iii. Write the Legendre equation, if $P_n(x)$ is Legendre's function of first kind then show that $\int_{-1}^{1} P_n(x)P_m(x)dx = 0$ for $m \neq n$. **5**

Q.5 Attempt any two:

i. The joint pdf of X and Y is: **5**

$$f(x,y)=\begin{cases}\frac{6}{5}\left(x+y^2\right); & 0\le x\le 1, 0\le y\le 1\\ 0; & \text{otherwise}\end{cases}$$

find the marginal density functions and $P\left(\frac{1}{4}<y<\frac{3}{4}\right)$.

ii. The life time of certain brand of an electric bulb may be considered as a random variable with mean 1200 hours and standard deviations of 250 hours. Find probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1250hrs, given probability of standard normal variable Z ; $P(0\le Z\le 1.55)=0.4394$. **5**

iii. Describe the random process concept, Wide sense stationary, Strict Sense Stationary, with example. **5**

Q.6 Attempt any two:

i. Calculate the rank correlation coefficient for the following data **5**

x:	81	78	73	73	69	68	62	58
y:	10	12	18	18	18	22	20	24

ii. Explain the concept of hypothesis testing, null hypothesis, alternate hypothesis, level of significance and critical region. **5**

iii. Test made on breaking strength of 9 pieces of metal gave the following result :45, 47,50,52,48,47,49,53 and 51.Test if the mean breaking strength of the wire can be assumed to be 47.5 (Given $t_{0.05,8}=2.31$). **5**

******

Total No. of Questions: 6 Total No. of Printed Pages:4

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2018

EE3BS03/EX3BS03 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: EE/EX

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The size of '6' regular graph of 'n' vertex is equal to **1**

(a) $6n$ (b) $3n$ (c) $12n$ (d) $24n$

ii. Two graphs $G=\langle V,E\rangle$ and $G'=\langle V',E'\rangle$ are said to be isomorphic if $P:|V(G)|=|V'(G)|, Q:|E(G)|=|E'(G)|$ **1**

(a) Only P is sufficient (b) Only Q is sufficient

(c) Insufficient conditions (d) None of these.

iii. The total number of pendant vertices in a complete binary tree with 9 vertices are **1**

(a) 6 (b) 5 (c) 4 (d) 3

iv. The maximum value of flow from source 'S' to sink 'T' in G is ---------- value of capacities of all cuts in G from S to T **1**

(a) Maximum (b) Minimum

(c) Greater than the (d) Less than the

v. If $P_n(x)$ is the solutions of Legendre's polynomial **1**

$\int_{-1}^{1} P_5^2(x)dx =$ ____

(a) 1 (b) 2/15 (c) 0 (d) 2/11

vi. For the equation $P_0(x)\frac{d^2y}{dx^2}+P_1(x)\frac{dy}{dx}+P_2(x)y=0$ **1**

if $P_0(x)\neq 0$ at $x=0$ then $x=0$ is called

(a) Regular singular point (b) Irregular singular point

(c) Ordinary point (d) Can't say.

vii. The expectation of the product of two independent variables X and Y is equal to: **1**

(a) E(X) E(Y) (b) E(X) ± E(Y)

(c) E(X +Y) (d) None of these

viii. If (X, Y) is two dimensional RV, then the value of joint characteristic function $\phi_{xy}(0,0) =$ **1**

(a) 0 (b) -1 (c) 1 (d) 1/2

ix. If the value of regression coefficients $b_{xy} = 0.45$ and $b_{yx} = 1.25$, then the value of correlation coefficient r is equal to **1**

