Q.No.		Question	Marks	BL	PO	CO	PSO
Q.6		Attempt any two:					
	i.	Show that the function $u = 3x^2y - y^3$ is harmonic and find the corresponding analytic function $f(z)$.	**5**	03	01 02 12	02	-
	ii.	Evaluate $\int_0^{2+i} \bar{z}^2\, dz$ along the real axis to 2 and then vertically to $2+i$.	**5**	04	01 02 12	04	-
	iii.	Evaluate using Cauchy's Integral formula $\int_C \frac{e^{2z}}{(z-1)(z-2)}\, dz$, if C is the circle $\lvert z\rvert = 3$.	**5**	04	01 02 12	04	-

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No....................................

Faculty of Engineering/Science
End Sem Examination Dec 2024
EN3BS11 / BC3BS01 Engineering Mathematics -I

Programme: B.Tech./ B.Sc. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.No.		Question	Marks	BL	PO	CO	PSO
Q.1	i.	A system of homogeneous equations in *n* unknowns has infinite many non-trivial solution if the rank of coefficient matrix A is- (a) Less than *n* (b) Greater than *n* (c) Equal to *n* (d) None of these	**1**	02	01	01	-
	ii.	A square matrix A is singular if $\lvert A\rvert$ equals to- (a) 0 (b) 1 (c) -1 (d) None of these	**1**	02	01	01	-
	iii.	If $f(x)$ is a function of the variable x. Then which of the following is not a necessary condition for Lagrange's mean value theorem- (a) $f(x)$ is continuous in $[a, b]$ (b) $f(x)$ is differentiable in (a, b) (c) $f(a) = f(b)$ (d) None of these	**1**	02	01	01	-
	iv.	If $f(x, y) = x^3 - 4xy + 2y^2$ then the value of f_{xy} is- (a) 1 (b) -1 (c) 4 (d) -4	**1**	02	01	01	-
	v.	The value of the integral $\int_0^1 \frac{1}{1+x^2}\, dx$ is- (a) 0 (b) $\pi/4$ (c) $\frac{\pi}{2}$ (d) π	**1**	02	01	01	-
	vi.	The value of $B(1,4) - B(4,1)$ is- (a) 1 (b) 4 (c) 0 (d) -1	**1**	02	01	01	-

vii. If roots of the differential equation $(D^2-4)y=x^2$ are 2 and -2 then Complementary function (C.F.) is given by- **1** 02 01 01 -

(a) $c_1e^{2x}+c_2e^{-2x}$ (b) $(c_1+c_2x)e^{-2x}$

(c) $(c_1+c_2x)e^{2x}$ (d) None of these

viii. The order and degree of the differential equation $(\frac{d^3y}{dx^3})^2+2\frac{d^2y}{dx^2}+8\frac{dy}{dx}+4y=x^2$ is given by- **1** 02 01 01 -

(a) 2,3 (b) 2,2 (c) 3,3 (d) 3,2

ix. Cauchy Riemann equations for the function $w=f(z)=u(x,y)+iv(x,y)$ is- **1** 02 01 01 -

(a) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=\frac{\partial v}{\partial x}$

(b) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$

(c) $\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\ \frac{\partial u}{\partial y}=-x\frac{\partial v}{\partial x}$

(d) None of these

x. A single valued function $f(z)$ which is defined and possesses a unique derivative with respect to z at each point of a domain D is known as- **1** 02 01 01 -

