

- OR iii. We have five jobs each of which must pass through three machines A, B, C in the order ABC. Processing time in hours are given below

Jobs	1	2	3	4	5
Machine A	5	7	6	9	5
Machine B	2	1	4	5	3
Machine C	3	7	5	6	7

Find optimal sequence for 5 jobs that will minimize the total elapsed time. Also find idle time for machine A, B and C.

- Q.6 i. Define Network, Activity and Event in PERT/CPM.
ii. Consider the following data for the activities of a project:

Activity	A	B	C	D	E	F
Predecessor	-	A	A	B, C	-	E
Duration	2	3	4	6	2	8

From the above information

- (a) Construct a network diagram,
 - (b) Compute earliest event time and latest event time,
 - (c) Determine the critical path and total project duration

- OR iii. A, B, C,, H, I constitute a project. Given $X < Y$ mean that Y cannot be started until X is completed i.e., X is preceding activity of Y. With the notation

$$A \leq D \ A \leq E \ B \leq F \ D \leq F \ C \leq G \ C \leq H \ F \leq I \ G \leq J$$

Construct the network diagram.

* * * * *

Enrollment No.....



Faculty of Management Studies

End Sem (Even) Examination May-2019

MS3CO05 Operations Research

Programme: BB

Branch/Specialisation: Management / DM

Duration: 3 Hrs

Maximum Marks: 60

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

[2]

- viii. The method used for solving an assignment problem is called 1
(a) Inverse method (b) MODI method
(c) Hungarian method (d) None of these

ix. Any activity which does not consume either any resource or time is 1
called _____ activity.
(a) Predecessor (b) Successor
(c) Dummy (d) End

x. An event that represent the joint completion of more than one activity 1
is known as
(a) Burst event (b) Joint event
(c) Merge event (d) None of these

Q.2

Attempt any two:

- i. What are Operations Research models? Explain any four Operations Research models with examples? 5

ii. Write any two definitions and three limitations of operations research? 5

iii. Explain any five Scope of Operations Research? 5

i. Define Feasible Solution, Optimal Solution and unbounded solution in context of Linear Programming Problem? 3

ii. Use the graphical method to solve the following L.P. problem 7

03

- Define Feasible Solution, Optimal Solution and unbounded solution in context of Linear Programming Problem? 3

Use the graphical method to solve the following L.P. problem 7

OR

- Use Simplex method to solve the Linear Programming Problem 7

Maximize $Z = 2x_1 + 5x_2$

Subject to

$$x_1 + 4x_2 \leq 24$$
$$3x_1 + x_2 \leq 21$$
$$x_1 + x_2 \leq 9$$

and $x_1, x_2 \geq 0$

Q.4 i. What is Unbalanced Transportation Problem? How do you start in this case? 3

[3]

- ii. Find only Initial Basic Feasible Solution by North West Corner Rule, 7
Least Cost Method and Vogel's Approximation Method

Distribution center						
Plant		D ₁	D ₂	D ₃	D ₄	Supply
	P ₁	19	30	50	12	7
	P ₂	70	30	40	60	10
	P ₃	40	10	60	20	18
	Demand	5	8	7	15	35

OR iii. An organization wants to decide the optimal shipments of a product to three retailers from its three warehouses. The cost of shipment per unit from each warehouse to each retailer and the requirements and availability of the product are given below in the table. 7

Retail Outlet					
Warehouses		R ₁	R ₂	R ₃	Availabilities
	W ₁	5	1	7	10
	W ₂	6	4	6	80
	W ₃	3	2	5	15
Requirements		75	20	50	

There are penalty costs for every unsatisfied demand unit. These penalty costs are Rs. 5, 3 and 2 respectively for retail outlets R_1 , R_2 and R_3 . Find the optimal shipping plan.

Q.5 i. Define Assignment problem. Write assumptions of Assignment problem. 3
ii. Solve the following Assignment problem for Minimization of cost. 7

	J ₁	J ₂	J ₃	J ₄	J ₅
P ₁	6	2	5	3	6
P ₂	2	5	8	7	7
P ₃	7	8	6	9	8
P ₄	6	2	3	4	5
P ₅	9	3	8	9	7
P ₆	4	7	4	6	8

P.T.O.

MCQ.

