Q.6 i. Give the formula to calculate χ^2 –statistic. **2**

ii. Write the assumptions in Fisher's F-test. **3**

iii. A dice is tossed 120 times with the following results : **5**

Number turned up :	1	2	3	4	5	6	Total
Frequency :	30	25	18	10	22	15	120

Test the hypothesis that the dice is unbiased $\left(\chi^2_{0.05,5} = 11.07\right)$.

OR iv. Find the student's t-statistic for the following variable values in a sample : **5**

-4, -2, -2, 0, 2, 2, 3, 3

taking the mean of the universe to be zero.

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No.....................................

Faculty of Science

End Sem (Odd) Examination Dec-2017

CA3CO11 Mathematics-III

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. An overflow is said to have occurred when the sum of two n digits occupies : **1**

(a) (n + 1) digits (b) (n – 1) digits

(c) n digits. (d) (n + 2) digits.

ii. Which of the following numbers have greatest precision? **1**

(a) 4.3201 (b) 4.32

(c) 4.320106 (d) 4.3

iii. Which among the following is incorrect? **1**

(a) $\Delta = E - 1$ (b) $\nabla = 1 - E^{-1}$

(c) $E = 1 + \Delta$ (d) $\nabla = 1 - E$

iv. If the values of x are given at unequal intervals, then which types of difference formulae are used for interpolation or extrapolation? **1**

(a) Forward difference formulae

(b) Backward difference formulae

(c) Central difference formulae

(d) Divided difference formulae.

v. The Runge-Kutta method of first order is known as **1**

(a) Euler's method (b) Picard's method

(c) Modified Euler's method (d) None of these.

vi. To apply Weddle's rule, the interval of integration must be divided into a : **1**

(a) 2-multiple number (b) 6-multiple number

(c) 3-multiple number (d) 4-multiple number

vii. A die is thrown, then the probability of getting a prime number is : **1**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

viii. In binomial distribution, standard deviation is : **1**

(a) $(npq)^{1/2}$ (b) $\sqrt[3]{npq}$ (c) $\left(\sqrt{npq}\right)^{-1}$ (d) None of these.

ix. The values of χ^2 are always : **1**

(a) Negative (b) Positive
(c) None of these (d) Both positive and negative.

x. A hypothesis which is tested for possible rejection under the assumption that is true is called : **1**

(a) Alternative hypothesis (b) Null hypothesis
(c) Both (a) and (b) (d) None of these.

Q.2 i. Round-off the number 537.261 to four significant digits and then calculate absolute error, relative error and percentage error. **2**

ii. Find the real root of the equation $x\log_{10} x - 1.2 = 0$ by the method of false position correct to three decimal places. **3**

iii. Evaluate $\sqrt{12}$ correct to four decimal places by Newton-Raphson method. **5**

OR iv. Solve the following equations by Gauss-Seidel method : **5**

$10x_1 - 2x_2 - x_3 - x_4 = 3$
$-2x_1 + 10x_2 - x_3 - x_4 = 15$
$-x_1 - x_2 + 10x_3 - 2x_4 = 27$
$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

Q.3 i. Prove that : $E = e^{hD}$. **2**

ii. Express $f(x) = x^3 - 2x^2 + x - 1$ into factorial notation and show that $\Delta^4 f(x) = 0$. **3**

iii. Find the number of men getting wages between ` 10 and ` 15 from the following data : **5**

Wages in `	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

OR iv. Given the values : **5**

x :	5	7	11	13	17
f(x) :	150	392	1452	2366	5202

Evaluate f(9) using Newton's divided difference formula.

