Q.6 Attempt any two:

i. The two regression equations of the variables x and y are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find **5**

(a) Mean of $x's$

(b) Mean of $y's$

(c) The correlation coefficient between x and y.

ii. Fit a second degree parabola to the following data **5**

x :	10	12	15	23	20
y :	14	17	23	25	21

iii. The mean weekly sales of the chocolate bar in candy stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a standard deviation of 17.2. Was the advertising campaign successful? **5**

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Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2019

EC3BS03 / EI3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: EC / EI

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated.Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. For a given graph G having 'v' vertices and 'e' edges which is connected and has no cycles, which of the following statement is true? **1**

(a) v = e (b) v = e+1 (c) v+1 = e (d) None of these

ii. What is the number of edges present in a complete graph having 'n' vertices? **1**

(a) $\frac{(n(n+1))}{2}$ (b) $\frac{(n(n-1))}{2}$ (c) n^2 (d) None of these.

iii. The depth of complete Binary tree is given by **1**

(a) nlog n (b) nlog $2n + 1$

(c) log $2n$ (d) log $2n + 1$

iv. In a 2 tree, nodes with '0' children are called- **1**

(a) Exterior node (b) Outside node

(c) Outer node (d) External node

v. The Rodrigue's Formula is given by **1**

(a) $P_n(x) = \frac{1}{2^n.n!}\frac{d^n}{dx^n}(x^2 - 1)^n$

(b) $P_n(x) = \frac{1}{2^n(2n)!}\frac{d^n}{dx^n}(x^2 - 1)^n$

(c) $P_n(x) = \frac{1}{2^n(n)!}\frac{d^{n-1}}{dx^{n-1}}(x - 1)^n$

(d) $P_n(x) = \frac{1}{2^n(n)!}\frac{d^n}{dx^{n-1}}(x - 1)^{2n}$

vi. If $P_n(x)$ is Legendre's polynomial of first kind, which is the correct option. **1**

(a) $P_{2n+1}(0) = 1$ (b) $P_{2n+1}(0) = 2$

(c) $P_{2n+1}(0) = 0$ (d) $P_{2n+1}(0) = 3$

vii. What is the value of an area under the conditional p.d.f. **1**

(a) Greater than '0' but less than 1.

(b) Greater than 1

(c) Equal to 1

(d) Infinite

viii. The joint cumulative density function CDF is a- **1**

(a) Non negative function

(b) Non decreasing function of x and y planes.

(c) Always a continuous function in xy plane.

(d) All of these

ix. The range of correlation coefficient is **1**

(a) -1 to 0 (b) 0 to 1 (c) -1 to 1 (d) None of these

x. What does the Z-Test consist of? **1**

(a) < 30 Samples (b) > 30 Samples

(c) = 30 Samples (d) None of these

Q.2 Attempt any two:

i. Define the following with examples: walk, path, circuit, Hamiltonian graph and isomorphic graph. **5**

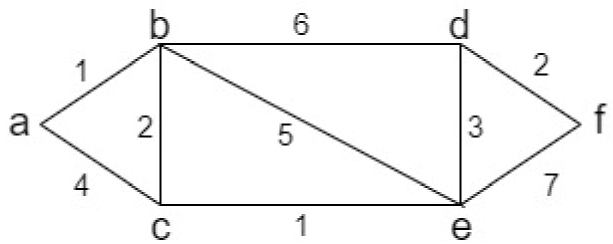
ii. State and prove Hand shaking theorem. Also prove that the maximum numbers of edges in a simple connected graph with n vertices is $\frac{n(n-1)}{2}$. **5**

iii. Define Incidence matrix and Adjacency matrix of a directed graph. Draw a directed graph with the given Adjacency matrix. **5**

