Q.5 Attempt any two:

i. Solve the game by using graphical method whose pay-off matrix is **5**

	B_1	B_2	B_3
A_1	4	-1	0
A_2	-1	4	2

ii. In a game of matching coins, player A wins rupees 5 if there are two tails, wins 1 rupee if there are two heads and losses rupees 2 when there is one head and one tail. Determine pay-off matrix and best strategies for each player. **5**

iii. In a game of matching coins, player A wins Rs. 5 if there are two tails, wins 1 Rupees of there are two heads and losses rupees 2 when there is one head and one tail. Determine pay-off-matrix and best strategies for each player. **5**

Q.6 Attempt any two:

i. The cost of a new machine is Rs. 5000. The maintenance cost during the nth year is given by Rn = Rs. 500 (n -1), where n = 1, 2,3…. If the rate per year is 0.05, after how many years will it be economical to replace the machine by a new one? **5**

ii. A machine costs Rs. 500. Operation and maintenance cost are zero for the first year and increase by Rs. 100 every year. If money is worth 5% every year, determine the best age at which the machine should be replaced. The resale value of the machine is neglibly small. What is the weighted average cost of owing and operating the machine? **5**

iii. A machine costs Rs. 10,000. Its operating cost and resale value are given below. At what year replacement due? **5**

Year	1	2	3	4	5	6	7	8
Operating costs	1000	1200	1400	1700	2000	2500	3000	3500
Resale value	6000	4000	3200	2600	2500	2400	2000	1600

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No....................................

Faculty of Management Studies

End Sem (Even) Examination May-2022

MS5CO11 Operations Research

Programme: MBA Branch/Specialisation: Management

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Which of the following is a component of a linear programming problem ____? **1**

(a) Constraints (b) Objective function
(c) Decision variables (d) All of these

ii. Operations Research attempts to find _______ solution to a problem. **1**

(a) Perfect (b) Optimal (c) Both (a) and (b) (d) None of these

iii. Minimum number of lines to cover all the zero in assignment problem is equal to number of- **1**

(a) Order of assignment (b) Row
(c) Column (d) All of these

iv. Transportation model is also known as- **1**

(a) Logistics model (b) Distribution model
(c) Both (a) and (b) (d) None of these

v. In Transition probability Matrices the sum of all elements in each row is equal to- **1**

(a) 2 (b) 0 (c) 1 (d) None of these

vi. If state probability does not change over a long period of time, then such condition is known as- **1**

(a) Transition probability matrix
(b) Steady state condition
(c) Tree diagram
(d) None of these

vii. A simulation model uses the mathematical expressions and logical relationships of the- **1**

(a) Real system (b) Computer model
(c) Performance measure (d) None of these

viii. A Game theory model is classified by the- **1**

(a) Number of players

(b) Sum of all payoffs

(c) Number of strategies

(d) None of these

ix. The sudden failure among items is seen as- **1**

(a) Progressive (b) Retrogressive

(c) Random (d) All of these

x. The group replacement policy is suitable for identical low-cost items which are likely to- **1**

(a) Fail over a period of lime

(b) Fail suddenly

(c) Fail completely and suddenly

(d) None of these

Q.2 Attempt any two:

i. Solve the given linear programming problem by graphical method. **5**

Max $z = 6x_1 + 4x_2$

subject to: $2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

and $x_1, x_2 \geq 0$

ii. Discuss the scope of operations research in business or management in any five key areas. **5**

iii. Solve the given linear programming problem by simplex method. **5**

Max $z = -2x_1 + 3x_2$

subject to: $x_1 \leq 5$

$2x_1 - 3x_2 \leq 6$

and $x_1, x_2 \geq 0$

Q.3 Attempt any two:

i. Solve the minimal Assignment problem whose effectiveness matrix is- **5**

	1	2	3	4
II	2	3	4	5
II	4	5	6	7
III	7	8	9	8
IV	3	5	8	4

ii. Find the basic feasible solution of transportation problem given below using Vogel's approximation method. **5**

	Distribution centres					
		D1	D2	D3	D4	Supply
Plants	P1	1	2	1	4	30
	P2	3	3	2	1	50
	P3	4	2	5	9	20
Demand		20	40	30	10	

iii. Find the basic feasible solution of transportation problem given below using least cost method. **5**

	Destination				
		D1	D2	D3	Supply
Source	S1	2	7	4	5
	S2	3	3	1	8
	S3	5	4	7	7
	S4	1	6	2	14
Demand		7	9	18	

Q.4 Attempt any two:

i. Define Markov Chain. Write any four properties of Markov Chain. **5**

ii. A market survey is made on two brands of T-shirts A and B. Every time a customer purchases the same brands or switches to another brand. The probability of the customers using A brand, again purchases A is 0.70 and the customer switching to brand B from A with probability 0.75, then **5**

(a) Construct transition probability matrix and explain the concept of retention and loss, retention and gain using matrix.