Q.4 i. Write the formulae for $\int_{x_0}^{x_n} y\,dx$ by Trapezoidal rule and Simpson's one-third rule. **2**

ii. Evaluate $\int_0^6 \frac{1}{1+x^2}dx$ by Weddle's rule. **3**

iii. Find by Taylor's series method, the value of y at x = 0.1 and x = 0.2 to four places of decimal from $\frac{dy}{dx} = x^2y - 1$ with y(0) = 1. **5**

OR iv. Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.2 in steps of 0.1 if $\frac{dy}{dx} = x + y^2$ given that y = 1 where x = 0. **5**

Q.5 i. Given $P(A\cup B) = \frac{7}{8}$, $P(A\cap B) = \frac{1}{4}$ and $P(A\cap \overline{B}) = \frac{5}{8}$, find P(A) and P(B). **2**

ii. A coin is tossed four times, what is the probability of getting two heads? **3**

iii. If 10% of the bolts produced by a machine are defective, determine the probability that out of 5 bolts chosen at random at least two will be defective. **5**

OR iv. Show that the mean deviation from the mean of the normal distribution is $\frac{4}{5}$ times its standard deviation. **5**

P.T.O.

MEDICAPS UNIVERSITY

FACULTY OF SCIENCE, DEC 2017

BCA – III (ODD)

MATHEMATICS – III (CA3CO11)

SOLUTION SET

Q.1 (i) (a) (n+1) digits

(ii) (c) 4.320106

(iii) (d) $\nabla = 1 - E$

(iv) (d) Divided Difference Formulae

(v) (a) Euler's Method

(vi) (b) 6 – multiple number

(vii) (a) 1/2

(viii) (a) $(npq)^{1/2}$

(ix) (b) Positive

(x) (b) Null hypothesis.

Q.2 (i) Given no. is $x = 537.261$

After rounded-off to four significant digits is $537.3 = x_1$ ——— +1

Absolute error $= E_a = |x - x_1| = 0.039$

Relative error $= E_r = \left|\frac{x - x_1}{x}\right| = \frac{.039}{537.261} = 7.25 \times 10^{-5}$ ——— +1

percentage error $= E_p = E_r \times 100 = 7.25 \times 10^{-3}$

(ii) $f(x) = x \log_{10} x - 1.2$

$f(2) = 0.23136$

$x_1 = 2$ $x_2 = 3$ $f(x_1) = -0.59794$, $f(x_2) = -0.23136$

$$x_3 = x_1 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_1) = 2.72102$$

$f(x_3) = -0.01709$

$\therefore$ root lie b/w 2.72102 and 3

Taking $x_1 = 2.72102$ $x_2 = 3$ $f(x_1) = -0.01709$ $f(x_2) = .23136$

$x_4 = 2.74021$ ——— +1

$f(x_4) = -0.00038$ ∴ root lie b/w 2.74021 and 3 (+1)

Take $x_1 = 2.74021$ and $x_2 = 3$ $f(x_1) = -0.00038$ $f(x_2) = \cancel{2.3}.23136$

$x_5 = 2.74064$ $f(x_5) = -.00001$ (+1)

$\boxed{x_6 = 2.74065}$

Q.2(iii) Let $x = \sqrt{12} \Rightarrow x^2 - 12 = 0 = f(x)$ $f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_{n+1} = \frac{1}{2}\left(x_n + \frac{12}{x_n}\right)$$

$f(3) = -3$ $f(4) = 4$ ∴ root lie b/w 3 and 4 (+2)

Take $x_0 = 3.5$

$$x_1 = \frac{1}{2}\left(3.5 + \frac{12}{3.5}\right) = 3.4643$$

(+1)

$$x_2 = \frac{1}{2}\left(3.4643 + \frac{12}{3.4643}\right) = 3.4641$$

$$x_3 = \frac{1}{2}\left(3.4641 + \frac{12}{3.4641}\right) = 3.4641$$ (+1)

$x_2 = x_3 \Rightarrow \sqrt{12} = 3.4641$ (+1)

