Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

CE3CO08 Fluid Mechanics

Programme: B.Tech. Branch/Specialisation: CE

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Bulk modulus is the ratio of: **1**
(a) Volumetric strain to Shear stress.
(b) Compressive stress to Volumetric strain.
(c) Shear stress to Volumetric strain.
(d) Volumetric strain to Compressive stress.

ii. What is the correct formula of Vacuum pressure? **1**
(a) PVAC = PATM - PGAUGE
(b) PVAC = PABS + PATM
(c) PVAC = PATM - PABS
(d) PVAC = PABS + PGAUGE

iii. Centre of Buoyancy always: **1**
(a) Coincides with the centre of gravity.
(b) Coincides with the centroid of the volume of fluid displaced.
(c) Remains above the centre of gravity.
(d) Remains below the centre of gravity.

iv. The increase of meta-centric height **1**
(I) Increase stability.
(II) Decrease stability.
(III) Increase comfort for passenger.
(IV) Decrease comfort for passenger.
The correct answer is:
(a) (I) and (III) (b) (I) and (IV)
(c) (II) and (III) (d) (II) and (IV)

v. Flow occurring in a pipeline when a valve is being is opened is called **1**
(a) Steady flow (b) Laminar flow
(c) Turbulent flow (d) Rotational flow

vi. Uniform flow occurs when: **1**
(a) The flow is steady
(b) The flow is streamline
(c) Size and shape of cross section in a particular length remains constant
(d) Flow occurs at constant rate

vii. For measuring flow by a venturimeter, it should be installed in: **1**
(a) Horizontal
(b) Vertical
(c) Inclined with flow upward
(d) In any direction and in any location

viii. According to Bernoulli's equation for steady ideal fluid flow: **1**
(a) Velocity and Pressure are inversely proportional
(b) Total energy is constant throughout
(c) The energy is constant along streamline but may vary across streamline
(d) None of the above

ix. Which of the following is a major loss **1**
(a) Frictional loss (b) Shock loss
(c) Entry loss (d) Exit loss

x. Which of the following is true **1**
(a) HGL will never be above EGL
(b) HGL will never be under EGL
(c) HGL will never coincide with EGL
(d) HGL will may or may not be above EGL

Q.2 i. Define the following terms: **4**
(a) Density (b) Specific weight.

ii. Explain the Phenomenon of Capillarity and obtain an expression for Capillary rise of a liquid. **6**

OR iii. A rectangular tank 3m X 2m in plain contains 0.75m depth of water underlying 1m depth of oil of specific gravity 0.90. Calculate the total pressure and its location on one of the longer side of tank. **6**

Q.3 i. Discuss the Stability of floating bodies. **4**

ii. Define Metacentre. Derive an expression for Meta-centric height. **6**

OR iii. A wooden block of specific gravity 0.80 and width 20cm X 30cm deep and length 200cm floated horizontally on the surface of sea water (sp. weight=10kN/m^3). Calculate the volume of water displaced, depth of immersion and position of centre of buoyancy. **6**

Q.4 i. Explain the different types of Fluid flow. **4**

ii. Write short note on Flow net. Explain applications and limitation of Flow net. **6**

OR iii. The Stream function for a two dimensional flow is given by Ψ = 8xy, calculate the velocity at the point (4,5) and Velocity potential function. **6**

Q.5 i. Write short note on- Notches and Weirs. **4**

ii. Derive Euler's equation of motion along a streamline, and hence derive the Bernoulli's theorem. **6**

OR iii. Sketch the Venturimeter and Manometer arrangement. Derive an expression for calculating discharge through it. **6**

Q.6 i. Define the terms: Hydraulic gradient line & Total energy line. **4**

ii. Obtain an expression for head loss in a sudden expansion in the pipe. **6**

OR iii. Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3.0 m/s using (a) Darcy formula and (b) Chezy's formula for which C = 60. Take v for water = 0.01 stoke. **6**

