| Characteristics | Facility A | Facility B |
|---|---|---|
| Installation Cost | Rs.4,00,000 | Rs. 6,00,000 |
| Labour Cost /month | Rs. 80,000 | Rs. 60,000 |
| Repair Rates | 60 trucks/month | 40 trucks/month |
| Arrival Rates | 24 trucks/month | 24 trucks /month |
| Economic life | 4 years | 4 years |

Determine which facility should be built by company.

OR iii. A small project consists of seven activities, the details of which are given below: **6**

| Activity | Duration (days) | | | Immediate Predecessor |
|---|---|---|---|---|
| | Optimistic | Most Likely | Pessimistic | |
| A | 1 | 3 | 7 | - |
| B | 2 | 6 | 14 | A |
| C | 3 | 3 | 3 | A |
| D | 4 | 10 | 22 | B, C |
| E | 3 | 7 | 15 | B |
| F | 2 | 5 | 14 | D, E |
| G | 4 | 4 | 4 | D |

(a) Draw the network, fine critical path and the expected project completion time.

(b) What project duration will have 95% and 90% confidence of completion?

Q.5 i. Write down any four limitations of decision tree. **4**

ii. Solve the following game by any method **6**

| | | Player B | | | |
|---|---|---|---|---|---|
| | | B1 | B2 | B3 | B4 |
| Player A | A1 | -1 | 7 | 6 | 4 |
| | A2 | 5 | -3 | 3 | 6 |
| | A3 | -2 | 4 | 2 | 3 |

OR iii. A bakery keeps stock of popular brand of cake. Previous experience shows the daily demand pattern for the cake with associated probabilities, as given below: **6**

| Daily Demand: | 0 | 15 | 25 | 35 | 45 | 50 |
|---|---|---|---|---|---|---|
| Probability: | 0.01 | 0.15 | 0.20 | 0.50 | 0.12 | 0.02 |

Consider the following sequence of random numbers to simulate the demand for next 10 days: -

*Total No. of Questions: 6* *Total No. of Printed Pages:5*

**Enrollment No......................................**

MEDI-CAPS UNIVERSITY
Knowledge is Power

# Faculty of Engineering

## End Sem (Odd) Examination Dec-2018

## ME3EI02 Operations Research

Programme: B.Tech. Branch/Specialisation: ME

**Duration: 3 Hrs.** **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Non–negativity condition is an important component of Linear Programming (LP) model because: **1**

(a) Variables value should remain under the control of the decision maker
(b) Value of variables make sense and corresponds to real world problems
(c) Variables are interrelated in terms of limited resources
(d) None of these

ii. If two constraints do not intersect in the positive quadrant of the graph, then: **1**

(a) The problem is infeasible (b) The solution is unbounded
(c) One of the constraint is redundant (d) None of these

iii. An unoccupied cell in the transportation method is analogous to a **1**

(a) $C_j$- $Z_j$ value in the simplex table
(b) Variable in the B- Column in the simplex table
(c) Variable not in the B- Column in the simplex table
(d) Value in the $X_B$ column in the simplex table

iv. Every basic feasible solution of a general assignment problem having a square pay off matrix of order n, should have assignment equal to: **1**

(a) 2n + 1 (b) 2n-1 (c) m+n+1 (d) m-n-1

v. In PERT the span of time between the optimistic and pessimistic time estimates of an activity is: (Where σ = Standard Deviation) **1**

(a) 3σ (b) 6σ (c) 2σ (d) None of these

vi. Customer behaviour in which, customer do not join the queue either by seeing the number of customers already in service system or by estimating the excessive waiting time for the desired service is: **1**

(a) Jockeying (b) Reneging (c) Balking (d) None of these

vii. Considering the following two-person game, what percentage of the time should Y play strategy Y1? **1**

| | Y1 | Y2 |
|---|---|---|
| X1 | 6 | 3 |
| X2 | 2 | 8 |

(a) 6/9 (b) 3/9 (c) 5/9 (d) 4/9

viii. While assigning random numbers in Monte Carlo simulation, it is **1**

(a) Not necessary to assign the exact range of random number interval as the probability
(b) Necessary to develop a cumulative probability distribution
(c) Necessary to assign the particular appropriate random number
(d) All of these

ix. If the unit cost decreases, then optimal order quantity **1**

(a) Increases (b) Decreases (c) Either (a) or (b) (d) None of these

x. If Economic Ordered Quantity (EOQ) is calculated, but an order is placed which is smaller than this, will the variable cost: **1**

(a) Increases (b) Decreases (c) Either (a) or (b) (d) No change

Q.2 i. Maximize $Z = 8x_1 + x_2$ **4**

Subjected to
$$2x_1 + x_2 \le 6$$
$$3x_1 + x_2 \le 6$$
$$8x_1 + x_2 \le 8$$
$$x_1 + 6x_2 \le 8$$
$$x_1, x_2 \ge 0$$

For above problem also justify, whether the infinite number of optimal solution exist or not.

ii. Use penalty method to solve following L.P. Problem **6**

Minimize $Z = x_1 + 2x_2 + x_3$

Subjected to
$$x_1 + x_2/2 + x_3/2 \le 1$$
$$3x_1/2 + 2x_2 + x_3 \ge 8$$
$$x_1, x_2, x_3 \ge 0$$

OR iii. Solve the following L.P. Problem by Simplex method **6**

Maximize $Z = 3x_1 + 5x_2$

Subjected to
$$x_1 - 2x_2 \le 6$$
$$x_1 \le 10$$
$$x_2 \ge 1$$
$$x_1, x_2 \ge 0$$



Q.3 i. Define following terms with respect to transportation problem **2**

(a) Non- degenerate basic feasible solution
(b) Degenerate basic feasible solution

ii. A company produces a small component for an industrial product and distributes it to five wholesalers at a fixed delivered price Rs. 250 per unit. Sales forecasts indicate that monthly deliveries will be 300,300,100,500 and 400 units to wholesalers 1,2,3,4 and 5 respectively. The direct costs of production of each unit are Rs. 100, Rs.90 and Rs. 80 at plants A, B, and C respectively. The transportation costs of shipping a unit from plants to wholesalers are given below: **8**

| | Wholesalers | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| Plant A | 5 | 7 | 10 | 15 | 15 |
| Plant B | 8 | 6 | 9 | 12 | 14 |
| Plant C | 10 | 9 | 8 | 10 | 15 |

Find how many components each plant must supply to each wholesaler to maximize the profit? What is the maximum total profit? Take monthly production capacities of plant A, B and C as 500, 100 and 1250 respectively.

