Q.6 Attempt any two:

i. A dice is tossed 120 times with the following results: **5** 04 02, 05 03

Number turned up	1	2	3	4	5	6
Frequency	30	25	18	10	22	15

Test the hypothesis that the dice is unbiased.
$(\chi^2_{0.05,5} = 11.07)$

ii. Two gauge operators are tested for precision in making measurements. One operator completes a set of 26 readings with standard deviation of 1.34 and the other does 34 reading with a standard deviation of 0.98. What is the level of significance of this difference? Given that for dof $v_1 = 25$ and $v_2 = 33$, the value of $z_{0.05} = 0.306$ and $z_{0.01} = 0.432$. **5** 04 02, 05 03

iii. The average number of articles produced by two machines per day are 200 and 250 with standard deviations 20 and 25 respectively on the basis of records of 25 days production. Can you regard both machine equally efficient at 1% level of significance? (Given that $t_{0.01,48} = 2.58$). **5** 04 02, 05 03

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

MEDI-CAPS UNIVERSITY
Knowledge is Power

Faculty of Engineering
End Sem Examination Dec 2024
EN3BS15 Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Marks BL PO CO PSO

Q.1 i. The averaging operator $\mu\{f(x)\}$ is equal to- **1** 01 01 01

(a) $f\left(x+\frac{h}{2}\right)+f\left(x-\frac{h}{2}\right)$

(b) $\frac{1}{2}\left[f\left(x+\frac{h}{2}\right)+f\left(x-\frac{h}{2}\right)\right]$

(c) $f\left(x+\frac{h}{2}\right)-f\left(x-\frac{h}{2}\right)$

(d) None of these

ii. Bessel's formula for equal interval is used when the value of u lies between- **1** 01 01 01

(a) $\frac{1}{4} < u < \frac{3}{4}$ (b) $0 < u < 1$

(c) $-1 < u < 0$ (d) None of these

iii. In Newton cote's quadrature formula, the Trapezoidal rule is obtained by putting $n =$ ___. **1** 01 01 01

(a) 1 (b) 2 (c) 3 (d) None of these

iv. Which of the following method is used to solve an ordinary differential equation? **1** 01 02 01

(a) Simpson's $\frac{3}{8}$ rule (b) Weddle's rule

(c) Picard's method (d) None of these

v. The area under normal curve is- **1** 01 01 01

(a) 0 (b) 1 (c) 2 (d) None of these

vi. Which of the following is a discrete distribution? **1** 01 01 01

(a) Binomial distribution

(b) Normal distribution

(c) Exponential distribution

(d) None of these

vii. If correlation coefficient $r = 1$, then correlation is- **1** 02 01 02
(a) Low degree correlation
(b) Perfect negative
(c) No correlation
(d) None of these

viii. The mean values of x and y for two regression lines $4x - 5y = -33$ and $20x - 9y = 107$ are- **1** 03 01 04
(a) (20, 13) (b) (13,17)
(c) (-13,-17) (d) None of these

ix. t- test is used when size of the sample is- **1** 01 01 02
(a) Less than or equal to 50
(b) Greater than or equal to 50
(c) Less than or equal to 30
(d) None of these

x. The value of F- statistic is always greater than- **1** 01 01 02
(a) -1 (b) 0 (c) 1 (d) None of these

Q.2 Attempt any two:

i. Use Stirling's formula to evaluate $f(1.22)$, given- **5** 05 01, 05 04

x	1.0	1.1	1.2	1.3
$f(x)$	8.403	8.781	9.129	9.451

ii. Find out the missing values from the following: **5** 03 01, 05 03

x	5	10	15	20	25	30
y	7	?	13	15	?	25

iii. Use Newton's divided difference formula, to find the value of $f(2)$ and $f(8)$ from the following table: **5** 03 01, 05 03

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Q.3 Attempt any two:

i. The differential equation $\frac{dy}{dx} = 1 + y^2$ satisfies the following sets of values of x and y: **5** 05 01, 02, 05 04

x	0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841

Compute $y(0.8)$ by using Milne's method.

