			Marks	BL	PO	CO	PSO
Q.4	i.	Write the statement of pigeonhole principle.	**3**	01	01, 02	01	01
	ii.	Solve the recurrence relation- $$a_r - 5a_{r-1} + 6a_{r-2} = 2 + r, r \geq 2$$ with boundary conditions $a_0 = 1, a_1 = 1$.	**7**	03	01, 02	02	01
OR	iii.	Using mathematical induction prove that: $11^{n+2} + 12^{n+1}$ is divisible by 133, $n \in N$	**7**	03	01, 02	02	01
Q.5	i.	Define Eulerian path and Hamiltonian path.	**4**	01	01, 02	01	01
	ii.	Prove that: 'The sum of the degree of all vertices in a simple graph is equal to twice the number of edges.'	**6**	02	01, 02	01	01
OR	iii.	State and prove Euler's formula.	**6**	02	01, 02	01	01
Q.6		Attempt any two:					
	i.	Prove that $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ is a tautology.	**5**	05	01, 02, 03	04	01
	ii.	Express the following formula into conjunctive normal form: $$\sim(p \vee q) \Leftrightarrow (p \wedge q)$$	**5**	05	01, 02, 03	04	01
	iii.	Define the following terms: (a) Compactness (b) Resolution (c) Soundness (d) Completeness (e) Validity	**5**	01	01, 02	01	01

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering
End Sem Examination Dec 2024
EN3BS06 Discrete Mathematics

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

			Marks	BL	PO	CO	PSO
Q.1	i.	A relation from set A to set B is- (a) A relation that maps elements of A to elements of B (b) A subset of the cartesian product A×B (c) A rule that defines multiplication on A (d) None of these	**1**	01	01, 02	01	01
	ii.	In a partially ordered set, which of the following is true about maximal elements? (a) They are greater than all the other elements in the set (b) They are smaller than all other elements (c) They have no element greater than them in the set (d) None of these	**1**	02	01, 02	01	01
	iii.	In a ring $(R, +, .)$ which of the following properties must hold- (a) Addition in R is not associative (b) R must have multiplicative inverse of each element (c) R must be closed under both addition and multiplication (d) None of these	**1**	01	01, 02	01	01

iv.	A lattice is complemented if- (a) Every element has a complement with respect to the lattice identity (b) Every element has an inverse (c) The lattice has only one element (d) None of these	**1**	01	01, 02	01	01
v.	How many ways can 6 different books be arranged on a shelf? (a) 36 (b) 120 (c) 720 (d) 5040	**1**	03	01, 02,	02	01
vi.	which of the following proof technique involves assuming the negation of the statement and deriving a contradiction? (a) Direct proof (b) Proof by contradiction (c) Proof by induction (d) None of these	**1**	02	01, 02	01	01
vii.	A tree with n vertices has- (a) $n-1$ edges (b) n edges (c) $n+1$ edges (d) None of these	**1**	01	01, 02	01	01
viii.	The independence number of a graph is: (a) The maximum number of edges that can be removed (b) The maximum degree of any vertex (c) The largest set of vertices between no edges between them (d) None of these	**1**	01	01, 02	01	01
ix.	Which of the following is not a standard logical connective? (a) Negation (b) Inclusion (c) Disjunction (d) Conjunction	**1**	02	01, 02	01	01
x.	The natural deduction system is primarily based on- (a) Axioms (b) Truth tables (c) Formal rules of inference (d) None of these	**1**	01	01, 02	01	01

Q.2	i.	Let R be a binary relation on the set of all positive integers, such that- $R = \{(a,b): a-b \textit{ is an odd positive integer}\}$ is R reflexive? Symmetric? Transitive?	**4**	03	01, 02	02	01
	ii.	A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games, 50 do not watch any of the three games. (a) How many people in the survey watch all the three games? (b) How many people watch exactly one of the three games?	**6**	03	01, 02	02	01
OR	iii.	Define composition mapping. And Let function f and g defined on set of real numbers and defined by $f(x) = 2x+1$, and $g(x) = x^2 - 2$ respectively. Find: (a) $(gof)(4)$ (b) $(fog)(4)$ (c) $(gof)(a+2)$ (d) $(fog)(a+2)$	**6**	03	01, 02	02	01
Q.3	i.	Show that the set $G = \{1, \omega, \omega^2\}$ is a group with respect to multiplication, ω being an imaginary cube root of unit.	**4**	04	01, 02, 03	03	01
	ii.	Let N be the set of positive integers. Let the meaning of $x \leq y$ in N be x divides y. Show that N is a lattice where the meet ($\wedge$) and join ($\vee$) are respectively defined by- $x \wedge y = H.C.F.(x,y)$ $and\ x \vee y = L.C.M.(x,y)$	**6**	03	01, 02	02	01
OR	iii.	Simplify the given Boolean expression using K-map- $f(A,B,C,D) = \bar{A}\bar{B}CD + \bar{A}BC\bar{D} + ABC\bar{D} + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}.$	**6**	03	01, 02	02	01