OR iii. Four jobs to be assigned to the different workers. Due to some constraints A cannot assigned to III, B to V, C to II and D to I respectively. Find the maximum cost solution for the given assignment problem whose cost coefficients are as given: **8**

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | -2 | -4 | - | -6 | -1 |
| B | 0 | -9 | -5 | -5 | - |
| C | -3 | - | -9 | -2 | -6 |
| D | - | -3 | -1 | 0 | -3 |

Q.4 i. Write down the limitation of queuing model. **4**

ii. A company is considering the construction of two repair facilities, each with different characteristics. On the average, 24 trucks require repairs each month and the arrival are Poisson distributed. The loss of revenue to the firm of having a truck in repairs is estimated to be Rs. 300 per month. The two facilities which are under consideration have the following characteristics. **6**

Random numbers: 48,78,09,51,56,77,15,14,68,09
Find the stock situation if the owner of the bakery decided to make 35 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data.

Q.6 Attempt any two:

i. The product is sold at the rate of 50 pieces per day and is manufactured at the rate of 250 pieces per day. The setup cost of the machine is Rs. 1000 and the storage cost is found to be Rs.0.0015 per piece per day. With labour charges of Rs. 3.20 per piece, material cost at Rs. 2.10 per piece and overhead cost of Rs.4.10 per piece, find the minimum cost batch size if the interest charges are 8 % (assume 300 working days in a year). Also compute the optimal number of orders in a year, maximum inventory, total cost per year, manufacturing time and total time. **5**

ii. The purchasing manager of a distillery company is considering three sources of supply for oak barrels. The first supplier offers any quantity of barrels at Rs.150 each. The second supplier offers barrels in lot of 150 or more at Rs.125 per barrel. The third supplier offers barrels in lots of 250 or more at Rs. 100 each. The distillery uses 1500 barrels a year at a constant rate. Carrying cost are 40 % and it costs the purchasing agent Rs.400 to place an order. Calculate the total annual cost for the order placed to the probable suppliers and find out the supplier to whom orders should be placed. **5**

iii. A company works 50 weeks in a year. For a certain part, included in the assembly of several parts, there is an annual demand of 10000 units. This part may be available from either an outside supplier or a subsidiary company. The following data relating to the part are given: **5**

| Description | From outside supplier (Rs.) | From subsidiary company (Rs.) |
|---|---|---|
| Purchase Price/unit | 12 | 13 |
| Cost of placing an order | 10 | 10 |
| Cost of receiving an order | 20 | 15 |
| Storage and all carrying cost including capital cost/unit/year | 2 | 2 |

(a) What purchase quantity form which source would you recommend?
(b) What would be the minimum total cost?

******

[5]

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(a) What purchase quantity form which source would you recommend?
(b) What would be the minimum total cost?

******

Marking Scheme
MS5CO05 Business Mathematics and Statistics for Managers

Q.1 MCQ

i) —(b) Value of variable make sense & corresponds to real world problem
ii) —(a) The problem is infeasible
iii) —(c) Variable not in the B-column in the simplex table
iv) —(b) 2n-1
v) —(b) $6\sigma$
vi) —(c) Balking
vii) —(c) 5/9
viii) —(b) necessary to develop a cumulative probability distribution
ix) —(a) Increases
x) —(d) No changes.

Q.2(i) Maximize $z = 8x_1 + x_2$

subjected to $2x_1 + x_2 \le 6$

$3x_1 + x_2 \le 6$

$8x_1 + x_2 \le 8$

$x_1 + 6x_2 \le 8$

$x_1, x_2 \ge 0$

obtaining extreme points

(i) $2x_1 + x_2 \le 6$

$x_1 = 0 \quad x_2 = 6$

$x_1 = 3 \quad x_2 = 0$

(ii) $3x_1 + x_2 \le 6$

$x_1 = 0 \quad x_2 = 6$

$x_1 = 2 \quad x_2 = 0$

(iii) $8x_1 + x_2 \le 8$

$x_1 = 0 \quad x_2 = 8$

$x_1 = 1 \quad x_2 = 0$

(iv) $x_1 + 6x_2 \le 8$

$x_1 = 0 \quad x_2 = 8/6$

$x_1 = 8 \quad x_2 = 0$

$$Z_{(0)} = 8x_1 + x_2$$
$$= 0$$

$$Z_A = 8x_1 + x_2$$
$$= 8 \times 0 + 1.33$$
$$= 1.33$$

$$Z_B = 0.8 \times 8 + 1.2$$
$$= 6.4 + 1.2$$
$$= 7.6 \simeq 8$$

$$Z_C = 8x_1 + x_2$$
$$= 8 \times 1 + 0$$
$$= 8$$

As point B and C gives the same maximum value $z = 8$. It follows that every point b/w B and C on the line BC also gives the same value of Z. The problem, therefore has multiple optimal solution.