Q1

- (i) (a) System orientation
- (ii) (c) Mathematical Model
- (iii) (b) Non-Negative
- (iv) (d) Non-basic Variables
- (v) (c) Non-degenerate
- (vi) (b) Demand
- (vii) (b) Transportation
- (viii) (c) Hungarian
- (ix) (c) Dummy
- (x) (c) Merge event

Q2

(i) OR is the application of scientific methods, technique and tools to problems involving the operations of a system so as to provide there in control of the system with optimum solution to the problem. A model is a representation of reality +1

The objective of model is to identify significant factors and their interrelationship.

Various OR models are. [do any four]

① Iconic model: is a pictorial representation of various aspects of a system Ex. Toys, miniature of building +1

② Anologue: are small physical systems that has similar characteristics and work like a system it represents. +1

Ex. maps in different colors represent water, desert and other geographical features +1

③ Mathematical model: employ a set of mathematical symbols to represent the decision variable of a system. The variable are related by mathematical system.

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4) Static model: does not take time into account

Ex. LPP, transportation, assignment

5) dynamic Model: consider time as important factor
Ex. a dynamic prog. problem

other Models are Deterministic, Stochastic, Analytics,
predictive, Simulation, Descriptive etc.

2

(ii) Any two definition

(a) OR is a scientific method of providing executive departments with a quantitative basis for decisions regarding the operations under their control.

(b) OR is art of winning war without actually fighting it

(c) The art of giving bad answers to problems
worse answers are given

(d) OR is scientific approach to problem solving for executive management

three limitation (Any three)

① Magnitude of Computation: Modern problem involve large number of variables and hence to find interrelationship, among makes it difficult.

② Analysis of quantifiable factors: It can evaluate only the effects of numeric and quantifiable factors does not consider human behaviour

(3) A wide gap between managers and operation researchers.

(4) Money and time costs: when the basic data are subjected to frequent changes, incorporating them in to OR is costly affair.

Q² (iii) Any five Scope of OR

- 1) In personnel management
- 2) In finance and accounting
- 3) In inventory Control
- 4) In Research and development
- 5) In production Management
- 6) In Marketing & industry
- 7) In agriculture
- 8) In L.I.C.

Explain in 2 lines every application.

75

Q³ (1) feasible solution:- A set of variables

x_1, x_2, \dots, x_n is called a feasible solution to LPP if it satisfy constraints as well as non-negative restrictions.

+1

Optimal Solution:- A feasible solution which optimizes (Max / Min) the value of objective function. Called optimal solⁿ.

+1

Unbounded Solution:- A solution which can increase or decrease the value of objective function of given LPP indefinitely. Called an unbounded solⁿ.

+1

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q³ (ii) use graphical Method to solve

$$\text{Min } Z = 3x_1 + 2x_2$$

$$\text{s.t. } 5x_1 + 2x_2 \geq 10$$

$$x_1 + x_2 \geq 6$$

$$2x_1 + 4x_2 \geq 12 \text{ and } x_1, x_2 \geq 0$$

Express each term (constraint) in term of x_1 , x_2 .

$$\text{i.e. } 5x_1 + 2x_2 = 10 \quad \text{--- (1)}$$

$$x_1 + x_2 = 6 \quad \text{--- (2)}$$

$$2x_1 + 4x_2 = 12 \quad \text{--- (3)}$$

for (1)

x_1	x_2
2	0
0	10

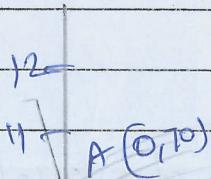
for (3)

x_1	x_2
12	0
0	3

for (2)

x_1	x_2
6	0
0	6

(12, 0) (0, 3)



feasible region

+2

B (1, 5)

C (4, 2)

+2

(0, 0)

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

Region common to all constraints is a feasible region. Optimal solution exists at one of the corner point of feasible region.

There are four corner point of feasible region A, B, C, D.

Find value of objective function at each of the corner point of feasible region.

Corner pt	Coordinate	Value of objective function
A	(0, 10)	$Z = 3x_1 + 2x_2$
B	(1, 5)	$Z = 20$
C	(4, 1)	$Z = 13$
D	(12, 0)	$Z = 16$

$\text{Min } Z$ occurs at (1, 5) i.e. $Z = 13$
Ans is $x_1 = 1$ $x_2 = 5$. $\text{Min } Z = 13$.

