(b) Obtain the discrete numeric function corresponding to the generating function $A(z)=\dfrac{2}{1-4z^2}$.

ii. Solve the recurrence relation $a_r-5a_{r-1}+6a_{r-2}=5^r$. **5**

iii. Solve $y_{h+1}-y_h=h$ with $y_0=1$ by using the method of generating function. **5**

******

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2019

CA5BS04 Mathematics of Computer Applications

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The first step of an indirect proof is to **1**

(a) Write the given
(b) Assume the negation of the given
(c) Assume the negation of what you are trying to prove
(d) write the conclusion.

ii. In Cricket League, in first round every team plays a match with every other team. 9 teams participated in the Cricket league. How many matches were played in the first round? **1**

(a) 36 (b) 72 (c) 9! (d) 9! – 1

iii. A graph without self-loop and parallel edges is known as: **1**

(a) Multi graph (b) Regular graph
(c) Complete graph (d) Simple graph.

iv. The total number of edges in a complete graph with 8 vertices is **1**

(a) 20 (b) 28 (c) 32 (d) None of these.

v. A binary tree is special class of rooted tree in which every vertex has **1**

(a) At most two children (b) At least two children
(c) Exactly two children (d) Any number of children.

vi. If *G* is a connected graph and *T* be a spanning tree of *G* addition of any _____ chord in *T* creates exactly one circuit, such a circuit is called fundamental circuit. **1**

(a) One (b) Two (c) Three (d) None of these

vii. If *(Z,*)* is a group defined by binary composition $a*b = a+b+1, \forall a,b \in Z$ then the value of identity element is **1**

(a) -2 (b) 0 (c) -1 (d) *a*-2.

viii. A cyclic group is always a/an **1**

(a) Abelian group (b) Subgroup

(c) Quotient (d) None of these

ix. For Recurrence relation $a_{r+3}+5a_{r+2}+16a_{r+1}-3a_r=0$, respective degree and order are: **1**

(a) 1 and 2 (b) 3 and 1 (c) 1 and 3 (d) 1 and 1.

x. For generating function $A(z)=\dfrac{5}{1-2z}$ the corresponding discrete numeric function a_r is **1**

(a) 2.5^r (b) $5/2^r$ (c) 2^r (d) None of these.

Q.2 Attempt any two:

i. Prove that $7^{2n}+2^{3n-3}.3^{n-1}$ is divisible by 25 for all $n \in N$ by method of induction. **5**

ii. Prove that ${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$. **5**

iii. State the Pigeonhole Principle, and hence find the minimum number of students required in a class to be sure that at least 10 will receive the same grade if there possible grades are A, B, and C. **5**

Q.3 Attempt any two:

i. Define with suitable examples - Circuit, Hamilton graph, connected graph, Circuit matrix representation, Pendant vertex. **5**

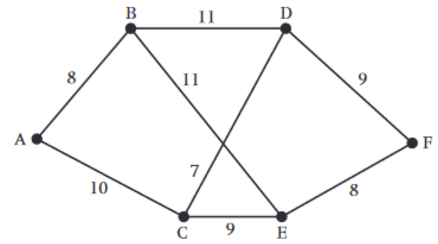
ii. Prove that the sum of degrees of all vertices in a graph is equal to twice the number of edges. **5**

iii. Let *G* be a simple graph with *n* vertices. If *G* has *k* components, then prove that the maximum number of edges that G can have are $(n-k)(n-k+1)/2$. **5**

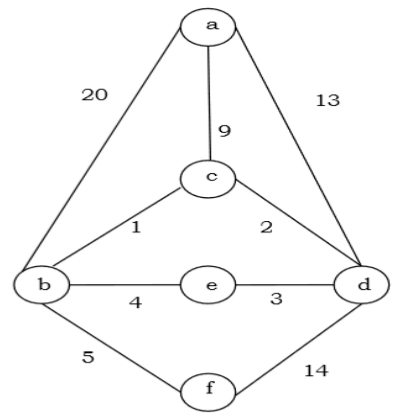
Q.4 Attempt any two:

i. Prove that a tree with *n* vertices has exactly *n-1* number of edges. **5**

ii. Use Dijkstra's algorithm to find the length of the shortest path between A to F in the following graph: **5**



iii. Find a minimal spanning tree for given weighted graph by using Prims method: **5**