(a) 0.65 (b) 0.85 (c) 0.75 (d) 0.95

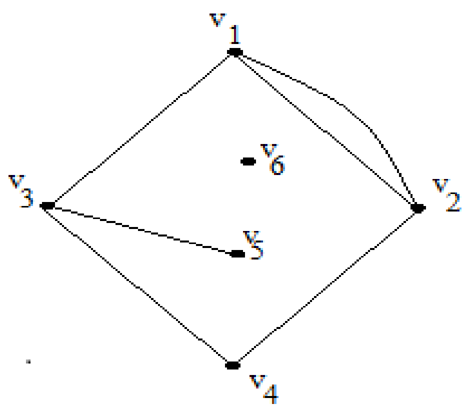
x. If we reject the null hypothesis when it is true , it is known as error of **1**

(a) Type I (b) Type II (c) Type III (d) Type IV

Q.2 i. Define Subgraph, vertex disjoint and edge disjoint subgraph of a graph G with example. **3**

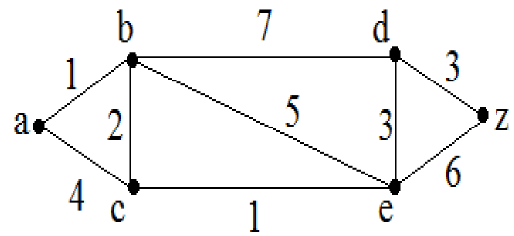
ii. Prove that a simple disconnected graph G with vertices $'n'$ and $'k'$ components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges. **7**

OR iii. Describe the use of matrix representation of graph and find the incidence matrix representation for the following graph **7**

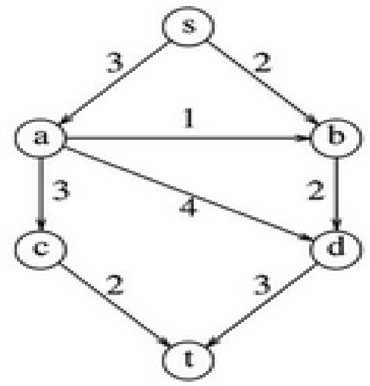


Q.3 i. Define minimally connected graph and prove that a graph is a tree if and only if it is minimally connected. **3**

ii. Define length of path in a weighted graph and find shortest path form vertex $'a'$ to $'z'$ using Dijkstra' algorithm for the graph: **7**



OR iii. In the following network, find out the cut set with minimum capacity and maximum possible flow using Ford Fulkerson's algorithm: **7**



Q.4 Attempt any two:

i. Find the series solution of the given differential equation **5**

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$$

ii. Prove that **5**

$$xJ_n' = nJ_n - xJ_{n+1} \text{ and } \frac{d}{dx}(x^{-n}J_n) = -x^{-n}J_{n+1}$$

Where $J_n(x)$ is Bessel's function of first kind.

Solution Set

End - Sem Examination May 2018

EE3BS03 / EX3BS03

Mathematics – III

Q.1

(i) (b). $3n$

(ii) (c). insufficient conditions

(iii) (b) 5

(iv) (b) minimum

(v) (d) $2/11$

(vi) (c) Ordinary point

(vii) (a) $E(X)\,E(Y)$

(viii) (c) 1

(ix) (c) 0.75

(x) (a) Type I

Q.2

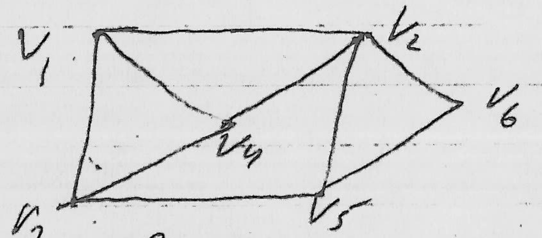
Definitions — 2 marks

Subgraph : $g(v, e)$ is subgraph of $G(V, E)$ if $v \subseteq V$ $e \subseteq E$

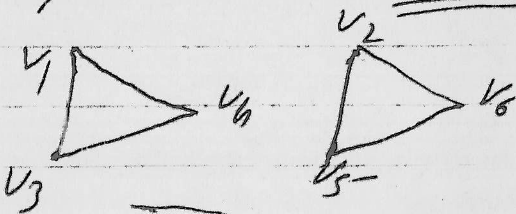
Vertex disjoint subgraph

$G_1(V_1, E_1)$, $G_2(V_2, E_2)$ are vertex disjoint subgraph of $G = \langle V, E\rangle$ if $V_1 \cap V_2 = \phi$

