Total No. of Questions: 6 | Total No. of Printed Pages:3

Enrollment No......................................

Faculty of Science / Engineering

End Sem Examination Dec-2023

CA3CO20 Mathematics -III

Programme: BCA / BCA-MCA (Integrated) | Branch/Specialisation: Computer Application

Duration: 3 Hrs. | **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Let x be the exact value and x_a be the approximate value then absolute error is- **1**

(a) $|x-x_a|$ (b) $|x+x_a|$ (c) $\left|\frac{x-x_a}{x}\right|$ (d) None of these

ii. For iterative method convergence condition is- **1**

(a) $|\phi'(x)|=1$ (b) $|\phi'(x)|<1$ (c) $|\phi'(x)|>1$ (d) None of these

iii. $(1+\Delta)(1-\nabla)$ =……… **1**

(a) E^2 (b) $(1^2-\Delta^2)$ (c) 1 (d) -1

iv. For Newton-Gregory Forward Interpolation method, as per standard formula- **1**

(a) $u=\frac{x-x_0}{h}$ (b) $u=\frac{x+x_0}{h}$

(c) $u=\frac{x-x_n}{h}$ (d) $u=\frac{x+x_n}{h}$

v. Picard's method is also called method of- **1**

(a) Successive differentiation (b) Successive integration

(c) Successive displacement (d) None of these

vi. The highest order of polynomial integrand for which Trapezoidal rule of integration is best- **1**

(a) First order (b) Second order

(c) Third order (d) Fourth order

vii. The probability of throwing an even number with a dice is- **1**

(a) 1/6 (b) 2/6

(c) 3/6 (d) 4/6

viii. When the number of trials n is very large and the probability of success p is very small (but mean is finite), we can use- **1**

(a) Binomial distribution (b) Poisson distribution
(c) Normal distribution (d) None of these

ix. Normal curve is symmetrical about the- **1**

(a) x- axis (b) $x = \mu$, the mean line
(c) both (a) and (b) (d) None of these

x. For Exponential distribution $f(x) = \lambda e^{-\lambda x}$, $x > 0$ value of variance is- **1**

(a) $1/\lambda^2$ (b) $1/\lambda$ (c) λ (d) None of these

Q.2 Attempt any two:

i. Find the root of the equation $xe^x = \cos x$ by Regula Falsi Method correct to three decimal places. **5**

ii. Solve by Gauss Elimination method- **5**

$6x + 3y + 2z = 6,\ 6x + 4y + 3z = 0,\ 20x + 15y + 12z = 0.$

iii. Find the cube root of 2 by Newton-Raphson method. **5**

Q.3 Attempt any two:

i. Apply Lagrange's formula to find the missing value: **5**

x:	1	2	3	4	5	7
$y=f(x)$:	2	4	8	16	-	128

ii. Express the polynomial $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation, the interval of differencing being unity. Also find successive differences. **5**

iii. Use Newton's divided difference formula to find $f(2)$ from the following table **5**

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

Q.4 Attempt any two:

i. Using Taylor's series find the solution of the following: **5**

$$\frac{dy}{dx} = y^2 + 1 \text{ at } x = 1,$$

Assuming that, $y = 0$ at $x = 0$.

ii. Evaluate- **5**

$$\int_0^6 \frac{1}{1+x^2}\,dx,$$

By using Simpson's 1/3 rule.

iii. Apply Runge-Kutta method to obtain $y(0.1)$, given that- **5**

$$\frac{dy}{dx} = x^2 - y,$$

with initial conditions $y(0) = 1$.

Q.5 Attempt any two:

i. Obtain the mean, variance and standard deviation of Binomial distribution. **5**

ii. Define the following- **5**

(a) Random variable with types
(b) Probability mass function

iii. If the probability that an individual suffers a bad reaction from a certain injection is 0.001, determine the probability that out of 2000 individuals **5**

(a) exactly 3
(b) more than 2 individuals
will suffer a bad reaction.

