ii. The joint pdf of X and Y is given by **7**

$$f(x,y)=\begin{cases} xy^2+\dfrac{x^2}{8}; & 0\le x\le 2, 0\le y\le 1 \\ 0; & otherwise \end{cases}$$

Find

(a) Marginal density functions

(b) $P(X>1/Y<1/2)$

(c) $P(Y<1/2/X>1)$.

OR iii. A coin is tossed 10 times. Find the probability of getting 3 or 4 or 5 heads using CLT. Given that $P(0<z<1.58)$=0.4429 and $P(0<z<0.316)=0.1217$. **7**

Q.6 i. If θ is the angle between the two regression lines, show that **3**

$$\tan\theta=\frac{1-r^2}{r}\cdot\frac{\sigma_x\sigma_y}{\sigma_x{}^2+\sigma_y{}^2}.$$

ii. Fit a parabola $y=a+bx+cx^2$ to the following data: **7**

x:	2	4	6	8	10
y:	3.07	12.85	31.47	57.36	91.29

OR iii. From a random sample of 10 pigs fed on diet A, the increases in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increases in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 lbs. Test whether diets A and B differ significantly as regards their effect on increases in weight ? (Given $t_{0.05,20}=2.09$). **7**

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Total No. of Questions: 6 Total No. of Printed Pages:4

Enrollment No....................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2018

EC3BS03/EI3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: EC/EI

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. A graph with all vertices having zero degree is known as a ______ **1**

(a) Null graph (b) Multi graph

(c) Simple graph (d) Complete Graph

ii. The sum of the in-degrees over all vertices in any directed graph is equal to **1**

(a) Twice the number of edges

(b) The sum of the edges

(c) The sum of the out-degrees over all vertices

(d) None of these

iii. The total number of vertices in a binary tree is always **1**

(a) Even (b) Odd (c) Prime (d) 0

iv. In a network for each vertex which is not source or sink, then inflow ------ outflow **1**

(a) Is less than to (b) Is greater than to

(c) Is not equal to (d) Is equal to the

v. When $x=a$ is an arbitrary points of the equation $P_0(x)\dfrac{d^2y}{dx^2}+P_1(x)\dfrac{dy}{dx}+P_2(x)y=0$, its every solution y can be expressed in the form **1**

(a) $\sum_{k=1}^{\infty}a_k(x+a)^k$ (b) $\sum_{k=1}^{\infty}a_kx^k$

(c) $\sum_{k=1}^{\infty}a_k(x-a)^k$ (d) None of these

vi. $\frac{d}{dx}(x^{-n}J_n) =$ **1**

(a) $-x^{-n}J_n$ (b) $-x^{n}J_{n+1}$ (c) $x^{-n}J_{n+1}$ (d) $-x^{-n}J_{n+1}$

vii. If (X, Y) is two dimensional RV, then the value of joint characteristic function $\phi_{xy}(0,0) =$ **1**

(a) 0 (b) -1 (c) 1/2 (d) 1

viii. If X and Y are two continuous random variables with pdf $f(x,y) = k, 0 < x < y < 1,$ then the value of k is **1**

(a) 1/3 (b) 1 (c) 2 (d) -3

ix. The correlation coefficient is perfect and positive if $r =$ --------. **1**

(a) 1 (b) 0.1 (c) 0.4 (d) -1

x. Test to be applied when number of observations are less than 30 and variance is not known, is said to be **1**

(a) Chai square test (b) t-test

(c) z-test (d) f-test

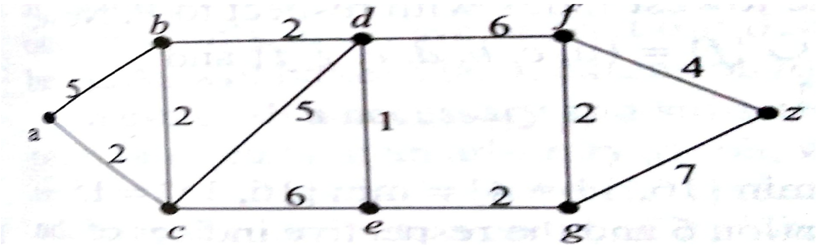
Q.2 i. Define a spanning subgraph with an example. **3**

ii. Prove that a simple disconnected graph G with vertices $'n'$ and $'k'$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges **7**

