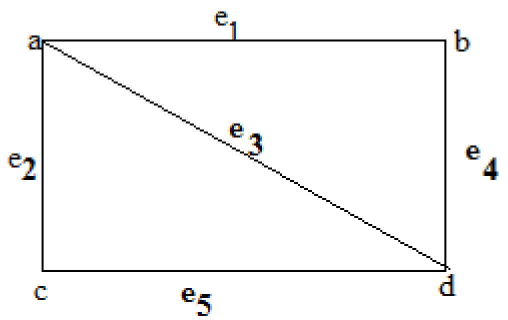
Q.5 Attempt any two:

i. Define cut-set, fundamental cut-sets, edge connectivity, vertex connectivity and separability. **5**

ii. Draw Kuratowski's first and second non-planner graph, Kuratowski's theorem and using it prove that K_6 is non-planner **5**

iii. Find the circuit subspace V_C, and its basis for the graph **5**



Q.6 Attempt any two:

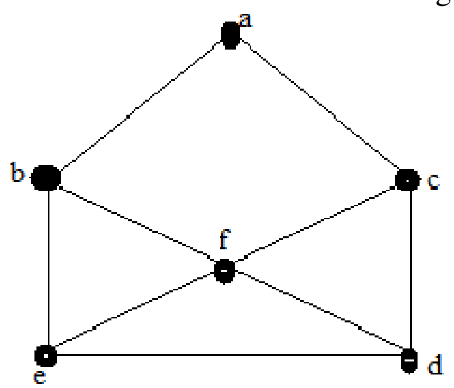
i. Define chromatic polynomial and prove that the chromatic polynomial of $K_{2,6}$ is **5**

$$P_n(\lambda)=\lambda(1-\lambda)^6+\lambda(\lambda-1)(\lambda-2)^6$$

ii. Write short note on: **5**

(a) Graph partitioning (b) Four colour problem

iii. Find chromatic number and minimal covering of the graph **5**



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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2018

BC3EM01 Graph Theory

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Maximum degree of any vertex in a simple graph with n vertices is **1**

(a) n-1 (b) n(n-1) (c) n (d) None of these

ii. A graph with all vertices having equal degree is known as **1**

(a) Multi Graph (b) Simple Graph

(c) Regular Graph (d) Complete Graph

iii. What is the dimensions of an Adjacency matrix? **1**

(a) Number of edges × number of edges

(b) Number of edges × number of vertices

(c) Number of vertices × number of vertices

(d) None of these

iv. The rank of incidence matrix of a disconnected graph of n-vertices and k-components is **1**

(a) n (b) n-k (c) k (d) None of these

v. A Binary Tree can have **1**

(a) Can have 2 children (b) Can have 1 children

(c) Can have 0 children (d) All of these

vi. Find the shortest path using **1**

(a) Dijkstra's algorithm (b) Krushkal's Algorithm

(c) Both (a) and (b) (d) None of these

vii. Edge connectivity of complete graph of n vertices is **1**

(a) n-1 (b) n-2 (c) 2n (d) None of these

P.T.O.

viii. There will be total _____ subgraph of G which can be represented by unique linear combination of five basis vector. **1**

(a) 25 (b) 16 (c) 32 (d) None of these

ix. The minimum number of colors are required to color the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same color as **1**

(a) 2 (b) 3 (c) 4 (d) 5

x. If the regions of any map are colored so that adjacent regions have different colors. Then no more than k colors are requires. What is the value of k **1**

(a) 1 (b) k (c) k-1 (d) k/2

Q.2 Solve any two:

i. Define following with example **5**

(a) Walk (b) Euler graph

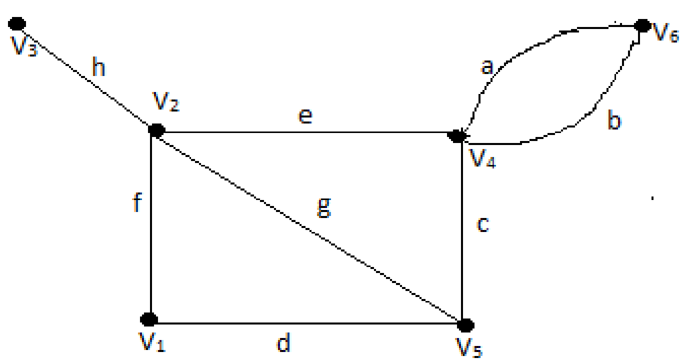
(c) Hamiltonian graph (d) Isomorphic graph

(e) Spanning sub graph.

ii. Prove that the sum of the degrees of all vertices in a graph is twice the number of edges. **5**

iii. A simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **5**

Q.3 i. Define circuit matrix. Represent circuit matrix for the given graph **4**



ii. Define the following with example: **6**

(a) Cut set matrix (b) Fundamental circuit matrix

(c) Path matrix.

