		Jobs				
		A	B	C	D	E
Machines	I	5	11	10	12	4
	II	2	4	6	3	5
	III	3	12	5	14	6
	IV	6	14	4	11	7
	V	7	9	8	12	5

OR iii. Five jobs are performed, first on machine X and then on Machine Y. The time taken, in hours by each job on each machine is given below: **6**

Jobs	1	2	3	4	5
Machine A	5	1	9	3	10
Machine B	2	6	7	8	4

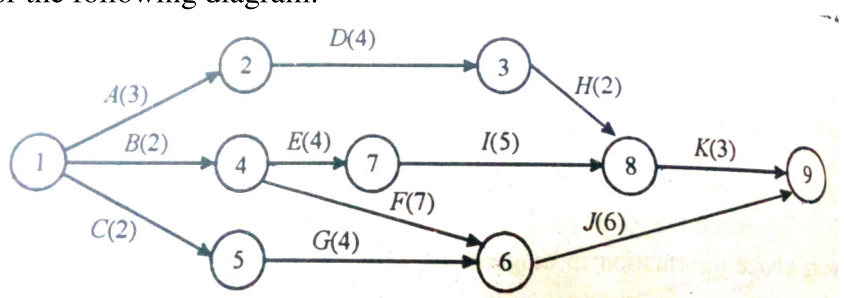
Determine the sequence for 5 jobs that will minimize the total elapsed time and idle time for machine A and B

Q.6 i. What are the three time estimates used in the context of PERT (Program Evaluation Review Technique)? Define them. **3**

ii. Draw the network for the data given below and compute: **7**
(a) Critical path (b) Early start and Late start times for each activity and (c) Total project duration.

Activity	A	B	C	D	E	F	G	H	I
Predecessor	-	-	-	A	B	C	D, E	B	H, F
Estimated time (weeks)	3	5	4	2	3	9	8	7	9

OR iii. For the following diagram: **7**



(a) Compute earliest event time and latest event time.
(b) Critical path and total project duration.

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Management
End Sem (Even) Examination May-2018
MS3CO05 Operations Research

Programme: BBA Branch/Specialisation: Management

Duration: 3 Hrs. **Maximum Marks: 60**

Note: (a) All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.
(b) Use of simple (non-programmable) calculator is allowed.

Q.1 i. Operations research is the application of ____________methods to arrive at the optimal solutions to the problems. **1**
(a) Economical (b) Scientific
(c) Both (a) and (b) (d) Artistic

ii. _______ Models are obtained by enlarging or reducing the size of an item. **1**
(a) Analogue (b) Symbolic (c) Iconic (d) None of these

iii. In graphical method to solve LPP, if there is no feasible region in the plotted graph, then it is case of **1**
(a) Unique solution (b) Many solution
(c) Bounded solution (d) No solution

iv. In linear programming problems, the objective function and objective constraints are: **1**
(a) Linear (b) Quadratic (c) Solved (d) Adjacent

v. Transportation model is also known as: **1**
(a) Logistics model (b) Distribution model
(c) Assignment model (d) None of these

vi. The Penalty in Vogel's Approximation Method represents difference between ________cost of respective row / column. **1**
(a) Two Largest (b) Largest and smallest
(c) Two smallest (d) None of these

vii. Minimum number of lines to cover all the zeroes in an Assignment problem is equal to number of **1**
(a) Assignments (b) Rows
(c) Columns (d) All of these

viii. The time interval between the starting the first job and completing the last job including idle time if any in a particular order by the given set of machines is called ______ time **1**

(a) Processing (b) Waiting
(c) Total elapsed (d) Idle

ix. Activity which starts only after finishing other activity is called ____ **1**

(a) Predecessor (b) Successor (c) Dummy (d) None of these

x. Network models have advantage in terms of project________ **1**

(a) Planning (b) Controlling
(c) Scheduling (d) All of these

Q.2 i. Discuss any four limitations of Operations Research. **4**

ii. Discuss in brief the scope of Operations Research in any three key areas of Business Management. **6**