ii. Apply Runge-Kutta method of fourth order to solve – **5** 03 01, 02, 05 04

$$10\frac{dy}{dx} = x^2 + y^2, y(0) = 1 \text{ for } x = 0.1$$

iii Use Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation- **5** 03 01, 02, 05 04

$$\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$$

Q.4 Attempt any two:

i. Derive the mean and variance for Exponential distribution. **5** 02 01 01

ii. Fit a Poisson distribution to the following data for calculating theoretical frequencies: **5** 03 01 02

x	0	1	2	3	4
f	122	60	15	2	1

iii. The mean yield per plot of a crop is 17 kg and standard deviation is 3 kg. If distribution of yield per plot is normal, find the percentage of plots giving yields: **5** 03 01, 02, 06 02
(a) Between 15.5 kg and 20 kg
(b) More than 20 kg.
(Given $P(0 < z < 0.5) = 0.1915$ and
$P(0 < z < 1) = 0.3413$)

Q.5 Attempt any two:

i. Fit a second-degree parabola to the following: **5** 03 01, 02, 05 02

x	0	1	2	3	4
y	-4	-1	4	11	20

ii. Calculate the Karl Pearson's correlation coefficient between x and y for the following data: **5** 03 01, 02, 05 02

x	150	153	154	155	157	160	163	164
y	65	66	67	70	68	53	70	63

iii. Write 5 properties of regression coefficient. **5** 02 01 02

Faculty of Engineering

Medi-Caps University

End Sem Examination, Dec-2024

Solution of EN3BS15 (23/12/2024)

Programme:- B.Tech

Q.1

(i)	(b)	$\frac{1}{2}\left[f\left(x+\frac{h}{2}\right)+f\left(x-\frac{h}{2}\right)\right]$	+1
(ii)	(a)	$\frac{1}{4} < u < \frac{3}{4}$	+1
(iii)	(a)	1	+1
(iv)	(c)	Picard's method	+1
(v)	(b)	1	+1
(vi)	(a)	Binomial Distribution	+1
(vii)	(d)	None of these	+1
(viii)	(b)	(13, 17)	+1
(ix)	(c)	Less than or equal to 30	+1
(x)	(c)	1	+1

Q.2 (i) Difference Table is

x	u	$y=f(x)$	$\delta f(x)$	$\delta^2 f(x)$	$\delta^3 f(x)$
1.0	−2	8.403			
			0.378		
1.1	−1	8.781		−0.030	
			0.348		0.004
1.2	0	9.129		−0.026	
			0.322		
1.3	1	9.451			

+2.5

Here $a = 1.2$, $h = 0.1$

$a + uh = 1.22$

$\Rightarrow 1.2 + u(0.1) = 1.22$

$\Rightarrow u = 0.2$ +1

By Stirling formula

$$y_u = y_0 + u\left[\frac{\Delta y_0 + \Delta y_{-1}}{2}\right] + \frac{u^2}{2!}\Delta^2 y_{-1} + \text{-----}$$ +1

$$= 9.129 + 0.2\left(\frac{0.348 + 0.322}{2}\right) + \frac{(0.2)^2}{2}(-0.026)$$

$$= 9.129 + 0.0670 - 0.00052$$

$$= 9.19548$$ +0.

Q.2 (ii) Since we are given four entries so y can be represented by the third degree polynomial.

Hence

$$\Delta^3 f(x) = \text{constant}$$

$$\Rightarrow \Delta^4 f(x) = 0 \text{ for all } x$$

$$\Rightarrow (E-I)^4 f(x) = 0$$

$$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) f(5) = 0$$

$$\Rightarrow f(25) - 4f(20) + 6f(15) - 4f(10) + f(5) = 0$$

$$\Rightarrow f(25) - 4 \times 15 + 6 \times 13 - 4f(10) + 7 = 0$$

$$\Rightarrow f(25) - 4f(10) = -25 \longrightarrow (1)$$ +2

Again,

$$(E-I)^4 f(10) = 0$$

$$\Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1) f(10) = 0$$

$$\Rightarrow f(30) - 4f(25) + 6f(20) - 4f(15) + f(10) = 0$$

$$\Rightarrow 25 - 4f(25) + 6 \times 15 - 4 \times 13 + f(10) = 0.$$

$$\Rightarrow -4f(25) + f(10) = -63 \longrightarrow (2)$$ +2

Solving eq^n (1) & (2), we get

$$f(10) = 10.87, \quad f(25) = 18.47$$ +1

Q.2 (iii)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
4	48				
		$\frac{100-48}{5-4}=52$			
5	100		$\frac{97-52}{7-4}=15$		
		$\frac{294-100}{7-5}=97$		$\frac{21-15}{10-4}=1$	
7	294		$\frac{202-97}{10-5}=21$		0
		202		1	
10	900		27		0
		310		1	
11	1210		33		
		409			
13	2028				

