Q.5 Solve any two:

i. A curve is down to pass through the points given by following table **5**

x	1	1.5	2	2.5	3
y	2	2.4	2.7	2.8	3

Estimate the area bounded by the curve x-axis and the lines $x=1$ to $x=3$ by using simpson $1/3$ rule.

ii. Using Taylor's series, find the solution of the differential equation $xy' = x - y;\ y(2) = 2$ at $x = 2.1$, correct to five place of decimal. **5**

iii. Solve $\frac{dy}{dx} = \frac{1}{x+y}$ for $x = 0.5$ by using Runge-Kutta method with $x_0 = 0, y_0 = 1$ (take h=0.5). **5**

Q.6 Solve any two:

i. Calculate the Karl pearson's coefficient of correlation between X and Y series: **5**

X	17	18	19	19	20	20	21	21	22	23
Y	12	16	14	11	15	19	22	16	15	20

ii. Fit the second degree parabola to the following: **5**

x	1	2	3	4	5
y	25	28	33	39	46

iii. In experiment on pea-breading, mendal obtained the following frequencies of seeds: **5**

Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total
315	101	108	32	556

Theory predicts that the frequencies should be in proportion 9:3:3:1. Examine the correspondence between theory and experiment. Given that the value of x^2 for 3 dof at 5% level of significance is 7.815.

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2018

EN3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: AU/CE/FT/ME

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. An analytic function with constant modulus is always: **1**

(a) Constant (b) Variable (c) Both (a) and (b) (d) None of these.

ii. If $z = a$ is a pole of $f(z)$ of order 3 then residue of $f(z)$ at $z = a$ is given by: **1**

(a) $\frac{1}{3!}\left[\lim_{z\to a}\frac{d^3}{dz^3}(z-a)^3 f(z)\right]$

(b) $\frac{1}{2!}\left[\lim_{z\to a}\frac{d^3}{dz^3}(z-a)^3 f(z)\right]$

(c) $\frac{1}{2!}\left[\lim_{z\to a}\frac{d^2}{dz^2}(z-a)^3 f(z)\right]$

(d) None of these.

iii. Let x be the exact value and x_a be the approximate value then relative error is : **1**

(a) $|x - x_a|$ (b) $\left|\frac{x-x_a}{x}\right|$ (c) $\left|\frac{x-x_a}{x_a}\right|$ (d) None of these.

iv. Gauss seidel method is known as method of : **1**

(a) Simultaneous Displacement

(b) Successive Displacement

(c) Both (a) and (b)

(d) None of these.

P.T.O.

v. In numerical analysis which of the following relation is correct **1**

(a) $\mu^2 = 1 + \frac{\delta^2}{4}$ (b) $\mu^2 = 1 + \frac{\delta}{4}$

(c) $\delta^2 = 1 + \frac{\mu^2}{4}$ (d) None of these.

vi. The relation between difference operator Δ and differential operator D is **1**

(a) $e^{hD} = 1 + \Delta$ (b) $e^{hD} = 1 - \Delta$

(c) $e^{hD} = \Delta$ (d) None of these.

vii. According to Trapezoidal rule $\int_{x_0}^{x_0+nh} y\,dx$ is equal to **1**

(a) $\frac{h}{2}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right]$

(b) $\frac{h}{2}\left[\left(y_0 + y_n\right) + 3\left(y_1 + y_2 + ... + y_{n-1}\right)\right]$

(c) $\frac{h}{3}\left[\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right]$

(d) None of these.

viii. According to Picard's method the n^{th} approximation given by **1**

(a) $y^{(n)} = y_1 + \int_{x_0}^{x} f\left(x, y^{(n-1)}\right)dx$ (b) $y^{(n)} = y_0 + \int_{x}^{x_0} f\left(x, y^{(n-1)}\right)dx$

(c) $y^{(n)} = y_0 + \int_{x_0}^{x} f\left(x, y^{(n-1)}\right)dx$ (d) None of these.

ix. The product of regression coefficient is **1**

(a) ≤ 1 (b) ≥ 1 (c) ≥ 2 (d) None of these.

x. Fisher's Z-test is applied when number of data is **1**

(a) $n = 30$ (b) $n > 30$ (c) $n < 30$ (d) None of these.