OR iii. What are the three major phases of Scientific Method in Operations Research? Explain them in brief. **6**

Q.3 i. A firm manufactures three products A, B and C. The profits are Rs.3, Rs.2 and Rs.4 respectively. The firm has two machines M_1 and M_2 and below is the required capacity processing time in minutes for each machine on each product. **4**

Machines	Product		
	A	B	C
M_1	4	3	5
M_2	2	2	4

Machines M_1 and M_2 have 2000 and 2500 machine minutes respectively. The firm must manufacture 100 A's, 200 B's and 50 C's but not more than 150 A's. Formulate an LPP to maximize the profit.

ii. Solve the given Linear Programming Problem by Graphical Method. **6**

$$
\begin{aligned}
Min\ z &= 6x_1 + 14x_2 \\
subject\ to{:}\ \ 5x_1 + 4x_2 &\geq 60 \\
3x_1 + 7x_2 &\leq 84 \\
x_1 + 2x_2 &\geq 18 \\
and \quad x_1, x_2 &\geq 0
\end{aligned}
$$

OR iii. Solve the given Linear Programming Problem by Simplex Method. **6**

$$
\begin{aligned}
Max\ z &= 4x_1 + 10x_2 \\
subject\ to{:}\ \ 2x_1 + x_2 &\leq 10 \\
2x_1 + 5x_2 &\leq 20 \\
2x_1 + 3x_2 &\leq 18 \\
and \quad x_1, x_2 &\geq 0
\end{aligned}
$$

Q.4 i. Define the following terms in a Transportation Model: **4**

(a) Feasible solution (b) Initial basic feasible solution
(c) Optimal solution (d) Non-degenerate solution

ii. Solve the following cost minimizing Transportation problem to get an optimal solution. **6**

	Destinations					
		D1	D2	D3	D4	Supply
Sources	S1	5	2	4	3	22
	S2	4	8	1	6	15
	S3	4	6	7	5	8
Demand		7	12	17	9	

OR iii. Obtain the initial basic feasible solution of the following Transportation problem using North West Corner Rule and Least Cost Method and find the transportation cost associated in both the cases. **6**

	Distribution centres					
		I	II	III	IV	Supply
Plants	A	2	3	11	7	6
	B	1	0	6	1	1
	C	5	8	15	9	10
Requirement		7	5	3	2	

Q.5 i. Explain the mathematical model of an Assignment Problem **4**

ii. A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning i^{th} $(i = I, II, III, IV, V)$ machines to the j^{th} job$(j = A, B, C, D, E)$. Assign the five jobs to the five machines so as to maximize the total expected profits. **6**

Medi-Caps University
Faculty of Management
End Sem (Even) Examination May-2017
MS3CO05 Operations Research
Programme: BBA M.M. 60

Solution

Q.1 i. (b) scientific (1)
ii. (c) Iconic (1)
iii. (d) No solution (1)
iv. (a) linear (1)
v. (b) Distribution model (1)
vi. (c) two smallest (1)
vii. (a) assignments (1)
viii. (c) total elapsed (1)
ix. (a) Predecessor (1)
x. (d) All of the above (1)

Q.2 (i) 1. Mathematical models, which are essence of O.R. do not take into account qualitative factors or emotional factors which are quite real. (1)

2. OR Models are applicable to only specific category of problems (+1)

3. OR techniques are usually very expensive as they require services of specialised persons and computers. (+1)

4. Many OR techniques having extremely lengthy techniques/computations to solve any mathematical problem (+1)

Note: Student may write other limitations also.

Q.2 (ii) Some of the areas of mgmt. where OR techniques have been successfully applied are as below :

1. Marketing :-
   a.) selection of advertising media planning
   b.) Sales effort allocation
   c.) Optimum policies for marketing. (2)
2. Personnel Mgmt. :-
   a.) Selection of personnel
   b.) Determination of retirement age
   c.) Wages administration
   d.) Scheduling of training pgm.s (+2)
3. Finance/Accounting :-
   a.) Capital requirement
   b.) Cash flow analysis
   c.) Profit plans
   d.) Capital Budgeting (+2)

(Note: Student may write other key areas, may explain the key points & also may give examples)

Q2 (OR) (iii) The most important feature in O.R. is use of scientific method. There are three major phases of scientific method on which OR practise is based.

