Q.6 Attempt any two:

i. A dice is tossed 120 times with the following results: **5**

Number turned up:	1	2	3	4	5	6	*Total*
Frequency:	30	25	18	10	22	15	120

Test the hypothesis that the dice is Unbiased (Given $\chi^2_{0.05,5} = 11.07$)

ii. Find the students t- statistics for the variable values $-4, -2, -2, 0, 2, 2, 3, 3$ taking the mean of universe to be zero. **5**

iii. Test whether the two sets of observations: **5**

I:	17	27	18	25	27	29	27	23	17
II:	16	16	20	16	20	17	15	21	

Indicates the samples drawn from the same universe. [The value of z at 5 % level of for 8 and 7 degree of freedom is 0.6575.]

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2022

EC3BS01 / EE3BS01 / EX3BS01

Engineering Mathematics III

Programme: B.Tech. Branch/Specialisation: EC/EE/EX

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The Arithmetic mean of 6 numbers is 12 if each number is increased by 4 what is new arithmetic mean- **1**

(a) 12 (b) 8 (c) 18 (d) None of these

ii. Which one of the following is same as median? **1**

(a) First Quartile Q_1 (b) Second Quartile Q_2

(c) Third Quartile Q_3 (d) None of these

iii. If a continuous random variable has density function, **1**

$$f(x) = \begin{cases} \dfrac{x^2}{9}, & 0 \le x \le 3 \\ 0, & Otherwise \end{cases}$$ then distribution function is-

(a) 1 (b) x^3 (c) $\dfrac{x^3}{27}$ (d) None of these

iv. If three coins are tossed simultaneously then probability of getting one or two heads is- **1**

(a) $\frac{3}{4}$ (b) $\frac{3}{6}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

v. If the moment generating function for binomial distribution is $\left(\frac{2}{5} + \frac{3}{5}e^t\right)^5$ than the variance is equals to- **1**

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{6}{5}$ (d) $\frac{6}{25}$

vi. If X is continuous random variable following normal distribution, then the odd moments about mean is equals to- **1**
(a) An odd number (b) Mean
(c) Zero (d) None of these

vii. Using the principle of least square method, from a set of given data points we can fit ___________ curve. **1**
(a) Linear (b) Non-linear
(c) Both (a) and (b) (d) None of these

viii. Arithmetic mean of coefficient of regression is ________ than-coefficient of correlation- **1**
(a) Greater (b) Less (c) Can't say (d) None of these

ix. Which one of the following test is used for comparison of two sample mean? **1**
(a) χ^2 test (b) F test (c) Z test (d) Student's t test.

x. A hypothesis which is tested for possible rejection under the assumption that it is true named as- **1**
(a) Composite hypothesis (b) Null hypothesis
(c) Both (a) and (b) (d) None of these

Q.2 Attempt any two:

i. Find the mean of the following frequency distribution- **5**

Class :	0-7	7-14	14-21	21-28	28-35	35-42	29-49
Frequency:	19	25	36	72	51	43	28

ii. Find the median for the following distribution- **5**

Wages in Rs. :	0-10	10-20	20-30	30-40	40-50
No. of workers:	22	38	46	35	20

iii. Find the standard deviation of the following series- **5**

Marks (Above):	0	10	20	30	40	50	60	70
No. of Students:	100	90	75	50	25	15	5	0

Q.3 i. Define discrete random variable, probability mass function and cumulative distribution function for it with example. **4**

ii. A random variable X has the following probability distribution Find out the expected value and variance of X. **6**

X	0	1	2	3	4	5	6	7
$P(X)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

OR iii. A continuous random variable X has a pdf **6**

$$f(x)=\begin{cases} ax & : 0\le x\le 1 \\ a & : 1\le x\le 2 \\ 3a-ax & : 2\le x\le 3 \\ 0 & : \text{otherwise} \end{cases}$$. Find

