OR iii. A coin is tossed 10 times. Find the probability of getting 3 or 4 or 5 heads using CLT. Given that $P(0<z<1.58)=0.4429$ and $P(0<z<0.316)=0.1217$. **7**

Q.6 i. Find the acute angle between the two regression lines. **3**

ii. Fit a second-degree parabola to the following data, x is an independent variable: **7**

x:	1	2	3	4	5	6	7	8	9
y:	2	6	7	8	10	11	11	10	9

OR iii. From a random sample of 10 pigs fed on diet A, the increases in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increases in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 lbs. Test whether diets A and B differ significantly as regards their effect on increases in weight? (Given $t_{0.05,20}=2.09$). **7**

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Enrollment No....................................

Faculty of Engineering

End Sem (Even) Examination May-2022

EC3BS03 / EE3BS03 / EX3BS03

Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: EC/EE/EX

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Two graphs $G=\langle V,E\rangle$ and $G'=\langle V',E'\rangle$ are said to be isomorphic if- **1**

$$P:|V(G)|=|V'(G)|, Q:|E(G)|=|E'(G)|$$

(a) Only P is sufficient (b) Only Q is sufficient
(c) Insufficient conditions (d) None of these.

ii. The sum of the in-degrees over all vertices in any directed graph is equal to- **1**

(a) Twice the number of edges
(b) The sum of the edges
(c) The sum of the out-degrees over all vertices
(d) None of these

iii. The total number of pendant vertices in a complete binary tree with 23 vertices are- **1**

(a) 1 (b) 12 (c) 10 (d) 0

iv. The maximum value of flow from source 'S' to sink 'T' in G is ---------- value of capacities of all cuts in G from S to T. **1**

(a) Maximum (b) Minimum
(c) Greater than the (d) Less than the

v. If $P_n(x)$ is the solutions of Legendre's polynomial $\int_{-1}^{1} P_7^2(x)dx =$ ____. **1**

(a) 2/13 (b) 2/11 (c) 0 (d) 2/15

vi. $\frac{d}{dx}(x^{-n}J_n)=$ ____. **1**

(a) $-x^{-n}J_n$ (b) $-x^{n}J_{n+1}$ (c) $x^{-n}J_{n+1}$ (d) $-x^{-n}J_{n+1}$

vii. Let (X, Y) be two random variables and P_{XY} denote the joint probability mass function of X, Y then the value of $\sum\sum P_{XY}(x_i, y_j)$ is- **1**

(a) 0 (b) 5 (c) 2 (d) 1

viii. If X and Y are two continuous random variables with pdf, $f(x,y) = k(x+y), 0 \le x \le 2,\ 0 \le y \le 1$ then the value of k is- **1**

(a) 1/3 (b) 1 (c) 2 (d) 2

ix. A coefficient of correlation is computed to be -0.95 means that- **1**

(a) The relationship between two variables is strong and but negative
(b) The relationship between two variables is strong and positive
(c) The relationship between two variables is weak
(d) Correlation coefficient cannot have this value

x. Two samples of sizes 25 and 35 are independently drawn from two normal populations, where the unknown variances are assumed to be equal. The number of degrees (of freedom for the equal-variances t-test statistic is: **1**

(a) 24 (b) 58 (c) 34 (d) None of these

Q.2 i. Draw graphs of the following chemical compounds: **3**

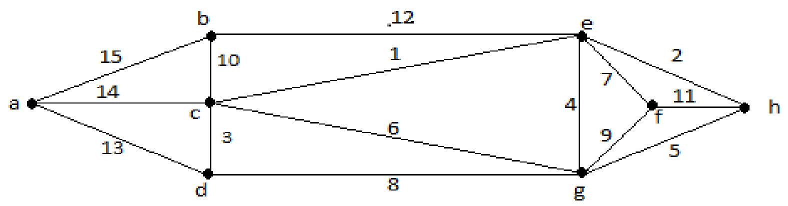
(a) CH_4 (b) C_2H_6 (c) C_6H_6

ii. Prove that a simple disconnected graph G with 'n' vertices and 'k' components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **7**

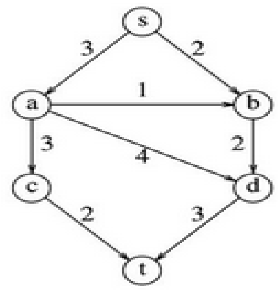
OR iii. Show that there is always a Hamiltonian path in a directed complete graph. **7**

