Q.5 i. Define context free and context sensitive grammar. **3**

ii. Construct a grammar for the language $L=\{a^i b^{2i} : i \geq 1\}$ is not a finite state language. **7**

OR iii. Describes the type of grammar with examples. **7**

Q.6 i. If $r = ab^*a$, describe $L(r)$ in the form of set. **3**

ii. Show that $L=\{0^i 1^i : i \geq 1\}$ is not regular. **7**

OR iii. Prove that the class of regular sets over Σ is the smallest class R contained $\{a\}$ for every $a \in \Sigma$ and closed under union, concatenation and closure. **7**

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Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2022

MA5CO10 Advanced Discrete Mathematics -II

Programme: M.Sc. Branch/Specialisation: Mathematics

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Which one of the following is complete non-planer graph with smallest number of vertices? **1**

(a) K_3 (b) K_5 (c) K_4 (d) K_6

ii. A graph containing circuit involving all the vertices of the graph is known as: **1**

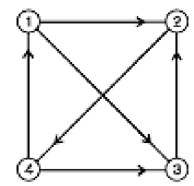
(a) Hamiltonian (b) Euler

(c) Spanning (d) None of these

iii. Fundamental cut sets are the cut set related to a given spanning tree that contains __________ one branch of the spanning tree. **1**

(a) At most (b) At least (c) Exactly one (d) More than one

iv. The following directed graph is ______________ connected. **1**



(a) Strongly (b) Weakly (c) Unilaterally (d) Can't say

v. In a Mealy machine if length of the input string is n then the length of output sequence is- **1**

(a) n (b) $n+1$ (c) $n-1$ (d) None of these

vi. A state of a finite state machine M, with output $O=\{0,1\}$ is called rejecting state if its output is: **1**

(a) 1 (b) 0

(c) Insufficient information (d) None of these

vii. Regular expression for all strings starts with ab and ends with bba is: **1**
(a) aba*b*bba (b) ab(ab)*bba
(c) ab(a+b)*bba (d) All of these

viii. Every regular language is also ________. **1**
(a) Context free (b) Context sensitive
(c) Insufficient information (d) None of these

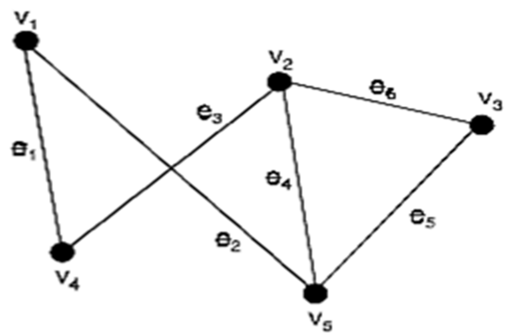
ix. Every finite set of words over Σ (including empty set) is a _______. **1**
(a) Regular set (b) Not regular
(c) Can't say (d) None of these

x. Polish notation is the form of an expression obtained from a ________ traversal of the tree representing this expression. **1**
(a) Preorder (b) Postorder
(c) Both (a) and (b) (d) None of these

Q.2 i. Prove that in a tree there exists one and only one path between any specified pair of vertices. **3**

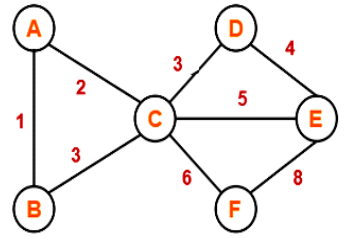
ii. For any connected planer graph G with n- vertices and e – edges the total number of regions R is equal to $e - n + 2$. **7**

OR iii. Represent the following graph using adjacency matrix representation and write any 4 observations on it. **7**

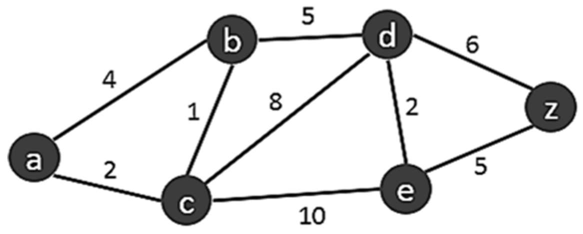


Q.3 i. Give the statement of Eulerian theorem for the existence of Eulerian circuit by explanation with an example and non-example. **3**

ii. Define spanning tree and find the minimum spanning tree for the following weighted graph: **7**



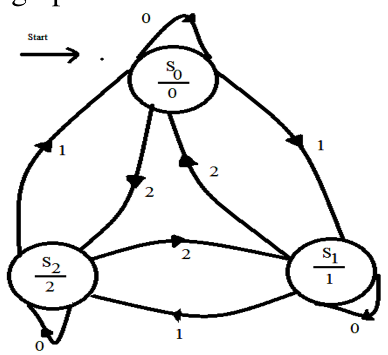
OR iii. Find the length of shortest path using Dijkstra's algorithm for the following graph **7**



Q.4 Attempt any two

i. Design a Turing Machine for adding two non-negative integers. **5**

ii. Define finite state machine and construct the state table for the following transition graph. **5**



iii. Construct a Mealy machine which is equivalent to the Moore machine given in the table **5**

Present State	Next State		Output
	$n\ \alpha = 0$	$\alpha = 1$	
A	D	B	0
B	B	C	1
C	C	D	0
D	D	A	0
E	B	A	1

P.T.O.