(a) Harmonic function

(b) Conjugate function

(c) Analytic function

(d) None of these

Q.2 i. Define rank and nullity of a matrix. **2** 02 01 01 -

ii. Find the normal form of the matrix and hence find its rank- **3** 03 01 02 12 02 -

$$A=\begin{bmatrix}8 & 1 & 3 & 6\\ 0 & 3 & 2 & 2\\ -8 & -1 & -3 & 4\end{bmatrix}$$

iii. Examine the following equations for consistency and if consistent then find the complete solution- **5** 03 01 02 12 02 -

$$x+2y-z=3$$
$$3x-y+2z=1$$
$$2x-2y+3z=2$$
$$x-y+z=-1$$

OR iv. Find the eigen values and eigen vectors of the matrix- **5** 03 01 02 12 02 -

$$A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$$

Q.3 Attempt any two:

i. Find the maxima or minima of the function- **5** 03 01 02 12 02 -

$$u=x^3-y^2-7x^2+4y+15x-13$$

ii. Expand $tan^{-1}x$ in ascending powers of x by Maclaurin's theorem. **5** 03 01 02 12 02 -

iii. If $x^xy^yz^z=c$ then show that- **5** 03 01 02 12 02 -

$$\frac{\partial^2 z}{\partial x\partial y}=-(xlog\ ex)^{-1}$$

when $x=y=z$.

Q.4 Attempt any two:

i. Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of beta function and hence, evaluate- **5** 03 01 02 12 02 -

$$\int_0^1 x^5(1-x^3)^{10}dx$$

ii. Find the area enclosed by the parabolas- **5** 03 01 02 12 02 -

$$y^2=4ax \text{ and } x^2=4ay$$

iii. Evaluate $\lim_{n\to\infty}[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+.....+\frac{1}{2n}]$. **5** 03 01 02 12 02 -

Q.5 i. Solve the following linear differential equation- **4** 03 01 02 12 02 -

$$(1+y^2)dx=(tan^{-1}y-x)dy$$

Where, $D=\frac{d}{dx}$.

ii. Solve the following differential equation- **6** 04 01 02 12 03 -

$$(D^2-4D+4)y=e^x+cos\ 2x$$

Where, $D=\frac{d}{dx}$.

OR iii. Solve the following differential equation- **6** 04 01 02 12 03 -

$$(x^3D^3+2x^2D^2+2)y=10(x+\frac{1}{x})$$

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Solution of EN3BS11 (4/Dec./2024)

Q. 1

(i) a) less than n +1

(ii) a) 0 +1

(iii) c) $f(a) = f(b)$ +1

(iv) d) -4 +1

(v) b) $\pi/4$ +1

(vi) c) 0 +1

(vii) a) $c_1 e^{2x} + c_2 e^{-2x}$ +1

(viii) d) 3, 2 +1

(ix) b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ +1

(x) c) Analytic function +1

Q. 2 (i) Rank : Let 'A' be any matrix. A number r is called the rank of the matrix 'A'. if it obeys the following properties

(i) there is atleast one minor of A of order r which does not vanish +1

(ii) every minor of A of order higher than r vanishes

Nullity : Let A be a square matrix of order n and if the rank of A is r, then $(n-r)$ is called the nullity of the matrix A and is usually denoted by $N(A)$ +1

i.e. $N(A) = n-r$

Q. 2 (ii) Normal form

$$A = \begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

$C_2 \leftrightarrow C_1$

$$\approx \begin{bmatrix} 1 & 8 & 3 & 6 \\ 3 & 0 & 2 & 2 \\ -1 & -8 & -3 & 4 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 + R_1$

$$\sim \begin{bmatrix} 1 & 8 & 3 & 6 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \to C_2 - 8C_1$, $C_3 \to C_3 - 3C_1$ $C_4 \to C_4 - 6C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -24 & -7 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_2 \to \frac{1}{-24} C_2$ $C_3 \to \frac{1}{-7} C_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -16 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$ +1

$C_3 \to C_3 - C_2$ $C_4 \to C_4 + 16C_2$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_4 \to \frac{1}{10} C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ +1

$C_3 \leftrightarrow C_4$ $$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{c:c} I_3 & 0 \end{array}\right]$$

$\rho(A) = 3$ +1

Q. 2(iii)

$$x + 2y - z = 3$$
$$3x - y + 2z = 1$$
$$2x - 2y + 3z = 2$$
$$x - y + z = -1$$

Augmented matrix $[A:B]$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$ +1

