Enrollment No....................................

Faculty of Engineering / Science

End Sem Examination Dec-2023

CS3BS04 / IT3BS01 / BC3BS05 Discrete Mathematics

Programme: B.Tech. / B.Sc.

Branch/Specialisation: CSE All / IT / Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Let a function $f: N \rightarrow N$ is defined as $f(x) = [x], where$ [ ] is greatest integer function is- **1**

(a) One- One only (b) Onto only

(c) One -One and Onto (d) None of these

ii. Let $A = \{1,2,3,4\}$ and $B = \{2,4,6,7\}$ are two sets then total number of proper subset of $A \cap B$ is- **1**

(a) 4 (b) 3 (c) 2 (d) None of these

iii. Which of the following is partial order relation defined on set of integers Z? **1**

(a) $R = \{(a, b): a \leq b, \forall a, b \in Z\}$

(b) $R = \{(a, b): a + b = 0, \forall a, b \in Z\}$

(c) $R = \{(a, b): a = b + 1, \forall a, b \in Z\}$

(d) None of these

iv. If $f(x, y) = x.y' + y'x$ is Boolean function in Boolean algebra $(B, +, .)$ then value of $f(0,1)$ is- **1**

(a) 0 (b) 1 (c) 2 (d) None of these

v. If $G = \{a, a^2\ a^3, a^4, a^5, a^6, a^7 = e\}$ is a cyclic group, then G has- **1**

(a) 5 generators (b) 7 generators

(c) 6 generators (d) None of these

vi. Which of the following is not Groupoid? **1**

(a) $(N, +)$ (b) $(Q, +)$

(c) $(Q^c, +)$ (d) None of these

vii. If $G = (V, E)$ is a graph with 7 edges, then sum of degrees of all vertices of G is- **1**

(a) 7 (b) 10

(c) 14 (d) 0

viii. A Graph G have neither self-loops nor parallel edges is known as- **1**
(a) Pseudograph (b) Simple Graph
(c) Multi-Graph (d) All of these

ix. How many 3 three digits numbers can be formed by using digits 2,5,9,7 when repetition of digits is allowed? **1**
(a) 3^4 (b) 4^3 (c) 12 (d) None of these

x. Number of subsets of the {1,2,3,4,5,6,7,8,9,10,11} having 4 elements- **1**
(a) 330 (b) 100 (c) 300 (d) None of these

Q.2 Attempt any two:

i. Define equivalence relation. Also prove that in the set of all triangles relation of "similarity" is equivalence relation. **5**

ii. If $f: R \rightarrow R$, $g: R \rightarrow R$ and $h: R \rightarrow R$ are three function defined as $f(x) = Sinx$, $g(x) = Cosx$ and $h(x) = \frac{1}{x}$ then find the value of $fog(x), ho(gof)(0)$ and $gof(x)$. And prove that $fog \neq gof$. **5**

iii. Write the statement of De- Morgan's theorem (set theory) and If A and B are two sets, then prove- **5**
(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

Q.3 Attempt any two:

i. Let S be a set of family of all sets which is closed under the operation union "∪ " and intersection " ∩" and complementary law then prove (S,∪,∩,') is Boolean Algebra. **5**

ii. Define Maximal and Minimal elements of POSET. **5**
If A = {1,2,3} and P(A) is power set of set A and relation of inclusion (a is subset of b) is defined in P(A) then find maximal and minimal elements of (PA)

iii. (a) Write the Expression E= $(xy' + xz)' + x'$ in conjunctive normal form. **5**
(b) Write the expression E= $(x + y).(x + z') + (y + z')$ in disjunctive normal form.

Q.4 Attempt any two:

i. If H_1 and H_2 are two subgroups of Group G then prove $H_1 \cap H_2$ is also a Subgroup of G. **5**

ii. Show that the set Q^+ of all positive rational numbers is a group with respect to the composition "*" defined as follows: **5**
$a * b = \frac{1}{4}ab \quad Where\ a, b \in Q^+$.

iii. Prove that any Right (Left) cosets of a subgroup of Group G are either disjoint or identical. **5**

