ii. Write the mathematical model of assignment problem. **5** 1 2 1

iii. Find the assignment of salesmen to district that will result in maximization profit. **5** 3 7 3

	District				
Salesmen	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Q.6 Attempt any two:

i. Solve the following game whose pay-off matrix is given by- **5** 3 7, 8 4

		B's Strategy				
		B1	B2	B3	B4	B5
A's Strategy	A1	8	10	-3	-8	-12
	A2	3	6	0	6	12
	A3	7	5	-2	-8	17
	A4	-11	12	-10	10	20
	A5	-7	0	0	6	2

ii. In a game of matching coins, player A wins Rs. 5 if there are two tails, wins Rupee 1 if there are two heads and losses rupee 2 when there is one head and one tail. Determine pay-off matrix and best strategies for each player. **5** 3, 4 7, 8 4

iii. For the given pay-off matrix, use graphical method to find the value of the game. **5** 3 7, 8 4

	B_1	B_2	B_3
A_1	4	-1	0
A_2	-1	4	2

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Science / Engineering
End Sem Examination Dec 2024
CA3CO21 Operations Research

Programme: BCA/BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Marks BL PO CO PSO

Q.1 i. The origin of operations research was during ______. **1** 1 2, 3 1

(a) World War I (b) World War II
(c) Civil War (d) None of these

ii. Operations research attempts to find ______ solution to a problem. **1** 1 2, 3 1

(a) Perfect (b) Optimum
(c) Both (a) and (b) (d) None of these

iii. In simplex method, we add _____ variable when the constraint is in $\geq$ sign. **1** 2 2, 3 1

(a) Surplus (b) Artificial
(c) Slack (d) None of these

iv. Which of the following is a part of linear programming problem? **1** 1 2, 3 1

(a) Objective function (b) Constraints
(c) Restrictions (d) All of these

v. In transportation problem, modified distribution method is used to find _______ solution. **1** 1 7 1

(a) Initial basic feasible (b) Optimal
(c) Both (a) and (b) (d) None of these

vi. In transportation problem, if total supply is equal to total demand, then the problem is said to be ______ problem. **1** 1 7 1

(a) Balanced (b) Unbalanced
(c) Both (a) and (b) (d) None of these

vii. Hungarian method is used to solve _______ problem. **1** 1 2, 3 1
(a) Transportation (b) Job sequencing
(c) Assignment (d) None of these

viii. _____ is the time required by a job on each machine. **1** 1 2, 3 1
(a) Total elapsed time (b) Ideal time
(c) Processing time (d) None of these

ix. In a mixed strategy game- **1** 1 2, 3 1
(a) Saddle point exist
(b) No saddle point exist
(c) Can't say
(d) None of these

x. In game theory, if maximin value = minimax value=0, then the game is- **1** 1 2, 3 1
(a) Fair (b) Unfair
(c) Can't say (d) None of these

Q.2 Attempt any two:

i. Explain any 5 models in operations research. **5** 1 2, 3 1

ii. Define operations research. Also explain the importance of operations research in decision making. **5** 1 2, 3 1

iii. Explain any 5 scientific methodology in operations research. **5** 1 2, 3 1

Q.3 Attempt any two:

i. Explain the advantages of linear programming problem. **5** 1 2 1

ii. Solve by simplex method **5** 3 2 2

$$Max\ z = 3x_1 + 4x_2$$

$$x_1 + x_2 \leq 450$$

$$2x_1 + x_2 \leq 600$$

and

$$x_1, x_2 \geq 0$$

iii. A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in table below: **5** 3 2 2

Food type	Yield per unit			Cost per unit (Rs.)
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Formulate linear programming model for the above problem.

