Total No. of Questions: 25

MU/1332

Enrollment No : ______________________

MEDICAPS UNIVERSITY

Faculty of Engineering / Science

End Semester Examination May 2025

EN3BS11 / BC3BS01 Engineering Mathematics -I

Programme	:	B.Tech. / B. Sc.	**Branch/Specialisation**	:	All
Duration	:	3 hours	**Maximum Marks**	:	60

Note: *All questions are compulsory. Internal choices, if any, are indicated. Assume suitable data if necessary. Notations and symbols have their usual meaning.*

Section 1 (Answer all question(s))

Marks CO BL

Q1. Which of the following statement is true for a rectangular matrix A of order $m \times n$? **1 1 2**

(A) Rank of $A \leq min(m,n)$

(B) Rank of $A \geq min(m,n)$

(C) Rank of $A \leq max(m,n)$

(D) None of these

Q2. Which of the following condition is correct when the given system $[A:B]$ is consistent and it has unique solution? **1 1 2**

(A) $\rho(A:B) = \rho(A) = r <$ number of variable

(B) $\rho(A:B) = \rho(A) = r =$ number of variable

(C) $\rho(A:B) \neq \rho(A)$

(D) None of these

Q3. If the rolle's theorem for the function $f(x) = x^2 - 5x + 6$ is verified on [2,3] then the value of c is- **1 2 3**

(A) 2

(B) 2.75

(C) 2.5

(D) None of these

Q4. If $u = 2xy + 2y$ then the partial derivative of u with respect to x is- **1 2 3**

(A) $2y$

(B) $2x$

(C) $2x + 2y$

(D) None of these

Q5. The sum of the series $1 + 2 + 3 + - - - - - - + n$ is- **1 1 1**

(A) $n(\frac{n+1}{2})$

(B) $n(\frac{n-1}{2})$

(C) $(\frac{n+1}{2})$

(D) None of these

Q6. Which of the statement is correct for beta gamma relation (where $m > 0, n > 0$)? **1 1 1**

(A) $B(m,n) = \dfrac{\Gamma(m+n)}{\Gamma m \Gamma n}$

(B) $B(m,n) = \dfrac{\Gamma m \Gamma n}{\Gamma(m-n)}$

(C) $B(m,n) = \dfrac{\Gamma m \Gamma n}{\Gamma(m+n)}$

(D) None of these

Q7.	The necessary and sufficient condition that the ordinary differential equation $M(x,y)dx + N(x,y)dy = 0$ be exact is- (A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (C) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ (D) None of these	**1**	**1**	**1**
Q8.	Which of the following is reducible to homogeneous differential equation? (A) $\frac{dy}{dx} = \frac{x^2+y+4}{y^2+x+4}$ (B) $\frac{dy}{dx} = \frac{x+y+2}{2x+4y+2}$ (C) $\frac{dy}{dx} = \frac{x+y}{x^2+y^2}$ (D) None of these	**1**	**2**	**3**
Q9.	The function $f(z) = \lvert z\rvert^2$ is analytic- (A) at $z = 0$ (B) Everywhere (C) Nowhere (D) None of these	**1**	**1**	**2**
Q10.	Polar form of Cauchy Riemann equation is- (A) $u_\theta = rv_r, v_\theta = -ru_r$ (B) $v_\theta = rv_r, u_\theta = -ru_r$ (C) $u_\theta = -rv_r, v_\theta = ru_r$ (D) None of these	**1**	**1**	**1**

	Section 2 (Answer any 2 question(s))	**Marks**	**CO**	**BL**
Q11.	Find the normal form of the matrix and hence find its rank $A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$	**5**	**2**	**3**
Q12.	Prove that the following equations are consistent and solve $2x + 4y - z = 9, 3x - y + 5z = 5, 8x + 2y + 9z = 19$	**5**	**3**	**4**
Q13.	Find the eigen values and eigen vectors of the matrix- $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$	**5**	**3**	**4**

