Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Commerce

End Sem (Even) Examination May-2018

CM3CO05 Business Mathematics

Programme: B.Com (Hons.) Branch/Specialisation: Commerce

Duration: 3 Hrs. **Maximum Marks: 60**

Note: (a) All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

(b) Use of simple (non-programmable) calculator is allowed.

Q.1 i. If the order of matrix A is $m\times n$ and the order of B is $p\times m$. Then the order of matrix BA is? **1**

(a) $p \times n$ (b) $n \times m$ (c) $m\times n$ (d) $n\times p$

ii. If $A = \begin{bmatrix} 3 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix}$, then $|A| = ?$ **1**

(a) 30 (b) 60 (c) 12 (d) 0

iii. If $A\ and\ B$ are any two sets and if $(A \cup B)' = A' \cap B'$ then this is known as **1**

(a) Distributive law (b) Associative Law

(c) Commutative law (d) De - Morgan's law

iv. If $f(x) = 4x^2 + 2x + 6$ then the function $f(x)$ is called: **1**

(a) Constant (b) Linear (c) Quadratic (d) Identity

v. The value of $\lim_{x\to a} \frac{x^n - a^n}{x-a}$ is? **1**

(a) nx^n (b) na^{n-1} (c) nx^{n-1} (d) na^n

vi. The derivative of $f(x) = c$ with respect to x, where c is a constant, is? **1**

(a) 0 (b) 1 (c) cx (d) x

vii. The value of $\frac{d}{dx}\int f(x)dx$ =? **1**

(a) $f'(x)$ (b) $f''(x)$ (c) $f(x)$ (d) None of these

viii. The integral of the type $\int_a^b f(x)dx$ is called? **1**

(a) Indefinite (b) Definite (c) Finite (d) Infinite

P.T.O.

ix. If a, b, c *and* d are any four quantities of same kind and if $a : b = c : d$ then the property $(a + b): b = (c + d) : d$ is called? **1**

(a) Dividendo (b) Componendo
(c) Alternendo (d) Invertendo

x. In how many ways the word PETROL can be arranged? **1**

(a) 6! (b) 720 (c) 36 (d) Both (a) and (b)

Q.2 Attempt any two.

i. If $= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$, then find the inverse of A. **5**

ii. If $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, and 't' stands for transpose of a matrix then prove that: $(AB)^t = B^t A^t$ **5**

iii. Solve the following system of equations using Cramer's Rule: **5**

$x - 2y = 4$
$-3x + 5y = -7$

Q.3 Attempt any two.

i. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 6, 8\}$, *and* $C = \{3, 4, 5, 6\}$ where U is universal set, then find: **5**

(a) $(A \cup B)'$ (b) $A \,\Delta\, B$
(c) $(A - C)$ (d) $A \cap B \cap C$ e. $(A')'$

ii. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. **5**

(a) How many like tennis only and not cricket?
(b) How many like tennis?

iii. The fixed cost of a product is Rs. 35000 and the variable cost per unit is Rs. 500. If the demand function is $p(x) = 5000 - 100x$, find the total Cost function, the total Revenue function and Break-Even point. **5**

Q.4 Attempt any two.

i. Differentiate the following functions with respect to x **5**

(a) $y = 10 \sin x + 2^x$.
(b) $y = x \log x$

ii. Find the maximum and minimum values of the function $f(x) = x^3 - 12x$ **5**

iii. The cost function of a firm is given by $C(x) = 4x^3 - 9x^2 + 10x + 10$. Find : **5**

(a) Average cost (b) Slope of Average Cost
(c) Marginal cost (d) Slope of Marginal Cost

Q.5 Attempt any two.

i. Evaluate: $\int_0^1 x^2 e^{2x}\, dx$ **5**

ii. If the demand curve is $p = 20 - 2x$ where p *and* x are respectively the price and quantity demanded of a commodity. Find the consumer's surplus when $p = 6$. **5**

iii. If the Marginal Revenue is given by the function (in rupees) $MR = 10 - 15x + x^2$ where x being the units sold. Determine the total revenue function, given that $R(0) = 0$. Also determine the Demand function. **5**

Q.6 Attempt any two.

i. (a) Find the value of n if $P(n, 3) = 60$ **5**

(b) Evaluate: $C(10, 5) + C(10,4)$

ii. If the n^{th} term of an Arithmetic Progression (A.P.) is $t_n = 10n - 3$. Find the corresponding A.P., its common difference and sum of first 20 terms. **5**

iii. Ram opened a book shop with initial investment of Rs.32000. In the first year he incurred a loss of 5%. However, during the second year he earned a profit of 10% which in the third year rises to 12.5%. Calculate the net profit for entire period of three years. **5**

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Medi-Caps University

Faculty of Commerce

End Sem (Even) Examination May-2017

CM3CO05 Business Mathematics

Programme : B.Com (Hons.) M.M. 60

Solution

Q1. i. (a) $p \times n$ (1)

ii. (b) 60 (1)

iii. (d) De-Morgan's law (1)

iv. (c) Quadratic (1)

v. (b) na^{n-1} (1)

vi. (a) 0 (1)

vii. (c) $f(x)$ (1)

viii. (b) Definite (1)

ix. (b) Componendo (1)

x. (d) both a & b (1)