(b) Draw probability tree diagram and hence find joint probabilities.

iii. In a service department manned by one server, on an average one customer arrives every ten minutes. It has been found out that each customer requires 6 minutes to be served. Find out **5**

(a) Average queue length.

(b) Average time spent in the system.

(c) Probability that there would be two customers in the queue.

Q.1 MCQ —

(i) (d) All of these +1

(ii) (b) Optimal +1

(iii) (a) Order of assignment +1

(iv) (c) Both (a) and (b) +1

(v) (c) 1 +1

(vi) (b) Steady state Condition +1

(vii) (a) Real System +1

(viii) All of these (+1 Bonus marks for each student)

(ix) (d) All of these +1

(x) (a) fail over a period of time +1

Q.2 (i) $\max Z = 6x_1 + 4x_2$

Sub. to $2x_1 + 4x_2 \leq 4$

$4x_1 + 8x_2 \geq 16$

and $x_1, x_2 \geq 0$

Express both the constraints in terms of eq^n

$2x_1 + 4x_2 = 4$ eq^n I

x_1	0	2
x_2	1	0

~~ie~~.

ie. (0,1) and (2,0). +1

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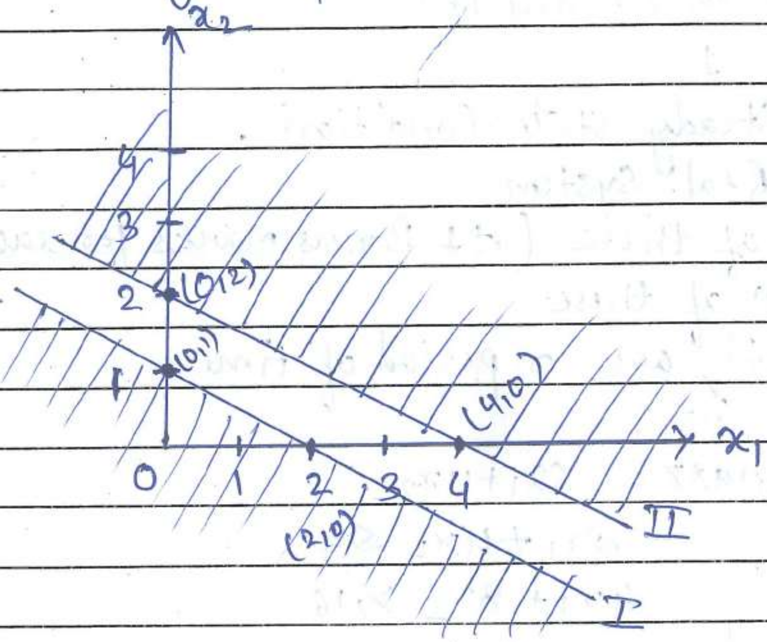
$4x_1 + 8x_2 = 16$ $\quad$ IInd eq^n

x_1	0	4
x_2	2	0

ie. $(0,2)$ and $(4,0)$ $\quad$ +1

plot the given points on graph, we get



+2

as each point below line I satisfy the 1st constraint $\therefore$ solution for line I will be below the line and similarly for line II the solution will be above the line II

clearly there is no common feasible region hence the solution is Infeasible for the given LPP.

+1

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2 (ii) Scope of Operations Research –

(1) In Marketing : With the help of OR techniques a marketing administrator can decide where to distribute the products for sale so that the total cost of transportation cost is minimum +1

(2) In finance and accounting : OR methods can be used to maximize the per capita income with minimum resources used, also credit policy analysis and investment analysis can be done using OR techniques. +1

(3) In Production Management : A Production manager can use OR techniques to find out the number and size of the items to be produced, to calculate optimum product mix. +1

(4) In Industry : OR is useful to the industry director in deciding optimum allocation of various limited resources such as men, machines, material, etc to arrive at the optimum decision. +1

(5) In agriculture : Optimum distribution of water from various resources like canal for irrigation purpose +1