1.) <u>Judgement Phase</u> :- This phase include (2)
   a.) identification of real-life problem
   b.) selection of an appropriate objective funcn
   c.) formulation of the problem etc.

2.) <u>Research phase</u> :- This phase includes (+2)
   a.) formulation of hypothesis, b.) analysis of info etc.

3.) <u>Action Phase</u> :- consist of recomendations for decision etc. (+2)

(Student should explain the three phase & may give exam...

Q. 3 (i) Let x_1, x_2, x_3 be the no. of units of product A, B, C resp. s.t. these quantities maximize the profit. Pg (3)

∴ Decision variables are x_1, x_2, x_3

Required LPP is :-

Objective func^n: Max $z = 3x_1 + 2x_2 + 4x_3$ (1)

Subject to constraints
$$4x_1 + 3x_2 + 5x_3 \le 2000$$
$$2x_1 + 2x_2 + 4x_3 \le 2500$$
$$100 \le x_1 \le 150$$
$$x_2 \geq 200$$
$$x_3 \geq 50$$ (+2)

& Non-negativity $x_1, x_2, x_3 \geq 0$. (+1)

Q-3 (ii) Replacing all inequalities of the constraints

$5x_1 + 4x_2 = 60$

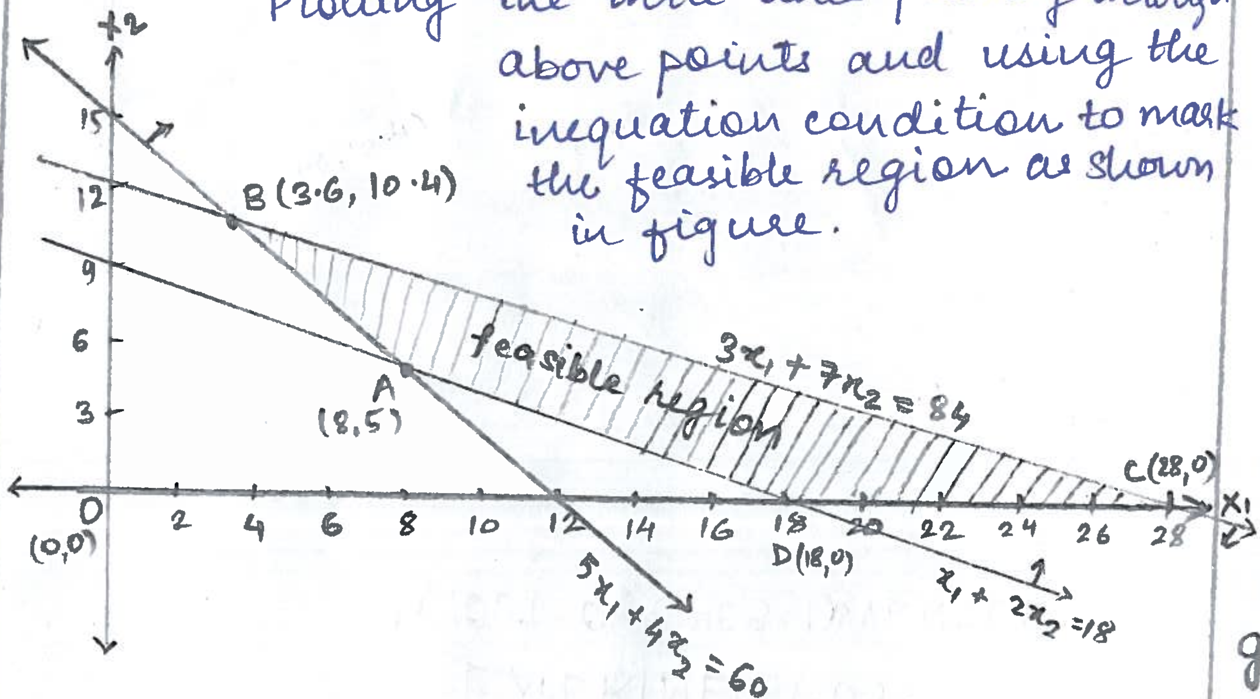
if $x_1 = 0 \Rightarrow x_2 = 15$ i.e. point (0,15)

if $x_2 = 0 \Rightarrow x_1 = 12$ i.e. point (12,0)

