Total No. of Questions: 6 *Total No. of Printed Pages:2*

Enrollment No......................................

Faculty of Agriculture

End Sem (Odd) Examination Dec-2019

AG3RC02 Elementary Mathematics

Programme: B.Sc. (Ag.) Branch/Specialisation: Agriculture

Duration: 3 Hrs. **Maximum Marks: 50**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The distance of the point P (1, – 3) from the line 2y – 3x = 4 is **1**

(a) 13 (b) $\frac{7}{13}\sqrt{13}$ (c) $\sqrt{13}$ (d) None of these

ii. The two lines ax + by = c and a′x + b′y = c′ are perpendicular if **1**

(a) aa′ + bb′ = 0 (b) ab′ = ba′

(c) ab + a′b′ = 0 (d) ab′ + ba′ = 0

iii. A line which connects any two points on a circle is known as **1**

(a) Perimeter (b) Diameter (c) Chord (d) Radius

iv. In terms of radius, a diameter is equals to **1**

(a) $2 + r$ (b) 2r (c) $\frac{2}{r}$ (d) $\frac{r}{2}$

v. $\lim_{x\to 0}\frac{sinx}{x} =$ **1**

(a) 0 (b) 1 (c) 2 (d) Not exist

vi. if $f(x) = e^x$ then $f'(x) =$ **1**

(a) e^x (b) e^{2x} (c) $\frac{1}{x}$ (d) None of these

vii. The value of $\int_0^2 x^2 dx$ is **1**

(a) $\frac{8}{3}$ (b) $\frac{x^3}{3}$ (c) $\frac{4}{3}$ (d) None of these

viii. $\int sinx\ dx =$ **1**

(a) $cosx$ (b) $-cosx$ (c) $tanx$ (d) $-tanx$

ix. If the order of matrix A is m×p. And the order of B is p×n. Then the order of matrix AB is? **1**

(a) n × p (b) m × n (c) n × p (d) n × m

x. If |A| = 0, then A is **1**

(a) Null matrix (b) Singular matrix

(c) Non-singular matrix (d) 0

Q.2 Attempt any two:

i. Find the equation of the straight line passing through (1, 2) and perpendicular to the line x + y + 7 = 0 **4**

ii. Find the coordinates of a point which divides the line joining the points (2, 1) and (5, 4) in the ratio 3:1 internally. **4**

iii. Find the acute angle between the lines 7x - 4y = 0 and 3x - 11y + 5 = 0. **4**

Q.3 Attempt any two:

i. Find the coordinates of the centre and radius of circle, if $x^2 + y^2 - 4x + 8y - 61 = 0$. **4**

ii. Find the length of tangent from a point which is at a distance of 5 cm from the centre of the circle of radius 3 cm. **4**

iii. Find the equation of the circle whose centre is (1, -2) and which may touch the line x + y +5 = 0 **4**

Q.4 Attempt any two:

i. Find $\lim_{x\to 2} f(x)$, where $f(x) = \frac{\sqrt{3-x}\ -1}{2-x}$ **4**

ii. Differentiate $x^4 sin3x$ with respect to x. **4**

iii. Differentiate x^n with respect to x by first principle. **4**

Q.5 Attempt any two:

i. Evaluate: **4**

(a) $\int xe^x\ dx$ (b) $\int_1^2 x^2 sinx\ dx$

ii. Find the area bounded by parabola $y^2 = 4ax$, x axis and latus rectum. **4**

iii. Evaluate $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$ **4**

Q.6 Attempt any two:

i. Evaluate: $\begin{vmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{vmatrix}$ **4**

ii. Find the inverse of a matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & 4 & -5 \end{bmatrix}$ **4**

iii. Show that $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **4**

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Marking Scheme

Programme BSc Ag. AG3RC02 Elementary Mathematics

Q.1 MCQ

i) $\sqrt{13}$ (c)

ii) $aa' + bb' = 0$ (a)

iii) (c) Chord

iv) (b) $2r = d$

v) (b) 1

vi) (a) e^x

vii) (a) $8/3$

viii) (b) $-\cos x$

ix) (b) $m \times n$

x) (b) Singular Matrix

Q.2 i) Given Point $(x_1, y_1) = (1, 2)$

line $x + y + 7 = 0$ —(1) (+1)

line Perpendicular to (1) is $y - x + k = 0$ —(2)

line (2) Passes through $(x_1, y_1) = (1, 2)$ (+2)

$y_1 - x_1 + k = 0$

$2 - 1 + k = 0$

$1 + k = 0$, $\boxed{k = -1}$

by (2) Equation of line $\perp$ to (1) is (+1)

