iii.	Find the basic feasible solution of the following problem using Vogel's Approximation method:	**5**

	D_1	D_2	D_3	D_4	Supply
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
Demand	7	5	3	2	

Q.5 Attempt any two:

i. Define the mathematical model for an assignment problem. **5**

ii. Write the steps of Hungarian method for solving an assignment problem. **5**

iii. Suppose that there are five jobs, each of which has to be processed on two machines A and B in the order AB. Processing times are given in the following table: **5**

Job	Machine A	Machine B
1	6	3
2	2	7
3	10	8
4	4	9
5	11	5

Determine a sequence in which these jobs should be processed so as to minimize the total processing time.

Q.6 i. Differentiate between pure and mixed strategy games. **4**

ii. Solve the game whose payoff matrix is given below: **6**

		Player B		
Player A		I	II	III
	I	-2	15	-2
	II	-5	-6	-4
	III	-5	20	-8

OR iii Write a short note on any three of the following terms: **6**

(a) Payoff (b) Strategy

(c) Saddle point (d) Value of the game

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrolment No......................................

Faculty of Science / Engineering
End Sem Examination Dec-2023
CA3CO21 Operations Research

Programme: BCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Which of the following is correct for operations research? **1**

(a) It is a branch of science

(b) It helps in decision making problems

(c) It helps in establishing the optimal solutions

(d) All of these

ii. Operations research models are- **1**

(a) Abstraction of real-life situations

(b) Not dependent on experts

(c) Hypothetical models

(d) None of these

iii. In a linear programming problem, the restrictions under which the objective function is to be optimised are called- **1**

(a) Constraints (b) Objective function

(c) Decision variables (d) None of these

iv. A feasible solution to a linear programming problem- **1**

(a) Must satisfy all of the problem's constraints simultaneously.

(b) Need not satisfy all of the constraints.

(c) Must be a corner point of the feasible region.

(d) None of these

v. How many occupied cells must a transportation matrix with 8 rows and 7 columns have so that it does not degenerate? **1**

(a) 15 (b) 55 (c) 56 (d) 14

vi.	Which of the following method is used for obtaining initial basic feasible solution to the transportation problem? (a) North-west corner method (b) Vogel's approximation method (c) MODI method (d) None of these	**1**
vii.	An assignment problem- (a) Does not assign a number of resources to an equal number of activities (b) Maximizes total cost and minimizes total profit (c) Is a particular case of transportation problem (d) All of these	**1**
viii.	In job sequencing problems, (a) The processing time on each machine is known. (b) No machine can process more than one job simultaneously. (c) Each job, once started on a machine is to be performed up to completion on that machine. (d) All of these	**1**
ix.	When strategies for both players are pure, then payoff matrix contains- (a) One saddle point (b) Two saddle points (c) Three saddle points (d) No saddle point	**1**
x.	In a two-person zero-sum game- (a) The losses of one player are greater than the gains of the other (b) The losses of one player are equivalent to the gains of the other (c) The losses of one player are lesser than the gains of the other (d) None of these	**1**

Q.2 Attempt any two:

i. Write five scopes of operations research and explain. **5**

ii. Describe five limitations of operations research. **5**

iii. Explain five models of operations research. **5**

Q.3 Attempt any two:

i. Write a short note on any three of the following terms: **5**

(a) Feasible solution (b) Basic feasible solution

(c) Slack variables (d) Infeasible solution

ii Solve the following linear programming problem by graphical method: **5**

Maximize $z = x + 3y$

subject to the constraints

$$2x + 5y \leq 120$$
$$x + y \leq 4$$
$$x \geq 0, y \geq 0.$$

iii Solve the following linear programming problem by simplex method: **5**

Maximize $z = 2x + 4y$

subject to the constraints

$$x + 2y \leq 5$$
$$4x + 2y \leq 80$$
$$x \geq 0, y \geq 0.$$

Q.4 Attempt any two:

i. Find the basic feasible solution of the following problem using the Least Cost method: **5**

	D_1	D_2	D_3	D_4	Supply
O_1	3	1	7	4	300
O_2	2	6	5	9	400
O_3	8	3	3	2	500
Demand	250	350	400	200	

ii. Determine basic feasible solution to the following transportation problem using North west Corner rule: **5**

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
Demand	3	3	4	5	6	

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MCQ

Q1 (i) (d) All of these +1
(ii) (d) None of these +1
(iii) (a) Constraints +1
(iv) (a) must satisfy all the problem's constraints simultaneously +1
(v) (d) 14 +1
(vi) (a) & (b) : N-W-CR method and Vogels both are correct +1
(vii) Only ~~(b)~~ (c) is correct. ~~Maximize~~ Particular case of Transportation +1
(viii) (d) All of these +1
(ix) (a) One saddle point +1
(x) (b) The losses of one player are equivalent to the gains of other. +1

Q2.
(i) Five Scope of OR.