(iii)

Simpson method.

$$\text{Max } Z = 2x_1 + 5x_2$$

$$\text{St. } x_1 + 4x_2 \leq 24$$

$$3x_1 + x_2 \leq 21$$

$$x_1 + x_2 \leq 9 \quad x_1, x_2 \geq 0$$

Solt.

Express problem in standard form of LPP

$$\text{Max } Z = 2x_1 + 5x_2 + 0s_1 + 0.s_2 + 0.s_3$$

$$\text{St. } x_1 + 4x_2 + s_1 + 0.s_2 + 0.s_3 = 24$$

$$3x_1 + x_2 + 0s_1 + s_2 + 0s_3 = 21$$

$$x_1 + x_2 + 0s_1 + 0s_2 + s_3 = 9$$

$$x_1, s_i \geq 0 \\ i = 1, 2, 3$$

+2

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Find IBFS using ($x_4 = x_5 = 0$)

i.e. non basic variable = 0

We get $S_1 = 24, S_2 = 21, S_3 = 9$ (basic variable)

prepare I Simplex table

	G_i	2	5	\bar{x}_3	0	0	0	x_0/x_3
C_B	x_B	x_0	x_4	x_5	\bar{x}_3	S_1	S_2	S_3
0	S_1	24	1	(4)		1	0	0
0	S_2	21	3	1		0	1	0
0	S_3	9	1	1		0	0	1

+2

$$(G_i - \bar{z}_j) \Rightarrow 2 \quad 5$$

all $G_i - \bar{z}_j \neq 0$ so not optimal ↑ KC
enter x_2

depart S_1 Key element = 4

KC = Key column

KR = Key row

II Simplex Table

	G_i	2	5	\bar{x}_3	0	0	0	x_0/x_4
C_B	x_B	x_0	x_4	x_5	\bar{x}_3	S_1	S_2	S_3
5	x_2	6	1/4	.1		1/4	0	0
0	S_2	15	11/4	0		-1/4	1	0
0	S_3	3	(3/4)	0		-1/4	0	1

+2

$$(G_i - \bar{z}_j) \Rightarrow 3/4 \quad 0 \leftarrow -1/4 \quad 0 \quad 0$$

↑ KC $G_i - \bar{z}_j$ are not regular
so not optimal.

$3/4$ = Key element

enter x_3

depart $x_3 S_3$

III Simplex Table

	C_j^0	2	5		0	0	0	
C_B	x_3	x_0	x_4	x_2	x_1	s_1	s_2	s_3
5	x_2	5	0	1		$\frac{1}{3}$	0	$-\frac{1}{3}$
0	s_2	4	0	0		$\frac{2}{3}$	1	$-\frac{11}{3}$
2	x_1	4	1	0		$-\frac{1}{3}$	0	$\frac{4}{3}$
	$(C_j^0 - Z_j^0) \Rightarrow$	0	0		-1	0	0	

Since all $C_j - Z_j \leq 0$ so current
 ΔP^n is optimal $x_4 = 4 \quad x_2 = 5$
 $\text{Max } Z = 33$

Q4 (i) Unbalanced transportation problem:

When total demand = Total supply.

problem is unbalanced. Then we add

dummy demand or dummy supply
 as per requirement of the problem.
 which one is short.

& we assume cost per unit for
 a dummy demand or supply is zero.

(ii) IBFS using NWCR, LCM, VAN.

problem is already balanced.

NWCR

	D_1	D_2	D_3	D_4		99
P_1	19(5)	30(2)	50	12		72
P_2	36	30(6)	40(4)	60		104
P_3	46	10	60(3)	20(15)		1815
	61	5	86	74	15	35

$$\begin{aligned} \text{total cost} &= 19 \times 5 + 30 \times 2 + 30 \times 6 + 40 \times 4 + 60 \times 3 + 20 \times 15 \\ &= 975 \end{aligned}$$

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(LCM)

	D_1	D_2	D_3	D_4	Supply
P_1	19	36	80	12	7
P_2	70	36	40	60	10.5
P_3	40	10	60	20	18.162
Demand.	5	8	7	15	35

Using LCM Cost

$$\begin{aligned}
 &= 70 \times 3 + 12 \times 7 + 40 \times 7 + 10 \times 8 + 40 \times 2 \\
 &\quad + 20 \times 8 = 894.
 \end{aligned}$$

Now VAM.

