Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2019

CS3BS03/IT3BS06 Discrete Mathematics

Programme: B.Tech. Branch/Specialisation: CS/IT

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If $f(x) = x^3$ and $g(x) = x^2 + 1$ then $(g \circ f)$ is **1**

(a) $x^6 + 1$ (b) $(x^2 + 1)^3$

(c) $(x^2 + 1)(x^3)$ (d) $x^3 + 1$.

ii. If there are n pigeons and m pigeon holes such that $(n > m)$, then by generalized Pigeonhole principle, some pigeonhole must contain how many pigeons in it (where $[x]$ denotes greatest integer function): **1**

(a) $\frac{(n-1)}{m}-1$ (b) $\left[\frac{(n-1)}{m}\right]+1$

(c) $\frac{(n+1)}{m}+1$ (d) $\frac{(n-1)}{m}$

iii. In Boolean Algebra (B,+, . ,') Absorption law states that **1**

(a) $a + a = a$ (b) $a + 1 = 1$

(c) $a + (a.(b) = a$ (d) $a.a'=0$.

iv. Let $P = \{2,3,6,12,24,36\}$ be a partial order set with "Divisibility". Then least upper bound and greatest lower bound for the subset $\{12,24,36\}$ are respectively **1**

(a) None, 12 (b) 36, 12 (c) 36, none (d) 12, 36.

v. The set of fourth roots of unity $(1)^{1/4} = \{1,-1,i,-i\}$ with respect to multiplication forms a/an **1**

(a) Group (b) Cyclic group

(c) Abelian group (d) All of these

vi. The generators of the group $\{a^1, a^2, a^3, a^4, a^5, a^6 = e\}$ are **1**

(a) a^6 (b) a^1, a^3, a^5 (c) a^1, a^5 (d) a .

vii. The total number of edges in a complete graph with n vertices is equal to **1**

(a) $n(n+1)/2$ (b) $2n-1$ (c) $n-1$ (d) $n(n-1)/2$.

viii. For any graph the sum of the degrees of all vertices is equal to **1**

(a) The number of edges (b) Twice the number of edges

(c) The number of vertices (d) Twice the number of vertices.

ix. For recursively defined function $u_{n+1} = \frac{u_n}{4} + 8$ with $u_0 = 32$ the value of u_2 will be **1**

(a) 12 (b) 14 (c) 16 (d) 18.

x. How many different arrangements can be made by taking five of the letters of the word EQUATION? **1**

(a) 6725 (b) 56 (c) 6720 (d) 50.

Q.2 Attempt any two:

i. Define relation and prove that if R is an equivalence relation then R^{-1} is also an equivalence relation in the set A. **5**

ii. For some sets A, B, C and D prove that **5**

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

$A - (B \cup C) = (A - B) \cap (A - C)$

iii. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% buy A and B, 3% buy B and C, 4% buy A and C. If 2% families buy all the three newspaper, then find the number of families **5**

(a) which buy A only (b) which buy none of these.

Q.3 Attempt any two:

i. In a Boolean algebra $(B, +, \cdot, ')$, prove that $(a+b)' = a'.b'$ **5**

ii. Obtain the simplified expressions in sum of product for the given Boolean function by using Karnaugh map. **5**

$F = \sum m(0,1,2,3,4,6,8,9,10,11,12,14)$

iii. Let N be the set of positive integers. Let the meaning of $x \le y$ in N *be "x divides y"*. Show that N is a lattice where the meet ($\wedge$) and join ($\vee$) are respectively defined by **5**

$x \wedge y = H.C.F.(x, y)$ and $x \vee y = L.C.M.(x, y)$.