Edge disjoint subgraph if $E_1 \cap E_2 = \phi$ — 1 marks



Graph G



Vertex disjoint and edge disjoint subgraphs

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PAGE ______

Q-2

(ii) $G(V,E)$ be a k component Graph. Let $n_1, n_2 \text{ --- } n_k$ be number of vertices in each of component respectively. If n: no. of vertices in G, then

$$n_1 + n_2 \text{ --- } + n_k = n \qquad \text{1 mark}$$

Consider

$$\sum_{i=1}^{k} (n_i - 1) = n - k$$

Squaring both the sides, we get

$$\sum_{i=1}^{k} (n_i^2 - 2n_i) + k + \text{non negative term} = n^2 + k^2 - 2nk$$

$$\Rightarrow \sum_{i=1}^{k} n_i^2 \le n^2 + k^2 - 2nk + 2n - k \qquad + 2 \text{ marks}$$

Now maximum possible edges in a simple connected graph with n vertices is $\frac{n(n-1)}{2}$

$\Rightarrow$ Maximum possible edges in G $\qquad + 2$ marks

$$= \sum_{i=1}^{k} \frac{n_i(n_i-1)}{2}$$

$$= \frac{1}{2} \sum n_i^2 - \frac{n}{2}$$

$$\le \frac{1}{2} (n^2 + k^2 - 2nk + 2n - k) - \frac{n}{2}$$

$$= \frac{1}{2} (n-k)(n-k+1) \qquad + 2 \text{ marks}$$

7 marks

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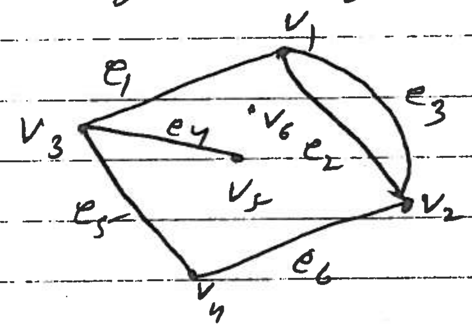
Q2 (iii)

Student will discribe the use of matrix representation from programming point of view with observation based on type, degree etc.

(2) marks



+(4)

$$
\begin{array}{c|cccccc}
 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 \\
\hline
e_1 & 1 & 0 & 1 & 0 & 0 & 0 \\
e_2 & 1 & 1 & 0 & 0 & 0 & 0 \\
e_3 & 1 & 1 & 0 & 0 & 0 & 0 \\
e_4 & 0 & 0 & 1 & 0 & 1 & 0 \\
e_5 & 0 & 0 & 1 & 1 & 0 & 0 \\
e_6 & 0 & 1 & 0 & 1 & 0 & 0
\end{array}
$$

7 marks

Q3 (i) Defination — 1 marks

Proof — 2 marks

Defination :- A graph G is minimally connected if removal of one edge makes the graph dis connected.

Pf. Let G (V, E) be minimally connected thus removal of any one edge makes it disconnected ⇒ there

5

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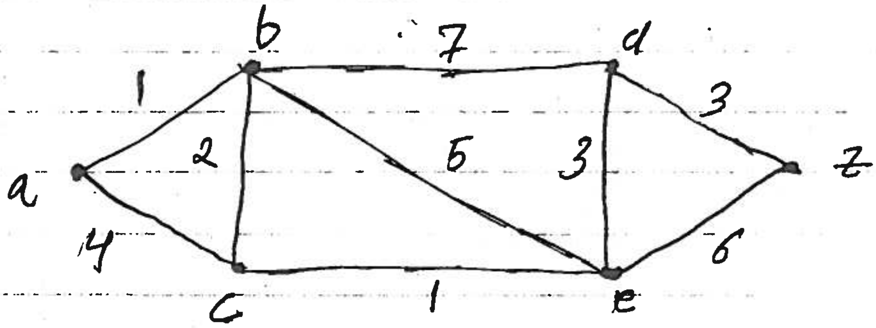
is no circuit the graph
$\Rightarrow$ G(V, E) be a tree

