iii. In a test given to two groups of students drawn from two normal populations, the marks obtained were as follows: **5**

Group A 18 20 36 50 49 36 34 49 41

Group B 29 28 26 35 30 44 46

Examine whether the two populations have the same variance.
(value of F for (8,6) degree of freedom at 5% level is 4.15)

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2019

CA5BS02 Computer Oriented Numerical & Statistical Methods

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Let x be the exact value and x_a be the approximate value then relative error is **1**

(a) $|x-x_a|$ (b) $|x+x_a|$ (c) $\left|\frac{x-x_a}{x}\right|$ (d) $\left|\frac{x-x_a}{x}\right|.100$

ii. In Bisection method Order of convergence is **1**

(a) 0 (b) 1 (c) 2 (d) 3

iii. If $y = f(x)$ and x can take the equispaced values $x_0, x_1, \dots\dots\dots\dots, x_n$ and h is interval of differencing the by Newton-Gregory backward interpolation formula: **1**

(a) $u = \frac{x-x_0}{h}$ (b) $u = \frac{x+x_0}{h}$ (c) $u = \frac{x-x_n}{h}$ (d) $u = \frac{x+x_n}{h}$

iv. In the Interpolation when the values of the arguments are not equispaced, we can define a more general class of differences which is called **1**

(a) Divided difference (b) Forward difference
(c) Backward difference (d) None of these.

v. In fourth order Runge-Kutta method the value for k is **1**

(a) $k_1 + 2k_2 + 2k_3 + k_4$ (b) $(k_1 + 2k_2 + 2k_3 + k_4)/6$
(c) $(k_1 + 2k_2 + 2k_3 + k_4)/2$ (d) None of these.

vi. One of the following methods is also called "Method of successive integration" **1**

(a) Euler's method (b) Runge-Kutta's method
(c) Taylor's series method (d) Picard's method

vii. The probability of occurring a king when a card is drawn from a pack of 52 cards is **1**

(a) 3/14 (b) 2/15 (c) 1/13 (d) 1/12

viii. If the mean and variance of binomial distribution are 8 and 2 respectively. Then the values of n and p will be **1**

(a) 32/3 and 3/4 (b) 30/3 and 1/4

(c) 33/3 and 2/4 (d) 35/3 and 5/4.

ix. For test of hypothesis if number of observations > 30 which of the following method is suitable **1**

(a) Z test (b) t test (c) F test (d) χ^2

x. In F test, for carrying out the test of significance the ratio F is defined as (where $S_1 > S_2$) **1**

(a) $\frac{S_1^2}{S_2^1}$ (b) $\frac{S_1^2}{S_2^2}$ (c) $\frac{S_1}{S_2}$ (d) None of these.

Q.2 Attempt any two:

i. Find the real root of the $x\log_{10} x = 1.2$ by Bisection method correct to three decimal places. **5**

ii. Find the real root of the equation $xe^x = \cos x$ using the Regula-False method correct to three decimal places. **5**

iii. Evaluate $\sqrt{12}$ to four decimal places by Newton-Raphson method. **5**

Q.3 Attempt any two:

i. (a) Represent the function $f(x) = x^3 + 4x^2 + 9x + 12$ in Factorial notation. **5**

(b) Given $\sin 45^0 = 0.7071$, $\sin 50^0 = 0.7660$, $\sin 55^0 = 0.8192$ and $\sin 60^0 = 0.8660$. Find $\sin 52°$ using Newton's Forward Interpolation formula.

ii. Find a polynomial satisfied by (- 4, 1245), (- 1, 33), (0, 5), (2, 9) and (5, 1335) by the use of Divided differences method. Also find $f(1)$. **5**

iii. Find $\int_0^1 \frac{1}{1+x^2}dx$ by Simpson's 1/3 rule, where the interval is divided in to six equal parts. **5**

Q.4 Attempt any two:

i. Solve the following system by Gauss-Seidal method **5**

$27x + 6y - z = 85$, $6x + 15y + 2z = 72$, $x + y + 54z = 110$

ii. Find by Taylor's series method, the value of y at $x = 0.2$ from $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$. **5**

iii. Apply Runge-Kutta fourth order method to find approximate value of y when $x = 0.2$. Given that $\frac{dy}{dx} = x + y$ and $y = 1$ when $x = 0$. **5**

Q.5 Attempt any two:

i. A random variable X has the probability density function **5**

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax + 3a & 2 \le x \le 3 \\ 0 & otherwise \end{cases}$$

Find the value of a and $P(X \le 1.5)$.

ii. Obtain the mean, variance and standard deviation of Binomial distribution. **5**

iii. The distribution of the number of road accidents per day in a city in Poisson with mean 4. Find the number of days out of 100 days when there will be: no accident, at least 2 accidents, at most 3 accidents, between 2 and 5 accidents. **5**

Q.6 Attempt any two:

i. Explain the terms – types of hypothesis, null hypothesis, level of significance, types I and type II errors. **5**

ii. A company can claim that the weight of their product is 10 kg. A sample of 10 items taken from a lot supplied by the company has shown the following weights: **5**

10.2, 9.7, 10.3, 10.0, 9.8, 9.7, 9.6, 9.6, 9.7, 9.4

Is there any statistical evidence to support the claim of the company about the weight of the item?

