Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2022

EN3BS07

Introductory Topics in Statistics, Probability & Calculus

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The word 'Statistics' have been derived from the Latin word- **1**

(a) Strata (b) Status (c) Statista (d) None of these

ii. What is the first stage in statistics- **1**

(a) Representation of data (b) Collection of data

(c) Analysis of data (d) None of these

iii. The arithmetic mean is 12 and the number of observations are 20 than the sum of all the value is- **1**

(a) 24 (b) 240 (c) 40 (d) None of these

iv. The relationship between the measure of central tendency mean, median and mode- **1**

(a) Mode=3median-2mean (b) Mode=median-2mean

(c) Mode=2median-3mean (d) None of these

v. When we throw a coin then what is the probability of getting head- **1**

(a) 1 (b) 0 (c) 1/2 (d) None of these

vi. In mathematical expectations E(X+Y) is equal to- **1**

(a) E(X)+E(Y) (b) X+Y

(c) E(XY) (d) None of these

vii. A variable which can assume only integer values is called: **1**

(a) Random (b) Discrete

(c) Continuous (d) None of these

viii. In a discrete probability distribution, the sum of all the probabilities is equal to: **1**

(a) Zero (b) One (c) Constant (d) None of these

P.T.O.

ix. The value of $\int \log x \, dx$ is- **1**

(a) $x + log\ x + c$ (b) $x\ log\ x - x + c$

(c) $\frac{\log x}{x} + c$ (d) None of these

x. Triple Integral $\int_0^1 \int_0^1 \int_0^1 dxdydz$ is equal to- **1**

(a) 0 (b) -1 (c) 1 (d) None of these

Q.2 Attempt any two:

i. What is statistics? What are its objectives, importance and limitations? **5**

ii. Write five differences between primary data and secondary data with examples. **5**

iii. Write any five applications of statistics in various branches of science with examples. **5**

Q.3 Attempt any two:

i. Calculate arithmetic mean from the following frequency: **5**

Marks	10	20	30	40	50	60	70	80
No. of students	15	35	60	84	96	127	198	250

ii. Find the mean, standard deviation and variance of first 'n' natural numbers. **5**

iii. Define mode. Write formula for continuous frequency distribution series. Write merits and demerits of mode. **5**

Q.4 Attempt any two:

i. A fair coin with '1'marked on one face and '6' on the other face and a fair die are both tossed. Find the probability that the sum of numbers that turn up is 3. **5**

ii. A bag I contains 4 white and 6 black balls while another Bag II contains 4 white and 3 black balls. One ball is drawn at random from one of the bags, and it is found to be black. Find the probability that it was drawn from Bag I. **5**

iii. Define Probability with example. Explain the following terms- **5**

(a) Random Experiment (b) Outcome

(c) Sample Space (d) Equally likely outcomes

(e) Events

Q.5 Attempt any two:

The mean height of 500 students is 151 cm, and the standard deviation is 15 cm. Assuming that the heights are normally distributed, find how many students have heights between 120 and 155 cms? **5**

Given: P(0≤z≤2.07) =0.4808 and P(0≤z≤0.27) =0.1085

i. If 10% of the bolts produced by a machine are defective, Determine the probability that out of 10 bolts chosen at random. **5**

(a) One (b) None (c) At most 2 bolts will be defective

ii. The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? Given $e^{-2.8} = 0.06$ **5**

Q.6 Attempt any two:

i. Evaluate double Integral $\int_0^1 \int_0^1 \left(\frac{1}{1+x^2}\right)\left(\frac{1}{1+y^2}\right) dxdy$. **5**

ii. Find the area of the circle of radius (r) and equation $x^2 + y^2 = r^2$ by using double integral with proper diagram. **5**

iii. Evaluate Triple Integral $\int_0^2 \int_0^x \int_0^{2x+2y} \left(e^{x+y+z}\right) dxdydz.$ **5**

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Introductory Topics in Statistics, Probability & Calculus.

Programme: B.Tech Branch: CSBS. Marks.

Q1

1. Status +1
2. Collection of data +1
3. 240 +1
4. Mode = 3 median − 2 mean +1
5. 1/2 +1
6. $E(X) + E(Y)$ +1
7. Discrete +1
8. One +1
9. $x \log x - x + c$ +1
10. 1 +1

Q2 (i)

Statistics is the science of learning from data and of measuring, controlling and communicating uncertainly; and it thereby provides the navigation essential for controlling the course of scientific and societal advance. +2

Objectives

① To draw conclusions about a populations followed by an analysis of information contained in sample data. +1

② To determine the type and quantity of data that must be collected.

