Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering / Science

End Sem Examination May-2023

EN3BS12 / BC3BS03 / SC3BS02

Engineering Mathematics -II

Programme: B.Tech. / B.Sc. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. If $L^{-1}\{f(s)\}=F(t)$ then $L^{-1}\{f(s-a)\}$ is - **1**

(a) $e^{at}F(t)$ (b) $e^{-at}F(t)$

(c) e^{at} (d) None of these

ii. If $L\{F(t)\} = f(s)$, then $L\{t.F(t)\}$ is - **1**

(a) $-f'(s)$ (b) $f'(s)$

(c) $sf'(s)-f(0)$ (d) None of these

iii. If f(x) is an even function of x in $(-l,l)$ then its Fourier series contains only- **1**

(a) Cosine terms (b) Sine terms

(c) Both (a) & (b) (d) None of these

iv. For a periodic function $f(x)$ with period T, which of the following is true- **1**

(a) $f(x)=f\left(x+T/2\right)$ (b) $f(x)=f(x+T)$

(c) $f(x)=f\left(x+T/3\right)$ (d) None of these

v. In partial differential equation the equation $Pp+Qq=R$ is called as- **1**

(a) Charpit's equation (b) Clairaut's equation

(c) Lagrange's equation (d) None of these

vi. Order and degree of a partial differential equation $p+r+s=1$ is- **1**

(a) Order 2 and degree 1 (b) Order 1 and degree 1

(c) Order 1 and degree 2 (d) None of these

P.T.O.

vii. If Vector function F(r) is solenoidal in a given range R if- **1**

(a) Curl F = 0 (b) divF = 0

(c) Curl F = 1 (d) None of these

viii. If $\emptyset, f, g$ are scalar point function, then which of the following is not correct (where ∇ is gradient)- **1**

(a) $\nabla\emptyset = 0$ (b) $\nabla(f \pm g) = \nabla f \pm \nabla g$

(c) $\nabla(fg) = f\nabla g + g\nabla f$ (d) None of these

ix. Rate of convergence of the Newton-Raphson method is generally- **1**

(a) Linear (b) Quadratic

(c) Cubic (d) None of these

x. If $f_n(x)$ is purely a polynomial in x, then $f_n(x) = 0$ is called- **1**

(a) An Algebraic equation

(b) Transcendental equation

(c) Both (a) & (b)

(d) None of these

Q.2 i. Find the Laplace transform of $\left\{\frac{1-cos2t}{t}\right\}$. **4**

ii. By convolution theorem, evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. **6**

OR iii. Using Laplace transform solve the equation $(D+1)^2 y = t$ given that $y(0) = -3, y(1) = -1$. **6**

Q.3 i. Write four Dirichlet's conditions for Fourier expansion. **4**

ii. Find a series of sines and cosines of multiples of x which will represent $x + x^2$ in the interval $-\pi < x < \pi$. **6**

OR iii. Find the Fourier transform of $f(x) = \begin{cases} x^2, |x| < a \\ 0, \ |x| > a \end{cases}$. **6**

Q.4 Attempt any two:

i. Find complete integral of the following using Charpit's method- **5**

$(p^2 + q^2)y = qz$

ii. Solve: $r - s - 2t = (y-1)e^x$. **5**

iii. Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ by method of separation of variables where $u(x,0) = 6e^{-3x}$. **5**

Q.5 Attempt any two:

i. Find the directional derivative of the function $\emptyset = xy^2 + yz^2 + x^2z$ along the curve $x = t, y = t^2, z = t^3$ at (1,1,1). **5**

ii. Find the value of $\int_C F.dr$, where$F = e^x siny\text{i} + e^x cosy\text{j}$. The vertices of the rectangle C are (0,0),(1,0),(1,$\pi/2$),(0,$\pi/2$). **5**

iii. Evaluate$\iint_S$ A.$\hat{\text{n}}\, dS$ where A $= 18z\text{i} - 12\text{j} + 3y\text{k}$ and S is a part (i.e. surface) of the plane $2x + 3y + 6z = 12$ which is in first octant. **5**

Q.6 Attempt any two:

i. Using Newton-Raphson method, find the real root of the equation $x^3 - x - 1 = 0$. **5**

ii. Apply Gauss elimination method to solve the equation- **5**

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

iii. Solve by Jacobi's iteration method, the equations- **5**

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

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End Sem Examination May - 2023 10/5/II
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Engineering Mathematics - II

Programme: B.tech Branch / specialisation: All.

