Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering
End Sem (Odd) Examination Dec-2019
EN3BS01 Engineering Mathematics-I

Programme: B .Tech. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated.Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Rank of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is: **1**

(a) 0 (b) 1 (c) 2 (d) None of these

ii. Characteristic values of the matrix $A=\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ is **1**

(a) 2 and 3 (b) 1 and 2 (c) 3 and 4 (d) None of these

iii. If $y=x^5$, then y_5 is : **1**

(a) $\underline{|5}$ (b) 20 (c) $\underline{|4}$ (d) 15

iv. If $u=\tan^{-1}\left(\frac{y}{x}\right)$, then the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is : **1**

(a) 0 (b) u (c) 2u (d) $\frac{y}{x}$

v. The value $\frac{\overline{|z+1}}{\overline{|z}}$ is equal to, where $z \geq 1$: **1**

(a) $\overline{|z-1}$ (b) $\overline{|z}$ (c) z (d) None of these

vi. Value of $\int_0^1\int_0^2 (x+y)\,dx\,dy$ is **1**

(a) $\frac{5}{2}$ (b) $\frac{7}{2}$ (c) 3 (d) -1

vii. Integrating factor of linear differential equation $\frac{dx}{dy}+p(y)\,x=Q(y)$ is: **1**

(a) $e^{\int P\,dx}$ (b) $e^{-\int P\,dx}$ (c) $e^{\int P\,dy}$ (d) $e^{-\int P\,dy}$

P.T.O.

viii. General solution of differential equation $\frac{d^2y}{dx^2} - y = 0$ is : **1**

(a) $y = e^{x}$ (b) $y = e^{-x}$

(c) $y = ae^{x} + be^{-x}$ (d) $y = ae^{2x} + be^{-2x}$

ix. If 1+P+Q=0, then which of the following will be a part of complementary function of $D^2y + PDy + Qy = R$? Where D= $\frac{d}{dx}$. **1**

(a) $y = x$ (b) $y = \frac{1}{x}$ (c) $y = e^{-x}$ (d) $y = e^{x}$

x. The complementary function of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^{x}$ is: **1**

(a) $c_1x + c_2x^{-1}$ (b) $c_1e^{x} + c_2e^{-x}$

(c) $c_1e^{x^2} + c_2e^{-x^2}$ (d) None of these

Q.2 Attempt any two:

i. Find the normal form of the matrix A and hence find its rank, where **5**

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

ii. Prove that the equations: **5**

$3x + 3y + 2z = 1,$

$x + 2y = 4,$

$10y + 3z = -2,$

$2x - 3y - z = 5$

are consistent and hence find the solution.

iii. Using Cayley Hamilton theorem, find the inverse of A, where **5**

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

Q.3 Attempt any two:

i. Using Maclaurin's series prove that: **5**

$\log(1 + e^{x}) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192}.$

ii. If $u = f(y - z,\ z - x,\ x - y)$, Prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$ **5**

iii. Discuss the maxima and minima of the function: **5**

$x^2y^2 - 5x^2 - 5y^2 - 8xy.$

Q4. Attempt any two:

i. Evaluate : $\int_0^\infty \sqrt{x}\ e^{-3\sqrt{x}}\ dx$. **5**

ii. Evaluate $\iint_R xy(x + y)\,dxdy$ over the region R bounded by the curve $y = x^2$ and $y = x$. **5**

iii. Evaluate $\iiint_R (x - 2y + z)\,dx\,dy\,dz$, where R is the region determined by $0 \le x \le 1,\ \ 0 \le y \le x^2,\ \ 0 \le z \le x + y.$ **5**

Q.5 Attempt any two:

i. Solve the differential equation $\text{Sec}x\frac{dy}{dx} = y + \text{Sin}x$ **5**

ii. Solve the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\,\text{Cos}^2x$ **5**

iii. Solve the differential equation $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \log x$ **5**

Q.6 Attempt any two:

i. Solve the differential equation $\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 5y = \text{Sec}x\ e^{x}$ **5**

ii. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ **5**

iii. Solve in series of the following equation $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ **5**

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Scheme / Solution

Q1 i.) (b) 1 +1
ii.) (a) 2 and 3 +1
iii) (a) 15 +1
iv.) (a) 0 +1
(v) (c) z +1
(vi) (c) 3 +1
(vii) (c) $e^{\int P dy}$ +1
(viii) (c) $y = ae^{x} + be^{-x}$ +1
(ix) (d) $y = e^{x}$ +1
(x) (a) $c_1 x + c_2 x^{-1}$ +1

