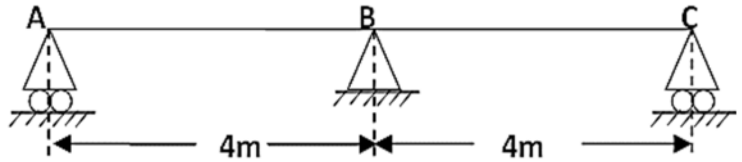
Q.5 i. State Muller Breslau principal for influence line diagram. Explain with a suitable example. **4**

ii. For a propped cantilever of span 10 m draw the influence line for moment at fixed end. Compute the ordinates at intervals of 1.25 m. **6**

OR iii. Draw I.L.D. for vertical reaction at point A of beam shown below and compute ordinates at every 1m internal. **6**

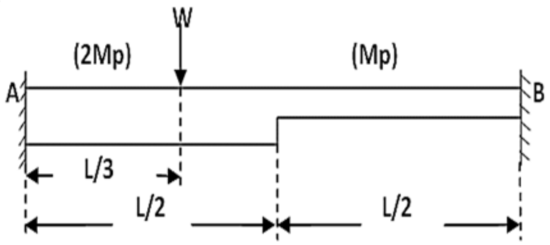


Q.6 i. Define the following: **2**

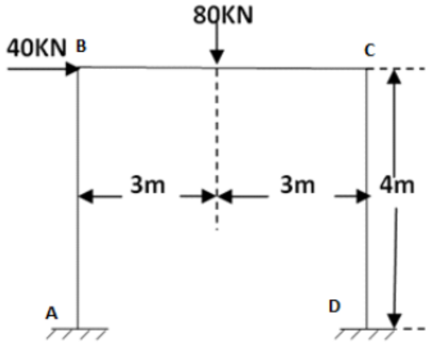
(a) Plastic moment capacity (b) Collapse mechanism

ii. Determine shape factor of a rectangular section **3**

iii. Find the collapse load of the beam shown below: **5**



OR iv. Determine the plastic moment capacity and real mechanism for the frame shown in fig. below. The loads are working loads and take load factor 1.75. Assume same plastic moment capacity for all the members. **5**



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Total No. of Questions: 6 *Total No. of Printed Pages: 4*

Enrollment No......................................

Faculty of Engineering

End Sem Examination May-2023

CE3CO11 Structural Analysis -II

Programme: B.Tech. Branch/Specialisation: CE

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. A Joint where several members meet, any applied moment will be distributed among the various members. **1**

(a) In proportion to their stiffnesses
(b) Equally among such members whose far ends are fixed
(c) Equally among such members whose far ends are hinged
(d) All of these

ii. When an end of continuous beam is fixed, in kani's method, its rotation contribution will be- **1**

(a) Zero (b) EI
(c) EI/L (d) 2 EI/L

iii. The ratio of the bending moment at the centre of a simply supported beam to the bending moment at the centre of a fixed beam, when both are of same span and both are subjected to same U.D.L. is- **1**

(a) 1.5 (b) 3 (c) 4.5 (d) 6

iv. Number of unknowns to be determined in the stiffness method is equal to- **1**

(a) Static indeterminacy
(b) Kinematic indeterminacy
(c) Sum of static and kinematic indeterminacy
(d) Difference of static and kinematic indeterminacy

v. In which method of tall buildings frame analysis, the horizontal shear taken by each interior column is double of that taken by exterior column. **1**

(a) Substitute frame method (b) Portal method
(c) Cantilever method (d) Factor method

vi. Point of contraflexure occurs at the middle of all members of tall frame in- **1**

(a) Portal method (b) Cantilever method

(c) Both (a) & (b) (d) None of these

vii. Muller Breslau principle for influence line is applicable for- **1**

(a) Simple beam (b) Continuous beam

(c) Redundant truss (d) All of these

viii. The influence line for any stress function are used for obtaining maximum value due to- **1**

(a) Single point load only (b) Uniform live load only

(c) Several points load (d) All of these

ix. The ratio of plastic moment capacity to the yield moment capacity of a rectangular section is- **1**

(a) 0.5 (b) 1.0 (c) 1.5 (d) 2.0

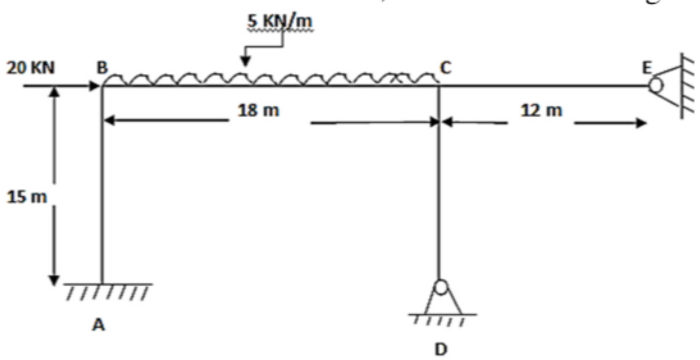
x. Plastic analysis of frames can be easily done by- **1**

(a) Static method (b) Kinetic method

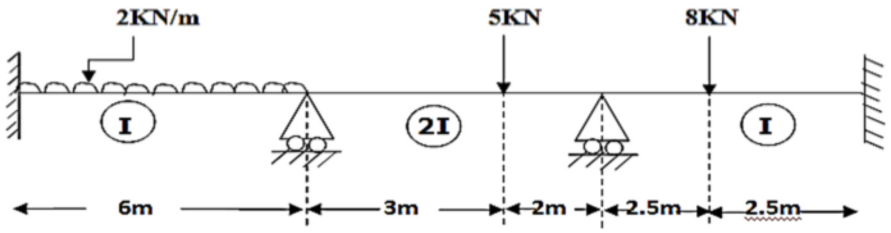
(c) Kinematic method (d) All of these

Q.2 i. Define carry over factor in moment distribution method of analysis. **2**

ii. Determine the moments at joints of the frame shown in figure below by moment distribution method for frame, EI is constant throughout. **8**

