iii. Solve the following simultaneous differential equations: **5**

$$\frac{dx}{dt}+2y=e^{t}\ ,\ \frac{dy}{dt}-2x=e^{-t}$$

Q.6 Attempt any two:

i. Solve the differential equation: $x\frac{d^2y}{dx^2}-(2x-1)\frac{dy}{dx}+(x-1)y=0$ **5**

ii. Apply the method of variation of parameters to solve: **5**

$$\frac{d^2y}{dx^2}+y=\cos ecx$$

iii. Solve in series: $\frac{d^2y}{dx^2}+xy=0$ **5**

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Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2019

EN3BS01 Engineering Mathematics-I

Programme: B.Tech. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If the normal form of a matrix is $\begin{bmatrix} I_3 \\ O \end{bmatrix}$, its rank is **1**

(a) 2 (b) 5 (c) 3 (d) 4

ii. A Homogeneous system of linear equations is **1**

(a) Always inconsistent

(b) Always consistent

(c) Solvable depending on the system

(d) None of these

iii. The Maclaurin's series of $f(x)=\sin x$ is **1**

(a) $1+x+\frac{x^2}{2!}+..........$ (b) $1-x-\frac{x^3}{3!}+..........$

(c) $x+\frac{x^2}{2!}+..........$ (d) $x-\frac{x^3}{3!}+..........$

iv. The value of $\frac{dy}{dx}$, if $x^2+y^2=1$, is **1**

(a) $-\frac{x}{y}$ (b) $\frac{x}{y}$ (c) $-\frac{y}{x}$ (d) $\frac{y}{x}$

v. The value of $\Gamma\left(\frac{3}{2}\right)$ is **1**

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$

vi. The double integral $\iint_R dxdy$, where R is a region in xy-plane, gives **1**

(a) Perimeter of R (b) Area of R
(c) Both (a) and (b) (d) None of these

vii. The differential equation $Mdx + Ndy = 0$ is an exact differential equation if and only if: **1**

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$
(c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (d) None of these

viii. The solution of the differential equation $\frac{dy}{dx} + y = 1$ is **1**

(a) $x = -\log(1-y) + c$ (b) $x = \log(1+y) + c$
(c) $x = -\log(1+y) + c$ (d) $x = \log(1-y) + c$

ix. A part of complementary function of $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ (P, Q and R are functions of x) when $P + Qx$ =0, is : **1**

(a) x (b) x^2 (c) e^x (d) e^{-x}

x. The point $x = 0$ is an ordinary point of the equation $P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2(x)y = 0$ if the value of $P_0(x)$ at $x = 0$ is **1**

(a) 0 (b) 1 (c) -1 (d) Any non-zero value

Q.2 Attempt any two:

i. Reduce the matrix $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ in to normal form and hence find its rank. **5**

ii. Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. **5**

iii. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ and hence find its inverse **5**

Q.3 Attempt any two:

i. Find the Taylor's series expansion of the function $f(x) = \log(\cos x)$ about the point $\pi/3$ up to fourth derivative term. **5**

ii. If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2} + y^{1/2}}{x^{1/3} + y^{1/3}}\right)^{1/2}$, prove that **5**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

iii. Discuss the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$. **5**

Q.4 Attempt any two:

i. Evaluate $\int_a^b x^2 dx$, using definition as limit of a sum. **5**

ii. Show that $\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ where $m > 0$. **5**

iii. Change the order of integration $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}} dydx$ and hence evaluate the same. **5**

Q.5 Attempt any two:

i. Solve the differential equation: $x^2ydx - (x^3 + y^3)dy = 0$ **5**

ii. Solve the differential equation: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x} + \sin 2x$ **5**

Total ... page

Faculty of Engineering

EN3BS01 Engineering Mathematics-I

B. Tech.

Examination May-2019

Q.1 (i) (c) 3

(ii) (b) Always consistent 1

(iii) (d) $x - x^3/\lfloor 3 + \cdots$ 1

(iv) (c) $-y/x$ 1

(v) (d) $\sqrt{\pi}/2$ 1

(vi) (b) Area of R 1

(vii) (c) $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ 1

(viii) (a) $x = -\log(1-y) + C$ 1

(ix) (a) x 1

(x) (d) Any non zero values 1

Q.2
$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$ 1

R_{31}
$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \\ \sim \end{array}$$ +1

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 6 & 2 & 4 \\ 0 & 1 & 5 & 10 \end{bmatrix}$$ +1 +1

after some elementry operations

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim [I_3 : O]$$ +1

Rank is 3.