Q.2 (ii) Adding Slack variables to the constraints

$\max Z = -2x_1 + 3x_2 + 0S_1 + 0S_2$

Subj. to

$x_1 + S_1 = 5$

$2x_1 - 3x_2 + S_2 = 6$

and $x_1, x_2, S_1, S_2 \geq 0$ +1

if $x_1 = x_2 = 0$ then $S_1 = 5$, $S_2 = 6$ {basic variables}

Initial Simplex table

	C_j	-2	3	0	0		Min. +ve
C_B	Basic Var.	x_1	x_2	S_1	S_2	b	Ratio
0	S_1	1	0	1	0	5	divide by 0 is not possible
0	S_2	2	-3	0	1	6	÷ by -ve is not possible
	Z_j	0	0	0	0		
	$C_j - Z_j$	-2	3	0	0		

KC

+3

$\because$ $C_j - Z_j$ is positive under some column so current sol^n is not optimal

Also most +ve column is of variable x_2
$\therefore$ x_2 is incoming variable but we are unable to find minimum +ve Ratio hence we cannot find outgoing variable
hence Solution of given LPP is unbounded

+1

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Q.3(i)

	1	2	3	4
I	2	3	4	5
II	4	5	6	7
III	7	8	9	8
IV	3	5	8	4

Step I : Row Reduction

	1	2	3	4
I	0	1	2	3
II	0	1	2	3
III	0	1	2	1
IV	0	2	5	1

+1

Step II : Column Reduction

	1	2	3	4
I	$\boxed{0}$	~~0~~	~~0~~	2
II	~~0~~	$\boxed{0}$	~~0~~	2
III	~~0~~	~~0~~	$\boxed{0}$	~~0~~
IV	~~0~~	1	3	$\boxed{0}$

+2

As we are unable to find exactly one zero in each row & each column
∴ we will assign arbitrary.

Now no. of assignment is equal to order of matrix

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hence current solution is optimal sol^n

Assignments : I → 1
II → 2
III → 3
IV → 4

+1

Total optimal cost = 2+5+9+4
= 20

+1

Q.3 (ii)

	D_1	D_2	D_3	D_4	Supply	Row diff^n
P_1	1 (20)	2	1 (10)	4	~~30~~ ~~10~~ 0	0, 0, 1, 1 (4)
P_2	3	3 (20)	2 (20)	1 (10)	~~80~~ ~~40~~ ~~20~~ 0	1, 1, 1, 1
P_3	4	2 (20)	5	9	~~20~~ 0	2, 2, 3 (3), –
Demand	~~20~~ 0	~~40~~ ~~20~~ 0	~~30~~ ~~20~~ 0	~~10~~ 0		
Column diff^n	2	0	1	3 (1)		
	2 (2)	0	1	–		
	–	0	1	–		
	–	1	1	–		
	–	–	–	–		

+4

total optimal cost = $1\times20 + 1\times10 + 3\times20 + 2\times20 + 1\times10 + 2\times20$

$= 20+10+60+40+10+40$
$= 180$

+1

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(iii)

	D_1	D_2	D_3	Supply
S_1	2	7 (2)	4 (3)	~~5~~ ~~2~~ 0
S_2	3	3	1 (8)	~~8~~ 0
S_3	5	4 (7)	7	~~7~~ 0
S_4	1 (7)	6	2 (7)	~~14~~ ~~7~~ 0
	~~7~~ 0	~~9~~ ~~2~~ 0	~~18~~ ~~10~~ ~~3~~ 0	

+4

total optimal cost $= 7\times2 + 4\times3 + 1\times8 + 4\times7 + 1\times7 + 2\times7$

$= 14+12+8+28+7+14$

$= 83$

+1

(i) A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event

+1

Properties —

* There should be a limited no. of states
* Each possible states should be identified
* Only one state at a time is possible.
* The probability of changing states remains same over the time.

