Q.6 i. Explain the condition for consistency and inconsistency of equations. **2**

ii. Find the rank of matrix $A=\begin{bmatrix}3 & 2 & -1\\ 4 & 2 & 6\\ 7 & 4 & 5\end{bmatrix}$ **3**

iii. Find the Eigen values and Eigen vectors of the matrix: **5**

$$A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$$

OR iv. Using Cayley Hamilton theorem find A^{-1} and hence evaluate the value of $A^5-4A^4-7A^3+11A^2-A-10I$ .where $A=\begin{bmatrix}1 & 4\\ 2 & 3\end{bmatrix}$ **5**

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2018

CA3CO04 Mathematics-I

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Negation of the statement: "If Dhoni looses the toss then the team wins", is **1**

(a) Dhoni does not lose the toss and the team does not win.
(b) Dhoni loses the toss but the team does not win.
(c) Either Dhoni loses the toss or the team wins.
(d) Dhoni loses the toss if the team wins.

ii. Which of the following is a statement? **1**

(a) Open the door. (b) Do your homework.
(c) Switch on the fan. (d) Two plus two is four.

iii. If A and B are two sets and $A\cup B=A\cap B$ then A = **1**

(a) $\varnothing$ (b) B (c) A^c (d) None of these

iv. Order of the power set of a set of order n is: **1**

(a) n (b) $2n$ (c) n^2 (d) 2^n

v. Every identity relation is: **1**

(a) Reflexive (b) Transitive (c) Symmetric (d) None of these

vi. Which of the following function from A to B is one - one onto where A = {1,2,3} and B = {2,5,7}: **1**

(a) $f_1=\{(1,2),(2,5),(3,7),(3,7)$
(b) $f_2=\{(1,3),(2,7),(3,7)$
(c) $f_3=\{(1,5)(2,8)(3,7)$
(d) None of these

vii. $\lim_{x\to 0}\frac{x}{|x|}$ is equal to **1**

(a) 1 (b) -1 (c) 0 (d) None of these

viii. If f is a function such that $\lim_{x\to a} f(x)$ does not exist then f is **1**

(a) Not continuous (b) Continuous

(c) Can't answer (d) None of these.

ix. If $A=\begin{bmatrix}2 & 0\\ 5 & 3\end{bmatrix}$ then Eigen values of matrix A is: **1**

(a) 2,3 (b) 2,5 (c) 2,0 (d) None of these

x. A square matrix A is said to be non-singular if **1**

(a) $|A|=0$ (b) $|A|\neq 0$ (c) $|A|=1$ (d) None of these

Q.2 i. Obtain the converse, inverse and contrapositive of the conditional statement $p \to q$. **2**

ii. Explain Conditional and Bi-conditional operator with truth table. **3**

iii. Prove that the statement $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p)$ is tautology. **5**

OR iv. Test the validity of the argument: If 8 is even then 2 does not divide 9.Either 7 is not prime or 2 divides 9. But 7 is prime, therefore 8 is odd. **5**

Q.3 i. If A = {1, 2}, B = {2, 3} and C = {3, 5} then find$(A \times B) \cap (A \times C)$. **2**

ii. If A, B, C are any three set, then prove that $A\times(B\cup C)=(A\times B)\cup(A\times C)$. **3**

iii. State and prove De Morgan's Law on set. **5**

OR iv. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects. **5**

(a) Find the number of students studying all the three subjects.

(b) Find the number of students studying exactly one of the three subjects.

Q.4 i. If A is the set of first ten natural numbers 1 to 20. If A relation R is defined as $x R y \Leftrightarrow 2x + y = 20,$ where $x, y\in A$ then evaluate **2**

(a) Domain of R (b) Range of R.

ii. State and prove trigonometric identities. **3**

iii. Show that the relation **5**

$R=\{(a,b): a,b\in I, (a-b)\text{is divisible by 3}\}$

is an equivalence relation.