Q.6 Attempt any two:

i. Find the value of k for the probability density function $f(x) = kx^2$ $(0 \le x \le 3)$ and compute $P(1 \le x \le 2)$. **5**

ii. The diameters of 1000 brass plugs taken consecutively from a machine with mean 0.7571 cm and standard deviation 0.002 cm. Find the number of plugs likely to be rejected if the approved diameter is 0.752 ± 0.004 cm. **5**

$[(0 < z < 1.75) = 0.4599$ and $(0 < z < 2.25) = 0.4878]$

iii. The life time of a certain kind of battery is a continuous random variable, which is as an exponential distribution with a mean of 200 hrs. Find the probability that such a battery will last at most 100 hrs. **5**

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112/II

End Sem Examination Dec 2023

Course Code: CA3CO20 M-III

Programme: BCA

Date :
P. No. : 1

Q 1

(i) (a) $|x - x_a|$ 1

(ii) (b) $|\phi'(x)| < 1$ 1

(iii) (c) $= 1$ 1

(iv) (a) $\frac{x - x_0}{h}$ 1

(v) (b) Successive integration. 1

(vi) (a) First order 1

(vii) (c) 3/6 1

(viii) (b) Poisson distribution 1

(ix) (B) $x = \mu$, the mean line. 1

(x) (a) $1/\lambda^2$ 1

Q 2

(i) given $x e^x = \cos x$

$\cos x - x e^x = 0$ —— (1)

let $f(x) = \cos x - x e^x$ —— (1)

$f(0) = \cos 0 - 0 = 1$

$f(1) = \cos 1 - 1.e = -2.1779$

∴ Root of eq^n (1) lies between 0, & 1. +1

Taking $x_0 = 0$, $x_1 = 1$, $f(x_0) = 1$, ~~$f(x_1) = -2.1779$~~

$f(x_1) = -2.1779$

By Regula falsi method :

First approximation :

$x = \frac{af(b) - bf(a)}{f(b) - f(a)}$ / $x = x_0 - \frac{x_1 - x_0}{[f(x_1) - f(x_0)]} \times f(x_0)$ —— (2)

$= 0 - \frac{(1-0)}{(-2.1779 - 1)} \times 1 = 0.3147$ +1

from (1)

$f(.3147) = .5198$

clearly $f(0.3147)$ and $f(1)$ are opposite sign,

so that next approximation root lies between

$(.3147, 1)$

Taking $x_0 = .3147$, $x_1 = 1$, $f(x_0) = .5198$

$f(x_1) = -2.1779$

Second approximation:

from ②, we get $x = .3147 - \dfrac{(1-0.3147) \times .5198}{(-2.17792 - 0.5198)}$

$= 0.4467$

from ① $f(0.4467) = 0.2036$

clearly next approximate root lies between

$(0.4467, 1)$

$x_0 = 0.4467$, $x_1 = 1$, $f(x_0) = .2036$, $f(x_1) = -2.1779$ +1

III - approximation:

from ② $x = .4467 - \dfrac{(1-0.4467) \times 0.2036}{(-2.1779 - 0.2036)}$

$x = 0.4940$

from ① we have

$f(0.4940) = 0.0709$

clearly root lie betn $(0.4940, 1)$

now taking $x_0 = .4940$, $x_1 = 1$,

$f(x_0) = 0.0709$, $f(x_1) = -2.1779$

IV - approximation:

by ② $x = 0.4940 - \dfrac{(1-0.4940) \times 0.0709}{(-2.1779 - 0.0709)}$

$x = 0.5091$

from ① $f(0.5091) = 0.0261$

Hence the lies between $(0.5091, 1)$

Taking $x_0 = 0.509$, $x_1 = 1$, $f(x_0) = 0.0261$, +1

$f(x_1) = -2.1779$

V - approximation - from (2)

$$x = \cancel{0.6}\ 0.5091 - \frac{[1-0.5091] \times 0.0261}{(-2.1779 - 0.0261)}$$

$$x = 0.5149$$

from (1)

$$f(0.5149) = 0.0087$$

clearly root lies between (0.5149, 1)

∴ VI - approximation -

from (2) $x = 0.5149 - \frac{(1-0.5149) \times (\cdot 0087)}{(-2.1779 - 0.0087)}$

$$x = 0.5168$$

from (1) we have

$$f(0.5168) = 0.0029$$

clearly root ~~lies~~ lies between (0.5168, 1)