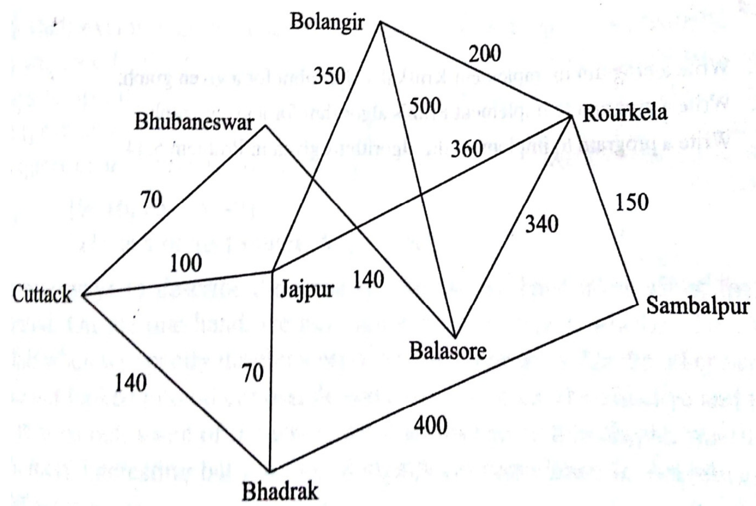
OR iii. Show that there is always a Hamiltonian path in a directed complete graph. **7**

Q.3 i. Show that a cut set and any spanning tree must have at least one edge in common. **3**

ii. Define length of path in a weighted graph and find shortest path form vertex $'a'$ to $'z'$ using Dijkstra' algorithm for the graph: **7**



OR iii. Using Prim's algorithm to find the minimum spanning tree for the weighted graph **7**



Q.4 i. Prove that **3**

$$J_{1/2} = \sqrt{\left(\frac{2}{\pi x}\right)} \sin x$$

ii. Obtain the series solution of the equation **7**

$$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0.$$

OR iii. Prove that **7**

$$\frac{d}{dx}(xber'x) = -xbei\ x \text{ and } \frac{d}{dx}(xbei'x) = xber\ x$$

Q.5 i. The joint probabilities of two discrete random variables X and Y have $P(X=0, Y=0) = 2/9, P(X=0, Y=1) = 1/9,$ **3**

$P(X=1, Y=0) = 1/9, P(X=1, Y=1) = 5/9.$

Test whether X and Y are independent.

EC3BS03 / EI3BS03.

Q.1 (i) (a) Null-graph +1

(ii) (C) the sum of the out-degree over all vertics +1

(iii) (b) odd +1

(iv) (d) is equal to the +1

(v) (d) None of these +1

(vi) (d) $-x^{-n} J_{n+1}(x)$ +1

(vii) (d) 1 +1

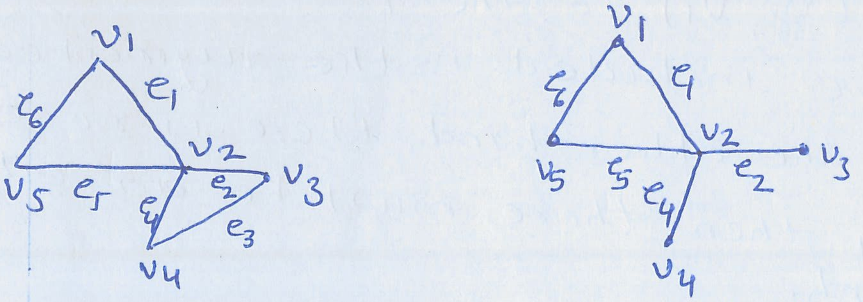
(viii) (c) 2 +1

(ix) (a) 1 +1

(x) (b) t-test +1

2 (i). A subgraph of a graph G is said to be a spanning subgraph if it contains all the vertices of G. (+2)

Ex.