Q.5 Attempt any two:

i. Prove that the sum of the degrees of all vertices in a graph is equal to the twice of the number of edges. **5**

ii. Define graph colouring and chromatic number with example. **5**

iii. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. **5**

Q.6 Attempt any two:

i. Solve by the method of generating functions the recurrence relation $a_r - 4a_{r-1} + 3a_{r-2} = 0, r \geq 2$ with the boundary condition $y_0 = 2$ $and y_1 = 4$. **5**

ii. Solve the recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = 0$ given that- **5**
$$a_0 = 1 \ and a_1 = 3$$

iii. Let n and r be non-negative integers such that $r \leq n$. Then prove that- **5**
$${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$$

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Medi-Caps University Indore (M.P.)
End Sem Examination. Dec-2023
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Date :
P. No. :

Programme - B.Tech Branch - CSE All / IT / B.Sc

Answer

Q.1

(i)	(c)	One-one & onto	+1
(ii)	(b)	3	+1
(iii)	(a)	$R = \{(a,b) : a \le b, \forall a, b \in Z\}$	+1
(iv)	(a)	0	+1
(v)	(c)	6 generators	+1
(vi)	(c)	$(Q^c, +)$	+1
(vii)	(c)	14	+1
(viii)	(b)	Simple Graph	+1
(ix)	(b)	4^3	+1
(x)	(a)	330	+1

Q.2 Q.2 (i)

Equivalence Relation :-
A relation R defined in set A is said to be equivalence Relation if R is

(i) Reflexive ie $(a,a) \in R \quad \forall a \in A$ 2
(ii) symmetric i.e $(a,b) \in R \Rightarrow (b,a) \in R \quad \forall a, b \in A$
(iii) transitive i.e $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$
~~$\forall a, b, c \in A$~~
$\forall a, b$ and $c \in A$

let $T = \{T_1, T_2, T_3 \ldots\}$ is a set of all triangles in a plane
~~and~~ $R = \{(T_1, T_2) : T_1 \cong T_2 \quad \forall T_1, T_2 \in T\}$
is relation defined in T.

②

Date :
P. No. :

Reflexive Relation

Let T_1 ~~and T_2 are~~ is ~~two~~ arbitrary triangles of set T

and $(T_1, T_1) \in R$ ①

$\Rightarrow T_1 \cong T_1$ (Law of similar triangle)

$\Rightarrow$ Every triangle is similar to itself.

$\Rightarrow (T_1, T_1) \in R$

$\Rightarrow R$ is Reflexive Relation

Symmetric Relation

Let T_1 and T_2 are two arbitrary triangle of set T

and $(T_1, T_2) \in R$ ①

$\Rightarrow T_1 \cong T_2$

$\Rightarrow T_2 \cong T_1$

$\Rightarrow (T_2, T_1) \in R$

$\Rightarrow R$ is symmetric Relation

Transitive Relation

let T_1, T_2 and T_3 are three arbitrary triangles of set T)

if $(T_1, T_2) \in R$ ①

$\Rightarrow T_1 \cong T_2$ ——— ①

and $(T_2, T_3) \in R$

$\Rightarrow T_2 \cong T_3$ ——— (II)

From (I) and (II)

$\Rightarrow T_1 \cong T_3$

$\Rightarrow (T_1, T_3) \in R$

$\Rightarrow$ R is transitive Relation

Hence, ~~set of all~~ Relation of similarity in the set of all triangle is 'Equivalence'

$\boxed{Q.(2)(ii)}$

Given

$f: R \rightarrow R$, $g: R \rightarrow R$ and $h: R \rightarrow R$ are three function such that

$f(x) = \sin x \quad g(x) = \cos x \quad h(x) = \frac{1}{x}$

$$fog(x) = f(g(x))$$
$$fog(x) = \sin(g(x))$$
$$fog(x) = \sin(\cos x) \quad \text{——— (I)}$$

$1\frac{1}{2}$

$$gof(x) = g(f(x))$$
$$(gof)(x) = \cos f(x)$$

$1\frac{1}{2}$

4

Date :
P. No. :

$$gof(x) = \cos(\sin x) \quad \text{---} (11)$$

from (I) and (II)

$$fog(x) \neq gof(x)$$

and

$$ho(gof)(0) = h(\cos(\sin 0)) \quad [\sin 0 = 0]$$

$$ho(gof)(0) = h(\cos 0) \quad \left[h(x) = \frac{1}{x}\right]$$

$$= h(1)$$

$$ho(gof)(0) = \frac{1}{1}$$

$$ho(gof)(0) = 1$$

2

Q.2(iii)