(table value of t for 9 degree of freedom at 5% level is 1.833)

Faculty of Engineering

End Sem (Even) Examination May - 2019

CA5BS02 - Computer Oriented Numerical & Statistical Methods

Programme - MCA Branch - Computer Application

Qu. ① 1. (c). $\left|\frac{x-x_a}{x}\right|$ (10)

2. (b). 1

3. (c). $u = \frac{x-x_n}{h}$

4. (a). Divided difference

5. (b). $(K_1+2K_2+2K_3+K_4)/6$

6. (d). Picard's method

7. (c). 1/13

8. (a). 32/3 and 3/4

9. (a) Z test

10. (b). S_1^2/S_2^2

Qu. ② (i.) $f(x) = x\log_{10}x - 1.2$ (1)

$f(0) = -1.2$, $f(1) = -1.2$, $f(2) = -0.59$, $f(3) = 0.23$

Root lies between (2, 3)

For Bisection method $x_2 = \frac{x_0+x_1}{2}$

Iteration I $x_2 = \frac{2+3}{2} = 2.5$

$f(x_2) = -0.205$ RLB (2.5, 3)

Iteration II $x_3 = \frac{2.5+3}{2} = 2.75$

$f(x_3) = 0.008$ RLB (2.5, 2.75)

Iteration III $x_4 = \frac{2.5+2.75}{2} = 2.625$ (3)

$f(x_4) = -0.099$ RLB (2.625, 2.75)

The root of given eq is approx 2.688 (1) (ii). $f(x) = xe^x - \cos x = 0$ or $\cos x - xe^x = 0$

$f(0) = -1$, $f(1) = +2.177$ 1

Root lies between $(0,1)$

For Regula False $x_2 = x_0 - \dfrac{(x_1 - x_0)}{f(x_1) - f(x_0)} \times f(x_0)$

Iteration I $x_2 = 0.315$ $f(x_2) = -0.520$

— RLB $(0.315, 1)$

Iteration II $x_3 = 0.447$ $f(x_3) = -0.204$

RLB $(0.447, 1)$

Iteration III $x_4 = 0.494$ $f(x_4) = -0.079$ 3

RLB $(0.494, 1)$

The root of given eq is approx 0.49 or 0.5 1

(iii) $x = \sqrt{12}$ $\Rightarrow$ $x^2 = 12$ $\Rightarrow$ $x^2 - 12 = 0 = f(x)$

$f(0) = -12$, $f(1) = -11$, $f(2) = -8$, $f(3) = -3$, $f(4) = 4$

Root lies between $(3, 4)$ 1

For Raphson method $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$x_1 = x_0 - \dfrac{(x_0^2 - 12)}{2x_0} = \dfrac{x_0^2 + 12}{2x_0}$ 1

Iteration I $x_0 = 3.5$ $x_1 = 3.4643$

Iteration II $x_1 = 3.4643$ $x_2 = 3.4641$

Iteration III $x_2 = 3.4641$ $x_3 = 3.4641$ 3

The root of given eq is approx 3.4641

Que. 3 (i). a). Let $f(x) = Ax^{(3)} + Bx^{(2)} + Cx^{(1)} + Dx^{(0)}$ be a required factorial notation 1

By synthetic Division method

1	1	4	9	12
	0	1	5	
2	1	5	14	
	0	2		
3	1	7		
	0			
	1			

Hence

$f(x) = x^{(3)} + 7x^{(2)} + 14x^{(1)} + 12$ 1 b).

x	y	Δy	Δy	$\Delta^3 y$
45	0.7071	0.0589	−0.0057	−0.0007
50	0.7660	0.0532	−0.0064	
55	0.8192	0.0458		
60	0.8660			

(1)

$x = 52$, $h = 5$, $x_0 = 45$, then $u = \frac{x - x_0}{h} = 1.4$

By Newton's Forward Interpolation formula (1)

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0.$$

(1)

$$= 0.788$$

(ii) (2)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
−4	1245	−404	94	−14	3
−1	33	−28	10	13	
0	5	2	88		
2	9	442			
5	1335				