~~Not~~ (*) Multiple optimal solution—

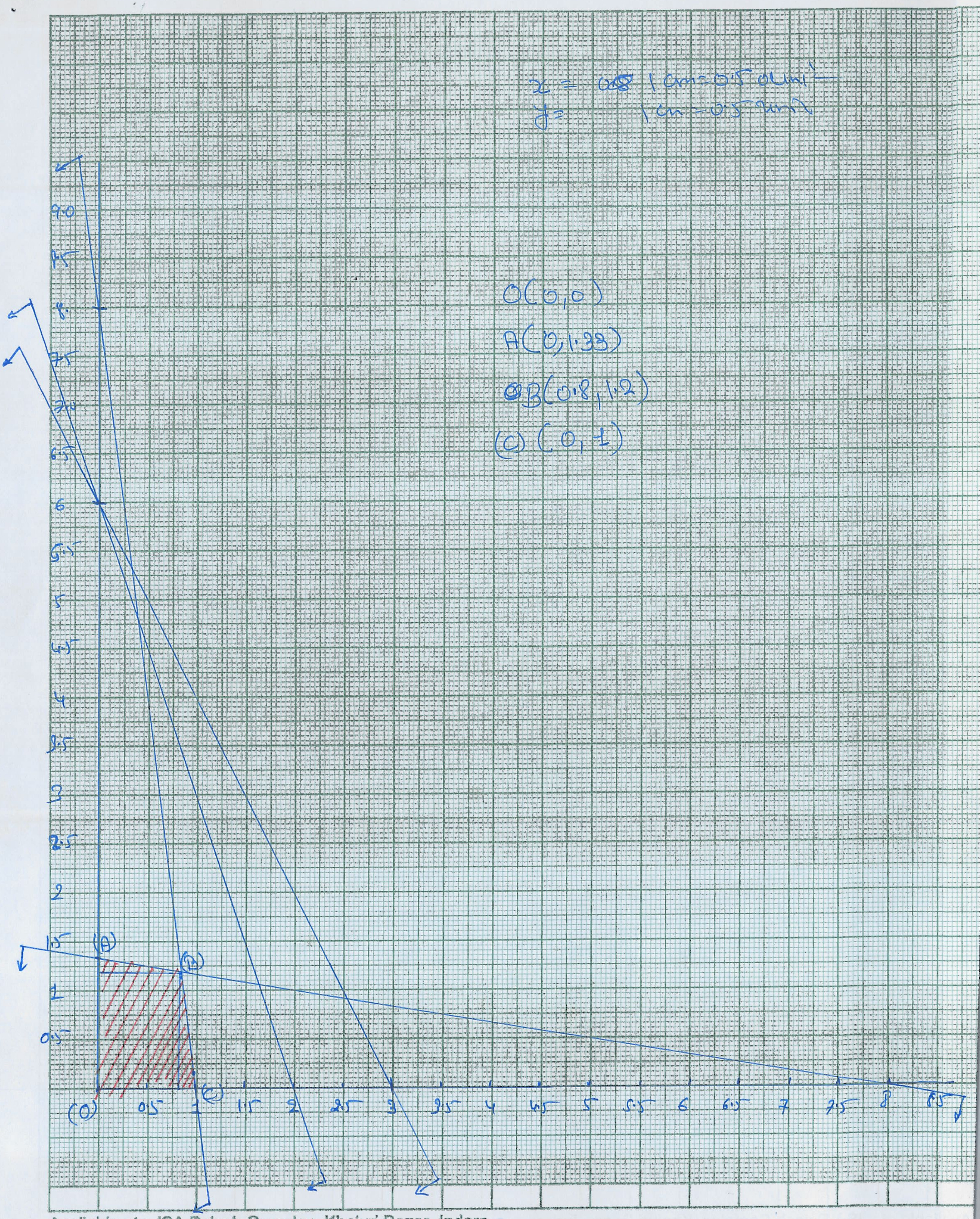
A L.P. Problem may have the same optimal value of the objective function at more than one extreme points.

(i) The objective function, when plotted, should be parallel to a constraint that forms the boundary of the feasible ~~reg~~ region.

Slope of objective function is same as binding constraint

$$Z = 8x_1 + x_2$$
subjected to $8x_1 + x_2 \leq 8$

(ii) Constraint should be an active constraint.


x = ~~08~~ 1 cm = 0.5 unit
y = 1 cm = 0.5 unit
O(0,0)
A(0,1.33)
B(0.8,1.2)
(C) (0, 1)
(A)
(B)
(C)
(O)

Q2 Use penalty method to solve

minimize $z = x_1 + 2x_2 + x_3$

subjected to $x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \leq 1$

$\frac{3}{2}x_1 + 2x_2 + x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

Sol:

Step 1: Set up the Problem in Standard form

minimize $z = x_1 + 2x_2 + x_3 + 0.S_1 + 0.S_2$

subjected to $x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + S_1 = 1$

$\frac{3}{2}x_1 + 2x_2 + x_3 - S_2 = 8$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

Step 2: Find an initial Basic feasible solution

Setting $x_1 = x_2 = x_3 = 0$, yields the initial solution

$S_1 = 1$ & $S_2 = -8$

which is not feasible since $S_2$ is negative.

Introducing artificial variable $A_1$, the problem takes form —

minimize $z = x_1 + 2x_2 + x_3 + 0.S_1 + 0.S_2 + MA_1$

Subjected to $x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + S_1 = 1$

$\frac{3}{2}x_1 + 2x_2 + x_3 - S_1 + A_1 = 8$

$x_1, x_2, x_3, S_1, S_2, A_1 \geq 0$

The initial Basic feasible solution, to problem is

$x_1 = x_2 = x_3 = 0$

$S_2 = 0$

then $S_1 = 1$ & $A_1 = 8$

& $z = 8M$

Following Table shows this solution

| | $C_j$ | 1 | 2 | 1 | 0 | 0 | M | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $A_1$ | b | θ |
| 0 | $S_1$ | 1 | 1/2 | 1/2 | 2 | 0 | 0 | 1 | 2 ⟸ |
| M | $A_1$ | 3/2 | 2 | 1 | 0 | -1 | 1 | 8 | 4 |
| | $Z_j$ | $\frac{3}{2}M$ | 2M | M | 0 | -M | M | 8M | |
| | $C_j - Z_j$ | $1-\frac{3}{2}M$ | 2-2M | 1-M | 0 | M | 0 | | |
| | | | ↑ | | | | | | |

**Step 3:** Perform optimality Test

Since $C_j - Z_j$ is negative under some column, above solution is not optimal

**Step 4:** Iterate Towards an optimal solution

$R_1 \to R_1 \times 2$

$R_2 \to R_2 - 2R_1$

| | $C_j$ | 1 | 2 | 1 | 0 | 0 | M | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | $A_1$ | b |
| 2 | $x_2$ | 2 | 1 | 1 | 2 | 0 | 0 | 2 |
| M | $A_1$ | -5/2 | 0 | -1 | -4 | -1 | 1 | 4 |
| | $Z_j$ | $4-\frac{5}{2}M$ | 2 | 2-M | 4-4M | -M | M | 4+4M |
| | $C_j - Z_j$ | $-3+\frac{5}{2}M$ | 0 | -1+M | -4+4M | M | 0 | |

Since $C_j - Z_j$ is non negative under all columns, above table is optimal. However, since $A_1$ appears in the basis at a positive value (4), the given problem has no feasible solution. Thus above table does not give an optimal solution but a pseudo-optimal solution.