1) Research and Development
* Projects selection
* Determination of area of R&D +1
* Reliability and alternative Design

2) In Marketing
* Where to distribute the products for sale so that total cost is minimized. +1
* Best time to launch new project
* Sale price per unit
* Size of stock to meet requirement

(3) Agriculture
* optimum allocation of land to various crops in accordance with climatic Conditions. +1
* optimum distribution of water from Various resources like canal for irrigation

(4) LIC
* to decide premium rates for the various modes of policies. +1
* How best the profits could be distributed in Case of with profit policies.

(5) Inventory Control
* optimal buying +1
* Quantities and timing of purchase
* Optimal reordering
other area of application are production management, Industry, finance accounting, Personal Management.

Q2 (ii) five Limitations of OR

a) Magnitude of Computation → (Explain). +1

b) Analysis of Only Quantifiable factors (Explain) +1

c) Gap between Managers & OR Team member (Explain) +1
(d) Expensive in Alternative Design required +1
(e) Time factor (need months to make Algo.) +1

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Q2(iii) five models of OR.

a) Iconic model : Pictorial representation of various aspects of System Eg. maps, globes. +1

b) Mathematical Model : These type of models employ a set of mathematical symbols to represent the decision variable of System. The variables are related by mathematical system. Ex. LPP, sequencing, replacement model. +1

c) Static Model : Static models does not take time in to account. assumes value of Variables do not change with time. Ex LPP, Transportation, Assignment. +1

d) Deterministic Model : These type of Model which does not take uncertainity into account. LPP. TP & assignment are Ex. +1

e) Descriptive model : It just describes a situation or system based on observations, survey, questionnaire results and other avaliable data.
Ex. an opinion Poll, any survey. +1

other models are Dynamic, predictive, Stochastic, Analysis, Simulation.
Consider any five model.

Q3.
(1) Short note on.

(a) feasible Solution : A set of values of Variables $x_1, x_2, \ldots x_n$ is called a feasible Solution to LPP if its satisfy all the constraints of given problem i.e. +2

constraints as well as non-negative restrictions. OR.
A Solution which satisfy constraints as well as non-negative restriction, called feasible solution.

(b) basic feasible solution:
First explain Basic Solution +2
For a set of m- simultaneous equation in n variables ($n > m$), a solution obtained by setting ($n-m$) variables equals to zero and solving for remaining m equations in m variables is called basic solution.
Now. A feasible solution to a LPP which is also basic solution is called the basic feasible solution. There are of two types Degenerate and Non-degenerate.

(c) Slack variables:
It is a variable that is added to the Left hand side of a $\leq$ (less than or equal to) type constraint to convert it into an equality constraint. In economic terms, slack variable represent left over or unused Capacity. +1
Ex. $2x + 3y \leq 10$
using slack variable S_1 it becomes.
$2x + 3y + S_1 = 10$. $S_1 \geq 0$

OR

(d) Infeasible Solution:
A set of values of variables $x_1, x_2, \ldots x_n$ is called infeasible solution to a LPP if it does not satisfy either constraints +1

or non-negative restrictions.
It is also called no-solution case.

Q3 (ii) LPP using graphical method.

Max $Z = x + 3y$

S.t. Constraints $2x + 5y \leq 120$

$x + y \leq 4$.

$x \geq 0 \quad y \geq 0$

First we express each Constraint in terms of equation. line & get points on each line i.e