Q.5 Attempt any two:

i. Prove that the set of all positive rational numbers forms an Abelian group under the composition defined by *a*b= ab/2*. **5**

ii. Define subgroup. If H_1 and H_2 are two subgroups of a group *G* then prove that $H_1 \cap H_2$ is also a subgroup of *G*. **5**

iii. State and prove Lagrange's theorem. **5**

Q.6 Attempt any two:

i. (a) Determine the generating function of the given discrete numeric function $a_r = \begin{cases} 2^r & if\ r\ is\ even \\ -2^r & if\ r\ is\ odd \end{cases}$ **5**

Medi-Caps University

Faculty of Engineering

End Sem (odd) Exam Dec-2019

CA5BS04 Mathematics of Comp. Application

Programme – MCA Branch – Comp. Application

Qu. 1. MCQ 10

1) c. Assume the negation of what you are trying to prove.
2) a. 36 $[n=9, r=2, {}^9C_2 = 36]$
3) d. simple graph.
4) b. 28 $[n=8, e=\frac{n(n-1)}{2} = \frac{8(7)}{2} = \frac{56}{2}]$
5) a. At most two children
6) a. one
7) c. −1 $[a*e=a, a+e+1=a, e=-1]$
8) a. Abelian group
9) c. 1 and 3
10) d. None of these.

Qu. 2 (i). # Basic step

$n=1$

$P(1) = 7^2 + 2^{3-3} \cdot 3^{1-1} = 49+1$

$= 50 = 25 \times 2$

which is divisible by 25 1

Inductive hypothesis

$n=m$

$P(m) = 7^{2m} + 2^{3m-3} \cdot 3^{m-1} = 25 \times k$

which is divisible by 25 1

Date: / / Page no:

Inductive step

$n = m+1$

$P(m+1) = 7^{2(m+1)} + 2^{3(m+1)-3} \cdot 3^{m+1-1}$

$= 49 \cdot 7^{2m} + 24 \, 2^{3m-3} \cdot 3^{m-1}$

$= (50-1)\, 7^{2m} + (25-1)\, 2^{3m-3}\, 3^{m-1}$

$= 50 \cdot 7^{2m} + 25\, 2^{3m-3} \cdot 3^{m-1} - 7^{2m} - 2^{3m-3}\, 3^{m-1}$

$= 25\,[2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{m-1}] - [7^{2m} + 2^{3m-3}\, 3^{m-1}]$

$= 25\,[2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{m-1}] - P(m)$

$= 25\,[2 \cdot 7^{2m} + 2^{3m-3} \cdot 3^{m-1} - K]$

which is also divisible by 25 (2)

conclusion - By the principle of mathematical induction the given proposition is true for all n. (1)

(ii). $^{n}C_{r} = {}^{n-1}C_{r} + {}^{n-1}C_{r-1}$

RHS $\dfrac{(n-1)!}{r!\,(n-1-r)!} + \dfrac{(n-1)!}{(r-1)!\,(n-\cancel{1}-r+\cancel{1})!}$ (1)

$= \dfrac{(n-1)!}{r(r-1)!\,(n-1-r)!} + \dfrac{(n-1)!}{(r-1)!\,(n-r)\,(n-r-1)!}$ (1)

$= \dfrac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\dfrac{1}{r} + \dfrac{1}{n-r}\right]$ (1)

$= \dfrac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\dfrac{n-\cancel{r}+\cancel{r}}{r(n-r)}\right]$

$= \dfrac{n\,(n-1)!}{r\,(r-1)!\,(n-r)\,(n-r-1)!}$ (1)

$= \dfrac{n!}{r!\,(n-r)!}$

$= {}^{n}C_{r}$ (1)

(3)

(iii). Pigeon hole principle. –
If n pigeons (objects) occupy m pigeonholes (categories) and $n>m$ then atleast one pigeonhole must contain $\frac{(n-1)}{m}+1$ pigeons in it.

