ii. Explain the principle of least square used for curve fitting of linear curve. Also fit a straight line for **5**

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

iii. The following data shows the distribution of digits in 10,000 numbers chosen at random from a telephone directory **5**

Digit :	0	1	2	3	4	5	6	7	8	9
Frequency :	1026	1107	997	996	1075	933	1107	972	964	853

Test at 5% level of significance, whether the digit may be taken to occur equally frequently in the directory (Given $\chi^2_{0.05,9} = 16.919$)

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No.....................................

Faculty of Engineering

End Sem (Even) Examination May-2019

EE3BS03 / EX3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: EE/EX

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The size of graph having 3 vertices of degree 3, one vertex of degree two, one pendent vertex and one isolated vertex will be **1**

(a) 3 (b) 6 (c) 12 (d) 24

ii. A graph containing a cycle involving all the vertices of a graph is known as **1**

(a) Peterson's graph (b) Hamiltonian graph

(c) Euler's Graph (d) None of these

iii. The total number of fundamental circuits in a graph with 7 vertices and 15 edges is **1**

(a) 6 (b) 14 (c) 9 (d) 8

iv. The maximum possible value of flow for the flow augmenting path **1**

is

(a) 3 (b) 2 (c) 1 (d) 5

v. The value of Bessels function $J_{1/2}(x)$ is **1**

(a) $\sqrt{\frac{2}{\pi x}}\sin x$ (b) $\sqrt{\frac{2}{\pi x}}\cos x$

(c) 0 (d) None of these

vi. For the equation $2x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+(x^2+1)y=0$ then $x=0$ is a **1**

(a) Regular singular point (b) Irregular singular point

(c) Ordinary point (d) Can't say.

vii. The covariance of two independent variables X and Y is equal to: **1**

(a) 0 (b) var(X) +var(Y)

(c) var(X +Y) (d) None of these

viii. If the future value of any sample function can be predicted exactly by the past values, it is called **1**
(a) Deterministic random process
(b) Non- deterministic random process
(c) Can't say
(d) Both (a) and (b)

ix. If the value of regression coefficients $b_{xy} = 0.15$ and $b_{yx} = 1.05$, then the value of correlation coefficient r is equal to **1**
(a) 0.65 (b) 0.15 (c) 0.39 (d) None of these

x. If we accept the null hypothesis when it is incorrect, it is known as error of **1**
(a) Type I (b) Type II (c) Type III (d) Type IV

Q.2 i. Define complete, regular and bipartite graph with example. **3**

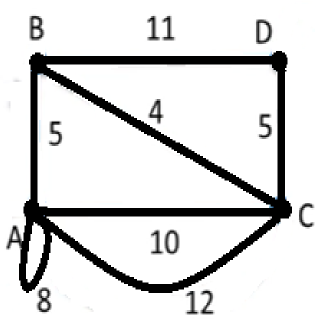
ii. Prove that a simple disconnected graph G with vertices $'n'$ and $'k'$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **7**

OR iii. Describe the use of matrix representation of graph and find the graph corresponding to following incidence matrix representation **7**

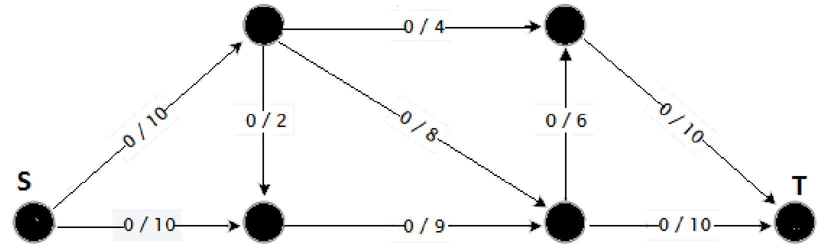
$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Q.3 i. Find out the number of pendent vertices in a complete binary tree with 'n' number of vertices. **3**

ii. Define minimum spanning tree and find out the minimum spanning tree using Prim's algorithm for the following graph **7**



OR iii. In the following network, find maximum possible flow using Ford Fulkerson's algorithm: **7**



Q.4 Attempt any two:

i. Solve in series $2x^2\frac{d^2y}{dx^2}+(2x^2-x)\frac{dy}{dx}+y=0.$ **5**

ii. Prove that: **5**
$\int_0^1 xJ_n(\alpha x).J_n(\beta x)\,dx=0, \alpha\neq\beta$ where α and β are roots of $J_n(x)=0$ and $J_n(x)$ is Bessel's function of first kind.