Q.2(ii) Solve the following L.P. Problem –

$$\text{Maximize } Z = 3x_1 + 5x_2$$

Subjected to
$$x_1 - 2x_2 \le 6$$
$$x_1 \le 10$$
$$x_2 \ge 1$$
$$x_1, x_2 \ge 0$$

Step-1: Set up the problem in standard form

$$\text{Maximize } Z = 3x_1 + 5x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1$$

Subjected to
$$x_1 - 2x_2 + S_1 = 6$$
$$x_1 + S_2 = 10$$
$$x_2 - S_3 + A_1 = 1$$
$$x_1, x_2, x_3, S_1, S_2, S_3, A_1 \ge 0$$

The initial solution to this is shown in table

$x_1 = x_2 = x_3 = 0$, $S_3 = 0$ $\quad S_1 = 6, S_2 = 10, A_1 = 1$

| | $C_j$ | 3 | 5 | 0 | 0 | 0 | $-M$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | b | θ |
| 0 | $S_1$ | 1 | $-2$ | 1 | 0 | 0 | 0 | 6 | (−3) |
| 0 | $S_2$ | 1 | 0 | 0 | 1 | 0 | 0 | 10 | — |
| $-M$ | $A_1$ | 0 | 1 | 0 | 0 | $-1$ | 1 | 1 | 1 ← |
| | $Z_j$ | 0 | $-M$ | 0 | 0 | $M$ | $-M$ | $Z=-M$ | |
| | $C_j - Z_j$ | 3 | $5+M$ | 0 | 0 | $-M$ | 0 | | |

↑

Step 3 Perform optimility Test

Since $C_j - Z_j$ is +ve under some column above solution is not optimal

| | $C_j$ | 3 | 5 | 0 | 0 | 0 | −M | | |
|---|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | b | θ |
| 0 | $S_1$ | 1 | 0 | 1 | 0 | −2 | 2 | 8 | −ve |
| 0 | $S_2$ | 1 | 0 | 0 | 1 | 0 | 0 | 10 | ~~−ve~~ ∞ |
| 5 | $x_2$ | 0 | 1 | 0 | 0 | −1 | 1 | 1 | −ve |
| | $Z_j$ | 0 | 5 | 0 | 0 | −5 | 5 | Z=5 | |
| | $C_j - Z_j$ | 3 | 0 | 0 | 0 | 5 ↑ | −M−5 | | |

Since $C_j - Z_j = 5$ is the largest positive value, so variable $S_3$ should enter in to the basis. But, coefficient in the $S_3$ column are all negative or zero. This indicate that $S_3$ cannot be entered into the basis.

However the value of $S_3$ can be increased infinitely without removing ~~the~~ any one of the basic variables. Further since $S_3$ is associated with $x_2$ in the third constraint, $x_2$ will also be increased infinitely because it can be expressed as $x_2 = 1 + S_3 - A_1$. Hence the solution to given problem is unbounded.

Q. 3 (ii) A Company Produces — — — — — — — — given below

| | | Wholesalers | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 |
| Plant | A | 5 | 7 | 10 | 15 | 15 |
| | B | 8 | 6 | 9 | 12 | 14 |
| | C | 10 | 9 | 8 | 10 | 15 |

Step 1: First of all, to construct a profit matrix, in which

Profit/unit = Sales price/unit − Production cost/unit − ~~trans~~ transportation cost/unit

Thus, Profit/unit from ~~plant~~ wholesaler 1 to plant A
= Rs. (250 − 100 − 5) = 145, and so on

The resulting profit matrix is shown in table

| ~~Plant~~ | | Wholesalers | | | | | Production capacity |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | |
| Plant | A | 250 − 100 − 5 = 145 | 250 − 100 − 7 = 143 | 250 − 100 − 10 = 140 | 250 − 100 − 15 = 135 | 250 − 100 − 15 = 135 | 500 |
| | B | 250 − 90 − 8 = 152 | 250 − 90 − 6 = 154 | 250 − 90 − 9 = 151 | 250 − 90 − 12 = 148 | 250 − 90 − 14 = 146 | 100 |
| | C | 250 − 80 − 10 = 160 | 250 − 80 − 9 = 161 | 250 − 80 − 8 = 162 | 250 − 80 − 10 = 160 | 250 − 80 − 15 = 155 | 1250 |
| Demand | | 300 | 300 | 100 | 500 | 400 | ~~1600~~ 1850 / 1600 |

Total production capacity = 1850
Total demand = 1600
Surplus capacity = 250 units

Therefore, introducing a dummy wholesaler to take up the excess capacity of 250 units. The ~~profit~~ associate profit is zero in each cell

| | | Wholesaler | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | Production capacity |
| Plant | A | 145 | 143 | 140 | 135 | 135 | 0 | 520 |
| | B | 152 | 154 | 151 | 148 | 148 | 0 | 100 |
| | C | 160 | 161 | 182 | 160 | 155 | 0 | 1250 |
| Demand | | 300 | 300 | 100 | 500 | 400 | 250 | 1850/1850 |

Balance maximization problem

Step 2:

Convert maximization problem to minimization problem:

By subtracting all elements from max element

| | | Wholesaler | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | Capacity |
| Plant | A | 17 | 19 | 22 | 27 | 27 | 162 | 520 |
| | B | 10 | 8 | 11 | 14 | 16 | 162 | 100 |
| | C | 2 | 1 | 0 | 2 | 7 | 162 | 1250 |
| | | 300 | 300 | 100 | 500 | 400 | 250 | |

Balanced minimization problem

Step (3) find initial Basic feasible solution -

| | | Wholesalers | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | | | | |
| Plant | A | 17 | 19 | 22 | 27 | 27 | 162 | 520 | 2 | 2 | 2 |
| | B | 10 | 8 | 11 | 14 | 16 | 162 | 100 | 2 | 2 | 2 |
| | C | 2(250) | 1 | 0(100) | 2(500) | 7(400) | 162 | 1250 / 350 / 350 / 250 | 1 | 1 | 1 |
| | | 300 / 50 | 300 | 100 / 0 | 500 / 0 | 400 / 0 | 250 | | | | |
| | | 8 | 7 | 11 | 12 | 9 | 0 | | | | |
| | | 8 | 7 | 11 | | 9 | 0 | | | | |
| | | 8 | 7 | | | | | | | | |

