Q.6 Attempt any two:

i. A solid round bar 3m long and 5 cm in diameter. Determine the crippling load take E= $2x10^5$ N/mm^2 used following end conditions: **5**
(a) Both ends are hinged
(b) Both ends are fixed.

ii. Derive the formulae for finding buckling load in column, if both ends of column pinned. **5**

iii. A 1.5 m long column has a circular cross section of 5cm diameter. One of the end fix other free take FOS=3 calculate safe load using: **5**
(a) Rankine's formula, take yield stress $560N/mm^2$and a= 1/1600.
(b) Euler's formula take E for C.I.=$1.2x10^5N/mm^2$.

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering
End Sem (Odd) Examination Dec-2017
CE3ES10 Strength of Material

Programme: B.Tech. Branch/Specialisation: CE

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Two parallel, equal and opposite forces acting tangentially to the surface of the body is called as _____ **1**
(a) Complementary stress (b) Compressive stress
(c) Shear stress (d) Tensile stress

ii. The relation between modulus of elasticity (E), modulus of rigidity (G) and bulk modulus (K) is given as ________ **1**
(a) K+G/(3K+G) (b)3KG/(3K+G
(c) 3KG/(9K+G (d) 9 KG / (3K+ G)

iii. In a simply supported beam, bending moment at the end **1**
(a) Is always zero if it does not carry couple at the end
(b) Is zero, if the beam has uniformly distributed load only
(c) Is zero if the beam has concentrated loads only
(d) May or may not be zero.

iv. For any part of a beam between two concentrated load, Bending moment diagram is a **1**
(a) Horizontal straight line (b) Vertical straight line
(c) Line inclined to x-axis (d) Parabola

v. For bending equation to be valid, radius of curvature of the beam after bending should be **1**
(a) Equal to its transverse dimensions
(b) Infinity
(c) Very large compared to its transverse dimensions
(d) Double its transverse dimensions

vi. If depth of a beam is doubled then changes in its section modulus **1**
(a) Will remain same (b) Will decrease
(c) Will be doubled (d) Will increase by 4 times

vii. In the relation ($T/J = G\theta/L = \tau/R$), the letter G denotes modulus of ______ **1**
(a) Elasticity (b) Plasticity (c) Rigidity (d) Resilience

viii. Hoop stress in a thin vessel is **1**
(a) pD/2t (b) pD/4t (c) pD/3t (d) None of these

ix. In Euler's theory, long columns having the ratio of (L_e /LLD) ≥ 12 fail due to _______ **1**
(a) Crushing (b) Buckling
(c) Both (a) and (b) (d) None of these

x. What is the relation between actual length and effective length while determining crippling load for a hollow rectangular cast iron column having both ends fixed? **1**
(where L= actual length and L_e =effective length)
(a) L_e=L (b) L_e=L/2 (c) L_e=2L (d) L_e = 4L

Q.2 i. Define longitudinal strain, Poisson's ratio. **2**

ii. Find the modulus of elasticity for a rod, which tapers uniformly from 30mm to 15mm diameter in a length of 350 mm. the rod, is subjected to an axial load of 5.5KN and extension of the rod is 0.025mm. **3**

iii. The tensile stresses at a point across two mutually perpendicular planes are 120N/mm^2 and 60N/mm^2. Determine the normal, tangential and resultant stresses on a plane inclined at 30^0 from vertical side of beam in anticlockwise direction. **5**

OR iv. A load of 2MN is applied on a short concrete column 500mmx500mm the column is reinforced with four steel bars of 10mm diameter one in each corner. Find the stress in the concrete and steel bars. Take E for steel as 2.1x10^{5}N/mm^2 and for concrete as 1.4x10^4 N/mm^2. **5**

Q.3 i. Define the following terms: **3**
(a) Shear force (b) Bending moment (c) Slope and deflection.

ii. A beam of length 10m is simply supported and carries point load of 50 KN and 40KN each at a distance of 2m and 6m respectively from left support and also a UDL of 10KN/M between the point loads. Draw SFD and BMD for the beam. **7**

OR iii. Derive an expression for slope at support and deflection for a simply supported beam of span "**L**" subjected to point load "W" at a distance "a" from left support and "b" from right support by Macaulay's method. **7**

Q.4 i. Define the terms: **3**
(a) Pure bending
(b) Neutral axis
(c) Section modulus.

ii. A timber beam of rectangular section is to support a load of 20KN uniformly distributed over a span of 3.6 m when beam is simply supported. If the depth of section is to be twice the breadth, and the stress in the timber is not to exceed 7N/mm^2, find the dimension of the cross- section. **7**

OR iii. Derive the bending equation ; $M/I = \sigma_b/y = E/R$ **7**

Q.5 i. A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm^2 determine. 1. Longitudinal and circumferential stress developed in the pipe. **4**

ii. A solid shaft of diameter 80mm is subjected to twisting moment of 8MN-mm and a bending moment of 5MN-mm at a point determine **6**
(a) Principal stresses (b) Position of plane on which they act.