	Section 3 (Answer any 2 question(s))	**Marks**	**CO**	**BL**
Q14.	Expand $sinx$ in power of $(x - \frac{\pi}{2})$ using taylor's series.	**5**	**2**	**3**
Q15.	Find the maximum and minimum value of the function- $F(x,y) = x^2 + 2xy + 2y^2 + 2x + y$	**5**	**3**	**4**
Q16.	Define homogeneous function and if - $u = sin^{-1}(\frac{x^2+y^2}{x+y})$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	**5**	**2**	**3**

	Section 4 (Answer any 2 question(s))	**Marks**	**CO**	**BL**
Q17.	Evaluate by expressing the following limit of a sum in the form of a definite integral- $\lim_{n\to\infty}\left\{\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + - - - - - - + \frac{1}{3n}\right\}$	**5**	**4**	**5**
Q18.	Define beta and gamma function and write three properties of beta function.	**5**	**1**	**1**

Q19.	Find the value of the integral $\int_0^1 \int_0^{2-x} xy\,dx\,dy$ by changing the order of integration.	5	3	4

Section 5 (Answer any 2 question(s))

		Marks	CO	BL
Q20.	Solve the following differential equation- $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$	5	2	3
Q21.	Solve the following- $(ycosx + sinny + y)dx + (sinx + xcosy + x)dy = 0$	5	2	3
Q22.	Solve the following: $\frac{dx}{dt} + wy = 0, \frac{dy}{dt} - wx = 0$	5	2	3

Section 6 (Answer any 2 question(s))

		Marks	CO	BL
Q23.	Find the analytic function $u + iv$ of which the real part $u = e^x(xcosy - ysiny)$.	5	2	3
Q24.	Evaluate $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$ using cauchy integral formula, where C is the circle $\lvert z \rvert = 3$.	5	2	3
Q25.	Evaluate $\int_0^{2+i} (\overline{z})^2 dz$, using complex line integral along the line $y = \frac{x}{2}$.	5	4	4

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Medicaps University

End Sem Exam May 2025

EN3BS11 , Engineering Maths-I

(Section 1)

Q1

(A) Rank of $A \leq \min(m,n)$ +)

Q2

(B) $\rho(A:B) = \rho(A) = \lambda =$ number of variable +)

Q3 (C) 2, 5

Q4 (A) $2y$

Q5 (A) $\frac{n(n+1)}{2}$

Q6 (C) $\frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}$

Q7 (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Q8 (B) $\frac{dy}{dx} = \frac{x+y+2}{2x+4y+2}$

Q.9 (C) No where.

Q10 (C) $u_\theta = -r v_r$

$v_\theta = r u_r$

Que 12

Given set of eqⁿ:

$$2x+4y-z=9$$
$$3x-y+5z=5$$
$$8x+2y+9z=19$$

The matrix eqⁿ is given as:

$$AX=B$$

$$\begin{bmatrix} 2 & 4 & -1 \\ 3 & -1 & 5 \\ 8 & 2 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 9 \\ 5 \\ 19 \end{bmatrix} \quad \text{---} (1)$$

Now its augmented matrix is

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 4 & -1 & 9 \\ 3 & -1 & 5 & 5 \\ 8 & 2 & 9 & 19 \end{array}\right] \qquad +1$$

$R_2 \to R_2 - R_1$

$$\sim \left[\begin{array}{ccc:c} 2 & 4 & -1 & 9 \\ 1 & -5 & 6 & -4 \\ 8 & 2 & 9 & 19 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & -5 & 6 & -4 \\ 2 & 4 & -1 & 9 \\ 8 & 2 & 9 & 19 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 8R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & -5 & 6 & -4 \\ 0 & 14 & -13 & 17 \\ 0 & 42 & 39 & 51 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & -5 & 6 & -4 \\ 0 & 14 & -13 & 17 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad +1$$