Step 3: Find initial Basic feasible solution

Wholesaler

| | | 1 | 2 | 3 | 4 | 5 | 6 | Capacity | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | A | 17 (50) | 19 (200) | 22 | 27 | 27 | 162 (250) | 500 | 2 | 2 | 2 | 2 | 2 |
| Plant | B | 10 | 8 (100) | 11 | 14 | 16 | 162 | 100 | 2 | 2 | 2 | 2 | 2 |
| | C | 2 (250) | 1 | 0 (100) | 2 (500) | 7 (400) | 162 | 1250/750/650/250/0 | 1 | 1 | 1 | 1 | x |
| Demand | | 300/50/0 | 300/200/0 | 100/0 | 500/0 | 400/0 | 250/0 | | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| 8 | 7 | 11 | 12 ↑ | 9 | 0 |
| 8 | 7 | 11 ↑ | x | 9 | 0 |
| 8 | 7 | x | x | 9 ↑ | 0 |
| | | | | | 0 |
| 8 ↑ | 7 | | | | |
| 7 | 7 | | | | |

Re Step Perform optimility Test

No. of allocation = m+n-1

8 = 6+3-1

8 = 8

there fore optimility test can be performed

Cost Matrix

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ | $V_6$ |
|---|---|---|---|---|---|---|
| $U_1$ | 17 | 19 | | | | 162 |
| $U_2$ | | 8 | | | | |
| $U_3$ | 2 | | 0 | 2 | 7 | |

$U_1+V_1=17$

$U_1+V_2=19$

$U_1+V_6=162$

$U_2+V_2=8$

$U_3+V_1=2$

$U_3+V_3=0$

$U_3+V_4=2$

$U_3+V_5=7$

let $U_3=0$

then $U_1=15$

$U_2=4$

$V_1=2$ $\quad V_4=2$

$V_2=4$ $\quad V_5=7$

$V_3=0$ $\quad V_6=147$

| $u_i \backslash v_j$ | 2 | 4 | 0 | 2 | 7 | 147 |
|---|---|---|---|---|---|---|
| 15 | • | • | 15 | 17 | 12 | • |
| 4 | 6 | • | 4 | 6 | 11 | 157 |
| 0 | • | 4 | • | • | • | 147 |

Implicit cost matrix

| | | | | | |
|---|---|---|---|---|---|
| • | • | 7 | 10 | 5 | • |
| 4 | • | 7 | 8 | 5 | 11 |
| • | -3 | • | • | • | 15 |

There is negative value present so, above solution is not optimal.

<u>Step:</u> Iterate towards optimility

| | | | | | |
|---|---|---|---|---|---|
| + 50 | − 200 | | | | 250 |
| | 100 | | | | |
| − 250 | ✓ + | 100 | 500 | 400 | |

Make new allocations

| | | | | | |
|---|---|---|---|---|---|
| 250 | | | | | 250 |
| | 100 | | | | |
| 50 | 200 | 100 | 500 | 400 | |

Total cost $= 250 \times 145 + 100 \times 154 + 50 \times 160$
$+ 200 \times 161 + 100 \times 162 + 160 \times 520 + 400 \times 155$

$= 250,355$ /—

(259,300)

~~Iterate to~~

Perform optimality Test

Cost matrix

| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ |
|---|---|---|---|---|---|---|
| $u_1$ | 17 | | | | | 162 |
| $u_2$ | | 8 | | | | |
| $u_3$ | 2 | 1 | 0 | 2 | 7 | |

$u_1+v_1=17$
$u_1+v_6=162$
$u_2+v_2=8$

$u_3+v_1=2$
$u_3+v_2=1$
$u_3+v_3=0$
$u_3+v_4=2$
$u_3+v_5=7$

let $u_3=0$
$u_2=7$
$u_1=15$

$v_1=2$
$v_2=1$
$v_3=0$
$v_4=2$
$v_5=7$
$v_6=147$

| | 2 | 1 | 0 | 4 | 7 | 147 |
|---|---|---|---|---|---|---|
| 15 | • | 16 | 15 | 19 | 22 | • |
| 7 | 9 | • | 7 | 11 | 14 | 154 |
| 0 | • | • | • | • | • | 147 |

Implicit cost

Cell evaluation matrix

| | | | | | |
|---|---|---|---|---|---|
| • | 3 | 7 | 8 | 5 | • |
| 1 | • | 4 | 3 | 2 | 8 |
| • | • | • | • | • | 15 |

all the element are +ve the solution is optimal.

$x_{11}=250$ , $x_{22}=100$
$x_{31}=50$ $x_{32}=200$ $x_{33}=100$
$x_{34}=500$ $x_{35}=400$

$Z_{max}=250,355/-$
$(253,300/-)$

Q2(iii)

Worker

| Job | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | -2 | -4 | - | -6 | -1 |
| B | 0 | -9 | -5 | -5 | - |
| C | -3 | - | -9 | -2 | -6 |
| D | - | -3 | -1 | 0 | -3 |

Maximization problem →

Step 1 Convert in to minimization

Worker

| Job | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 2 | 4 | - | 6 | 1 |
| B | 0 | 9 | 5 | 5 | - |
| C | 3 | - | 9 | 2 | 6 |
| D | - | 3 | 1 | 0 | 3 |

Step 2 → Unbalance problem
(Balance the problem by adding dummy job E)

Worker

| Job | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 2 | 4 | M | 6 | 1 |
| B | 0 | 9 | 5 | 5 | M |
| C | 3 | M | 9 | 2 | 6 |
| D | M | 3 | 1 | 0 | 3 |
| E | 0 | 0 | 0 | 0 | 0 |

Step 3

(Now problem is of minimization solve by Hungarian method.