∴

VII - approximation : from (2)

$$x = 0.5168 - \frac{(1-0.5168) \times 0.0029}{(-2.1779 - 0.0029)}$$

$$x = 0.5174$$

we have $f(0.5174) = 0.0010$

Hence the root of the given eq^n is

$$\boxed{x = 0.5174}$$

Q.2 (ii) Gauss elimination method

given that $6x+3y+2z=6$

$6x+4y+3z=0$

$20x+15y+12z=0$

The given system of eq^n can be written as in matrix form

$$AX=B \quad \text{--- (1)}$$

$$\begin{bmatrix} 6 & 3 & 2 \\ 6 & 4 & 3 \\ 20 & 15 & 12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

its augmented matrix :-

$$[A:B] = \left[\begin{array}{ccc:c} 6 & 3 & 2 & 6 \\ 6 & 4 & 3 & 0 \\ 20 & 15 & 12 & 0 \end{array}\right]$$ +1

operating $R_2 \to R_2 - R_1$

$R_3 \to R_3 - \frac{20}{6} R_1$

$$\sim \left[\begin{array}{ccc:c} 6 & 3 & 2 & 6 \\ 0 & 1 & 1 & -6 \\ 0 & 0 & \frac{16}{3} & -20 \end{array}\right] \quad \text{--- (2)}$$ +1

which is in upper triangular form

from (2), we get

$6x+3y+2z=6$

$y+z=-6$

$\frac{16}{3}z=-20$

using back substitution, we get

$\frac{16}{3}z=-20 \Rightarrow z=-20\times\frac{3}{16}=\frac{-15}{4}=(-3.75)$ +1

$y=-6-z$

$=-6-\left(\frac{-15}{4}\right)=-6+\frac{15}{4}$

$y=\frac{-24+15}{4}=\frac{-9}{4}$ $\boxed{y=\frac{-9}{4}}=-2.25$ +1

Q2 (ii)

$$6x + 3y + 2z = 6$$

$$6x = 6 - 3y - 2z$$

$$= 6 - 3\left(\frac{-9}{4}\right) - 2\left(\frac{-15}{4}\right)$$

$$= 6 + \frac{27}{4} + \frac{30}{4}$$

$$= \frac{24 + 27 + 30}{4}$$

$$6x = \frac{81}{4} \Rightarrow x = \frac{\cancel{81}^{27}}{4} \times \frac{1}{\cancel{6}_{2}}$$

$$\boxed{x = \frac{27}{8}} = 3.375$$ +1

Q2 (iii)

Let $x = (2)^{1/3}$

$$x^3 = 2$$

$$x^3 - 2 = 0$$

Let $f(x) = x^3 - 2$ —— (1)

then $f'(x) = 3x^2$.

Now

$$f(1) = -1 \quad \text{and} \quad f(2) = 6$$

$\Rightarrow$ root lies between 1 & 2. +1

By Newton Raphson method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^3 - 2)}{3x_n^2}$$

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right)$$ (2) +1

taking initial approximation : $x_0 = 1.5$

Q.2 (iii) continue

$x_0 = 1.5$

putting $n=0$ in (2), we get

∴ First approximation:

$$x_1 = \frac{2}{3}\left[1.5 + \frac{1}{(1.5)^2}\right]$$

$$\boxed{x_1 = 1.29630}$$

putting $n=2$ in (2), we get

∴ second approximation:

$$x_2 = \frac{2}{3}\left[1.29630 + \frac{1}{(1.29630)^2}\right]$$

$$\boxed{x_2 = 1.26093}$$ +1

putting $n=2$ in (2), we get

∴ third approximation:

$$x_3 = \frac{2}{3}\left[1.26093 + \frac{1}{(1.26093)^2}\right]$$

$$\boxed{x_3 = 1.25992}$$

putting $n=3$ in (2), we get

∴ Fourth approximation:

$$x_4 = \frac{2}{3}\left[1.25992 + \frac{1}{(1.25992)^2}\right]$$

$$\boxed{x_4 = 1.25992}$$ +1

∵ $x_3 = x_4 = 1.2599$ correct to four decimal places.

∴ root of given equation is

1.2599 Ans +1

Que (3) (i) ~~Given~~

Given that

x	:	1	2	3	4	5	7
$y=f(x)$	:	2	4	8	16	–	128
		y_0	y_1	y_2	y_3		y_4

Since the given data is unevenly spaced, therefore we use Lagrange's interpolation formula:

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \times y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \times y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} \times y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \times y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \times y_4$$

+1

Here we have ~~$x_0=0$, $x_1=2$, $x_2=3$, $x_4=4$,~~ x

$x_0=1$ $x_1=2$ $x_2=3$, $x_3=4$, $x_4=7$

$y_0=2$ $y_1=4$ $y_2=8$, $y_3=16$ $y_4=128$

$$\therefore y = \frac{(x-2)(x-3)(x-4)(x-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$

$$+ \frac{(x-1)(x-3)(x-4)(x-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(x-1)(x-2)(x-4)(x-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

