ii. The following table gives the result of series of controlled exponent. Discuss whether the treating may be considered to have any positive effect: **5**

	Positive	No Effect	Negative
Treatment	9	2	1
Control	3	6	3

Given the value of χ^2 for 2 degree of freedom at 5% level of significance is 5.99.

iii. 10 boys are selected at random from a class and their marks in Maths out of 100 are found to be 70, 67, 62, 68, 61, 68, 70, 69, 64, 66. In the light of these marks, discuss the statement the marks in mean marks of the subject in class was 64. Given value of '*t*' for 2 degree of freedom and 5% level of significance is 1.833. **5**

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No.....

Faculty of Engineering

End Sem Examination Dec-2023

EC3BS01 / EE3BS01 / EX3BS01

Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: EC/EE/EX

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Geometric Mean (GM) of 2, 4, 16, 32 is …….. **1**

(a) 6 (b) 12 (c) 8 (d) None of these

ii. The sum of Deviations from the …… is always zero. **1**

(a) Arithmetic Mean (b) Median

(c) Mode (d) None of these

iii. A Discrete random variable may take ……. in a random experiment. **1**

(a) Finite

(b) Countably infinite

(c) Either finite or countably infinite

(d) None of these

iv. If A and B are two events and are not mutually exclusive then we have- **1**

(a) $P(A \cup B) = P(A) - P(B) - P(A \cap B)$

(b) $P(A \cup B) = P(A) - P(B) + P(A \cap B)$

(c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(d) None of these

v. For the Poisson distribution with parameter λ, the value of mean and variance is same as- **1**

(a) λ (b) λ^2 (c) $\frac{1}{\lambda}$ (d) None of these

vi. Which one of the following is a continuous probability distribution? **1**

(a) Binomial (b) Poisson

(c) Exponential (d) All of these

vii. The value of Karl Pearson's coefficient r satisfies the range- **1**

(a) $r \geq 1$ (b) $-1 \leq r \leq 1$

(c) $r \leq -1$ (d) None of these

viii. If one of the regression coefficient $b_{xy} > 1$ then b_{yx} is- **1**

(a) > 1 (b) < 1

(c) $= 1$ (d) None of these

ix. The region of sample space in which null hypothesis is rejected is called- **1**

(a) Level of significance

(b) Critical region

(c) Both significant and critical regions

(d) None of these

x. χ^2 test is also known as- **1**

(a) Test of independence attributes

(b) Test of goodness of fit

(c) Test of significance of population variance

(d) All of these

Q.2 i. Show that arithmetic mean of first n natural numbers is given by $\frac{(n+1)}{2}$. **2**

ii. Calculate the mode of following distribution: **3**

Class	0-7	7-14	14-21	21-28	28-35	35-42	42-49
Frequency	19	25	36	72	51	43	28

iii. Find the Standard deviation of the following series: **5**

Marks(Above):	0	10	20	30	40	50	60	70
No. of Students:	100	90	75	50	25	15	5	0

OR iv. Describes merits and demerits of various measure of central tendencies. **5**

Q.3 i. A bag contains 3 red and 4 white balls. Two draws are made without replacement. What is the probability that both the balls are red? **2**

ii. A random variable X has following distribution **8**

X:	-2	-1	0	1	2	3
$P(X)$:	0.1	k	0.2	$2k$	0.3	$3k$

Find

(a) The value of k (b) $P(X < 2)$

(c) The mean value of X (d) Standard deviation of X.

OR iii. A continuous random variable X has probability density function given by **8**

$$f(x) = \begin{cases} ax & ;0 \leq x \leq 1 \\ a & ;1 \leq x \leq 2 \\ 3a - ax & ;2 \leq x \leq 3 \\ 0 & ;\text{otherwise} \end{cases}$$

Find

(a) The value of a

(b) Cumulative distribution function

(c) $P(1 \leq X \leq 2.5)$

Q.4 i. Describe the normal distribution and the properties of the normal curve. **3**

ii. Fit the Poisson distribution for the following frequency distribution. **7**

x:	0	1	2	3	4
y:	122	60	15	2	1

OR iii. Derive the formula for the variance of Binomial distribution. **7**

Q.5 Attempt any two:

i. Fit a second-degree parabola for the following data **5**

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

ii. Find the Rank correlation for the following two games score out of 50 points: **5**

$Game1$:	35	23	47	35	10	43	9	6	28
$Game2$:	30	33	23	23	20	23	12	4	31

iii. Find the Equation of two regression lines and correlation coefficient between the following to correlated quantities: **5**

x	25	22	28	35	20	22	40	20	18	26
y	18	15	20	22	14	16	21	15	14	17

Also estimate *y* when *x=30.*

Q.6 Attempt any two:

i. Describe the concept of hypothesis testing with the definition of null hypothesis, critical region, level of significance type 1 and type 2 error. **5**