Worker

| Job | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 3 | M | 6 | $\boxed{0}$ |
| B | $\boxed{0}$ | 9 | 5 | 5 | M |
| C | 1 | M | 7 | $\boxed{0}$ | 4 |
| D | M | 3 | 1 | $\cancel{0}$ | 3 |
| E | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ | $\cancel{0}$ | $\cancel{0}$ |

No. of assignment are less than n=5, so is not a optimal solution, finding minimum number of lines crossing all zero

Worker

| Job | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 3 | M | 6 | $\boxed{0}$ |
| B | $\boxed{0}$ | 9 | 5 | 5 | M |
| C | 1 | M | 7 | $\boxed{0}$ | 4 ✓ |
| D | M | 3 | 1 | $\boxed{0}$ | 3 ✓ |
| E | $\cancel{0}$ | $\boxed{0}$ | $\cancel{0}$ | $\cancel{0}$ | $\cancel{0}$ |

lines crossing all zero = 4 < 5 = n

Step 4 - Iterate towards optimality

~~Worker I II III IV V~~

~~Job A 0 2 M 7 0~~

~~B~~

~~C~~

~~D~~

~~E~~

| Job | Worker I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 1 | 3 | M | 7 | $\boxed{0}$ |
| B | $\boxed{0}$ | 9 | 5 | 6 | M |
| C | ~~0~~ | M | 6 | $\boxed{0}$ | 3 |
| D | M | 2 | $\boxed{0}$ | ~~0~~ | 2 |
| E | ~~0~~ | $\boxed{0}$ | ~~0~~ | 1 | ~~0~~ |

No. of assingment 5 →

~~A-V~~ job A — worker V (A-V)
B~~-~~I job B — worker I (B-I)
job C — worker IV (C-IV)
job D — worker III (D-III)
& worker II will not get any job
He remains unalloted.

**Total Time:**

$= -1 + 0 + -2 + -1$

$= -4$

Q.4(ii)

Ans

Total annual cost = $\frac{1}{4}$ (total installation cost) + annual labour cost + annual cost of lost revenue due to down trucks.

Facility A :

Annual installation cost : $\frac{1}{4} \times 4,00,000$ = Rs. 1,00,000

Annual labour cost : $12 \times 80,000$ = ~~96,00~~ 9,60,000

Computation of cost of down time of trucks

$$L_s = \frac{\lambda}{\mu_1 - \lambda} = \frac{24}{60-24} = \frac{24}{36} = \frac{2}{3}$$

$\therefore$ Cost/year of lost revenue = $\frac{2}{3} \times 12 \times \cancel{300}^{100}$ = 2400/-

$\therefore$ Total annual cost = 1,00,000 + 9,60,000 + 2400

$T_A$ = 10,62,400/-

Facility B

Annual installation cost : $\frac{1}{4} \times 6,00,000$ = 1,50,000/-

Annual labour cost : $12 \times 60,000$ = 7,20,000/-

Computation of cost of down time of trucks

$$L_s = \frac{\lambda}{\mu_2 - \lambda} = \frac{24}{40-24} = \frac{24}{16} = 3/2$$

$\therefore$ Cost/year of lost revenue = $\frac{3}{\cancel{2}} \times \cancel{12}^{6} \times 300$ = 5400/-

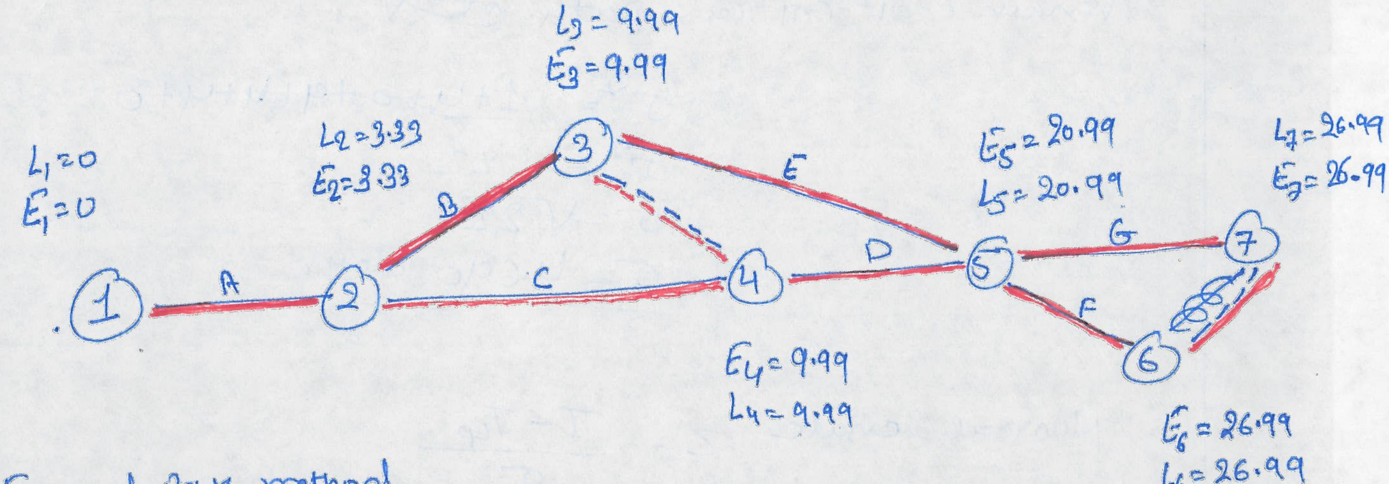
$\therefore$ Total annual cost = 1,50,000 + 7,20,000 + 5400

$T_B$ = 875400/-

As cost/year for facility B is lesser, the Company should Construct Facility B.

P. 4(iii)

| Activity | $t_o$ | $t_m$ | $t_p$ | $t_e = \frac{t_o + 4t_m + t_p}{6}$ | $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$ |
|---|---|---|---|---|---|
| A | 1 | 3 | 7 | 3.33 | 1 |
| B | 2 | 6 | 14 | 6.66 | 4 |
| C | 3 | 3 | 3 | 3 | 0 |
| D | 4 | 10 | 22 | 11 | 9 |
| E | 3 | 7 | 15 | 7.66 | 4 |
| F | 2 | 5 | 14 | 6 | 4 |
| G | 4 | 4 | 4 | 4 | 0 |




Forward pass method.