iii. If $P_n(x)$ is Legendre's function of first kind, then show that $nP_n(x)=(2n-1)xP_{n-1}(x)-(n-1)P_{n-2}(x)$ **5**

Q.5 Attempt any two:

i. The joint probability density function of X and Y is **5**
$f(x,y)=\begin{cases} k(6-x-y); & 0\le x\le 2, 2\le y\le 4 \\ 0; & otherwise \end{cases}$ find k and $P(X+Y<3)$.

ii. A coin is tossed 10 times. Find the probability of getting 3 or 4 or 5 heads using central limit theorem. **5**
(Area against $z = 1.58$ is 0.4429 and against $z = 0.316$ is 0.1217)

iii. Explain and classify random process. **5**

Q.6 Attempt any two:

i. Calculate the correlation coefficient and obtain the lines of regression for the following data **5**

x:	1	2	3	4	5	6	7	8	9
y:	9	8	10	12	11	13	14	16	15

Faculty of Engineering

End Sem (Even) Examination May-2019

EE3BS03/EX3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: EE/EX

Marking Scheme

Q.1

i. b) 6
ii. b) Hamiltonian graph
iii. c)9
iv. d) 5
v. a) $\sqrt{\frac{2}{\pi x}}\sin x$
vi. a)regular singular point
vii. a) 0
viii. a)Deterministic random process
ix. c)0.39
x. b)Type II

10 marks

Q.2

(i) 1 definition with example 1 no (1+1+1) = (3)

(ii) Pf Let $n_1, n_2, n_3 \dots n_k$ be the no. of vertices in k connected components of disconnected graph G with n vertices. +1

$$\Rightarrow n_1 + n_2 + \dots + n_k = n$$

$$\Rightarrow \sum_{i=1}^{k}(n_i - 1) = n-k$$

Squaring both the sides, we get +1

$$\Rightarrow \sum(n_i^2 - 2n_i) + k + \text{positive term} = n^2 + k^2 - 2nk$$

$$\Rightarrow \sum n_i^2 \le n^2 + k^2 - 2nk + 2n - k \quad \text{—(1)}$$ +1

Now as the maximum number of edges in a simple connected graph is $\frac{n(n-1)}{2}$, (2)

the maximum possible edge in k components

$$= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}$$ +1

$$= \frac{1}{2} \cdot \sum n_i^2 - \sum n_i$$ +1

$$= \frac{1}{2}\left(\sum n_i^2 - n\right)$$

$$\leq \frac{1}{2}\left(n^2+k^2-2nk+2n-k-n\right)$$ +1

$$= \frac{(n-k)(n-k+1)}{2}$$ +1

Hence proved

7 marks

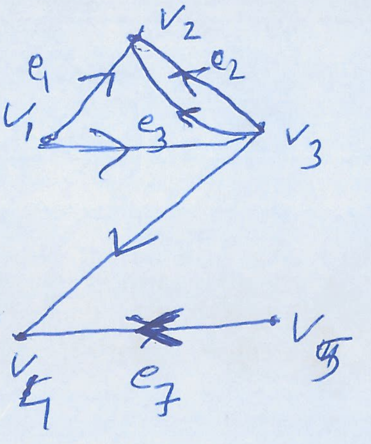
2(iii) Use of matrix representation +3 marks

The graph for the given matrix is directed with incident

out +1

incident in −1 thus for

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}
\begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \Rightarrow$$



+4

7 marks

Q 3 (i) Let P be the number of pendant vertices with a complete binary tree of n vertices.

In a binary tree there is exactly one vertex of degree two and each of the remaining vertices is of degree one or three.

thus

One 2 deg vertex

P deg one vertex

$(n-P-1)$ degree 3 vertex

+1

total degree $= 2 \times 1 + 1 \times P + 3 \times (n-P-1)$

Now the no of edges in tree with n vertices $= n-1$

+1

$\Rightarrow$ total degree $= 2(n-1)$

$\Rightarrow 2 + P + 3(n-P-1) = 2(n-1)$

+1

$\Rightarrow \boxed{P = \frac{n+1}{2}}$

3 marks.