Q.2(i) Ans:- $\boxed{x_1 = 1 \quad x_2 = 2, x_3 = 3, x_4 = 0}$

For First iteration $x_2 = 0$, $x_3 = 0$, $x_4 = 0$

$x_1 = \left[\frac{3 - (2x_2 + x_3 + x_4)}{10}\right]$ $x_1^{(1)} = 0.3$

$x_2 = \left[\frac{15 - (2x_1 + x_3 + x_4)}{10}\right]$ $x_2^{(1)} = \cancel{2} 1.56$

$x_3 = \left[\frac{27 - (2x_4 + x_1 + x_2)}{10}\right]$ $x_3^{(1)} = 2.886$

$x_4 = \left[\frac{-9 - (x_1 + x_2 + 2x_3)}{10}\right]$ $x_4^{(1)} = -.1368$

(+1)

for second iteration :-

$$x_1^{(2)} = \left[\frac{3+2(1.56)+2.886-.1368}{10}\right] = 8.8692 = .88692$$

$$x_2^{(2)} = \left[\frac{15+2(.88692)+2.886+.1368}{10}\right] = \frac{19.52204}{10} = 1.9523$$

$$x_3^{(2)} = \left[\frac{27+2(-.1368)+.88692+1.9523}{10}\right] = \frac{29.56562}{10} = 2.956562$$

$$x_4^{(2)} = \left[\frac{-9+.88692+1.9523+2(2.95656)}{10}\right] = \frac{-.24766}{10} = -.02476$$

+1

for Third iteration

$$x_1^{(3)} = \left(\frac{3+2(1.9523)+2.9565-.02476}{10}\right) = .983634$$

$$x_2^{(3)} = \left(\frac{15+2(.9836)+2.9565+(-.02476)}{10}\right) = 1.967$$

$$x_3^{(3)} = \left[\frac{27+2(-.02476)+.9836+.1967}{10}\right] = 2.81307$$

+1

$$x_4^{(3)} = \left[\frac{-9+.9836+1.967+2(2.81307)}{10}\right] = -.042326$$

for Fourth iteration

$x_1^{(4)} = .997$ $x_2^{(4)} = 1.998$ $x_3^{(4)} = 2.999$ $x_4^{(4)} = -.0007$

+1

for fifth iteration

$x_1^{(5)} = .999$, $x_2^{(5)} = 1.999$, $x_3^{(5)} = 2.999$, $x_4^{(5)} = -.00044$

+1

$\therefore$ required solⁿ is

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

$x_4 = 0$

Q.3 (i) By def of differential coeff

$$Df(x) = \frac{d}{dx}f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

By Taylor's Theorem

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \cdots$$ +1

$$= f(x) + hDf(x) + \frac{h^2}{2!}D^2 f(x) + \cdots$$

$$= \left[1 + hD + \frac{h^2}{2!}D^2 + \cdots\right] f(x)$$

$$= e^{hD} f(x)$$

Also $f(x+h) = Ef(x)$

$$\Rightarrow Ef(x) = e^{hD} f(x)$$ +1

$$\Rightarrow \boxed{E = e^{hD}}$$

(ii) $f(x) = x^3 - 2x^2 + x - 1$

Let $f(x) = Ax^{[3]} + Bx^{[2]} + Cx^{[1]} + D$ in factorial

$$x^3 - 2x^2 + x - 1 = Ax(x-1)(x-2) + Bx(x-1) + Cx + D$$ +1

Put $x = 0$
$D = -1$

Put $x = 1$
$C + D = -1 \Rightarrow C = 0$

Put $x = 2$ $2B + 2C + D = 1 \Rightarrow B = 1$

$\therefore \boxed{A = 1}$ +1

$\therefore f(x) = x^{[3]} + x^{[2]} - 1$

$\Delta f(x) = 3x^{[2]} + 2x^{[1]}$

$\Delta^2 f(x) = 6x^{[1]} + 2$

$\Delta^3 f(x) = 6$

$\Delta^4 f(x) = 0$ +1

Q.3 (iii)

Wages Less than (x)	Frequency y	Δy	$\Delta^2 y$	$\Delta^3 y$
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

