Enrollment No......................................

Faculty of Engineering

End Sem Examination Dec-2023

EN3BS06 Discrete Mathematics

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. If P = {1,2,3,4,5} is partially ordered set by the relation $\leq$, where $\leq$ has its usual meaning 'less than equal to' then the maximal element of P is: **1**

(a) 3 (b) 4 (c) 5 (d) None of these

ii. Which of the following function is also referred to as surjective function? **1**

(a) Many to one (b) Onto

(c) One to one (d) None of these

iii. Which of the following is a generator of the group $(\{0,1,2,3,4,5\}, +_6)$: **1**

(a) 5 (b) 0 (c) 3 (d) None of these

iv. Let $(P, \leq)$ be a lattice. For any $a, b \in P$ absorption law is: **1**

(a) $a \wedge (a \wedge b) = a$ (b) $a \wedge (a \vee b) = a$

(c) $a \wedge (a \vee b) = b$ (d) None of these

v. The order of the recurrence relation $y_{x+2} - 5y_{x+1} + 6y_x = 0$, where x is independent variable is: **1**

(a) 0 (b) 1 (c) 2 (d) None of these

vi. An ordered arrangement of objects from a finite set of objects is called a- **1**

(a) Combination (b) Permutation

(c) Both (a) and (b) (d) None of these

vii. In a graph $G(V, E)$ the degree of a vertex v_1 is one, then v_1 is: **1**

(a) Isolated vertex (b) Even vertex

(c) Pendent vertex (d) None of these

viii. In a graph, sum of the degrees of all vertices can be: **1**
(a) 11 (b) 100 (c) 1 (d) None of these

ix. If p is 'True', q is 'False', then $p \vee q$ is: **1**
(a) True (b) False
(c) Neither true nor false (d) None of these

x. Let $p \equiv$ I run fast, $q \equiv$I shall win. Then symbolic notation of the statement 'I shall win iff I run fast' is: **1**
(a) $p \Rightarrow q$ (b) $q \Rightarrow p$ (c) $q \Leftrightarrow p$ (d) None of these

Q.2 i. If R and R' are equivalence relations in a set A, show that $R \cap R'$ is an equivalence relation in A. **4**

ii. Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **6**

OR iii. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects. **6**
(a) Find the number of students studying all the three subjects.
(b) Find the number of students studying exactly one of the subjects.

Q.3 Attempt any two:

i. Let $\mathbb{N}$ be the set of positive integers. Let the meaning of $x \leq y$ in $\mathbb{N}$ be x divides y. Show that $\mathbb{N}$ is lattice where the meet $(\wedge)$ and join $(\vee)$ are respectively defined by **5**
$x \wedge y = H.C.F.(x, y)$ and $x \vee y = L.C.M.(x, y)$

ii. Show that the set of four fourth roots of unity (namely 1,-1,i,-i) forms an abelian group with respect to multiplication. **5**

iii. Prove that the following is tautology: **5**
(a) $(p \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p)$
(b) $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$

Q.4 Attempt any two:

i. (a) State the Pigeonhole Principle. **5**
(b) Prove that among 1,00,000 people there are two who are born on same time.
(c) Given any five points in the interior of an equilateral triangle of side x cms, show that there exists two points within a distance of at most $^{x}/_{2}$ cms.

ii. Use mathematical induction to prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for all $n > 0$. **5**

iii. Solve the recurrence relation: **5**
$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2 . r \geq 2.$$

Q.5 Attempt any two:

i. (a) Define Eulerian circuit and draw a graph with six vertices containing a Hamiltonian circuit but not a Eulerian circuit. **5**
(b) Define Hamiltonian circuit and draw a graph with six vertices containing a Eulerian circuit but not Hamiltonian circuit.

