Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2022

EN3BS08 Linear Algebra

Programme: B.Tech. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The inverse of the matrix $\begin{bmatrix} -3 & 5 \\ 2 & 1 \end{bmatrix}$ is- **1**

(a) $\begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & -\frac{3}{13} \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{13} & -\frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{13} & \frac{5}{13} \\ -\frac{2}{13} & \frac{3}{13} \end{bmatrix}$

ii. If A is a real symmetric matrix, then AA^T is- **1**

(a) Always symmetric (b) Unsymmetric

(c) Skey symmetric (d) None of these

iii. The rank of every singular matrix of order n is- **1**

(a) $\leq n$ (b) $< n$ (c) $> n$ (d) $= n$

iv. Which method can be applied to solve systems of linear equations? **1**

(a) LU decomposition method (b) Runge - Kutta method

(c) Newton Raphson method (d) None of these

v. If S_1, S_2 are two bases of a finite dimensional vector space, then which of the following is correct- **1**

(a) $|S_1| > |S_2|$ (b) $|S_1| < |S_2|$ (c) $|S_1| \neq |S_2|$ (d) $|S_1| = |S_2|$

vi. If w_1 and w_2 are two vector subspace of Vector space V(F) with $\dim(w_1) = 12, \dim(w_2) = 8$ and $\dim(w_1 \cap w_2) = 4$ then $\dim(w_1 + w_2)$ is equal to- **1**

(a) 8 (b) 12 (c) 16 (d) 20

vii. Let $V(R)$ be the vector space of all polynomials in indeterminate x with coefficients on R of the form $f(x) = a_0 x^0 + a_1 x^1 + a_2 x^2$. If differential operator D is a linear transformation on $V(R)$, then dimension of null space of D is- **1**

(a) 1 (b) 0 (c) 3 (d) 2

viii. If P is a projection on vector space $X(F)$, then which of the following is true- **1**

(a) $P^2 = P + I$ (b) $P^2 = P - I$ (c) $P^2 = P$ (d) None of these

ix. Singular Value Decomposition is some sort of generalisation of ______ decomposition. **1**

(a) Singular (b) Eigen value (c) Eigen vector (d) None of these

x. Which of the following is an example of Digital Image Processing? **1**

(a) Computer graphics (b) Pixels

(c) Camera mechanism (d) All of these

Q.2 i. Show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and skew-Hermitian matrix. **3**

ii. Define matrix and determinant. What is the difference between them? Explain types of matrices. **7**

OR iii. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using inverse matrix method. **7**

Q.3 i. Define rank of the matrix with examples. **3**

ii. Apply Gauss elimination method to solve the following equations: **7**

$$x + 4y - z = -5;\ \ x + y - 6z = -12;\ \ 3x - y - z = 4$$

OR iii. Find the eigen values and eigen vectors of the matrix: **7**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q.4 i. Show that $W = \{(x, 2y, 3z) : x, y, z \in R\}$ is a subspace of $V_3(R)$. **3**

ii. Show that (2,1,4), (1,-1,2) (3,1,-2) from a basis for R^3. **7**

OR iii. Define dependent and independent vectors of a vector space. If w_1, w_2, w_3 are independent vectors, then show that the difference $w_2 - w_3, w_1 - w_3, w_1 - w_2$, and addition $w_2 + w_3, w_1 + w_3, w_1 + w_2$, are linearly independent. **7**

Q.5 i. Define orthogonal vector. Show that $x - y$ is orthogonal to $x + y$ if and only if $\|x\| = \|y\|$. **3**

ii. Define inner product space. If $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in R^2, let $\langle \alpha, \beta \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 4 x_2 y_2$. Show that $\langle , \rangle$ is an inner product on R^2. **7**

OR iii. Apply the Gram-Schmidt process to- **7**

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

and write the result in the form $A = QR$.