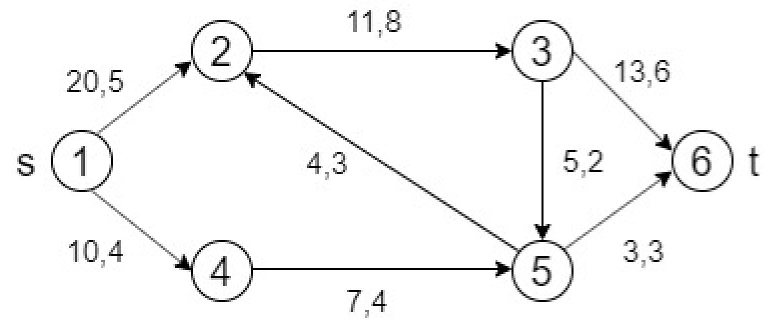
$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Q.3 Attempt any two:

i. Apply Dijkstra's algorithm to the graph given below and find the shortest path from 'a' to 'f'. **5**



ii. Define flow augmenting path. Apply the Ford Fulkerson algorithm to determine the maximum flow for the network given below. **5**



iii. Write the steps of Prim's and Kruskal algorithm for finding minimal spanning tree. **5**

Q.4 Attempt any two:

i. Solve in series the equation **5**

$$2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$$

ii. Prove that: (a) $x\,J'_n(x) = nJ_n(x) - xJ_{n+1}(x)$ **5**

(b) $x\,J'_n(x) = -\,nJ_n(x) + xJ_{n+1}(x)$

iii. Prove that $P_n(0) = \begin{cases} 0, if\ n\ is\ odd \\ \frac{(-1)^n n!}{2^n\{(\frac{n}{2})!\}^2}, if\ n\ is\ even \end{cases}$ **5**

Q.5 Attempt any two:

i. The Joint probability mass function of (x, y) is given by $p(x, y) = k(2x + 3y), x = 0,1,2;\ y = 1,2,3.$ Find all the marginal and conditional probability distribution. Also find the probability distribution of $(x + y)$. **5**

ii. If $f(x, y) = 2 - x - y$, when $0 \le x \le 1, 0 \le y \le 1$ and $f(x, y) = 0$ for all other values of x and y. Find **5**

(a) The marginal density $g(x)$ and $h(y)$

(b) The mean $\bar{x}\ and\ \bar{y}$.

(c) The variance ${\sigma_x}^2$ and ${\sigma_y}^2$

iii. Define random process. Give the classification of random process. **5**

Solution End sem Exam Dec-19

EC3BS03 mathematics III

BTech EC/EE

Q.1 (i) b. $v = e+1$ 1

(ii) b. $\frac{n(n-1)}{2}$ 1

(iii) d. $\log_2(n+1)$ 1

(iv) d. External node 1

(V) a. 1

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2-1)^n$$

(Vi) 1

c. $P_{2n+1}(0) = 0$

1

(Vii) c. Equal to 1

1

(Viii) d. All of these

(ix) c. -1 to 1 1

(X) b. > 30 Samples 1

Q.2 (i) One mark for each definition with example. (5)

(ii) Hand shaking theorem :-

The sum of degrees of the vertices of a graph is twice the number of edges (+1)

Pf/ Let us consider a graph $G = \langle V, E \rangle$ with 'n' vertices $v_1, v_2, \ldots, v_n$ and 'e' edges to prove

$$\sum_{i=1}^{n} \deg(v_i) = 2e$$

Since each edge is associated with two vertices and hence contributes two degree in G. (+1)

So if 'e' is the numbers of edges in G, the total degree will be equal to '2e'

$$\Rightarrow \sum_{i=1}^{n} deg(V_i) = 2e \rightarrow (1)$$ (+0.5)

Now if the graph is simple connected with 'n' vertices, the maximum possible degree of any vertex is $(n-1)$ (+1)

thus maximum possible total degree of graph will be $n(n-1)$ (+0.5)

Using (1) maximum possible no. of edges will be

(+1)

$$\frac{n(n-1)}{2}$$

5 marks

(iii) Consider a graph 'G' with 'n' vertices and 'e' edges,

Incidence matrix of 'G' (with no self loops) is defined as

$$A = [a_{ij}]_{n \times e}$$

$$a_{ij} = \begin{cases} +1 & \text{'j' th edge is incident out of 'i' th vertex} \\ 0 & \text{not incident} \\ -1 & \text{'j' edge is incident in of 'i' vertex} \end{cases}$$

(+1.5)

(3)

