Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Commerce

End Sem Examination May-2024

CM3CO18 Business Mathematics

Programme: B.Com. (Hons.) Branch/Specialisation: Commerce

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. The matrix $\begin{bmatrix} 6 & 1 & 7 \\ 0 & 6 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ is a- **1**

(a) Upper triangular matrix
(b) Lower triangular matrix
(c) Diagonal matrix
(d) None of these.

ii. If $|A| = 0$ then matrix A is called- **1**

(a) Non-singular matrix (b) Singular matrix
(c) Can't say (d) None of these

iii. Cardinality of set $A = \{1,3,4,6,7\}$ is- **1**

(a) 7 (b) 6 (c) 5 (d) None of these

iv. An equation that relates price per unit and quantity demand at that price is called the_____. **1**

(a) Demand function (b) Supply function
(c) Cost function (d) None of these

v. Derivative of x^2 is- **1**

(a) 2 (b) $2x$ (c) 0 (d) None of these

vi. Profit function $P(x) =$ **1**

(a) Revenue function-Cost function
(b) Cost function- Revenue function
(c) Revenue function-marginal cost
(d) None of these

P.T.O.

vii. The value of $\int 1dx =$ **1**

(a) 0 (b) x (c) $x + c$ (d) None of these

viii. The value of $\int_0^{\pi} cosxdx =$ **1**

(a) 0 (b) 1 (c) -1 (d) None of these

ix. If a student gets 360 marks out of 400 in exam, then what percentage of marks did he get- **1**

(a) 70% (b) 80% (c) 90% (d) None of these

x. A ratio equivalent to 3:7 is- **1**

(a) 3:9 (b) 6:10 (c) 9:21 (d) None of these

Q.2 Attempt any two:

i. Solve the following determinant and also find the cofactors of every element. $A = \begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix}$ **5**

ii. Solve the equations using Cramer's rule: **5**

$$x + y = 5, \quad 2x - 3y = -4$$

iii. If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ $and\ B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ Find the product of A and B. **5**

Q.3 Attempt any two:

i. Write the following laws: **5**

(a) Idempotent law (b) Associative law

(c) Commutative law (d) Distributive law

(e) De-Morgan's law

ii. Define the following: **5**

(a) Even and odd function

(b) Algebraic and transcendental function

iii. A company sells x tins of chocolate powder each day at Rs. 20 a tin. The cost of manufacturing these tins is Rs. 15 per tin plus a fixed daily overhead cost of Rs. 900. Find: **5**

(a) Cost function (b) Revenue function

Q.4 Attempt any two:

i. Using quotient rule differentiate $\frac{x+sinx}{x+cosx}$. **5**

ii. If the cost of product $c(x)$ is given by $c(x) = 20 + 2x + \frac{x^2}{2}$ where x is output. Evaluate- **5**

(a) Average cost (b) Marginal cost

(c) Variable cost

iii. The manufacturer cost of an item consists of Rs. 1000 as overheads. Material cost Rs.2 per item and the labour cost $\frac{x^2}{90}$ for x items produce. How many items must be produced to have a minimum average cost? ($c(x)$=Material cost + Labour Cost + Fixed Cost) **5**

Q.5 Attempt any two:

i. Evaluate: $\int x^2 logxdx$. Using integration by parts. **5**

ii. Evaluate the following: **5**

(a) $\int_0^1 \frac{x}{x+1} dx$ (b) $\int_1^2 \frac{x^3}{3}$

iii. Evaluate $\left(2x^2 + e^x - \frac{1}{x}\right) dx$. **5**

Q.6 Attempt any two:

i. (a) Evaluate $100_{C_{98}}$ **5**

(b) If $2P(5,3) = P(n,4)$, find the value of n.

ii. Show that the sequence 9,12,15,18, _____ is an arithmetic progression. Find its 16^{th} term and the general term. **5**

iii. Write the difference between ratio and proportion. **5**

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Q1) (MCQs)

i) a) Upper triangular Matrix (1)

ii) b) & Singular Matrix. (1)

iii) c) 5 (1)

iv) a) Demand function. (1)

v) b) $2x$ (1)

vi) a) Revenue function – Cost function. (1)

vii) c) $x + c$. (1)

viii) a) 0 (1)

ix) c) 90% (1)

x) c) 9:21 (1)

Q2(i) $A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}$

$|A| = (-4 + 3) = -1$ (1)

Cofactors

$C_{11} = (-1)^{1+1}(-2)$

$C_{11} = 1 \times -2 = \boxed{-2}$

$C_{12} = (-1)^{1+2}(1)$

$C_{12} = -1 \times 1 = \boxed{-1}$

$C_{21} = (-1)^{2+1}(-3)$

$= -1 \times -3$

$\boxed{C_{21} = 3}$ (2)

$C_{22} = (-1)^{2+2}(2)$

$C_{22} = 1 \times 2 = 2$

$\boxed{C_{22} = 2}$ (2)

Q2)(ii) $x + y = 5$

$2x - 3y = -4$.

