Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2019

CA3CO08 Mathematics-II

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Which of the theorem is used to evaluate n^{th} derivate of product of two functions? **1**

(a) Rolle's Theorem (b) Lagrange's Theorem

(c) Leibnitz's Theorem (d) None of these

ii. The value of 'c' by Lagrange's mean value Theorem for the function $f(x) = x^3 - 2x^2 - x + 3$ in the interval [0,1] is **1**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) None of these

iii. The function $f(x, y)$ is minimum, when **1**

(a) $rt - s^2 > 0, r > 0$ (b) $rt - s^2 > 0, r < 0$

(c) $rt - s^2 \geq 0$ (d) None of these

iv. If X is the true value of a quantity and X' is its approximate value then absolute error is equal to **1**

(a) $\left|\frac{X-X'}{X}\right| \times 100\%$ (b) $\left|\frac{X-X'}{X}\right|$

(c) $|X - X'|$ (d) None of these

v. The value of $\beta(3,4)$ is

(a) $\frac{1}{12}$ (b) 60 (c) $\frac{1}{30}$ (d) $\frac{1}{60}$ **1**

vi. If n is positive integer then value of Γn is equal to **1**

(a) $\int_0^\infty e^{-x} x^n \, dx$ (b) $\int_0^\infty e^{-x} x^{n-1} \, dx$

(c) $\int_0^\infty e^{-x} x^{n+1} \, dx$ (d) None of these

vii. $\int_0^1 \int_0^x e^x \, dx dy$ is equal to **1**

(a) -1 (b) 0 (c) 2 (d) None of these

viii. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is **1**

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) None of these

ix. The differential equation $Mdx + Ndy = 0$ is an exact differential equation if **1**

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (b) $\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$

(c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (d) None of these

x. The integrating factor of $\frac{dy}{dx} + Py = Q$ is **1**

(a) $e^{\int Qdx}$ (b) $e^{\int Qdy}$ (c) $e^{\int Pdy}$ (d) None of these

Q.2 Attempt any two:

i. Expand the function $\frac{e^x}{1+e^x}$ by Maclaurin's theorem as far as the term x^3. **5**

ii. Expand $\tan x$ in power of $\left(x - \frac{\pi}{4}\right)$ by Taylor's theorem. **5**

iii. State Rolle's theorem and Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ in $[-\sqrt{2}, \sqrt{2}]$. **5**

Q.3 i. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$ **2**

ii. If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then find the value of $\frac{\partial^2 z}{\partial y \partial x}$. **3**

iii. Discuss the maximum or minimum values of the function $f(x, y) = x^3 - 4xy + 2y^2$. **5**

OR iv. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that **5**

(a) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

(b) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$

Q.4 Attempt any two:

i. Evaluate $\lim_{n\to\infty}\left[\frac{(n+1)(n+2)\ldots\ldots\ldots\ldots(n+n)}{n^n}\right]^{1/n}$. **5**

ii. If m is positive real number, then prove that $\Gamma(m)\Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$ **5**

iii. Show that $\int_0^{\pi/2}\frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2}\sqrt{\sin x}\ dx = \pi$. **5**

Q.5 Attempt any two:

i. Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the same. **5**

ii. Calculate the volume of the solid bounded by the following surfaces $z = 0, x^2 + y^2 = 1, x + y + z = 3$. **5**

iii. If the region is bounded by $x + y + z = 1, x = 0, y = 0\ and\ z = 0$ then find the value of $\iiint\frac{dxdydz}{(1+x+y+z)^3}$. **5**

Q.6 Attempt any two:

i. Solve $(1 + y^2)dx = (\tan^{-1} y - x)dy$. **5**

ii. Solve $[1 + \log(xy)]dx + \left[1 + \frac{x}{y}\right]dy = 0$ **5**

iii. Solve $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$. **5**

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Q.1. (i) (c) Leibnitz's theorem +1