Q.4 Attempt any two:

i. Define unbalanced problem, feasible solution, degenerate solution, non-degenerate solution and optimum solution. **5** 1 2 1

ii. Find initial basic feasible solution by Vogel's approximations Method. **5** 3 7 2

	D_1	D_2	D_3	D_4	Supply
O_1	5	2	4	3	22
O_2	4	8	1	6	15
O_3	4	6	7	5	8
Demand	7	12	17	9	45

iii. Solve the following transportation problem by North-West Corner rule and Least cost Method. **5** 3 7 2

	D_1	D_2	D_3	D_4	D5	Supply
S_1	5	10	2	5	8	100
S_2	3	7	10	15	3	50
S_3	8	5	7	2	10	200
S_4	15	20	5	10	5	400
S_5	18	13	12	7	6	250
Demand	250	100	200	200	250	

Q.5 Attempt any two:

i. There are 5 jobs, each of which is to be processed through three machines A, B and C in order ABC. Processing time in hours are: **5** 5 7 4

Job	1	2	3	4	5
Machine A	3	8	7	5	4
Machine B	4	5	1	2	3
Machine C	7	9	5	6	10

Determine the optimum sequence for the five jobs and the minimum elapsed time. Also, find the ideal time for the three machines.

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1. Programme - BCA/BCA-MCA

	Marks
Q.1 MCQ	
i. b World War - II	+1
ii. b optimum	+1
iii. a ~~Sa Slack~~ Surplus	+1
iv. d All of these	+1
v. b Optimal	+1
vi. a Balanced	+1
vii. c Assignment	+1
viii. c Processing time	+1
ix. b No saddle point exist	+1
x. a Fair.	+1

Marks

Q.2 Attempt any two:-

Sol^n 2(i)

1. Iconic Models:- Iconic model is a pictorial representation of various aspects of a system. eg - Toys, miniature etc. +1

2. Analogue Models:- Analog models are small physical systems that has similar characteristics & work like a system it represent. eg - map. +1

3. Mathematical Models:- Mathematical model employ a set of mathematical symbols to represent the decision variables of the system. +1

4. Static Models:- Static model does not take time into account. eg - LPP. +1

5. Dynamic Model:- Dynamic models considers time as one of the important variable eg - replacement model. +1

* other models can also be considered.

Sol^n 2(ii)

Def^n → OR is the art of winning war without actually fighting it. +1

* other def^n can also be considered.

Marks

1. Importance of OR in decision-making.

(a) Improved / Better decisions :- OR models frequently yield actions ~~that~~ that do improve an intuitive decision making. Some times a situation may be so complicated that the human mind can ~~develop~~ never hope to assimilate all the important factors without the help of OR. +1

(b) Improved Control :- The management of big concerns find it much costly to provide continuous executive supervisions over routine decisions. An OR approach directs the executives to devote their attention to more pressing matters. +1

(c) Improved coordination :- Sometimes OR has been very useful in maintaining the law & order situation out of Chaos. +1

(d) Improved system :- OR study is also initiated to analyze a particular problem of decision making such as establishing a new warehouse. Later OR can be further developed into a system to be employed repeatedly. +1

Marks

Sol^n 2(iii)

(a) Linear Programming :- Linear programming deals with the optimization of a fut^n of Variables Known as objective fut^n, subject to a set of linear equ^{ns} or inequalities Known as constraints. The objective fut^n may be profit, cost, etc. +1

(b) Transportation Problem :- The transportation problem is Sub class of linear programming problem which deals with the distribution of goods from several points of supply to a no. of points of demand. +1

(c) Assignment Problem :- The problem deals with the allocation problem in which the objective is to assign 'n' no. of jobs to 'n' no. of persons at a minimum cost or time +1

(d) Job Sequencing problem; The Sequencing model involves the determination of an optimal order or sequence of performing a series of jobs by a no. of facilities that are arranged in a specific order so as to optimize the total time or cost +1

Marks

(e) Replacement Models:- These models are concerned with the problem of replacement of machines, individuals, capital assets, etc. due to their deteriorating efficiency, failure or break down +1

Q3 Attempt any two:-

Soln 3(i)

Advantages of LPP

1. It helps us in making the optimum utilization of productive resources. +1
2. The quality of decisions may also be improved by linear programming technique. +1
3. Provide practically soln. +1
4. In production processes, high lighting of bottlenecks is the most significant advantage of this technique. +2

