

Q.6 Attempt any two:

- i Two Judges in a beauty contest rank the ten competitors in the following order: 5

|                      |   |   |   |   |   |   |    |   |    |   |
|----------------------|---|---|---|---|---|---|----|---|----|---|
| First judge opinion  | 6 | 4 | 3 | 1 | 2 | 7 | 9  | 8 | 10 | 5 |
| Second judge opinion | 4 | 1 | 6 | 7 | 5 | 8 | 10 | 9 | 3  | 2 |

Find the correlation between their judgements.

- ii Fit a second-degree parabola to the following data regarding  $x$  as an independent variable: 5

|     |   |   |    |    |    |
|-----|---|---|----|----|----|
| $x$ | 0 | 1 | 2  | 3  | 4  |
| $y$ | 1 | 5 | 10 | 22 | 38 |

- iii 200 digits were chosen at random from a set of tables. The frequencies of the digits were: 5

|           |   |    |    |    |    |    |    |    |    |    |    |
|-----------|---|----|----|----|----|----|----|----|----|----|----|
| Digit     | : | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| Frequency | : | 18 | 19 | 23 | 21 | 16 | 25 | 22 | 20 | 21 | 15 |

Use the  $\chi^2$  test to assess the correctness of the hypothesis that the digits were distributed in equal numbers in the tables from which these were chosen. The 5% value of  $\chi^2$  for 9 d.f. is 16.919.

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Enrollment No.....



Faculty of Engineering

End Sem (Even) Examination May-2022

EN3BS03 Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: AU/ME/CE/FT/RA

Duration: 3 Hrs.

Maximum Marks: 60

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

- Q.1 i.  $f(z) = \left| \frac{-1}{z} \right|^2$  is 1
- (a) Differentiable and analytic everywhere  
 (b) Not differentiable and not analytic at  $z = 0$   
 (c) Differentiable at  $z = 0$  but not analytic at  $z = 0$   
 (d) None of these
- ii. A point at which a function ceases to be analytic is called a 1
- (a) Singular point (b) Non-singular point  
 (c) Regular point (d) None of these
- iii. Order of convergence of Regula-Falsi method is 1
- (a) 1.321 (b) 1.618 (c) 2.231 (d) None of these
- iv. Jacobi's method is also known as 1
- (a) Simultaneous method  
 (b) Displacement method  
 (c) Simultaneous displacement method  
 (d) None of these
- v. Let  $h$  be the finite difference, then forward difference of  $f(x)$  is 1
- (a)  $\Delta f(x) = f(x+h) - f(x)$  (b)  $\Delta f(x) = f(x-h) - f(x)$   
 (c)  $\Delta f(x) = f(x+h)$  (d) None of these
- vi. The Bessel's interpolation formula gives best interpolate values of  $y$  for  $u$  when 1
- (a)  $\frac{1}{4} < u < \frac{3}{4}$  (b)  $-\frac{1}{4} < u < \frac{1}{4}$   
 (c)  $-1 < u < 0$  (d) None of these

P.T.O.

[2]

- vii. In trapezoidal rule, the curve  $y = f(x)$  is assumed to be **1**  
 (a) Straight line (b) Circle  
 (c) Parabola (d) None of these
- viii. Picard's method is applicable to find the solution of **1**  
 (a) Transcendental equation only  
 (b) System of linear equations  
 (c) Ordinary differential equation of first order  
 (d) None of these
- ix. If the value of any regression coefficient is zero, then two variables **1**  
 are  
 (a) Dependent (b) Independent  
 (c) Correlated (d) None of these
- x. If the correlation coefficient have positive value, then the slope of the **1**  
 regression line  
 (a) Must also be positive (b) Can be either negative or positive  
 (c) Can be zero (d) None of these

Q.2 Attempt any two:

- i. Find the imaginary part of the analytic function whose real part is **5**  
 $x^3 - 3xy^2 + 3x^2 - 3y^2$
- ii. Using Cauchy's integral formula, prove that  $\int_C \frac{e^{2z}}{(z+1)^4} dz = \frac{8\pi e^{-2}}{3} i$ , **5**  
 where  $C$  is the circle  $|z| = 3$ .
- iii. Find the poles, order of poles and residues at it for the function **5**  
 $f(z) = \frac{1}{z^4 + 1}$ .