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Q.1
(i) (b) k_5 [1]
(ii) (a) Hamiltonian [1]
(iii) (c) exactly one [1]
(iv) (a) Strongly [1]
(v) (a) n [1]
(vi) (b) 0 [1]
(vii) (c) $ab(a+b)^*bba$ [1]
(viii) (a) Context free language [1]
(ix) (a) Regular set [1]
(x) (a) Precedes [1]

Q.2
b
(i) As graph is a tree, so it is a connected graph with no circuits in it. Circuit in a [1] graph means there exist a pair of vertices such that there exist more than one path in between them and as graph is [2] circuitless, so there exist one and only one path between every pair of vertices of it. [3]

(ii) We shall prove the theorem by induction on number of edges of the graph G, where G is a connected planar graph.

If $e=1$ then $n=1$ or $n=2$

 [2]

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In each case $r = e - n + 2$ is satisfied
Hence result is true for $e = 1$
Now suppose that the result holds for all the graphs with atmost $e-1$ edges
Assume that G is a connected planar graph with e edges and r regions. In case G is a tree then $e = n - 1$ & no. of regions is 1. So $r = e - n + 2 = n - 1 - n + 2 = 1$ [4]
Hence result is true. Now consider the case when G is not a tree, then G has some circuits. Consider an edge c say in some circuit. By removing this edge 'c' from the plane representation of G, the regions are merged in to a new region. Therefore $G - \{c\}$ is a connected graph with n vertices, $e-1$ edges and $r-1$ regions. Thus by induction hypothesis, we have

$$r - 1 = e - 1 - n + 2 \quad \text{or} \quad r = e - n + 2$$

[7]

H.P//

(iii)

$$X = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

[3]

Observations:
a) Number of ones in a row represent degree of that vertex. [[4]
b) Number of ones in the principle diagonal represent number of self loops in the graph [5]

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(c) Permutation of rows and of the correspond-ing columns imply reordering the vertices. [6]

(d) A graph G is disconnected & is in two components g_1 and g_2 iff its adjacency matrix $X(G)$ can be partitioned as [7]

$$X(G) = \left[\begin{array}{c|c} X(g_1) & 0 \\ \hline 0 & X(g_2) \end{array}\right]$$

where $X(g_1)$ is the adjacency matrix of component g_1 and $X(g_2)$ is adjacency matrix of component g_2.

Q. 3

(i) The necessary and sufficient condition for a connected graph to be an Euler graph is that "all vertices of G are of even degree [1]

[2]

Example of Euler graph

[3]

Non Euler graph.

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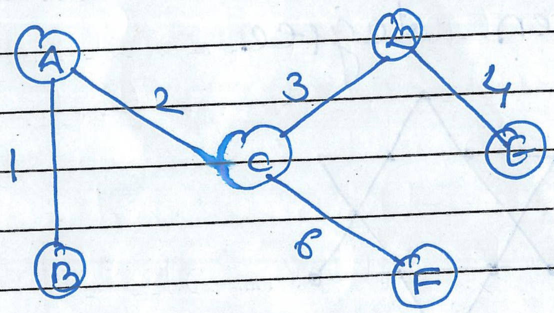
ii) A subgraph of a connected graph which is a tree and contains all the vertices of the graph is called spanning tree. [2]

Edges in increasing order of weights	weight of the edge	Selection the edge
AB	1	yes
AC	2	yes
BC	3	No
CD	3	yes [4]
DE	4	yes
CE	5	No
CF	6	yes
EF	8	— [5]



This is the required minimal spanning tree whose weight is 16. [7]

iii) Assign $P(L(a)) = 0$
set $v = a$
& $T(L(b)) = T(L(c)) = T(L(d)) = T(L(e)) = T(L(z)) = \infty$ [1]

Now

$T(L(b)) = \min\{\infty, 0+4\} = 4$

$T(L(c)) = \min\{\infty, 0+2\} = 2$

$T(L(d)) = \min\{\infty, 0+\infty\} = \infty$

$T(L(e)) = \min\{\infty, 0+\infty\} = \infty$

$T(L(z)) = \min\{\infty, 0+\infty\} = \infty$ [3]