$R_2 \to \frac{R_2}{14}$

$$\left[\begin{array}{ccc:c} 1 & -5 & 6 & -4 \\ 0 & 1 & -13/14 & 17/14 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{--- (2)}$$

which is in echelon form

clearly $\rho(A:B) = 2$ & $\rho(A) = 2$

$\Rightarrow \rho(A:B) = \rho(A) = 2$ ~~< 3 (no.~~

Hence the given system of eq^n is consistent. +1

$\because \rho(A:B) = \rho(A) = 2 < 3$ (no. of unknowns)

$\therefore$ the system of eq^n have infinite many solution.

Now we shall assign arbitrary value to $n - r$ ie $3 - 2 = 1$ variable.

Now using (2), eq^n (1) becomes +1

$$\begin{bmatrix} 1 & -5 & 6 \\ 0 & 1 & -13/14 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 17/14 \\ 0 \end{bmatrix}$$

$$x - 5y + 6z = -4$$

$$y - \frac{13}{14} z = 17/14$$

let $\boxed{z = k}$

$$\therefore \boxed{y = \frac{17}{14} + \frac{13}{14} k = \left(\frac{17 + 13k}{14}\right)}$$

+1

$$\& \quad x = -4 + 5y - 6z$$

$$= -4 + 5\left(\frac{17 + 13k}{14}\right) - 6k$$

$$\boxed{x = \frac{29 - 19k}{14}}$$

Que 13

The given matrix is

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

The characteristic eq^n of A is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(2-\lambda)(5-\lambda) = 0$$

$$\Rightarrow \lambda = 2, 3, 5 \quad \text{are (eigen values.)} \qquad (+1)$$

For Eigen vector:

let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigen vectors corresponding to eigen values λ is given by

$$[A - \lambda I] X = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{---} (1) \qquad (+1)$$

(i) when $\lambda = 2$, then (1) becomes:

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \to \frac{R_2}{2} \sim \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3 - R_2 \sim \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{---} (2)$$

$\because$ the rank of the coeff. matrix is 2

i.e $\rho(A) = 2 < 3$ (no. of unknown)

$\therefore$ arbitrary value will be given to $n - \lambda$ i.e

$3 - 2 = 1$ variable.

from ①

$$x + y + 4z = 0$$

$$3z = 0 \Rightarrow \boxed{z = 0}$$

let $y = K$

$\therefore x = -K$

Hence eigen vector for $\lambda = 2$ is $\begin{bmatrix} -K \\ K \\ 0 \end{bmatrix}$ OR $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ (+1)

(ii) When $\lambda = 3$, then ① becomes:

$$\begin{bmatrix} 3-3 & 1 & 4 \\ 0 & 2-3 & 6 \\ 0 & 0 & 5-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$$\sim \begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 10 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \to \frac{R_2}{5} \sim \begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_3 \to R_3 - R_2 \sim \begin{bmatrix} 0 & 1 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow y + 4z = 0$$

$$2z = 0$$

$\because \rho(A) = 2 < 3$ (no. of arbitrary constant)

$\therefore$ arbitrary value will be given to
$n-1$ ie $3-2=1$ variable

let $x = k$

$z = 0$

$y = 0$

Hence corresponding to $\lambda = 3$ the eigen vector is

$$X = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad (f1)$$

(iii) when $\lambda = 5$, then eq^n 1 becomes

$$\begin{bmatrix} 3-5 & 1 & 4 \\ 0 & 2-5 & 6 \\ 0 & 0 & 5-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -2x + y + 4z = 0$$

$$-3y + 6z = 0$$

$\because \rho(A) = 2 < 3$ (no of unknown)

$\therefore$ arbitrary value will be given to $n-1$ ie $3-2=1$ variable.