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CE3CO08 Fluid Mechanics

Marking Scheme

Q.1	i.	Bulk modulus is the ratio of:	**1**
		(b) Compressive stress to Volumetric strain.	
	ii.	What is the correct formula of Vacuum pressure?	**1**
		(c) PVAC = PATM - PABS	
	iii.	Centre of Buoyancy always:	**1**
		(b) Coincides with the centroid of the volume of fluid displaced.	
	iv.	The increase of meta-centric height	**1**
		(I) Increase stability.	
		(II) Decrease stability.	
		(III) Increase comfort for passenger.	
		(IV) Decrease comfort for passenger.	
		The correct answer is:	
		(b) (I) and (IV)	
	v.	Flow occurring in a pipeline when a valve is being is opened is called	**1**
		(d) Rotational flow	
	vi.	Uniform flow occurs when:	**1**
		(c) Size and shape of cross section in a particular length remains constant	
	vii.	For measuring flow by a venturimeter, it should be installed in:	**1**
		(d) In any direction and in any location	
	viii.	According to Bernoulli's equation for steady ideal fluid flow:	**1**
		(c) The energy is constant along streamline but may vary across streamline	
	ix.	Which of the following is a major loss	**1**
		(a) Frictional loss	
	x.	Which of the following is true	**1**
		(a) HGL will never be above EGL	
Q.2	i.	Define the following terms:	**4**
		(a) Density - 2 marks	
		(b) Specific weight - 2 marks	
	ii.	Phenomenon of Capillarity -2 marks	**6**
		Expression for Capillary rise of a liquid – 4 marks	
OR	iii.	Total pressure - 3 marks	**6**
		Its location - 3 marks	
Q.3	i.	Stability of floating bodies.	**4**
	ii.	Metacentre definition – 2 marks	**6**
		Derivation of Meta-centric height – 4 marks	
OR	iii.	Volume of water displaced – 2 marks	**6**
		Depth of immersion - 2 marks	
		Position of centre of buoyancy – 2 marks	
Q.4	i.	Types of Fluid flow	**4**
	ii.	Flow net definition – 2 marks	**6**
		Applications - 2 marks	
		Limitation – 2 marks	
OR	iii.	Velocity at the point (4,5) - 3 marks	**6**
		Velocity potential function – 3 marks	
Q.5	i.	Notches – 2 marks	**4**
		Weirs – 2 marks	
	ii.	Euler's equation of motion - 4 marks	**6**
		Bernoulli's theorem – 2 marks	
OR	iii.	Venturimeter description – 2 marks	**6**
		Discharge through Venturimeter – 4 marks	
Q.6	i.	Hydraulic gradient line – 2 marks	**4**
		Total energy line - 2 marks	
	ii.	Expression for head loss	**6**
OR	iii.	Head lost by (a) Darcy formula – 3 marks	**6**
		Head lost by (b) Chezy's formula - 3 marks	

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1.2.1 **Density or Mass Density.** Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho). The unit of mass density in SI unit is kg per cubic metre, *i.e.*, kg/m^3. The density of liquids may be considered as constant while that of gases changes with the variation of pressure and temperature.

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}. \qquad \text{mass} = \frac{w}{g}$$

The value of density of water is 1 gm/cm^3 or 1000 kg/m^3.

Specific Weight:-

1.2.2 **Specific Weight or Weight Density.** Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically,
$$w = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(\text{Mass of fluid}) \times \text{Acceleration due to gravity}}{\text{Volume of fluid}}$$
$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$
$$= \rho \times g \qquad \left\{\because \frac{\text{Mass of fluid}}{\text{Volume of fluid}} = \rho\right\}$$
$$\therefore \quad w = \rho g \qquad \text{...(1.1)}$$

Ans.2 (ii)

Capillarity and Expression for capillary rise:-

1.6.4 Capillarity. Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise. Consider a glass tube of small diameter '*d*' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

