iii. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = xy$ for $x = 1.4$ initially $x = 1$, $y = 2$ and take $h = 0.2$. **5**

Q.6 Attempt any two:

i. Calculate the Karl Pearson's coefficient of correlation between X and Y series: **5**

X	17	18	19	19	20	20	21	21	22	23
Y	12	16	14	11	15	19	22	16	15	20

ii. Fit a second degree parabola to the following data: **5**

X	1	2	3	4	5
Y	1090	1220	1390	1625	1915

iii. Two random samples drawn from two normal populations are: **5**

Sample 1 : 20 , 16 , 26 , 27 , 23 , 22 , 18 , 24 , 25 , 16

Sample 2 : 27 , 33 , 42 , 35 , 32 , 37 , 38 , 28 , 41 , 43 , 30 , 37 .

By F- test obtain the estimates of the variances of the population and test whether the two populations have the same variance (for degree of freedom $v_1 = 9, v_2 = 11$, the 5% value of F is 3.112).

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Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2019

EN3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: AU/CE/FT/ME

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The value of $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$ is **1**

(a) $\left(\frac{1}{3}-\frac{1}{2}i\right)$ (b) $\left(\frac{1}{2}-\frac{1}{3}i\right)$ (c) $\left(\frac{5}{6}-\frac{1}{6}i\right)$ (d) $\left(\frac{1}{6}-\frac{5}{6}i\right)$.

ii. Harmonic conjugate of $x^3 - 3xy^2$ is **1**

(a) $3x^2y - y^3 + c$ (b) $3xy - y^3 + c$

(c) $3xy^2 - y^3 + c$ (d) $3xy - y^2 + c$

iii. If $f(x) = 0$ is an algebraic equation, then Newton-Raphson method is given by $x_{n+1} = x_n - \frac{f(x_n)}{?}$ **1**

(a) $f(x_n - 1)$ (b) $f'(x_n - 1)$

(c) $f'(x_n)$ (d) $f''(x_n)$

iv. As soon as a new value of a variable is found by iteration, it is used immediately in the following equations, this method is called **1**

(a) Gauss-Jordan method (b) Gauss-Seidel method

(c) Jacobi's method (d) Relaxation method

v. If $y = x(x-1)(x-2)(x-3)$, then Δy is **1**

(a) $x(x-1)(x-3)$ (b) $3x(x-1)(x-2)$

(c) $3x(x-1)(x-3)$ (d) $x(x-1)(x-2)(x-3)$

vi. The nth divided difference of a polynomial of degree n is **1**

(a) 0 (b) A constant (c) A variable (d) None of these

vii. By Trapezoidal rule, the value of $\int_{x_0}^{x_0+6h} y\,dx$ is **1**

(a) $\frac{1}{2}h[(y_0+y_6)+4(y_1+y_2+y_3+y_4+y_5)]$

(b) $\frac{1}{2}h[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)]$

(c) $\frac{1}{2}h[(y_0+y_6)+2(y_1+y_2+y_4+y_5)]$

(d) $\frac{1}{2}h[(y_0+y_5)+2(y_1+y_2+y_3+y_4+y_6)]$

viii. The nth approximation $y^{(n)} = y_0 + \int_{x_0}^{x} f(x, y^{(n-1)})\,dx$ is given for the differential equation $\frac{dy}{dx} = f(x,y)$ with initial conditions $y = y_0$ at $x = x_0$. This formula for **1**

(a) Taylor's series method (b) Euler's method

(c) Runge's method (d) Picard's method

ix. The equation of regression line of y on x is given by **1**

(a) $y-\overline{y} = r\frac{\sigma_y}{\sigma_x}(x-\overline{x})$ (b) $y-\overline{y} = r\frac{\sigma_x}{\sigma_y}(x-\overline{x})$

(c) $x-\overline{x} = r\frac{\sigma_y}{\sigma_x}(y-\overline{y})$ (d) $x-\overline{x} = r\frac{\sigma_x}{\sigma_y}(y-\overline{y})$

x. The formula for χ^2-test is given by: (where f_o and f_e are observed an expected frequency respectively) **1**