Q6 i. Define Image? **3**

ii. Given data = {2, 3, 4, 5, 6, 7; 1, 5, 3, 6, 7, 8}. Compute the principal component using Principal Component Analysis Algorithm. **7**

iii. Find singular value decomposition of **7**

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

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End Sem Exam 2022

Solution

B Tech (CSBS) Linear Algebra EN3BS08

Q.1 MCQs

1 b $\begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{2}{13} & \frac{3}{13} \end{bmatrix}$ 1

2 a always symmetric 1

3 b $< n$ 1

4 a LU decomposition Method 1

5 d $|S_1| = |S_2|$ 1

6 c 16 1

7 a 1 1

8 c $P^2 = P$ 1

9 b Eigen value 1

10 d All of the mentioned 1

Q.2

i) Let A be a given Square Matrix

then $A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$ 1

Now

$(A+A')' = A' + A = A + A'$ 1

$\therefore$ $A+A'$ is a symmetric Matrix

Also $A - A'$ or $\frac{1}{2}(A-A')$ is a antisymmetric 1

$$\boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$$

Symmetric + Skew symmetric

ii) A system of numbers arranged in a rectangular array in rows and Columns and bounded by the brakets is called a matrix (1/2)

Ex $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

The numerical value of a given matrix is called determinant. For a matrix A it is denoted by $|A|$ and written as det(A) (1 1/2)

Ex $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$ $|A| = 2\times1 - 3\times4 = 2-12 = -10$

Types (6)

1) Column matrix
2) row matrix
3) null matrix
4) Square matrix
5) Diagonal matrix
6) Identity matrix
7) Symmetric matrix
8) Skew symmetric matrix

iii) Let the numbers be a, b and c

ATQ $a+b+c = 6$ — (1) (2)

$b+3c = 11$ — (2)

$a+c = 2b$

$a-2b+c = 0$ — (3)

Algebraic expression of the given problem

Now matrix form of the given system of equation (1)

$AX = B$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ $X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

$X = A^{-1}B$

we have to find out A^{-1}

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix}$$ 1

$$= 1(1+6) - 1(0-3) + 1(-2-1) = 9$$

$\boxed{|A| = 9}$

$$\text{Cofactor matrix} = \begin{bmatrix} 7 & 3 & -1 \\ -3 & 0 & 3 \\ 2 & -3 & 1 \end{bmatrix}$$ 1

$$\text{adj } A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 1

$$X = A^{-1}B$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$ 1

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\boxed{a = 1}$ $\boxed{b = 2}$ $\boxed{c = 3}$

Q.3

1) Rank of the matrix:

The rank of a matrix is said to be r if 2

a) It has at least one non zero minor of order r

b) Every minor of A of order higher than r is zero.

Rank = Number of non zero rows in upper triangular matrix (1)

Example - Any matrix of order 2 or 3

iii) Given System of Equations (1)

$x + 2y - z = -5$ ——— (1)

$x + y - 6z = -12$ ——— (2)

$3x - y - z = 4$ ——— (3)

By method of Gauss Elimination, first eliminate x from (2) & (3) with (1) (2)

we get $3y + 5z = 7$, $y + \frac{5}{3}z = \frac{7}{3}$ — (4)

$13y - 2z = -19$, $y - \frac{2}{13}z = \frac{-19}{13}$ — (5)

Now eliminating y from (4) & (5)

we get $71z = 148$ $\boxed{z = \frac{148}{71}}$ = ~~0.207~~ = 2.08 (4)

By back substitution in (4)

$\boxed{y = \frac{243}{213}}$ = ~~1.140~~ -1.13

Again back substitution of y and z in (1)

we get $\boxed{x = 1.64}$

iii) Given Matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

The Charact equation of the Matrix is $|A - \lambda I| = 0$ — 1

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\{(3-\lambda)^2 - 1\} + 2\{-2(3-\lambda)+2\}$$
$$+2\{2-2(3-\lambda)\} = 0$$ — 2

$$-\lambda^3 + 12\lambda^2 - 36\lambda + 32 = 0$$

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\lambda = 2, 2, 8$$ — 1

Eigen vector for λ is X given by

$$AX = \lambda X$$
$$(A - \lambda I)X = 0$$

for $\lambda = 2$ $\quad (A - 2I)X = 0$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$ $\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to 2R_2 + R_1$ $\begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Rank of a matrix $r=1$ $\therefore$ equations have $3-1=2$ linearly independent solutions