+3

By Newton's divided difference interpolation formula

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3)$$ +1

$$\Rightarrow f(x) = 48 + (x-4) \times 52 + (x-4)(x-5) \times 15 + (x-4)(x-5)(x-7) \times 1$$

$\therefore f(2) = 48 + (2-4)\times 52 + (2-4)(2-5)\times 15$

$+ (2-4)(2-5)(2-7)\times 1$

$\Rightarrow f(2) = 4$ +0.5

& $f(8) = 48 + (8-4)\times 52 + (8-4)(8-5)\times 15$

$+ (8-4)(8-5)(8-7)\times 1$

$\Rightarrow f(8) = 448$ +0.5

Q.3 (i) $\frac{dy}{dx} = 1 + y^2 = f(x, y)$

Here, $x_0 = 0$, $x_1 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$

$y_0 = 0$, $y_1 = 0.2027$, $y_2 = 0.4228$, $y_3 = 0.6841$
& $h = 0.2$ then $x_4 = x_3 + h = 0.8$

Since $y_0' = f(x_0, y_0) = 1 + y_0^2 = 1$

$y_1' = 1 + y_1^2 = 1 + (0.2027)^2 = 1.0411$

$y_2' = 1 + y_2^2 = 1 + (0.4228)^2 = 1.1787$

$y_3' = 1 + y_3^2 = 1 + (0.6841)^2 = 1.4681$ +1

Now using Milne Predictor formula

$$y_4 = y_0 + \frac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right) \quad +1$$

$$= 0 + \frac{4\times0.2}{3}\left(2\times1.0411 - 1.1787 + 2\times1.4681\right)$$

$$y_4 = 1.0239$$

Then $y_4' = f(x_4, y_4) = 1 + y_4^2$

$$= 1 + (1.0239)^2$$

$$= 2.0480 \quad +1$$

Using Milne corrector formula

$$y_4 = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right] \quad +1$$

$$= 0.4228 + \frac{(0.2)}{3}\left[1.1787 + 4\times1.4681 + 2.0480\right]$$

$$y_4 = 1.0294$$

Hence, $y_4 = y(x_4) = y(0.8) = 1.0294 \quad +1$

Q.3 (ii) Here $\frac{dy}{dx} = \frac{x^2+y^2}{10} = f(x, y)$

$x_0 = 0, \quad y_0 = 1 \quad \& \quad x = 0.1$

$$\therefore h = \frac{x - x_0}{n} = \frac{0.1 - 0}{1} = 0.1$$

[as taking $n=1$ i.e; one step]

Then, $x_1 = x_0 + h = 0.1$
we have

$$K_1 = hf(x_0, y_0)$$
$$= (0.1)\, f(0, 1)$$
$$= (0.1)\left(\frac{0^2 + 1^2}{10}\right) = 0.01$$ +1

$$K_2 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_1}{2}\right)$$
$$= (0.1)\, f\,(0.05,\ 1.005)$$
$$= (0.1)\left[\frac{(0.05)^2 + (1.005)^2}{10}\right]$$
$$= 0.01012525$$ +1

$$K_3 = hf\left(x_0 + \frac{h}{2},\ y_0 + \frac{K_2}{2}\right)$$
$$\cancel{= (0.1)\, f\, (0.10,\ 1.0102}$$
$$= (0.1)\, f\,(0.05,\ 1.0050626)$$
$$= (0.1)\left[\frac{(0.05)^2 + (1.0050626)^2}{10}\right]$$
$$= 0.010126508$$ +1

$$K_4 = hf(x_0 + h,\ y_0 + K_3)$$
$$= (0.1)\, f\,(0.10,\ 1.010126508)$$
$$= 0.010303555$$ +1