Q2.(i) Applying the following operations stepwise,
1) $R_1 \leftrightarrow R_2$
2) $C_2 \to C_2 + C_1$; $C_3 \to C_3 + 2C_1$; $C_4 \to C_4 + 4C_1$
3) $R_4 \to R_4 - R_3$
4) $R_4 \to R_4 - R_2$ (+2)
5) $R_2 \to R_2 - R_3$
6) $R_3 \to R_3 - 4R_2$
7) $C_3 \to C_3 + 6C_2$; $C_4 \to C_4 + 3C_2$
8.) $C_3 \to (1/33) C_3$ & $C_4 \to (1/22) C_4$
9.) $C_4 \to C_4 - C_3$, we get the normal form (+2)

as $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ or $\begin{bmatrix} I_3 & O \\ O & O \end{bmatrix}$

$\therefore \rho(A) = 3$ Ans. (+1)

2.1

Q.2(ii) Write the system in the form AX = B and find the augmented matrix

$$[A:B] = \left[\begin{array}{ccc:c} 3 & 3 & 2 & 1 \\ 1 & 2 & 0 & 4 \\ 0 & 10 & 3 & -2 \\ 2 & -3 & -1 & 5 \end{array}\right]$$ (+1)

Applying operations stepwise to obtain Echelon form

1) $R \leftrightarrow R_2$
2) $R_2 \to R_2 - 3R_1$, $R_4 \to R_4 - 2R_1$
3) $R_3 \to 3R_3 + 10R_2$, $R_4 \to 3R_4 - 7R_2$
4) $R_4 \to 29R_4 + 17R_3$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 0 & 4 \\ 0 & -3 & 2 & -11 \\ 0 & 0 & 29 & -116 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ (+1.5)

Thus, $r(A:B) = 3$ and $r(A) = 3$

Since, $r(A:B) = r(A) = 3$ (= no. of variables)

$\therefore$ System is consistent & it has a unique solution (+1)

Rewrite system as:

$$x + 2y = 4$$
$$-3y + 2z = -11$$
$$29z = -116$$ (+1)

using back substitution, we get

$$z = -4,\ y = 1,\ x = 2$$

Ans $\boxed{x = 2,\ y = 1,\ z = -4}$ (+0.5)

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Q.2 (iii) The characteristic eqⁿ is

$|A - \lambda I| = 0$ (+1)

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 3-\lambda & -3 \\ -2 & -4 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 20\lambda + 8 = 0$ — eqⁿ(1) (+1)

By CHT, we have (putting $\lambda = A$)

$A^3 - 20A + 8 = 0$

mult by A^{-1},

$A^2 - 20I + 8A^{-1} = 0$

$A^{-1} = \frac{1}{8}[-A^2 + 20I]$ — eqⁿ(2) (+1)

$$A^2 = \begin{bmatrix} -4 & -8 & -12 \\ 10 & 22 & 6 \\ 2 & 2 & 22 \end{bmatrix}$$ put in eqⁿ(2) (+1)

Then, $$A^{-1} = \frac{1}{8}\begin{bmatrix} 24 & 8 & 12 \\ -10 & -2 & -6 \\ -2 & -2 & -2 \end{bmatrix}$$ (+1)

Ans.

Q.3 (i) Given

$f(x) = \log(1+e^x)$ so $f(0) = \log 2$

$f'(x) = \frac{e^x}{1+e^x}$; $f'(0) = \frac{1}{2}$

$f''(x) = \frac{e^x}{(1+e^x)^2}$; $f''(0) = \frac{1}{4}$

$f'''(x) = -\frac{e^x(e^x-1)}{(1+e^x)^3}$; $f'''(0) = 0$

$f^{IV}(x) = -\frac{(1+e^x)(2e^{2x}-e^x) - (e^{2x}-e^x).3e^x}{(1+e^x)^4}$; (+3)

$f^{IV}(0) = -\frac{1}{8}$

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w.k.t, Maclaurin's series is

$$f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \cdots \quad (+1)$$

putting all the required values,

$$\log(1+e^x) = \log 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots \quad (+1)$$

