iii. Find the eigen values and eigen vector of the matrix **5**

$$A = \begin{bmatrix} 2 & 2 & 1 \\ -4 & 8 & 1 \\ -1 & -2 & 0 \end{bmatrix}$$

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2018

CA3CO15 Algebra

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated.Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The statement of the type '$p \Rightarrow q$ and $q \Rightarrow p$' i.e. '$p \Rightarrow q \wedge q \Rightarrow p$' is called **1**

(a) Conditional (b) Conjunction

(c) Negation (d) Bi-conditional

ii. The statement of the type $p \downarrow q$, read as 'neither p nor q' is called **1**

(a) Negation (b) Bi-conditional

(c) Conditional (d) Joint Denial

iii. If $(G,.)$ is a group such that 2a=e, $\forall a \in G$, then G is **1**

(a) Semi group (b) Abelian group

(c) Non-Abelian group (d) None of these

iv. The inverse of -i in the multiplicative group {1, -1, i, -i} is **1**

(a) 1 (b) -1 (c) i (d) -i

v. A commutative ring R which contains no proper zero – divisors is called an **1**

(a) Ring (b) Kinds of rings

(c) Integral Domain (d) None of these

vi. The intersection P of all sub fields of a field F is called the **1**

(a) Ring of F (b) Prime field of F

(c) Sub field of F (d) Prime Sub field of F

vii. Let P_2 be the vector space of polynomials of degree at most 2, and let B be the basis $B = \{1, 1+x, 1+\}$. Find the B coordinate $[P]_B$ of the polynomial $P(x) = (1-x)^2$ **1**

(a) $\begin{bmatrix}1\\2\\3\end{bmatrix}$ (b) $\begin{bmatrix}2\\-2\\1\end{bmatrix}$ (c) $\begin{bmatrix}3\\2\\1\end{bmatrix}$ (d) $\begin{bmatrix}1\\1\\-1\end{bmatrix}$

viii. Let V be the vector space of all function $f(x)$ where $f: R \to R$. Which of the following are spaces of V? **1**

I. The constant functions;

II. Functions with $\lim_{x\to\infty} f(x) = 3$;

III. Functions with f(1)=1;

IV. Functions with f(0)=0;

(a) I,II and III (b) I, II, III and IV

(c) II, III and IV only (d) I and IV only

ix. The vector $\begin{bmatrix}1\\0\\1\end{bmatrix}$ is an eigen vector for the matrix $\begin{bmatrix}2 & 5 & 1\\1 & 7 & -1\\1 & 0 & 2\end{bmatrix}$ **1**

What is the corresponding eigen value?

(a) 3 (b) 2 (c) 0 (d) -1

x. What are the eigen values of the matrix $\begin{bmatrix}5 & 4\\-2 & -1\end{bmatrix}$? **1**

(a) 0,1 (b) 5,-1 (c) 1,3 (d) -1,-3

Q.2 i. For every element of Boolean Algebra B , prove that **2**

(a) $a + a.b = a$ (b) $a.(a+b) = a$

ii. Is the statement $(\sim p) \vee q$ is a tautology? **3**

iii. In a Boolean algebra $(B, +, ., ')$, prove that **5**

(a) $(a+b).(a'+c) = a.c + a'.b$

(b) $(a+b).(b+c).(c+a) = a.b + b.c + c.a$

OR iv. State and prove the De-Morgan's Law for any statements p and q. **5**

Q.3 i. Write the definition of Groups. **2**

ii. A subgraph H of a graph G is normal if and only if $g^{-1}hg \in H$ for every $h \in H, g \in G$. **8**

OR iii. Show that a finite group of even order has at least one element of order 2. **8**

Q.4 i. Consider z, the additive group of integers and $G = \{2^n : n \in z\}$ then show that G is a cyclic w.r.t. multiplication. **3**

ii. Let G and G' be any two groups, e and e', their respective identities. If f is a homomorphism of G and G', then **7**

(a) $f(e) = e'$ (b) $f(x^{-1}) = f(x)^{-1} \forall x \in G$

(c) Ker f is normal subgroup of G.