Q1. (i) (a) $e^{at}F(t)$ +1
(ii) (a) $-f'(s)$ +1
(iii) (a) cosine terms +1
(iv) (b) $f(x) = f(x+T)$ +1
(v) (c) Lagrange's equation +1
(vi) (a) Order 2 and degree 1. +1
(vii) (b) div $F = 0$ +1
(viii) (d) ~~$p=\infty$~~ None of these +1
(ix) (b) Quadratic +1
(x) (a) An Algebraic equation. +1

Q2. (i) Let $F(t) = 1-\cos 2t$

Now $\lim_{t\to 0} \frac{1-\cos 2t}{t} = \lim_{t\to 0} \frac{2\sin 2t}{1} = 0$, exists +1

$L\{1-\cos 2t\} = \frac{1}{s} - \frac{s}{s^2+4} = f(s)$. $+\frac{1}{2}$

$\therefore L\left\{\frac{1-\cos 2t}{t}\right\} = \int_s^\infty f(x)\,dx$ $+\frac{1}{2}$

$= \int_s^\infty \left[\frac{1}{x} - \frac{x}{x^2+4}\right] dx$

$= \left[\log x - \frac{1}{2}\log(x^2+2^2)\right]_s^\infty$

$$= \frac{1}{2}\left[\lim_{x\to\infty} \log \frac{1}{1+(4/x^2)} - \log\frac{s^2}{s^2+4}\right]$$

$$= -\frac{1}{2}\log\frac{s^2+4}{s^2} = \frac{1}{2}\log\frac{s^2}{s^2+4}$$ +1

(ii) $L^{-1}\left\{\frac{s}{s^2+a^2}\right\} = \cos at = F(t)$, $L^{-1}\left\{\frac{s}{s^2+b^2}\right\} = \cos bt = G(t)$ +1

$$\therefore \cancel{L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\} = \int_0^t \cos ax \cos b(t-x)\,dx}$$

$$\cancel{= \frac{1}{2}\int_0^t \left[\frac{\cos\{(a-b)x+bt\}}{a-b} + \frac{\cos\{(a+b)x-bt\}}{a+b}\right]_0^t}$$

According to Convolution theorem,

$$L^{-1}\{f(s)\,g(s)\} = \int_0^t F(x)\,G(t-x)\,dx = F * G.$$ +1

$$\therefore L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\} = \int_0^t \cos ax \cos b(t-x)\,dx$$ +1

$$= \frac{1}{2}\int_0^t [\cos\{(a-b)x+bt\} + \cos\{(a+b)x-bt\}]\,dx$$

$$= \frac{1}{2}\left[\frac{\sin at - \sin bt}{a-b} + \frac{\sin at + \sin bt}{a+b}\right]_0^t \qquad +1$$

$$= \frac{1}{2(a^2-b^2)}\left[(a+b)\sin at - (a+b)\sin bt + (a-b)\sin at + (a-b)\sin bt\right]$$

$$= \frac{a\sin at - b\sin bt}{a^2-b^2} \qquad +1$$

(iii) The given equation is $(D+1)^2 y = t$

$\Rightarrow (D^2+2D+1)y = t$

or $y'' + 2y' + y = t$.

taking Laplace transform of Both Sides, we get

$$L\{y''\} + 2L\{y'\} + L\{y\} = L\{t\}$$

$$s^2\bar{y} - sy(0) - y'(0) + 2[s\bar{y} - y(0)] + \bar{y} = L\{t\} \qquad +1$$

$$(s^2+2s+1)\bar{y} + 3s - A + 6 = 1/s^2$$

, where $A = y'(0)$.

$$\Rightarrow (s+1)^2\bar{y} = \frac{1}{s^2} - (3s+6) + A$$

$$\bar{y} = \frac{1}{s^2(s+1)^2} - \frac{(3s+6)}{(s+1)^2} + \frac{A}{(s+1)^2} \qquad +1$$

$$= \frac{1}{s^2(s+1)^2} - \frac{3(s+1)+3}{(s+1)^2} + \frac{A}{(s+1)^2}$$

$$= \frac{-2}{s} + \frac{1}{s^2} + \frac{2}{s+1} + \frac{1}{(s+1)^2} - \frac{3}{(s+1)} - \frac{3}{(s+1)^2} + \frac{A}{(s+1)^2}$$