OR

must contain more than one pigeon in it. (2)

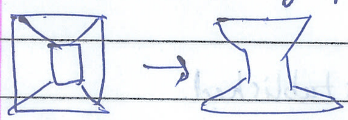
$\frac{(n-1)}{m}+1 = PHP$ (1)

$n = ?$ $\quad m=3$

$\frac{(n-1)}{3}+1 = 10 \implies n = 28$ Ans (2)

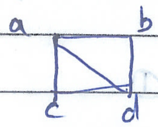
Que. (3) (i). circuit – circuit is a closed walk in which all vertices are distinct except terminal vertices. (1)

Hamilton graph – Hamiltonian circuit is defined to be a closed walk which traverses every vertex exactly once except the starting vertex. A graph is called Hamiltonian graph if it contains a hamiltonian circuit. (1)



Connected graph – A graph is called connected if there is atleast one path between every pair of vertices. (1)

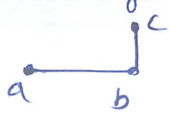
Circuit matrix – Matrix representation between circuit and vertices known as circuit matrix (1)



$C_1 = a, c, d$
$C_2 = a, b, d$
$C_3 = a, b, c, d$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline C_1 & 1 & 0 & 1 & 1 \\ C_2 & 1 & 1 & 0 & 1 \\ C_3 & 1 & 1 & 1 & 1 \end{array}$$

Pendant vertex – A vertex of degree one is called a pendant vertex. (1)



(ii). $\Sigma \deg(v) = 2e$ (1)

Proof. Let $G = \{V, E\}$ be a graph with $o(V) = n$ & $o(E) = e$. By mathematical induction

Step I. $e = 0$ (1)

$\Sigma \deg(v) = 0 = 2 \cdot 0 = 2e$

Step II $e = 1$ (1)

$\Sigma \deg(v) = 1 + 1 = 2 = 2 \cdot 1 = 2e$

Step III = $e = e - 1$ (1)

By hypothesis we assume that theorem is true for all graphs having less than e nu of edges

$\Sigma \deg(v) = 2(e-1)$

Step IV $e = e + 1$ (1)

If we add one more edge then degree of each vertex will be increase by one.

$\Sigma \deg(v) = 2(e-1) + 2 = 2e$

Hence theorem is completely established.

Note:- Alternative proof is also correct.

(iii). Given that graph is simple and has k components.

i.e. $G = G_1, G_2, G_3 \ldots G_k$

where $n_1, n_2, n_3 \ldots n_k$ are nu of vertices for each component. (1)

$n_1 + n_2 + \ldots n_k = \sum_{i=1}^{k} n_i = n$ — (1)

$\because$ Max nu of edges in any simple graph is $\frac{n(n-1)}{2}$.

3

$\therefore \frac{n_1(n_1-1)}{2} + \frac{n_2(n_2-1)}{2} + \cdots \frac{n_k(n_k-1)}{2} = \frac{1}{2}\sum_{i=1}^{k} n_i(n_i-1)$

$= \frac{1}{2}\sum_{i=1}^{k} n_i^2 - n_i$ — (2)

(2)

$\sum_{i=1}^{k}(n_i-1) = n-k$

$\sum_{i=1}^{k}(n_i-1)^2 = (n-k)^2$

$\sum n_i^2 + k - 2n = n^2+k^2-2nk$

$\sum n_i^2 - \sum n_i = n^2+k^2-2nk-k+2n-\sum n_i$

$= (n-k)^2 - k + n$

$\frac{1}{2}\sum n_i^2 - n_i = \frac{(n-k)}{2}[(n-k)+1]$ (2)

Hence proved.