OR iii. For all a, b in a ring R, prove that **7**

(a) $a0 = 0a = 0$ (b) $a(-b) = (-a)b = -ab$

(c) $(-a)(-b) = (ab)$

Q.5 i. Let V be a vector space. If U and W be subspace of V, then $U \cap W$ is also a subspace of V. **4**

ii. Determine whether the set $S = \{1 + x, x + x^2, x^2 + 1\}$ of P_2 vector space of polynomials of degree ≤ 2 is linearly independent or linearly dependent. **6**

OR iii. Find the coordinates of $x^2 + 2x - 1$ with respect to $B = \{x + 1, x^2 + x + 1, x^2 - x + 1\}$ ordered basis of P_2. **6**

Q.6 Attempt any two:

i. Let $U = V_3$ and $V = V_2$, let $T: V_3 \to V_2$ be defined by $T[x_1, x_2, x_3] = [x_1 - x_3, x_2 + x_3] \forall x_1, x_2, x_3 \in V_3$ then show that T is a linear transformation. **5**

ii. Let $T: U \to V$ be a linear map which is one – one. A subset $\{u_1, u_2, \ldots\ldots\ldots u_n\}$ of U is linearly independent, if and only if $\{T(u_1), T(u_2) \ldots\ldots T(u_n)\}$ is linearly independent. **5**

B.C.A.

CA3C015
Algebra

Q.1 (i) (d) Bi-conditional +1

(ii) (d) Joint Denial +1

(iii) (b) Abelian group +1

(iv) (c) i +1

(v) (c) Integral domain +1

(vi) (c) sub-field of F +1

(vii) (b) $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$ +1

(viii) (d) I and IV only +1

(ix) (a) 3 +1

(x) (c) 1, 3 +1

Q.2 (i) (a) $a + a \cdot b = a \cdot 1 + a \cdot b$

$= a(1+b) = a \cdot 1 = a$ +1

(b) $a \cdot (a+b) = a \cdot a + a \cdot b$ [Distributive law]

$= a + a \cdot b$ [$\because a \cdot a = a$

$= a$ +1

(ii)

p	~p	q	~p ∨ q
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	T

+2

+1

~p ∨ q is not a Tautology

(iii) L.H.S. $= (a+b) \cdot (a' + c)$

$= (a+b) \cdot a' + (a+b)c$ [Distr. Law]

$= a \cdot a' + b \cdot a' + ac + bc$ [Dist. law]

$= 0 + a' \cdot b + a \cdot c + b \cdot c$ [By commutative. +1

$= a'.b + a.c + (b.c).1$ [$\because y = y.1$

$= a'.b + a.c + (b.c)(a+a')$ [$\because a+a'=1$

$= a'.b + a.c + (b.c).a + (bc).a'$ [By D.L]

$= [a'.b + (a'.b).c] + [(a.c) + (a.c).b]$

$= a'.b + a.c$ [$\because 1+c=1, 1+b=1$ [+1.5]

(b) ~~R.H.S. = a.b + b.c + c.a~~

L.H.S. $= (a+b)(b+c)(c+a)$

$= (a+b)[(c+b)(c+a)]$

$= (a+b)[c + b.a]$ [By Distributive law]

$= (a+b).c + (a+b).ba$ [+1]

$= ac + b.c + a.ba + b.ba$

$= ac + bc + (a.a)b + (b.b)a$

$= ac + bc + ab + ba$ [$\because a.a = a$
$b.b = b$

$= a.c + b.c + ab + a.b$ [$\because ba = ab$]

$= a.c + b.c + ab$ [$\because x + x = x$ [+1.5]

$= ab + bc + ca$

OR.

(iv) For any two elements a and b of Boolean algebra $(B, +, .,')$, we have

(i) $(a+b)' = a'.b'$ (ii) $(a.b)' = a'+b'$ [+1]

Proof: (i) To prove $(a+b)' = a'.b'$

i.e. To show that the complement of $a+b$ is $a'.b'$, for which it is sufficient to prove

$(a+b) + a'.b' = 1$

$(a+b).(a'.b') = 0$

Now $(a+b)+(a'.b')$

$= [(a+b)+a'].[(a+b)+b']$ [by Distri. law]

$= [(a+a')+b].[a+(b+b')]$

$= [1+b].[a+1]$

$= 1.1 = 1$ [$\because 1+x=1$ [+2]

and

$(a+b).(a'.b') = a.(a'.b')+b.(a'.b')$

$=$ ~~a+b~~ [By Distributive law]

$= (a.a')b + (b.b')a'$ [Asso. law

$= 0.b+0.a$ [$\because x.x'=0$

$= 0+0=0$ [+2]

Q.3 (i) An algebraic structure (G, o), where G is non-empty set with binary operation o, is called a group if

G_1 closure law:

If $a, b \in G \Rightarrow aob \in G, \forall a, b \in G$

G_2 Associative law:

$\forall a, b, c \in G$,

$ao(boc) = (aob)oc, \forall a, b, c \in G$ [+1]

G_3 $\exists$ of identity element:

$\forall a \in G \ \exists \ e \in G$ s.t

$aoe = eoa = a \quad \forall a \in G$

where e is called an identity element in G

G_4: $\exists$ of an inverse element of an element in G:

If $a \in G$ $\exists$ $a' \in G$ s.t.
$a \circ a' = a' \circ a = e$,
where a' is called an inverse of a in G. [+1]

(ii) If H is a normal subgroup of G
i.e. $xH = Hx$, $\forall x \in G$ —— (1)
To show that $g^{-1}hg \in H$ $\forall h \in H, g \in G$ [+1]

If $g \in G$, then by (1)

$gH = Hg$
$\Rightarrow g^{-1}(gH) = g^{-1}Hg$

$\Rightarrow (g^{-1}g)H = g^{-1}Hg$

$\Rightarrow e.H = g^{-1}Hg$

$\Rightarrow g^{-1}Hg = H$
$\Rightarrow g^{-1}Hg \subseteq H$ [+3]
$\Rightarrow g^{-1}hg \in H$, $\forall h \in H$

Only if: Suppose $g^{-1}hg \in H$, $\forall g \in G$, $h \in H$

$\Rightarrow g^{-1}Hg \subseteq H$

$\Rightarrow Hg \subseteq gH$ —— (1) [+1]

Again $g^{-1}hg \in H$
$\Rightarrow$ ~~$h \in gHg^{-1}$~~ $\Rightarrow (g^{-1}hg)H = H$
$\Rightarrow$ ~~$H \subseteq gHg^{-1}$~~ $\Rightarrow hgH = gH$
$\Rightarrow$ ~~$Hg \subseteq$~~ $\Rightarrow gH = Hg$ [+3]

OR

3 (iii) let G be a group of even order ~~and~~ ~~a∈G~~,

i.e. $|G| = 2n$ $n \in \mathbb{N}$

Since G is finite, there exists

$a \in G$ such that $a^p = e$ and by Langranges theorem, $p \mid 2n$. [+4]

By Euclid's Lemma, since p does not divide 2,

$\Rightarrow p \mid n.$ [+2]

$\Rightarrow n = pk.$

Hence $(a^n)^2 = (a^{pk})^2$

$= ((a^p)^k)^2$

$= (e^k)^2$

$= e$

There fore, $o(a^n) = 2$ [+2]

(i) Given $G = \{2^n \mid n \in \mathbb{Z}\}$

clearly G is closed w.r.t '$\circ$'
and G has associative, identity and
inverse w.r.t '$\circ$'
Every element of G is of the
form 2^n

$\Rightarrow G = \langle 2 \rangle$

$\Rightarrow$ G is cyclic [+3]

4(ii) Given $f: G \to G'$ is a homom.
(a) If $a \in G$ $\exists$ $a' \in G$ s.t

$a \cdot a^{-1} = e$

Now $f(a \cdot a') = f(e)$

$\Rightarrow f(a) \cdot f(a^{-1}) = f(e)$

$\Rightarrow f(a) \cdot f(a)^{-1} = f(e)$

$\Rightarrow f(a)^{-1}$ is an inverse of
$f(a)$ in G'

$\Rightarrow \boxed{f(e) = e'}$ [+2]

(b) $f(x^{-1}) = f(x)^{-1}$

~~$f(x) \cdot f(x)^{-1}$~~ $=$ $\because x^{-1}x = e$

$\Rightarrow f(x^{-1} \cdot x) = f(e)$

$\Rightarrow f(x^{-1}) \cdot f(x) = e'$

$\Rightarrow$

DATE
PAGE 07

Inverse of $f(x)$ is $f(x^{-1})$

$\Rightarrow f(x)^{-1} = f(x^{-1})$ [+2]

(c) Now $\ker f = \{a \in G \mid f(a) = e'\}$

Since $e \in G$ s.t. $f(e) = e'$

$\Rightarrow e \in \ker f$

$\Rightarrow \ker f \neq \phi$

If $x, y \in \ker f$

$\Rightarrow f(x) = e', \ f(y) = e'$

Now $f(xy^{-1}) = f(x) \cdot f(y^{-1})$

$= f(x) \cdot f(y)^{-1}$

$= e' \cdot e'^{-1} = e$

$\Rightarrow xy^{-1} \in \ker f$

$\Rightarrow \ker f$ is ~~normal~~ subgroup of f.

let $x \in G$, $y \in \ker f$

To show $xyx^{-1} \in \ker f$

$\Rightarrow f(xyx^{-1}) = f(x) \cdot f(y) \cdot f(x^{-1})$ [f is hom.]

