Q.6 Attempt any two:

i. Without expanding the determinant, prove that **5**

$$\begin{vmatrix} s & a^2 & b^2+c^2 \\ s & b^2 & c^2+a^2 \\ s & c^2 & a^2+b^2 \end{vmatrix} = 0$$

ii. Solve the following system of 2x2 equations by using Cramer's rule: $x + y = 5$ and $2x - 3y = -4$. **5**

iii. Find out the area of the triangle whose vertices are given by A(0,0) , B (3,1) and C (2,4) by using determinant method. **5**

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Enrollment No......................................

Knowledge is Power

Faculty of Science / Engineering

End Sem (Odd) Examination Dec-2022

CA3CO17 Mathematics -I

Programme: BCA, BCA-MCA(Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Which of the following two sets are equal? **1**

(a) A = {1, 2} and B = {1}
(b) A = {1, 2} and B = {1, 2, 3}
(c) A = {1, 2, 3} and B = {2, 1, 3}
(d) A = {1, 2, 4} and B = {1, 2, 3}

ii. If A, B and C are any three sets, then $A \times (B \cup C)$ is equal to- **1**

(a) $(A \times B) \cup (A \times C)$ (b) $(A \cup B) \times (A \cup C)$
(c) $(A \times B) \cap (A \times C)$ (d) None of these

iii. A relation R in a set A is called _______, if $(a_1, a_2) \in R$ implies $(a_2, a_1) \in R$, for all $a_1, a_2 \in A$. **1**

(a) Symmetric (b) Transitive (c) Equivalence (d) Non-symmetric

iv. A function is said to be ______ if and only if f(a) = f(b) implies that a = b for all a and b in the domain of f. **1**

(a) One-to-many (b) One-to-one
(c) Many-to-many (d) Many-to-one

v. What is the value of derivative of $(\sin x^3 \cos x^2)$? **1**

(a) $3x^2 \cos x^2 \cos x^3 + 2x \sin x^3 \sin x^2$
(b) $3x^2 \cos x^2 \cos x^3 - 2x \sin x^3 \sin x^2$
(c) $2x \cos x^2 \cos x^3 - 2x \sin x^3 \sin x^2$
(d) $2x \cos x^2 \cos x^3 + 3x^2 \sin x^3 \sin x^2$

vi. Find the derivative of e^{x^2}. **1**

(a) e^x (b) 2x (c) $2\,e^{x^2}$ (d) $2xe^{x^2}$

vii. Integration of $\cot^2 x\, dx$ equals to- **1**

(a) $\cot x - x + C$ (b) $-\cot x - x + C$
(c) $\cot x + x + C$ (d) $-\cot x + x + C$

P.T.O.

viii. If (d/dx) f(x) is g(x), then the antiderivative of g(x) is- **1**

(a) f(x) (b) f'(x) (c) g'(x) (d) None of these

ix. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$ then the value of x is- **1**

(a) 3 (b) ±3 (c) ±6 (d) 6

x. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is- **1**

(a) 4 (b) -4 (c) ±4 (d) 0

Q.2 i. If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following: **2**

A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C)

ii. Given three sets P, Q and R such that: **3**

P = {x: x is a natural number between 10 and 16},

Q = {y: y is a even number between 8 and 20} and

R = {7, 9, 11, 14, 18, 20}

(a) Find the difference of two sets P and Q

(b) Find Q - R

(c) Find R - P

iii. In a class of 200 students who appeared certain examinations, 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT entrance,15 in MHT-CET and IIT entrance and 5 failed in all three examinations. Find how many students. **5**

(a) Did not fail in any examination.

(b) Failed in AIEEE or IIT entrance

OR iv. Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, **5**

(a) How many students are in neither class?

(b) How many are in either class?

Q.3 i. Let 'R' be the relation from, **2**

A = {1, 2, 3, 4} to B = {x, y, z} defined by

R = {(1, y), (1, z), (3, y), (4, x), (4, z)}

Determine the domain and range of R.

ii. Let A = {1, 2, 3, 4, 6} and let R be the relation on A defined by "x divides y" written x/y. **8**

(a) Write R as a set of ordered pairs.

(b) Draw its directed graph.

(c) Find the inverse relation of R.