Q.2 Solve any two:

i. Find imaginary part and construct the analytic function whose real part is $u = e^{2x}\left(x\cos 2y - y\sin 2y\right)$. **5**

ii. Evaluate $\int_C \frac{e^z}{\left(z^2 + \pi^2\right)^2}dz$ where C is $|z| = 4$ using cauchy's integral formula. **5**

iii. Apply calculus of residues, to prove that $\int_0^{\pi} \frac{d\theta}{17 - 8\cos\theta} = \frac{\pi}{15}$. **5**

Q.3 Solve any two:

i. By using Newton-Raphson's method find the root of $x^4 - x - 10 = 0$ correct to three places of decimal **5**

ii. Solve the equations by Gauss-Jordan method **5**

$2x - 3y + z = -1; x + 4y + 5z = 25; 3x - 4y + z = 2$

iii. Apply Crout's triangularization method to solve the equations: **5**

$x_1 + 2x_2 + 3x_3 = 14; 2x_1 + 5x_2 + 2x_3 = 18; 3x_1 + x_2 + 5x_3 = 20$

Q.4 Solve any two:

i. The population of a country in the decennial censuses were as under. Estimate the population for the year 1925 by Newton backward interpolation formula. **5**

Year x	1891	1901	1911	1921	1931
Population y	46	66	81	93	101

ii. Find a polynomial satisfied by(-4,1245),(-1,33),(0,5)(2,9) and (5,1335) by the use of Newton's interpolation formula with divided difference. **5**

iii. Given: **5**

x	0.1	0.2	0.3	0.4
$y = f(x)$	1.10517	1.22140	1.34986	1.49182

Find $\frac{dy}{dx}$ at $x = 0.4$

'Solution Set'

Engg. Mathematics – III

[ME, CE, AU, FT]

MCQ's

Q.1. (i) a) Constant

(ii) c) $\frac{1}{2i}\left[\lim_{z\to a}\frac{d^2}{dz^2}(z-a)^3 f(z)\right]$

(iii) b) $\left|\frac{x-x_a}{x}\right|$

(iv) b) successive Displacement

(v) a) $\mu^2 = 1+\frac{\delta^2}{4}$

(vi) a) $e^{hD}=1+\Delta$

(vii) a) $\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$

(viii) c) $y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})\,dx$

(ix) a) ≤ 1

(x) b) $n > 30$

②

Q. 2 (i) Given $u = e^{2x}(x\cos 2y - y\sin 2y)$ —①

$v = ?$ $u + iv = ?$

By C-R eqⁿ we have

$u_x = v_y$ and $u_y = -v_x$

from ①

$u_x = 2e^{2x}(x\cos 2y - y\sin 2y) + e^{2x}(\cos 2y) = v_y$ +1 —②

integrating ② w.r. to y

$$V = \frac{2x e^{2x}\sin 2y}{2} - 2e^{2x}\left[-y\frac{\cos 2y}{2} + \frac{\sin 2y}{4}\right]$$ +1

$$+ e^{2x}\frac{\sin 2y}{2} + \phi(x) \quad —③$$

Now diffⁿ ③ w.r. to x

$V_x = 2xe^{2x}\sin 2y + e^{2x}\sin 2y + y\cos 2y\, 2e^{2x}$

$- \frac{1}{2}2e^{2x}\sin 2y + e^{2x}\sin 2y + \phi'(x)$

since $u_y = -V_x$ $\left[u_y = xe^{2x}\sin 2y(-2) - e^{2x}\sin 2y - e^{2x}y2\cos 2y\right]$