(Breaking into partial fractions)

$$= \frac{-2}{s} + \frac{1}{s^2} - \frac{1}{(s+1)} + \frac{A-2}{(s+1)^2}$$ +1

Taking inverse Laplace transform of Both sides, we have

$$y = -2 + t - e^{-t} + (A-2)e^{t} \cdot t \text{ ——— (1)}$$ +1

Now given that $y(1) = -1$ so putting values in (1), we get

$$-1 = -2 + 1 - e^{-1} + (A-2)\,e^{-1} \cdot 1$$

$$\Rightarrow A = 3$$ +1

putting $A = 3$ in (1), the Required solution is

$$y = -2 + t - e^{-t} + te^{-t}$$

$$\Rightarrow y = (t-2) + (t-1)e^{-t}.$$ +1

Q.3. (i) Dirichlet's conditions.

(2) The given function $f(x)$ has a finite number of discontinuities in any one period. +1

(3) The function $f(x)$ has at the most a finite number of maxima and minima. +1

(4) The series $a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$. is +1

Uniformly convergent, so.

Under the above conditions, the series converges:

(a) to $f(x)$ if x is a point of continuity of.

(b) to $\frac{f(x+0)+f(x-0)}{2}$ if x is a point of discontinuity.

(ii) Let $f(x) = x + x^2$

suppose $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ ——— (1) +1

Now $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} (x+x^2)\,dx = \frac{1}{2\pi}\left[\frac{x^2}{2}+\frac{x^3}{3}\right]_{-\pi}^{\pi}$

$= \frac{\pi^2}{3}$ +1

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} (x+x^2) \cos nx\,dx.$

$$= \frac{2}{\pi}\left[(x^2)\left(\frac{\sin nx}{n}\right) - (2x)\cdot\left(-\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[2\pi\left(\frac{\cos n\pi}{n^2}\right)\right] = \frac{4}{n^2}(-1)^n$$ +1

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x+x^2)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} x\sin nx\,dx$$

($\because x^2 \sin nx$ is an odd function)

$$= \frac{2}{\pi}\left[(x)\left(-\frac{\cos nx}{n}\right) - (1)\cdot\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi\left(-\frac{\cos n\pi}{n}\right) + 0\right] = \frac{-2}{n}(-1)^n.$$ +1

Now, puttingvalues of a_0, a_n, b_n in (1), we have.

$$f(x) = (x+x^2) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left\{\frac{4}{n^2}(-1)^n\cos nx - \frac{2}{n}(-1)^n \sin nx\right\}$$ +1

$$\Rightarrow x+x^2 = \frac{\pi^2}{3} - 4\left(\cos x - \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x - \cdots\right)$$

$$+ 2\left(\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \cdots\right)$$ +1.

This is the Required Fourier Series.

(iii) $f(x) = \begin{cases} x^2, & |x| \leq a \\ 0, & |x| > a \end{cases} \Rightarrow f(x) = \begin{cases} x^2, & -a < x < a \\ 0, & x \notin (-a, a) \end{cases}$

Now, $F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{isx} dx$ +1

$$\Rightarrow F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} x^2 e^{isx} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[x^2 \cdot \frac{e^{isx}}{is} - \int 2x \frac{e^{isx}}{is} dx \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \left[x^2 \cdot \frac{e^{isx}}{is} - \left(2x \frac{e^{isx}}{(is)^2} - \int \frac{2e^{isx}}{(is)^2} dx \right) \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{x^2 e^{isx}}{is} - \frac{2x e^{isx}}{i^2 s^2} + \frac{2e^{isx}}{(is)^3} \right]_{-a}^{a}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{x^2 e^{isx}}{is} + \frac{2x e^{isx}}{s^2} - \frac{2e^{isx}}{is^3} \right]_{-a}^{a}$$ +2

