OR iii. A small project consists of seven activities for which the relevant data is given below: **7**

Activity	Preceding OR activity	Duration (Days)
A	-	4
B	-	7
C	-	6
D	A,B	5
E	A,B	7
F	C,D,E	6
G	C,D,E	5

Draw the network diagram and find the project completion time and total float.

Q.6 Attempt any two:

i. Discuss the basic element of waiting line model. **5**

ii. Auto vehicles arrive at a petrol pump, having one petrol unit, in poisson fashion with an average of 10 units per hour. The service time is distributed exponentially with a mean of 3 minutes. Find the following: **5**

(a) Average number of units in the sytem
(b) Average waiting time for customers
(c) Average length of queue.
(d) Probability that a customer arriving at the pump will have to wait
(e) The utilisation factor
(f) Probability tht the numberofcustomer in the system is 2

iii. A book store wishes to carry a particular book in stock. The demand of the book is not certain and there is a lead time of 2 days for stock replenishment. The probabilities of demand are given below: **5**

Each time an order is placed, the store incurs an ordering cost of Rs 10 per order. The store also incurs a carrying cost of Re 0.5 per book per day. The inventory carrying cost is calculated on the basis of stock at the end of each day.

Order 5 books when the present inventory plus any outstanding order falls below 8 books.

Currently (beginning of 1st day) the store has a stock of 8 books plus 5 books ordered two days ago and are expected to arrive the next day. Carryout simulation run for 10 days to recommend an appropriate option. You may use random numbers in the sequences, using the first number for day one. 89, 34, 78, 63, 61, 81, 39, 16, 13, 73.

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering
End Sem Examination May-2024
EN3ES15 Operations Research

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. _______ represent a relationship between dependent and independent variables and hence measure 'cause and effect' due to changes in independent variables. **1**

(a) Descriptive models (b) Predictive models
(c) Normative models (d) Dynamic models

ii. Observations and data collection for a better understanding of the problem is carried out in- **1**

(a) Judgement phase (b) Action phase
(c) Research phase (d) None of these

iii. Non-negativity condition in an LP model implies- **1**

(a) A positive coefficient of variables in objective function
(b) A positive coefficient of variables in any constraint
(c) Non-negative value of resources
(d) None of these

iv. If a non-redundant constraint is removed from an LP problem, then- **1**

(a) Feasible region will become larger
(b) Feasible region will become smaller
(c) Solution will become infeasible
(d) None of these

v. The solution to a transportation problem with m-sources & n-destinations is feasible if the number of allocations are- **1**

(a) m + n (b) m x n (c) m – n (d) m + n - 1

vi. If there were n workers and n jobs there would be- **1**

(a) $(n\,!)^n$ solutions (b) (n – 1)! solutions
(c) n! solutions (d) n solutions

vii. The activity that can be delayed without affecting the execution of the immediate succeeding activity is determined by- **1**

(a) Total float (b) Free float (c) Independent float (d) None of these

viii. Operating decisions in an inventory system are concerned with- **1**

(a) Order quantity (b) Reorder level
(c) Customer service level (d) All of these

ix. Which of the following relationships is not true? **1**

(a) $W_s = W_q + 1/\mu$ (b) $L_s = \lambda W_s$
(c) $L_s = L_q + 1/\lambda$ (d) $L_q = \lambda W_q$

x. As simulation is not an analytical model, therefore, the result of simulation must be viewed as- **1**

(a) Unrealistic (b) Exact
(c) Approximation (d) All of these

Q.2 i. Describe any two model based on method of solution. **2**

ii. What are various phases of operations research? Explain them briefly. **3**

iii. Describe the characteristics of operations research. **5**

OR iv. What is model? State the advantages of model building. **5**

Q.3 i. Write down the assumption of a linear programming model. **2**

ii. A firm manufactures two types of shafts A and B. For any month it must produce 250 shafts A and 100 shafts B. The maximum total requirment of shafts A and B is 1250 and minimum total requirement is 500. **8**

Both shafts are to be processed on machines M_1 and M_2. Total number of machines M_1 and M_2 available are 15 each. Processing times in hours for each shaft on machines M_1 and M_2 are as follows:

	A	B
M_1	1.5	2
M_2	1	1.5
Profit/unit(Rs)	400	600

If the firm has 25 working days a month, each of 8 hours, formulate the mathematical model for the problem and solve it by graphical method.