Adjacency matrix of 'G' (without parallel edges) is $X = [x_{ij}]_{n \times n}$

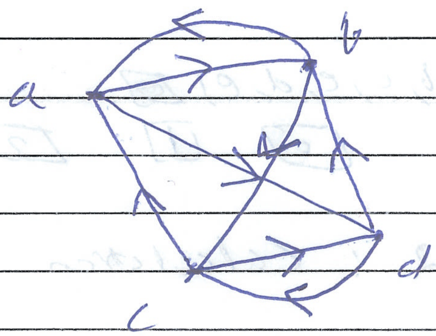
$$x_{ij} = \begin{cases} 1 : \text{if } \exists \text{ a directed edge } \text{~~between~~ from 'i'th to 'j'th vertex} \\ 0 : \text{otherwise} \end{cases}$$

(+1.5)

Now the Given matrix

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

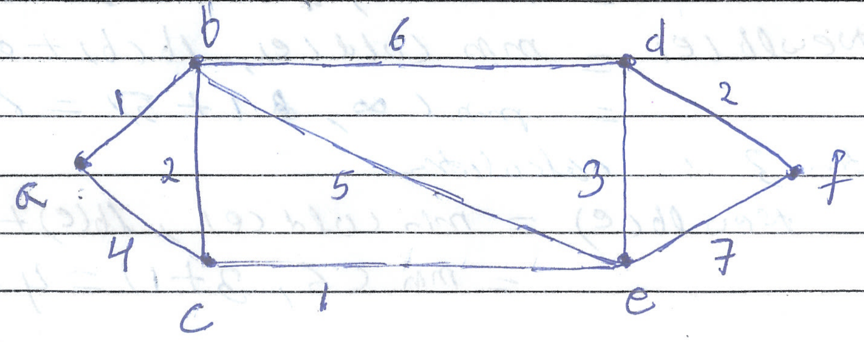
Graph :-



(+2)

5 marks

Q.3 (i)



4

Iteration no.	set	a	b	c	d	e	f
1	$PL = \{a\}$ $TL = \{b, c, d, e, f\}$	$\boxed{0}$	1	4	∞	∞	∞
2	$PL = \{a, b\}$ $TL = \{c, d, e, f\}$	$\boxed{0}$	$\boxed{1}$	3	7	6	∞
3	$PL = \{a, b, c\}$ $TL = \{d, e, f\}$	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	7	4	∞
4)	$PL = \{a, b, c, e\}$ $TL \{d, f\}$	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	7	$\boxed{4}$	11
5)	$PL = \{a, b, c, e, d\}$ $TL = \{e\}$	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{7}$	$\boxed{4}$	9
(6)	$PL = \{a, b, c, e, d, e\}$	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{7}$	$\boxed{4}$	9

(+2)

Ans

Iteration 2 : calculation

$$\text{New}\,lb(c) = \min(\text{old}(c),\ lb(b) + d_1(b, c))$$
$$= \min(4,\ 1+2) = 3$$
$$\text{New}\,lb(d) = \min(\text{old}(d),\ lb(b) + d_1(b, d))$$
$$= \min(\infty,\ 1+6) = 7$$
$$\text{New}\,lb(e) = \min(\text{old}(e),\ lb(b) + d_1(b, e))$$
$$= \min(\infty,\ 1+5) = 6$$

Iteration 3 : calculation

$$\text{New}\,lb(e) = \min(\text{old}(e),\ lb(c) + d_1(c, e))$$
$$= \min(6,\ 3+1) = 4$$

Iteration 4 : Calculation

$$New\ lb(d) = min(old(d),\ lb(e) + dis(e,d))$$
$$= min(7,\ 4+3) = 7$$

$$New\ lb(f) = min(old(f),\ lb(e) + dis(e,f))$$
$$= min(\infty,\ 4+7) = 11$$

Iteration 5 : Calculation

$$New\ lb(f) = min(old(f),\ lb(d) + dis(d,f))$$
$$= min(11,\ 7+2) = 9 \quad +2$$

The shortest path length from 'a' to 'f'
$= \underline{\underline{9}}$

$a \xrightarrow{1} b \xrightarrow{2} c \xrightarrow{1} e \xrightarrow{3} d \xrightarrow{3} f$ +1

5 marks)