Similarly in $3x_1 + 7x_2 = 84$, points are (28,0) (0,12)

& in $x_1 + 2x_2 = 18$, points are (18,0), (0,9) (2)

Plotting the three lines passing through above points and using the inequation condition to mark the feasible region as shown in figure.



(+2) for graph

The feasible region is ABCD
Determine the values of objective function z at the extreme points A, B, C & D.

Extreme point	x_1	x_2	$Z = 6x_1 + 14x_2$
A	8	5	118
B	3.6	10.4	168
C	28	0	148
D	18	0	108 ← Minimum

Since z is to be minimized, ∴ the min^m value of z occurs at D (18, 0)
Hence the optimal sol^n to the LPP is
$x_1 = 18$, $x_2 = 0$, $Z = 108$ (min value) (+2)

Q.3 (OR) (iii) Converting the given LPP into standard form

Max $z = 4x_1 + 10x_2 + 0S_1 + 0 \cdot S_2 + 0 \cdot S_3$
sub to $2x_1 + x_2 + S_1 = 10$
$2x_1 + 5x_2 + S_3 = 20$
$2x_1 + 3x_2 + S_3 = 18$
Non-neg. $x_1, x_2, S_1, S_2, S_3 \geq 0$

(2) for std. form

		$C_j \rightarrow$	4	10	0	0	0	
Basic variable ↓	C_B ↓	↓ X_B	x_1	x_2	S_1	S_2	S_3	Min^m non-neg. Ratio X_B/x_j
1st iteration S_1	0	10	2	1	1	0	0	$10/1 = 10$
S_2	0	20	2	$\boxed{5}$	0	1	0	$20/5 = 4$ ←
S_3	0	18	2	3	0	0	1	$18/3 = 6$
	$z_j = \Sigma C_B X_B = 0$		−4	−10 ↑	0	0	0	$\Delta_j = \Sigma C_B x_j - C_j$
2nd iteration S_1	0	6	8/5	0	1	−1/5	0	
x_2	10	4	2/5	1	0	1/5	0	
S_3	0	6	4/5	0	0	3/5	1	
	$z_j = \Sigma C_B X_B = \boxed{40}$		0	0	0	+2	0	$\Delta_j = \Sigma C_B x_j - C_j$ ←

(+3) ↓ for table

In 2nd iteration,
New R_2 → old $R_2/5$
New R_1 → old R_1 − 1·New R_2
New R_3 → Old R_3 − 3·New R_2

Pg (5)

we see that in 2^{nd} iteration, all Δ_j's are zero or positive, so the optimal sol^n is

$$x_1 = 0 \;\&\; x_2 = 4 \;\&\; \text{Max } z = 4\times 0 + 10\times 4 = 40 \quad \text{Ans.}$$ (+1)

Q.4 (i) a.) <u>Feasible sol^n</u> → A feasible sol^n to a T.P. is a set of non-negative allocations x_{ij} that satisfies the rim (row & col^m) restrictions (1)

b.) <u>Initial B.F.S.</u> → A feasible sol^n to a T.P. is said to be I.B.F.S if it contains not more that $(m+n-1)$ non-negative allocations, where m is no. of rows & n is no. of columns. (+1)

c.) <u>Optimal Sol^n</u> → A feasible sol^n (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution. (+1)

d.) <u>Non-degenerate Sol^n</u> → A B.F.S. to a T.P. is said to be non-degenerate if,