	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3	P_4	Penalty (Column)
Plant	19(5)	36	58	12	7	(7)	(18)	(38)	-	(38) + 3
P_1	70	36	40	60	10	(10)	(10)	(20)	(20)	
P_2	40	10	60	20	18.162	(10)	(10)	(40)	(40)	
Demand.	5	8	7	15	35					

	P_1	P_2	P_3	P_4
Row	(21)	(20)	(10)	(8)
P_1	-	(20)	(10)	(8)
P_2	-	-	(10)	(8)
P_3	-	-	20	48

$$\begin{aligned}
 &19 \times 5 + 12 \times 2 + 40 \times 7 + 60 \times 3 + 20 \times 10 \\
 &\quad + 10 \times 8 = 859.
 \end{aligned}$$

4. (iii)

Total Supply ≠ Total demand

$$105 \neq 145$$

So we add dummy source w_4 with penalty cost5, 3, 2 for R_1, R_2, R_3 respectively.and Supply = $145 - 105 = 40$ (unsatisfied demand)

Retail outlet

	<u>R_1</u>	<u>R_2</u>	<u>R_3</u>	<u>a_i (available)</u>
w_1	5	1	7	10
w_2	6	4	6	80
w_3	3	2	5	15
(dummy) w_4	5	3	2	40
bj (requirement)	75	20	50	145

Now problem is balanced.

IBFS By VAM.

w.	R_1	R_2	R_3	a_i
w_1	5	1(10)	7	10
w_2	6(60)	4(10)	6(10)	80
w_3	3(15)	2	5	15
dummy w_4	5	3	2(40)	40
bj	75	20	50	145

by VAM Cost = 595

Test for optimality

No of positive independent allocation.

$$\chi_j = m+n-1 = 6$$

∴ So solution is non-degenerate

Apply Modi Method.

- Set up cost matrix for occupied cells.

10

calculate u_i , v_j using $c_{ij} = u_i + v_j$

-	1 (10)	-	$u_1 = -3$
6 (6)	4 (10)	6 (10)	$u_2 = 0$
3 (18)	-	-	$u_3 = -3$
-	-	2 (40)	$u_4 = -4$

$v_1 = 6$ $v_2 = 4$ $v_3 = 6$

2) Calculate opportunity cost $a_{ij} = c_{ij} - (u_i + v_j)$
for unoccupied cell

5 (8)	*	7 (4)	$u_1 = -3$
*	*	*	$u_2 = 0$
*	2 (1)	5 (2)	$u_3 = -3$
5 (6)	3 (0)	*	$u_4 = -4$

$v_1 = 6$ $v_2 = 4$ $v_3 = 6$

c_{ij}	A_{ij}
$u_i + v_j$	

+2

Since all $A_{ij} \geq 0$ solution is ~~not~~ optimal.
So. Min cost = 595.

Q5. (i) Assignment problem.

The assignment problems deal with the allocation problem in which the objective is to assign ' n ' number of jobs to ' n ' number of persons at minimum cost

Assumptions of Assignment problem.

* Number of persons is equal to number of jobs

* Each person is assigned to only one job

* Assignment criteria are clearly specified

+1

+2

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(ii)

	J_1	J_2	J_3	J_4	J_5	J_6
O_1	6	2	5	3	6	0
O_2	2	5	8	7	7	0
O_3	7	8	6	9	8	0
O_4	6	2	3	4	5	0
O_5	9	3	8	9	7	0
O_6	4	7	4	6	8	0

+1

using row transformation (Reduction)
matrix would be same as above

	J_1	J_2	J_3	J_4	J_5	J_6
O_1	1	0	2	0	1	0
O_2	0	3	6	4	2	0
O_3	5	6	4	6	3	0
O_4	4	6	1	1	0	0
O_5	7	1	6	6	2	0
O_6	2	5	2	3	3	0

+1

(Using column reduction)

Now. Sol^m is not optimal
we draw V st line covering maximum zeros.

min.

