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Enrollment No.....



Programme: B.Tech.

End Sem (Even) Examination May-2019
EN3BS02 Engineering Mathematics-II

Branch/Specialisation: All

Duration: 3 Hrs.

Maximum Marks: 60

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

- Q.1** i. The value of the integral $\int_0^{\infty} e^{-3t} \sin t dt$ is 1
 (a) 9/10 (b) 1/10 (c) 3/10 (d) 7/10

ii. The inverse Laplace transform of the function $\frac{1}{s(s+1)}$ is 1
 (a) $1 + e^{-t}$ (b) $1 - e^t$ (c) e^{-t} (d) $1 - e^{-t}$

iii. The constant term in Fourier series of $f(x) = x^3$ in the interval $(-l, l)$ is 1
 (a) $l/2$ (b) l (c) 0 (d) None of these

iv. If $\bar{f}(s)$ is the complex Fourier transform of $f(x)$, then the complex Fourier transform of $f(x-a)$ is 1
 (a) $e^{isa} \bar{f}(s)$ (b) $e^{-isa} \bar{f}(s)$
 (c) $e^{-isa} \bar{f}(s/a)$ (d) $e^{isa} \bar{f}(s/a)$

v. The partial differential equation from the relation $z = ax + by$, where a and b are arbitrary constants is 1
 (a) $px + qy = 0$ (b) $z = p + qy$
 (c) $z = px + qy$ (d) $z = pq$

vi. The complete solution of partial differential equation $pq = 5$ is 1
 (a) $z = ax + (5/a)y + b$ (b) $z = ax + 5ay + b$
 (c) $z = ax + (a/5)y + b$ (d) None of these

P.T.O.

MEDI-CAPS UNIVERSITY, INDORE

SUBJECT CODE - EN3BS02

SUB. NAME - ENGINEERING MATHEMATICS-

BRANCH / SPECIALISATION - ALL

PROGRAMME: B.Tech.

Q. 1

- (i) (b) 1/10. +1
- (ii) (d) $1 - e^{-t}$. +1
- (iii) (c) 0. +1
- (iv) (a) $e^{isa} \bar{f}(s)$ +1
- (v) (c) $\vec{z} = p\vec{x} + q\vec{y}$. +1
- (vi) (a) $\vec{z} = ax + (5/a)\vec{y} + b$. +1
- (vii) (b) $\vec{0}$. +1
- (viii) (c) $\hat{i} + \hat{j} + \hat{k}$. +1
- (ix) (d) All of these. +1
- (x) (a) Binomial distribution. +1

Q.2(i) Let $F(t)$ be a piecewise continuous on every finite interval in the range $t \geq 0$ and satisfies.

$$|F(t)| \leq M e^{-at}, \text{ for all } t \geq 0. \quad +1$$

and for some constant ' a ' and M . Then the Laplace transform of $F(t)$ exist for all $s > a$. +1

If the Laplace transform of a given function exists, it is uniquely determined, i.e., if two continuous function have the same transformation, then they are identical. +1

(ii) Since $L\{1 - \cos t\} = \frac{1}{s} - \frac{s}{s^2 + 1}$. +1

$$\therefore L\left\{\frac{1 - \cos t}{t}\right\} = \int_{s=s}^{\infty} \left\{ \frac{1}{s} - \frac{s}{s^2 + 1} \right\} ds$$

$$= -\frac{1}{2} \log \left(\frac{s^2}{s^2 + 1} \right) \quad \text{if} \quad +2$$

$$\text{Now, } L\left\{\frac{1 - \cos t}{t^2}\right\} = L\left\{\frac{(1 - \cos t)/t}{t}\right\} = L\left\{\frac{F(t)}{t}\right\} \quad \text{if} \quad +1$$

$$\text{where } F(t) = \frac{1 - \cos t}{t}$$

$$\therefore L\{F(t)\} = -\frac{1}{2} \log \left(\frac{s^2}{s^2 + 1} \right) = f(s)$$

equation (1) becomes.

