Total No. of Questions: 6 *Total No. of Printed Pages:2*

Enrollment No......................................

Faculty of Agriculture

End Sem Examination Dec-2023

AG3RC02 Elementary Mathematics

Programme: B.Sc. (Hons.) Branch/Specialisation: Agriculture

Duration: 3 Hrs. **Maximum Marks: 50**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Distance of the point (2,0) from X- axis is______. **1**

(a) 0 (b) 2 (c) 4 (d) None of these

ii. Two straight lines whose slopes are m_1 and m_2 are parallel if- **1**

(a) $m_1 = m_2$ (b) $m_1 = -m_2$

(c) $m_1 . m_2 = -1$ (d) None of these

iii. Radius of the circle $x^2 + y^2 = 36$ is- **1**

(a) 3 (b) 9 (c) 6 (d) None of these

iv. In the equation of the circle $(x-3)^2 + (y+3)^2 = 49$ the coordinates of centre is- **1**

(a) (3,3) (b) (3,-3) (c) (-3,3) (d) None of these

v. The value of $\frac{d}{dx}(1+\log x)$: **1**

(a) 1/x (b) 1+1/x (c) 1 (d) None of these

vi. $\frac{d}{dx}(\tan x) =$______. **1**

(a) $\sec^2 x$ (b) $\tan^2 x$ (c) $\operatorname{cosec}^2 x$ (d) None of these

vii. If $\int f(x)dx = \frac{d}{dx} f(x)$ then $f(x) =$ _____? **1**

(a) e^x (b) e^{5x}

(c) Both (a) and (b) (d) None of these

viii. The value of $\int_1^a \frac{2}{x} dx$ is- **1**

(a) 0 (b) 2 (c) 2 log a (d) None of these

ix. The value of $\begin{vmatrix} 10 & 2 \\ 5 & 10 \end{vmatrix}$ is ______. **1**

(a) 10 (b) 100 (c) 90 (d) None of these

x. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \end{bmatrix}$ then A is the type of matrix- **1**

(a) Skew symmetric matrix (b) Symmetric matrix

(c) Idempotent matrix (d) None of these

Q.2 Attempt any two:

i. Find the value of a if the points (3,4), (-12, -6) and (9,a) are collinear. **4**

ii. Prove that the points (0, -1), (2,1), (0,3), (-2,1) are the vertices of a square. **4**

iii. Find the equation of the line passing through the points (3,3) and (7,6). **4**

Q.3 Attempt any two:

i. Find the equation of the circle which touches both the coordinate axes and has centre (1,1). **4**

ii. Find the equation of the circle passing through the points (1,1), (2,-1) and (3,-2). **4**

iii. Find the coordinates of centre and length of radius of the circle. **4**

$$x^2 + y^2 - 2x + 4y - 11 = 0$$

Q.4 Attempt any two:

i. Find $\frac{dy}{dx}$ if $y = \frac{1+\tan x}{1-\tan x}$. **4**

ii. Evaluate $\lim_{x\to 0} \frac{\sin 5x}{x}$. **4**

iii. Discuss the continuity of the function $f(x) = e^x$ at $x = 0$. **4**

Q.5 Attempt any two:

i. Evaluate $\int x.\cos x \; dx$. **4**

ii. Evaluate $\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$. **4**

iii. Evaluate $\int x^2 \log x \; dx$. **4**

Q.6 Attempt any two:

i. If $A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$ then evaluate 2A + 3B. **4**

ii. If $A = \begin{bmatrix} 3 & 3 & 5 \\ 4 & 4 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \\ 0 & 0 \end{bmatrix}$ then evaluate AB. **4**

iii. Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. **4**

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Medicaps University
End Sem - Dec. 2023
AG3RC02 Elementary mathematics
Solution.