let $\boxed{z = k}$

$-3y = -6k$

$\boxed{y = 2k}$

& $2x = y + 4z$

$2x = 2k + 4k$

$2x = 6k$

$\boxed{x = 3k}$

Hence $\lambda = 8$, the eigen vector is

$$X = \begin{bmatrix} 3k \\ 2k \\ k \end{bmatrix} \text{ or } \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$ +1

(Section - 3)

Que 14

let $f(x) = \sin x$

so we can write

$$f(x) = f\left[\frac{\pi}{2} + (x - \pi/2)\right] = f(a+h)$$

here $a = \pi/2$, $h = (x - \pi/2)$

Hence by Taylor's theorem, we have

$$f(x) = f\left[\frac{\pi}{2} + (x - \pi/2)\right] = f(\pi/2) + (x - \tfrac{\pi}{2}) f'(\pi/2) + \frac{(x-\pi/2)^2}{\lfloor 2} f''(\pi/2) + \cdots \quad (1)$$ (+1)

Now $f(x) = \sin x$ $\qquad f(\pi/2) = 1$

$f'(x) = \cos x$ $\qquad f'(\pi/2) = 0$

$f''(x) = -\sin x$ $\qquad f''(\pi/2) = -1$

$f'''(x) = -\cos x$ $\qquad f'''(\pi/2) = 0$ (+2)

$f^{iv}(x) = +\sin x$ $\qquad f^{iv}(\pi/2) = 1$

put this values in (1), we get

$$\sin x = 1 + (x - \pi/2)(0) + \frac{(x-\pi/2)^2}{\lfloor 2}(-1) + \frac{(x-\pi/2)^3}{\lfloor 3}(0) + \frac{(x-\pi/2)^4}{\lfloor 4}(1) + \cdots$$ (+1)

$$\Rightarrow \boxed{\sin x = 1 - \frac{(x-\pi/2)^2}{\lfloor 2} + \frac{(x-\pi/2)^4}{\lfloor 4} - \cdots}$$ (+1)

Ques 15

given that $F(x,y) = x^2 + 2xy + 2y^2 + 2x + y$

$\therefore \frac{\partial F}{\partial x} = 2x + 2y + 2$, $\frac{\partial F}{\partial y} = 2x + 4y + 1$

Also $r = \frac{\partial^2 F}{\partial x^2} = 2$, $s = \frac{\partial^2 F}{\partial x \partial y} = 2$, $t = \frac{\partial^2 F}{\partial y^2} = 4$. (+1)

For maxima or minima we have

$\frac{\partial F}{\partial x} = 0$ & $\frac{\partial F}{\partial y} = 0$

$\Rightarrow 2x + 2y + 2 = 0$ $2x + 4y + 1 = 0$

$\Rightarrow x = -3/2$, $y = 1/2$ (+1)

At point $(-3/2, 1/2)$ we have

$rt - s^2 = 4 > 0$ and $r > 0$ (+1)

$\Rightarrow F(x,y)$ has minimum value at $(-3/2, 1/2)$ (+1)

$\therefore F_{min} = \left(\frac{-3}{2}\right)^2 + 2\left(\frac{-3}{2}\right)\left(\frac{1}{2}\right) + 2\left(\frac{1}{2}\right)^2 + 2(-3/2) + (1/2)$.

$\boxed{F_{min} = -5/4}$ (+1)

Que 16

Def^n of Homogeneous fun :-

A function in which every term is of same degree is known as homogeneous fun of that degree.
Thus every homogeneous fun of x & y of degree 'n' can be expressed in the form

$$x^n F(y/x).$$ (+2)

Now given that $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$

Clearly u is not homogeneous fun.

$$\therefore \quad \sin u = \frac{x^2+y^2}{x+y} = f(u) \text{ say}$$ (+1)

$$\text{ie} \quad f(u) = \frac{x^2[1+(y/x)^2]}{x[1+(y/x)]} = x^1 F(y/x)$$

$\Rightarrow$ $f(u) = \sin u$ is a Homogeneous fun of x & y of degree $n=1$. (+1)