Soln :- $D = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix} = (-3-2) = -5$ (1)

$Dx = \begin{bmatrix} 5 & 1 \\ -4 & -3 \end{bmatrix}$, $Dy = \begin{bmatrix} 1 & 5 \\ 2 & -4 \end{bmatrix}$ (2)

$X = Dx/D$

$Dx = (-15+4) = -11$ (1)

$Dy = (-4-10) = -14$.

$x = Dx/D = -11/-5 \Rightarrow 11/5$ (1)

$y = Dy/D = -14/-5 = 14/5$

(Q2) iii

$$A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$$ (2)

$$AB = \begin{bmatrix} 2+3-0 & 3+6+0 & 4+9+0 \\ -2+2-1 & -3+4+1 & -4+6+2 \\ 0+0+(-2) & 0+0+2 & 0+0+4 \end{bmatrix}_{3\times 3}$$ (2)

$$AB = \begin{bmatrix} 5 & 9 & 13 \\ -1 & 2 & 4 \\ -2 & 2 & 4 \end{bmatrix}$$ Ans (1)

(Q3)(i) a) Idempotent law :-

(1) $A \cup A = A$ (2) $A \cap A = A$ (1)

b) Associative law :-

1) $(A \cup B) \cup C = A \cup (B \cup C)$

2) $(A \cap B) \cap C = A \cap (B \cap C)$ (1)

c) Commutative law.

1) $A \cup B = B \cup A$ 2) $A \cap B = B \cap A$ (1)

d) Distributive law:-

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (1)

2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap B)$

e) De-Morgan's law.

1) $(A \cup B)^c = A^c \cap B^c$ (1)

2) $(A \cap B)^c = A^c \cup B^c$

Q3)(ii)

a) Even and Odd function

A function $f(x)$ is said to be an even function if $f(-x) = f(x)$. (1)

Example :- (i) $f(x) = x^2$ (ii) $f(x) = 4x^2 + 2x^2 + 4$

A function $f(x)$ is said to be an odd function if $f(-x) = -f(x)$ (1.5)

example :- (i) $f(x) = 3x^3 - x$

(ii) $f(x) = 2x$.

b) Algebraic and transcendental function :-

Any expression of the variable x, containing power of x are connected by four fundamental operations of arithmatic, & constant quantities are termed as Algebraic function. (1.5)

Ex :- $f(x) = 5x^2 + 2x + 4$.

function that is not algebraic is called

~~true~~ transcedental function & Contain term like Sin, Cos, $\log x$ etc. (1)

Ex → $y = \sin x + 2\log x + e^{2x+3}$,

Q3(iii) Let $R(x)$ be revenue received by the Company per day. Since the Company Sells x tins of Chocolate powder each day at Rs 20 a tin. Therefore, (1)

$R(x) = 20x$.

a) Cost function

$C(x)$ = Fixed Cost + Variable Cost (2)

$C(x) = 900 + 15x$.

(b) Revenue function.

$R(x) = 20x$. (2)

Q4)

(i) $\frac{d}{dx}\left(\frac{x+\sin x}{x+\cos x}\right)$

$\Rightarrow \frac{d}{dx}\left(\frac{f_1(x)}{f_2(x)}\right) = \frac{f_2(x)\frac{d}{dx}f_1(x) - f_1(x)\frac{d}{dx}f_2(x)}{(f_2(x))^2}$ (1)

$\Rightarrow \frac{d}{dx}\left(\frac{(x+\sin x)}{x+\cos x}\right) = \frac{(x+\cos x)\frac{d}{dx}(x+\sin x) - (x+\sin x)\frac{d}{dx}(x+\cos x)}{(x+\cos x)^2}$ (2)

$= \frac{(x+\cos x)[1+\cos x] - (x+\sin x)[1-\sin x]}{(x+\cos x)^2}$

$$\frac{x + x\cos x + \cos x + \cos^2 x - x + x\sin x - \sin x + \sin}{(x + \cos x)^2}$$ (1)

$$\Rightarrow \boxed{\frac{x\cos x + \cos x + x\sin x - \sin x + 1}{(x + \cos x)^2}} \text{ Ans} \quad (1)$$

Q4 (ii)

Given $C(x) = 20 + 2x + \frac{x^2}{2}$

a) Average Cost $= \frac{C(x)}{x}$ (1)

$$= \frac{20 + 2x + x^2/2}{x}$$

$$\boxed{AC = \frac{20}{x} + 2 + \frac{x}{2}}$$ (1)

b) Marginal Cost $= C'(x) = \frac{d}{dx} C(x)$ (1)

$$= \frac{d}{dx}\left(20 + 2x + \frac{1}{2}x^2\right)$$

$$= 2 + 2x \times \frac{1}{2}$$

$$\boxed{MC = 2 + x}$$ (1)

c) Variable Cost (VC) =

$$\boxed{(VC) = 2x + \frac{1}{2}x^2}$$ (1)

Q4) iii.