$$L\left\{\frac{1 - \cos t}{t^2}\right\} = \int_{x=s}^{\infty} f(s) ds = -\frac{1}{2} \int_{x=s}^{\infty} \log \left(\frac{s^2}{s^2 + 1} \right) ds \quad +1$$

on integration by parts

$$= -\frac{1}{2} \left[\log \left(\frac{s^2}{s^2+1} \right) \cdot s - \int \frac{1}{s^2(s^2+1)} \cdot \frac{(s^2+1) \cdot 2s - 2s^3}{(s^2+1)^2} \cdot s ds \right]_s^\infty + 1$$

$$= \cot^{-1}s + \frac{s}{2} \log \left(\frac{s^2}{s^2+1} \right).$$

$$\Rightarrow L \left\{ \frac{1 - \cos t}{t^2} \right\} = \cot^{-1}s + \frac{s}{2} \log \left(\frac{s^2}{s^2+1} \right). \quad +2$$

(iii)

Given Differential Equat.

$$(D^2 + 9)y = \cos 2t, \quad y(0) = 1, \quad y(\frac{\pi}{2}) = -1$$

Rewrite:

$$D^2y + 9y = \cos 2t$$

$$\Rightarrow y''(t) + 9y(t) = \cos 2t$$

Taking the Laplace transform on both sides and using:

$$1. L\{y''(t)\} = s^2 L\{y(t)\} - sy(0) - y'(0)$$

$$2. L\{\cos 2t\} = \frac{s}{s^2+4}$$

we get

$$(s^2+9)L\{y(t)\} - s - y'(0) = \frac{s}{s^2+4} \quad +2$$

$$L\{y(t)\} = \frac{s+A}{s^2+9} + \frac{s}{(s^2+9)(s^2+4)}$$

where $y'(0) = A$

$$\Rightarrow L\{y(t)\} = \frac{s}{s^2+9} + \frac{A}{s^2+9} + \frac{s}{5(s^2+4)} - \frac{s}{5(s^2+9)} \quad +2$$

(1)

Taking the inverse Laplace transform on both sides, we get

$$y(t) = \frac{4}{5} \cos 3t + \frac{1}{3} A \sin 3t + \frac{1}{5} \cos 2t \quad \text{--- (2)} \quad +1$$

Now given that $y(\frac{\pi}{2}) = -1$, so putting $t = \frac{\pi}{2}$, and $y = -1$ in (2), we get

$$A = \frac{12}{5}$$

Putting $A = \frac{12}{5}$ in (2), the required solution is

$$y(t) = \frac{4}{5} \cos 3t + \frac{4}{5} \sin 3t + \frac{1}{5} \cos 2t \quad +1$$

Q. 3(i) Any functions $f(x)$ can be expressed as a Fourier series $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$

in the interval, $(0, 2\pi)$ or $(-\pi, \pi)$, where a_0 , a_n , and b_n are constants, provided:

(i) $f(x)$ is periodic, single valued and finite $+1$

(ii) $f(x)$ has a finite number of discontinuities in any one period, $+1$

(iii) $f(x)$ has a finite number of maxima and minima $+1$

(ii) The Fourier Series of $f(x)$ over the interval $(-\pi, \pi)$:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx] \quad \text{--- (1)}$$

Where

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

+1

$$\text{Now } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{1}{\pi} \left[\int_{-\pi}^0 dx + \int_0^{\pi} x dx \right] = -\frac{\pi}{2}$$

$$a_n = \frac{1}{\pi} \left[\int_{-\pi}^0 \cos nx dx + \int_0^{\pi} x \cos nx dx \right]$$

$$\Rightarrow a_n = \frac{1}{\pi} \left[\frac{(-1)^n - 1}{n^2} \right] = \begin{cases} 0, & n \text{ is even} \\ -\frac{2}{\pi n^2}, & n \text{ is odd} \end{cases}$$

+2

Finally,

$$b_n = \frac{1}{\pi} \left[\int_{-\pi}^0 \pi \sin nx dx + \int_0^{\pi} x \sin nx dx \right]$$

$$\Rightarrow b_n = \frac{1}{n} [1 - 2(-1)^n] = \begin{cases} -\frac{1}{n}, & n \text{ is even} \\ \frac{3}{n}, & n \text{ is odd} \end{cases}$$

+2

Hence, eq. (1) becomes

$$f(x) = -\frac{\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{\pi} \left\{ \frac{(-1)^n - 1}{n^2} \right\} \cos nx + \sum_{n=1}^{\infty} \frac{1}{n} [1 - 2(-1)^n] \sin nx$$

O.R.