Now

$$4x-2y+2z=0$$

$y=0$ $\quad 4x+2z=0$

$$4x=-2z$$

$$\frac{x}{2}=\frac{-z}{4}=\frac{1}{-2}$$

$$x=1 \quad y=0 \quad z=-2$$

One Eigen vector for $\lambda=2$ $X_1=\begin{bmatrix}1\\0\\-2\end{bmatrix}$

$z=0$ $\quad 4x-2y=0 \quad \frac{x}{y}=\frac{1}{2}$

$$x=1,\ y=2 \quad z=0$$

Another eigen vector for $\lambda=2$ $X_2=\begin{bmatrix}1\\2\\0\end{bmatrix}$

Eigen vector for $\lambda=8$

$$(A-8I)X=0$$

$$\begin{bmatrix}-2&-2&2\\-2&-5&-1\\2&-1&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_3\to R_3+R_1$ $\begin{bmatrix}-2&-2&2\\-2&-5&-1\\0&-3&-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$R_2\to R_2-R_1$ $\begin{bmatrix}-2&-2&2\\0&-3&-3\\0&-3&-3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$R_3\to R_3-R_2$ $\begin{bmatrix}-2&-2&2\\0&-3&-3\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

Rank of Matrix $r = 2$

$\therefore$ Equations will have $3 - 2 = 1$ Linearly Independent Solution

$$-2x - 2y + 3z = 0$$

$$x + y - z = 0 \quad \text{---} (1)$$

$$-3y - 3z = 0 \quad \text{---} (2)$$

$$y + z = 0$$

$$y = -z$$

$$\frac{y}{z} = \frac{-1}{1}$$

$$\boxed{y = -1} \quad \boxed{z = 1}$$

by (1)

$$x - 1 - 1 = 0 \quad \boxed{x = 2}$$

Eigen vector for $\lambda = 8$ is $X_3 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ (2)

Q4, i) Given

$$W = \{(x, 2y, 3z) : x, y, z \in R\}$$

To Prove W is a subspace we show that (1)

$$a, b \in R, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$$

Let $\alpha = (x_1, 2y_1, 3z_1), \beta = (x_2, 2y_2, 3z_2) \in R$

$$a\alpha + b\beta = a(x_1, 2y_1, 3z_1) + b(x_2, 2y_2, 3z_2)$$

$$= (ax_1 + bx_2, 2ay_1 + 2by_2, 3az_1 + 3bz_2) \quad \text{---} (2)$$

$$= (ax_1 + bx_2, 2[ay_1 + by_2], 3[az_1 + bz_2])$$

$\{ax_1 + bx_2 \in R\}$

$\therefore a, b \in R, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$

Hence W is a subspace of $V_3(R)$

ii) Let $S = \{(2,1,4), (1,-1,2), (3,1,-2)\}$

Again let $a_1, a_2, a_3 \in R$ be such that

$a_1(2,1,4) + a_2(1,-1,2) + a_3(3,1,-2) = 0$ — (1)

$\Rightarrow (2a_1 + a_2 + 3a_3,\ a_1 - a_2 + a_3,\ 4a_1 + 2a_2 - 2a_3) = (0,0,0)$

$\therefore$ $2a_1 + a_2 + 3a_3 = 0$ — (1)

$a_1 - a_2 + a_3 = 0$ — (2) — (2)

$4a_1 + 2a_2 - 2a_3 = 0$ — (3)

$\therefore$ The coefficient matrix of these equations is

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 4 & 2 & -2 \end{bmatrix}$$ — (2)

$|A| = 2(2-2) - 1(-2-4) + 3(2+4) = 24 \neq 0$

$\therefore$ Rank$(A) = 3$ i.e. the number of unknowns a_1, a_2, a_3 — (1)

Hence the only solution of these equations is $a_1 = 0$ $a_2 = 0$ $a_3 = 0$ therefore the set is L.I

Also the dimension of R^3 is 3. Hence any set of 3 L.I. vectors is a basis R^3 — (1)

therefore the set S is a basis for R^3

iii) Dependent and Independent vectors

Consider a vector space $V(F)$.

Let $S = \{\alpha_1, \alpha_2 - \alpha_n\}$ be a finite set of vectors of $V(F)$. The set S is called L.D if $\exists$ scalars $a_1, a_2 - a_n \in F$ not all zero (some of them may be zero) such that $a_1\alpha_1 + a_2\alpha_2 + - + a_n\alpha_n = 0$ — (2)

