Q.6 i. A branch of Punjab National Bank has only one typist. Since the typing work varies in length, the typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour .The letters arrive at a rate of 5 per hour during the entire 8 hour work day. If the typewriter is valued at rupees 150 per hour. Determine **4**

(a) Equipment utilization and the percent time that an arriving letter has to wait.

(b) Average cost due to waiting on the part of the typewriter i.e. it remaining idle.

ii. The price of an equity share of a company may increase, decrease or remain constant on any given day. It is assumed that change in price in any day affects the change on the following day as described by the following transition matrix **6**

Change tomorrow

$$\text{change today}\quad \begin{array}{c c} & \begin{array}{ccc} I & D & C \end{array} \\ \begin{array}{c} I \\ D \\ C \end{array} & \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.7 & 0.1 & 0.2 \\ 0.4 & 0.5 & 0.1 \end{bmatrix} \end{array}$$

Here I-increase, D-decrease, C-constant. Now find

(a) If the price of share increased today, what are these chances it will increase decrease or remain unchanged tomorrow?

(b) If the price of share remains unchanged today, what are the chances it will increase, decrease or remain unchanged the day after tomorrow?

OR iii Define following **6**

(a) Service disciplines of a queuing system

(b) Kendall's notations for classification of queuing system.

(c) Markov Process, Transition Probability Matrix.

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2018

CS3BS06 Engineering Mathematics-III

Programme: B.Tech. Branch/Specialisation: CS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If u and v are harmonic functions then function $f(z)=u+iv$ is **1**

(a) Analytic (b) Need not be analytic

(c) Analytic only at z = 0 (d) None of these

ii. If in Laurrent's expansion, the principal part of f(z) contains an infinite number of non zero terms of (z-(a) then z = a is known **1**

(a) Poles (b) Isolated Singularity

(c) Essential Singularity (d) Removable Singularity

iii. In Sterling's central difference formula if $\frac{x-x_0}{h}=u$ where x_0 is central value and h is interval ,then **1**

(a) $0<u<1$ (b) $\frac{-1}{4}<u<\frac{1}{4}$

(c) $\frac{1}{4}<u<\frac{3}{4}$ (d) None of these

iv. If Δ is forward difference operator then , Δk equals to , here k is constant, **1**

(a) 1 (b) 0 (c) f(k)- f(0) (d) None of these

v. The convergence of which of the following method is sensitive to starting value?

(a) False position (b) Gauss seidal method

(c) Newton-Raphson method (d) None of these

vi. In the Gauss elimination method for solving a system of linear algebraic equations, triangularzation leads to **1**

(a) Diagonal matrix (b) Lower triangular matrix

(c) Upper triangular matrix (d) Singular matrix

vii. The innovative science of Operations Research was discovered during --- **1**

(a) World War I (b) World War II

(c) Civil War (d) Industrial Revolution

viii. Any feasible solution which minimizes or maximizes the objective function of the Linear Programming Problem is called its ---------------- **1**

(a) Basic solution (b) Optimum solution

(c) Solution (d) None of these

ix. Which of the following is not one of the assumptions of Markov analysis **1**

(a) There are a limited number of possible states

(b) A future state can be predicted from the preceding one

(c) There are limited number of future periods

(d) All of these

x. The operations Research technique which helps in minimizing total waiting and service costs is **1**

(a) Queuing Theory (b) Decision Theory

(c) Both (a) and (b) (d) None of these

Q.2 i. State and prove Cauchy integral formula. **4**

ii. Define regular function .If $f(z)$ is regular function of z, Prove that **6**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

OR iii. Apply the calculus of residues, evaluate **6**

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}dx$$

Q.3 i. Find the missing values in the following table **4**

x	45	50	55	60	65
y	3.0	----	2.0	----	-2.4

ii From the following table, estimate the number of students who obtained marks between 40 and 45 **6**

Marks	30-40	40-50	50-60	60-70	70-80
Number of Students	31	42	51	35	31

OR iii Use Newton's divided difference interpolation formula to fit a polynomial to the data : **6**

x :	-4	-1	0	2	5
f(x) :	1245	33	5	9	1335

Q.4 i. Perform two iterations of Picard's method to find an approximate solution of $y' = x + y^2, y(0) = 1$ **4**

ii. Apply Gauss-Seidel method to solve the system of equation **6**

$6x + y + z = 105;\ 4x + 8y + 3z = 155;\ 5x + 4y - 10z = 65.$

OR iii. Given the initial value problem: $y' = x - y^2, y(0) = 0$, Find $y(1)$ by Milne's Predictor corrector method taking step size as 0.2. **6**