+1

$$f(x) = -\frac{\pi}{4} - \frac{2}{\pi} \left[\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots \right] + \left[3 \sin x - \frac{\sin 2x}{2} + \frac{3 \sin 3x}{3} - \frac{\sin 4x}{4} + \dots \right]$$

(iii) By definition of Fourier transform of $f(x)$:

$$F\{f(x)\} = \overline{f}(s) = \int_{-\infty}^{\infty} f(x) e^{isx} dx$$

since $f(x) = \begin{cases} 1 & \text{for } |x| < a, \text{i.e. } -a < x < a \\ 0, & \text{otherwise} \end{cases}$

$$\therefore \overline{f}(s) = \int_{-a}^a f(x) \cdot e^{isx} dx = \int_{-a}^a 1 \cdot e^{isx} dx$$

$$F\{f(x)\} = \overline{f}(s) = \frac{2 \sin sa}{s}$$

— 1 + 3

(2)

By the inverse Fourier transform of $\overline{f}(s)$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{f}(s) e^{-isx} ds$$

+1

$$\rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \sin sa}{s} e^{-isx} ds = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases} \quad \text{--- (2)}$$

Put $x=0$ in (2), we get

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \sin sa}{s} ds = 1 \Rightarrow \frac{2}{2\pi} \int_0^{\infty} \frac{2 \sin sa}{s} ds = 1$$

$$\rightarrow \int_0^{\infty} \frac{\sin sa}{s} ds = \frac{\pi}{2} \Rightarrow \left| \int_0^{\infty} \frac{\sin ax}{x} dx = \frac{\pi}{2} \right| \quad \text{Take } a=1$$

+1

(b) From eq. (2)

$$\rightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \sin as}{s} \{ \cos sx + i \sin sx \} ds = \begin{cases} 1, & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$$

On Equating

$$\frac{1}{2\pi} \cdot 2 \cdot \int_{-\infty}^{\infty} \frac{\sin as}{s} \cos sx ds = \begin{cases} 1 & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$$

and $\frac{2}{2\pi} \int_{-\infty}^{\infty} \frac{\sin as}{s} \sin sx ds = 0$

$$\rightarrow \int_{-\infty}^{\infty} \frac{\sin as}{s} \cos sx ds = \begin{cases} \pi & \text{for } |x| < a \\ 0, & \text{for } |x| > a \end{cases}$$

4(i)

Given equation becomes

$$p^2 - x = y - q^2.$$

$$\text{Putting } p^2 - x = a, y - q^2 = a$$

$$\Rightarrow p = \sqrt{(x+a)}, q = \sqrt{(y-a)}$$

Putting these values of p and q in $dz = pdx + qdy$, we get

$$(z+b) = \frac{2}{3} (x+a)^{3/2} + \frac{2}{3} (y-a)^{3/2}$$

(ii)

The subsidiary equations are

$$\frac{dx}{1} = \frac{dy}{3} = \frac{dz}{5z + \tan(y-3x)} \quad (1)$$

+1

Taking first two fractions of (1) and integrating, we get

$$y = 3x + a \quad \text{--- (2)} \quad +2$$

Taking first and third fractions of (1), we get

$$\begin{aligned} \frac{dx}{1} &= \frac{dz}{5z + \tan(y-3x)} \\ \Rightarrow \frac{dx}{1} &= \frac{dz}{5z + \tan a} \\ \Rightarrow \frac{dz}{dx} - 5z &= \tan a \end{aligned}$$

which is linear in z , therefore its solution is

$$\begin{aligned} z e^{-5x} &= \tan a \cdot \frac{e^{5x}}{-5} + b \\ \Rightarrow b &= e^{-5x} [5z + \tan(y-3x)] \quad \text{--- (3)} \end{aligned} \quad +2$$