$R_2 \to R_3 - R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right]$$

$R_3 \to R_3 + 6R_2$, $R_4 \to R_4 + 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & 2 & 8 \end{array}\right]$$

+1

$R_3 \to \frac{1}{5} R_3$, $R_4 \to \frac{1}{2} R_4$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_4 \to R_4 - R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

+1

Here

$\rho(A:B) = \rho(A) = 3 =$ number of variable

system is consistent and it has unique solution +1

$x + 2y - z = 3$

$y = 4$

$z = 4$

$\Rightarrow \quad x = -1$ +1

Q. 2 (iv)
$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

given matrix is upper triangular matrix so the eigen values are
$$3, 2, 5$$

or $|A - \lambda I| = 0 \Rightarrow (3-\lambda)(2-\lambda)(5-\lambda) = 0 \Rightarrow \lambda = 3, 2, 5$ +1

eigen vectors are given by the equation
$$[A - \lambda I] X = 0$$ +1

$$\begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Case I - for $\lambda = 3$

$$\begin{bmatrix} 0 & 1 & 4 \\ 0 & -1 & 6 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_2 + 4x_3 = 0, \quad -x_2 + 6x_3 = 0$$
$$2x_3 = 0$$

on solving these eq^n, we have

$$x_3 = 0, \quad x_2 = 0, \quad x_1 = 1$$

$X_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, is the eigen vector for $\lambda = 3$ +1

Q. 4(i) $\int_0^1 x^m (1-x^n)^p \, dx$ —— (1)

putting $x^n = y$

i.e. $x = y^{1/n}$

$dx = \frac{1}{n} y^{(1/n)-1} dy$. +1

then from (1) we have

$$\int_0^1 x^m (1-x^n)^p dx = \frac{1}{n} \int_0^1 y^{(m/n)} (1-y)^p \cdot y^{\frac{1}{n}-1} dy$$ +1

$$= \frac{1}{n} \int_0^1 y^{[(m+1)/n]-1} \cdot (1-y)^{(p+1)-1} dy$$

$$= \frac{1}{n} B\left(\frac{m+1}{n}, p+1\right)$$ +1

putting $m=5$, $n=3$, $p=10$ in (1), we have

$$\int_0^1 x^5 (1-x^3)^{10} dx = \frac{1}{3} B\left(\frac{5+1}{3}, 10+1\right)$$ +1

$$= \frac{1}{3} B(2, 11)$$

$$= \frac{1}{3} \frac{\Gamma 2 \, \Gamma 11}{\Gamma 13}$$

$$= \frac{1}{396}$$ +1

Case - II $\lambda = 2$

eigen vector corresponding to $\lambda = 2$

$$\begin{bmatrix} 3-2 & 1 & 4 \\ 0 & 2-2 & 6 \\ 0 & 0 & 5-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 6 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + 4x_3 = 0$$
$$6x_3 = 0$$
$$x_2 = -x_1$$ +1

$X_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, is the eigen vecter for $\lambda = 2$

Case III $\lambda = 5$

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x_1 + x_2 + 4x_3 = 0$$
$$-3x_2 + 6x_3 = 0$$

$$\frac{x_2}{2} = \frac{x_3}{1} = k \text{ (say)}$$

$$\frac{x_1}{3} = \frac{x_2}{2} = \frac{x_3}{1}$$ +1

$X_3 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, is the eigen vecter for $\lambda = 5$

$$= \int_0^{4a} \left[\sqrt{4ax} - \frac{x^2}{4a} \right] dx =$$ +1

$$= 2a^{1/2} \left[\frac{2}{3} x^{3/2} \right]_0^{4a} - \frac{1}{4a} \left[\frac{x^3}{3} \right]_0^{4a}$$

$$= \frac{16}{3} a^2$$ +1

Q.4 (iii) $\lim_{n \to \infty} \left[\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \cdots + \frac{1}{2n} \right]$

$$\lim_{n \to \infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$$ +1

$$\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} \left[\frac{1}{1+\left(\frac{r}{n}\right)^2} \right] \Rightarrow \lim_{n \to \infty} \frac{1}{n} \sum f\left(\frac{r}{n}\right)$$ +1

— (1)

Replace Σ by ~~dx~~ $\int$, $\frac{r}{n}$ by x and $\frac{1}{n}$ by dx.