Contin

$$+\frac{(x-1)(x-2)(x-3)(x-7)}{(4-1)(4-2)(4-3)(4-7)}\times 16$$

$$+\frac{(x-1)(x-2)(x-3)(x-4)}{(7-1)(7-2)(7-3)(7-4)}\times 128$$ +1

when $x=5$ then

$$y=\frac{(5-2)(5-3)(5-4)(5-7)}{(-1)(-2)(-3)(-6)}\times 2$$

$$+\frac{(5-1)(5-3)(5-4)(5-7)}{(1)(-1)(-2)(-5)}\times 4$$

$$+\frac{(5-1)(5-2)(5-4)(5-7)}{(2)(1)(-1)(-4)}\times 8$$

$$+\frac{(5-1)(5-2)(5-3)(5-7)}{(3)(2)(1)(-3)}\times 16$$

$$+\frac{(5-1)(5-2)(5-3)(5-4)}{(6)(5)(4)(3)}\times 128$$ +1

$$y=\frac{3.2.1.(-2)}{36}\times 2+\frac{4.2.1.(-2)}{-10}\times 4$$

$$+\frac{4.3.1.(-2)}{8}\times 8+\frac{4.3.2.(-2)}{-18}\times 16$$ +1

$$+\frac{4.3.2.1}{240}\times 128$$

$$=\frac{-24}{36}+\frac{64}{10}-\frac{192}{8}+\frac{768}{18}+\frac{3072}{240}$$

$$=-\frac{2}{3}+\frac{32}{5}-\frac{24}{1}+\frac{128}{3}+\frac{192}{15}$$

$$=-.67+6.4-24+42.67+12.8$$

$$y=38.54$$

Hence the missing term for $x=5$ is $y=38.54$ +1

≈ 39

Q.3 (ii)

Given $f(x) = 2x^3 - 3x^2 + 3x - 10$

let $f(x) = Ax^{[3]} + Bx^{[2]} + Cx^{[1]} + D$ — ① +1

in factorial notation.

$\Rightarrow 2x^3 - 3x^2 + 3x - 10 = Ax(x-1)(x-2) + Bx(x-1) + Cx + D$ — ② +1

now putting $x=0$ on both side of ②, we get

$D = -10$

Again putting $x=1$ in ②, we get

$C + D = -8 \Rightarrow \boxed{C = 2}$ +1

Finally, putting $x=2$ in ②, we get

$B = \frac{1}{2}(-2C - D) \Rightarrow \boxed{B = 3}$

Also, equating the coefficient of x^3 on both side, we get

$\boxed{A = 2}$ +1

Thus, polynomial in factorial notation ② becomes;

$f(x) = 2x^{[3]} + 3x^{[2]} + 2x^{[1]} - 10$

Hence

$\Delta f(x) = 6x^{[2]} + 6x^{[1]} + 2$

$\Delta^2 f(x) = 12x^{[1]} + 6$

$\Delta^3 f(x) = 12$

$\Delta^4 f(x) = 0$ +1

Q. 3 (iii)

given that

x:	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

The divided difference table is:

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
$x_0 = 4$	$y_0 = 48$				
		$\frac{100-48}{5-4} = 52$			
$x_1 = 5$	$y_1 = 100$		$\frac{97-52}{7-4} = 15$		
		$\frac{294-100}{7-5} = 97$		$\frac{21-15}{10-4} = 1$	
$x_2 = 7$	$y_2 = 294$		$\frac{202-97}{10-5} = 21$		$\frac{1-1}{11-4} = 0$
		$\frac{900-294}{10-7} = 202$		$\frac{27-21}{11-5} = 1$	
$x_3 = 10$	$y_3 = 900$		$\frac{310-202}{11-7} = 27$		$\frac{1-1}{13-5} = 0$
		$\frac{1210-900}{11-10} = 310$		$\frac{33-27}{13-7} = 1$	
$x_4 = 11$	$y_4 = 1210$		$\frac{409-310}{13-10} = 33$		
		$\frac{2028-1210}{13-11} = 409$			
$x_5 = 13$	$y_5 = 2028$				