Q 3 (ii) A tree containing all the vertices of the graph with minimum weight is called minimum spanning tree.

+2

the adjacency matrix for the given graph

$$
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
A & 8 & 5 & 10 & 6 \\
B & 5 & - & 4 & 11 \\
C & 10 & 4 & - & 5 \\
D & - & 11 & 5 & -
\end{array}
$$

+1

1)

(M)

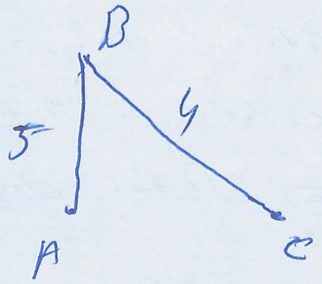
Considering 'A' as starting vertex, we select the near by neighbour as 'B', cut the colour for A & B we get.

$$
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
A & 8 & 5 & 10 & \cancel{2} \\
B & 5 & \cancel{8} & 4 & 11 \\
C & 10 & 4 & - & 5 \\
D & - & 11 & 5 & -
\end{array}
$$

+1

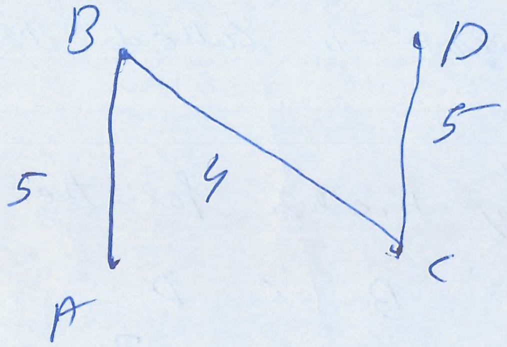
Selecting A B subgraph, we find 'C' as nearly neighbour of 'B', cut col. of C by selecting 'C', then

$$
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
A & 8 & 5 & 10 & - \\
B & 5 & - & 4 & 11 \\
C & 10 & 4 & - & 5 \\
D & - & 11 & 5 & -
\end{array}
$$



+1

Similarly select 'D' as nearly neighbour of subgraph ABC, we get minimum spanning tree



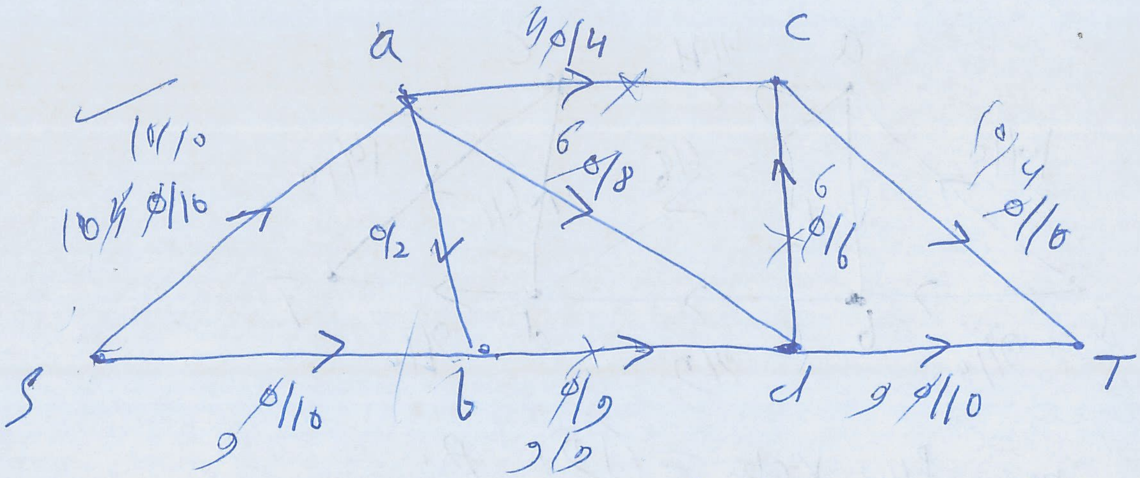
+1

+1

Weight $= 5+4+5 = \underline{\underline{14}}$

7 marks

3 (iii)



Considering flow Augmenting path $S \to a \to c \to T$ +1

for forward edge

$d_1 = \min(10, 4, 10) = 4$ +1

for backward edge

d_2 : N.A.