Q.3 i. Define minimally connected graph and prove that a graph is a tree if and only if it is minimally connected. **3**

ii. Define length of path in a weighted graph and find shortest path form vertex 'a' to 'h' using Dijkstra' algorithm for the graph: **7**



OR iii. In the following network, find out the cut set with minimum capacity and maximum possible flow using Ford Fulkerson's algorithm: **7**



Q.4 i. Prove that: **3**

$$J_{-1/2} = \sqrt{\left(\frac{2}{\pi x}\right)} \cos x$$

ii. Obtain the series solution of the equation: **7**

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$$

OR iii. Prove that: **7**

$$\frac{d}{dx}(xber'x) = -xbei\ x \text{ and } \frac{d}{dx}(xbei'x) = xber\ x$$

Q.5 i. If $f(x,y) = \begin{cases} 4xy; 0<x<1, 0<y<1 \\ 0; \quad \text{otherwise} \end{cases}$ **3**

Show that X and Y are independent.

ii. The joint pdf of X and Y is **7**

$$f(x,y) = \begin{cases} \frac{6}{5}\left(x+y^2\right); & 0 \le x \le 1, 0 \le y \le 1 \\ 0; & \textit{otherwise} \end{cases}$$

Find the marginal density functions of X & Y and also find $P\left(\frac{1}{4} < y < \frac{3}{4}\right)$.

P.T.O.

Faculty of Engineering

End Sem (Even) Examination May-2022

EC3BS03/ EE3BS03 / EX3BS03

Engineering Mathematics - II

Programme: B.Tech Branch: EC/EE/EX

Date: 18-5-22

Q.1

i) (c) Insufficient conditions [1]

ii) (c) The sum of the out-degrees over all vertices [1]

iii) (b) 12 [1]

iv) (b) Minimum [1]

(v) (d) 2/15 [1]

(vi) (d) $-x^{-n} J_{n+1}(x)$ [1]

(vii) (d) 1 [1]

(viii) (b) 1 [1]

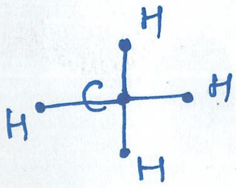
(ix) (a) The relationship between two variables is strong but negative. [1]

(X) (b) 58 [1]

Q.2

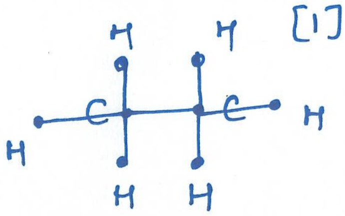
i) (a)

[1]



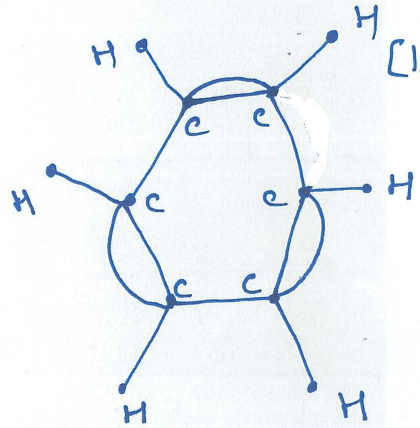
CH_4

[1]



C_2H_6

[1]



C_6H_6

Q 2(ii) Let $G_1, G_2, \ldots G_k$ be the k components of the disconnected graph G having $n_1, n_2, \ldots n_k$ number of vertices in each component respectively.

$$\therefore \sum_{i=1}^{k} n_i = n$$

[1]

So,

$$\sum_{i=1}^{k}(n_i - 1) = n_1 - 1 + n_2 - 1 + \cdots + n_k - 1$$
$$= n - k$$

$$\therefore \left[\sum_{i=1}^{k}(n_i-1)\right]^2 = [n-k]^2$$

[2]

$$\Rightarrow \sum_{i=1}^{k}(n_i-1)^2 + \sum_{\substack{i,j=1\\ i\neq j}}^{k}(n_i-1)(n_j-1) = (n-k)^2$$

$$\Rightarrow \sum_{i=1}^{k}(n_i-1)^2 + \text{some non -ve terms} = (n-k)^2$$

$$\Rightarrow \sum_{i=1}^{k}(n_i-1)^2 \leq (n-k)^2$$

[3]

$$\Rightarrow \sum_{i=1}^{k}(n_i^2 + 1 - 2n_i) \leq (n-k)^2$$

$$\Rightarrow \sum_{i=1}^{k} n_i^2 + k - 2n \leq (n-k)^2$$

$$\Rightarrow \sum_{i=1}^{k} n_i^2 \leq (n-k)^2 + 2n - k \quad \text{—(1)}$$

[4]

