Total No. of Questions: 25 **Enrollment No :** ____________________

Faculty of Engineering / Science

End Semester Examination May 2025

CS3BS04 / EC3BS02 / BC3BS05 Discrete Mathematics

Programme	:	B.Tech. / B.Sc.	**Branch/Specialisation**	:	CSE/EC/ CS
Duration	:	3 hours	**Maximum Marks**	:	60

Note: *All questions are compulsory. Internal choices, if any, are indicated. Assume suitable data if necessary. Notations and symbols have their usual meaning.*

Section 1 (Answer all question(s))

Marks CO BL

Q1. If $f(x) = (3x^2 + 1)$ and $g(x) = \log x$ then $fog\ (x)$ is- **1 3 3**

Rubric	Marks
option b	1

- ○ $\log(3x^2 + 1)$
- ◉ $(3(\log x)^2 + 1)$
- ○ $3(\log x)^2$
- ○ None of these

Q2. Let $A = \{2,3,4,5\}, B = \{3,4,5,6\}$ and $C = \{5,6,7,8\}$ are three sets then number of elements in power set of $(A - B) \cup C$? **1 3 3**

Rubric	Marks
option c	1

- ○ 2^8
- ○ 2^7
- ◉ 2^5
- ○ None of these

Q3. In a Boolean Algebra B, the law $x + x = x\ ;\ \forall x \in B$ is known as______ law. **1 2 2**

Rubric	Marks
option d	1

- ○ Associative
- ○ Distributive
- ○ Involution
- ◉ Idempotent

Q4. Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and relation 'divides' be a partial ordering relation on D_{30}. The greatest lower bound of 10 and 15 respectively is **1 3 3**

Rubric	Marks
option a	1

- ◉ 5
- ○ 10
- ○ 15
- ○ None of these

Q5. If $G = \{(0, 1, 2, 3, 4), +_5\}$ then order of element 2 is- **1** **3** **1**

Rubric	Marks
option c	1

- ○ One
- ○ Three
- ◉ Five
- ○ None of these

Q6. Every cyclic group is _________. **1** **2** **2**

Rubric	Marks
option a	1

- ◉ Abelian group
- ○ Coset
- ○ Non abelian
- ○ None of these

Q7. Every tree with two or more vertices is _______ chromatic. **1** **2** **2**

Rubric	Marks
option b	1

- ○ 1
- ◉ 2
- ○ 3
- ○ 4

Q8. An Euler graph is one in which all the vertices are of ________ degree. **1** **1** **1**

Rubric	Marks
option d	1

- ○ Prime
- ○ Composite
- ○ Odd
- ◉ Even

Q9. The number of ways in which 8 students can be seated in a line is- **1** **1** **1**

Rubric	Marks
option b	1

- ○ 7!
- ◉ 8!
- ○ 2x 7!
- ○ None of these

Q10. What is the recurrence relation for 1, 7, 31, 127, 499 ? **1** **1** **1**

Rubric	Marks
option c	1

- ○ $b_n=b_{n-1}+3$
- ○ $b_n=4b_{n-1}$
- ◉ $b_n=4\ b_{n-1}+3$
- ○ None of these

Section 2 (Answer any 2 question(s))

Marks CO BL

Q11. In a group of 265 persons, 200 like singing, 110 like dancing and 55 like painting. If 60 persons like both singing and dancing, 30 like both singing and painting and 10 like all three activities then, Find the number of students who like (i) only dancing and painting and (ii) only singing. **5 3 3**

Rubric	Marks
Represent data 1 mark Part(1) 2 marks Part (2) 2 marks	5

Q12. Given any five points in the interior of an equilateral triangle of side x cm, show that there exists two points within a distance of at most x / 2 cm. **5 3 3**

Rubric	Marks
Use Pigeon hole principle to evaluate the result.	5

Q13. Prove that $R = \{(x, y); x, y \in I\, and\, x - y\, is\, divisible\, by\, 3\}$ is an equivalence relation. **5 3 3**

Rubric	Marks
Reflexive 1 mark Symmetric 2 marks Transitive 2 marks	5

Section 3 (Answer any 2 question(s))

Marks CO BL

Q14. Show that the set of integers which are divisors of 90 is a partial order-set. **5 3 3**

