Enrollment No......................................

Faculty of Engineering / Science
End Sem (Even) Examination May-2022
EN3BS12 / SC3BS02 Engineering Mathematics -II

Programme: B.Tech. / B.Sc. Branch/Specialisation: All / AIML/CS/CTIS/MAIS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. $L^{-1}\{f(s)\}=F(t)$ then $L^{-1}\{f(s-a)\}$ is - **1**

(a) $e^{at}F(t)$ (b) $e^{-at}F(t)$ (c) e^{at} (d) None of these

ii. $L\{\cosh 2t\}$ = ____. **1**

(a) $\frac{2}{(s^2-4)}$ (b) $\frac{s}{(s^2+4)}$ (c) $\frac{s}{(s^2-4)}$ (d) None of these

iii. The Fourier series expansion of $f(x)=x\sin x$ in the range $-2\le x\le 2$ contains: **1**

(a) Only sine terms (b) Only cosine terms

(c) Both (a) and (b) (d) None of these

iv. As per the Dirichlet's conditions for the uniform convergence of a Fourier series, discontinuity in any one period of function should be: **1**

(a) Finite (b) Infinite (c) No discontinuity (d) None of these

v. The solution of the equation $xp+yq=z$ is: **1**

(a) $f\left(\frac{x}{y},\frac{y}{z}\right)=0$ (b) $f\left(\frac{x}{z},\frac{y}{x}\right)=0$

(c) $f\left(\frac{y}{z},\frac{y}{x}\right)=0$ (d) All of these

vi. The general solution of $\left(D^2+D'^2-2DD'\right)z=0$ is: **1**

(a) $f_1(y+x)+xf_2(y+x)$ (b) $f_1(y-x)+f_2(y-x)$

(c) $f_1(y-x)+x\,f_2(y-x)$ (d) None of these

vii. If $\vec{F}$ is a constant vector, then $Curl\,\vec{F}$ is- **1**

(a) 1 (b) 2 (c) 0 (d) None of these

viii. As per the Stoke's theorem, the $\oint_c \vec{F}.d\vec{r} = ...$ where $\vec{F}$ is continuous differentiable vector function S is surface enclosed by Closed curve C, $\vec{n}$ is unit normal vector at any point of S. **1**

(a) $\iint_S \left(curl\vec{F}\right).\overrightarrow{n}\, dS$ (b) $\iint_S \left(div\vec{F}\right).\overrightarrow{n}\, dS$

(c) $\iint_S \left(grad\vec{F}\right).\overrightarrow{n}\, dS$ (d) None of these

ix. Which one of the following methods not surely converges, to find root of equation $f(x)=0$? **1**

(a) Newton Raphson Method
(b) Regula Falsi Method
(c) Both (a) and (b)
(d) None of these

x. The aim of elimination steps in Gauss elimination method is to reduce the coefficient matrix to ____. **1**

(a) Diagonal matrix (b) Lower Triangular matrix
(c) Upper Triangular matrix (d) None of these

Q.2 Attempt any two:

i. Find the Laplace Transform of $\dfrac{e^{-3t}\sin t}{t}$. **5**

ii. State convolution theorem for Laplace and use it to evaluate $L^{-1}\left\{\dfrac{1}{(s-1)(s-2)}\right\}$. **5**

iii. Using Laplace Transform solve $\dfrac{d^2y}{dt^2}+y=0$, given that $y=1$ and $\dfrac{dy}{dt}=0$ when $t=0$. **5**

Q.3 i. Mention Dirichlet's conditions for the uniform convergence of a Fourier series and explain how to calculate value of series at a point of discontinuity. **3**

ii. Find the Fourier series for $f(x)=x^2$ defined periodic in $-\pi \le x \le \pi$ and hence deduce that $\dfrac{\pi^2}{8}=\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+---$. **7**

OR iii. Define Fourier transform and Inverse Fourier transform also find the Fourier Transform of $f(x)=\begin{cases}1-x^2 & |x|\le 1\\ 0 & |x|>0\end{cases}$ **7**

Q.4 Attempt any two:

i. Solve the Lagrange's Partial differential equation $\left(y^2+z^2-x^2\right)\dfrac{\partial z}{\partial x}-2xy\dfrac{\partial z}{\partial y}+2zx=0$. **5**

ii. Solve $\dfrac{\partial^2 z}{\partial x^2}-2\dfrac{\partial^2 z}{\partial x\partial y}+\dfrac{\partial^2 z}{\partial x^2}=e^{x+2y}+x^3$. **5**

iii. Using Separation of variables solve $3\dfrac{\partial u}{\partial x}+2\dfrac{\partial u}{\partial y}=0$ with $u(x,0)=4e^{-x}$. **5**