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Q1) (I)

I (b) 2
II (a) $m_1 = m_2$
III (c) 6
IV (b) $(3, -3)$
V (a) $1/x$ +10
VI (a) $\sec^2 x$
VII (a) e^x
VIII (c) $2\log a$
IX (c) 90 [instead of matrix it should be determinant]
X (b) symmetric matrix

Q2 (I) Given points are

$A(3,4) = (x_1, y_1)$, $B(-12,-6) = (x_2, y_2)$

$C(9, a) = (x_3, y_3)$

$x_1 = 3$, $x_2 = -12$, $x_3 = 9$ +1

$y_1 = 4$, $y_2 = -6$, $y_3 = a$

If the points are collinear than area of $\Delta ABC = 0$

$$\frac{1}{2}\left[y_1(x_2 - x_3) + y_2(x_3 - x_1) + y_3(x_1 - x_2)\right] = 0$$ +1

Can use $[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$

$$\frac{1}{2}\left[4(-12-9) + (-6)(9-3) + a(3+12)\right] = 0$$

$$\frac{1}{2}\left[-84-36+15a\right]=0$$

$$-120+15a=0$$

$$15a=120$$

$$\boxed{a=8} \quad \underline{Ans} \qquad +2$$

Q2) (II) Given points are
$A(0,-1)$, $B(2,1)$, $C(0,3)$, $D(-2,1)$

$AB = \sqrt{(2-0)^2+(1+1)^2} = \sqrt{4+4} = \sqrt{8}$

$BC = \sqrt{(0-2)^2+(3-1)^2} = \sqrt{4+4} = \sqrt{8}$ +1

$CD = \sqrt{(-2-0)^2+(1-3)^2} = \sqrt{4+4} = \sqrt{8}$ +1

$AD = \sqrt{(-2-0)^2+(1+1)^2} = \sqrt{4+4} = \sqrt{8}$ +1

as $AB = BC = CD = AD$
$\therefore$ A, B, C, D are vertices of a square. +1

Q2) (III) Given points are $A(3,3)$ & $B(7,6)$
$x_1 = 3$, $y_1 = 3$, $x_2 = 7$, $y_2 = 6$

eq. of line

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$ +1

$$y - 3 = \frac{6-3}{7-3}(x-3)$$

$$y - 3 = \frac{3}{4}(x-3)$$ +1

$$4(y-3) = 3(x-3)$$

$$4y - 12 = 3x - 9$$ +1

$$= 3x - 4y - 9 + 12 = 0$$

$$3x - 4y + 3 = 0$$ Ans +1

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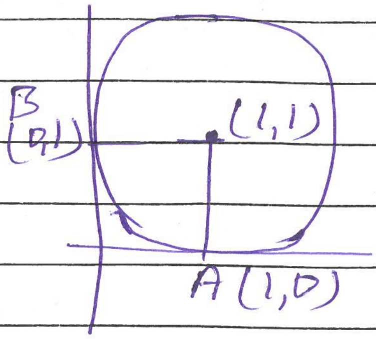
Q3) (I) As centre of the circle is (1,1) and it touches both the axes ∴ radius of the circle is $a=1$



+1

∴ eq. of the circle is

$(h,k) = (1,1)$ +1

$(x-h)^2+(y-k)^2 = a^2$

$(x-1)^2 + (y-1)^2 = 1$ +1

$x^2-2x+1+y^2-2y+1 = 1$

$x^2+y^2-2x-2y+1 = 0$ Ans +1

Q3) (III) Given points are

$A(1,1)$, $B(2,-1)$, $C(3,-2)$

General eq. of the circle is

$x^2+y^2+2gx+2fy+c=0$ — (A) +1

As it passes through $A(1,1)$ ∴ we get

$1^2+1^2+2g\cdot1+2f\cdot1+c = 0$

$2g+2f+c = -2$ — (1)