Hence proved

Q.3 (ii) Given $u = f(y-z, z-x, x-y)$

~~where~~ let $X = y-z$,

$Y = z-x$

$Z = x-y$

Since u is a composite function (+1)

i.e. $u = f(X, Y, Z)$

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial u}{\partial X} \cdot \frac{\partial X}{\partial x} + \frac{\partial u}{\partial Y} \cdot \frac{\partial Y}{\partial x} + \frac{\partial u}{\partial Z} \cdot \frac{\partial Z}{\partial x}$$

$$= \frac{\partial u}{\partial X}(0) + \frac{\partial u}{\partial Y} \cdot (-1) + \frac{\partial u}{\partial Z}(1)$$

$$= -\frac{\partial u}{\partial Y} + \frac{\partial u}{\partial Z} \quad \text{— eq}^n(1) \quad (+1)$$

Similarly,

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial X} - \frac{\partial u}{\partial Z} \quad \text{— eq}^n(2) \quad (+1)$$

$$\& \quad \frac{\partial u}{\partial z} = -\frac{\partial u}{\partial X} + \frac{\partial u}{\partial Y} \quad \text{— eq}^n(3) \quad (+1)$$

Adding eq^n (1), (2) and (3)

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0 \quad (+1)$$

Hence proved

Q.3(iii) $u = x^2y^2 - 5x^2 - 5y^2 - 8xy$

For maxima & minima,

$$\frac{\partial u}{\partial x} = 2xy^2 - 10x - 8y = 0 \quad eq^n (1)$$

$$\frac{\partial u}{\partial y} = 2x^2y - 8x - 10y = 0 \quad eq^n (2)$$

subtracting eq^n (2) from (1)

$$2xy(y-x) + 2(y-x) = 0$$

$$\Rightarrow (y-x)(xy+1) = 0$$

$$\Rightarrow y = x \quad \text{or} \quad y = -\frac{1}{x}$$ (+1)

when $y = x$ then from eq^n(1),

$$2x^3 - 18x = 0$$

$$\Rightarrow x = 0, \pm 3$$ (+1)

& when $y = -\frac{1}{x}$ then from eq^n(1)

$$\frac{2}{x} - 10x + \frac{8}{x} = 0$$

$$\Rightarrow 1 - x^2 = 0$$

$$\Rightarrow x = \pm 1$$ (+1)

Hence the solutions are

$x = y = 0$; $x = y = 3$; $x = y = -3$;

$x = 1, y = -1$; $x = -1, y = 1$

Thus, critical points are

$(0,0), (3,3), (-3,-3), (1,-1)$ & $(-1,1)$

Now,

$$r = \frac{\partial^2 u}{\partial x^2} = 2y^2 - 10, \quad s = \frac{\partial^2 u}{\partial x \partial y} = 4xy - 8$$

$$\& \quad t = \frac{\partial^2 u}{\partial y^2} = 2x^2 - 10$$ (+0.5)

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At $x = y = 0$;
$r = -10$, $s = -8$, $t = -10$
so $rt - s^2 = 100 - 64 = 36 > 0$
& $r < 0$
$\therefore$ we have maximum at $x = y = 0$ (+0.5)

At $x = y = \pm 3$, we have
$r = 8$, $s = 28$, $t = 8$
$\therefore rt - s^2 = 64 - (28 \times 28)$ = a negative quantity
& $r > 0$
$\therefore$ we have neither max^m nor min^m at $x = y = \pm 3$ (+0.5)

At $x = \pm 1$ & $y = \mp 1$,
we have,
$r = -8$, $s = -12$, $t = -8$
we have $rt - s^2 = 64 - 144$ = a -ve quantity
& $r < 0$
Hence in this case we have neither max^m nor min^m. (+0.5)

Q.4 (i) $I = \int_0^\infty \sqrt{x}\, e^{-3\sqrt{x}}\, dx$

~~let~~ $\cancel{I} = \cancel{\int_0^\infty}$

put $3\sqrt{x} = y \Rightarrow 9x = y^2$
$\Rightarrow dx = \frac{2}{9} y\, dy$ we have, (+1)