Q2. (i) $|A| = -1 \neq 0 \therefore A^{-1}$ exists (1)

Matrix of Co-factors $C = \begin{bmatrix} -2 & 1 & 4 \\ 0 & 0 & -1 \\ 1 & -1 & -2 \end{bmatrix}$ (+1)

Adjoint A $= C^T = \begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & -1 \\ 4 & -1 & -2 \end{bmatrix}$ (+1)

$A^{-1} = \frac{\text{Adj. } A}{|A|}$ (formula) (+1)

$\therefore A^{-1} = \frac{1}{-1}\begin{bmatrix} -2 & 0 & 1 \\ 1 & 0 & -1 \\ 4 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ -1 & 0 & 1 \\ -4 & 1 & 2 \end{bmatrix}$ Ans. (+1)

Q2 (ii) $A^t = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ $B^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ (1)

$AB = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 11 & 16 \end{bmatrix}$ (+1)

$(AB)^t = \begin{bmatrix} 3 & 11 \\ 4 & 16 \end{bmatrix}$ — (1) (+1)

Now $B^t A^t = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 4 & 16 \end{bmatrix}$ — (2) (+2)

By eq^n (1) & (2) $(AB)^t = B^t A^t$ proved.

Q2. (iii) $D = \begin{vmatrix} 1 & -2 \\ -3 & 5 \end{vmatrix} = 5-6 = -1 \neq 0$ (1)

Now By Cramer's rule,

$x = \dfrac{D_1}{D}$ where $D_1 = \begin{vmatrix} 4 & -2 \\ -7 & 5 \end{vmatrix} = 20-14 = 6$ (+2)

$\therefore\ x = \dfrac{6}{-1} = -6$

and $y = \dfrac{D_2}{D}$ where $D_2 = \begin{vmatrix} 1 & 4 \\ -3 & -7 \end{vmatrix} = 5$

$\therefore\ y = \dfrac{5}{-1} = -5$ (+2)

Q.3 (i) a) $A \cup B = \{1,2,3,4,6,8\}$

$(A \cup B)' = \{5,7,9\}$ (1)

b) $A \Delta B = (A-B) \cup (B-A)$ (+1)

$= \{1,4\} \cup \{6,8\} = \{1,4,6,8\}$

c.) $A - C = \{1,2\}$ (+1)

d.) $A \cap B \cap C = \{3\}$ (+1)

e.) $(A')' = \{1,2,3,4\}$ (+1)

Q3 (ii) Let A be the set of people who like cricket, & B be the set of people who likes tennis

Then, $n(A \cup B) = 65$

$n(A) = 40$

$n(A \cap B) = 10$ (1)

(b) W.k.t

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ (+1)

$65 = 40 + n(B) - 10$

$n(B) = 65 - 40 + 10 = 35$ (+1)

$\therefore$ The no. of people who like tennis are 35 (+1)

(c) Now, $n(B - A) = n(B) - n(A \cap B)$

$= 35 - 10$

$= 25$ (+1)

$\therefore$ The no. of people who like tennis only & not cricket are 25

3 (iii) Given: $F = 35000$, $V(x) = 500x$, $p(x) = 5000 - 100x$

$\therefore$ Cost func^n $C(x) = F + V(x) = 35000 + V(x)$

$= 35000 + 500x$ (+1.5)

Revenue func^n $R(x) = x \cdot p(x)$

$= x(5000 - 100x)$

$= 5000x - 100x^2$ (+1.5)

Break Even point $C(x) = R(x)$

$\Rightarrow 35000 + 500x = 5000x - 100x^2$

$\Rightarrow 100x^2 - 4500x + 35000 = 0$

$\Rightarrow x^2 - 45x + 350 = 0$

$\Rightarrow (x - 10)(x - 35) = 0$

$\Rightarrow x = 10$ or $x = 35$ (+2)

B.E. Values are $x = 10$ or $x = 35$

Q4 (i) a) $y = 10 \sin x + 2^x$

$\frac{dy}{dx} = 10 \cos x + 2^x \log 2$ (2)

(b) $y = x \log x$

Differentiating w.r.t 'x' using product rule

$$\frac{dy}{dx} = x \frac{d}{dx} \log x + \log x \frac{d}{dx} x$$

$$= x \cdot \frac{1}{x} + \log x \cdot 1$$

$$= 1 + \log x$$ (+3)

Q4 ii) $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$, $f''(x) = 6x$ (1)

For maxima or minima, $f'(x) = 0$

$\Rightarrow 3x^2 - 12x = 0 \Rightarrow x = \pm 2$ (+1)

At $x = 2$, $f''(x) = 12 > 0$

$\therefore f(x)$ is $\min^m$ at $x = 2$ & the $\min^m$ (+1)

value is $f(2) = 2^3 - 12(2) = -16$

At $x = -2$ $f''(x) = -12 < 0$ (+1)

$\therefore f(x)$ is $\max^m$ at $x = -2$ & the $\max^m$

value is $f(-2) = 16$ (+1)