De-Morgan's law

If A and B are two subsets of a universal set U then

(i) $(A \cup B)' = A' \cap B'$ — 1

(ii) $(A \cap B)' = A' \cup B'$

Proof:-

Let $x \in (A \cup B)'$ $\quad [\because x \in A \Rightarrow x \notin A']$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

2

$\Rightarrow x \in A' \cap B'$

$\therefore (A \cup B)' \subseteq A' \cap B'$ — (1)

5

Date :
P. No. :

Now, $x \in A' \cap B'$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin A \cup B$

$\Rightarrow x \in (A \cup B)'$

$\Rightarrow \therefore A' \cap B' \subseteq (A \cup B)'$ ——— (ii)

From (i) and (ii)

$(A \cup B)' = A' \cap B'$

Similarly, for second case

let $x \in (A \cap B)'$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

2

$\Rightarrow x \in A' \cup B'$

$\Rightarrow \therefore (A \cap B)' \subseteq A' \cup B'$ ——— (iii)

and

let $x \in A' \cup B'$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow x \in (A \cap B)'$

$\therefore A' \cup B' \subseteq (A \cap B)'$ ——— (iv)

6

Date :
P. No. :

From (iii) and (iv)

$$(A \cap B)' = A' \cup B'$$

Hence De-Morgan's law proved.

Q. 3 (i)

The set $P(S)$ is a Boolean Algebra since all the axioms hold true.

(B_1) Since $\cap$ and $\cup$ are commutative operations. Therefore

$$A, B \in P(S) \Rightarrow \begin{cases} A \cap B = B \cap A \\ A \cup B = B \cup A \end{cases}$$

1

(B_2) Distributive law

let $A, B, C \in P(S)$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

1

(B_3) Identity law

since $A \cup \phi = A$ $\quad A \cap S = A$

ϕ is the least or zero element

and S is the unit element

1

(B_4) Complement laws

$A \in P(S)$

$$A \cup \bar{A} = S = \bar{A} \cup A, \quad A \cap \bar{A} = \phi = \bar{A} \cap A$$

1

(8)

Date :
P. No. :

ϕ, is minimal element because:
No element ~~such~~ is related to ϕ. other than it'self.

Q.3 (iii)

Given $f(x,y,z) = (xy' + xz)' + x'$

Conjunctive Normal form is

$f(x,y,z) = (xy' + xz)' + x'$

$= (xy')' \cdot (xz)' + x'$ $\left[\because (a+b)' = a' \cdot b'\right]$

$= (x' + (y')') \cdot (x' + z') + x'$ $\left[(a \cdot b)' = a' + b'\right]$

$= (x' + y)(x' + z') + x'$

2.5

$= x' + (x' + y) \cdot (x' + z')$ [Distributive law]
$[a + bc = (a+b)(a+c)]$

$= (x' + x' + y) \cdot (x' + x' + z')$

$= (x' + y) \cdot (x' + z')$

$= (x' + y + zz') \cdot (x' + y \cdot y' + z')$

$= (x' + y + z) \cdot (x' + y + z') \cdot (x' + y + z')(x' + y' + z')$

$= (x' + y + z)(x' + y + z')(x' + y' + z')$

Which is required conjunctive normal form

(1)

Date :
P. No. :

$\bar{A} = S - A$ 1+

$\bar{\phi} = S$ and $\bar{S} = \phi$

If S has n elements then $P(S)$ has 2^n elements

Hence $P(S)$ is Booleon Algebra.

Q. 3 (ii)

Maximal elements

Let $(P, \leq)$ be partially ordered set (poset). An element m in P is said to be a maximal element if

$m \leq x \Rightarrow m = x$ (for any $x \in P$) 2+

Minimal Element

Let $(P, \leq)$ be a partially ordered set An element ~~a~~ n in P is said to be a minimal element if for any $x \in P$ 2+

$x \leq n \Rightarrow x = n$

Example :-

Let $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

is power set of $A = \{1,2,3\}$

maximal element of $P(A)$ is $\{1,2,3\}$ 1+

Because $\{1,2,3\}$ is not related to any other element of $P(A)$ other than itself

(9)

Date :
P. No. :

(b) $E = (x+y)\cdot(x+z') + (y+z')$

$E = x + yz' + y + z'$ $\quad [(a+b)\cdot(a+c) = a+bc]$

$E = x + y + z' + yz'$

$E = x+y + z'(1+y)$ $\quad$ [Distributive law]