Hence,
$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$
$$= \frac{1}{6}(0.06080 7071)$$
$$= 0.0101345$$

$$\therefore \quad y_1 = y_0 + k$$

$$\Rightarrow y(0.1) = 1 + 0.0101345$$
$$= 1.0101345$$ +1

Q-3 (iii) Given
$$\frac{dy}{dx} = 2y + 3e^x$$
$$x_0 = 0, \quad y_0 = 0$$

Since, $y' = 2y + 3e^x \Rightarrow y_0' = 2y_0 + 3e^0 = 3$

$y'' = 2y' + 3e^x \Rightarrow y_0'' = 2y_0' + 3e^0$
$= 2 \times 3 + 3 = 9$

$y''' = 2y'' + 3e^x \Rightarrow y_0''' = 2y_0'' + 3e^0$
$= 2 \times 9 + 3 = 21$

$y^{iv} = 2y''' + 3e^x \Rightarrow y_0^{iv} = 2y_0''' + 3e^0$ +2
$= 2 \times 21 + 3$
$= 45$

& differentiating so on,

Using Taylor's Series:

$$y(x) = y_0 + (x-x_0)\, y_0' + \frac{(x-x_0)^2}{2!}\, y_0'' + \frac{(x-x_0)^3}{3!}\, y_0''' + \cdots$$ +1

$$\Rightarrow y(x) = 0 + x(3) + \frac{x^2}{2!}(9) + \frac{x^3}{3!}(21) + \frac{x^4}{4!}(45) + \cdots$$

$$\Rightarrow y(x) = 3x + \frac{9x^2}{2} + \frac{7}{2}x^3 + \frac{15}{8}x^4 + \cdots$$ +1

when $x = 0.2$

$$y(0.2) = 3(0.2) + \frac{9}{2}(0.2)^2 + \frac{7}{2}(0.2)^3 + \frac{15}{8}(0.2)^4 + \cdots$$

$$\Rightarrow y(0.2) = 0.8110$$ +1

Q.4 (i) The p.d.f. of exponential distribution is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0, \lambda > 0 \\ 0 & ; \text{otherwise} \end{cases} \rightarrow (1) \quad +1$$

To find Mean:-

we have Mean $= \mu_1' = E(x)$

$$= \int_{-\infty}^{\infty} x f(x)\, dx \quad \text{(By definition)}$$

$$= \int_{0}^{\infty} x \lambda e^{-\lambda x}\, dx \quad \text{(By eq}^n\text{ (1))}$$

$$= \lambda\left[\left(x \frac{e^{-\lambda x}}{-\lambda}\right)_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{-\lambda}\, dx\right]$$

$$= 0 + \int_0^{\infty} e^{-\lambda x}\, dx$$

$$= \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^{\infty}$$

$$= -\frac{1}{\lambda}(0-1) = \frac{1}{\lambda}$$

$$\therefore \text{Mean} = \mu_1' = \frac{1}{\lambda} \longrightarrow (2) \qquad +1.5$$

To find variance:-

The second moment about the origin

$$\mu_2' = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$$

$$\Rightarrow \mu_2' = \int_0^\infty x^2 . \lambda e^{-\lambda x} dx \quad [\text{By eq}^n \text{ 1}]$$

$$= \lambda \int_0^\infty x^2 e^{-\lambda x} dx$$

$$= \lambda \left[\left(x^2 \frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty - \int_0^\infty 2x . \frac{e^{-\lambda x}}{-\lambda} dx \right]$$

$$= 0 + 2 \int_0^\infty x e^{-\lambda x} dx$$

$$= 2 \left[\left(x \frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty - \int_0^\infty 1 . \frac{e^{-\lambda x}}{-\lambda} dx \right]$$

$$= \frac{-2}{\lambda} \left[0 - \left(\frac{e^{-\lambda x}}{-\lambda} \right)_0^\infty \right]$$

$$= \frac{-2}{\lambda^2} (0 - 1)$$

$$\mu_2' = \frac{2}{\lambda^2} \longrightarrow 3$$ +1.5

we have,

$$\text{Variance} = \mu_2'^2 - (\mu_1')^2$$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda} \right)^2 \quad [\text{By 2 \& 3}]$$