Since each component is a connected subgraph. So ith component of G has $\frac{n_i(n_i-1)}{2}$ max. no of edges in it. So disconnected graph G has maximum number of edges

[5]

$$\sum_{i=1}^{k} \frac{n_i(n_i-1)}{2} = \frac{1}{2}\sum_{i=1}^{k}(n_i^2 - n_i)$$

Using equation ① we have

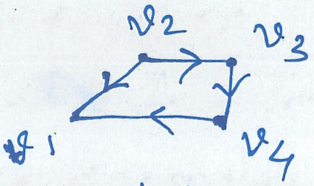
$$\sum_{i=1}^{k} \frac{n_i(n_i-1)}{2} \leq \frac{1}{2}\left[(n-k)^2+2n-k-n\right]$$

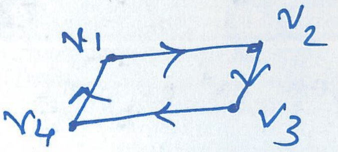
$$\leq \frac{1}{2}\left[(n-k)(n-k+1)\right] \quad -[6]$$

Hence maximum number of edges in a disconnected graph with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$ -[7]

OR

ii) Proof:
We have two types of complete Digraphs. [1]
Complete assymmetric & Complete symmetric.
If digraph is complete asymmetric then there is exactly one edge between every pair of vertices, so there is a possiblity that we get a directed circuit or a semi circuit. [2]
In both the cases we get a Hamiltonian circuit & removing any one edge from Hamiltonial circuit we get Hamiltonian path. [4]
If digraph is complete symmetric then there exist a directed Hamiltonian circuit in it [5]
and removing ~~of~~ any one edge from Hamiltonian circuit we get Hamitonian path. [6]





[7]

Complete assymmetric graph with semi circuit

Complete assymmetric graph with directed circuit

Que 3

(i) Minimally connected Graph: A connected graph G is said to be minimally connected if removal of any one edge from it disconnects the graph. [1]

Proof:

If part : Given: A graph is a tree.

T.P.T : Graph is minimally connected.

Proof: As graph is a tree. So it is connected graph. As we know that in a tree there exist one and only one path between every pair of vertices of it. So removal of any one edge from it disconnects the graph. [2]

H.P//

Only if part : Given: A graph is minimally connected.

T.P.T: Graph is a tree.

Proof: As graph is minimally connected so it is a connected graph such that removal of any one edge from it disconnects the graph. So there exist one and only one path between every pair of vertices of the graph. Hence graph is a tree. [3]

H.P//

(ii) Length of the path in a weighted graph:
" " " " " " " " is the sum of the weights of the edges present in the path. [1]

Assign $P(L(a)) = 0$
set $v = a$

$T(L(b)) = T(L(c)) = T(L(d)) = T(L(e)) = T(L(f))$
$= T(L(g)) = T(L(h)) = \infty$.

Now,

$T(L(b)) = \min\{\infty, 0+15\} = 15$

$T(L(c)) = \min\{\infty, 0+14\} = 14$

$T(L(d)) = \min\{\infty, 0+13\} = 13$

$T(L(e)) = \min\{\infty, 0+\infty\} = \infty$

$T(L(f)) = \min\{\infty, 0+\infty\} = \infty$

$T(L(g)) = \min\{\infty, 0+\infty\} = \infty$

$T(L(h)) = \min\{\infty, 0+\infty\} = \infty$ [2]

Now, Assign $P(L(d)) = 13$
set $v = d$

$T(L(b)) = \min\{15, 13+\infty\} = 15$

$T(L(c)) = \min\{14, 13+3\} = 14$

$T(L(e)) = \min\{\infty, 13+\infty\} = \infty$

$T(L(f)) = \min\{\infty, 13+\infty\} = \infty$

$T(L(g)) = \min\{\infty, 13+8\} = 21$

$T(L(h)) = \min\{\infty, 13+\infty\} = \infty$ [3]