$$2x + 5y = 120 \quad \text{—— (1)}$$

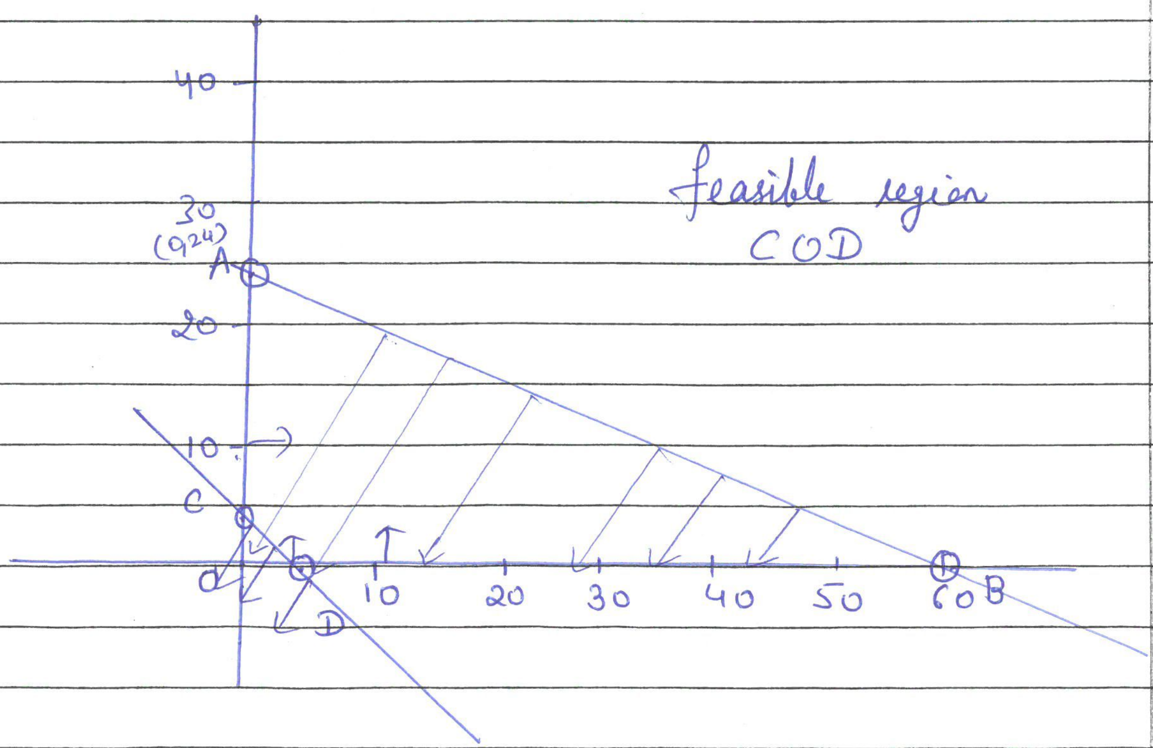
put $x = 0$ $5y = 120$ $y = 24$. $(0, 24)$ A. +1

put $y = 0$ $2x = 120$ $x = 60$ $(60, 0)$ B.

$$x + y = 4 \quad \text{—— (2)}$$

$x = 0$ $y = 4$ $(0, 4)$ C +1

$y = 0$ $x = 4$ $(4, 0)$ D.



+1

To get optimized output

Corner pt	Value	$Z = x + 3y$
C	(0,4)	$Z = 0 + 3 \times 4 = 12$
O.	(0,0)	$Z = 0 \times 0 = 0$
D	(4,0)	$Z = 4 + 0 = 4$

+2

Max $Z = 12$ at $x = 0$ $y = 4$.

Q3 (iii) Simplex method.

Max $Z = 2x + 3y$

S.t. $x + 2y \leq 5$

$4x + 2y \leq 80$

$x \geq 0, y \geq 0$

using two slack variables S_1, S_2 The revised LPP will be as followed.

Max $Z = 2x + 3y + 0.S_1 + 0.S_2$

St $x + 2y + S_1 + 0.S_2 = 5$

$4x + 2y + 0.S_1 + S_2 = 80$ +1.5

$x, y, S_1, S_2 \geq 0$.

Now we find initial Basic feasible solution

Set $x_1 = 0$. $x_2 = 0$ | $x = 0$, $y = 0$.

We get $S_1 = \cancel{80}\ 5$, $\cancel{S_2 = 0}$ $S_2 = 80$

S_1, S_2 are basic Variables

I Simplex table

C_B	x_B	c_j / x_0	2 / x	4 / y	0 / s_1	0 / s_2	x_0/y
0	s_1	5	1	2	1	0	$5/2 \rightarrow$
0	s_2	80	4	2	0	1	$80/2=40$
		Z_j	0	0	0	0	
	$C_j - Z_j$		2	3	0	0	