$$\therefore \text{Variance} = \frac{1}{\lambda^2}$$ +1

Q.4 (ii):-

x	f	fx
0	122	0
1	60	60
2	15	30
3	2	6
4	1	4
Total	200	100

+1

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{100}{200} = \frac{1}{2}$$

Poisson Distribution is

$$P(r) = \frac{e^{-m} \cdot m^{r}}{r!}$$

+1

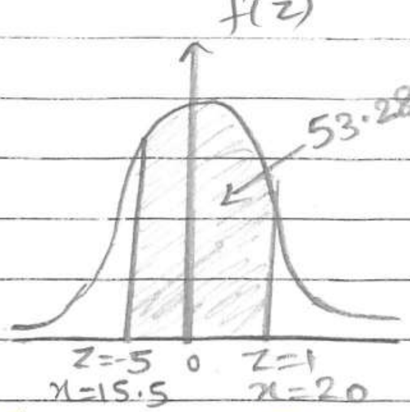
r	$P(r) = \frac{e^{-1/2}(1/2)^{r}}{r!}$	Theoretical Frequency	Given Frequency	
(1) 0	$P(0) = \frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^{0}}{0!} = 0.6065$	$0.6065 \times 200 = 121.3$	121	+1
(2) 1	$P(1) = \frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^{1}}{1!} = 0.3033$	$0.3033 \times 200 = 60.7$	61	
(3) 2	$P(2) = \frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^{2}}{2!} = 0.0758$	$0.0758 \times 200 = 15.2$	15	+1
(4) 3	$P(3) = \frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^{3}}{3!} = 0.0126$	$0.0126 \times 200 = 2.5$	2	
(5) 4	$P(4) = \frac{e^{-\frac{1}{2}}\left(\frac{1}{2}\right)^{4}}{4!} = 0.0016$	$0.0016 \times 200 = 0.32$	1	+1

Q.4 (iii) Mean $= \mu = 17$ kg

S.D. $= \sigma = 3$ kg

Standard Normal Variable

$$Z = \frac{x-\mu}{\sigma}$$



+1

(a) When $x_1 = 15.5$

$$Z_1 = \frac{x_1-\mu}{\sigma} = \frac{15.5-17}{3} = -0.5$$

When $x_2 = 20$,

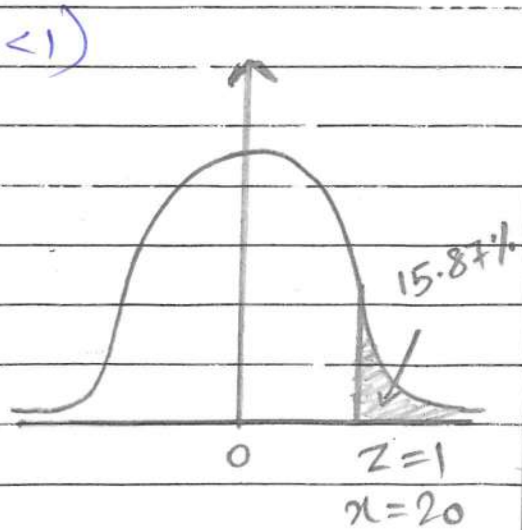
$$Z_2 = \frac{x_2-\mu}{\sigma} = \frac{20-17}{3} = 1$$ +1

$$\therefore P(15.5 < x < 20) = P(-0.5 < Z < 1)$$
$$= P(0 < Z < +0.5) + P(0 < Z < 1)$$
$$= 0.1915 + 0.3413$$
$$= 0.5328$$ +1

$\therefore$ Required percentage of plots $= 53.28\%$

(b) When $x = 20$, $Z = \frac{20-17}{3} = 1$ +1

$$P(x > 20) = P(Z > 1)$$
$$= 0.5 - P(0 < Z < 1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$