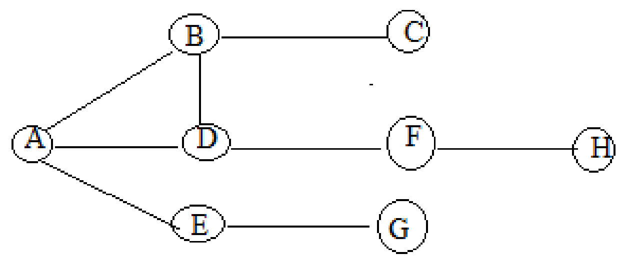
OR iii. Define adjacency matrix. Draw a graph for the following adjacency matrix **6**

$$\begin{bmatrix} 2 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

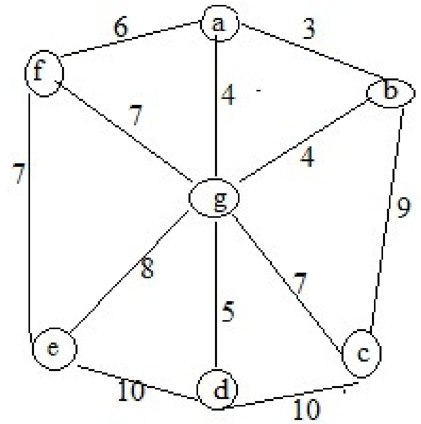
Q.4 Attempt any two:

i. Define a tree, and prove that a graph is a tree if and only if it is minimally connected. **5**

ii. Traverse the graph using Breadth First search algorithm **5**



iii. Find minimal spanning tree using Prim's algorithm **5**



Marking Scheme

BC3EM01 Graph Theory

Q.1 i. Maximum degree of any vertex in a simple graph with n vertices is
a) n-1

ii. A graph with all vertices having equal degree is known as
c) Regular Graph

iii. What is the dimensions of an Adjacency matrix
c) Number of vertices $\times$ number of vertices

iv. The rank of incidence matrix of a disconnected graph of n-vertices and k-components is
b) n-k

v. A Binary Tree can have
(a) Can have 2 children

vi. Find the shortest path using
a) Dijkstra's algorithm

vii. Edge connectivity of complete graph of n vertices is
a) n-1

viii. There will be total _____ subgraph of G which can be represented by unique linear combination of five basis vector.
c) 32

ix. The minimum number of colors are required to color the vertices of a cycle with n nodes in such a way that no two adjacent nodes have the same color
b) 3

x. If the regions of any map are colored so that adjacent regions have different colors. Then no more than k colors are requires. What is the value of k
d) k/2

Q.2 (i) a) Walk Def. +1/2
eg: +1/2
b) Euler graph +1/2
eg: +1/2

c) Hamiltonian graph +1/2
eg: +1/2

d) Isomorphic graph +1/2
eg: +1/2

e) Spanning subgraph +1/2
eg: +1/2

5

Q.2 (ii) Proof Let $G=(V,E)$ be a graph and let no. of edges in G be e i.e. order of $E=e$. Then we are to prove that

$$\sum_{v\in V} deg(v) = 2e$$ where $v\in V$ is any vertex (+1)

we shall prove by induction on the no. of edges

step 1 : If no. of edges in G is zero i.e. there is no edge in G i.e. $e=0$
Also in this case degree of each vertex $v\in V$ is zero then

$$\sum_{v\in V} deg(v) = 0$$ (+1)

$$\sum_{v\in V} deg(v) = 2e = 2\times 0 = 0.$$

∴ the theorem is true in this case

Page No.:

Step 2 If $e=1$ i.e. if there is only one edge in G. In this case the graph G has only two vertices and the degree of each vertex is one

$$\sum_{v\in V} deg(v) = 1+1 = 2 = 2\times 1 = 2e \quad i.e.$$

$$\sum_{v\in V} deg(v) = 2e.$$

$\therefore$ the theo. is true in this case (+1)