OR iii. Solve by simplex method- **8**

Maximize $Z = 107x_1 + x_2 + 2x_3$

Subjected to- $14x_1 + x_2 - 6x_3 + 3x_4 = 5$

$$16x_1 + \frac{1}{2}x_2 - 6x_3 \le 5$$

$$3x_1 - x_2 - x_3 \le 0$$

$$x_1, x_2, x_3, x_4 \ge 0$$

Q.4 i. Define following: **3**

(a) Feasible solution
(b) Basic feasible solution
(c) Optimal solution

ii. A company produces a small components for an industrial prodcut and distrubutes it to five wholsalers at a fixed delivered price of Rs 250 per unit. Sales forecasts indicate that monthly deliveries will be 300,300,100,500 and 400 units to wholesalers W_1, W_2, W_3, W_4 and W_5 repectively. The direct costs of production of each unit are Rs 100, Rs 90 and Rs 80 at plant P1, P2 and P3 respectively. The transportation cost of shipping a unit from plants to wholesalers are given below. **7**

		Wholesalers				
		W_1	W_2	W_3	W_4	W_5
Plants	P_1	5	7	10	15	15
	P_2	8	6	9	12	14
	P_3	10	9	8	10	15

Find how many components each plant must supply to each wholesaler to maximize the profit? What is the maximum profit? Take the monthly production capacities of plant P_1, P_2 and P_3 as 500,100 and 1250 respectively.

OR iii. Find the minimum cost for the assignment problem whose cost coefficient are as given below: **7**

	1	2	3	4	5
I	-2	-4	-8	-6	-1
II	0	-9	-5	-5	-4
III	-3	-8	-9	-2	-6
IV	-4	-3	-1	0	-3
V	-9	-5	-8	-9	-5

Q.5 i. Explain briefly ABC analysis. **3**

ii. A shopkeeper has a uniform demand of an item at the rate of 50 items per month. He buys it from a supplier at a cost of Rs 6 per item and the cost of ordering is Rs.10 each time. If the stock holding cost are 20% per year of the stock value, how frequently should he replenish his stock? Suppose the supplier offers a 5% discount on orders between 200 and 900 itmes and a 10 % discount on orders exceeding or equal to 1000 items, can the shopkeeper reduce his costs by taking advantage of either of these discounts? **7**

Marking Scheme
Operations Research EN3ES15

Q.1	i)	b) Predictive models	1
	ii)	c) Research phase	1
	iii)	c) Non negative value of resources	1
	iv)	a) feasible region will become larger	1
	v)	d) m+n-1	1
	vi)	c) n! solutions	1
	vii)	b) free float	1
	viii	d) all of the above	1
	ix)	c) Ls = Lq + 1/λ	1
	x)	c) approximation	1

Q.2 i. Any two of following : — 2

a) Heuristic Model
b) Analytical Model
c) Simulation Model
* 1 mark for each explanation 1*2 = 2 marks

ii. Briefly describing following phases — 3

a) Judgement Phase
b) Research Phase
c) Action Phase

* 1 mark for each explanation 1*3 = 3 marks

iii. Explaining following characteristics — 5

a) Interdisciplinary Approach
b) Scientific Approach
c) Holistic Approach
d) Objective Oriented Approach
e) System Approach
* 1 mark for each explanation 1*5 = 5 marks

OR iv. Definition – 1 marks — 5

Adavantages – 4 marks

Q.3 i. Assumption of LPP — 2

a) Certainty
b) Additivity
c) Linearity
d) Divisibility
0.5 Marks each 0.5*4 = 2 marks

ii. — 8

$$\text{Max } Z = 400x_1 + 600x_2,$$

$$\begin{aligned}
\text{subject to} \quad x_1 &\geq 250, \\
x_2 &\geq 100, \\
x_1 + x_2 &\leq 1{,}250, \\
x_1 + x_2 &\geq 500, \\
1.5x_1 + 2x_2 &\leq 3{,}000, \\
x_1 + 1.5x_2 &\leq 3{,}000, \\
x_1, x_2 &\geq 0.
\end{aligned}$$

$x_1 = 250,\ x_2 = 1{,}000;\ Z_{max} = ₹\ 7{,}00{,}000.)$

Formulation of LPP - 3 marks
Constructing graph - 3 marks
Finding optimum answer – 2 marks

OR iii. — 8

Step 1. Set up the Problem in the Standard Form

The first constraint is of equality form. It can be divided throughout by 3 to give unit coefficient to x_4.

i.e., $$\frac{14}{3}x_1 + \frac{1}{3}x_2 - 2x_3 + x_4 = \frac{7}{3} \rightarrow 5/3$$

Variable x_4, now, occurs in constraint 1 with unit coefficient and it occurs in no other constraint and hence can be treated as a slack variable. Introducing slack variables, say x_5 and x_6 in second and third constraints, the problem can be expressed in standard form as

$$\text{maximize } Z = 107x_1 + x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6,$$

$$\begin{aligned}
\text{subject to} \quad & \frac{14}{3}x_1 + \frac{1}{3}x_2 - 2x_3 + x_4 + 0x_5 + 0x_6 = \frac{7}{3}, \rightarrow 5/3 \\
& 16x_1 + \frac{1}{2}x_2 - 6x_3 + 0x_4 + x_5 + 0x_6 = 5, \\
& 3x_1 - x_2 - x_3 + 0x_4 + 0x_5 + x_6 = 0, \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0.
\end{aligned}$$

Step 2. Find an Initial Basic Feasible Solution

Setting decision variables x_1, x_2, x_3 each equal to zero, the basic (degenerate) feasible solution is

$$\begin{aligned}
x_1 = x_2 = x_3 &= 0 \text{ (non-basic)}, \\
x_4 &= 7/3 \text{ (basic)}, \\
x_5 &= 5 \text{ (basic)}, \\
x_6 &= 0 \text{ (basic)}, \\
Z &= 0.
\end{aligned}$$

This solution is represented in table 2.89.