③ To organies and summarise the information

Importance of Statistics.

Statistics helps in gathering information about the appropriate

Marks

0. quantitative data. It provides the exact descriptions and better understanding. It helps in designing the effective and proper planning of the statistical inquiry in any field. +1

Limitations

1. Statistics is not concerned with individual observations
2. Statistics do not analyse qualitative phenomenon. +1
3. Statistical generalisation are true only on average.

ii)

	Primary Data	Secondary Data
1. Definition	The data that is collected for the first time by the user himself	The data that is previously collected by others and used by another +1
2. Method of collection	These are collected via physical testing, observation, surveys, questionnaires, photographs, case studies, diary entries. etc.	They are collected from published data by the state or central govt. magazines. journals, etc +1

			Marks.
3) Purpose of Collection	The are mainly collected for a specific purpose and are involved in direct usage without any manipulation	They may be collected for multiple purpose as required by the users to derive various kinds of inferences from them.	+1
4) Authenticity	They are collected by user directly so they are original and devoid of any kind of alterations	The data are collected by other for their usage. so it is not original.	+1
5. Example	Researcher, Survey, case studies	Internet, magazines,	+1

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Marks

(Q.i)

Ans. Applications of Statistics — 1

1. Astrostatistics - It is the discipline that applies statistical analysis to the understanding of astronomical data. +1

2. Business - Business analytics is a rapidly developing business process that applies statistical methods to data sets to develop new insights and understanding of business performance and opportunities +1

3. Biostatistics - It is a branch of biology that studies biological phenomena and observation by means of statistical analysis and includs medical statistics +1

4. Environment Statistics - It is the application of statistical method to environmental scienes - Weather climate, air and water quality are included as are studies of plant and animal populations +1

5. Epidemiology - It is the study of factors affecting the health and illness of populations and serves as the foundation & logic of interventions made in the interest of public health and preventive medicine. +1

Q3 (i)

Marks (x)	No. of students (frequency)	$x-A$ $(x-50)$	$u=\frac{x-A}{i}$	fu	Marks.
10	15	-40	-4	-60	
20	35	-30	-3	-105	+2
30	60	-20	-2	-120	
40	84	-10	-1	-84	
50	96	0	0	0	
60	127	10	1	127	
70	198	20	2	396	
80	250	30	3	750	
Total	$N=865$			$\Sigma fu=904$	

Here

assumed mean $A = 50$ & $i = 10$ +1

Arithmetic mean $M = A + i \cdot \frac{\Sigma fu}{N}$ +1

$= 50 + 10 \times \frac{904}{865}$

$= 50 + 10.45$

$= 60.45$ Ans +1

ii)

Sol We know that

First n natural numbers are $1, 2, 3 \cdots n$.

Sum of first n natural number are +1

$(1+2+3+4 \cdots +n) = \frac{n(n+1)}{2}$.

Now

Mean $(\bar{x}) = \Sigma x/n$

Q.

Marks

$$= \frac{n(n+1)}{2n}$$

$$= (n+1)/2 \quad +1$$

Variance

$$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 \quad +1$$

$$= \frac{\Sigma n^2}{n} - \left(\frac{n+1}{2}\right)^2$$

$$= \frac{1}{n} \frac{n(n+1)(2n+1)}{6} - \frac{1}{4}(n+1)^2$$

$$\left\{\because \Sigma n^2 = \frac{1}{6} n(n+1)(2n+1)\right\}$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{1}{4}(n+1)^2$$

$$= (n+1)\left[\frac{(2n+1)}{6} - \frac{(n+1)}{4}\right]$$

$$= (n+1)\left[\frac{2(2n+1) - 3(n+1)}{12}\right]$$

$$= (n+1)\left[\frac{4n+2-3n-3}{12}\right]$$

$$= \frac{(n+1)(n-1)}{12}$$

$$\sigma^2 = \frac{n^2-1}{12} \quad \{(a+b)(a-b) = a^2-b^2\} \quad +1$$

Standard deviation $\sigma = \sqrt{\frac{n^2-1}{12}}$

$$= \frac{1}{2}\sqrt{\frac{n^2-1}{3}} \quad \text{Ans} \quad +1$$

iii) Marks.