and if $a_1 = 0, a_2 = 0 - - a_n = 0$ then S is linearly independent set

Given w_1, w_2, w_3 are L.I vectors then

$$aw_1 + bw_2 + cw_3 = 0$$

$$\Rightarrow a=0, b=0, c=0$$

For Given set $w_2-w_1, w_1-w_3, w_1-w_2$

$$a(w_2-w_1) + b(w_1-w_3) + c(w_1-w_2) = 0$$

$$(-a+b+c)w_1 + (a-c)w_2 + (-b)w_3 = 0 \quad -(1)$$ — 1

but w_1, w_2, w_3 are L.I

therefore (1) implies that

$$-a + b + c = 0$$

$$a + 0b - c = 0$$

$$0a + \cancel{0b} + 0c = 0$$

The Coefficient Matrix is

$$A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$ — 2

$$|A| = -1(0-1) - 1(0) + 1(-1)$$
$$= 1-1 = 0$$ — 1

Rank $A = 2 \neq$ no of unknowns

Therefore $w_2-w_1, w_1-w_3, w_1-w_2$ is L.D

For Given set $w_2+w_3, w_1+w_3, w_1+w_2$

$$a(w_2+w_3) + b(w_1+w_3) + c(w_1+w_2) = 0$$ — 1

$$(b+c)w_1 + (a+c)w_2 + (a+b)w_3 = 0$$

Coefficient Matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(2)

$$|A| = 0(0-1) - 1(0-1) + 1(1-0)$$

$$= 1 + 1 = 2 \neq 0$$

Rank A = 3 i.e number of unknown
Hence only solution of the equation
a=0, b=0, c=0 Therefore given
set of vectors are LI

Q.5
i) Orthogonal vector - Two vectors are orthogonal if they are perpendicular to each other the dot product of the two vectors is zero (2)

$$\bar{V}_i \cdot \bar{V}_j = 0 \quad \text{for all } i \neq j$$

Example:
The set of vectors $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$
is mutually orthogonal

$$(1, 0, -1) \cdot (1, \sqrt{2}, 1) = 0$$

$$(1, 0, -1)\ (1, -\sqrt{2}, 1) = 0$$

$$(1, \sqrt{2}, 1)\ (1, -\sqrt{2}, 1) = 0$$

$x-y$ is orthogonal to $x+y$ then (1)

$$\langle x-y, x+y \rangle = 0 \quad \text{if } \|x\| = \|y\|$$

$$= x \cdot x + x \cdot y - y \cdot x - y \cdot y$$

$$= \|x\|^2 + \cancel{xy} - \cancel{xy} - \|y\|^2$$ (1)

$$= \|x\|^2 - \|y\|^2 \quad \{xy = yx\}$$

$$= 0 \quad \{\|x\| = \|y\|\}$$

11) Given $\alpha = (x_1 x_2)$ $\beta = (y_1, y_2) \in R^2$

$\langle \alpha, \beta \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 4 x_2 y_2$

It is an inner Product if

1) $\|\alpha\|^2 = \langle \alpha, \alpha \rangle \geq 0$ (-2)

2) $\|\alpha\|^2 = 0 \Rightarrow \|\alpha\| = 0$ if $\alpha = 0$

3) $\langle \alpha, \beta \rangle = \langle \beta, \alpha \rangle$

4) $\langle \alpha + \beta, \gamma \rangle = \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle$

$\langle \alpha, \alpha \rangle = \|x_1\|^2 - x_0 x_2 - x_2 x_1 + 4\|x_2\|^2$ (-1)

$= \|x_1\|^2 \geq 0, \ \|x_2\|^2 \geq 0$

$\langle \alpha, \alpha \rangle = \|\alpha\|^2 \geq 0$

If $\|x_1\| = 0$ then

$\|x_2\| = 0$ $\langle \alpha, \alpha \rangle = 0$ (-1)

$\langle \alpha, \beta \rangle = x_1 y_1 - x_1 y_2 - x_2 y_1 + 4 x_2 y_2$ (-1)

$= y_1 x_1 - y_2 x_1 - y_1 x_2 + 4 y_2 x_2$

$= \langle \beta, \alpha \rangle$

$\langle \alpha + \beta, \gamma \rangle = (x_1 + y_1) z_1 - (x_1 + y_1) z_2$

$- (x_2 + y_2) z_1 + 4 (x_2 + y_2) z_2$ (-2)

$= x_1 z_1 - x_1 z_2 - x_2 z_1 + 4 x_2 z_2$

$+ y_1 z_1 - y_1 z_2 - y_2 z_1 + 4 y_2 z_2$

$= \langle \alpha, \gamma \rangle + \langle \beta, \gamma \rangle$

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iii) Given vectors

$$a = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Performing the Gram Schmidth Procedure

