Q.5 Attempt any two:

i. Expand $\log x$ in powers of $(x-1)$ by Taylor's theorem and hence find the value of $\log(1.1)$. **5**

ii. If $y=\sin^{-1}x$ then using Leibnitz theorem, prove that **5**

$(1-x^2)\,y_{n+2}-(2n+1)\,xy_{n+1}-n^2y_n=0.$

iii. Find out the radius of curvature at the point (4,8) of the parabola $y^2=16x$. **5**

Q.6 Attempt any two:

i. Evaluate: **5**

$$\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+---+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)$$

ii. Prove that: **5**

$$\int_0^1\frac{x^2}{\sqrt{1-x^5}}dx=\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right).$$

iii. Evaluate using reduction formula **5**

$$\int\sin^5 x\,dx$$

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2018

BC3CO03 Mathematics-I

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. How many imaginary roots are there for the equation $x^6+3x^5+4x^2+8=0$: **1**

(a) 2 (b) 4 (c) 0 (d) 6

ii. The equation whose roots are negative to the roots of the equation $x^4-4x^3+8x^2+4x+1=0$ is **1**

(a) $x^4+4x^3-8x^2-4x+1=0$

(b) $x^4+4x^3+8x^2-4x+1=0$

(c) $x^4+4x^3+8x^2+4x+1=0$

(d) None of these

iii. If the rank of augmented matrix $[A:B]=\text{Rank}[A]<n$, the number of unknowns than the system of equation $AX=B$ has **1**

(a) No solution

(b) Unique solution

(c) Infinite number of solutions

(d) n number of solutions

iv. If $A=\begin{bmatrix}1&2\\2&1\end{bmatrix}$ then the value of $A^2-2A=$ **1**

(a) 0 (b) 3 (c) -3 (d) $3I_2$

v. A function continuous at a point is **1**

(a) Differentiable there

(b) Not differentiable

(c) May or may not be differentiable

(d) None of these

vi. If $y = \log((1+x^3))$ then using chain rule of differentiation the value of $\frac{dy}{dx} =$ **1**

(a) $\frac{(1+x^3)}{3x^2}$ (b) $\frac{3x^2}{(1+x^3)}$ (c) $\frac{x^2}{(1+x^3)}$ (d) $\frac{1+3x^2}{(1+x^3)}$

vii. The curvature at every point of circle is **1**

(a) Constant

(b) Equal to reciprocal of its radius

(c) Both (a) and (b)

(d) Can't say without knowing circle

viii. Asymptote parallel to the x axis for the curve $(y-4)x^2 + x + 2 = 0$ is **1**

(a) $y=0$ (b) $y=-2$ (c) $y=4$ (d) $x=4$

ix. $\int_0^\infty e^{-x}x^{n-1}dx, n>0$, is equal to **1**

(a) $(n+1)!$ (b) $n!$ (c) $\overline{|n}$ (d) None of these

x. $\lim_{n\to\infty}\left\{\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}\right\}$ is equals to **1**

(a) $\log_e 2$ (b) $\log_e 4$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Q.2 Attempt any two:

i. Find the equation whose roots are the roots of equation $x^4 + x^3 - 3x^2 - x + 2 = 0$ each diminished by 3. **5**

ii. Solve the equation $8x^5 - 22x^4 - 55x^3 + 55x^2 + 22x - 8 = 0$. **5**

iii Solve by Cardans method $x^3 - 3x + 1 = 0$ **5**

Q.3 Attempt any two:

i. Find the rank and nullity of the following matrix by reducing it into the normal form **5**

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

ii. Discuss the consistency of the system of equations **5**

$$2x+3y+4z=11$$
$$x+5y+7z=15$$
$$3x+11y+13z=25$$

If found consistence, solve it.

iii. Find the inverse of the following matrix using Cayley – Hamilton theorem **5**

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

Q.4 Attempt any two:

i. Discuss the continuity and differentiability of **5**

$$f(x) = \begin{cases} x & , 0 \le x < \frac{1}{2} \\ 1 & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x < 1 \end{cases}$$

At $x = \frac{1}{2}$.

ii. Find the derivative of $\log\left(\sqrt{\sin e^x}\right) + x^x$ using chain rule. **5**

iii. Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 10x + 29$ in $[2,7]$. **5**

Solutions

End sem Examination Dec 2018

DC3CO03 - Mathematics-I

Q.1

(i) b : 4 | 1

(ii) a : $x^4+4x^3-8x^2-4x+1=0$ | 1

(iii) c : Infinite number of solution | 1

(iv) d : $3I_2$ | 1

(v) c : may or may not be differentiable | 1

(vi) b : $\dfrac{3x^2}{(1+x^3)}$ | 1

(vii) c : Both (a) and (b) | 1

(viii) c : $y=4$ | 1

(ix) c : $\sqrt{\pi}$ | 1

(x) a : $\log_e 2$ | 10

Q.2 (i) To find the required eq, whose roots are each diminished by 3, to the given ~~given~~ equation : $x^4 + x^3 - 3x^2 - x + 2 = 0$, we divide the equation by $x - 3$, successively and remainder gives the coefficient of the required equation +1

