Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2019

BC3CO11 Mathematics –III

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if **1**

(a) $P>1$ (b) $P\leq 1$ (c) $P\geq 1$ (d) None of these.

ii. In a positive series $\sum u_n$, if $\lim_{n\to\infty}(u_n)^{1/n}=\lambda$ then for $\lambda>1$ the series is **1**

(a) Convergent (b) Divergent

(c) Oscillatory (d) None of these.

iii. If $J_n(x)$ is Bessel's function of first kind then $J_{-n}(x)=$ **1**

(a) $[J_n(x)]^{-1}$ (b) $[(-1)^n J_n(x)]$

(c) $[J_{2n}(x)]$ (d) None of these

iv. A point $x=a$ is called singular point of the equation $P_0(x)\frac{d^2y}{dx^2}+P_1(x)\frac{dy}{dx}+P_2(x)y=0$ if **1**

(a) $P_0(x)$ Vanishes for $x=a$ (b) $P_1(x)$ Vanishes for $x=a$

(c) $P_2(x)$ Vanishes for $x=a$ (d) None of these

v. The equation of the form $Pp+Qq=R$ where P, Q, R are functions of x, y, z, is called as **1**

(a) Charpit's equation (b) Clairaut's equation

(c) Lagrange's equation (d) None of these.

P.T.O.

vi. The general solution of $\left(D^2 + D'^2 - 2DD'\right)z = 0$ is **1**

(a) $f_1(y+x) + xf_2(y+x)$ (b) $f_1(y-x) + f_2(y-x)$

(c) $f_1(y-x) + x f_2(y-x)$ (d) None of these

vii. A vector $\vec{F}$ is said to be solenoidal vector if **1**

(a) $grad\,\vec{F} = 0$ (b) $div\,\vec{F} = 0$

(c) $Curl\,\vec{F} = 0$ (d) None of these.

viii. Which of the following theorem gives the relation between surface integral and volume integral? **1**

(a) Gauss Divergence theorem

(b) Stokes theorem

(c) Green's theorem

(d) None of these

ix. The residue of the function $f(z) = \dfrac{e^z}{z^4}$ is **1**

(a) 0 (b) $\dfrac{1}{12}$ (c) $\dfrac{1}{6}$ (d) None of these.

x. The necessary condition for the function $f(z) = u(x,y) + iv(x,y)$ to be analytic in domain D are **1**

(a) $u_x = v_y, u_y = -v_x$ (b) $u_x = u_y, v_y = -v_x$

(c) $u_x = -v_y, u_y = v_x$ (d) None of these.

Q.2 i. Define sequence of partial sums and convergence for the series $\sum_{n=1}^{\infty} u_n$. **2**

ii. Test the convergence of the series $\sum_{n=1}^{\infty} u_n$, where $u_n = \dfrac{x^n}{(2n-1)2n}$ **8**

OR iii. Prove that the series $\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{2n-1}$ conditionally convergent. **8**

Q.3 i. Write Rodrigue's Formula. **2**

ii. Prove that Bessels function of first kind $J_n(x)$ is the coefficient of z^n in the expression of $e^{\frac{x}{2}\left(z-\frac{1}{z}\right)}$. **8**

OR iii. Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre's polynomials. **8**

Q.4 i. Solve $z = px + qy + p^2 + q^2$ by Charpit's method. **4**

ii. Solve the partial differential equation $\left(D^2 - 2DD' + D'^2\right)z = e^{(x+2y)} + x^3$. **6**

OR iii. Solve the partial differential equation $\left(\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2}\right) = y\cos x$. **6**

Q.5 i. Find the total work done in moving a particle in the force field given by $\vec{F} = 3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$ **4**

ii. If $\vec{F} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x - y)\hat{k}$ then find $grad\,\vec{F}$, $div\,\vec{F}$, $curl\,\vec{F}$. **6**

OR iii. Using Stoke's theorem evaluate $\int_C \left[(x+y)\,dx + (2x-y)\,dy + (y+z)\,dz\right]$ where c is the boundary of the triangle with vertices $(2,0,0),(0,3,0),(0,0,6)$. **6**

Q.6 i. Determine the analytic function whose real part is $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. **4**

ii. Evaluate the integral $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is the circle $|z| = 3$. **6**

OR iii. Prove that $\int_0^{2\pi} \dfrac{d\theta}{2 - \cos\theta} = \dfrac{2\pi}{\sqrt{3}}$ using contour integral. **6**

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FACULTY OF SCIENCE
END SEM ODD EXAMINATION DEC-2019
BC3CO11 MATHEMATICS-III