$d = \min(d_1, d_2) = 4$ (bottle neck cap) +1

Resultant flow

$S \xrightarrow{4/10} a \xrightarrow{4/4} c \xrightarrow{4/10} T$ +1

Consider path $S \xrightarrow{0/10} b \xrightarrow{0/9} d \xrightarrow{0/10} T$

$d_1 = 9 \quad d_2 = N.A$

bottle neck cap $d = d_1 = 9$

Resultant flow +1

$S \xrightarrow{9/10} b \xrightarrow{9/9} d \xrightarrow{9/10} T$

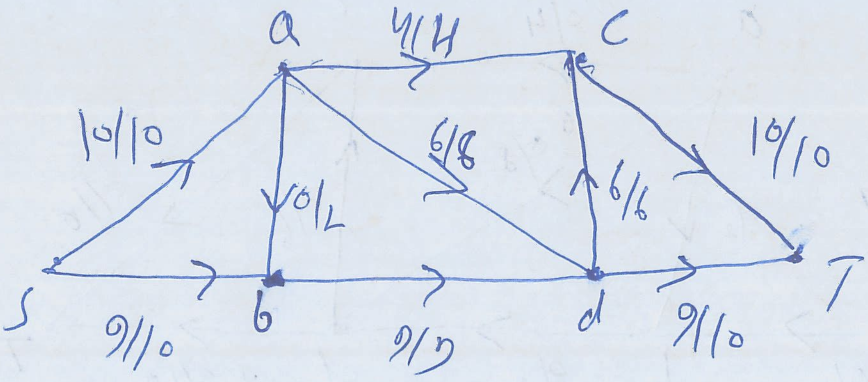
Consider $S \xrightarrow{4/10} a \xrightarrow{0/8} d \xrightarrow{0/6} c \xrightarrow{4/10} T$

$d_1 = \min(10-4, 8, 6, 10-4) = 6$ +1

$d_2 = NA \quad d = d_1 = 6$

Res. $S \xrightarrow{10/10} a \xrightarrow{6/8} d \xrightarrow{6/6} c \xrightarrow{10/10} T$

(6) Saturated flow Network (6)



+1

7 marks

max flow value = 19 Ans

Q. 4

(i) Solve in series

$$2x^2 \frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0$$

$x = 0$ is regular singular point.

+1

Let $y = \sum_{k=0}^{\infty} a_k x^{m+k}$ be the solution.

$$\Rightarrow 2x^2 \sum_{k=0}^{\infty} a_k (m+k)(m+k-1) x^{m+k-2}$$

$$+ (2x^2 - x) \sum_{k=0}^{\infty} a_k (m+k) x^{m+k-1} + \sum_{k=0}^{\infty} a_k x^{m+k} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} [2(m+k)^2 - 3(m+k) + 1] a_k x^{m+k}$$

$$+ \sum_{k=0}^{\infty} 2 a_k (m+k) x^{m+k+1} = 0$$

+1

Equating coeff of lowest term $(x^m) = 0$

$\Rightarrow 2m^2 - 3m + 1 = 0 \Rightarrow$ indicial eq

+1

$$\Rightarrow \boxed{m = 1, \ 1/2}$$

(7)

Eq coeff of $x^{m+k+1} = 0$

$$\Rightarrow \quad a_{k+1} = \frac{-2a_k(m+k)}{2(m+k+1)^2 - 3(m+k+1)+1}$$

(+1)

$k=0$

$$a_1 = \frac{-2m\, a_0}{2(m+1)^2 - 3(m+1)+1}$$

$k=1$

$$a_2 = \frac{-2(m+1)\,a_1}{2(m+2)^2 - 3(m+2)+1}$$

$$= \frac{4m(m+1)\,a_0}{[2(m+1)^2 - 3(m+1)+1]\,[2(m+2)^2 - 3(m+2)+1]}$$

and so on

(+1)

Soln $y = c_1(y)_{m=1} + c_2(y)_{m=1/2}$

$$(y)_m = x^m \sum a_k x^k$$

$$= x^m\left[a_0 + a_1 x + a_2 x^2 \cdots\right]$$

	$m=1$	$m=1/2$
a_1	$-2/3\, a_0$	$-a_0$
a_2	$-2/15\, a_0$	$-\frac{a_0}{2}$
$\vdots$		$-$