→ the total no. of non-negative allocations is exactly $m+n-1$, and

→ these $m+n-1$ allocations are in independent positions. (+1)

Q.4 (ii) ∵ total supply = total demand = 45

∴ problem is balanced, so, By VAM

	D_1	D_2	D_3	D_4	Supply	Row penalties			
S_1	5	[12] 2	[2] 4	[8] 3	22/10/8	1	1	1	2
S_2	4	8	[15] 1	6	15/0	3	3	–	–
S_3	[7] 4	6	7	[1] 5	8	1	1	1	1
Demand	7	12/0	17/2/0	9					
Column Penalties	0	4^*	3	2					
	0	–	3^*	2					
	0	–	3^*	2					
	1	–	–	2^*					

Pg 6

Thus, By VAM, initial B.F.S is

$T.C. = 2\times12 + 4\times2 + 3\times8 + 1\times15 + 4\times7 + 5\times1$

$= 24 + 8 + 24 + 15 + 28 + 5$

$= Rs. 104.$

(3)

Applying test of optimality by MODI method

$\because$ no. of occupied cells = 6

& $m+n-1 = 6$

$\therefore$ I.B.F.S. is non-degenerate. Thus the test of optimality is non-degenerate.

1.) Set the cost matrix & set the no.s u_i & v_j (for occupied cells) s.t $c_{ij} = u_i + v_j$

	D_1	D_2	D_3	D_4	u_i
S_1		2	4	3	$u_1 = 3$
S_2			1		$u_2 = 0$
S_3	4			5	$u_3 = 5$
v_j	$v_1 = -1$	$v_2 = -1$	$v_3 = 1$	$v_4 = 0$	

$(i = 1, 2, 3)$

$(j = 1, 2, 3, 4)$

(let $v_4 = 0$)

2.) Compute c_{ij} for unoccupied cells

5			
4	8		6
	6	7	

3.) Compute $u_i + v_j$ for unoccupied cells

2			
-1	-1		0
	4	6	

4.) Construct $\Delta_{ij} = c_{ij} - (u_i + v_j)$

3			
5	9		6
	2	1	

Since all the entries in Δ_{ij} are positive or zero the current IBFS is optimal. Hence IBFS obtained by VAM is optimal & Min^m T.C. is Rs 104

Ans

(+3)

Q 4 (OR) (iii) By NWCR rule, the IBFS is

	I	II	III	IV	Supply
A	$\boxed{6}$ 2	3	11	7	6/0
B	$\boxed{1}$ 1	0	6	1	1/0
C	5	$\boxed{5}$ 8	$\boxed{3}$ 15	$\boxed{2}$ 9	10/5/2/0
Demand	7/1/0	5/0	3/0	2/0	

$$T.C. = 2\times6 + 1\times1 + 8\times5 + 3\times15 + 2\times9$$
$$= 12 + 1 + 40 + 45 + 18$$
$$= Rs.\ 116$$

(3)

By LCM, the IBFS is

	I	II	III	IV	Supply
A	$\boxed{6}$ 2	3	11	7	6/0
B	1	$\boxed{1}$ 0	6	1	1/0
C	$\boxed{1}$ 5	$\boxed{4}$ 8	$\boxed{3}$ 15	$\boxed{2}$ 9	10/9/5/3/0
Demand	7/1/0	5/4/0	3/0	2/0	

$$T.C. = 6\times2 + 1\times0 + 1\times5 + 4\times8 + 3\times15 + 2\times9$$
$$= Rs.\ 112.$$

(+3)

Q.5 (i) Suppose there are n jobs & n persons are available for doing these jobs.
Assume that each person can do each job at a time through varying degree of efficiency. The problem is: assigning each person to one & only one job so that total cost of performing all jobs is minimum

Let c_{ij} be the cost of assigning i^{th} person to j^{th} job. The assignment matrix as shown in the table as.