OR iii. A hollow shaft of external diameter 120mm transmits 300KW power at 200r.p.m determine the internal diameter. If maximum stress in the shaft is not to exceed 60N/mm^2. **6**

CE3ES10 Strength of Material

Marking Scheme

Q.1	i.	C	**1**
	ii.	D	**1**
	iii.	A	**1**
	iv.	C	**1**
	v.	C	**1**
	vi.	D	**1**
	vii.	C	**1**
	viii.	A	**1**
	ix.	B	**1**
	x.	B	**1**

Q.2

i Each definition allot 1 marks **2**

ii Find the modulus of elasticity for a rod, which tapers uniformly from 30mm to 15mm diameter in a length of 350mm . the rod is subjected to an axial load of 5.5KN and extension of the rod is 0.025mm **3**

Sol. Given :

Larger diameter, $D_1 = 30$ mm

Smaller diameter, $D_2 = 15$ mm

Length of rod, $L = 350$ mm

Axial load, $P = 5.5$ kN $= 5500$ N

Extension, $dL = 0.025$ mm

Using equation (1.10), we get

$$dL = \frac{4PL}{\pi E D_1 D_2}$$

1 mark

or
$$E = \frac{4PL}{\pi D_1 D_2 \, dL} = \frac{4 \times 5500 \times 350}{\pi \times 30 \times 15 \times 0.025}$$

$= \mathbf{217865\ N/mm^2}$ or $\mathbf{2.17865 \times 10^5\ N/mm^2}$. **Ans.**

2 mark

iii The tensile stresses at a point across two mutually perpendicular planes are $120 N/mm^2$ and $60 N/mm^2$. Determine the normal ,tangential and resultant stresses on a plane inclined at 30^0 from vertical side of beam in anticlockwise direction. **5**

Sol. Given :

Major principal stress, $\sigma_1 = 120$ N/mm^2

Minor principal, $\sigma_2 = 60$ N/mm^2

Angle of oblique plane with the axis of minor principal stress,

$\theta = 30°$.

Normal stress

The normal stress (σ_n) is given by equation (3.6),

$$\therefore \quad \sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{120 + 60}{2} + \frac{120 - 60}{2} \cos 2 \times 30°$$

2 mark

$$= 90 + 30 \cos 60° = 90 + 30 \times \tfrac{1}{2}$$

$= \mathbf{105\ N/mm^2}$. **Ans.**

Tangential stress

The tangential (or shear stress) σ_t is given by equation (3.7).

$$\therefore \quad \sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta$$

$$= \frac{120 - 60}{2} \sin (2 \times 30°)$$

2 mark

$$= 30 \times \sin 60° = 30 \times 0.866$$

$$= \mathbf{25.98\ N/mm^2}. \quad \textbf{Ans.}$$

Resultant stress

The resultant stress (σ_R) is given by equation (3.8)

$$\therefore \quad \sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{105^2 + 25.98^2}$$

$$= \sqrt{11025 + 674.96} = \mathbf{108.16\ N/mm^2}. \quad \textbf{Ans.}$$

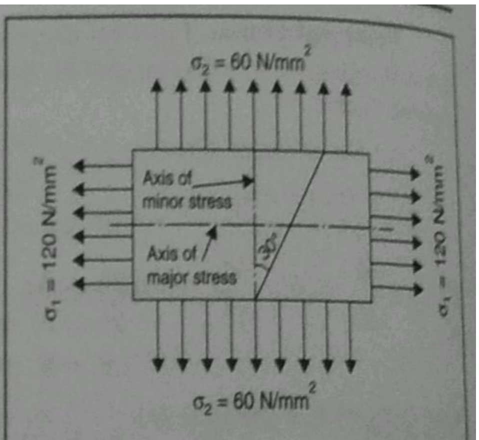


Fig. 3.5

1 mark

iv A load of 2MN is applied on a short concrete column 500mmx500mm the column is reinforced with four steel bars of 10mm diameter one in each corner. Find the stress in the concrete and steel bars. Take E for steel as $2.1\text{x}10^5\text{N/mm}^2$ and for concrete as $1.4\text{x}10^4$ N/mm^2. **5**

Sol. Given :

Total load applied, $P = 2\text{ MN} = 2 \times 10^6\text{ N}$

Area of column $= 500 \times 500 = 250000\text{ mm}^2$

Area of 4 steel bars, $A_s = 4 \times \frac{\pi}{4}(10)^2 = 314.159\text{ mm}^2$

Area of concrete, A_c = Area of column − Area of steel bars

$= 250000 - 314.159$

$= 249685.841\text{ mm}^2$

E for steel, $E_s = 2.1 \times 10^5\text{ N/mm}^2$

E for concrete, $E_c = 1.4 \times 10^4\text{ N/mm}^2$

Let σ_s = Stress in steel bar in N/mm²

σ_c = Stress in concrete in N/mm²

1 mark

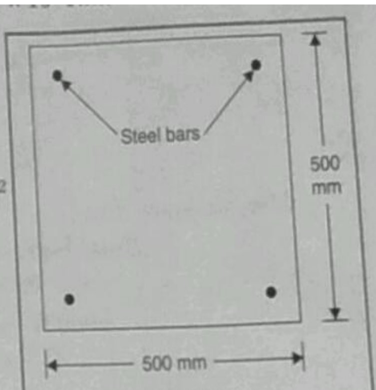


Fig. 1.18

Now strain in steel = Strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \qquad \left(\because \text{ Strain} = \frac{\text{Stress}}{E}\right)$$

$$\therefore \quad \sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2.1 \times 10^5}{1.4 \times 10^4}\sigma_c = 15\,\sigma_c \qquad ...(i)$$