Rubric	Marks
Prove all the conditions for the set to be POSET	5

Q15. Minimize the Boolean function using K Map (Karnaugh map) method $F(P,\ Q,\ R) = \prod(0,\ 3,\ 6,\ 7)$. **5 3 3**

Rubric	Marks
Form K map, Group cells and write minimized function	5

Q16. Let S be a set of family of all subsets of set $A = \{\ a,b,c\ \}$ which is closed under the operation union " ∪ " and intersection " ∩ " and complementary law then prove (S, ∪ , ∩ ') is Boolean algebra. **5 3 3**

Rubric	Marks
Prove all the laws for the algebraic structure to be Boolean Algebra each law is of 1 mark	5

Section 4 (Answer any 2 question(s))

Marks CO BL

Q17. Prove that intersection of two subgroups of a group is always a subgroup of that group. **5 3 3**

Rubric	Marks
Use def and necessary condition for existence of a subgroup to prove the result.	5

Q18. The algebric structure (G,.) is a group then prove reversal law that is- **5 2 2**

$$(ab)^{-1}=b^{-1}a^{-1}$$

Rubric	Marks
Use concept of inverses in a group.	5

Q19. Is the set of fourth root of unity {-1, 1, i, -i } forms a group with respect to the multiplication composition ? 5 3 3

Rubric	Marks
Prove all the laws for the algebraic structure to be a group and conclude the result.	5

Section 5 (Answer any 2 question(s))

Marks CO BL

Q20. Define: (a) Bipartite Graph (b) Isomorphism of graphs with example. 5 2 2

Rubric	Marks
Bipartite graph def and example 2.5 marks Isomorphism of graphs def and example 2.5 marks	5

Q21. Prove that number of vertices of odd degree in a graph are always present in even number. 5 2 2

Rubric	Marks
Use the concept of degree to prove the result	5

Q22. State and prove Euler's Theorem for planar graph. 5 3 3

Rubric	Marks
Use concept of regions and boundaries to prove the result.	5

Section 6 (Answer any 2 question(s))

Marks CO BL

Q23. Solve the recurrence relation $y_{h+2} - 7y_{h+1} + 10y_h = 0$ *with* $y_0 = 0$, $y_1 = 3$. 5 3 3

Rubric	Marks
Find the recurrence relation and then find constants present in the solution	5

Q24. A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women? 5 3 3

Rubric	Marks
part i 2.5 marks part ii 2.5 marks	5

Q25. Solve the recurrence relation $y_{h+2} - 2y_{h+1} + y_h = 3h + 4$. 5 4 4

Rubric	Marks
Find homogeneous solution 2 marks particular solution 3 marks	5

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END-SEMESTER EXAMINATION May-2025

Course Name: Discrete Mathematics

Course Code: CS3BS04/EC3BS02/BC3BS05

"solution"

			Marks
Q.1	Q.1	b) $(3(\log x)^2+1)$	1
2.	Q.2	c) 2^5	1
3.		d) Idempotent	1
4.		a) 5	1
5.		c) Five	1
6.		a) Abelian Group	1
7.		b) 2	1
8.		d) Even	1
9.		b) 8!	1
10.		c) $b_n = 4b_{n-1}+3$	1

(ii) only singing

$|S_1| = |S - D - P|$

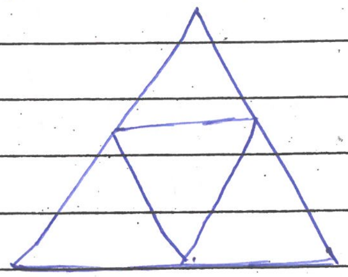
$= |S| - |S \cap D| - |S \cap P| + |S \cap D \cap P|$ +1

$= 200 - 60 - 30 + 10$

$= 120$ +1

Q. 12

We divide the given equilateral triangle into four equal triangles.



+1

The each side of the small triangles is $x/2$ cms. +1

Now the given five points (pigeons) will be placed in the four small triangles (pigeon-holes). +1

Hence by pigeonhole principle some small triangle must contain at least two points.

The distance b/w these two points can not exceed $x/2$ cms. (the side of small triangle) +1

Hence there exists two points within a distance of almost $x/2$ cms. +1

Q.13 Given

$$R = \{(x,y) : x, y \in I \text{ and } x-y \text{ is divisible by } 3\}$$

~~is an eq~~ where I is set of integers.