Q.4 Attempt any two:

i. Show that the set of all rational numbers other than 1, forms an infinite abelian group with the operation '*' defined by the rule $a * b = a + b - ab$. **5**

ii. Prove that a necessary and sufficient condition for a non-empty subset H of a group G to be a subgroup is that $a \in H, b \in H \Rightarrow ab^{-1} \in H$.(where b^{-1} is the inverse of b in G) **5**

iii. Prove that the set of numbers of the form $a + b\sqrt{2}$, with a and b as rational numbers is a field with usual addition and multiplication operation. **5**

Q.5 Attempt any two:

i. (a) Differentiate Planar graph and Map. If G is a graph with 1000 vertices and 3000 edges how can you concluded about G is planar or not? **5**

(b) Define Tree with suitable example. Prove that in any tree with two or more vertices there are at least two pendant vertices.

ii. Prove that in graph the number of vertices of odd degree is always even. **5**

iii. Define: **5**

Regular graph, Binary tree, Adjacency matrix representation of graph, Euler graph, Chromatic number in graph colouring.

Q.6 Attempt any two:

i. (a) Write the generating function for the numeric function (1, -2, 3, -4, 5, ……). **5**

(b) Obtain the discrete numeric function corresponding to the generating function $A(z) = \frac{1}{1-9z^2}$

ii. Solve the recurrence relation $a_r - 2a_{r-1} = 7r$. **5**

iii. Solve $y_{h+2} - 7y_{h+1} + 10y_h = 0$ with $y_0 = 0, y_1 = 3$ by the method of generating function. **5**

******

(1)

Date: / / Page no:

Faculty of Engineering

End Sem Exam Dec - 2019

CS3BS03 / IT3BS06 Discrete Mathematics

Programme - B.Tech. Branch - CS-IT

Qu. 1 (10)

1) a. x^6+1

2) b. ~~$\left[\frac{(n+1)}{m}\right]+1$~~ $\left[\frac{(n-1)}{m}\right]+1$

3) c. $a+(a\cdot b)=a$

4) a. None, 12

5) d. All of these

6) c. a^1, a^5

7) d. $n(n-1)/2$

8) b. twice the number of edges

9) a. 12 $\{u_1=16, u_2=12\}$

10) c. 6720 $\{n=8, r=5, {}^nP_r=6720\}$

Qu. 2 (i) Relation - (2)

A statement which connects the elements of two different sets said to be relation.

A relation from set A to set B is a subset of $A\times B$ and symbolically denoted as $R\subseteq A\times B$.

$R=\{(x,y): xRy$ where $x\in A$ and $y\in B\}$

Given that R is an equivalence relation. To show R^{-1} is also an equivalence -

R^{-1} is reflexive - (1)

$\forall\ a\in A \Rightarrow (a,a)\in R$

$\because (a,a)\in R \Rightarrow (a,a)\in R^{-1}$ {by definition}

now $(a,a)\in R^{-1}$ $\forall\ a\in A$

Hence R^{-1} is reflexive.

R^{-1} is symmetric - $a, b \in A$ ①

$(a,b) \in R \Rightarrow (b,a) \in R^{-1}$ {by def.}

and $(a,b) \in R \Rightarrow (b,a) \in R \Rightarrow (a,b) \in R^{-1}$

$\therefore$ $(b,a) \in R^{-1} \Rightarrow (a,b) \in R^{-1}$

or $(a,b) \in R^{-1} \Rightarrow (b,a) \in R^{-1}$

Hence R^{-1} is symmetric.

R^{-1} is transitive - $a,b,c \in A$ ①

$\because$ $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \Rightarrow (c,a) \in R^{-1}$

$\therefore$ $(b,a) \in R^{-1}$ & $(c,b) \in R^{-1}$

or $(c,b) \in R^{-1}, (b,a) \in R^{-1} \Rightarrow (c,a) \in R^{-1}$

Hence R^{-1} is ~~sym~~ transitive.

It is proved that R^{-1} is also an equivalence relation.