Hence the required general solution is

$$\phi(y-3x, e^{-5x}(5z + \tan(y-3x))) = 0$$

OR

$$e^{-5x}[5z + \tan(y-3x)] = \phi(y-3x)$$

OR

$$y-3x = \phi(e^{-5x}(5z + \tan(y-3x)))$$

$$(iii) \quad \text{Given : } (D^3 - 4D^2D^1 + 5D^1D^2 - 2D^3)z = e^{2x+y}$$

For C.F. : Its A.E. is $m^3 - 4m^2 + 5m - 2 = 0$

$$\therefore m = 1, 1, 2$$

$$\therefore \text{C.F.} = \phi_1(y+x) + x\phi_2(y+x) + \phi_3(y+2x) \quad +1$$

where ϕ_1, ϕ_2, ϕ_3 are arbitrary functions

Now

$$\text{P.I.} = \frac{1}{D^3 - 4D^2 D' + 5D D'^2 - 2D'^3} [e^{2x+y}]$$

$$= \frac{1}{(D-D')(D-D')(D-2D')} [e^{2x+y}]$$

[Putting $D \rightarrow 2$, $D' \rightarrow 1$, $\therefore \frac{1}{D-2D'} = \frac{e^{2x+y}}{0}$ i.e. fails]

Θ

$$\boxed{\text{P.I.} = \frac{1}{(2-1)^2} \cdot \frac{1}{(D-2D')} [e^{2x+y}]} \quad +1$$

$$\text{Using } \frac{1}{(bD-aD')^n} [\phi(ax+by)] = \frac{x^n}{b^n n!} \phi(ax+by)$$

$$= 1 \cdot \frac{1}{(1 \cdot D-2D')} [e^{2x+y}] = \frac{x!}{1! 1!} e^{2x+y}$$

$$\boxed{\text{P.I.} = x \cdot e^{2x+y}} \quad +2$$

\therefore The general solution is,

$$\vec{z} = C.F. + P.I.$$

$$\Rightarrow \boxed{\vec{z} = \phi_1(y+z) + \alpha \phi_2(y+z) + \phi_3(y+2z) + x e^{2x+y}} \quad +1$$

5(i) If \vec{F} is a solenoidal vector, then

$$\operatorname{div} \vec{F} = 0$$

$$\Rightarrow 1+1+\alpha = 0$$

$$\Rightarrow \alpha = -2$$

+1

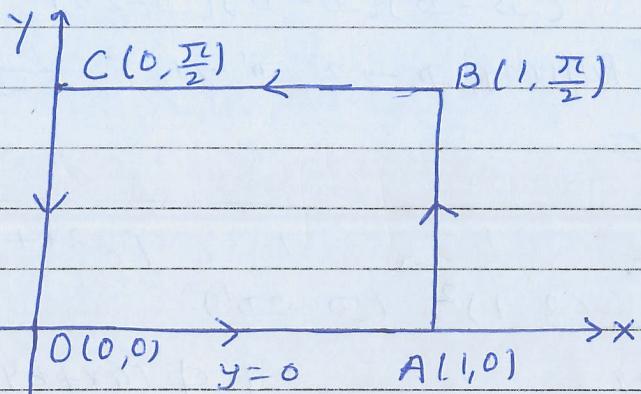
+1

+1

4(ii) Here $\vec{r} = x\hat{i} + y\hat{j} \Rightarrow d\vec{r} = dx\hat{i} + dy\hat{j}$

$$\therefore \vec{F} \cdot d\vec{r} = e^x \sin y \, dx + e^x \cos y \, dy \quad \text{--- (1)} \quad +1$$

Draw the rectangle C as in figure.