Q.5 Attempt any two:

i. Find directional derivative of the function $\phi = xy + yz + zx$ at the point (1, 2, 0) in the direction of the vector $i + 2j + 2k$. **5**

ii. Apply Stoke's theorem to evaluate $\oint_c \vec{F}.d\vec{r}$ where $\vec{F}=x^2\hat{i}+xy\hat{j}$ is integrated round the square in xy plane whose sides are along the lines $x=0, y=0, x=a, y=a$. **5**

iii. Use Gauss divergence theorem in Cartesian form to evaluate $\iint_S (xdydz+ydzdx+zdxdy)$ where S is the surface of sphere $x^2+y^2+z^2=a^2$. **5**

Q.6 Attempt any two:

i. Apply Regula Falsi method to find real root of equation $x\log_{10}x-1.2=0$, correct to two places of decimals. **5**

ii. Solve the following system of equation by Gauss Elimination $2x-3y+z=-1,\quad x+4y+5z=25,\quad 3x-4y+z=2$. **5**

iii. Solve the following system of equation by Gauss Seidel Method $10x+y+z=12,\quad 2x+10y+z=13,\quad 2x+2y+10z=14$. **5**

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Medi-Caps University

Faculty of Engg

End Sem (Even) Exam – May (2022)

EN3BS12 Engg Mathematics – II

Qu ① 1. (a) $e^{at} F(t)$ +1

2 (c) s/s^2-4 +1

3 (b) Only cosine terms +1

4 (a) finite +1

5 (d) All of these. +1

6 (a) $f_1(y+x) + x f_2(y+x)$ +1

7 (c) 0 +1

8 (a) $\iint (\text{curl } \vec{F}) \cdot \vec{n}\, ds$ +1

9 (a) Newton Raphson method +1

10 (c) Upper triangular matrix +1

10 marks

Qu ② (i). $L\left[\frac{e^{-3t} \sin t}{t}\right]$ +1

$L\{\sin t\} = \frac{1}{s^2+1}$

Sol $L[e^{-3t} \sin t] = \frac{1}{(s+3)^2+1}$ +1

$L\left[\frac{e^{-3t} \sin t}{t}\right] = \int_s^\infty \frac{1}{(s+3)^2+1}\, ds$ +1

$= \left[\tan^{-1}(s+3)\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}(s+3)$ +1

$= \cot^{-1}(s+3)$ +1

(ii). Convolution theorem –

If $L^{-1}[f(s)] = F(t)$ and $L^{-1}[g(s)] = G(t)$ then

$L^{-1}[f(s) \cdot g(s)] = \int_0^t F(x)\, G(t-x)\, dx = F * G.$ +1

5 marks

$L^{-1}\left[\frac{1}{(s-1)(s-2)}\right] \Rightarrow$ Let $f(s)=\frac{1}{s-1} \Rightarrow F(t)=e^{t}$ +1

$g(s)=\frac{1}{s-2} \Rightarrow G(t)=e^{2t}$ +1

By convolution

$F*G=\int_0^t e^{x}\cdot e^{2(t-x)}\,dx = e^{2t}\int_0^t e^{x}\cdot e^{-2x}\cdot dx$ +1

$= e^{2t}\int_0^t e^{-x}\,dx = e^{2t}\left[\frac{e^{-x}}{-1}\right]_0^t$ +1

$= -e^{2t}\left[e^{-t}-1\right] = \underline{e^{2t}-e^{t}}$ Ans

5 marks

(iii) Solve $y''+y=0$, $y(0)=1$, $y'(0)=0$

Apply LT on both sides

$L[y'']+L[y]=L[0]$ +1

$\left[s^2\bar{y}-s\,y(0)-y'(0)\right]+\bar{y}=0$ +1

$s^2\bar{y}-s-0+\bar{y}=0$

$(s^2+1)\bar{y}=s$ +1

$\bar{y}=s/s^2+1$

By ILT on both sides +1

$L^{-1}[\bar{y}]=L^{-1}\left[s/s^2+1\right]$

$y=\cos t.$ +1

5 marks

Date: / /
Page: 2

Qu. (3) (i). The dirichlet's conditions are sufficient conditions for real valued, periodic function $f(x)$ to be equal the sum of its fourier series at each point where $f(x)$ is continuous. The condition are –

1. $f(x)$ must be periodic, single variable, finite.
2. $f(x)$ must have finite no. of extremes in interval
3. $f(x)$ must have finite no. of discontinuities in interval
4. $f(x)$ must be integrable over the period.
5. $f(x)$ must be bounded.

