Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2018

BC3CO07 Mathematics-II

Programme: B.Sc.(CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1	i.	If $x^2 + y^2 = 1$ then $\frac{dy}{dx} =$	**1**
		(a) $-\frac{x^3}{y^2}$ (b) $-\frac{x}{y}$ (c) $\frac{x^3}{y^2}$ (d) None of these	
	ii.	The Euler's theorem is for	**1**
		(a) Homogeneous functions (b) Non homogenous functions	
		(c) Both (a) and (b) (d) None of these	
	iii.	Which of the following double integrals represent area of a bounded region R in *xy*-plane	**1**
		(a) $\iint_R x^2 y^2 dxdy$ (b) $\iint_R xydxdy$ (c) $\iint_R dxdy$ (d) $\iint_R x^3 y^3 dxdy$	
	iv.	The triple integral $\iiint_D dxdydz$, where D is a bounded surface in 3-dimensional space, represents	**1**
		(a) Length of D (b) Mass of D	
		(c) Density of D (d) Volume of D	
	v.	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^4 + \frac{dy}{dx} = \sin x$ is	**1**
		(a) 3 (b) 4 (c) 5 (d) None of these	
	vi.	The solution of the Clairaut's equation $y = px + f(p)$, where $p = \frac{dy}{dx}$ is :	**1**
		(a) $y = cx + f(c)$ (b) $y = c^2x + f^2(c)$	
		(c) $y = cx^3 + f^3(c)$ (d) None of these.	

vii. If for the differential equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, $P+xQ=0$ then a part of complementary function is **1**

(a) e^x (b) x (c) e^{-x} (d) None of these

viii. While solving the equation $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$ by removal of the first derivative method a part of the complete solution u is determined by which of the following formula: **1**

(a) $u=e^{\frac{dP}{dx}}$ (b) $u=e^{\int Qdx}$ (c) $u=e^{-\frac{1}{2}\int Pdx}$ (d) None of these.

ix. If $L\{f(t)\}=\bar{f}(s)$ then $L\left\{\int_0^t f(t)dt\right\}=$ **1**

(a) $\bar{f}(s)$ (b) $s^2\bar{f}(s)$ (c) $\frac{\bar{f}(s)}{s}$ (d) None of these

x. $L^{-1}\left\{\frac{1}{(s-2)^2}\right\}=$? , Here $L^{-1}=$ Inverse Laplace operation. **1**

(a) t (b) te^{2t} (c) $\frac{t^2}{2}e^{-2t}$ (d) None of these

Q.2 Solve any two:

i. Prove that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist where **5**

$$f(x,y)=\begin{cases}\frac{xy^2}{x^2+y^4} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

ii. If $u=\sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, prove that **5**

(a) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$

(b) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=-\frac{\sin u\cos 2u}{4\cos^3 u}$

iii. Find maxima and minima of the function $u=x^3+y^3-3x-12y+20$. **5**

Q.3 Solve any two:

i. Trace the curve $y^2(2a-x)=x^3$. **5**

ii. Change the order of integration $\int_0^\infty\int_0^x xe^{-\frac{x^2}{y}}dydx$ and hence evaluate. **5**

iii. Evaluate $\int_1^3\int_{\frac{1}{x}}^1\int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$. **5**

Q.4 Solve any two:

i. Solve $\frac{dy}{dx}+\frac{y}{x}=y^3$. **5**

ii. Solve $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$. **5**

iii. Solve $(D^2-4D+4)y=x^2+e^x+\cos 2x$ **5**

Q.5 Solve any two:

i. Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. **5**

ii. Solve $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$. **5**

iii. Solve by method of variation of parameters $\frac{d^2y}{dx^2}+4y=4\tan 2x$. **5**

Q.6 Solve any two:

i. Find $L\left\{\frac{e^{-at}-e^{-bt}}{t}\right\}$. **5**

ii. Using convolution theorem find $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. **5**

iii. Solve using Laplace transform **5**

$\frac{d^2y}{dt^2}-2\frac{dy}{dt}+y=e^t$, $y(0)=2$, $y'(0)=-1$.

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BC3CO07 Mathematics II (i)

Prog:- B.Sc. M.M:- 60

Solution Set.