$C(x)$ = Material Cost + labour Cost + fixed Cost

$$C(x) = 2x + \frac{x^2}{90} + 1000. \quad (1)$$

$\therefore$ Average Cost $= \frac{C(x)}{x}$

$$= \frac{2x + \frac{x^2}{90} + 1000}{x} \quad (2)$$

$$AC. = 2 + \frac{x}{90} + \frac{1000}{x}$$

Now, $(AC)' = \frac{d(AC)}{dx} = 0$

$$= \frac{1}{90} - \frac{1000}{x^2} = 0. \quad (2)$$

$$x^2 = 90000.$$

$$\boxed{x = \pm 300}$$

but $x = -300$ is not admissible.

Hence $x = 300$. the average Cost will be Minimum.

(Q5) i.

$$\int x^2 \log x \, dx$$

$$f(x) = \log x \qquad g(x) = x^2$$

$$\int f(x) g(x) \, dx = f(x) \int g(x) dx - \int \left[f'(x) \int g(x) dx \right] dx$$ (1)

$$= \log x \int x^2 dx - \int \left(\frac{d}{dx} \log x \int x^2 dx \right) dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \left(\frac{1}{x} \times \frac{x^3}{3} \right) dx$$ (2)

$$= \frac{x^3}{3} \log x - \int \frac{x^2}{3} dx$$ (1)

$$\boxed{\int x^2 \log x \, dx = \frac{x^3}{3} \log x - \frac{1}{9} x^3 + C}$$ Ans (1)

(Q5) ii (a) $$\int_0^1 \frac{x}{1+x} \, dx$$

$$\int_0^1 \frac{(x+1)-1}{1+x} \, dx$$

$$\int_0^1 \frac{1+x}{1+x} dx - \int_0^1 \frac{1}{1+x} dx$$ (1)

$$\int_0^1 1 \, dx - \int_0^1 \frac{1}{1+x} dx$$

$= [x]_0^1 - [\log(1+x)]_0^1$ (Put $1+x = t$, $dx = dt$)

$= [1-0] - [\log(1+1) - \log(1+0)]$ (1)

$= 1 - (\log 2) + \log 1 +$ Any (0.5)

or

$= \boxed{1 - \log 2} +$

(b) $\int_1^2 \frac{x^3}{3}\, dx$

$\Rightarrow \frac{1}{3}\left(\frac{x^4}{4}\right)_1^2$ (1)

$\Rightarrow \frac{1}{12}[x^4]_1^2$ (1)

$\Rightarrow \frac{1}{12}[2^4 - 1^4]$ (.5)

$= \frac{1}{12}(16-1) \Rightarrow \frac{15}{12} \Rightarrow \boxed{\frac{5}{4}}$ Any

Q5) iii) $\int \left(2x^2 + e^x - \frac{1}{x}\right) dx$

(Note → Misprint in the quest^n so if any One attempted this quest^n then full mark will be given)

$\int 2x^2\, dx + \int e^x\, dx - \int \frac{1}{x}\, dx$ (2)

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$\int e^x dx = e^x$, $\int \frac{1}{x} = \log x$.

$= \boxed{\frac{2x^3}{3} + e^x - \log x + c}$ Ans (2)

Q6)

(i) a) ${}^{100}C_{98}$

${}^nC_r = \frac{n!}{r!(n-r)!}$ (1)

$= \frac{100!}{98!(100-98)!}$ (1)

$= \frac{100 \times 99 \times 98!}{98! \, 2!} \Rightarrow \frac{\overset{50}{\cancel{100}} \times 99}{\cancel{2} \times 1}$ (0.5)

$= \boxed{4950}$ Ans

b) $2P(5,3) = P(n,4)$

$n = ?$

$2 \times \frac{5!}{(5-3)!} = \frac{n!}{(n-4)!}$ (1)

$n(n-1)(n-2)(n-3) = 5(5-1)(5-2)(5-3)$ (1)

On comparing, we get (0.5)

$\boxed{n = 5}$

Q6) ii

We have,

$(12-9) = (15-12) = (18-15) = 3$.

Therefore, the given sequence is an A.P with common difference 3. (1)

a = first term = 9.

$\therefore$ 16th term $= T_{16} = a + (16-1)d$ (2)

$= a + 15d$.

$= 9 + 15 \times 3$

$\boxed{T_{16} = 54}$

General term = n^{th} term.

$= a + (n-1)d$

$= 9 + (n-1)3$ (2)

$\boxed{\text{General term} = 3n + 6}$

Q6) iii

	Ratio	Proportion
i)	Defines the quantitative relationship between two or more quantities	Indicates the equality of two ratios. (1)
a)	Can be expressed as a fraction, using the colon symbol (:), or in decimal form.	Usually represented using the "=" sign. (1)

3)	Can have an infinite number of possible values.	Generally, a proportion has only one solution. (1)
4)	The Order of the terms in a ratio matters (ex - 2:1 is different from 1:2)	The Order of the term in a proportion does not matter (ex - 2:1 = 1:2) (1)
5)	Does not guarantee an equal distribution	Ensures a balanced distribution. (1)