Now Assign $P(L(c)) = 14$
set $v = c$

$T(L(b)) = \min\{15, 14+10\} = 15$
$T(L(e)) = \min\{\infty, 14+1\} = 15$
$T(L(f)) = \min\{\infty, 14+\infty\} = \infty$
$T(L(g)) = \min\{21, 14+6\} = 20$
$T(L(h)) = \min\{\infty, 14+\infty\} = \infty$. [4]

Assign $P(L(e)) = 15$
set $v = e$

$T(L(b)) = \min\{15, 15+12\} = 15$
$T(L(f)) = \min\{\infty, 15+7\} = 22$
$T(L(g)) = \min\{20, 15+4\} = 19$
$T(L(h)) = \min\{\infty, 15+2\} = 17$ [5]

Assign $P(L(b)) = 15$
set $v = b$

$T(L(f)) = \min\{22, 15+\infty\} = 22$
$T(L(g)) = \min\{19, 15+\infty\} = 19$
$T(L(h)) = \min\{17, 15+\infty\} = 17$ [6]

Assign $P(L(h)) = 17$
set $v = h$

So length of the shortest path from vertex a to h is 17. [7]

iii) Define a vertex set P in the graph as follows:

a) $s \in P$

[1]

b) If vertex $i \in P$ & either $f_{ij} < c_{ij}$ or $f_{ij} > 0$ then $j \in P$. Any vertex not in P belongs to $\bar{P}$. t must be in the vertex set $\bar{P}$. Cut $C(P, \bar{P})$ seperates s from t. [2]

Vertex set P	$C(P, \bar{P})$
$\{s\}$	5
$\{s, b\}$	5
$\{s, a\}$	10
$\{s, b, d\}$	6
$\{s, a, c\}$	9
$\{s, a, b\}$	9
$\{s, a, b, d\}$	6
$\{s, a, b, d, c\}$	5

[5]

The cut with the minimum capacity among these is the one in which $P = \{s\}$ and $\bar{P} = \{a, b, c, d, t\}$ or $P = \{s, b\}$ and $\bar{P} = \{a, c, d, t\}$ or $P = \{s, a, b, d, c\}$ and $\bar{P} = \{t\}$. The maximum possible flow in s to t in the network is therefore 5 units. [7]

4(i)

$$J_{-n}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{-n+2r}}{r!\,\Gamma(-n+r+1)}$$ —[1]

Put $n = 1/2$

$$J_{-1/2}(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (x/2)^{-1/2+2r}}{r!\,\Gamma(r+1/2)}$$

$$= \left(\frac{x}{2}\right)^{-1/2} \sum_{r=0}^{\infty} \frac{(-1)^r x^{2r}}{2^{2r}\, r!\,\Gamma(r+1/2)}$$

$$= \left(\frac{x}{2}\right)^{-1/2} \left[\frac{1}{\Gamma 1/2} - \frac{x^2}{2^2 . 1!\,\Gamma 3/2} + \frac{x^4}{2^4 . 2!\,\Gamma 5/2} - - - \right]$$ [2]

$$= \sqrt{\frac{2}{x}} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{2^2\, 1/2\sqrt{\pi}} + \frac{x^4}{2^4 . 2 . \frac{3}{2} . \frac{1}{2}\sqrt{\pi}} - - - \right]$$

$$= \sqrt{\frac{2}{\pi x}} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + - - - \right]$$

$$= \sqrt{\frac{2}{\pi x}} \cos x .$$ [3]

(ii)

As $P_0(x) = 0$ at $x = 0$

also $P_1(x)$ & $P_2(x)$ are analytic at $x = 0$

So $x = 0$ is a regular singular points.

