Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem Examination Dec-2023

EN3BS07

Introductory Topics in Statistics, Probability & Calculus

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. To which of the following options individual respondents, focus groups, and panels of respondents belong? **1**

(a) Primary data sources (b) Secondary data sources

(c) Itemized data sources (d) Pointed data sources

ii. In statistics, a population consists of- **1**

(a) All people living in a country

(b) All people living in the area under study

(c) All subjects or objects whose characteristics are being studied

(d) None of these

iii. The mean of a distribution is 14 and the standard deviation is 5. What is the value of coefficient of variation? **1**

(a) 60.4% (b) 48.3% (c) 35.7% (d) None of these

iv. Data must be arranged in ascending or descending order if someone wants to compute- **1**

(a) Mean (b) Median

(c) Mode (d) None of these

v. The number of accidents in a city during 2010 is- **1**

(a) Discrete variable (b) Continuous variable

(c) Quality variable (d) None of these

vi. The second moment about mean (central moment) is equal to- **1**

(a) 0 (b) Mean (c) Variance (d) 1

vii. What is the mean of the binomial distribution, if n=16 and p = 0.85? **1**

(a) 6 (b) 7.4 (c) 12.4 (d) 13.6

viii. The shape of normal curve is- **1**

(a) Bell shaped (b) Triangular (c) Rectangular (d) Circle

P.T.O.

ix. The value of $\int_0^1 \int_0^1 dydx$ is- **1**

(a) ½ (b) 1

(c) 2 (d) None of these

x. Value of $\int logx$ is- **1**

(a) $x + log\ x + c$ (b) $\frac{\log x}{x}$

(c) $x\ log\ x - x + c$ (d) None of these

Q.2 Attempt any two:

i. Write any five differences between primary and secondary data. **5**

ii. What is statistics? Write any four applications of statistics in various branches of science with example. **5**

iii. Define population and sample. Write any three differences between population and sample. **5**

Q.3 Attempt any two:

i. Define median and mode. Write its formula for continuous frequency distribution. **5**

ii. Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 1.46. **5**

X	0	1	2	3	4	5	Total
F	46	?	?	25	10	5	200

iii. Find standard deviation and its coefficient from the following data: **5**

Life (in hours)	0-8	8-16	16-24	24-32	32-40
No. of bulbs	3	5	10	12	2

Q.4 Attempt any two:

i. Define mathematical expectation. Write any four properties of mathematical expectation. **5**

ii. One bag contains 4 white balls and 2 black balls; another bag contains 3 white balls and 5 black balls. If one ball is drawn at random from each bag, find the probability that: **5**

(a) Both are white (b) One is black and other is white

iii. A candidate is selected for interview of management trainees for 3 companies. For the first company there are 12 candidates, for the second there are 15 candidates and for the third there are 10 candidates. What is the probability of his getting job at least at one of the companies? **5**

Q.5 Attempt any two:

i. Write any five characteristics of normal distribution. **5**

ii. The incidence of a certain disease is such that on an average 20% of workers suffer from it. If 10 workers are selected at random, find the probability that- **5**

(a) Exactly two workers suffer from the disease

(b) Not more than two workers suffer from the disease

iii. Suppose that on an average a manufactured product has 2 defects per unit. Using Poisson's distribution, calculate the probabilities of finding a product (Given: $e^{-2} = 0.135$). **5**

(a) Without any defect

(b) 3 defects

(c) 4 defects

Q.6 Attempt any two:

i. Find the area of the circle of radius (r) and equation $x^2 + y^2 = r^2$ by using double integral with proper diagram. **5**

ii. Solve double integral $\int_0^1 \int_0^1 \left(\frac{1}{1+x^2}\right)\left(\frac{1}{1+y^2}\right) dxdy$. **5**

iii. Solve triple integral $\int_0^2 \int_0^x \int_0^{2x+2y} (e^{x+y+z})\, dxdydz$. **5**

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Introductory topics in Statistics, Probability and Calculus - EN3BS07
B.Tech (CSBS)