1mark

Now load on steel + Load on concrete = Total load

$$\sigma_s \cdot A_s + \sigma_c \cdot A_c = P \qquad (\because \text{ Load = Stress} \times \text{Area})$$

1mark

or $15\sigma_c \times 314.159 + \sigma_c \times 249685.841 = 2000000 \qquad (\because\ \sigma_s = 15\sigma_c)$

or $254398\sigma_c = 2000000$

$$\therefore \quad \sigma_c = \frac{2000000}{254398} = \mathbf{7.86\ N/mm^2}. \quad \textbf{Ans.}$$

1mark

Substituting this value in equation (*i*), we get

$$\sigma_s = 15 \times 7.86 = \mathbf{117.92\ N/mm^2}. \quad \textbf{Ans.}$$

1mark

Q.3 i Each definition allot 1 marks **3**

ii A beam of length 10m is simply supported and carries point load of 50 KN and 40KN each at a distance of 2m and 6m respectively from left support and also a **7**

UDL of 10KN/M between the point loads. Draw SFD and BMD for the beam.

Sol. First calculate the reactions R_A and R_B.

Taking moments of all forces about A, we get

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2+4)$$

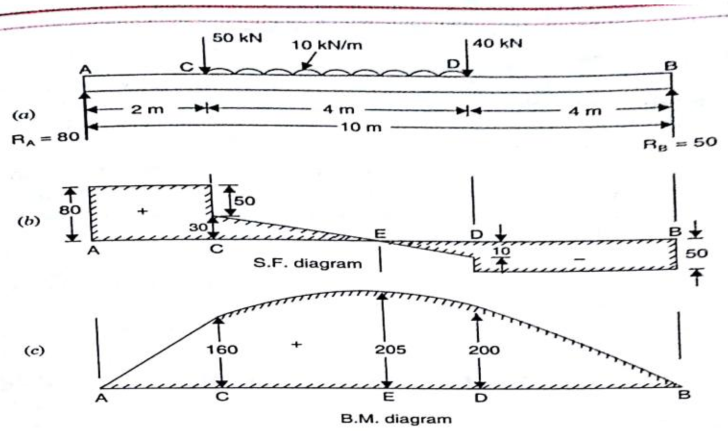
$$= 100 + 160 + 240 = 500$$

1 mark

$$\therefore \quad R_B = \frac{500}{10} = 50 \text{ kN}$$

and $\quad R_A = \text{Total load on beam} - R_B$

$$= (50 + 10 \times 4 + 40) - 50 = 130 - 50 = 80 \text{ kN}$$



1 mark

1 mark

Fig. 6.31

S.F. Diagram

The S.F. at A, $\quad F_A = R_A = +80$ kN

The S.F. will remain constant between A and C and equal to + 80 kN

The S.F. just on R.H.S. of $C \quad = R_A - 50 = 80 - 50 = 30$ kN

The S.F. just on L.H.S. of $D \quad = R_A - 50 - 10 \times 4 = 80 - 50 - 40 = -10$ kN

The S.F. between C and D varies according to straight line law.

The S.F. just on R.H.S. of $D \quad = R_A - 50 - 10 \times 4 - 40 = 80 - 50 - 40 - 40 = -50$ kN

The S.F. at $B = -50$ kN

The S.F. remains constant between D and B and equal to -50 kN

The shear force diagram is drawn as shown in Fig. 6.31 (b).

The shear force is zero at point E between C and D.

Let the distance of E from point A is x.

Now shear force at $\quad E = R_A - 50 - 10 \times (x - 2)$

$$= 80 - 50 - 10x + 20 = 50 - 10x$$

But shear force at $\quad E = 0$

$$\therefore \quad 50 - 10x = 0 \quad \text{or} \quad x = \frac{50}{10} = 5 \text{ m}$$

S.F. calculation

2 mark

B.M. Diagram

B.M. at A is zero

B.M. at B is zero

B.M. at C, $\quad M_C = R_A \times 2 = 80 \times 2 = 160$ kNm

B.M. at D, $\quad M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times \frac{4}{2}$

$$= 80 \times 6 - 200 - 80 = 480 - 200 - 80 = 200 \text{ kNm}$$

BM calculation

2 mark

OR	iii	Derive an expression for slope at support and deflection for a simply supported beam of span "**L**" subjected to point load " W" at a distance " **a**" from left support and " **b**" from right support by Macaulay's method.	7

... expressed and in the manner in which the integration ... carried out.

12.7.1. Deflection of a Simply Supported Beam with an Eccentric Point Load

A simply supported beam AB of length L and carrying a point load W at a distance 'a' from left support and at a distance 'b' from right support is shown in Fig. 12.7. The reactions at A and B are given by,

$$R_A = \frac{W.b}{L} \quad \text{and} \quad R_B = \frac{W.a}{L}$$

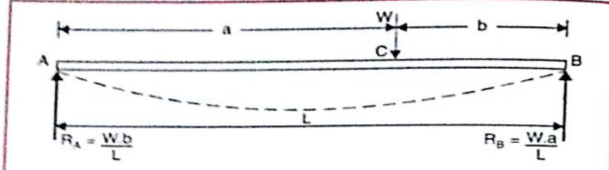


Fig. 12.7

The bending moment at any section between A and C at a distance x from A is given by,

$$M_x = R_A \times x = \frac{W.b}{L} \times x$$

The above equation of B.M. holds good for the values of x between 0 and 'a'. The B.M. at any section between C and B at a distance x from A is given by,

$$M_x = R_A.x - W \times (x - a)$$

$$= \frac{W.b}{L} . x - W(x - a)$$

The above equation of B.M. holds good for all values of x between $x = a$ and $x = b$.