PROG. B.Sc (CS) Branch :- COMPUTER SCIENCE

SCHEME / SOLUTION

Q.1 (i) (a) $P > 1$ (+1)
(ii) (b) Divergent (+1)
(iii) (b) $[(-1)^n J_n(x)]$ (+1)
(iv) (a) $P_0(x)$ vanishes for $x = a$ (+1)
(v) (c) Lagrange's equation (+1)
(vi) (a) $f_1(y+x) + x f_2(y+x)$ (+1)
(vii) (b) $\operatorname{div} \vec{F} = 0$ (+1)
(viii) (a) Gauss Divergence Theorem (+1)
(ix) (c) 1/6 (+1)
(x) (a) $u_x = v_y$, $u_y = -v_x$ (+1)

Q.2 (i) Consider the infinite series
$\sum u_n = u_1 + u_2 + \cdots + u_n + \cdots \infty$
Let the sum of first n terms be
$S_n = u_1 + u_2 + \cdots + u_n$ which is called as sequence of partial sums. (+1)

clearly S_n is a function of n ~~and as n increases indef~~

If S_n tends to a finite limit as $n \to \infty$ the series $\sum u_n$ is said to be convergent. (+1)

Q.2(ii) we have $u_n = \frac{x^n}{(2n-1)(2n)}$

and

$$u_{n+1} = \frac{x^{n+1}}{(2n+1)(2n+2)} \quad (+2)$$

Now

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \lim_{n\to\infty} \frac{x^n}{(2n-1)(2n)} \times \frac{(2n+1)(2n+2)}{x^{n+1}}$$

$$= \lim_{n\to\infty} \left[\frac{(2+1/n)(2+2/n)}{(2-1/n)(2)}\right] x^{-1}$$

$$= x^{-1} \quad (+2)$$

Hence Σu_n converges if $x^{-1} > 1$ i.e $x < 1$ and diverges for $x > 1$.

If $x = 1$ then $u_n = \frac{1}{(2n-1)(2n)} = \frac{1}{2n^2(2-\frac{1}{n})}$

Taking $v_n = \frac{1}{n^2}$ (+2)

we get

$$\lim_{n\to\infty} \frac{u_n}{v_n} = \lim_{n\to\infty} \frac{1}{2n^2(2-\frac{1}{n})} \cdot n^2 = \frac{1}{4} = \text{finite}$$

∴ Both Σu_n & Σv_n converges or diverges together

But $\Sigma v_n = \Sigma \frac{1}{n^2}$ is convergent

∴ Σu_n is also convergent.

Hence given series is convergent if $x \le 1$ and diverges if $x > 1$. (+2)

Q. 2 (iii) (OR) Given series is

$$\sum u_n = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots \infty$$

This is an alternating series of which terms go on decreasing and

$$\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{2n-1} = 0 \qquad (+2)$$

$\therefore$ By Leibnitz's Rule, $\sum u_n$ Converges

The Series of absolute terms is

$$1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots \infty \qquad (+2)$$

Here $u_n = \frac{1}{2n-1}$ Take $v_n = \frac{1}{n}$

we have $\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2} \neq 0$ & finite (+2)

$\therefore$ By Comparison test $\sum u_n$ diverges [$\because \sum v_n$ diverg]

Hence the given series Converges and the series of absolute term diverges, therefore the given series Converges Conditionally. (+2)

Q. 3 (i)

The Rodrigues formula is given by

$$P_n(x) = \frac{1}{n!\,2^n} \frac{d^n}{dx^n} (x^2-1)^n \qquad (+2)$$

Q.3 (ii) To prove $e^{\frac{x}{2}(z-\frac{1}{z})} = \sum_{n=-\infty}^{\infty} z^n J_n(x)$

we have

$$e^{\frac{1}{2}x(z-\frac{1}{z})} = e^{\frac{xz}{2}} \times e^{\frac{-x}{2z}}$$

$$= \left[1+\left(\frac{xz}{2}\right)+\frac{1}{2!}\left(\frac{xz}{2}\right)^2+\cdots\right] \times$$

$$\left[1-\left(\frac{x}{2z}\right)+\frac{1}{2!}\left(\frac{x}{2z}\right)^2-\frac{1}{3!}\left(\frac{x}{2z}\right)^3+\cdots\right] \quad (+2)$$