Solution Set - Dec 2023

EC3BS01 / EE3BS01 / EX3BS01
Engineering Maths - III

Date :
P. No. : 1

Q.1

(i) (C) 8 (1)

(ii) (a) Arithmetic mean (1)

(iii) (c) Either finite or countably infinite (1)

(iv) (c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (1)

(v) (a) λ (1)

(vi) (c) Exponential (1)

(vii) (b) $-1 \leq r \leq 1$ (1)

(viii) (b) < 1 (1)

(ix) (b) Critical region (1)

(x) (d) All of these (1)

Q.2

(i) Sum of first 'n' natural numbers 1, 2, 3 ... n

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$ 1

Arithmetic mean

$$= \frac{\text{Sum}}{\text{Total no.}} = \frac{\left(\frac{n(n+1)}{2}\right)}{n}$$ +1

$$= \frac{n+1}{2}$$

2 marks

(ii) Max fre 72 is in class 21–28 1

Modal class 21–28

$$M_0 = l + \frac{f_1}{f_{-1} + f_1} \times h$$

$f = 72$ $l = 21$ $f_{-1} = 36$ $f_1 = 51$ $h = 7$ +1

$$\boxed{M_0 = 25.103}$$

OR $$M_0 = l + \frac{f - f_{-1}}{2f - f_{-1} - f_1} \times h$$ +1

$$\boxed{M_0 = 25.4}$$

3 marks

(iii)

Marks (Above)	0	10	20	30	40	50	60	70
No. of St.	100	90	75	50	25	15	5	0

Class	Freq f_x	mid val x	$f_x x$	$(x-m)^2$	$f(x-m)^2$
0-10	10	5	50	676	6760
10-20	15	15	225	256	3840
20-30	25	25	625	36	900
30-40	25	35	875	16	400
40-50	10	45	450	196	1960
50-60	10	55	550	576	5760
60-70	5	65	325	1156	5780
	$N=100$		$\Sigma f_x x = 3100$		25400

2 +1

$$\text{Mean } m = \frac{\Sigma x f_x}{N}$$

$$= \frac{3100}{100} = 31 \quad +1$$

St. dev.

$$\sigma = \sqrt{\frac{\Sigma f (x-m)^2}{N}}$$

$$= \sqrt{\frac{25400}{100}} \quad +1$$

$$= 15.94$$

5 marks

(iv)

Merits 2.5

Demerits of central Tendency 2.5

5 marks

Q 3

(i) Bag Contains 3 Red 4 white balls

Total Balls = 7

No. of way of selecting two white ball = $^7C_2$

No of way of selecting 2 Red ball = $^3C_2$ +1

P(Both ball are red) = $\frac{^3C_2}{^7C_2}$

$= \frac{3 \times 2}{7 \times 6_2} = \frac{1}{7}$ +1 (2 marks)

(ii)

(a) As P(X) is P.m.f

$\sum_x P(x) = 1$

$\Rightarrow 0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$

$0.6 + 6k = 1$

$\Rightarrow 6k = 0.4$

$k = \frac{0.4}{60} = \frac{1}{15}$

$\boxed{k = \frac{1}{15}}$ 2

(b) $P(X < 2) = ?$

$\Rightarrow P(X<2) = P(X=-2) + P(X=-1) + P(X=0) + P(X=1)$

$= 0.1 + k + 0.2 + 2k$

$= 0.3 + 3k$

$= 0.3 + \frac{3}{15} = 0.5$ +2

(c) The mean of X

$= E(X)$

$= \sum x P(X)$

$= -2 \times 0.1 + -1 \times \frac{1}{15} + 0 \times 0.2 + 1 \times \frac{2}{15}$

$+ 2 \times 0.3 + 3 \times \frac{3}{15}$

$= 1.0666 \quad (1.0666)$ +2

(d) St. dev $= \sqrt{Var(X)}$

$Var(X) = E(X^2) - [E(X)]^2$ +1

Now $E(X) = 1.0666$

$E(X^2) = \sum x^2 P(X)$

$= 3.6$

$Var(X) = E(X^2) - [E(X)]^2$

$= 3.6 - (1.0666)^2$

$= 2.46222$

$S.D \approx 1.5691$ +1

8 marks

(ii) P.d.f

$$f(x) = \begin{cases} ax & : 0 \le x \le 1 \\ a & : 1 \le x \le 2 \\ 3a - ax & : 2 \le x \le 3 \\ 0 & : \text{otherwise} \end{cases}$$