$\because$ Newton divided difference formula. +2

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + \cdots \quad \text{—} (1)$$ +1

at $x = 2$ then (1) becomes

$$f(2) = 48 + (2-4)52 + (2-4)(2-5)15 + (2-4)(2-5)(2-7)\cdot 1 + 0$$ +1

$$= 48 + (-2)52 + (-2)(-3)15 + (-2)(-3)(-5)\cdot 1$$

$$= 48 - 104 + 90 - 130$$

$$f(2) = 138 - 234 = -96 \quad \text{Ans}$$ +1

$$\boxed{f(2) = -96}$$

Q.4 (i)

given that $\frac{dy}{dx} = y^2+1$, $x_0=0$, $y_0=0$

ie $y' = y^2+1$ —— (1)

On putting $x_0=0$, $y_0=0$ in (1) .

$y'(0) = y(0)^2+1 = 0+1$

$y'(0) = 1$ +1

$y'' = 2yy'$ $(y'')_0 = 2.(0).1 = 0$

$y''' = 2[yy''+(y')^2]$ $(y''')_0 = 2[0+1^2] = 2$ +1

$y^{iv} = 2[y.y'''+3y'y'']$ $(y^{iv})_0 = 2[0+3.0] = 0$

$y^{v} = 2[yy^{iv}+4y'''.y'+3(y'')^2]$ $(y^v)_0 = 2[0+4.2.1+3.0]$

$(y^v)_0 = 16$ +1

By taylor series expansion about $x=0$

$$y = y_0 + x(y')_0 + \frac{x^2}{2!}(y'')_0 + \frac{x^3}{3!}(y''')_0 + \frac{x^4}{4!}(y^{iv})_0 + \frac{x^5}{5!}(y^v)_0$$

$+ \ldots$ +1

$$= 0 + x.1 + \frac{x^2}{2!}.0 + \frac{x^3}{3!}2. + \frac{x^4}{4!}.0 + \frac{x^5}{5!}.16 + \ldots$$

$$= x + \frac{x^3}{3.2.1}2 + \frac{x^5}{5.4.3.2.1}16 + \ldots$$

$$y = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots$$

Put $x=1$

$y(1) = 1 + \frac{1}{3} + \frac{2}{15} + \ldots = 1+0.33+0.133+\ldots$ +1

$\simeq 1.463$ Ans

Q 4. (ii)

Divide the interval (0, 6) into six parts each of width $h=1$.

The value of $f(x) = \frac{1}{1+x^2}$ are given below +1

x	0	1	2	3	4	5	6
$f(x)=y$	1	0.5	0.2	0.1	0.0588	0.0385	0.027
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

+1

By simpson's 1/3 rule

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$ +1

$$= \frac{1}{3}\left[(1+0.27)+4(0.5+0.1+0.0385)+2(0.2+0.0588)\right]$$ +1

$$= 1.3662$$ +1

Q.4 (iii) Here $f(x,y) = x^2 - y$, $x_0 = 0$, $y_0 = 1$

taking $h = .1$

then

$$x_1 = x_0 + h = 0 + .1 = .1$$ +1

Now

$$k_1 = h f(x_0, y_0) = .1 f(0, 1) = .1[0^2 - 1]$$

$$k_1 = -.1$$

$$k_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right) = .1 f\left(0 + \frac{.1}{2}, \frac{1+(-.1)}{2}\right)$$

$$= .1 f(0.05, 0.95)$$

$$= .1[(.05)^2 - .95] = .1[.0025 - 0.95]$$ +1

$$= .1[-0.975]$$

$$= -0.0925$$

Date :
P. No. : 14

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= .1 \cancel{f[0(\frac{.1}{2}), 1 + \frac{(-.1)}{2}]}$

$= .1 f[\frac{.1}{2}, 1 - \frac{.0925}{2}]$

$= .1 f(.05, 1 - 0.04625)$

$k_3 = .1 f(.05, .95375)$ f)

$= .1 [(.05)^2 - (.95375)]$

$= .1 [.0025 - .95375]$

$= .1 [-0.95125]$

$k_3 = -0.092875$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= .1 f(.1, 1 + (-0.092875))$

$= .1 f(.1, .907125)$

$= .1 [(.1)^2 - (.907125)]$

$= .1 [0.01 - .907125]$

$= .1 [-0.897125]$

$k_4 = -0.0807125$ f)

According to Runge kutta (4^{th} order) method

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 1 + \frac{1}{6}[-.1 + 2(-.0925) + 2(-0.092875)$
$+ (-0.0807125)]$

