Q.6 Attempt any two:

i. Solve the recurrence relation $y_{h+2} - 2y_{h+1} + y_h = 3h + 4$. **5**

ii. Solve the recurrence relation $y_{h+2} - 7y_{h+1} + 10y_h = 0$ with $y_0 = 0, y_1 = 3$ by the method of generating function. **5**

iii. Determine the discrete numeric function corresponding to the generating function $A(z) = \frac{1}{5-6z+z^2}$. **5**

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*Total No. of Questions: 6* *Total No. of Printed Pages:4*

**Enrollment No......................................**

Faculty of Engineering

End Sem Examination Dec-2023

CA5BS04 Mathematics of Computer Application

Programme: MCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

**Duration: 3 Hrs.** **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. To prove that a certain result is true we start by assuming that it is false this technique of proving the result is called----- **1**

(a) Direct proof (b) Indirect proof
(c) Method of contradiction (d) None of these

ii. There are 12 points in a plane, 5 only of which are in the straight line, then the number of triangles formed by joining the points is ____. **1**

(a) 210 (b) 120 (c) 228 (d) None of these

iii. If sum of degree of all the vertices in a connected graph G is 24 then number of edges in G is ____. **1**

(a) 10 (b) 11 (c) 12 (d) None of these

iv. A simple disconnected graph G with 10 vertices and 3 components has maximum number of edges ____. **1**

(a) 12 (b) 28 (c) 15 (d) None of these

v. A tree is connected graph without any ____. **1**

(a) Walk (b) Path (c) Circuit (d) None of these

vi. Number of vertices in a spanning tree of a connected graph G with 10 vertices is ____. **1**

(a) 9 (b) 10 (c) 11 (d) None of these

vii. If (G,.) is a group then $\forall a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$ is called ____ law. **1**

(a) Absorption (b) Reversal
(c) Commutative (d) None of these

viii. Every cyclic group is ____. **1**

(a) Abelian (b) Group

(c) Semigroup (d) All of these

ix. Order of the recurrence relation $y_{h+3} - 2y_{h+1} + y_h = 25h^2$ is ____. **1**

(a) 3 (b) 2 (c) 1 (d) None of these

x. Homogeneous solution of the recurrence relation $y_h - 6y_{h-1} + 8y_{h-2} = 0$ is ____. **1**

(a) $c_1 2^r + c_2 4^r$ (b) $c_1 2^r + c_2 8^r$

(c) $c_1 2^r + c_2 6^r$ (d) None of these

Q.2 Attempt any two:

i. Prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133, $n \in N$ using mathematical induction. **5**

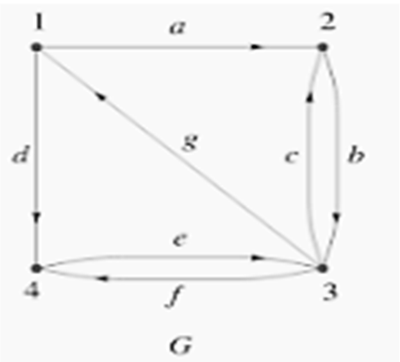
ii. State and prove Pigeonhole principle. **5**

iii. A men has 7 friends, 3 of them are gentlemen and 4 ladies; his wife has 7 friends 4 of them are gentlemen and 3 ladies. In how many ways can they invite a dinner party of 3 gentlemen and 3 ladies so that there are 3 of man's friends and 3 of wife's friends? **5**

Q.3 Attempt any two:

i. Prove that number of vertices of odd degree in a graph are always present in even number. **5**

ii. Represent adjacency matrix for the given digraph: **5**




iii. Define: **5**

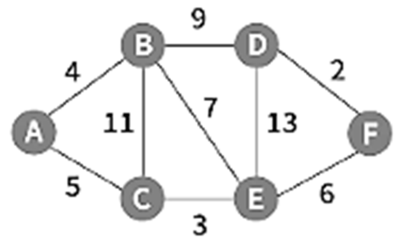
(a) Strongly connected digraph.

(b) Hamiltonian graph with example.