$G=(V,E)$ $\qquad\qquad$ $H=(V',E')$ +(1)

Here $H=(V',E')$ is a spanning subgraph of G

(ii) Let the number of vertices in each of the K components of a graph G be $n_1, n_2, n_3, \cdots, n_K$. Thus we have $\sum_{i=1}^{K} n_i = n$, $n_i \geq 1$. (+1)

We have

$$\sum_{i=1}^{K} n_i^2 \leq n^2 - (K-1)(2n-K) \quad \text{—— (1)}$$ (+2)

Now the maximum number of edges in the i^{th} component of G (which is a simple connected graph) is $\frac{n_i(n_i-1)}{2}$. Therefore, the maximum number of edges in G is equal to +(1)

$$= \frac{1}{2}\sum_{i=1}^{K} n_i(n_i-1) = \frac{1}{2}\sum_{i=1}^{K} n_i^2 - \frac{n}{2}$$

$$\leq \frac{1}{2}(n-K)(n-K+1) \quad \text{(From (1))}$$ (+3)

2 (iii) Let there be a path with (P-1) edges in a directed complete graph which meets the sequence of vertices $(v_1, v_2, \cdots, v_p)$. Let v_x be a vertex that is not included in this path. If there is an edge (v_x, v_1)

in the graph.

We can augment the original path by adding the edge (v_x, v_1) to the path so that the vertex v_x will be included in the augmented path. (+1

If, on the other hand, there is no edge from v_x to v_1 then there must be an edge (v_1, v_x) in the graph. (+1

Suppose that (v_x, v_2) is also an edge in the graph. We can replace the edge (v_1, v_2) in the original path with the two edges (v_1, v_x) and (v_x, v_2) so that the vertex v_x will be included in the augmented path. On the other hand, if there is no edge from v_x to v_2, then there must be an edge (v_2, v_x) in the path and we can repeat the argument. +(2)

Eventually if we find that it is not possible to include the vertex v_x in any augmented path by replacing an edge (v_k, v_{k+1}) in the original path with two edges (v_k, v_x) and (v_x, v_{k+1}) with $1 \leq k \leq p-1$ then we conclude that there must be an edge (v_p, v_x) in the graph. +(2)

We can, therefore, augment the original path by adding to it the edge (v_p, v_x) so that the vertex v_x will be included in the augmented path.

We can repeat the argument until all vertices in the graph are included in a path. (+1)

(03)

.3

i) If there is a cut-set that has no common edge with a spanning tree, the removal of the cut-set will leave the spanning tree intact. However, this means that the removal of the cut-set will not separate the graph into two components which is in contradiction to the definition of a cut-set. +2 +1

ii) Initially, let $P = \{a\}$, $T = \{b, c, d, e, f, g, z\}$ Now

$l(b) = w(a,b) = 5$, $l(c) = w(a,c) = 2$, $l(d) = w(a,d) = \infty$,

$l(e) = w(a,e) = \infty$, $l(f) = w(a,f) = \infty$, $l(g) = w(a,g) = \infty$ +1

$l(z) = w(a,z) = \infty$

Iteration 1: Consider the vertex c of T as $l(c)$ is the lowest index w.r.t to P. Now