$E = x+y+z'$ $\quad [1+a=1]$

2. $E = x(y+y') + (x+x')y + (y+y')z'$

$[a+a'=1]$

$E = xy + xy' + xy + x'y + yz' + y'z'$

$E = xy + xy' + x'y + yz' + y'z'$

$E = xy(z+z') + xy'(z+z') + x'y(z+z')$

$+ (x+x')yz' + (x+x')y'z'$

$E = xyz + xyz' + xy'z + xy'z' + x'yz + x'yz'$

$+ xyz' + x'yz' + xy'z'$

$+ x'y'z'$

$E = xyz + xyz' + xy'z + xy'z' + x'yz$

$+ x'yz' + x'y'z'$

~~Which is disjunctive Normal form.~~

Q. 4 (i)

let H_1 and H_2 are two subgroup of Group G

$1\frac{1}{2}$ We have to prove that $H_1 \cap H_2$ is also subgroup of G

For this we will prove ~~xy~~ $\cdot$ If

$x \in H_1 \cap H_2$ and $y \in H_1 \cap H_2$

then

$xy^{-1} \in H_1 \cap H_2$

proof:-

since H_1 and H_2 are subgroup of G

$\Rightarrow H_1 \cap H_2 \subseteq G$

$1\frac{1}{2}$ let

$x \in H_1 \cap H_2$ and $y \in H_1 \cap H_2$

$\Rightarrow x \in H_1$ and $x \in H_2$ and $y \in H_1$ and $y \in H_2$

$\Rightarrow xy^{-1} \in H_1$ and $xy^{-1} \in H_2$

2 [since H_1 and H_2 are subgroup of G]

$\Rightarrow xy^{-1} \in H_1 \cap H_2$

$\Rightarrow H_1 \cap H_2$ is subgroup of G.

Date :
P. No. :

Alternate Method

Suppose H_1 and H_2 are two subgroup of G

then $H_1 \cap H_2 \neq \phi$

Because at least 'e' (identity element) will be in common.

If $H_1 \cap H_2 = \{e\}$ then $H_1 \cap H_2$ is trivial subgroup

In order to prove $H_1 \cap H_2$ is subgroup of G ~~it is sufficient to prove that~~

$a \in H_1 \cap H_2$, $b \in H_1 \cap H_2 \Rightarrow a \cdot b^{-1} \in H_1 \cap H_2$

Now,

Let

$a \in H_1 \cap H_2 \Rightarrow a \in H_1$ and $b \in H_2$

$b \in H_1 \cap H_2 \Rightarrow b \in H_1$ and $b \in H_2$

But H_1 and H_2 are subgroup, therefore

$a \in H_1$, $b \in H_2 \Rightarrow ab^{-1} \in H_1$,

$a \in H_2$; $b \in H_2 \Rightarrow ab^{-1} \in H_2$

finally,

$ab^{-1} \in H_1$; $ab^{-1} \in H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$

$H_1 \cap H_2$ is subgroup of G.

(12)

Date :
P. No. :

Q. 4 (ii)

Let $Q^+ = \left\{ \frac{p}{q} \ ; \ p, q \in Z^+ \right\}$

is a set of positive Rational number

and $a * b = \frac{1}{4} a.b \quad \forall a, b \in Q^+$

1

Closure law

Let a and b $\in Q^+$

$$a * b = \frac{1}{4} a.b \in Q^+$$

1

$\therefore$ Q^+ satisfy closure law

Associative law

let a, b and c $\in Q^+$

$$(a*b)*c = \left(\frac{ab}{4}\right) * c$$

$$= \frac{(abc)}{16} \quad \text{--- (i)}$$

1

$$a*(b*c) = a * \frac{(bc)}{4}$$

$$= \frac{abc}{16} \quad \text{--- (ii)}$$

from (i) and (ii)

$$(a*b)*c = a*(b*c)$$

Q^+ satisfy Associative law

(13)

Date :
P. No. :

Existence of
Identity law

Let e is identity element for all $a \in \mathbb{Q}^+$ with respect to '$*$' then

$$\Rightarrow a * e = e * a = a$$

$$\Rightarrow \frac{a \cdot e}{4} = a$$

$$\Rightarrow \boxed{e = 4}$$

$$\Rightarrow e = 4 \in \mathbb{Q}^+$$

Identity element exist in $\mathbb{Q}^+$

Existence of inverse element

Let a^{-1} is inverse element for $a \in \mathbb{Q}^+$ then

$$a * a^{-1} = a^{-1} * a = e$$

$$\Rightarrow \frac{a \cdot a^{-1}}{4} = \frac{a^{-1} \cdot a}{4} = e$$

$$\Rightarrow \frac{a \cdot a^{-1}}{4} = 4$$

$$\Rightarrow a^{-1} = \frac{16}{a} \in \mathbb{Q}^+$$

14

Date :
P. No. :

