OR iii. A thin plate of two cross sections as shown in Figure 4 is subjected to a load P= 500 N at node point 2. The thickness of plate is 10 mm and for the plate material E= 200 GPa and density= 7850kg/m^3. Using a FE model of two bar elements, determine- **7**

(a) Nodal displacements (b) Stresses induced in each section.

(Use either elimination approach or penalty approach to handle the boundary conditions).

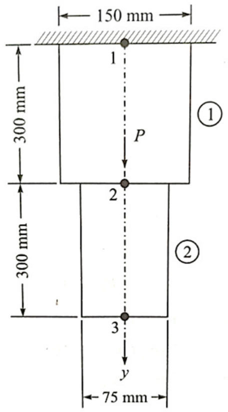

Figure 4

Q.5 i. Write the equations for stresses and strain for both plane stress and plan strain conditions for an 2-D element. Evaluate the Shape functions for the point `P' for the triangular element shown in Figure 5. **4**

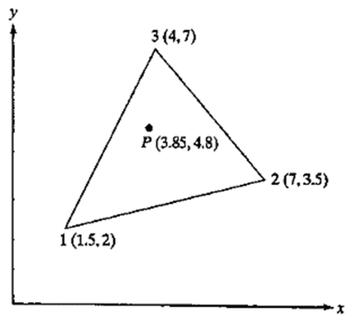

Figure 5

ii. A plate as shown in Figure 6, is discretized using two triangular elements. Determine the strain displacement matrices (B) for the two elements. **6**

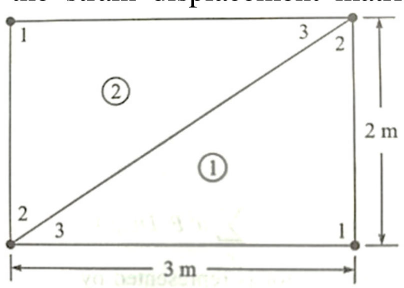

Figure 6

Total No. of Questions: 6 *Total No. of Printed Pages:5*

Enrollment No......................................

Faculty of Engineering
End Sem Examination May-2024
ME3EL05 Finite Element Method

Programme: B.Tech. Branch/Specialisation: ME

Duration: 3 Hrs. Maximum Marks: 60

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Which one of these is weighted residual method – **1**

I. Point collocation method
II. Method of least squares
III. Galerkin's method

(a) Only I (b) Only II (c) Both I & II (d) All of these

ii. Which one is correct about Rayleigh Ritz method? **1**

I. It requires problem as expression of potential energy in Integral form
II. It yields an exact solution

(a) Only I (b) Only II (c) Both I & II (d) None of these

iii. The principle of minimum potential energy involves ______ that represents a physical phenomenon. **1**

(a) Minimization of a function
(b) Maximization of a function
(c) Minimization of a differential operator
(d) None of these

iv. Which amongst these is correct? **1**

I. Essential boundary conditions are sufficient to solve the problem completely
II. Natural boundary conditions are not sufficient to solve the problem completely

(a) Only I (b) Only II (c) Both I & II (d) None of these

v. For a one-dimensional domain, discretized with `N' number of two node bar elements, the global stiffness matrix will be- **1**

(a) It is N x N matrix (b) It is (N+1) x (N+1) matrix
(c) It is (N-1) x (N-1) (d) More information is required

vi. In penalty approach of handling boundary conditions, a spring of ______ stiffness is used to model the support at which boundary condition is specified. **1**

(a) Very low (b) Normal (c) Very high (d) None of these

vii. For a constant strain triangle, the shape function N_1, N_2 and N_3 are given by- **1**

(a) $N_1=\xi$, $N_2=\eta$, $N_3= 1-\xi-\eta$

(b) $N_1=1$, $N_2=\eta$, $N_3=\xi$

(c) $N_1=\xi$, $N_2=\eta$, $N_3=\xi+\eta$

(d) None of these

viii. Which one is an example of Plain strain condition? **1**

I. A thin ring press fitted on a cylinder

II. A long cylinder carrying a pressurized liquid

(a) Only I (b) Only II (c) Both I & II (d) None of these

ix. The connvective heat loss in the fin is accounted for by adding the term ______ to d/dx(k dT/dx). **1**

(a) $-Plh/Ac(T-T_\infty)$ (b) $Plh/Ac(T-T_\infty)$

(c) $Ph/Ac(T-T_\infty)$ (d) $-Ph/Ac(T-T_\infty)$

x. For a one dimensional fluid element of length `L', the hydraulic gradient matrix is given by- **1**