Note:- Alternate proof is also correct.

Qu. 4. (i) By mathematical induction

$n=1$ $e=0$

$n=2$ $e=1$

$n=3$ $e=2$  or (1)

i.e $e=n-1$

By induction hypothesis assume that theorem is true for all trees having less than n vertices

Consider a tree with n no. of vertices and no. of edges. If we remove one edge then obtained two components.

$T-e = T_1 + T_2$

T_1 has n_1 no. of vertices

T_2 has n_2 — " — such that $n_1+n_2=n$

$\because n_1 < n$ & $n_2 < n$ and induction hypothesis

$\therefore$ No. of edges in $T_1 = n_1-1$

— " — $T_2 = n_2-1$ (2)

Total no. of edges $T_1 + T_2 = n_1 - 1 + n_2 - 1$

— " — $T - e = n - 2$

— " — $T = n - 2 + 1 = n - 1$ 2

Hence it is proved that tree with n vertices has exactly $(n-1)$ edges.

Note:- Alternate way is also correct.

(ii). Dijkstra's algorithm —

Step 1. $V = \{A, B, C, D, E, F\}$

$l(A) = 0$ $l(B) = \infty$ $l(C) = \infty$

$l(D) = \infty$ $l(E) = \infty$ $l(F) = \infty$

Step 2 $P = \{A\}$ $V = \{B, C, D, E, F\}$

$l(B) = \min\{\infty, 0+8\} = 8$ ✓ $l(C) = \{\infty, 0+10\} = 10$

$l(D) = \min\{\infty, 0+\infty\} = \infty$ $l(E) = \{\infty, 0+\infty\} = \infty$

$l(F) = \min\{\infty, 0+\infty\} = \infty$ 1

Step 3 $P = \{A, B\}$ $V = \{C, D, E, F\}$

$l(C) = \min\{10, 8+\infty\} = 10$ ✓ ~~(not considered)~~.

$l(D) = \min\{\infty, 8+11\} = 19$ ✓

$l(E) = \min\{\infty, 8+11\} = 19$ 1

$l(F) = \min\{\infty, 8+\infty\} = \infty$

Step 4. $P = \{A, B, C\}$ $V = \{D, E, F\}$

$l(D) = \min\{19, 10+7\} = 17$ ✓

$l(E) = \min\{19, 10+9\} = 19$ 1

$l(F) = \min\{\infty, 10+\infty\} = \infty$

Step 5 $P = \{A, B, C, D\}$ $V = \{E, F\}$

$l(E) = \min\{19, 17+\infty\} = 19$ ✓

$l(F) = \min\{\infty, 17+9\} = 26$ 1

Step 6 $P = \{A, B, C, D, E\}$ $V = \{F\}$

$l(F) = \min\{26, 19+8\} = \{26, 27\} = 26$ 1

The length of the shortest path is 26.

Note :- Path will be ACDF

7

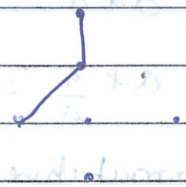
Date: / / Page no:

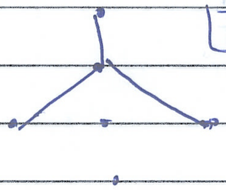
(iii). Prim's algorithm –

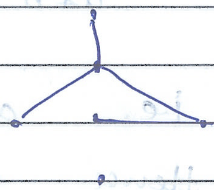
Step 1

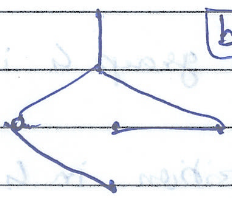
$$\begin{array}{c|cccccc|}
 & a & b & c & d & e & f \\
\hline
a & - & 20 & 9 & 13 & \infty & \infty \\
b & 20 & - & \cancel{1} & \infty & 4 & 5 \\
c & \cancel{9} & 1 & - & 2 & \infty & \infty \\
d & 13 & \infty & \cancel{2} & - & 3 & 14 \\
e & \infty & 4 & \infty & \cancel{3} & - & \infty \\
f & \infty & \cancel{5} & \infty & 14 & \infty & -
\end{array}$$ (1)