~~Hence inverse element exist in Q^+~~

Therefore Q^+ is a Group under the operation '$\times$'

Q.4 (iii)

Proof:-

Suppose H is subgroup of G and Ha and Hb are any two right cosets of H in G then

we are to prove that Ha and Hb are either disjoint or identical

2

i.e either $Ha \cap Hb = \phi$

or

$Ha = Hb$

Suppose Ha and Hb are not disjoint Then there exist at least one element say c, such that $c \in Ha$ and $c \in Hb$

1

Let $c = h_1 a$ and $c = h_2 b$ where $h_1, h_2 \in H$

Now $h_1 a = h_2 b \Rightarrow h_1^{-1} h_1 a = h^{-1} h_2 b$

[$\because$ H is subgroup. $\therefore h_1 \in H \Rightarrow h_1^{-1} \in H$]

1

$\Rightarrow ea = (h^{-1} h_2) b$

$\Rightarrow a = (h^{-1} h_2) b$ [$\because h_1^{-1} h_1 = e$]

$\Rightarrow Ha = H(h_1^{-1} \cdot h_2) b$

15

Date :
P. No. :

$\Rightarrow Ha = (H h_1^{-1} h_2)(b)$

$\Rightarrow Ha = Hb$

[$\because$ H is subgroup and $h_1 \in H, h_2 \in H$

$\Rightarrow h_1^{-1} \in H, h_2 \in H$

$\Rightarrow h_1^{-1} h_2 \in H \Rightarrow H h_1^{-1} h_2 = H$]

If the two right cosets are not disjoint then they are identical

Therefore either $Ha \cap Hb = \phi$

or $Ha = Hb$

similarly, we can prove that

either $aH \cap bH = \phi$

or $aH = bH$

If H be a subgroup of G, then any two left cosets of H are either disjoint or Identical.

Date :
P. No. :

Q.5 (i)

Proof:-

Let $G=(V,E)$ be a graph and let number of edges in G be e

We have to prove $\sum\limits_{v \in V} \deg(v) = 2e$

where $v \in V$ is any vertex

We shall prove the theorem by induction on the number of edges in the following steps

Let Graph has no edge

i.e $e=0$

~~then G has only one isolated vertex~~ $\overset{\bullet}{v}$

$\deg(v) = 0$

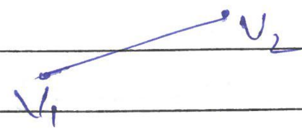
$\sum \deg v = 2 \cdot e$

Statement is true for $e=0$

Let Graph G has only one edge

i.e $e=1$

then no. of vertices is 2 say v_1 and v_2



$\deg v_1 + \deg v_2 = 1+1$

$\sum \deg v = 2$

(17)

Date :
P. No. :

$\sum \deg V = 2.e$

Statement is true for $e=1$

(1) Now, we assume that the theorem is true for edges $e-1$

$\Rightarrow \sum \deg V = 2(e-1)$

We if we add one more edge to the graph then no. of edges become is 'e' and sum of all degree is increased by 2

(1)

$\Rightarrow \sum \deg V = 2(e-1)+2$

$\Rightarrow \sum \deg V = 2e-2+2$

$\Rightarrow \sum \deg V = 2e$

$\Rightarrow$ Hence statement is true for all value of e

$\Rightarrow$ Hence theorem is proved

(18)

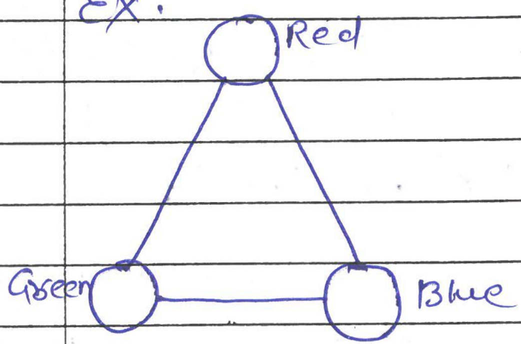
Date :
P. No. :