(a) as $f(x)$ is P.d.f

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$$

Date :
P. No. :

$$\Rightarrow \left[\frac{a x^2}{2}\right]_0^1 + (ax)_1^2 + \left(3ax - \frac{ax^2}{2}\right)_2^3 = 1$$

$$\Rightarrow \frac{a}{2} + a + 3a - a\left(\frac{9}{2} - 2\right) = 1$$

$$\Rightarrow \frac{a}{2} + 4a - \frac{5a}{2} = 1$$

$$\Rightarrow \frac{4a}{2} = 1 \Rightarrow \boxed{a = \frac{1}{2}} \quad (2)$$

(b) Cummulative dist. function

$$F(x) = \int_{-\infty}^{x} f(x)\,dx \quad (+1)$$

$$F(x) = 0 \quad ; x \le 0$$

If $0 < x \le 1$

$$F(x) = \int_0^x ax\,dx$$

$$= a\left(\frac{x^2}{2}\right)_0^x = a\frac{x^2}{2} = \frac{x^2}{4} \quad 0 \le x \le 1$$

(+1)

If $1 \le x \le 2$

$$F(x) = \int_0^1 f(x)\,dx + \int_1^x a\,dx$$

$$= \left(\frac{ax^2}{2}\right)_0^1 + a(x-1)$$

$$= \frac{x}{2} - \frac{1}{4} \quad 0 \le x \le 2 \quad (+1)$$

$a = \frac{1}{2}$

If $2 \le x \le 3$

$$F(x) = \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4} \quad ; 2 \le x \le 3 \quad (+1)$$

(c) $P(1 \le X \le 2.5)$

$$= \int_1^{2.5} f(x)\,dx = \int_1^2 f(x)\,dx + \int_2^{2.5} f(x)\,dx$$

$$= \frac{11}{16} \quad (+2)$$

8 marks

Q4

(i) Normal Distribution +1
Properties of Normal curve +2
3 marks

(ii)

x	0	1	2	3	4
$\Sigma f = y$	122	60	15	2	1

$N = \Sigma f = 200$ (+1)

$$\text{mean } (\lambda) = \frac{\Sigma xf}{N} = \frac{60+30+8+4}{200} = \frac{1}{2} = 0.5$$

$\Rightarrow \lambda = 0.5$ (+1)

Poisson's Freq. is given by

$= N\ P(X=x) \quad x = 0, 1, 2, 3, 4$

$= N\ e^{-\lambda} \frac{\lambda^x}{x!}$ (+2)

$= 200 \times e^{-0.5} \frac{\lambda^x}{x!} \quad (e^{-0.5} \simeq 0.61)$

Expected

$x=0 \Rightarrow 200 \times 0.61 \times \frac{(0.5)^0}{0!} = 122$

$x=1 \Rightarrow 61$

$x=2 \Rightarrow 15$

$x=3 \Rightarrow 2$

$x=4 \Rightarrow 0$

(+3)

7 marks

OR (ii) Binomial with parameter n, p, q

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$Var(X) = E(X^2) - [E(X)]^2 \qquad +1$$

$$E(X) = \sum_{r=0}^{n} r\, P(X=r)$$

$$= \sum_{r=0}^{n} r\, {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= \sum_{r=0}^{n} np^{r-1}\, {}^{n-1}C_{r-1}\, p^{r-1} q^{(n-1)-(r-1)} \qquad (+2)$$

$$= np\,(p+q)^{n-1} = np$$

$$E(X^2) = \sum_{r=0}^{n} r^2 P(X=r)$$

$$= \sum_{r=0}^{n} [r(r-1)+r]\, P(X=r) \qquad (+2)$$

$$= \sum_{r=0}^{n} r(r-1)\, P(X=r) + \sum_{r=0}^{n} r\, P(X=r)$$

$$= n(n-1) \sum_{r=0}^{n} {}^{n-2}C_{r-2}\, p^{r} q^{n-r} + np$$

$$= n(n-1)\, p^2\, (p+q)^{n-2} + np$$

$$= n^2p^2 - np^2 + np \qquad (+2)$$

$$Var(X) = E(X^2) - [E(X)]^2 = n^2p^2 - np^2 + np - n^2p^2$$

$$= np(1-p)$$

$$= npq$$

7 Marks

Q5 (i)

x	y	xy	x^2	x^3	x^4	x^2y
0	1	0	0	0	0	0
1	1.8	1.8	1	1	1	1.8
2	1.3	2.6	4	8	16	5.2
3	2.5	7.5	9	27	81	22.5
4	6.3	25.2	16	64	256	100.8
Σ 10	12.9	36.9	30	100	354	130.3

(+2)

$y = a + bx + cx^2$

N.E.