The coeff of z^n in this product is

$$= \frac{1}{n!}\left(\frac{x}{2}\right)^n - \frac{1}{(n+1)!}\left(\frac{x}{2}\right)^{n+2} + \frac{1}{2!(n+2)!}\left(\frac{x}{2}\right)^{n+4} \quad (+2)$$

$$= J_n(x)$$

As all the integral powers of z, both positive and negative occur, we have (+2)

$$e^{\frac{x}{2}(z-\frac{1}{z})} = J_0(x) + zJ_1(x) + z^2 J_2(x) + \cdots$$

$$\cdots + z^{-1}J_{-1}(x) + z^{-2}J_{-2}(x) + \cdots$$

$$= \sum_{n=-\infty}^{\infty} z^n J_n(x). \quad (+2)$$

Q.3 (iii) (OR) Given $f(x) = 4x^3 - 2x^2 - 3x + 8$

Since w.k.T

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3) \quad (+1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x) \quad (+1)$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1) \quad (+1)$$

$$P_1(x) = x \quad (+1)$$

$$P_0(x) = 1 \quad (+1)$$

$$\therefore f(x) = \frac{8}{5}P_3(x) - \frac{4}{3}P_2(x) - \frac{3}{5}P_1(x) + \frac{22}{3}P_0(x) \quad (+3)$$

Q.4 (i) Let $f = px + qy + p^2 + q^2 - Z$

$$\frac{\partial f}{\partial x} = p \quad \frac{\partial f}{\partial y} = q \quad \frac{\partial f}{\partial z} = -1, \quad \frac{\partial f}{\partial p} = x + 2p$$

$$\frac{\partial f}{\partial q} = y + 2q \quad (+2)$$

Charpit's Auxillary eq^n is

$$\frac{dP}{\frac{\partial f}{\partial x} + p\left(\frac{\partial f}{\partial z}\right)} = \frac{dq}{\left(\frac{\partial f}{\partial y}\right) + q\left(\frac{\partial f}{\partial z}\right)} = \frac{dz}{-p\left(\frac{\partial f}{\partial p}\right) - q\left(\frac{\partial f}{\partial q}\right)} \quad (+1)$$

$$= \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\left(\frac{\partial f}{\partial q}\right)} = \frac{df}{0}$$

$$\Rightarrow \frac{dp}{p-p} = \frac{dq}{q-q} = \frac{dz}{-p(x+2p)-q(y+2q)} = \frac{dx}{-(x+2p)} = \frac{dy}{-(y+2q)} = \frac{dF}{0}$$

$dp = 0 \Rightarrow p = a$

$dq = 0 \Rightarrow q = b$

$\therefore z = ax + by + a^2 + b^2$ (+1)

Q. 4(ii) $(D-D')^2 z = e^{x+2y} + x^3$

$m = 1, 1$

$C.F = \phi_1(y+x) + x\phi_2(y+x)$ (+3)

$$P.I = \frac{1}{(D-D')^2} e^{x+2y} + \frac{1}{(D-D')^2} x^3$$

Put $D=1$ $D'=2$

$$= \frac{1}{(1-2)^2} e^{x+2y} + \frac{1}{D^2\{1-D'/D\}^2} x^3$$

$$= e^{x+2y} + \frac{1}{D^2}\left[1-\frac{D'}{D}\right]^{-2} x^3$$

$$= e^{x+2y} + \frac{1}{D^2}\left[1+\frac{2D'}{D}+\frac{3D'^2}{D^2}+\cdots\right] x^3$$

$$P.I = e^{x+2y} + \frac{x^5}{20}$$ (+2)

$\therefore z = C.F + P.I$

$$= \phi_1(y+x) + x\phi_2(y+x) + e^{x+2y} + \frac{x^5}{20}.$$ (+1)

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Q.4 (iii) (OR)

$(D^2 + DD' - 6D'^2)z = y\cos x$

$m = 2, -3$

$C.F = \phi_1(y+2x) + \phi_2(y-3x)$ (+3)

$P.I = \frac{1}{(D-2D')(D+3D')} y\cos x$ $(y-3x = a)$

$= \frac{1}{D-2D'} \int (3x+a)\cos x\,dx$

$= \frac{1}{(D-2D')} \left[(3x+a)\sin x + 3\cos x\right]$

$= \frac{1}{(D-2D')} \left[y\sin x + 3\cos x\right]$ $y+2x = b$

$= \int \left[(b-2x)\sin x + 3\cos x\right] dx$

$= -(b-2x)\cos x - 2\sin x + 3\sin x$

$P.I = -y\cos x + \sin x$ (2.5)

$z = C.F + P.I$

$= \phi_1(y+2x) + \phi_2(y-3x) - y\cos x + \sin x$ (0.5)