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{a^2 e^{isa}}{is} + \frac{2a e^{isa}}{s^2} - \frac{2e^{isa}}{is^3} - \frac{a^2 e^{-isa}}{is} \right.$$

$$\left. + \frac{2a e^{-isa}}{s^2} + \frac{2e^{-isa}}{is^3} \right]$$ +1

$$-\frac{2}{is^3}\left(e^{isa}-e^{-isa}\right)\Big] \quad +1$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}}\left[\frac{a^2}{s}(2\sin sa)+\frac{2a}{s^2}(2\cos sa)-\frac{2}{is^3}(2i\sin sa)\right]$$

$$=\frac{1}{\sqrt{2\pi}}\left[\frac{2a^2}{s}(\sin sa)+\frac{4a}{s^2}\cos sa-\frac{4}{s^3}\sin sa\right] \quad +1$$

$$=\frac{1}{s^3\sqrt{2\pi}}\left[2a^2s^2\sin sa+4as\cos sa-4\sin sa\right]$$

Q4 (i) Let $f \equiv (p^2+q^2)y - qz = 0.$ ——— (1)

$$\frac{\partial f}{\partial x} = 0,\ \frac{\partial f}{\partial y} = p^2+q^2,\ \frac{\partial f}{\partial z} = -q,\ \frac{\partial f}{\partial p} = 2py,\ \frac{\partial f}{\partial q} = 2qy - z \quad +1$$

Hence from Charpit's Auxiliary equations,

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}} = \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dF}{0} \quad +1$$

we have

$$\frac{dp}{-pq} = \frac{dq}{(p^2+q^2) - q^2} = \frac{dz}{-p.(2py) - q(2qy - z)} = \frac{dx}{-2py} = \frac{dy}{-2qy+z} = \frac{dF}{0}$$

——— (2)

Taking 1st and 2nd ratios, we get

$$\frac{dp}{-pq} = \frac{dq}{p^2} \quad \text{or} \quad p\,dp + q\,dq = 0.$$

On Integration, $p^2+q^2 = a^2$ ——— (3) $\quad +1$

where a is any arbitrary Constant

Using (3) in (1), we get

$$a^2 y = qz \Rightarrow q = \frac{a^2 y}{z} \quad \text{——— (4)}$$

$$p = \sqrt{\left(a^2 - \frac{a^4y^2}{z^2}\right)} = \frac{a}{z}\sqrt{z^2 - a^2y^2} \quad \text{—— (5)} \qquad +1$$

Now, Substituting the Values of p and q from (4) and (5) in $dz = p\,dx + q\,dy$, we get

$$dz = \frac{a}{z}\sqrt{(z^2 - a^2y^2)}\,dx + \frac{a^2y}{z}\,dy$$

$$\text{or} \quad \frac{z\,dz - a^2y\,dy}{\sqrt{(z^2 - a^2y^2)}} = a\,dx.$$

On integration, we have

$\sqrt{z^2 - a^2y^2} = ax + b$, where b is any arbitrary constant.

$$z^2 - a^2y^2 = (ax+b)^2$$

$$z^2 = a^2y^2 + (ax+b)^2 \qquad +1$$

which is the Required Complete integral of the given equation (1)

(ii) The given equation is,

$$(D^2 - DD' - 2D'^2)z = (y-1)e^x.$$

its A.E. is, $m^2 - m - 2 = 0$

$$\Rightarrow (m-2)(m+1) = 0 \Rightarrow m = 2, -1. \qquad +1$$

$$\text{C.F.} = \phi_1(y+2x) + \phi_2(y-x) \qquad +1$$

$$= \frac{1}{(D-2D')} \cdot \frac{1}{(D+D')} \cdot (y-1)e^x$$

$$= \frac{1}{(D-2D')} \int (x+a-1)e^x\,dx \quad \left(\text{where } y-x=a \Rightarrow y=x+a\right)$$

$$= \frac{1}{(D-2D')} \left[(a-1)\int e^x\,dx + \int x e^x\,dx\right]$$

$$= \frac{1}{(D-2D')} \left[(a-1)e^x + (x-1)e^x\right]$$

$$= \frac{1}{(D-2D')} \left[(y-x-1+x-1)e^x\right] \quad (\because a=y-x) \qquad +1$$

$$= \frac{1}{(D-2D')} (y-2)e^x.$$

$$= \int (b-2x-2)e^x\,dx. \quad \left(\text{where } y+2x=b \Rightarrow y=b-2x\right)$$