$\therefore$ Required percentage of plots $= 15.87\%$ +1

Q.5 (i) Let the equation of the parabola be

$y = a + bx + cx^2 \longrightarrow (1)$ +0.5

x	y	xy	x^2	x^2y	x^3	x^4
0	-4	0	0	0	0	0
1	-1	-1	1	-1	1	1
2	4	8	4	16	8	16
3	11	33	9	99	27	81
4	20	80	16	320	64	256
$\Sigma x=10$	$\Sigma y=30$	$\Sigma xy=120$	$\Sigma x^2=30$	$\Sigma x^2y=434$	$\Sigma x^3=100$	$\Sigma x^4=354$

+1.5

Normal equations are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$
$$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$ +1

Putting the values

$$\left.\begin{aligned} 30 &= 5a + 10b + 30c \\ 120 &= 10a + 30b + 100c \\ 434 &= 30a + 100b + 354c \end{aligned}\right\} \rightarrow (2)$$

On solving above equations we get

$a = -4, \ b = 2, \ c = 1$ +1

$\therefore$ Required eq^n of second degree parabola is

$y = -4 + 2x + x^2$ [By eq^n (1)] +1

Q.5 (ii) Let $u_i = x_i - 155$, $v_i = y_i - 68$

x_i	y_i	$u_i = x_i - 155$	$v_i = y_i - 68$	$u_i v_i$	u_i^2	v_i^2
150	65	−5	−3	15	25	9
153	66	−2	−2	4	4	4
154	67	−1	−1	1	1	1
155	70	0	2	0	0	4
157	68	2	0	0	4	0
160	53	5	−15	−75	25	225
163	70	8	2	16	64	4
164	63	9	−5	−45	81	25
		$\Sigma u_i = 16$	$\Sigma v_i = -22$	−84	204	272

+2

Here $n = 8$

we have

$$r(x,y) = \frac{n\Sigma u_i v_i - (\Sigma u_i)(\Sigma v_i)}{\sqrt{n\Sigma u_i^2 - (\Sigma u_i)^2} \times \sqrt{n\Sigma v_i^2 - (\Sigma v_i)^2}}$$ +1

$$= \frac{8(-84) - (16)(-22)}{\sqrt{8(204) - (16)^2} \times \sqrt{8(272) - (-22)^2}}$$

$$= \frac{-320}{\sqrt{1376}\sqrt{1692}}$$ +1

$$= \frac{-320}{1525.84}$$

$r(x,y) = -0.2097$ +1

Q-5 (iii) 5 Properties of Regression coefficient are:

(1) The coefficient of correlation is the geometric mean of the coefficients of regression +1

i.e; $r = \pm \sqrt{b_{xy} \times b_{yx}}$

(2) If one of the regression coefficients is greater than unity, then the other is less than unity. +1

i.e; if $b_{yx} > 1 \Rightarrow b_{xy} < 1$

Or if $b_{yx} < 1 \Rightarrow b_{xy} > 1$

(3) Both regression coefficients and correlation coefficient are of the same sign +1

i.e; $r > 0$ if b_{xy} & $b_{yx} > 0$

$r < 0$ if b_{xy} & $b_{yx} < 0$

(4) Regression coefficients are independent of change of origin but not of scale. +1

(5) Arithmetic mean of the regression coefficient is greater than the correlation coefficient. +1

Q. 6 (i):- Step 1:- Null Hypothesis H_0 : The dice is unbiased one. +1

Step 2:- Calculation of Expected Frequency (f_e)

Here $N = 120$

$\therefore$ Expected Frequency

$$f_e(x) = N \cdot p(x)$$

$$= 120 \times \frac{1}{6} \quad \left[\because p(x) = \frac{1}{6} \text{ for } x = 1,2,3,4,5,6\right]$$

$$= 20$$ +1

Step 3:- Calculation of χ^2-statistic:

$$\chi^2 = \Sigma \left\{ \frac{(f_o - f_e)^2}{f_e} \right\}$$

x	f_o	f_e	$(f_o-f_e)^2$	$\frac{(f_o-f_e)^2}{f_e}$
1	30	20	100	5
2	25	20	25	1.25
3	18	20	4	0.20
4	10	20	100	5
5	22	20	4	0.20
6	15	20	25	1.25
Total	$N = 120$	$N = 120$	—	$\chi^2 = 12.90$

+2

Also degree of freedom $\nu = 6 - 1 = 5$.