Soln 3(iii)

$$\text{Max} Z = 3x_1 + 4x_2 + 0s_1 + 0s_2$$

s.t.

$$x_1 + x_2 + s_1 = 450$$

$$2x_1 + x_2 + s_2 = 600$$

& $x_1, x_2, s_1, s_2 \geq 0$. +1

Initial basic feasible soln:-

$x_1 = x_2 = 0$

$s_1 = 450, \ s_2 = 600$ +1

C_B	C_j / Basis	3 x_1	4 x_2	0 s_1	0 s_2	value of B.V	Min ratio	Marks
0	S_1	1	1	1	0	450	450	
0	S_2	2	1	0	1	600	600	+1
	Z_j	0	0	0	0			
	$C_j - Z_j$	3	4 ↑	0	0			

C_B	C_j / Basis	3 x_1	4 x_2	0 s_1	0 s_2		
0	x_2	0	1/2	1	-1/2	450	
4	S_2	2	1	0	1	600	+1
	Z_j	8	4	0	4		
	$C_j - Z_j$	-5	0	0	-4		

$\because$ all $C_j - Z_j \leq 0$,

So optimal sol^n is

$x_1 = 0, \quad x_2 = 450$

Max $Z = 1800$ Ans +1

No. | Marks

Sol^n 3(iii)

Let x_1, x_2, x_3, ~~x_4~~ be the number of units of ~~food type 1, 2, 3, 4~~ yield. +1

and

$$x_1, x_2, x_3, \cancel{x_4} \geq 0$$ +1

objective fut^n:-

$$\text{Min } Z = 800x_1 + 200x_2 + 700x_3$$

$$\cancel{\text{Min } Z = 45x_1 + 40x_2 + 85x_3 + 65x_4}$$ +1

s.t.

$$3x_1 + 2x_2 + 6x_3 \leq 45$$

$$4x_1 + 2x_2 + 4x_3 \leq 40$$ +2

$$8x_1 + 7x_2 + 7x_3 \leq 85$$

$$6x_1 + 5x_2 + 4x_3 \leq 65$$

& $x_1, x_2, x_3, \cancel{x_4} \geq 0$.

Q.4 Attempt any two:-

Sol^n 4(i)

(1) Unbalanced problem:- If total supply is not equal to total demand, then problem is said to be an unbalanced transportation problem. +1

$$\Sigma a_i \neq \Sigma b_j$$

(2) Feasible sol^n:- A set of non-negative allocations $x_{ij} \geq 0$ which satisfies the supply and demand limitations is called feasible sol^n to a transportation problem. +1

Marks

(3) Degenerate Solⁿ :- If the no. of positive allocations is less than m+n-1, then it is called as degenerate solⁿ +1

(4) Non-degenerate solⁿ :- If the no. of positive allocations $x_{ij} = m+n-1$ and are in independent position then it is called as Non-Degenerate Basic feasible Solⁿ. +1

(5) OPtimum solⁿ :- A feasible solⁿ which minimizes the total transportation cost is called optimal solⁿ. +1

Solⁿ 4 (ii)

	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3	P_4	P_5	P_6
O_1	5	2(12)	4(2)	3(8)	22/10/8/0	1	1	1	2	4	
O_2	4	8	1(15)	6	15/0	3	3				
O_3	4(7)	6	7	5(1)	8/7/0	1	1	1	1	4	
	7/0	12/0	17/2/0	9/1/0	45						
P_1	0	4 1	3	2							
P_2	0		3 2	2							
P_3	1		3 3	2							
P_4	1			2							
P_5	4			5 5							
P_6	4										

+4

opti

Marks

optimal soln is

$= (2\times12) + (4\times2) + (3\times8) + (1\times15) + (4\times7) + (5\times1)$ +1

$= 24 + 8 + 24 + 15 + 28 + 5$

$=$ Rs 104 Ans

Soln 4(iii)