$$= (b-2)\int e^x\,dx - 2\int x e^x\,dx$$

$$= (b-2)e^x - 2(x-1)e^x$$

$$= (y+2x-2)e^x - 2(x-1)e^x \quad (\because b=y+2x) \qquad +1$$

$$= ye^x.$$

Hence the general solution is,

(iii) The given equation is

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u \quad \text{--- (1)}$$

Let the solution of (1) be

$$u(x,t) = X(x)T(t)$$

$$\therefore \quad \frac{\partial u}{\partial x} = X'(x)T(t), \quad \frac{\partial u}{\partial t} = X(x)T'(t) \quad \text{--- (2)} \quad +1$$

Substituting above values in (1), we get

$$X'T = 2XT' + XT$$

$$\Rightarrow (X' - X)T = 2XT'$$

$$\Rightarrow \frac{X' - X}{2X} = \frac{T'}{T} \quad \text{--- (3)}$$

Let $\frac{X'-X}{2X} = \frac{T'}{T} = \lambda$

$$\Rightarrow \frac{X'-X}{2X} = \lambda \quad \text{--- (4)} \quad \text{and} \quad \frac{T'}{T} = \lambda \quad \text{--- (5)} \quad +1$$

Now from (4), we have

$$X' - X = 2\lambda X \Rightarrow \frac{X'}{X} = 1 + 2\lambda$$

Integrating, $\log X = (1+2\lambda)x + \log c_1$

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Again integrating (5), we get $T = c_2 e^{\lambda t}$. +1

putting values of X and T in (2), we have

$$u(x,t) = c_1 c_2 e^{(1+2\lambda)x} \cdot e^{\lambda t} \quad \text{--- (6)}$$ +1

Given $u(x,0) = 6e^{-3x}$. --- (7)

$\therefore$ putting $t=0$ in (6) and using (7), we get

$$6e^{-3x} = c_1 c_2 e^{(1+2\lambda)x}$$

$\Rightarrow c_1 \cdot c_2 = 6$ & $-3 = 1+2\lambda$

$\Rightarrow c_1 \cdot c_2 = 6$ & $\lambda = -2$.

putting these values in (6), the required solution is.

$u(x,t) = 6e^{-3x} \cdot e^{-2t}$.

or $u(x,t) = 6e^{-(3x+2t)}$. +1.

Q.5.(i) If $\vec{r}$ be the position vector of any point on the given curve, then

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = t\hat{i} + t^2\hat{j} + t^3\hat{k}$$

~~Here~~ at (1,1,1).

$$\vec{r} = \hat{i} + \hat{j} + \hat{k}.$$ +1

$$= (y^2+2zx)\hat{i} + (z^2+2xy)\hat{j} + (x^2+2yz)\hat{k}$$

$$= 3(\hat{i}+\hat{j}+\hat{k}) \text{ at } (1,1,1)$$ +2

$$\vec{r} = \hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow \hat{r} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{1^2+1^2+1^2}} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$ +1

$\therefore$ directional derivatives along the curve at $(1,1,1)$

$$= \hat{r}\cdot \text{grad}\,\phi = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}} \cdot 3(\hat{i}+\hat{j}+\hat{k})$$

$$= \frac{3}{\sqrt{3}}(1+1+1) = \frac{3\cdot 3}{\sqrt{3}}$$

$$= 3\sqrt{3}$$ +1

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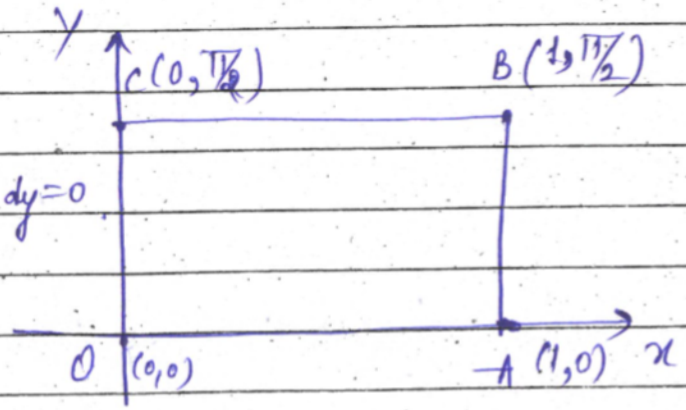
(ii) Here $\vec{r} = x\hat{i} + y\hat{j} \Rightarrow dr = dx\,\hat{i} + dy\,\hat{j}$.