as circle (A) passes through $B(2,-1)$ ∴ we get

$4+1+4g-2f+c=0$

$4g-2f+c = -5$ — (2) +1

Similarly for point $(3,-2)$

$$9+4+6g-4f+c=0$$

$$6g-4f+c=-13 \quad \text{--- } 3$$

eq. 2 $-$ eq. 1

$$2g-4f=-3 \quad \text{--- } 4$$

eq. 3 $-$ eq. 1

$$4g-6f=-11 \quad \text{--- } 5$$

eq. 5 $-$ $2\times$ eq. 4

$$2f=-5$$

$$\boxed{f=-\frac{5}{2}}$$

from eq. 4

$$2g-\cancel{4}^{2}\left(-\frac{5}{\cancel{2}}\right)=-3$$

$$2g+10=-3$$

$$2g=-13$$

$$\boxed{g=-\frac{13}{2}}$$

from eq. 1

$$-\cancel{2}\times\frac{13}{\cancel{2}}-\cancel{2}\times\frac{5}{\cancel{2}}+c=-2$$

$-18 + c = -2$

$\boxed{c = 16}$ +1

Putting the values of g, f & c in eq - (A)

$x^2 + y^2 + 2\left(\frac{-13}{2}\right)x + 2\left(\frac{-5}{2}\right)y + 16 = 0$

$x^2 + y^2 - 13x - 5y + 16 = 0$ Ans +1

Q3) III eq. of the circle is

$x^2 + y^2 - 2x + 4y - 11 = 0$

on comparing it with

$x^2 + y^2 + 2gx + 2fy + c = 0$

$g = -1, \; f = 2, \; c = -11$ +1

centre $= (-g, -f) = (1, -2)$ Ans

Radius $= \sqrt{g^2 + f^2 - c}$ +1

$= \sqrt{1 + 4 + 11}$

$= \sqrt{16}$

$= 4$ unit Ans +2

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Q4(I) $y = \frac{1+\tan x}{1-\tan x}$

$$\frac{dy}{dx} = \frac{(1-\tan x)\frac{d}{dx}(1+\tan x) - (1+\tan x)\frac{d}{dx}(1-\tan x)}{(1-\tan x)^2}$$

+1

$$\frac{dy}{dx} = \frac{(1-\tan x)(\sec^2 x) - (1+\tan x)(-\sec^2 x)}{(1-\tan x)^2}$$

+1

$$\frac{dy}{dx} = \frac{\sec^2 x - \cancel{\sec^2 x \tan x} + \sec^2 x + \cancel{\sec^2 x \tan x}}{(1-\tan x)^2}$$

+1

$$\frac{dy}{dx} = \frac{2\sec^2 x}{(1-\tan x)^2}$$ Ans +1

Q4(II) $\lim_{x\to 0} \frac{\sin 5x}{x}$

$\lim_{x\to 0} 5 \cdot \frac{\sin 5x}{5x}$ +1

$5 \lim_{5x\to 0} \frac{\sin 5x}{5x}$ +1

$= 5 \times 1$ $\because \lim_{x\to 0} \frac{\sin x}{x} = 1$ +1

$= 5$ +1

Q4) (III) Given function is

$$f(x) = e^x$$

$$f(0) = e^0 = 1 \qquad +1$$

LHL

$$\lim_{x \to 0-h} f(0-h)$$

$$\Rightarrow \lim_{x \to 0-h} e^x$$

$$\lim_{h \to 0} e^{-h}$$

$$\Rightarrow \lim_{h \to 0} \frac{1}{e^h} = 1 \qquad +1$$

RHL

$$\lim_{x \to 0+h} f(0+h)$$

$$\Rightarrow \lim_{x \to 0+h} e^x$$

$$\Rightarrow \lim_{h \to 0} e^h = 1) \qquad +1$$

As at $x = 0$

LHL = RHL = $f(0)$

$\therefore f(x) = e^x$ is continues at $x=0$

+1

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Q5) (I) Given

$$I = \int \underset{\mathrm{I}}{x} \cdot \underset{\mathrm{II}}{\cos x}\, dx$$

$$I = x \int \cos x\, dx - \int \left[\frac{d}{dx}(x) \int \cos x\, dx\right] dx$$