The B.M. for all sections of the beam can be expressed in a single equation written as

$$M_x = \frac{W.b}{L} x \quad \vdots \quad - W(x - a) \qquad ...(i)$$

Stop at the dotted line for any point in section AC. But for any point in section CB, add the expression beyond the dotted line also.

The B.M. at any section is also given by equation (12.3) as

$$M = EI \frac{d^2y}{dx^2} \qquad ...(ii)$$

Hence equating (i) and (ii), we get

$$EI \frac{d^2y}{dx^2} = \frac{W.b}{L} . x \quad \vdots \quad - W(x - a) \qquad ...(iii)$$

Integrating the above equation, we get

$$EI \frac{dy}{dx} = \frac{W.b}{L} \frac{x^2}{2} + C_1 \quad \vdots \quad - \frac{W(x - a)^2}{2} \qquad ...(iv)$$

where C_1 is a constant of integration. This constant of integration should be written after the first term. Also *the brackets are to be integrated as a whole*. Hence the integration of $(x - a)$ will be $\frac{(x-a)^2}{2}$ and not $\frac{x^2}{2} - ax$.

Integrating equation (*iv*) once again, we get

$$EIy = \frac{W.b}{2L} \cdot \frac{x^3}{3} + C_1 x + C_2 \quad \vdots \quad - \frac{W}{2} \frac{(x-a)^3}{3} \qquad ...(v)$$

3 mark

where C_2 is another constant of integration. This constant is written after $C_1 x$. The integration of $(x - a)^2$ will be $\left(\frac{x-a}{3}\right)^3$. This type of integration is justified as the constant of integrations C_1 and C_2 are valid for all values of x.

The values of C_1 and C_2 are obtained from boundary conditions. The two boundary conditions are :

(*i*) At $x = 0$, $y = 0$ and (*ii*) At $x = L$, $y = 0$

(*i*) At A, $x = 0$ and $y = 0$. Substituting these values in equation (*v*) upto dotted line only, we get

$$0 = 0 + 0 + C_2$$

$$\therefore \quad C_2 = 0$$

(*ii*) At B, $x = L$ and $y = 0$. Substituting these values in equation (*v*), we get

$$0 = \frac{W.b}{2L} \cdot \frac{L^3}{3} + C_1 \times L + 0 - \frac{W}{2} \frac{(L-a)^3}{3}$$

($\because$ $C_2 = 0$. Here complete Eq. (*v*) is to be taken)

$$= \frac{W.b.L^2}{6} + C_1 \times L - \frac{W}{2} \frac{b^3}{3} \qquad (\because \quad L - a = b)$$

$$\therefore \quad C_1 \times L = \frac{W}{6} . b^3 - \frac{W.b.L^2}{6} = - \frac{W.b}{6} (L^2 - b^2)$$

$$\therefore \quad C_1 = - \frac{W.b}{6L} (L^2 - b^2) \qquad ...(vi)$$

1 mark

Substituting the value of C_1 in equation (*iv*), we get

$$EI \frac{dy}{dx} = \frac{W.b}{L} \frac{x^2}{2} + \left[- \frac{W.b}{6L} \left(L^2 - b^2\right) \right] \quad \vdots \quad - \frac{W(x-a)^2}{2}$$

$$= \frac{W.b.x^2}{2L} - \frac{W.b}{6L} \left(L^2 - b^2\right) \quad \vdots \quad - \frac{W(x-a)^2}{2} \qquad ...(vii)$$

Equation (*vii*) gives the slope at any point in the beam. Slope is maximum at A or B. To find the slope at A, substitute $x = 0$ in the above equation upto dotted line as point A lies in AC.

$$\therefore \quad EI.\theta_A = \frac{W.b}{2L} \times 0 - \frac{Wb}{6L} (L^2 - b^2) \qquad \left(\because \frac{dy}{dx} \text{ at } A = \theta_A\right)$$

$$= - \frac{Wb}{6L} (L^2 - b^2)$$

$$\therefore \quad \theta_A = - \frac{Wb}{6EIL} (L^2 - b^2) \qquad \text{(as given before)}$$

$x = L$, $\theta_B = \frac{Wb}{6LEI}(3L^2 - a^2 - b)$

2 mark

Substituting the values of C_1 and C_2 in equation (*v*), we get

$$EIy = \frac{W.b}{6L} . x^3 + \left[- \frac{Wb}{6L} (L^2 - b^2) \right] x + 0 \quad \vdots \quad - \frac{W}{6} (x-a)^3 \qquad ...(viii)$$

Equation (*viii*) gives the deflection at any point in the beam. To find the deflection y_c under the load, substitute $x = a$ in equation (*viii*) and consider the equation upto dotted line (as point C lies in AC). Hence, we get

$$EIy_c = \frac{W.b}{6L} . a^3 - \frac{W.b}{6L} (L^2 - b^2)a = \frac{W.b}{6L} . a (a^2 - L^2 + b^2)$$

$$= - \frac{W.a.b}{6L} (L^2 - a^2 - b^2)$$

$$= - \frac{W.a.b}{6L} [(a + b)^2 - a^2 - b^2] \qquad (\because \quad L = a + b)$$

$$= - \frac{W.a.b}{6L} [a^2 + b^2 + 2ab - a^2 - b^2]$$

$$= - \frac{W.a.b}{6L} [2ab] = - \frac{Wa^2 . b^2}{3L}$$

1 mark

$$\therefore \quad y_c = - \frac{Wa^2 . b^2}{3EIL} \qquad \text{...(same as before)}$$

Note. While using Macaulay's Method ...