~~for~~ Now find Limit $\int$

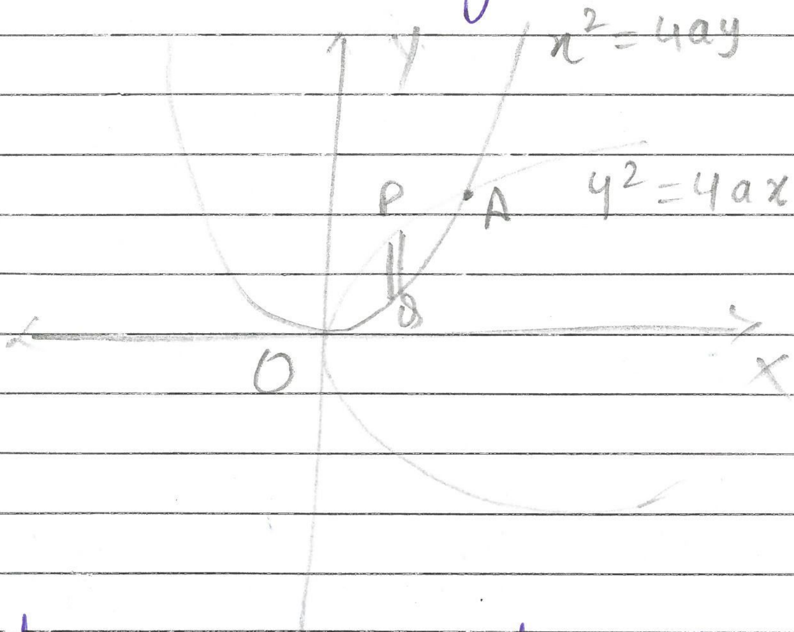
for upper limit $r = n$

$$\lim_{n \to \infty} \frac{r}{n} = \lim_{n \to \infty} \frac{n}{n} = 1$$

for lower limit $r = 1$

$$\lim_{n \to \infty} \frac{r}{n} = \lim_{n \to \infty} \frac{1}{n} = 0$$ +1

Q. 4(ii) $y^2 = 4ax$, and $x^2 = 4ay$



The points of intersection of parabolas are obtained by solving $y^2=4ax$ and $x^2=4ay$.

we get

$$y=0 \text{ or } y=4a$$

Hence, when $y=0 \Rightarrow x=0$

and when $y=4a \Rightarrow x=4a$ +1

Thus points of intersection are $O(0,0)$, $A(4a,4a)$ +1

$\therefore$ The required are

$$= \int_0^{4a} \int_{x^2/4a}^{\sqrt{4ax}} dx\,dy.$$ +1

$$= \int_0^{4a} \left[y \right]_{x^2/4a}^{\sqrt{4ax}}$$

from 1, we get

$$\int_0^1 \frac{1}{1+x^2}\,dx$$ +1

$$\left[\tan^{-1}x\right]_0^1$$

$= \tan^{-1}1 - \tan^{-1}0$

$= \pi/4$ +1

$r = \frac{\partial^2 u}{\partial x^2} = 6 \times \frac{5}{3} - 14 = -4$

$s = \frac{\partial^2 u}{\partial x \partial y} = 0$

$t = \frac{\partial^2 u}{\partial y^2} = -2$

$\therefore rt - s^2 = (-4)(-2) - 0 = 8 =$ positive

and $r = -4$ is negative

$\Rightarrow u$ has a maxima at $\left(\frac{5}{3}, 2\right)$ +1

$\therefore U_{max} = \left(\frac{5}{3}\right)^3 - (2)^2 - 7\left(\frac{5}{3}\right)^2 + 4 \times 2$