Q-1 MCQs

Q-1(i) (a) Primary data sources +1

(ii) (c) All subjects or objects whose characteristics are being studied +1

(iii) (c) 35.7% +1

(iv) (b) Median +1

(v) (a) Discrete Variable +1

(vi) (c) Variance +1

(vii) (d) 13.6 +1

(viii) (a) Bell shaped +1

(ix) (b) 1 +1

(x) (c) $x \log x - x + c$ +1

Q-2

(i)

	Primary Data	Secondary Data	
1.	They are collected by the measearcher himself and are original	They are not collected by the researcher and are not real. They are in existence by someone or by any organization.	+1
2.	In collecting primary data time and money are involved.	No much money and time are involved in secondary data	+1

3.	They are obtained by the individuals from the field of investigation.	They are obtained from the researcher, research journals or organizations.	+1
4.	These are collected according to the need and problem of the researcher.	These are collected by someone else.	+1
5.	No extra precautions are required.	They need proper precautions before use.	+1

Q-2 (ii) **Statistics :—** Statistics may be defined as Science of collection, presentation, analysis and interpretation of numerical data. +1

Applications of Statistics.

1. Statistics and physical Science :— Statistical methods are very effective in the analysis of quantitative expresions in all the fields of most physical science like physics, geology, meterology, astronomy etc. +1

2. Statistics and natural Science :— Statistical methods such as Sampling techniques, estimation, design of experiments are very much useful in the study of medical Science, Zoology, biology etc. +1

3. Statistics and State :- In ancient time state collected data for maintaing economy, miliatary strength, Law and analyze data for framing suitable policies. +1

4. Statistics and Economy :- Statistical techniques are extremely used in the study of all branches of economics like demand analysis, index numbers etc. +1

5. Statistics and Computers :- It is widely used tool in the feild of computers. Computers are capable of performing a large no. of calculations in few seconds which is of great benefit to business and other organizations maintaing records of their operations. +1 (Any four)

5

Q-2

(iii) Population :- A population is the totality of objects under consideration. +1

or

A population may be defined as an aggregate of items possesing a common characteristics.

Sample :- A part of the population selected to know some thing about the population is called a sample. It is a subset of the population. +1

	Population	Sample	
1.	It is collection of all the units that posses common characterstics.	It is a subset of the population.	+1
2.	It includes each and every unit of a group.	It included a handful units of population.	+1
3.	Characterstic of population is parameter.	Characterstic of Sample is Statistic.	+1

Q-3.

(i) Median :- Median means middle value of distribution. It is the value in the series which divides the series into two equal parts, one part consisting the values equal to median or smaller than it and the other part having the value equal to median or larger than it. +1

Mode :- Mode is that value of the variable, which occurs or repeats itself the greatest numbes of times. It is most frequent value in a series. +1

Formula for median :-

$$\text{Median } (M) = l_1 + \left[\frac{\frac{N}{2} - F}{f}\right](l_2 - l_1)$$ +1

Where, l_1 = Lower limit of median class
l_2 = Upper limit of median class +0.5
F = Preceding c.f of median class
f = Frequency of median class.

Formula for mode :-

$$\text{Mode}(z) = l_1 + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right](l_2 - l_1) \quad +1$$

Where,
l_1 = Lower limit of modal class
l_2 = Upper limit of modal class +0.5
f_1 = frequency of model class
f_0 = frequency of the preceding modal class.
f_2 = Frequency of post modal class.

Q-3 (ii) Let the missing frequencies by f_1 and f_2

x	f	fx
0	46	0
1	f_1	f_1
2	f_2	$2f_2$
3	25	75
4	10	40
5	5	25
Total	$86 + f_1 + f_2 = \Sigma f$	$\Sigma fx = 140 + f_1 + 2f_2$

+2

We have $N = 200 = 86 + f_1 + f_2$

$f_1 + f_2 = 114$

$f_1 = 114 - f_2$ } +1

$\therefore$ Mean $= \frac{\Sigma fx}{\Sigma f}$ } +0.5

$1.46 = \frac{140 + f_1 + 2f_2}{200}$

$292 = 140 + f_1 + 2f_2$

$292 = 140 + (114 - f_2) + 2f_2$

$292 = 254 + f_2$

$\boxed{f_2 = 38}$ } +1

$f_1 = 114 - f_2$

$= 114 - 38$

$\boxed{f_1 = 76}$ } +0.5

Q-3 (iii)

