Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2022

BC3CO15 Mathematics -IV

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If $(G,*)$ is a group with multiplication, then order of an element implies that there exist an positive integer n such that- **1**

(a) $a^n \neq e$ (b) $a^n = e$ (c) $a^n = a^{-1}$ (d) $a^n \neq a^{-1}$

ii. A group of prime order is always- **1**

(a) Simple (b) Cyclic

(c) Both (a) and (b) (d) None of these

iii. The permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ is an ------------permutation. **1**

(a) Odd

(b) Even

(c) Can't be expressed as disjoint cycles

(d) None of these

iv. If $f: G_1 \to G_2$ is an isomorphism, then kernel of f is **1**

(a) $\{\ \}$ (b) G_1 (c) $\{e\}$ (d) G_2

v. Which one of the following statements is true? **1**

P: Every field is an integral domain

Q: Every integral domain is a field.

(a) Only Q (b) Only P

(c) Both (a) and (b) (d) None of these

vi. The degree of polynomial $f(x) = 3x^4 + 0x^3 + 2x + 10$ over ring of integers is equal to- **1**

(a) 0 (b) 4 (c) 2 (d) 1

vii. Any set containing a single nonzero vector is linearly- **1**
(a) Independent (b) Dependent
(c) Both (a) and (b) (d) Neither (a) nor (b)

viii. If $V(F)$ is a finite dimensional vector space over filed F, then the number of elements in any two basis is always- **1**
(a) Even (b) Odd (c) Equal (d) Unequal in numbers

ix. The kernel of linear transformation T from vector space U to vector space V over the field F is a subspace of $U(F)$- **1**
(a) True (b) False (c) Can't say (d) None of these

x. If V be finite dimensional vector space and w be a subspace of V then dimension of quotient space V/w is- **1**
(a) $\dim(V)+\dim(w)$ (b) $\dim(V)-\dim(w)$
(c) $\dim(V)/\dim(w)$ (d) Can't say

Q.2 Attempt any two:

i. If G is an abelian group and Z is a set of integers, then prove that $(ab)^n = a^n b^n \ \forall a,b \in G$ and $\forall n \in Z$ **5**

ii. State and prove Lagrange's Theorem. **5**

iii. Show that the order of cyclic group is same as that of its generator. **5**

Q.3 Attempt any two:

i. Show that any two right cosets of subgroup H of a group (G,o) are either disjoint or identical. **5**

ii. Show that the group $\{(1,2,3,4,5,6),\times_7\}$ is cyclic and also find all possible generators. **5**

iii. State and prove Cayley's theorem. **5**

Q.4 Attempt any two:

i. Prove that $\{(0,2,4,6,8),+_{10},\times_{10}\}$ is an integral domain. **5**

ii. Find sum and product of polynomials $f(x)=2x^4+3x^3+2$ and $g(x)=2x^3+3x^2+5$ over field $Z_5=(\{0,1,2,3,4\},+_5,\times_5)$. **5**

iii. Prove that every finite integral domain is a field. **5**

Q.5 Attempt any two:

i. Show that the set $W=\{(a,b,c): a-3b+4c=0; \forall a,b,c \in R\}$ is a vector subspace of $V_3(R)$. **5**

ii. If W_1 , W_2 are two subspaces of a finite dimensional vector space over the field F then $dim(W_1+W_2)=dim W_1+dim W_2-dim(W_1 \cap W_2)$ **5**

iii. Show that the vectors $\alpha_1=(1,0,-1), \alpha_2=(1,2,1)$ and $\alpha_3=(0,-3,2)$ form a basis of $V_3(R)$. **5**

Q.6 Attempt any two:

i. Show that the mapping $T: R_2 \to R_3$ defined by $T(a,b)=(a-b,b-a,-a) \forall a,b \in R$ is a linear transformation from R_2 to R_3 also find the nullity of T. **5**

ii. State and prove "Rank–Nullity theorem ". **5**

iii. Find the matrix representation of linear transformation T on vector space V over the field R defined as $T(a,b,c)=(ab+c,a-4b,3a)$ corresponding to the basis $B=\{(1,1,1),(1,1,0),(1,0,0)\}$ **5**

******

Faculty of Science

End sem (Even) Examination May 2022

BC3CO15 Mathematics -IV

Program B.Sc (CS)

Q1.