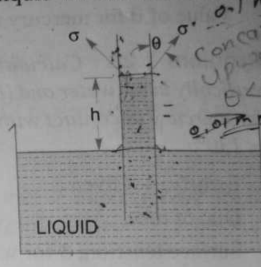


Fig. 1.13 *Capillary rise.*

Adhesion more than Cohesion

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

The weight of liquid of height h in the tube = (Area of tube $\times h) \times \rho \times g$

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \quad \text{...(1.17)}$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \quad \text{...(1.18)}$$

Stress = $\frac{F}{A}$

$\sigma = \frac{F}{\text{Circum.} \times \cos\theta}$

$F = \sigma \times \text{Circum.} \times \cos\theta$

For equilibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4\sigma\cos\theta}{\rho \times g \times d} \quad \text{...(1.19)}$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos\theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \quad \text{... (1.20)}$$

Expression for Capillary Fall. If the ...

Ans.2 (iii)

Num.-5 :- A rectangular tank 3m × 2m in plain contains 0.75 m depth of water underlaying 1m depth of oil of sp. gr. 0.90. Calculate the total pressure and its location on one of the longer side of tank.

Soln. :- Given that.

length = l = 3m

width = b = 2 m

ht. of oil = h_1 = 1 m

ht. of water = h_2 = 0.75 m.

Spe. gr. of oil = 0.9 = 0.9 × 1000 = 900 kg/m³

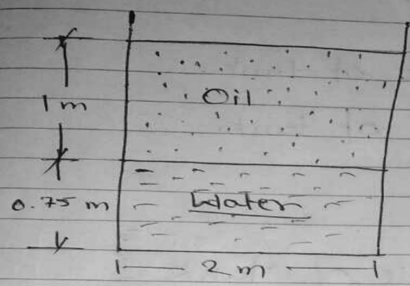


Fig (a)

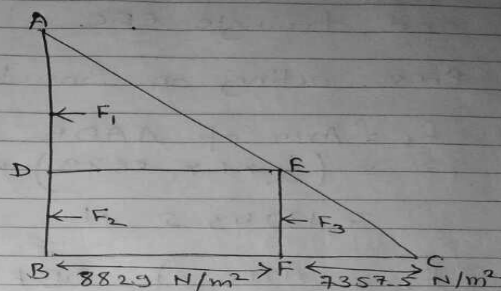


Fig (b)

The pressure dia. for the longer side of the tank is shown in fig (b)

Intensity of pressure on top = $P_A = 0$

Intensity of pressure 1 m from the top 'P_D'

$$P_D = \rho_1 g h_1 = 900 \times 9.81 \times 1$$
$$= 8829 \text{ N/m}^2$$

Pressure at 1.75 m from the top or at the base

$$P_B = \rho_2 g h_2 + \rho_1 g h_1$$
$$= 1000 \times 9.81 \times 0.75 + 8829$$
$$= 16186.5 \text{ N/m}^2$$

To determinate total pressure on the side of tank, pressure dia. can be splitted into three components viz. tangle ADE, rect. DBFE and triangle EFC.

Press. acting on 3m long side of tank

F_1 = Area of $\Delta ADE \times$ side of tank

$= \left(\frac{1}{2} \times 1 \times 8829\right) \times 3$

$= 13243.5$ N

F_2 = Area of DBFE $\times$ side of tank

$= (0.75 \times 8829) \times 3 = 19865.25$ N

F_3 = Area of $\Delta EFC \times$ side of tank

$= \left(\frac{1}{2} \times 0.75 \times 7357.5\right) \times 3$

$= 8277.2$ N.

Total press. acting $= F = F_1 + F_2 + F_3 = \underline{41386\ N}$

Let total pressure is acting at a distance $\bar{h}$ from the top of the tank.
Taking moment of all forces about A

$$F \times \bar{h} = F_1 \times \left(\frac{2}{3} AD\right) + F_2 \left(AD + \frac{1}{2} BD\right) + F_3 \left(AD + \frac{2}{3} BD\right)$$

$$41386 \times \bar{h} = 13243.5 \times \frac{2}{3} \times 1 + 19865.25 \left(1 + \frac{1}{2} \times 0.75\right) + 8277.2 \times \left(1 + \frac{2}{3} \times 0.75\right)$$