$\boxed{y - x - 1 = 0}$

ii) Given Points are

$A(x_1, y_1) = (2, 1)$, $B(x_2, y_2) = (5, 4)$ (+1)

ratio $m : n = 3 : 1$

Let Point $P(x, y)$ divide the line Joining internally in the ratio $m : n$ is (+1)

$x = \frac{mx_2 + nx_1}{m+n}$ $y = \frac{my_2 + ny_1}{m+n}$

$= \frac{3 \times 5 + 1 \times 2}{4} = \frac{17}{4}$, $y = \frac{3 \times 4 + 1 \times 1}{4} = \frac{13}{4}$ (+2)

Point is $P(x, y) = (17/4, 13/4)$

iii) Given lines are, $7x-4y=0$, $3x-11y+5=0$

$4y=7x$, $y=\frac{7}{4}x$ — (1)

$11y=3x+5$, $y=\frac{3}{11}x+\frac{5}{11}$ — (2) +1

Slope of (1) & (2)

$\boxed{m_1=7/4}$, $\boxed{m_2=3/11}$

Acute Angle between (1) & (2)

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$ +1

$$=\left|\frac{\frac{7}{4}-\frac{3}{11}}{1+\frac{7}{4}\times\frac{3}{11}}\right|$$

$$=\left|\frac{77-12/44}{44+21/44}\right|=\left|\frac{65}{65}\right|$$ +2

$\boxed{\tan\theta=1}$ $\boxed{\theta=45^\circ \text{ or } \pi/4}$

OR Compare Given Equation by

$a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$

$a_1=7$, $b_1=-4$, $c_1=0$ +1

$a_2=3$, $b_2=-11$, $c_2=5$

Acute Angle between lines (1) & (2)

$$\tan\theta=\left|\frac{a_2b_1-a_1b_2}{a_1a_2+b_1b_2}\right|$$ +2

$$=\left|\frac{-12+77}{21+44}\right|=\left|\frac{65}{65}\right|=1$$ +1

$\boxed{\tan\theta=1}$ $\boxed{\theta=\pi/4 \text{ or } 45^\circ}$

Q3 i) Given Equation of the Circle is

$$x^2 + y^2 - 4x + 8y - 61 = 0 \quad \text{— (1)}$$

On comparing by

$$x^2 + y^2 + 2gx + 2fy + c = 0$$ (+2)

$2g = -4, \boxed{g = -2}$

$2f = -8, \boxed{f = 4}$

$\boxed{c = -61}$

Centre of the circle is $(-g, -f) = (2, -4)$ (+1)

radius of the circle is $r = \sqrt{g^2 + f^2 - c}$

$r = \sqrt{4 + 16 + 61}$

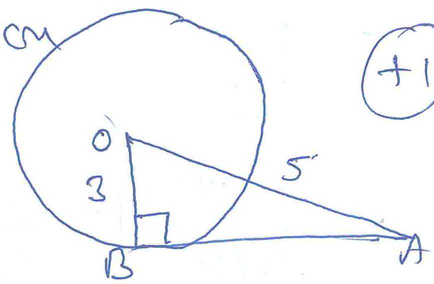
$r = \sqrt{81}$ (+1)

$\boxed{r = 9}$

ii) Let $OA = 5\,cm$, $OB = 3\,cm$

$\angle OBA = 90°$ (+1)

AB is tangent and OB is $\perp$ to AB then



Pythagoras theorem

$OA^2 = AB^2 + OB^2$ (+1)

$25 = AB^2 + 9$

$AB^2 = 25 - 9$

$AB^2 = 16$ (+2)

$\boxed{AB = 4\,cm}$

iii) Given Centre of Circle

$(h, k) = (1, -2)$ (+1)

Equation of line $x + y + 5 = 0$

radius of circle = length drawn from centre to line

$$r = \left|\frac{x+y+5}{\sqrt{1+1}}\right| = \left|\frac{h+k+5}{\sqrt{2}}\right| \quad (+1)$$

$$= \left|\frac{1-2+5}{\sqrt{2}}\right| = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{r = 2\sqrt{2}}$$