Hence atleas one solution of the given differential equation is of the form

$$y = \sum_{r=0}^{\infty} a_r x^{m+r}$$ [1]

i.e

$$y = a_0 x^m + a_1 x^{m+1} + - - - + a_n x^{m+n} + - - -$$

$$\frac{dy}{dx} = m a_0 x^{m-1} + (m+1) a_1 x^m + - - - + (m+n) a_n x^{m+n-1} + - - -$$

$$\frac{d^2y}{dx^2} = m(m-1)a_0x^{m-2} + (m+1)m a_1 x^{m-1} + \cdots + (m+n)(m+n-1)a_n x^{m+n-2} + \cdots$$

[2]

So given eq. can be written as

$$9[m(m-1)a_0x^{m-1} + (m+1)m a_1 x^{m} + \cdots + (m+n)(m+n-1)a_n x^{m+n-1} + \cdots]$$
$$-9[m(m-1)a_0x^{m} + (m+1)m a_1 x^{m+1} + \cdots + (m+n)(m+n-1)a_n x^{m+n} + \cdots]$$
$$-12[m a_0 x^{m-1} + (m+1)a_1x^{m} + \cdots + (m+n)a_n x^{m+n-1} + \cdots]$$
$$+4[a_0x^m + a_1x^{m+1} + \cdots + a_n x^{m+n} + \cdots] = 0.$$

Its indical eq. is [3]

$$[9m^2 - 9m - 12m]a_0 = 0$$

$$m[9m - 21]a_0 = 0$$

$$\Rightarrow m = 0, \frac{21}{9} \quad \text{as } a_0 \neq 0$$

i.e. $m = 0, 7/3$ [4]

Now comparing the coefficient of x^{m+n} both the sides we get

$$9(m+n+1)(m+n)a_{n+1} - 9(m+n)(m+n-1)a_n$$
$$-12(m+n+1)a_{n+1} + 4a_n = 0$$

$$3(m+n+1)a_{n+1}[3(m+n)-4] = a_n[9[(m+n)^2 - (m+n)] - 4]$$

$$a_{n+1} = \frac{[9[(m+n)^2-(m+n)]-4]a_n}{(3m+3n+3)(3m+3n-4)}$$ [5]

Put $n=0$

$$a_1 = \frac{[9(m^2-m)-4]a_0}{(3m+3)(3m-4)} = \frac{(3m+1)\cancel{(3m-4)}a_0}{(3m+3)\cancel{(3m-4)}}$$

$$a_1 = \frac{3m+1}{3m+3} a_0$$

Put $n=1$

$$a_2 = \frac{(3m+4)(3m+1)}{(3m+6)(3m+3)} a_0$$

and so on...

$$\therefore \quad y = a_0 x^m + \frac{3m+1}{3m+3} a_0 x^{m+1} + \frac{(3m+4)(3m+1)}{(3m+6)(3m+3)} a_0 x^{m+2} + \cdots$$

[6]

$$\therefore \quad C.S \quad y = A(y)_{m=0} + B(y)_{m=7/3}.$$

$$y = A\left[a_0 + \frac{1}{3} a_0 x + \frac{4}{18} a_0 x^2 + \cdots\right]$$
$$+ B\left[a_0 x^{7/3} + \frac{8}{10} a_0 x^{10/3} + \frac{11 \cdot 8}{13 \cdot 10} a_0 x^{13/3} + \cdots\right]$$ [7]

OR

(iii)

$$\frac{d}{dx}(x\, ber' x) = -x\, bei\, x$$

$$ber\, x = 1 + \sum_{m=1}^{\infty} (-1)^m \frac{x^{4m}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m)^2}$$

$$x\, ber' x = x \sum_{m=1}^{\infty} \frac{4m\, (-1)^m\, x^{4m-1}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m)^2}$$ [2]

$$\frac{d}{dx}[x\, ber' x] = \frac{d}{dx} \sum_{m=1}^{\infty} \frac{4m\, (-1)^m x^{4m}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2 (4m)^2}$$

$$= \frac{x}{x} \sum_{m=1}^{\infty} \frac{\cancel{(4m)^2} (-1)^m x^{4m-1}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2 \cancel{(4m)^2}} = -\left[x \sum_{m=1}^{\infty} \frac{(-1)^m x^{4m-2}}{2^2 \cdot 4^2 \cdots (4m-2)^2}\right]$$

$$= -x \,\mathrm{bei}\, x$$

$$\mathrm{bei}\, x = -\sum_{m=1}^{\infty} \frac{(-1)^m x^{4m-2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2}$$

$$x \,\mathrm{bei}' x = -x \sum_{m=1}^{\infty} \frac{(-1)^m (4m-2)\, x^{4m-3}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2}$$