(a) $\chi^2 = \sum\frac{(f_e-f_0)^2}{f_e}$ (b) $\chi^2 = \sum\frac{(f_0-f_e)^2}{f_e}$

(c) $\chi^2 = \sum\frac{(f_0-f_e)^2}{f_0}$ (d) $\chi^2 = \sum\frac{(f_e-f_0)^2}{f_0}$

Q.2 i. Define harmonic conjugate functions. **2**

ii. If z is a complex function, then determine whether $1/z$ is analytic or not in the entire plane. **3**

iii. Using Cauchy's integral formula evaluate $\int_C \frac{\cos \pi z}{z^2-1}dz$ around a rectangle C with vertices $2\pm i$, $-2\pm i$. **5**

OR iv. Find the poles, order of the poles and residue at it for the function $\frac{1}{z^4+1}$ **5**

Q.3 Attempt any two:

i. Find the cube root of 2 approximately by Newton-Raphson method correct to five decimal places up to fourth approximation. **5**

ii. Using Gauss's elimination method solve the linear simultaneous equations $10x-7y+3z+5u=6$, $-6x+8y-z-4u=5$, $3x+y+4z+11u=2$ and $5x-9y-2z+4u=7$. **5**

iii. Solve the following system of equations by Gauss-Seidel iteration method: $10x+2y+z=9$, $2x+20y-2z=-44$, $-2x+3y+10z=22$ **5**

Q.4 Attempt any two:

i. Using Newton's forward interpolation formula, from the following data estimate the number of students who obtained marks between 40 and 45: **5**

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students:	31	42	51	35	31

ii. Find the polynomial of fifth degree by Lagrange's interpolation formula from the given data $u_0 = -18$, $u_1 = 0$, $u_2 = 0$, $u_3 = -248$, $u_4 = 0$, $u_5 = 13104$. **5**

iii. Using Newton's backward formula find dy/dx at x=0.4 from the following data: **5**

x:	0.1	0.2	0.3	0.4
$y = f(x)$:	1.10517	1.22140	1.34986	1.49182

Q.5 Attempt any two:

i. Using Simpson's 1/3rd rule, find the value of log 2 from $\int_0^1 \frac{x^2}{1+x^3}dx$ by dividing the range into four equal parts. **5**

ii. Employ Taylor's series method to obtain approximate value of y at $x=0.2$ for $\frac{dy}{dx} = 2y+3e^x$, $y(0)=0$. Compare the numerical solution with the exact solution. **5**

P.T.O.

Medi-Caps University

Sub. Code:- EN3BS02

Sub. Name:- Engineering mathematics - III

Solution.

Q.1

(i) (c) $\left(\frac{5}{6} - \frac{i}{6}\right)$ +1

(ii) (a) $3x^2y - y^3 + c$ +1

(iii) (c) $f'(x_n)$ +1

(iv) (b) Gauss-Seidel method +1

(v) (b) $3x(x-1)(x-2)$ +1

(vi) (b) A constant +1

(vii) (b) $\frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$ +1

(viii) (d) Picard's method +1

(ix) (a) $y - \bar{y} = r \frac{\sigma_y}{\sigma_x}(x - \bar{x})$ +1

(x) (b) $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$ +1

Q.2 (i) If the function $f(z) = u(x,y) + iv(x,y)$, is analytic in a domain D, then the function u and v are said to conjugate function. +2

