Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

EN2BS01 Mathematics-I

Programme: Diploma Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If $A=\{1,2,3\}$ and $B=\{3,4,5\}$, then the value of $A\cap B$ is **1**

respectively

(a) $\{3\}, \{1,2\}$ (b) $\{1,2\}, \{3\}$

(c) $\{3\}, \{4,5\}$ (d) $\{4,5\}, \{1,2\}$

ii. If $f(x)=x^2-3x+5$, then the value of $f(-4)$ is **1**

(a) 9 (b) 1 (c) 33 (d) 23

iii. The value of ${}^5P_0$ is **1**

(a) 5 (b) 0 (c) 6 (d) 1

iv. The coefficient of xy in the expansion of $(x+y)^2$ is **1**

(a) 1 (b) 2 (c) -2 (d) None of these

v. What is the conjugate of $z=2+3i$ **1**

(a) $\bar{z}=2+3i$ (b) $\bar{z}=\sqrt{13}$ (c) $\bar{z}=-2-3i$ (d) $\bar{z}=2-3i$

vi. The series $2,4,6,\cdots$ is in **1**

(a) H.P. (b) G.P. (c) A.P. (d) None of these

vii. Transpose of a column matrix is **1**

(a) Column matrix (b) Zero matrix

(c) Diagonal matrix (d) Row matrix

viii. If $|A| = 0$, then A is **1**

(a) 0 (b) Singular matrix

(c) Non-singular matrix (d) Null matrix

P.T.O.

ix. Equation of straight line with slope -8 and y intercept (0,16) is written as **1**

(a) $-8x = 16y$ (b) $y = 16x + (-8)$

(c) $y = -8x + 16$ (d) None of these

x. If two straight lines have same slope then these two lines are classified as **1**

(a) Parallel lines (b) Perpendicular lines

(c) Horizontal lines (d) Vertical lines

Q.2 i. How many degrees are contained in the arc of $35\pi/12$ radians? **2**

ii. Find the domain and range of the $f(x) = 1/x - 2$. **3**

iii. In a recent survey of 400 students in a school, 100 were listed as smokers and 150 as chewers of gum; 75 were listed as both smokers and gum chewers. How many students are neither smokers nor gum chewers? How many students are smokers but not chewers? How many students are chewers but not smokers?. **5**

OR iv. Evaluate $\tan\left(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{5}{13}\right)$. **5**

Q.3 i. Write down formulas for the number of permutations and combinations. **2**

ii. In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together? **3**

iii. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there? **5**

OR iv. If the coefficient of $(2r + 4)^{th}$ and $(r - 2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal then find the value of r. **5**

Q.4 Attempt any two

i. Express $z = 1 + i\sqrt{3}$ in polar form **5**

ii. If 10^{th} term of an A. P. is – 15 and 31^{st} term is –57, then find the 15^{th} term. **5**

iii. Insert three geometric means between 1 and 256. **5**

Q. 5 Attempt any two.

i. Find the value of B, if it is given that $A + B = A^2$ where **5**

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

ii. Find the inverse of the matrix **5**

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

iii. Solve the following system of equations by using Cramer's rule: **5**

$x + y + z = 16$

$x - y + z = 2$

$2x + y - z = 1$

Q 6 i. Find the distance between two points (5, 3) and (7, 4) **3**

ii. Find the distance from the point $(-3, 7)$ to the line $y = \frac{6}{5}x + 2$. **7**

OR iii. Find the acute angle between the lines $7x - 4y = 0$ and $3x - 11y + 5 = 0$. **7**

******

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

EN2BS01 Mathematics-I

SOLUTION

Programme: Diploma | Branch/Specialisation: All

Q.1	i.	A {3}	1
	ii.	C 33	1
	iii.	D 1	1
	iv.	B 2	1
	v.	A 2+3i	1
	vi.	C A.P.	1
	vii.	D Row matrix	1
	viii.	B Singular matrix	1
	ix.	C $y=-8x+16$	1
	x.	A Parallel lines	1
Q.2	i.	π Radians = 180°	+(1)
		$35\pi/12$ radians $=\frac{35}{12}\times 180^\circ = 525^\circ$	+(1)
	ii.	$f(x) = 1/x-2$	
		Domain D = the set of all real numbers other than 2	+(1.5)
		Range = The set of all real numbers	+(1.5)
	iii.	The number of students are neither smokers nor gum chewers = 325	+(3)
		The number of students are smokers but not chewers = 25.	
		The number of students are chewers but not smokers = 75.	+(1)