Q.5 i. Explain any two characteristics and two applications of Operations Research. **4**

ii. Solve the following linear programming problem by simplex method : **6**

Maximize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to
$$2x_1 + 3x_2 + 2x_3 \le 440$$
$$4x_1 + 3x_3 \le 470$$
$$2x_1 + 5x_2 \le 430 \quad \text{and } x_1, x_2, x_3 \ge 0$$

OR iii. Use Big-M method to solve the following linear programming problem : **6**

Minimize $Z = 12x_1 + 20x_2$

Subject to $6x_1 + 8x_2 \ge 100$

$7x_1 + 12x_2 \ge 120$ *and* $x_1, x_2 \ge 0$

Total No. of Questions: 6	Total No. of Printed Pages:4
	Enrollment No......................................
MEDI-CAPS UNIVERSITY Knowledge Is Power	Faculty of Engineering End Sem (Even) Examination May-2018 CS3BS06 Engineering Mathematics-III
Programme: B.Tech.	Branch/Specialisation: CS

Duration: 3 Hrs. **Maximum Marks: 60**

Solutions

Marking Scheme

Q.1	i.	a) analytic	1
	ii.	c) Essential Singularity	1
	iii.	b) $\frac{-1}{4} < u < \frac{1}{4}$	1
	iv.	b) 0	1
	v.	c) Newton-Raphson method	1
	vi.	c) Upper triangular matrix	1
	vii.	b) World War II	1
	viii.	b) optimum solution	1
	ix.	c) There are limited number of future periods	1
	x.	a) Queuing Theory	1

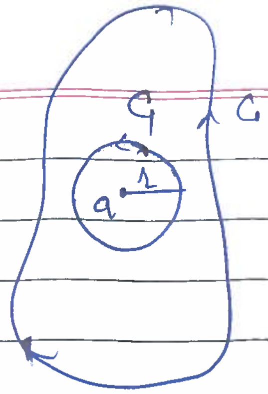
Q2.(1) Cauchy Integral formula.

If $f(z)$ is analytic within and on a closed curve C, then $f(a) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)}\,dz$ where a is any pt inside C. +1

Consider $\frac{f(z)}{z-a}$, which is analytic at every pt within C. except at $z=a$

Let us consider a circle C_1 ; $|z-a| = r$

s.t. C_1 lies inside C.



$\because \frac{f(z)}{z-a}$ is analytic in region between C and C_1.

By using Cauchy Extension Th.

$$\int_C \frac{f(z)}{(z-a)}\,dz = \int_{C_1} \frac{f(z)}{(z-a)}\,dz$$

$\because C_1 : |z-a| = r$ or $z = a + re^{i\theta}$.

$\Rightarrow dz = ire^{i\theta}d\theta$ & $\theta \to 0$ to 2π

$$\int_C \frac{f(z)}{(z-a)}\,dz = \int_{C_1} \frac{f(z)}{(z-a)}\,dz$$

$$= \int_0^{2\pi} \frac{f(a+re^{i\theta})}{re^{i\theta}}\, ire^{i\theta}\,d\theta. \quad \text{--- (2)}$$

+2

in limiting form C_1 shrink to point 'a' i.e. $r \to 0$

(2) becomes $\int_C \frac{f(z)}{(z-a)}\,dz = i\,f(a) \int_0^{2\pi} d\theta$

Hence

$$f(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)}\,dz$$

+1

(ii) regular function:

A single valued function $f(z)$ in a domain D is said to be analytic at a point $z=a$ if there exist neighbourhood $|z-a| < \delta$ at all point of which $f'(z)$ exists.

If $f'(z)$ exist at every pt of domain D except a finite number of exceptional pairs then $f(z)$ is said to analytic in D analytic/regular.