$P_1 = \{a, c\}$, $T_1 = T - \{c\} = \{b, d, e, f, g, z\}$

Then we compute

$l'(b) = 4$, $l'(d) = 7$, $l'(e) = 8$, $l'(f) = \infty$, $l'(g) = \infty$, +1

$l'(z) = \infty$

Iteration 2: $P_2 = \{a, c, b\}$, $T_2 = \{d, e, f, g, z\}$

Then we compute: $l'(d) = 6$, $l'(e) = 8$, $l'(f) = \infty$, $l'(g) = \infty$ +1

$l'(z) = \infty$

Iteration 3: $P_3 = \{a, c, b, d\}$, $T_3 = \{e, f, g, z\}$

Then we compute +1

$l'(e) = 7$, $l'(f) = 12$, $l'(g) = \infty$, $l'(z) = \infty$

Iteration 4 $P_4 = \{a, c, b, d, e\}$ $T_4 = \{f, g, z\}$

Then we compute $l'(f) = 12$, $l'(g) = 9$, $l'(z) = \infty$ +1

In a similar manner,

We get $l'(z) = 15$

Thus the minimum distance between a to z is 15 +2

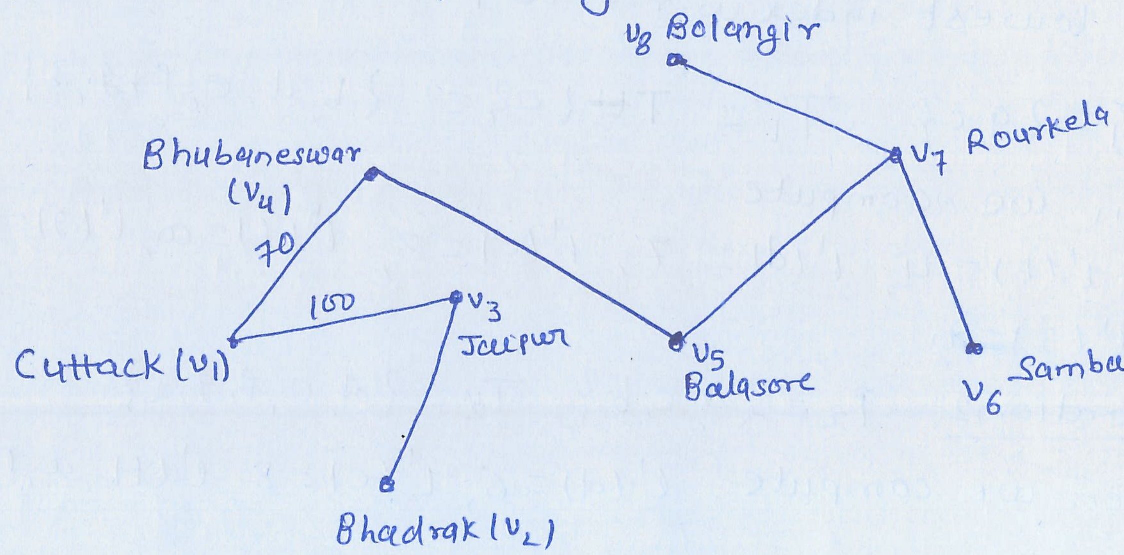
(04)

(iii) ~~Let~~ If ~~Bha~~ Cuttack, Bhadrak, Jaipur, Bhubaneswar, Balasore, Sambalpur, Rourkela, Bolangir are denoted by $v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8$ respectively +1

Now Adjacency matrix is

$$
\begin{array}{c}
\\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8
\end{array}
\begin{array}{c}
\begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\
\begin{bmatrix}
- & 140 & \cancel{100} & 70 & - & - & - & - \\
140 & - & 70 & - & - & 400 & - & - \\
100 & \cancel{70} & - & - & - & - & 360 & 350 \\
\cancel{70} & - & - & - & 140 & - & - & - \\
- & - & - & \cancel{140} & - & - & 340 & 500 \\
- & 400 & - & - & - & - & \cancel{150} & - \\
- & - & 360 & - & \cancel{340} & 150 & - & 200 \\
- & - & 350 & - & 500 & - & \cancel{200} & -
\end{bmatrix}
\end{array}
$$
+3

The minimum spanning tree is



+3

(i) The Bessel function of the first kind of order n denoted by

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\frac{x}{2}\right)^{n+2r} \frac{1}{r!\,\Gamma(n+r+1)}$$ +1

Put $n = \frac{1}{2}$. We have

$$J_{1/2}(x) = \frac{\sqrt{x}}{\sqrt{2}\,\Gamma(\frac{1}{2})} \left\{ \frac{2}{1!} - \frac{2x^2}{3!} + \frac{2x^4}{5!} - \cdots \right\} = \sqrt{\frac{2}{\pi x}}\, \sin x$$ +2

1(iii) Given diff. Eq. (5)

$$x(1-x)y'' - (1+3x)y' - y = 0 \quad \text{——} (1)$$

Here $x=0$ is a regular singular point

Substituting $y = \sum_{r=0}^{\infty} a_r x^{m+r}$, $y' = \sum_{r=0}^{\infty} a_r (m+r) x^{m+r-1}$

$y'' = \sum_{r=0}^{\infty} a_r (m+r)(m+r-1) x^{m+r-2}$ in (1). (+1)

We have

$$\sum_{r=0}^{\infty} a_r (m+r)(m+r-2) x^{m+r-1} - \sum_{r=0}^{\infty} a_r [(m+r)(m+r-1)$$

$$+ 3(m+r) + 1] x^{m+r} = 0 \quad \text{——} (2)$$ (+1)

Equating to zero, the coefficients of the lowest power of x, and $x^{m+\cancel{r}-1}$