if no of line = no of assignm. = order of mat^m

= Sol^m is optimal

again not optm

	I	II	III	IV	V	VI	
I	5	4	0	0	9	1	
II	0	3	5	3	2	0	
III	5	6	3	5	3	0	
IV	4	0	0	0	0	0	
V	7	1	5	5	2	0	
VI	2	5	1	2	3	0	

+2

J₁ J₂ J₃ J₄ J₅ J₆

O₁ 0, 6 1 2 10 2 2

O₂ 0 2 4 2 1 *

O₃ 5 5 2 4 2 10 +2

O₄ 5 * * * 10 1

O₅ 7 10 4 4 1 *

O₆ 2 4 10 1 2 *

O₁ → J₄

O₂ → J₁

O₃ → J₆

O₄ → J₅

O₅ → J₂

O₆ → J₃

$$\text{Min Cost} = 2+2+0+5+3+4 = 16$$

Q 5 (iii) Here $\text{Min}(A_i) = 5$ $\text{Max}(B_i) = 5$, $\text{Min}(C_i) = 3$.

Since $\text{Min}(A_i) \geq \text{Max}(B_i)$ satisfied

The problem can be converted into a problem of
5 jobs and two machines we define
two fictitious machines C_r and H.

$$C_i = A_i + B_i \quad H_i = B_i + C_i$$

Job.	1	2	3	4	5
Machine C _r	7	8	10	14	8
Machine H	5	8	9	11	10

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Continue

Q5 (iii)

Jobs.	A	B	C	D	E
A	5	7	6	9	5
B	2	1	4	5	3
C	3	7	5	6	7

So far G.H. we find optimal sequen.
using Johnson's Algo

+2

[5 | 2 | 4 | 3 | 1]

The total elapsed time and idle times for
Machi A, B, C are as follows.

for Idle time

Job	Machine A		Machine B		Machine C		Idle time		
	In	Out	In	Out	In	Out	A	B	C
5	0	5	5	8	8	15	0	5	8
2	5	12	12	13	15	22	0	4	0
4	12	21	21	26	26	32	0	8	4
3	21	37	27	31	32	37	0	1	0
1	27	32	32	34	37	40	0	1	0

Now total elapsed time = 40 hrs.

Idle time for Machi A = 40 - 32 = 8 hrs.

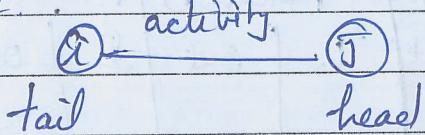
Idle time for Machi B = 40 - 34 + 19 = 25 hrs +1

Idle time for Machi C = 40 - 40 + 12 = 12 hrs.

Q6 (i) Network ; A network is defined as a graphic representation with a flow of some type in its arrow. It represents arrow and nodes. Arrow represents activity and node represents event.

Ex. Highways system Node arrow flow
 Fluid Supply Sys. Intersection Roads vehicles
 Pumping station Pipes fluid.

(ii) Activity ; An individual operation which utilises resources and takes certain amount of time for completion and has and end and a beginning called activity.



Three type - predecessor, Successor, dummy.

(iii) Event ; An event represents a point in time satisfying (signifying) the completion of some activities and the beginning of new ones i.e. It represents project milestone.

This is usually represented by a circle in a Network.

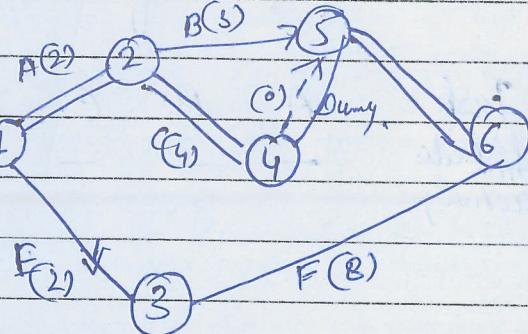
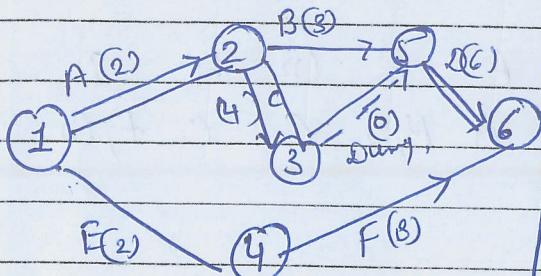
Type - Merge, Burst, Merge and burst



Q6 (ii) First we prepare network diagram

OK

Two ways to prepare



+3

Earliest event time
forward Pass Calculation.