			+(1)
OR	iv.	To find $\tan\left(\arcsin\frac{3}{5}+\arccos\frac{5}{13}\right)$	
		Let	
		$x=\arcsin\frac{3}{5}, y=\arccos\frac{5}{13}$	+(1)
		$\sin x=\frac{3}{5}, \cos y=\frac{5}{13}$	+(1)
		$\cos x=\sqrt{1-\sin^2 x}=\frac{4}{5}, \sin y=\frac{12}{13}$	+(1)
		$\tan x=\frac{3}{4}, \tan y=\frac{12}{5}$	+(1)
		so, $\tan\left(\arcsin\frac{3}{5}+\arccos\frac{5}{13}\right)=\tan(x+y)$	
		$=\frac{\tan x+\tan y}{1+\tan x\tan y}=\frac{63}{56}$	+(1)
Q.3	i.	The number of Permutation $= {}^nP_r = n!/(n-r)!$	+(1)
		The num. of Combination $= {}^nC_r = n!/r!(n-r)!$	+(1)
	ii.	The word 'LEADING' has 7 different letters.	
		When the vowels EAI are always together, they can be supposed to form one letter.	
		Then, we have to arrange the letters LNDG (EAI).	+(1)
		Now, 5 (4 + 1 = 5) letters can be arranged in 5! = 120 ways.	
		The vowels (EAI) can be arranged among themselves in 3! = 6 ways.	+(1)
		∴ Required number of ways = (120 x 6) = 720.	+(1)

	iii.	In a group of 6 boys and 4 girls, four children are to be selected such that at least one boy should be there. Hence we have 4 options as given below We can select 4 boys ...(option 1) Number of ways to this = ${}^6C_4$ We can select 3 boys and 1 girl ...(option 2) Number of ways to this = ${}^6C_3 \times {}^4C_1$ We can select 2 boys and 2 girls ...(option 3) Number of ways to this = ${}^6C_2 \times {}^4C_2$ We can select 1 boy and 3 girls ...(option 4) Number of ways to this = ${}^6C_1 \times {}^4C_3$ Total number of ways $= {}^6C_4 + {}^6C_3 \times {}^4C_1 + {}^6C_2 \times {}^4C_2 + {}^6C_1 \times {}^4C_3$ $= {}^6C_2 + {}^6C_3 \times {}^4C_1 + {}^6C_2 \times {}^4C_2 + {}^6C_1 \times {}^4C_1 [\because {}^nC_r = {}^nC_{(n-r)}]$ $=6\times52\times1+6\times5\times43\times2\times1\times4$ $+6\times52\times1\times4\times32\times1+6\times4$ $=15+80+90+24=209$	+(1) +(1) +(1) +(1)
OR	iv.	The general term of $(1+x)^n$ is $T_{r+1} = C_r x^r$ Hence coefficient of $(2r+4)^{th}$ term will be $T_{2r+4} = T_{2r+3+1} = {}^{18}C_{2r+3}$ and coefficient or $(r-2)^{th}$ term will be $T_{r-2} = T_{r-3+1} = {}^{18}C_{r-3}$. $\Rightarrow {}^{18}C_{2r+3} = {}^{18}C_{r-3}$. $\Rightarrow (2r+3)+(r-3) = 18$ ($\because {}^nC_r = {}^nC_K \Rightarrow r = k$ or $r+k=n$) $\therefore \quad r = 6$	+(1) +(1) +(1) +(2)