Equation of Circle $(x-h)^2 + (y-k)^2 = r^2$

$$(x-1)^2 + (y+2)^2 = (2\sqrt{2})^2 \quad (+2)$$

$$x^2 + 1 - 2x + y^2 + 4 + 4y = 8$$

$$\boxed{x^2 + y^2 - 2x + 4y - 3 = 0}$$

4

Q.4 i) Given $f(x) = \frac{\sqrt{3-x} - 1}{2-x}$

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{\sqrt{3-x} - 1}{2-x} \times \frac{\sqrt{3-x}+1}{\sqrt{3-x}+1} \quad (+1\tfrac{1}{2})$$

rationalizing

$$= \lim_{x \to 2} \frac{(\sqrt{3-x})^2 - 1^2}{(2-x)\sqrt{3-x}+1} \quad (+1\tfrac{1}{2})$$

$$= \lim_{x \to 2} \frac{3-x-1}{(2-x)\sqrt{3-x}+1}$$

$$= \lim_{x \to 2} \frac{\cancel{2-x}}{(\cancel{2-x})\sqrt{3-x}+1} = \lim_{x \to 2} \frac{1}{\sqrt{3-x}+1} \quad (+2)$$

Putting $x = 2$ we get $= \frac{1}{2}$

Q.4 ii) Given

$y = x^4 \sin 3x$

Differentiating with respect to x

$\frac{dy}{dx} = \frac{d}{dx} \underset{\text{I}}{x^4} \underset{\text{II}}{\sin 3x}$ (+2)

$\frac{dy}{dx} = \text{I} \frac{d}{dx}\text{II} + \text{II} \frac{d}{dx}\text{I}$

$= x^4 \frac{d}{dx} \sin 3x + \sin 3x \frac{d}{dx} x^4$ (+1)

$= x^4 . 3\cos 3x + \sin 3x . 4x^3$ (+1)

$= x^3(3x\cos 3x + 4\sin 3x)$

iii) Let $f(x) = x^n$

$f(x+h) = (x+h)^n$

By First Principle Method (+1)

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} = \lim_{h\to 0} \frac{x^n(1+\frac{h}{x})^n - \cancel{x^n}}{h}$

using $= x^n \lim_{h\to 0} \frac{\cancel{1} + n\frac{h}{x} + \frac{h^2}{x^2}\frac{n(n-1)}{2} \cancel{-1}}{h}$ (+2)

$= x^n \lim_{h\to 0} \frac{\cancel{h}(\frac{1}{x} + n(n-1)h/x^2 + \cdots)}{\cancel{h}}$

$= x^n \cdot \frac{n}{x}$ Putting $h = 0$ in rest term (+1)

$\boxed{\frac{d}{dx} f(x) = n x^{n-1}}$

Q.5 1) a) $I = \int \underset{\text{I}}{x} \, \underset{\text{II}}{e^x} \, dx$

$$\int u v \, dx = u \int v \, dx - \int \left[\frac{d}{dx} u \cdot \int v \, dx \right] dx \quad (+1)$$

$$I = \int x e^x \, dx = x \int e^x \, dx - \int \left[\frac{d}{dx} x \int e^x \, dx \right] dx \quad (+1)$$

$$= x e^x - \int 1 \cdot e^x \, dx \quad (+1)$$

$$= x e^x - e^x + c$$

$$\int x e^x \, dx = (x-1) e^x + c \quad (+1)$$

b) $I = \int_1^2 \underset{\text{I}}{x^2} \, \underset{\text{II}}{\sin x} \, dx$

$$I = \left[x^2 \int \sin x \, dx - \int \left[\frac{d}{dx} x^2 \int \sin x \, dx \right] dx \right]_1^2 \quad (+1)$$

$$= \left[-x^2 \cos x - \int 2x (-\cos x) \, dx \right]_1^2$$

$$= \left[-x^2 \cos x + \int 2x \cos x \, dx \right]_1^2 \quad (+1)$$

$$= \left[-x^2 \cos x + 2x \sin x - \int 2 \sin x \, dx \right]_1^2$$

$$= \left[-x^2 \cos x + 2x \sin x + 2 \cos x \right]_1^2 \quad (+1)$$

$$= \left[(-4 \cos 2 + 4 \sin 2 + 2 \cos 2) - (-\cos 1 + 2 \sin 1 + 2 \cos 1) \right]$$

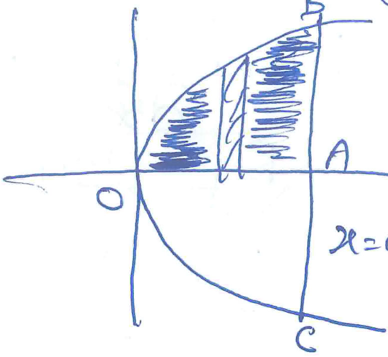
$$= \left[(-2 \cos 2 + 4 \sin 2) - (\cos 1 + 2 \sin 1) \right] \quad (+1)$$