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Q1(i)(b) A subset of the Cartesian Product $A \times B$.
(ii) (c) They have no element greater than them in the set.
(iii) (c) R must be closed under both addition and multiplication.
(iv) (a) Every element has a Complement with Respect to the lattice identity.
(v) (c) 720
(vi) (b) Proof By Contradiction
(vii) (a) n-1 edges.
(viii) (c) The Largest set of Vertices between no edges between them.
(ix) (b) Inclusion
(x) (c) Formal rules of inference.

Q2. (i) Let I^+ denote the set of all positive integers, and $R = \{(a,b) : a-b$ is an odd positive integer$\}$

Reflexive: Let $a \in I^+$, then $a-a=0$, which is an even integer.
$\therefore (a,a) \notin R$
Hence, R is not Reflexive.

Symmetric: Let $(a,b) \in R$, where $a,b \in I^+$
Now, $(a,b) \in R \Rightarrow a-b$ is an odd positive integer
$\Rightarrow -(a-b)$ is an odd Negative integer
$\Rightarrow (b-a)$ is an odd negative integer.
$\Rightarrow (b,a) \notin R$
Hence, R is not symmetric.

Transitive: let $a, b, c \in I^+$
if $a-b$ and $b-c$ are both odd positive integers
then $(a-b)+(b-c)$ is an even positive integer
i.e., $a-c$ is an even positive integer.
$\therefore (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \notin R$

Hence, R is not transitive Relation.

(ii) Let ~~denote~~ H, B & F denote the set of all people who watch Hockey game, Basketball game and Football game. and X denote all the people. then, we have
$|X| = 500$, $|H| = 195$, $|B| = 115$, $|F| = 285$,
$|F \cap B| = 45$, $|F \cap H| = 70$, $|H \cap B| = 50$,
$|(F \cup B \cup H)'| = 50$.
$\therefore |F \cup B \cup H| = |X| - |(F \cup B \cup H)'| = 500 - 50 = 450$

(a) By the principle of inclusion & Exclusion,

$$|F \cap B \cap H| = |F \cup B \cup H| - |F| - |B| - |H| + |F \cap B| + |F \cap H| + |B \cap H|$$
$$= 450 - 285 - 115 - 195 + 45 + 70 + 50$$
$$= 20$$

(b) Let H_1, B_1 & F_1 be the set of all peoples who watches only Hockey, Basketball and football respectively. then,

$H_1 = H - B - F$, $B_1 = B - F - H$, $F_1 = F - B - H$

$$\therefore |H_1| = |H - B - F| = |H| - |H \cap B| - |H \cap F| + |H \cap B \cap F|$$
$$= 195 - 50 - 70 + 20$$
$$= 95$$

$|B_1| = |B-F-H| = |B| - |B\cap F| - |B\cap H| + |B\cap H\cap F|$

$= 115 - 45 - 50 + 20$

$= \cancel{70}\ 40$

$|F_1| = |F-B-H| = |F| - |F\cap B| - |F\cap H| + |F\cap B\cap H|$

$= 285 - 45 - 70 + 20$

$= 190.$

Hence, the no. of people who watches exactly one of the three games $= |H_1| + |B_1| + |F_1|$

$= 95 + 40 + 190 = 325.$

(iii) Composition Mapping :- Let A, B, C be three non-empty sets. Consider two mappings $f: A \to B$ and $g: B \to C$, The composition of mappings f and g is the composite mapping denoted by $g\circ f$ and is a function $A \to C$ defined by.

$g\circ f : A \to C$ where $(g\circ f)(x) = g\{f(x)\}, \forall x \in A.$

Now, $f(x) = 2x+1$, $g(x) = x^2 - 2$.