$$E_1 = 0$$

$$E_2 = E_1 + d_{1,2} = 0 + 2 = 2$$

$$E_3 = 6$$

$$E_4 = 2$$

$$E_5 = \max [2+3, 6+0] = 6$$

$$E_6 = \max [6+6, 2+8] = 12$$

Latest event time

Backward Pass Calculation

$$L_6 = 12$$

$$L_5 = 6$$

$$L_4 = 4$$

$$L_3 = 6$$

$$L_2 = 2$$

$$L_1 = 0$$

$$\text{Now } E_1 = L_1 = 0$$

$$E_2 = L_2 = 2$$

$$E_3 = L_3 = 6$$

$$E_5 = L_5 = 6$$

$$E_6 = L_6 = 12$$

So Path is 1-2-3-5-6

Project duration

$$2 + 4 + 0 + 6 = 12 \text{ weeks}$$

Earliest event time
forward pass calculation.

$$E_1 = 0$$

$$E_2 = E_1 + d_{1,2} = 0 + 2 = 2$$

$$E_3 = E_1 + d_{1,3} = 0 + 2 = 2$$

$$E_4 = E_2 + d_{2,4} = 2 + 4 = 6$$

$$E_5 = \max [E_2 + d_{2,5}; E_4 + d_{4,5}] = \max [2+3, 6+0] = 6$$

$$E_6 = \max [6+6, 2+8] = 12$$

Latest event time

Backward Pass Calculation

$$L_6 = E_6 = 12$$

$$L_5 = L_6 - d_{5,6} = 12 - 6 = 6$$

$$L_4 = L_5 - d_{4,5} = 6 - 0 = 6$$

$$L_3 = L_6 - d_{3,6} = 4$$

$$L_2 = \min [L_5 - d_{2,5}, L_4 - d_{4,2}] = \min [3, 2] = 2$$

$$L_1 = \min [2 - 2, 4 - 2] = 0$$

at critical path L values = E values.

critical Path = 1-2-4-5-6

Critical activity (A, C, D)

Project duration

$$= 2 + 4 + 6$$

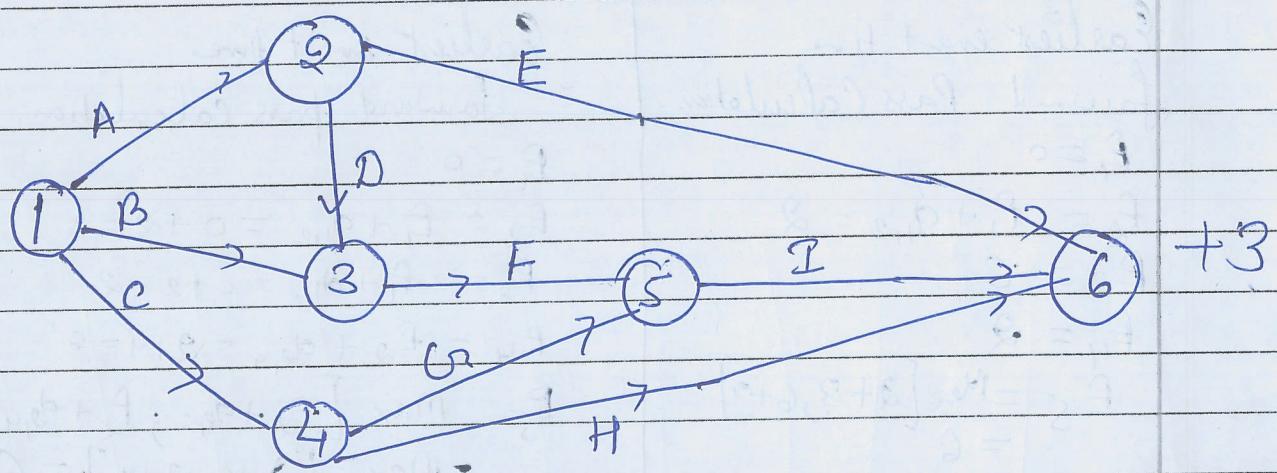
$$= 12 \text{ weeks.}$$

+2

Q 6

(ii) we prepare following table using information given.

Task	A	B	C	D	E	F	G	H	I	J
Preceding activity	-	-	-	A	A, B, D	C	C	F, G	J	



C is predecessor of G and H.

There is no predecessor for A, B, C activity

A is predecessor of D and E activity

B, D are predecessor of F activity

F, G are predecessor of I activity.

Complete: