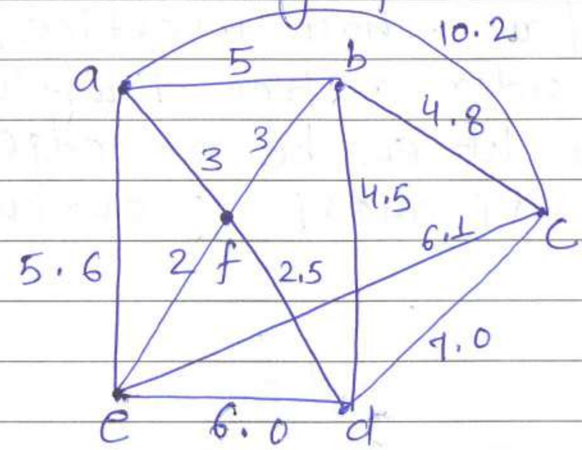
Assume that the tho. holds for all ~~ver~~ trees with fewer than n-vertice.
Let us now consider a tree T with n vertices. In T let e_k be an edge with end vertices v_i and v_j as shown +1



According the tho. [There is one and only one path b/w every pair of vertices in a tree.]
there is no other path b/w v_i and v_j except e_k. Therefore deletion of e_k from T will disconnect the graph as shown in above graph. +1
Furthermore $T - e_k$ consists of exactly two Components and since there were no Circuits in T to begin with, each of these Component is a tree. +1

Both there tree t_1 and t_2, have fewer than n-vertices each, and therefore, by the induction hypothesis, each contain one less edge than the no. of vertices in it.
Thus T-ek ~~contains of~~ Consists of n-2 edges (and n-vertice). Hence T has exactly n-1 edges. +1

Q.4 (ii) Given graph

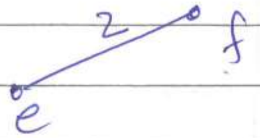


Solution: Step I: List all the edges of graph in order of increasing weight.

edge	weight
(e, f)	2
(f, d)	2.5
(f, a)	3.0
(f, b)	3.0
(b, d)	4.5
(b, c)	4.8
(a, b)	5.0
(a, e)	5.6
(e, d)	6.0
(e, c)	6.1
(d, c)	7.0
(a, c)	10.2

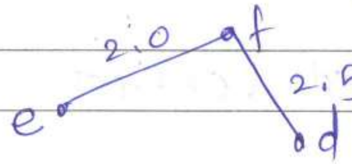
+1

Step 2: select the edge with minimum weight i.e. first edge from the list.
So select (e,f) edge with weight 2



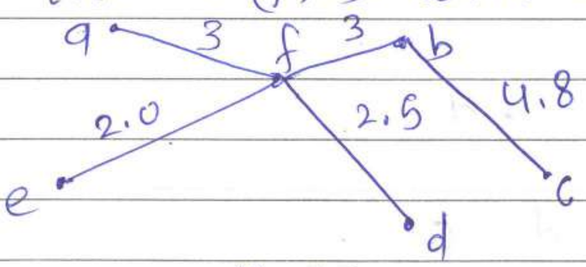
+1

Step 3: Select next minimum weight edge and which doest not Creat a circuit
i.e. select (f,d) edge with weight 2.5



+1

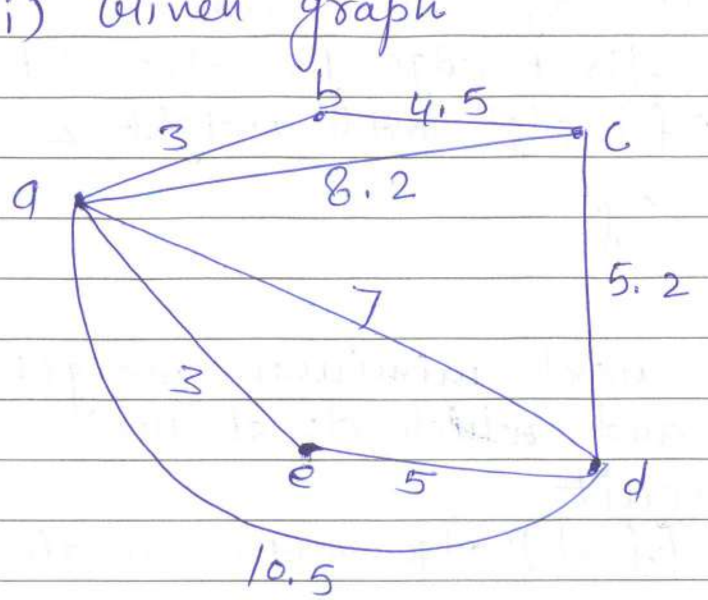
according this process. we select an edge with minimum weight
i.e. (f, a) with 3.0