$= 1 + \frac{1}{6}[-.1 - .185 - .18575 - 0.0807125]$

$= 1 + .17[-0.5514625]$

$y(.1) = 1 - 0.09374862 = .90625$ Ans f)

Q.5 (i)

Mean of Binomial distribution is given as

$$m = \sum_{r=0}^{n} r \cdot P(r)$$ +1

$$= 0 + \sum_{r=1}^{n} r\, P(r)$$

$$= \sum_{r=1}^{n} r \cdot {}^{n}C_{r}\, p^{r} q^{n-r} \qquad \because P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= \sum_{r=1}^{n} n\, {}^{n-1}C_{r-1}\, p^{r} q^{n-r} \qquad \because r\, {}^{n}C_{r} = n \cdot {}^{n-1}C_{r-1}$$

$$= \sum_{r=1}^{n} n\, {}^{n-1}C_{r-1}\, p^{r-1} p \cdot q^{(n-1)-(r-1)}$$

$$= np \sum_{r=1}^{n} {}^{n-1}C_{r-1}\, p^{r-1} q^{(n-1)-(r-1)}$$

$$= np\,(q+p)^{n-1}$$

$$m = \boxed{\text{mean} = np} \qquad \because (q+p)^{n} = 1$$ +1

Variance of Binomial distribution is ~~giv~~ defined as

$$\text{Variance } v = \sum_{r=0}^{n} r^{2} P(r) - (\text{mean})^{2}$$ +1

$$= \sum_{r=0}^{n} [r(r-1)+r]\, P(r) - m^{2} = \sum_{r=0}^{n} r(r-1)P(r) + \sum_{r=0} r P(r) - m^{2}$$

$$= 0 + 0 + \sum_{r=2}^{n} r(r-1)\, P(r) + m - m^{2}$$

$$= \sum_{r=2}^{n} r(r-1)\, {}^{n}C_{r}\, p^{r} q^{n-r} + m - m^{2}$$

$$\because r(r-1)\, {}^{n}C_{r} = n(n-1)\, {}^{n-2}C_{r-2}$$

$$= \sum_{r=2}^{n} n(n-1)\, {}^{n-2}C_{r-2}\, p^{r} q^{n-r} + m - m^{2}$$

$$= n(n-1)\, p^{2} \sum_{r=2}^{n} {}^{n-2}C_{r-2}\, p^{r-2} q^{n-r} + m - m^{2}$$

$$= n^2p^2 - np^2 \sum_{r=2}^{n} {}^{n-2}C_{r-2}\, p^{r-2} q^{(n-2)-(r-2)} + m - m^2$$

$$= n^2p^2 - np^2 (q+p)^{n-2} + m - m^2$$

$$= n^2p^2 - np^2 + np - (np)^2 \qquad \because (q+p)^{n-2} = 1$$

$$= n^2p^2 - np^2 + np - n^2p^2 \qquad \because m = np$$

$$= np - np^2$$

$$= np(1-p)$$

$$= npq \qquad \because p+q=1 \therefore q = 1-p$$

$$\boxed{\text{Variance } V = npq}$$

+1

Standard deviation of Binomial distribution

$$\sigma^2 = V = \text{Variance}$$

$$\sigma = \sqrt{V}$$

$$\sigma = \sqrt{npq} \qquad \because V = npq$$

+1

Q. 5 (ii)

(a) Random variable :- Let S be the sample space associated with a given random experiment. Then a real valued function X which assigns to each outcome $x \in S$ to a unique real No $X(x)$ is called a random variable, or

random variable is a real valued function having domain as the sample space associated with a given random experiment.

+1

There are two types of random variable:

(i) <u>Discrete Random variable</u>:— A variable, when real valued function defined on a discrete sample space is called a discrete random variable.

ex: The marks obtained in a test, number of successes in n-trials etc +1

(ii) <u>Continuous Random variable</u>: A Random variable X is said to be continuous random variable, if it can take all possible values between certain limits.

ex: Age, weight, height and so on. +1

(b) <u>Probability mass function</u>: If X is a discrete random variable then its probability fun $p(x)$ is a discrete probability function. It is also called probability mass function. In other word " If $x_1, x_2 -- x_n$ are n different values of a discrete random variable X & $p(x_1), p(x_2) --- p(x_n)$ be their respective probabilities such that

(i) $P(x_i) \geq 0$ $i = 1, 2, 3 -- n$

(ii) $\Sigma P(x_i) = 1$ $i = 1, 2, 3 -- n$ +2

Then $p(x)$ is known as the probability mass function of the variable X.