Q.4

i	Each definition allot 1 marks	**3**
ii	A timber beam of rectangular section is to support a load of 20KN uniformly	**7**

distributed over a span of 3.6 m when beam is simply supported. If the depth of section is to be twice the breadth, and the stress in the timber is not to exceed 7N/mm^2, find the dimension of the cross- section.

Sol. Given :

Total load, $W = 20 \text{ kN} = 20 \times 1000 \text{ N}$

Span, $L = 3.6 \text{ m}$

Max. stress, $\sigma_{max} = 7 \text{ N/mm}^2$

Let b = Breadth of beam in mm

Then depth, $d = 2b$ mm

Section modulus of rectangular beam $= \dfrac{bd^2}{6}$

$$\therefore \quad L = \frac{b \times (2b)^2}{6} = \frac{2b^3}{6} \text{ mm}^3$$ 1 mark

Maximum B.M., when the simply supported beam carries a U.D.L. over the entire span, is at the centre of the beam and is equal to $\dfrac{wL^2}{8}$ or $\dfrac{WL}{8}$.

$$\therefore \quad M = \frac{WL}{8} = \frac{20000 \times 3.6}{8} = 9000 \text{ Nm}$$ 1 mark

$$= 9000 \times 1000 \text{ Nmm}$$

Now using equation (7.6), we get

$$M = \sigma_{max} \cdot Z$$

$$\text{or} \quad 9000 \times 1000 = 7 \times \frac{2b^3}{3}$$

$$\text{or} \quad b^3 = \frac{3 \times 9000 \times 1000}{7 \times 2} = 1.92857 \times 10^6$$

$$\therefore \quad b = (1.92857 \times 10^6)^{1/3}$$

$= 124.47$ mm say **124.5 mm. Ans.** 2 mark

and $d = 2b = 2 \times 124.5 =$ **249 mm. Ans.**

Dimension of the section when the beam carries a point load at the centre.

B.M. is maximum at the centre and it is equal to $\dfrac{W \times L}{4}$ when the beam carries a point load at the centre. 1 mark

$$\therefore \quad M = \frac{W \times L}{4} = \frac{20000 \times 3.6}{4} = 18000 \text{ Nm}$$

$$= 18000 \times 1000 \text{ Nmm}$$

$$\sigma_{max} = 7 \text{ N/mm}^2$$

and $Z = \dfrac{2b^3}{3}$ ($\because$ In this case also $d = 2b$)

Using equation (7.6), we get

$$M = \sigma_{max} Z$$

$$\text{or} \quad 18000 \times 1000 = 7 \times \frac{2b^2}{3}$$

$$\therefore \quad b^3 = \frac{3 \times 18000 \times 1000}{7 \times 2} = 3.85714 \times 10^6$$

$\therefore \quad b = (3.85714 \times 10^6)^{1/3} =$ **156.82 mm. Ans.** 2 mark

and $d = 2 \times 156.82 =$ **313.64 mm. Ans.**

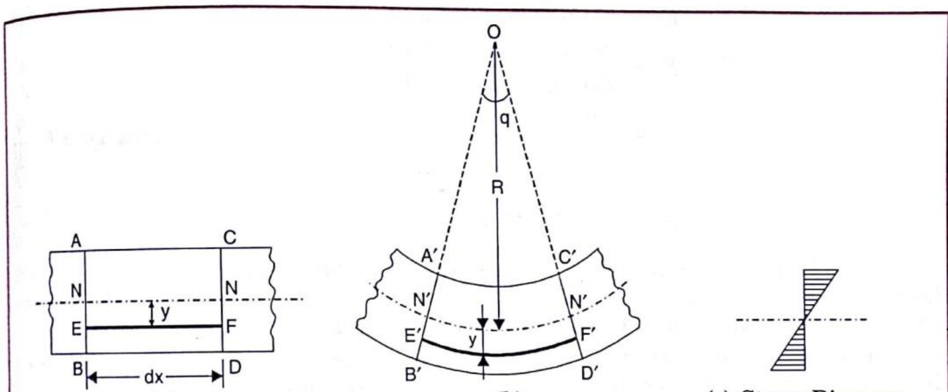
OR iii Derive the bending equation ; $M/I = \sigma_b/y = E/R$ **7**

7.4. EXPRESSION FOR BENDING STRESS

Fig. 7.3 (a) shows a small length δx of a beam subjected to a simple bending. Due to the action of bending, the part of length δx will be deformed as shown in Fig. 7.3 (b). Let $A'B'$ and $C'D'$ meet at O.

Let R = Radius of neutral layer $N'N'$

θ = Angle subtended at O by $A'B'$ and $C'D'$ produced.



(a) (b) (c) Stress Diagram

Fig. 7.3

1 mark

7.4.1. Strain Variation Along the Depth of Beam. Consider a layer EF at a distance y below the neutral layer NN. After bending this layer will be elongated to $E'F'$.

Original length of layer $EF = \delta x$.