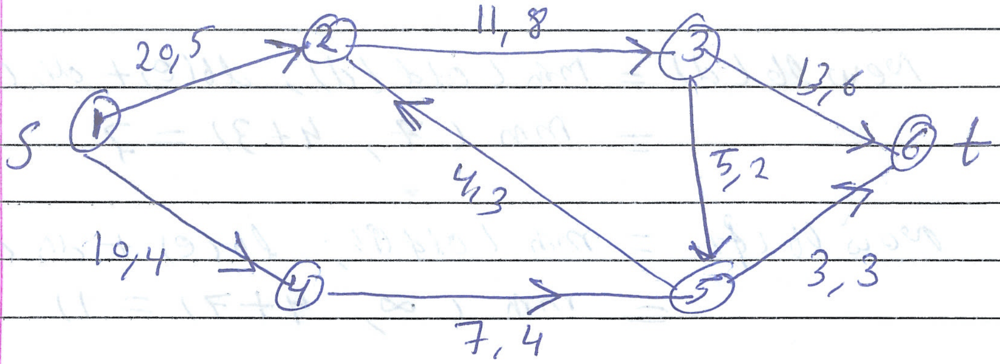
(ii) Flow Augmenting path :- In a network graph 'G' involving source 'S' and sink 'T', a flow augmenting path is a path from source 'S' to sink 'T', satisfying following conditions

(i) For forward edge E_i in path
$0 \leq flow(E_i) < capacity(E_i)$

(ii) For backward edge E_j in path
$0 < flow(E_j)$

+1

Given Network graph



Consider flow augmenting path

$s \; (1) \xrightarrow{20,5} (2) \xrightarrow{11,8} (3) \xrightarrow{13,6} (6)$

For forward edges

$\Delta_1 = \min(20-5,\ 11-8,\ 13-6) = 3$

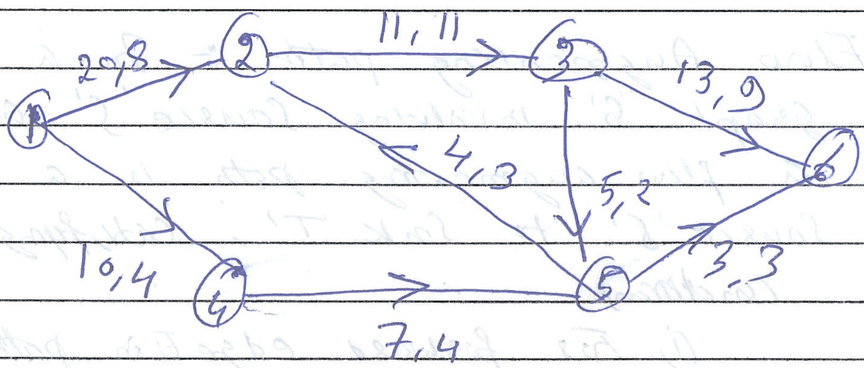
For Backward edges: NA

Δ_2 NA

Bottle neck cap. $\Delta = \min(\Delta_1, \Delta_2)$

$= 3$

Maximized flow for the path in Network



+2

7

Consider flow augmenting path

$$S\ (1) \xrightarrow{20,8} (2) \xleftarrow{4,3} (5) \xleftarrow{5,2} (3) \xrightarrow{13,9} (6)$$

For forward Edges

$\Delta_1 = \min(20-8,\ 13-9) = 4$

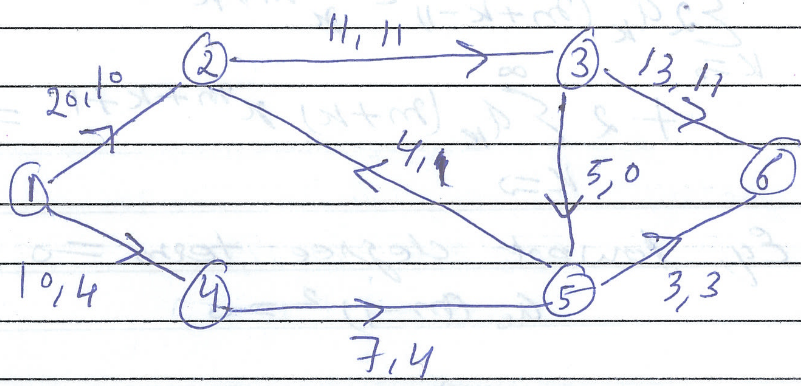
For Backward Edges

$\Delta_2 = \min(3, 2) = 2$

$\Delta = \min(\Delta_1\ \Delta_2)$

$\Delta = \min(4, 2) = 2$

maximized flow for the network path



+2

Now the graph is saturated

max flow = 14 Ans

5 marks

(iii) Prims Algorithm — Steps +2.5

Kruskals Algorithm — Steps +2.5

Q4 (i)

$$2x^2 \frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$$

$x=0$ is regular singular point. (+1)