$+ 15 \times \frac{5}{3} - 13$

$= \frac{32}{27}$ +1

(ii) Expand $\tan^{-1} x$ in ascending powers of x by Maclaurin's theorem

Solⁿ:- Let $y = \tan^{-1} x$

$(y)_0 = \tan^{-1} 0 = 0$

Differentiating w.r.t. 'x'

$\therefore y_1 = \frac{1}{1+x^2}$ $\quad (y_1)_0 = \frac{1}{1+0} = 1$

or $(1+x^2) y_1 = 0 \longrightarrow (1)$

Again differentiating w.r.t. 'x'

$(1+x^2) y_2 + 2x y_1 = 0$ $\quad \therefore (y_2)_0 = 0$ +1

Differentiating equation (1) n times by Leibnitz's theorem, we get

Q.3(i) Find the maxima or minima of the function
$$u = x^3 - y^2 - 7x^2 + 4y + 15x - 13$$

Sol^n We have $u = x^3 - y^2 - 7x^2 + 4y + 15x - 13$

$\Rightarrow \dfrac{\partial u}{\partial x} = 3x^2 - 14x + 15$

and $\dfrac{\partial u}{\partial y} = -2y + 4$

$\therefore r = \dfrac{\partial^2 u}{\partial x^2} = 6x - 14, \quad s = \dfrac{\partial^2 u}{\partial x \partial y} = 0,$

$t = \dfrac{\partial^2 u}{\partial y^2} = -2$ +1

For maxima or minima, we have

$\dfrac{\partial u}{\partial x} = 0$ and $\dfrac{\partial u}{\partial y} = 0$

$\Rightarrow 3x^2 - 14x + 15 = 0$ and $-2y + 4 = 0$.

On solving these equations, we get

$x = 3, \dfrac{5}{3}$ and $y = 2, 2$

$\Rightarrow (3, 2)$ and $\left(\dfrac{5}{3}, 2\right)$ are critical points +1

At point $x = 3, y = 2$

$r = \dfrac{\partial^2 u}{\partial x^2} = 18 - 14 = 4$

$s = \dfrac{\partial^2 u}{\partial x \partial y} = 0$

$t = \dfrac{\partial^2 u}{\partial y^2} = -2$

$\therefore rt - s^2 = 4 \times (-2) - 0 = -8 = -ve$

~~Clearly~~ $\Rightarrow u$ has neither maxima nor minima at $(3, 2)$ +1

At point $x = \dfrac{5}{3}, y = 2$

(iii) If $x^x y^y z^z = c$ then show that
$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$
when $x = y = z$

Sol^n:- Given relation is
$$x^x y^y z^z = c \longrightarrow (1)$$
Taking logarithm on both sides, we get
$$x \log x + y \log y + z \log z = \log c \longrightarrow (2) \quad +1$$
Differentiating eq^n (2) partially w.r.t. 'x' by noting that z is a function of x and y, we get
$$\left[\cancel{x} \times \frac{1}{\cancel{x}} + \log x \times 1\right] + \left[\cancel{z} \times \frac{1}{\cancel{z}} + \log z \times 1\right] \frac{\partial z}{\partial x} = 0$$
$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{(1+\log x)}{(1+\log z)} \longrightarrow (3) \quad +1$$
Similarly,
$$\frac{\partial z}{\partial y} = -\frac{(1+\log y)}{(1+\log z)} \quad +1$$
Now,
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$
$$= \frac{\partial}{\partial x}\left[-\frac{(1+\log y)}{(1+\log z)}\right] \quad +1$$
$$= -(1+\log y) \frac{\partial}{\partial x}\left[(1+\log z)^{-1}\right]$$
$$= -(1+\log y)\left\{-(1+\log z)^{-2}\right\} \times \frac{1}{z} \times \frac{\partial z}{\partial x}$$
$$= \frac{(1+\log y)}{(1+\log z)^2} \times \frac{1}{z} \times \left\{-\frac{(1+\log x)}{(1+\log z)}\right\}$$
(using eq^n (3))