Northwest Corner Rule

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	5(100)	10	2	5	8	100/0
S_2	3(50)	7	10	15	3	50/0
S_3	8(100)	5(100)	7	2	10	200/100/0
S_4	15	20	5(200)	10(200)	5	400/200/0
S_5	18	13	12	7	6(250)	250
Demand	250/150/100/0	100/0	200/0	200/0	250	1000

+2

Total cost NWCR $= (5\times100) + (3\times50) + (8\times100) + (5\times100) + (5\times200) + (10\times200) + (6\times250)$

$= 500 + 150 + 800 + 500 + 1000 + 2000 + 1500$

$= 6450$ +5

Marks

LCM

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	5	10	2(100)	5	8	100/0
S_2	3(50)	7	10	15	3	50/0
S_3	8	5	7	2(200)	10	200/0
S_4	15(50)	20	5(100)	10	5(250)	400/300/50/0
S_5	18 (150)	13 (100)	12	7	6	250/150
Demand	250/200/150	100/0	200/100/0	200/0	250/0	1000

+2

Total cost by LCM

$= (2\times100) + (3\times50) + (2\times200) + (15\times50) + (5\times100) + (5\times250) + (18\times150) + (13\times100)$

$= 200 + 150 + 400 + 750 + 500 + 1250 + 2700 + 1300$

$= 7250$

+0.5

Q.5 Attempt any two :-

Sol^n 5 (i)

Job	1	2	3	4	5
Machine A	3	8	7	5	4
Machine B	4	5	1	2	3
Machine C	7	9	5	6	10

Here $\text{Min } A_i = 3$, $\text{Max } B_i = 5$,
$\text{Min } C_i = 5$

+0.5

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[or]

Marks

Sol^n 4 (iii) Least cost Method

	D_1	D_2	D_3	D_4	D_5	
S_1	5	10	2(100)	5	8	100
S_2	3	7	16	15	3(50)	50
S_3	8	5	7	2(200)	10	200
S_4	15(100)	20	5(100)	10	(200) 5	400
S_5	18(50)	13(100)	12	7	6	250
	250	100	200	200	250	1000

Ans = 7750

Marks

Marks

$\because$ Min C_i = Max B_i

We can convert it into two machines and n jobs problem.

Consider two fictitions machines G and H.

So,

$G_i = A_i + B_i$, $H_i = B_i + C_i$

The optimal sequence ~~is~~ can be any one of the following.

| 4 | 1 | 5 | 2 | 3 | or | 1 | 4 | 5 | 2 | 3 |

or

| 1 | 5 | 4 | 2 | 3 | or | 5 | 1 | 4 | 2 | 3 | +1

or | 5 | 4 | 1 | 2 | 3 |

Job	Machine A Time in	Machine A Time out	Machine B Time in	Machine B Time out	Machine C Time in	Machine C Time out
4	0	5	5	7	7	13
1	5	8	8	12	13	20
5	8	12	12	15	20	30
2	12	20	20	25	30	39
3	20	27	27	28	39	44

+2

Minimum elapsed time = 44 hrs   +0.5

Idle time for machine A = 44 − 27 = 17 hrs

" " " B = (5 + 1 + 5 + 2) + (44 − 28) = 29 hrs   +1

" " " C = 7 hrs

Marks

Sol^n 5(ii) Let C_{ij} be the cost of assigning i^{th} person to j^{th} job.

Jobs → / persons ↓	D_1	D_2 ...	D_n	supply
S_1	C_{11}	C_{12}	C_{1n}	1
S_2	C_{21}	C_{22}	C_{2n}	1
⋮	⋮	⋮	⋮	⋮
S_m	C_{n1}	C_{n2}	C_{nm}	1
Demand.	1	1	1	N

+1

$$x_{ij} = \begin{cases} 0 = \text{if } i^{th} \text{ person is not assign to } j^{th} \text{ job.} \\ 1 = \text{if } i^{th} \text{ person is assign to } j^{th} \text{ job.} \end{cases}$$

+1

The problem now becomes.

$$\text{Min } Z = C_{11}x_{11} + C_{12}x_{12} + \dots + C_{1n}x_{1n} + C_{21}x_{21} + C_{22}x_{22} + \dots + C_{2n}x_{2n} + \dots + C_{m1}x_{m1} + \dots + C_{nn}x_{nn}.$$