Q.2 (ii) let $f(z) = u + iv = \frac{1}{z}$

$$\Rightarrow u + iv = \frac{1}{x+iy} = \frac{x-iy}{x^2+y^2}$$ +1

Equating real and imaginary parts, we get

$$u = \frac{x}{x^2+y^2}, \quad v = -\frac{y}{x^2+y^2}$$

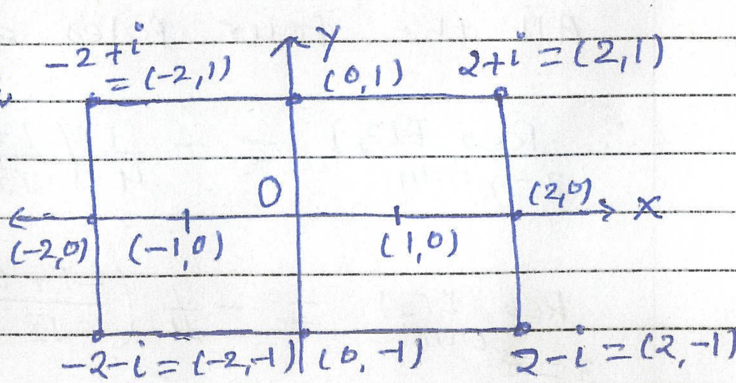
$$\frac{\partial u}{\partial x} = \frac{y^2-x^2}{(x^2+y^2)^2}, \text{ and } \frac{\partial u}{\partial y} = -\frac{2xy}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial x} = \frac{2xy}{(x^2+y^2)^2}, \text{ and } \frac{\partial v}{\partial y} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

clearly $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ +1

Thus Cauchy-Reimann equations are satisfied. Also partial derivatives are continuous except at (0,0). Therefore, $\frac{1}{z}$ is analytic everywhere except at $z=0$. +1

Q.2 (iii) Take $f(z) = \cos \pi z$, which is analytic in the region bounded by the given rectangle



+1

The singular points given by:

$$z^2 - 1 = 0 \Rightarrow z = \pm 1$$ +2

~~Clearly~~ clearly lie within the given rectangle

We have $\frac{1}{z^2-1} = \frac{1}{(z-1)(z+1)} = \frac{1}{2}\left[\frac{1}{z-1} - \frac{1}{z+1}\right]$

$\therefore \int_C \frac{f(z)}{z^2-1}\,dz = \frac{1}{2}\int_C \left(\frac{1}{z-1} - \frac{1}{z+1}\right) f(z)\,dz$ +1

$= \frac{1}{2}\int_C \frac{f(z)}{z-1}\,dz - \frac{1}{2}\int \frac{f(z)}{z+1}\,dz$

$= \frac{1}{2}\cdot 2\pi i\, f(1) - \frac{1}{2}\cdot 2\pi i\, f(-1)$ +1

$= \pi i \cos\pi - \frac{1}{2}\cdot 2\pi i \cos(-\pi)$

$= -\pi i - \pi i\cdot(-1) = 0$ +1

(iv) Let $f(z) = \frac{1}{z^4+1}$

The poles of $f(z)$ are given by

$f(z) = 0$ i.e. $z^4+1=0$

i.e. $z = e^{i\pi/4}, e^{3i\pi/4}, e^{5i\pi/4}, e^{7i\pi/4}$ +1

All the four poles are simple poles. +1

$\therefore \underset{z=e^{i\pi/4}}{\text{Res}} f(z) = -\frac{1}{4}\left(\frac{1+i}{\sqrt{2}}\right)$ +1

$\underset{z=e^{3i\pi/4}}{\text{Res}} f(z) = -\frac{1}{4}\left(\frac{-1+i}{\sqrt{2}}\right)$ +1

$\underset{z=e^{5i\pi/4}}{\text{Res}} f(z) = -\frac{1}{4}\left(\frac{-1-i}{\sqrt{2}}\right)$ +1

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$$\operatorname*{Res}_{z=e^{7i\pi/4}} f(z) = -\frac{1}{4}\left(\frac{1-i}{\sqrt{2}}\right)$$ +1

Q.3 (i) Let $x = (2)^{1/3} \Rightarrow x^3 = 2 \Rightarrow x^3 - 2 = 0$

Let $f(x) = x^3 - 2$, then $f'(x) = 3x^2$ ——(1) +1

Now $f(1) = -1$ and $f(2) = 6 \Rightarrow$ root of eq.(1) lies between 1 and 2

By Newton – Raphson formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^3 - 2}{3x_n^2}$$

$$\Rightarrow x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2}$$

$$\Rightarrow x_{n+1} = \frac{2}{3}\left[\frac{x_n^3 + 1}{x_n^2}\right] = \frac{2}{3}\left[x_n + \frac{1}{x_n^2}\right]$$

$$\Rightarrow x_{n+1} = \frac{2}{3}\left[x_n + \frac{1}{x_n^2}\right] \quad \text{—— (2)}$$ +1

Taking initial approximation: $x_0 = 1.5$

$\therefore$ The first approximation:

$$x_1 = \frac{2}{3}\left[1.5 + \frac{1}{(1.5)^2}\right] = 1.2930$$

The Second Approximation: $x_2 = 1.26093$

The Third Approximation: $x_3 = 1.25992$ +2

The Fourth Approximation: $x_4 = 1.25992$

$\therefore x_3 = x_4 = 1.25992$ correct to four decimal places.