$\Leftarrow$ Let G(V, E) be a tree
$\Rightarrow$ there exist one and only one path between every pair of vertices
$\Rightarrow$ Removal of any one edge makes it disconnected
$\Rightarrow$ G(V, E) is minimally connected

(ii)



Step I PL = {a}   TL = {b, c, d, e, z}   (1)

	a	b	c	d	e	z
lb	$\boxed{0}$	1	4	∞	∞	∞

min lb : b is 1

Step II PL = {a, b}   TL = {c, d, e, z}
* New lb (u) = min { old lb(u), latest per lb(V) + dis (u, V) }

$\Rightarrow$ New lb (c) = min (old lb (c), lb (b) + dis (b, c))   (+1)
= min (4, 1+2) = 3
lb (d) = min (old lb (d), lb (b) + dis (b, d))

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$= \min(\infty, 1+7) = 8$

$lb(e) = \min(\infty, 1+5) = 6$
$lb(z) = \min(\infty, 1+\infty) = \infty$

a	b	c	d	e	z
$\boxed{0}$	$\boxed{1}$	3	8	6	∞

(+1)

min lb : c is 3

Step III $PL = \{a, b, c\}$ $TL = \{d, e, z\}$

$lb(d) = \min(8, 3+\infty) = 8$
$lb(e) = \min(6, 3+1) = 4$
$lb(z) = \min(\infty, \infty) = \infty$

a	b	c	d	e	z
$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	8	4	∞

min lb : e is 4 (+1)

Step IV $PL = \{a, b, c, e\}$ $TL = \{d, z\}$

$lb(d) = \min(8, 4+3) = 7$
$lb(z) = \min(\infty, 4+6) = 10$

a	b	c	d	e	z
$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	7	$\boxed{4}$	10

(+1)

min lb : d is 8

Step V $PL = \{a, b, c, e, d\}$ $TL = \{z\}$

$lb(z) = (10, 8+3) = 10$

a	b	c	d	e	z
$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{7}$	$\boxed{4}$	10

(+1)

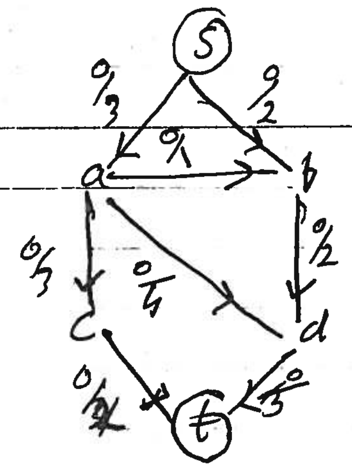
min z : 10 PL : $\{a, b, c, d, e, z\}$

Ans (+1)

Shortest distance between 'a' to 'z' is $\underline{10}$.

7 marks

(iii)



Assume initial flow is zero (1)

Step I : Consider flow augmenting path

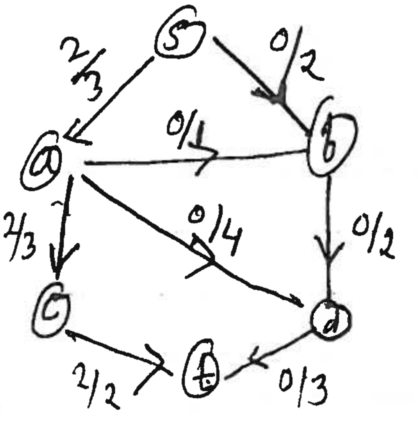
$$S \xrightarrow{0/3} a \xrightarrow{0/3} c \xrightarrow{0/2} t$$