$$(1+x^2)\, y_{n+1} + {}^nc_1\, y_n (2x) + {}^nc_2\, y_{n-1} (2) = 0$$

$$\Rightarrow (1+x^2)\, y_{n+1} + 2nx\, y_n + \frac{n(n-1)}{\cancel{2}}\, y_{n-1} \cancel{(2)} = 0$$ +1

Put $x = 0$

$$(y_{n+1})_0 = -n(n-1)(y_{n-1})_0 \longrightarrow (2)$$

Put $n = 2, 3, 4, ---$ in eq^n (2)

$$(y_3)_0 = -1\cdot 2\cdot (y_1)_0 = -1\cdot 2\cdot 1 = -(2)!$$

$$(y_4)_0 = -2\cdot 3\cdot (y_2)_0 = 0$$

$$(y_5)_0 = -3\cdot 4\cdot (y_3)_0 = 3\cdot 4\cdot 2! = 4!$$

$$(y_6)_0 = 0$$

$$(y_7)_0 = -5\cdot 6\cdot (y_5)_0 = -5\cdot 6\cdot 4! = -(6)!$$ etc +1

$\therefore$ By Maclaurin's theorem, we get

$$f(x) = f(0) + x\cdot f'(0) + \frac{x^2}{2!} f''(0) + \text{———}$$ +1

$$\Rightarrow \tan^{-1} x = 0 + x\cdot 1 + \frac{x^2}{2!} \times 0 + \frac{x^3}{3!} \times -(2)!$$

$$+ \frac{x^4}{4!} \times 0 + \frac{x^5}{5!} \times 4! + \frac{x^6}{6!} \times 0 + \frac{x^7}{7!} \times -(6)!$$

$$+ ----$$

$$\Rightarrow \tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ----$$ +1

Q.5 (i) Solve $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

Sol^n:- The given equation can be written as

$$(1+y^2)\,dx + x\,dy = \tan^{-1}y\,dy$$

$$\Rightarrow \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$$ +1

(On dividing eq^n by $(1+y^2)dy$)

which is a linear equation in x.

On comparing given eq^n with

$\frac{dx}{dy} + Px = Q$ we get

$$P = \frac{1}{1+y^2}, \quad Q = \frac{\tan^{-1}y}{1+y^2}$$

~~$\therefore \int P$~~

$$\therefore I.F. = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy}$$ +1

$$= e^{\tan^{-1}y}$$

Hence the required solution is

$$x \times I.F. = \int (Q \times I.F.)\,dy + c$$

$$\Rightarrow x e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} \times e^{\tan^{-1}y}\,dy + c$$

Put $\tan^{-1}y = t$

$$\Rightarrow \frac{1}{1+y^2}dy = dt$$

$$\therefore x e^{\tan^{-1}y} = \int t e^{t}\,dt + c$$ +1

$$= te^t - e^t + c$$

when $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{\cancel{(1+\log x)^2}}{x(1+\log x)^3}$$

$$= -[x(1+\log x)]^{-1}$$

$$= -[x(\log e+\log x)]^{-1} \quad (\because \log e = 1)$$

$$= -[x \log ex]^{-1}$$

+1

$\therefore$ General solution is

$$y = C.F. + P.I.$$
$$y = (C_1 + C_2 x)e^{2x} + e^{x} - \frac{\sin 2x}{8} \quad +1$$

(iii) solve $(x^3 D^3 + 2x^2 D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$