$$I = x \cdot \sin x - \int 1 \cdot \sin x\, dx \quad +1 \; +1$$

$$I = x \cdot \sin x - (-\cos x) + c \quad +1$$

$$I = x \cdot \sin x + \cos x + c \quad \text{Ans} \quad +1$$

Q5) (II) Given

$$I = \int_0^{\pi/4} \tan^2 x\, dx$$

$$I = \int_0^{\pi/4} (\sec^2 x - 1)\, dx \quad +1$$

$$I = \int_0^{\pi/4} \sec^2 x\, dx - \int_0^{\pi/4} 1 \cdot dx \quad +1$$

$$I = [\tan x]_0^{\pi/4} - [x]_0^{\pi/4}$$

$$I = \tan\frac{\pi}{4} - \frac{\pi}{4} \quad +1$$

$$= 1 - \pi/4 \quad \text{Ans} \quad +1$$

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Q5) (III) Given

$$I = \int \underset{\mathrm{I}}{x^2} \underset{\mathrm{II}}{\log x}\, dx \quad \text{--- (1)}$$

$$I = x^2 \int \log x\, dx - \int \left[\frac{d}{dx}(x^2) \int \log x\, dx\right] dx$$

+1

$$I = x^2 \left[x \log x - x\right] - \int 2x \cdot (x \log x - x)\, dx$$

$$I = x^3 \log x - x^3 - 2\int x^2 \log x\, dx + 2\int x^2\, dx$$

$$I = x^3 \log x - x^3 - 2I + \frac{2x^3}{3}$$

+2

$$I + 2I = x^3(\log x - 1) + \frac{2x^3}{3} + c$$

$$I = \frac{1}{3}\left[x^3(\log x - 1) + \frac{2x^3}{3} + c\right]$$

+1

(11)

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Q6) (I) Given

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 6 & 3 \\ 1 & 4 & 5 \end{bmatrix}$$

$$2A = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix}, \quad 3B = \begin{bmatrix} 21 & 18 & 9 \\ 3 & 12 & 15 \end{bmatrix} \; +1$$

$$2A+3B = \begin{bmatrix} 0 & 4 & 6 \\ 4 & 2 & 8 \end{bmatrix} + \begin{bmatrix} 21 & 18 & 9 \\ 3 & 12 & 15 \end{bmatrix} \; +1$$

$$2A+3B = \begin{bmatrix} 21 & 22 & 15 \\ 7 & 14 & 23 \end{bmatrix} \; \underline{\underline{\text{Ans}}} \quad +2$$

Q6) (II) Given

$$A = \begin{bmatrix} 3 & 3 & 5 \\ 4 & 4 & 4 \end{bmatrix}_{2\times3} \; \& \; B = \begin{bmatrix} 1 & 3 \\ -1 & 2 \\ 0 & 0 \end{bmatrix}_{3\times2}$$

$$AB = \begin{bmatrix} 3-3+0 & 9+6+0 \\ 4-4+0 & 12+8+0 \end{bmatrix}_{2\times2} \quad +3$$

$$AB = \begin{bmatrix} 0 & 15 \\ 0 & 18 \end{bmatrix} \; \cancel{AB}\, 2\times2 \quad +1$$

96) (III) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

$A^{-1} = \dfrac{Adj\ A}{|A|}$ —— (1)

$|A| = 1(24-25) - 2(12-15) + 3(10-12)$

$|A| = -1 + 6 - 6$

$|A| = -1$ $\qquad +1$

$C_{ij} = (-1)^{i+j} M_{ij}$

$C_{11} = (-1)^2 \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24-25$

$= -1$

$C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} = -(12-15)$

$= 3$

$C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 10-12 = -2$

$$c_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = -(12-15) = 3$$

$$c_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} = (6-9) = -3$$

$$c_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 5 \end{vmatrix} = -(5-6) = 1$$

$$c_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10-12 = -2$$

$$c_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = -(5-6) = 1$$

$$c_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} = 4-4 = 0$$

+2

$$\text{Adj } A = \begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

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$$A^{-1} = \frac{Adj\,A}{|A|} = \frac{Adj\,A}{-1}$$

$$= -\,Adj\,A$$

$$= -\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$ +1

Ans