We know that R will be an equivalence relation if its is reflexive, symmetric and transitive +1

(i) R is reflexive: let $x \in I$ then

$x - x = 0$, which is divisible by 3

$\therefore (x,x) \in R, \ \forall x \in I$

Hence R is reflexive +1

(ii) R is symmetric: let $x, y \in I$

If $(x-y)$ is divisible by 3 then $y - x = -(x-y)$ is also divisible by 3

$\therefore (x,y) \in R \Rightarrow (y,x) \in R$ is true +1.5

Hence R is symmetric

R is transitive : let $x, y, z \in I$

If $x-y$ and $y-z$ are divisible by 3 then

$x-z = (x-y) + (y-z)$ is also divisible by 3

$\therefore$ $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$ is true

Hence R is transitive relation

Therefor R is an equivalence relation +1.5

Q.16. let S be a set of family of all subsets of $A = \{a,b,c\}$
i.e S is a power set $P(A)$

If $(S, \cup, \cap, ')$ satisfy following laws then it is Boolean algebra

Sol^n

1) closure law : For any two element B and C of S, we know that $B \cup C$ and $B \cap C$ are unique element of S, i.e

$B \cup C \in S$ and $B \cap C \in S \;\; \forall\; B, C \in S$ +1

2) Commutative law :

$B \cup C = C \cup B$ and $B \cap C = C \cap B$, $\forall B, C \in S$ +1

3) Identity law : ϕ (empty set) and U (universal set) exist in S such that

(i) $B \cup \phi = B$

(ii) $B \cap U = B$, $\forall B \in S$

Hence identity element for $\cup$ is ϕ and for $\cap$ is U +1

4) Distributive law : For any three elements B, C, D of S, we have

(i) $B \cup (C \cap D) = (B \cup C) \cap (B \cup D)$

(ii) $B \cap (C \cup D) = (B \cap C) \cup (B \cap D)$ +1

5) Complement law : $\forall B \in S$, there exists an element B' in S such that

(i) $B \cup B' = U$ (ii) $B \cap B' = \phi$

Hence $(B, \cup, \cap, ')$ is a Boolean algebra +1

~~Q. 14~~

~~let set A = { set of all integer }~~ {1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90}

~~To prove A is Partial order set w.r.t. the~~

Q.14 $A = \{ x \in I \mid x \mid 90 \}$ +1

To prove A is partial order set.

For this

(1) Reflexive:

$\forall x \in A$

$x \mid x$.

+1

(2) Anti symmetric:

If $x \mid y$ & $y \mid x \Rightarrow x = y$, $\forall x, y \in A$

+1

(3) Transitive:

$\forall x, y, z \in A$,

If $x \mid y$ & $y \mid z \Rightarrow x \mid z$. +1

Hence A is POSET. +1

7

Q.15.

A/BC	00	01	11	10
0	1	–	1	–
1	–	–	1	1

$A'B'C' + BC + AB$

+5

0	000
3	011
6	110
7	111

Q. 17 let H_1 and H_2 are any two subgroup of Group G.
then

$$H_1 \cap H_2 \neq \phi$$ +1

Since at least the identity element e is common to both H_1 and H_2
In order to prove that $H_1 \cap H_2$ is a subgroup ~~of~~ ~~G~~. it is sufficient to prove that

$$a \in H_1 \cap H_2,\ b \in H_1 \cap H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$$

$$a \in H_1 \cap H_2 \Rightarrow a \in H_1 \text{ and } a \in H_2$$

$$b \in H_1 \cap H_2 \Rightarrow b \in H_1 \text{ and } b \in H_2$$ +2

But H_1 and H_2 are subgroup,

$\therefore$

$$a \in H_1,\ b \in H_1 \Rightarrow ab^{-1} \in H_1$$

$$a \in H_2,\ b \in H_2 \Rightarrow ab^{-1} \in H_2$$

Finally $ab^{-1} \in H_1$; $ab^{-1} \in H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$

Thus

$$a \in H_1 \cap H_2,\ b \in H_1 \cap H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2.$$