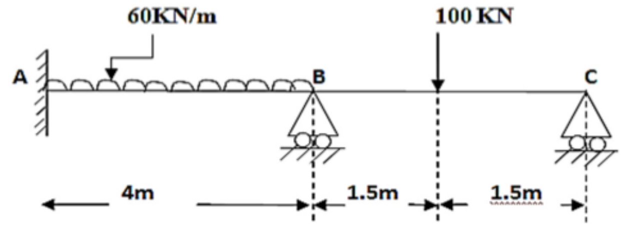


OR iii. Draw bending moment diagram of the continuous beam loaded as shown below. Use kani's method. Assume constant E throughout the beam. **8**



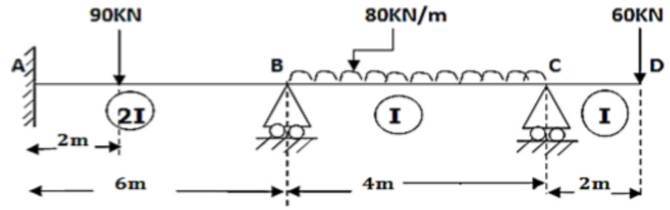
Q.3 i. Write down the steps to get the required solution by direct flexibility method. **4**

ii. Analyse the continuous beam shown in figure below by flexibility matrix method. Take moments at A and B as redundants and EI constant throughout the beam. **6**



OR iii. Analyse the beam shown in figure below by stiffness matrix method. Take modulus of elasticity constant. **6**

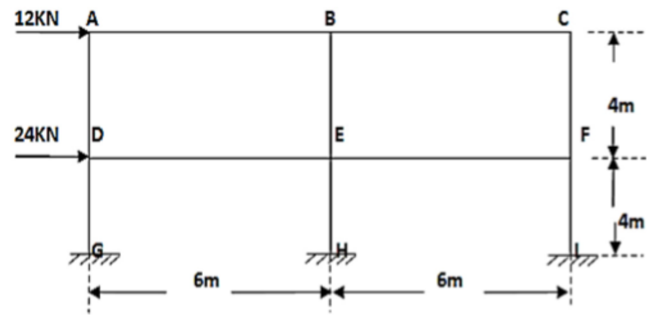
Select rotations at B and C as coordinates.



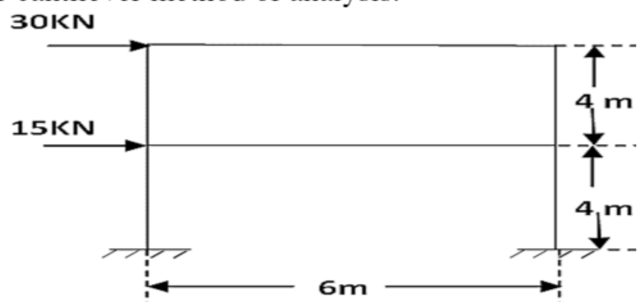
Q.4 i. Write down the assumptions made in portal method of frame analysis. **2**

ii. Write down the assumptions made in cantilever method of frame analysis. **2**

iii. In figure below, wind loads transferred to joints A and D are 12 KN & 24 KN respectively. Analyse the frame by portal method of analysis. **6**



OR iv. Determine the reactions at the base of the columns of the frame shown below. Use cantilever method of analysis. **6**



SCHEME END SEM QP JUNE 2023

STRUCTURE ANALYSIS II

CE3CO11

Q1)

1) A

2)A

3)B

4)B

5)B

6)C

7)D

8)D

9)C

10)C

Q2) I) For correct definition give 2 marks

Q2)II)For Correct FEM give 2 marks

For correct Moment distribution table give 3 marks

For Correct Moment distribution table with sway give 3 marks

Q2)III

Step 1. Computation of fixed end moments (kN-m units)

$$M_{FAB} = -\frac{2 \times 6^2}{12} = 6.0 \qquad M_{FBA} = +\frac{2 \times 6^2}{12} + 6$$

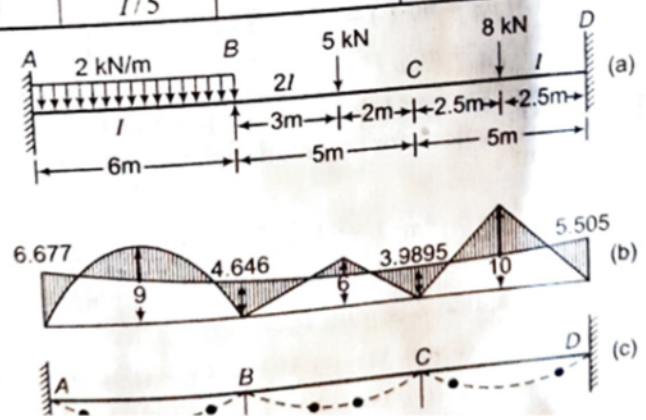
$$M_{FBC} = -\frac{5 \times 3 \times 2^2}{5^2} = -2.4\,; \qquad M_{FCB} = +\frac{5 \times 2 \times 3^2}{5^2} = +3.6$$

$$M_{FCD} = -\frac{8 \times 5}{8} = -50\,; \qquad M_{FDC} = +\frac{8 \times 5}{8} = +5.0$$

Step 2. Rotation factors : As pointed out earlier, rotation factor is equal to – 0.5 times the distribution factor used in moment distribution. The relative stiffness, distribution factors and rotation factors are calculated in Table 2.1