+1

s.t.

$$\left.\begin{aligned} x_{11} + x_{12} + \dots + x_{1n} &= 1 \\ x_{21} + x_{22} + \dots + x_{2n} &= 1 \\ &\vdots \\ x_{n1} + x_{n2} + \dots + x_{nn} &= 1 \end{aligned}\right\} \text{1 job is assigned to } i^{th} \text{ person}$$

+1

$$\left.\begin{aligned} x_{11} + x_{21} + \dots + x_{n1} &= 1 \\ x_{12} + x_{22} + \dots + x_{nn} &= 1 \\ &\vdots \\ x_{1n} + x_{2n} + \dots + x_{nn} &= 1 \end{aligned}\right\} \text{1 person is assigned to } j^{th} \text{ job.}$$

+1

& $x_{ij} = 0$ or 1

Marks

Sol^n 5 (iii) District

Salesmen	A	B	C	D	E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

The given matrix is a square matrix
i.e. no. of rows = no. of col^ms. +0.5

First, convert maximization matrix into minimization matrix by substracting all the elements of profit matrix from highest profit i.e. 41.
So,

	A	B	C	D	E
1	9	3	1	13	1
2	1	17	13	20	5
3	0	14	8	11	4
4	19	3	0	5	5
5	12	8	1	6	2

+1

Reduce the matrix
Row Reduction

	A	B	C	D	E
1	8	2	0	12	0
2	0	16	12	19	4
3	0	14	8	11	4
4	19	3	0	5	5
5	11	7	0	5	1

+0.5

Marks

Column Reduction

	A	B	C	D	E
1	8	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$
2	$\boxed{0}$	14	12	14	4
3	$\cancel{0}$	12	8	6	4
4	19	1	$\boxed{0}$	$\cancel{0}$	5
5	11	5	$\cancel{0}$	$\boxed{0}$	1

∵ no. of assignment < order of matrix

A	B	C	D	E	
8	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$	
$\boxed{0}$	14	12	14	4	✓ 3
$\cancel{0}$	12	8	6	4	✓ 1
19	1	$\boxed{0}$	$\cancel{0}$	5	
11	5	$\cancel{0}$	$\boxed{0}$	1	
✓ 2					

+1

A	B	C	D	E
12	$\boxed{0}$	$\cancel{0}$	7	$\cancel{0}$
$\boxed{0}$	10	8	10	$\cancel{0}$
$\cancel{0}$	8	4	2	0
23	1	$\boxed{0}$	$\cancel{0}$	5
15	5	$\cancel{0}$	$\boxed{0}$	1

+1

no. of assignment = 5 = order of matrix

Marks

Salesmen	District	Profit
1	B	38
2	A	40
3	E	37
4	C	41
5	D	35
		191 Ans

+1

Q.6 Attempt any two:-

Soln. 6 (i)

	B_1	B_2	B_3	B_4	B_5	
A_1	$\boxed{8}$	10	-3	-8	-12	-12
A_2	3	6	$\boxed{0}$	6	12	0 Maximin value.
A_3	7	5	-2	-8	17	-8
A_4	-11	$\boxed{12}$	-10	$\boxed{10}$	$\boxed{20}$	-11
A_5	-7	0	0	6	2	-7
	8	12	0	10	20	

Minimax value

+3

Saddle point (A_2, B_3) +1

Strategy of player A is A_2
" " " B is B_3 +1

Value of the game = 0 = fair.