Assign $P(L(c)) = 2$

set $v = c$

$T(L(b)) = \min\{4, 2+1\} = 3$

$T(L(d)) = \min\{\infty, 2+8\} = 10$

$T(L(e)) = \min\{\infty, 2+10\} = 12$

$T(L(z)) = \min\{\infty, 2+\infty\} = \infty$ [4]

Assign $P(L(b)) = 3$

set $v = b$

$T(L(d)) = \min\{10, 3+5\} = 8$

$T(L(e)) = \min\{12, 3+\infty\} = 12$

$T(L(z)) = \min\{\infty, 3+\infty\} = \infty$ [5]

Assign $P(L(d)) = 8$

set $v = d$

$T(L(e)) = \min\{12, 8+2\} = 10$

$T(L(z)) = \min\{\infty, 8+6\} = 14$

[6]

Assign $P(L(e)) = 10$

set $v = e$

$T(L(z)) = \min\{14, 10+5\} = 14$

Length of the shortest path from a to z is 14. [7]

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Q4
(i) Let m, n be two non negative integers. We have to construct Turing machine T which computes the function $f(m,n) = m+n$. The pa[ir] (m,n) is represented by a string $m+1$ 1s followed by an asterik followed by $n+1$ [1s]. The machine take this as input and produces a tape with $m+n+1$ as outpu[t]. This can be done as follows: [2]

The machine starts at the left most [1] of the input string and carries ou[t] steps to erase this 1, halting if $m=0$ s[o] that there are no more 1's before th[e] asterik replace the asterik with the le[ft] most remaining 1 and then halts. The following quintuples can be used to do this: [4]

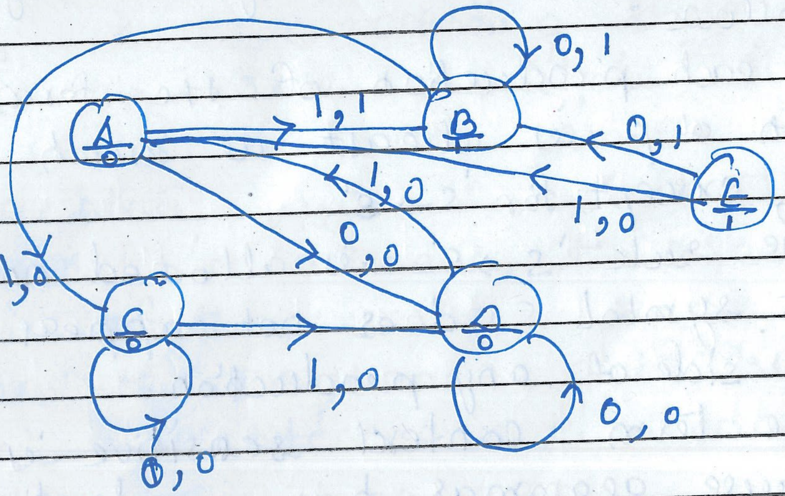
$(s_0, 1, s_1, B, R)$, $(s_1, *, s_3, B, R)$, $(s_1, 1, s_2, B, R)$,
$(s_2, 1, s_2, 1, R)$, $(s_2, *, s_3, 1, R)$. [5]

(ii) A finite state machine M is defined as 6-tuple (S, A, O, f, g, s_0) where,
$A = \{a_1, a_2, \ldots\}$ is a finite set of input alph[abet]
$S = \{s_0, s_1, s_2, \ldots\}$ is a finite set of internal sta[tes]
$s_0 \in S$ is initial state
$O = \{o_1, o_2, \ldots\}$ is finite set of output alph[abet]
f is a function from $S \times A$ is to S called t[he] next state function
$g : S \times A \rightarrow O$ is called the output funct[ion]
[3]

State	Input 0	Input 1	Input 2	Output
→ S_0	S_0	S_1	S_2	0
S_1	S_1	S_2	S_0	1
S_2	S_2	S_0	S_1	2

[5]

iii)



[2]

Mealy Machine:

Present state	Next state $\alpha=0$	Next state $\alpha=1$
A	D, 0	B, 1
B	B, 1	C, 0
C	C, 0	D, 0
D	D, 0	A, 0
E	B, 1	A, 0

[5]

Q. 5
(i) Context free grammar: The only allowed type of production is $A \rightarrow \alpha$, where A is non terminal and α is in sentential

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form i.e. $\alpha \in (V \cup T)^*$ & α may also be equals to ϵ. The start symbol of grammar can also appear on the right hand side.
[1.5]