(i) (b) $a^n = e$ +1

(ii) (b) Cyclic +1

(iii) (b) Even +1

(iv) (c) $\{e\}$ +1

(v) (b) Only P +1

(vi) (b) 4 +1

(vii) (a) Independent +1

(viii) (c) equal +1

(ix) (a) True +1

(x) (b) $\dim V - \dim W$ +1

Q2

(i) Sol: We are using mathematical induction to Solve this

Case I: When n is positive integer +2

When $n=1$

$(ab)^1 = a^1 b^1 = ab$ Hence true for $n=1$

Suppose for $n=k$ relation is true then

$(ab)^k = a^k b^k$ — (i)

Now for $n = k+1$

$(ab)^{k+1} = (ab)^k . (ab)$

$= (a^k b^k)(ab)$ by (i)

$= a^k (b^k a) b$ [Associative property]

$= a^k (a b^k) b$ [$\because$ G is Abelian]

$= (a^k a)(b^k . b)$ [Associative property]

$= a^{k+1} b^{k+1}$

Hence it is true for all positive integers

Case II When $n=0$ +1

$(ab)^0 = a^0 b^0$

$1 = 1 \cdot 1$ Hence true

Case III When n is negative integer +2

Let $n = -m$ (m is positive)

$(ab)^n = (ab)^{-m}$

$= [(ab)^m]^{-1}$ By Case I property is true for

$= [(ba)^m]^{-1}$ (Abelian group) positive integers

$= [b^m a^m]^{-1}$

$= a^{-m} b^{-m}$ [By reversal law of group] Hence proved

Q2(ii)

Langrange's theorem :- The order of each Subgroup of a finite group is a divisor of the order of group

Pf:- Let G be a group of finite order n. Let H be Subgroup of G and $O(H) = m$ Suppose $h_1, h_2, \ldots h_m$ are the m members of H Let $a \in G$. Then Ha is right coset of H in G and we have

$Ha = \{h_1 a, h_2 a, \ldots h_m a\}$

Ha has m distinct members, Since $h_i a = h_j a \Rightarrow h_i = h_j$

Therefore each right coset of H in G has m distinct member Any two distinct right cosets of H in G are disjoint ie they have no elements in Common. Since G is a finite group, the number of distinct right Cosets of H in G will be finite, Say equal to K. The union of these K distinct right Cosets of H in G is equal to G. Thus if $Ha_1, Ha_2, \ldots Ha_K$ are the K distinct right Cosets of H in G then

$G = Ha_1 \cup Ha_2 \cup \ldots Ha_K$

$\Rightarrow$ the number of elements in G = the number of elements in Ha_1 + ... + the number of elements in Ha_K [$\because$ two distinct right Cosets are mutually disjoint

$\Rightarrow O(G) = Km \Rightarrow n = km$

$\Rightarrow k = n/m \Rightarrow m$ is divisor of n

$\Rightarrow O(H)$ is a divisor of $O(G)$ Hence proved

+1

+2

+2

Q2(iii)

Pf: Let the order of a generator of a cyclic group be n then $a^n = e$ while $a^s \neq e$ for $0 < s < n$

when $s > n$, $s = nq + r$, $0 \leq r < n$ (say)

$a^s = a^{nq+r} = (a^n)^q \cdot a^r = e^q \cdot a^r$

$= e \cdot a^r = a^r$

Thus there are exactly n elements in the group by a^r, where $0 \leq r < n$. Therefore there are n and only n distinct elements in the cyclic group i.e the order of the group is n.

+1

+2

+2

Q3

(i) Let H be a subgroup of a group G and let aH and bH be two left cosets. Suppose these cosets are not disjoint. Then they possess an element, say C in common. Then C may be written as $C = ah$, and also $C = ah'$, where h and h' are in H

Therefore $ah = bh'$

$a = bh'h^{-1}$

Since H is a subgroup $hh^{-1} \in H$

$h'h^{-1} = h''$

Then $a = bh''$

$aH = (bh'')H = b(h''H) = bH$

+2

+2

Therefore the two left cosets are either identical if they are not disjoint. Thus either
$aH \cap bH = \phi$ or $aH = bH$
A similar result can be shown to hold for right cosets