$\uparrow$

+1

Most positive from $C_j - Z_j$ will give entering vector i.e 'y' here

Min $x_0/y \rightarrow$ gives departing vector

introducing y, departing s_1

and Key element 2, The revised form & next table will be

II Simplex table

C_B	x_B	c_j / x_0	2 / x	4 / y	0 / s_1	0 / s_2	$x_0/$
4	y	$5/2$	$1/2$	1	$1/2$	0	
0	s_2	75	3	0	-1	1	
		Z_j	2	4	2	0	
	$C_j - Z_j$		0	0	-2	0	

+2

All $C_j - Z_j \leq 0$, the current Solⁿ is optimal Solⁿ is $x = 0$ $y = 4$

Maximum Value of Z

$2x + 4y = 2 \times 0 + 4 \times 5/2$ +0.5

$= 10.$

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Q4 (i) BFS using Least Cost method.

	D_1	D_2	D_3	D_4	Supply
O_1	3x	1 (300)	7x	4x	~~300~~
O_2	2 (250)	6 (50)	5 (100)	9x	~~400~~ ~~150~~ ~~100~~
O_3	8x	3x	3 (300)	2 (200)	~~500~~ ~~300~~
Demand	~~250~~	~~350~~ ~~50~~	~~400~~ ~~100~~	~~200~~.	1200

+3

As total supply = total Demand = 1200
So problem is balanced. & can Continue with given cost matrix
So minimum cost is

$1 \times 300 + 2 \times 250 + 6 \times 50 + 5 \times 100$
$+ 3 \times 300 + 2 \times 200 = 2900$

+2

Note:
Here there two case when there is tie is among minimum cost.
marks may be allocated if arbitrary cost is chased. Like in above problem.
Cost may be (2850 if random cost is chosed).

(ii) NWCR.

	D_1	D_2	D_3	D_4	D_5	Supply.
O_1	2	11	10	3	7	4
O_2	1	4	7	2	1	8
O_3	3	9	4	8	12	9
Demand.	3	3	4	5	6	21

As total supply = total Demand = 21
We can continue with N-W C R

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	2 (3)	11 (1)	10 x	3 x	7 x	~~4~~ ~~1~~
O_2	1 x	4 (2)	7 (4)	2 (2)	1 x	~~8~~ ~~6~~ ~~2~~
O_3	3 x	9 x	4 x	8 (3)	(6) 12	~~9~~ ~~6~~
Demand	~~3~~	~~3~~ ~~2~~	~~4~~	~~5~~ ~~3~~	~~6~~	21

+3

Total cost

$2 \times 3 + 11 \times 1 + 4 \times 2 + 7 \times 4 + 2 \times 2 + 8 \times 3 + 6 \times 12$ +2

$6 + 11 + 8 + 28 + 4 + 24 + 72 = 153$

(iii) Least cost method.

	D_1	D_2	D_3	D_4	Supply	Penalty P_1
O_1	2	3	11	7	6	(1)
O_2	1 x	0 x	6 x	1 (1)	~~1~~	(1)
O_3	5	8	15	9	10	(3)
Demand	7	5	3	~~2~~ 1	17	
Penalty P_1	(1)	(3)	(5)	(6)		

+1

Note * After every allocation we calculate another penalty row & column for next allocation.

eleminat O_2 (as supply zero)

	D_1	D_2	D_3	D_4	Supply	P_2
O_1	2	3 (5)	11	7	~~6~~ 1	(1)
O_3	5	8 x	15	9	10	(3)
	7	~~5~~	3	1		
Penalty P_2	(3)	(5)	(4)	(2)		

+1

Eleminate D_2

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	D_1	D_3	D_4	S_4	Penalty (P_3)
O_1	2 (1)	11	7	~~1~~	(5)
O_3	5 (6)	15 (3)	9 (1)	10	(4)
Demand	~~7~~ 6	3	1		
Penalty (P_3)	(3)	(4)	(2)		

+2

Now only one row left So we use LCM

$$\text{Min cost} = 1\times1 + 5\times3 + 2\times1 + 5\times6 + 15\times3 + 9\times1$$ +1
$$= 1 + 15 + 2 + 30 + 45 + 9 = 102$$

Q5(i) Mathematical Model for assignment problem
n jobs, n persons.
Assumption.
* Number of persons = number of jobs.
* Each person is assigned with only one job +1

Let C_{ij} be the cost of assigning i^{th} person to job

Job. persons. ↓	D_1	D_2	...	D_n	Supply
S_1	C_{11}	C_{12}	...	C_{1n}	1
S_2	C_{21}	C_{22}	...	C_{2n}	1
⋮	⋮				
S_n	C_{n1}	C_{n2}	...	C_{nn}	1
Demand	1	1	...	1	N