Context sensitive grammar:
The restrictions on this type of grammar are as follows:
a) for each production of the form $\alpha \rightarrow \beta$ the length of β is atleast as much as the length of α, except for '$S \rightarrow \epsilon$'
b) The rule '$S \rightarrow \epsilon$' is allowed only if the start symbol S does not appear on right hand side of any production.
c) The term context sensitive is used because grammar has production of the form $\alpha_1 A \alpha_2 \longrightarrow \alpha_1 \beta \alpha_2$; $(\beta \neq \epsilon)$
where the replatement of a non terminal by β is allowed only if α_1 preceeds A and α_2 succeeds A: α_1 & α_2 may or may not empty.
[3]

ii) Define a Grammar G
$G = \{N, T, P, S\}$
$\& N = \{S, A\}$, $T = \{a, b\}$

$P = \{S \rightarrow S_2, S_2 \rightarrow S_2 b, S_2 \rightarrow bA, A \rightarrow aAb,$
$A \rightarrow ab\}$
[2]

Suppose if possible, that a finite state machine exist that accepts the sentences in the given language L. Suppose this machine has i states clearly the machine accepts $a^i b^{2i}$.. Starting from the initial state the machine will visit i states after receiving the i a's in the input sequence. [4]

Also $S_{j_{3i}}$, is the state of machine is in after receiving the sequence $a^i b^{2i}$. Clearly $S_{j_{3i}}$ is the accepting state. So by pigeon hole principle at least there are two states that are same. Suppose the machine visit S_K state twice. Let there are x b's between the second and first visit of the machine to the state S_K. [6]

Then the sequence $a^i b^{2i-x}$ is accepted by L, which is not a sentence in L. Hence it is not a finite state Language. [7]

(iii) Types of Grammar.

a) Type 0 unrestricted grammar:

There are no restriction on the production of a grammar of this type. $\alpha \to \beta$

eg. $V = \{A, B, C\}$, $T = \{a, b, c\}$

$G = \{V, T, P, A\}$

$P = \{A \to AB,\ AB \to BC,\ B \to ACD,$
$Ac \to abc,\ D \to \epsilon\}$ [2]

b) Type 1 context sensitive grammar.

Define above

eg.

$G = \{(A, B, C), (a, b), P, A\}$

$P = \{A \to AB,\ AB \to Ac,\ AC \to ab\}$ [4]

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c) Type 2 Context free grammar:
Defined above.
eg. $G = \{(S), \{a, b\}, P, S\}$

$P = \{S \to aSa \mid bSb \mid a \mid b\}$ [5]

d) Type 3 Regular grammar:
The LHS of each product contains one non terminal. The RHS can contain at most one non-terminal symbol, which is allowed to appear as the rightmost symbol or the left most symbol.

eg. $G = \{(S, B, C), (a, b), P, S\}$
$P = \{S \to Ca \mid Bb,\ C \to Bb,\ B \to Ba \mid b\}$
[7]

Q 6
(i) $L(G) = \{aa, aba, abba, abbba, \ldots\}$ [1]
The string aa is obtained considering zero occurences of the middle string b. The remaining string aba, abba, --- are obtained considering one, two or three occurences of b respectively. [3]

(ii) Let, if possible L be a regular language. Then there exists a finite state machine that accepts the sentences in L.
Assume that this finite machine has N states. Let $w = 0^N 1^N$. Then $w \in L$ and the

the length of w is $2N > N$. By pumping lemmas we can write $w = xyz$ with y non empty. We wish to find q such that $xy^qz \notin L$ for getting contradiction. Note that the string y can be in one of the following forms: [3]

Case I: $y = 0^k$ for some $k \geq 1$

We take $q = 0$. Then, by Pumping lemma $xy^0z = xz = 0^{N-k}1^N$ must belong to L. But $k \geq 1$ gives $N-k \neq N$ & so $xz \notin L$, a contradiction. [4]

Case II: $y = 0^k 1^j$ for some $k, j \geq 1$

We take $q = 2$. Then by Pumping lemma, xy^2z must belong to L. But $xy^2z = 0^{N-k}(0^k 1^j)^2 1^{N-j} = 0^N 1^j 0^k 1^N \notin L$ because xy^2z is not of the form $0^i 1^i$. [6]

Case III: $y = 1^k$ for some $k \geq 1$

We take $q = 0$. Then by pumping lemma xy^0z must belong to L. But $xy^0z = xz = 0^N 1^{N-k} \notin L$ because $N \neq N-k$. [7]

Thus in all the cases we arrive at contradiction. Hence L is not regular.

ii) The set $\{a\}$ is represented by the regular expression a. So $\{a\}$ is regular for every $a \in S$. As the class of regular sets is closed under union, concatenation and closure, R is contained in the class of regular sets. Let L be a regular set [3]

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then $L = T(M)$ for some DFA, $M = \{S_1, S_2, \ldots$ Σ, δ, S_0, F. But know that any set L accepted by finite automaton M is represented by a regular expression. [S

$$L = \bigcup_{j=1}^{n} P_1^{m} f_j$$

where $F = \{S_{f_1}, S_{f_2}, \ldots, S_{f_n}\}$ and $P_1^{m} f_j$ is obtained by applying union, concatenation and closure to singleton in Σ. Thus L is i R.

[7]