+1

Q3(ii)
Sol: Here we find that
Closure property: $1 \cdot 2 = 2$ $1 \cdot 3 = 3$ $1 \cdot 4 = 4$ $1 \cdot 5 = 5$
$1 \cdot 6 = 6$, $2 \cdot 3 = 6$
$2 \cdot 4 = 8 \equiv 1 \pmod 7$ $3 \cdot 4 = 12 \equiv 5 \pmod 7$
$2 \cdot 5 = 10 \equiv 3 \pmod 7$ $3 \cdot 6 \equiv 18 \equiv 4 \pmod 7$
$2 \cdot 6 = 12 \equiv 5 \pmod 7$ $4 \cdot 6 \equiv 24 \equiv 3 \pmod 7$
$4 \cdot 5 = 20 \equiv 6 \pmod 7$ $5 \cdot 6 \equiv 30 \equiv 2 \pmod 7$

+2

Hence closure is satisfied
Associative property: $2 \cdot (3 \cdot 4) = 2 \cdot 5$
$= 3$ [$\because 3 \cdot 4 = 12 = 5 \pmod 7$
$\because 2 \cdot 5 = 10 = 3 \pmod 7$
$\because 6 \cdot 4 = 24 = 3 \pmod 7$
and $(2 \cdot 3) \cdot 4 = (6) \cdot 4 = 3$
$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4)$ Hence the associative
~~Inverse~~
Identity The element exsist and is equal to 1.

+1

Inverse: As in closure property the inverse of each element exsists and inverse of 1, 2, 3, 4, 5 and 6 are 1, 4, 5, 2, 3, 6 respectively

+1

Hence G is Group. Now we are to prove that it is cyclic for this let there exsist an element $a \in G$ Such that $O(a) = O(G) = 6$ then the group will be cyclic

Here we find that

$3^1 = 3 \cdot 3^2 = 3 \cdot 3 = 9 \equiv 2 (\text{Mod } 7)$

$3^3 = 3^2 \cdot 3 = 2 \cdot 3$

$3^2 = 2 (\text{mod } 7)$ or $3^3 = 6$

$= O(G)$

$3^4 = 3^3 \cdot 3 = 6 \cdot 3$

$3^4 = 18 \equiv 4 (\text{mod } 7)$

$3^5 = 3^4 \cdot 3 = 4 \cdot 3$

$3^5 = 12 \equiv 5 (\text{mod } 7)$

$3^6 = 3^5 \cdot 3 = 15 \equiv 1 (\text{mod } 7)$

From (i) we observe that $O(3) = 6 = O(G)$ and so 3 is a generator of G and we have also found that $3^6 = 1$, $3^2 = 2$, $3^1 = 3$

$3^4 = 4$, $3^5 = 5$ $3^3 = 6$

Hence G is a cyclic group.

+1

Q3(iii)

Statement: Every finite group is isomorphic to a permutation group. +1

Pf: Let G be a finite group of order n. If $a \in G$ then $\forall x \in G \Rightarrow ax \in G$. Now consider a function from G into G defined by

$f_a(x) = ax \ \forall \ x \in G$

for $x, y \in G$, $f_a(x) = f_a(y) \Rightarrow ax = ay$

$\Rightarrow x = y$ (by left Cancellation law)

Therefore function f_a is one-one

The function f_a is also onto because if x is any element of G then $\exists$ an element f_a such that $f_a(a^{-1}x) = a(a^{-1}x) = (aa^{-1})x = ex = x$

Thus f_a is one-one from G onto G. Therefore f_a is a permutation on G. Let G' denote the set of all such one-to one onto functions defined on G Corresponding to every element of G i.e

$G' = \{f_a ; a \in G$

+1

Now we show that G' is group w.r.t product of f^{ns}

(i) Closure : Let $f_a, f_b \in G'$ where $a, b \in G$, then

+1

$(f_a \circ f_b)x = f_a[f_b(x)] = f_a(bx) = a(bx)$

$= (ab)x = f_{ab}(x) \ \forall \ x \in G$

Hence $f_a \circ f_b = f_{ab}$ — (i)

$\because ab \in G$, so $f_{ab} \in G'$ and thus G' is closed

(ii) Associative: Let $f_a, f_b, f_c \in G'$ where $a, b, c \in G'$

$f_a \circ (f_b \circ f_c) = f_a \circ f_{bc}$ from [i]

+1

$= f_a(bc)$ [from i]

$= (ab)c$ by associative law

$= f_{ab} \circ f_c$ [from i]

$= (f_a f_b) \circ f_c$ [from i]

Product of functions is associative in G'

(iii) identity axiom: If e is the identity element in G, then f_e is the identity of G' because $\forall f_x \in G'$ we have $f_e \circ f_x = f_{ex} = f_x$ &