$= f(x) \cdot e' \cdot f(x)^{-1}$ [$\because f(x^{-1}) = f(x)^{-1}$]

$= e' \cdot e' \cdot e'^{-1}$

$= e' \cdot e'^{-1} = e'$

$\Rightarrow xyx^{-1} \in \ker f$

$\ker f$ is normal subgroup of G [+3]

OR.

4(iii) (a) $0+0=0$

Now $a(0+0)=a\cdot 0$

$\Rightarrow a\cdot 0+a\cdot 0=a\cdot 0$ [By distr. law

$\Rightarrow a\cdot 0$ is an additive identity

$\Rightarrow a\cdot 0=0$

Similarly $0\cdot a=0$ [+2.5]

(b) Now $b+(-b)=0$

$a(b+(-b))=a\cdot 0$

$\Rightarrow a\cdot b+a(-b)=0$ [$\because a\cdot 0=0$

$\Rightarrow$ additive inverse of ab is $a(-b)$

$\Rightarrow a(-b)=-ab$

Similarly $(-a)b=-ab$ [+2.5]

(c) $(-a)(-b)=-(-a)b=-[-(ab)]$

$=(ab)$. [+2]

(i) ~~Let α,β~~ If U and W be subspace of V

$\Rightarrow 0\in U$, and $0\in W$

$\Rightarrow 0\in U\cap W$

$\Rightarrow U\cap W\neq\phi$ [+1]

If $\alpha,\beta\in U\cap W$ and a,b are scalar

$\Rightarrow \alpha,\beta\in U$ & $\alpha,\beta\in W$, a,b are scalar

$\Rightarrow a\alpha+b\beta\in U$ and $a\alpha+b\beta\in W$ [+2]

[$\because U$ and W are vector subspace]

$\Rightarrow a\alpha+b\beta\in U\cap W$, $\forall\,\alpha,\beta\in U\cap W$, a,b are scalare [+1]

$U\cap W$ are subspace of V

5 (ii) If a, b, c are scalar's, then let $a(1+x) + b(x+x^2) + c(x^2+1) = 0$

$\Rightarrow (a+c) + (a+b)x + (b+c)x^2 = 0 + 0x + 0x^2$ — ① [+1]

~~Equating to ze~~

$a + c = 0$ — ②

$a + b = 0$ — ③

$b + c = 0$ — ④ [+1]

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The rank of coefficient matrix = 3

i.e. $a + c = 0$, $b = 0$, $c = 0$ $\Rightarrow a = 0$

The set S is L.I [+3]

Q. 5(iii) Let $v = x^2+2x-1$,

$u_1 = x+1,\ u_2 = x^2+x+1,\ u_3 = x^2-x+1$

$\exists\ a, b, c$ s.t

$v = au_1 + bu_2 + cu_3$

$$\Rightarrow x^2+2x-1 = a(x+1) + b(x^2+x+1) + c(x^2-x+1)$$

$$\Rightarrow x^2+2x-1 = (b+c)x^2 + (a+b-c)x + a+b+c$$

[+2]

$b+c = 1$ —— (1)

$a+b-c = 2$ —— (2)

$a+b+c = -1$ —— (3)

[+1]

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$R_1 \longleftrightarrow R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$$

$a + b - c = 2$ — (4)

$b + c = 1$ — (5)

$2c = -3$

$c = -3/2$

from (5)

$b = 1 - c = 1 + 3/2$

$b = 5/2$

from (4)

$a = 2 - b + c$

$a = 2 - \frac{5}{2} - \frac{3}{2}$

$a = 2 - 4 = -2$ [+2]

So $v = -2u_1 + \frac{5}{2}u_2 - \frac{3}{2}u_3$

$[v]_S = \begin{bmatrix} -2 \\ 5/2 \\ -3/2 \end{bmatrix}$ [+1]

5(1) If a, b are scalar and $\alpha, \beta \in V_3$, where
$\alpha = (x_1, x_2, x_3)$, $\beta = (y_1, y_2, y_3)$ [+1]

Now $T(a\alpha + b\beta) = T(ax_1 + by_1, ax_2 + by_2, ax_3 + by_3)$ [+1]

$= (ax_1+by_1-ax_3-by_3,\ ax_2+by_2+ax_3+by_3)$

$= a(x_1-x_3,\ x_2+x_3)+b(y_1-y_3,\ y_2+y_3)$ [+1]

$= aT(\alpha)+bT(\beta)$ [+1]

$\Rightarrow$ T is a Linear transformation

6(ii) let $S_1=\{u_1,u_2,\dots,u_n\}\subseteq U$ is L.I.