(ii) (a) $\frac{4}{3}$ +1

(iii) (a) $rt-s^2>0, r>0$ +1

(iv) (c). $|X-X'|$ +1

(v) (d) $\frac{1}{60}$ +1

(vi) (b) $\int_0^\infty e^{-x}x^{n-1}dx$ +1

(vii) (d) None of these +1

(viii) (b) $\frac{1}{6}$ +1

(ix) (c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ +1

(x) (d) None of these. +1

Q.2(i) Let $y = \frac{e^x}{1+e^x}$

differentiate w.r.t x, we get

$$y_1 = \frac{e^x}{(1+e^x)^2} = \frac{e^x}{1+e^x}\left(1 - \frac{e^x}{1+e^x}\right)$$

$$\Rightarrow y_1 = y - y^2$$

$$y_2 = y_1 - 2yy_1$$

$$y_3 = y_2 - 2y \cdot y_2 - 2y_1^2$$

$$y_4 = y_3 - 2y \cdot y_3 - 6y_1 \cdot y_2$$ +1 1/2

Putting $x = 0$, we get

$y_0 = \frac{1}{2}, (y_1)_0 = \frac{1}{4}, (y_2)_0 = 0, (y_3)_0 = \frac{-1}{8}$ + 1 1/2

$(y_4)_0 = 0$

Maclaurin's series is given by +1

$$y = y_0 + \frac{x}{\lfloor 1}(y_1)_0 + \frac{x^2}{\lfloor 2}(y_2)_0 + \frac{x^3}{\lfloor 3}(y_3)_0 + \frac{x^4}{\lfloor 4}(y_4)_0 + \cdots$$

$$\frac{e^x}{1+e^x} = \frac{1}{2} + \frac{x}{4} - \frac{x^3}{48} + \cdots$$ +1

Q-2(ii) By Taylor's theorem

$$f(x) = f(a) + \frac{(x-a)}{\lfloor 1} f'(a) + \frac{(x-a)^2}{\lfloor 2} f''(a) + \cdots \quad -(1)$$ +1

$f(x) = \tan x \qquad \therefore f(\pi/4) = 1$

$f'(x) = \sec^2 x \qquad \therefore f'(\pi/4) = 2$ +1

$f''(x) = 2\sec^2 x \tan x \qquad f''(\pi/4) = 4$ +1

$f'''(x) = 2[\sec^4 x + 2\sec^2 x \tan^2 x]; \quad f'''(\pi/4) = 16$ +1

Put the values in equ(1), we get

$$\tan x = 1 + 2(x - \pi/4) + 2(x - \pi/4)^2 + \frac{8}{3}(x - \pi/4)^3 + \cdots$$ +1

Q-2(iii) If $f(x)$ be a real valued fut^n of x

Such that

(i) $f(a) = f(b)$

(ii) $f(x)$ is continuous fut^n in the closed interval $[a,b]$, i.e. $a \leq x \leq b$.

(iii) $f(x)$ is differentiable in the open interval (a,b) i.e. $a < x < b$.

Then $\exists$ at least one real value of $c \in (a,b)$ Such that

$$\boxed{f'(c) = 0.}$$ +2

Given function is

$$f(x) = 2x^3 + x^2 - 4x - 2$$

in $[-\sqrt{2}, \sqrt{2}]$

(i) Put $x = -\sqrt{2}$ and $x = \sqrt{2}$ in given fun^n.

Clearly, $f(-\sqrt{2}) = 0 = f(\sqrt{2})$

(ii) $\because$ $f(x)$ is a polynomial fun^n in x, +1
then $f(x)$ is continuous in $[-\sqrt{2}, \sqrt{2}]$

(iii) $\because$ $f(x)$ is polynomial fun^n in x, then it can differentiable such that

$$f'(x) = 6x^2 + 2x - 4.$$ +1

then by Rolle's theorem $\exists$ $c \in (-\sqrt{2}, \sqrt{2})$ s.t.

$$f'(c) = 0$$

$$6c^2 + 2c - 4 = 0$$

$$\Rightarrow (3c-2)(c+1) = 0.$$

$$\Rightarrow c = \frac{2}{3}, -1 \in (-\sqrt{2}, \sqrt{2})$$ +1

Hence Rolle's theorem is verified for the given fun^n.