Life (inhrs)	No. of bulbs (f)	Mid-value (x)	fx	$(x-\bar{x})$	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
0-8	3	4	12	-17.25	297.562	892.686
8-16	5	12	60	-9.25	85.562	427.81
16-24	10	20	200	-1.25	1.562	15.62
24-32	12	28	336	6.75	45.562	546.744
32-40	2	36	72	14.75	217.562	435.124
Total	32		680			2317.984

+2

+1 +1

Arithmetic mean $\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{680}{32}$ +1

$\boxed{\bar{x} = 21.25}$

Standard deviation $(\sigma) = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$ +1

$= \sqrt{\frac{2317.984}{32}}$

$= \sqrt{72.437}$

$\boxed{\sigma = 8.51}$

Coefficient of Variation $= \frac{\sigma}{\bar{x}} \times 100$ +1

$= \frac{8.51}{21.25} \times 100$

$\boxed{C.V. = 40.04\%}$

Q-4

(i) Mathematical Expectation :— The average value of random phenomena is known as its mathematical expection or expected value. +1

OR

The mean of a probability distribution is known as mathematical expection. It is denoted by $E(x)$

Properties of Mathematical expection.

1. $E(a) = a$ +1
2. $E(ax) = a\,E(x)$ +1
3. $E(a+x) = a + E(x)$ +1
4. $E(x+y) = E(x) + E(y)$ +1
5. $E(xy) = E(x) \cdot E(y)$

Q-4 (ii) Probability of drawing a white ball from
(a) the first bag $P(A) = \frac{4}{6}$ } +0.5

Prob. of drawing a white ball from the second bag $P(B) = \frac{3}{8}$ } +0.5

Prob. that both the balls are white =

$$P(A \cap B) = P(A) \cdot P(B)$$
$$= \frac{4}{6} \times \frac{3}{8}$$
$$P(A \cap B) = \frac{12}{48} = \boxed{\frac{1}{4}}$$

} +1 } 2

(b) Prob. that one is white and one is black

$$P(A \cap B) + P(B \cap A) = P(A) \cdot P(B) + P(B) \cdot P(A)$$ } +1
$$= \left(\frac{4}{6} \times \frac{5}{8}\right) + \left(\frac{2}{6} \times \frac{3}{8}\right)$$ } +1
$$= \frac{20}{48} + \frac{6}{48}$$
$$= \frac{26}{48} = \boxed{\frac{13}{24}}$$

} +1 } 3

Q-4 (iii) Let A, B and C be the events that the candidate is selected in the first, second and third company. Then.

$P(A) = \frac{1}{12}$ and $P(\bar{A}) = 1 - \frac{1}{12} = \frac{11}{12}$

$P(B) = \frac{1}{15}$ and $P(\bar{B}) = 1 - \frac{1}{15} = \frac{14}{15}$ +2

$P(C) = \frac{1}{10}$ and $P(\bar{C}) = 1 - \frac{1}{10} = \frac{9}{10}$

Now,

Prob. that he will not get job in any company is

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$$

$$= \frac{11}{12} \cdot \frac{14}{15} \cdot \frac{9}{10} \quad +2$$

$$= \frac{77}{100}$$

Prob. that he will get job at least at one comany =

$$P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) \quad +1$$

$$= 1 - \frac{77}{100}$$

$$\boxed{P(A \cup B \cup C) = \frac{23}{100}} \quad \text{Ans.}$$

Q-5

(i) Characteristics of normal distribution

1. The normal curve is symmetrical about the mean. It is bell-shaped. +1

2. The mean, median and mode have the same value i.e. mean = median = mode. +1

3. The height of the normal curve is maximum at the mean value. This ordinate divide the curve in two equal parts. +1

4. The first and third quartile are equidistant from median. +1

5. For normal distribution $\beta_1 = 0$ and $\beta_2 = 3$. +1

Q-5 (ii) It is given that no. of workers is 10.
i.e $n = 10$

Prob. of workers suffes from disease. +1

$$p = \frac{20}{100} = \frac{1}{5}$$

$$\therefore q = 1 - \frac{1}{5} = \frac{4}{5}$$

Using Binomial distribution, the prob. of x success is +0.5

$$P(x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

Let x be the random variable, showing the number of workers suffer from disease

(a) Prob. that exactly 2 workers suffer from disease.