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$z = x + iy \qquad \bar{z} = x - iy$

Then. $x = \frac{z+\bar{z}}{2} \qquad y = \frac{1}{2i}(z-\bar{z})$

$\frac{\partial x}{\partial z} = \frac{1}{2} \quad \frac{\partial x}{\partial \bar{z}} = \frac{1}{2} \quad , \frac{\partial y}{\partial z} = \frac{1}{2i} \quad \frac{\partial y}{\partial \bar{z}} = \frac{-1}{2i}$

$\frac{\partial f}{\partial z} = \frac{1}{2} \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y}$

$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y}\right)$

$\frac{\partial^2 f}{\partial z \partial \bar{z}} = \frac{1}{4}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f$

$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$ +4

using L.H.S. $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2$

$= 4 \frac{\partial^2}{\partial z \partial \bar{z}} |f(z)|^2$

$= 4 \frac{\partial}{\partial z} [f(z) f(\bar{z})]$

$= 4 |f(z)|^2$ +2

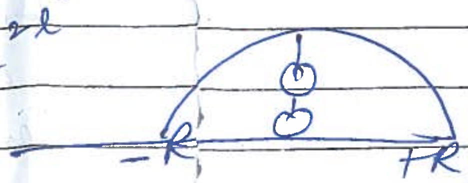
OR
(iii)

$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx.$

Consider. $\int_C \frac{z^2}{(z^2+1)(z^2+4)} dz.$

$C_1 =$ upperhalf plane

poles. $\pm i, \pm 2i$



C: Contour considering a large semi circle C_R of radius R together with the part of real axis from -R to R.

using residue th

$$\int_C f(z)\,dz = \int_{-R}^{R} \frac{x^2}{(x^2+1)(x^2+4)}\,dx + \int_{C_R} \frac{z^2\,dz}{(z^2+1)(z^2+4)} = 2\pi i \,\Sigma R^+$$ +2

Simple poles $z = i, 2i$)

Now $\lim_{R\to\infty} z f(z) = 0$

$$\therefore \int_{C_R} \frac{z^2\,dz}{(z^2+1)(z^2+4)} = 0$$

$$\therefore \lim_{R\to\infty} \int_{-R}^{R} \frac{x^2}{(x^2+1)(x^2+4)} = \int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)} = 2\pi i \,\Sigma R^+$$ +2

$z = +i, +2i$ lies in upper half plane

Residue at simple pole $\lim_{z\to a} (z-a) f(z)$

$$\therefore \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)}\,dx = \pi/3$$ +2

Q3 (i) missing value. $y_1, y_3 = ?$

only three entries are known.
By fundamental Th. on diff Calculus

$\Delta^3 y_0 = 0$ $\quad\quad$ $\Delta^3 y_1 = 0$.

$(E-1)^3 y_0 = 0$ $\quad\quad$ $(E-1)^3 y_1 = 0$

$E^n y_x = y_{x+n}$ (using) $\quad\quad$ $y_1 = 2.925$ +2

$y_3 = 0.225$ +2

(ii) Marks less than (x)

Marks less than (x)	y_x	Δ	Δ^2	Δ^3	Δ^4
40	31				
		42			
50	73		9		
		51		-25	
60	124		-16		37
		35		12	
70	159		-4		
		31			
80	190				

+3

$$y_{45} = y_0 + 4\Delta y_0 + \frac{4(4-1)\Delta^2 y_0}{L2} + \cdots$$ +1

$= 47.87 \approx 48.$

betn 40 and 45

$= 48 - 31 = 17$ +2

(iii) Newton D.D. formula

x	y	$\Delta f(x)$	Δ^2	Δ^3	Δ^4
-4	1245				
		-404			
			94		
-1	33			-14	
		-28			3
			10		
0	5			13	
		2			
			88		
2	9				
		442			
5	1335				

