Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Pharmacy

End Sem (Odd) Examination Dec-2019

PY3RC02 Remedial Mathematics

Programme: B. Pharma Branch/Specialisation:

Duration: 3 Hrs. **Maximum Marks: 75**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The value of $\lim_{x\to 0}\frac{\tan x}{x}$ is _____________. **2**

ii. The partial fraction of polynomial $\frac{x+5}{(x-3)^2}$ can be written as ________. **2**

iii. The Characteristic Values of A are 1,3,5 then the characteristic values of A^{-1} is _____________. **2**

iv. If $|A| \neq 0$ then matrix A is called as _____________. **2**

v. The differential coefficient of $\cos ecx$ with respect to x is _____________. **2**

vi. If $f''(a) > 0$ then the point $x = a$ is called as point of _____________. **2**

vii. The point $(-5,2)$ lies in which of the following quadrant. **2**

viii. The value of the integral $\int \sin x\,dx$ is _____________. **2**

ix. The degree of the differential equation $\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}+y=0$ is _______. **2**

x. The Laplace transform of $\cos at$ is _____________. **2**

Q.2 Attempt any two:

i. Resolve into partial fraction $\frac{2x+1}{(x-2)(x+1)^2}$. **10**

ii. If $A=\begin{bmatrix}1 & 2\\ 3 & 5\end{bmatrix}, B=\begin{bmatrix}3 & 4\\ -1 & 2\end{bmatrix}$ then find $A+B, 2A-5B, A^{-1}, B^{-1}$. **10**

P.T.O.

iii. (a) Evaluate $\lim_{x\to 5}\frac{1-\sqrt{x-4}}{x-5}$. **5**

(b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ **5**

Q.3 Attempt any seven: Two questions from each section is compulsory.

Section - A

i. Find the differential coefficient of the function $y = x^3 + \tan x + e^{5x}$ with respect to x. **5**

ii. Find the differential coefficient of the function $y = x^2$ with respect to x using first principle. **5**

iii. Find the maximum and minimum value of the function $x^3 - 3x$. **5**

Section - B

iv. Find the equation of a line passing through the points $(2,-3)$ and $(4,6)$. **5**

v. Evaluate the integral $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx$. **5**

vi. Evaluate the integral $\int x^2 \sin x\,dx$. **5**

Section - C

vii. Solve the differential equation $(x^2 - y^2)dx + 2xydy = 0$ **5**

viii. Solve the differential equation $x\frac{dy}{dx} - 2y = x^2$. **5**

ix. Find the Laplace transform of the function $te^{3t} + \sin 4t$. **5**

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Scheme / Solution

Q.1

(i) The value of $\lim_{x \to 0} \frac{\tan x}{x}$ is $\underline{1}$ (+2)

(ii) $\frac{A}{x-3} + \frac{B}{(x-3)^2}$ (+2)

(iii) 1, 1/3, 1/5 (+2)

(iv) Non-singular Matrix (+2)

(v) $-\operatorname{Cosec}x \operatorname{Cot}x$ (+2)

(vi) minima (+2)

(vii) second quadrant (+2)

(viii) $-\operatorname{Cos}x + C$ (+2)

(ix) 1 (+2)

(x) $\frac{s}{s^2+a^2}$ (+2)

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Q. 2 (i) $\frac{2x+1}{(x-2)(x+1)^2}$

$$\frac{2x+1}{(x-2)(x+1)^2} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad \text{---(i)} \qquad (+2)$$

$$2x+1 = A(x+1)^2 + B(x-2)(x+1) + C(x-2) \quad \text{---(ii)} \qquad (+2)$$

Putting $x=2$ in (ii)

$$A = 5/9 \qquad (+2)$$

Putting $x=-1$ in (ii)

$$C = 1/3 \qquad (+2)$$

Putting $x=0$ in (ii)

$$1 = A - 2B - 2C$$

$$1 = \frac{5}{9} - 2B - 2/3 \Rightarrow B = -5/9 \qquad (+1)$$

Hence

$$\frac{2x+1}{(x-2)(x+1)^2} = \frac{5}{9(x-2)} - \frac{5}{9(x+1)} + \frac{1}{3(x+1)^2} \qquad (+1)$$