Step 3 : Now assume that the theo is true for all graphs having $e-1$ edges. Let G be a graph having e edges. Delete one edge, say, $e'=(a,b)$ from G. Thus a new graph G', say is obtained having $e-1$ edges where $G' = G - \{e'\}$. Therefore by hypothesis; we have in G', (+1)

$$\sum deg(v) = 2(e-1)$$

Now if we replace the edge $e'=(a,b)$ to obtain the graph G, then the degree of each of vertices a and b will be increased by one. Therefore adding the edge $e'=(a,b)$ to G' to obtain G, we have in G

$$\sum deg(v) = 2(e-1)+2$$ (+1)

$$= 2e$$

(iii) Let G be a simple graph i.e. $G_1, G_2, \ldots G_K$ be K components of G. Let $n_1, n_2, \ldots n_K$ be the no. of vertices in $G_1, G_2, \ldots G_K$ resp. so that

$$n_1 + n_2 + \cdots + n_K = n$$

$$\sum_{i=1}^{K} n_i = n \quad n_i \geq 1 \ \forall\, i = 1, 2, \ldots K$$ +1

where n is the no. of vertices in G.

we have $\sum_{i=1}^{K} (n_i - 1) = (n_1 - 1) + (n_2 - 1) + \cdots + (n_K - 1)$

$$= \sum_{i=1}^{K} n_i - K = n - K$$

Squaring on both side

$$\left[\sum_{i=1}^{K} (n_i - 1)\right]^2 = (n-K)^2$$ +1

$$\sum_{i=1}^{K} n_i^2 - 2n + K \leq n^2 + K^2 - 2nK$$

$$\sum_{i=1}^{K} n_i^2 \leq n^2 - (2n-K)(K-1)$$ +1

Since the max. no. of edges in a simple graph with n vertice $\frac{1}{2} n(n-1)$.

$\therefore$ the i^{th} component of G has the most $\frac{1}{2} n_i(n_i - 1)$ edges. Hence the max. no. of edges in the graph G.

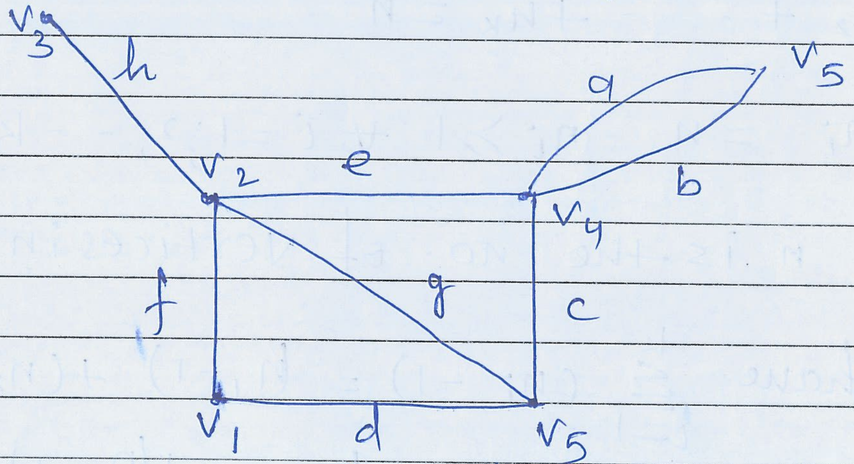
$$\sum_{i=1}^{K} \frac{1}{2} n_i(n_i - 1) = \frac{1}{2}\left[\sum_{i=1}^{K} n_i^2 - \sum_{i=1}^{K} n_i\right]$$ +1

$$= \frac{1}{2}\left[\sum_{i=1}^{K} n_i^2 - n\right]$$

$\leq \frac{1}{2}(n-k)(n-k+1)$ +1

Q.3 (i) <u>Circuit Matrix</u> : +1

For defination



there are 4 different circuit

(a, b), (c, e, g), (d, f, g) (c, d, f, e) +1

$$B(G) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccccc} a & b & c & d & e & f & g & h \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$ +2