(ii). a). $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

$\Leftrightarrow$ $(x,y) \in (A \times B) \cap (C \times D)$ ①

$\Leftrightarrow$ $(x,y) \in (A \times B)$ and $(x,y) \in (C \times D)$

$\Leftrightarrow$ $(x \in A$ and $y \in B)$ and $(x \in C$ and $y \in D)$

$\Leftrightarrow$ $(x \in A$ and $x \in C)$ and $(y \in B$ and $y \in D)$

$\Leftrightarrow$ $(x \in A \cap C)$ and $(y \in B \cap D)$

$\Leftrightarrow$ $(x,y) \in (A \cap C) \times (B \cap D)$ $1\frac{1}{2}$

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

b). $A - (B \cup C) = (A - B) \cap (A - C)$

$\Leftrightarrow$ $x \in A - (B \cup C)$ ①

$\Leftrightarrow$ $x \in A$ and $x \notin (B \cup C)$

$\Leftrightarrow$ $x \in A$ and $(x \notin B$ and $x \notin C)$

(2)

Date: / / Page no:

$\Leftrightarrow$ $(x \in A \text{ and } x \in A)$ and $(x \notin B \text{ and } x \notin C)$

$\Leftrightarrow$ $(x \in A \text{ and } x \notin B)$ and $(x \in A \text{ and } x \notin C)$

$\Leftrightarrow$ $x \in (A - B)$ and $x \in (A - C)$

$\Leftrightarrow$ $x \in (A - B) \cap (A - C)$ (1½)

$$A - (B \cup C) = (A - B) \cap (A - \cancel{B})$$

(iii) Given $|U| = 10{,}000$

$|A| = 40\% \text{ of } 10{,}000 = 4000$

$|B| = 20\%$ — " — $= 2000$

$|C| = 10\%$ — " — $= 1000$

$|A \cap B| = 5\%$ — " — $= 500$

$|B \cap C| = 3\%$ — " — $= 300$

$|A \cap C| = 4\%$ — " — $= 400$

$|A \cap B \cap C| = 2\%$ — " — $= 200$ (1)

which buy A only

$|A \cap B' \cap C'| = |A - B - C| = |A| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

$= 4000 - 500 - 400 + 200 = 3300$ (2)

Which buy none of these

$|A' \cap B' \cap C'| = |(A \cup B \cup C)'| = |U| - |A \cup B \cup C|$

$= |U| - \{|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|\}$

$= 10000 - 4000 - 2000 - 1000 + 500 + 300$

$+ 400 - 200 = 4000$ (2)

Qu. (3) (i). $(a+b)' = a'.b'$

For these we need to show

$(a+b)' + a'.b' = 1$ $\Rightarrow$ as $a + a' = 1$

$(a+b)' . a'.b' = 0$ $\quad a.a' = 0$ (1)

$$(a+b) + a'.b' = (a+b+a').(a+b+b')$$
$$= (a+a'+b).(a+b+b')$$
$$= (1+b).(a+1)$$
$$= (1.1) = 1 \quad 2$$

$$(a+b).a'.b' = (a.a'.b') + (b.a'.b')$$
$$= (a.a'.b') + (b.b'.a')$$
$$= (0.b') + (0.a')$$
$$= 0+0 = 0 \quad 2$$

Hence proved.

(ii). $\vec{F} = \Sigma m(0,1,2,3,4,6,8,9,10,11,12,14)$

$$= \bar{x}\bar{y}\bar{z}\bar{w} + \bar{x}\bar{y}\bar{z}w + \bar{x}\bar{y}z\bar{w} + \bar{x}y\bar{z}\bar{w} + \bar{x}yz\bar{w} +$$
$$x\bar{y}\bar{z}\bar{w} + \bar{x}\bar{y}zw + x\bar{y}\bar{z}w + x\bar{y}z\bar{w} +$$
$$x\bar{y}zw + xy\bar{z}\bar{w} + xyz\bar{w} \quad 2$$

zw \ xy	$\bar{x}\bar{y}$	$\bar{x}y$	xy	$x\bar{y}$
$\bar{z}\bar{w}$	1	1	1	1
$\bar{z}w$	1			1
zw	1			1
$z\bar{w}$	1	1	1	1