Q.3 (i) $f(x) = 3x^2 + 5x + 3$

$f'(x) = 6x + 5$

we know that

$$f(x+\delta x) = f(x) + f'(x)\,\delta x \text{ (Approx)}.$$ +1

$$f(x+\delta x) = [3x^2 + 5x + 3] + [6x+5]\,\delta x \quad \text{—(1)}.$$

Taking $x = 3$, $\delta x = 0.02$,

then equ(1) becomes.

$$f(3.02) = [3(3)^2 + 5(3) + 3] + [6(3)+5](0.02)$$

$$f(3.02) = \underline{\underline{45.46}}$$ +1

Q.3(ii) Given $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ — (1)

differentiate partially equ (1) w.r.t. x.

$$\frac{\partial z}{\partial x} = 2x \tan^{-1}\left(\frac{y}{x}\right) - \frac{y}{x^2+y^2}(x^2+y^2)$$

$$\frac{\partial z}{\partial x} = 2x \tan^{-1}\left(\frac{y}{x}\right) - y \quad \text{— (2)}$$ $+1\frac{1}{2}$

again differentiate partially equ (2) w.r.t. y

$$\frac{\partial^2 z}{\partial y \partial z} = \frac{2x^2}{x^2+y^2} - 1$$

$$\frac{\partial^2 z}{\partial y \partial z} = \frac{x^2-y^2}{x^2+y^2}$$ $+1\frac{1}{2}$

Q.3(iii) Given $f(x,y) = x^3 - 4xy + 2y^2$ — (1)

$\frac{\partial f}{\partial x} = 3x^2 - 4y$, $\frac{\partial f}{\partial y} = -4x + 4y$

$\frac{\partial f}{\partial x} = 0 \Rightarrow 3x^2 - 4y = 0$ — (2)

$\frac{\partial f}{\partial y} = 0 \Rightarrow -4x + 4y = 0$ $+1$

$\Rightarrow x = y$

put in equ (2) we get

$x = 0,\ x = 4/3$

$y = 0,\ y = 4/3.$ $+1$

We will discuss the maxima or minima at $(0,0)$ and $\left(\frac{4}{3}, \frac{4}{3}\right)$

$r = \frac{\partial^2 f}{\partial x^2} = 6x$, $s = \frac{\partial^2 f}{\partial x \partial y} = -4$, $t = \frac{\partial^2 f}{\partial t^2} = 4$

Case I at $(0,0)$

$r = 0$, $s = -4$, $t = 4$.

$rt - s^2 = -16 < 0$. +1

Hence, we conclude that $(0,0)$ is a saddle pt. of $f(x,y)$

Case II. at $\left(\frac{4}{3}, \frac{4}{3}\right)$

$r = 8 > 0$, $s = -4$, $t = 4$

and $rt - s^2 = 16 > 0$

$\therefore rt - s^2 > 0$ & $r < 0$, +1

$\therefore f(x,y)$ has minimum at $\left(\frac{4}{3}, \frac{4}{3}\right)$

$$\boxed{f_{min} = \frac{-32}{27}}$$ +1

Q.3(iv) Given $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

$\Rightarrow \sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$.

Let $z = \sin u$

t-test $z(x(t), y(t)) = \frac{tx+ty}{\sqrt{tx}+\sqrt{ty}}$

$= t^{1/2} \cdot z(x,y)$ +1

Clearly, z is a homogeneous fut^n of x & y of degree $\frac{1}{2}$.

Now, by Euler's theorem

(a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\cdot z$ +1

$x\frac{\partial(\sin u)}{\partial z} + y\frac{\partial}{\partial y}(\sin u) = \frac{1}{2}\sin u$

$\Rightarrow \boxed{x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u}$ +1

(b) By Euler's II deduction

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = g(u)\cdot[g'(u)-1]$

Let $g(u) = \frac{1}{2}\tan u$, $g'(u) = \frac{1}{2}\sec^2 u$. +1

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{2}\tan u\left[\frac{1}{2}\sec^2 u - 1\right]$

$= -\frac{\sin u\cdot\cos 2u}{4\cos^3 u}$. +1

Q.4 (i) Let $P = \lim_{n\to\infty}\left[\frac{(n+1)(n+2)\cdots(n+n)}{n^n}\right]^{1/n}$.

$P = \lim_{n\to\infty}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right]^{1/n}$.

