Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2022

CS3EA09 Graph Theory

Programme: B.Tech. Branch/Specialisation: CSE

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. What is a bipartite graph? **1**

(a) A graph which contains only one cycle
(b) A graph which consists of more than 3 number of vertices
(c) A graph which has odd number of vertices and even number of edges
(d) A graph which contains no cycles of odd length

ii. The number of edges in a regular graph of degree 46 and 8 vertices is _____. **1**

(a) 347 (b) 230 (c) 184 (d) 186

iii. There is a region in a plane that cannot be subdivided further because it is bounded by _____? **1**

(a) Vertices (b) Edges
(c) Endpoints (d) None of these

iv. Which of the following is false in the case of a spanning tree of a graph G? **1**

(a) It is tree that spans G
(b) It is a subgraph of the G
(c) It includes every vertex of the G
(d) It can be either cyclic or acyclic

v. A matching that matches all the vertices of a graph is called- **1**

(a) Cardinality matching (b) Good matching
(c) Simplex matching (d) Perfect matching

P.T.O.

vi. What is chromatic number? **1**

(a) The maximum number of colors required for proper edge coloring of graph

(b) The maximum number of colors required for proper vertex coloring of graph

(c) The minimum number of colors required for proper vertex coloring of graph

(d) The minimum number of colors required for proper edge coloring of graph

vii. If $^{16}P_{r-1} : {}^{15}P_{r-1} = 16 : 7$ then find r- **1**

(a) 10 (b) 12 (c) 7 (d) 8

viii. In the expansion of $(a+b)^n$, if n is even then the middle term is- **1**

(a) $(\frac{n}{2})^{th}$ term (b) $(\frac{n}{2}+1)^{th}$ term

(c) $(\frac{n+1}{2})^{th}$ term (d) $(\frac{n+3}{2})^{th}$ term

ix. What is the generating function for generating series 1,2,3,4,5,…? **1**

(a) $\frac{2}{(1-3x)}$ (b) $\frac{1}{(1-x)}$ (c) $\frac{1}{(1-x)^2}$ (d) $\frac{1}{(1-x^2)}$

x. Find the value of a_4 for the recurrence relation $a_n = 2a_{n-1} + 3$, with $a_0 = 6$- **1**

(a) 141 (b) 221 (c) 320 (d) 65

Q.2 i. What is binary tree and its properties? **2**

ii. Define the following terms with a suitable example: **3**

(a) Subgraph (b) Connected graph

iii. Prove that a tree with n vertices has (n-1) edges. **5**

OR iv. Define the following terms with an example for each: **5**

(a) Euler graph (b) Hamiltonian circuit

Q.3 i. What is 1-isomorphism and 2-isomorphism in graph theory? **2**

ii. What do you understand by fundamental circuit explain with an example? **3**

iii. Prove that the ring sum of any two cut-sets in a graph is either a third cut-sets or an edge-disjoint union of cut-sets. **5**

OR iv. What is minimum spanning tree? Write the steps of Kruskal's algorithm and explain with the help of an example. **5**

Q.4 i. Explain digraph and how to find indegree and outdegree of a graph? **3**

ii. What is chromatic number? Explain the four-colour problem in graph theory. **7**

OR iii. What do you understand by matching? Define and explain the problem of distinct representation. **7**

Q.5 i. What do you understand by the fundamental principles of counting, explain with the help of the examples? **4**

ii. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has **6**

(a) No girls

(b) At least one boy and one girl

(c) At least three girls

OR iii. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken a course in at least one of Spanish French and Russian. Then calculate how many students have taken a course in all 3 languages and draw a Venn diagram related to the information given in the problem. **6**

Q.6 Attempt any two:

i. What do you understand by the term generating function? Find the generating function for $a_r = 3^r, r \geq 0$. **5**

ii. Solve by the method of generating function the recurrence relation $a_r - 4a_{r-1} + 3a_{r-2} = 0, r \geq 2$ With the boundary conditions $y_0 = 2$ and $y_1 = 4$. **5**

iii. Solve the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 3$. **5**

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Scheme of Marking

MEDI-CAPS UNIVERSITY Knowledge is Power	Faculty of Engineering End Sem (Odd) Examination Dec-2022 Graph Theory-CS3EA09(T) Programme: B.Tech. | Branch/Specialisation:

Note: The Paper Setter should provide the answer wise splitting of the marks in the scheme below.