+3

Start with $x_0 = y_0 = z_0 = 0$

$x_1 = \frac{1}{6}[105 - y_0 - z_0] = 17.5$

$y_1 = \frac{1}{8}[155 - 3z_0 - 4x_1] = 10.625$

$z_1 = \frac{1}{10}[5x_1 + 4y_1 - 65] = 6.5$

+2

Seri 2nd iteration

$x_2 = \frac{1}{6}[105 - 10.625 - 6.5] = 14.64$

$y_2 = 9.6175$

$z_2 = 4.66$

$x_3 = 15.12$, x_5 14.98

$y_3 = 10.30$, 9.98

$z_3 = 5.04$, 4.98

+3

finally → $x = 15$, $y = 10$, $z = 5$

(iii) $\frac{dy}{dx} = x - y^2$, $y(0) = 0$

using picard or taylor seri $h = 0.2$

	x	y	
x_0	0	0	$y_0' = 0$
			$y_1' = 0.1996$
x_1	0.2	0.2000	$y_2' = 0.3937$
x_2	0.4	0.3795	$y_3' = 0.5690$
x_3	0.6	0.1762	$y_4' = x_4 - y_4^2$

+3

x_4

$y_4^P = y(0.8) = y_0 + \frac{4h}{3}[2y_1' - y_2' + 2y_3'] = 0.3049$

$y_4' = 0.7072$, $y_4^C = 0.3046$

+1

$y_5^P = y(1) = y_1 + \frac{4h}{3}[2y_2' - y_3' + 2y_4'] = 0.4556$

+1

y_5 at $x = 1$ $y_5^C = 0.4555$

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$$y = f(x_0) + (x - x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) + \dots$$

$$f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$$

+3

Q4 (i) Picard's.

$$\frac{dy}{dx} = x + y^2 \quad ; \quad y(0) = 1$$

two iteration

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx.$$

$$y_1 = 1 + \int_0^1 (x+1)\, dx$$

$$= 1 + x + x^2/2$$

+2

$$y_2 = 1 + \int_0^1 x + \left[1 + x + \frac{x^2}{2}\right]^2 dx$$

$$= 1 + \int_0^1 \left[x + 1 + x^2 + \frac{x^4}{4} + 2x + x^2 + \frac{\cancel{2}x^3}{\cancel{2}}\right] dx$$

$$= 1 + \int_0^1 \left(1 + 3x + 2x^2 + x^3 + \frac{x^4}{4}\right) dx$$

$$y_2 = 1 + x + \frac{3x^2}{2} + \frac{2x^3}{3} + \frac{x^4}{4} + \frac{x^5}{20}$$

+2

(ii) gauss seidal method.

$$|6| > |1| + |1|$$

$$|8| > |4| + |3|$$

$$|-10| > |5| + |4|$$

Solution → Converges.

+1

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Q5 (i) characteristics of O.R. (any two) ANY TWO
(a) Interdisciplinary team approach
(b) Systematics approach.
(c) increases creative ability of decision maker.
(d) Scientific approach.
(e) Improves quality of sol^n.

+2

Application. (any two)
(a) In personnel management
(b) In finance and accounting
(c) In Research and development
(d) In production management
(e) In marketing

+2

(ii) L.P.P. revised LPP using slack variable

$$\text{Max } Z = 4x_1 + 3x_2 + 6x_3 + 0.s_1 + 0.s_2 + 0s_3$$

S.t. $2x_1 + 3x_2 + 2x_3 + S_1 + 0.S_2 + 0.S_3 = 440$

$4x_1 + 0.x_2 + 3x_3 + 0.S_1 + S_2 + 0S_3 = 470$

$2x_1 + 5x_2 + 0.x_3 + 0.S_1 + 0S_2 + 0S_3 = 430$

$i = 1, 2, 3$ $x_i, S_i \geq 0$

+1

I table

			4	3	6	0	0	0	
C_B	x_B	x_0	x_1	x_2	x_3	S_1	S_2	S_3	Min x_0/x_3
0	S_1	440	2	3	2	1	0	0	440/2
0	S_2	470	4	0	3	0	1	0	470/3 →
0	S_3	430	2	5	0	0	0	1	430/0
	$C_j - Z_j$		4	3	6 ↑	0	0	0	

+1

entre → x_3 depart - S_2

3 - Key

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II table

C_B	x_B	C_j / x_0	4 / x_1	3 / x_2	6 / x_3	0 / S_1	0 / S_2	0 / S_3	$x_0/$
0	S_1	380/3	−2/3	3	0	1	−2/3	0	380/9 →
6	S_2	470/3	4/3	0	1	0	1/3	0	∞
0	S_3	430	2	5	0	0	0	1	86
		$C_j - Z_j$	−4	3	0	0	−2	0	

↑

entre x_2, depart S_1 Key 3

(f2)