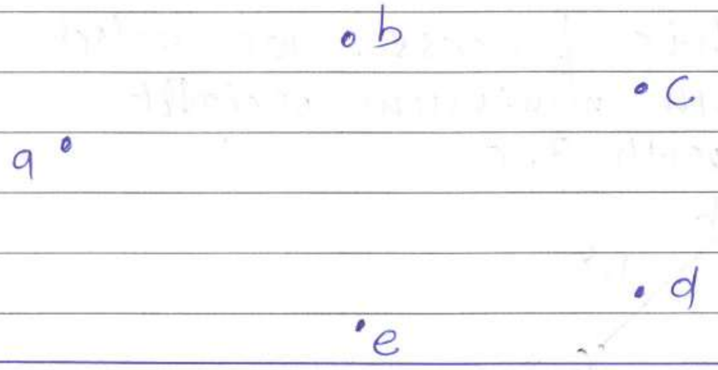
+1

Select (f, b) with 3,
and (b, c) with 4.8, If we consider (b, d) with 4.5 Creat a circuit so (b, d) is not consider
Next if we consider next edges from the list so the graph T have so many circuit which does not correct
Hence above graph is a minimal spanning tree of given graph +1

Q.4 (iii) Given graph



~~step I~~ Draw 5 isolated vertices



$$
\begin{array}{c|ccccc}
 & a & b & c & d & e \\
\hline
a & - & 3 & 8.2 & 7 & 3 \\
b & \cancel{3} & - & 4.5 & - & - \\
c & 8.2 & \cancel{4.5} & \cancel{5.2} & 5.2 & - \\
d & 7 & - & 5.2 & - & \cancel{5} \\
e & \cancel{3} & - & - & 5 & -
\end{array}
$$

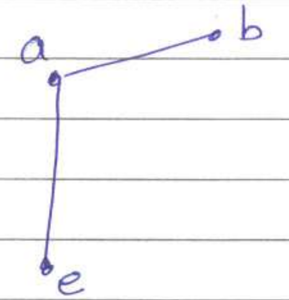
+1

step II we start with vertex a and pick the smallest entry in row one. which is either (a, b) or (a, e). we select any of them let us pick (a, b)

step III Now we find the closest neighbour of the subgraph (a, b) by selecting the vertex which has smallest entry in the rows corresponding to vertices a, and b

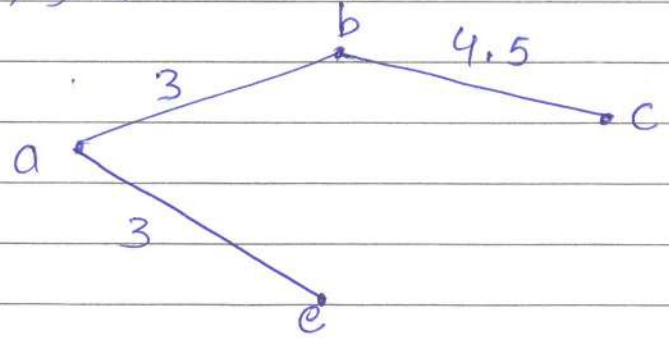
+1

Now we find vertex e as it has the smallest weight in row 1 and 2 other than vertice a and b



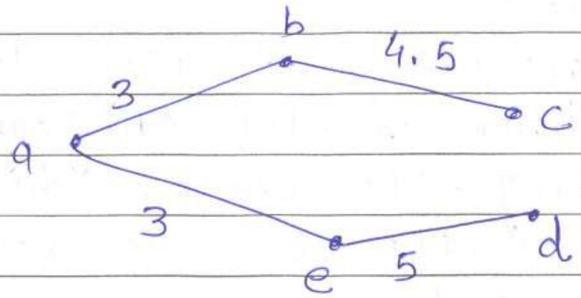
step - IV Now consider the vertices a, b, e as one subgraph and connect it to the vertex c as it has the smallest entry in rows 1, 2, 5.

+1



we consider the vertices a, b, e, c as one subgraph and connect it to the vertex d as it has the smallest entry in row 1, 2, 5, 3.

+1



Hence the above graph is show the minimum spanning tree with 5 vertices and 4 edges.

Q. 5.(i) Abelian Group : A group (G, o) is said to be abelian group if it satisfies following postulates:
where 'o' is a binary operation

G_1 Closure Property : If a and belongs to G then aob also belongs to G . i.e.

$$a \in G,\ b \in G \Rightarrow aob \in G \quad \forall\ a, b \in G$$

G_2 Associativity : If a, b, c are the elements in G

then
$$ao(boc) = (aob)oc \quad \forall\ a, b, c \in G$$ +1

G_3 Existence of identity : $\exists$ an element e in G such that $eoa = a = aoe \quad \forall\ a \in G$. The element e is called the identity.