Q1 Multiple Choice questions

(i) b) $-x/y$ 1

(ii) a) homogeneous functions 1

(iii) c) $\iint_R dx\,dy$ 1

(iv) d) volume of D 1

(v) b) 4 1

(vi) a) $y = cx + f(c)$ 1

(vii) b) x 1

(viii) c) $u = e^{-\frac{1}{2}\int P dx}$ 1

(ix) c) $\frac{f(s)}{s}$ 1

(x) b) te^{2t} 1

Q2. Solve any two

(i) $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$

let $(x,y)\to(0,0)$ along $y=x$

$= \lim_{x\to 0} \frac{x^3}{x^2+x^4}$

$= \lim_{x\to 0} \frac{x}{1+x^2}$

$= \frac{0}{1} = 0$ (2.5)

let $(x,y)\to(0,0)$ along $x=y^2$

$= \lim_{y\to 0} \frac{y^2 y^2}{y^4+y^4} = \lim_{y\to 0} \frac{1}{1+1} = \frac{1}{2}$ (2.5)

$\therefore$ limit is different along different paths

$\therefore$ limit does not exist

ii) a) $\sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}} = z \text{ (say)} = x^{1/2} f(y/x) \to ①$

$\therefore$ degree $= n = \frac{1}{2}$

$\therefore$ by Euler's theorem.

$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}z$ ~~let~~ $z = \sin u$

$\therefore x\frac{\partial}{\partial x}\sin u + y\frac{\partial}{\partial y}\sin u = \frac{1}{2}\sin u$

$x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u$

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u = g(u)$ (2.5)

b) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)[g'(u)-1]$

Q: $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{2}\tan u\left(\frac{1}{2}\sec^2 u - 1\right)$

$= \frac{1}{2}\frac{\sin u}{\cos u}\left[\frac{1}{2\cos^2 u} - 1\right]$

$= \frac{1}{2}\left[\frac{\sin u}{\cos u}\right]\left[\frac{1 - 2\cos^2 u}{2\cos^2 u}\right]$

$= \frac{-\sin u\cos 2u}{4\cos^3 u}$ (2)

iii) Find maxima and minima of

$u = x^3 + y^3 - 3x - 12y + 20$

Critical pts are

$(-1, 2) \rightarrow$ maxima

$(1, 2) \rightarrow$ minima (2.5)

$(-1, 2) \rightarrow$ neither maxima nor minima

$(1, -2) \rightarrow$ neither maxima nor minima (2.5)

4

3. Solve any two

) Trace $y^2(2a - x) = x^3$

Symmetry about x-axis

Curve passes thro' origin, origin is cusp and passes thro' origin only

Asymptote $\parallel$lel to axis:-

$x = 2a$ (2.5)

4. $y^2 = x^3/(2a-x)$

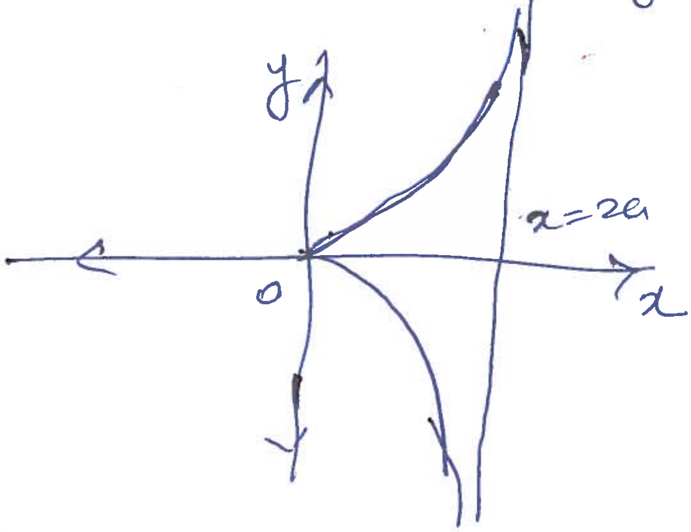
if x is +ve, then when

$0 < x < 2a$, $y^2 = +ve$ i.e. y is real

$x > 2a$ $y^2 = -ve$ i.e. y is imaginary

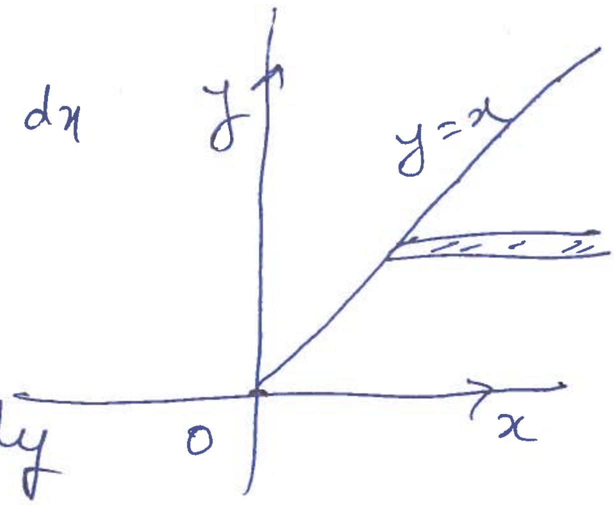
when x is negative $y^2 = -ve \Rightarrow y$ is imagi-