$a_0 \cdot m \cdot (m-2) = 0 \Rightarrow m = 0, 2$ $[\because a_0 \neq 0]$

$$a_r (m+r)(m+r-2) - a_{r-1} [(m+r-1)(m+r-2) + 3(m+r-1) + 1] = 0$$

$$a_r = \frac{(m+r-1)(m+r-2) + 3(m+r-1) + 1}{(m+r)(m+r-2)} a_{r-1}$$

$$\Rightarrow a_r = \frac{(m+r-1)(m+r+1) + 1}{(m+r)(m+r-2)} a_{r-1}$$

$$\Rightarrow a_r = \frac{(m+r)^2}{(m+r)(m+r-2)} a_{r-1} = \frac{m+r}{m+r-2} a_{r-1} \quad \text{—} (3)$$

Put $r = 1, 2, 3, \ldots$, we get

(6)

$a_1 = \frac{m+1}{m-1} a_0, \quad a_2 = \frac{(m+1)(m+2)}{(m-1)m} a_0, \quad a_3 = \frac{(m+1)(m+2)(m+3)}{(m-1)m(m+1)} a_0$ etc (+2

Since $y = \sum_{r=0}^{\infty} a_r x^{m+r}$

$$y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + \cdots$$

$$y = a_0 x^m \left[1 + \frac{m+1}{m-1} x + \frac{(m+1)(m+2)}{(m-1)m} x^2 + \frac{(m+1)(m+2)(m+3)}{(m-1)m(m+1)} x^3 + \cdots \right]$$

If we put $m=0$ in this series, the coefficients become infinite. Put $a_0 = b_0(m-0)$ so that

$$y_1 = b_0 x^m \left[m + \frac{m(m+1)}{m-1} x + \frac{(m+1)(m+2)}{m-1} x^2 + \frac{(m+1)(m+2)(m+3)}{(m-1)(m+1)} x^3 + \cdots \right]$$ is a solution if $m=0$ (+1)

This gives only one solution and the second solution is $y_2 = \left(\frac{\partial y_1}{\partial m} \right)_{m=0}$

$$= y_1 \log x + b_0 x^m \left[1 + \frac{m^2-2m-1}{(m-1)^2} x + \frac{m^2-m-5}{(m-1)^2} x^2 + \frac{m^2-2m-11}{(m-1)^2} x^3 + \cdots \right]$$ when $m=0$

$$y_2 = (y_1)_0 \log x + b_0 \left[1 + x - 5x^2 - 11x^3 + \cdots \right]$$ (+1)

Hence the complete solution is

$$\boxed{y = c_1 y_1 + c_2 y_2}$$ (+1)

(iii) We know that

$$\operatorname{ber} x = 1 + \sum_{m=1}^{\infty} (-1)^m \frac{x^{4m}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m)^2} \quad \text{--- (1)} \qquad (+1)$$

$$\operatorname{bei} x = - \sum_{m=1}^{\infty} (-1)^m \frac{x^{4m-2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2} \quad \text{--- (2)} \qquad (+1)$$

Now

$$x \operatorname{ber}' x = \sum_{m=1}^{\infty} (-1)^m \frac{x^{4m}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m-2)^2 \, 4m} = -\int_0^{\infty} x \operatorname{bei} x \, dx \qquad (+2)$$

$$(+0.5)$$

$$\Rightarrow \boxed{\frac{d}{dx}[x \operatorname{ber}' x] = -x \operatorname{bei} x}$$

Again

$$\int_0^x x \operatorname{ber} x \, dx = \frac{x^2}{2} + \sum_{m=1}^{\infty} (-1)^m \frac{x^{4m+2}}{2^2 \cdot 4^2 \cdot 6^2 \cdots (4m)^2 (4m+2)}$$

$$(+2)$$

$$= x \operatorname{bei}' x$$

$$(+0.5)$$

$$\Rightarrow \frac{d}{dx}[x \operatorname{bei}' x] = x \operatorname{ber} x$$

(i)

$$E(X) = \sum x \, P(x) = 2/3 \qquad (+\tfrac{1}{2})$$

$$(+\tfrac{1}{2})$$

$$E(Y) = \sum y \cdot P(y) = 2/3$$

$$E(XY) = \sum xy \, P(x, y)$$

$$= 5/9 \qquad (+1)$$

Here $E(XY) \neq E(X) \cdot E(Y)$ (+1)

$\Rightarrow$ X, Y are not independent

5(ii)

Given

$$f(x,y) = \begin{cases} xy^2 + \frac{x^2}{8} ; & 0 \le x \le 2 ; 0 \le y \le 1 \\ 0 ; & \text{otherwise} \end{cases}$$