+1

$$\therefore \int_C \vec{F} \cdot d\vec{r} = \int_{OA} \vec{F} \cdot d\vec{r} + \int_{AB} \vec{F} \cdot d\vec{r} + \int_{BC} \vec{F} \cdot d\vec{r} +$$

$$\int_{CO} \vec{F} \cdot d\vec{r} \quad \text{--- (2)}$$

1. Along OA: $y=0 \Rightarrow dy=0$

$$\therefore \int_{OA} \vec{F} \cdot d\vec{r} = \int_0^1 [e^x \cdot \sin 0 \cdot dx + 0] = 0 \quad +1$$

2. Along AB: $x=1 \Rightarrow dx=0$

$$\therefore \int_{AB} \vec{F} \cdot d\vec{r} = \int_0^{\pi/2} [e^1 \cdot \sin y \cdot 0 + e^1 \cdot \cos y \, dy]$$

$$= e$$

+1

3. Along BC: $y=\pi/2 \Rightarrow dy=0$

$$\therefore \int_{BC} \vec{F} \cdot d\vec{r} = 1 - e$$

+1

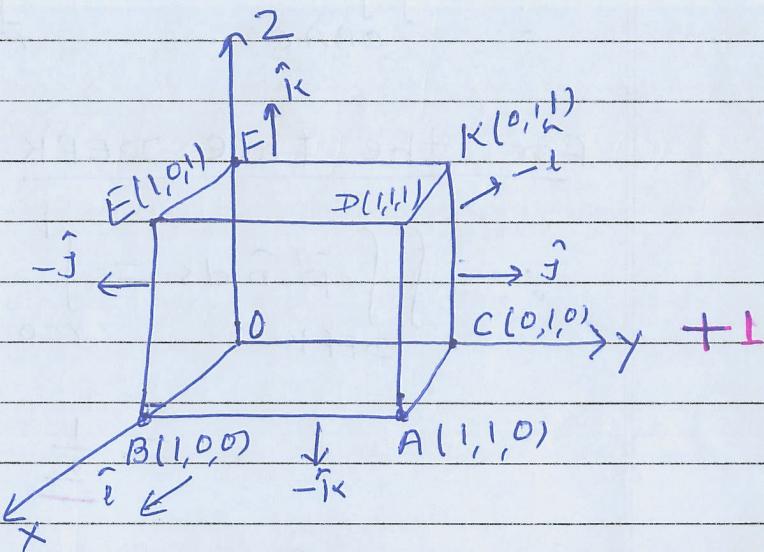
4. Along CO: $x=0 \Rightarrow dx=0$

$$\therefore \int_{CO} \vec{F} \cdot d\vec{r} = \int_{\pi/2}^0 \cos y dy = -1 \quad +1$$

$$\therefore \boxed{\int_C \vec{F} \cdot d\vec{r} = 0 + e + (1-e) + (-1) = 0} \quad +1$$

4(iii)

Ques



By Gauss-divergence theorem

$$\iint_S \vec{F} \cdot \hat{n} ds = \iiint_V \text{div } \vec{F} dv \quad -(i) \quad +1$$

Given $\vec{F} = x^2 \hat{i} + xy \hat{j} + yz \hat{k}$

$$\text{div } \vec{F} = 2x + y$$

$$\begin{aligned} \therefore \text{R.H.S.} &= \iiint_V (2x+y) dv = \int_0^1 \int_0^1 \int_0^1 (2x+y) dx dy dz \\ &= \frac{3}{2} \quad -(2) \end{aligned} \quad +1$$

To evaluate $\iint_S \vec{F} \cdot \hat{n} dS$, where S consists of six planes surfaces

For the face $ABCO$: $\hat{n} = -\hat{k}$, $z=0$, $dS = \frac{dx dy}{|\hat{n} \cdot \hat{k}|}$

$$\Rightarrow dS = \frac{dx dy}{1} = dx dy$$

$$\therefore \iint_{OBAC} \vec{F} \cdot \hat{n} dS = - \int_{z=0}^1 \int_{y=0}^1 y z dx dy = 0 \quad (\because z=0)$$

For the face $DEFK$: $\hat{n} = \hat{k}$, $z=1$, $dS = dx dy$

$$\therefore \iint_{DEFK} \vec{F} \cdot \hat{n} dS = \int_{z=0}^1 \int_{y=0}^1 y z dx dy = \int_{z=0}^1 \int_{y=0}^1 y dx dy$$

$$= \frac{1}{2}$$

+1

For the face $ABED$: $\hat{n} = \hat{i}$, $x=1$, $dS = \frac{dy dz}{|\hat{n} \cdot \hat{i}|}$

$$dS = dy dz$$

$$\therefore \iint_{ABED} \vec{F} \cdot \hat{n} dS = \int_{y=0}^1 \int_{z=0}^1 x^2 dy dz = \underline{1} \quad (\because x=1)$$