$f_x \circ f_e = f_{xe} = f_x$

(iv) Inverse axiom: If $\bar{a}'$ is the inverse of a in G, then $f_{\bar{a}'}$ is the inverse of f_a in G' because

$f_a^{-1} \circ f_a = f_{a^{-1}a} = f_e$ and $f_a \circ f_a^{-1} =$

$f_{aa^{-1}} = f_e$

Hence G' is group w.r.t to composite of f^n

Now consider the f^n g from G to G' defined by

$g(a) = f_a \ \forall \ a \in G$

$\Rightarrow ax = bx \Rightarrow a = b \ \forall \ x \in G$

g is onto because if $f_a \in G'$ then for $a \in G$ we have $g(a) = f_a$

Hence g preserves composition in G and G' because if $ab \in G$ then

$g(ab) = f_{ab}$

$= f_a \circ f_b$

$= g(a) \circ g(b)$

$\therefore G \cong G'$ Hence proved

+1

Q4(i) Clearly $\{(0,2,4,6,8), +_{10}, \times_{10}\}$ is an Ring with unity. Also it has no zero divisors hence it is integral domain.

+5

Q4(ii) $f(x) = 2x^4 + 3x^3 + 0 \cdot x^2 + 2 \quad g(x) = 0 \cdot x^4 + 2 \cdot x^3 + 3x^2 + 5$

Sum. $(2 +_5 0)x^4 + (3 +_5 2)x^3 + (0 +_5 3)x^2 + (2 +_5 5)$

$= 2x^4 + x^3 + 3x^2 + 2$

+2.5

$f(x) \cdot g(x) = (2 \times_5 0)x^4 + (3 \times_5 2)x^3 + (0 \times_5 3)$
$+ (2 \times_5 5)$

$= 0 + x^3 + 0 + 0$

$= x^3$

+2.5

Q4(iii) Let D be an integral domain with finite number of distinct elements $a_1, a_2, \ldots a_n$

i.e $D = \{a_1 \ldots a_n\}$

Let a_i be any fixed element of D. Now multiplying each element of D by a_i then we get same set in different order which is

$a_i a_1, a_i a_2, a_i a_3, \ldots a_i a_n$

Let $a_i a_k = a_i a_j$

$\Rightarrow a_k = a_j$ (by left cancellation law)

Which is Contradiction

So $a_k \neq a_j$

Therefore $a_i a_k \neq a_i a_j$

Hence all elements of the form $a_j a_k$ are distinct

Now one element is unity. Therefore we must have $a_j a_k = 1$ for some k such that $1 \leq k \leq n$ since a_i is an arbitrary non zero element of D. Therefore every element of the type a_i has a multiplicative inverse therefore D is a field.

(1) +2 +2

Q5(i)

Set $W = \{(a,b,c) : a-3b+4c=0 \ \forall\, a,b,c \in R\}$
is a vector subspace of $V_3(R)$

Here it is sufficient to show that
$a\alpha + b\beta \in W \quad \forall\ \alpha, \beta \in W$ +1

Here take $\alpha = (a_1, b_1, c_1)$ Here by condition
$\beta = (a_2, b_2, c_2)$ $\quad a_1 - 3b_1 + 4c_1 = 0$
$a_2 - 3b_2 + 4c_2 = 0$

then $a\alpha + b\beta = a(a_1, b_1, c_1) + b(a_2, b_2, c_2)$
$= (aa_1 + ba_2, ab_1 + bb_2, ac_1 + bc_2)$

By condition $aa_1 + ba_2 - 3(ab_1 + bb_2) + 4(ac_1 + bc_2)$ +2

$= aa_1 + ba_2 - 3ab_1 - 3bb_2 + 4ac_1 + 4bc_2$

$= a(a_1 - 3b_1 + 4c_1) + b(a_2 - 3b_2 + 4c_2)$

$= 0 + 0$

$= 0$

Hence this clearly shows that by def^n $a\alpha + b\beta \in W$ Hence the given set is vector subspace of $V_3(R)$ +2

Q5 (ii)

Pf: Let the set $S = \{r_1, r_2, \dots r_k\}$ is a basis of $W_1 \cap W_2$, So that $\dim(W_1 \cap W_2) = k$

Then $S \subseteq W_1$ and $S \subseteq W_2$. Since S is linearly independent and $S \subseteq W_1$ and therefore by extension theorem S can be extended to form a basis of W_1. Let

$$\{r_1, r_2, \dots r_k, \alpha_1, \alpha_2, \dots \alpha_l\}$$

be a basis of W_1. Then

$\dim W_1 = k + l$

Similarly let $\{r_1, r_2 \dots r_k, \beta_1, \beta_2, \dots \beta_m\}$ is a basis of $W_1 + W_2$. First we show that S_1 is linearly independent.