Ans Mode is a measure of central tendency of statistical series. Mode is the most frequently occuring value in series. It is typical value around which of the item stand to cluster It is the representative value of a series around which there is maximum concentration +1

Formula for continuous frequency distribution

$$M_o = L + \frac{f_1 - f_o}{2f_1 - f_o - f_2} \times i$$ +1

where

M_o = Mode
L = lower limit of modal class
f_1 = frequency modal class.
f_o = frequency of pre-modal class
f_2 = frequency of class succeeding modal class
i = class interval

Merits of Mode

① It is very easy to calculate. In some cases it can be determined just by observation or inspections +1½
② It is a value around which there is maximum concentration of observation. It is best respresentative of the data
③ It is not affected by extreme values of the given data. It can be calculated

Q. Marks

even if there extreme observation are not known.

(4) It can be used to describe quantitative as well as qualitative data.

Demerits of Mode.

(1) The value of mode is not based on each and every items of the series as it considers only the highest concentration of frequency.

(2) Value of mode may not be determined always. Some distributions can be Bi-modal, Tri-modal or Multi-modal. +1½

(3) Mode is affected by sampling fluctuation to a great extent

(4) Grouping of data is desirable for correct computations but it is a complex process and involves so much calculations.

Q4

(i) Since the fair coin has 1 marked on one face and 6 on the other and the die has six faces that are numbered 1, 2, 3, 4, 5 and 6 the sample space is given by

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (6,1), (6,2), (6,3), (6,4) (6,5) (6,6)\}$. +1

$n(S) = 12$. +1

Let A be the event in which the sum of numbers that turn up is 3.

Marks.

$\Rightarrow A = \{(1,2)\}$. +1

$\therefore P(A) = \frac{\text{No. of outcomes favourable to } A}{\text{Total number of possible outcomes}}$ +1

$P(A) = \frac{n(A)}{n(S)}$

$= 1/12$. Ans// +1

ii)

Sol Let

E_1 be the event of choosing bag I

E_2 be the event of choosing bag II

and A be the event of drawing a black ball.

Then

$P(E_1) = P(E_2) = 1/2$. +1

Also.

$P(A/E_1) = 6/10 = 3/5$ +1

$P(A/E_2) = 3/7$ +1

By using Baye's theorem

$P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ +1

$= \frac{1/2 \times 3/5}{1/2 \times 3/5 + 1/2 \times 3/7}$

$= 7/12$ Ans// +1

Marks

Q.(iii)

Ans Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only chance of an event to occure. i.e. Probability can range from 0 to 1, where 0 means the event to be an impossible one and 1 indicates a certain event. +1

Example. - When we toss a coin, either we get Head or Tail, only two possible outcomes are possible But when two coins are tossed then there will be four possible outcomes.

$$P(E) = \frac{\text{No. of fav. outcomes}}{\text{Total no. of outcomes}}$$

(a) Random Experiment

An activity that produces a result or an outcome is called an experiment. It is an element of uncertainty as to which one of these occurs when we perform an activity or experiment. +1
When an experiment satisfies the following two conditions, it is called a random experiment.

(i) It has more than one possible outcome.
(ii) It is not possible to predict the outcome in advance.

Marks.

(b) Outcome — When dealing with probability the outcomes of a process are the possible results. For example, when a die is rolled, the possible outcomes are 1, 2, 3, 4, 5 and 6. In mathematical language, an event is a set of outcomes. +1/2

(c) Sample Space. — A sample space is a collection or a set of possible outcomes of a random experiment. The sample space is represented using the symbol, "S". ~~The subset of possible outcomes of an experiment is called events~~ A sample space may contain a number of outcomes that depends on the experiment. +1

(d) Equally likely Outcomes — It refers to two or more possible outcomes of a given situation that have the same probability or the same likelihood to occur. +1/2

(e) Events - Events in probability are outcomes of random experiments. Any subset of the sample space will form events in probability. The likelihood of occurrence of events in probability can be calculated by dividing the number of favorable outcomes by the total number of outcomes of that experiment +1

Marks

Q.5 (ii)

Sol. Given

The number of defective bolts $n = 10$

and

Probability of defective bolt $p = 10\% = 0.1$

Probability of non-defective bolt $q = 1 - p$ +1

$= 1 - 0.1$

$= 0.9$

1. The probability of 1 defective bolt $= P(X=1)$

$= {}^{10}C_1 p^1 q^9$

$= 10 (0.1)^1 (0.9)^9$ +1

$= 0.3874$

2. The probability of none defective bolt $= P(X=0)$

$= {}^{10}C_0 p^0 q^{10}$

$= 10 (0.1)^0 (0.9)^{10}$ +1

$= 0.3486$

3. The probability of at most 2 defective bolts

$= P(X=0) + P(X=1) + P(X=2)$ +1

$= {}^{10}C_0 p^0 q^{10} + {}^{10}C_1 p^1 q^9 + {}^{10}C_2 p^2 q^8$

$= 0.3486 + 0.3874 + 0.1937$

$= 0.9297$ Ans +1

Q5. (i) Sol

Marks.