$\therefore \vec{F}\cdot d\vec{r} = (e^x \sin y\,\hat{i} + e^x \cos y\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$

$= e^x \sin y\,dx + e^x \cos y\,dy.$ +1

Draw the Rectangle C.



Now, ~~Along~~ Along OA, $y=0$, $dy=0$

Along AB, $x=1$, $dx=0$

Along BC, $y=\pi/2$, dy 0

Along CO, $x=0$, $dx=0$. +2

So, the Circulation of $\vec{F}$ along C ie, along the Rectangle

$= \int_C \vec{F}\cdot d\vec{r} = \int_C (e^x \sin y\,dx + e^x \cos y\,dy)$

$= 0 + \int_0^{\pi/2} e \cos y\,dy + \int_1^0 e^x \sin(\pi/2)\,dx + \int_{\pi/2}^0 \cos y\,dy$

$= e[\sin y]_0^{\pi/2} + [e^x]_1^0 + [\sin y]_{\pi/2}^0$ +1

$= e + (1-e) + (0-1) = 0$ +1

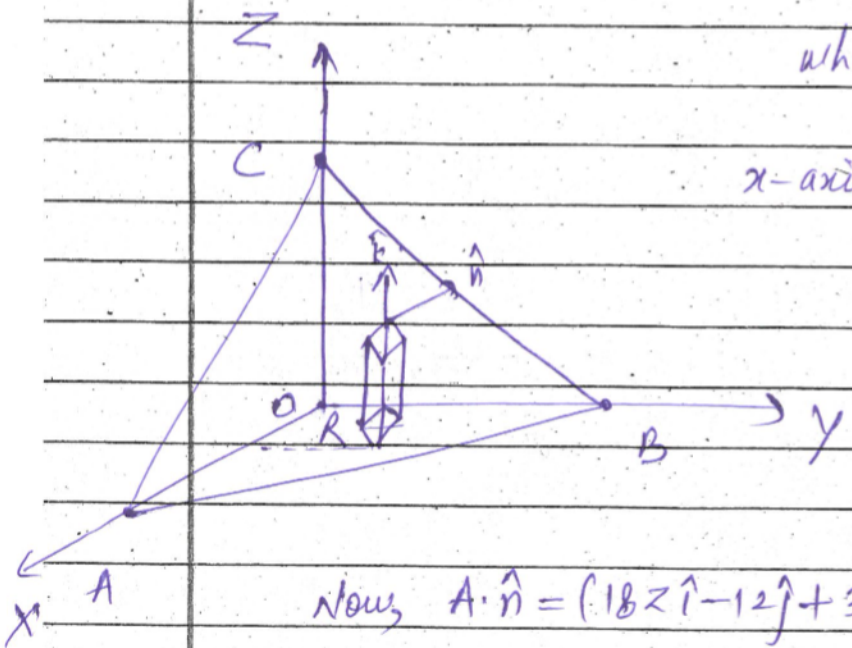
(iii) Let $f \equiv 2x + 3y + 6z - 12$

if $\hat{n}$ is the Outward drawn Unit normal Vector to the surface S of the given plane, then

$$\hat{n} = \frac{\text{grad } f}{|\text{grad } f|} = \frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}$$ +1

Now, $\hat{k}\cdot\hat{n} = \hat{k}\cdot\left(\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}\right) = \frac{6}{7}$

$$\therefore \iint_S A\cdot\hat{n}\, ds = \iint_R A\cdot\hat{n}\, \frac{dx\, dy}{|\hat{k}\cdot\hat{n}|} \quad \text{—— (1)}$$ +1



where R is the triangle Bounded By the lines x-axis, y-axis & $2x+3y=12$, $z=0$

Now, $A\cdot\hat{n} = (18z\hat{i} - 12\hat{j} + 3y\hat{k})\cdot\left(\frac{2\hat{i}+3\hat{j}+6\hat{k}}{7}\right)$