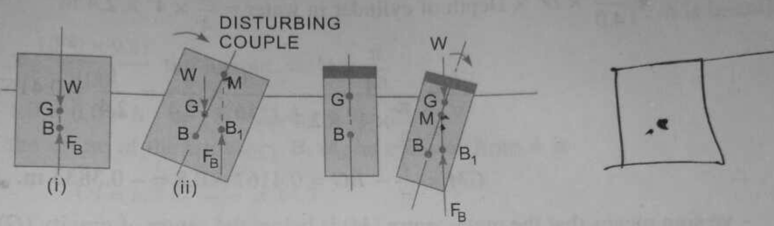
$\bar{h} = \underline{1.175\ m}$ from top Ans

Ans.3 (i)

Stability of floating body:-

4.7.2 Stability of Floating Body. The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(*a*) **Stable Equilibrium.** If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(*a*) Stable equilibrium M is above G (*b*) Unstable equilibrium M is below G.

Fig. 4.13 *Stability of floating bodies.*

(*b*) **Unstable Equilibrium.** If the point M is below G, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Ans.3 (ii)

Metacenter and Expression for Metacentric Height.

▶ 4.4 META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (a). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

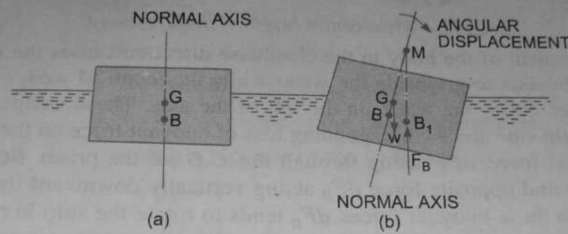


Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (b). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (b). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

▶ 4.5 META-CENTRIC HEIGHT

The distance MG, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

▶ 4.6 ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (*a*) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at G and B. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (*b*). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at M. Hence M is the meta-centre and GM is meta-centric height.

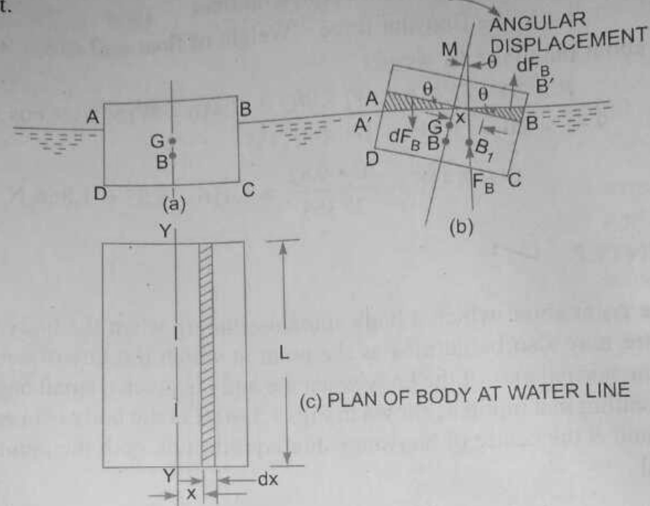


Fig. 4.6 *Meta-centre height of floating body.*

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism BOB' on the right of the axis to go inside the water while the identical wedge-shaped prism represented by AOA' emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the $C.G.$ of the prism BOB' while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of AOA'. The couple due to these buoyant forces dF_B tends to rotate the ship in the counter clockwise direction. Also the moment caused by the displacement of the centre of buoyancy from B to B_1 is also in the counter clockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness dx at a distance x from O as shown in Fig. 4.5 (*b*). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$$

$\therefore$ Area of strip $= \text{Height} \times \text{Thickness} = x \times \theta \times dx$

If L is the length of the floating body, then

Volume of strip $= \text{Area} \times L$

$= x \times \theta \times L \times dx$

$\therefore$ Weight of strip $= \rho g \times \text{Volume} = \rho g x\, \theta L\, dx$