2

$F = \bar{y} + \bar{w}$ Simplified function. 1

(iii). To show N is a lattice —

\# Reflexivity — $x \in N \quad xRx$

$\because$ x divides x

$\therefore$ $x \leq x$

Relation is reflexive. 1

Anti symmetric $xRy, yRx \Rightarrow x = y$

$x, y, z \in N$

$x \le y,\ y \le x \Rightarrow c_1 = \frac{y}{x},\ c_2 = \frac{x}{y}$

$\Rightarrow c_1 \cdot c_2 = \frac{y}{x} \frac{x}{y} = 1$

$\Rightarrow c_1 = c_2$

$\Rightarrow \frac{y}{x} = \frac{x}{y}$

$\Rightarrow x = y$

Relation is anti symmetric. 1

Transitivity $xRy, yRz \Rightarrow xRz$

$x, y, z \in N$

$x \le y,\ y \le z \Rightarrow c_1 = \frac{y}{x},\ c_2 = \frac{z}{y}$.

$\Rightarrow c_1 \cdot c_2 = \frac{y}{x} \cdot \frac{z}{y}$

$\Rightarrow c_3 = \frac{z}{x}$

$\Rightarrow x \le z$ 1

Relation is transitive

Given $x \wedge y$ = HCF (x, y), $x \vee y$ = LCM (x, y)

Clearly, for every pair of positive integer elements their HCF & LCM are again positive integer quanties and belong in N.

i.e $x \wedge y \in N$ & $x \vee y \in N$

We can say that binary operation is closed. 2

Hence N is Lattice. 5

Date: / / Page no:

Qu. 4 (i) Given $Q = \{a : \text{ration nu} \neq 1\}$

Closure property $a, b \in Q$
where $a \neq 1$ & $b \neq 1$

To show $a*b \in Q$ use contradiction method

let $a*b = 1$ and $a*b \notin Q$

$a+b-ab=1 \Rightarrow a+b-ab-1=0$

$(a-1)(1-b)=0 \Rightarrow a=1$ & $b=1$

As our assumption becomes fail, it is clear that $a*b \neq 1 \in Q$ or $a*b \in Q$ 1

Associative property $a, b, c \in Q$

$a*(b*c) = a+b+c-ab-ac-bc+abc$

$(a*b)*c = a+b+c-ab-ac-bc+abc$

$\therefore a*(b*c) = (a*b)*c$ 1

Existence of identity

$a*e=a \Rightarrow a+e-ae=a$

$e(1-a)=0 \Rightarrow e=0$ & $a=1$

but $a \neq 1$ therefore $e=0 \in Q$

And $a*0 = a+0-0=a$

Hence 0 is identity element 1

Existence of inverse

$a*a^{-1}=0 \Rightarrow a+a^{-1}-aa^{-1}=0$

$a^{-1} = \frac{-a}{1-a}$ or $\frac{a}{a-1}$

but $a^{-1} \neq 1$ therefore $a^{-1} = \frac{a}{a-1} \in Q$

And $a*a^{-1} = a+\frac{a}{a-1} - \frac{a^2}{a-1} = 0$ 1

Hence for every element there exist inverse element.

4

Commutative property $a, b \in Q$

$a * b = a + b - ab$

$b * a = b + a - ba$

$\therefore a * b = b * a$ 1

Hence proved. Q forms an abelian group.

(ii) # Necessary condition –

Let H be a subgroup of group G.

We have to prove that $a \in H, b \in H \Rightarrow ab^{-1} \in H$

As H is a subgroup, by closure & inverse property

$a \in H, b \in H \Rightarrow a \in H, b^{-1} \in H \Rightarrow ab^{-1} \in H$ 2

Sufficient condition

Let $a \in H, b \in H \Rightarrow ab^{-1} \in H$

We have to prove that H is a subgroup.

