			Marks	BL	PO	CO	PSO
OR	iii.	State the Binomial Theorem and use it to expand $(x+y)^6$. Also, find the general term in the expansion of $(x+y)^n$.	**5**	04	2	4	3
Q.6		Attempt any two:					
	i.	Discuss how generating functions can be used to solve recurrence relations. Solve the recurrence relation $a_n=2a_{n-1}+n$ using generating functions, given $a_0=1$.	**5**	04	2	5	2
	ii.	Explain what is meant by a first-order and second order recurrence relation. Solve the first-order recurrence relation $a_n=5a_{n-1}+6$ with $a_0=3$ and explain each step.	**5**	03	5	5	2
	iii.	Define the exponential generating function (EGF) of a sequence and explain how it differs from the ordinary generating function. Illustrate with an example by finding the EGF for the sequence of factorials $\{n!\}$.	**5**	03	5	5	2

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering
End Sem Examination Dec 2024
IT3EA09 Graph Theory

Programme: B.Tech. Branch/Specialisation: IT

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

			Marks	BL	PO	CO	PSO
Q.1	i.	Which of the following is true about connected graphs? (a) Every vertex has at least one edge (b) There exists a path between every pair of vertices (c) Every edge is part of a cycle (d) The graph contains no circuits	**1**	01	2	1	2
	ii.	A Hamiltonian circuit in a graph is defined as: (a) A circuit that visits every vertex exactly once (b) A path that uses each edge exactly once (c) A circuit that uses all vertices and edges exactly once (d) A circuit that visits each vertex at least twice	**1**	01	2	1	2
	iii.	In a weighted graph, the weight of a spanning tree is: (a) The total number of vertices in the tree (b) The sum of all edge weights in the tree (c) The maximum edge weight in the tree (d) The number of edges in the tree	**1**	02	2	2	2
	iv.	If two graphs are 1-isomorphic, they have: (a) Identical structure and same edge weights (b) The same adjacency list representation (c) The same structure and degrees of corresponding vertices (d) Different structures and different degrees	**1**	02	2	2	2

	v.	The chromatic number of a complete graph with n vertices is: (a) n (b) n-1 (c) 1 (d) n/2	**1**	01	2	2	2
	vi.	A graph that can be colored with two colors and has no odd cycles is called: (a) Bipartite (b) Complete (c) Connected (d) Weighted	**1**	02	2	2	2
	vii.	How many five-digit numbers can be made from the digits 1 to 7 if repetition is allowed? (a) 16807 (b) 54629 (c) 23467 (d) 32354	**1**	03	2	3	2
	viii.	Using the inclusion-exclusion principle, find the number of integers from a set of 1-100 that are not divisible by 2, 3 and 5. (a) 22 (b) 25 (c) 26 (d) 33	**1**	03	5	2	2
	ix.	In the context of generating functions, a partition of an integer n refers to: (a) Representing n as a sum of distinct positive integers (b) Representing n as a sum of positive integers in multiple ways (c) A unique prime factorization of n (d) Grouping all divisors of n	**1**	01	2	4	2
	x.	A non-homogeneous recurrence relation differs from a homogeneous recurrence relation because: (a) It includes a non-zero constant or function term (b) It always has a linear solution (c) It has no initial conditions (d) It must be of first order	**1**	01	2	2	2
Q.2	i.	Explain the term sub graph, walks, path and circuit with example	**2**	01	2	1	2
	ii.	Explain Konigsberg bridge problem in detail.	**3**	01	5	2	2
	iii.	Prove that the number of vertices of odd degree in graph G is always even.	**5**	03	5	2	2