$$P(x=2) = {}^{10}C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{10-2}$$

$$= \frac{10!}{8!\cdot 2!} \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^8$$

$$= \frac{45}{9} \times \frac{1}{25} \left(\frac{4}{5}\right)^8 \qquad +1.5$$

$$= \boxed{\frac{9}{5}\left(\frac{4}{5}\right)^8}$$

$$= \frac{9}{5} \times \frac{65536}{390625}$$

$$= \frac{589824}{1953125} = \boxed{0.3019}$$

(b) Not more than 2 workers suffer from disease.

$$P(x = \text{not more than } 2) = P(0) + P(1) + P(2)$$

$$P(x \le 2) = {}^{10}C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{10-1} + {}^{10}C_3 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^{10-2}$$

$$+2$$

$$= 1 \times 1 \times \left(\frac{4}{5}\right)^{10} + 10 \times \frac{1}{5}\left(\frac{4}{5}\right)^9 + 45 \times \frac{1}{25}\left(\frac{4}{5}\right)^8$$

$$= \left(\frac{4}{5}\right)^{10} + 2\left(\frac{4}{5}\right)^9 + \frac{9}{5}\left(\frac{4}{5}\right)^8$$

$$= \left(\frac{4}{5}\right)^8 \left[\frac{16}{25} + \frac{8}{5} + \frac{9}{5}\right]$$

$$= \left(\frac{4}{5}\right)^8 \left(\frac{16+40+45}{25}\right)$$

$$= \boxed{\frac{101}{25}\left(\frac{4}{5}\right)^8}$$

$$= \frac{101}{25} \times \frac{65536}{390625}$$

$$= \frac{6619136}{9765625}$$

$$= \boxed{0.677}$$

Q-5

(iii) It is given that on an average a manfuctured product has 2 defects } +0.5

$\therefore \lambda = \text{mean} = 2$

(*) Let x denotes the no. of defective products. They by using poission distribution } +1

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

(a) Without any defect

$$P(x=0) = \frac{e^{-2}(2)^0}{0!} \quad +0.5$$

$$= e^{-2}$$

$$= 0.135$$

(b) 3 defects.

$$P(x=3) = \frac{e^{-2}(2)^3}{3!}$$

$$= \frac{0.135 \times 8}{6}$$ +1.5

$$= \frac{0.54}{3}$$

$$= 0.18$$

(c) 4 defects.

$$P(x=4) = \frac{e^{-2}(2)^4}{4!}$$ +1.5

$$= \frac{0.135 \times 16}{4 \times 3 \times 2 \times 1}$$

$$= \frac{0.54}{6}$$

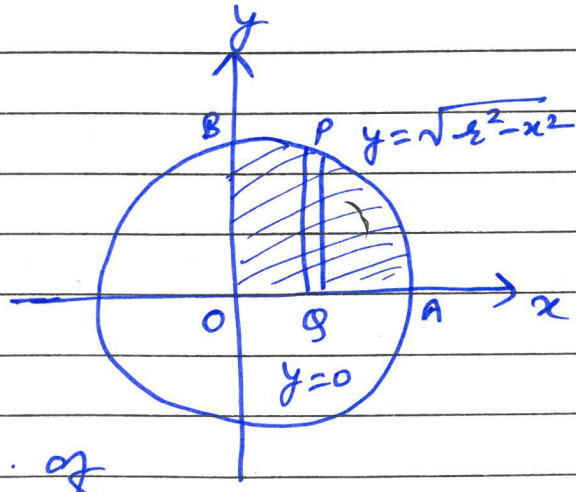
$$= 0.09$$

Q-6

(i) $x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y = \sqrt{r^2 - x^2}$



+1

Since, the circle is symmetric about the coordinate axes, area of circle is 4 times the area OAB.