By Newton's Divided Difference method

$$f(x) = y + (x - x_0)\Delta y_0 + (x - x_0)(x - x_1)\Delta^2 y_0 + (x - x_0)(x - x_1)(x - x_2)$$
$$\Delta^3 y_0 + (x - x_0)(x - x_1)(x - x_2)(x - x_3)\Delta^4 y_0$$ (2)

$$= 3x^4 - 5x^3 + 6x^2 - 14x + 5$$ (1)

$f(1) = -5$

(1)

(iii) Given $n = 6$ then $h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	1
$f(x)$	1	0.973	.9	.8	0.692	0.590	.5

(2)

By Sim 1/3 rule

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$ (1)

(1)

$$= 0.785$$

4 Qu (4)·(i). $x = \frac{1}{27}(85 - 6y + z)$, $y = \frac{1}{15}(72 - 6x - 2z)$, $z = \frac{1}{54}(110 - x - y)$

We start with $y = z = 0$ (2)

Iteration I $x_1 = 3.15$, $y_1 = 3.54$, $z_1 = 1.91$

Iteration II $x_2 = 2.43$, $y_2 = 3.57$, $z_2 = 1.92$

Iteration III $x_3 = 2.426$, $y_3 = 3.572$, $z_3 = 1.925$ (3)

(ii). Given $x = 0.2$, $x_0 = 0$, $y_0 = 1$

By Taylor Series method (4)

$$y = y_0 + (x - x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \frac{(x-x_0)^3}{3!}y_0''' + \frac{(x-x_0)^4}{4!}y_0^{iv} + \cdots$$

$y' = x^2y - 1$ $\quad y_0' = -1$

$y'' = 2xy + x^2y'$ $\quad y_0'' = 0$

$y''' = 2xy' + 2y + 2xy' + x^2y''$ $\quad y_0''' = 2$

$y^{iv} = 6y' + 6xy'' + x^2y'''$ $\quad y_0^{iv} = -6$ (2)

Hence $y(0.2) = 0.8022$ (2)

(iii) Given $x = 0.2$, $x_0 = 0$, $y_0 = 1$, $h = 0.2$ (3)

$K_1 = h\, f(x_0, y_0) = 0.2000$ (1)

$K_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}) = 0.2 \times f(0.1, 1.1) = 0.2400$ (1)

$K_3 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}) = 0.2 \times f(0.1, 1.12) = 0.2440$ (1)

$K_4 = h\, f(x_0 + h, y_0 + K_3) = 0.2 \times f(0.2, 1.244) = 0.2888$ (1)

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.2428$ (1)

Qu. (5). (i). For value of a –

$\because \int_{-\infty}^{\infty} f(x)\,dx = 1$ (1)

$\int_{-\infty}^{0} + \int_{0}^{1} + \int_{1}^{2} + \int_{2}^{3} + \int_{3}^{\infty} = 1$

$\int_{0}^{1} ax\,dx + \int_{1}^{2} a\,dx + \int_{2}^{3} (-ax+3a)\,dx + \int_{3}^{\infty} 0 = 1$ (1)

Hence $a = \frac{1}{2}$ (1)

For $P(X \le 1.5)$ –

$P(-\infty < X \le 1.5) = \int_{-\infty}^{1.5} f(x)\,dx = \int_{0}^{1.5} f(x)\,dx = \int_{0}^{1} ax\,dx + \int_{1}^{1.5} a\,dx$

$= \frac{a}{2} + (0.5)a = a = \frac{1}{2}$ (2)

(ii) Binomial distribution $P(r) = {}^{n}C_r\, p^r q^{n-r}$

Mean. $\mu_1' = \sum_{r=0}^{n} r\,P(r) = \sum_{r=0}^{n} r.\,{}^{n}C_r\, p^r q^{n-r}$ (2)

$= \sum_{r=1}^{n} n\,{}^{n-1}C_{r-1}\, p^{r-1+1}\, q^{(n-1)-(r-1)}$

$= np\,(q+p)^{n-1} = np$

Variance $\mu_2 = \sigma^2 = \mu_2' - (\mu_1')^2$ (2)

$\mu_2' = \sum_{r=0}^{n} r^2\,P(r) = \sum_{r=0}^{n} (r^2-r)\,P(r) + \sum_{r=0}^{n} r\,P(r)$

$= \sum_{r=2}^{n} n(n-1)\,{}^{n-2}C_{r-2}\, p^r q^{n-r} + np$

$= n(n-1)p^2 \sum_{r=2}^{n} {}^{n-2}C_{r-2}\, p^{r-2}\, q^{(n-2)-(r-2)} + np$

$= n(n-1)p^2 + np = n^2p^2 + npq$

Hence $\mu_2 = (n^2p^2 + npq) - (np)^2 = npq$

Standard deviation $\sigma = \sqrt{\text{variance}} = \sqrt{npq}$ (1)