Jobs

	1	2	......	n	a_i Supply
1	c_{11}	c_{12}	.....	c_{1n}	1
2	c_{21}	c_{22}	...	c_{2n}	1
Persons ⋮	⋮	⋮	⋱	⋮	⋮
n	c_{n1}	c_{n2}	----	c_{nn}	1
b_j Demand	1	1	......		

(2)

. Mathematically, the assignment model can be expressed as

Let x_{ij} denote the assignment of person i to job j s.t $x_{ij} = \begin{cases} 1 \; ; \text{ if } i^{th} \text{ is assigned } j^{th} \text{ job} \\ 0 \; ; \text{ '' } i^{th} \text{ is not '' } j^{th} \text{ job} \end{cases}$

Then, model is given by

$$\min Z = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij}$$

sub. to. $\sum_{j=1}^{n} x_{ij} = 1 \quad , i = 1, 2, 3 \ldots, n$

(one job is done by i^{th} person)

& . $\sum_{i=1}^{n} x_{ij} = 1 \quad , j = 1, 2, \ldots n$

(one person is assign the j^{th} job) (+2)

and $x_{ij} = 0$ or $1 \; \forall \; i \; \& \; j$

Q.5 (ii) Converting the above maximization prob. into minimization problm by subtracting all the elem. from highest elem. 14.

9	3	4	2	10
12	10	8	11	9
11	2	9	0	8
8	0	10	3	7
7	5	6	2	9

(1)

Subtracting the smallest elem. of each row from all the elem.s of that row, we have Pg (9)

$$\begin{array}{ccccc} 7 & 1 & 2 & 0 & 8 \\ 4 & 2 & 0 & 3 & 1 \\ 11 & 2 & 9 & 0 & 8 \\ 8 & 0 & 10 & 3 & 7 \\ 7 & 5 & 6 & 2 & 9 \end{array}$$

(+1)

Subtracting the smallest elem. of each colm from all the elem.s of that colm & them determine an optimal assignment

$$\begin{array}{ccccc} 3 & 1 & 2 & \boxed{0} & 7 \\ \boxed{0} & 2 & \cancel{0} & 3 & \cancel{0} \\ 7 & 2 & 9 & \cancel{0} & 7 \\ 4 & \boxed{0} & 10 & 3 & 6 \\ 1 & 3 & 6 & \cancel{0} & 6 \end{array}$$

(+2)

Since 3rd & 5th row have no assignment, draw minm no. of lines covering all zeroes (as above) as current soln is not optimal

The new revised matrix is constructed by selecting the smallest elem. among all uncov. elem. & subtract it from all uncov. elem & add it to the elem.s at intersection of lines & again determine an optimal assign. & repeating the process if necessary.

$$\begin{array}{ccccc} 2 & \cancel{0} & 1 & \cancel{0} & 6 \\ \cancel{0} & 2 & \boxed{0} & 4 & \cancel{0} \\ 6 & 1 & 8 & \boxed{0} & 6 \\ 4 & \boxed{0} & 10 & 4 & 6 \\ \boxed{0} & 2 & 5 & \cancel{0} & 5 \end{array}$$

(+1)

$$\begin{array}{ccccc} 1 & \cancel{0} & \boxed{0} & \cancel{0} & 5 \\ \cancel{0} & 3 & \cancel{0} & 5 & \boxed{0} \\ 5 & 1 & 7 & \boxed{0} & 5 \\ 3 & \boxed{0} & 9 & 4 & 5 \\ \boxed{0} & 3 & 5 & 1 & 5 \end{array}$$

(+1)

Pg (10)

This table gives optimum assignment as
$1 \to C$, $2 \to E$, $3 \to D$, $4 \to B$, $5 \to A$
and max. profit = Rs. (10 + 5 + 14 + 14 + 7)
= Rs. 50 Ans

Q5 (OR) (iii) Step-I
The smallest processing in the given problem is 1 on machine A for job 2 so perform job 2 first as shown below:

Job

2				

Delete the job 2 & reduce list of processing time becomes:

Job	1	3	4	5
Machine A	5	9	3	10
Machine B	2	7	8	4

(3)