Table 2.1

Joint	*Member*	*Relative Stiffness*	*Sum*	*Distribution factor D_F*	*Fotation factor $R = -0.5 \times D_F$*
B	BA	$I/6$	$17I/30$	5/17	– 0.147
	BC	$2I/5$		12/17	– 0.353
C	CB	$2I/5$	$3I/5$	2/3	– 0.353
	CD	$I/5$		1/3	– 0.167



It is to be noted that the sum of rotation factors at a joint is equal to -0.5

Thus $\qquad R_{BA} + R_{BC} = -(0.147 + 0.353) + -0.5$

Step 3. Resultant restraint moments : Compute resultant restraint moment a each joint by equation 5

$$M_{Fi} = \sum_j M_{Fij}$$

Thus
$$M_{FB} = M_{FBA} + M_{FBC} = +6 - 2.4 = +3.6 \text{ kn} - \text{m}$$
$$M_{FC} = M_{FCB} + M_{FCD} = +3.6 - 5 = -1.4 \text{ kN} - \text{m}$$

Enter these values within the small square ([Fig. 2.3 (*d*)])

Step 4. Kani's Iteration cycles

Cycle 1. Kani's iteration procedure can now be commenced, assuming all rotational components (m_{ij}) to be zero at all joints which will indirectly mean that $\theta_i = 0$. Note that m_{AB} and m_{DC} are permanently zero since ends *A* and *D* are fixed.

Applying equation 8 at joint *C* and assuming $m_{BC} = 0$, we get

$$m_{CB} = R_{CB}\,(M_{FC}) = -0.333\,(-1.4) = +0.466$$

and
$$m_{CD} = R_{CD}\,(M_{FC}) = -0.167\,(-1.4) = +0.234$$

Thus values are now used in equation 8 for computing rotational components (m_{ij}) at joint *B*.

Thus, $m_{BC} = R_{BC}\,(M_{FB} + m_{CB}) = -0.353\,(+3.6 + 0.466) = -1.435$
$$m_{BA} = R_{BA}\,(M_{FB} + m_{CB}) = -0.147\,(+3.6 + 0.466) = -0.598$$

Cycle 2. The values of the four rotational components found i... be used to get better appr...

$$m_{BA} = R_{BA}\,(M_{FB} + m_{CB}) = -0.147\,(+3.6 + 0.466) = -0.598$$

Cycle 2. The values of the four rotational components found in cylce 1 will now be used to get better approximations for the rotational components at joint *C*. Applying equation 8 at joint *C*,

$$m_{CB} = R_{CB}\,(M_{FC} + m_{BC}) = -0.333\,(-1.4 - 1.435) = +(0.944)$$
$$m_{CD} = R_{CD}\,(M_{FC} + m_{BC}) = -0.167\,(-1.4 - 1.435) = +0.473$$

Similarly, applying equation 8 at joint *B*,

$$m_{BC} = R_{BC}\,(M_{FB} + m_{CB}) = -0.353\,(+3.6 + 0.944) = -1.604$$
$$m_{BA} = R_{BA}\,(M_{FB} + m_{CB}) = -0.147\,(+3.6 + 0.944) = -0.668$$

Cycle 3. Applying equation 8 at joint *C*,

$$m_{CB} = -0.333\,(-1.4 - 1.604) = +1.000$$
$$m_{CD} = -0.167\,(-1.4 - 1.604) = +0.502$$

At joint *B*,
$$m_{BC} = -0.353\,(+3.6 + 1.000) = -1.624$$
$$m_{BA} = -0.147\,(+3.6 + 1.000) = -0.676$$

Cycle 4.

At, *C*,
$$m_{CB} = -0.333\,(-1.4 - 1.624) = +1.007$$
$$m_{CD} = -0.167\,(-1.4 - 1.624) = +0.505$$

At *B*,
$$m_{BC} = -0.353\,(+3.6 + 1.007) = -1.626$$
$$m_{BA} = -0.147\,(= 3.6 + 1.007) = -0.677$$

Cycle 5.

At. C, $\quad m_{CB} = -0.333\,(-1.4-1.626) \quad = +1.008$

$\quad m_{CD} = -0.167\,(-1.4-1.626) \quad = +0.505$

At. B, $\quad m_{BC} = -0.353\,(+3.6+1.008) \quad = -1.627$

$\quad m_{BA} = -0.147\,(+3.6+1.008) \quad = -0.677$

The iteration is terminated at the end of 5th cycle as there is no change in the values of m_{ij} as compared to the corresponding values of 4th cycle. Hence the final values of the rotational components are

$$m_{BA} = -0.677\,;\; m_{BC} = -1.627\,;\; m_{CB} = +1.008 \text{ and } m_{CD} = +0.505$$

It should be clearly noted that we have actually solved the displacement equations in the above iteration, since from equation 3, we observe that

$$\theta_B = \frac{m_{BA}}{2E\,K_{BA}} = \frac{-0.677}{2E\,(I/6)} = -\frac{2.031}{EI}$$

and

$$\theta_C = \frac{m_{CB}}{2\,EK_{CB}} = \frac{+1.008}{2E\,(2I/5)} = +\frac{1.26}{EI}$$

Step 5. Computation of final momments at joint : The final moments (M_{ij}) are computed from equation 2. The computations have been arranged in Table 2.2.

Step 5. Computation of final momments at joint : The final moments (M_{ij}) are computed from equation 2. The computations have been arranged in Table 2.2.

Table 2.2 : Final Moments

M_{ij}	M_{Fij}	$2m_{ij}$	m_{ji}	*Sum (Kn-m)*
M_{AB}	− 6.0	0	− 0.677	− 6.677
M_{BA}	+ 6.0	− 1.354	0	+ 4.646
M_{BC}	− 2.4	− 3.254	+ 1.008	− 4.646
M_{CB}	+ 3.6	+ 2.016	− 1.627	+ 3.989
[illegible]	− 5.0	+ 1.010	0	− 3.990

Q3)1) For Each Steps give 1 marks.