G_4 Existence of inverse : For each element $a \in G$, $\exists$ an element of G called the inverse of a and denoted by a^{-1} such that

$$a^{-1} o a = e = a o a^{-1}$$

G_5: Commutativity : If $a, b \in G$ +1

$$aob = boa \quad \forall\ a, b \in G$$

Given $G = \{1, 3, 7, 9\}$, '$*$' is ~~binary~~ multiplication operation

write the compostition table

$*_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

+1

G_1 : Closure property : let $a, b \in G$
then
$$a *_{10} b \in G \quad \forall\, a, b \in G$$
for eg. : $1 \in G$, $3 \in G$
$$1 *_{10} 3 \in G \Rightarrow 3 \in G$$

G_2 : Associativity : let $a, b, c \in G$
then
$$a *_{10} (b *_{10} c) = (a *_{10} b) *_{10} c$$
$$\forall\, a, b, c \in G$$
for eg. :

G_3 : Existence of identity :
$1 \in G$, and we have
$$1 *_{10} a = a *_{10} 1 = a \quad \forall\, a \in G$$
so '1' is the identity

+1

G_4 : Existence of inverse: for all $a \in G$ $\exists$ an unique element $a^{-1} \in G$ such that

$$a *_{10} a^{-1} = 1 = a^{-1} *_{10} a$$

Hence

a	a^{-1}
1	1
3	7
7	3
9	9

G_5 : Commutativity: let $a, b \in G$ such that
$a *_{10} b = b *_{10} a \quad \forall a, b \in G$
we can see from composition table commutative law holds +1

Hence $(G, *_{10})$ is an abelian group.

Q.5. (iii) Lagrange's Theorem:
The order of each subgroup of a finite group is a divisor of the order of the group. +1

Proof Let H be any subgroup of order m of a finite group G of order n. Consider the left coset decomposition of G relative to H. let $a \in G$, then aH is the left coset of H in G.
First we shall prove that each coset aH, has as many distinct elements as the subgroup of H.

Suppose $h_1, h_2, h_3 - - - h_n$ are the m elements of H (since $o(H)=m$), then

$$aH = \{ah_1, ah_2, ah_3, - - - ah_m\}$$

Since for the two distinct elements h_i and h_j of H

$$ah_i = ah_j$$

$$\Rightarrow \quad h_i = h_j$$ +2

$\therefore$ each left coset of H in G has m distinct members

Now we know that the group G can be decomposed into disjoint left coset of H in G and their number will be finite, since G is a finite group. Let the no. of left coset of H in G be equal to k (say)

then

$$G = a_1H \cup a_2H \cup a_3H \cup - - - \cup a_kH$$ +1

Now each coset has m members

$$\therefore \quad o(G) = mk$$

$$\Rightarrow n = mk$$

$$k = n/m$$

$$k = \frac{o(G)}{o(H)}$$

$\Rightarrow$ $o(H)$ is divisor of $o(G)$ +1

Proved

Q.5 (ii) Suppose H is a subgroup of G and let $a*H$ and $b*H$ be any two ~~right~~ left cosets in G, then we are to prove that $a*H$ and $b*H$ are either disjoint or identical ie +1

$$a*H \cap b*H = \phi \text{ or } a*H = b*H$$

Suppose, $a*H$ and $b*H$ are not disjoint. Then, ∃ at least one element, say c, Such that

$$c \in a*H \text{ and } c \in b*H$$

Let, +1

$$c = a*h_1 \text{ and } c = b*h_2$$

where $h_1, h_2 \in H$

Now,

$$a*h_1 = b*h_2 \Rightarrow a*h_1*h_1^{-1} = b*h_2*h_1^{-1}$$

[∵ H is a subgroup, ∴ $h_1 \in H \Rightarrow h_1^{-1} \in H$, $h_1^{-1}h_2 = e$] +1

$$\Rightarrow a*e = b*(h_2*h_1^{-1})$$
$$\Rightarrow a = b*(h_2*h_1^{-1})$$
$$\Rightarrow a*H = b*(h_2*h_1^{-1})*H$$
$$\Rightarrow a*H = b*(h_2h_1^{-1}H)$$
$$\Rightarrow a*H = b*H.$$ +1

[∵ H is a subgroup and $h_1 \in H, h_2 \in H$
$\Rightarrow h_1^{-1} \in H, h_2 \in H$
$\Rightarrow h_2h_1^{-1} \in H \Rightarrow h_2h_1^{-1}H = H$]

Thus, two ~~of~~ left cosets are not disjoint, then they are identical. +1

∴ either $Ha \cap Hb = \phi$ or $Ha = Hb$.