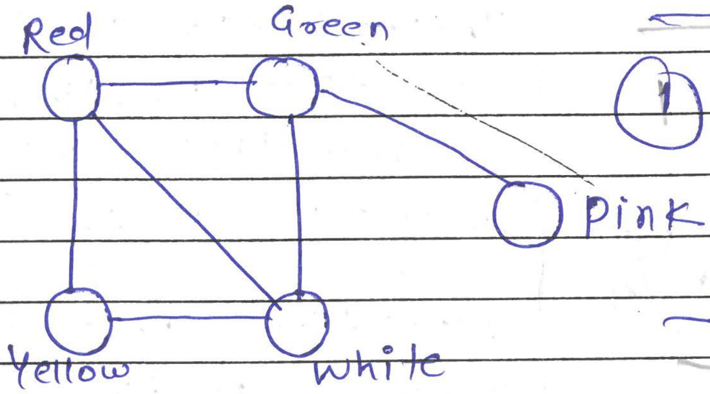
Q·5 (ii)

Graph Colouring/proper Colouring

The process of colouring the vertices of a Graph G in such a manner that no two adjacent vertices have same colour. (2)

Ex.





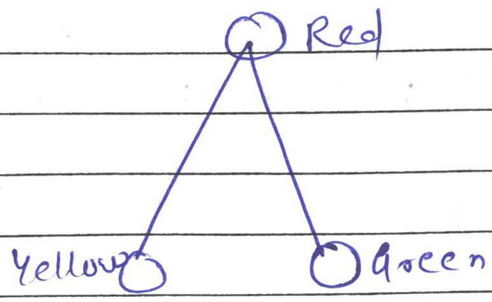
(1)

Chromatic Number

Least number of colour required for proper colouring is called chromatic number

it is denoted by χ (2)

chromatic Number of triangle is 3



(19)

Date :
P. No. :

5(iii)

Let G be a simple graph with n vertices $v_1, v_2, v_3, \ldots v_n$ say.

$1\frac{1}{2}$ The vertex v_1, can be joined to the remaining $n-1$ vertices, $v_2, v_3, v_4, v_5 \ldots v_5$. to obtain a maximum number $n-1$ of edges namely $(v_1, v_2), (v_1, v_3) \ldots (v_1, v_n)$

$1\frac{1}{2}$ The vertex v_2 can be joined to $n-2$ vertices $v_3, v_4, v_5 \ldots v_n$ to obtain a maximum number $n-2$ of edges namely $(v_2, v_3)(v_2 v_4), \ldots (v_2, v_n) \ldots$

Note that in this case we have not joined v_2 to v_1 since this edges (v_1, v_2) has already obtained.

proceeding in this manner the vertex v_{n-1} will give us only one new edge namely (v_{n-1}, v_n)

(2) Hence maximum number of edges in the graph G is.

$$=(n-1)+(n-2)+(n-3)\cdots+2+1$$

$$=\frac{n(n-1)}{2}$$

Hence proved.

20

Date :
P. No. :

Q.6 (i)

Given,

$a_r - 4a_{r-1} + 3a_{r-2} = 0$ — (I) $r \geq 2$.

~~and boundary condition~~ $y_0 = 2$, ~~and~~ $y_1 = 4$

~~Let $a_r = m^r$~~

$\Rightarrow m^r - 4m^{r-1} + 3m^{r-2} = 0$

$\Rightarrow m^2 - 4m + 3 = 0$

2 $\Rightarrow m^2 - 3m - m + 3 = 0$

$\Rightarrow (m-3)(m-1) = 0$

$\Rightarrow m = 1, 3$

$a_r^{(h)} = c_1 m_1^r + c_2 m_2^r$

$a_r^{(h)} = c_1 + c_2 3^r$ — (II)

put $r = 0$ in eq (II)

2 $a_0 = 2$

$c_1 + c_2 = 2$ — (III)

put $r = 1$ in eq. (II)

$a_1 = 4$

$c_1 + 3c_2 = 4$ — (IV)

21

Date :
P. No. :

Solving eq (III) and (IV)

$C_1 = 1, \quad C_2 = 1$

1

Hence Required solution is

$$\boxed{a_r^{(h)} = 1 + 3^r}$$

Alterate Method

Solution by the method of generating function.