Similarly, if a small strip of thickness dx at a distance x from O towards the left of the axis is considered, the weight of strip will be $\rho g x \theta\, L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = Weight of each strip × Distance between these two weights

$$= \rho g x \, \theta L \, dx \, [x + x]$$

$$= \rho g x \, \theta L \, dx \times 2x = 2\rho g x^2 \, \theta L \, dx$$

∴ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2 \, \theta L \, dx \qquad \text{...(4.1)}$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$$= F_B \times BB_1$$

$$= F_B \times BM \times \theta \qquad \{\because BB_1 = BM \times \theta \text{ if } \theta \text{ is very small}\}$$

$$= W \times BM \times \theta \qquad \{\because F_B = W\} \text{ ...(4.2)}$$

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2 \, \theta \, L dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$$

$$W \times BM = 2\rho g \int x^2 L dx$$

Now Ldx = Elemental area on the water line shown in Fig. 4.6 (c) and = dA

$$\therefore \quad W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 (c) it is clear that $2 \int x^2 \, dA$ is the second moment of area of the plan of the body at water surface about the axis y-y. Therefore

$$W \times BM = \rho g I \qquad \{\text{where } I = 2 \int x^2 \, dA\}$$

$$\therefore \quad BM = \frac{\rho g I}{W}$$

But W = Weight of the body

= Weight of the fluid displaced by the body

= ρg × Volume of the fluid displaced by the body

= ρg × Volume of the body sub-merged in water

= $\rho g \times \forall$

$$\therefore \quad BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \qquad \text{...(4.3)}$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

$$\therefore \text{ Meta-centric height} = GM = \frac{I}{\forall} - BG. \qquad \text{...(4.4)}$$

Ans.3 (iii)

Num. - 8. A wooden block of sp. Gravity 0.8 and width 20 cm x 30 cm deep and length 200 cm floats horizontally on the surface of sea water of sp. wt. 10 kN. Calculate the volume of water displaced, depth of immersion and position of the centre of buoyancy.

Soln:

Given that: sp. Gr. of block = 0.8

$b = 20\text{ cm} = 0.2\text{ m}$

$d = 30\text{ cm} = 0.3\text{ m}$

$l = 200\text{ cm} = 2\text{ m}$

sp. wt. of sea water = 10 kN = 10000 N.

(I) Volume of water displaced

Volume of wooden block $= b \times d \times l = 0.2 \times 0.3 \times 2 = 0.12\ m^3$

$wt. = \rho \times g \times V = 0.8 \times 9.81 \times 1000 \times 0.12 = 941.76\ N.$

Since wt. of water displaced = wt. of block = 941.76

Volume of water displaced $= \dfrac{\text{wt. of water displace}}{\text{sp. wt. of Sea water}}$

$= \dfrac{941.76}{10000} = 0.094176\ m^3$ Ans

(II) Depth of immersion:

let h = Depth of immersion

then

Volume of wooden block in water = Volume of water displace.

$b \times h \times l = 0.094176$

$0.2 \times h \times 2 = 0.094176$

$h = 0.235\text{m} = 23.5\ m$ Ans

(III) Position of centre of buoyancy

Position from base of wooden block

$= \dfrac{h}{2} = \dfrac{23.5}{2} = 11.75\text{ cm}$ Ans

Ans.4 (i)

Different types of fluid flow:-

5.3 TYPES OF FLUID FLOW

The fluid flow is classified as :

(i) Steady and unsteady flows ; नियमित

(ii) Uniform and non-uniform flows ;

(iii) Laminar and turbulent flows ;

(iv) Compressible and incompressible flows ;

(v) Rotational and irrotational flows ; and

(vi) One, two and three-dimensional flows.

5.3.1 Steady and Unsteady Flows. Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. at a point do not change with time. Thus for steady flow, mathematically, we have

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where (x_0, y_0, z_0) is a fixed point in fluid field.

Unsteady flow is that type of flow, in which the velocity, pressure of density at a point changes with respect to time. Thus, mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

5.3.2 Uniform and Non-uniform Flows. Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (*i.e.*, length of direction of the flow). Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

where ∂V = Change of velocity

∂s = Length of flow in the direction S.