+4

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Q.4(ii) Transition prob. matrix —

(a)

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{Next State } (n=1)} \\ & & A & B \\ \text{current state} & A & 0.70 & 0.30 \\ (n=0) & B & 0.75 & 0.25 \end{array}$$

(+1)

Now P_{11} = prob. that customer using brand A will again use brand A $= 0.70$
This show Retention to A

P_{12} = prob. that customer using brand A will switch to Brand B $= 0.30$
This show loss to A

hence Rowewise we have Retention & loss

P_{21} = prob that customer using brand B will switch to Brand A $= 0.75$
this shows gain to A & loss to B

P_{22} = prob. that customer using brand B will again use brand B $= 0.25$
This shows Retention to B

hence, Column wise $\begin{bmatrix} P_{11} \\ P_{21} \end{bmatrix}$ we have Retention & gain to Brand A

(+2)

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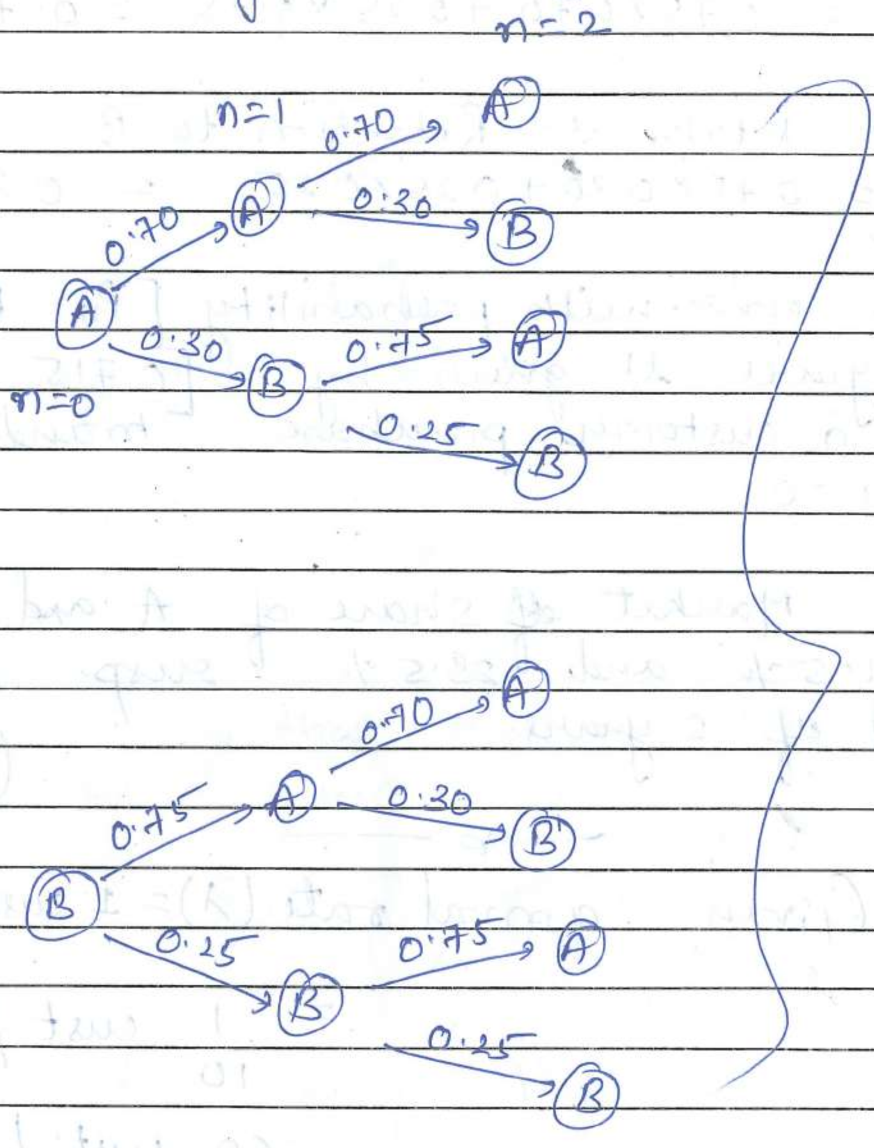
(b) Probability tree diagram.



+1

Joint probability —

P_{11} = A to A i.e. Retention to A

$= 0.70 \times 0.70 + 0.30 \times 0.75 = 0.715$

P_{12} = A to B i.e. loss to A

$= 0.70 \times 0.30 + 0.30 \times 0.25 = 0.285$

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P_{21} = B to A i.e. ~~B~~ gain to A
$= 0.75 \times 0.70 + 0.25 \times 0.75 = 0.7125$

P_{22} = B to B i.e. Retention to B
$= 0.75 \times 0.30 + 0.25 \times 0.25 = 0.2875$

The rone with probability $[P_{11} \quad P_{12}]$ after 2 years is given by $[0.715 \quad 0.285]$, if a customer purchase brand A at $n=0$

ie. Market ~~of~~ share of A and B are 71.5% and 28.5% resp. at the end of 2 years. +1