$u_1 = a_1 = (0, 0, 1)$

$e_1 = \frac{u_1}{\|u_1\|} = (0, 0, 1)$ — 1

$u_2 = a_2 - (a_2 \cdot e_1) e_1$

$= (0, 1, 1) - 1 \cdot (0, 0, 1) = (0, 1, 0)$

$e_2 = \frac{u_2}{\|u_2\|} = (0, 1, 0)$ — 1

$u_3 = a_3 - (a_3 \cdot e_1) e_1 - (a_3 \cdot e_2) e_2$

$= (1, 1, 1) - 1 (0, 0, 1) - 1 (0, 1, 0)$

$= (1, 1, 0) - (0, 1, 0) = (1, 0, 0)$ — 1

$e_3 = \frac{u_3}{\|u_3\|} = (1, 0, 0)$

Thus $Q = [e_1 \mid e_2 \mid e_3] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ — 2

$R = \begin{bmatrix} a_1 \cdot e_1 & a_2 \cdot e_1 & a_3 \cdot e_1 \\ 0 & a_2 \cdot e_2 & a_2 \cdot e_3 \\ 0 & 0 & a_3 \cdot e_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ — 2

$A = QR = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q6

i) Image: An image is an artifact that depicts visual perception such as a photograph or other two dimensional picture that resembles a subject usually a physical object thus provides a depiction of it in the context of signal processing. An image is a distributed amplitude of color. — 3

ii)

x	y	A: $x_i-\bar{x}$ ($x_i-4.5$)	B: $y_i-\bar{y}$ (y_i-5)	AB	A^2	B^2
2	1	-2.5	-4	10	6.25	16
3	5	-1.5	0	0	2.25	0
4	3	0.5	-2	-1	0.25	4
5	6	0.5	1	1.5	2.25	1
6	7	1.5	2	5	6.25	4
7	8	2.5	3	10.5	12.25	9

$\Sigma x = 27$ $\Sigma y = 30$ $\Sigma AB = \cancel{26}\ 22$ $\Sigma A^2 = \cancel{29.5}\ 17.5$ $\Sigma B^2 = 34$ — 2

$n = 6$

$\bar{x} = \frac{\Sigma x}{n} = \frac{27}{6} = 4.5$

$\bar{y} = \frac{30}{6} = 5$ — 2

$Cov(x,x) = \frac{\Sigma A^2}{n-1} = 5.9$

$Cov(x,y) = Cov(y,x) = \frac{\Sigma AB}{n-1} = \cancel{5.2}\ 4.4$

$Cov(y,y) = \frac{\Sigma B^2}{n-1} = 6.8$

Now S = Cov mat

$$S = \begin{array}{c|cc} & x & y \\ \hline x & 3.5 & 4.4 \\ y & 4.4 & 6.8 \end{array}$$

$$|S - \lambda I| = 0$$

$$\begin{vmatrix} 3.5-\lambda & 4.4 \\ 4.4 & 6.8-\lambda \end{vmatrix} = 0$$ 1

$$(3.5-\lambda)(6.8-\lambda) - (4.4)^2 = 0$$

$$\lambda^2 - 10.3\lambda + 17.06 = 0$$

By quadratic formula

$$\lambda_1 = 10.71, \quad \lambda_2 = \cancel{1.53}\ 0.41$$

find eigenvector for $\lambda_1 = 10.71$, $\begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}$ 2
and $\lambda_2 = \cancel{1.53}\ 0.41$ $\begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}$ then

Principal Components are given by

$$Z_1 = a_{11}x_1 + a_{12}x_2$$

and

$$Z_2 = a_{21}x_1 + a_{22}x_2$$

iii) Singular value Decomposition of

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$ with rank

2 thus A has two Singular values σ_1 & σ_2. To find σ_1 & σ_2 Begin with 2

$$A^T A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad A A^T = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

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Find eigen values of A^TA and AA^T

$|A^TA - \lambda I| = 0$ and $|AA^T - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} = 0 \qquad \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$ — 2

we get $\lambda = 0, 1, 3$

σ_1 is larger than $\lambda_{max} = 3$

σ_2 is smaller then $\lambda_{min} = 1$

Trace of A^TA and $AA^T = 4$ — 1

Singular values are $\sigma_1 = \sqrt{3}$ $\sigma_2 = 1$

Eigenvector for $\lambda = 3$

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$ — 2

$v_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $v_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Singular value Matrix

$$\Sigma = \begin{bmatrix} \sqrt{3} & & \\ & \sqrt{1} & \\ & & 0 \end{bmatrix}$$