OR iv. Define one-one, many-one and one-one onto function with example and show that the function $f: R \to R$ defined by $f(x) = e^x, x \in R$ is one one, where R is the set of real numbers. **5**

Q.5 i. Evaluate $\lim_{x\to 2}\frac{x^2-5x+6}{x^2-4}$. **2**

ii. Let $f(x)$ be a function defined by **3**

$$f(x)=\begin{cases}4x-5 & ; \ if \ \ x\le 2\\ x-\lambda & ; \ if \ \ x>2\end{cases}$$

Then find the value of λ, if $\lim_{x\to 2} f(x)$ exists.

iii. Differentiate the function $\log\sqrt{\frac{1+\sin x}{1-\sin x}}$. w.r.t. x. **5**

OR iv. Define continuity at a point. Show that **5**

$$f(x)=\begin{cases}5x-4 & ; \ 0<x\le 1\\ 4x^3-3x & ; \ 1<x<2\end{cases}$$ is continuous at $x=1$.

Faculty of science

End sem (odd) Examination Dec-2018

CA3CO04 Mathematics - I

Q1. MCQs

(i) (a) Dhoni does not lose the toss and the team does not win

(ii) (d) Two plus two is four

(iii) (b) B

(iv) (d) 2^n

(V) (a) Reflexive

(Vi) (a) $f_1 = \{(1,2),(2,5),(3,7),(3,7)\}$

(Vii) (d) None of these

(Viii) (a) Not continuous

(ix) (a) 2, 3

(X) (b) $|A| \neq 0$

Q.2 (i) Converse of $p \to q$ is $q \to p$ 1/2

Inverse is $\sim p \to \sim q$ 1/2

contrapositive $\sim q \to \sim p$ 1

Q.2 (ii) conditional operator: Let p and q be two statements.

If p then q

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

1.5

Bi-conditional operator (2)

$$p \Leftrightarrow q$$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1½

2 i) Prove that $A = (p \Leftrightarrow q \wedge r) \Rightarrow \underbrace{(\sim r \Rightarrow \sim p)}_{B}$ 1

p	q	r	$q \wedge r$	$p \Rightarrow q \wedge r$	$\sim r$	$\sim p$	B	A
T	T	T	T	T	F	F	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	F	F	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	F	T	T	T
F	F	F	F	T	T	T	T	T

3

Hence Tautology 1

2 ii) Let p: 8 is even

q: 2 divides 9 1

r: 7 is prime

The given argument is valid if the statement

$(\underbrace{p \to \sim q}_{a}) \wedge \underbrace{(\sim r \vee q)}_{b} \wedge r \to \sim p$ is a 1

tautology we construct the truth table

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \to \sim q$	$\sim r \vee q$	$a \wedge b$	$a \wedge b \wedge r = f$	$f \to \sim p$
T	T	T	F	F	F	F	T	F	F	T
T	T	F	F	F	T	F	T	F	F	T
T	F	T	F	T	F	T	F	F	F	T
T	F	F	F	T	T	T	T	F	F	T
F	T	T	T	F	F	T	T	T	T	T
F	T	F	T	F	T	T	T	F	F	T
F	F	T	T	T	F	T	F	F	F	T
F	F	F	T	T	T	T	T	F	F	T
									F	T

2

since last column contains only T's hence the given argument is valid. 1

i) Let $A = \{1, 2\}$ $B = \{2, 3\}$ $C = \{3, 5\}$

$A \times B = \{(1,2)\ (1,3)\ (2,2)\ (2,3)\}$ 1

$A \times C = \{(1,3)\ (1,5)\ (2,3)\ (2,5)\}$

$(A \times B) \cap (A \times C) = \{(1,3), (2,3)\}$ 1

ii) Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Let $(x, y) \in A \times (B \cup C) = x \in A,\ y \in B \cup C$ 1.5

$= x \in A,\ y \in B$ or $y \in C$

$= (x, y) \in (A \times B) \cup (A \times C)$

$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — (1)