Hence by using deduction of Eulers Th we have

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$$

$$= 1\,\frac{\sin u}{\cos u}$$

$$\boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u}$$ (+1)

Section (4)

Que 17

given that

$$\lim_{n\to\infty}\left\{\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{3n}\right\}$$

The ~~gen~~ $(r+1)^{th}$ term of the series is
OR general

$$=\frac{1}{n+r}$$ where r varies from 0 to $2n$

$$=\frac{1}{n}\frac{1}{(1+r/n)}$$ (+1)

Now the lower limit of integration

$$=\lim_{n\to\infty}\left(\frac{0}{n}\right)=0$$

upper limit of integration

$$\lim_{n\to\infty}\frac{2n}{n}=2$$ (+1)

∴ limit of summation of the given series

$$=\lim_{n\to\infty}\sum_{r=0}^{2n}\frac{1}{n}\left\{\frac{1}{1+r/n}\right\}$$ (+1)

$$=\int_0^2\frac{dx}{1+x}$$ ∵ $\frac{r}{n}\to x$, $\frac{1}{n}\to dx$ & $r\to 0$ to $2n$ (+1)

$$=\left[\log_e(1+x)\right]_0^2$$

$$=\log_e 3-\log 1$$

$$=\log_e 3$$ Ans (+1)

Que 18

Def^n:- The Beta function denoted by $\beta(m,n)$ is defined by the definite integral

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

where m, n are positive numbers (may be integral or fractional) (1)

Properties of Beta fun

① Symmetrical property of Beta function

$$\beta(m,n) = \beta(n,m)$$ (1)

② Integral def^n

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$ (1)

③ Relation with Gamma fun

$$\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$ (1)

Def^n of Gamma function ⇒ The Gamma function is denoted by Γn and defined as the definite integral

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\,dx \quad , \text{for } x > 0$$

where n is the number (may be integral or fractional) (1)

Note → other forms of Beta fun may be considered as correct answer.

Que 19

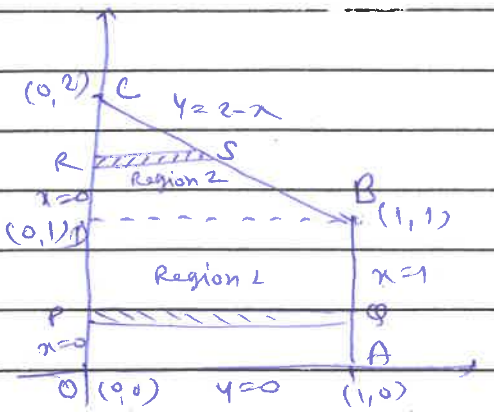
Let $I = \int_{x=0}^{1} \int_{y=0}^{2-x} xy \, dx\,dy$ ——— (1)

Firstly we draw the bounded region from the given curves : $x=0$; $x=1$

$y=0$ $y=2-x$ ie $x+y=2$.
straight line.

we get the possible points for the bounded region

(0,0), (0,2), (2,0), (1,1), (+1)



For changing the order of integration, the given integral is integrate first w.r.to 'x'. Now taking two strips || to x-axis say PQ & RS.

∴ The limit of integration will be two regions. (+1)

Region 1 :
(OABCD)
x varies from ~~Pa~~ $x=0$ to $x=1$
y varies from $y=0$ to $y=1$

Region II :
(BCD)
x varies from $x=0$ to $x=2-y$
& y varies from $y=1$ to $y=2$. (+1).