$\therefore 2^{1/3} = 1.25992.$ +1

(ii) $$[A \mid B] = \left[\begin{array}{cccc|c} 10 & -7 & 3 & 5 & 6 \\ 5 & -9 & -2 & 4 & 7 \\ 3 & 1 & 4 & 11 & 2 \\ -6 & 8 & -1 & -4 & 5 \end{array}\right]$$ [After partial pivoting] +1

Applying elementary Row Operation, +1

Reduce the above matrix to upper triangular form, then +2

On solving upper triangular matrix,

$x = 5,\ y = 4,\ z = -7,\ u = 1$ +1

(iii) Since in each equation one of the coefficient is larger than the other, satisfying the condition for Gauss-Siedel method, write the given equations in the following form:

$$x = \frac{1}{10}[9 - 2y - z]$$

$$y = \frac{1}{10}[-22 - x + 2z] = \frac{1}{20}[-44 - 2x + 2z]$$

$$z = \frac{1}{10}[22 + 2x - 3y]$$ +1

Start with $y = 0 = z$ and using the most recent values of x, y, z, we get

First iteration: $x_1 = 0.9,\ y_1 = -2.29,\ z_1 = 3.067$ +1

Second iteration: $x_2 = 1.0513,\ y_2 = -1.9984,\ z_2 = 3$ +1

Third Approximation: $x_3 = 0.99968,\ y_3 = -1.9999,\ z_3 = 2.999$ +1

Fourth iteration: $x_4 = 1,\ y_4 = -2,\ z_4 = 3$

Fifth iteration: $x_5 = 1,\ y_5 = -2,\ z_5 = 3$ +1

$\therefore$ The required solution is:

$x = 1,\ y = -2,\ z = 3$

4 (i) The difference table is.

marks less than (x)	$y = f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
$x_0 = 40$	$31 = y_0$				
		$\Delta y_0 = 42$			
$50 = x_1$	$73 = y_1$		$\Delta^2 y_0 = 9$		
		$\Delta y_1 = 51$		$\Delta^3 y_0 = -25$	
$60 = x_2$	$124 = y_2$		$\Delta^2 y_1 = -16$		$\Delta^4 y_0 = 37$
		$\Delta y_2 = 35$		$\Delta^3 y_1 = 12$	
$70 = x_3$	$159 = y_3$		$\Delta^2 y_2 = -4$		
		$\Delta y_3 = 31$			
$80 = x_4$	$190 = y_4$				

+2

Here $h = 10$, then $p = \frac{x_p - x_0}{h}$

$$\Rightarrow p = \frac{45-40}{10} = 0.5 \qquad (\because x_p = 45$$

$\therefore$ Using Newton's forward interpolation formula

$$y_p = y(x_p) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$ +1

Substituting the all required values, we get

$\Rightarrow y(45) = 47.87 \approx 48$ +1

Hence the number of students getting marks between 40 and 45

$= y(45) - y(40) = 48 - 31 = 17.$ +1

(ii) Here $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 4$
$u_0 = -18$, $u_1 = 0$, $u_2 = 0$, $u_3 = -248$, $u_4 = 0$,
$x_5 = 5$
$u_5 = 13104$

The Lagrange's interpolation Polynomial is given by

$$L_5(x) \approx u(x) = \sum_{i=0}^{5} l_i(x) u_i \quad \text{(a)}$$

$$= l_0(x)u_0 + l_1(x)u_1 + l_2(x)u_2 + l_3(x)u_3 + l_4(x)u_4 + l_5(x)u_5 = l_0 u_0 + l_3(x)u_3 + l_5(x)u_5 \quad \text{(1)}$$ +1

where

$$l_i(x) = \frac{(x-x_0)\cdots(x-x_{i-1})(x-x_{i+1})\cdots(x-x_5)}{(x_i-x_0)\cdots(x_i-x_{i-1})(x_i-x_{i+1})\cdots(x_i-x_5)}$$