(+1)

and so on

for $(y)_{m=1} = x\left[a_0 - \frac{2}{3}x a_0 - \frac{2}{15}x^2 a_0 - \cdots\right]$

$(y)_{m=1/2} = x^{1/2}\left[a_0 - a_0 x - \frac{a}{2}x^2 - \cdots\right]$

(+1)

$y = c_1(y)_{m=1} + c_2(y)_{m=1/2}$

7 marks

(8) (8)

Q4 (ii)

Pf let $y = J_n(\alpha x)$ be the solution of

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (\alpha^2 x^2 - n^2) y = 0 \quad -(1)$$

and $z = J_n(\beta x)$ be the solution of

$$x^2 \frac{d^2 z}{dx^2} + x\frac{dz}{dx} + (\beta^2 x^2 - n^2) z = 0 \quad -(2)$$

(+1)

and α & β are zero of $J_n(x) = 0$ (given)

$$\Rightarrow J_n(\alpha) = 0 \quad J_n(\beta) = 0$$

$eq(1) \times \frac{z}{x} - eq(2) \times y/x$

$$\Rightarrow x\left(z\frac{d^2y}{dx^2} - y\frac{d^2z}{dx^2}\right) + \left(z\frac{dy}{dx} - y\frac{dz}{dx}\right) + (\alpha^2 \cancel{x^2} - \beta^2 \cancel{x^2}) xyz = 0$$

(+1)

$$\Rightarrow \frac{d}{dx}\left[x\left(z\frac{dy}{dx} - y\frac{dz}{dx}\right)\right] + (\alpha^2 - \beta^2) xyz = 0$$

Integrating w.r.to x between 0 to 1

$$\Rightarrow \left[x\left(z\frac{dy}{dx} - y\frac{dz}{dx}\right)\right]_0^1 + (\alpha^2 - \beta^2)\int_0^1 xyz\,dx = 0$$

(+1)

$$\Rightarrow (\beta^2 - \alpha^2)\int_0^1 xyz\,dx = \left[x\left(z\frac{dy}{dx} - y\frac{dz}{dx}\right)\right]_{x=1}$$

(+1)

now $y = J_n(\alpha x) \qquad z = J_n(\beta x)$

$\frac{dy}{dx} = \alpha J'(\alpha x) \qquad \frac{dz}{dx} = \beta J'(\beta x)$

$J_n(\alpha) = 0 \quad J_n(\beta) = 0$

If $\beta \neq \alpha$

(+1)

$$\Rightarrow \int_0^1 x J_n(\alpha x) J_n(\beta x)\,dx = 0$$

5 marks

4 (iii)

We know $(1-2xz+z^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x) z^n$ (+1)

differentiating w.r.to z

$$-\frac{1}{2}(1-2xz+z^2)^{-3/2}(-2x+2z) = \sum_{n=0}^{\infty} n P_n(x) z^{n-1}$$ (+1)

multiplying both sides by $(1-2xz+z^2)$ we get

$$(x-z)\sum_{n=0}^{\infty} P_n(x) z^n = (1-2xz+z^2)\sum_{n=0}^{\infty} n P_n(x) z^n$$ (+1)

Comparing coeff of z^{n-1} on both sides we get

$$x P_{n-1}(x) - P_{n-2}(x) = n P_n(x) - 2x(n-1)P_{n-1}(x) + (n-2)P_{n-2}(x)$$ (+1)

$$\Rightarrow n P_n(x) = (2n-1)x P_{n-1}(x) - (n-1)P_{n-1}(x)$$ (+1)

Hence.

5 marks

Q.5

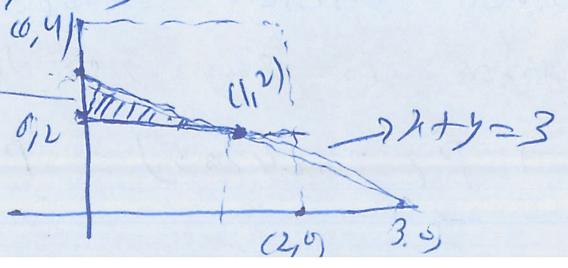
As $f(x,y)$ is P.d.f

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\Rightarrow \int_0^2\int_2^4 k(6-x-y)\,dy\,dx = 1$$

$$\Rightarrow \boxed{k = 1/8}$$ (+2)