$E_1 = 0$

$E_2 = 3.33$

$E_3 = 3.33 + 6.66 = 9.99$

$E_4 = \text{Max of} \begin{cases} 3.33 + 6.66 + 0 = 9.99 \checkmark \\ 3.33 + 3 = 6.66 \end{cases}$

$E_5 = \text{Max of} \begin{cases} 9.99 + 7.66 = 17.65 \\ 9.99 + 11 = 20.99 \checkmark \end{cases}$

$E_6 = 20.99 + 6 = 26.99$

$E_7 = \text{max of} \begin{cases} 20.99 + 4 = 24.99 \\ 26.99 + 0 = 26.99 \checkmark \end{cases}$

Backward pass method

$L_7 = E_7 = 26.99$

$L_6 = 26.99$

$L_5 = \text{min of} \begin{cases} 26.99 - 4 = 22.99 \\ 26.99 - 6 = 20.99 \checkmark \end{cases}$

$L_4 = 20.99 - 11 = 9.99 \checkmark$

$L_3 = \text{min of} \begin{cases} 9.99 - 0 = 9.99 \checkmark \\ 20.99 - 7.66 \end{cases}$

$L_2 = \text{min of} \begin{cases} 9.99 - 3 = 6.99 \\ 9.99 - 6.66 = 3.33 \checkmark \end{cases}$

$L_1 = 3.33 - 3.33 = 0$

Critical Path

1-2-3-4-5-6-7 & Project completion time = 26.99 days

1-2-4-5-7

1-2-3-5-7

(b) What project duration will have 95% & 90% Confidence of Completion

variance of critical path $\sigma^2$

$\sigma^2 = 1+4+0+9+4+4+0$

$\sigma^2 = 22$

$\sigma = \sqrt{22}$

$\sigma = 4.690$ days

Normal deviate $Z = \dfrac{T - T_{cp}}{\sigma}$

From table (Hcam chapter)

For 90% $Z = 1.29$

$1.29 = \dfrac{T - 26.99}{4.69}$

$T = 33.04$ days

For 95% $Z = 1.65$

$1.65 = \dfrac{T - 26.99}{4.69}$

$T = 34.9161$ days

From table G JK from

For 90% = Z =

Q.5(i) Limitations :

(1) Decision tree diagrams becomes more complicated as the number of decision alternatives increases and more variables are introduced.

(2) It becomes highly complecated when interdependent alternatives and dependent variables ~~are~~ are present in the problem.

(3) It assumes that utility of money is linear with money.

(4) It analyses the problem in terms of expected values and thus yield an 'average' valued solution.

(5) There is often inconsistency in assigning probabilities for different events

———— x ———— x ————

Q.5(ii)

| | | Player B | | | |
|---|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
| Player A | $A_1$ | -1 | 7 | 6 | 4 |
| | $A_2$ | 5 | -3 | 3 | 6 |
| | $A_3$ | -2 | 4 | 2 | 3 |

Step 1: Find saddle point

| | | Player B | | | | |
|---|---|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ | $B_4$ | min of row |
| Player A | $A_1$ | -1 | 7 | 6 | 4 | (-1) maximin |
| | $A_2$ | 5 | -3 | 3 | 6 | -3 |
| | $A_3$ | -2 | 4 | 2 | 3 | -2 |
| max of colum | | 5 minimax | 7 | 6 | 6 | |

* No saddle point

Rule 2: Applying Dominance

1) Column 4 is dominated by Column 1
2) Row 3 is dominated by Row 1

Thus, eliminating the Column 4 & Row 3, the Reduced game matrix is

Player B

| Player A | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|
| $A_1$ | -1 | 7 | 6 |
| $A_2$ | 5 | -3 | 3 |

Step: 7 Find saddle point

* No saddle point, therefore solving game by methods of subgame, graphical method etc, Algebraic method

* Method of sub game:

Subgame 1

Player B

| Player A | $B_1$ | $B_2$ |
|---|---|---|
| $A_1$ | -1 | 7 |
| $A_2$ | 5 | -3 |

Subgame 2

Player B

| Player A | $B_1$ | $B_3$ |
|---|---|---|
| $A_1$ | -1 | 6 |
| $A_2$ | 5 | 3 |

Subgame 3

Player B

| Player A | $B_2$ | $B_3$ |
|---|---|---|
| $A_1$ | 7 | 6 |
| $A_2$ | -3 | 3 |

Find saddle point:

Subgame – I

Player B

| Player A | $B_1$ | $B_2$ | | |
|---|---|---|---|---|
| $A_1$ | -1 | 7 | 8 | 8/14 |
| $A_2$ | 5 | -3 | 6 | 6/14 |
| | 10 | 4 | | |
| | $\frac{10}{14}$ | $\frac{4}{14}$ | | |

Subgame II

Player B

| Player A | $B_1$ | $B_3$ | | |
|---|---|---|---|---|
| $A_1$ | -1 | 6 | 2 | 2/7 |
| $A_2$ | 5 | 3 | 5 | 5/7 |
| | 3 | 4 | | |
| | $\frac{3}{7}$ | $\frac{4}{7}$ | | |

Sub game - 3

|  |  | Player B |  |  |
|---|---|---|---|---|
|  |  | $B_1$ | $B_2$ | Row mi |
| Player A | $A_1$ | 7 | 6 | (6) |
|  | $A_2$ | -3 | 3 | 3 |
|  |  | 7 | (6) |  |

For sub game - I

Strategy for A $\left(\frac{8}{14}, \frac{6}{14}\right)$

Strategy for B $\left(\frac{10}{14}, \frac{4}{14}, 0\right)$

game value $= -1 \times \frac{8}{14} + 5 \times \frac{6}{14} = \frac{-8}{14} + \frac{30}{14} = \frac{22}{14} = 1.57$

For sub game - II

Strategy for A $= \left(\frac{2}{7}, \frac{5}{7}\right)$

Strategy for B $= \left(\frac{3}{7}, 0, \frac{4}{7}\right)$

value of game $= -1 \times \frac{3}{7} + 6 \times \frac{4}{7} = \frac{-3}{7} + \frac{24}{7} = \frac{21}{7} = 3$

Sub game - III

Strategy for A : $(1, 0)$

Strategy for B $= (0, 1, 0)$

This game has saddle point $(1, 2)$

& value of game is 6

Ans Now, since player B has flexibility to play any two out of these course of action available to it, it will play those strategies for which loss occurring is minimum. As the values for all subgames are +ve A is winner. Hence player B will play sub game 1 for which loss is minimum 1.57