$(g\circ f)(x) = g(f(x)) = g(2x+1) = (2x+1)^2 - 2$

$= 4x^2 + 1 + 4x - 2$

$= 4x^2 + 4x - 1$

$(f\circ g)(x) = f(g(x)) = f(x^2-2) = 2(x^2-2) + 1$

$= 2x^2 - 4 + 1$

$= 2x^2 - 3$

(a) $g\circ f)(4) = 4(4)^2 + 4(4) \cancel{+3} - 1$
$= 79$

(b) $(f\circ g)(4) = 2(4)^2 - 3 = 29$

(c) $(g\circ f)\ (a+2) = 4(a+2)^2 + 4(a+2) - 1$
$= 4(a^2 + 2a + 4) + 4a + 8 - 1$
$= 4a^2 + 8a + 16 + 4a + 8 - 1$
$= 4a^2 + 12a + 23$

(d) $(f\circ g)\ (a+2) = 2(a+2)^2 - 3 = 2(a^2 + 2a + 4) - 3$
$= 2a^2 + 4a + 8 - 3$
$= 2a^2 + 4a + 5$

P3(i) Composition table.

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

closure property: Since all the entries in the composition table are the elements of the set G, thus G is closed with respect to multiplication.

Associativity: Since the elements of G are all complex numbers and we know that the multiplication of complex numbers is associative because
$(1\cdot\omega)\cdot\omega^2 = 1\cdot(\omega\cdot\omega^2)$

Existence of identity: from the composition table we see that

$$1(1) = 1, \quad 1(\omega) = \omega = \omega(1), \quad 1(\omega^2) = \omega^2 = \omega^2(1)$$

$\therefore$ 1 is the identity element.

Existence of inverse:- From the composition ~~to~~ table, we find that $1.1 = 1$.(identity)

$\therefore$ the inverse of the element $1 \in G$ is 1 itself.

$$\omega \cdot \omega^2 = \omega^2 \cdot \omega = 1$$

$$\therefore \omega^{-1} = \omega^2 \in G$$

and $(\omega^2)^{-1} = \omega \in G$

Thus, the inverse of every element of G is in G.

Therefore, G is an group with respect to multiplication.

(ii) Let N = set of positive integers. Then

Reflexivity: Let $x \in N$ be arbitrary. then x divides x.

i.e, $x \leq x \quad \forall x \in N$.

Hence the relation $\leq$ is reflexive on N.

Anti-symmetry: let $x, y \in N$ be arbitrary and let $x \leq y$, $y \leq x$. then

$x \leq y, y \leq x \Rightarrow$ x divides y and y divides x.

$\Rightarrow \exists\ n_1, n_2$ (positive integers) such that

$$x n_1 = y, \quad y n_2 = x.$$

$$\Rightarrow y n_2 n_1 = y$$

$$\Rightarrow n_2 n_1 = 1$$

$$\Rightarrow n_1 = 1, n_2 = 1$$

$$\Rightarrow x = y.$$

Hence, the ~~rel~~ relation $\leq$ is anti-symmetric on N.

Transitivity: Let $x, y, z \in N$ be arbitrary, and let $x \leq y, y \leq z$. Then,

$x \leq y, y \leq z \Rightarrow x$ divides y and y divides z

$\Rightarrow \exists\, n_1', n_2'$ (positive integers) such that $xn_1' = y$, $yn_2' = z$

$\Rightarrow xn_1'n_2' = z$

$\Rightarrow \exists\, n$ (positive integer) such that $xn = z$, where $n = n_1' \cdot n_2'$

$\Rightarrow x$ divides z

$\Rightarrow x \leq z$

Hence, the relation $\leq$ is transitive on N.

Since, the relation $\leq$ is reflexive, anti-symmetric and transitive on N and so $(N, \leq)$ is a partially ordered set.

In N, meet and join, which are denoted by $\wedge$ and $\vee$, are respectively defined by

$$x \wedge y = H.C.F(x, y)$$
$$x \vee y = L.C.M(x, y)$$

Clearly, every pair of elements x, y in N has their H.C.F and L.C.M in N itself. i.e, for each pair of elements $x, y \in N$,

$x \wedge y = H.C.F(x, y)$ and $x \vee y = L.C.M(x, y)$

exits in N.

Hence, N is a lattice.