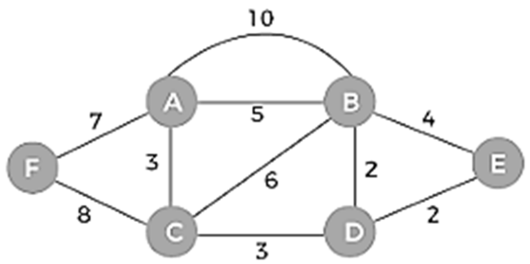


Q.4 Attempt any two:

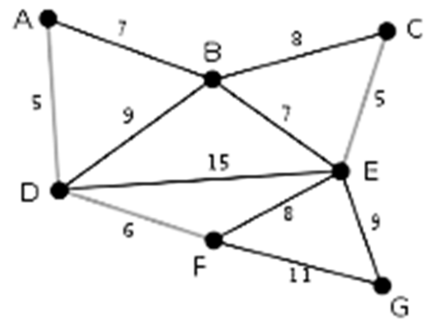
i. Find length of the shortest path from A to E for the given graph by using Dijkstras algorithm: **5**




ii. Find minimal spanning tree by Prims algorithm for the given graph: **5**




iii. Find minimal spanning tree by Kruskals algorithm for the given graph: **5**




Q.5 Attempt any two:

i. Show that the set of all integers $I$ forms a group with respect to the binary operation '$*$' defined by rule $a * b = a + b + 1, \forall a, b \in I$. **5**

ii. Prove that intersection of two subgroups of a group $G$ is a subgroup of $G$. **5**

iii. Prove that the order of each subgroup of a finite group is divisor of the order of the group. **5**

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Q. 1

i) c) Method of contradiction [1]

ii) a) 210 [1]

iii) c) 12 [1]

iv) b) 28 [1]

v) c) circuit [1]

vi) b) 10 [1]

vii) b) Reversal [1]

viii) d) ~~All of~~ these [1]

ix) a) 3 [1]

x) a) $c_1 2^r + c_2 4^r$ [1]

Q. 2

i) Let $P(n) = 11^{n+2} + 12^{2n+1}$

If $n=1$

$P(1) = 11^3 + 12^3 = 3059 = 133 \times 23$

So, $P(1)$ is divisible by 133.

Hence result is true for $n=1$ [1]

Let us assume that result is true for $n=m$.

i.e. $P(m) = 11^{m+2} + 12^{2m+1}$ is divisible by 133

i.e. $P(m) = 133 \cdot k$ ; k is some integer. [2]

Now, $P(m+1) = 11^{m+3} + 12^{2m+3}$

$= 11 \cdot 11^{m+2} + 12^2 \cdot 12^{2m+1}$

$= 11 \cdot 11^{m+2} + 144 \cdot 12^{2m+1}$

$= 11 \cdot 11^{m+2} + (11+133) \cdot 12^{2m+1}$ [3]

$= 11 \cdot [11^{m+2} + 12^{2m+1}] + 133 \cdot 12^{2m+1}$

$= 11 \cdot (133k) + 133 \cdot 12^{2m+1}$

$= 133[11k + 12^{2m+1}]$ $\left\{ \begin{array}{l} \text{as } k, m \in I \text{ so} \\ 11k + 12^{2m+1} \in I \end{array} \right\}$

Let $11k + 12^{2m+1} = t \in I$ [4]

[2]

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So, $P(m+1) = 133.t$
Hence divisible by 133
So; result is true for $n = m+1$
Hence by induction hypothesis result is true for all values of n. [5]

H.P//

ii) Statement: If n pigeons are assigned to m pigeonholes, where $m < n$, then atleast one pigeonhole contains two or more pigeons. [2]

Proof: Let us assume that each pigeonhole contains at the most one pigeon. Therefore the total number of pigeons assigned to m pigeonholes is at the most m. Since $m < n$ [3] therefore $n-m$ or more pigeons are left without having assigned a pigeon hole. Hence if all pigeons are to be assigned [4] pigeonholes, then atleast one pigeonhole contains two or more pigeons. [5]

iii) Four possibilities are there:

a) 3 gentlemen from husband's side & 3 ladies from wifes side:

$^3C_3 \times {^3C_3} = 1 \times 1 = 1$ [1]

b) 3 gentlemen from wifes side & 3 ladies from husbands side;

$^4C_3 \times {^4C_3} = 4 \times 4 = 16$ [2]

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c) 2 ladies & 1 gentlemen from husbands side & 1 lady & 2 gentlemen from wifes side.