Q: 2 (ii)

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

The characteristic eq^n of A is given by

$$|A - \lambda I| = 0$$

$$(1-\lambda)\left[(2-\lambda)^2 - 1\right] = 0$$ 1

$$(1-\lambda)(3-\lambda)(1-\lambda) = 0$$ 1

$$\lambda = 1, 1, 3$$

Eigen vectors corresponding to

$\lambda = 1$ is $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ 2

$\lambda = 3$ $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ 1

Q.2 (iii)

Cayley - Hamilton theorem.

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 1 \\ 0 & 1-\lambda & -1 \\ 3 & -1 & 1-\lambda \end{vmatrix} = 0$$ 1

$$\lambda^3 - 3\lambda^2 - \lambda + 9 = 0$$

by Cayley Hamilton theorem

$$A^3 - 3A^2 - A + 9I = 0$$ 1

$$A^2 = \begin{bmatrix} 4 & 3 & 2 \\ -3 & 2 & -2 \\ 6 & 4 & -1 \end{bmatrix} \qquad A^3 = \begin{bmatrix} -14 & 5 & -7 \\ -9 & -2 & -7 \\ 31 & 17 & -11 \end{bmatrix}$$ 1 1

$$A^3 - 3A^2 + 5A + 3I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ 2

Hence A verfies the cayley Hamilton theorem.

$$A^{-1} = \frac{-1}{3}\begin{bmatrix} 0 & -1 & -1 \\ -3 & 4 & 1 \\ -3 & 7 & 1 \end{bmatrix}$$ 1

3(i) Taylor's series expansion of $f(x) = \log(\cos x)$ about the point $x = \pi/3$

$f(x) = \log(\cos x)$

$$f(x) = f[a + (x-a)] = f(a) + (x-a)f'(x) + \frac{(x-a)^2}{\lfloor 2} f''(x) + \cdots \quad (1)$$ 1

$f(x) = \log \cos x$

$f'(x) = \dfrac{-\sin x}{\cos x} = -\tan x$ 1

$f''(x) = -\sec^2 x$

$f''(x) = y_2 = -(1 + \tan^2 x) = -(1 + (-y_1)^2)$

$= -(1 + y_1^2)$

$y_3 = -2y_1 y_2$

$y_4 = -2[y_1 y_3 + y_2^2]$ 1

$f(\pi/3) = \log \cos \pi/3 = \log 1/2$

$f'(\pi/3) = -\tan \pi/3 = -\sqrt{3}$ 1

$f''(\pi/3) = -\sec^2 \pi/3 = -4$

Hence the series is

$$\log \cos x = \log 1/2 + (x-\pi/3)(-\sqrt{3}) + (x-\pi/3)^2(-4) + \cdots$$ 1

3(ii)

$$u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right)^{1/2}$$

$$\operatorname{cosec} u = \left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$$ 1

$$= \frac{x^{1/4}\left[1+y^{1/2}/x^{1/2}\right]^{1/2}}{x^{1/6}\left[1+y^{1/3}/x^{1/3}\right]}$$

$$= x^{1/12} f(y/x) = f(x,y)$$

Hence $f(x,y)$ is a homogeneous fun^n of degree $1/12$ hence by euler's deduction 1

$$x^2\frac{\partial^2 u}{\partial x} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

$$= \cancel{\frac{1}{12}\left(\frac{1}{12}-1\right)}$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = +\frac{1}{12}f = \frac{1}{12}\operatorname{cosec} u$$

$$x\frac{\partial \operatorname{cosec} u}{\partial x} + y\frac{\partial \operatorname{cosec} u}{\partial y} = \frac{1}{12}\operatorname{cosec} u \quad -(1)$$ 1

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{12}\frac{\operatorname{cosec} u}{-\operatorname{cosec} u \cot u} = \frac{-1}{12}\tan u$$

Partially diff^n wrt. x.