When these conditions are satisfied, the F.S. converges to $f(x)$ at every point of continuity. Hence, the series converges if (1) $f(a)$ if $x=a$ is point of continuity. +2

(2) At a point of discontinuity, the sum of the series is equal to the mean of the limits of i.e.

$$f(a) = \frac{1}{2}\left[f(a+0) + f(a-0)\right]$$ +1

3 marks

(ii). $f(x) = x^2$ $\quad -\pi \le x \le \pi$

Sol $f(x)$ is even function then $b_n = 0$ +1

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{—(1)}$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{3}\pi^2$$ +1

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx$$

$$= \frac{2}{\pi}\left[(x^2)\cdot\left(\frac{\sin nx}{n}\right) - (2x)\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left\{0 + \frac{2\pi(-1)^n}{n^2} - 0\right\} - \left\{0 + 0 - 0\right\}\right]$$

$$= \frac{4}{n^2}(-1)^n$$ +2

Date: __/__/__

Page: ____

By ① $x^2 = \frac{\pi^2}{3} + 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \frac{\cos 4x}{4^2} + \cdots\right)$ +1

Put $x=\pi$ $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

Put $x=0$ $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$

Adding both $\frac{\pi^2}{4} = 2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$

Hence $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2}$ +2

7 marks

(iii) Fourier transform or complex fourier transform of $f(x)$ is given by $F(f(x)) = \bar{f}(s) = \int_{-\infty}^{\infty} f(x).\, e^{isx}.dx$ +1

$s>0,\ x\in[-\infty,\infty]$

While the function ~~f(x)~~ $f(x)$ define by

$F^{-1}[\bar{f}(s)] = f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(s).\, e^{-isx}.ds$ +1

is known as inverse fourier transform of $\bar{f}(s)$.

By the definition, fourier transform of $f(x)$ is

$$F[f(x)] = \int_{-\infty}^{\infty} f(x).\, e^{isx}\, dx$$

$$= \int_{-1}^{1} (1-x^2).\, e^{isx}\, dx$$ +1

$$= \left[(1-x^2)\frac{e^{isx}}{is} - (-2x)\frac{e^{isx}}{i^2s^2} + (-2)\frac{e^{isx}}{i^3s^3}\right]_{-1}^{1}$$ +1

$$= \frac{2}{i^2s^2}\left[x.\,e^{isx} - \frac{e^{isx}}{is}\right]_{-1}^{1}$$

$$= \frac{-2}{s^2}\left[\left\{e^{is} - \frac{e^{is}}{is}\right\} - \left\{-e^{-is} - \frac{e^{-is}}{is}\right\}\right]$$ +1

$$= \frac{-2}{s^2}\left[\left(e^{is}+e^{-is}\right) - \frac{1}{is}\left(e^{is} - e^{-is}\right)\right] \times \frac{2}{2}$$

$$= \frac{-4}{s^2}\left[\left(\frac{e^{is}+e^{-is}}{2}\right) - \frac{1}{s}\left(\frac{e^{is}-e^{-is}}{2i}\right)\right]$$ +2

$$= \frac{-4}{s^2}\left[\cos s - \frac{1}{s}\sin s\right]$$

Qu. ④ (i) $(y^2+z^2-x^2)\frac{\partial z}{\partial x} - 2xy\frac{\partial z}{\partial y} + 2zx = 0$

Sol By Lagrange's method

$$\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2zx}$$ +1

By grouping method

$$\frac{dy}{-2xy} = \frac{dz}{-2zx} \Rightarrow \frac{dy}{y} = \frac{dz}{z}$$ On integrating $\Rightarrow c_1 = y/z$ +1