Non-uniform flow is that type of flow in which the velocity at any given time changes with respect to space. Thus, mathematically, for non-uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

5.3.3 Laminar and Turbulent Flows. Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all the stream-lines are straight and parallel. Thus the particles move in laminas or layers gliding smoothly over the adjacent layer. This type of flow is also called stream-line flow or viscous flow.

Turbulent flow is that type of flow in which the fluid particles move in a *zig-zag* way. Due to the movement of fluid particles in a *zig-zag* way, the eddies formation takes place which are responsible for high energy loss. For a pipe flow, the type of flow is determined by a non-dimensional number $\frac{VD}{\nu}$ called the Reynold number.

where D = Diameter of pipe

V = Mean velocity of flow in pipe

and ν = Kinematic viscosity of fluid.

If the Reynold number is less than 2000, the flow is called laminar. If the Reynold number is more than 4000, it is called turbulent flow. If the Reynold number lies between 2000 and 4000, the flow may be laminar or turbulent.

5.3.4 Compressible and Incompressible Flows. Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for incompressible flow

$$\rho = \text{Constant.}$$

5.3.5 Rotational and Irrotational Flows. Rotational flow is that type of flow in which the fluid particles while flowing along stream-lines, also rotate about their own axis. And if the fluid particles while flowing along stream-lines, do not rotate about their own axis that type of flow is called irrotational flow.

5.3.6 One, Two and Three-Dimensional Flows. **One-dimensional flow** is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say x. For a steady one-dimensional flow, the velocity is a function of one-space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible. Hence mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \text{ and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates say x and y. For a steady two-dimensional flow the velocity is a function of two space co-ordinates only. The variation of velocity in the third direction is negligible. Thus, mathematically for two-dimensional flow

$$u = f_1(x, y)\ v = f_2(x, y) \text{ and } w = 0.$$

Three-dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. But for a steady three-dimensional flow the fluid parameters are functions of three space co-ordinates (x, y and z) only. Thus, mathematically, for three-dimensional flow

$$u = f_1(x, y, z),\ v = f_2(x, y, z),\ w = f_3(x, y, z).$$

Ans.4 (ii): Flow net:-

Flow net :- A grid obtained by drawing a series of equipotential line and stream line is Called flow net.

The flow net is an important tool in analysing two-dimensional irrotational flow problem.

usually, flow net is a square mesh However in regions, where the boundaries converge or diverge or bent, the flow net dones not contain squares.

Application - The flow net have the following applications

1./ for given boundaries of flow, the velocity and pressure distribution can be determined, if the velocity distribution and pressure at any refrence section are known.

2./ Loss of flow due to seepage in earth dam and unlined canals. can be evaluated.

3./ Uplift pressure on the bottom of dam can be worked out.

4.) outlets can be designed for their stream lining.

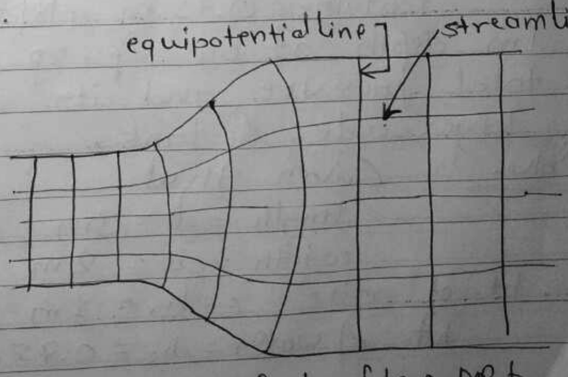


fig:- flow net

Limitations :- (I) A flow net when applied to real fluid always indicate some velocity at the boundary, but a a real fluid must have zero velocity adjacent to the boundary on account of the fluid friction. As such the flow net always connot be applied in the region close to the boundary where effect of viscosity are predominant.

(II) Flow net analysis also can not be applied to a sharply diverging flow.