Let $(x', y') \in (A \times B) \cup (A \times C)$ 1.5

$\Rightarrow (x', y') \subseteq A \times (B \cup C)$

$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ — (2)

from (1) and (2) $(A \times (B \cup C)) = (A \times B) \cup (A \times C)$

(iii) De Morgan's Law

If U is a universal set and A and B are its two subset then by De Morgan's Law

(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$ 1

To prove $(A \cup B)' = A' \cap B'$ we have to prove

$(A \cup B)' \subseteq A' \cap B'$ and ~~$(A \cap B')$~~ 1.5

$A' \cap B' \subseteq (A \cup B)'$

by taking an arbitrary element one has to prove above.

To prove $(A \cap B)' = A' \cup B'$

we have to prove that 1

$(A \cap B)' \subseteq A' \cup B'$ 1.5

$A' \cup B' \subseteq (A \cap B)'$

by taking an arbitrary element one has to prove above..

2 (v). Let X: The set of all the students

M: students studying mathematics

P: " " Physics

S: " " Biology 1

$|X| = 100$, $|M| = 32$, $|P| = 20$, $|B| = 45$

$|M \cap B| = 15$ $|M \cap P| = 7$ $|P \cap B| = 10$ $(M \cup P \cup B)' = 30$.

Page (5)

Now we have

$$|M \cup P \cup B| = |X| - |(M \cup P \cup B)'| = 100 - 30 = 70$$

$$|M \cap P \cap B| = |M \cup P \cup B| - |M| - |P| - |B| + |M \cap P|$$ 1

$$+ |M \cap B| + |P \cap B|$$

$$= 70 - 32 - 20 - 45 + 7 + 15 + 10 = 5$$

All three subjects = 5

exactly one subject

$$|M_1| = |M - P - B| = 32 - 7 - 15 + 5 = 15$$ 1

$$|P_1| = |P - M - B| = |P| - |P \cap M| - |P \cap B|$$

$$+ |P \cap M \cap B|$$

$$= 20 - 7 - 10 + 5 = 8$$ 1

$$|B_1| = \cancel{15 + 8 + 25} = 45 - 15 - 10 + 5 = 25$$

$$= |P_1| + |M_1| + |B_1| = 15 + 8 + 25 = 48$$ 1

4(i) Let $A = \{1, 2 - - 10\}$

The value of x & y satisfying the eqn

$2x + y = 20$ are given as. 1

$$R_1 = \{(x, y) : x \in A, y \in A \text{ and } 2x + y = 20\}$$

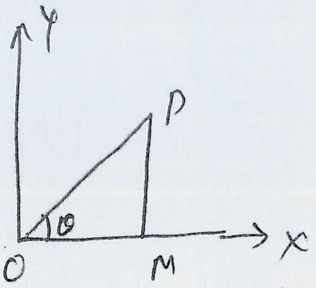
$$= \{(5,10), (6,8), (7,6), (8,4) (9,2)\}$$

Domain of $R = \{5, 6, 7, 8, 9\}$ 1

Range of $R = \{10, 8, 6, 4, 2\}$

4(iii) To prove $\sin^2\theta + \cos^2\theta = 1$

L.H.S.



$$\left(\frac{PM}{OP}\right)^2 + \left(\frac{OM}{OP}\right)^2$$ 1

$$= \frac{(PM)^2 + (OM)^2}{OP^2} = \frac{OP^2}{OP^2} = 1$$

$1 + \tan^2\theta = \sec^2\theta$

$$1 + \left(\frac{PM}{OM}\right)^2 = \frac{OP^2}{OM^2} = \sec^2\theta$$ 1

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$1 + \frac{(OM)^2}{(PM)^2} = \frac{(PM)^2 + (OM)^2}{(PM)^2} = \operatorname{cosec}^2\theta$$ 1