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = -\left[\frac{1}{12}\sec^2 u + 1\right]\frac{\partial u}{\partial x} \quad -(2)$$ 1

$$y\frac{\partial^2 u}{\partial y^2} + x\frac{\partial^2 u}{\partial x \partial y} = -\left(\frac{1}{12}\sec^2 u + 1\right)\frac{\partial u}{\partial y} \quad (3)$$

adding (2) and (3)

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$ 1

3(iii) $f(x,y) = x^3 + y^3 - 3x - 12y + 20.$

$p = \frac{\partial f}{\partial x} = 3x^2 - 3$ 1

$q = \frac{\partial f}{\partial y} = 3y^2 - 12$

$r = \frac{\partial^2 f}{\partial x^2} = 6x$ 1

$s = \frac{\partial^2 f}{\partial x \partial y} = 0$

$t = \frac{\partial^2 f}{\partial y^2} = 6y$

$\frac{\partial f}{\partial x} = 0 \qquad 3x^2 - 3 = 0 \qquad x^2 = 3/3 = 1$

$\cancel{x^2 = 0} \qquad x = \pm 1$

$\frac{\partial f}{\partial y} = 0 \qquad 3y^2 - 12 = 0 \qquad y^2 = +12/3$ 1

$y^2 = +4$

$y = \pm 2$

(1, 2) (-1, -2) ~~(0,0)~~ critical point
(-1, +2) (1, -2) are

~~$r = 0$ at $(0,0)$~~

at (1, 2)

$r = 6(1) = 6$

$s = 0$

$t = 12$

at (-1, -2)

$r = -6$ 1

$s = 0$

$t = -12$

If $rt-s^2$ is +ive & $r>0$ then $f(x,y)$ has maximum

at $(1,2)$ 1/2

$rt-s^2 = 72-0>0$ and $r=6>0$ hence

~~maxima~~ minima.

at $(-1,-2)$

$rt-s^2 = 72-0>0$ & $r=-6<0$ hence 1/2

maxima. ~~so~~ on check for remaining

(i) Evaluate $\int_a^b x^2 dx$

Here $f(x) = x^2$

$$\int_a^b x^2 dx = \lim_{n\to\infty} h\,[a^2 + (a+h)^2 + (a+2h)^2 + \cdots$$ 1

$$\cdots + \{a+(n-1)h\}^2]$$

$h \to 0$ as $n\to\infty$ and $nh = b-a$ 1

$$= \lim_{n\to\infty} h[\,na^2 + 2ah\,\{1+2+\cdots(n-1)\} + h^2\,(1^2+2^2 \cdots (n-1)^2)]$$ 1

$$= \lim_{n\to\infty} h\,[\,na^2 + 2ha\frac{(n-1)n}{2} + h^2\frac{(n-1)\,n\,(2n-1)}{6}]$$ 2

$$= (b-a)\,[a^2 + a(b-a) + \frac{1}{3}(b-a)^2]$$

$$= \frac{1}{3}[b^3-a^3]$$

Hence

$$\int_a^b x^2 dx = \frac{1}{3}[b^3-a^3]$$

.4(ii)

$$\Gamma M \, \Gamma M+1/2 = \frac{\sqrt{\pi}}{2^{2M-1}} \Gamma 2M \qquad M > 0$$

we know that

$$\int_0^{\pi/2} \sin^{2M-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{\Gamma M \, \Gamma n}{2 \, \Gamma M+n} \quad -(1)$$

(1)

putting $2n-1 = 0 \Rightarrow n = 1/2$

$$\int_0^{\pi/2} \sin^{2M-1}\theta \, d\theta = \frac{\Gamma M \, \Gamma 1/2}{2 \, \Gamma M+1/2} \quad -(2)$$

again putting $n = m$ in (1)

(1)

$$\int_0^{\pi/2} \sin^{2M-1}\theta \cos^{2M-1}\theta \, d\theta = \frac{(\Gamma M)^2}{2 \, \Gamma 2M}$$

$$\Rightarrow \frac{1}{2^{2M-1}} \int_0^{\pi/2} (2\sin\theta\cos\theta)^{2M-1} d\theta = \frac{(\Gamma M)^2}{2 \, \Gamma 2M} \quad (1)$$

$$\frac{1}{2^{2M}} \int_0^{\pi/2} (\sin 2\theta)^{2M-1} . 2 \, d\theta = \frac{(\Gamma M)^2}{2 \, \Gamma 2M}$$

(1)

Now putting $2\theta = \phi \Rightarrow 2d\theta = d\phi$

$$\frac{1}{2^{2M}} \int_0^{\pi} (\sin\phi)^{2M-1} d\phi = \frac{(\Gamma M)^2}{2 \, \Gamma 2M}$$

$$\Rightarrow \frac{1}{2^{2M}} \int_0^{\pi} (\sin\theta)^{2M-1} d\theta = \text{''} \quad (1)$$