(ii) $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$

find $A+B$, $2A-5B$, A^{-1}, B^{-1}

$$A+B = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix} \qquad (+1)$$

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$$A+B = \begin{bmatrix} 4 & 6 \\ 2 & 7 \end{bmatrix}$$ (+2)

$$2A-5B = 2\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix} - 5\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$$ (+1)

$$= \begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix} - \begin{bmatrix} 15 & 20 \\ -5 & 10 \end{bmatrix}$$ (+1)

$$2A-5B = \begin{bmatrix} -13 & -16 \\ 11 & 0 \end{bmatrix}$$ (+1)

Now to find A^{-1} & B^{-1}

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = 5-6 = -1 \neq 0$$ (+1)

Hence A^{-1} will exist.

we have

$$C_{11} = (-1)^{1+1}\, 5 = 5$$

$$C_{12} = (-1)^{1+2}\, 3 = -3$$

$$C_{21} = -2, \quad C_{22} = 1$$

$$\text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{Adj} A}{|A|}$$

$$= \frac{\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}}{-1}$$

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$$ (+1)

given $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$

$$|B| = \begin{vmatrix} 3 & 4 \\ -1 & 2 \end{vmatrix} = 6+4 = 10 \neq 0$$ (+1)

B^{-1} will exists

$C_{11} = 2 \quad C_{12} = 1 \quad C_{21} = -4, \quad C_{22} = 3$

$$\text{Adj } B = \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{\text{Adj } B}{|B|}$$

$$= \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} / 10$$

$$= \begin{bmatrix} 1/5 & -2/5 \\ 1/10 & 3/10 \end{bmatrix}$$ (+1)

(iii) (a) $\lim_{x \to 5} \frac{1-\sqrt{x-4}}{x-5}$

$\lim_{x \to 5} \left[\frac{1-\sqrt{x-4}}{x-5} \left(\frac{1+\sqrt{x-4}}{1+\sqrt{x-4}}\right)\right]$ (+2)

$= \lim_{x \to 5} \left[\frac{1-(x-4)}{(x-5)(1+\sqrt{x-4})}\right]$ (+1)

$= \lim_{x \to 5} \left[\frac{5-x}{(x-5)(1+\sqrt{x-4})}\right]$

$= \lim_{x \to 5} \frac{-1}{1+\sqrt{x-4}}$ (+1)

$= \frac{-1}{1+\sqrt{5-4}} = -\frac{1}{2}$ (+1)

(b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Characteristic eqn

$|A - \lambda I| = 0$

$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$ (+2)

$(1-\lambda)(4-\lambda) - 6 = 0$

$4 - 5\lambda + \lambda^2 - 6 = 0$

$\lambda^2 - 5\lambda - 2 = 0$ (+1)

Put $\lambda = A$ and verify

$$A^2 - 5A - 2I = 0$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$ (+1)

$$A^2 - 5A - I$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ (+1)

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section A

Q.3

(i) $y = x^3 + \tan x + e^{5x}$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(\tan x) + \frac{d}{dx}(e^{5x}) \quad (+1)$$

$$\frac{d}{dx} x^n = n x^{n-1}, \quad \frac{d}{dx} e^{ax} = a e^{ax} \quad (+2)$$

$$\frac{dy}{dx} = 3x^{3-1} + \sec^2 x + 5e^{5x} \quad (+1)$$

$$\frac{dy}{dx} = 3x^2 + \sec^2 x + 5e^{5x} \quad (+1)$$

(ii) $y = x^2$

By definition of first principle

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (+2)$$

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \quad (+1)$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2}{h} \quad (+1)$$

$= \lim_{h \to 0} (2x+h)$

$= 2x$ (+1)

(iii) $y = x^3 - 3x$

$f'(x) = \frac{dy}{dx} = 3x^2 - 3$ (+1)

$f''(x) = \frac{d^2y}{dx^2} = 6x$ (+1)

For maxima and minima of $f(x)$

Put $f'(x) = 0$ i.e.