Q.3 Attempt any two:

- i. Find the cube root of 2 approximately by Newton Raphson method **5**  
 correct to five decimal places.
- ii. Solve the following simultaneous equations by Gauss seidel iteration **5**  
 method  
 $10x + y + z = 12; 2x + 10y + z = 13; 2x + 2y + 10z = 14$

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- iii. Solve the following simultaneous equations by Gauss elimination **5**  
 method  $10x + y + 2z = 13; 3x + 10y + z = 14; 2x + 3y + 10z = 15$

Q.4 Attempt any two:

- i. Using Newton's Gregory Backward Difference formula, find the **5**  
 cubic polynomial which takes the following values  
 $x: 0 \quad 1 \quad 2 \quad 3$   
 $y: 1 \quad 2 \quad 1 \quad 10$
- ii. Using Newton's Divided Difference Formula find the value of  $f(6)$  **5**  
 from the following data  
 $x: 1 \quad 2 \quad 7 \quad 8$   
 $y: 4 \quad 5 \quad 5 \quad 4$
- iii. Using Stirling formula, find  $y_{28}$ ; given **5**  
 $x: 20 \quad 25 \quad 30 \quad 35 \quad 40$   
 $y: 49225 \quad 48316 \quad 47326 \quad 45926 \quad 44306$

Q.5 Attempt any two:

- i. A river is 80 metres wide. The depth  $d$  (in metres) of the river at a **5**  
 distance  $x$  from the bank is given by the following table  
 $x: 0 \quad 10 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80$   
 $y: 0 \quad 4 \quad 7 \quad 9 \quad 12 \quad 15 \quad 14 \quad 8 \quad 3$   
 Find approximately the area of cross section of the river using  
 Simpson's three-eighth rule.
- ii. Employ Taylor's series methods to obtain approximate value of  $y$  at **5**  
 $x = 0.2$  for the differential equation  $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ . Compare  
 the numerical solution with exact solution
- iii. Using Runge-Kutta method, solve **5**  
 $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}, y(0) = 1$  at  $x = 0.2$  and  $0.4 (h = 0.2)$

P.T.O.

- Q.1 (i) c) Differentiable at  $z=0$  but not analytic at  $z=0$  +1
- (ii) a) singular point +1
- (iii) b) 1.618 +1
- (iv) c) simultaneous displacement method +1
- (v) a)  $\Delta f(x) = f(x+h) - f(x)$  +1
- (vi) a)  $\frac{1}{4} < u < \frac{3}{4}$  +1
- vii) a) straight line +1
- viii) c) ordinary differential eq<sup>n</sup> +1
- ix) b) Independent +1
- x) a) must also be positive +1



Q.2 (i) given  $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$  — (1)

First we note that  $u(x, y)$  is harmonic and hence it is feasible to construct an analytic function  $f(z) = u(x, y) + iV(x, y)$ . Since  $f(z)$  is analytic, the first order partial derivative of  $u$  &  $V$  satisfy the C-R eq<sup>n</sup>, i.e.

$$u_x = v_y \quad \text{and} \quad u_y = -v_x$$

from (1)  $u_x = 3x^2 - 3y^2 + 6x$   
i.e.  $v_y = 3x^2 - 3y^2 + 6x$  — (2) +

integrating (2) w.r.to  $y$

$$V = 3x^2y - y^3 + 6xy + \phi(x) \text{ — (3) +}$$

where  $\phi(x)$  is a func<sup>n</sup> of  $x$   
Now, differentiating (3) w.r.to  $x$

$$V_x = 6xy + 6y + \phi'(x)$$

since

$$u_y = -v_x$$

Hence from (1)

$$-6xy - 6y = -6xy - 6y - \phi'(x)$$

$$\phi'(x) = 0$$

and

$$\phi(x) = C \text{ where } C \text{ is some real constant}$$

Now put in (3)

$$V = 3x^2y - y^3 + 6xy - 6y^2 + C$$

$$\boxed{V = 3x^2y + 6xy - y^3 + C}$$

Ans.