Given Equation can be written as

1 Given $y_{k+2} - 4y_{k+1} + 3y_k = 0$ —— (1)

with $y_0 = 2, \quad y_1 = 4$

Consider the generating function

$$y(t) = \sum_{k=0}^{\infty} y_k t^k = y_0 + y_1 t + y_2 t^2 + \cdots$$

Multiply (1) by t^k and summing

1 from $k=0$ to $k=\infty$

$$\Rightarrow \sum_{k=0}^{\infty} y_{k+2} t^k - 4\sum_{k=0}^{\infty} y_{k+1} t^k + 3\sum_{k=0}^{\infty} y_k t^k = 0$$

$$\Rightarrow (y_2 + y_3 \cancel{k} + y_4 t^2 + \cdots) - 4(y_1 + y_2 t + y_3 t^2 + \cdots) + 3 \cancel{y(t)}\, y(t) = 0$$

$$\frac{y(t)-y_0-y_1 t}{t^2} - 4\left[\frac{y(t)-y_0}{t}\right] + 3\,y(t) = 0$$

1

From condition $y_0 = 2,\ y_1 = 4$

$$\Rightarrow \frac{y(t)-2-4t}{t^2} - 4\left[\frac{y(t)-2}{t}\right] + 3\,y(t) = 0$$

$$\Rightarrow \frac{y(t)-2-4t-4t\,y(t)+8t+3t^2 y(t)}{t^2} = 0$$

1

$$\Rightarrow (3t^2-4t+1)\,y(t) = 2-4t$$

$$\Rightarrow y(t) = \frac{2-4t}{3t^2-4t+1}$$

$$\Rightarrow y(t) = \frac{2-4t}{3t^2-3t-t+1}$$

$$\Rightarrow y(t) = \frac{2-4t}{3t(t-1)-1(t-1)}$$

1

$$\Rightarrow y(t) = \frac{2-4t}{(3t-1)(t-1)}$$

23

Date :

P. No. :

$$Y(t) = \frac{2-4t}{(1-3t)(1-t)}$$

$$Y(t) = \frac{1}{1-t} + \frac{1}{1-3t}$$

$$Y(t) = (1-t)^{-1} + (1-3t)^{-1}$$

$$Y(t) = (1+t+t^2+\cdots) + (1+3t+(3t)^2 + \cdots)$$

$$= \sum_{k=0}^{\infty} t^k + \sum_{k=0}^{\infty} 3^k t^k$$

$$Y(t) = \cancel{\infty}$$

$$Y(t) = \sum_{k=0} (1+3^k)\, t^k$$

Equating Coefficient

$$Y(t) = \sum_{k=0}^{\infty} Y_k t^k = \sum_{k=0}^{\infty} (1+3^k) t^k$$

Hence solution

$$\boxed{Y_k = 1+3^k}$$

24

Date :
P. No. :

Q.6 (ii)

Given

$a_r - 4a_{r-1} + 4a_{r-2} = 0$ given that

$a_0 = 1,\ a_1 = 3$

Auxiliary Equations

Let $a_r = m^r \quad a_{r-1} = m^{r-1} \quad a_{r-2} = m^{r-2}$

$\Rightarrow m^r - 4m^{r-1} + 4m^{r-2} = 0$

2

$\Rightarrow m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0$

$\Rightarrow m = 2, 2$

homogeneous solution

$a_r^{(h)} = (c_1 + c_2 r)\, 2^r$

put $r = 0$,

$\Rightarrow a_0 = (c_1 + c_2 0)\, 2^0$

$\Rightarrow 1 = c_1$.

put $r = 1$

2

$a_1 = (c_1 + c_2)\, 2^1$

$\Rightarrow 3 = 2c_1 + c_2$

$\Rightarrow 3 = 2 + 2c_2$

$\Rightarrow 2c_2 = 3 - 2 \Rightarrow c_2 = \frac{1}{2}$

25

Date :
P. No. :

Hence solution

1 $$\boxed{a_r^{(h)} = \left(1+\frac{r}{2}\right)2^r}$$

$$a_r^{(h)} = (2+r)\,2^{r-1}$$

Q. 6 (iii)

Given n and r be non-negative integers such that $r \le n$.

Then

L.H.S.

1 $$^nC_r + {}^nC_{r-1} = \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

1 $$= \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)\,(n-r)!}$$

$$= \frac{n!}{r\,(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)(n-r)!}$$

1 $$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n-r+1+r}{r\,(n-r+1)}\right]$$

26

Date :
P. No. :

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{(n+1)}{r(n-r+1)}\right]$$

$$= \frac{(n+1)n!}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!}$$

$${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}$$