Now

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)}$$

$$l_0(x) = \frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{-1\cdot(-2)(-3)(-4)(-5)}$$

$$\Rightarrow l_0(x) = -\frac{1}{1\cdot2\cdot3\cdot4\cdot5}(x-1)(x-2)(x-3)(x-4)(x-5)$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)(x-x_5)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)(x_3-x_5)}$$

$$\Rightarrow l_3(x) = \frac{x(x-1)(x-2)(x-4)(x-5)}{3\cdot2\cdot1\cdot(-1)\cdot(-2)}$$

$$l_5(x) = \frac{x(x-1)(x-2)(x-3)(x-4)}{5\cdot4\cdot3\cdot2\cdot1}$$

Now eq. (1) becomes.

$$L_5(x) = -\frac{1}{1\cdot2\cdot3\cdot4\,5}(x-1)(x-2)(x-3)(x-4)(x-5)\times(-18)$$

$$+\frac{x(x-1)(x-2)(x-4)(x-5)}{12}\times(-248)$$

$$+\frac{x(x-1)(x-2)(x-3)(x-4)}{5\cdot4\cdot3\cdot2\cdot1}\times 13104$$

$$=(x-1)(x-2)(x-4)\left[(x-3)\cdot(x-5)\times\left(-\frac{3}{20}\right)\right.$$

$$\left.-\frac{62x}{3}(x-5)+\frac{546}{5}x(x-3)\right]$$

$$=\left[x^2-3x+2)(x-4)\left[-\frac{3}{20}(x^2-8x+15)-\frac{62}{3}x^2+\frac{310}{3}x\right.\right.$$ +3

$$\left.+\frac{546}{5}x^2-\frac{1638}{5}x\right]$$

$$=(x^2-3x+2)(x-4)\left[-\frac{3}{20}x^2+\frac{12x}{5}+\frac{9}{4}-\frac{62}{3}x^2\right.$$

$$\left.+\frac{310}{3}x+\frac{546}{5}x^2-\frac{1638}{5}x\right]$$

$$=(x^3-3x^2-4x^2+14x-8)\left(\frac{5303}{60}x^2-\frac{1106}{5}x-\frac{9}{4}\right)$$

$$=\cancel{x^5-9x^4+18x^3-x^2+9x-18}$$

$$=88.3833x^5-839.88333x^4-313.284x^3+2373.984x^2$$

$$-1738.x - 18.\cancel{18}$$ +1

4(iii)

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$
$x_0=0.1$	$y_0=1.10517$			
		0.11623		
$x_1=0.2$	$y_1=1.22140$		0.01223	
		0.12846		0.00127
$x_2=0.3$	$y_2=1.34986$		0.0135	
		0.14196		
$x_3=0.4$	$y_3=1.49182$			

+2

here $h=0.1$, $x_3=0.4$, $\&$ x

By Newton's Backward Formula

$$\frac{dy}{dx}=\frac{1}{h}\left[\nabla y_3+\frac{\nabla^2 y_3}{2}+\frac{\nabla^3 y_3}{3}+\cdots\right]$$ +1

So $\left(\frac{dy}{dx}\right)_{x=0.4}=1.4913$ +2

5(i) let $y=f(x)=\frac{x^2}{1+x^3}$, $h=\frac{1}{4}$

x	$x_0=0$	$x_1=1/4$	$x_2=2/4$	$x_3=3/4$	$x_4=1$
$y=f(x)$	0	0.06154	0.22222	0.39560	0.5