(a) The marginal density function of X is

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \frac{x}{3} + \frac{x^2}{8} ; \quad 0 \le x \le 2$$ (+1)

The marginal density function of Y is

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = 2y^2 + \frac{1}{3} ; \quad 0 \le y \le 1$$ +(1)

Now

(b) $$P(X>1) = \int_1^2 f(x)\,dx = \frac{19}{24}$$

and $$P(Y<\tfrac{1}{2}) = \int_0^{1/2} f(y)\,dy = \frac{1}{4}$$

~~(b)~~ $\cancel{P(X>1/Y<\tfrac{1}{2}) = \int_0^{1/2}\int_1^2 f(x,y)\,dx\,dy = \frac{5}{24}}$ ~~(+2)~~

(b)

$$P(X>1 | Y<\tfrac{1}{2}) = \frac{P(X>1, Y<\frac{1}{2})}{P(Y<\frac{1}{2})}$$

Now

$$P(X>1, Y<\tfrac{1}{2}) = \int_0^{1/2}\int_1^2 f(x,y)\,dx\,dy = \frac{5}{24}$$ (+3)

(+1)

$$P(X>1/Y<\tfrac{1}{2}) = \frac{5/24}{1/4} = \frac{5}{6}$$

(c) $$P(Y<\tfrac{1}{2}/X>1) = \frac{P(X>1, Y<1/2)}{P(X>1)} = \frac{5/24}{19/24} = \frac{5}{19}$$ (+1)

OK

(9)

(iii) Let p be the probability of getting head

$\therefore p = \frac{1}{2}, q = 1/2, n = 10$ (+1)

Thus, the random variable X is binomially distributed

$\therefore$ mean $\mu = np = 5$ and variance $\sigma^2 = npq = 2.5$

$\therefore \sigma = \sqrt{2.5}$ (+1)

Here $X \sim N(\mu, \sigma)$.

The standard normal variate $Z = \frac{x-5}{\sqrt{2.5}}$

$\therefore P(3 \text{ or } 4 \text{ or } 5) = P(2.5 < X < 5.5)$ (+1)

$= P\left(\frac{2.5-5}{\sqrt{2.5}} < Z < \frac{5.5-5}{\sqrt{2.5}}\right)$

$= P(0 < Z < 1.58) + P(0 < Z < 0.316)$

$= 0.4429 + 0.1217 = 0.5646$ (+4)

(i) The regression line of y on x is

$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$y = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) + \bar{y}$ — (1)

The line of regression of x on y is

$x - \bar{x} = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$

$\Rightarrow y - \bar{y} = \frac{1}{r} \cdot \frac{\sigma_y}{\sigma_x}(x - \bar{x})$ — (2) +2

So we have

$m_1 = r\frac{\sigma_y}{\sigma_x}, \quad m_2 = \frac{1}{r}\frac{\sigma_y}{\sigma_x}$.

If θ is the ~~ag~~ angle between (1) & (2)

then

$\tan\theta = \frac{m_1 \sim m_2}{1 + m_1 \cdot m_2}$

$= \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$ +1

(10)

6(ii) The eq. of parobola $y = a + bx + cx^2$ —(1)

The normal eq. are $\sum y = 5a + b\sum x + c\sum x^2$ —(2)

$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$ —(3)

$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$ —(4) (+1)

from the given data, we get $a = 0.34$,

$b = -0.78$, $c = 0.99$. +(5)

so $y = 0.34 - 0.78x + 0.99x^2$ +(1)

6(iii) $\bar{x} = 12$ lbs, $\bar{y} = 15$ lbs, $n_1 = 10$, $n_2 = 12$

$\sigma_s^2 = \left[\sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2\right] / (n_1 + n_2 - 2) = 21.1$

$\sigma_s = 4.65$ (+3)

Assuming that the samples do not differ in weight so far as the two diets are concerned i.e. (+1)

$\mu_1 - \mu_2 = 0$.

Hence $t = \dfrac{(\bar{y} - \bar{x}) - 0}{\sigma_s \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 1.6$ nearly (+1)

Here df $v = n_1 + n_2 - 2 = 20$

For $v = 20$, we find $t_{0.05} = 2.09$

$\therefore$ The calculated value of $t < t_{0.05}$

Hence the difference between the sample means is not significant i.e. the two diets do not differ significantly as regards their effect on increase in weight (+2)