To find $P(X+Y<3)$ (+1)



$$\Rightarrow P(X+Y<3) = \int_0^1 \int_2^{3-x} f(x,y)\, dy\, dx$$

(from diagram)

$$= \int_0^1 \int_2^{3-x} \frac{1}{8}(6-x-y)\, dy\, dx$$ (+1)

$$= \frac{1}{8} \int_0^1 \left(6y - xy - \frac{y^2}{2}\right)_2^{3-x} dx$$

$$= \frac{1}{8} \int_0^1 \left(\frac{x^2}{2} - 4x - \frac{7}{2}\right) dx$$ (+1)

$$= \frac{5}{24} \text{ Ans}$$

5 marks

ii) Let P be the probability of getting head (+1)

$\therefore P = \frac{1}{2} \quad q = \frac{1}{2}, \; n = 10$

X is binomially distributed mean $= np = 5$

$Var = npq = 2.5$

$\sigma = \sqrt{2.5}$

$X \sim N(\mu, \sigma)$ (+1)

$X \sim N(5, \sqrt{2.5})$

St. Normal Variable $Z = \frac{X-\mu}{\sigma} = \frac{X-5}{\sqrt{2.5}}$

Convert discrete distribution of selecting 3 or 4 or 5 to continuous in range 2.5 to 5.5 (+1)

$\Rightarrow P[3 \text{ or } 4 \text{ or } 5] = P[2.5 < X < 5.5]$

(11)

$$= P\left[\frac{1.5-5}{\sqrt{2.5}} < Z < \frac{5.5-5}{\sqrt{2.5}}\right]$$

(+1)

$$\Rightarrow P(-1.58 < Z < 0.316)$$

$$= P(0 < Z < 1.58) + P(0 < Z < 0.316)$$

$$= 0.4429 + 0.1217$$

$$= 0.5646$$

(+1)

5 marks

i) Random process explanation

classification

(+2)

(+3)

5 marks

6(i) Correlation coefficient

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \, \Sigma Y^2}}$$

$$X = x - m_x \qquad Y = y - m_y$$

$$m_x = 5 \qquad m_y = 12$$

$r = 0.95$ Ans

+2 marks

Regression line

$$Y - m_y = r\frac{\sigma_y}{\sigma_x}(X - m_x) \qquad \sigma_x = 1, \ \sigma_y = 1$$

$$\Rightarrow (Y - 12) = 0.95(X - 5)$$

(+1.5)

$$X - m_x = r\frac{\sigma_x}{\sigma_y}(Y - m_y)$$

$(X - 5) = 0.95(Y - 12)$ Ans (+1.5)

5 marks

Q. 6 (ii) Principle of least square +2

Let $y = a + bx$ be the line of fit

N. E

$\Sigma y = na + b\Sigma x$ +1

$\Sigma xy = a\Sigma x + b\Sigma x^2$

x	y	x^2	xy
0	1	0	0
1	1.8	1	1.8
2	3.3	4	6.6
3	4.5	9	13.5
4	6.3	16	25.2
$\Sigma x = 10$	$\Sigma y = 16.9$	$\Sigma x^2 = 30$	$\Sigma xy = 47.1$

$\Rightarrow$ $16.9 = 5a + 10b$

$47.1 = 10a + 30b$

$\Rightarrow y = 0.72 + 1.33x$ Ans +2

5 marks

H_0 :- The digits occur equally frequently in directory

+1

Total observed frequency $= N = 10{,}000$

$\therefore$ Expected frequency $= N \times P(x)$

$= 10000 \times \frac{1}{10} = 1000$

Now

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

We have

x	f_o	f_e	$\frac{(f_o - f_e)^2}{f_e}$
0	1026	1000	0.676
1	1107	1000	11.449
2	997	1000	0.009
3	966	1000	1.156
4	1075	1000	5.625
5	933	1000	4.489
6	1107	1000	11.449
7	972	1000	0.784
8	964	1000	1.296
9	853	1000	21.609

+2

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} = 58.542$$

Degree of freedom $v = 10 - 1 = 9$

$\chi^2_{tabulate}$ at 5% & 9 d.o.f $= 16.919$

+1

cal $\chi^2 >$ tabulated χ^2

⇒ Null hypothesis is rejected

⇒ The digit do not occure equally frequenently.

(+1)

5 marks