Hence curve lies only betn $0 < x < 2a$



(2.5)

ii) $$I = \int_{x=0}^{\infty} \int_{0}^{x} x e^{-x^2/y} \, dy \, dx$$



$$= \int_{y=0}^{\infty} \int_{x=y}^{\infty} x e^{-x^2/y} \, dx \, dy$$

(2)

$$= \frac{1}{2} \int_{0}^{\infty} y e^{-y} \, dy =$$

$$= 1/2$$

(3)

iii). Evaluate $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$

$I = \int_1^3 \int_{1/x}^1 xy\left[\frac{z^2}{2}\right]_0^{\sqrt{xy}} dy\,dx$

$= \int_1^3 \int_{1/x}^1 xy\,\frac{xy}{2}\,dy\,dx = \frac{1}{2}\int_1^3 \int_{1/x}^1 x^2y^2\,dy\,dx$

$= \frac{1}{2}\int_1^3 x^2\left(\frac{y^3}{3}\right)_{1/x}^1 dx = \frac{1}{6}\int_1^3 x^2\left[1-\frac{1}{x^3}\right]dx$ (2.5)

$= \frac{1}{6}\int_1^3\left[x^2-\frac{1}{x}\right]dx = \frac{1}{6}\left[\frac{x^3}{3}-\log x\right]_1^3$

$= \frac{1}{6}\left[\left(\frac{27}{3}-\frac{1}{3}\right)-(\log 3-\log 1)\right]$

$= \frac{1}{6}\left[\frac{26}{3}-\log 3\right]$ (2.5)

Solve any two

Solve $\frac{dy}{dx}+\frac{y}{x}=y^3$

$y^{-3}\frac{dy}{dx}+\frac{y^{-2}}{x}=1$

$y^{-2}=z \qquad -2y^{-3}\frac{dy}{dx}=\frac{dz}{dx}$

$y^{-3}\frac{dy}{dx}=-\frac{1}{2}\frac{dz}{dx}$

$$-\frac{1}{2}\frac{dz}{dx}+\frac{z}{x}=1 \qquad \frac{dz}{dx}-\frac{2z}{x}=-2$$ (2.5)

$$I.F.=e^{\int -\frac{2}{x}dx}=e^{-2\log x}=e^{+\log_e x^{-2}}=\frac{1}{x^2}$$

$$\therefore z\times\frac{1}{x^2}=\int\frac{-2}{x^2}dx+c$$

or $\frac{z}{x^2}=-2\frac{x^{-1}}{-1}+c$ or $\frac{y^{-2}}{x^2}=\frac{2}{x}+c$

or $\frac{1}{x^2y^2}=\frac{2}{x}+c$ (2.5)

Solve $\frac{dy}{dx}-\frac{dx}{dy}=\frac{x}{y}-\frac{y}{x}$

$\frac{dy}{dx}=p$, $p-\frac{1}{p}=\frac{x}{y}-\frac{y}{x}$

$$p^2-1=\left(\frac{x^2-y^2}{xy}\right)p.$$

or $p^2-p\left(\frac{x}{y}-\frac{y}{x}\right)-1=0$ (2.5)

$\Rightarrow p=\frac{x}{y}, -\frac{y}{x}$

$xy=c \qquad x^2-y^2=c$

$\therefore (xy-c)(x^2-y^2-c)=0$ (2.5)

(7)

iii). $(D^2-4D+4)y = x^2+e^x+\cos 2x$

$m^2-4m+4=0 \Rightarrow m=2,2$ (2)

$C.F = (c_1+c_2x)e^{2x}$

$P.I = \frac{1}{(D^2-4D+4)}[x^2+e^x+\cos 2x]$

$= \frac{e^x}{1} + \frac{-1}{4D}\cos 2x + \frac{1}{4}\left[1-\left(D-\frac{D^2}{4}\right)\right]^{-1}x^2$

$= e^x - \frac{1}{8}\sin 2x + \frac{1}{4}\left[1+\left(D-\frac{D^2}{4}\right)+\left(D-\frac{1}{4}D^2\right)^2 - -\right]x^2$

$= e^x - \frac{1}{8}\sin 2x + \frac{1}{4}\left[x^2+2x+\frac{3}{2}\right]$ (3)

Q5 Solve any two

(i) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$

Let $x = e^z$ $\qquad x\frac{dy}{dx} = Dy$, $x^2\frac{d^2y}{dx^2} = D(D-1)y$

and $x^3\frac{d^3y}{dx^3} = D(D-1)(D-2)y$

$D \equiv \frac{d}{dz}$

$\therefore [D(D-1)(D-2) + 2D(D-1) + 2]\,y = 10[e^z + e^{-z}]$

$[D^3 - D^2 + 2]\,y = 10[e^z + e^{-z}]$

$m = -1, 1 \pm i$

$\cancel{cf = 5e^z + 2ze^{-z}}$

$C.F = c_1 e^{-z} + e^z(c_2\cos z + c_3\sin z)$

$= c_1 x^{-1} + x\,[c_2\cos(\log x) + c_3\sin(\log x)]$ (2.5)