Using synthetic division

1	1	-3	-1	2	3
x	3	12	27	78	
1	4	9	26	<u>80</u>	
x	3	21	90		
1	7	30	<u>116</u>		
x	3	30			
1	10	<u>60</u>			
x	3				
<u>1</u>	<u>13</u>				

+1

+1

+1

+1

Ans $x^4 + 13x^3 + 60x^2 + 116x + 80 = 0$

(5)

(ii) Solve $\cancel{8x^5 - 22x^4 + 97?}$

$$8x^5 - 22x^4 - 55x^3 + 55x^2 + 22x - 8 = 0$$

Reciprocal equation of odd degree with opposite sign $\therefore x = 1$ is root +1

dividing by $x - 1$, we get

$$8x^4 - 14x^3 - 69x^2 - 14x + 8 = 0$$ +1

dividing by x^2

$$8\left(x^2 + \tfrac{1}{x^2}\right) - 14\left(x + \tfrac{1}{x}\right) - 69 = 0$$ +1

3

$x+\frac{1}{x}=y$ and $x^2+\frac{1}{x^2}=y^2-2$, we get

$$8(y^2-2)-14y-69=0$$

i.e. $8y^2-14y-85=0$

or $y=\frac{14\pm\sqrt{2196}}{2}$

i.e $y=\frac{17}{4}$ and $y=-\frac{5}{2}$ +1

$\Rightarrow x+\frac{1}{x}=\frac{17}{4}$ $\qquad \Rightarrow x+\frac{1}{x}=-\frac{5}{2}$

$\Rightarrow x=4,-\frac{1}{4}$ $\qquad \Rightarrow x=2,-\frac{1}{2}$

+1

Ans $x=1,\ 4,\ -\frac{1}{4},\ 2,\ -\frac{1}{2}$ 5

Q Solve by cardon's method

$$x^3-3x+1=0$$

to solve by cardon's method

Put $x=u+v$ +1

$$x^3-3uvx-(u^3+v^3)=0$$

Comparing we get

$uv=1 \quad u^3+v^3=-1$ +1

$\therefore u^3, v^3$ is root of equation

$$t^2+t+1=0$$ +1

$\Rightarrow u^3=\frac{-1+i\sqrt{3}}{2},\ v^3=\frac{-1+i\sqrt{3}}{2}$ +1

$u=\cos\frac{\theta+2n\pi}{3}+i\sin\frac{\theta+2n\pi}{3}$ +1

$v=\cos\frac{\theta+2n\pi}{3}-i\sin\frac{\theta+2n\pi}{3}$

5

$x = 0, 1, 2$

$x = u + v$

$= 2\cos\frac{2\pi}{9}, \quad 2\cos\frac{8\pi}{9}, \quad 2\cos\frac{14\pi}{9}$ Ans

Q. 3 (i)

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$R_2 \leftarrow R_2 - R_1 \quad R_3 \leftarrow R_3 - 2R_1 \quad R_4 \leftarrow R_4 - R_1$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -3 & 1 \\ 0 & -1 & -2 & 0 \end{bmatrix}$$

$R_4 \leftarrow R_4 + R_2$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & -2 & 0 \end{bmatrix}$$

~~R_4~~ $R_3 (-1/3), \quad R_4 \leftarrow R_4 + 2R_3$

$$\approx \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & 0 & 2/3 \end{bmatrix}$$ +2

$R_4 (3/2)$

$, C_2 \leftarrow C_2 - C_1 \quad C_3 \leftarrow C_3 - 3C_1 \quad C_4 = C_4 - C_1$

$$\approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ +2

$''\;\;'' \approx [I_4] \quad \rho(A) = 4 \quad$ Nullity $= 4 - 4 = 0$ +1

(5)

Q.3.
(ii) $2x+3y+4z=11$

$x+5y+7z=15$

$3x+11y+13z=25$

$$[A:B]=\left[\begin{array}{ccc|c} 2 & 3 & 4 & 11 \\ 1 & 5 & 7 & 15 \\ 3 & 11 & 13 & 25 \end{array}\right]$$

Applying Row transformation, we get

$$\begin{bmatrix} 1 & 5 & 7 & 15 \\ 0 & 1 & \frac{10}{7} & \frac{19}{7} \\ 0 & 0 & 4/7 & \frac{16}{7} \end{bmatrix}$$ +2