$\Delta_1 = \min(3-0,\ 3-0,\ 2-0) = 2$

$\Delta_2 = NA$

$\Delta = \min(\Delta_1, \Delta_2) = 2$ (Δ Bottle neck capacity)

Out put



(+2)

Step II Consider flow augmenting path

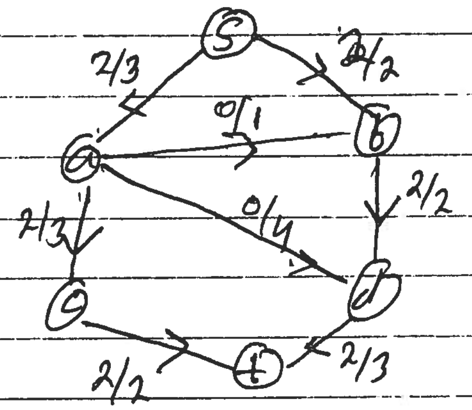
$$S \xrightarrow{0/2} b \xrightarrow{0/2} d \xrightarrow{0/3} t$$

$\Delta_1 = \min(2-0,\ 2-0,\ 3-0) = 2$

$\Delta_2 = NA$

$\Delta = \min(\Delta_1, \Delta_2) = 2$

Output



+2

Step III Consider flow augmenting path

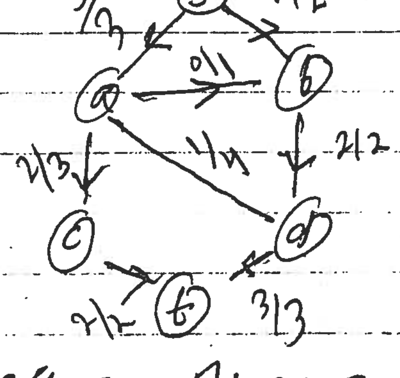
$$S \xrightarrow{2/3} a \xrightarrow{0/4} d \xrightarrow{2/3} t$$

$\Delta_1 = \min(3-2,\ 4-0,\ 3-2) = 1$

$\Delta_2 = NA$

$\Delta = \min(\Delta_1, \Delta_2) = 1$

Output



No further Augmentation is possible

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$\Rightarrow$ max possible flow of the given network $= \underline{\underline{5}}$

min cut set
$\{S\} \; \{a, b, c, d, t\}$ (+2)

min capacity $= \text{cap}(s, a) + \text{cap}(s, b)$
$= 3 + 2 = 5$

max flow $=$ min cap.

7 marks

Q4 (1) Solve $9x(1-x)\dfrac{d^2y}{dx^2} - 12\dfrac{dy}{dx} + 4y = 0$

$x = 0$ is regular singular point.
Let the soln be
$$y = \sum_{k=0}^{\infty} a_k x^{m+k}$$

substituting y, y' and y'' we get

$$9x(1-x) \sum_{k=0}^{\infty} (m+k)(m+k-1)\, x^{m+k-2}$$
$$-12 \sum_{k=0}^{\infty} (m+k)\, x^{m+k-1} + 4 \sum_{k=0}^{\infty} a_k x^{m+k} = 0$$

(+1)

$$\Rightarrow \sum_{k=0}^{\infty} a_k (3m+3k-4)(3m+3k+1)\, x^{m+k}$$
$$- \sum_{k=0}^{\infty} a_k\, 3(m+k)(3m+3k-7)\, x^{m+k-1} = 0$$