Given

Total no. of students = 500

The mean height of students = 151 cm

and Standard Deviation = 15 cm.

also given

$P(0 \leq z \leq 2.07) = 0.4808$

and $P(0 \leq z \leq 0.27) = 0.1085$.

$\mu = 151$ and $\sigma = 15$

Now

$$z = \frac{X - \mu}{\sigma} = \frac{X - 151}{15}$$ +1

when

$X = 120$

then

$$z = \frac{120 - 151}{15}$$

$$= -\frac{31}{15}$$

$$= -2.067 \simeq -2.07$$ +1/2

when

$X = 155$

then

$$z = \frac{155 - 151}{15}$$

$$= \frac{04}{15}$$

$$= 0.2667 \simeq 0.27$$ +1/2

Marks

Q. Now.

$$P(120 < x < 155) = P(-2.07 < Z < 0.27) \quad +1$$

$$= P(-2.07 \leq Z \leq 0) + P(0 \leq Z \leq 0.27)$$

$$= P(0 \leq Z \leq 2.07) + P(0 \leq Z \leq 0.27)$$

$$= 0.4808 + 0.1085.$$

$$= 0.5892 \quad +1$$

$\therefore$ The required no. of students $= 0.5892 \times 500$

$= 294.$ $+1$

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Marks.

iii)

Sol Since

The mortality rate for a certain disease is 7 in 1000

$\therefore \quad p = \frac{7}{1000}$ and $n = 400$ +1

The value of mean $\lambda = np$ $\lambda = np$

$= 400 \times \frac{7}{1000}$

$\lambda = 2.8$ +1

Let x be a random variable following distribution with $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ +1

$\therefore$ The distribution is $P(X=2) = \frac{e^{-2.8}(2.8)^2}{2!}$ +1

$= \frac{0.06 \times 7.84}{2}$

$= 0.2352$ +1

Ans//

Marks

Q6 i

Sol Let

$$I = \int_0^1 \int_0^1 \left(\frac{1}{1+x^2}\right)\left(\frac{1}{1+y^2}\right) dx\,dy \quad +1$$

$$I = \int_0^1 \left(\frac{1}{1+y^2}\right) dy \int_0^1 \left(\frac{1}{1+x^2}\right) dx.$$

$$= \int_0^1 \frac{1}{1+y^2}\, dy \,[\tan^{-1}x]_0^1 \quad +1$$

$$= \int_0^1 \frac{1}{1+y^2}\, dy \,[\tan^{-1}1 - \tan^{-1}0] \quad +1$$

$$= \frac{\pi}{4}\int_0^1 \frac{1}{1+y^2}\, dy$$

$$= \frac{\pi}{4}\,[\tan^{-1}y]_0^1 \quad +1$$

$$= \frac{\pi}{4} \times \frac{\pi}{4}$$

$$= \pi^2/16 \quad \text{Ans//} \quad +1$$

Marks.

ii)

Sol Given

Radius of circle be γ and equation $x^2+y^2=\gamma^2$.

Let the limit be

$$y = -\sqrt{\gamma^2-x^2}$$
$$y = \sqrt{\gamma^2-x^2}$$

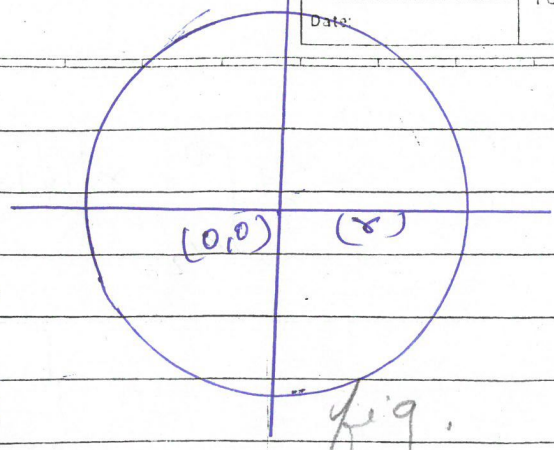

fig. +1

and

$$x = -\gamma \text{ to } \gamma$$

$$I = \int_{-\gamma}^{\gamma}\int_{-\sqrt{\gamma^2-x^2}}^{\sqrt{\gamma^2-x^2}} dx\,dy$$