$(4C_2 \times 3C_1) \times (3C_1 \times 4C_2) = 6 \times 3 \times 3 \times 6 = 324$ [3]

d) 1 lady & 2 gentlemen from husbands side & 2 ladies & 1 gentleman from wifes side.

$(4C_1 \times 3C_2) \times (3C_2 \times 4C_1) = 4 \times 3 \times 3 \times 4 = 144$ [4]

Hence total no. of ways are:

$= 1 + 16 + 324 + 144 = 485$ Ans// [5]

Q. 3

i) Let $G = (V, E)$ be the given graph.
Let $V_0, V_e$ be the set of vertices having odd & even degree respectively. [1]

so $V_0 \cup V_e = V$ & $V_0 \cap V_e = \emptyset$

As, we know that

$$\sum_{v_i \in V} \deg v_i = 2e; \text{ e is no. of edges in } G.$$

$$\Rightarrow \sum_{v_i \in V_0} \deg v_i + \sum_{v_i \in V_e} \deg v_i = 2e$$ [2]

As we know that sum of even numbers is always an even no.

Let $\sum_{v_i \in V_e} \deg v_i = 2k$ ; $k \in I_+$ [3]

$$\Rightarrow \sum_{v_i \in V_0} \deg v_i = 2e - 2k$$

So, we get sum of odd numbers an

[4]

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even number which is possible only when number of terms in the sum is even. [4]

Hence no. of vertices of odd degree in a graph are always present in even number. [5]

H.P//

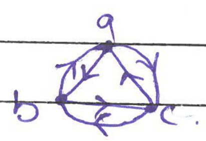
ii) Adjacency matrix for $G_1$ :

$$X = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

[1.25]
[2.5]
[3.75]
[5.00]

iii)

a) Strongly connected digraph:
A digraph is said ~~to~~ be strongly connected if there exist at least one directed path from every vertex to every other vertex. [1.25]




[2.5]

G.

b) Hamiltonian Graph : A closed walk in a graph which covers all the vertices of the graph exactly once except the terminal vertices is called

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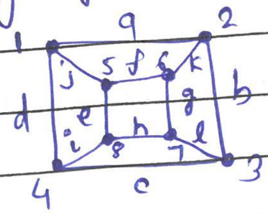
hamiltonian circuit of graph that contains hamiltonian circuit in it is called hamiltonian graph. [3.75]




1 j 5 f 6 k 2 b 3 l 7 h 8 i 4 d 1 - hamiltonian circuit [5.0]

Q. 4

i) Assign $P(L(A)) = 0$
set $v = A$.

$T(L(B)) = \infty$, $T(L(C)) = \infty$, $T(L(D)) = \infty$, $T(L(E)) = \infty$, $T(L(F)) = \infty$ [1]

Now,

$T(L(B)) = \text{Min}\{\infty, 0+4\} = 4$

$T(L(C))] = \text{Min}\{\infty, 0+5\} = 5$

$T(L(D)) = \text{Min}\{\infty, 0+\infty\} = \infty$

$T(L(E)) = \text{Min}\{\infty, 0+\infty\} = \infty$ [2]

$T(L(F)) = \text{Min}\{\infty, 0+\infty\} = \infty$.

Assign $P(L(B)) = 4$
set $v = B$.

$T(L(C)) = \text{Min}\{5, 4+11\} = 5$

$T(L(D)) = \text{Min}\{\infty, 4+9\} = 13$

$T(L(E)) = \text{Min}\{\infty, 4+7\} = 11$ [3]

$T(L(F)) = \text{Min}\{\infty, 4+\infty\} = \infty$.

Assign $P(L(C)) = 5$
set $v = C$.

$T(L(D)) = \text{Min}\{13, 5+\infty\} = 13$

$T(L(E)) = \text{Min}\{11, 5+3\} = 8$.