Q.1	i)	d) a graph which contains no cycles of odd length	**1**
	ii)	c) 184	**1**
	iii)	c) Edges	**1**
	iv)	d) It can be either cyclic or acyclic	**1**
	v)	c) The minimum number of colors required for proper vertex coloring of graph	**1**
	vi)	d) Perfect matching	**1**
	vii)	a) 10	**1**
	viii)	$b)\ (\frac{n}{2}+1)^{th}$ term	**1**
	ix)	$c)\ \frac{1}{(1-x)^2}$	**1**
	x)	a) 141	**1**
Q.2	i.	Definition 1 mark Properties 1 mark	**2**
	ii.	a. Definition + example(1+0.5=1.5marks) b. Definition+ example (1+0.5=1.5 marks)	**3**
	iii.	result is true for n=1 and n=2 by the mathematical induction method 2 marks	**5**
		Assume the result is true for (n-1) and prove for the n vertices 3 marks	
OR	iv.	a. Definition + example(1.5+1=2.5marks) b. Definition+ example (1.5+1=1.5 marks)	
Q.3	i.	Definition of 1-isomorphism 1 mark Definition of 2-isomorphism 1 mark	**2**
	ii.	Definition 2 marks Example 1 mark	**3**
	iii.	For initial notation and assumption 2 marks And 3 marks for the further result 3 marks	**5**
OR	iv.	Definition 1 mark Steps of Kruskal's Algorithm 2 marks Example 2 marks	
Q.4	i.	Definition 1 mark Indegree and out degree with example 2 marks	**3**
	ii.	Definition of chromatic number 2 marks Proof of four colour problem 5 marks	**7**
OR	iii.	Definition of matching 2 marks Statement of problem of distinct representation 2 marks Explanation for the problem of distinct representation 3 marks	**7**
Q.5	i.	Fundamental principles of counting 2 marks Example 2 marks	**4**
	ii.	(i) no girls=21 2 marks (ii) at least one boy and one girl =441 2 marks (iii) at least three girls=91 2 marks	**6**
OR	iii.	$\|S\| = 1232, \|F\| = 879, \|R\| = 114, \|S\cap F\| = 103, \|S\cap R\| = 23,$ $\|F\cap R\| = 14$, and $\|S\cup F\cup R\| = 23$. 1mark inclusion-exclusion formula for three sets 1 mark $(S \cap F \cap R)$=7 2 marks Venn diagram 2 marks	**6**

Q.6			
	i.	Definition of generating function 2 marks $A(z) - 1 = 3z + (3z)^2 + (3z)^3 + \cdots$ 1 mark $A(z) = \frac{1}{1-3z}$ 2 marks	**5**
	ii.	$G(z) - a_1 z - a_0 - 4z[G(z) - a_0] + 3z^2 G(z) = 0$ 2 marks $G(z) = \frac{1}{1-z} + \frac{1}{1-3z}$ 2 marks $a_r = 1 + 3^r$, $r \geq 0$ 1 mark	**5**
	iii.	$\frac{1}{(1-x)(1-2x)} = \frac{-1}{(1-x)} + \frac{2}{(1-2x)}$ 2 marks $\frac{-1}{(1-x)}$ generates -1,-1,-1,... 1 mark $\frac{2}{(1-2x)}$ generates 2,4,8,16,.... 1 mark The solution is $a_n = 2^{n+1} - 1$ 1 mark	**5**

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