2)

Fig. 5.3

Sol. Number of reaction components $= 5$

Number of independent equations of equilibrium $= 3$

Degree of static indeterminacy $= 5 - 3 = 2$

Select M_A and M_B as redundant forces.

Therefore, the released structures are the two independent. Simply supported beam AB and BC as shown in fig.

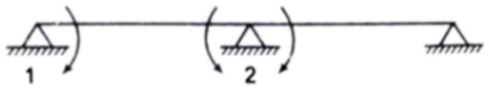


Fig. 5.4 Released beam.

Thus, coordinates selected are shown in figure. The bending moment diagram in released structure known as free moment diagram is :

(i) A symmetric parabolic curve with maximum ordinates

$$= \frac{Wl^2}{8} = \frac{60 \times 4^2}{8} = 120 \text{ kN - m in portion } AB$$

(ii) In portion BC, the free moment diagram is a symmetric triangle with maximum ordinate.

$$= \frac{WL}{4} = \frac{100 \times 3}{4} = 75 \text{ KN - m in portion } BC$$

The Conjugate beam for the released structure has two simply supported beams $A'B'$ and $B'C''$ with $\frac{M}{EI}$ diagram as loads. This is shown in figure.

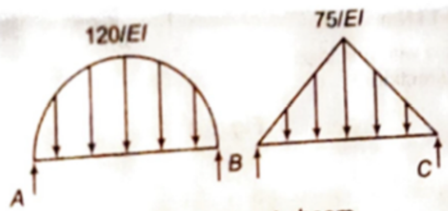


Fig. 5.5 Conjugate beam.

Δ_{1L} = Shear in Conjugate beam at A

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{120}{EI} \times 4 = \frac{160}{EI}$$

$$\Delta_{2L} = \text{Rotation at } B \text{ in } A'B' + \text{Rotation at } B \text{ in } B'C'$$

$$= \text{Shear at } B \text{ in } A'B' + \text{Shear at } B \text{ in } B'C'$$

$$= \frac{1}{2} \times \left(\frac{2}{5} \times \frac{120}{EI}\right) \times 4 + \frac{1}{2}\left(\frac{1}{2} \times \frac{75}{EI} \times 4\right) = \frac{216.25}{EI}$$

To find the flexibility matrix, a unit force is applied in each of the coordinate directions and the resulting displacements in coordinate directions 1 and 2 are found. Applying unit force in coordinate direction 1 the resulting bending moment diagram is found : this divided by $E2$ value is shown in fig.

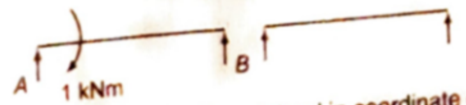


Fig. 5.6 Released structure with unit load in coordinate direction-1.

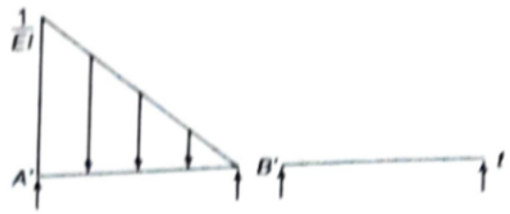


Fig. 5.7 Conjugate beam.

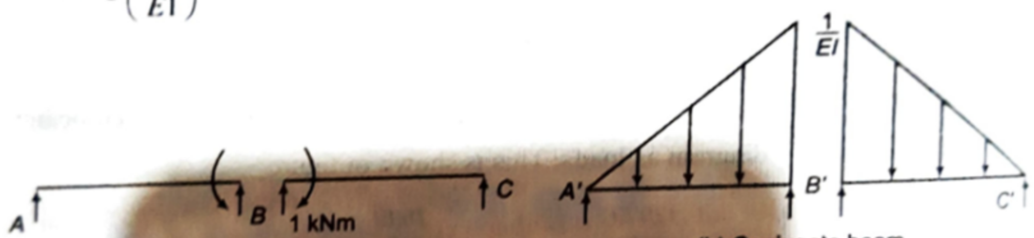
Since, conjugate beam is a simply supported beam

$$\delta_{11} = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{E2} \times 4 = \frac{4}{3EI}$$

and

$$\delta_{21} = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{EI} \times 4 = \frac{2}{3EI}$$

Similarly, a unit load is applied in coordinate direction 2 as shown in fig. and the resulting $\left(\frac{M}{EI}\right)$ diagram is as shown in fig.



(a) Released structure with unit load in coordinate direction-2

(b) Conjugate beam

Fig. 5.8

$$\delta_{12} = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{EI} \times 4 = \frac{2}{3EI}$$

and

$$\delta_{22} = \frac{2}{3} \times \frac{1}{2} \times 4 \times \frac{1}{EI} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{EI} \times 3 = 7/3EI$$

from consistency condition, final displacements.

$$\Delta_1 = 0, \Delta_2 = 0$$

Note : Δ_2 is the relatives between beam BA and BC hence, it is zero.

Therefore, the matrix equation is

$$[\delta][P]=[\Delta]-[\Delta_L]$$

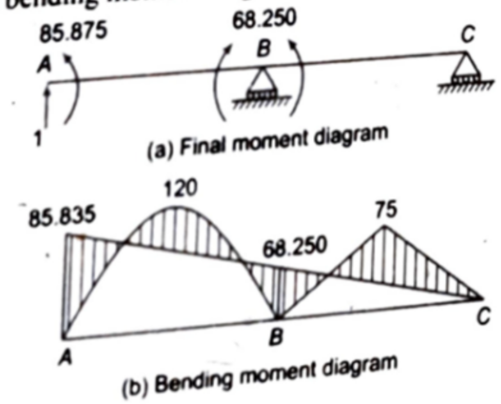
$$\begin{bmatrix} \frac{4}{3EI} & \frac{2}{3EI} \\ 2 & 7 \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}-\begin{bmatrix} \frac{160}{EI} \\ 216.5 \end{bmatrix}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix}=3\begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}^{-1}\begin{bmatrix} -160 \\ -216.5 \end{bmatrix}$$

$$=\frac{3}{4\times 7-2\times 2}\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} -160 \\ -216.5 \end{bmatrix}$$

$$=\frac{1}{8}\begin{bmatrix} -687 \\ -546 \end{bmatrix}=\begin{bmatrix} -85.875 \\ -68.250 \end{bmatrix}$$

nal moments and bending moment diagrams are as shown in fig. 5.9 (a) and (b).