$\Sigma y = na + b\Sigma x + c\Sigma x^2$

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$

$\Rightarrow$ $12.9 = 5a + 10b + 30c$

$36.9 = 10a + 30b + 100c$

$130.3 = 30a + 100b + 354c$ (+2)

$\Rightarrow$ $a = 1.42$ $b = -1.07$ $c = 0.55$

$y = 1.42 - 1.07x + 0.55x^2$ Ans (+1)

5 marks

(ii)

Game 1 x	Game 2 y	Rank x	Rank y	d_i^2
35 3	30 3	$(3+4)/2 = 3.5$	3	0.25
23 6	33 1	6	1	25
47 1	23 4	1	5	16
35 4	23 5	$(4+3)/2 = 3.5$	5	2.25
10 7	20 7	7	7	0
43 2	23 6	2	5	9
9 8	12 8	8	8	0
6 9	4 9	9	9	0
28 5	31 2	5	2	9

(3)

$\Sigma d_i^2 = 61.5$

Date :
P. No. :

Rank repeats for 35 (game1) 2 times
$\Rightarrow m_1 = 2$

Rank repeats for 23 (game2) 3 time
$m_2 = 3$

Rank correlation

$$R = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}(m_1^3 - m_1) + (m_2^3 - m_2)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[61.5 + \frac{1}{12}((6) + 24)\right]}{9\times(80)}$$

$$= 1 - 6\,\text{[illegible]}$$

$R = 0.533$

$\boxed{R = 0.533}$ Ans

+2

5 marks

(iii)

	x	$u = x-26$	u^2	y	$v = y-17$	v^2	uv
	25	-1	1	18	1	1	-1
	22	-4	16	15	-2	4	8
	28	2	4	20	3	9	6
*	26	0	0	17	0	0	0
	35	9	81	22	5	25	45
	20	-6	36	14	-3	9	18
	22	-4	16	16	-1	1	4
	40	14	196	21	4	16	56
	20	-6	36	15	-2	4	12
	18	-8	64	14	-3	9	24
Σ	256	-4	450	172	2	78	172

(+2)

Date : 10
P. No. :

$\bar{x} = 25.6 \quad \bar{y} = 17.2$

$$b_{yx} = \frac{\Sigma uv - \frac{\Sigma u \Sigma v}{n}}{\left(\Sigma u^2 - \frac{(\Sigma u)^2}{n}\right)}$$

$\boxed{b_{yx} = 0.385}$ +1

$$b_{xy} = \frac{\Sigma uv - \frac{\Sigma u \Sigma v}{n}}{\left[\Sigma v^2 - \frac{(\Sigma v)^2}{n}\right]}$$

$\boxed{b_{xy} = 2.23}$ +1

y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y = 0.385x + 7.34$$

x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$\Rightarrow x = 2.23y - 12.76$$

for $x = 30$ $\boxed{y \simeq 19}$

$r = \pm\sqrt{b_{xy} b_{yx}} = \pm\sqrt{0.385 \times 2.23}$ +1

$\boxed{r = 0.927}$ $\because b_{xy}, b_{yx} > 0$

5 marks

Date : 11
P. No. :

Q6

(i) Hypothesis Testing Concept +1
Null hypothesis +1
Critical Region +1
Level of Significance +1
Type I Type II +1
5 marks

(ii) H_0 : Treating may be considered for +ve effect

(iii) Observed freq

	Positive	No effect	Negative	Total
Treatment	9	2	1	12
Control	3	6	3	12
Total	12	8	4	24

Expected freq

	+ve	≈	-ve	+2
Tre.	6	4	2	
Cont	6	4	2	

$$\chi^2 = \frac{(9-6)^2}{6} + \frac{(2-4)^2}{4} + \frac{(1-2)^2}{2} \quad (+1)$$
$$+ \frac{(3-6)^2}{6} + \frac{(6-4)^2}{4} + \frac{(3-2)^2}{2}$$

$$= 1.5 + 1 + 0.5 + 1.5 + 1 + 0.5$$

$$\chi^2 = 6$$

dof = 2 Cal χ^2 for 2 dof = 6 (+1)

Tab. χ^2 for 2 dof 5% leve = 5.99

Cal > Tabu. We Reject H_0 5

(11) (i) $H_0 = \bar{x} \approx \mu$

$$t = \frac{\bar{x} - \mu}{s'} \sqrt{n} \qquad +1$$

$\mu = 64$

$\bar{x} = \frac{\Sigma x}{n} = 66$

$s' = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n-1}}$ $\qquad +1$

$= \sqrt{10}$

$\Rightarrow t = \frac{66 - 64}{\sqrt{10}} \sqrt{10} = 2$ $\qquad +1$

$dof = n - 1 = 9$

cal t for 9 dof $= 2$

tabulated $t_{0.05, 9} = 1.83$ $\qquad +1$

cal $t >$ tabulated t for 5% level of significance. $\qquad +1$

H_0 is rejected.

5 marks

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7/11/23