$\rho(A:B)=3$

$\rho(A)=3$ +1

$\rho(A:B)=\rho(A)=3=$ no of unknowns

$\Rightarrow$ unique soln

From IIIrd eq. $\Rightarrow z=4$

$7y+10z=19$

$\Rightarrow y=-3$

$x+5y+7z=15$

$\Rightarrow x=2$ +2

Ans $x=2,\ y=-3,\ z=4$ 5

(6)

Q3 (iii) $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

Characteristic equation is given by $|A - \lambda I| = 0$

$$\begin{vmatrix} 4-\lambda & 3 & 1 \\ 2 & 1-\lambda & -2 \\ 1 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0$$ +2

Using Cayley Hamilton theorem

$$\Rightarrow A^3 - 6A^2 + 6A - 11I = 0$$

$$A^2 = \begin{bmatrix} 23 & 11 & -1 \\ 8 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 125 & 84 & -12 \\ 36 & 23 & 0 \\ 48 & 30 & -7 \end{bmatrix}$$ (no need) +1

To find A^{-1} : multiply both sides by A^{-1}

$$A^{-1} = \frac{1}{11}(A^2 - 6A + 6I)$$

$$= \frac{1}{11}\begin{bmatrix} 5 & -1 & -7 \\ -4 & 3 & 10 \\ 3 & -5 & -2 \end{bmatrix}$$ Ans +2

(5)

(1)

Q.4 (i) $f(x)$ is continuous at $x=\frac{1}{2}$ if

$$\lim_{h\to 0^+} f(1/2+h) = \lim_{h\to 0^-} f(1/2-h) = f(1/2)$$ +1

Given $f(1/2) = 1$

Now

$$\lim_{h\to 0^+} f(1/2+h) = \lim_{h\to 0} (1-(1/2+h))$$

$$= 1/2$$ +1

$$\lim_{h\to 0^-} f(1/2-h) = \lim_{h\to 0} (1/2-h)$$

$$= 1/2$$ +1

$$f(1/2+h) = f(1/2-h) \neq f(1/2)$$ +1

$\Rightarrow f(x)$ is not continuous at $x=1/2$ +1

$\Rightarrow f(x)$ is not differentiable at $x=1/2$ +1

(5)

(ii) $y = \log\sqrt{\sin e^x} + x^x$

Let $y = Y + Z$

$Y = \log\sqrt{\sin e^x}$

$e^x = u$

$Y = \sqrt{\sin u}$

$\sin u = v \qquad \sqrt{v} = w$ +1

$Y = \log\sqrt{v}$

$= \log w$

$$\frac{dY}{dx} = \frac{1}{w}\frac{dw}{dx} = \frac{1}{\sqrt{v}}\left(-\frac{1}{2}\frac{1}{\sqrt{v}}\right)\frac{dv}{dx}$$

$$= -\frac{1}{2v}\cdot \cos u\cdot\frac{du}{dx}$$

$\frac{dY}{dx} = -\frac{1}{2v} \cos u \, e^x$ ⑧

$= -\frac{1}{2 \cdot \sin u} \cdot \cos u \cdot e^x$

$= -\frac{1}{2 \sin e^x} \cdot \cos e^x \cdot e^x$ +2

$Z = x^x$

$\log Z = x \log x$

$\frac{1}{Z} \frac{dZ}{dx} = (1 + \log x)$

$\Rightarrow \frac{dZ}{dx} = x^x (1 + \log x)$ +2

$\Rightarrow y = Y + Z$

$= -\frac{1}{2} (\cot e^x) \cdot e^x + x^x (1 + \log x)$ ⑤

(iii) Lagranges mean value theorem :

$f(x) = 2x^2 - 10x + 29$ in $[2, 7]$

function is continuous in $[2, 7]$ +1

function is differentiable in $(2, 7)$

$f(2) = 17$ $f(7) = 57$ +1

$f(2) \neq f(7)$ conditions for

Lagranges mean value theorem is satisfied.