Step 4:- The tabulated value of χ^2 at 5% level of significance and for dof $v = 5$ is 11.07 ie; $\chi^2_{0.05,5} = 11.07$

Step 5:- Decision: clearly calculated value of $\chi^2 = 12.90$ & tabulated value of $\chi^2_{0.05,5} = 11.07$

$\Rightarrow$ the Null Hypothesis is rejected
$\Rightarrow$ the dice is a biased one. +1

Q.6 (ii) Given $n_1 = 26$, $n_2 = 34$
S.D. $\sigma_1 = 1.34$, $\sigma_2 = 0.98$

Since $\sigma_1^2 = \frac{\sum(x-\bar{x})^2}{n_1}$ & $\sigma_2^2 = \frac{\sum(y-\bar{y})^2}{n_2}$

$\Rightarrow (1.34)^2 = \frac{\sum(x-\bar{x})^2}{26}$ & $(0.98)^2 = \frac{\sum(y-\bar{y})^2}{34}$

$\Rightarrow \sum(x-\bar{x})^2 = 46.68$ & $\sum(y-\bar{y})^2 = 32.6536$ +1

Step I:- Null Hypothesis H_0: The difference between variances is not significant ie; $\sigma_1^2 = \sigma_2^2$ +1

Step 2:- Calculation of Z-statistic

$$S_1^2 = \frac{\sum(x-\bar{x})^2}{n_1 - 1} = \frac{46.68}{25} = 1.8674$$

& $$S_2^2 = \frac{\sum(y-\bar{y})^2}{n_2 - 1} = \frac{32.6536}{33} = 0.9895$$

Clearly, $S_1^2 > S_2^2$

$$\therefore Z = \frac{1}{2}\log_e\left(\frac{S_1^2}{S_2^2}\right)$$

$$= \frac{1}{2}\log_e\left(\frac{1.8674}{0.9895}\right)$$

$$= \frac{1}{2}\log_e(1.8770)$$

$$Z = 0.3148$$ +2

Step 3:- The tabulated value of z at 5% & 1% level of significance and for d of 25 and 33 are $Z_{0.05} = 0.306$ & $Z_{0.01} = 0.432$

Step 4:- Decision:

(i) Since calculated value of $Z = 0.3148 >$ tabulated value of $Z_{0.05} = 0.306$.
$\Rightarrow$ the null hypothesis H_0 is rejected
$\Rightarrow$ the difference between variances is significant at 5% level of significance.

(ii) Since calculated value of $Z = 0.3148 <$ tabulated value of $Z_{0.01} = 0.432$

$\Rightarrow$ the null hypothesis H_0 is accepted

$\Rightarrow$ the difference between variances is not significant at 1% level of significance. +1

Q.6 (iii):- Given $n_1 = 25$, $\overline{x_1} = 200$, S.D. $\sigma_1 = 20$

$n_2 = 25$, $\overline{x_2} = 250$, S.D. $\sigma_2 = 25$

$$S^2 = \frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2 - 2}$$
$$= \frac{25 \times (20)^2 + 25 \times (25)^2}{25 + 25 - 2}$$
$$= 533.85$$
$$\therefore S = \sqrt{533.85} = 23.1$$

Degrees of freedom $v = n_1 + n_2 - 2 = 48$ +1.5

Step 1:- Null Hypothesis H_0: both the machines are equally efficient +1
i.e; $\mu_1 = \mu_2$

Step 2:- Calculation of t-statistic:

$$t = \frac{(\overline{x_1} - \overline{x_2})}{S} \sqrt{\frac{n_1 n_2}{n_1 + n_2}}$$

$$= \frac{-50}{23.1} \times \sqrt{\frac{25 \times 25}{25 + 25}}$$
$$= -7.65$$

$$\Rightarrow |t| = 7.65$$ +1.5

Step 3:- The tabulated value of t at 1% level of significance and dof $\nu = 48$ is 2.58 i.e; $t_{0.01, 48} = 2.58$

Step 4:- Decision: Since ~~tabulated~~ calculated value of $|t| = 7.65 \nless$ tabulated value of $t_{0.01, 48} = 2.58$

$\Rightarrow$ the null hypothesis is rejected.

$\Rightarrow$ the two machines are not equally efficient at 1% level of significance. +1