+1

By Simpson's $\frac{1}{3}$rd rule

$$\int_0^1 f(x)\,dx=\frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$ +1

$$\Rightarrow \int_0^1 \frac{x^2}{1+x^3}\,dx = 0.23108 \quad \text{--- (1)}$$ +1

$$\frac{1}{3}\left[\ln(1+x^3)\right]_0^1 = 0.23108$$ | $\ln 2 = 0.6932$ +2

$$\Rightarrow [\ln 2-\ln 1] = 3\times 0.23108$$

(ii) Here $\frac{dy}{dx} = 2y + 3e^x$, $y(0)=0$

$\Rightarrow y' = 2y + 3e^x$, $x_0 = 0$, $y_0 = 0$ —①

Now Taylor's series arround $x = x_0 = 0$ is

$y(x) = y_0 + (x-0)y_0' + \frac{(x-0)^2}{2!}y_0'' + \frac{(x-0)^3}{3!}y_0''' + \cdots$ —② +1

Differentiating (1) successively,

$y'' = 2y' + 3e^x$, $y''' = 2y'' + 3e^x$, $y^{iv} = 2y''' + 3e^x$ +1

$\ldots\ldots$,

put $x = x_0 = 0$ in all above eq.

$y_0' = 3$, $y_0'' = 9$, $y_0''' = 21$, $y_0^{iv} = 45$, $y_0^{(v)} = 93$ +1

then eq. (2) becomes.

$y(x) = 3x + \frac{9}{2}x^2 + \frac{7}{2}x^3 + \frac{15}{8}x^4 + \frac{31}{40}x^5 + \cdots$ —③ +1

To find the approximate value of y at $x=0.2$ putting $x = 0.2$ in eq. ③, we have

+1

$y(0.2) = 0.81125$

5(iii) $f(x,y) = xy$, $x_0 = 1$, $y_0 = 2$, $h = 0.2$

$k_1 = hf(x_0,y_0)$, $k_2 = hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2})$, $k_3 = hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2})$, $k_4 = hf(x_0+h, y_0+k_3)$ +1

Now $k_1 = hf(x_0, y_0) = h \cdot x_0 \cdot y_0 = 0.4$

$k_2 = h \cdot f(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}) = 0.2 f(1.1, 2.2)$

$k_2 = 0.2 \times 1.1 \times 2.2 = 0.484$

$k_3 = 0.2 \times 1.1 \times 2.242 = 0.49324$

$k_4 = hf(1.1, 2.49324)$

$k_4 = 0.5485$

Now

$$k = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$$

$k = 0.48383$

$y_1 = y_0 + k = 2.48383$ +2

Now $x_1 = x_0 + h = 1 + 0.2 = 1.20$

To find y at $x = 1.4$

Now $k_1 = h \cdot f(x_1, y_1)$

$= h \cdot x_1 \cdot y_1$

$= 0.2 \times 1.2 \times 2.48383 = \cancel{0.6955}\ 0.59612$

$k_2 = h f(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}) = 0.7233$

$k_3 = 0.7398, \quad k_4 = 0.9026$

$k = 0.7375$

$y_2 = y_1 + k = 2.4838 + 0.7375$

$y_2 = 3.2213$ +2

6(ii) correlation formula

$$r = \frac{\Sigma xy}{n \sigma_x \cdot \sigma_y}, \text{ where } \sigma_x = \sqrt{\Sigma x^2 / n}, \ \sigma_y = \sqrt{\frac{\Sigma y^2}{n}}$$

$x = X - M_x, \ y = Y - M_y$

OR.

$$r = \frac{\Sigma xy - \frac{\Sigma x \cdot \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}} \cdot \sqrt{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}}$$

OR.

$$r_{xy} = r_{uv} = \frac{\sum uv - \frac{\sum u \cdot \sum v}{n}}{\sqrt{\sum u^2 - \frac{(\sum u)^2}{n}} \cdot \sqrt{\sum v^2 - \frac{(\sum v)^2}{n}}}$$

$$u = \frac{x-A}{h}, \quad v = \frac{y-B}{k}$$

$$= \frac{\frac{\sum uv}{n} - \frac{\sum u}{n} \cdot \frac{\sum v}{n}}{\sqrt{\frac{\sum u^2}{n} - \left(\frac{\sum u}{n}\right)^2} \sqrt{\frac{\sum v^2}{n} - (\sum v/n)^2}}$$