$$I = 2\int_{-\gamma}^{\gamma}\int_{-\sqrt{\gamma^2-x^2}}^{\sqrt{\gamma^2-x^2}} dx\,dy \quad \left\{\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx\right.$$

$$I = 2\int_{-\gamma}^{\gamma} [y]_{-\sqrt{\gamma^2-x^2}}^{\sqrt{\gamma^2-x^2}}\, dx.$$

$$= 2\int_0^{\gamma} \left(\sqrt{\gamma^2-x^2}+\sqrt{\gamma^2-x^2}\right) dx.$$

$$= 4\int_0^{\gamma} \sqrt{\gamma^2-x^2}\, dx. \quad +1$$

put

$$x = \gamma\cos\theta$$
$$dx = -\gamma\sin\theta\, d\theta$$

when $x=0$, $\theta=\pi/2$ +1

$x=\gamma$, $\theta=0$

$$= -4\int_{\pi/2}^{0} \sqrt{\gamma^2-\gamma^2\cos^2\theta}\cdot\gamma\sin\theta\, d\theta$$

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Q.

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$$= -4 \int_{\pi/2}^{0} r^2 \left(\sqrt{1-\cos^2\theta}\right) \sin\theta \, d\theta.$$

$$\left\{ 1 - \cos^2\theta = \sin^2\theta. \right\}$$

$$= -4 \int_{\pi/2}^{0} r^2 \left(\sqrt{\sin^2\theta}\right) \sin\theta \, d\theta.$$

$$= -4 \int_{\pi/2}^{0} r^2 \sin^2\theta \, d\theta \qquad +1$$

$$\left\{ \because \sin^2\theta = \frac{1-\cos 2\theta}{2} \right.$$

$$= -4r^2 \int_{\pi/2}^{0} \frac{1-\cos 2\theta}{2} \, d\theta$$

$$= -2r^2 \left[\theta - \frac{\sin 2\theta}{2} \right]_{\pi/2}^{0}$$

$$= -2r^2 \left[0 - \frac{\sin 2\theta}{2} - \frac{\pi}{2} + \frac{\sin 2 \times \pi/2}{2} \right.$$

$$= \frac{-2r^2}{2} \left[0 - \sin 0 - \pi + \sin \pi \right.$$

$$= -r^2 \left[0 - 0 - \pi + 0 \right]$$

$$I = +\pi r^2 \quad \text{Ans//} \qquad +1$$

(iii) Marks.

Sol Let

$$I = \int_0^2 \int_0^x \int_0^{2x+2y} (e^{x+y+z})\, dx\, dy\, dz. \quad +1$$

$$= \int_0^2 \int_0^x \left(e^{x+y+z}\right)_0^{2x+2y} dx\, dy \quad +1$$

$$= \int_0^2 \int_0^x \left(e^{x+y+2x+2y} - e^{x+y}\right) dx\, dy$$

$$= \int_0^2 \int_0^x \left(e^{3(x+y)} - e^{x+y}\right) dx\, dy$$

$$= \int_0^2 \left[\frac{e^{3(x+y)}}{3} - e^{x+y}\right]_0^x dy \quad +1$$

$$= \int_0^2 \left[\frac{e^{6x}}{3} - e^{2x} - \frac{e^{3x}}{3} + e^x\right] dx.$$

$$= \left[\frac{e^{6x}}{18} - \frac{e^{2x}}{2} - \frac{e^{3x}}{9} + e^x\right]_0^2 \quad +1$$

$$= \left[\frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^6}{9} + e^2 - \frac{e^0}{18} + \frac{e^0}{2} + \frac{e^0}{9} - e^0\right]$$

$$= \left[\frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^6}{9} + e^2 - \frac{1}{18} + \frac{1}{2} + \frac{1}{9} - 1\right]$$

$$I = \left[\frac{e^{12}}{18} - \frac{e^4}{2} - \frac{e^6}{9} + e^2 - \frac{1}{2}\right] \quad +1$$

Ans