[6]

$T(L(F)) = \text{Min}\{\infty, 5+\infty\} = \infty$. [4]

Assign $P(L(F)) = 8$.

set $V = E$

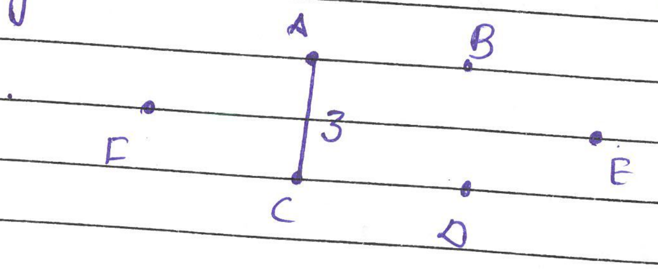
So length of the shortest path from A to E is 8. [5]

ii) Adjacency Matrix :

$$X = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} - & \cancel{5} & 3 & - & - & 7 \\ \cancel{5} & - & \cancel{6} & \cancel{2} & \cancel{4} & - \\ \cancel{3} & \cancel{6} & - & 3 & - & 8 \\ - & 2 & \cancel{3} & - & 2 & - \\ - & \cancel{4} & - & \cancel{2} & - & - \\ 7 & - & 8 & - & - & - \end{bmatrix} \end{array}$$

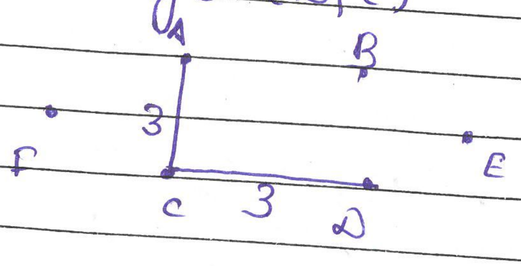
[1]

Starting from vertex A select the edge (A, C) & cross the edge (C, A) we get




[2]

Now, in row A, C select the edge (C, D) with next minimum wt & cross the edge (D, C)



[17]

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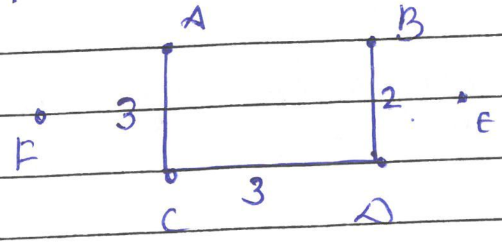
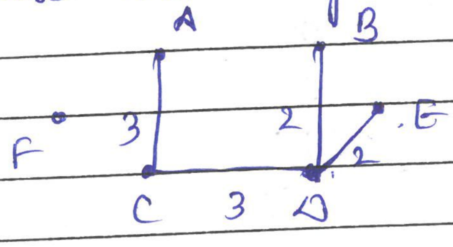
Now, in row A, C, D select the edge (D,B) and cross the edge (B,D) as its selection does not creates a circuit.

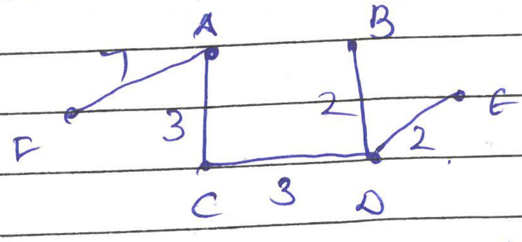



[3]

Now in row A, B, C, D select the edge (D, E) & cross the edge (E, D) we get




Now in row A, B, C, D, E edge with minimum wt is (B, E) or (E, B) don't select this edge as its selection creates a circuit. Edge with next minimum wt is (A, B) or (B, A) but its selection creates a circuit don't select this edge. Edge with next minimum wt. is (B, C) or (C, B) but its selection creates a circuit, don't select this edge. Select the edge (A, F) with next minimum wt. as its selection does not creates circuit. [4]




As 6 vertices are there & already 5 edges are selected so this

[8]

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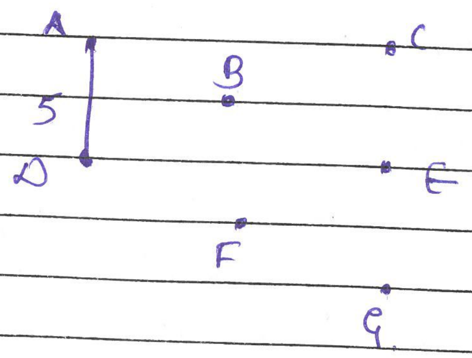
is the required minimal spanning tree whose weight is 17. [5]

iii)

| Edge | weight of the edge | Selection of edge. |
| --- | --- | --- |
| (A, D) | 5 | Yes |
| (C, E) | 5 | Yes |
| (D, F) | 6 | Yes |
| (A, B) | 7 | Yes |
| (B, E) | 7 | Yes. |
| (B, C) | 8 | No |
| (E, F) | 8 | No. |
| (E, G) | 9 | Yes |
| (B, D) | 9 | — |
| (F, G) | 11 | — |
| (D, E) | 15 | — |