(III) when fluid flow pasts a solid body, the flow net gives a fairly accurate picture of flow pattern while it may be give very little or no information about the rear end due to separation of eddies.

Ans.4 (iii)

Prob. 03 The Stream function for a two-dim flow is given by $\psi = 8xy$, calculate the velocity at the point (4, 5) and velocity potential function.

Solⁿ: (I) Velocity at the point (4, 5)

Given that

$$\psi = 8xy$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(8xy) = 8x$$

$$v = \frac{-\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(8xy) = -8y$$

At (4, 5), $u = 8x = 8 \times 4 = 32$

$$v = -8y = -8 \times 5 = -40$$

the Resultant Velocity at (4, 5)

$$V = \sqrt{u^2 + v^2}$$

$$= \sqrt{(32)^2 + (-40)^2} = 51.22 \text{ unit/sec.}$$

(II) Velocity potential function

$$u = \frac{\partial \phi}{\partial x} \quad \& \quad v = \frac{\partial \phi}{\partial y}$$

$$8x = \frac{\partial \phi}{\partial x} \quad \& \quad -8y = \frac{\partial \phi}{\partial y}$$

$$\int \partial \phi = \int 8x \cdot \partial x$$

$$\phi = \frac{8x^2}{2} + f(y)$$

$$\phi = 4x^2 + f(y) \quad \text{——— (1)}$$

where $f(y)$ is a constant of integration which can be a function of y only.

Differentiating eqⁿ (1) with r.t. y, we get

$$\frac{\partial \phi}{\partial y} = f'(y)$$

But $$\frac{\partial \phi}{\partial y} = -8y$$

then $$f'(y) = -8y$$

$$f(y) = -\frac{8y^2}{2} + C = -4y^2 + C$$

where C is a constant of integration, which can be a numeric value, hence it may be considered as zero.

the $f(y) = -4y^2$

Substituting value of $f(y)$ in eq^n (1)

$$\phi = 4x^2 - 4y^2$$

$$\phi = 4(x^2 - y^2) \quad - \text{Ans}$$

Ans.5 (i)

Notches and Weirs:-

A notch is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A weir is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

Ans.5 (ii)

Euler's equation of motion:-

6.3 EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are :

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element $\times$ acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s \qquad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.

Now $\quad a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v \partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{dS}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v \partial v}{\partial s}$$

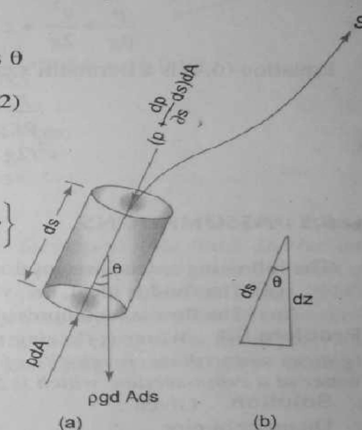


Fig. 6.1 *Forces on a fluid element.*

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{v \partial v}{\partial s}$$

Dividing by $\quad \rho ds dA, \; -\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v \partial v}{\partial s}$

or $\quad \frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{v \partial v}{\partial s} = 0$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{v \partial v}{\partial s} = 0 \quad \text{or} \quad \frac{\partial p}{\rho} + gdz + vdv = 0$$

or $\quad \frac{\partial p}{\rho} + gdz + vdv = 0 \qquad \text{...(6.3)}$

Equation (6.3) is known as Euler's equation of motion.