4(iii) To Prove equivalance

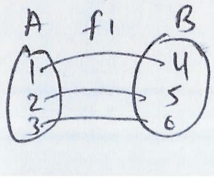
(i) R is Reflexive : $a \in I$ then $a - a = 0 \mid 3$ $(a, a) \in R \ \forall a \in I$ 1.5

(ii) R is Symmetric : $(a, b) \in R \Rightarrow (b, a) \in R$ [explaination] 1.5

(iii) R is transitive : $(a, b) \in R$ $(b, c) \in R \Rightarrow (a, c) \in R$. 2
[Explaination required]

4(iv) One one function:- A mapping $f: A \to B$ is called one one if different elements in A have different f image in B i.e. 1

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad x_1, x_2 \in A$$



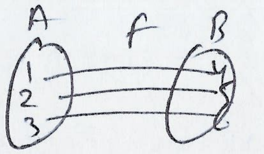
Many one mapping : $f: A \to B$ if two or more than two different element in A have the same image in B.

$$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2) \quad \forall x_1, x_2 \in A$$ 1

One one onto : $f: A \to B$ If f is one one and onto

$A = \{1,2,3\}$ $\quad B = \{4,5,6\}$ 1

$f(1) = 4 \quad f(2) = 5 \quad f(3) = 6.$



To prove $f(x) = e^x$, $x \in R$ is one one

let x_1, x_2 be any two element of R

Then $f(x_1) = e^{x_1}$ and $f(x_2) = e^{x_2}$ 2

$f(x_1) = f(x_2) = e^{x_1} = e^{x_2}$

$\log e^{x_1} = \log e^{x_2}$

$x_1 = x_2$

5(i) we have $\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4} = \lim_{x \to 2} \dfrac{(x-2)(x-3)}{(x-2)(x+2)}$ 2

$= -1/4$

5(ii) $$f(x) = \begin{cases} 4x - 5 & ; \text{ if } x \leq 2 \\ x - \lambda & ; \text{ if } x > 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} 4(2-h)-5 = 3$$ 1

and $$\lim_{x\to 2^+} f(x) = (2-\lambda)$$ 1

If $$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$3 = 2-\lambda \Rightarrow \lambda = -1$$ 1

(iii) Differentiate the funⁿ $\log\sqrt{\dfrac{1+\sin x}{1-\sin x}}$

$$y = \frac{1}{2}\{\log(1+\sin x) - \log(1-\sin x)\}$$ 1

1

$$\frac{dy}{dx} = \frac{1}{2}\cos x\left\{\frac{2}{1-\sin x}\right\} = \frac{\cos x}{\cos^2 x} = \sec x.$$ 3

5 (iv) We have

$$\lim_{x\to 1^-} f(x) = 5\times 1 - 4 = 1$$ 1.5

$$\lim_{x\to 1^+} 4(1)^3 - 3(1) = 1$$ 1.5

$$f(1) = 5\times 1 - 4 = 1$$

$$\lim_{x\to 1^-} = f(1) = \lim_{x\to 1^+}$$ so that 2

$f(x)$ is continuous at $x = 1$

(iii) consistency of eqn

If rank of A = Rank of [A : B] then eqn 1

are known as consistently.

Inconsistent :- If rank of A = Rank of [A : B] 1

5(ii) $A = \begin{bmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{bmatrix}$

$R_2 \to R_2 - R_1$ 1.5

$\begin{bmatrix} 3 & 2 & -1 \\ 1 & 0 & 7 \\ 7 & 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -11 \\ 0 & 0 & 0 \end{bmatrix}$ 1.5

which Echelon form $\rho(A) = 2$

6(iii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ 1 & -2 & 0 \end{bmatrix}$

2

$\lambda = 5, -3, -3$

$X_1 = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ 3

86. (iii)

page (9) (10)

$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

$A^3 = \begin{bmatrix} 41 & 48 \\ 42 & 83 \end{bmatrix} \quad A^2 = \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix}$ 2

1

$A^{-1} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$

1

$\lambda^2 - 4\lambda - 5 = 0$

$A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$

Value of polynomial 1