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Eq coefficient of lowest degree term (i.e. x^{m-1}) = 0 we get Indicial eq

$$-3a_0(m)(3m-7)=0$$

$$\Rightarrow m=0 \quad m=7/3$$ +2

the solution is given by

$$y = c_1 (y)_{m=0} + c_2 (y)_{m=7/3}$$

Eq coeff of ~~highest~~ deg term = 0 we get

$$a_{k+1} = \frac{(3m+3k-4)(3m+3k+1)}{3(m+k+1)(3m+3k-4)} a_k$$

$$\Rightarrow \boxed{a_{k+1} = \frac{(3m+3k+1)}{3(m+k+1)} a_k}$$ +1

$$\Rightarrow a_1 = \frac{3m+1}{3(m+1)} a_0$$

~~$m=0$ $m=7/3$~~
~~$a_1 = \frac{1}{3} a_0$~~

$$a_2 = \frac{(3m+4)(3m+1)}{3^2(m+2)(m+1)} a_0$$

~~$y=$~~

	$m=0$	$m=7/3$
a_1	$\frac{1}{3} a_0$	$8/10\, a_0$
a_2	$\frac{1}{3}\frac{4}{6} a_0$	$8/10 \cdot \frac{11}{13} a_0$
a_3	$\frac{1}{3}\cdot\frac{4}{6}\cdot\frac{7}{6} a_0$	$8/10 \cdot 11/13 \cdot 14/16\, a_0$

+1

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$$y = a_0\left[1 + \frac{1}{3}x + \frac{1\cdot 4}{3\cdot 6}x^2 \dots\right]$$
$$+ a_0 x^{7/3}\left[1 + \frac{8}{10}x + \frac{8}{10}\cdot\frac{11}{13}x^2 \dots\right]$$

(4)

$\frac{1}{5}$ marks

(ii) Prove that

$$x J_n' = n J_n - x J_{n+1}$$

and $\frac{d}{dx}(x^{-n} J_n) = -x^{-n} J_{n+1}$

We know

$$J_n(x) = \sum_{r=0}^{\infty} (-1)^r \frac{1}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

(1) mark

$$x J_n'(x) = \sum_{r=0}^{\infty} \frac{(-1)^r (n+2r)}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$$

$$= n J_n(x) + \sum_{r=0}^{\infty} \frac{(-1)^r\, 2r}{r!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)\left(\frac{x}{2}\right)^{n+2r-1}$$

(+1)

$$= n J_n(x) + x \sum_{r=1}^{\infty} \frac{(-1)^r}{(r-1)!\,\Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r-1}$$

put $r-1 = s$

(+1)

$$= n J_n(x) - x \sum_{s=0}^{\infty} \frac{(-1)^s}{s!\,\Gamma(n+1+s+1)} \left(\frac{x}{2}\right)^{(n+1)+2s}$$

(+1)

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$\Rightarrow x J_n'(x) = n J_n(x) - x J_{n+1}(x)$. (+3)

Multiplying by x^{-n-1}, we get

$$x^{-n} J_n'(x) = n x^{-n-1} J_n(x) - x^{-n} J_{n+1}(x)$$

$$\Rightarrow x^{-n} J_n'(x) - n x^{-n-1} J_n(x) = - x^{-n} J_{n+1}(x)$$

$$\Rightarrow \frac{d}{dx}\left(x^{-n} J_n(x) \right) = - x^{-n} J_{n+1}(x)$$

+2

5 marks

(ii) Eq: $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ (+1)

To prove

$$\int_{-1}^{1} P_m(x) P_n(x)\, dx = 0 \quad , n \neq m$$

We know

$y = P_n(x)$ is soln of

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 \quad -(1)$$

and

$z = P_m(x)$ is soln of

$$(1-x^2)\frac{d^2z}{dx^2} - 2x\frac{dz}{dx} + n(n+1)z = 0 \quad -(2)$$

(+1)