+1

OK

(i) By Milne Thomson

Given  $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$

By C-R eq<sup>n</sup> we have

$$u_x = v_y \text{ and } u_y = -v_x$$

+1

Consider  $f'(z) = u_x + i v_x$

$$f'(z) = u_x - i u_y$$

+1

$$= 3x^2 - 3y^2 + 6x - i(-6xy - 6y)$$

$$= 3x^2 - 3y^2 + 6x + i6xy + i6y$$

Put  $x = z$  and  $y = 0$

$$f'(z) = 3z^2 + 6z$$

+1

integrating w.r to  $z$

$$f(z) = z^3 + 3z^2$$

$$= (x + iy)^3 + 3(x + iy)^2$$

+1

$$= x^3 + (iy)^3 + 3xy(x + iy) + 3(x^2 + i^2y^2 + 2ixy)$$



$$= x^3 + i^3 y^3 + 3ix^2y + 3x^2 + 3i^2 y^2 + 6ixy + 3i^2 xy^2$$

$$= x^3 - y^3 + 3ix^2y + 3x^2 - 3y^2 - 3xy^2 + 6ixy$$

$$= x^3 + 3x^2 - 3xy^2 - 3y^2 - i(y^3 + 3xy^2 - 6xy)$$

$$= (x^3 + 3x^2 - 3xy^2 - 3y^2) + i(3x^2y + 6xy - y^3)$$

Hence

$$\boxed{v = 3x^2y + 6xy - y^3} \quad \text{Ans.} \quad +1$$

Q.2 (ii) given  $\int_c \frac{e^{2z}}{(z+1)^4} dz$

By Cauchy's Integral formula

$$f^{(n)}(a) = \frac{n!}{2\pi i} \int_c \frac{f(z)}{(z-a)^{n+1}} dz \quad \text{--- (1)} \quad +1$$

Put  $a = -1, n = 3$ , then

$$f^{(3)}(-1) = \frac{3!}{2\pi i} \int_c \frac{f(z)}{(z+1)^4} dz \quad \text{--- (2)} \quad +2$$

Now take  $f(z) = e^{2z}$

$$f^{(3)}(z) = 8e^{2z}$$

$$f^{(3)}(-1) = 8e^{-2} \quad +1$$

Put in (2)

$$8e^{-2} = \frac{3!}{2\pi i} \int_C \frac{e^{2z}}{(z+1)^4} dz$$

$$\text{or } \int_C \frac{e^{2z}}{(z+1)^4} dz = \frac{8\pi e^{-2}}{3} i \quad +1$$

Proved

Q. 2 (iii)  $f(z) = \frac{1}{z^4 + 1}$

Poles of  $f(z)$  are given by  $z^4 + 1 = 0$

$$z^4 = -1 = e^{(2n+1)i\pi/4}$$

where

$$n = 0, 1, 2, 3$$

$$z = e^{i\pi/4}, e^{3i\pi/4}, e^{5i\pi/4}, e^{7i\pi/4} \quad +1$$

All the four poles are simple poles.

$$\text{Res}_{z=e^{i\pi/4}} f(z) = \lim_{z \rightarrow e^{i\pi/4}} -\frac{1}{4} \left( \frac{1+i}{\sqrt{2}} \right) \quad +1$$

$$\text{Res}_{z=e^{3i\pi/4}} f(z) = -\frac{1}{4} \left( \frac{-1+i}{\sqrt{2}} \right) \quad +1$$

$$\text{Res}_{z=e^{5i\pi/4}} f(z) = -\frac{1}{4} \left( \frac{-1-i}{\sqrt{2}} \right) \quad +1$$

$$\text{Res}_{z=e^{7i\pi/4}} f(z) = -\frac{1}{4} \left( \frac{1-i}{\sqrt{2}} \right) \quad +1$$

$$\left\{ f(z) = \frac{-1}{4} z \right\}$$



6

Q.3 (i) Let  $x = 2^{1/3}$   
 So that  $x^3 = 2$   
 Hence the eq<sup>n</sup>  
 $x^3 - 2 = 0$