For Observations ~~+1~~

4

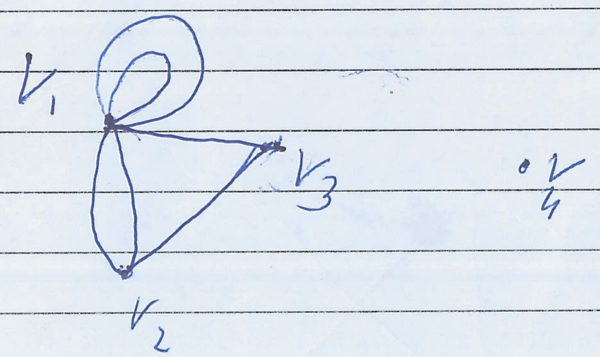
Q.3

ii Cut set matrix +2
Fundamental circuit matrix +2
Path matrix +2

6

(iii) Definition of Adjcency Matrix +2

$$\begin{array}{c c} & \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} & \begin{bmatrix} 2 & 2 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



Page No.:

Q.4

(i) Tree is a simple connected circuitless graph (+1)

Let G be a minimally connected graph, i.e. removal of one edge disconnects the graph

$\Rightarrow \exists$ no circuit in the graph (+2)

$\Rightarrow$ G is tree

Conversely, let G be tree i.e. $\exists$ one and only one path between every pair of vertices, removal of one edge disconnects the path, hence G is minimally connected (+2)

Q.4 (ii) For display (+1)

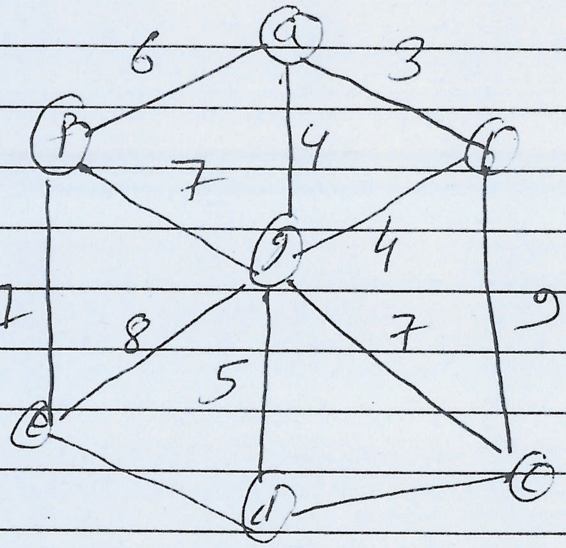
For explaining algorithm using queue discipline, entry and exist of node from queue, of given graph (+3)

For output : ABDECFGH (+1)

OR ABBCFGH

Q.4 (iii)

Q4 (iii) Prim's Algorithm

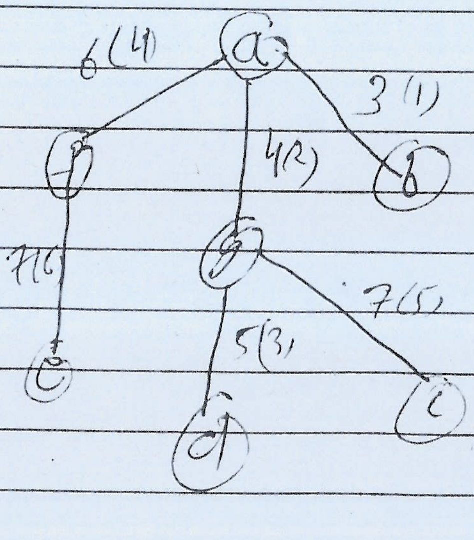


Matrix

	a	b	c	d	e	f	g
a	–	3	–	–	–	6	4
b	~~3~~	–	9	–	–	–	4
c	–	9	–	10	–	–	7
d	–	–	10	–	10	–	~~5~~
e	–	–	–	10	–	~~7~~	8
f	~~6~~	–	–	–	7	–	7
g	~~4~~	4	7	5	8	7	–

+2

Steps for algorithm +1



+2

Total weight = 32

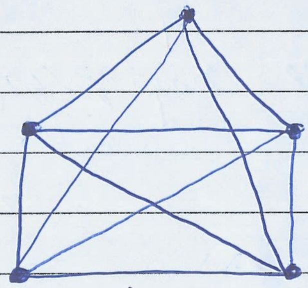
5 marks

Q5 (i) Cut set +1
Fundamental cut set +1
Edge connectivity +1
Vertex connectivity +1
Separability +1