Taking log on both sides

~~$\log P = \lim_{n\to\infty}$~~

$\log P = \lim_{n\to\infty}\frac{1}{n}\left\{\log\left(1+\frac{1}{n}\right) + \log\left(1+\frac{2}{n}\right) + \cdots + \log\left(1+\frac{n}{n}\right)\right\}$ +1

$$\log P = \lim_{n\to\infty} \sum_{r=1}^{n} \log\left(1+\frac{r}{n}\right)\cdot \frac{1}{n} \qquad +1$$

Then by Summation of series, $\frac{1}{n} \to dx$, $\frac{r}{n}$ by x, Σ by $\int$.

$$\log P = \int_0^1 \log(1+x)\,dx \qquad +1$$

$$= \left[x\log(1+x)\right]_0^1 - \int_0^1 \frac{x}{1+x}\,dx$$

$$= 2\log 2 - 1$$

$$\log P = \log\left(\frac{4}{e}\right) \qquad +1$$

$$\Rightarrow P = \frac{4}{e}$$

$$\therefore \lim_{n\to\infty}\left[\frac{(n+1)(n+2)\cdots(n+n)}{n^n}\right]^{1/n} = \frac{4}{e} \qquad +1$$

Q 4(ii) We know that

$$B(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n} \qquad +1$$

$$\Rightarrow 2\int_0^{\pi/2} \sin^{2m-1}\theta\cdot\cos^{2n-1}\theta\,d\theta = \frac{\Gamma m\,\Gamma n}{\Gamma m+n} \quad (1) \qquad +1$$

Put $2n-1=0$, ie $n=1/2$ in (1) we get

$$2\int_0^{\pi/2} \sin^{2m-1}\theta\,d\theta = \frac{\Gamma m\,\Gamma \frac{1}{2}}{\Gamma(m+\frac{1}{2})} = \frac{\Gamma m\,\sqrt{\pi}}{\Gamma(m+\frac{1}{2})} \quad (2) \qquad +1$$

Again put $n=m$ in equ (1).

$$2\int_0^{\pi/2} \sin^{2m-1}\theta\cdot\cos^{2m-1}\theta\,d\theta = \frac{\Gamma m\,\Gamma m}{\Gamma 2m}$$

$$\Rightarrow \frac{2}{2^{2m-1}}\int_0^{\pi/2} (\sin 2\theta)^{2m-1}\,d\theta = \frac{\Gamma m\,\Gamma n}{\Gamma 2m}$$

Put $2\theta = t \Rightarrow d\theta = \frac{dt}{2}$

$$\frac{2}{2^{2m-1}} \int_0^{\pi} (\sin t)^{2m-1} \frac{dt}{2} = \frac{\Gamma m \Gamma m}{\Gamma 2m}$$

$$\frac{1}{2^{2m-1}} \int_0^{\pi} (\sin t)^{2m-1} dt = \frac{\Gamma m \Gamma m}{\Gamma 2m}$$

$$\Rightarrow \frac{2}{2^{2m-1}} \int_0^{\pi/2} \sin^{2m-1} t \, dt = \frac{\Gamma m \Gamma m}{\Gamma 2m}$$

$$\Rightarrow 2 \int_0^{\pi/2} \sin^{2m-1} \theta \, d\theta = \frac{\Gamma m \Gamma m}{\Gamma 2m} \cdot 2^{2m-1} \quad (3)$$ +1

Now, from equ (2) & (3).

$$\frac{\Gamma m \sqrt{\pi}}{\Gamma(m+\frac{1}{2})} = \frac{\Gamma m \Gamma m}{\Gamma(2m)} \cdot 2^{2m-1}$$

$$\Rightarrow \Gamma m \, \Gamma(m+\tfrac{1}{2}) = \frac{\sqrt{\pi}\, \Gamma(2m)}{2^{2m-1}}$$ +1