u_y

$\phi'(x) = 0 \cancel{e^{2x}\sin 2y - e^{2x}}$ +1

$\phi(x) = C$

$V = xe^{2x}\sin 2y + ye^{2x}\cos 2y - \cancel{\frac{1}{2}e^{2x}\sin 2y}$ +1

$\cancel{+ \frac{1}{2}e^{2x}}$ $+ C$

$u + iv = e^{2x}(x\cos 2y - y\sin 2y) + i\, e^{2x}$ +1
$(x \sin 2y + y\cos 2y)$

Q.2 (ii) $\int_C \frac{e^z}{(z^2+\pi^2)^2} dz$ where C is $|z|=4$

By Cauchy's integral formula we have

$$f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$$ +1

$$\frac{e^z}{(z^2+\pi^2)^2} = \frac{e^z}{(z+\pi i)^2 (z-\pi i)^2}$$ +1

both $z = \pm \pi i$ lie within the circle $|z|=4$

Now

$$\frac{1}{(z+\pi i)^2(z-\pi i)^2} dz = \frac{7}{2\pi^3 i}\left\{\int_C \frac{e^z}{z+\pi i} dz - \int_C \frac{e^z}{z-\pi i} dz\right\}$$

$$- \frac{1}{4\pi^2}\left\{\int_C \frac{e^z}{(z+\pi i)^2} dz + \int \frac{e^z}{(z-\pi i)^2} dz\right\}$$

$$= \frac{7}{2\pi^3 i}\left[2\pi i f(-\pi i) - 2\pi i f(\pi i)\right] - \frac{1}{4\pi^2}$$ +2

$$\left[2\pi i f'(-\pi i) + 2\pi i f'(\pi i)\right]$$

$$= \frac{-14 i}{\pi^2} \sin \pi - \frac{i}{\pi} \cos \pi = \frac{i}{\pi}.$$ +1

Q.2 (iii)

$$\frac{1}{2}\int_0^{2\pi}\frac{d\theta}{17-8\cos\theta} = I$$

Put $z = e^{i\theta}$

$dz = i e^{i\theta} d\theta$

$$\cos\theta = \frac{z+z^{-1}}{2} = \frac{z^2+1}{2z}$$

$$d\theta = \frac{dz}{iz}$$

$$I = \frac{1}{2}\int_C \frac{dz}{iz\left(\frac{34z-8z^2-8}{2z}\right)}$$ +1

$$= \frac{\cancel{2}}{\cancel{2}}\frac{1}{i}\int_C \frac{dz}{-8(z-1/4)(z-4)} = \frac{1}{i}\int_C G(z)\,dz$$ +1

Now $G(z)$ has simple pole at $z=4, 1/4$

only $z=1/4$ lie within the circle

$$\text{Res } f(1/4) = \lim_{z\to 1/4}(z-1/4)\frac{1}{-8(z-\frac{1}{4})(z-4)}$$

$$= \frac{1}{-8\left(\frac{-15}{4}\right)}$$ +1

$$\text{Res } f(1/4) = \frac{1}{30}$$

$$\int_C G(z)\,dz = 2\pi i\, f(1/4) = 2\pi i\left(\frac{1}{30}\right) = \frac{\pi i}{15}$$ +1

$$\frac{1}{2}\int_0^{2\pi}\frac{d\theta}{17-8\cos\theta} = \frac{1}{i}\,\frac{\pi i}{15} = \frac{\pi}{15}$$ +1

Q.3 (i) Let $f(x) = x^4 - x - 10 \quad \therefore f'(x) = 4x^3 - 1$

By Newton Raphson formula,

we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad +1$$

$$= x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$= \frac{3x_n^4 + 10}{4x_n^3 - 1} \qquad +1$$

~~taking~~ $x_0 = 1.5$. $\quad x_1 = 2$

$f(x_0) = -6.4375$, $f(x_1) = 4$

at $n = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{3x_0^4 + 10}{4x_0^3 - 1} = \frac{3(1.5)^4 + 10}{4(1.5)^3 - 1}$$

$= 2.015$

$f(x_2) = 4.4704270506$

$$x_2 = \frac{3x_1^4 + 10}{4x_1^3 - 1} = \frac{3(2.015)^4 + 10}{4(2.015)^3 - 1} = \frac{59.4562811519}{31.7254135} \qquad +1$$