Q.5 (ii) Suppose H is a subgroup of G and let H*a & H*b be any two right cosets of H in G then we are to prove that H*a and H*b.

Q.6. (i) a) $A(z) = \frac{7}{1-3z}$

$= 7(1-3z)^{-1}$

$= 7[1 + 3z + 3^2z^2 + \cdots + 3^r z^r + \cdots]$ +1

Hence the numeric function a_r corresponding to $A(z)$ is

$a_r = 7 \cdot 3^r$ +1

b) $A(z) = (1+z)^n + (1-z)^n$

$A(z) = 1 + {}^nC_1 z + {}^nC_2 z^2 + \cdots + {}^nC_r z^r + \cdots + {}^nC_n z^n) + (1 - {}^nC_1 z + {}^nC_2 z^2 + \cdots + (-1)^r\, {}^nC_r z^r + \cdots + (-1)^n\, {}^nC_n z^n)$

$= 2 + 2\cdot{}^nC_2 z^2 + 2\cdot{}^nC_4 z^4 + \cdots + 2\cdot{}^nC_{2r} z^{2r} + \cdots$ +1

Hence, the numeric function a_r corresponding to $A(z)$ is given by

$$a_r = \begin{cases} 0, & \text{if } r \text{ is odd or } r > n \\ 2\cdot{}^nC_r, & \text{if } r \text{ is even} \end{cases}$$ +2

Note that ${}^nC_r = 0$ when $r > n$

Q.6 (ii) $a_r - 2a_{r-1} + a_{r-2} = 7$ — ①

Put

$$a_r = m^r$$

$$m^r - 2m^{r-1} + m^{r-2} = 0$$

divided by m^{r-2}

$$m^2 - 2m + 1 = 0$$

$$m = 1, 1$$ +1

then

$$a_r^{(h)} = (C_1 + C_2 r)\,1^r$$

or

$$= (C_1 + C_2 r)$$ — ② +1

Let the particular solution corresponding to the term '7' be Ar^2

i.e.

$$a_r^{(P)} = Ar^2$$ — ③ +1

substituting ③ in ①, we get

$$Ar^2 - 2A(r-1)^2 + A(r-2)^2 = 7$$

$$2A = 7$$

$$\Rightarrow A = 7/2$$

$$\therefore a_r^{(P)} = \frac{7}{2} r^2$$ +1

Hence total solution of ① is

$$a_r = a_r^{(h)} + a_r^{(P)}$$

$$a_r = C_1 + C_2 r + \frac{7}{2} r^2$$ +1

Ans.

Q.6 (iii) given eq^n is

$$a_r - 5a_{r-1} + 6a_{r-2} = 2^r + r, \quad r \geq 2 \quad (1)$$

with $a_0 = 1$ and $a_1 = 1$

Multiplying both sides of eq^n (1) by z^r and summing from $r = 2$ to ∞, we have +1

$$\sum_{r=2}^{\infty} a_r z^r - 5\sum_{r=2}^{\infty} a_{r-1} z^r + 6\sum_{r=2}^{\infty} a_{r-2} z^r = \sum_{r=2}^{\infty} 2^r z^r + \sum_{r=2}^{\infty} r z^r$$ +1

where $\sum_{r=0}^{\infty} a_r z^r = A(z)$

$$\therefore A(z) - a_0 - a_1 z - 5(A(z) - a_0)z + 6z^2 A(z) = \sum_{r=2}^{\infty} 2^r z^r + \sum_{r=2}^{\infty} r z^r$$ +1

$$A(z) - a_0 - a_1 z - 5zA(z) + 5a_0 z + 6z^2 A(z)$$
$$= [2^2z^2 + 2^3z^3 + \cdots + 1 + 2z - 1 - 2z] + (2z^2 + 3z^3 + \cdots + z - z)$$

$$A(z)(1 - 5z + 6z^2) - 1 - z + 5z = \frac{1}{1-2z} - 1 - 2z + \frac{z}{(1-z)^2} - z \quad \left\{ \begin{array}{l} \text{given} \\ a_0 = 1, a_1 = 1 \end{array} \right.$$ +1

$$A(z) = \frac{14z^4 - 35z^3 - 8z + 27z^2 + 1}{(1-2z)(1-z)^2(3z-1)(2z-1)}$$

On solving
we get

$$a_r = 2a_{r-1} + a_{r-2}$$

$a_1 = 3$, $a_2 = 7$
and
$$A(z) = \frac{3z + z^2}{1 - 2z - z^2}$$

+1