(a) Final moment diagram

(b) Bending moment diagram

3)

Sol. Overhanging portion being the determinate portion, it may be dropped from the indeterminate structure analysis after accounting, its effect on the rest of the beam. Hence, the beam considered for the analysis is as shown in fig. (b) in which the cantilever moment 120 kN-m is the final moment at end C. The coordinates selected are shown in the fig. (c). Thus, final force vector is

$$P = \begin{bmatrix} 0 \\ 120 \end{bmatrix}$$

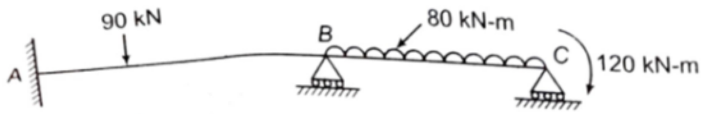


Fig. 5.22(b) Cantilever portion replaced by 120 kN-m moment.

Fig. 5.22(b) Cantilever portion replaced by 120 kN-m moment.

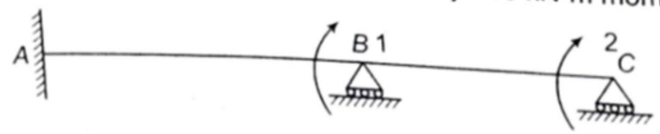


Fig. 5.22(c) Coordinates selected.

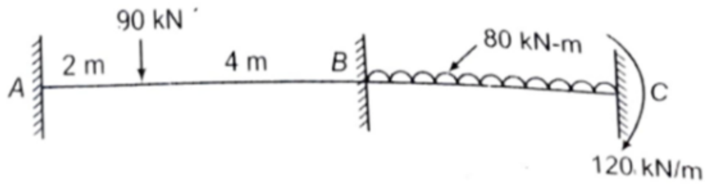


Fig. 5.22(d) shows a fully restrained structure to be considered.

Now,
$$M_{FAB} = \frac{-90 \times 2 \times 4^2}{6^2} = -80 \text{ kN - m}$$

$$M_{FBA} = \frac{+90 \times 2^2 \times 4}{6^2} = +40 \text{ kN - m}$$

$$M_{FBC} = \frac{-80 \times 4^2}{12} = -106.67 \text{ kN - m}$$

$$M_{FCB} = \frac{+80 \times 4^2}{12} = +106.67 \text{ kN - m}$$

$$[P_2] = \begin{bmatrix} M_{FBA} + M_{FBC} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} 40 - 106.67 \\ 106.67 \end{bmatrix} = \begin{bmatrix} -66.67 \\ 106.67 \end{bmatrix}$$

ıess Matrix

(a) Unit displacement in coordinate direction 1 :

k_{11} = Moment required to rotate BA and BC (joints) by 1 radians

$$= \frac{2E\,(21)}{6}(\theta + 2 \times 7 - 0) + \frac{2EI}{4}(2 \times 1 + 0 - 0)$$

$$= \frac{8EI}{6} + EI = \frac{7}{3}EI$$

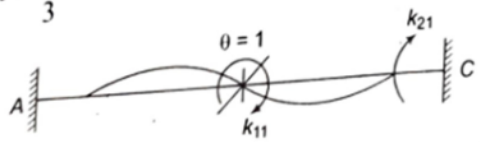


Fig. 5.23(a) Restrained beam with unit load in coordinate direction 1.

(b) Unit rotation at 'C'

$$k_{21} = \frac{2EI}{4} = 0.5EI$$

$$k_{22} = \frac{4EI}{4} = EI$$

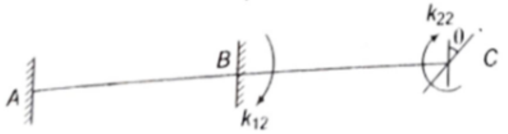


Fig. 5.23(b) Restrained structure with unit displacement in coordinate direction 2.

Therefore, the stiffness matrix equation is

$$[k][\Delta] = [P - P_2]$$

$$\begin{bmatrix} \frac{7EI}{3} & 0.5EI \\ 0.5EI & EI \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 0 + 66.7 \\ 120 - 106.67 \end{bmatrix}$$

$$\frac{EI}{3} \begin{bmatrix} 7 & 1.5 \\ 1.5 & 3 \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \frac{3}{EI} \begin{bmatrix} 7 & 1.5 \\ 1.5 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$= \left(\frac{3}{EI}\right)\left(\frac{1}{7 \times 3 - 1.5^2}\right) \begin{bmatrix} 3 & -1.5 \\ -1.5 & 7 \end{bmatrix} \begin{bmatrix} 66.67 \\ 13.33 \end{bmatrix}$$

$$= \left(\frac{3}{EI} \times \frac{1}{18.7}\right) \begin{bmatrix} 180.015 \\ -66.95 \end{bmatrix}$$

$$= \frac{1}{EI} \begin{bmatrix} 28.802 \\ -1.017 \end{bmatrix}$$

$$\theta_B = \frac{28.802}{EI} \text{ and } \theta_C = \frac{-1.017}{EI}$$

28.85

-1.09

From slope deflection equations.

$$M_{AB} = 40 + \frac{2E \times 23}{6} (2\theta_A + \theta_B - 6)$$

$$= -80 + \frac{4}{6} EI \left[\frac{28.80^2}{EI}\right]$$

$$= -60.80 \text{ kN - m}$$

$$M_{BA} = 40 + \frac{2E \times 21}{6} \left[0 + 2 \times \frac{28.80^2}{EI} - 6\right] = 78.403 \text{ kN - m}$$

$$M_{BC} = -106.67 + \frac{2EI}{4} \left[2 \times \frac{28.802}{EI} - \frac{1.071}{EI}\right] = -78.403 \text{ kN - m}$$

$$M_{CB} = -106.67 + \frac{2EI}{4}\left[\frac{28.80^2}{EI} - \frac{2 \times 1.071}{EI}\right] = 120 \text{ kN - m}$$

nple 5.11 : Analyse the continuou...