(d) Verify that R is Reflexive, Antisymmetric and Transitive

OR iii. If f, g : R → R, defined by f(x) = x + 1 and g(x) = x^2 , find **8**

(a) (fog) (x) (b) (gof) (x) (c) (fof) (x)

(d) (gog) (x) (e) (fog) (3) (f) (gof) (3)

(g) $f^{-1}(x)$ (h) $g^{-1}(x)$

Q.4 i. Find the derivative of the given function: **3**

$$R(z) = \frac{6}{\sqrt{z^3}} + \frac{1}{8z^4} - \frac{1}{3z^{10}}$$

ii. Find the first derivative of the given function: **7**

$$f(x) = \cos 3x^2 \cdot \sqrt[3]{5x^3 - 1}$$

OR iii. Find the first derivative of the given function: **7**

$$f(x) = \frac{4x^4 + 5}{\tan 3x^5}$$

Q.5 i. Evaluate the following: **4**

$$\int t^3 - \frac{e^{-t} - 4}{e^{-t}}\, dt.$$

ii. Evaluate the following: **6**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sec^2(w) - 8\csc(w)\cot(w)\, dw$$

OR iii. Evaluate the following: **6**

$$\int_0^4 f(t)\, dt \text{ where } f(t) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \le 1 \end{cases}$$

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Q1)

(I) (c) $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$

(II) (a) $(A \times B) \cup (A \times C)$

(III) (a) symmetric

(IV) (b) one-to-one

(V) (b) $3x^2 \cos x^2 \cdot \cos x^3 - 2x \sin x^3 \sin x^2$

(VI) (d) $2x \cdot e^{x^2}$

(VII) (b) $-\cot x - x + c$

(VIII) (d) ~~none of these~~ $f(x)$

(IX) (~~a~~) (c) $\cancel{x=3}$ ± 6

(X) (c) $2k^2 - 32 = 0$

$k^2 = 16$

$k = \pm 4$

Marks

Q2) I) Given

$A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$, $C = \{4,5,6,7,8\}$

and

$X = \{1,2,3,4,5,6,7,8,9,10\}$ then

LHS

$A \cup (B \cap C) = \{1,2,3,4\} \cup [\{3,4,5,6\} \cap \{4,5,6,7,8\}]$ +1

$= \{1,2,3,4\} \cup \{4,5,6\}$

$A \cup (B \cap C) = \{1,2,3,4,5,6\}$ —— (1)

RHS

$A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$

$A \cup B = \{1,2,3,4,5,6\}$ —— (2)

$A \cup C = \{1,2,3,4\} \cup \{4,5,6,7,8\}$ +1

$= \{1,2,3,4,5,6,7,8\}$ —— (3)

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\} \cap \{1,2,3,4,5,6,7,8\}$

$(A \cup B) \cap (A \cup C) = \{1,2,3,4,5,6\}$ —— (4)

from eqs (1) and (4)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ H.P.

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Marks.

32) Given that

II) $P = \{x : x \text{ is a natural no. b/w } 10 \,\&\, 16\}$

$P = \{11, 12, 13, 14, 15\}$ — (1)

$Q = \{y : y \text{ is an even no. between } 8 \,\&\, 20\}$

$Q = \{10, 12, 14, 16, 18\}$ — (2)

$R = \{7, 9, 11, 14, 18, 20\}$ — (3)

$P - Q = \{x \mid x \in P \text{ and } x \notin Q\}$

$P - Q = \{11, 13, 15\}$ Ans +1

$Q - R = \{x \mid x \in Q \text{ and } x \notin R\}$

$Q - R = \{10, 12, 16\}$ Ans +1

$R - P = \{x \mid x \in R \text{ and } x \notin P\}$

$R - P = \{7, 9, 18, 20\}$ Ans +1

Marks

Q.2 III) Let set

Total no. of students = 200

A = Students failed in MHT-CET = 35

B = Students failed in AIEEE = 40

C = Students failed in IIT = 40

$A \cap B$ = Students failed in MHT-CET and AIEEE = ~~17~~ 20

+1

$B \cap C$ = Students failed in AIEEE and IIT = 17

$A \cap C$ = Students failed in MHT-CET and IIT = 15

$A \cap B \cap C$ = Students failed in all three examination = 05

Now wkt

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$n(A \cup B \cup C) = 35 + 40 + 40 - 20 - 17 - 15 + 5$

$n(A \cup B \cup C) = 68$

= Total number of students failed in one or more examinations

+1

Marks.