Let $y = \sum_{k=0}^{\infty} a_k x^{m+k}$ be the solution.

substituting the values of y, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

in the given eq, we get

$$\sum_{k=0}^{\infty} 2a_k (m+k-1)^2 x^{m+k}$$
$$+ 2\sum_{k=0}^{\infty} a_k (m+k) x^{m+k+1} = 0$$ (+1)

Equating lowest degree term $= 0$

$$a_0[2m^2 - 3m + 1] = 0$$
$$\Rightarrow m = 1, 1/2$$

$\Rightarrow$ The solu "

$$y = c_1 (y)_{m=1} + c_2 (y)_{m=1/2}$$

Eq. higher degree term coeff $= 0$

$$a_1 = \frac{-2}{2m+1} a_0$$

$$a_2 = \frac{4}{(2m+1)(2m+3)} a_0$$

$$a_3 = \frac{-8}{(2m+1)(2m+3)(2m+5)} a_0$$

at $m=1$ $\Rightarrow$ $a_1 = -2/3\, a_0$ $\quad a_2 = \frac{4}{3\cdot 15} a_0$ $\quad a_3 = \frac{-8}{105} a_0$

(9)

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at $m = 1/2$

$$a_1 = -a_0 \qquad a_2 = \frac{a_0}{2} \qquad a_3 = -\frac{1}{6} a_0$$

$$y = c_1 (y)_{m=1} + c_2 (y)_{m=1/2}$$

$$(y)_{m=1} = a_0 x \left(1 - \frac{2}{3}x + \frac{4}{15}x^2 - \frac{8}{105}x^3 \cdots\right)$$

$$(y)_{m=1/2} = a_0 x^{1/2} \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6} \cdots\right)$$

$$\Rightarrow y = c_1 x^{1/2}\left(1 - x + \frac{x^2}{2} - \frac{x^3}{6}\right) + c_2 x \left(1 - \frac{2}{3}x + \frac{4}{15}x^2 - \frac{8x^3}{105} \cdots\right)$$

(ii)

(a) We know

$$J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

diff w.r.to x, we get

$$J_n'(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (n+2r)}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1} \left(\frac{1}{2}\right)$$

$$\Rightarrow x J_n'(x) = n \sum_{r=0}^{\infty} \frac{(-1)^r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r} \qquad (+1)$$

$$+ x \sum_{r=0}^{\infty} \frac{(-1)^r}{(r-1)!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1} \qquad (+1)$$

$$= n J_n(x) + x \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{k!\,\Gamma(k+1+r+1)} \left(\frac{x}{2}\right)^{k+1+2r}$$

$$= n J_n(x) - x J_{n+1}(x) \qquad +0.5$$

2.5 marks

(b)

$$J_n(x) = \sum_{r=0}^{n} \frac{(-1)^r}{r! \, \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

$$J_n'(x) = \sum_{r=0}^{n} \frac{(-1)^r \, (n+2r)}{r! \, \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1} \left(\frac{1}{2}\right)$$

(+1)

$$x J_n'(x) = \sum_{r=0}^{n} \frac{(-1)^r (2n+2r-n)}{r! \, \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

(+1)

$$= \sum_{r=0}^{n} \frac{(-1)^r \, 2}{r! \, \Gamma(n+r)} \left(\frac{x}{2}\right)^{n+2r} - n J_n(x)$$

$$= x \sum_{r=0}^{\infty} \frac{(-1)^r \, \cancel{2}}{r! \, \Gamma(n-1+r+1)} \left(\frac{x}{2}\right)^{(n-1)+2r} - n J_n(x)$$