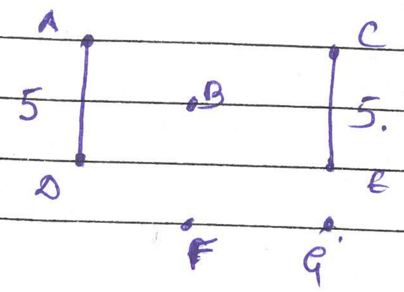
[1]

Select the edge (A, D) with minimum wt

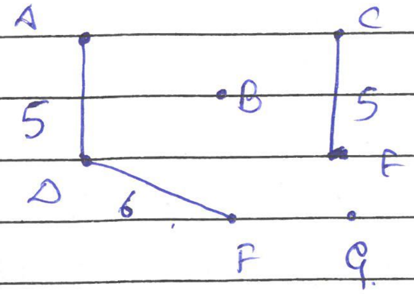



[2]

Now select the edge (C, E) with next minimum wt.

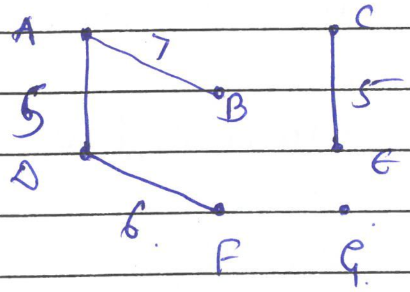


Now select the edge (D,F) with next minimum wt.

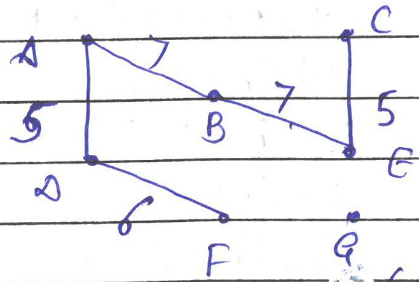



[3]

Now select the edge (A,B) with next minimum wt.

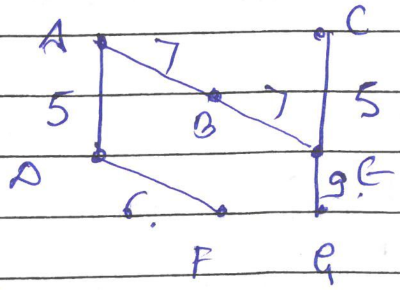



Now select the edge (B,E) with next minimum wt.




[4]

Don't select the edge (B,C) (E,F) as its selection creates a circuit
Select the edge (E,G) as its selection does not creates a circuit.




As 7 vertices are there and already 6.

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edges are selected so this is the required minimal spanning tree whose wt is 39. [5]

Q.5

(i) To prove that $(I, *)$ is a group we are required to prove these four properties:

a) closure law:

$$\forall a, b \in I \Rightarrow a+b+1 \in I \Rightarrow a*b \in I.$$

as sum of two integers is an integer adding 1 to it still represents an integer.

Hence closure law is satisfied. [1]

b) Associative law:

Consider, $\forall a, b, c \in I$

$$(a*b)*c = (a+b+1)*c$$
$$= (a+b+1)+c+1$$
$$= a+(b+c+1)+1$$
$$= a+(b*c)+1$$
$$= a*(b*c)$$

Hence associative law is satisfied. [2]

c) Identity law:

$\forall a \in I$, $e \in I$ is said to be identity element if

$$a*e = e*a = a.$$

Consider

$$a*e = a$$
$$\Rightarrow a+e+1 = a$$
$$\Rightarrow e = -1$$

as $-1 \in I$, so identity law is satisfied. [3]

d) Inverse law:

$\forall a \in I$, $a^{-1} \in I$ is said to be inverse of a if $a * a^{-1} = a^{-1} * a = -1$

Consider

$a * a^{-1} = -1$

$a + a^{-1} + 1 = -1$

$a^{-1} = -2 - a$. [4]

as $-2-a \in I \Rightarrow \forall a \in I \Rightarrow a^{-1} \in I$.