(a) $g=1/L\ [-1\ \ 1]\ \{p_1\ p_2\}^T$ (b) $g=1/2L\ [-1\ \ 1]\ \{p_1\ p_2\}$

(c) $g=1/3L\ [-1\ \ 1]\ \{p_1\ p_2\}^T$ (d) $g=1/4L\ [-1\ \ 1]\ \{p_1\ p_2\}^T$

Q.2 i. What do you mean by a governing equation for a physical phenomenon? Give two examples. **2**

ii. Briefly discuss the solution methodologies used to solve an engineering problem. **3**

iii. Using Rayleigh –Ritz method, determine the displacement of mid-point of a bar subjected to axial load as shown in Figure 1. The potential energy of the rod, neglecting body and traction forces, is given by- **5**

$$\Pi = \frac{1}{2}\int_0^L EA\left(\frac{du}{dx}\right)^2 dx - 2u_1$$

where $u_1= u$ (at x=1).

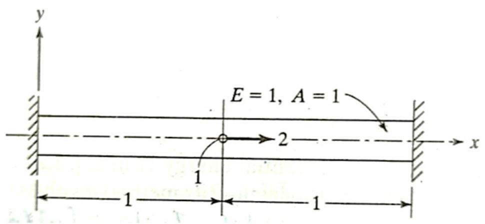

Figure 1

Use two-degree polynomial as approximate function and consider the units of various parameters as consistent.

OR iv. Discuss atleast six application fields of FEM and write its atleast two advantages and limitations. **5**

Q.3 i. What is discretization of domain? Explain the concept of degree of freedom of a node for a discretized domain with one example. **2**

ii. With neat sketches, explain the different types of elements used in a finite element model. **3**

iii. Discuss various approaches used to solve problems in FEM. Give example for each approach. **5**

OR iv. State principle of minimum potential energy. Using it, determine the displacements of the nodes of the spring system, shown in Figure 2. **5**

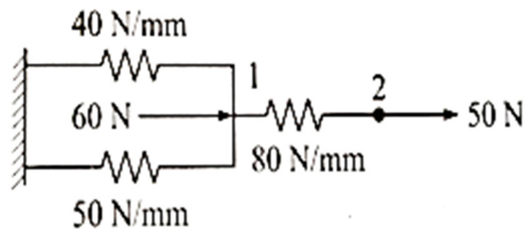

Figure 2

Q.4 i. What is a one-dimensional problem? For a one-dimensional bar element, what are the shape functions? Express shape functions of bar element in terms of local coordinate system. **3**

ii. A two step bar is subjected to a point load P=200 kN as shown in Figure 3. Using any of the two approaches to handle the boundary conditions, determine- **7**

(a) Nodal displacements (b) Stresses in each section

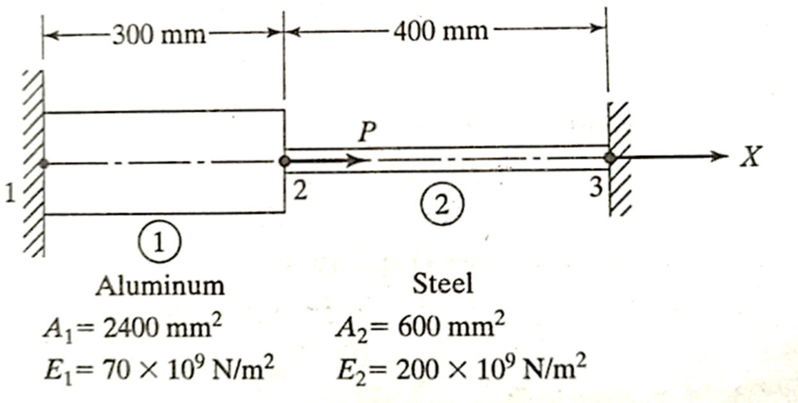

Figure 3

OR iii. A two-dimensional plate is shown in Figure 7. Determine the traction loads at nodes 3 and 4 for linearly distributed pressure load acting on the edge 3-4. The thickness of plate is 10 mm. **6**

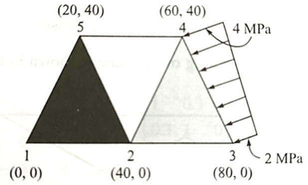

Figure 7

Q.6 Attempt any two:

i. Determine the temperature distribution through a composite wall as shown in Figure 8. When the convection heat loss occurs on the left surface assuming unit area of cross section. Assume wall thickness t_1= 4 cm and t_2= 2cm, k_1=0.5W/cm°C., k_2=0.05W/cm°C, h= 0.1 W/cm^2°C and T_∞= - 5°C. **5**