Hence (1) becomes

$$I = \int_0^1 \int_0^{2-x} xy \, dx\,dy$$

$$= \int_{y=0}^{1} \int_{x=0}^{1} xy \, dx\,dy + \int_{y=1}^{2} \int_{x=0}^{2-y} xy \, dx\,dy$$

$$= \int_{y=0}^{1} y \left[\frac{x^2}{2}\right]_0^1 dy + \int_{y=1}^{2} y \left[\frac{x^2}{2}\right]_0^{2-y} dy$$

$$= \frac{1}{2}\int_0^1 y\,dy + \frac{1}{2}\int_1^2 (2-y)^2 y\,dy$$

$$= \frac{1}{2}\left[\frac{y^2}{2}\right]_0^1 + \frac{1}{2}\int_1^2 (4-4y+y^2)y\,dy$$

$$= \frac{1}{4} + \frac{1}{2}\int_1^2 (4y-4y^2+y^3)\,dy$$

$$= \frac{1}{4} + \frac{1}{2}\left[\frac{4y^2}{2} - \frac{4y^3}{3} + \frac{y^4}{4}\right]_1^2$$

$$= \frac{1}{4} + \frac{1}{2}\left[\left(8-\frac{32}{3}+4\right) - \left(2-\frac{4}{3}+\frac{1}{4}\right)\right]$$

$$= \frac{1}{4} + \frac{1}{2}\left(\frac{5}{12}\right)$$

$$= \frac{11}{24} \quad \text{Ans} \qquad (+2)$$

Section 05

Que 20

The given diff eq^n is –

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$$

its auxiliary eq^n is given by –

$$m^2 + m + 1 = 0$$

$$\Rightarrow m = -\frac{1}{2} \pm \frac{1}{2} i\sqrt{3}$$

$$CF = e^{-x/2}\left[C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right]$$

$$P.I = \frac{1}{(D^2+D+1)} e^{-x} = \frac{1}{[(-1)^2+(-1)+1]} e^{-x}$$

$$= e^{-x}$$

Hence general sol^n is

$$y = CF + P.I = e^{-x/2}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right) + e^{-x}$$

Note: In Q. 21 instead of $\sin y$, $\sin ny$ Printed.

Que 21

given diff eq^n

$$(y\cos x + \sin y + y)\,dx + (\sin x + x\cos y + x)\,dy = 0$$

Here $M = y\cos x + \sin y + y$ $\qquad N = \sin x + x\cos y + x$

$\frac{\partial M}{\partial y} = \cos x + \cos y + 1$ $\qquad \frac{\partial N}{\partial x} = \cos x + \cos y + 1$. (+1)

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ the given diff. eq^n is exact. (+1)

Now

$\int_{y\text{-}\cancel{\text{Constant}}} M\,dx = \int_{y\,\text{const}} (y\cos x + \sin y + y)\,dx = y\sin x + x\sin y + xy$ (+1)

and $\int_{x\,\text{Constant}} N\,dy = \int_{x\text{-Constant}} (\sin x + x\cos y + x)\,dy = \cancel{\cos}\ y\sin x + x\sin y + xy$ (+1)

$\because$ we do not get any new term in the integration of N, therefore the required sol^n is

$$\boxed{y\sin x + x\sin y + xy = C}$$ Ans (+1)

Que 22 write $D = \frac{d}{dt}$, the given eq^n is

$Dx + wy = 0$ — (1)

$-wx + Dy = 0$ — (2)

(1) $\times D$ & (2) $\times w$ & subtracting, we get

$(D^2 + w^2)x = 0$ — (3) +1

A.E. $\Rightarrow m^2 + w^2 = 0 \Rightarrow m = \pm wi$ +1

Hence its sol^n is $x = (A\cos wt + B\sin wt)$ Ans +1

$\Rightarrow \frac{dx}{dt} = -Aw\sin wt + Bw\cos wt$ +1

$\therefore$ from (1) $\cancel{x} = y = -\frac{1}{w}\frac{dx}{dt}$ +1

$= -\frac{1}{w}[-Aw\sin wt + Bw\cos wt]$

$y = [A\sin wt - B\cos wt]$ Ans

(Section : D)