$$= \frac{1}{7}(36z - 36 + 18y) = \frac{18}{7}(2z - 2 + y)$$

$$= \frac{18}{\;}\left[2(12 - 2x - 3y) - 2 + y\right]$$

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$= \frac{6}{67}(6-2x) = \frac{12}{7}(3-x)$ +1

putting values in (1), we have

$$\iint_S A\cdot\hat{n}\,ds = \int_0^6\int_0^{\frac{12-2x}{3}} \frac{12}{7}(3-x)\frac{dx\,dy}{(6/7)} = 2\int_0^6 (3-x)\left(\frac{12-2x}{3}\right)dx$$

$$= \frac{4}{3}\int_0^6 (3-x)(6-x)\,dx = \frac{4}{3}\int_0^6 (18-9x+x^2)\,dx$$

$$= \frac{4}{3}\left[18x - \frac{9x^2}{2} + \frac{x^3}{3}\right]_0^6$$

$$= \frac{4}{3}\left[18\times 6 - \frac{9\times 36}{2} + \frac{36\times 6}{3}\right]$$

$= 24.$ +1

Q.6 (i) Let $f(x) = x^3 - x - 1 = 0$ —— ①

$\Rightarrow f'(x) = 3x^2 - 1$ —— ②

then $f(0) = -1$

$f(1) = -1 = (-ve)$

$f(2) = 5 = +ve$

Clearly roots lie b/w 1 & 2 +1

taking initial approximation

$x_0 = \frac{1+2}{2} = 1.5$ +1

By Newton Raphson Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$ +1

$$x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$

$$x_{n+1} = \frac{2x_n^3 + 1}{3x_n^2 - 1} \quad \text{—— ③}$$

Putting $n=0$ in ③, we get

first approx

$$x_1 = \frac{2x_0^3 + 1}{3x_0^2 - 1} = \frac{2(1.5)^3 + 1}{3(1.5)^2 - 1}$$

Putting $n=1$, in 3

second appro.

$$x_2 = 1.3252$$

at $n=2$

third app.

$$x_3 = 1.3247$$

fourth

$$x_4 = 1.3247$$

Hence

$x_3 = x_4 = 1.3247$ correct to four decimal places

$\therefore$ roots of given eq^n is 1.3247 +2

(ii)

$$2x + y + z = 10$$
$$3x + 2y + 3z = 18$$
$$x + 4y + 9z = 16$$

Interchange the first and last eq^n.

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{array}\right]$$ +1

$R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 2R_1$

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$R_2 \to (-\frac{1}{2})R_2$; $R_3 \to (-1) R_3$, we get

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 17 & 22 \end{array}\right]$$ +1

$R_3 \to 5R_3 - 7R_2$, we get

$$[A:B] \sim \left[\begin{array}{ccc|c} 1 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & 0 & 1 & 5 \end{array}\right]$$ +1

$\Rightarrow x+4y+9z = 16$

$5y + 12z = 15$

$z = 5$ +1

Using Back Substitution, we get

$z = 5, \; y = -9, \; x = 7$ +1

(can also solve Using Back Substitution without using matrix method)

(iii) Since $|20| > |1| + |-2|$

$|20| > |3| + |1|$

$|20| > |2| + |-3|$

So, Gauss-jacobi method can be used for the system of equations.

∴ given system can be written as:

$$\left.\begin{aligned} x &= \frac{1}{20}[17 - y + 2z] \\ y &= \frac{1}{20}[-18 - 3x + z] \\ z &= \frac{1}{20}[25 - 2x + 3y \end{aligned}\right\} \quad -(1)$$ +1

Let the initial value be $x = y = z = 0$, we have +1

$x^{(1)} = 17/_{29} = 0.85$, $y^{(1)} = -0.9$, $z^{(1)} = 1.25$

Second iteration $x^{(2)} = 1.02$

$y^{(2)} = -0.965$

$z^{(3)} = 1.03$

third iteration

$x^{(3)} = 1.001$, $y^{(3)} = -1.001$, $z^{(3)} = 1.003$ +1

fourth iteration

$x^{(4)} = 1.000$, $y^{(4)} = -1.000$, $z^{(4)} = 1.000$

Hence, solution is $x = 1, y = -1, z = 1$. +1