Step 2  $\boxed{a-c}$

Step 3  $\boxed{c-b}$ (2)

Step 4  $\boxed{c-d}$

Step 5  $\boxed{d-e}$

Step 6  $\boxed{b-f}$ (2)

Qu. 5 (i) Let $Q = \{$ set of all + rational nu. $\}$

Closure property – $a, b \in Q$ (1)

$a * b = \frac{ab}{2} \in Q$

Associative property $a, b, c \in Q$ (1)

$a*(b*c) = a*\left(\frac{bc}{2}\right) = \frac{abc}{4}$

$(a*b)*c = \left(\frac{ab}{2}\right)*c = \frac{abc}{4}$

i.e. $a*(b*c) = (a*b)*c$

8

Date: / / Page no:

Identity property - 1

Let e is identity element

$a * e = \frac{ae}{2} = a \Rightarrow e = 2 \in Q$

$a * 2 = \frac{a2}{2} = a$

i.e $e = 2 \in Q$ There exist identy element

Inverse property - 1

Let a^{-1} is inverse element

$a * a^{-1} = 2 \Rightarrow \frac{aa^{-1}}{2} = 2 \Rightarrow a^{-1} = \frac{4}{a} \in Q$

$a * \frac{4}{a} = \frac{a\,4}{2a} = 2$ There exist inverse element

Commutative property - 1

$a * b = \frac{ab}{2}$ & $b * a = \frac{ba}{2}$

i.e. $a * b = b * a$

Hence Q is Abelian group.

(ii) A non empty subset H of a group G is called a subgroup of G if 2

- H is stable for the composition in G
- H is group "

Let H_1 & H_2 are two subgroups therefore 2

$a \in H_1, b \in H_1 \Rightarrow ab^{-1} \in H_1$

$a \in H_2, b \in H_2 \Rightarrow ab^{-1} \in H_2$

Finally $a \in H_1 \cap H_2$, $b \in H_2 \cap H_2 \Rightarrow ab^{-1} \in H_1 \cap H_2$

This the necessary & sufficient condition for a non-empty subset to a subgroup.

Hence $H_1 \cap H_2$ is also a subgroup. 1

90

Date: / / Page no:

(iii). Lagrange's theorem - (2)

The order of each subgroup of a finite group is a divisor of the order of the group. $\frac{o(G)}{o(H)}$

Proof. Let G is group & H is its subgroup

$\because o(G) = n \quad o(H) = m$

$\because o(H) = m$ i.e. $h_1, h_2, h_3 \dots h_m \in H$ (1)

For some elements in G $a_1, a_2, a_3 \dots a_c \in G$

Left & Right cosets would be

$a_1H, a_2H, \dots a_cH$

therefore $a_1H \cup a_2H \cup \dots a_cH = G$

$c \cdot o(H) = o(G)$

$c = \frac{o(G)}{o(H)} = \frac{n}{m}$ (2)

Note:- Alternate proof is also correct.

Qu. 6 (i). (a) $A(z) = \sum_{r=0}^{\infty} a_r z^r = a_0 z^0 + a_1 z^1 + a_2 z^2 + \dots$ (1)

$\because a_r = \begin{cases} 2^r & r = \text{even} \\ -2^r & r = \text{odd} \end{cases}$

$\therefore a_0 = 1 \quad a_1 = -2 \quad a_2 = 4$

$a_3 = -8 \quad a_4 = 16 \quad a_5 = -32 \dots$ (1)