$x_2 = 1.8740900304$

$$x_3 = \frac{3(x_2)^4 + 10}{4(x_2)^3 - 1} = \frac{\cancel{22.3356432635}}{\cancel{5.5822041964}} \quad \frac{47.0069297874}{25.3288167855} \qquad +1$$

$= 1.8558675751$

$$x_4 = \frac{3(x_3)^4 + 10}{4(x_3)^3 - 1} = \frac{45.5884596244}{24.5682464287}$$

$= 1.855584596 \qquad +1$

Since $x_3 = x_4$. The desired root is 1.856

Q.3 (ii) $2x-3y+z=-1,\ x+4y+5z=25$

$3x-4y+z=2$

By Gauss Jordan Method

$$3x-4y+z=2$$
$$2x-3y+z=-1$$
$$x+4y+5z=25$$

$$\left[\begin{array}{ccc:c} 3 & -4 & 1 & 2 \\ 2 & -3 & 1 & -1 \\ 1 & 4 & 5 & 25 \end{array}\right]$$

$R_1 \to \frac{1}{3}R_1$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 2 & -3 & 1 & -1 \\ 1 & 4 & 5 & 25 \end{array}\right]$$

+1

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & -1/3 & 1/3 & -7/3 \\ 0 & 16/3 & 14/3 & 73/3 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & 16/3 & 14/3 & 73/3 \\ 0 & -1/3 & 1/3 & -7/3 \end{array}\right]$$

+1

$R_2 \to \frac{3}{16} R_2$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & 1 & 7/8 & \frac{73}{16} \\ 0 & -1/3 & 1/3 & -7/3 \end{array}\right]$$

$R_3 \to R_3 + \frac{1}{3} R_2$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & 1 & 7/8 & \frac{73}{16} \\ 0 & 0 & 15/24 & -\frac{39}{48} \end{array}\right]$$

$R_3 \to \frac{24}{15} R_3$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & 1 & 7/8 & \frac{73}{16} \\ 0 & 0 & 1 & \frac{-39}{30} \end{array}\right]$$ +1

$\cancel{R_1 \to R_1 + \frac{4}{3} R_2}$

$\cancel{1} \; 0$

$R_2 \to R_2 - \frac{7}{8} R_3$

$$\left[\begin{array}{ccc:c} 1 & -4/3 & 1/3 & 2/3 \\ 0 & 1 & 0 & \end{array}\right]$$

Reduce in identity matrix +2

$x = 8.7, \; y = 5.7, \; z = -1.3$

3 (iii) $x_1 + 2x_2 + 3x_3 = 14, \quad 2x_1 + 5x_2 + 2x_3 = 18$

$3x_1 + x_2 + 5x_3 = 20$

By Crout's triangularization

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 14 \\ 18 \\ 20 \end{bmatrix}$$

let $A = LU$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$ +1

Solve it

we get

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 3 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -4 \\ 0 & 0 & -24 \end{bmatrix}$$ +1

Now the given system of eq^n, in matrix form

$AX = B$

$\Rightarrow$ $LUX = B$

Put $UX = Y$ so we have $LY = B$ +1

$$Y = \begin{bmatrix} 14 \\ -10 \\ -72 \end{bmatrix}$$ +1

again in $UX = Y$

$$X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ +1

1.4 (i)

Yr.	Population	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
1891	46				
		20			
1901	66		−5		
		15		2	
1911	81		−3		−3
		12		−1	
1921	93		−4		
		8			
1931	101				

+2

$x_n = a + nh = 1931, \quad h = 10, \quad x = 1925$

$f(a+nh) = y_n = 101$

$u = \dfrac{x-(a+nh)}{10} = -0.6$ +1

Applying Newton beackward ~~d~~interpolation for.

$f(x) = y = y_n + u\nabla y_n + \dfrac{u(u+1)}{2!}\nabla^2 y_n + \dfrac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$ +1

$f(1925) = 101 - 0.48 + 0.056 + 0.1008$ +1

$= 96.8368$ ~~the~~ Ans.