To show that
$S_2=\{Tu_1,Tu_2,\dots,Tu_n\}$ is L.I.

if there exist $a_1,a_2,\dots,a_n$ are scalar's s.t
$a_1Tu_1+a_2Tu_2+\dots+a_nTu_n=0'$

$\Rightarrow T(a_1u_1+a_2u_2+\dots+a_nu_n)=T(0)$

$\Rightarrow a_1u_1+a_2u_2+\dots+a_nu_n=0$ [$\because$ T is 1-1]

$\Rightarrow a_1=0=a_2=\dots=a_n=0$
[$\because S_1$ is L.I]

$\Rightarrow$ ~~S~~ [+2.5]

$\Rightarrow S_2$ is a L.I.

<u>Conversly:</u> If $S_2=\{Tu_1,Tu_2,\dots,Tu_n\}$ is L.I.

Show that $S_1=\{u_1,u_2,\dots,u_n\}$ is L.I

$\exists$ scalar $b_1,b_2,\dots,b_n$ s.t

$b_1u_1+b_2u_2+\dots+b_nu_n=0$ [+1]

$\Rightarrow T(b_1u_1+b_2u_2+\dots+b_nu_n)=T(0)$

$\Rightarrow b_1Tu_1+b_2Tu_2+\dots+b_nTu_n=0'$ [$\because$ T is L.T. and 1-1]

$\Rightarrow b_1=b_2=b_3=\dots=b_n=0$ [+1.5]

$\Rightarrow S_1$ is L.I

6 (iii) The characteristic equation is given by

$$|A-\lambda I|=0$$

$$\lambda^3-10\lambda^2+27\lambda-18=0$$

$$\lambda^2(\lambda-1)-9\lambda(\lambda-1)+18(\lambda-1)=0$$

$$\Rightarrow (\lambda-1)(\lambda^2-9\lambda+18)=0$$

$$\Rightarrow (\lambda-1)(\lambda-3)(\lambda-6)=0$$

$$\lambda=1,3,6$$ [+2]

<u>At $\lambda=1$:</u> Let $X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ be an eigen vector corresponding to $\lambda=1$.

$$[A-\lambda I]X=0$$

$$\begin{bmatrix}1&2&1\\-4&7&1\\-1&-2&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_2\to R_2+4R_1$, $R_3\to R_3+R_1$

$$\begin{bmatrix}1&2&1\\0&15&5\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Rank of the coefficient matrix $=2$
no. of variable $=3$
so $3-2=1$ variable will be free
say $x_3=k\neq 0$

$$x_1+2x_2+x_3=0 \quad \text{--- (1)}$$

$$15x_2+5x_3=0$$

$$x_2=-\frac{1}{3}x_3 \Rightarrow x_2=-\frac{k}{3}$$

from (1)

$$x_1-\frac{2k}{3}+k=0\Rightarrow x_1=-\frac{k}{3}$$

$$X=k\begin{bmatrix}-1/3\\-1/3\\1\end{bmatrix}$$ [+1]

<u>At $\lambda = 3$</u> let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$(A - \lambda I) X = 0$$

$$\begin{bmatrix} -1 & 2 & 1 \\ -4 & 5 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 4R_1, \quad R_3 \longrightarrow R_3 - R_1$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & -3 & -3 \\ 0 & -4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \longrightarrow R_3 - \frac{4}{3} R_2$

$$\begin{bmatrix} -1 & 2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank = 2, variable = 3, so 3 − 2 = 1 variable will be free

$-x_1 + 2x_2 + x_3 = 0$ — (1)

$-x_2 + x_3 = 0$ — (2)

Say $x_3 = K \neq 0$

$x_2 = K$

~~from~~ (1)

$x_1 = 2x_2 + x_3$

$x_1 = 3K$

$$X = \begin{bmatrix} 3K \\ K \\ K \end{bmatrix} = K \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$ [+1]

<u>At $\lambda = 6$:</u> Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be an eigen

vector corresponding to $\lambda = 6$.

$$\begin{bmatrix} -4 & 2 & 1 \\ -4 & 2 & 1 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1$ of $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -1 & -2 & 0 \\ -4 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank $= 2$, no of variable $= 3$

So $3 - 2 = 1$ variable will be free

$x_2 = k$ (say)

$-x_1 - 2x_2 = 0$

$-4x_1 + 2x_2 + x_3 = 0$ —— (1)

$x_1 = -2k$

from (1)

$8k + 2k + x_3 = 0$

$x_3 = -10k$

$$X = k\begin{bmatrix} -2 \\ 1 \\ -10 \end{bmatrix}$$

[+1]