(iii) $f(A,B,C,D) = \bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}\bar{D}$
$+ \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$

AB \ CD	00	01	11	10
00		1	1	
01	1			
11	1	1	1	
10				1

$\therefore$ The Simplified Boolean expression is:

$$f(A,B,C,D) = \bar{A}\bar{B}D + B\bar{C}\bar{D} + AB\bar{D} + A\bar{B}C\bar{D}$$

Q4(i) Pigeonhole Principle :- If the number of pigeons is more than the number of pigeonholes, then some pigeonhole must be occupied by two or more than two pigeons.

(ii) The given equation is

$$a_r - 5a_{r-1} + 6a_{r-2} = 2 + r \quad \text{——} \quad (1)$$

The ch.E is $m^2 - 5m + 6 = 0 \Rightarrow m = 2, 3$

$$a_r^{(h)} = C_1 \cdot 2^r + C_2 \cdot 3^r \quad \text{——} \quad (2)$$

The particular solution (trial solution) corresponding to the term $2+r$ on R.H.S of (1) is $A_0 + A_1 r$

$$a_r^{(P)} = A_0 + A_1 r \quad \text{—— (3)}$$

Substituting (3) in (1), we get

$$(A_0 + A_1 r) - 5\{A_0 + A_1(r-1)\} + 6\{A_0 + A_1(r-2)\} = 2 + r$$

$$\Rightarrow A_0 = 11/4, \; A_1 = 1/2$$

Putting for A_0 and A_1 in (3), we get

$$a_r^{(P)} = \frac{11}{4} + \frac{1}{2} r$$

$\therefore$ Total solution of (1) is given by

$$a_r = a_r^{(h)} + a_r^{(P)}$$

$$a_r = c_1 . 2^r + c_2 . 3^r + \frac{11}{4} + \frac{1}{2} r . \quad \text{—— (5)}$$

Using Boundary Conditions in (5), we get

$$c_1 = -3, \; c_2 = 5/4$$

$\therefore$ The required solution is

$$a_r = -3.2^r + 5/4 . 3^r + 11/4 + 1/2 \, r$$

(iii) $p(n) = 11^{n+2} + 12^{n+1}$

for $n = 1$.

$$p(1) = 11^{1+2} + 12^{1+1}$$
$$= 11^3 + 12^2$$
$$= 1331 + 144$$
$$= 1475$$

which is not divisible by 133.

$\therefore$ the statement can't be proved.

Q5(i) Eulerian Path :- Eulerian Path in graph $G=(V,E)$ is defined as a path which traverses each edge in the graph G once and only once.

Hamiltonian path :- A path that passes through each of the verteices in a graph G exactly once, is called a Hamiltonian path.

(ii) Let $G=(V,E)$ be a graph and let number of edges in G be e i.e, Order of $E = e$. then we are to prove that $\sum_{v \in V} deg(v) = 2e$ where $v \in V$ is any vertex

we shall prove the theorem by induction on the number of edges in the following steps;

Step 1: If number of edges in G is zero. then, in this Case degree of each Vertex $v \in V$ is zero.

$$\sum_{v \in V} deg(v) = 0$$

$$\therefore \sum_{v \in V} deg(v) = 2e = 2 \times 0 = 0$$

$\therefore$ the theorem is true in this case.

Step 2: if $e=1$. i.e., if there is only one edge in G. in this case the graph G has only two vertices & the degree of each vertex is one

$$\sum_{v \in V} deg(v) = 1+1 = 2 = 2 \times 1 = 2e$$

i.e., $\sum_{v \in V} deg(v) = 2e.$

$\therefore$ the theorem is true in this case.

Step 3:- Now assume that the theorem is true for all graphs having $e-1$ edges.
Let G be the graph having e edges. Delete one edge, say $e' = (a, b)$ from G. Thus a new graph G', say, is obtained having $e-1$ edges where $G' = G - \{e'\}$.
$\therefore$ by hypothesis, we have in G'

$$\sum deg(v) = 2(e-1) \quad \text{——— } (1)$$

Now, if we replace the edge $e' = (a, b)$ to obtain the graph G, then the degree of each of vertices a & b will be increased by one.
$\therefore$ adding the edge $e' = (a, b)$ to G' to obtain G.
we have in G,

$$\sum_{v \in V} deg(v) = 2(e-1) + 2 \qquad (\text{using } (1))$$
$$= 2e$$

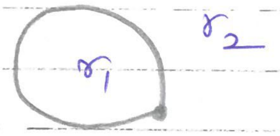
Hence, the theorem is proved.