now

(a) number of students not failed in any examination =

Total number of students − $n(A \cup B \cup C)$

$= 200 - 68$ +½

$= 132$ Ans

(b) number of students failed in AIEEE or IIT =

$n(B \cup C) = n(B) + n(C) - n(B \cap C)$ +2

$n(B \cup C) = 40 + 40 - 17$

$= 63$ Ans +½

Q2) IV) Total number of students = 40

Let

$n(A)$ = Number of students taking English-composition = 14

+1

$n(B)$ = Number of students taking chemistry = 29

$A \cap B$ = ~~Al~~ students taking both subjects = 05 +1

Marks

now wkt

No. of students taking either class

(b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 29 + 14 - 5$

$= 38$ Ans +$\frac{1}{2}$

(a) number of students taking neither subjects =

Total number of students − $n(A \cup B)$ +2

$= 40 - 38$

$= 02$ Ans +$\frac{1}{2}$

Marks

Q3) I Given that

$A = \{1, 2, 3, 4\}$, $B = \{x, y, z\}$

given relation is

$R = \{(1,y), (1,z), (3,y), (4,x), (4,z)\}$ +1

domain of R is the set of first element of ordered pair and range of R is the set of second elements of ordered pair ∴

domain of $R = \{1, 3, 4\}$ Ans

Range of $R = \{x, y, z\}$ Ans +1

Q3) II Given

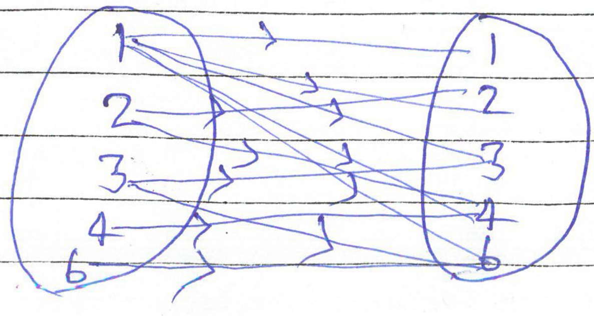
$A = \{1, 2, 4, 3, 6\}$

relation R on set A is defined as

$R = \{x \mid x \text{ divides } y \ \forall\ x, y \in A\}$

(a) $R = \{(1,2), (1,3), (1,4), (1,6), (1,1),$ +1
$(2,4), (2,6), (2,2), (3,3), (4,4), (3,6), (6,6)\}$ ~~+~~ +2

(b)



+2

Marks.

(c) $R^{-1} = \{ y \mid y$ ~~divi~~ is multiple of $x \ \forall x, y \in A \}$

$R^{-1} = \{ (1,1), (2,2), (3,3), (4,4), (6,6)$
$(2,4), (2,6), (3,6) \}$ +2

from relation R it is clear that

$(1,1), (2,2), (3,3), (4,4), (6,6) \in R$

$\therefore$ Relation R is Reflexive. +1

from Relation R it is clear that

If xRy and $yRz \Rightarrow xRz \ \forall x, y, z \in A$
$\therefore$ relation R is transitive +1

from relation R it is clear that
If $(a,b) \in R$ and $(b,a) \in R$

$\Rightarrow a = b$
because $(1,1), (2,2), (3,3) \cdots \in R$

$\therefore$ Relation R is antisymmetric. +1

Marks

Q3) III) Given $f, g : R \to R$ where

$f(x) = x+1$ and $g(x) = x^2 \quad \forall x \in R$

(a) $(f \circ g)(x) = f\{g(x)\}$

$= f\{x^2\}$ +1

$= x^2+1$ Ans

(b) $(g \circ f)(x) = g\{f(x)\}$

$= g\{x+1\}$ +1

$= (x+1)^2$ Ans

c) $(f \circ f)(x) = f\{f(x)\}$

$= f\{x+1\}$ +1

$= (x+1)+1$

$= x+2$ Ans

d) $(g \circ g)(x) = g\{g(x)\}$

$= g\{x^2\}$ +1

$= (x^2)^2$

$= x^4$ Ans

(e) $(f \circ g)(3) = f\{g(3)\}$

$= f\{3^2\}$

$= f(9)$ +1

$= 9+1$

$= 10$ Ans

Marks.