Hence inverse law is satisfied.

So $(I, *)$ is a group. [5]

ii) Let $(G_1, o)$, $(G_2, o)$ be two subgroups of $(G, o)$ [1]

To prove that $(G_1 \cap G_2, o)$ is a subgroup of G we are required to prove that

$G_1 \cap G_2 \neq \phi$, $G_1 \cap G_2 \subseteq G$ and

$\forall a, b \in G_1 \cap G_2 \Rightarrow a o b^{-1} \in G_1 \cap G_2$. [2]

As $e \in G_1$ & $e \in G_2 \Rightarrow e \in G_1 \cap G_2 \Rightarrow G_1 \cap G_2 \neq \phi$ [3]

& as $G_1 \subseteq G$ & $G_2 \subseteq G \Rightarrow G_1 \cap G_2 \subseteq G$. [3]

Now,

$\forall a, b \in G_1 \cap G_2 \Rightarrow a, b \in G_1$ & $a, b \in G_2$.

$\forall a, b \in G_1 \Rightarrow a o b^{-1} \in G_1$ [as $G_1$ is subgroup of G] [4]

$\forall a, b \in G_2 \Rightarrow a o b^{-1} \in G_2$ [as $G_2$ is subgroup of G]

$\Rightarrow a o b^{-1} \in G_1 \cap G_2$

$\Rightarrow (G_1 \cap G_2, o)$ is a subgroup of G. [5]

H.P//

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iii) Let $(G,\cdot)$ be the group & $(H,\cdot)$ be subgroup of $G$.
Let $O(G) = n$
& $O(H) = m$.
To prove that $m | n$ [1]
Let $H = \{a_1, a_2, \dots a_m\}$
Let $a \in G$ arbitrarily, so $aH$ is left coset of $H$ in $G$.
So $aH = \{aa_1, aa_2, \dots aa_m\}$ [2]
we claim that all these are distinct elements of $aH$ for if
$$aa_i = aa_j \quad ; 1 \le i, j \le m \ \& \ i \ne j$$
$$\Rightarrow a_i = a_j \quad \text{[by cancelation law in G]}$$ [3]
which is a contradiction as $a_i, a_j$ are distinct elements of $H$
Hence $aa_i \ne aa_j$.
So, $O(aH) = m$.
Let no. of distinct left cosets of $H$ in $G$ be $k$, [4]
So, $\therefore O(G) = mk$.
$$\Rightarrow n = mk$$
$$\Rightarrow \frac{n}{m} = k.$$
as $k \in I \Rightarrow m | n$.
Hence order of subgroup divides order of the group. [5]
H.P//.

Q. 6
i) $y_{n+2} - 2y_{n+1} + y_n = 3n + 4$.

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$\Rightarrow E^2(y_h) - 2Ey_h + y_h = 3h^{(1)} + 4h^{(0)}$

$\Rightarrow (E^2 - 2E - 1)\, y_h = 3h^{(1)} + 4h^{(0)}$

$\Rightarrow (E-1)^2 y_h = 3h^{(1)} + 4h^{(0)}$ [1]

Its characteristic eq. is

$(m-1)^2 = 0$

$\Rightarrow m = 1, 1$

So homogeneous sol. is

$(c_1 + c_2 h)(1)^h = c_1 + c_2 h$ [2]

Its particular sol. is

$$\frac{1}{(E-1)^2}\, 3h^{(1)} + 4h^{(0)}$$

$$= \frac{1}{(1+\Delta-1)^2}\, 3h^{(1)} + 4h^{(0)}$$ [3]

$$= \Delta^{-2}\left[3h^{(1)} + 4h^{(0)}\right]$$

$$= \Delta^{-1}\left[\frac{3h^{(2)}}{2} + \frac{4h^{(1)}}{1}\right]$$

$$= \frac{3h^{(3)}}{2\cdot 3} + \frac{4h^{(2)}}{1\cdot 2}$$

$$= \frac{h^{(3)}}{2} + 2h^{(2)}$$ [4]

$$= \frac{h(h-1)(h-2)}{2} + 2h(h-1)$$

$$= \frac{h(h-1)}{2}\left[h-2+4\right]$$

$$= \frac{h(h-1)(h+2)}{2}$$

So, total sol.