Hence $H_1 \cap H_2$ is a subgroup of G. +2

Q. 18 Form K map, Group cells and write minimized function 05

8

9

Q.18. Suppose a and b are any element of group G and a^{-1}, b^{-1} are respectively the inverse of a and b, then

$$a^{-1}a = e = aa^{-1}$$
$$b^{-1}b = e = bb^{-1}$$
{where e is the identity} +1

Now,

$(ab)(b^{-1}a^{-1}) = [(ab)b^{-1}]a^{-1}$ {associative law}

$= [a(bb^{-1})]a^{-1}$ { " }

$= [ae]a^{-1}$

$= aa^{-1}$

$= e$ — (1) +1

Similarly

$(b^{-1}a^{-1})ab = b^{-1}[a^{-1}(ab)]$

$= b^{-1}[(a^{-1}a)b]$

$= b^{-1}[eb]$

$= b^{-1}b$

$= e$ — (2) +1

From (1) & (2) we have

$(ab)(b^{-1}a^{-1}) = e = (b^{-1}a^{-1})ab$ +1

$\therefore$ by the definition of inverse +1

$$(ab)^{-1} = b^{-1}a^{-1}.$$

Q.19 Given
$G = \{1, -1, i, -i\}$

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

+1

(1) Closure property: Since all the entries in the composition table are the elements of set G.
$\therefore$ set G is closed with respect to multiplication +1

(2) Associativity: The elements of G are all complex numbers and multiplication of complex number obeys associative law

$(1.i).(-i) = 1.(i.(-i)) = 1$
$(1.i)(-1) = 1.(i.(-1)) = i$
$\therefore$ Associative law hold. +1

(3) Existence of identity: From the composition table, we see that the identity law hold
$1(1) = 1$ $1(-1) = -1$, $1(i) = i$, $1(-i) = -i$ +1
i.e 1 is the identity
or $1 \in G$ such that $1.a = a \ \forall a \in G$

4) Existence of inverse: we know that the identity element is its own inverse.
From composition table

a	a'
1	1
-1	-1
i	$-i$
$-i$	i

+1

~~ref~~ $a \in G$ then inverse of a i.e $a' \in G$ if $a \cdot a' = e$ (identity) $\forall\ a \in G$

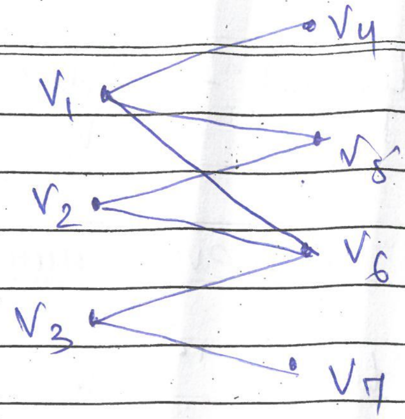
Q. 20 Bipartite Graph: let $G(V,E)$ be graph. The graph G is called bipartite graph if its set V of vertices can be partitioned into two subsets H and S such that each edge of G connects a vertex of H to a vertex of S. +1.5

for eg: let $G(V,E)$ be graph.

where $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

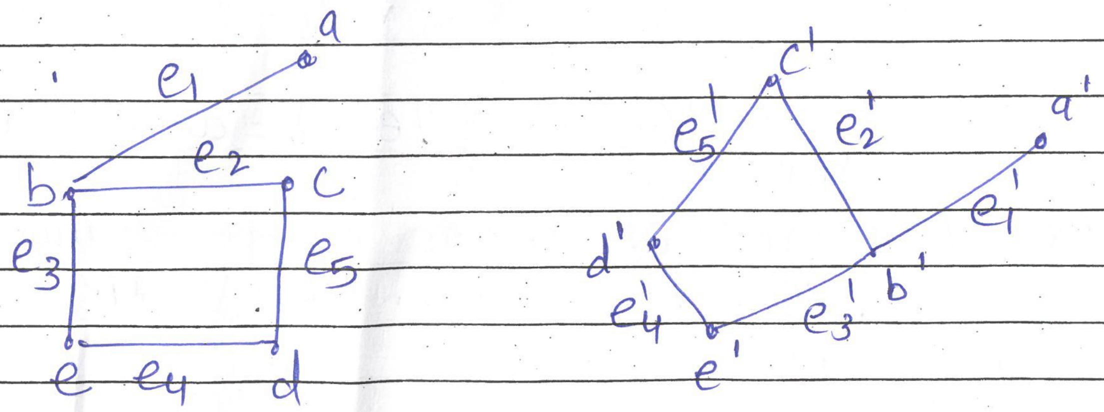
Such that

$H = \{v_1, v_2, v_3\}$, $S = \{v_4, v_5, v_6, v_7\}$



+1

Isomorphism of graph : Two graph are called isomorphic if there is a one to one correspondence between their vertices and edges such that their incidence relationship is presered. +1.5