(f) $(g \circ f)(3) = g\{f(x)\}$

$= g(3+1)$ +1

$= g(4)$

$= 4^2$

$= 16$ Ans

g) $f^{-1}(x)$

$=$ Let $f(x) = y = x+1$

$\Rightarrow y-1 = x$

$\Rightarrow f^{-1}(x) = x-1$ Ans +1

(h) $g^{-1}(x)$

Let

$g(x) = x^2 = y$

$\Rightarrow x = \sqrt{y}$

$\Rightarrow g^{-1}(x) = \sqrt{x}$ Ans +1

Marks

Q4) I) Given that

$$R(Z) = \frac{6}{\sqrt{Z^3}} + \frac{1}{8Z^4} - \frac{1}{3Z^{10}}$$

$$R(Z) = \frac{6}{Z^{3/2}} + \frac{1}{8}Z^{-4} - \frac{1}{3}Z^{-10}$$

$$R(Z) = 6 \cdot Z^{-3/2} + \frac{1}{8}Z^{-4} - \frac{1}{3}Z^{-10}$$ +1

on differentiating w.r.t. Z

$$R'(Z) = \frac{d}{dZ}(6Z^{-3/2}) + \frac{d}{dZ}(\tfrac{1}{8}Z^{-4}) - \frac{1}{3}\frac{d}{dZ}(Z^{-10})$$

$$R'(Z) = 6 \cdot \frac{d}{dZ}(Z^{-3/2}) + \frac{1}{8}\frac{d}{dZ}(Z^{-4}) - \frac{1}{3}\frac{d}{dZ}(Z^{-10})$$ +1

$$R'(Z) = \overset{3}{\cancel{6}}\left(\frac{-3}{\underset{1}{\cancel{2}}}\right) Z^{-5/2} + \frac{1}{8}(-4)Z^{-5} - 3\cdot(-10)Z^{-11}$$

$$R'(Z) = \frac{-9}{Z^{5/2}} - \frac{1}{2Z^5} + \frac{30}{Z^{11}}$$ Ans +1

Marks.

Q4) II) Given

$$f(x) = \cos 3x^2 \cdot \sqrt[3]{5x^3-1}$$

$$f(x) = \underset{\text{I}}{\cos 3x^2} \cdot \underset{\text{II}}{(5x^3-1)^{\frac{1}{3}}}$$ +1

on diff. wrt. x

$$f'(x) = \cos 3x^2 \times \frac{1}{\cancel{3}}(5x^3-1)^{-\frac{2}{3}} \times \cancel{15}^{5}x^2$$

$$+ (5x^3-1)^{\frac{1}{3}}(-\sin 3x^2) \times 6x$$ +1

$$f'(x) = \frac{5x^2 \cdot \cos 3x^2}{(5x^3-1)^{2/3}} - 6x \cdot \sin 3x^2 (5x^3-1)^{\frac{1}{3}}$$

$$f'(x) = \frac{5x^2 \cdot \cos 3x^2 - 6x \cdot \sin 3x^2 (5x^3-1)}{(5x^3-1)^{2/3}}$$ +2

$$f'(x) = \frac{5x^2 \cdot \cos 3x^2 - (30x^4 - 6x)\sin 3x^2}{(5x^3-1)^{2/3}}$$

$$f'(x) = \frac{x\left[5x\cos 3x^2 - 30x^3\sin 3x^2 + 6\sin 3x^2\right]}{(5x^3-1)^{2/3}}$$ +3

Ans

Q4) III) Given that

$$f(x) = \frac{4x^4+5}{\tan 3x^5}$$

on diff. wrt. x

$$f'(x) = \frac{\tan 3x^5 \frac{d}{dx}(4x^4+5) - (4x^4+5)\frac{d}{dx}(\tan 3x^5)}{\tan^2 3x^5}$$ +1

Marks

$$f'(x) = \frac{\tan 3x^5 (16x^3) - (4x^4+5)\sec^2 3x^5 \times 15x^4}{\tan^2 3x^5}$$

$$f'(x) = \frac{16x^3 \tan 3x^5 - (60x^8 + 75x^4)\sec^2 3x^5}{\tan^2 3x^5} + 2$$

$$f'(x) = \frac{16x^3 \tan 3x^5 - x^4(60x^4+75)\sec^2 3x^5}{\tan^2 3x^5}$$

$$f'(x) = \frac{x^3\left[16\tan 3x^5 - (60x^4+75)x \cdot \sec^2 3x^5\right]}{\tan^2 3x^5} + 2$$

$$f'(x) = \frac{x^3\left[16\tan 3x^5 - 60x^5 \sec^2 3x^5 - 75x \cdot \sec^2 3x^5\right]}{\tan^2 3x^5} + 2$$