$I = \int_0^3 \frac{1}{3} y\, e^{-y} \cdot \frac{2}{9} y\, dy$

$$I = \frac{2}{27}\int_0^\infty e^{-y} y^{3-1}\,dy \quad (+1)$$

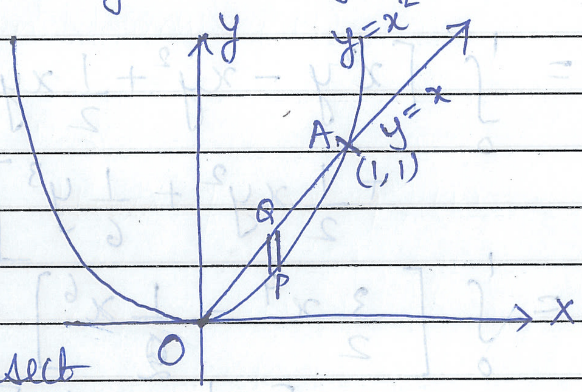
$$\boxed{\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx} \quad (+1)$$

$$= \frac{2\,\Gamma 3}{27}$$

$$= \frac{2}{27} \times 2 \times 1$$

$$= \frac{4}{27} \quad \text{Ans.} \quad (+2)$$

Q.4 (ii) Given curves are : $y = x^2$ & $y = x$



At, pt. of intersection, we have

$x = x^2$

$\Rightarrow x(1-x) = 0$

$\Rightarrow x = 0, 1.$

Thus, curves intersect at O(0,0) & A(1,1).

Thus the region R is the area OPAQO & the limits can be found by the strip PQ. (+2)

Then,

$$\iint_R xy(x+y)\,dx\,dy = \int_{x=0}^{1}\int_{y=x^2}^{x} (x^2y + xy^2)\,dx\,dy \quad (+1)$$

$$= \int_0^1 \left[\frac{x^2y^2}{2} + \frac{xy^3}{3}\right]_{x^2}^{x} dx$$

$$= \int_0^1 \left[\left(\frac{x^4}{2} + \frac{x^4}{3}\right) - \left(\frac{x^6}{2} + \frac{x^7}{3}\right)\right] dx$$

$$= \left[\frac{x^5}{10} + \frac{x^5}{15} - \frac{x^7}{14} - \frac{x^8}{24}\right] = \frac{1}{10} + \frac{1}{15} - \frac{1}{14} - \frac{1}{24} = \boxed{\frac{3}{56} \text{ Ans}} \quad (+2)$$

Q.4 (iii)

Let $I = \iiint_R (x-2y+z)\,dx\,dy\,dz$

$$= \int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{x+y} (x-2y+z)\,dx\,dy\,dz \quad \text{— } (+1)$$

$$= \int_0^1\int_0^{x^2} \left[(x-2y)z + \frac{z^2}{2}\right]_0^{x+y} dy\,dx$$

$$= \int_0^1\int_0^{x^2} \left[(x-2y)(x+y) + \frac{1}{2}(x+y)^2\right] dy\,dx \quad \text{— } (+1.5)$$

$$= \int_0^1 \left[x^2y - xy^2 + \frac{1}{2}xy^2 - \frac{2}{3}y^3 + \frac{1}{2}x^2y + \frac{1}{2}xy^2 + \frac{1}{6}y^3\right]_0^{x^2} dx$$

$$= \int_0^1 \left[\frac{3}{2}x^4 - \frac{1}{2}x^6\right] dx \quad \text{— } (+1.5)$$

$$\cancel{= \frac{3}{2}\left[\frac{x^5}{5}\right]_0^1 - \frac{1}{6}\left[\frac{x^7}{7}\right]_0^1} = \frac{8}{35}$$

$$\cancel{= \frac{3}{10} - \frac{1}{42} = \frac{29}{105}} \quad \text{Ans} \quad (+1)$$

Q.5 (i) Given eq^n can be written as

$$\frac{dy}{dx} - (\cos x)\cdot y = \sin x \cdot \cos x$$

which is LDE where

$P = -\cos x, \quad Q = \sin x \cos x$

$I.F = e^{\int P dx} = e^{\int -\cos x\, dx} = e^{-\sin x}$ (+1)

Hence sol^n is

$$y(IF) = \int Q\cdot(IF)\,dx + c \quad (+1)$$

i.e; $y \cdot e^{-\sin x} = \int e^{-\sin x} \cdot \sin x \cos x \, dx + C$

⇒ putting $\sin x = t$ in RHS
$\Rightarrow \cos x \, dx = dt$

$\therefore y \cdot e^{-\sin x} = \int t e^{-t} dt + C$

$= t e^{-t}(-1) - \int e^{-t} \cdot (-1) dt + C$
$= -t e^{-t} - e^{-t} + C$ (+2)

$\therefore y \cdot e^{-\sin x} = (-\sin x - 1) e^{-\sin x} + C$

$\Rightarrow y = -(\sin x + 1) + C e^{\sin x}$ (+1)