By multipliers method, using x, y, z as multipliers

$$\frac{x\,dx + y\,dy + z\,dz}{xy^2+xz^2-x^3-2xy^2-2z^2x} = \frac{dz}{-2zx}$$ +1

$$\frac{x\,dx + y\,dy + z\,dz}{-x^3-xy^2-xz^2} = \frac{dz}{-2zx}$$

$$\frac{x\,dx + y\,dy + z\,dz}{-x(x^2+y^2+z^2)} = \frac{dz}{-2zx}$$

$$\frac{2x\,dx + 2y\,dy + 2z\,dz}{x^2+y^2+z^2} = \frac{dz}{z}$$ +1

On integrating

$$\log(x^2+y^2+z^2) = \log z + \log c_2$$

$$c_2 = x^2+y^2+z^2/z$$

Hence general sol $f\left(y/z, \frac{x^2+y^2+z^2}{z}\right) = 0$ +1

5 marks

(ii) $2\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} = e^{x+2y} + x^3$

$(2D^2 - 2DD')z = e^{x+2y} + x^3$

CF $2m^2 - 2m = 0 \Rightarrow 2m(m-1) = 0$

$m = 0, 1$ (real & distinct)

CF $= f_1(y) + f_2(y+x)$ +2

Date: __/__/__
Page: ______

$$PI\ \frac{e^{x+2y}}{2D^2-2DD'} + \frac{x^3}{2D^2-2DD'}$$

$$= \frac{e^{x+2y}}{2(1)^2-2(1)(2)} + \frac{x^3}{2D^2\left[1-D'/D\right]}$$ +1

$$= \frac{e^{x+2y}}{2-4} + \frac{1}{2D^2}\left[1-\frac{D'}{D}\right]^{-1} \qquad \because (1-x)^{-1} = 1+x+x^2+x^3+\cdots$$

$$= -\frac{e^{x+2y}}{2} + \frac{1}{2D^2}\left[1+\frac{D'}{D}+\left(\frac{D'}{D}\right)^2+\cdots\right]x^3$$ +1

$$= \frac{-e^{x+2y}}{2} + \frac{1}{2D^2}\left[x^3+0+0+\cdots\right]$$

$$= \frac{-e^{x+2y}}{2} + \frac{1}{2D}\left[\frac{x^4}{4}\right] = \frac{-e^{x+2y}}{2} + \frac{x^5}{40} = PI$$ +1

$$C.S = CF + PI = \left[f_1(y)+f_2(y+x)\right] + \left[-\frac{e^{x+2y}}{2} + \frac{x^5}{40}\right]$$

5 marks

(iii) Seperation of variable method —

Trial sol $u(x,y) = X_{(x)} \cdot Y(y)$ — (1)

By given eq $3X'Y + 2XY' = 0$

$3X'Y = -2XY'$

Divide by XY

$$\frac{3X'}{X} = \frac{-2Y'}{Y} = K \quad (\text{any constant})$$ +1

Or $3X' = KX$ & $-2Y' = KY$

By $\frac{3X'}{X} = K \Rightarrow \frac{X'}{X} = \frac{K}{3} \Rightarrow \log X = \frac{K}{3}x + \log C_1$

$X = C_1 e^{K/3\,x}$ +1

By $-2\frac{Y'}{Y} = K \Rightarrow \frac{Y'}{Y} = \frac{-K}{2} \Rightarrow \log Y = -\frac{K}{2}y + \log C_2$

$Y = C_2 e^{-\frac{K}{2}y}$ +1

By (1) $u = C_1C_2 e^{\frac{K}{3}x} \cdot e^{-\frac{K}{2}y}$ — (2) +1

By given condition $4e^{-x} = C_1C_2 e^{\frac{K}{3}x}$

Hence $C_1C_2 = 4$, $\frac{K}{3} = -1$ means $K = -3$

By (2) $u = 4e^{-\frac{3}{3}x} \cdot e^{-\left(-\frac{3}{2}\right)y} = 4e^{-x} \cdot e^{\frac{3}{2}y}$ +1

5 marks

Qu(5) (i). D.D. of $\phi = \hat{a} \cdot \nabla\phi$ —— (1) +1

$\nabla\phi = i(y+z) + j(x+z) + k(y+x)$ +1

$= 2i + j + 3k$ [at (1,2,0)] +1

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{i+2j+2k}{\sqrt{1+4+4}} = \frac{i+2j+2k}{3}$ +1

By (1) $D.D = \frac{10}{3}$ +1

5 marks

(ii). By Stoke's theorem $\int_C \vec{F} \cdot d\vec{r} = \iint \text{curl}\, \vec{F} \cdot \hat{n}\, ds$ —— (1) +1

$\text{curl}\, \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & xy & 0 \end{vmatrix} = y\hat{k}$ +1

Since the surface is on xy-plane then $\hat{n} = k$ +1

and $ds = dx\,dy$.