For this – 1

Existence of identity –

$a \in H, b \in H \Rightarrow ab^{-1} \in H$

$a \in H, a \in H \Rightarrow aa^{-1} \in H \Rightarrow e \in H$

Existence of inverse

$a \in H, b \in H \Rightarrow ab^{-1} \in H$

$e \in H, a \in H \Rightarrow ea^{-1} \in H \Rightarrow a^{-1} \in H$

Closure property

$a \in H, b \in H \Rightarrow a \in H, b^{-1} \in H \Rightarrow a(b^{-1})^{-1} \in H \Rightarrow ab \in H$

Associative property

$a, b, c \in H \Rightarrow a, b, c \in G$ $\{\because H \subseteq G\}$

$\because a(bc) = (ab)c$ in G

$\therefore a(bc) = (ab)c$ also in H. 2

Hence H itself a group and a subgroup of G.

(iii) To show $R = \{a + b\sqrt{2} : a, b \in Q\}$ form a field following properties must be satisfied.

1. $(R, +)$ is abelian group
   - closure for (+) addition operation
   - Associative —"—
   - Identity —"—
   - Inverse —"—
   - commutative —"—

2

2. $(R, \cdot)$ is abelian group {for non zero elements}
   - closure for ($\cdot$) multiplication operation
   - Associative —"—
   - Identity —"—
   - Inverse —"—
   - commutative —"—

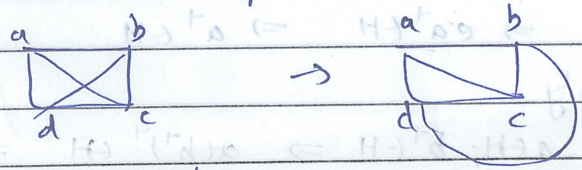
2

3. Distribution with respect to multiplication
   - left distribution
   - Right distribution

1

Ans 5 (i). a) Planar graph – If graph can be drawn on a paper without intersecting edges

Map – After embedding, obtained graph is called map



planar graph → map

1

Given $n = 1000$, $e = 3000$

$3n - 6 \geq e$ 1

$2994 \geq 3000$ 1

Hence graph is not planar graph.

b). Tree - A simple connected graph without any circuit is called tree  1

To prove the given statement -

$\because \sum d(v) = 2e$

$= 2(n-1) = 2n-2$ (Total number of degreees)

When we assign first degree to each vertex remaining degreees $= (2n-2)-n = n-2$

When we assign second degreee to each vertex remaining degrees $= (n-2)-n = -2$ 1

Clearly atleast two vertices not getting more than one degreee and they are pendant vertices.

(ii). Let $G=(V,E)$ be a graph

V_o = set of vertices of odd degreees 1

V_e = —"— even —"—

$V_o \cap V_e = \phi$

$$\sum d(v) = \sum_{V_e} d(V) + \sum_{V_o} d(V)$$

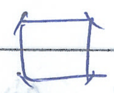
$$2e = 2K + \sum_{V_o} d(V)$$

$$\sum_{V_o} d(V) = 2(e-K) = \text{even quantity}$$ 2

To make the sum an even quantity, the number of terms in the sum must be even.

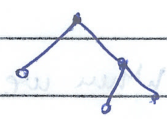
Hence the number of vertices in V_o is even, the number of vertices of odd degreee is always even 2 (iii). **Regular graph** — 1

A graph in which all vertices are of equal degrees.



Binary tree — 1

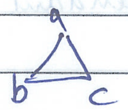
A rooted tree in which each vertex has atmost two children.



Adjacency matrix representation — 1

Let G be a graph with n vertices. Then the adjacency matrix of G is an $n \times n$ symmetric matrix $A = [a_{ij}]$ defined by

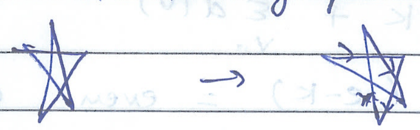
$$a_{ij} = \begin{cases} 1 & \text{edge between the vertices} \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

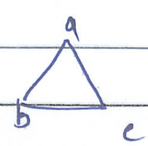
Euler graph — 1

A closed walk in a graph which includes all the edges of the graph, is called euler circuit & the graph is called an euler graph