Q.5 (i) $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \Rightarrow d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

work done $= \int_C \vec{F}\cdot d\vec{r}$

$= \int_C (3xy\hat{i} - 5z\hat{j} + 10x\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$ (+1)

$$= \int_C (3xy\,dx - 5z\,dy + 10x\,dz)$$

$$= \int_1^2 [3(t^2+1)\cdot 2t^2 \cdot 2t - 5t^3 \cdot 4t + 10(t^2+1)\cdot 3t^2]\,dt$$ (+2)

$$= \int_1^2 (12t^5 + 10t^4 + 12t^3 + 30t^2)\,dt$$

W.D = 303 units (+1)

Q5 (ii) $\vec{F} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x-y)\hat{k}$

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\vec{F}$$

$$= \frac{\partial}{\partial x}(\sin y + z) + \frac{\partial}{\partial y}(x\cos y - z) + \frac{\partial}{\partial z}(x-y)$$

$$= -x\sin y$$ (+3)

$$\operatorname{curl}\vec{F} = \nabla\times\vec{F} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\times\vec{F}$$

$$= 0$$ (+3)

grad $\vec{F}$ cannot be evaluated as gradient can be find only of scalar function and F is a vector function.

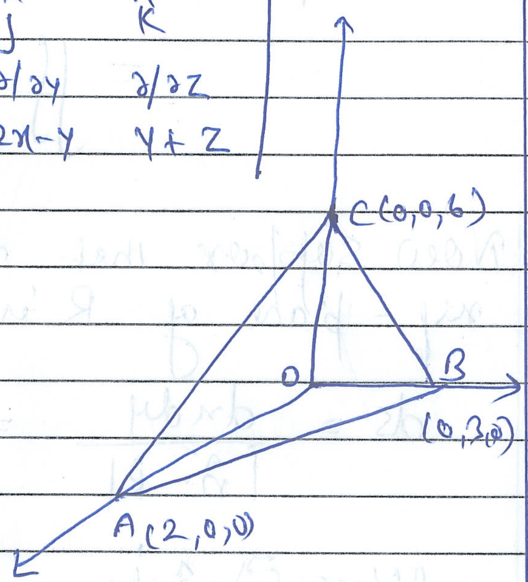
Q5 (iii) Given

Let $\vec{F} = (x+y)\hat{i} + (2x-y)\hat{j} + (y+z)\hat{k}$

W.K.T By Stokes Theorem

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times F)\cdot\hat{n}\,ds \quad (+1)$$

$$\nabla\times\vec{F} = \text{curl}\,\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+y & 2x-y & y+z \end{vmatrix}$$



$$= \hat{i} + \hat{k} \quad (+1)$$

where C is denotes the boundary of triangle ABC, S denotes the surface of ΔABC and n is unit vector normal to the surface S in outward direction.

The eqn of plane of ΔABC is

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1 \quad \text{or} \quad 3x+2y+z = 6$$

Let $f = 3x+2y+z-6$

$$\therefore \text{grad}\, f = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right)f$$

$$= 3\hat{i} + 2\hat{j} + \hat{k} \quad (+1)$$

$$\hat{n} = \frac{\text{grad} f}{|\text{grad} f|} = \frac{3\hat{i}+2\hat{j}+k}{\sqrt{14}} \quad (+1)$$

$$\therefore \iint_S (\nabla\times F)\cdot\hat{n}\, ds = \iint_S (\hat{i}+\hat{k})\cdot\left(\frac{3\hat{i}+2\hat{j}+\hat{k}}{\sqrt{14}}\right) ds$$

$$= \iint \frac{4}{\sqrt{14}}\, ds \quad (+1)$$

Now suppose that the projection of S on xy-plane of R is ΔOAB so

$$ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} = \frac{dx\,dy}{1/\sqrt{14}} = \sqrt{14}\,dx\,dy$$

$$\therefore \iint (\nabla\times\vec{F})\cdot\hat{n}\, ds = \iint_R \frac{4}{\sqrt{14}}\cdot\sqrt{14}\,dx\,dy$$

$$= 4\iint_R dx\,dy$$

$$= 4(\text{area of } \Delta OAB)$$

$$= 4\times\frac{1}{2}\times 2\times 3$$

$$= 12 \quad (+1)$$

$$\therefore \int_C \vec{F}\cdot d\vec{r} = \int_C (x+y)dx + (2x-y)dy + (y+z)dz = 12$$

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Q. 6 (i) Given $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$

$u_x = 3x^2 - 3y^2 + 6x \; ; \; u_y = -6xy - 6y$ (+1)

$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$ (+1)

$= (6xy + 6y)dx + (3x^2 - 3y^2 + 6x)dy$

$= M dx + N dy$

$\therefore v = 3x^2y + 6xy - y^3 + c$ (+1)

i.e. analytic function is given by

$f(z) = u + iv$

$f(z) = z^3 + 3z^2 + C$ (+1)

Q. 6 (ii) Let $f(z) = \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$

The poles of $f(z)$ are given by $z = 1, z = 2$ which lies inside the circle $|z| = 3$.