OR	iv.	Explain the difference between a Eulerian circuit and a Hamiltonian circuit, with examples.	**5**	02	2	2	2
Q.3	i.	Explain the properties of cut sets and give an example.	**3**	02	2	2	2
	ii.	Explain spanning tree and fundamental circuit with example. Also prove that if the graph G has e edges and n vertices in spanning tree T then there is exactly (e-n+1) fundamental circuit.	**7**	03	2	3	2
OR	iii.	Explain Isomorphism, 1-isomorphism and 2-isomorphism with example.	**7**	02	2	3	2
Q.4	i.	Define the chromatic number of a graph. How would you determine the chromatic number of a complete graph Kn?	**2**	01	2	3	2
	ii.	What is a matching in a graph? Differentiate between a perfect matching and a maximum matching in a graph, providing examples.	**3**	02	2	3	2
	iii.	Explain the greedy coloring algorithm for graph coloring. How does the greedy algorithm work? Provide an example.	**5**	02	2	3	2
OR	iv.	Explain different types of digraphs with example.	**5**	02	2	3	2
Q.5	i.	State and explain the fundamental counting principle with an example	**2**	01	2	1	2
	ii.	What is the chromatic polynomial of a graph? How does it help in graph coloring? Illustrate with an example.	**3**	01	2	1	2
	iii.	What is the difference between permutations and combinations? Also find in how many ways can 4 students be arranged in a line if two specific students must be in the middle positions? Explain the method used to calculate this.	**5**	04	2	4	3

Marking Scheme
IT3EA09 (T) Graph Theory (T)

Q.1	i)	b) There exists a path between every pair of vertices		**1**
	ii)	a) A circuit that visits every vertex exactly once		**1**
	iii)	b) The sum of all edge weights in the tree		**1**
	iv)	c) The same structure and degrees of corresponding vertices		**1**
	v)	a) n		**1**
	vi)	a) Bipartite		**1**
	vii)	a) 16807		**1**
	viii)	c) 26		**1**
	ix)	b) Representing n as a sum of positive integers in multiple ways		**1**
	x)	a) It includes a non-zero constant or function term		**1**
Q.2	i.	Sub Graph, Walks, Path and Circuit with example each	.5 mark	**2**
	ii.	Konigsberg bridge problem	3 marks	**3**
	iii.	Explanation (Proof)	5 marks	**5**
OR	iv.	difference	4 marks	**5**
		examples	1 mark	
Q.3	i.	properties of cut sets	2 marks	**3**
		example	1 marks	
	ii.	Spanning tree and fundamental circuit with example	3 marks	**7**
		Explanation (proof)	3 marks	
		example	1 marks	
OR	iii.	Isomorphism, 1-isomorphism and 2-isomorphism with example	2 mark each 1 marks example	**7**
Q.4	i.	chromatic number of a graph	1 mark	**2**
		chromatic number of a complete graph Kn	1 mark	
	ii.	matching in a graph	1 mark	**3**
		perfect matching and a maximum matching in a graph, providing examples	2 marks	
	iii.	greedy coloring algorithm for graph coloring	2 marks	**5**
		How does the greedy algorithm work with example	3 marks	
OR	iv.	different types of Digraph with example	5 marks	**5**
Q.5	i.	Fundamental Counting Principle with an example	2 marks	**2**
	ii.	chromatic polynomial of a graph	1 mark	**3**
		how it helps in graph coloring with an example	2 marks	
	iii.	difference between permutations and combinations	3 marks	**5**
		Solution of question	2 marks	
OR	iv.	Binomial Theorem	2 marks	**5**
		use it to expand $(x+y)^6$	1.5 marks	
		find the general term in the expansion of $(x+y)^n$	1.5 marks	
Q.6		Attempt any two:		
	i.	generating functions used to solve recurrence relations	2.5 marks	**5**
		recurrence relation $a_n=2a_{n-1}+n$ solution	2.5 marks	
	ii.	first-order and second order recurrence relation	2 marks	**5**
		recurrence relation $a_n=5a_{n-1}+6$ solution	3 marks	
	iii.	exponential generating function (EGF)	2 marks	**5**
		how it differs from the ordinary generating function	1 marks	
		with an example by finding the EGF for the sequence of factorials {n!}.	2 marks	

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