+2

Let us define a variable x_{ij} where x_{ij} is assignment of i^{th} person to j^{th} job

$$x_{ij} = \begin{cases} 0 ; & \text{if } i^{th} \text{ person is not assign to j Job} \\ 1 ; & \text{if } i^{th} \text{ person is assign to j Job} \end{cases}$$ +2

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The Problem now, is to assign non-negative allocation x_{ij} So as to Min $Z = \sum_i \sum_j C_{ij} x_{ij}$

s.t.

$$\sum_{j=1}^{n} x_{ij} = 1 \quad , \quad \sum_{i=1}^{n} x_{ij} = 1$$ +0.5

and $x_{ij} = 0$ or 1

Q 5(ii)

Hungarian method.

I Prepare Square matrix

II Reduce matrix

(a) Row Reduction; Subtract smallest element of each row from all the elements of respective row.

(b) Reduce column for column in same way.

(c) In reduced matrix there should be one zero element in each row and each column. +2

III Make an assignment in the reduced matrix.

(a) Rowwise assignment: check all rows from top to bottom until a row with exact zero is found. Make an assignment to this single zero by making a square around it and cross all zeroes in the corresponding column. +2

(b) Columnwise assignment :- Same process as rowwise process. change role of Row & column

IV optimality Procedure

* If number of assignment = order of matrix then solⁿ is optimal

* If number of assignment < order of matrix, not optimal

V Revise New Cost matrix. +2

Draw minimum number of Horizontal & vertical lines necessary to cover all zero. and do next step.

* Examine the uncovered elements, select minimum of uncovered element
* Subtract this minimum element from all uncovered element, Add this element to intersection pt element
* Remaining element on line will be as it is
Repeat Step 3 to 6 until reached optimality.

Q5. (iii) Job sequencing

Jobs →	1	2	3	4	5
Machine A	6	2	10	4	11
Machine B	3	7	8	9	5

We find that the smallest processing time is 2 on machine A corresponding to job 2, So job 2 will be processed first in the sequence

2				

+2

Now eleminate job 2 from table
Then next smallest processing time is 3 on machine B with time '3', job 1, According rule it will be processed Last in sequence

2				1

+2

Now eleminate job 1
in same way next is job '4' by machine A.

2	4			1

Similar way we find optimal sequence

+0.5

2	4	3	5	1

Q6 (i) Pure strategy; If a player knows exactly what the other player is going to do, a deterministic situation is obtained and the objective is to maximize gain or minimize loss, is a rule always selects a particular course of action +2

(ii) Mixed: If a player is guessing as to which activity is to be selected by the other on any particular occasion A probabilistic situation is obtained and objective function is to maximize expected gain or the minimize expected loss: Thus the mixed strategy is a selection among pure strategies with fixed probabilities. +2

Q6 (ii) Solve the game with payoff

Player B

		I	II	III	Row Min
	I	−2	15	−2	−2
Player A.	II	−5	−6	−4	−6
	III	−5	20	−8.	−8
Col max		−2	20	−2	Maximin value

minimax values. +2.5

We always try to search a (saddle point) method. If it exist Then it is the Sol^n.
here Maximin = Minimax = −2 +2.5
So Saddle point (AI, BI) OR (AI BIII) Exists
& value of game is −2.
Strategy for Player A is AI, B is BI or BIII +1
It is pure strategic game.

Payoff :- The gains and loss in a game when players select their particular course of action, can be represented

in the form of matrix, called pay-off matrix.
Suppose A has m activities $A_1, A_2, \ldots A_m$ & B has n $B_1, B_2 \ldots B_n$
a_{ij} represents the payoff Then payoff matrix for Player A.

$$\begin{array}{cc} & \text{Player B's strategies} \\ \text{Player A's strategies} & \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & & a_{mn} \end{bmatrix} \end{array}$$

+2

Strategy: It is the predetermined rule by which Player decides his course of action from his own list during the game. (Two type pure & mixed)

Saddle point: A saddle point of a matrix is the position which is minimum in its row and max^m in its cols. i.e. maximin = minimax value gives Location (saddle pt) +2

Value of game: Value of the game is the maximum guaranteed gain to the Player A (maximizing Player) if both the Player uses their best strategies. It is generally denoted by v and It is unique. +2

"END of the Solution"

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verified
7 DEC 2023
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07/12/23