ii. (a) Draw the graph (undirected graph) represented by the following adjacency matrix A: **5**
$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
(b) The maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

iii. State and prove Euler's Formula. **5**

Q.6 Attempt any two:

i. (a) Express the following into disjunctive normal form: **5**
$$\sim (p \vee q) \leftrightarrow (p \wedge q).$$
(b) Express the following into conjunctive normal form:
$$\sim (p \vee q) \leftrightarrow (p \wedge q).$$

ii. Simplify the following: **5**
(a) $(P \wedge Q) \vee P$
(b) $(P \wedge Q) \wedge \sim P$
(c) $\sim (P \vee \phi) \vee (\sim P \wedge \phi)$
(d) $(P \vee Q) \vee \sim P$
(e) $\sim (P \vee Q) \vee (\sim P \wedge Q)$
where π is a tautology.

iii. (a) If $p$and q are two statements, show that the implication $p \Rightarrow q$ and its contrapositive $(\sim q) \Rightarrow (\sim p)$ are logically equivalent. **5**
(b) Show that $\sim (p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

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Date :
P. No. :

Faculty of Engineering
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EN3BS06 Discrete Mathematics

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Q1. (i) (c) 5 +1
(ii) (b) onto +1
(iii) (a) 5 +1
(iv) (b) $a \wedge (a \vee b) = a$ +1
(v) (c) 2 +1
(vi) (b) permutation +1
(vii) (c) Pendent Vertex +1
(viii) (b) 100 +1
(ix) (a) True +1
(x) (c) $q \Leftrightarrow p$

~~Q2~~.

Q2. (i) Since R and R' are relations in A, so $R \subseteq A \times A$ and $R' \subseteq A \times A$.
Hence $R \cap R' \subseteq A \times A$ and thus $R \cap R'$ is also a relation in A. +1

a) $R \cap R'$ is reflexive :-
Since R is reflexive, $\therefore (a,a) \in R \ \forall a \in A$
Also, R' is reflexive, $\therefore (a,a) \in R' \ \forall a \in A$

$\therefore \forall a \in A$, we have
$(a,a) \in R \cap R'$
$\therefore R \cap R'$ is reflexive.
+1

b) $R \cap R'$ is Symmetric :- Let $(a,b) \in R \cap R'$.

Date :
P. No. :

Now $(a,b) \in R \cap R' \Rightarrow (a,b) \in R$ and $(a,b) \in R'$
$\Rightarrow (b,a) \in R$ and $(b,a) \in R'$
(Since R and R' are symmetric).
$\Rightarrow (b,a) \in R \cap R'$
$\therefore$ $R \cap R'$ is symmetric. +1

c) $R \cap R'$ is transitive :- we have
$(a,b) \in R \cap R'$ and $(b,c) \in R \cap R'$
$\Rightarrow [(a,b) \in R$ and $(a,b) \in R']$ and $[(b,c) \in R$ and $(b,c) \in R']$
$\Rightarrow [(a,b) \in R$ and $(b,c) \in R]$ and $[(a,b) \in R'$ and $(b,c) \in R']$
$\Rightarrow (a,c) \in R$ and $(a,c) \in R'$ (Since R and R' are transitive)
$\Rightarrow (a,c) \in R$ and R'
$\therefore$ $R \cap R'$ is transitive.
Since $R \cap R'$ is reflexive, symmetric and transitive, hence $R \cap R'$ is an equivalence relation. +1

(ii) Let (x,y) be any arbitrary element of $A \times (B \cup C)$.
$\therefore (x,y) \in A \times (B \cup C) \Rightarrow x \in A, y \in (B \cup C)$
$\Rightarrow x \in A, (y \in B$ or $y \in C)$ +1
$\Rightarrow (x \in A, y \in B)$ or $(x \in A, y \in C)$
$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$ +1
$\Rightarrow (x,y) \in (A \times B) \cup (A \times C)$

$\therefore$ $A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ ——— (1) +1

Again let $(x',y') \in (A \times B) \cup (A \times C)$, then
$(x',y') \in (A \times B) \cup (A \times C) \Rightarrow (x',y') \in (A \times B)$ ~~and~~
or $(x',y') \in (A \times C)$
$\Rightarrow (x' \in A, y' \in B)$ or $(x' \in A, y' \in C)$ +1

$\Rightarrow x' \in A, (y' \in B \text{ or } y' \in C)$

$\Rightarrow x' \in A, \; y' \in (B \cup C)$ +1

$\Rightarrow (x', y') \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ ——— (2)

Hence from (1) and (2)

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$ +1

(iii) (i) Let X denote the set of all students. Let M, P and B denote the set of students studying Mathematics, Physics and Biology respectively. Then we have

$|X| = 100, \; |M| = 32, \; |P| = 20, \; |B| = 45,$

$|M \cap B| = 15, \; |M \cap P| = 7, \; |P \cap B| = 10, \; |(M \cup P \cup B)'| = 30$

$\therefore |M \cup P \cup B| = |X| - |(M \cup P \cup B)'| = 100 - 30 = 70.$ +1

By the Principle of inclusion and exclusion, we have

$$|M \cap P \cap B| = |M \cup P \cup B| - |M| - |P| - |B| + |M \cap P| + |M \cap B| + |P \cap B|$$

$$= 70 - 32 - 20 - 45 + 7 + 15 + 10$$

$$= 5$$

Hence the number of students studying all the three subjects is 5. +1

(ii) Let M_1, P_1 and B_1 be the set of students who study only mathematics, Physics and Biology respectively then

$M_1 = M - P - B, \; P_1 = P - M - B,$

$B_1 = B - M - P.$ +1

Date :
P. No. :

$\therefore$ The number of students who study only mathematics.

$= |M_1| = |M - P - B|$

$= |M| - |M \cap P| - |M \cap B| + |M \cap P \cap B|$

$= 32 - 7 - 15 + 5 = 15$ +1

The number of students who study only Physics.

$= |P_1| = |P - M - B|$

$= |P| - |P \cap M| - |P \cap B| + |P \cap M \cap B|$

$= 20 - 7 - 10 + 5 = 8$ +1

And the number of students who study only Biology.

$= |B_1| = |B - M - P|$

$= |B| - |B \cap M| - |B \cap P| + |B \cap M \cap P|$

$= 45 - 15 - 10 + 5 = 25$

Hence, the number of students studying exactly one of the three subjects

$= |M_1| + |P_1| + |B_1| = 15 + 8 + 25 = 48.$ +1

Q.3. (i) Let N = Set of positive integers. then we see that

Reflexivity :- Let $x \in N$ be arbitrary. then x divides x.

i.e., $x \leq x \ \forall x \in N$

Hence the relation $\leq$ is reflexive on N. +1

Anti-Symmetry :- Let $x, y \in N$ be arbitrary and let $x \leq y$, $y \leq x$. then

$x \leq y, y \leq x \Rightarrow x$ divides y and y divides x.

$\Rightarrow \exists n_1, n_2$ (positive integers) such that

$x n_1 = y, \ y n_2 = x.$ —— (1)

$\Rightarrow y n_2 n_1 = y$
$\Rightarrow n_2 \cdot n_1 = 1$
$\Rightarrow n_1 = 1, n_2 = 1$
$\Rightarrow x = y$

Hence the relation $\leq$ in Anti-Symmetric on N +1

Transitivity:- Let $x, y, z \in N$ be arbitrary, and let $x \leq y$, $y \leq z$. then,

$x \leq y, y \leq z \Rightarrow x$ divides y and y divides z.
$\Rightarrow \exists n_1', n_2'$ (positive integers) such that
$x n_1' = y$, $y n_2' = z$

$\Rightarrow x n_1' n_2' = z$
$\Rightarrow \exists n$ (positive integers) such that $xn = z$, where $n = n_1' n_2'$
$\Rightarrow x$ divides z
$\Rightarrow x \leq z$ +1

Hence the relation $\leq$ is transitive on N.
Since the relation $\leq$ is reflexive, anti-Symmetric and transitive on N. and so, $(N, \leq)$ is a partially Ordered set (poset) +1

Now, In N, meet and join, which are denoted by $\wedge$ and $\vee$ are respectively defined by
$x \wedge y = H.C.F(x, y)$
$x \vee y = L.C.M(x, y)$

clearly, every pair of elements x, y in N has their H.C.F and L.C.M in N itself. ie, for each pair of elements $x, y \in N$,

Date :
P. No. :

$$x \wedge y = H.C.F(x, y) \quad \text{and} \quad x \vee y = L.C.M(x, y)$$ +1

exist in N.

Hence. N is a lattice.

(ii) ~~Let x =~~ To show that multiplication is a Composition in G, we form a Composition table as Under:

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1.

+1

1.) closure Property: Since all the entries in the Composition table are the elements of the set G. $\therefore$ the set G is closed with respect to multiplication and the Operation of Multiplication is binary in G. +1

2.) Associativity: The elements of G are all Complex numbers and the multiplication of complex numbers obeys associative Law. i.e,

$$(1.i).(-i) = 1.\{i.-i\} = 1$$
$$\{1.i\}.(-1) = 1.\{i(-1)\} = i \text{ etc}.$$

Therefore, the given Composition is associative in G.

3). Existence of Identity: from the Composition table we see that the row headed by 1. just Coincides with the top row of the Composition +1 table. thus we have,

$$1(1) = 1, \quad 1(-1) = -1, \quad 1(i) = i, \quad 1(-i) = -i$$

Existence of inverse:- We know that the identity element is its own inverse. from the composition table. It is clear that +1

$$(1)^{-1} = 1; \ (-1)^{-1} = -1; \ (i)^{-1} = i \ \& \ (-i)^{-1} = -i.$$

$\therefore$ the inverse of every element of G is in G.
Hence G is a group.

Commutative Law: The multiplication of complex numbers is commutative i.e, +1

$$1(-1) = (-1)(1), \ (-1).i = i(-1) \ \text{etc}.$$

Since the number of elements in the set G is 4.
$\therefore (G, \cdot)$ is a finite abelian group of order 4.

(iii)(a) Let $A \equiv (P \Rightarrow q) \wedge (q \Rightarrow P)$

$$B \equiv (P \Leftrightarrow q) \Leftrightarrow (p \Rightarrow q) \wedge (q \Rightarrow p).$$

Then,

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	A	B
T	T	T	T	T	T	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T

+2.5

Date :
P. No. :

∵ all the entries in the last column is T.
∴ it is a tautology.

(b)

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$	$(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

∵ all the entries in the column of $(p \Rightarrow q) \Leftrightarrow (\sim p \vee q)$ are T.
∴ the statement is Tautology. +2.5

Q4. (i)(a) pigeonhole principle :- if the number of pigeons is more than the number of pigeonholes, then some pigeonhole must be occupied by two or more than two pigeons. +1

(ii)(b) Assume 1,00,000 people are pigeon~~hole~~.

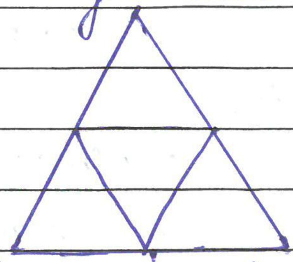
A person can be born at any second of $24 \times 60 \times 60 = 86400$ seconds of a day.
Taking 86,400 as pigeonholes. By generalized pigeonhole principle, atleast

$$\left[\frac{1,00,000 - 1}{86,400}\right] + 1 = [1.16] + 1 = 2.$$ +2

people are born on same time.

(C). we divide the equilateral triangle into four equal triangles.

The each Side of the small triangle is $x/2$.



Now, the given five points (pigeons) will be placed in the four small triangles (pigeonholes). Hence by pigeonhole principle some small triangles must Contain atleast two points. the distance between these two points cannot exceed $x/2$ cms. (the Side of a small triangle)

Hence there exists two points within a distance of almost $x/2$ cms. +2

(ii) Let $P(n) = 2.7^n + 3.5^n - 5$.

$\therefore P(1) = 2.7 + 3.5 - 5 = 14 + 15 - 5 = 24$ &

24 is divisible by 24.

$\Rightarrow P(1)$ is true. +1

Let for some natural number $n=m$, $p(m)$ is divisible by 24.

ie, $p(m) = 2.7^m + 3.5^m - 5 = 24k$, where $k \in N$. —— (1) +1

Now $p(m+1) - p(m) = 2(7^{m+1} - 7^m) + 3.(5^{m+1} - 5^m) - 5 + 5$

$= 2.7^m.6 + 3.5^m.4 = 12(7^m + 5^m)$ +1

Date :
P. No. :

As both 7^m and 5^m are Odd integers for all $m \in N$. their sum must be an even integer,

$$7^m + 5^m = 2p \text{ (say)}, \ p \in N.$$

ie,

$$\therefore p(m+1) - p(m) = 12 \cdot 2p = 24p.$$

$$\text{or } p(m+1) = p(m) + 24p = 24k + 24p. \quad \text{(from (1))}$$ +1

$$\text{or } p(m+1) = 24(p+k) \text{ where } p+k \in N.$$

$\Rightarrow p(m+1)$ is divisible by 24, if $p(m)$ is divisible By 24.

+1

$\therefore$ By Mathematical induction $p(n)$ is divisible by 24 for all natural numbers n.

(iii). The Given equation is

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1)^2, \ r \geq 2 \quad \text{——— (1)}$$

The A.E. is

$$m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2$$ +1

$\therefore$ the Homogeneous solution is,

$$a_r^{(h)} = (C_1 + C_2 r) 2^r \quad \text{——— (2)}$$ +1

The particular solution Corresponding to the form $1 + 2r + r^2$ on the R.H.S of (1) is

$$A_0 + A_1 r + A_2 r^2$$ +1

$$\therefore a_r^{(P)} = A_0 + A_1 r + A_2 r^2 \quad \text{—— (3)}$$

Substituting (3) in (1), we have

$$(A_0 + A_1 r + A_2 r^2) - 4[A_0 + A_1(r-1) + A_2(r-1)^2] + 4[A_0 + A_1(r-2) + A_2(r-2)^2] = (r+1)^2$$

$\Rightarrow A_0 = 29,\ A_1 = 10,\ A_2 = 1$ +1

Putting the values in (3), the particular solution is given by

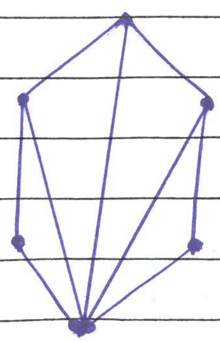
$$a_r^{(P)} = 29 + 10r + r^2$$

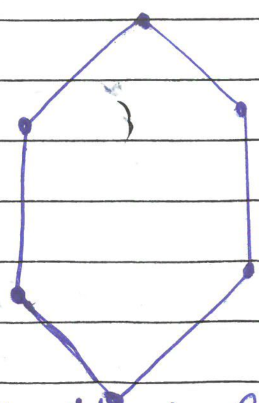
Hence, the total solution of (1) is.

$$a_r = a_r^{(h)} + a_r^{(P)}$$

$$a_r = (C_1 + C_2 r).2^r + r^2 + 10r + 29.$$ +1

Q5. (i) (a) Eulerian Circuit :- A closed walk in a graph which includes all the edges of the graph is called an Eulerian Circuit. +2.5



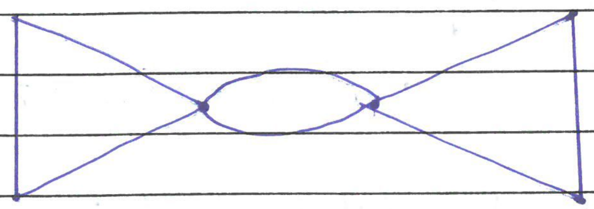


Hamiltonian Circuit

+2.5

The Graph has a Hamiltoniah Circuit But it does not have an eulerian Circuit.

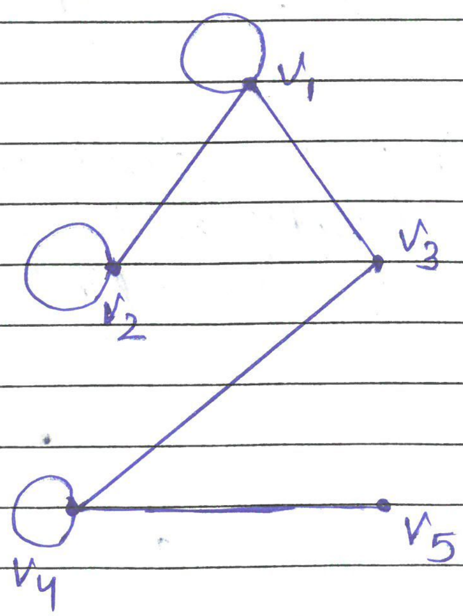
(b) Hamiltonian Circuit :- In a graph $G=(V,E)$, a Hamiltonian Circuit is defined to be a closed walk which traverses every vertex of G exactly once except the starting vertex. +2.5



+2.5

The above graph Contain Eulerian Circuit But not hamiltonian Circuit.

(ii) (a)



+2

(b) Let G be a Simple graph with n vertices $v_1, v_2, \ldots, v_n$ say. the vertex v_1 can be joined to the remaining $(n-1)$ vertices $v_2, v_3, \ldots, v_n$ to

Obtain a maximum number $(n-1)$ of edges namely $(v_1, v_2), (v_1, v_3), \ldots, (v_1, v_n)$. +1

The vertex v_2 can be joined to $(n-2)$ vertices $v_3, v_4, \ldots, v_n$ to obtain a maximum number $(n-2)$ of edges namely $(v_2, v_3), (v_2, v_4), \ldots, (v_2, v_n)$. Note in this case we have not joined v_2 to v_1 since this edge (v_1, v_2) has already obtained.

proceeding in this manner, the vertex v_{n-1} will give us only one new edge namely ~~(v_n, v_{n-1})~~ (v_{n-1}, v_n) +1

Hence, maximum number of edges in the graph G is.

$$= (n-1)+(n-2)+\cdots+2+1$$ +1

$$= \frac{1}{2} n(n-1).$$

(iii) Euler's formula:- A connected planar graph with n vertices and e edges has r regions given by $r = e-n+2$ +2

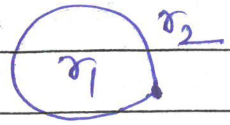
proof:- we shall prove the theorem by induction on the number of edges e of G, where G is a connected planar graph.

Suppose $e=1$ then n may be equal to 1 or 2. In case $e=1, n=2$, then number of regions $r = 1-2+2 = 1$ clearly below graph has one region. +1

$e=1, n=2$

Date :
P. No. :

Again in Case $e=1$, $n=1$, then $r=1-1+2=2$ the Below graph has two regions r_1 and r_2.



$e=1$, $n=1$.

Hence, the Result is true for $e=1$.

Now suppose that the Result holds for all graphs with atmost $e-1$ edges.
Assume that G is a Connected graph with e edges and r regions. In Case G is a tree then $e=n-1$. and number of regions is 1.
In this Case By the formula, we have.

$$r = e-n+2 = (n-1)-n+2 = 1.$$ +1

Hence the theorem holds in Case G is a tree.
Now Consider the Case when G is not a tree, then G has some Circuits. Consider an edge 'c' say, in some Circuit. By removing this edge 'c' from the plane representation of G, the regions are merged into a new region. Therefore G-{c} is a Connected graph with n vertices, e-1 edges and r-1 regions. (where the number of regions in G is r). Thus By induction hypothesis, we have

$$r-1 = (e-1)-n+2 \quad \text{or} \quad r = e-n+2.$$ +1

This proves the theorem.

Date :
P. No. :

(i)(a)
Let

$P = -(p \vee q)$ and $Q = (p \wedge q)$ +0.5

$\therefore \quad -(p \vee q) \leftrightarrow (p \wedge q)$

$\leftrightarrow (P \leftrightarrow Q)$

$\leftrightarrow (P \wedge Q) \vee (-P \wedge -Q)$

$\leftrightarrow (-(p \vee q) \wedge (p \wedge q)) \vee ((p \vee q)$

$\leftrightarrow (-p \wedge -q \wedge p \wedge q) \vee ((p \vee q) \wedge$ (By De Morgan's laws) +1

$\leftrightarrow (-p \wedge -q \wedge p \wedge q) \vee ((p \vee q) \wedge -p) \vee ((p \vee q) \wedge -q)$
(By distributive law)

$\leftrightarrow (-p \wedge -q \wedge p \wedge q) \vee (p \wedge -p) \vee (q \wedge -p) \vee (p \wedge -q)$
$\vee (q \wedge -q)$ +1

which is the required disjuctive normal form.

(b) Let $P = -(p \vee q)$ and $Q = (p \wedge q)$
We know that

$P \leftrightarrow Q.$ +0.5

$\leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

Putting for P and Q in (1), we get

$-(p \vee q) \leftrightarrow (p \wedge q))$

$\leftrightarrow [-(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow -(p \vee q)]$

$\leftrightarrow [(p \vee q) \vee (p \wedge q)] \wedge [-(p \wedge q) \vee -(p \vee q)]$

$[\because x \rightarrow y \leftrightarrow [-x \vee y]]$ +1

$\leftrightarrow [(p \vee q) \vee (p \wedge q)] \wedge [(-p \vee -q) \vee (-p \wedge -q)]$

Date :
P. No. :

$\Leftrightarrow (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$
[By distributive laws] +1

Which is the required conjunctive normal form.

(ii)

(a) $(P \wedge Q) \vee P$

Sol. $(P \wedge Q) \vee P = P \vee (P \wedge Q)$ $\{\because (P \wedge Q) \vee P = P \vee (P \wedge Q)\}$

$= (P \wedge \pi) \vee (P \wedge Q)$ $\{\because P \wedge \pi = P\}$

$= P \wedge (\pi \vee Q)$ $\{\because P \wedge (\pi \vee Q) \equiv (P \wedge \pi) \vee (P \wedge Q)\}$

$= P \wedge \pi$ $\{\because \pi \vee Q = \pi\}$

$= P$ $\{\because P \wedge \pi = P\}$ +1

(b) Sol. $(P \wedge Q) \wedge \sim P \equiv \sim P \wedge (P \wedge Q)$ {By Commutative law}

$\equiv (\sim P \wedge P) \wedge (\sim P \wedge Q)$

$\equiv \phi \wedge (\sim P \wedge Q)$ where ϕ is contradiction

$= \phi.$ +1

Verification by Truth Table:

P	Q	$P \wedge Q$	$\sim P$	$(P \wedge Q) \wedge \sim P$
T	T	T	F	F
T	F	F	F	F
F	T	F	T	F
F	F	F	T	F

(c) Sol. $\sim (P \vee \phi) \vee (\sim P \wedge \phi) \equiv (\sim P \wedge \sim \phi) \vee (\sim P \wedge \phi)$

$\equiv \sim P \wedge (\pi \vee \phi)$

$\equiv \sim P \wedge \pi \equiv \sim P.$ +1

Date :
P. No. :

(9)

d) sol.

$$(P \vee Q) \vee \sim P \equiv \sim P \vee (P \vee Q)$$
$$\equiv (\sim P \vee P) \vee Q$$
$$\equiv T \vee Q$$
$$\equiv T$$

+1

e) sol

$$\sim(P \vee Q) \vee (\sim P \wedge Q) \equiv (\sim P \wedge \sim Q) \vee (\sim P \wedge Q)$$
$$\equiv \sim P \wedge (\sim Q \vee Q)$$
$$\equiv \sim P \wedge T$$
$$\equiv \sim P.$$

+1

Date :
P. No. :

(iii) (a)

Sol. Truth table:

p	q	$p \Rightarrow q$	$-p$	$-q$	$(-q) \Rightarrow (-p)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

+2

The entries of third and sixth columns are identical. Hence $p \Rightarrow q$ and $(-q) \Rightarrow (-p)$ are logically equivalent. +0.5

(b)

Sol. If $-(p \Rightarrow q)$ and $p \wedge (-q)$ are equivalent, then

$$-(p \Rightarrow q) \Leftrightarrow \{p \wedge (-q)\}$$

will be tautology, which is clear from the following truth table: +0.5

p	q	$-q$	$p \Rightarrow q$	$-(p \Rightarrow q)$	$p \wedge (-q)$	$-(p \Rightarrow q) \Leftrightarrow (p \wedge (-q))$
T	T	F	T	F	F	T
T	F	T	F	T	T	T
F	T	F	T	F	F	T
F	F	T	T	F	F	T

+2