Q4) 1) FoR Each Assumption give 1 marks

2) For each assumption give 1 marks

3) For Correct Calculation of load P give 1 marks and For correct calculation of moments at each story give 5 marks (one for each span)

4)For Correct Calculation of neutral axis give 2 marks and for axial forces in each column give 2 marks and for shear in beams give 2 marks

Q5)1) For statement give 2 marks and for explaination give 2 marks

2)

(i) Reaction at B

(ii) Moment at A for the propped cantilever shown in fig. 6.12. C ordinates at intervals of 1.25 m.

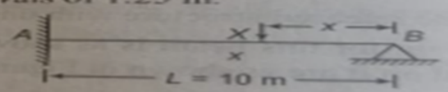


Fig. 6.12

Sol.

(i) Influence Line for R_B

We know that,

$$R_B = \frac{y_{XB}}{y_{BB}}$$

To compute y_{XB}, apply a unit vertical load at B.

We know that at any section X distant x from, B we have

$$EI\frac{d^2y}{dx^2} = -M_x$$

Moment = Force × Perpendicular distance = $1 \times x = x$ (Downward)

$$EI\frac{d^2y}{dx^2} = 1.x$$

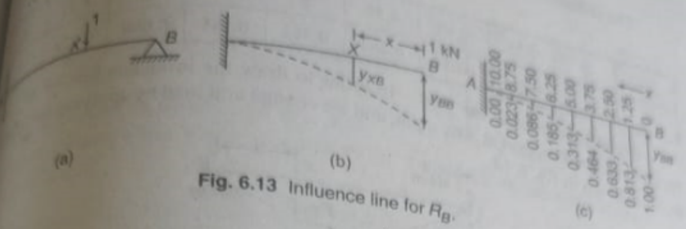


Fig. 6.13 Influence line for R_B.

Integrating,

$$EI\frac{dy}{dx} = \frac{x^2}{2} + X_1$$

At $x = L$, $\frac{dy}{dx} = 0$

$$C_1 = -\frac{L^2}{2}$$

Hence

$$EI\frac{dy}{dx} = \frac{x^2}{2} - \frac{L^2}{2}$$

We again integrating this equation

$$EI\ y = \frac{x^3}{6} - \frac{L^2}{2}x + C_2$$

At $x = L$, $y = 0$

$$C_2 = \frac{L^3}{2} - \frac{L^3}{6} = \frac{L^3}{3}$$

Hence

$$EI\ y = \frac{x^3}{6} - \frac{L^2}{2}\cdot x + \frac{L^3}{3} \qquad \text{...(ii)}$$

At $x = 0$, $y = y_{BB} = \frac{L^3}{3EI}$

At $x = x$,

$$y = y_{XB} = \frac{1}{EI}\left[\frac{x^3}{6} - \frac{L^2}{2}\cdot x + \frac{L^3}{3}\right]$$

Substituting these values in equation (i), we get

$$R_B = \left(\frac{x^3}{6} - \frac{L^2}{2}\cdot x + \frac{L^3}{3}\right) \times \frac{3}{L^3}$$

$$R_B = \frac{1}{2}\left(\frac{x^3}{L^3} - \frac{3x}{L} + 2\right)$$

The ordinates of I.L. for R_B are computed in table 6.1.

Table 6.1

x(m)	0	1.25	2.50	3.75	5	6.25	7.5	[illegible]
R_B	1	0.813	0.633	0.464	0.313	0.185	0.086	[illegible]

(ii) Influence Line for M_A—In order to draw the influence line for M_A replace the fixed support at A by a pin, and we change unit load by applying unit moment at A as shown in fig. 6.14.

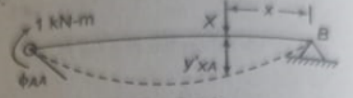


Fig. 6.14

We know that,

$$M_A = \frac{y'_{XA}}{\phi_{AA}}$$

where y'_{XA} = Vertical deflection at X due to unit couple at A

ϕ_{AA} = Slope at A due to unit couple at A.

Let R'_B = Reaction at B, when unit moment is acting at $A = \frac{1}{L}(\uparrow)$

We know that,

$$EI\frac{d^2y}{dx^2} = -M_x$$

$$EI\frac{d^2y}{dx^2} = -R'_B \cdot X = -\frac{x}{L}$$

Integrating,

$$EI\frac{dy}{dx} = \frac{-x^2}{2L} + C_1$$

Again integrating,

$$EIy = \frac{-x^3}{6L} + C_1 + C_2$$

At $x = 0,\ y = 0$

$\therefore$

$$C_2 = 0$$

At $x = L,\ y = 0$

$\therefore$

$$C_1 = L/6$$

Hence

$$EI\frac{dy}{dx} = -\frac{x^2}{2L} + \frac{L}{6}$$

$$EIy = -\frac{x^3}{6L} + \frac{Lx}{6}$$

At $x = L$,

$$\frac{dy}{dx} = \phi_{AA} = \frac{1}{EI}\left(-\frac{L^2}{2L} + \frac{L}{6}\right) = \frac{-L}{3} \times \frac{1}{EI}$$

At $x = x$,

$$y = y'_{XA} = \frac{1}{\ }\left(-\frac{x^3}{\ } + \frac{Lx}{\ }\right)$$

Putting these values in equation (i), we get

$$M_A = \left(\frac{x^3}{6L} - \frac{Lx}{6}\right) \times \frac{3}{L}$$

$$M_A = \frac{1}{2}\left(\frac{x^3}{L^2} - x\right)$$

This is the general equation of M_A. The values of influence line for M_A are given in table 6.2.