5/3

TABLE 2.89

	c_j	107	1	2	0	0	0		
c_B	Basis	x_1	x_2	x_3	x_4	x_5	x_6	b	θ
0	x_4	14/3	1/3	−2	1	0	0	7/3	1/2
0	x_5	16	1/2	−6	0	1	0	5	5/16
0	x_6	(3)	−1	−1	0	0	1	0	0 ←
	Z_j	0	0	0	0	0	0	0	
	$c_j - Z_j$	107	1	2	0	0	0		
		↑							

Initial basic feasible solution

Step 3. Perform Optimality Test

Since $c_j - Z_j$ is positive under some variable columns, table 2.89 is not optimal.

Step 4. Iterate Towards an Optimal Solution

In table 2.89, x_1 is incoming variable, x_6 is outgoing variable and (3) is the key element. In table 2.90, x_6 is replaced by x_1.

TABLE 2.90

	c_j	107	1	2	0	0	0		
c_B	Basis	x_1	x_2	x_3	x_4	x_5	x_6	b	θ
0	x_4	0	$17/9$	$-4/9$	1	0	$-14/9$	$7/3$	$-21/4$
0	x_5	0	$35/6$	$-2/3$	0	1	$-16/3$	5	$-15/2$
107	x_1	1	$-1/3$	$-1/3$	0	0	$1/3$	0	-0
	Z_j	107	$-107/3$	$-107/3$	0	0	$107/3$	0	
	$c_j - Z_j$	0	$110/3$	$113/3$ ↑	0	0	$-107/3$		

Second basic feasible solution

Step 5. Perform Optimality Test

Since $c_j - Z_j$ is positive under some variable columns, table 2.90 is not optimal.

Step 6. Iterate Towards an Optimal Solution

In table 2.90, x_3 is the incoming variable. However, since all replacement ratios (in θ-column) are negative, the problem has an unbounded solution.

Standard form – 2 marks
Initial feasible solution and initial table- 3 marks
Optimality check and second feasible solution and finding that solution is unbounded – 3 marks

Q.4 i. 3

1. **Feasible Solution.** A feasible solution to a transportation problem is a set of non-negative allocations, x_{ij} that satisfies the rim (row and column) restrictions.
2. **Basic Feasible Solution.** A feasible solution to a transportation problem is said to be a basic feasible solution if it contains no more than $m + n - 1$ non-negative allocations, where m is the number of rows and n is the number of columns of the transportation problem.
3. **Optimal Solution.** A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

Explaining each one above 1 marks each 1*3 = 3 marks

ii. (*Ans.* $x_{11} = 250$, $x_{22} = 100$, $x_{31} = 50$, $x_{32} = 200$, $x_{33} = 100$, $x_{34} = 500$, $x_{35} = 400$; $Z_{max} = ₹\ 2{,}53{,}300$.) 7

Finding initial solution by VAM - 3 marks
Finding optimum solution by MODI – 4 marks

OR iii. (*Ans.* 1 – 3, 2 – 2, 3 – 5, 4 – 1, 5 – 4; $Z_{min} = -36$.) 7

Initial solution - 3 marks
Optimality check and optimal solution – 4 marks

Q.5 i. Explaining the three categories 1 mark each 1*3 = 3 marks 3

ii. (*Ans.* (*i*) 20 days, C.P. : B-E-F (1-3-5-6-7.) 7

Drawing network diagram 2 marks
Calculating ES- EF, LS – LF timings – 2 marks
Calculation of Total float – 2 marks
Critical path – 1 mark

OR iii. (*Ans.* 5% discount should be accepted.) 7

Calculation of cost without discount - 2 marks
Calculation of cost with 5% discount – 3 marks
Calculation of cost with 10 % discount – 2mark

Q.6 i. Explaining 5

A queuing system is specified completely by seven main elements:

1. Input or arrival (inter-arrival) distribution
2. Output or departure (service) distribution
3. Service channels
4. Service discipline
5. Maximum number of customers allowed in the system
6. Calling source or population
7. Customer's behaviour.

First 2 point – 1 mark
3 point – 1 mark
4 point – 1 mark
5 and 6 point – 1 mark
7 point – 1 mark

ii. a) Average number of units in the sytem =1 (0.5 marks) 5
b) Average waiting time for customers = 3 min (0.5 marks)
c) Average length of queue. = 0.5 (0.5 marks)
d) Probability that a customer arriving at the pump will have to wait= 0.5 (1.5 mark)
e) The utilisation factor =0.5 (0.5 mark)
f) Probability tht the numberofcustomer in the system is 2 =0.125 (1.5 mark)

iii. Assume probabilities 1 Mark 5
Cumulative probability 1 Mark
Expected demand calculation 3 Marks

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