Ans

Marks

Q35) I) Let

$$I = \int \left[t^3 - \frac{(e^{-t} - 4)}{e^{-t}} \right] dt$$

$$I = \int [t^3 - (1 - 4e^t)] dt \qquad +1$$

$$I = \int (t^3 - 1 + 4e^t) dt \qquad +2$$

$$I = \frac{t^4}{4} - t + 4e^t + c \qquad \text{Ans} \qquad +1$$

Q35) II) Let

$$I = \int_{\pi/6}^{\pi/3} [2\sec^2 w - 8 \operatorname{cosec} w \cdot \cot w] dt$$

$$I = [2\tan w - (-8 \operatorname{cosec} w)]_{\pi/6}^{\pi/3} \qquad +1$$

$$I = [2\tan w + 8 \operatorname{cosec} w]_{\pi/6}^{\pi/3} \qquad +1$$

$$I = \left[2\tan\frac{\pi}{3} + 8\operatorname{cosec}\frac{\pi}{3} - \left(2\tan\frac{\pi}{6} + 8\operatorname{cosec}\frac{\pi}{6}\right)\right] \qquad +1$$

$$I = \left[2\sqrt{3} + 8 \times \frac{2}{\sqrt{3}} - \left(2 \times \frac{1}{\sqrt{3}} + 8 \times 2\right)\right] \qquad +2$$

$$I = 2\sqrt{3} + \frac{14}{\sqrt{3}} - 16$$

$$I = \frac{6 + 14 - 16\sqrt{3}}{\sqrt{3}} = \frac{20 - 16\sqrt{3}}{\sqrt{3}} \qquad \text{Ans}$$

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Marks.

Q5) III Let

$$I = \int_0^4 f(t)\,dt$$

and given function is

$$f(t) = \begin{cases} 2t & t > 1 \\ 1 - 3t^2 & t \le 1 \end{cases}$$

$$I = \int_0^1 f(t)\,dt + \int_1^4 f(t)\,dt$$ +1

$$I = \int_0^1 (1-3t^2)\,dt + \int_1^4 2t\,dt$$

$$I = \left[t - t^3\right]_0^1 + \left[t^2\right]_1^4$$ +2

$$I = \left[\cancel{1} - \cancel{1} - (0)\right] + \left[4^2 - 1^2\right]$$ +2

$$I = 0 + 16 - 1$$

$$I = 15$$ Ans +1

Marks

Q.6) I) Given

$$\begin{vmatrix} 5 & a^2 & b^2+c^2 \\ 5 & b^2 & c^2+a^2 \\ 5 & c^2 & a^2+b^2 \end{vmatrix}$$

taking common 5 from c_1

$$5\begin{vmatrix} 1 & a^2 & b^2+c^2 \\ 1 & b^2 & c^2+a^2 \\ 1 & c^2 & a^2+b^2 \end{vmatrix}$$ +1

$c_3 \to c_2 + c_3$

$$5\begin{vmatrix} 1 & a^2 & a^2+b^2+c^2 \\ 1 & b^2 & a^2+b^2+c^2 \\ 1 & c^2 & a^2+b^2+c^2 \end{vmatrix}$$ +1

taking common $(a^2+b^2+c^2)$ from c_3 +1

$$5(a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & 1 \\ 1 & b^2 & 1 \\ 1 & c^2 & 1 \end{vmatrix}$$ +1

$5(a^2+b^2+c^2) \times 0$

$= 0$ H.P.

because by the property of determinant $c_1 \equiv c_3$ ∴ it's value is 0 +1

	Marks.
36) II) Given eqs are	
$x+y=5 \quad -①$	
$2x-3y=-4 \quad -②$	
on converting in to determinant	
$D = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3-2 = -5$	+1
$D_1 = \begin{vmatrix} 5 & 1 \\ -4 & -3 \end{vmatrix} = -15+4 = -11$	+1
$D_2 = \begin{vmatrix} 1 & 5 \\ 2 & -4 \end{vmatrix} = -4-10 = -14$	+1
Now by cramer's rule	
$x = \frac{D_1}{D} = \frac{-11}{-5} = \frac{11}{5}$ Ans	+1
$y = \frac{D_2}{D} = \frac{-14}{-5} = \frac{14}{5}$ Ans	+1
36) III) Given vertices of triangle are A(0,0), B(3,1) and C(2,4)	
now area of triangle will be	
$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3 & 1 & 1 \\ 2 & 4 & 1 \end{vmatrix}$ then	+1
$C_2 \longleftrightarrow C_3$ and $C_1 \longleftrightarrow C_2$	

Marks

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 1 & 3 & 1 \\ 1 & 2 & 4 \end{vmatrix}$$ +1

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$ +2

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 2 & 4 \end{vmatrix}$$

on solving from R_1

$\Delta = \frac{1}{2}(12-2) = \frac{1}{2} \times 10$ +1

$\Delta = 5$ sq. unit

can be solve by direct expansion also.