Q.4	i.	Given $z = 1 + i\sqrt{3}$ Put $1 = r\cos\theta, \sqrt{3} = r\sin\theta$ $r = 2, \theta = \frac{\pi}{3}$ $z = 1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$	+(2) +(3)
	ii.	Let a be the first term and d be the common difference of the A. P. Then from the formula: $t_n = a + (n-1)d$, we have $t10 = a + (10-1)d$ $= a + 9d$ $t31 = a + (31-1)d$ $= a + 30d$ We have, $a + 9d = -15$ ...(1) $a + 30d = -57$ ...(2) Solve equations (1) and (2) to get the values of a and d. Subtracting (1) from (2), we have $21d = -57 + 15 = -4211$ $\therefore d = -42/21 = -2$ Again from (1), $a = -15 - 9d$ $= -15 - 9(-2) = -15 + 18 = 3$ Now $t15 = a + (15-1)d$ $= 3 + 14(-2) = -25$	+(1) +(1) +(2) +(1)
	iii.	Let G_1, G_2, G_3, be the three geometric means between 1 and 256. Then $1, G_1, G_2, G_3, 256$ are in G. P. If r be the common ratio, then $T_5 = 256$ i.e, $ar^4 = 256$	+(1)

		$\Rightarrow 1.r^3 = 256$ or, $r^2 = 16$ or, $r = \pm 4$ When $r = 4$, $G_1 = 1.4 = 4$, $G_2 = 1.(4)^2 = 16$ and $G_3 = 1.(4)^3 = 64$ When $r = -4$, $G_1 = -4$, $G_2 = (1)(-4)^2 = 16$ and $G_3 = (1)(-4)^3 = -64$ ∴ G.M. between 1 and 256 are 4, 16, 64, or, −4, 16, −64.	+(2) +(1) +(1)
Q.5	i.	Given that $A + B = A^2$ and $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ To find the value of B, $A^2 = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ $A^2 = \begin{bmatrix} 13 & 12 \\ 9 & 14 \end{bmatrix}$ Now $A + B = A^2$ $B = A^2 - A$ $B = A^2 - A = \begin{bmatrix} 13 & 12 \\ 9 & 14 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ $= \begin{bmatrix} 12 & 8 \\ 6 & 12 \end{bmatrix}$	+(3) +(2)
	ii.	$\lvert A \rvert = 1(1-4) - 2(2-4) + 2(4-2) = 5$ Now $A_{11} = -3$, $A_{12} = 2$, $A_{13} = 2$. $A_{21} = 2$, $A_{22} = -3$, $A_{23} = 2$	+(1)

		$A_{31} = 2$, $A_{32} = 2$, $A_{33} = -3$ So $\text{Adj}\,A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ Now $A^{-1} = \frac{\text{Adj}\,A}{\lvert A \rvert} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$	+(2) +(1) +(1)
	iii.	Here $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix} = 6$ $D_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = 6$, $D_y = \begin{vmatrix} 1 & 1 & 6 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 42$ $D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & -1 & 2 \\ 2 & 1 & 1 \end{vmatrix} = 48$ Solution is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$ $x = 1$, $y = 7$, $z = 8$	+(1) +(2) +(1) +(1)
Q 6	i.	$P(x_1, y_1) = (5, 3)$, $Q(x_2, y_2) = (7, 4)$ $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(7-5)^2 + (4-3)^2} = \sqrt{4+1}$ $= \sqrt{5}$	+(1) +(1)

	ii.	First we write $y=\frac{6}{5}x+2$ $6x-5y+2=0$ The distance from (m, n) to the line $Ax+By+C=0$ is given by $\frac{\lvert Am+Bn+C\rvert}{\sqrt{A^2+B^2}}$ Thus the distance from the point (-3, 7) to the line $=5.506$	+(2) +(3) +(2)
OR	iii.	First we need to find the slope of both the lines. $7x - 4y = 0$ $\Rightarrow y = 7x/4$ Therefore, the slope of the line $7x - 4y = 0$ is 7/4 Again, $3x - 11y + 5 = 0$ $\Rightarrow y = 3x/11 + 5/11$ Therefore, the slope of the line $3x - 11y + 5 = 0$ is $= 3/11$ Now, let the angle between the given lines $7x - 4y = 0$ and $3x - 11y + 5 = 0$ is θ Now, $\tan\theta = \lvert (m2-m1)/(1+m1m2)\rvert = \pm 1$ Since θ is acute, hence we take, $\tan\theta = 1 = \tan 45°$ Therefore, $\theta = 45°$ Therefore, the required acute angle between the given lines is 45°	+(1) +(1) +(1) +(1) +(1)

******