Step-II : Again the smallest processing in the reduced list is 2 on machine B for job 1. So place job 1 last in the list

2				1

Proceeding same way, the optimal job seq. is

2	4	3	5	1

i.e. (2-4-3-5-1) (+2)

Computation of elapsed time:

Jobs	Machine A IN	Machine A OUT		Machine B IN	Machine B OUT		Idle time Machine A	Idle time Machine B
2	0	1		1	7	(6)	0	1
4	1	4	(1+3)	7	15	(8)	0	0
3	4	13	(9+4)	15	22	(7)	0	0
5	13	23	(10)	23	27	(4)	0	1
1	23	28	(5)	28	30	(2)	30-28 =2	1
							2	3

(+2)

Note: A job is assign on a machine A first and after it has been completely process on machine A it is assigned to machine B. If the machine B is not free at the moment for processing the same job, the job has to get in a waiting time for its turn on machine B i.e. passing is not allowed.

Total elapsed time – The min^{m} elaps time is the time from start up job to the ending of last job is 30. (+1)

Ideal time on machine A → It is given by 30 – 28 = 2 unit.

Ideal time on machine B → Machine B remains ideal for 3 units (i.e. 0-1, 22-23, 27-28) (+1)

Q.6 (i) The three time estimates used in the context of PERT are :-

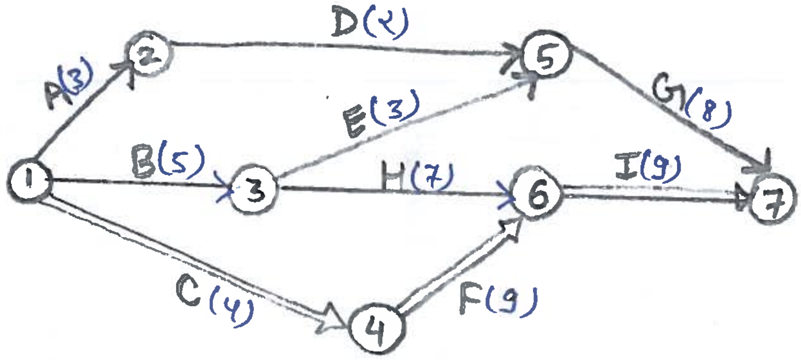
a) optimistic time estimate - It is the min^{m} possible time (ideally). The optimistic time that an activity will take if everything goes well. The prob. that the time activity will take less than this time estimate is 0.01, which is every little chance that activity can be done in time less than optimistic time. It is denoted by t_o or (a). (1)

b) Most likely time: It is a subjective estimate of most frequently time. It refers to the estimate of the normal time that the activity would take (under normal conditions). In most likely time, the most realistic time available & assumes normal delays.

It is denoted by t_m or (m). (+1)

c.) <u>Pessimistic time estimate</u> : It is a (max time) longest time possible if everything goes wrong. It does not include acts of nature like earth quakes, floods etc. The prob. that the activity will exceed this time estimate is 0.01. It is denoted by t_p or b. (+1)

Q.6 (ii)



(+2)

(i) Critical path $C \rightarrow E \rightarrow I$ (+1)

(ii) & (iii)

~~Activity~~ Events :	1	2	3	4	5	6	7	[illegible]	[illegible]
Early start (E_i) :	0	3	5	4	8	13	22	[illegible]	[illegible]
Late start (L_i) :	0	12	6	4	14	13	22	[illegible]	[illegible]

(+2)

Project duration = 22 weeks. (+2)

Q.6 (iii) a.) Determination of earliest event time & latest event time :

Fwd Pass computations. In this section we calculate earliest starting time of event E_i and earliest occurrence time for node j,

$$E_j = \max_i (E_i + d_{ij})$$

Event 1. Set $E_1 = 0$ as E_1 is in initial node

Event 2. $E_2 = \max(E_1 + d_{12}) = 0 + 3 = 3$

Event 3. $E_3 = \max(E_2 + d_{23}) = 3 + 4 = 7$

Event 4. $E_4 = \max(E_1 + d_{14}) = 0 + 2 = 2$

Event 5. $E_5 = Max(E_1 + d_{15}) = 0 + 2 = 2$

Event 6. $E_6 = Max(E_4 + d_{46}, E_5 + d_{56}) = Max(2+7, 2+4)$
$= Max(9, 6) = 9$