Now $A(z) = 1 - 2z + 4z^2 - 8z^3 + 16z^4 - 32z^5 + \dots$

$= 1 - 2z + (2z)^2 - (2z^3) + (2z)^4 - (2z)^5 + \dots$

$= (1+2z)^{-1}$

$= \frac{1}{1+2z}$ (1)

10

Date: __/__/__ Page no: ____

(b). $A(z) = \frac{2}{1-4z^2}$

Sol $A(z) = \frac{2}{1-4z^2} = \frac{1}{1-2z} + \frac{1}{1+2z}$ 1

$= (1-2z)^{-1} + (1+2z)^{-1}$

$= [1+2z+(2z)^2+(2z)^3 - - -] + [1-2z+(2z)^2-(2z)^3+ - - -]$

$= 2 + 2(2z)^2 + 2(2z)^4 + - - -$

$a_r = \begin{cases} 0 & r = \text{odd} \\ 2 \cdot 2^r & r = \text{even} \end{cases}$

OR

$a_r = 2^r + (-2)^r$ 1

(ii). $a_r - 5a_{r-1} + 6a_{r-2} = 5^r$

Sol Total sol $= a^{(h)} + a^{(p)}$ — ①

Homogeneous sol —

$\frac{m^r - 5m^{r-1} + 6m^{r-2}}{m^{r-2}} = 0$

$m^2 - 5m + 6 = 0$

$m = 2, 3$

$a_r^{(h)} = c_1 2^r + c_2 3^r$ — ② 2

Particular sol —

Trial sol $a^{(t)} = A \cdot 5^r$ 1

To obtain the value of $A \Rightarrow$

$A \cdot \cancel{5^r} - 5\cancel{A}\, 5^{r-1} + 6A\, 5^{r-2} = 5^r$

$6A\, 5^{r-2} = 5^r$

$A = \frac{5^r}{5^{r-2} \cdot 6} = \frac{5^2}{6} = \frac{25}{6}$

Now $a_r^{(p)} = \frac{25}{6} 5^r = \frac{5^{r+2}}{6}$ — ③ 2

By ①, ②, ③

$a_r = c_1 2^r + c_2 3^r + \frac{5^{r+2}}{6}$

(11)

Date: / / Page no:

(iii). Generating function

$$Y(t) = \sum_{n=0}^{\infty} y_n t^n = y_0 t^0 + y_1 t^1 + y_2 t^2 + \cdots$$

$$\frac{\sum_{n=0}^{\infty} y_{n+1} t^{n+1}}{t} - \sum_{n=0}^{\infty} y_n t^n = \sum_{n=0}^{\infty} n t^n$$

$$\frac{(y_1 t^1 + y_2 t^2 + \cdots)}{t} - Y(t) = t + 2t^2 + 3t^3 + \cdots$$

$$\frac{(Y(t) - y_0 t^0)}{t} - Y(t) = t(1 + 2t + 3t^2 + \cdots)$$

$$\frac{(Y(t) - 1)}{t} - Y(t) = t(1 + 2t + 3t^2 + \cdots)$$

$$Y(t) - 1 - tY(t) = t^2(1-t)^{-2}$$

$$Y(t)[1-t] = 1 + \frac{t^2}{(1-t)^2}$$

$$Y(t) = \frac{1}{1-t} + \frac{t^2}{(1-t)^3}$$

$$\sum_{n=0}^{\infty} y_n t^n = (1-t)^{-1} + t^2\left[(1-t)^{-3}\right]$$

$$= (1-t)^{-1} + t^2\left[1 + 3t + 6t^2 + \cdots \frac{(n+1)(n+2)}{2} t\right.$$

$$= [1 + t + t^2 + \cdots] + \left[t^2 + 3t^3 + 6t^4 + \cdots + \frac{(n+1)(n+2)}{2}\right.$$

compair coefficient of t^n.

$$y_n = 1^n + \frac{1}{2}(n-1)(n)$$

or $$y_n = 1 + \frac{n(n-1)}{2}$$

Approved
5/12/19