Q.4(iii) Given arrival rate (λ) = 1 cust / 10 min

$= \frac{1}{10}$ cust / min

$= \frac{60}{10}$ cust / hr

$= 6$ cust / hr +1

Service Rate (μ) = 1 cust / 6 min

$= \frac{1}{6}$ cust / min

$= \frac{60}{6}$ cust / ~~min~~ hr

$= 10$ cust / hr. +1

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(i) Average queue length $= \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu-\lambda}\right)$

$$L_q = \frac{6}{10}\left(\frac{6}{4}\right)$$

$$L_q = 0.9 \simeq 1 \text{ customer.}$$

+1

(ii) Average time spent in the system $= \frac{1}{\mu-\lambda}$

$$W_s = \frac{1}{10-6} = 0.25 \text{hr or}$$

15 mins.

+1

(iii) Probability that there would be 2 customer in the queue

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right)$$

$$= \left(\frac{6}{10}\right)^2 \left(1-\frac{6}{10}\right)$$

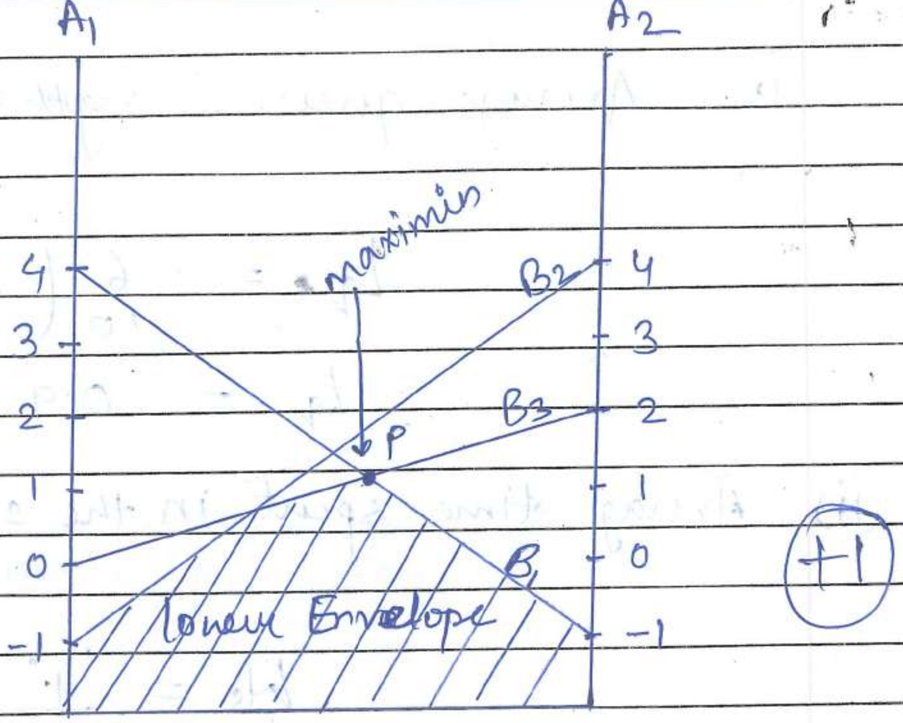
$$= \frac{144}{1000}$$

$$= 0.144 \text{ or } 14.4\%$$

+1

—— x ——

(i) Graph of ~~abo~~ given matrix will be as follows :-



here P represent the maximin point in the graph which is obtained by the intersection of stratigies B_1 and B_3

hence the Reduced pay-off matrix will be –

$$\begin{array}{c|cc} & B_1 & B_3 \\ \hline A_1 & 4 & 0 \\ A_2 & -1 & 2 \end{array}$$

+1

Now using Algebraic method to find value of game and Stratigies

~~Strat~~ $\cancel{x_1 = \dfrac{a_{11} \times a_{22} - a_{12} \times a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}}$

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

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$$x_1 = \frac{2-(-1)}{(4+2)-(-1+0)} = \frac{3}{7}$$

~~x_2~~ as $x_1 + x_2 = 1$

$$x_2 = 1 - x_2 = 1 - \frac{3}{7} = \frac{4}{7}$$ +1

Similarly $y_1 = \dfrac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$

$$= \frac{2-0}{(4+2)-(-1+0)} = \frac{2}{7}$$

as $y_1 + y_2 = 1$

$$y_2 = 1 - \frac{2}{7} = \frac{5}{7}$$

∴ strategy of Player A is $\left(\frac{3}{7}, \frac{4}{7}\right)$

& strategy of Player B is $\left(\frac{2}{7}, \frac{5}{7}\right)$ +1

and value of the game $(V) = \dfrac{a_{11} \times a_{22} - a_{12} \times a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$

$$= \frac{8-0}{7} = \frac{8}{7}$$ +1

—— x ——

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Q.5 (ii) Pay off matrix of A is —