Ans

Q.5 (ii) Write the given eq^n as
$(D^2 + 3D + 2) y = 4\cos^2 x$
For C.F,
the auxillary eq^n is
$m^2 + 3m + 2 = 0$
$\Rightarrow m = -1, -2$
$\therefore CF = c_1 e^{-x} + c_2 e^{-2x}$ (+2)

For P.I. $= \dfrac{1}{D^2+3D+2} (4\cos^2 x)$

$= 2 \cdot \dfrac{1}{D^2+3D+2} [1 + \cos 2x]$

$= 2 \left[\dfrac{1}{D^2+3D+2} e^{0x} + \dfrac{1}{D^2+3D+2} \cos 2x \right]$

put $D = 0$ & put $D^2 = -4$

$= 2 \left[\dfrac{1}{2} + \dfrac{1}{3D-2} \cos 2x \right]$

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$$= 1 + 2\left(\frac{1}{3D-2}\cos 2x\right)$$

$$= 1 + 2\left[\frac{3D+2}{9D^2-4}\cos 2x\right]$$

put $D^2 = -4$

$$= 1 + 2\left[\frac{(3D+2)}{-40}\cos 2x\right]$$

$$= 1 - \frac{2}{40}\left[3D\cos 2x + 2\cos 2x\right]$$

$$= 1 - \frac{2}{40}\left[-6\sin 2x + 2\cos 2x\right]$$

$$= 1 + \frac{1}{10}\left[3\sin 2x - \cos 2x\right]$$ (+3)

Thus the complete solⁿ is

$y = CF + PI$

Q.5 (iii) put $x = e^z$

$\Rightarrow z = \log x$

and $x\frac{dy}{dx} = Dy$ & $x^2\frac{d^2y}{dx^2} = D(D-1)y$

where $D = \frac{d}{dz}$

Hence eqⁿ becomes

$D(D-1)y + 2Dy = z$ (+1)

$\Rightarrow (D^2 + D)y = z$

$\therefore$ For CF, $m^2 + m = 0$

$m = 0, -1$

$CF = c_1 e^{0z} + c_2 e^{-z}$

$= c_1 + c_2 x^{-1}$

$= c_1 + \frac{c_2}{x}$ (+2)

For P.I.,

$$PI = \frac{1}{D^2+D} z = \frac{1}{D}[1+D]^{-1} z$$

$$= \frac{1}{D}[1 - D + D^2 + \cdots] z$$

$$= \frac{1}{D}[z-1]$$

$$= \frac{z^2}{2} - z$$

$$= \frac{(\log x)^2}{2} - \log x$$ (+2)

Thus complete solⁿ is $y = CF + PI$

Q.6 (i) Comparing with std. form,

$P = -2\tan x$, $Q = 5$, $R = \sec x \cdot e^x$

we choose,

$$u = e^{-\frac{1}{2}\int P dx} = e^{\int \tan x\, dx} = \sec x$$ (+1)

Let the general solⁿ be $y = uv$

Then v is given by normal eqⁿ

$$\frac{d^2v}{dx^2} + Iv = \frac{R}{u}$$ (+1)

where, $I = Q - \frac{1}{2}\left(\frac{dP}{dx}\right) - \frac{1}{4}P^2$

$$= 5 - \tan^2 x + \sec^2 x$$

$$= 6$$ (+1)

Hence eqⁿ becomes :

$$\frac{d^2v}{dx^2} + 6v = e^x \cdot \sec x \cdot \left(\frac{1}{\sec x}\right)$$

$$\Rightarrow \frac{d^2v}{dx^2} + 6v = e^x$$

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$\Rightarrow (D^2+6)v = e^x$

A.E is $m^2+6=0$
$m = \pm\sqrt{6}\,i$

$CF = c_1\cos\sqrt{6}x + c_2\sin\sqrt{6}x$ (+1)

and,

$PI = \frac{1}{D^2+6}e^x = \frac{1}{1+6}e^x = \frac{1}{7}e^x$

$\therefore v = CF + PI$
$= c_1\cos\sqrt{6}x + c_2\sin\sqrt{6}x + \frac{1}{7}e^x$ (+1)

Thus the complete solⁿ is
$y = (c_1\cos\sqrt{6}x + c_2\sin\sqrt{6}x + \frac{1}{7}e^x)\cdot \boxed{\sec x}$

Q.6(iii) Let the complete solⁿ of given eqⁿ is $y = Au + Bv$ (+0.5)
where u & v are the two parts of CF & A & B are determined as:-

$A = -\int \frac{Rv}{uv_1 - u_1v}dx + c_1$

& $B = \int \frac{Ru}{uv_1 - u_1v}dx + c_2$ (+1)

Now A.E is $m^2+4=0 \Rightarrow m = \pm 2i$
Thus $\cos 2x$ & $\sin 2x$ are two parts of CF.