Hence the complete solution —

strategies $A\left(\frac{8}{14}, \frac{6}{14}\right)$

strategies $B\left(\frac{10}{14}, \frac{4}{14}, 0\right)$

game value $v = \frac{22}{14} = 1.57$

---

Q.5(iii) Sol. →

(i) The simulated demand for the cakes for the next 10 days can be obtained from the table below -

(*) According to given distribution of demand, the random number coding for various demand levels _

| Demand | Probability | Cumulative Probability | Random No. interval | Random No. fitted |
|---|---|---|---|---|
| 0 | 0.01 | 0.01 | 00 | |
| 15 | 0.15 | 0.16 | 01 – 15 | 09(3), 15(7), 14(8), 09(10) |
| 25 | 0.20 | 0.36 | 16 – 35 | |
| 35 | 0.50 | 0.86 | 36 – 85 | 48(1), 78(2), 57(4), 56(5), 77(6), 68(9), |
| 45 | 0.12 | 0.98 | 86 – 97 | |
| 50 | 0.02 | 1.00 | 98 – 99 | |

In order to simulate the demand, the Number 00 is assigned to zero demand -- number 1–15 are assigned to demand of 15 cakes -- & so on

Determination of Demand & Stock levels

~~Day~~ ~~Demand~~ ~~No. of Cakes made~~

| Day | Random No. | Demand | No. of cakes made | Stock |
|---|---|---|---|---|
| 1 | 48 | 35 | 35 | — |
| 2 | 78 | 35 | 35 | — |
| 3 | 09 | 15 | 35 | 20 |
| 4 | 57 | 35 | 35 | 20 |
| 5 | 56 | 35 | 35 | 20 |
| 6 | 77 | 35 | 35 | 20 |
| 7 | 15 | 15 | 35 | 40 |
| 8 | 14 | 15 | 35 | 60 |
| 9 | 68 | 35 | 35 | 60 |
| 10 | 09 | 15 | 35 | 80 |

∴ Average daily demand $= \frac{270}{10} = 27$ units per day

Q. 6(i) ~~S.P.~~

No. of unit sold = 50/day

Production = 250/day

Set up cost = 1000/-

Storage cost = 0.0015/ piece/ day

Labour charge = 3.20/piece

Material cost = 2.10/piece

Overhead cost = 4.10/piece

interest = 8 %

Working day = 300

Total No. of unit sold = 50 × 300 = 15000/year

Production/year = 250×300 = 75,000 pieces/year

$C_0$ = 1000/- setup

$C_h = [0.0015 \times 300 + 0.08(3.20 + 2.10 + 4.10)$

$= 12.02$/year

$$EBQ = \sqrt{\frac{P}{P-D}} \sqrt{\frac{2DC_0}{C_h}}$$

$$= \sqrt{\frac{75000}{75000-15000}} \cdot \sqrt{\frac{2 \times 1000 \times 15000}{12.02}}$$

EBQ = 5,586 pieces

$$\text{no of orders} = \frac{R}{q_0} = \frac{15000}{5,586} \simeq 3 \text{ cycles/year}$$

Q. 6 (ii) annual demand $R = 1500$ barrels

inventory rate $I = 40\% = 0.4$

Ordering cost = 400/order

The cost/unit is shown in table below —

| Supplier | Quantity of barrels | Cost/unit |
|---|---|---|
| First | Any quantity | Rs. 150 |
| Second | 150 and above | Rs. 125 |
| Third | 250 and above | Rs 100 |

Total annual cost —

First supplier —

$$q_0 = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2\times1500\times400}{150\times0.4}} = 141.4 \text{ barrels (feasible)}$$

$$\text{Total annual cost} = CR + \sqrt{2DC_0C_h}$$

$$= 150\times1500 + \sqrt{2\times1500\times400\times150\times0.4}$$

$$= 2,25,000 + 8484 = 2,33,484$$

Second supplier

$$q_0 = \sqrt{\frac{2\times1500\times400}{125\times0.4}} = 154.92 \text{ barrel feasible}$$

$$\text{Total annual cost} = CR + \sqrt{2DC_0C_h}$$

$$= 125\times1500 + \sqrt{2\times1500\times400\times125\times0.4}$$

$$= 187,500 + 7746 = 1,95,246$$

Third supplier

$$q_0 = \sqrt{\frac{2\times1500\times400}{100\times0.4}} = 173.2 \text{ barrel}$$

Not feasible since min qty is 250 barrel.

∴ minimum batch size for which order can be placed = 250 barrels.

Total annual cost = $\cancel{CR + \sqrt{2DC_0C_h}}$

$= \cancel{100\times1500 + \sqrt{2\times1500\times400\times100\times0.4}}$

$= CR + \frac{q}{2}C\cdot I + C_0 \times \frac{D}{q}$

$$= 100 \times 1500 + \frac{250}{2} \times 100 \times 0.4 + 400 \times \frac{1500}{250}$$

$$= 1,50,000 + 5000 + 2400 = 1,57,400$$

Thus the order for 250 oak barrels each time should be placed with the third supplier as it involves lowest annual cost

Q. 5 (iii)

(a)

(a) Given Cost/unit = 12

Ordering cost = 10+20 = 30

Carrying cost = $I \times C$ = 2

For outside Company

$$EOQ_{outside} = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 10000 \times 30}{2}} = 547.72 \text{ units}$$

Total Cost = Material cost + ordering cost + carrying cost

$$= C \times D + C_0 \times \frac{D}{Q} + C_h \times \frac{Q^*}{2}$$

$$= 10000 \times 12 + 30 \times \frac{10000}{548} + 2 \times \frac{548}{2} = \underline{1,21,095.45}$$

For subsidiary company

$C = 13$, $C_0 = 25$ $D = 10000$ $C_h = 2$

$$EOQ = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 10000 \times 25}{2}} = 500 \text{ units}$$

$$TC = 13 \times 10000 + 25 \times \frac{10000}{500} + 2 \times \frac{500}{2}$$

$$TC = 1,31,000.$$

548 units purchased from the outside company the total cost is Rs. 1,21,095.45.