For the region OAB,

y varies from 0 to $\sqrt{r^2-x^2}$

x varies from 0 to r } +1

$\therefore$ Area of the circle $= 4\int_0^r \int_0^{\sqrt{r^2-x^2}} dy\,dx$

$= 4\int_0^r [y]_0^{\sqrt{r^2-x^2}}\,dx$ +1

$= 4\int_0^r \sqrt{r^2-x^2}\,dx$

$= 4\left[\frac{r}{2}\sqrt{r^2-x^2} + \frac{r^2}{2}\sin^{-1}\frac{x}{r}\right]_0^r$

$= 4\left[0 + \frac{r^2}{2}\sin^{-1}\frac{r}{r} - \left(0 + \frac{r^2}{2}\sin^{-1}\frac{0}{r}\right)\right]$ +2

$= 4\left[\frac{r^2}{2}\sin^{-1}(1) - 0\right]$

$= 2r^2 \cdot \frac{\pi}{2}$ $\left\{\because \sin^{-1}(1) = \frac{\pi}{2}\right\}$

$= \pi r^2$

Q-6 (ii) $\int_0^1 \int_0^1 \frac{1}{(1+x^2)}\cdot\frac{1}{(1+y^2)}\,dx\,dy$

$\int_0^1 \frac{1}{1+x^2}\left[\int_0^1 \frac{1}{1+y^2}\,dy\right]dx$

$$= \int_0^1 \frac{1}{1+x^2} \left(\tan^{-1} y\right)_0^1 dx$$

$$= \int_0^1 \frac{1}{1+x^2} \left[\tan^{-1}(1) - \tan^{-1}(0)\right] dx$$

$$= \int_0^1 \frac{1}{1+x^2} \left[\frac{\pi}{4} - 0\right] dx \quad \left\{\because \tan^{-1}(1) = \frac{\pi}{4}\right\} \left\{\tan^{-1}(0) = 0\right\}$$

$$= \frac{\pi}{4} \int_0^1 \frac{1}{1+x^2} dx$$

$$= \frac{\pi}{4} \left[\tan^{-1} x\right]_0^1$$

$$= \frac{\pi}{4} \left[\tan^{-1}(1) - \tan^{-1}(0)\right]$$

$$= \frac{\pi}{4} \left[\frac{\pi}{4} - 0\right]$$

$$= \frac{\pi^2}{16}$$

+2 +2 +1

Q-6(iii) $$\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z} \, dx\, dy\, dz$$

$$= \int_{x=0}^{x=2} \int_{y=0}^{y=x} \int_{z=0}^{z=2x+2y} e^{x+y+z} \, dx\, dy\, dz$$

$$= \int_0^2 \int_0^x e^{x+y} \left[e^z\right]_0^{2x+2y} dx\, dy$$

$$= \int_0^2 \int_0^x e^{x+y} \left[e^{2x+2y} - e^0\right] dx\, dy$$

$$= \int_0^2 \int_0^x \left[e^{3x+3y} - e^{x+y} \right] dx\,dy \longrightarrow +2$$

$$= \int_0^2 \left[\frac{e^{3x} \cdot e^{3y}}{3} - e^x \cdot e^y \right]_0^x dx$$

$$= \int_0^2 \left[\frac{e^{3x}}{3} (e^{3x} - e^0) - e^x (e^x - e^0) \right] dx$$

$$= \int_0^2 \left[\frac{e^{6x} - e^{3x}}{3} - e^{2x} + e^x \right] dx \longrightarrow +1$$

$$= \frac{1}{3} \left[\left(\frac{e^{6x}}{6} - \frac{e^{3x}}{3} \right) \right]_0^2 - \left(\frac{e^{2x}}{2} \right)_0^2 + \left(e^x \right)_0^2$$

$$= \frac{1}{3} \left[\frac{e^{12}}{6} - \frac{e^6}{3} - \frac{1}{6} + \frac{1}{3} \right] - \frac{1}{2} (e^4 - e^0) + (e^2 - e^0)$$

$$= \frac{1}{3} \left[\frac{e^{12}}{6} - \frac{e^6}{3} - \frac{1}{6} + \frac{1}{3} \right] - \frac{1}{2} (e^4 - 1) + (e^2 - 1) \longrightarrow +2$$

$$= \frac{e^{12}}{18} - \frac{e^6}{9} - \frac{1}{18} + \frac{1}{9} - \frac{e^4}{2} + \frac{1}{2} + e^2 - 1$$

$$= \frac{e^{12}}{18} - \frac{e^6}{9} - \frac{e^4}{2} + e^2 - \frac{4}{9}$$