+2

+2

Let $c_1r_1 + c_2r_2 + \dots + c_kr_k + a_1\alpha_1 + a_2\alpha_2 + \dots$

$+ a_l\alpha_l + b_1\beta_1 + \dots + b_m\beta_m = 0$

$\Rightarrow b_1\beta_1 + b_2\beta_2 + \dots + b_m\beta_m$

$= -(c_1r_1 + c_2r_2 + \dots + c_kr_k + a_1\alpha_1 + \dots a_l\alpha_l)$

$\in W_1 \cap W_2$

[$\because c_1r_1 + c_2r_2 + \dots + c_kr_k + a_1\alpha_1 + a_2\alpha_2 + \dots + a_l\alpha_l \in W_1$ $\because$ it is a linear Combination of a basis of W_1 and $b_1\beta_1 + b_2\beta_2 + \dots + b_m\beta_m \in W_2$ Since it is linear Combination of elements belonging to a basis of W_2. Also $b_1\beta_1 + b_2\beta_2 + \dots + b_m\beta_m \in W_1$]

Therefore $b_1\beta_1 + b_2\beta_2 + \dots + b_m\beta_m = W_1 \cap W_2$ can be expressed as L.C of basis of $W_1 \cap W_2$

Thus we have relation of the form

$b_1 B_1 + b_2 B_2 + \cdots + b_m \beta_m$

$= d_1 \gamma_1 + d_2 \gamma_2 + \cdots + d_k \gamma_k$

$\Rightarrow b_1 = 0 \quad b_2 = 0 \cdots b_m = 0$

as $\beta_1 \ldots \beta_m$ and $\gamma_1 \ldots \gamma_m$ are linearly independent vectors

Putting this values in (1), it reduces to

$c_1 \gamma_1 + c_2 \gamma_2 + \cdots + c_k \gamma_k + a_1 \alpha_1 + \cdots + a_l \alpha_l$

$\Rightarrow c_1 = 0, \; c_2 = 0, \cdots c_k = 0$

$a_1 = 0, \; a_2 = 0, \cdots a_l = 0$

Hence $c_1 \gamma_1 + \cdots c_k \gamma_k + a_1 \alpha_1 + \cdots a_l \alpha_l +$

$b_1 \beta_1 + \cdots b_m \beta_m = 0$

$\Rightarrow c_1 = 0, c_2 = 0 \ldots c_k = 0, a_1 = 0 \ldots a_l = 0$ &

$b_1 = 0 \ldots b_m = 0$

$\therefore$ The set S_1 of vectors $\gamma_1 \ldots \gamma_k, \alpha_1 \ldots \alpha_l$ & $B_1 \ldots B_m$ are linearly independent

Now we shall show $L(S_1) = W_1 + W_2$

Since $W_1 + W_2$ is a subspace of V and each element of S_1 belong to $W_1 + W_2$ therefore

$L(S_1) \subseteq W_1 + W_2$ — (2)

Again let α be any element of $W_1 + W_2$

then α = Some element of W_1 + Some element of W_2

= a linear combination of basis of W_1 + a linear combination of basis of W_2 = linear combination of S_1

$\therefore \alpha \in L(S_1)$

Hence $W_1 + W_2 \subseteq L(S_1)$ — (3)

Thus from (2) & (3) we have

$L(S_1) = W_1 + W_2$

$\therefore$ S_1 is a basis of $W_1 + W_2$ Consequently

so $\dim(W_1 + W_2) = k + l + m$

Hence $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

+1

Sol (iii) Let $S = \{\alpha_1, \alpha_2, \alpha_3\}$

$\alpha_1 = (1, 0, -1)$ $\alpha_2 = (1, 2, 1)$ $\alpha_3 = (0, -3, 2)$

$\Rightarrow a_1(1, 0, -1) + a_2(1, 2, 1) + a_3(0, -3, 2) = (0, 0, 0)$

$a_1 + a_2 + 0a_3 = 0$

$0a_1 + 2a_2 - 3a_3 = 0$

$-a_1 + a_2 + 2a_3 = 0$

+1

Coeff. matrix of above eq is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$

+1

$|A| = 1(4-3) + 1(-3-0) + 0(0+2) = -2 \neq 0$

$\therefore$ $\rho(A) = 3$ = the number of unknown constants

Hence $a_1 = 0$, $a_2 = 0$ & $a_3 = 0$ is the only solution of these eq. Therefore $\alpha_1, \alpha_2, \alpha_3$ are l. independent in $V_3(R)$

+2

(ii) Now Clearly $L(S) = V_3(R)$. Hence it form a basis of $V_3(R)$.