$$\frac{d}{dx}[x \,\mathrm{bei}' x] = -\frac{d}{dx} \sum_{m=1}^{\infty} \frac{(-1)^m (4m-2)\, x^{4m-2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2} \qquad [5]$$

$$= -\sum_{m=1}^{\infty} \frac{(-1)^m (4m-2)^2 x^{4m-3}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2}$$

$$= -\sum_{m=1}^{\infty} \frac{(-1)^m x^{4m-3}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-4)^2}$$

$$= x \,\mathrm{ber}\, x \qquad [7]$$

or

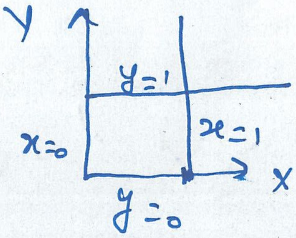
$$\int_0^x x \,\mathrm{ber}\, x \, dx = \frac{x^2}{2} + \sum_{m=1}^{\infty} \frac{(-1)^m x^{4m+2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m)^2 (4m+2)}$$

$$= -\sum_{m=1}^{\infty} \frac{(-1)^m x^{4m-2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-4)^2 (4m-2)} = x \,\mathrm{bei}' x$$

$$\therefore \frac{d}{dx}[x \,\mathrm{bei}' x] = x \,\mathrm{ber}\, x$$

Q5 (i)

$$f(x) = \int_0^1 4xy \, dy$$



$$= \left(4x \frac{y^2}{2}\right)_0^1 = 2x \;;\; 0 < x < 1 \qquad [1]$$

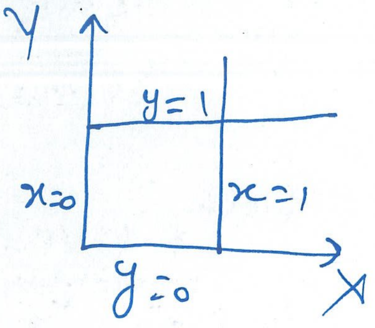
$$f(y) = \int_0^1 4xy \, dx = \left(4y \frac{x^2}{2}\right)_0^1 = 2y \;;\; 0 < y < 1 \qquad [2]$$

$$f(x,y) = f(x)f(y)$$

$\therefore$ X & Y are independent random variable. [3]

(ii)

$$f(x) = \int_0^1 \frac{6}{5}(x+y^2)\,dy$$



$$= \left[\frac{6}{5}xy + \frac{6}{5}\frac{y^3}{3}\right]_0^1$$ [1]

$$= \frac{6}{5}\left[x + \frac{1}{3}\right] \; ; \; 0 \leq x \leq 1$$ [2]

$$f(y) = \int_0^1 \frac{6}{5}(x+y^2)\,dx$$

$$= \frac{6}{5}\left[\frac{x^2}{2} + y^2 x\right]_0^1$$

$$= \frac{6}{5}\left[\frac{1}{2} + y^2\right] \; ; \; 0 \leq y \leq 1$$ [4]

$$P\left(\tfrac{1}{4} \leq y \leq \tfrac{3}{4}\right) = \int_{1/4}^{3/4} \frac{6}{5}\left[\frac{1}{2} + y^2\right] dy$$ [5]

$$= \frac{6}{5}\left[\frac{1}{2}y + \frac{y^3}{3}\right]_{1/4}^{3/4}$$

$$= \frac{6}{5}\left[\frac{3}{8} + \frac{9}{64} - \frac{1}{8} - \frac{1}{192}\right]$$

$$= \frac{6}{5}\left[\frac{(16+9)}{64} \times \frac{3}{3} - \frac{1}{192}\right]$$

$$= \frac{6}{5}\left[\frac{75-1}{192}\right] = \frac{\cancel{74}^{\,37}}{5 \times \cancel{32}_{\,16}} = \frac{37}{80}$$ [7]

(iii)

Let p be the probability of getting head

$\therefore p = \frac{1}{2}, q = \frac{1}{2}$

$n = 10$ [1]

The random variable X is binomially distributed

mean $\mu = np = 5$

$\sigma^2 = npq = 2.5$

$\therefore \sigma = \sqrt{2.5}$ [2]

$X \sim N(\mu, \sigma)$

$\therefore X \sim N(5, \sqrt{2.5})$ [3]

Standard normal variate $z = \frac{X-\mu}{\sigma} = \frac{X-5}{\sqrt{2.5}}$ [4]

$\therefore P[3 \text{ or } 4 \text{ or } 5] = P[2.5 < X < 5.5]$ [5]

$= P[-1.58 < z < \cdot 316]$

$= P[0 < z < 1.58] + P[0 < z < \cdot 316]$

$= \cdot 442 + \cdot 1217$

$= \cdot 5646.$ [7].