By Cauchy's Residue theorem

$$\int_C f(z)dz = 2\pi i \left[\underset{z=1}{\text{Res}} f(z) + \underset{z=2}{\text{Res}} f(z) \right] \quad \text{— (I)}$$

Here $z = 1$ is a pole of order 2 and $z = 2$ is simple pole (+2)

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Now Residue of $f(z)$ at $z=2$ is

$$\operatorname*{Res}_{z=2} f(z) = \lim_{z\to 2} (z-2) f(z)$$

$$= \lim_{z\to 2} (z-2) \times \left[\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\right]$$

$$= 1$$ (+1)

Now Residue of $f(z)$ at $z=1$ (pole of order 2) is given by

$$\operatorname*{Res}_{z=1} f(z) = \lim_{z\to 1} \frac{1}{(2-1)!} \frac{d}{dz} (z-1)^2 f(z)$$

$$= \lim_{z\to 1} \frac{1}{(2-1)!} \frac{d}{dz} (z-1)^2 \frac{(\sin \pi z^2 + \cos \pi z^2)}{(z-1)^2 (z-2)}$$

$$= \lim_{z\to 1} \frac{d}{dz} \left[\frac{\sin \pi z^2 + \cos \pi z^2}{z-2}\right]$$

$$= \lim_{z\to 1} \left\{\frac{\left[(z-2)[\cos \pi z^2 (2z) - \sin(\pi z^2)(2z)] - (\sin \pi z^2 + \cos \pi z^2)(1)\right]}{(z-2)^2}\right\}$$

$$= (-1)[(-2) - 0] + (1)$$

$$= 3$$ (+2)

$\therefore$ eq^n (1) becomes

$$\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz = 2\pi i [1 + 3]$$

$$= 8\pi i$$ (+1)

Q. 6 (iii)
(OR)

Let C be the unit circle $|z| = 1$

$$\text{Given } I = \int_0^{2\pi} \frac{d\theta}{2-\cos\theta}$$

put $z = e^{i\theta}$

$$dz = e^{i\theta} i\, d\theta = iz\, d\theta$$

$$\frac{dz}{iz} = d\theta$$

(+1)

$$\therefore I = \frac{1}{i}\int_C \frac{dz}{z\left[2 + \frac{1}{2}(-1)\left(z + \frac{1}{z}\right)\right]}$$

$$= \frac{2}{i}\int_C f(z)\, dz$$

where

$$f(z) = \cancel{\frac{1}{az^2+2az+b}} \quad \frac{1}{-z^2+4z-1}$$

poles of $f(z)$ are given by $-z^2+4z-1=0$

i.e $z = \frac{-4 \pm \sqrt{16-4}}{2(-1)} = \frac{-4 \pm \sqrt{12}}{-2}$

$$= \frac{-2 \pm \sqrt{3}}{-1}$$

Let $\alpha = \frac{-2+\sqrt{3}}{-1}$ $\quad \beta = \frac{-2-\sqrt{3}}{-1}$

(+2)

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Since $\alpha > \beta > 0$ we have $|\beta| > 1$

Since $|\alpha||\beta| = |\alpha\beta| = 1$

we have $|\alpha| < 1$

Here $z = \alpha$ is the only simple pole lie inside c.

Also $f(z) = \dfrac{1}{(-1)(z-\alpha)(z-\beta)}$

$\therefore \operatorname{Res}_{z=\alpha} f(z) = \lim_{z \to \alpha} (z-\alpha) \dfrac{1}{(-1)(z-\alpha)(z-\beta)}$

$= \dfrac{1}{2\sqrt{4-1}} = \dfrac{1}{2\sqrt{3}}$ (+2)

Hence

$I = \displaystyle\int_0^{2\pi} \frac{d\theta}{2-\cos\theta}$

$= \dfrac{2}{i} \displaystyle\int_c f(z)\, dz$

$= \dfrac{2}{i}\, 2\pi i \,\{\text{Residue at } z = \alpha\}$

$= 4\pi \times \dfrac{1}{2\sqrt{3}}$

$I = \dfrac{2\pi}{\sqrt{3}}$ (+1)