(+0.5)

$$= x J_{n-1} - n J_n(x)$$

~~2+5 marks~~

5 marks

(iii) We know

$$(1-2xt+t^2)^{-1/2} = \sum t^n P_n(x) \qquad (+1)$$

Put $x=0$, we get

$$(1+t^2)^{-1/2} = \sum t^n P_n(0) \qquad (+1)$$

$$\Rightarrow \sum t^n P_n(0) = 1 - \frac{1}{2}t^2 + \frac{1}{2}\cdot\frac{3}{4}t^4 \cdots - $$

$$+ (-1)^r \frac{(1.3.5 \cdots 2r+1)}{2.4.6 \cdots 2r} t^{2r} + \cdots$$

(+2)

Eq coeff of t^{2m+1}, we get $P_{2m+1}(0) = 0$

Eq coeff of t^{2m} we get

$$P_{2m}(0) = (-1)^m \frac{1.3.5 \cdots (2m-1)}{2.4.6 \cdots 2m} \qquad (+1)$$

5 marks

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Q-5 (i) Probability distribution table is given by

pmf $p(x,y) = k(2x+3y)$ $x = 0,1,2$
$y = 1,2,3$

$x \backslash y$	1	2	3
0	3k	6k	9K
1	5k	8k	11K
2	7K	10k	13k

Now as $\sum_j \sum_i P_{ij} = 1$

$\Rightarrow 72k = 1 \Rightarrow k = 1/72$ (+0.5)

Marginal distribution function $x_i = \sum_j P_{ij}$

for $x=0$ $P_{01}+P_{02}+P_{03} = 18K = \frac{18}{72}$

$x=1$ $P_{11}+P_{12}+P_{13} = 24k = \frac{24}{72}$

$x=2$ $P_{21}+P_{22}+P_{23} = 30k = \frac{30}{72}$

Marginal distribution for $y_j = \sum_i P_{ij}$

for $y=1$: $P_{10}+P_{11}+P_{12} = 15k = \frac{15}{72}$

$y=2$ $P_{20}+P_{21}+P_{22} = 24k = \frac{24}{72}$

$y=3$ $P_{30}+P_{31}+P_{32} = 33k = \frac{33}{72}$ (+1)

Conditional distribution of $X = x_i$ given $Y = y_j$

$= \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)}$ (+0.5)

(12)

Conditional for X given Y = 1

x

0 $3k/15k = 1/5$

1 $5k/15k = 1/3$

2 $7k/15k = 7/15$

Conditional for X given Y = 2

$x = 0$: $1/4$

$x = 1$: $1/3$

$x = 2$: $5/12$

Conditional for X given Y = 3

$x = 0$: $3/11$

$x = 1$: $1/3$

$x = 2$: $13/33$

Conditional for Y given $x = 0$

$y = 1$: $1/6$

$y = 2$: $1/3$

$y = 3$: $1/2$

Conditional for Y given $x = 1$

$y = 1$: $5/24$

$y = 2$: $1/3$

$y = 3$: $11/24$

Conditional for Y given $x = 2$

$y = 1$: $7/30$

$y = 2$: $1/3$

$y = 3$: $13/30$

(A 2)

Probability distribution for $X+Y = 1, 2, 3, 4, 5$

$P(X+Y=1) = 3/72$ $P(X+Y=2) = 11/72$

$P(X+Y=3) = 24/72$ $P(X+Y=4) = 21/72$

(A 1)

$P(X+Y=5) = 13/72$

(5 marks)

Q-5

(ii) Given P.d.f $f(x,y) = 2-x-y \quad 0 \le x \le 1, \; 0 \le y \le 1$

$= 0 \quad$ otherwise

Marginal density

$$g(x) = \int_0^1 f(x,y)\,dy$$

$$= \int_0^1 (2-x-y)\,dy$$

$$= \left(2y - xy - \frac{y^2}{2}\right)_0^1 = 2 - x - \tfrac{1}{2}$$

$$h(y) = \int_0^1 f(x,y)\,dx$$

$$= 2 - y - \tfrac{1}{2}$$

mean of X : $E(X) = \int_0^1 x\, g(x)\,dx$

$$= \int_0^1 x\left(2 - x - \tfrac{1}{2}\right) dx$$

$$= \int_0^1 \left(2x - x^2 - \frac{x}{2}\right) dx$$

$$= \left(x^2 - \frac{x^3}{3} - \frac{x^2}{4}\right)_0^1$$

$$= 1 - \frac{1}{3} - \frac{1}{4} = 5/12$$

mean of Y : $E(Y) = \int_0^1 y\, h(y)$

$$= 5/12$$

$$\sigma_x^2 = E(X^2) - [E(X)]^2$$
$$\sigma_y^2 = E(Y^2) - [E(Y)]^2$$

Now $E(x^2) = \int_0^1 x^2 g(x)\, dx$

$$= \int_0^1 x^2 (2 - x - 1/2)\, dx$$

$$= \left(\frac{2x^3}{3} - \frac{x^4}{4} - \frac{x^3}{6} \right)_0^1 = \frac{3}{12} = \frac{1}{4}$$