Q.4 (ii) Given $x_0 = -4,\ x_1 = -1,\ x_2 = 0,\ x_3 = 2,\ x_4 = 5$

$f(x_0) = 1245,\ f(x_1) = 33,\ f(x_2) = 5,\ f(x_3) = 9,\ f(x_4) = 1335$

By Newton's divided diff^n formula.

$$f(x) = f(-4) + (x+4) \underset{-1}{\Delta} f(-4) + (x+4)(x+1) \underset{-1,0}{\Delta^2} f(-4)$$

$$+ (x+4)(x+1)(x-0) \underset{-1,0,2}{\Delta^3} f(-4) + (x+4)(x+1)$$ +1

$$(x-0)(x-2) \underset{-1,0,2,5}{\Delta^4} f(-4)$$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
-4	1245	$\frac{33-1245}{-1+4} = -404$			
-1	33		94		
		-28		-14	
0	5				
			10		
		2		13	3
2	9				
		442	88		
5	1335				

+2

$$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$ +2

.4 (iii) Diffⁿ table [By Newton backward formula]

x	y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$
0.1	1.10517			
		0.11623		
0.2	1.22140		0.01223	
		0.12846		0.00127
0.3	1.34986		0.01350	
		0.14196		
0.4	1.49182 $= y_n$			

+2

$$y = f(a+nh+uh) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots \quad (1)$$

+1

$h = 0.1$, y

diff (1) w.r. to u

$$h f'(a+nh+uh) = \nabla y_n + \frac{(2u+1)}{2}\nabla^2 y_n + \frac{3(u^2+6u+2)}{6}\nabla^3 y_n$$

+1

$$f'(0.4) = \left(\frac{dy}{dx}\right)_{x=0.4} = \frac{1}{0.1}\left[0.14196 + \frac{1}{2}(0.01350) + \frac{1}{3}(0.00127)\right]$$

+1

$= 1.4913$ Ans.

x	1	1.5	2	2.5	3
y	2	2.4	2.7	2.8	3

Q 5 (i) Let $y = f(x)$ be the eq^n of the given curve then required area

$$= \int_1^3 f(x)\,dx$$

$$= \int_1^3 y\,dx$$ Here the range of integration +1

(1, 3) is divided by six equal part.

$h = 0.3$ +1

By Simpson's 1/3 rule

$$\int_1^3 y\,dx = \frac{1}{3}h\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$ +1

$$= \frac{0.3}{3}\left[(2+3)+4(2.4+2.8)+2(2.7)\right]$$

$$= \frac{1}{10}\left[5+20.8+5.4\right] = \frac{31.2}{10} = 3.12$$ +2

Ans.

Q.5 (ii) $xy' = x - y$

$y' = 1 - y/x = 0$ at $x=2, y=2$, i.e. $y'(2)=0$

diff^n $y'' = -\frac{y'}{x} + \frac{y}{x^2} = \frac{1}{2}$ at $x=2, y=2$, i.e.

$y''(2) = 1/2$

$$y''' = -\frac{y''}{x} + \frac{2y'}{x^2} - \frac{2y}{x^3} = -3/4 = y'''(2)$$ +2

$$y^{iv} = -\frac{y'''}{x} + \frac{3y''}{x^2} - \frac{6y'}{x^3} + \frac{6y}{x^4} = 3/2 = y^{iv}(2)$$ +1

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By taylor's expansion about $x=2$

$$y(x) = y\{2+(x-2)\} = y(2) + (x-2)y'(2) + \frac{(x-2)^2}{2!}y''(2) + \frac{(x-2)^3}{3!}y'''(2) + \frac{(x-2)^4}{4!}y^{iv}(2) + \cdots$$ +1

$$= 2 + (2.1-2)\cdot 0 + \frac{1}{4}(2.1-2)^2 - \frac{1}{8}(2.1-2)^3 + \frac{1}{16}(2.1-2)^4 + \cdots \text{ at } x = 2.1$$ +1

$$= 2.00238$$

Q. 5 (iii) Here $x_0 = 0$, $y_0 = 1$, $h = 0.5$ and $f(x,y) = 1/x+y$.

for the first interval, we have

$$k_1 = hf(x_0, y_0) = 0.5$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = 0.5\left(\frac{1}{0.25+1.25}\right) = 0.33333$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right) = 0.35294$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.26984$$