Let  $f(x) = x^3 - 2$   
 and  $f'(x) = 3x^2$

Now  $f(1) = -1$  and  $f(2) = 6$

So that

roots of  $x^3 - 2 = 0$  lies b/w 1 and 2 +1

By Newton Raphson method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad +1$$

$$= x_n - \frac{x_n^3 - 2}{3x_n^2}$$

$$= \frac{2}{3} \left( x_n + \frac{1}{x_n^2} \right)$$

taking  $x_0 = 1.5$  +1

first approximation  $x_1$  is

$$x_1 = \frac{2}{3} \left( 1.5 + \frac{1}{(1.5)^2} \right) = 1.29630$$

Similarly, successive approximation are



$$x_2 = 1.26093$$

$$x_3 = 1.25992$$

$$x_4 = 1.25992$$

Since  $x_3 = x_4$

$$2^{1/3} = 1.25992 \quad \underline{\underline{\text{Ans.}}}$$

+1

+1

Q.3 (ii) Given

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|a_{22}| > |a_{21}| + |a_{23}|$$

$$|a_{33}| > |a_{31}| + |a_{32}|$$

+1

above condition is satisfied. Now system of eq<sup>n</sup> can be written as

$$\left. \begin{aligned} x &= \frac{1}{10} (12 - y - z) \\ y &= \frac{1}{10} (13 - 2x - z) \\ z &= \frac{1}{10} (14 - 2x - 2y) \end{aligned} \right\} \text{--- (1)}$$

+1

Starting with initial approximation

$$x=0, y=0, z=0$$

For first approximation

Putting  $y=0, z=0$  in (1) we have

$$x^{(1)} = 1.2$$

Putting  $x=1.2, z=0$  in (1) for finding  $y$

$$y^{(1)} = 1.06$$

Putting  $x=1.2, y=1.06$  in (1) for finding  $z$

$$z^{(1)} = 0.948$$

+1

For second iteration

$$x^{(2)} = 0.999 \quad \left\{ \begin{array}{l} \text{taking} \\ y = 1.06, z = 0.948 \end{array} \right\}$$

$$y^{(2)} = 1.005 \quad \left\{ \text{since } x = 0.999, z = 0.948 \right\}$$

$$z^{(2)} = 0.999$$

+1

For third iteration

$$x^{(3)} = 1.000$$

$$y^{(3)} = 1.000$$

$$z^{(3)} = 1.000$$

for fourth

$$x^{(4)} = 1.000, y^{(4)} = 1.000, z^{(4)} = 1.000$$

+1

Hence  $x=y=z=1$

Aus.



Q. 3  
(iii) Given

$$10x + y + 2z = 13$$

$$3x + 10y + z = 14$$

$$2x + 3y + 10z = 15$$

By Gauss elimination Method.

~~Given eq<sup>n</sup> is~~

Given eq<sup>n</sup> is written in matrix form

$$\left[ \begin{array}{ccc|c} 10 & 1 & 2 & 13 \\ 3 & 10 & 1 & 14 \\ 2 & 3 & 10 & 15 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 3R_2$$

$$\left[ \begin{array}{ccc|c} 1 & -29 & -1 & -29 \\ 3 & 10 & 1 & 14 \\ 2 & 3 & 10 & 15 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 3R_1$$

$$R_3 \rightarrow R_3 - 2R_1$$

+

$$\left[ \begin{array}{ccc|c} 1 & -29 & -1 & -29 \\ 0 & 97 & 4 & 101 \\ 0 & 61 & 12 & 73 \end{array} \right]$$

$$R_2 \rightarrow R_2 - R_3$$

$$\begin{bmatrix} 1 & -29 & -1 & : & -29 \\ 0 & 36 & -8 & : & 28 \\ 0 & 61 & 12 & : & 73 \end{bmatrix}$$

$$R_2 \rightarrow \frac{1}{4} R_2$$

$$\begin{bmatrix} 1 & -29 & -1 & : & -29 \\ 0 & 9 & -2 & : & 7 \\ 0 & 61 & 12 & : & 73 \end{bmatrix}$$

$$R_2 \rightarrow \frac{1}{9} R_2$$

+1

$$\begin{bmatrix} 1 & -29 & -1 & : & -29 \\ 0 & 1 & -2/9 & : & 7/9 \\ 0 & 61 & 12 & : & 73 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 61 R_2$$

+1

$$\begin{bmatrix} 1 & -29 & -1 & : & -29 \\ 0 & 1 & -2/9 & : & 7/9 \\ 0 & 0 & \frac{230}{9} & : & \frac{230}{9} \end{bmatrix}$$

$$R_3 \rightarrow \frac{9}{230} R_3$$

+1

$$\begin{bmatrix} 1 & -29 & -1 & : & -29 \\ 0 & 1 & -2/9 & : & 7/9 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$



$$x - 2y - z = -29$$

$$y - \frac{2}{9}z = \frac{7}{9}$$

$$z = 1$$

$$y = 1, x = 1$$

Ans

Q.4 (i) Given

$$\begin{array}{cccc} x: & 0 & 1 & 2 & 3 \\ y: & 1 & 2 & 1 & 10 \end{array}$$

Newton Gregory Backward difference formula

$$y = y_n + u \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \dots + \frac{u(u+1)(u+2)\dots(u+n-1)}{n!} \nabla^n y_n$$