+1

Q. 21 let $G(V, E)$ be a graph, where V is the set of verties ~~of~~ ~~odd~~ in G and E is the set of edges in G

let V_e be the set of vertices of evendegree

V_o be ——— " ——— odd degree

So that

$$V_e \cup V_o = V, \quad V_e \cap V_o = \phi$$ +1

$$\sum_{v \in V} \deg(v) = \sum_{v \in V_e} \deg(v_-) + \sum_{v \in V_0} \deg(v)$$ +1

$\therefore \sum_{v \in V_e} \deg(v)$ is an even number, 2K (say)

Also, we know that "The sum of the degree of all vertices in a graph is equal to twice the no. of edges.

i.e.

$$\sum_{v \in V} \deg(v) = 2e$$ {e is the no. of edges. +1

Hence $2e = 2k + \sum_{v \in V_0} \deg(v)$ +1

$$\sum_{v \in V_0} \deg(v) = 2(e-k) = \text{an even no.}$$

i.e. the no. of terms in the sum must be even. Hence the no. of vertices in V_0 is even. +1

Q.2.2 Statement: A connected planar graph with n vertices and e edges has r regions given by +1

$r = e - n + 2$

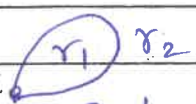
Proof we shall the theorem by induction on the number of edges e of G, where G is a connected planar graph suppose $e=1$ then n may be equal to 1 or 2 +1

In Case $e=1, n=2$ then

$e=1, n=2$

$r = 1-2+2 = 1$ +1

In case $e=1, n=1$:



$e=1, n=1$ +1

then

$r = 1-1+2 = 2$

Hence result is true for $e=1$

Now suppose that the result holds for all graphs with at most $e-1$ edges. Assume G is a connected graph with e edges, r-regions. In a case G is tree. then $e=n-1$ and $r=1$ by formula $r = e-n+2 = n-1-n+2$
$= 1$ +1

Hence theorem is true in case G is tree if G is not a tree then it has some circuit. Consider an edge 'c' say in some circuit. if we remove this edge region are merged i.e. $G-\{c\}$ is a connected graph with n vertice, $e-1$ edges and $r-1$ region on by induction

$r-1 = (e-1) - n+2$ +1

$r = e-n+2$

15

Q. 23 $y_{n+2} - 7y_{n+1} + 10y_n = 0$ with
$y_0 = 0,\ y_1 = 3$

1.P.

$m^2 - 7m + 10 = 0$

$m = 2, 5$ +1

$y_n = C_1 2^n + C_2 5^n$ — (1) +1

given $y_0 = 0$ $y_1 = 3$

from (1)

$y_0 = C_1 2^0 + C_2 5^0 = 0$

$\Rightarrow C_1 + C_2 = 0$ — (2) +1

and

$y_1 = C_1 2 + C_2 5 = 1$

$\Rightarrow 2C_1 + 5C_2 = 1$ — (3)

on solving (2) & (3)

$C_2 = 1/3$ $C_1 = -1/3$ +1

So, the required solution

$y_n = -\frac{1}{3} 2^n + \frac{1}{3} 5^n$ +1

Q.24 total member ~~of~~ in committee = 3 person

these following cases are possible

(i)

1 man × 2 women

2 Man × 1 women

0 Man × 3 women +1

$= 2C_1 \times 3C_2 + 2C_2 \times 3C_1 + 2C_0 \times 3C_3$ +1

= 10 ways +0.5

(ii) $2C_1 \times 3C_2 = 6$ +2

6 Committes would comprises of 1 man & 2 women. +0.5

Q.25 $y_{n+2} - 2y_{n+1} + y_n = 3n+4$ — (1)

$m^2 - 2m + 1 = 0$

$m = 1, 1$

Homogeneous solution $(C_1 + C_2 n) 1^n$

$= C_1 + C_2 n$ — (2) +2

Particular solution is

$y_n = A_0 n^2 + A_1 n^3$ — (3)

Substituting the value of y_n from (3) to (1) and find A_0 & A_1 then put in (3)

Thus the required solution is

$$y_n = C_1 + C_2 n + \frac{1}{2} n(n-1)(n+2)$$