(iii) Euler's formula :- A connected planar graph with n vertices and e edges has r regions given by $r = e - n + 2$.
We shall prove the theorem by induction on the number of edges e of G, where G is a connected planar graph.

Suppose $e=1$, then n may be equal to 1 or 2.
In case $e=1, n=2$. the number of regions $r = 1-2+2 = 1$
clearly graph (a) has one region.

$e=1, n=2$

(a)



$e=1, n=2$.

(b)

Again in case $e=1, n=1$, then $r = 1-1+2 = 2$. The graph (b) has two regions r_1 and r_2.
Hence, the result is true for $n=1$.

Now, suppose that the result holds for all graphs with atmost $e-1$ edges.
Assume that G is a connected graph with e edges and r- regions. in case G is a tree than $e=n-1$ & no. of regions is 1.
In this case by the formula, we have

$$r = e-n+2 \quad = (n-1)-n+2 = 1$$

Hence the theorem holds in case G is a tree. Now consider the case when G is not a tree, then G has some circuits. Consider an edge 'e' in some circuit. By removing the edge 'c' from the plane representation of G, the regions are merged into a new region. therefore $G-\{c\}$ is a connected graph with n vertices, $e-1$ edges and $r-1$ regions. thus By induction Hypothesis we have

$$r-1 = (e-1)-n+2$$

$$\Rightarrow r = e-n+2$$

Q6(i) Let $p \Leftrightarrow r = A$ & $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) = B$

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	A	B	$B \Rightarrow A$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	F	T	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	F	T	F	T
F	F	T	T	F	F	F	T
F	F	F	T	T	T	T	T

$\therefore$ it is a Tautology.

(ii) Let $P = \sim(p \vee q)$ & $Q = (p \wedge q)$

$\therefore \sim(p \vee q) \Leftrightarrow (p \wedge q)$

$\Leftrightarrow (P \Leftrightarrow Q)$

$\Leftrightarrow (P \wedge Q) \vee (\sim P \wedge \sim Q)$

$\Leftrightarrow (\sim(p \vee q) \wedge (p \wedge q)) \vee ((p \vee q) \wedge \sim(p \wedge q))$

$\Leftrightarrow (\sim p \wedge \sim q \wedge p \wedge q) \vee ((p \vee q) \wedge (\sim p \vee \sim q))$

$\Leftrightarrow (\sim p \wedge \sim q \wedge p \wedge q) \vee ((p \vee q) \wedge \sim p) \vee ((p \vee q) \wedge \sim q)$

$\Leftrightarrow (\sim p \wedge \sim q \wedge p \wedge q) \vee (p \wedge \sim p) \vee (q \wedge \sim p) \vee (p \wedge \sim q) \vee (q \wedge \sim q)$

~~which~~

(ii) Let $P = \sim(p \vee q)$ and $Q = (p \wedge q)$
we know that

$P \Leftrightarrow Q$

$$\Leftrightarrow (P \Rightarrow Q) \wedge (Q \Rightarrow P) \quad \text{--- } (1)$$

putting for P and Q in (1), we get

$$\sim(p \vee q) \Leftrightarrow (p \wedge q)$$

$$\Leftrightarrow [\sim(p \vee q) \Rightarrow (p \wedge q)] \wedge [(p \wedge q) \Rightarrow \sim(p \vee q)]$$

$$\Leftrightarrow [(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)]$$

$$(\because x \Rightarrow y \Leftrightarrow (\sim x \vee y))$$

$$\Leftrightarrow [(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$$

$$\Leftrightarrow (p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$$

(By distributive laws).

which is the required Conjunctive normal form.

(iii) (a) Compactness :- The compactness theorem states that if every finite subset of a set of logical formulas is satisfiable, then the entire set is also satisfiable.

(b) Resolution :- Resolution in logic is a rule of inference used to determine the satisfiability of a set of logical statements. It works by combining pairs of clauses to eliminate a literal, eventually deriving a conclusion or a contradiction.

(c) Soundness :- Soundness ensures that if a statement can be proven within a logical system, it is also true in all interpretations.

(d) Completeness :- Completeness ensures that if a statement is true in all models, it can also be proven within the logical system.

(e) Validity:- A logical formula is said to be valid if it is true in every possible interpretation. this means that no matter how you assign truth values to it's components. the formula will always evaluate to true.