Here  $a=0$ ,  $n=3$ ,  $h=1$ ,  $x=x$ ,

$$u = \frac{x - (a + nh)}{h} = x - 3$$

we have

$$y = 10 + (x-3)9 + \frac{(x-3)(x-2)}{2 \times 1} (10) + \frac{(x-3)(x-2)}{3 \times 2 \times 1}$$

$$= 2x^3 - 7x^2 + 6x + 1$$

Ans.

Q.4 (ii) Given

$$x: 1 \quad 2 \quad 7 \quad 8$$

$$y: 4 \quad 5 \quad 5 \quad 4$$

find  $f(6) = ?$

By Divided Difference formula

$$f(x) = f(1) + (x-1) \Delta_{2,1} f(1) + (x-1)(x-2) \Delta_{2,7,1}^2 f(1) + (x-1)(x-2)(x-7) \Delta_{2,7,8}^3 f(1)$$

$$+ 1$$

Difference Table

| $x$ | $f(x)$ | $\Delta f(x)$         | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|-----|--------|-----------------------|-----------------|-----------------|
| 1   | 4      |                       |                 |                 |
| 2   | 5      | $\frac{5-4}{2-1} = 1$ |                 |                 |
| 7   | 5      | 0                     | $-\frac{1}{6}$  |                 |
| 8   | 4      | -1                    | $-\frac{1}{6}$  | 0               |

Putting the values

$$f(x) = \frac{1}{6} (-x^2 + 9x + 16)$$

taking  $x=6$

$$f(6) = 5.667$$

Ans.



Q. 4 (iii) Given

|    |       |       |       |       |       |
|----|-------|-------|-------|-------|-------|
| x: | 20    | 25    | 30    | 35    | 40    |
| y: | 49225 | 48316 | 47326 | 45926 | 44306 |

$$y_{28} = ?$$

taking  $a = 30$  as origin and  $h = 5$   
the required value of  $y$  is  
for  $x = 28$  i.e. for

$$u = \frac{x-a}{h} = \frac{28-30}{5} = -0.4$$

Difference table

| x  | u  | $y_u$ | $\Delta y_u$ | $\Delta^2 y_u$ | $\Delta^3 y_u$ | $\Delta^4 y_u$ |
|----|----|-------|--------------|----------------|----------------|----------------|
| 20 | -2 | 49225 |              |                |                |                |
|    |    |       | -909         |                |                |                |
| 25 | -1 | 48316 |              | -171           |                |                |
|    |    |       | -1080        |                | -59            | +2             |
| 30 | 0  | 47236 |              | -230           |                | -21            |
|    |    |       | -1310        |                | -80            |                |
| 35 | 1  | 45926 |              | -310           |                |                |
|    |    |       | -1620        |                |                |                |
| 40 | 2  | 44306 |              |                |                |                |

Stirling formula

$$y_u = y_0 + u \left( \frac{\Delta y_0 + \Delta y_{-1}}{2} \right) + \frac{u^2}{2!} \Delta^2 y_{-1} + \frac{u(u^2-1^2)}{3!} \Delta^3 y_{-1} + \frac{u^2(u^2-1^2)}{4!} \Delta^4 y_{-2}$$

$u = -0.4$  we get-

$$y_{2/8} = 47692$$

Ans.

$$= 47236 + (-0.4) \left( \frac{-1310 - 1080}{2} \right) + \frac{0.16}{2} (-230) \\ + \frac{(-0.4) (-0.84) (-80 - 59)}{6} + \frac{0.16 (-0.84) (-21)}{24} + 1$$

$$= 47692 \quad \text{Ans.} \quad +1$$

Q.5 (i) By Simpson's 3/8 rule

$$\int_{x_0}^{x_0+3h} y dx = \frac{3h}{8} \left[ (y_0 + y_n) + 3(y_1 + y_2 + y_4 + 2y_5 + \dots + y_{n-2} + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3}) \right]$$