25

ii) Given Equation of Parabola

$y^2 = 4ax$, $y = 2\sqrt{a}\sqrt{x}$ (+1)

Req. area $= 2\int_0^a y\,dx$



$= 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$

$= 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_0^a$ (+2)

$= \frac{8\sqrt{a}}{3}\,a^{3/2} = \frac{8}{3}a^2$ units (+1)

iii) $I = \int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

$= \int e^x \cdot \frac{1}{x}\,dx - \int e^x \frac{1}{x^2}\,dx$ (+2)

$= \frac{e^x}{x} - \int e^x\left(-\frac{1}{x^2}\right)dx + \int e^x \cdot \frac{1}{x^2}\,dx$

$= \frac{e^x}{x} + \cancel{\int e^x \cdot \frac{1}{x^2}\,dx} - \cancel{\int e^x \frac{1}{x^2}\,dx}$ (+1)

$= \frac{e^x}{x} + c$ (+1)

OR By Property $\int e^x\left(f(x) + f'(x)\right]dx = e^x f(x) + c$ (+1)

Here $f(x) = \frac{1}{x}$ $f'(x) = -\frac{1}{x^2}$ then (+2)

$\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx = \frac{e^x}{x} + c$ (+1)

Q.6 i) $\Delta = \begin{vmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{vmatrix}$ (8)

$= 1\begin{vmatrix} 4 & 10 \\ 8 & 4 \end{vmatrix} - 3\begin{vmatrix} 2 & 10 \\ 3 & 4 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 8 \end{vmatrix}$ (+1)

$= 1(16-80) - 3(8-30) + 3(16-12)$ (+2)

$= -64 + 66 + 12$

$= 14$ (+1)

ii) Given Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & 4 & -5 \end{bmatrix}$

$|A| = 1\begin{vmatrix} 5 & 7 \\ 4 & -5 \end{vmatrix} - 2\begin{vmatrix} 2 & 7 \\ -2 & -5 \end{vmatrix} + 3\begin{vmatrix} 2 & 5 \\ -2 & 4 \end{vmatrix}$

$= 1(-25-28) - 2(-10+14) + 3(8+10)$ (+1)

$= -53 - 8 + 54 = -7$

Cofactors of elements of Matrix

$C_{11} = (-1)^{1+1}\begin{vmatrix} 5 & 7 \\ 4 & -5 \end{vmatrix} = -53$ $\quad C_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 4 & -5 \end{vmatrix} = -22$

$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 7 \\ -2 & -5 \end{vmatrix} = -4$ $\quad C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ -2 & -5 \end{vmatrix} = 1$

$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 5 \\ -2 & 4 \end{vmatrix} = 18$ $\quad C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ -2 & 4 \end{vmatrix} = -8$ (+1)

$C_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 5 & 7 \end{vmatrix} = -1$

$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = -1$

$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = 1$

Inverse of Matrix is $A^{-1} = \frac{\text{Adj } A}{|A|}$

$\text{Adj } A = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} -53 & -22 & -1 \\ -4 & 1 & -1 \\ 18 & -8 & 1 \end{bmatrix}$ (+1)

$A^{-1} = -\frac{1}{7}\begin{bmatrix} -53 & -22 & -1 \\ -4 & 1 & -1 \\ \end{bmatrix}$ (+1)

Q. 6

iii) $$\Delta = \begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 2(a+b+c) & b+c & c+a \\ 2(a+b+c) & c+a & a+b \\ 2(a+b+c) & a+b & b+c \end{vmatrix}$$ +1

$$= 2(a+b+c) \begin{vmatrix} 1 & b+c & c+a \\ 1 & c+a & a+b \\ 1 & a+b & b+c \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= 2(a+b+c) \begin{vmatrix} 1 & b+c & c+a \\ 0 & a+b & b+c \\ 0 & a+b & b+a \end{vmatrix}$$ +1

$= 2(a+b+c)\, 1.\{(a-b)(b-a) - (a-c)(b-c)\}$

$= 2(a+b+c)\,(-a^2-b^2-c^2+ab+ac+bc)$

RHS

$$\Delta = 2 \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$R_1 \to C_1 + C_2 + C_3$

$$= 2 \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$ +1

$$= 2(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= 2(a+b+c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$= 2(a+b+c)\, 1\{(c-b)(b-c) - (a-b)(a-c)\}$ +1

$= 2(a+b+c)\,(-a^2-b^2-c^2+ab+bc+ac)$

$\boxed{\text{LHS} = \text{RHS}}$