Chromatic number — 1

Minimum number of colours required for proper coloring such that no two adjacent vertices have same color.



this is $\chi(G) = 3$

6

Date: / / Page no:

Qu. 6 (i). a). Generating function - 1

$A(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + \cdots$

$= 1 - 2z + 3z^2 - 4z^3 + 5z^4 - \cdots$

$= (1+z)^{-2}$ or $\frac{1}{(1+z)^2}$ 1

b). $A(z) = \frac{1}{1-9z^2}$

1

$A(z) = \frac{1}{(1-3z)(1+3z)} = \frac{A}{1-3z} + \frac{B}{1+3z} = \frac{1}{2(1-3z)} + \frac{1}{2(1+3z)}$

$= \frac{1}{2}(1-3z)^{-1} + \frac{1}{2}(1+3z)^{-1}$

$= \frac{1}{2}\left[\{1+3z+(3z)^2+(3z)^3+\cdots\} + \{1-3z+(3z)^2-(3z)^3+\cdots\}\right]$

$= \frac{1}{2}\left[2 + 2(3z)^2 + 2(3z)^4 + \cdots\right]$ 1

$= 1 + (3z)^2 + (3z)^4 + \cdots$

$a_r = \begin{cases} 0 & r = \text{odd} \\ 3^r & r = \text{even} \end{cases}$ 1

(ii). $a_r - 2a_{r-1} = 7r$

Total sol $= a^{(h)} + a^{(p)}$ — ①

Homogeneous sol -

$\frac{m^r - 2m^{r-1}}{m^{r-1}} = 0$

$m - 2 = 0 \Rightarrow m = 2$ 1

$a^{(h)} = C_1 m^r = C_1 2^r$ — ② 1

Particular sol -

Trial sol $a^{(t)} = A_0 + A_1 r$ 1

Date: / / Page no:

To obtain $A_0 = ?$, $A_1 = ?$

$$(A_0 + A_1 r) - 2[A_0 + A_1(r-1)] = 7r$$

$$(-A_0 + 2A_1) + (-A_1 r) = 7r$$

comparing both sides.

$-A = 7$, $-A_0 + 2A_1 = 0 \Rightarrow A_0 = -14$, $A_1 = -7$ 1

Now $a^{(P)} = -14 - 7r$ — (3)

By (1), (2), (3)

$$a_r = c_1 2^r - 14 - 7r$$ 1

(iii) $y_{n+2} - 7y_{n+1} + 10y_n = 0$ $y_0 = 0$, $y_1 = 3$

$Y(t) = y_0 t^0 + y_1 t^1 + y_2 t^2 + \cdots$

$$\frac{\sum y_{n+2} t^{n+2}}{t^2} - \frac{7\sum y_{n+1} t^{n+1}}{t} + 10\sum y_n t^n = 0$$ 1

$$\frac{(y_2 t^2 + y_3 t^3 + \cdots)}{t^2} - 7\frac{(y_1 t^1 + y_2 t^2 + \cdots)}{t} + 10(y_0 t^0 + y_1 t^1 + \cdots) = 0$$

$$\frac{Y(t) - y_0 t^0 - y_1 t^1}{t^2} - 7\frac{(Y(t) - y_0 t^0)}{t} + 10\,Y(t) = 0$$

$$\frac{Y(t) - 3t}{t^2} - 7\frac{Y(t)}{t} + 10\,Y(t) = 0$$

$$Y(t) = \frac{3t}{1 - 7t + 10t^2} = \frac{3t}{(1-2t)(1-5t)} = \frac{1}{1-5t} - \frac{1}{1-2t}$$ 2

$$\sum y_n t^n = (1-5t)^{-1} - (1-2t)^{-1}$$

$$= [1 + 5t + (5t)^2 + \cdots] - [1 + 2t + (2t)^2 + \cdots]$$

Equating coefficient of t^n

$$y_n = 5^n - 2^n.$$ 2

Note:- Alternate solutions, definitions are also correct