Given

|       |   |    |    |    |    |    |    |    |    |
|-------|---|----|----|----|----|----|----|----|----|
| $x$ : | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| $y$ : | 0 | 4  | 7  | 9  | 12 | 15 | 14 | 8  | 3  |

$$\int_0^{80} f(x) dx = \int_0^{80} y dx = \frac{3h}{8} \left[ (y_0 + y_{80}) + 3(y_{10} + y_{20} + y_{40} + y_{50} + y_{60} + y_{70}) + 2(y_{30} + y_{60}) \right]$$

$$= \frac{3}{8} \times 10 \left[ (0 + 3) + 3(4 + 7 + 12 + 15 + 8) + 2(9 + 14) \right] + 1$$

$$= 701.25 \text{ sq meter} \quad \text{Ans.} \quad +1$$



Q.5

(ii) Here  $\frac{dy}{dx} = 2y + 3e^x$  — (1)

$$y' = 2y + 3e^x \quad \text{given } x=0, y_0=0$$

$$y'_0 = 3$$

diff<sup>n</sup> w.r.to (1)

$$y'' = 2y' + 3e^x, \quad y''_0 = 9$$

$$y''' = 2y'' + 3e^x, \quad y'''_0 = 21$$

$$y^{iv} = 2y''' + 3e^x, \quad y^{iv}_0 = 45$$

$$y^v = 2y^{iv} + 3e^x, \quad y^v_0 = 93$$

By Taylor's Series

$$y(x) = y_0 + (x-x_0)y'_0 + \frac{(x-x_0)^2}{2!}y''_0 + \frac{(x-x_0)^3}{3!}y'''_0 + \dots$$

here  $x_0=0$

$$= 0 + 3x + \frac{9x^2}{2!} + \frac{21x^3}{3!} + \frac{45x^4}{4!} + \frac{93x^5}{5!} + \dots$$

find approximate value of  $y$  at  $x=0.2$   
putting  $x=0.2$ , we have

$$y(0.2) = 3(0.2) + \frac{9}{2}(0.2)^2 + \frac{7}{2}(0.2)^3 + \frac{15}{8}(0.2)^4 + \frac{31}{40}(0.2)^5 + \dots$$

$$y(0.2) = 0.81125$$

Now exact sol<sup>n</sup>

$$\frac{dy}{dx} - 2y = 3e^x$$

which L.D.B

So I.F.  $e^{-\int 2dx} = e^{-2x}$

$$\text{Sol}^n \quad y e^{-2x} = \int 3e^x e^{-2x} dx + C \quad +2$$

$$= -3e^{-x} + C$$

$$y = -3e^x + Ce^{2x}$$

Since  $y=0$  when  $x=0$  so  $C=3$

$$y = 3e^{2x} - 3e^x$$

$$y(0.2) = 0.81126 \quad \text{Ans.}$$

Q.5 (iii) Here  $f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$ ,  $x_0 = 0, y_0 = 1$   
taking  $h = 0.2$

find value of  $y$  at  $x = 0.2$

we have

$$S_1 = h f(x_0, y_0) = 0.2 f(0, 1) = 0.2$$

$$S_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{S_1}{2}\right) = 0.2 \frac{((1.1))^2 - (0.1)^2}{((1.1))^2 + (0.1)^2} + 1$$

$$= 0.19672$$



$$s_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{s_2}{2}\right)$$

$$= 0.2 \left[ \frac{(1.09836)^2 - (0.1)^2}{(1.09836)^2 + (0.1)^2} \right] = 0.19671$$

$$s_4 = hf(x_0 + h, y_0 + s_3)$$

+1

$$= 0.2 \left[ \frac{(1.19671)^2 - (0.2)^2}{(1.19671)^2 + (0.2)^2} \right] = 0.18913$$

$$y_1 = y_0 + \frac{1}{6} (s_1 + 2s_2 + 2s_3 + s_4)$$

+1

$$y_1 = 1.19600 \quad \text{at } x=0.2$$

Now find  $y$  at  $x=0.4$

taking  $x_1 = 0.2, y_1 = 1.196$

$$s_1 = hf(x_1, y_1) = (0.2)f(0.2, 1.196) = 0.18912$$

$$s_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{s_1}{2}\right) = 0.17949$$

+1

$$s_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{s_2}{2}\right) = 0.17935$$

$$s_4 = hf(x_1 + h, y_1 + s_3) = 0.16880$$

Hence

$$y_2 = 1.37527 \quad \text{at } x=0.4$$

+1

Ans.