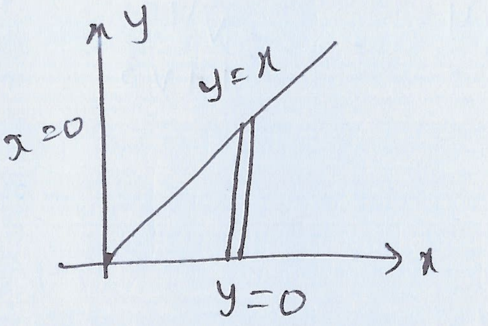
$$\Rightarrow \int_0^{\pi/2} \sin^{2M-1} d\theta = \frac{2^{2M-1} (\Gamma M)^2}{2 \, \Gamma 2M} \quad -(3)$$

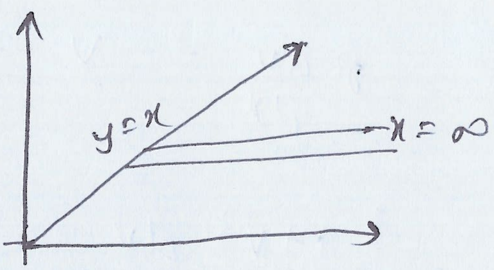
Equating (2) and (3) gives

$$\frac{2^{2M-1}(\Gamma M)^2}{2 \cdot \Gamma 2M} = \frac{\Gamma M \sqrt{\pi}}{2\,\Gamma M+1/2}$$

$$\Rightarrow \quad \Gamma M \; \Gamma M+1/2 = \frac{\sqrt{\pi}}{2^{2M-1}} \Gamma 2M$$ Hence proved.

4 (iii) $\int_0^\infty \int_0^x x e^{-x^2/y}\, dy\, dx$





1

$y=0$ & $y=x$ $\qquad$ $x=y$ & $x=\infty$ 1

$x=0$ & $x=\infty$ $\qquad$ $y=0$ & $y=\infty$

Thus

$$\int_0^\infty dy \int_y^\infty x e^{-x^2/y} dx = \int_0^\infty dy \left[-\frac{y}{2} e^{-x^2/y}\right]_y^\infty$$ 2

$$= \int_0^\infty \frac{y}{2} e^{-y} dy = 1/2$$ 1

Q(9)
.Si $x^2 y\,dx - (x^3+y^3)\,dy = 0.$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3+y^3} \quad \text{Homogeneous Eq}^n \text{ of order } 3$$

$y = vx$ (1)

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x^2 \cdot vx}{x^3+v^3x^3} = \frac{v}{1+v^3}$$

$$x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$ 1

$$x\frac{dv}{dx} = \frac{v - v - v^4}{1+v^3} = \frac{-v^4}{1+v^3}$$

$$\frac{1+v^3}{v^4}\,dv = \frac{-dx}{x}$$ 1

$$\int\left(\frac{1}{v^4} + \frac{1}{v}\right)dv = -\frac{dx}{x}$$

$$\frac{v^{-3}}{-3} + \log v = -\log x + C.$$ 1

$v = y/x$

$$\frac{x^3}{-3y^3} + \log y/x = -\log x + C$$

$$\frac{x^3}{-3y^3} + \log y - \log x = -\log x + C$$ 1

$$\frac{-x^3}{3y^3} + \log y = C$$

1.5 ii) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x} + \sin 2x$

A.E

$(m^2 - 3m + 2) = 0$

$m^2 - m - 2m + 2 = 0$ 1

$m(m-1) - 2(m-1) = 0$

$(m-2)(m-1) = 0$ 1

$m = 1, 2$

$C.F. = c_1 e^{x} + c_2 e^{2x}$

$P.I. = \frac{e^{3x}}{D^2 - 3D + 2} + \frac{\sin 2x}{D^2 - 3D + 2}$ 1

$= \frac{e^{3x}}{9 - 9 + 2} + \frac{\sin 2x}{-4 - 3D + 2}$ 1

$= \frac{e^{3x}}{2} + \frac{\sin 2x}{-3D - 2}$

$= \frac{e^{3x}}{2} - \frac{\sin 2x}{(3D+2)} = \frac{e^{3x}}{2} - \frac{(3D-2)\sin 2x}{(9D^2 - 4)}$

$= \frac{e^{3x}}{2} - \frac{(3D-2)\sin 2x}{(9(-4) - 4)} = \frac{e^{3x}}{2} + \frac{(3D-2)\sin 2x}{40}$ 1

$= \frac{e^{3x}}{2} + \frac{1}{40}\left[6\cos 2x - 2\sin 2x\right)$

$y = C.F. + PI$ is the sol^n.