Now $\iint \text{curl}\, \vec{F} \cdot \hat{n}\, ds = \int_0^a \int_0^a y\, k \cdot k \, dx\, dy =$ +1

$= \int_0^a \left(\frac{y^2}{2}\right)_0^a dx = \frac{a^2}{2} [x]_0^a = \frac{a^3}{2}$ +1

5 marks

(iii). By Gauss Div. theorem $\iint \vec{F} \cdot \hat{n}\, ds = \iiint \text{div}\, \vec{F}\, dV$ +1

Here $\vec{F} = xi + yj + zk$ +1

then $\text{div}\, \vec{F} = 1+1+1 = 3$ +1

Now $\iiint \text{div}\, \vec{F} \cdot dv = 3 \iiint dx\, dy\, dz$ +1

$= 3$ (volume of sphere)

$= 3 \cdot \frac{4}{3} \pi (a^3) = 4\pi a^3$ +1

5 marks

Date: __/__/__
Page: ______

Qu. 6 (i). Regula false method

$f(x) = x \log_{10} x - 1.2 = 0$

$f(0) = -1.2$, $f(1) = -1.2$, $f(2) = -0.59$, $f(3) = 0.23$ +1

RLB (2,3)

By regula false $x_{n+1} = x_{n-1} - \frac{(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \times f(x_{n-1})$ +1

I $x_0 = 2$ $x_1 = 3$

$x_2 = 2.72$ $f(x_2) = -0.01$ +1

RLB (2.72, 3)

II $x_3 = 2.74$ $f(x_3) = -0.0003$ +1

RLB (2.74, 3)

III $x_4 = 2.74$ $f(x_4) = -0.00001$ +1

Ans Hence root of eq is 2.74. 5 marks

(ii). $\left[\begin{array}{ccc:c} 2 & -3 & 1 & -1 \\ 1 & 4 & 5 & 25 \\ 3 & -4 & 1 & 2 \end{array}\right] \Rightarrow$ +1 $R_1 \leftrightarrow R_2$ $\left[\begin{array}{ccc:c} 1 & 4 & 5 & 25 \\ 2 & -3 & 1 & -1 \\ 3 & -4 & 1 & 2 \end{array}\right]$ +1

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - 3R_1$

$\left[\begin{array}{ccc:c} 1 & 4 & 5 & 25 \\ 0 & -11 & -9 & -51 \\ 0 & -16 & -14 & -73 \end{array}\right]$ +1

$R_2 \leftarrow R_2 / -11$

$\left[\begin{array}{ccc:c} 1 & 4 & 5 & 25 \\ 0 & 1 & 9/11 & 51/11 \\ 0 & -16 & -14 & -73 \end{array}\right]$

$R_3 \leftarrow R_3 + 16R_2$

$\left[\begin{array}{ccc:c} 1 & 4 & 5 & 25 \\ 0 & 1 & 9/11 & 51/11 \\ 0 & 0 & -10/11 & 13/11 \end{array}\right]$ +1

$x + 4y + 5z = 25$

$y + \frac{9}{11} z = \frac{51}{11}$

$\frac{-10}{11} z = \frac{13}{11}$

Ans $z = -1.3$

$y = 5.7$ +1

$x = 8.7$

5 marks

Page: 5

(iii) Gauss Seidel method

$10x+y+z=12$ $\qquad x^{(i+1)} = \frac{1}{10}(12-y^{(i)}-z^{(i)})$

$2x+10y+z=13 \Rightarrow \qquad y^{(i+1)} = \frac{1}{10}(13-2x^{(i+1)}-z^{(i)})$ +1

$2x+2y+10z=14$ $\qquad z^{(i+1)} = \frac{1}{10}(14-2x^{(i+1)}-2y^{(i+1)})$

I let $y=z=0$ +1 +1

$x_1 = 1.2$, $y_1 = 1.06$, $z = 0.948$

II $x_2 = 0.999$, $y_2 = 1.005$, $z_2 = 0.999$ +1

III $x_3 = 1$, $y_3 = 1$, $z_3 = 1$ +1

Ans $x=1$, $y=1$, $z=1$ 5 marks