Q 6(i) Regression line of y on x is

$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$ —(1)

& Regression line of x on y is

$x - \bar{x} = \frac{r\sigma_x}{\sigma_y}(y - \bar{y}) \Rightarrow y - \bar{y} = \frac{\sigma_y}{r\sigma_x}(x - \bar{x})$ —(2)

[1]

$\therefore$ from 1

$$\frac{dy}{dx} = m_1 = \frac{r\sigma_y}{\sigma_x}$$

& from 2

$$\frac{dy}{dx} = m_2 = \frac{\sigma_y}{r\sigma_x} \qquad [2]$$

Let θ be the acute angle between the two regression lines.

$$\therefore \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\frac{\sigma_y}{r\sigma_x} - \frac{r\sigma_y}{\sigma_x}}{1 + \frac{r\sigma_y}{\sigma_x} \cdot \frac{\sigma_y}{r\sigma_x}}$$

$$= \frac{\sigma_y}{\sigma_x}\left[\frac{1 - r^2}{r}\right] \cdot \frac{1}{1 + \frac{\sigma_y^2}{\sigma_x^2}}$$

$$= \frac{(1 - r^2)}{r}\left[\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right] \qquad [3]$$

(ii) The second degree parabolic curve is given by

$$y = a + bx + cx^2$$

Its equation of normal are

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\,\Sigma x^2 + b\,\Sigma x^3 + c\,\Sigma x^4 \quad [1]$$

x	y	x^2	x^3	x^4	xy	x^2y
1	2	1	1	1	2	2
2	6	4	8	16	12	24
3	7	9	27	81	21	63
4	8	16	64	256	32	128
5	10	25	125	625	50	250
6	11	36	216	1296	66	396
7	11	49	343	2401	77	539
8	10	64	512	4096	80	640
9	9	81	729	6561	81	729

[4]

$\Sigma x = 45$, $\Sigma y = 74$, $\Sigma x^2 = 285$, $\Sigma x^3 = 2025$

$\Sigma x^4 = 15333$, $\Sigma xy = 421$, $\Sigma x^2 y = 2771$ [5]

$$74 = 9a + 45b + 285c$$

$$421 = 45a + 285b + 2025c$$

$$2771 = 285a + 2025b + 15333c$$

Solving them

$a = -1$, $b = 3.55$, $c = -.27$ [6]

$\therefore \quad y = -1 + 3.55x - .27x^2$ [7]

(iii)

H_0: The difference between the sample mean is not significant. [1]

Here $n_1 = 10$, $n_2 = 12$

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{120}{10} = 12\text{ kg}$$

$$\bar{y} = \frac{\Sigma y}{n_2} = \frac{180}{12} = 15\,kg \qquad [2]$$

Diet A			Diet B		
x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
10	-2	4	7	-8	64
6	-6	36	13	-2	4
16	4	16	22	7	49
17	5	25	15	0	0
13	1	1	12	-3	9
12	0	0	14	-1	1
8	-4	16	18	3	9
14	2	4	8	-7	49
15	3	9	21	6	36
9	-3	9	23	8	64
			10	-5	25
			17	2	4

[4]

$$\Sigma(x-\bar{x})^2 = 120, \quad \Sigma(y-\bar{y})^2 = 314$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2}$$

$$= \frac{120+314}{10+12-2} = \frac{434}{20} = 21.7$$

$$\therefore\ s = \sqrt{21.7} = 4.65 \qquad [5]$$

$$t = \left(\frac{\bar{x}-\bar{y}}{s}\right)\sqrt{\left[\frac{n_1 n_2}{n_1+n_2}\right]} = 1.5067721$$

$$\nu = n_1+n_2-2 = 20 \qquad [6]$$

Calculated value of $|t|$ < tabulated value of t

so accept the null hypothesis. [7]