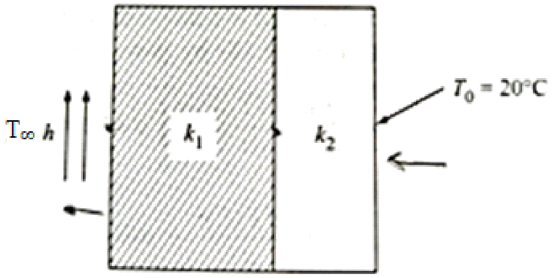

Figure 8

ii. A smooth pipe of variable cross section as shown in Figure 9, carries a liquid. Determine the potentials at the junctions, the velocities in each pipe and the volumetric flow rate. The potentials at the left end is 10 m and that at the right end is 2 m. The permeability coefficient is 1 m/s. **5**

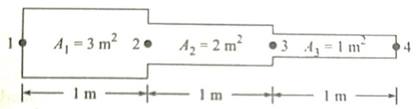

Figure 9

iii. What is a Eigen value-Eigen vector problem for a un-damped free vibration? Write equations of- consistent mass matrix, damped matrix and stiffness matrix for a single degree of freedom vibrating body and hence prove that for a vibrating bar element, the consistent mass matrix- **5**

$$me = \frac{\rho AL}{6}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$$

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[5]

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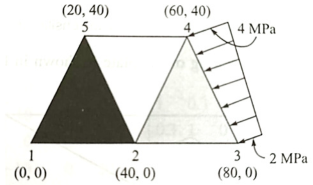
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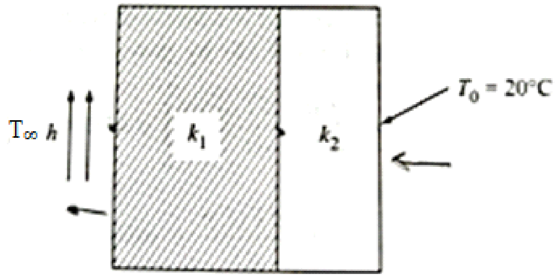
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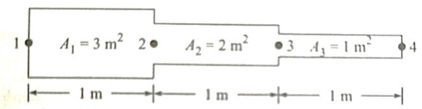
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Marking Scheme

Finite Element Method (T) - ME3EL05 (T)

Q.1	i)	d) All the three	1
	ii)	a) Only 1)	1
	iii)	b) minimization of a functional	1
	iv)	c) Both 1) and 2)	1
	v)	b) It is (N+1) x (N+) matrix	1
	vi)	c) very high	1
	vii)	a) $N_1=\xi$, $N_2= \eta$, $N_3= 1- \xi- \eta$	1
	viii)	b) Only 2)	1
	ix)	d) - Ph/Ac(T-T_∞)	1
	x)	a) g=1/L [-1 1] $\{p_1\ p_2\}^T$	1
Q.2	i.	Governing equation for a physical phenomenon?	1
		Two examples.	1
	ii.	Three solution methodologies	3
	iii.	Expression of Potential Energy	2
		Determination of displacement	2
		Expression of stress	1
OR	iv.	Six application fields	3
		Two advantages and limitations.	2
Q.3	i.	Dicretization	1
		Concept of degree of freedom of a node	1
	ii.	Six sketches of each of the three types of elements	6x1/2
	iii.	Four Approaches	4
		Examples	1
OR	iv.	Principle of minimum potential energy.	1
		Displacements of the nodes	4
Q.4	i.	One Dimensional problem?	1
		Shape functions for bar element.	1
		Shape functions in local coordinate system	1
	ii.	a) Nodal displacements	5
		b) Stresses in each section	2
OR	iii.	a) Nodal displacements	5
		b) Stresses induced in each section	2
Q.5	i.	Equations for stresses and strain	2
		Evaluate the Shape functions	2
	ii.	Determination of two Strain Displacement Matrices (B)	3x2
OR	iii.	Determination of Traction loads at two nodes	3x2
Q.6			
	i.	Two Element stiffness matrices	2
		Global Stiffness matrix	1
		Global Load Vector	1
		Temperatures	1
	ii.	Three Elemental Stiffness Matrices and assembly	3
		Potentials at junctions	1
		Velocities at junctions	1
	iii.	Eigen value-Eigen vector problem for a un-damped free vibration?	1
		Equations	
		Proof of consistent mass matrix for a vibrating bar element,	1
			3

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