Eq(1) $\times z$ − Eq(2) $\times y$ $\Rightarrow$

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$$\frac{d}{dx}\left[(1-x^2)\left(z\frac{dy}{dx}-y\frac{dz}{dx}\right)\right]$$

$$+[n(n+1)-m(m+1)]\,yz=0 \quad (+1)$$

Integrating between -1 to 1, we get

$$(1-x^2)\left(z\frac{dy}{dx}-y\frac{dz}{dx}\right)\Big|_{-1}^{1}+[n(n+1)-m(m+1)]\int_{-1}^{1}yz\,dx=0$$

$$\Rightarrow\ 0+[n(n+1)-m(m+1)]\int_{-1}^{1}yz\,dx=0 \quad (+1)$$

$$\Rightarrow\ [n(n+1)-m(m+1)]\int_{-1}^{1}yz\,dx=0$$

$n \neq m$

$$\Rightarrow\ \int_{-1}^{1}yz\,dx=0$$

i.e $\int_{-1}^{1}P_n(x)\,P_m(x)\,dx=0 \quad n\neq m$

(+1)

5 marks

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Q 5
(i)

P.d.f of X and Y is

$$f(x,y) = \begin{cases} 6/5\,(x+y^2) & ; \; 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ; \; \text{otherwise} \end{cases}$$

marginal function

: for x

$$f(x) = \int_0^1 6/5\,(x+y^2)\,dy$$

$$= 6/5\,(x + 1/3)$$ (+1)

; for y

$$f(y) = \int_0^1 6/5\,(x+y^2)\,dx$$

$$= 6/5\,(1/2 + y^2)$$ (+1)

$$P(1/4 < y < 3/4)$$

$$= \int_{1/4}^{3/4} f(y)\,dy$$

$$= 6/5 \int_{1/4}^{3/4} (1/2 + y^2)\,dy$$

$$= 0.4625$$ (+3)

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Q.5

(ii) Let $\bar{x}$ be the average lifetime of 60 bulbs.

given $n = 60$ $\mu = 1200$ $\sigma = 250$

$\bar{X}$ Normally distributed with mean μ and S.D $\sigma/\sqrt{n}$

$$\bar{X} \sim N(\mu, \sigma/\sqrt{n}) \quad (+1)$$

$$\sim N(1200, 32.27)$$

Now $z = \frac{x - \mu}{\sigma} = \frac{X - 1200}{32.27}$ (+1)

To find

$$P(\bar{X} > 1250)$$

$X = 1250 \Rightarrow z = 1.55$ (+1)

$\Rightarrow$

$$P(\bar{X} \geq 1250) = P(z > 1.55)$$

$$= 0.5 - P(0 < z < 1.55)$$

$$= 0.5 - 0.4394$$

$$= 0.0606 \quad (+2)$$

5 marks

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(iii) Random process concept +2.5

Wide sense stationary
strict sense stationary +2.5

5 marks

Q.6

Formula

$$r = 1 - \frac{6\left[\Sigma d_i^2 + \frac{1}{12}\Sigma(m_i^3 - m_i)\right]}{n(n^2-1)}$$

x	y	Ran X	Ran y	d_i^2
81	10	1	8	49
78	12	2	7	25
73	18	3.5	5	2.25
73	18	2.5	5	2.25
69	18	5	5	0
68	22	6	2	16
62	20	7	3	16
58	24	8	1	49

$\Sigma d_i^2 = 159.5$

$$r = 1 - \frac{6[159.5 + 2.5]}{8(8^2-1)}$$

$$= 1 - \frac{6 \times 162}{8 \times 63} = -$$

$$= -0.92$$

H_0 :- Let the population = sample mean

(iii)

x	$x-\bar{x}$	$(x-\bar{x})^2$
45	-4.1	16.81
47	-2.1	4.41
50	0.9	0.81
52	2.9	1.21
48	-1.1	4.41
47	-2.1	0.01
49	-0.1	15.21
53	3.9	3.61

$\bar{x} = 49.1$ $\qquad \Sigma(x-\bar{x})^2 = 54.81$

$$t = \frac{x-\mu}{s}\sqrt{n} \qquad \mu = 47.5$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = 6.8612$$

$$s = 2.6193$$

$$t = 1.8325$$

$$< t_{tab.} = 2.31$$

Accept the null Hypothesis