(iii) Given $m=4$, Poisson Distribution $P(X=x) = \frac{e^{-m} m^r}{r!}$ (1)

1). No accident $(r=0)$

$P(X=0) = \frac{e^{-4}4^0}{0!} = e^{-4} = 0.0183 \times 100 = 1.83 \simeq$ 2 days (1)

2). At least 2 accident ($r = 2, 3, 4, 5, \ldots$)

$$P(2 \leq X) = 1 - P(0) - P(1)$$ (1)

$$= 1 - e^{-4} - \frac{e^{-4}4}{1!} = 0.9085 \times 100 = 90.85 \simeq 91 \text{ days}$$

3). At most 3 accidents ($r = 0, 1, 2, 3$)

$$P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$$

$$= e^{-4} + \frac{e^{-4}4}{1!} + \frac{e^{-4}4^2}{2!} + \frac{e^{-4}4^3}{3!}$$

$$= 0.4331 \times 100 = 43.31 \simeq 43 \text{ days}$$ (1)

4). Between 2 and 5 accidents ($r = 3, 4$)

$$P(2 < X < 5) = P(3) + P(4)$$ (1)

$$= \frac{e^{-4}4^3}{3!} + \frac{e^{-4}4^4}{4!} = 0.3904 \times 100 = 39.04 \simeq 39 \text{ days}$$

Que. (6)(i) Types of hypothesis – (1)

A statistical hypothesis which specifies the population completely is called sample hypothesis.

A statistical hypothesis which does not specify the population completely is called composite hypothesis.

Null hypothesis – H_0 (1)

A statistical hypothesis which is stated for the purpose of possible acceptance is known as null hypothesis. It is tested for possible rejection under the assumption that is true.

Level of significance – (generally 5% or 1%) (1)

It is the maximum probability of rejecting the null hypothesis when it is true. Desired level of significance is always fixed in advance before applying the test.

Type I & II errors – (2)

	Accept H_0	Reject H_0
H_0 is true	No error	Type I error (α)
H_0 is false	Type II error (β)	No error

(ii) 1. Null hypothesis - H_0 (1)

The weight of product is 10 kg ($\mu = 10$ kg)

Alternate hypothesis - H_1

The weight of product is not 10 kg ($\mu \neq 10$ kg)

2. Calculation for mean & standard deviation (2)

x	$(x-\bar{x})$	$(x-\bar{x})^2$
10.2	0.4	0.16
9.7	-0.1	0.01
10.3	0.5	0.25
10.0	0.2	0.04
9.8	0	0
9.7	-0.1	0.01
9.6	-0.2	0.04
9.6	-0.2	0.04
9.7	-0.1	0.01
9.4	-0.4	0.16
98.0		0.72

$$\text{mean} = \bar{x} = \frac{\Sigma x}{n} = \frac{98}{10} = 9.8$$

$$SD = \sqrt{\frac{(x-\bar{x})^2}{n-1}} = \sqrt{\frac{0.72}{9}} = 0.283$$

3. The value of t - test statistics (1)

$$t = \frac{|\bar{x}-\mu|}{s} \cdot \sqrt{n} = \frac{|9.8-10|}{0.283} \sqrt{10} = \frac{0.632}{0.283} = 2.23$$

4. Level of significance - 5%

5. Degree of freedom - $v = n-1 = 9$

6. Table value - 1.833

7. Decision. - computed value > tabular value (1)

Then null hypothesis will be rejected. Hence there is no significance evidence to support the company's claim

(iii) 1. Null hypothesis - H_0 (1)

the variance do not differ significantly

Alternative hypothesis - H_1

The variance differ significantly

8:

2. Calculation for variance (2)

x	$x-\bar{x}$	$(x-\bar{x})^2$
18	−19	361
20	−7	289
36	−1	1
50	13	169
49	12	144
36	−1	1
34	−3	9
49	12	144
41	4	16

y	$y-\bar{y}$	$(y-\bar{y})^2$
29	5	25
28	6	36
26	8	64
35	1	1
30	4	16
44	10	100
46	12	144

$$\bar{x} = \frac{\Sigma x}{n} = \frac{333}{9} = 37$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{238}{7} = 34$$

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{1134}{8} = 141.75$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n-1} = \frac{386}{6} = 64.33$$ (1)

3. The value of F- statistics

$$F = \frac{S_1^2}{S_2^2} = \frac{141.75}{64.33} = 2.203$$

4. level of significance - 5%.

5. Degree of freedom - $V_1 = 9-1 = 8$, $V_2 = 7-1 = 6$

6. Table value - 4.15

7. Decision- computed value < table value (1)

The null hypothesis will be accepted.
Hence the populations from where the samples have been taken have the same variances.