III table

C_B	x_B	C_j / x_0	4 / x_1	3 / x_2	6 / x_3	0 / S_1	0 / S_2	0 / S_3	$x_0/$
3	x_2	380/9	−2/9	1	0	1/3	−2/9	0	
6	x_3	470/3	4/3	0	1	0	1/3	0	
0	S_3	1970/9	28/9	0	0	−5/3	10/9	0	
		$C_j - Z_j$	−10/3	0	0	−1	−4/3		

all $C_j - Z_j \leq 0$ optimal

$x_1 = 0, x_2 = 380/9 \quad x_3 = 470/3$

$Z_{max} = 3200/3 = 1066.67$

(f3)

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Q8 (iii) Big-M

~~Max z~~

$\text{Min } Z = 12x_1 + 20x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

s.t. $6x_1 + 8x_2 - s_1 + A_1 = 100$

$7x_1 + 12x_2 - 0s_1 - s_2 + A_2 = 120$

Table 1.

C_B	x_B	x_0 \ C_j	12	20	0	0	-M	-M	Min x_0/x_2
			x_1	x_2	s_1	s_2	A_1	A_2	
-M	A_1	100	6	8	-1	0	1	0	100/8 →
-M	A_2	120	7	12	0	-1	0	1	200/12
		$C_j - Z_j$	12+13M	20+20M	M	M	0	0	

entre x_2, depart A_1, key 8

Table 2

C_B	x_B	x_0 \ C_j	12	20	0	0	-M	Min $x_0/$
			x_1	x_2	s_1	s_2	A_2	
20	x_2	25/2	3/4	1	-1/8	0	0	—
-M	A_2	50	-2	0	3/2	-1	1	100/3
		$C_j - Z_j$	-3-2M	0	$\frac{3M}{2} - \frac{5}{2}$	-M	0	

$\frac{3}{2}$ → Key

Entre → s_1

depart → A_2

Key → $\frac{3}{2}$

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table III

			12	20	0	0	
C_B	X_B	x_0	x_1	x_2	s_1	s_2	$x_0/$
20	x_2	100/6	7/12	1	0	-1/12	
0	s_1	100/3	-4/3	0	1	-2/3	
	$c_j - z_j$		-1/3	0	0	-5/3	

all $c_j - z_j \le 0$ optimal.

$x_1 = 0 \quad x_2 = 100/6 \quad Z_{min} = \frac{1000}{3}$

2

Q6 (i)

$\lambda = 5$ per hr $\mu = 8$ per hour.

$\rho = \frac{\lambda}{\mu} = \frac{5}{8} = 0.625$

% time 62.5%

+2

(ii) cost

$8 \times \left(1 - \frac{5}{8}\right) \times 1.50 = 4.50$ R,

+2

(iii) (a)

$R_1 = R_0 \times P \qquad R_0 = [1 \ 0 \ 0]$

$R_1 = [0.5 \quad 0.2 \quad 0.3]$

+3

(b) $R_0 = [0 \ 0 \ 1]$

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$R_1 = R_0 P = [0.4 \quad 0.5 \quad 0.1]$

$R_2 = R_1 \times P = [0.59 \quad 0.18 \quad 0.23]$ +3

OR (ii)

(i) service dicipline

FCFS. LCFS PRIORITY SIRO +1

(ii) Kendall's Notation.

A/B/C : D/E

Max^m no of customers allowed in system

queue dicipline

arrival Pattern, service distⁿ, no of servers in system +1

(iii) (a) Markov Process :- Markov Process is a sequence of n experiments in which each experiment has n possible outcomes. $x_1, x_2, \ldots x_n$. Each individual outcome is called a state and the probability (that a particular outcome occurs) depends only on the probability of the outcome of the preceding experiment. +2

(b) Transition Probability matrix:

Let S_i = state i of a random process $i = 1, 2, \ldots n$

P_{ij} = conditional probability of moving from state S_i to state S_j at some later step. All conditional one step state probabilities can be expressed as elements square matrix called transition prob. matrix +2

$$P = [P_{ij}]_{m \times m} = \begin{array}{c} \\ S_1 \\ S_2 \\ \vdots \\ S_m \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & \cdots & S_m \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & & & \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{bmatrix} \end{array}$$

END OF PAPER