For the Face $OCKF$: $\hat{n} = -\hat{i}$, $x=0$, $dS = dy dz$

$$\therefore \iint_{OCKF} \vec{F} \cdot \hat{n} dS = \underline{0}$$

+1

For the Face $ACKD$: $\hat{n} = \hat{j}$, $y=1$, $dS = \frac{dx dz}{|\hat{n} \cdot \hat{j}|}$

$$\iint_{ACKD} \vec{F} \cdot \hat{n} dS = \int_{z=0}^1 \int_{x=0}^1 \frac{1}{2} dx dz = \underline{\frac{1}{2}}$$

+1

For the face OBEF: $\hat{n} = -\hat{j}$, $y=0$, $ds = dx dz$

$$\therefore \iint_{OBEF} \vec{F} \cdot \hat{n} ds = -\frac{1}{2} \quad +1$$

$$\begin{aligned} \text{Hence L.H.S.} &= \iint_S \vec{F} \cdot \hat{n} ds = 0 + \frac{1}{2} + 1 + 0 + \frac{1}{2} - \frac{1}{2} \\ &= \underline{\frac{3}{2}} \quad \text{--- (3)} \end{aligned} \quad +1$$

From (2) and (3), we get

Hence verified Gauss's divergence theorem

6(i) Given: mean = $np = 12$ — (1)

standard deviation = 2

$$\Rightarrow \sqrt{npq} = 2 \Rightarrow npq = 4 \quad \text{--- (2)} \quad +1$$

We have

$$\frac{npq}{np} = \frac{4}{12} \quad [\text{by (1) and (2)}]$$

$$\Rightarrow q = \frac{1}{3}$$

$$\text{Since } p+q = 1 \Rightarrow p = 1-q \Rightarrow p = 2/3 \quad +1$$

$$\text{Now, from (1) } np = 12 \Rightarrow n = 18. \quad +1$$

6(ii) Given :

$$x : 0 \quad 1 \quad 2 \quad 3$$

$$f : 200 \quad 75 \quad 20 \quad 5$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{0 \times 200 + 1 \times 75 + 2 \times 20 + 3 \times 5}{300} = \frac{130}{300} \quad +2$$

$$\Rightarrow \text{mean (m)} = 0.43.$$

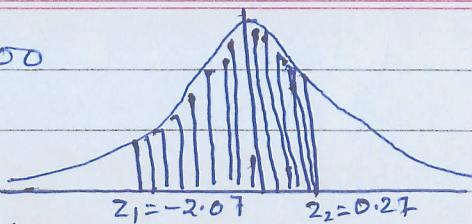
The probability of r successes in a Poisson distribution is given by

6(iii)

Number of students = 5,00

$$\therefore N = 5,00$$

$$x_1 = 120, x_2 = 155$$

Mean $\mu = 151$ c.m., and S.D. $\sigma = 15$ 

+1

By standard normal variable $z = \frac{x-\mu}{\sigma}$ When $x_1 = 120$ Standard normal variable $z_1 = \frac{x_1-\mu}{\sigma}$

$$z_1 = \frac{120-151}{15} = -2.07$$

+1

When $x_2 = 155$ c.m.

+1

Standard normal variable $z_2 = \frac{x_2-\mu}{\sigma}$

$$z_2 = \frac{155-151}{15} = 0.27$$

+1

$$\therefore P(120 < x < 155) = P(-2.07 < z < 0.27)$$

+1

$$= P(-2.07 \leq z \leq 0) + P(0 \leq z \leq 0.27)$$

+1

$$= P(0 \leq z \leq 2.07) + P(0 \leq z \leq 0.27)$$

+1

$$= 0.4808 + 0.1085$$

$$= 0.5892$$

+1

————— x —————