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cc} H & T \end{array} \\ \text{Player A} \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \end{array}$$

Using oddment method —

$$\begin{array}{ccccc} & & P_B & & \\ & H & T & \text{odds} & \text{Probabilit} \\ P_A \; H & 1 & -2 & |-2+5|=7 & 7/10 \\ \quad\;\; T & -2 & 5 & |1-(-2)|=3 & 3/10 \\ \text{odds} & |-2-5|=7 & |1-(-2)|=3 & & \\ \text{Prob.} & 7/10 & 3/10 & & \end{array}$$

+3

Strategy for Player A — $\left(\frac{7}{10} \;\; \frac{3}{10}\right)$

Strategy for Player B — $\left(\frac{7}{10} \;\; \frac{3}{10}\right)$

+2

Q.5(iii) Note: 5 Marks Bonus must given to each student whether he attempted the question or not.

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(i) Given Capital Cost = 5000
Interest rate = 0.05 or 5%

$\therefore V = \frac{1}{1+r} = \frac{1}{1+0.05} = 0.9523$

+1

and $R_n = 500(n-1)$; $n = 1, 2, ---$

We will write the running cost until it becomes half of the capital cost

Year (n)	1	2	3	4	5	6
Running cost (R_n)	0	500	1000	1500	2000	2500

Calculation of Weighted average cost –

Year (n)	Running cost (R_n)	Discounted Factor (V^{n-1})	$R_n \times V^{n-1}$	$\Sigma R_n \times V^{n-1}$	ΣV^{n-1}	Total Cost = $C + \Sigma R_n \cdot V^{n-1}$	Weighted average cost
1	0	1	0	0	1	5000	5000
2	500	0.9523	476.15	476.15	1.9523	5476.15	2804.9
3	1000	0.9069	906.9	1383.05	2.8592	6383.05	2232.46
4	1500	0.8636	1295.4	2678.45	3.7228	7678.45	2062.54
5	2000	0.8224	1644.8	4323.25	4.5452	9323.25	2051.22
6	2500	0.7831	1957.75	6281	5.3283	11281	2117.1

+3

Replacement Policy :– Replace the machine at the end of 5th year because the weighted avg. cost is minimum at the end of 5th year i.e. 2051.22.

+1

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Q. 6 (ii) Given Capital Cost = 500
Running cost = 100 per year.
Interest rate = 5%

$\therefore\ V = \frac{1}{1+r} = \frac{1}{1+0.05} = 0.9523$

+1

Year	1	2	3	4
Running cost (R_n)	0	100	200	300

Calculation of weighted average cost

Year (n)	Running cost (R_n)	Discounted factor V^{n-1}	$R_n \times V^{n-1}$	$\Sigma R_n V^{n-1}$	ΣV^{n-1}	$C + \Sigma R_n V^{n-1}$	Weig avg. co
1	0	1	0	0	1	500	50
2	100	0.9523	95.23	95.23	1.9523	595.23	304.
3	200	0.9069	181.38	276.61	2.8592	776.61	271.6
4	300	0.8636	259.14	535.75	3.7228	1035.75	278.

+3

Replacement Policy: Replace the machine at the end of 3rd year as the weighted avg. cost is minimum i.e. 271.61

+1

6 (iii)

Given, Capital cost = 10,000

Year (n)	Running cost $f(t)$	Cum. Run. cost $\Sigma f(t)$	Capital cost (C)	Scrap Value (S)	C−S	Total Cost = $C-S+\Sigma f(t)$	Average Annual cost
1	1000	1000	10000	6000	4000	5000	5000
2	1200	2200	10000	4000	6000	8200	4100
3	1400	3600	10000	3200	6800	12400	4133.33
4	1700	5300	10000	2600	7400	12700	3175
5	2000	7300	10000	2500	7500	14800	2960
6	2500	9800	10000	2400	7600	17400	2900
7	3000	12800	10000	2000	8000	20800	2971.42
8	3500	16300	10000	1600	8400	24700	3087.5

+4

Replacement Policy : Replace the machine at the end of 6^{th} year as the average annual is minimum at the end of 6^{th} year i.e. 2900