Q.4 (ii)

$$\frac{dx}{dt} + 2y = e^{t}$$

$$\frac{dy}{dt} - 2x = e^{-t}$$

Let $\frac{d}{dt} = D$

$Dx + 2y = e^{t} \quad \times D$

$Dy - 2x = e^{-t} \quad \times 2$ 1

$D^2x + 2Dy = e^{t}$

$2Dy - 4x = 2e^{-t}$

$- \quad + \quad -$

$(D^2+4)x = e^{t} - 2e^{-t}$ 1

A.E. $m^2+4=0$

$m = \pm 2i \qquad x = (c_1\cos 2t + c_2\sin 2t)$

$$P.I. = \frac{e^{t} - 2e^{-t}}{D^2+4}$$

$$= \frac{e^{t}}{5} - 2\frac{e^{-t}}{1+4}$$

$x = C.F + P.I$ 1

$$x = (c_1\cos 2t + c_2\sin 2t) + \frac{e^{t}}{5} - \frac{2e^{-t}}{5}$$

Similarly find y 2

6 (ii) $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$

$$\frac{d^2y}{dx^2} - \left(2 - 1/x\right)\frac{dy}{dx} + (1 - 1/x)y = 0 \quad -① $$

Comparing ① with $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Here $1 + P + Q = 0$ so that $y = u = e^x$ $-②$ +1

is the part of the complementary funⁿ C.F. of the solution ①

Let the complete solⁿ be

$$y = uv \quad -③$$ 1

then v is given by

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{dy}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$$ 1

$$\frac{d^2v}{dx^2} + \frac{1}{x}\frac{dv}{dx} = 0 \quad -④$$

Let $\frac{dv}{dx} = t$ so that $\frac{d^2v}{dx^2} = \frac{dt}{dx}$ 1

then ④ becomes

$$\frac{dt}{dx} + \frac{t}{x} = 0 \Rightarrow \log t + \log x = \log c_1$$

$$t = c_1/x$$ 1

$$\frac{dv}{dx} = \frac{c_1}{x} \quad \text{or} \quad v = c_1\log x + c_2$$

Hence $y = e^x . v = e^x(c_1 \log x + c_2)$

6(ii) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

A.E. $m^2+1=0 \quad m = \pm i$ 2

$y = c_1\cos x + c_2 \sin x \quad = c_1 y_1 + c_2 y_2$

$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$ $y = uy_1 + vy_2$ be complete sol^n 1

$\Rightarrow \cos^2 x + \sin^2 x = 1$

$u = \int \frac{-y_2 X}{W} dx = \int \frac{-\sin x \operatorname{cosec} x}{1} dx$ 1

$= -x$

$v = \int \frac{y_1 X}{W} dx = \log \sin x$

$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log \sin x$ 1

6(iii) $\frac{d^2y}{dx^2} + xy = 0$ — (1)

Let the sol^n be $y = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots$

$\frac{dy}{dx} = a_1 + 2a_2 x + 3a_3 x^2 + \dots$ (1)

$\frac{d^2y}{dx^2} = 2a_2 + 2.3\, a_3 x + \dots$

Substituting in (1)

$2a_2 + 2.3a_3 x + 4.3 a_4 x^2 + \dots n(n-1)a_n x^{n-2} + \dots$ (1)

$+ x[a_0 + a_1 x + \dots a_n x^n] = 0$

Equating to zero the co-efficient of the various powers of x (13) (14)

$$a_2 = 0$$

$$a_3 = -a_0/\lfloor 3 \qquad a_4 = \frac{-2a_1}{\lfloor 4}$$ (2)

In general $a_{n+2} = \dfrac{-a_{n-1}}{(n+2)(n+1)}$

Hence the solⁿ

$$y = a_0\left[1 - \frac{x^3}{\lfloor 3} + \frac{1.4x^6}{\lfloor 6} - \frac{1.4.7\, x^9}{\lfloor 9} - \dots\right]$$ 1

$$+a_1\left[x - \frac{2x^4}{\lfloor 4} + \frac{2\cdot 5x^7}{\lfloor 7} - \dots\right]$$

—— xx ——