Q. 5 (iii)

given $p = .001$ (very small)

$n = 2000$ (very large)

then mean $m = np$

$= 2000 \times .001$

$= 2$

using poisson distribution

$$P(X=r) = e^{-m} \frac{m^r}{\lfloor r}, \quad r = 0, 1, 2, \ldots 2000$$ +1

(a) P(exactly 3 suffer a bad reaction) = P(3)

$$\Rightarrow P(3) = \frac{m^3 e^{-m}}{\lfloor 3} = \frac{2^3 e^{-2}}{6} = \frac{8e^{-2}}{6}$$

$$= \frac{4}{3e^2} = 0.180$$ +2

(b) P(more than 2 suffer a bad reaction)

$$= P(r>2) = 1 - [P(0) + P(1) + P(2)]$$

$$= 1 - \left[e^{-m} + \frac{m^1 e^{-m}}{\lfloor 1} + \frac{m^2 e^{-m}}{\lfloor 2}\right]$$

$$= 1 - \left[\frac{1}{e^2} + \frac{2}{e^2} + \frac{2}{e^2}\right]$$

$$= 1 - \frac{5}{e^2}$$

$$= 0.323$$ +2

Q.6. (i) since $f(x)$ is a p.d.f, so that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$ 1

$$\Rightarrow \int_0^3 kx^2\,dx = 1$$

$$k\left[\frac{x^3}{3}\right]_0^3 = 1$$

$$\Rightarrow \frac{k}{3}[27-0] = 1$$

$$\Rightarrow k = 1/9$$ 1

Thus p.d.f: $f(x) = \frac{1}{9}x^2$ for $(0 \le x \le 3)$ 1

To compute $P(1 \le x \le 2)$?

we have $P(1 < x \le 2) = \int_1^2 f(x)\,dx$ 1

$$= \int_1^2 \frac{1}{9}x^2\,dx$$

$$= \frac{1}{9}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{27}[8-1]$$

$$= \frac{7}{27}$$ 1

—×—×—

Date :
P. No. : 20

Q 6. (ii)

since the given range of diameter are

$x_1 = 0.752 - 0.004$

$x_1 = 0.748$ cm

& $x_2 = .752 + .004 = .756$ cm +1

Mean $\mu = 0.7571$ & S.D $\sigma = 0.002$ cm

$\because$ The S. Normal variate is

$$z = \frac{x-\mu}{\sigma}$$ +1

$\therefore$ at $x_1 = .748$ then $z_1 = \frac{0.748 - 0.7571}{.002}$

$= \frac{-0.0091}{.002}$

$z_1 = -4.55$ +1

Also at $x_2 = .756^c$ then $z_2 = \frac{.756 - 0.7571}{.002}$

$z_2 = \frac{-0.0011}{.002} = -0.55$

$z_2 = -0.55$ +1

$\therefore P(x_1 < x < x_2) = P(z_1 < z < z_2)$

$= P(-4.55 < z < -0.55)$

$= P(0 < z < 4.55) + P(0 < z < 0.55)$ +1

this value is not specified + 0.2088

in the que. paper Hence full marks should be awarded to the students if she/he solved this que upto this level.

Date :
P. No. : 21

$\therefore$ number of plugs likely to be rejected

$= 1000(1- \quad)$

$= 1000 \times$

$=$

Que 6 (iii)

The p.d.f of exponential distribution:

$f(x) = \lambda e^{-\lambda x}, \quad x > 0, \; \lambda > 0$ +1

Given mean = 200 hrs

$\Rightarrow \dfrac{1}{\lambda} = 200$

$\Rightarrow \lambda = \dfrac{1}{200}$ +1

Probability that a battery will last at most 100 hrs

$= P(x \leq 100)$ +1

$= \int_0^{100} f(x)\, dx$

$= \int_0^{100} \lambda e^{-\lambda x} dx$ +1

$= \lambda \left(\dfrac{e^{-\lambda x}}{-\lambda} \right)_0^{100}$

$= -(e^{-100\lambda} - 1) = 1 - e^{-100\lambda}$

$= 1 - e^{-(100/200)} = 1 - e^{-0.5}$

$= .3935$ Ans +1