$||^{ly}$

$$E(Y^2) = \frac{1}{4}$$

$$\sigma_x^2 = E(x^2) - (E(x))^2$$

$$= \frac{1}{4} - \left(\frac{5}{12}\right)^2$$

$$= \frac{1}{4} - \frac{25}{144} = \frac{11}{144}$$

$||^{ly}$ $\sigma_y^2 = \frac{11}{144}$

Q.5

(iii) Defination of Random Process (+2)

Classification of Random Process (+3)

5 marks

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(iii) Given $n = 22$

mean $\bar{x} = 153.7$

S.D $\sigma = 17.2$

dof $n-1 = 21$ (+1)

H_0 :- There is no difference between sample mean and population mean.

i.e advertisement is not effective

$H_0 : \mu = 146.3$

Now t statistic

$$t = \frac{\bar{x} - \mu}{\sigma} \sqrt{(n-1)}$$ (+1)

$$= \frac{153.7 - 146.3}{17.2} \sqrt{21}$$ (+1)

$$t = 1.97$$

Comparing calculated t value with tabulated t value we can say at 5% level of significance for 21 dof, that

if calculated t < tabulated t

$\Rightarrow$ H_0 accepted (advertisement not effective)

else

H_0 rejected (advertisement is effective) (+2)

5 marks

16

Q6 (i) Two regression lines

$$x = 19.13 - 0.87y$$
$$y = 11.64 - 0.50x$$

The solution of two regression lines give mean of x's and y's

Solving $x + 0.87y = 19.13$
$0.50x + y = 11.64$

We get mean $y's = 3.67$ (b)
Mean $x's = 15.94$ (a)

(+2.5)

(C) Comparing x on y
$x = 19.13 - 0.87y$
We get regression coeff
$b_{xy} = -0.87$

Comparing y on x

$y = 11.64 - 0.50x$
We get regression coeff
$b_{yx} = -0.50$

Also correlation coeff $r = \pm\sqrt{b_{xy}b_{yx}}$
$= \pm\sqrt{-0.87 \times -0.50}$
$= \pm 0.6595$
+2.5

5 marks

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Q6 (i)

x	y	$u=x-15$	$v=y-23$	uv	u^2	u^2v
10	14	−5	−9	45	25	−225
12	17	−3	−6	18	9	−54
15	23	0	0	0	0	0
23	25	8	2	16	64	128
20	21	5	−2	−10	25	−50
		$\Sigma=5$	$\Sigma=-15$	$\Sigma=69$	$\Sigma=123$	$\Sigma=-201$

u^3	u^4
−125	625
−27	81
0	0
512	4096
125	625
$\Sigma=485$	$\Sigma=5427$

Let the second degree parabola to fit

$$y=a+bx+cx^2$$

Shifting $x \to x-15=u$ and $y \to y-23=v$

we fit

$$v=a+b'u+c'u^2$$

N.E

$$\Sigma v = 5a + b'\Sigma u + c'\Sigma u^2$$
$$\Sigma uv = a\Sigma u + b\Sigma u^2 + c\Sigma u^3$$
$$\Sigma u^2 v = a\Sigma u^2 + b\Sigma u^3 + c\Sigma u^4$$

$$\Rightarrow -15 = 5a + 5b + 123c$$
$$69 = 5a + 123b + 485c$$
$$-201 = 123a + 485b + 5427c$$

Solving $a=-2.215$ $b=0.925$ $c=-0.0694$

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$\Rightarrow v = -2.215 + 0.9254t - 0.0694 t^2$

$\Rightarrow (y-23) = -2.215 + 0.925(x-15)$
$- 0.0694 (x-15)^2$

$\Rightarrow y = -0.0694x^2 + 3.007x - 8.705$

Ans