Also length of neutral layer $NN = \delta x$.

After bending, the length of neutral layer $N'N'$ will remain unchanged. But length of layer $E'F'$ will increase. Hence

$$N'N' = NN = \delta x.$$

Now from Fig. 7.3 (*b*),

$$N'N' = R \times \theta$$

and $$E'F' = (R + y) \times \theta \qquad (\because \text{ Radius of } E'F' = R + y)$$

But $$N'N' = NN = \delta x.$$

Hence $$\delta x = R \times \theta$$

$\therefore$ Increase in the length of the layer EF

$$= E'F' - EF = (R + y)\,\theta - R \times \theta \qquad (\because EF = \delta x = R \times \theta)$$

$$= y \times \theta$$

$\therefore$ Strain in the layer EF

$$= \frac{\text{Increase in length}}{\text{Original length}}$$

$$= \frac{y \times \theta}{EF} = \frac{y \times \theta}{R \times \theta} \qquad (\because EF = \delta x = R \times \theta)$$

2 mark

$$= \frac{y}{R}$$

As R is constant, *hence the strain in a layer is proportional to its distance from the neutral axis.* The above equation shows the variation of strain along the depth of the beam. The variation of strain is linear.

7.4.2. Stress Variation

Let σ = Stress in the layer EF

E = Young's modulus of the beam

Then $$E = \frac{\text{Stress in the layer } EF}{\text{Strain in the layer } EF}$$

$$= \frac{\sigma}{\left(\frac{y}{R}\right)} \qquad \left(\because \text{ Strain in } EF = \frac{y}{R}\right)$$

$\therefore$ $$\sigma = E \times \frac{y}{R} = \frac{E}{R} \times y \qquad ...(7.1)$$

Since E and R are constant, *therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer.* The equation (7.1) shows the variation of stress along the depth of the beam. The variation of stress is linear.

In the above case, all layers below the neutral layer are subjected to tensile stresses whereas the layers above neutral layer are subjected to compressive stresses. The Fig. 7.3 (*c*) shows the stress distribution.

Equation (7.1) can also be written as

$$\frac{\sigma}{y} = \frac{E}{R} \qquad ...(7.2)$$

2 mark

7.5.1. Moment of Resistance. Due to pure bending, the layers above the N.A. are subjected to compressive stresses whereas the layers below the N.A. are subjected to tensile stresses. Due to these stresses, the forces will be acting on the layers. These forces will have moment about the N.A. The total moment of these forces about the N.A. for a section is known as moment of resistance of that section.

The force on the layer at a distance y from neutral axis in Fig. 7.4 is given by equation (*i*), as

Force on layer $$= \frac{E}{R} \times y \times dA$$

Moment of this force about N.A.

$$= \text{Force on layer} \times y$$

$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} \times y^2 \times dA$$

Total moment of the forces on the section of the beam (or moment of resistance)

$$= \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R}\int y^2 \times dA$$

Let M = External moment applied on the beam section. For equilibrium the moment of resistance offered by the section should be equal to the external bending moment.

$$\therefore \quad M = \frac{E}{R}\int y^2 \times dA.$$

But the expression $\int y^2 \times dA$ represents the moment of inertia of the area of the section about the neutral axis. Let this moment of inertia be I.

$$\therefore \quad M = \frac{E}{R} \times I \quad \text{or} \quad \frac{M}{I} = \frac{E}{R} \qquad ...(7.3)$$

But from equation (7.2), we have

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\therefore \quad \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R} \qquad ...(7.4)$$

2 mark

Equation (7.4) is known as bending equation.

In equation (7.4), the different quantities are expressed in consistent units as given below :

M is expressed in N mm ; I in mm^4

σ is expressed in N/mm^2 ; y in mm

and E is expressed in N/mm^2 ; R in mm.

Q.5

i A cylindrical pipe of diameter 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm^2 determine. 1. Longitudinal and circumferential stress developed in the pipe. **4**

Sol. Given :

Dia. of pipe,	$d = 1.5$ m
Thickness,	$t = 1.5$ cm $= 1.5 \times 10^{-2}$ m
Internal fluid pressure,	$p = 1.2$ N/mm^2

As the ratio $\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$, hence this is a case of thin cylinder.

Here unit of pressure (p) is in N/mm^2. Hence the unit of σ_1 and σ_2 will also be in N/mm^2.

(*i*) The longitudinal stress (σ_2) is given by equation (17.2) as,

$$\sigma_2 = \frac{p \times d}{4t}$$

$$= \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = \mathbf{30\ N/mm^2}. \quad \textbf{Ans.}$$

2 mark

(*ii*) The circumferential stress (σ_1) is given by equation (17.1) as

$$\sigma_1 = \frac{pd}{2t}$$

$$= \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}} = \mathbf{60\ N/mm^2}. \quad \textbf{Ans.}$$

2 mark

ii A solid shaft of diameter 80mm is subjected to twisting moment of 8MN-mmand a bending moment of 5MN-mm at a point determine 1. Principal stresses, 2. Position of plane on which they act. **6**

Sol. Given :

Diameter of shaft, $D = 80$ mm

Twisting moment, $T = 8$ MN-mm $= 8 \times 10^6$ N-mm

Bending moment, $M = 5$ MN-mm $= 5 \times 10^6$ N-mm.