Sol^n Put $x = e^{z}$ and $\frac{d}{dz} \equiv D'$

Then the given differential equation becomes

$$[D'(D'-1)(D'-2) + 2D'(D'-1) + 2]y = 10(e^{z} + e^{-z}) \quad +1$$

$$\Rightarrow (D'^3 - D'^2 + 2)y = 10(e^{z} + e^{-z})$$

Its auxiliary equation is

$$m^3 - m^2 + 2 = 0$$
$$\Rightarrow (m+1)(m^2 - 2m + 2) = 0$$
$$\Rightarrow m = -1, \quad m = 1 \pm i \quad +1$$

$$\therefore C.F. = C_1 e^{-z} + e^{z}(C_2 \cos z + C_3 \sin z) \quad +1$$

$$C.F. = C_1 \frac{1}{x} + x(C_2 \cos\log x + C_3 \sin\log x)$$

Now,

$$P.I. = \frac{1}{D'^3 - D'^2 + 2} 10e^{z} + \frac{1}{D'^3 - D'^2 + 2} 10e^{-z}$$

$$= \frac{1}{2}(10e^{z}) + \frac{1}{(D'+1)(D'^2 - 2D' + 2)} 10e^{-z} \quad +1$$

$\Rightarrow x e^{\tan^{-1}y} = \tan^{-1}y \, e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$

$\Rightarrow x = (\tan^{-1}y - 1) + ce^{-\tan^{-1}y}$ +1

(ii) Solve $(D^2 - 4D + 4)y = e^x + \cos 2x$

Solⁿ The auxiliary equation is

$m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0$

$\Rightarrow m = 2, 2$ +1

$\therefore$ C.F. $= (C_1 + C_2 x) e^{2x}$ +1

Now,

$$P.I. = \frac{1}{(D^2 - 4D + 4)} (e^x + \cos 2x)$$

$$= \frac{1}{D^2 - 4D + 4} e^x + \frac{1}{D^2 - 4D + 4} \cos 2x$$ +1

$$= \frac{1}{1^2 - 4 \times 1 + 4} e^x + \frac{1}{-2^2 - 4D + 4} \cos 2x$$

$$= e^x - \frac{1}{4D} \cos 2x$$

$$= e^x - \frac{1}{4} \int \cos 2x \, dx$$ +1

$$= e^x - \frac{1}{4} \frac{\sin 2x}{2}$$

$$P.I. = e^x - \frac{\sin 2x}{8}$$ +1

Q.6 (i) $u = 3x^2y - y^3 \longrightarrow (1)$

Differentiating eqⁿ (1) partially w.r.t. x and y respectively, we get

$$\frac{\partial u}{\partial x} = 6xy = \phi_1(x,y) \text{ (say)} \longrightarrow (2)$$

& $$\frac{\partial u}{\partial y} = 3x^2 - 3y^2 = \phi_2(x,y) \text{ (say)} \longrightarrow (3)$$

Also, $\frac{\partial^2 u}{\partial x^2} = 6y$ and $\frac{\partial^2 u}{\partial y^2} = -6y$ +1

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$ +1

Hence u is a harmonic function.

Putting $x = z$, $y = 0$ in eqⁿ (2) & (3) we get

$\phi_1(z,0) = 0$, $\phi_2(z,0) = 3z^2$ +1

I Method to find f(z)

Hence by Milne-Thompson method,

$$f(z) = \int [\phi_1(z,0) - i\,\phi_2(z,0)]\,dz + c$$ +1

$$= \int 0 - i\,3z^2\,dz + ci$$

$$= -i \times \cancel{3}\frac{z^3}{\cancel{3}} + ci$$

$$f(z) = -iz^3 + ci \Rightarrow \text{Ans}$$ +1

$$= -i(x+iy)^3 + ci$$

$$= -i(x^3 + 3ix^2y - 3xy^2 - iy^3) + ci$$

$$= (3x^2y - y^3) + i(3xy^2 - x^3 + c)$$