Q. 6 (i)

| First Judge<br>opinion X | Second Judge<br>opinion Y | Rank diff<br>$X - Y = d$ | $d^2$ |    |
|--------------------------|---------------------------|--------------------------|-------|----|
| 6                        | 4                         | 2                        | 4     |    |
| 4                        | 1                         | 3                        | 9     |    |
| 3                        | 6                         | -3                       | 9     |    |
| 1                        | 7                         | -6                       | 36    |    |
| 2                        | 5                         | -3                       | 9     |    |
| 7                        | 8                         | -1                       | 1     | +2 |
| 9                        | 10                        | -1                       | 1     |    |
| 8                        | 9                         | -1                       | 1     |    |
| 10                       | 3                         | 7                        | 49    |    |
| 5                        | 2                         | 3                        | 9     |    |
| $\Sigma d^2 = 128$       |                           |                          |       |    |

Here  $n = 10$

The required correlation coeff.  
is given by

$$r = 1 - \frac{6 \Sigma d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 128}{10(100 - 1)} \quad +2$$

$$r = 0.224$$

Ans.

+1



Q.6 (ii) Given

|   |   |   |    |    |    |
|---|---|---|----|----|----|
| x | 0 | 1 | 2  | 3  | 4  |
| y | 1 | 5 | 10 | 22 | 38 |

Let the eq<sup>n</sup> of second degree parabola to be fitted to the given data

$$y = a + bx + cx^2$$

then, its normal eq<sup>n</sup>s are

$$\sum y = na + b \sum x + c \sum x^2$$

$$\sum xy = a \sum x + b \sum x^2 + c \sum x^3$$

$$\sum x^2 y = a \sum x^2 + b \sum x^3 + c \sum x^4$$

+1

| x | y  | x <sup>2</sup> | x <sup>3</sup> | x <sup>4</sup> | xy  | x <sup>2</sup> y |
|---|----|----------------|----------------|----------------|-----|------------------|
| 0 | 1  | 0              | 0              | 0              | 0   | 0                |
| 1 | 5  | 1              | 1              | 1              | 5   | 5                |
| 2 | 10 | 4              | 8              | 16             | 20  | 40               |
| 3 | 22 | 9              | 27             | 81             | 66  | 198              |
| 4 | 38 | 16             | 64             | 256            | 152 | 608              |

+2

$$\sum x = 10 \quad \sum y = 76 \quad \sum x^2 = 30 \quad \sum x^3 = 100 \quad \sum x^4 = 354 \quad \sum xy = 243 \quad \sum x^2 y = 851$$

Substituting these values in normal eq<sup>n</sup>

$$76 = 5a + 10b + 30c$$

$$243 = 10a + 30b + 100c$$

$$851 = 30a + 100b + 354c$$

+1

on solving

$$a=1.43, b=0.24, c=2.21$$

So the required eq<sup>n</sup> of second degree parabola

$$y = 1.43 + 0.24x + 2.21x^2$$

+1

Q.6 (iii) Here total frequency = 200

The theoretical frequency for each digit =  $200/10$  [individual fe may be calculated]  
 = 20

By def<sup>n</sup>, of  $\chi^2$

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

+1

$$= \frac{(18-20)^2}{20} + \frac{(9-20)^2}{20} + \frac{(23-20)^2}{20} +$$

$$\frac{(21-20)^2}{20} + \frac{(16-20)^2}{20} + \frac{(25-20)^2}{20} +$$

+2

$$\frac{(22-20)^2}{20} + \frac{(20-20)^2}{20} + \frac{(21-20)^2}{20} + \frac{(15-20)^2}{20}$$

$$= \frac{1}{20} [4 + 1 + 9 + 16 + 25 + 4 + 0 + 1 + 25]$$

$$= 4.30$$



$$d.f (v) = 10 - 1 = 9$$

Since calculate value (4.30) of  $\chi^2 <$  the tabulated value (16.919) of  $\chi^2$  at 5% level for 9 d.f. Hence the calculated value is significant and consequently the hypothesis is correct.

+2