The major principal stress is given by equation (16.14), as

$$\text{Major principal stress} = \frac{16}{\pi D^3}\left(M + \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left(5 \times 10^6 + \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

$$= \frac{16 \times 10^6}{\pi \times 80^3}\left(5 + \sqrt{25 + 64}\right) = \mathbf{143.57\ N/mm^2}.\quad \textbf{Ans.} \longrightarrow$$

2 mark

Minor principal stress is given by equation (16.15).

$\therefore$ Minor principal stress

$$= \frac{16}{\pi D^3}\left(M - \sqrt{M^2 + T^2}\right)$$

$$= \frac{16}{\pi \times 80^3}\left(5 \times 10^6 - \sqrt{(5 \times 10^6)^2 + (8 \times 10^6)^2}\right)$$

$$= \frac{16 \times 10^6}{\pi \times 80^3}\left(5 - \sqrt{25 + 64}\right) = -44.1\ \text{N/mm}^2$$

$$= \mathbf{44.1\ N/mm^2\ (tensile).}\quad \textbf{Ans.} \longrightarrow$$

2 mark

Position of plane is given by equation (16.13), as

$$\tan 2\theta = \frac{T}{M} = \frac{8 \times 10^6}{5 \times 10^6} = 1.6$$

$\therefore$ $2\theta = \tan^{-1} 1.6 = 57° \, 59.68'$, or $237° \, 59.68'$

or $\theta = 28° \, 59.84'$ or $\mathbf{118° \, 59.84'}$. **Ans.** $\longrightarrow$ 2 mark

OR iii A hollow shaft of external diameter 120mm transmits 300KW power at 200r.p.m determine the internal diameter. If maximum stress in the shaft is not to exceed 60N/mm^2 **6**

to exceed 60 N/mm^2.

Sol. Given :

External dia., $D_0 = 120$ mm

Power, $P = 300$ kW $= 300{,}000$ W

Speed, $N = 200$ r.p.m.

Max. shear stress, $\tau = 60$ N/mm^2

Let D_i = Internal dia. of shaft

Using equation (16.7),

$$P = \frac{2\pi NT}{60} \quad \text{or} \quad 300{,}000 = \frac{2\pi \times 200 \times T}{60}$$

$$\therefore \quad T = \frac{300{,}000 \times 60}{2\pi \times 200} = 14323.9\ \text{N-m}$$

$$= 14323.9 \times 1000\ \text{Nmm} = 14323900\ \text{N-mm} \longrightarrow$$

3 mar**k**

Now using equation (16.6),

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_0^4 - D_i^4)}{D_0}$$

or
$$14323900 = \frac{\pi}{16} \times 60 \times \frac{(120^4 - D_i^4)}{120}$$

or
$$\frac{14323900 \times 16 \times 120}{\pi \times 60} = 120^4 - D_i^4$$

or
$$145902000 = 207360000 - D_i^4$$

$$\therefore \quad D_i^4 = 207360000 - 145902000 = 61458000$$

$$D_i = (61458000)^{1/4} = \mathbf{88.5\ mm.}\quad \textbf{Ans.} \longrightarrow$$

3 mark

Q.6

i A solid round bar 3m long and 5 cm in diameter . determine the crippling load take E= 2x10^5 N/mm^2 used following end conditions. **5**

1. Both the end are hinged. 2. Both end are fixed

Sol. Given :

Length of bar, $l = 3\text{ m} = 3000\text{ mm}$

Diameter of bar, $d = 5\text{ cm} = 50\text{ mm}$

Young's modulus, $E = 2.0 \times 10^5\text{ N/mm}^2$

Moment of inertia, $I = \dfrac{\pi}{64} \times 5^4 = 30.68\text{ cm}^4 = 30.68 \times 10^4\text{ mm}^4$

Let P = Crippling load.

As both the ends of the bar are hinged, hence the crippling load is given by equation

$$\therefore \quad P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2 \times 10^5 \times 30.68 \times 10^4}{3000^2}$$

$= 67288\text{ N} =$ **67.288 kN. Ans.** → 2 mark

Alternate Method

Using equation (19.5), $P = \dfrac{\pi^2 EI}{L_e^2}$

1 mark

where L_e = Effective length

$= \dfrac{l}{2}$ (when both the ends are fixed)

$= \dfrac{3000}{2}$ $(\because \quad l = 3000)$

$= 1500\text{ mm}$

$\therefore \quad P = \dfrac{\pi^2 \times 2.0 \times 10^5 \times 30.68 \times 10^4}{1500^2} =$ **269152 N. Ans.** → 2 mark

ii Derive the formulae for finding buckling load in column , if both ends of column pinned **5**

19.5. EXPRESSION FOR CRIPPLING LOAD WHEN BOTH THE ENDS OF THE COLUMN ARE HINGED

The load at which the column just buckles (or bends) is called crippling load. Consider a column AB of length l and uniform cross-sectional area, hinged at both of its ends A and B. Let P be the crippling load at which the column has just buckled. Due to the crippling load, the column will deflect into a curved form ACB as shown in Fig. 19.4.

Consider any section at a distance x from the end A.

Let y = Deflection (lateral displacement) at the section.