(a) The value of $'a'$ (b) $P(X\ge 1.5)$

Q.4 Attempt any two:

i. Derive the formula for mean and variance of binomial distribution. **5**

ii. If X is a Poisson variate with $P(X=1)=P(X=2)$ then find probability of at least $X=4$. **5**

iii. A sample of 100 dry battery cell tested to find the life length of life produced the following results **5**
Mean $\mu=12Hrs$, Standard deviation $\sigma=3Hrs$
Assuming the data to be normally distributed, what percentage of battery cell are expected to have life
(a) More than 15 Hrs
(b) Between 10 to 14 Hrs
Given $P(0<Z<1)=0.1587, P(0<Z<0.67)=0.2487$

Q.5 Attempt any two:

i. Fit a second-degree parabola to the following: **5**

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y:	1.1	1.3	1.6	2.0	2.7	3.4	4.1

ii. Find Karl Pearson's coefficient of correlation between x and y for the following data **5**

x:	6	2	4	9	1	3	5	8
y:	13	8	12	15	9	10	11	16

iii. If the regression equation of X on $Y: 5x-y=22$ and Y on $X: 64x-45y=24$ Find **5**
(a) Mean values for X and Y
(b) Regression coefficients
(c) Coefficient of correlation between X and Y

Dec 22

Solution Engineering maths - III
EC3BS011/EE3BS011/EX3BS01

Q.1 (i) d) None of these
(ii) b) Second Quartile Q_2
(iii) (c) $\frac{x^3}{27}$
(iv) a) $\frac{3}{4}$
(v) c) $\frac{6}{5}$

(vi) c) zero
(vii) c) Both (a) and (b)
(viii) a) Greater
(ix) d) Students t test
x) b) Null hypothesis

10

Q.2
(i)

Class	Mid x	Frequency f_x	$x f_x$
0-7	3.5	19	66.5
7-14	10.5	25	262.5
14-21	17.5	36	630
21-28	24.5	72	1764
28-35	31.5	51	1606.5
35-42	38.5	43	1655.5
42-49	45.5	28	1274
		$N = 274$	$\Sigma x f_x = 7259$

+3

$$\text{Mean} = \frac{1}{N} \Sigma x f_x$$ +1

$$= \frac{7259}{274} = \boxed{26.4}$$ +1

5

* Students can use short cut or step deviation method also

Q.2

(ii)

Wages in Rs	No of workers f	Cumulative frequency (CF)
0-10	22	22
10-20	38	60
20-30	46	106
30-40	35	141
40-50	20	161

(2)

$N = 161$, thus median is the measure of $\frac{1}{2}(N+1)$ ie. of 81^{th} term situated in class 20-30

$\Rightarrow$ Median class: 20-30 (+1)

$$\text{Median } M_d = l + \frac{\frac{1}{2}N - F}{f} \times h$$

$h = 10$

$l = 20$ (lower limit of M_d class)

$f = 46$ (frequence of Md class)

$F = 60$ (CF before Md)

$$Md = 20 + \frac{\left[\frac{1}{2}(161) - 60\right]}{46} \times h$$

$$= 20 + \frac{205}{46} = 20 + 4.456$$

$$= \boxed{24.46}$$ (+2)

$\boxed{\text{5 marks}}$

* Altemately

$$M_d = l_1 + \frac{m - F}{f}(l_2 - l_1)$$

$$= \boxed{24.57}$$

Q.2

(iii)

Class	mid value x	f_x	xf_x	$x-m$	$(x-m)^2$	$f_x(x-m)^2$
0-10	5	10	50	-26	676	6760
10-20	15	15	225	-16	256	3840
20-30	25	25	625	-6	36	900
30-40	35	25	875	4	16	400
40-50	45	10	450	14	196	1960
50-60	55	10	550	24	576	5760
60-70	65	5	325	34	1156	5780
Total		$N=100$	$\Sigma f_x x = 3100$			$\Sigma = 25400$

(3)

$$M = \frac{\Sigma x f_x}{N}$$

(+1)

$$= \frac{3100}{100} = 31$$

$$S.D = \sigma = \sqrt{\frac{\Sigma f_x (x-m)^2}{N}}$$

(+1)

$$= \sqrt{\frac{25400}{100}} = \boxed{15.94}$$

$\boxed{5 \text{ marks}}$

Alternate : with assumed mean A, $\xi = x - A$

$$M = A + \frac{\Sigma f \xi}{N}$$

$$S.D = \sigma \sqrt{\frac{\Sigma f \xi^2}{N} - \left(\frac{\Sigma f \xi}{N}\right)^2}$$

$$= 15.94$$

OR

Step deviation method

Ans 15.94

Q.3

(i) Each defination of Pmf, cdf, discrete RV +3

Examples +1

4

(ii)