BERNOULLI'S EQUATION FROM EULER'S EQUATION

rnoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

low is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

ation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

Ans.5 (iii)

Venturimeter and menometer arrangement and expression for discharge through it:-

6.7.1 **Venturimeter.** A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for Rate of Flow Through Venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) $= \frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

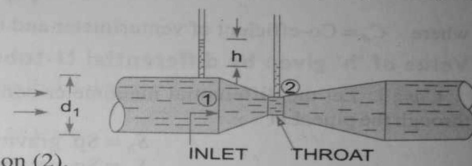


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$\therefore \quad v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ Discharge, $Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \quad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Ans.6 (i)

Hydraulic gradient line and total energy line:-

▶ 11.5 HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

11.5.1 Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

11.5.2 Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

Ans.6 (ii)

Expression for head loss in sudden expansion (enlargement) :-

11.4.1 Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. 11.1. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

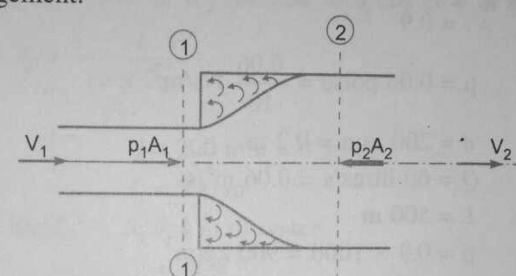


Fig. 11.1 *Sudden enlargement.*

Let p_1 = pressure intensity at section 1-1,
V_1 = velocity of flow at section 1-1,
A_1 = area of pipe at section 1-1,

p_2, V_2 and A_2 = corresponding values at section 2-2.

Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown in Fig. 11.1. The loss of head (or energy) takes place due to the formation of these eddies.

Let p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

h_e = loss of head due to sudden enlargement

Applying Bernoulli's equation to sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{loss of head due to sudden enlargement}$$

But $z_1 = z_2$ as pipe is horizontal

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

or

$$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{...(i)}$$

Consider the control volume of liquid between sections 1-1 and 2-2. Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = p_1A_1 + p'(A_2 - A_1) - p_2A_2$$

But experimentally it is found that $p' = p_1$

$$\therefore \quad F_x = p_1A_1 + p_1(A_2 - A_1) - p_2A_2 = p_1A_2 - p_2A_2$$

$$= (p_1 - p_2)A_2 \quad \text{...(ii)}$$

Momentum of liquid/sec at section 1-1 = mass × velocity

$$= \rho A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 = $\rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$

$\therefore$ Change of momentum/sec = $\rho A_2 V_2^2 - \rho A_1 V_1^2$

But from continuity equation, we have

$$A_1V_1 = A_2V_2 \text{ or } A_1 = \frac{A_2V_2}{V_1}$$

$$\therefore \quad \text{Change of momentum/sec} = \rho A_2 V_2^2 - \rho \times \frac{A_2V_2}{V_1} \times V_1^2 = \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2[V_2^2 - V_1V_2] \quad \text{...(iii)}$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating (ii) and (iii)

$$(p_1 - p_2)A_2 = \rho A_2[V_2^2 - V_1V_2]$$

or

$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1V_2$$

Dividing by g to both sides, we have $\frac{p_1 - p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$ or $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g}$

Substituting the value of $\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right)$ in equation (i), we get

$$h_e = \frac{V_2^2 - V_1 V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} = \left(\frac{V_1 - V_2}{2g}\right)^3$$

$$\therefore \qquad h_e = \frac{(V_1 - V_2)^2}{2g}.$$

Ans.6 (iii)

Problem 11.1 *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν for water = 0.01 stoke.

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke} = 0.01 \text{ cm}^2\text{/s}$

$= 0.01 \times 10^{-4} \text{ m}^2\text{/s}.$

(*i*) **Darcy Formula** is given by equation (11.1) as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynold number, R_e

But R_e is given by $R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$

$\therefore$ Value of $f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$

$\therefore$ Head lost, $h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} =$ **.7828 m. Ans.**

(*ii*) **Chezy's Formula.** Using equation (11.4)

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \frac{d}{4} = \frac{0.30}{4} = 0.075 \text{ m}$

$\therefore$

$$3 = 60\sqrt{.075 \times i} \text{ or } i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$$

But $i = \frac{h_f}{L} = \frac{h_f}{50}$

Equating the two values of i, we have $\frac{h_f}{50} = .0333$

$\therefore$ $h_f = 50 \times .0333 =$ **1.665 m. Ans.**

Problem 11.2 *Find the diameter of a pipe of l...*