+1

The data may be arranged in the following form

X	Y	$x = X - M_x$	$y = Y - M_y$	x^2	y^2	xy
17	12	-3	-4	9	16	12
18	16	-2	0	4	0	0
19	14	-1	-2	1	4	2
19	11	-1	-5	1	25	5
20	15	0	-1	0	1	0
20	19	0	3	0	9	0
21	22	1	6	1	36	6
21	16	1	0	1	0	0
22	15	2	-1	4	1	-2
23	20	3	4	9	16	12
200	160	0	0	30	108	35

+2

If the mean of X's and Y's are M_x and M_y respectively, then

$$M_x = \frac{\sum x}{n} = 20, \quad M_y = \frac{160}{10} = 16$$

if the s.d. of x's and y's are σ_x, and σ_y, then

$$\sigma_x = \sqrt{\frac{\sum x^2}{n}} = 1.73, \quad \sigma_y = \sqrt{\frac{\sum y^2}{n}} = 3.28$$

Now $\cancel{r = \frac{\sum xy}{\sqrt{\sum x^2}}}$ $r = \frac{\sum xy}{n \sigma_x \sigma_y} = 0.616.$

+2

(ii) Taking $x = X-3$ and $y = \frac{Y-1450}{5}$, $m=5$

Let the equation of parabola be
$$y = a + bx + cx^2 \quad \text{—(1)}$$
The normal equations are

$$\Sigma y = ma + b\Sigma x + c\Sigma x^2 \quad \text{—(2)}$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3 \quad \text{—(3)}$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4 \quad \text{—(4)}$$

+1

X	Y	$x=X-3$	$y=\frac{Y-1450}{5}$	x^2	x^3	x^4	$x.y$	x^2y
1	1090	-2	-72	4	-8	16	144	-288
2	1220	-1	-46	1	-1	1	46	-46
3	1390	0	-12	0	0	0	0	0
4	1625	1	35	1	1	1	35	35
5	1915	2	-2	4	8	16	186	372
		0	-2	10	0	34	411	73

+2

Put in (2), (3), (4) we get
$$5a + 0.b + 10.c = -2 \quad \text{—(5)}$$
$$0.a + 10.b + 0.c = 411 \Rightarrow b = 41.1$$
$$10a + 0.b + 34.c = 73 \quad \text{—(6)}$$
Solving (5) & (6)
$$a = -11.4, \quad c = 5.5$$

+1

Put in (1)
$$y = -11.4 + 41.1x + 5.5x^2$$

$$\boxed{Y = 1024 + 40.5X + 27.5X^2}$$

+1

6 (iii) Given that $n_1 = 10$, and $n_2 = 12$ with degrees of freedom $v = n_1 - 1 = 9$, and $v = n_2 - 1 = 11$

Step 1: Null hypothesis H_0: Let $\sigma_1^2 = \sigma_2^2$ +1

i.e; the two samples have the

Step-2: Calculation of F-statistic

Sample X / Sample Y

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
20	-1.7	2.89	27	-8.25	68.0625
16	-5.7	32.49	33	-2.25	5.0625
26	4.3	18.49	42	6.75	45.5625
27	5.3	28.09	35	-0.25	0.0628
23	1.3	1.69	32	-3.25	10.5625
22	0.3	0.09	37	1.75	3.0625
18	-3.7	13.69	38	2.75	7.5625
24	2.3	5.29	28	-7.25	52.5625
25	3.3	10.89	41	5.75	33.0625
16	-5.7	32.49	43	7.75	60.0625
			30	-5.25	27.5625
			37	1.75	3.0625
$\Sigma x = 217$		146.1	$\Sigma y = 423$		316.28

+2

$$\therefore \bar{x} = \frac{217}{10} = 21.7 \qquad \bar{y} = \frac{423}{12} = 35.25$$

$$\therefore S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{146.1}{9} = 16.233$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{316.25}{11} = 28.7504$$

Hence $F = \dfrac{S_2^2}{S_1^2} = \dfrac{28.7504}{16.233} = 1.771$ +1

Hence calculated value of F is 1.771

Given $F_{0.05} = 3.112$

Decision: Calculated value of $F = 1.771 <$ tabulated value of $F_{0.05} = 3.112$

$\Rightarrow$ The null hypotheas H_0 is accepted

$\Rightarrow$ the two samples have the same variance +1