Fig. 6.15 Influence line for M_A

Table 6.2

	0	1.25	2.5	3.75	5	6.25	7.5	8.75	10
	0	−0.615	−1.172	−1.61	−1.875	−1.9	−1.64	−1.025	0

6.8. Calculate the ordinates of influence line diagram for ...

3)

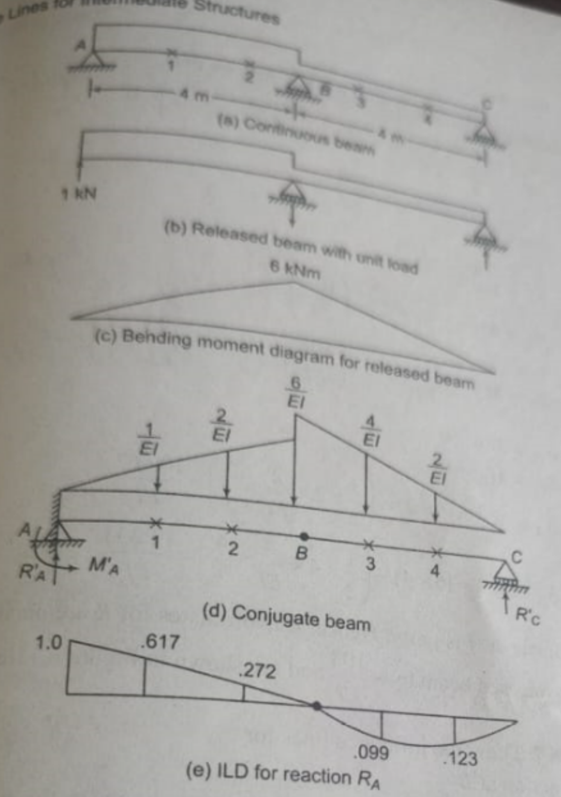


Fig. 6.11

Sol. Figure 6.11(b) shows the released structure with unit load in the direction of R_A. The bending moment diagram for this beam is as shown in Figure 6.11(c). and the conjugate beam and the load on it are as shown in Figure 6.11(d).

In conjugate beam

$\Sigma M'_B = 0$, gives

$$R'_C \times 6 = \frac{1}{2} \times \frac{6}{EI} \times 6 \times 2$$

$$\therefore \quad R'_C = \frac{6}{EI}$$

$\Sigma V = 0$, gives

$$R'_A = \left(\frac{1}{2} \times 6 \times \frac{3}{EI}\right) + \left(\frac{1}{2} \times 6 \times \frac{6}{EI}\right) - R'_C = \frac{21}{EI}$$

Calculating moment at B from the left end, we get

$$\frac{21}{EI}\times 6 = M'_A - \left(\frac{1}{2}\times 6\times\frac{3}{EI}\times 2\right) = 0$$

$$M'_A = \frac{108}{EI}$$

$$\delta_A = -M'_A = \frac{108}{EI}$$

$$\delta_1 = M_1 = \left(\frac{21}{EI}\times 2\right) - \left(\frac{108}{EI}\right) - \left(\frac{1}{2}\times 2\times\frac{1}{EI}\times\frac{2}{3}\right) = -\frac{66.67}{EI}$$

$$\delta_2 = M_2 = \left(\frac{21}{EI}\times 4\right) - \left(\frac{108}{EI}\right) - \left(\frac{1}{2}\times 4\times\frac{2}{EI}\times\frac{4}{3}\right) = -\frac{29.333}{EI}$$

$$\delta_B = M_B = 0$$

$$\delta_C = M_C = 0$$

$$\delta_4 = M_4 = (6\times 2) - \left(\frac{1}{2}\times 2\times\frac{2}{EI}\times\frac{2}{3}\right) = \frac{10.667}{EI}$$

$$\delta_3 = M_3 = (6\times 4) - \left(\frac{1}{2}\times 4\times\frac{4}{EI}\times\frac{4}{3}\right) = \frac{13.333}{EI}$$

ILD ordinate at A is unity. Hence, *ILD* ordinates for R are obtained by deflections in released beam by $-\frac{108}{EI}$ and are shown in Figure 6.11(e).

❐ **Example 6.7.** Draw the influence lines for :

(i) Reaction at B

(ii) Moment at A for the propped cantilever shown in fig. 6.12. C... ordinates at intervals of 1.25 m.

Q6)1) For Each Defination give 1 marks

2) For Correct Shape factor give 3 marks

3)

Sol. At collapse, plastic hinges will form at the ends and the third hinge may form under the load where positive moment is high or it may form at mid-span where, though moment is less than that under the load, plastic moment capacity is less. Hence, both mechanisms are to be investigated.

Mechanism I : This is shown in Figure 3.15(b). Let Δ be the virtual displacement given to the hinge under the load and θ_1 and θ_2 be the rotations at the ends as shown in the figure.

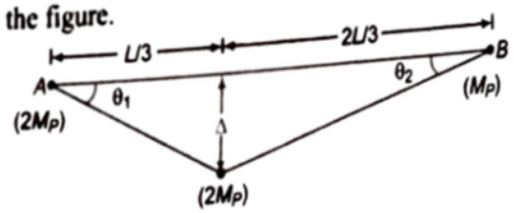


Fig. 3.15(b) Mechanism-I.