X	0	1	2	3	4	5	6	7
P(X)	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

$E(X) = \sum x_i \, P(X=x_i)$

$= 0\times 0 + 1\times 0.1 + 2\times 0.2 + \cdots + 7\times 0.17$

$= \boxed{3.66}$ 2

$Var(X) = E(X^2) - [E(X)]^2$

$E(X^2) = \sum x_i^2 \, P(X=x_i)$ +2

$= 0^2\times 0 + 1^2\times 0.1 + 2^2 \times 0.2 + \cdots + 7^2 \times 0.17$

$= \boxed{16.8}$

$V(X) = \boxed{3.4044}$ +2

6

OR

(iii) As f(x) is pdf

(a) $\int_{-\infty}^{\infty} f(x)\, dx = 1$ 1

$\Rightarrow \int_0^1 ax\, dx + \int_1^2 a\, dx + \int_2^3 (3a-ax)\, dx = 1$ +1

$\Rightarrow a\left(\frac{x^2}{2}\right)_0^1 + a(x)_1^2 + \left(3ax - \frac{ax^2}{2}\right)_2^3 = 1$

$\Rightarrow \frac{a}{2} + a + 3a - a\left(\frac{9}{2} - 2\right) = 1$

$\Rightarrow \boxed{a = \frac{1}{2}}$ +1 b)

$P(X \geq 1.5)$

$$= \int_{1.5}^{\infty} f(x)\,dx \qquad +1$$

$$= \int_{1.5}^{2} a\,dx + \int_{2}^{3} (3a - ax)\,dx$$

$$= \frac{a}{2} + a\left[\left(3x - \frac{x^2}{2}\right)\right]_2^3$$

$$= \frac{a}{2} + \frac{a}{2} = \boxed{a = \frac{1}{2}} \qquad +1$$

6 marks

Q.4 (i) Mean of Binomial 2.5

Var of Binomial +2.5

5 marks

(ii) X : Poisson variates

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} \qquad \lambda : \text{parameter}$$

Given $P(X=1) = P(X=2)$

$$\Rightarrow e^{-\lambda}\lambda = \frac{e^{-\lambda}\lambda^2}{2}$$

$$\Rightarrow \lambda^2 - 2\lambda = 0$$

$$\Rightarrow \boxed{\lambda = 0, 2}$$

Probability of at least K success

$$= \sum_{r=k}^{\infty} P(X=r)$$

Probability of at least $X=4$

$$= \sum_{r=4}^{\infty} P(X=r)$$

Now as $\sum_{r=0}^{\infty} P(X=r) = 1$

$$\Rightarrow \sum_{r=4}^{\infty} P(X=r)$$

$$= 1 - \left[\sum_{r=0}^{3} P(X=r)\right.$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right]$$

Put $\lambda = 2$ as $\lambda = 0 \Rightarrow P(X=r) = 0 \ \forall r$

$$= 1 - e^{-2}\left[1 + 2 + 2 + \frac{8}{6}\right]$$

$$= 0.1428$$

Q.4 (ii) Standard Normal Variable

$$z = \frac{X-\mu}{\sigma}$$

Given $\mu = 12$ hrs

$\sigma = 3$ hrs

$$z = \frac{x-12}{3}$$

(a) $P(x > 15) = ?$

$x = 15 \Rightarrow z = \frac{15-12}{3} = 1$

$P(x > 15) = P(z > 1)$

$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.1587$

$= \boxed{0.3413}$

Out of 100 dry battery cell

≈ 34 have life length > 15

$\boxed{34\%}$

2.5

(b)

$P(10 < x < 14) = ?$

$x = 10 \Rightarrow z = \frac{10-12}{3}$

$= \cancel{-0.33}\ -0.67$

$x = 14 \Rightarrow z = \frac{14-12}{3}$

$= \cancel{0.33}\ 0.67$

$P(10 < x < 14)$

$= P(-0.67 < x < 0.67)$

$= 2 \times P(0 < z < 0.67)$

$= 2 \times 0.2485$

$= 0.4970 = \boxed{49.70\%}$

2.5

5 marks

Q 5

(i) Fit a second degree Parabola

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.3	1.6	2	2.7	3.4	4.1

to fit $y = a + bx + cx^2$

$n = 7$

Let $u = \frac{x - 2.5}{0.5}$ $v = y$

to fit

$v = a_1 + b_1 u + c_1 u^2$

N.E.