5

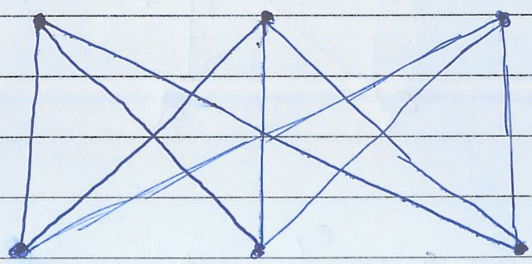
(ii)

Kuratowski's First graph



K_5 +1

Kuratowski's Second graph



$K_{3,3}$ +1

Theorem :- A necessary and sufficient conditions for graph G to be planar is that G does not contain either of Kuratowski's graph or homeomorphic to either of them as a subgraph of G. +1

Page No.:

For K_6: Simple graph with 6 vertices
i.e. $n = 6$

e = no. of edges $= \frac{1}{2} n(n-1)$
$= 15$

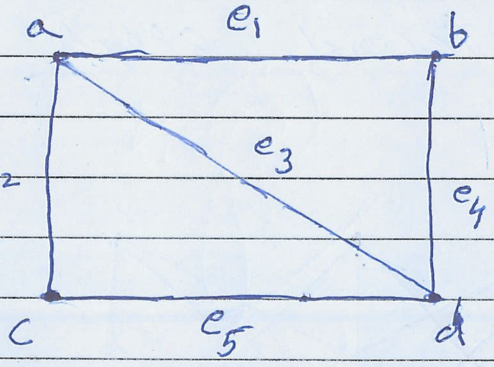
Also $3n - 6 = 3 \times 6 - 6$
$= 12 < 15 = e$

thus
$e \leq 3n - 6$ is not satisfied +2

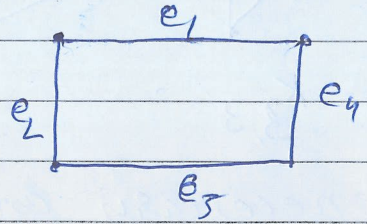
$\Rightarrow K6$ is non planar.

5

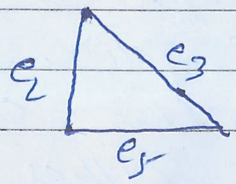
Q5 (iii)



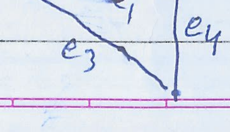
Circuit subspace



$(1, 1, 0, 1, 1)$ +1



$(0, 1, 1\ 0\ 1)$ +1



$(1, 0, 1, 1, 0)$ +1

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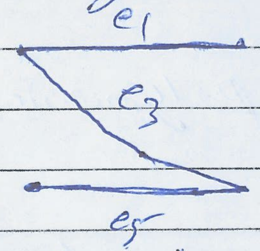
Consider spaning Tree



Fundamental circuits are (e_1, e_4, e_3) and (e_2, e_3, e_5)

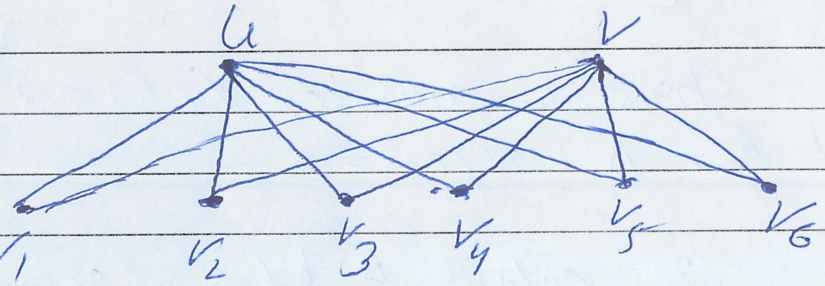
thus the basis corresponding to circuit vector subspace is

$(1, 0\ 1, 1, 0)$ and $(0, 1, 1, 0, 1)$ +2

5 marks

Q.6 (i) Defination: Chromatic polynomial +1

$K_{2,6}$



u and v are two non adjacent vertices,

If u and v are colored with different colors then chromatic polynomial for proper coloring is given by 1.5

$\lambda(\lambda-1)(\lambda-2)^6$

If u and v are colored with different colors then chromatic poly