Here $x_1 = 0+0.5 = 0.5$

So the value of y at $x_1 = 0.5$ is given by +2

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1 + \frac{1}{6}(2.1423)$$ +2

$$= 1 + 0.35705 = 1.35705$$

at $x = 0.5$, $y = 1.35705$ +1

Q.6 (i) Given

X :	17	18	19	19	20	20	21	21	22	23
Y :	12	16	14	11	15	19	22	16	15	20

Mean of X & Y series

$$M_x = \frac{\Sigma x}{n} = 20, \quad M_y = \frac{\Sigma y}{n} = 16$$ +1

then the table

X	Y	$x = X - M_x$	$y = Y - M_y$	x^2	y^2	xy
17	12	−3	−4	9	16	12
18	16	−2	0	4	0	0
19	14	−1	−2	1	4	2
19	11	−1	−5	1	25	5
20	15	0	−1	0	1	0
20	19	0	3	0	9	0
21	22	1	6	1	36	6
21	16	1	0	1	0	0
22	15	2	−1	4	1	−2
23	20	3	4	9	16	12
$\Sigma x = 200$	$\Sigma y = 160$	0	0	30	108	35

+2

$\Sigma x^2 = 30 \quad \Sigma y^2 = 108 \quad \Sigma xy = 35$

The Karl Pearson's Coeff. of Correlation is

$$r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2 \Sigma y^2)}} = \frac{35}{\sqrt{108 \times 30}} = 0.616$$ +2

Q6.(ii) Let the eqⁿ of second degree parabola to fitted to the given data be

$$y = a + bx + cx^2$$ +1

then its normal eqⁿs are

$$\Sigma y = ma + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$ +1

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	x^2	x^3	x^4	xy	x^2y
1	25	1	1	1	25	25
2	28	4	8	16	56	112
3	33	9	27	81	99	297
4	39	16	64	256	156	624
5	46	25	125	625	230	1150
$\Sigma x = 15$	$\Sigma y = 171$	$\Sigma x^2 = 55$	$\Sigma x^3 = 225$	$\Sigma x^4 = 979$	$\Sigma xy = 566$	$\Sigma x^2y = 2208$

+2

substituting in normal eqⁿ, $m = 5$

$$171 = 5a + 15b + 55c$$

$$566 = 15a + 55b + 225c$$

$$2208 = 55a + 225b + 979c$$

on solving +1

$a =$

Q.6 (iii)

Step 1 : Null Hypothesis H_0 : there is a correspondence between theory and experiment

step 2 : Calculation of expected frequencies (f_e) :

Given frequencies in proportion 9 : 3 : 3 : 1

total sum of proportion = 9 + 3 + 3 + 1 = 16,

$\therefore$ (i) Expected frequency of Round and Yellow seed is $= \frac{9}{16} \times 556$

≈ 313

(ii) ———— of wrinkled and ———— $= \frac{3}{16} \times 556$

≈ 104

(iii) ———— Round and green seed is $= \frac{3}{16} \times 556$

≈ 104

(iv) wrinkled and green ———— $= \frac{1}{16} \times 556$

$\approx 35.$

step 3 : Calculation of χ^2 - statistic :

+2

$$\chi^2 = \Sigma \left(\frac{(f_o - f_e)^2}{f_e} \right)$$

+1

$$\chi^2 = \frac{(315-313)^2}{313} + \frac{(101-104)^2}{104} + \frac{(108-104)^2}{104} + \frac{(32-35)^2}{35}$$

$\chi^2 \simeq 0.5$

Also the dof $\nu = n-1 = 3$

Step 4: The tabulated value of χ^2 at 5% level of significance and for dof $\nu=3$ is 7.815 i.e. $\chi^2_{0.05,3} = 7.815$ +1

Step 5: Calculated value of $\chi^2 = 0.5 <$ tabulated value of $\chi^2_{0.05,3} = 7.815$ +1

$\Rightarrow$ Null hypothesis is accepted

$\Rightarrow$ there is very high degree of correspondence b/w theory and experiment.