Hence proved

Q. 4(iii) On taking L.H.S

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} \, dx$$

$$= \int_0^{\pi/2} \sin^{-1/2} x \, dx \times \int_0^{\pi/2} \sin^{1/2} x \, dx$$ +1

$$= \int_0^{\pi/2} \sin^{2(1/4)-1} x \cdot \cos^{2(1/2)-1} dx \times \int_0^{\pi/2} \sin^{2(3/4)-1} x \cos^{2(1/2)-1} x \, dx$$

$$= \frac{1}{2} B\left(\frac{1}{4}, \frac{1}{2}\right) \times \frac{1}{2} B\left(\frac{3}{4}, \frac{1}{2}\right)$$ +1

$$= \frac{1}{2}\frac{\Gamma\frac{1}{4}\,\Gamma\frac{1}{2}}{\Gamma\frac{1}{4}+\frac{1}{2}} \times \frac{1}{2}\frac{\Gamma\frac{3}{4}\,\Gamma\frac{1}{2}}{\Gamma\frac{3}{4}+\frac{1}{2}}$$ +1

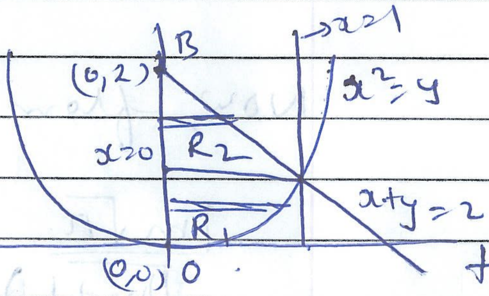
$$= \frac{1}{4}\frac{\Gamma\frac{1}{4}\sqrt{\pi}\cdot\Gamma\frac{3}{4}\cdot\sqrt{\pi}}{\Gamma\frac{3}{4}\,\Gamma\frac{5}{4}}$$

$$= \frac{\pi}{4}\frac{\Gamma\frac{1}{4}\,\Gamma\frac{3}{4}}{\Gamma\frac{3}{4}\,\Gamma\frac{5}{4}}$$ +1

$$= \frac{\pi}{4}\frac{\Gamma\frac{1}{4}}{\frac{1}{4}\Gamma\frac{1}{4}} = \underline{\pi = R.H.S}$$ +1

Q-5(i) Let $I = \int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$

$I = R_1 + R_2$



+1

In the Region R_1

The change of order of integration

$$I_1 = \int_0^1\int_0^{\sqrt{y}} xy\,dy\,dx$$ +1

$$= \int_0^1 \frac{y^2}{2}\,dy$$

$$= \frac{1}{6}$$ $+\frac{1}{2}$

In the Region R_2

The change of order of integration

$$I_2 = \int_1^2\int_0^{2-y} xy\,dy\,dx$$ +1

$$= \frac{1}{2}\int_1^2 y(2-y)^2\,dy$$

$$I_2 = \frac{5}{24} \qquad +\tfrac{1}{2}$$

$\therefore$ $I = \int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$

$$= \int_0^1 \int_0^{\sqrt{y}} xy\, dy\, dx + \int_1^2 \int_0^{2-y} xy\, dy\, dx$$

$$= \frac{1}{6} + \frac{5}{24}$$

$$= \frac{3}{8} \text{ Ans} \qquad +1$$

Q. 5(ii) Given surfaces are

$x^2+y^2=1,\ x+y+z=3,\ z=0$

Limits of z are from $z=0$ to $3-x-y$

Limits of y are from $y=-\sqrt{1-x^2}$ to $\sqrt{1-x^2}$

limits of x are from $x=-1$ to $x=1$ +1

$\therefore$ required volume

$$V = \int_{x=-1}^{1} \int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{z=0}^{z=3-x-y} 1\, dx\, dy\, dz \qquad +1$$

$$= \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3-x-y)\, dy\, dx \qquad +1$$

$$= \int_{-1}^{1} (3-x)\, 2\sqrt{1-x^2}\, dx \qquad +1$$

$$= 2\left[\int_{-1}^{1} 3\sqrt{1-x^2}\, dx - \int_{-1}^{1} x\sqrt{1-x^2}\, dx\right]$$