Q4 (iii) Given : $C(x) = 4x^3 - 9x^2 + 10x + 10$ (1)

Avg. cost $= \frac{C(x)}{x} = 4x^2 - 9x + 10 + \frac{10}{x} = AC$ (+1)

Slope of AC $= \frac{d}{dx}(AC) = 8x - 9 + 0 - \frac{10}{x^2}$ (+1)

Marginal cost $= \frac{d}{dx} C(x) = 12x^2 - 18x + 10 = MC$ (+1)

Slope of MC $= \frac{d}{dx}(MC) = 24x - 18$. (+1)

Q5 (i) $I = \int_0^1 x^2 e^{2x}\,dx$ $\quad \left[\begin{array}{l}\text{Rule}\\ \int I\cdot II\,dx = I\int II - \int\left(\frac{d}{dx}I\cdot\int II\,dx\right)dx\end{array}\right.$ (1) formula

$$= \left[x^2\cdot\frac{e^{2x}}{2} - \int\left\{2x\cdot\frac{e^{2x}}{2}\right\}dx\right]_0^1$$ (+1)

$$= \left[\frac{x^2e^{2x}}{2} - \int x\,e^{2x}\,dx\right]_0^1$$

$$= \left[\frac{x^2e^{2x}}{2} - \left\{x\,\frac{e^{2x}}{2} - \int 1\cdot\frac{e^{2x}}{2}\,dx\right\}\right]_0^1$$

$$= \left[\frac{x^2e^{2x}}{2} - \frac{x\,e^{2x}}{2} + \frac{e^{2x}}{4}\right]_0^1$$

$$= \left(\frac{e^2}{2} - \frac{e^2}{2} + \frac{e^2}{4}\right) - \left(0 - 0 + \frac{1}{4}\right)$$

$$= \frac{e^2}{4} - \frac{1}{4} = \frac{(e^2-1)}{4}$$ Ans. (+3)

Q5 (ii) Given $p = 20 - 2x$

when $p = 6$ we have $6 = 20 - 2x$

$\Rightarrow x = 7$. (1)

$\therefore$ Let $x_0 = 7$, $p_0 = 6$

Consumer Surplus $= \int_0^{x_0} f(x)\,dx - p_0x_0$ (+1)

$$= \int_0^7 (20 - 2x)\,dx - 6\times 7$$

$$= \left[20x - x^2\right]_0^7 - 42$$

$$= (140 - 49) - 42$$

$$= 49$$ Ans. (+3)

Q5 (iii) w.k.t. $R(x) = \int (M \cdot R)\,dx + k$ (1)

given, M.R $= 10 - 15x + x^2$

$\therefore\ R(x) = \int (10 - 15x + x^2)\,dx + k$

(Revenue function)

$= 10x - \frac{15}{2}x^2 + \frac{x^3}{3} + k$ (+1)

because, $R(0) = 0$, when there is no output

$\Rightarrow k = 0$ (+1)

$\therefore\ R(x) = 10x - \frac{15}{2}x^2 + \frac{x^3}{3}$

Now we also know $R(x) = p \cdot x$ w/r (+1)

p is demand func^n

$\Rightarrow p = \frac{R(x)}{x} = 10 - \frac{15}{2}x + \frac{x^2}{3}$ Ans. (+1)

Q6. (i) a) if $P(n,3) = 60$

$\Rightarrow \frac{n!}{(n-3)!} = 60$ $\left[\because\ ^nP_r = \frac{n!}{(n-r)!}\right]$

$\Rightarrow n(n-1)(n-2) = 5 \times 4 \times 3$

$\Rightarrow n(n-1)(n-2) = 5(5-1)(5-2)$ (2)

$\therefore\ \boxed{n = 5}$

b) $C(10,5) + C(10,4)$

$= \frac{10!}{5! \times 5!} + \frac{10!}{4!\,6!}$

$= 252 + 210$

$= \boxed{462}$ (+2)

Q6. ii. Given $t_n = 10n - 3$

$\therefore\ t_1 = 7,\ t_2 = 17,\ t_3 = 27,\ t_4 = 37$ etc (1)

Corresponding A.P is 7, 17, 27, 37, ------

Common difference $d = t_2 - t_1 = 10$ (+1)

Now $S_n = \frac{n}{2}[2a + (n-1)d]$ (+1

here $n = 20, d = 10, a = t_1 = 7$

$\therefore S_{20} = \frac{20}{2}[14 + 19 \times 10]$

$= 10[204]$

$= 2040$ Ans. (+2

Q6 (iii) Initial investment = Rs. 32,000

Amount left at the end of 1st year

$= 32000 \times \frac{95}{100}$ ($\because$ 5% loss)

= Rs. 30,400

which is investment of 2nd year

Amt. left at end of 2nd year $= 30,400 \times \frac{110}{100}$

($\because$ 10% profit)

= Rs 33,400 which is the investment of 3rd year

Amt. left at the end of 3rd year

$= 33400 \times \frac{112.5}{100}$ ($\because$ 12.5% profit)

= Rs. 37,620

$\therefore$ Net profit = 37,620 − 32,000

= Rs. 5620 (Ans.)

End