(+2)

Here at $x=10$ then $f(10) = 9$

Also $y_0 = 9 \quad \Delta y_0 = 30 \quad \Delta^2 y_0 = 5 \quad \Delta^3 y_0 = 2$

To find $f(15)$ Here $x = 15 \quad x_0 = 10, h = 10$ —— (+1)

$$u = \frac{x - x_0}{h} = \frac{15-10}{10} = 0.5$$

By Newtons Forward diff formula

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots \quad (+1)$$

$$= 9 + (0.5)(30) + \frac{(0.5)(-.5)(5)}{2} + \dots$$

$$= 9 + 15 - 0.625 + 0.125 = 23.5 \approx 24 \text{ men.}$$

Hence no. of men getting wages b/w 10 and 15 's

$$= f(15) - f(10) \quad —— (+1)$$

$$= 24 - 9 = 15.$$

Q.3 (iv)

x_0	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 5$	150				
		121			
$x_1 = 7$	392		24		
		265		1	
$x_2 = 11$	1452		32		0
		457		1	
$x_3 = 13$	2366		42		
		709			
$x_4 = 17$	5202				

(+2)

By Newton's divided formula

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + \text{---} \quad +1$$

Put $x = 9$

$$f(9) = 150 + (9-5)(121) + (9-5)(9-7)(24) + \text{---} \quad +1$$

$$(9-5)(9-7)(9-11)(1) + 0$$

$$\text{---} \quad +1$$

$$= 810$$

P. T. O

Q.4(i) Trapezoidal Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right] \quad \text{— } +1$$

Simpson's one-Third Rule

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

— +1

Q.4 (ii) $\displaystyle\int_0^6 \frac{dx}{1+x^2} = 1.3735$

$h = \dfrac{6-0}{6} = 1 \qquad y = \dfrac{1}{1+x^2}$

x_k	y_k
$x_0 = 0$	$y_0 = 1$
$x_1 = 1$	$y_1 = 0.5$
$x_2 = 2$	$y_2 = 0.2$
$x_3 = 3$	$y_3 = 0.1$
$x_4 = 4$	$y_4 = .05882$
$x_5 = 5$	$y_5 = .03846$
$x_6 = 6$	$y_6 = .027027$

— +2

$$\int_{x_0}^{x_0+6h} y\,dx = \int_0^6 \frac{dx}{1+x^2} = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right] \quad \text{— } +1$$

$$= \frac{3(1)}{10}\left[1 + 5(0.5) + 0.2 + 6(0.1) + .05882 + 5(.03846) + .027027\right]$$

$$= 1.37344 \quad \text{— } +1$$

Q.4 (iii) $\frac{dy}{dx} = x^2y - 1$

$y' = x^2y - 1 \quad y_0' = x_0^2 y_0 - 1 = -1$

$y'' = 2xy + x^2y' \Rightarrow y_0'' = 2\times 0\times 1 + 0\times(-1) = 0$

$y''' = 2y + 2xy' + 2xy' + x^2y'' \Rightarrow y_0''' = 2$

$y^{iv} = 2y' + 4y' + 4xy'' + 2xy'' + x^2y''' \Rightarrow (y^{iv})_0 = 6$

+2

By Taylor's series

$y(x) = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \cdots$ — +1

At $x = 0.1$

$y(0.1) = 1 + (0.1)(-1) + \frac{(0.1)^2}{2}(0) + \frac{(0.1)^3}{3!}(2) + \frac{(0.1)^4}{4!}(-6) + \cdots$

$= 0.90031$ +1

At $x = 0.2$

$y(0.2) = 1 + (0.2)(-1) + \frac{(0.2)^2}{2}(0) + \frac{(0.2)^3}{3!}(2) + \frac{(0.2)^4}{24}(-6)$