$P.I = \frac{1}{D^3 - D^2 + 2}\,10[e^z + e^{-z}] = 5e^z + 2ze^{-z}$

$= 5x + \frac{2}{x}\log_e x$ (2.5)

(8)

(9)

ii). Solve $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$

sol $\frac{d^2y}{dx^2}-\left(2-\frac{1}{x}\right)\frac{dy}{dx}+\left(1-\frac{1}{x}\right)y=0$

$P=-\left(2-\frac{1}{x}\right),\ Q=1-\frac{1}{x},\ R=0$

$1+P+Q=0 \quad \therefore u=e^{x}$ (1.5)

$y=uv$

$\frac{dp}{dx}+\left[\frac{2}{u}\frac{du}{dx}+P\right]\cancel{\frac{du}{dx}}p=\frac{R}{u}$, $p=\frac{dv}{dx}$

$\therefore \frac{dp}{dx}+\left[\frac{2}{e^{x}}\times e^{x}+-2+\frac{1}{x}\right]p=0$

$\frac{dp}{dx}+\frac{1}{x}p=0$ or $\frac{dp}{p}=-\frac{1}{x}dx$

$\log p+\log x=\log c_1 \quad \therefore p=c_1/x$ (2)

or $\frac{dv}{dx}=\frac{c_1}{x}$ or $v=\int\frac{c_1}{x}+c_2$

$\therefore v=c_1\log x+c_2$

$\therefore y=uv=e^{x}\left[c_1\log x+c_2\right]$ (1.5) ii). $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$

$m^2 + 4 = 0 \Rightarrow m = \pm 2i$ (2)

$C.F = c_1 \underset{u}{\cos 2x} + c_2 \underset{v}{\sin 2x}$

$A = \int \frac{-Rv}{uv' - u'v}\, dx + d_1$

$uv' - u'v$
$= \cos 2x (2\cos 2x) - (-2\sin 2x)\sin 2x$
$= 2[\cos^2 2x + \sin^2 2x]$
$= 2.$

$= \int -\frac{4\tan 2x \times \sin 2x}{2}\, dx + d_1$

$= -2\int \frac{\sin^2 2x}{\cos 2x}\, dx + d_1$

$= -\log(\sec 2x + \tan 2x) + d_1$

$B = \int \frac{Ru}{uv' - u'v}\, dx + d_2$

$= \int \frac{4\tan 2x \times \cos 2x}{2}\, dx + d_2$

$= 2\int \sin 2x\, dx + d_2$

$= -\cos 2x + d_2$

$y = Au + Bv$ (3)

Q6. Solve any two (11)

(i) $\mathcal{L}\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\}$

$$= \int_s^\infty \mathcal{L}\{e^{-at}-e^{-bt}\}\,ds = \int_s^\infty \frac{1}{s+a} - \frac{1}{s+b}\,ds$$ (2.5)

$$\left[\log\frac{(s+a)}{(s+b)}\right]_s^\infty = \log\left(\frac{s+b}{s+a}\right)$$ (2.5)

(ii) Using Convolution find

$$\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at = f(t), \quad \mathcal{L}^{-1}\left\{\frac{s}{s^2+b^2}\right\} = \cos bt = g(t)$$ (1)

$$\mathcal{L}^{-1}\{\bar{f}(s)\cdot\bar{g}(s)\} = \int_0^t f(u)\,g(t-u)\,du$$ (1)

$$= \int_0^t \cos au \cos b(t-u)\,du$$

$$= \frac{1}{2}\left[\frac{(\sin at-\sin bt)}{a-b} + \frac{(\sin at+\sin bt)}{a+b}\right]$$ (2)

$$= \frac{a\sin at - b\sin bt}{a^2-b^2}$$ (1)

(12)

(i) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = e^t \quad y(0) = 2, \; y'(0) = -1$

$\mathcal{L}\{y''(t) - 2\mathcal{L}\{y'(t)\} + \mathcal{L}\{y\} = \mathcal{L}\{e^t\}$

$s^2\bar{y} - s\,y(0) - y'(0) - 2((s\bar{y} - y(0)) + \bar{y} = \frac{1}{s-1}$

$(s^2 - 2s + 1)\bar{y} = 2s - 5 + \frac{1}{s-1}$ (2.5)

$\bar{y} = \frac{2}{s-1} - \frac{3}{(s-1)^2} + \frac{1}{(s-1)^3}$

$\therefore y = 2e^t - 3te^t + e^t\frac{t^2}{2}$

$= \left(2 - 3t + \frac{t^2}{2}\right)e^t$ (2.5)