Marks

Sol^n 6(ii) The pay-off matrix ~~is~~ of A is

$$
\begin{array}{cc}
 & \text{Player B} \\
 & \begin{array}{cc} H & T \end{array} \\
\text{Player A} \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}
\end{array}
$$

Using oddments.

$$
\begin{array}{lccc}
 & \text{Player B} & & \\
 & \begin{array}{cc} H & T \end{array} & \text{odds} & \text{prob.} \\
\text{Player A} \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} & \begin{array}{c} 7 = |-2-5| \\ 3 = |1-(-2)| \end{array} & \begin{array}{c} 7/10 \\ 3/10 \end{array} \\
\text{odds} & \begin{array}{cc} 7 & 3 \\ = |-2-5| & = |1-(-2)| \end{array} & & \\
\text{Prob.} & \begin{array}{cc} 7/10 & 3/10 \end{array} & &
\end{array}
$$

+2

Strategy for Player A is $\left(\frac{7}{10}, \frac{3}{10}\right)$ +1

" " " B is $\left(\frac{7}{10}, \frac{3}{10}\right)$ +1

Value of the game $= \left(1 \times \frac{7}{10}\right) + \left(\frac{-2 \times 3}{10}\right)$

$= \frac{7}{10} - \frac{6}{10}$

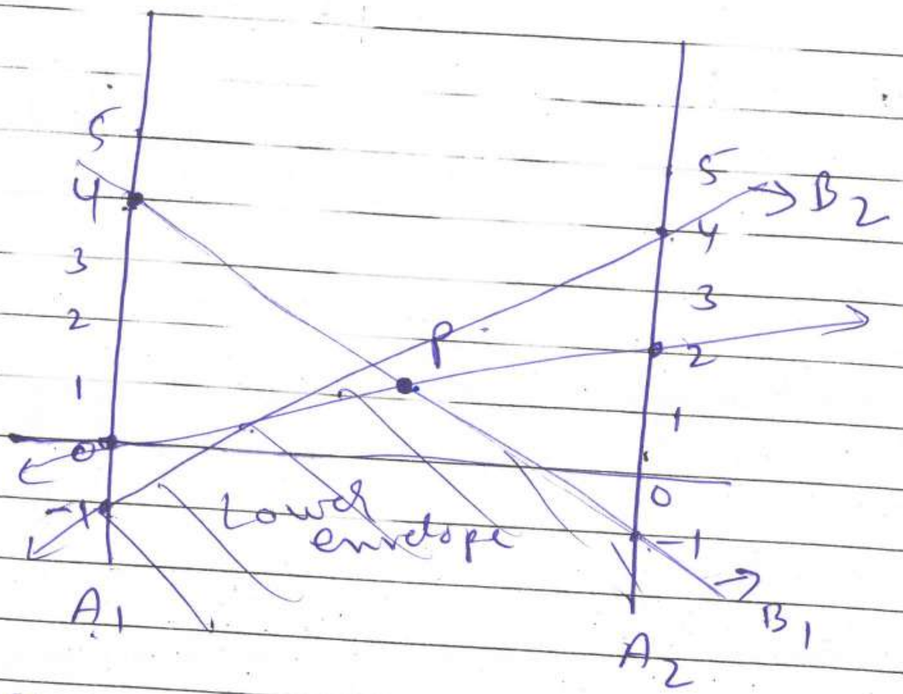
$= \frac{1}{10}$ Ans +1

Marks

Sol^n 6 (iii) The game has no saddle point

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 4 & -1 & 0 \\ A_2 & -1 & 4 & 2 \end{array}$$

1



+2

The highest expect gain is found at point P.

So, the pay-off matrix reduces to

$$\begin{array}{c|cc|c|c} & B_1 & B_3 & \text{odds} & \text{prob.} \\ \hline A_1 & 4 & 0 & |-1-2|=3 & 3/7 \\ A_2 & -1 & 2 & |4-0|=4 & 4/7 \end{array}$$

Odds $|0-2| = 2$ $\quad |4-(-1)| = 5$

+1

prob $\frac{2}{7}$ $\quad \frac{5}{7}$

Marks

optimal Strategy of Player $A(S_A) = \left(\frac{3}{7}, \frac{4}{7}\right)$

" " Player $B(S_B) = \left(\frac{2}{7}, 0, \frac{5}{7}\right)$ +1

Value of the game $= \left(\frac{4\times 2}{7}\right) + \left(\frac{0\times 5}{7}\right)$

$= \frac{8}{7}$ Ans +1