$\Sigma v = 7a_1 + b_1 \Sigma u + c_1 \Sigma u^2$

$\Sigma uv = a_1 \Sigma u + b_1 \Sigma u^2 + c_1 \Sigma u^3$

$\Sigma u^2 v = a \Sigma u^2 + b_1 \Sigma u^3 + c_1 \Sigma u^4$ +1

x	y	u	v	u^2	uv	u^2v	u^3	u^4
1	1.1	-3	1.1	9	-3.3	9.9	-27	81
1.5	1.3	-2	1.3	4	-2.6	5.2	-8	16
2	1.6	-1	1.6	1	-1.6	1.6	-1	1
2.5	2	0	2	0	0	0	0	0
3	2.7	1	2.7	1	2.7	2.7	1	1
3.5	3.4	2	3.4	4	6.8	13.6	8	16
4	4.1	3	4.1	9	12.3	36.9	27	81

Total $\Sigma u = 0$ $\Sigma v = 16.2$ $\Sigma u^2 = 28$ $\Sigma uv = 14.3$

$\Sigma u^2 v = 69.6$ $\Sigma u^3 = 0$

$\Sigma u^4 = 196$ +2

$\Rightarrow$ N.E for u & v

$7a_1 + 28c_1 = 16.2$

$28b_1 = 14.3 \Rightarrow b_1 = 0.51$

$28a_1 + 196c_1 = 69.9$

$\Rightarrow a_1 = 2.07$ $c_1 = 0.061$ +1

$v = 2.07 + 0.51u + 0.061u^2$

Substituting

$u = \frac{x-2.5}{0.5} = 2x-5$

$v = y$

we get

$\boxed{y = 1.04 - 0.193x + 0.243x^2}$ Ans +1

* Student may solve it directly

$\boxed{5 \text{ marks}}$

Q.5

(ii)

x	y	$X = x - m_x$	$Y = y - m_y$	X^2	Y^2	XY
6	13	1.25	1.25	1.5625	1.5625	1.5625
2	8	-2.75	-3.75	7.5625	10.5625	10.9375
4	12	-0.75	0.25	0.5625	0.0625	-0.1875
9	15	4.25	3.25	18.0625	10.5625	13.8125
1	9	-3.75	-2.75	14.0625	7.5625	10.3125
3	10	-1.75	-1.75	3.0625	3.8262	3.0625
5	11	0.25	-0.75	0.0625	0.5625	-0.1875
8	16	3.25	4.25	10.5625	18.0625	13.8125