Que. 23

Here $u = e^x(x\cos y - y\sin y)$

Now $dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy$

$= -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ (by C-R eqn)

$dv = -e^x(-x\sin y - y\cos y - \sin y)\,dx$
$+ e^x(x\cos y - y\sin y + \cos y)\,dy$

This is an exact diff eqn

$\therefore$ on integrating we get

$v = \int_{y\,\text{const}} e^x(x\sin y + y\cos y + \sin y)\,dx$
$+ \int \text{term free from } 'x'\ dy + c$

$= [(x-1)\sin y + y\cos y + \sin y]\,e^x + c$
$= (x\sin y + y\cos y)e^x + c$

Hence $f(z) = u + iv$

$= e^x(x\cos y - y\sin y) + i[e^x(x\sin y + y\cos y) + c]$

$= x e^x(\cos y + i\sin y) + iy\,e^x(\cos y + i\sin y) + c$

$= x e^x e^{iy} + iy\,e^x e^{iy} + c$

$= (x+iy)\,e^x e^{iy} + c$

$= (x+iy)\,e^{x+iy} + c$

$f(z) = z e^z + c$, where c is constant.

Que 24

given that $\int_c \frac{e^{2z}}{(z-1)(z-2)} dz$; $c: |z|=3$

Cauchy's integral formula at $z=a$

$$f(a) = \frac{1}{2\pi i} \int_c \frac{f(z)}{(z-a)} dz \quad \text{---} (1)$$

let $f(z) = e^{2z}$

$f(z)$ is analytic within and on a closed curve 'c' given by $c: |z|=3$.

And the singular point $z=1, z=2$ lies inside the circle c.

$$\therefore \int_c \frac{\cancel{f(z)}\, e^{2z}}{(z-1)(z-2)} dz = \int_c e^{2z} \left\{ \frac{1}{(z-2)} - \frac{1}{(z-1)} \right\} dz$$

$$= \int_c \frac{e^{2z}}{(z-2)} dz - \int_c \frac{e^{2z}}{(z-1)} dz$$

$$= \cancel{2\pi i\, e^{2z} - 2\pi i\, e^{2z}}$$

$$= 2\pi i\, f(2) - 2\pi i\, f(1) \quad \text{using } (1) \quad \text{---} (2)$$

$\because f(z) = e^{2z} \Rightarrow f(2) = e^4, \; f(1) = e^2$

$$= 2\pi i\, e^4 - 2\pi i\, e^2$$

$$f(z) = \; = 2\pi i\,(e^4 - e^2)$$

Que 25 Given that

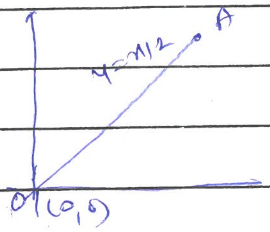
$$\int_0^{2+i} (\bar{z})^2 \, dz$$

$\because z = x + iy \therefore \bar{z} = x - iy$

& $dz = dx + i\,dy$ (+1)

$$\therefore \int_0^{2+i} (\bar{z})^2 = \int_0^{2+i} (x-iy)^2 (dx + i\,dy)$$

$$= \int_0^{2+i} (x^2 - y^2 - 2ixy)(dx + i\,dy) \quad \text{---} (1)$$



Along the line OA, $y = \frac{x}{2} \Rightarrow x = 2y$

so that $dx = 2dy$ (+1)

and y varies from 0 to 1.

Hence by (1) we have

$$\int_0^{2+i} (\bar{z})^2 dz = \int_0^1 (4y^2 - y^2 - 2i\,2y^2)(2+i)\,dy \quad (+1)$$

$$= (3-4i)(2+i) \int_0^1 y^2 \, dy$$

$$= (10 - 5i) \left(\frac{y^3}{3} \right)_0^1 \quad (+1)$$

$$= \frac{5}{3}(2 - i) \quad \text{Ans.} \quad (+1)$$