Now $f'(x) = 4x - 10$ +1

$f'(c) = 4c - 10$ +1

But $f'(c) = \frac{f(b) - f(a)}{b - a}$ +1

$\Rightarrow 4c - 10 = 8 \Rightarrow c = 4.5 \in [2, 7]$. 5

9

Q.5 (i) Let $f(x) = \log x$

We ~~by~~ know by Taylor's theorem, around a point 'a', we can write

$$f(x) = f(a) + \frac{(x-a)}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \cdots \infty$$ +1

$a = 1$,

$f(x) = \log x \qquad f(1) = \log 1 = 0$

$f'(x) = \frac{1}{x} \qquad f''(1) = 1$

$f''(x) = -\frac{1}{x^2} \qquad f''(1) = -1$ +2

$f'''(x) = \frac{2}{x^3} \qquad f'''(1) = 2$

$f^{iv}(x) = \frac{-6}{x^4} \qquad f'''(1) = -6$

$$\Rightarrow \log(x) = (x-1) - \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \cdots \infty$$ +1

Put $x = 1.1$ +1

$\log(1.1) = 0.095308$: Ans

(5)

Q.5 (ii) $y = \sin^{-1} x$

$y_1 = \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\, y_1 = 1$

$\Rightarrow (1-x^2) y_1^2 = 0$

$\Rightarrow (1-x^2) y_2 - x y_1 = 0$ +1

Now using Lebnitz th

$D^n(uv) = D^n u . v + n_{c_1} D^{n-1} u Dv + \cdots + n_{c_n} u D^n v$ +1

5 (iii). To find radius of curvature ρ at point $(4, 8)$ for the curve $y^2 = 16x$ (1)

$$\rho = \frac{[1 + y_1^2]^{3/2}}{y_2}$$ +1

$$y_1 = \frac{dy}{dx} : \quad 2y\frac{dy}{dx} = 16$$

$$y_1 = \frac{dy}{dx} = \frac{16}{2y}$$ +1

$$(y_1)_{(4,8)} = \frac{16}{16} = 1$$

$$y_2 = \frac{d^2y}{dx^2}$$

$$y\frac{dy}{dx} = 8$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

$$y y_2 + y_1^2 = 0$$ +1

$$\Rightarrow y_2 = \frac{8 - y_1^2}{y}$$

$$(y_2)_{(4,8)} = \frac{8-1}{8} = \frac{-\cancel{7}}{8} = -\frac{1}{8}$$

$$\rho = \frac{[1+1]^{3/2}}{7/8} = \frac{-2^{3/2}}{\cancel{8}} \times 8$$

$$= \frac{-2^{3/2} \cdot 2^3}{\cancel{8}}$$ +2

$$= \frac{2^{9/2}}{\cancel{7}} = \frac{16\sqrt{2}}{\cancel{8}}$$ (5)

$$\rho = |-16\sqrt{2}| = 16\sqrt{2}$$ Ans

(11)

5 (i) To find

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+\cdots+\frac{1}{\sqrt{n^2-(n-1)^2}}\right]$$

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{\sqrt{n^2-r^2}}$$ +1

$$=\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}\frac{1}{\sqrt{1-(r/n)^2}}$$ +2

$$=\int_0^1\frac{1}{\sqrt{1-x^2}}\,dx=(\sin^{-1}x)_0^1$$ +2

$$=\pi/2$$ Ans

(5)

5 (ii) To prove

$$\int_0^1\frac{x^2}{\sqrt{1-x^5}}\,dx=\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$$

Put $x^5=t \Rightarrow x=t^{1/5}$ +1

$$dx=\frac{1}{5}t^{1/5-1}$$

$$\Rightarrow\int_0^1\frac{x^2}{\sqrt{1-x^5}}\,dx=\int_0^1\frac{1}{5}\,t^{2/5}\cdot t^{1/5-1}(1-t)^{1/2-1}\,dt$$ +1

$$=\frac{1}{5}\int_0^1 t^{3/5-1}(1-t)^{1/2-1}\,dt$$ +1

Comparing with $\beta(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}dx$ +1

we get +1

$$\int_0^1\frac{x^2}{\sqrt{1-x^5}}\,dx=\frac{1}{5}\beta\left(\frac{3}{5},\frac{1}{2}\right)$$

(5)

6 (iii) To evaluate $\int \sin^5 x \, dx$ using Reduction formula

We use

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{5} + \frac{n-1}{n} \int \sin^{n-2} x \, dx \qquad n \geq 2$$ +1

put $n = 5$

$$\int \sin^5 x \, dx = -\frac{\sin^4 x \cos x}{5} + \frac{4}{5} \int \sin^3 x \, dx$$ +1

now put $n=3$, in right hand integrant. +1

$$\Rightarrow \int \sin^5 x \, dx = -\frac{\sin^4 x \cos x}{5} + \frac{4}{5}\left[-\frac{\sin^2 x \cos x}{3} + \frac{2}{3}\int \sin x \, dx\right]$$

+1

$$= -\frac{\sin^4 x \cos x}{5} \cancel{+} \frac{4}{15} \sin^2 x \cos x \cancel{+} \frac{8}{15} \cos x + c$$

$$\Rightarrow \text{Ans} \quad -\frac{\sin^4 x \cos x}{5} - \frac{4}{15} \sin^2 x \cos x - \frac{8}{15} \cos x + c$$ +1

5