$n = 8$

$m_x = \frac{\Sigma x}{8} = 4.75$

$m_y = \frac{\Sigma y}{8} = 11.75$

$\boxed{2.5}$

Total $\Sigma X^2 = 55.5$

$\Sigma Y^2 = 52$

$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \Sigma Y^2}}$

$\Sigma XY = 55.55$

$= 0.9759$

+2.5

$\boxed{5 \text{ marks}}$

(iii) Regression Equations

X on Y : $5x - y = 22$

Y on X : $64x - 45y = 24$

a) Mean values for X and Y is point of intersection of two regression lines

$5x - y = 22$

$64x - 45y = 24$

Solving we get

$$\boxed{\begin{aligned} m_x &= 6 \\ m_y &= 8 \end{aligned}}$$

2

b) $5x - y = 22$

$\Rightarrow x = \frac{1}{5}y + \frac{22}{5}$

$\Rightarrow \boxed{b_{xy} = \frac{1}{5}}$

$64x - 45y = 24$

$\Rightarrow y = \frac{64}{45}x - \frac{24}{45}$

$\boxed{b_{yx} = \frac{64}{45}}$

+2

c) Correlation Coefficient

$r^2 = b_{xy} \times b_{yx}$

$= \frac{1}{5} \times \frac{64}{45}$

$= \frac{64}{225}$

+1

$\boxed{r = \frac{8}{15}} = 0.533$ + as $b_{xy} > 0$ $b_{yx} > 0$

≈ 0.54

$\boxed{\text{5 marks}}$

Q.6
(i) dice is tossed 120 times

Number turned up	1	2	3	4	5	6	Total
Frequency	30	25	18	10	22	15	120

Null H_0 ⇒ The dice is unbiased

As it is Assumed that dice is unbiased
Expected frequency
$= 120 \times \frac{1}{6} = 20$ for each face
+1

Now
$$\chi^2 = \sum \left\{ \frac{(f_e - f_0)^2}{f_e} \right\}$$

x	f_0	f_e	$(f_0 - f_e)^2$
1	30	20	100
2	25	20	25
3	18	20	4
4	10	20	100
5	22	20	4
6	15	20	25

+2

$\boxed{\chi^2 = 12.90}$

dof : $\nu = 6 - 1 = 5$

Given tabulated χ^2 for 5% and 5 dof
$= 11.07$
Calculated $\chi^2 = 12.09 >$ tabulated χ^2
at 5% level of significance
Null hypothesis reject at 5% risk +3
⇒ Dice is biased.

[5 marks]

Q4

(ii) t - statistics

$$t = \left(\frac{\bar{x} - \mu}{S}\right)\sqrt{n}$$

$\bar{x}$: sample mean $= \frac{\Sigma x}{n}$

μ : mean of universe $= 0$ (Given)

$$S^2 = \frac{1}{n-1}\Sigma(x-\bar{x})^2$$ +1

$n = 8$ $\Sigma x = 2$ $\bar{x} = \frac{2}{8} = 0.25$

x	$x-\bar{x}$	$(x-\bar{x})^2$
-4	-4.25	18.0625
-2	-2.25	5.0625
-2	-2.25	5.0625
0	-0.25	0.0625
2	1.75	3.0625
2	1.75	3.0625
3	2.75	7.5625
3	2.75	7.5625

+2

$\Sigma(x-\bar{x})^2 = 49.500$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{49.5}{7}}$$

$= 2.659$

$$t = \left(\frac{0.25 - 0}{2.659}\right)\sqrt{8}$$

+2

$= 0.27$

(iii)- $n_1 = 9$ $\qquad$ $n_2 = 8$

$\nu_1 = 8$ $\qquad$ $\nu_2 = 7$

$\bar{x} = \frac{210}{9}$ $\qquad$ $\bar{y} = \frac{141}{8}$

$= 23.33$ $\qquad$ $= 17.625$ $\qquad$ +1

H_0 = Pop. Sample variance are same

ie $H_0 : \sigma_1^2 = \sigma_2^2$

I Sample			II Sample		
x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
17	-6.33	40.0689	16	-1.625	2.6406
27	3.67	13.4689	16	-1.625	2.6406
18	-5.33	28.4089	20	2.375	5.6406
25	1.67	2.7889	16	-1.625	2.6406
27	3.67	13.4689	20	2.375	5.6406
29	5.67	32.1489	17	-0.625	0.3906
27	3.67	13.4689	15	-2.625	6.8906
23	-0.33	0.1089	21	3.375	11.3906
17	-6.33	40.0689			

+2

Total $\Sigma(x-\bar{x})^2 = 184.0001$

$\Sigma(y-\bar{y})^2 = 37.8748$

$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1-1} = \frac{184.0001}{8} = 23$

$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2-1} = \frac{37.8748}{7} = 5.4107$

$$S_1^2 > S_2^2$$

$$Z = \frac{1}{2} \log_e \left(\frac{S_1^2}{S_2}\right)$$

$$= \frac{1}{2} \log_e (4.2508)$$

$$= \underline{0.7236}$$

Given

$$Z_{0.05,(8,7)} = 0.6575$$

Calculated Z value > tabulated Z value at 5% level of significance for 8 & 7 dof

Thus Null hypothesis is rejected at 5% l.o.s.

=) Two sample variances are not same.

+2

$\boxed{5 \text{ marks}}$