$$y_h = c_1 + c_2 h + \frac{h(h-1)(h+2)}{2}$$ Ans// [5]

ii) $y_{n+2} - 7y_{n+1} + 10y_n = 0$

Multiply above eq. by $t^n$ & take $\sum_{n=0}^{\infty}$ both the sides we get.

$$\sum_{n=0}^{\infty} y_{n+2} t^n - 7\sum_{n=0}^{\infty} y_{n+1} t^n + 10\sum_{n=0}^{\infty} y_n t^n = 0 \qquad [1]$$

$$\Rightarrow y_2 + y_3 t + \cdots - 7[y_1 + y_2 t + \cdots] + 10\,Y(t) = 0$$

$$\Rightarrow \frac{-y_0 - y_1 t + y_0 + y_1 t + y_2 t^2 + \cdots}{t^2} - 7\frac{[-y_0 + y_0 + y_1 t + y_2 t^2 + \cdots]}{t}$$

$$+ 10\,Y(t) = 0$$

$$\Rightarrow \frac{Y(t) - y_0 - y_1 t}{t^2} - 7\frac{[Y(t) - y_0]}{t} + 10\,Y(t) = 0 \qquad [2]$$

$$\Rightarrow \frac{Y(t) - 3t}{t^2} - 7\frac{Y(t)}{t} + 10Y(t) = 0 \qquad \left[\begin{array}{l} \text{as } y_0 = 0 \\ y_1 = 3 \end{array}\right]$$

$$\Rightarrow Y(t) - 3t - 7t\,Y(t) + 10t^2 Y(t) = 0$$

$$\Rightarrow Y(t)[10t^2 - 7t + 1] = 3t$$

$$\Rightarrow Y(t) = \frac{3t}{10t^2 - 5t - 2t + 1} \qquad [3]$$

$$\Rightarrow Y(t) = \frac{3t}{5t[2t-1] - 1[2t-1]}$$

$$\Rightarrow Y(t) = \frac{3t}{(5t-1)(2t-1)}$$

$$\Rightarrow Y(t) = \left[\frac{1}{2t-1} - \frac{1}{5t-1}\right]$$

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$$\Rightarrow Y(t) = \frac{1}{1-5t} - \frac{1}{1-2t}.$$ [4.]

$$\Rightarrow Y(t) = (1-5t)^{-1} - (1-2t)^{-1}$$

$$\Rightarrow Y(t) = [1+5t+(5t)^2+\cdots+(5t)^n+\cdots] - [1+2t+(2t)^2+\cdots+(2t)^n+\cdots]$$

Comparing the co-efficient of $t^n$ both the sides we get

$$\boxed{y_n = 5^n - 2^n}.$$ Ans// [5]

iii>

$$A(z) = \frac{1}{5-6z+z^2}$$

$$= \frac{1}{5-5z-z+z^2}$$ [1]

$$= \frac{1}{5(1-z)-z(1-z)}$$

$$= \frac{1}{(5-z)(1-z)}$$ [2]

$$= \frac{1}{4}\left[\frac{1}{1-z} - \frac{1}{5-z}\right]$$

$$= \frac{1}{4}\left[(1-z)^{-1} - \frac{1}{5}\left[1-\frac{z}{5}\right]^{-1}\right]$$ [3]

$$= \frac{1}{4}[1+z+z^2+\cdots+z^r+\cdots] - \frac{1}{20}\left[1+\frac{z}{5}+\left(\frac{z}{5}\right)^2+\cdots+\left(\frac{z}{5}\right)^r+\cdots\right]$$ [4]

Comparing the coefficient of $z^r$ both the sides we get

$$a_r = \frac{1}{4} - \frac{1}{20}\left(\frac{1}{5}\right)^r ; \quad r \geq 0.$$

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$$= \frac{1}{4}\left[1 - \frac{1}{5^{s+1}}\right];\ s \geq 0$$ [5]

ANS//