Let $u = \cos 2x$ & $v = \sin 2x$
$\Rightarrow u_1 = -2\sin 2x$ & $v_1 = 2\cos 2x$ (+0.5)

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Comparing with std. form $R = 4\tan 2x$

Now $A = -\int \frac{4\tan 2x \cdot \sin 2x}{2\cos^2 2x + 2\sin^2 2x} dx + c_1$

$= -2\int \frac{\sin^2 2x}{\cos 2x} dx + c_1$

$= -2\int \frac{(1-\cos^2 2x)}{\cos 2x} dx + c_1$

$= -2\int (\sec 2x - \cos 2x) dx + c_1$

$= -2\int \sec 2x\, dx + 2\int \cos 2x\, dx + c_1$

$A = -\log(\sec 2x + \tan 2x) + \sin 2x + c_1$ (+1)

Similarly,

$B = +\int \frac{4\tan 2x \cdot \cos 2x}{2\cos^2 2x + 2\sin^2 2x} dx + c_2$

$= 2\int \sin 2x \; dx + c_2$

$= -\frac{2\cos 2x}{2} + c_2$

$= -\cos 2x + c_2$ (+1)

Thus, complete solⁿ is

$y = Au + Bv$

i.e. $y = c_1 \cos 2x + c_2 \sin 2x$
$- [\log(\sec 2x + \tan 2x)] \cdot \cos 2x$ (+0.5)

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Q.6 (iii) Given eq^n is

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0 \quad \text{— eq}^n(1)$$

Here $P_0(x) = 1 + x^2$

at $x=0$, $P_0(0) = 1 \neq 0$

Hence $x=0$ is an ordinary pt. (+0.5)

Let its series sol^n be

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_kx^k + \cdots \quad \text{eq}^n(2)$$

then $y' = a_1 + 2a_2x + \cdots + a_k \cdot k \cdot x^{k-1} + \cdots$

&

$y'' = 2a_2 + 6a_3x + \cdots + k(k-1)a_kx^{k-2} + \cdots$ (+1)

put values of y, y' & y'' in (1), we have

$$(1+x^2)\left[2a_2 + 6a_3x + \cdots + k(k-1)a_kx^{k-2} + \cdots\right]$$
$$+ x\left[a_1 + 2a_2x + \cdots + ka_kx^{k-1} + \cdots\right]$$
$$- \left[a_0 + a_1x + a_2x^2 + \cdots + a_kx^k + \cdots\right] = 0 \quad (+1)$$

Equating to zero, the coeff. of x^0, x^1, x^2 & so on x^k etc.

$2a_2 - a_0 = 0 \Rightarrow a_2 = \frac{a_0}{2}$

$6a_3 + a_1 - a_1 = 0 \Rightarrow a_3 = 0$

Similarly, for x^k

$(k+2)(k+1)a_{k+2} + k(k-1)a_k + ka_k - a_k = 0$

or $a_{k+2} = \frac{-(k^2+1)}{(k+2)(k+1)}a_k = -\frac{(k-1)a_k}{(k+2)}$

put $k = 2, 3, 4$ etc.

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$$a_4 = -\frac{1}{4} a_2 = -\frac{a_0}{8}$$

$$a_5 = -\frac{2}{5} a_3 = 0$$

$$a_6 = -\frac{1}{2} a_4 = \frac{a_0}{16} \quad \& \text{ so on}$$ (+1.5)

put all these values in eqn (2)
the series soln is.

$$y = a_0 + a_1 x + \left(\frac{a_0}{2}\right)x^2 + \left(\frac{-a_0}{8}\right)x^4 + \left(\frac{a_0}{16}\right)x^6 + \cdots$$

or

$$y = a_0\left(1 + \frac{x^2}{2} - \frac{x^4}{8} + \frac{x^6}{16} - \cdots\right) + a_1 x$$ (+1)

Ans.

—x—x—x—