The moment due to the crippling load at the section $= -P \cdot y$

(– ve sign is taken due to sign convention given in Art. 19.4.1)

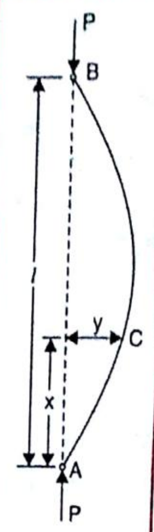


Fig. 19.4

But moment $= EI \dfrac{d^2y}{dx^2}$. → 1 mark

Equating the two moments, we have

$$EI \frac{d^2y}{dx^2} = -P \cdot y \quad \text{or} \quad EI \frac{d^2y}{dx^2} + P \cdot y = 0$$

or
$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

The solution* of the above differential equation is

$$y = C_1 \cdot \cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2 \cdot \sin\left(x\sqrt{\frac{P}{EI}}\right) \qquad \text{...}(i)$$

→ 1 mark

where C_1 and C_2 are the constants of integration. The values of C_1 and C_2 are as follows :

(i) At A, $x = 0$ and $y = 0$ (See Fig. 19.4)

Substituting these values in equation (i), we get

$$0 = C_1 . \cos 0 + C_2 \sin 0$$
$$= C_1 \times 1 + C_2 \times 0 \qquad (\because \cos 0 = 1 \text{ and } \sin 0 = 0)$$
$$= C_1$$
$$\therefore \qquad C_1 = 0. \qquad \text{...(ii)}$$

(ii) At B, $x = l$ and $y = 0$ (See Fig. 19.4).

Substituting these values in equation (i), we get

$$0 = C_1 . \cos\left(l \times \sqrt{\frac{P}{EI}}\right) + C_2 . \sin\left(l \times \sqrt{\frac{P}{EI}}\right)$$
$$= 0 + C_2 . \sin\left(l \times \sqrt{\frac{P}{EI}}\right) \qquad [\because C_1 = 0 \text{ from equation (ii)}]$$
$$= C_2 \sin\left(l\sqrt{\frac{P}{EI}}\right) \qquad \text{...(iii)}$$

1 mark

From equation (iii), it is clear that either $C_2 = 0$

or $\sin\left(l\sqrt{\frac{P}{EI}}\right) = 0.$

As $C_1 = 0$, then if C_2 is also equal to zero, then from equation (i) we will get $y = 0$. This means that the bending of the column will be zero or the column will not bend at all. Which is not true.

$$\therefore \qquad \sin\left(l\sqrt{\frac{P}{EI}}\right) = 0$$
$$= \sin 0 \text{ or } \sin \pi \text{ or } \sin 2\pi \text{ or } \sin 3\pi \text{ or } ...$$

or $$l\sqrt{\frac{P}{EI}} = 0 \text{ or } \pi \text{ or } 2\pi \text{ or } 3\pi \text{ or } ...$$

Taking the least practical value,

$$l\sqrt{\frac{P}{EI}} = \pi$$

or $$P = \frac{\pi^2 EI}{l^2}. \qquad \text{...(19.1)}$$

2 mark

iii A 1.5 m long column has a circular cross section of 5cm diameter. One of the end fix other free take FOS=3 calculate safe load using. 1. Rankine's formula , take yield stress 560N/mm^{2}and a= 1/1600 **5**

2. euler's formula take E for C.I.=1.2x10^{5}N/mm^2

Sol. $l = 1500$ mm ; $d = 5$ c.m.

$\therefore$ Area, $A = \frac{\pi}{4} \times 5^2 = 19.635 \text{ cm}^2 = 19.635 \times 10^2 \text{ mm}^2$

Moment of inertia, $I = \frac{\pi}{64} \times 5^4 = 30.7 \text{ cm}^4 = 30.7 \times 10^4 \text{ mm}^4$

and least radius of gyration, $k = \sqrt{\frac{I}{A}} = \sqrt{\frac{30.7 \times 10^4}{19.635 \times 10^2}} = 12.5$ mm.

End conditions = One end is fixed and other end is free.

$\therefore$ Effective length, $L_e = 2l = 2 \times 1500 = 3000$ mm

Factor of safety = 3.

(*a*) *Safe load by Rankine's formula*

Yield stress, $\sigma_c = 560 \text{ N/mm}^2$

Rankine's constant, $a = \frac{1}{1600}$

Let P = Crippling load by Rankine's formula

Using equation (19.9), we have

$$P = \frac{\sigma_c \times A}{1 + a\left(\frac{L_e}{k}\right)^2}$$

$$= \frac{560 \times 1963.5}{1 + \frac{1}{1600} \times \left(\frac{3000}{12.5}\right)^2} \quad (\because \; L_e = 3000 \text{ mm and } k = 12.5)$$

$= 29708.1$ N

1.5 mark

$\therefore$ Safe load $= \frac{\text{Crippling load}}{\text{Factor of safety}}$

$= \frac{29708.1}{3} = $ **9902.7 N. Ans.**

1 mark

(*b*) *Safe load by Euler's formula*

Young's Modulus, $E = 1.2 \times 10^5 \text{ N/mm}^2$

Let P = Crippling load by Euler's formula

Using equation (19.5), $P = \frac{\pi^2 EI}{L_e^2}$

$$= \frac{\pi^2 \times 1.2 \times 10^5 \times 30.7 \times 10^4}{3000^2} \quad (\because \; L_e = 3000 \text{ mm})$$

$= 40200$ N

1.5 mark

$\therefore$ Safe load $= \frac{\text{Crippling load}}{\text{Factor of safety}} = \frac{40200}{3} = $ **1340 N. Ans.**

1 mark

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