$3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

at $x = 1$

$f''(1) = 6 = +ve$ +1

$f(x)$ is min at $x = 1$

at $x = -1$

$f''(-1) = -6 = -ve$ +1

$f(x)$ is maximum at $x = -1$

minimum value of $f(x) = f(1) = 1^3 - 3 \times 1$

$= -2$

maximum value of $f(x) = f(-1) = (-1)^3 - 3(-1)$ (+1)

$= +2$

Section B

(iv) Equation of the line passing through (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad (+2)$$

$x_1 = 2$, $y_1 = -3$

$x_2 = 4$, $y_2 = 6$ (+1)

Putting values

$$y + 3 = \frac{6+3}{4-2}(x-2) \quad (+1)$$

$$y + 3 = \frac{9}{2}(x-2)$$

$$2y + 6 = 9x - 18$$

$$9x - 2y - 24 = 0 \quad (+1)$$

(v) $\displaystyle\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

let $I = \displaystyle\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$ (+1)

Put $\tan^{-1}x = t$

so $\dfrac{1}{1+x^2}\,dx = dt$ (+2)

$I = \int e^t dt$ (+1)

$= e^t + c$

$= e^{\tan^{-1} x} + c$ (+1)

(vi) $\int x^2 \sin x \, dx$

Integrating by parts taking x^2 as 1st funⁿ (u) $\int u.v\,dx = u\int v\,dx - \int\left(\frac{d}{dx}u \int v\,dx\right)dx$ (+1)

$= x^2(-\cos x) - \int 2x(-\cos x)\,dx$ (+1)

$= -x^2\cos x + 2\int x\cos x\,dx$

$= -x^2\cos x + 2\left[x\sin x - \int 1\sin x\,dx\right]$ (+1)

$= -x^2\cos x + 2\left[x\sin x + \cos x\right]$ (+1)

$= -x^2\cos x + 2x\sin x + 2\cos x + C$ (+1)

Section - C

(vii) $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \quad \text{—— (i)}$$ (+1)

This is a homogeneous differential equation

Put $y = vx$ (+1)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{—— (ii)}$$

from (i) and (ii)

$$v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x^2v}$$

$$v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\frac{2v}{v^2 + 1}\,dv = \frac{-1}{x}\,dx$$ (+1)

Integrating both sides

$$\int \frac{2v}{v^2+1}\,dv = -\int \frac{1}{x}\,dx \cancel{dx}$$

$v^2+1 = t$

$2v\,dv = dt$

$$\int \frac{dt}{t} = -\int \frac{1}{x}\,dx + \log c \qquad (+1)$$

$$\log t = -\log x + \log c$$

$$\log(v^2+1) = -\log x + \log c$$

$$x(v^2+1) = c$$

$v = y/x$

$$x\left(\frac{y^2+x^2}{x^2}\right) = c$$

$$y^2+x^2 = xc \qquad (+1)$$

(viii) $x\dfrac{dy}{dx} - 2y = x^2$

$$\frac{dy}{dx} - \frac{2y}{x} = x \quad \text{--- (1)} \qquad (+1)$$

eqn (1) is in linear differential eqn form $\dfrac{dy}{dx} + Py = Q$ (+1)

(ix)

$$L\{te^{3t}+\sin 4t\}$$

$\Rightarrow L\{te^{3t}\}+L\{\sin 4t\} = ?$ (+1)

$= L\{te^{3t}\}$

so $L\{t\} = 1/s^2 = f(s)$ (+1)

By first shifting we have
If $L\{F(t)\} = f(s)$ then

$$L\{e^{at}F(t)\} = f(s-a)$$ (+1)

Hence

$$L\{e^{3t}\,t\} = f(s-3) = \frac{1}{(s-3)^2}$$ (+1)

And

$$L\{\sin 4t\} = \frac{4}{s^2+4^2}$$

$$= \frac{4}{s^2+16} \qquad \left\{\because L\{\sin at\} = \frac{a}{s^2+a^2}\right.$$ (+1)

where

$$P = -\,{}^{2}/_{x} \quad Q = x$$

I.F. $e^{\int P\,dx} = e^{-\int {}^{2}/_{x}\,dx}$

$= e^{-2\log x}$

$= {}^{1}/_{x^{2}}$ (+1)

solution

$$y \cdot I.F. = \int Q.\,I.F.\,dx + C$$ (+1)

$$y \cdot {}^{1}/_{x^{2}} = \int x \frac{1}{x^{2}}\,dx + C$$

$$= \int \frac{1}{x}\,dx + C$$

$${}^{y}/_{x^{2}} = \log x + C$$

$$y = x^{2}\log x + x^{2}C$$ (+1)