$\because x\sqrt{1-x^2}$ is odd fun^n & $\sqrt{1-x^2}$ is even fun^n

$= 12 \int_0^1 \sqrt{1-x^2}\,dx$

$= 12 \left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x \right]_0^1$

$= \frac{12}{2}\sin^{-1}(1)$

$\boxed{V = 3\pi}$ +1

Q. 5(iii) In the bounded region $x=0, y=0, z=0$ and $x+y+z=1$

limits of integration are as

z varies from 0 to $1-x-y$ +1

y varies from 0 to $1-x$

x varies from 0 to 1

$I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dz}{(1+x+y+z)^3}\,dy\,dx$ +1

$= \int_0^1 \int_0^{1-x} \left[\frac{1}{4} - \frac{1}{(1+x+y)^2} \right] dy\,dx$ +1

$= -\frac{1}{2}\int_0^1 \left[\frac{1-x}{4} + \frac{1}{2} - \frac{1}{1+x} \right] dx$ +1

$= -\frac{1}{2}\left[\frac{1}{2} - \log 2 + \frac{1}{8} \right]$

$I = \frac{1}{2}\left[\log 2 - \frac{5}{8} \right]$

$\Rightarrow \iiint_R \frac{dx\,dy\,dz}{(1+x+y+z)^3} = \frac{1}{2}\left[\log 2 - \frac{5}{8} \right]$ +1

Q.6(i) $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2} \qquad +1$$

which is a linear differential eqn

$$I.F = e^{\int \frac{1}{1+y^2}dx} = e^{\tan^{-1}x}. \qquad +1$$

Hence, the soln is given.

$$x \times I.F = \int Q.\,dy + c. \qquad +1$$

$$\Rightarrow x e^{\tan^{-1}y} = \int e^{\tan^{-1}y} \cdot \frac{\tan^{-1}y}{1+y^2}\,dy + c \qquad +1$$

Put $\tan^{-1}y = t$.

$$\Rightarrow \frac{1}{1+y^2}\,dy = dt.$$

$$\cancel{xe^{\tan^{-1}}}\; xe^t = \int e^t.\,t\,dt$$

$$xe^t = te^t - e^t + c$$

$$\Rightarrow x = (\tan^{-1}y - 1) + ce^{-\tan^{-1}y} \qquad +1$$

Q.6(ii) $[1+\log(xy)]\,dx + \left[1+\frac{x}{y}\right]dy = 0.$

On comparing with $M\,dx + N\,dy = 0$

$M = 1+\log(xy)$, $N = 1+\frac{x}{y}$

$$\frac{\partial M}{\partial y} = \frac{1}{y}, \quad \frac{\partial N}{\partial x} = \frac{1}{y} \qquad +1$$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$, $\therefore$ given eqn is exact diff. eqn +1

Hence the sol^n is given by

$$\int_{y\text{-const}} M\,dx + \int N(\text{terms not containing } x)\,dy = c \qquad +1$$

$$\int [1+\log x + \log y]\,dx + \int dy = c \qquad +1$$

$$x + \int \log x + \int \log y\,dx + y = c$$

$$x + x\log x - x + x\log y + y = c$$

$$\Rightarrow x\log xy + y = c \qquad +1$$

Q. 6(iii) $(D^2 - 4D + 4)y = 8x^2e^{2x}\sin 2x$

A·E is given

$$m^2 - 4m + 4 = 0$$

$$\Rightarrow m = 2, 2. \qquad +1$$

$$C\cdot F = (c_1 + c_2 x)e^{2x}. \qquad +1$$

$$P\cdot I = \frac{1}{f(D)} 8x^2e^{2x}\sin 2x$$

$$= 8\frac{1}{(D-2)^2} e^{2x} x\sin 2x$$

$$D \rightarrow D+2$$

$$= [8e^{2x}\frac{1}{D^2}(x\sin 2x) \qquad +1$$

$$= 8e^{2x}\frac{1}{D}\left[\int x\sin 2x\,dx\right]$$

$$= 8e^{2x}\frac{1}{D}\left[\frac{-x^2}{2}\cos 2x + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right] \qquad +1$$

$$= 8e^{2x}\int\left[-\frac{x^2}{2}\cos 2x + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]dx$$

$$P.I = -e^{2x}\left[4x\cos 2x + (2x^2-3)\sin 2x\right] + 1$$

Complete soln is given by

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^{2x} - e^{2x}\left[4x\cos 2x + (2x^2-3)\sin 2x\right] + 1$$