$= 0.8023$ +1

Q.4 (iv) $h = 0.1 \quad f(x,y) = x + y^2 \quad x_0 = 0, \; y_0 = 1$

$x_1 = 0.1 \quad x_2 = 0.1$

$k_1 = hf(x_0 y_0) = 0.1$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = .11525$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = .11686$

$k_4 = hf(x_0 + h, y_0 + k_3) \; .1165$

+2

$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = .1165$

$y_1 = y_0 + k = 1.1165$ — +1

Now $x_1 = 0.1$ $y_1 = 1.1165$ $h = 0.1$

$K_1 = .13466$

$K_2 = 0.15514$

$K_3 = .15758$ (+1)

$K_4 = .18233$

$K = 0.1571$

$y_2 = y_1 + K = 1.2736$ — (+1)

Q.5(i) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

$\frac{5}{8} = P(A) - \frac{1}{4} \Rightarrow \boxed{P(A) = \frac{7}{8}}$ — (+1)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{7}{8} = \frac{7}{8} + P(B) - \frac{1}{4} \Rightarrow \boxed{P(B) = \frac{1}{4}}$ (+1)

(ii) $n = 4$ $p = 1/2$ $q = \frac{1}{2}$ — (+1)

By Binomial distribution $P(r) = {}^nC_r p^r q^{n-r}$ (+1)

$= 0.375$ (+1)

Q.5(iii) $p = \frac{1}{10}$ $q = \frac{9}{10}$ $n = 5$ — (+1)

$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$ (+1)

$P(r \geq 2) = P(r=2) + P(r=3) + P(r=4) + P(r=5)$ (+2)

$= 0.0814$ (+1)

Q.5(iv) $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ — (+1)

$M.D = \int_{-\infty}^{\infty} |x-\mu| f(x)\, dx$

Put $t = \frac{x-\mu}{\sigma\sqrt{2}} = t$ — (+2)

$= \int_{-\infty}^{\infty} |\sqrt{2}\sigma t| \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-t^2} \sigma\sqrt{2}\, dt$

$= \frac{\sigma}{\sqrt{\pi}} \sqrt{2} \int_{-\infty}^{\infty} |t| e^{-t^2} dt$

— (+1)

$u = t^2 \Rightarrow du = 2t\,dt$

$= (0.8)\sigma$

$= \frac{4}{5}\sigma$ (+1)

Q.6 (i) $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$ —— +2

where f_o = observed frequency $f_e \rightarrow$ expected frequency

(ii) (i) The value in each group should be normally distributed +1

(ii) The error should be independent of each value +1

(iii) The variance within each group should be equal for all groups. +1

(iii) $f_e(x) = 20$ —— +1

$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$ —— +1

x	f_o	f_e	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
1	30	20	100	5
2	25	20	25	1.25
3	18	20	4	.20
4	10	20	100	5.00
5	22	20	4	.20
6	15	20.	25	1.25
	120	$N = 120$		$\chi^2 = 12.90$

+2

$\nu = 6 - 1 = 5$ —— +1

$\Rightarrow$ dice is biased one

Q. 6 (iv)

$$t = \frac{(\bar{x} - \mu)}{S}\sqrt{n} \qquad \text{— } +1$$

$$S^2 = \frac{1}{n-1}\sum (x - \bar{x})^2 \qquad \text{— } +1$$

$n = 8$, $\mu = 0$

S NO	x	$x - \bar{x}$	$(x - \bar{x})^2$
1	-4	-4.25	18.0625
2	-2	-2.25	5.0625
3	-2	-2.25	5.0625
4	0	-0.25	0.0625
5	2	1.75	3.0625
6	2	1.75	3.0625
7	3	2.75	7.5625
8	3	2.75	7.5625
	$\boxed{\Sigma x = 2}$		49.5

+2

$$\bar{x} = \frac{\Sigma x}{n} = \frac{2}{8} = .25$$

$$\left.\begin{aligned} S &= 2.659 \\ t &= 0.27 \end{aligned}\right\} \quad +1$$