Event 7. $E_7 = Max(E_4 + d_{47}) = 2 + 4 = 6$

Event 8. $E_8 = Max(E_3 + d_{38}, E_7 + d_{78}) = Max(9, 11) = 11$

Event 9. $E_9 = Max(E_8 + d_{89}, E_6 + d_{69})$
$= Max(11 + 3, 9 + 6)$
$= Max(14, 15) = 15$

(2.5)

<u>Backward Pass Computation</u>: In this section we calculate, the latest finish time & latest start time. The latest time for an event i is given by

$$L_i = Min(L_j - d_{ij})$$

Event 9. set $L_9 = E_9 = 15$.

Event 8. $L_8 = Min(L_9 - d_{89}) = 15 - 3 = 12$

Event 7. $L_7 = Min(L_8 - d_{78}) = 12 - 5 = 7$

Event 6. $L_6 = Min(L_9 - d_{69}) = 15 - 6 = 9$

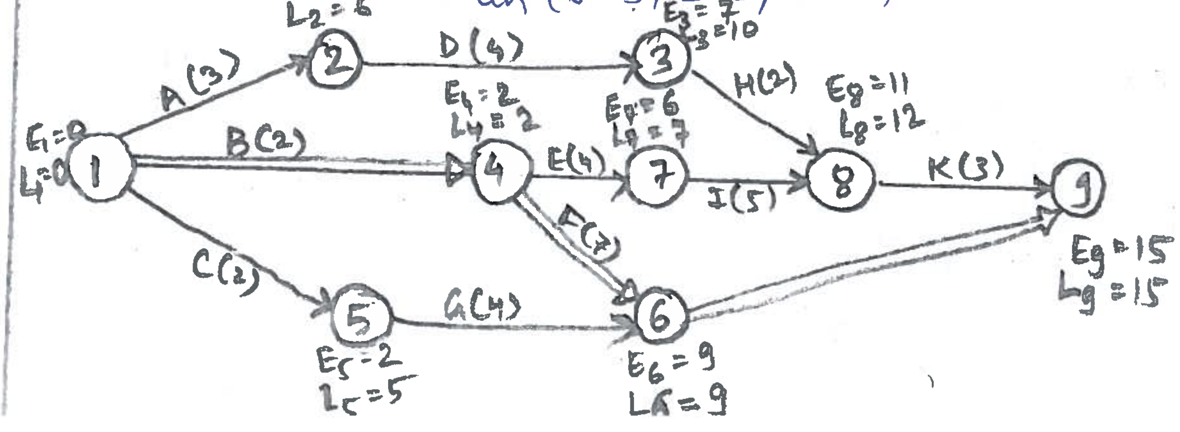
Event 5. $L_5 = Min(L_6 - d_{56}) = 9 - 4 = 5$

Event 4. $L_4 = L_4 = \underset{j=6,7}{Min}(L_6 - d_{46}, L_7 - d_{47})$
$= Min(9 - 7, 7 - 4) = Min(2, 3) = 2$

Event 3. $L_3 = Min(L_8 - d_{38}) = 12 - 2 = 10$

Event 2. $L_2 = Min(L_3 - d_{23}) = 10 - 4 = 6$

Event 1. $L_1 = \underset{j=2,4,5}{Min}(L_2 - d_{12}, L_4 - d_{14}, L_5 - d_{15})$ (2.5)
$= Min(6 - 3, 2 - 2, 5 - 2) = 0$



The critical path shown by double line in the figure is the largest path through the network. The critical path is 1-4-6-9 for which E-values are equal to L-values i.e. $E_1 = L_1$, $E_4 = L_4$, $E_6 = L_6$, $E_9 = L_9$. (+2)

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