+1

5

Q.6

(i) T is a linear Transformation:

Let $\alpha = (a_1, b_1)$, $\beta = (a_2, b_2)$ be two arbitrary elements of R^2 and $x, y \in R$, then

+2

$$T(x\alpha + y\beta) = T[x(a_1, b_1) + y(a_2, b_2)]$$
$$= T(xa_1 + ya_2, xb_1 + yb_2)$$
$$= (xa_1 + ya_2 - xb_1 - yb_2, xb_1 + yb_2 - xa_1 - ya_2, -xa_1 - ya_2)$$
$$= (xa_1 - xb_1, xb_1 - xa_1, -xa_1) + (ya_2 - yb_2, yb_2 - ya_2, -ya_2)$$

$$= x(a_1 - b_1, b_1 - a_1, -a_1) + y(a_2 - b_2, b_2 - a_2, -a_2)$$
$$= xT(a_1, b_1) + yT(a_2, b_2)$$

$$= xT(\alpha) + yT(\beta)$$

$\therefore$ T is a linear transformation.

Nullity of T: By the definition, the nullity of T

+2

$N(T) = \{(a, b) \in R^2 : T(a, b) = 0\}$, where 0 is zero vector of R^3

$= \{(a, b) \in R^2 : (a - b, b - a, -a) = (0, 0, 0)\}$

$= \{(a,b) \in R^2 ; a-b=0, b-a=0, -a=0\}$

$= \{(a,b) \in R^2 ; a=0, b=0\}$

$= \{(0,0)\}$

Thus there exists only one element viz, zero vector of R^2 in the null-space of T. +1

$\therefore$ nullity of $T = V(T) = \dim N(T) = 0$

Q.6

(ii) Rank-Nullity Theorem:

By fundamental Theorem of vector space homomorphism, 1

we have

$$U / \ker T \cong \operatorname{Im} T$$

$$\dim U / \ker T = \dim \operatorname{Im} T$$ 2

$$\dim U - \dim \ker(T) = \dim \operatorname{Im}(T)$$

$$\dim \operatorname{Im}(T) + \dim \ker(T) = \dim U$$

$$\dim R(T) + \dim N(T) = \dim U$$ 2

$$\operatorname{rank}(T) + \text{nullity of } T = \dim U$$

Q.6. (iii) Let

$T: V_3(R) \to V_3(R)$ be a Linear transformation defined by

$T(a,b,c) = (ab+c, a-4b, 3a)$

and $V_3(R)$ has a basis set

$B = \{(1,1,1), (1,1,0), (1,0,0)\}$

we have

$T(1,1,1) = (2, -3, 3)$

Now we wish to express $(2,-3,3)$ as a Linear ~~transformation~~ Combination of vectors of B

Let $(a,b,c) = x(1,1,1) + y(1,1,0) + z(1,0,0)$

$= (x+y+z, x+y, x)$

$\Rightarrow x+y+z = a, \; x+y = b, \; x = c$

$x = c, \; y = b-c, \; z = a-b$ —— ①

Putting $a=2, b=-3, c=3$ in ①

$x = 3, \; y = -6 \; z = 5$

$T(1,1,1) = (2,-3,3)$

$= 2(1,1,1) - 6(1,1,0) + 5(1,0,0)$

again $T(1,1,0) = (1, -3, 3)$

Thus putting $a=1, b=-3, c=3$ in ①

$x=3, \; y=-6 \; z=+4$

$T(1,1,0) = 3(1,1,1) - 6(1,1,0) + 4(1,0,0)$

Finally

2

2

$T(1,0,0) = (0, 1, 3)$

Putting $a=0,\ b=1,\ c=3$ in (1)

$x=3,\ y=-2,\ z=-1$

$T(1,0,0) = (0,1,3)$

$= 3(1,1,1) - 2(1,1,0) - 1(1,0,0)$

$$[T:B] = \begin{bmatrix} 2 & 3 & 3 \\ -6 & -6 & -2 \\ 5 & 4 & -1 \end{bmatrix}$$

1