Then,
$$\left(\frac{L}{3}\right)\theta_1 = \left(\frac{2L}{3}\right)\theta_2$$

or
$$\theta_1 = 2\theta_2$$

Internal work done $= 2M_p\theta_1 + 2M_p(\theta_1+\theta_2) + M_p\theta_2$

$= 2M_p \times 2\theta_2 + 2M_p(2\theta_2+\theta) + M_p\theta_2$ [Since, $\theta_1 = 2\theta_2$]

$= 11M_p\theta_2$

External work done $= W_c\,\Delta = W_c\left(\frac{2L}{3}\right)\theta_2$

Equating internal work to external work, we get

$$11M_p\,\theta_2 = W_c\left(\frac{2L}{3}\right)\theta_2$$

$\therefore$
$$W_c = \frac{16.5\,M_p}{L} \quad \ldots(i)$$

Mechanism II : It is shown in Figure 3.15(c). It is having at mid-span. Let virtual displacement at this point be Δ and the rotations at ends be θ_1 and θ_2, respectively as shown in figure. Then,

$$\frac{L}{2}\theta_1 = \Delta = \frac{L}{2}\theta_2$$

$$\theta_1 = \theta_2 = \theta$$

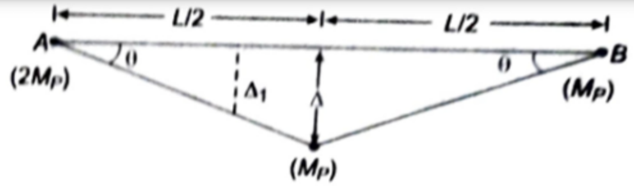


Fig. 3.15(c) Mechanism-II.

Displacement under the load $\Delta_1 = \dfrac{\left(\frac{L}{3}\right)}{\left(\frac{L}{2}\right)} \times \Delta = \dfrac{2}{3}\Delta$

$$= \frac{2}{3} \times \frac{L}{2}\theta = \frac{L}{3}\theta$$

Internal work done $= 2M_p\,\theta + M_p(2\theta) + M_p\theta$

$$= 5M_p\,\theta$$

Note : When there is a sudden change in the section, the hinge will form weaker side and the necessary rotations take place.

External work done $= W_c\,\Delta_1$

$$= W_c\left(\frac{L}{3}\theta\right)$$

Equating internal work to external work, we get

$$5M_p\,\theta = W_c\left(\frac{L}{3}\theta\right)$$

$$W_c = \frac{15M_p}{L}$$

From (i) and (ii), we conclude Mechanism II is the real Mechanism and th collapse load is ...(i

$$W_c = \frac{15M_p}{L}$$

4)

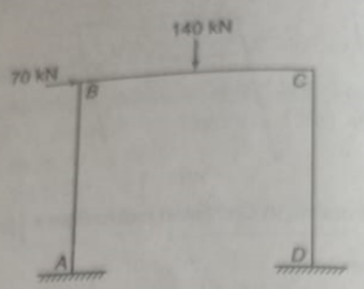


Fig. 3.20(b) Frame with collapse loads.

1. **Beam Mechanism :** This mechanism with virtual displacement Δ under the load is shown in Figure 3.20(c). Equating internal work done to external work, we get

$$M_p\theta + M_p\theta + M_p\,(\theta+\theta) + M_p\theta = 140\Delta$$

$$4M_p\theta = 140 \times 3\theta$$

or

$$M_p = 150 \text{ kNm} \quad \text{...(i)}$$

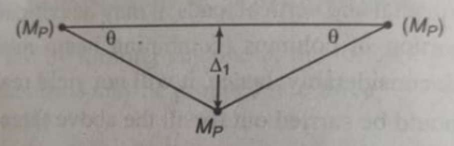


Fig. 3.20(c) Beam mechanism.

2. **Sway Mechanism :** This is shown in Figure 3.20(d) with virtual displacement Δ of the beam BC. Equating internal work to external work, we get

$$M_p\theta + M_p\theta + M_p\theta = 70\Delta$$

$$4M_p\,\theta = 70 \times 4\theta$$

or

$$M_p = 70 \text{ kNm} \quad \text{...(ii)}$$

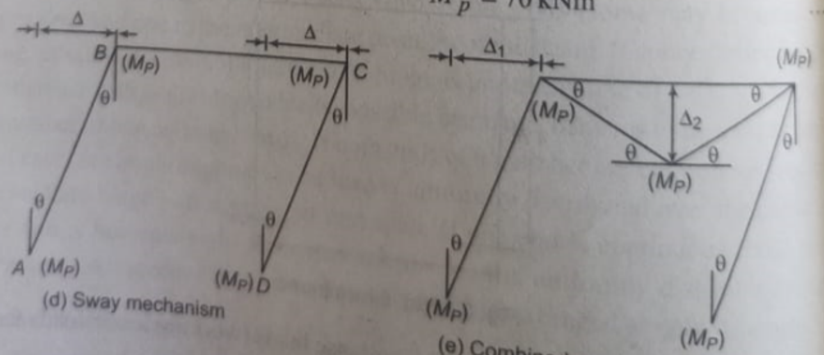


Fig. 3.20(d, e)

3. **Combined Mechanism :** This is shown in Figure 3.20(e). Equilibrium equation is

$$M_p\theta + M_p(\theta+\theta) + M_p(\theta+\theta) + M_p\theta = 70\,\Delta_1 + 140\Delta_2$$

$$6M_p\,\theta = 70\times 40 + 140\times 30$$

$$M_p = \frac{700}{6} = 11.67 \text{ kNm} \quad \dots(iii)$$

From (i), (ii), (iii), we conclude,

$$M_p = 116.67 \text{ kNm}$$

...bined mechanism is the real mechanism.