Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering / Science

End Sem (Odd) Examination Dec-2019

CA3CO11 Mathematics-III

Programme: BCA-MCA (Integrated) / BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Order of Convergence of Regula Falsi Method is **1**

(a) 2.231 (b) 1.321 (c) 1.681 (d) None of these

ii. The Algebraic and Transcendental equation can be solved by **1**

(a) Iterative method (b) Newton Raphson Method

(c) Both (a) and (b) (d) None of these

iii. In the Interpolation when the values of the argument are not equispaced, we can define a more general class of differences which is called as **1**

(a) Divided Difference (b) Forward Difference

(c) Backward difference (d) None of these

iv. Sterling's Formula to interpolate y gives best result when the value of u lies between **1**

(a) $0 < u < 1$ (b) $-.25 < u < .25$

(c) $-1 < u < 0$ (d) None of these

v. To apply Weddle's Rule the interval of integration must be divided into ____ equal parts. **1**

(a) 2 (b) 3 (c) 6 (d) None of these

vi. Which of the following method is used to find the solution of Ordinary differential equation? **1**

(a) Simpson's Rule (b) Lagrange's Method

(c) Picard's Method (d) None of these

vii. Variance of exponential distribution is equal to **1**

(a) $2/\lambda$ (b) $1/\lambda^2$ (c) $1/\lambda$ (d) None of these

P.T.O.

viii. In Standard Normal distribution the mean and variance are respectively **1**
(a) 0,1 (b) 1,0 (c) 1,1 (d) None of these

ix. The value of χ^2 are always **1**
(a) Positive (b) Negative
(c) Both (a) and (b) (d) None of these

x. If n is the no. of observation, then in Binomial distribution degree of freedom is given by **1**
(a) $n-1$ (b) $n-2$ (c) $n-3$ (d) None of these

Q.2 i. Define Relative error and Truncated error. **2**

ii. Find a real root of the equation $x^3-2x-5=0$ by Regula-Falsi Method correct to three decimal places. **8**

OR iii. Solve the following system of equation by Gauss-Seidal method **8**
$83x+11y-4z=95$; $7x+52y+13z=104; 3x+8y+29z=71$

Q.3 Attempt any two:

i. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=4$, using Lagrange's interpolation formula. **5**

x	1	2	4	8	10
y	0	1	5	21	27

ii. Using Newton's divided difference formula, find $f(8)$ if **5**

x	4	5	7	10	11	13
$y=f(x)$	48	100	294	900	1210	2028

iii. Express the function $f(x)=x^3-2x^2+x-1$ in factorial notation and show that $\Delta^4 f(x)=0$. **5**

Q.4 i. Evaluate $\int_0^6 \frac{dx}{(1+x^2)}$ by using Simpson's 1/3 Rule. **4**

ii. Using Taylor's series method find the solution of the differential equation xy′=x-y , y(2)=2 at x=2.1. **6**

OR iii. Use Runge-Kutta method to solve the equation $\frac{dy}{dx}=(1+y^2)$ for x=0.2 and x=0.4. **6**

Q.5 i. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six? **3**

ii. Fit a Poisson distribution to the following data **7**

X	0	1	2	3	4
f	46	38	22	9	1

OR iii. Derive the formula for mean and variance of Normal distribution. **7**

Q.6 Attempt any two:

i. From the table given below, whether the colour of son's eyes is associated with that of father's eyes? Given that the value of χ^2 for 1 degree of freedom at 5 % level of significance is 3.841. **5**

Eye Colour in Son			
Eye Colour in Father		Not light	light
	Not light	230	148
	light	151	471

ii. Ten objects are chosen at random from a population and their heights are found to be in inches 63, 63, 64, 65, 66, 69, 69, 70, 70, 71. Discuss the suggestion that the mean height in the universe is 65 inches, given that for 9 degree of freedom the value of t and 5% level of significance is 2.262. **5**

iii. Random samples are drawn from two populations and the following results were obtained: **5**

X	20	16	26	27	23	22	18	24	25	16	-	-
Y	27	33	42	35	32	34	38	28	41	43	30	37

Obtain the estimate of the variances of the population and test whether the two populations have the same variance.

******

Faculty of Science
End Sem (odd) Examination Dec-2019
CA3CO11 Mathematics - III

prog. BCA-MCA Integrated (BCA) Branch :- Computer Application.

SCHEME / SOLUTION

Q.1 (i) (c) 1.681 (+1)
(ii) (c) Both a and b (+1)
(iii) (a) Divided Difference (+1)
(iv) (b) $-.25 < u < .25$ (+1)
(v) (c) 6 (+1)
(vi) (c) Picard's Method (+1)
(vii) (b) $1/\lambda^2$ (+1)
(viii) (a) $(0,1)$ (+1)
(ix) (a) Positive (+1)
(x) (a) $n-1$ (+1)

Q.2 (i) Relative error :- Let x be the exact value of the quantity and x_a be the approximate value as obtained by measurement or calculation then Relative error is defined as $E_r = \left|\frac{x - x_a}{x}\right| = \left|1 - \frac{x_a}{x}\right|$ (+1)

Truncation Error :- Truncation error in a Problem arise when an infinite mathematical Process is approximated by a finite Process. (+1)

Q.2 (ii) Let $f(x) = x^3 - 2x - 5 = 0$

As $f(2) = -1$ $f(3) = 16$

$\therefore$ root lie ~~btw~~ between 2 and 3 (+2)

First approximation :- $x_0 = 2$ $x_1 = 3$

$f(x_0) = f(2) = -1$ $f(x_1) = 16$ (+1)

Formula

$$x_2 = x_1 + \frac{(x_1 - x_0)}{(f(x_1) - f(x_0))} f(x_0)$$ (+1)

$\therefore$ $x_2 = 2.059$

$f(2.059) = -0.389$ and $f(3) = +16$

$\therefore$ Next approximation root lie b/w $(2.059, 3)$ (+1)

Second approximation

$x_0 = 2.059$, $x_1 = 3$ $f(x_0) = 2.059$, $f(x_1) = 16$

$x_3 = 2.081$

$f(2.081) = -0.150$ $f(3) = 16$

$\therefore$ root lie between (2.081 and 3) (+1)

Third approximation

$x_0 = 2.081$ $x_1 = 3$

$f(x_0) = -.150$ $f(x_1) = 16$

$\therefore$ $x_3 = 2.090$

$f(2.090) = -0.051$ $f(3) = 16$

$\therefore$ next root lie b/w $(2.093, 3)$ (+1)

Fourth approximation

$x_0 = 2.090$, $x_1 = 3$ $f(x_0) = -0.51$

$f(x_1) = 16$

$x_4 = 2.093$

Hence root of eq^n is 2.093 (+1)

Q.2 (iii) (OR)

$$x = \frac{1}{83}(95 - 11y + 4z)$$

$$y = \frac{1}{52}(104 - 7x - 13z)$$ (+2)

$$z = \frac{1}{29}(71 - 3x - 8y)$$

We start with $y = 0, z = 0$, & using most recent values of x, y, z we get

First iteration

$$x^{(1)} = \frac{1}{83}(95 - 11y^{(0)} + 4z^{(0)}) = 1.145$$

$$y^{(1)} = \frac{1}{52}(104 - 7x^{(1)} - 13z^{(0)}) = 1.846$$

$$z^{(1)} = \frac{1}{29}(71 - 3x^{(1)} - 8y^{(1)}) = 1.821$$ (+2)

Second iteration

$$x^{(2)} = \frac{1}{83}(95 - 11y^{(1)} + 4z^{(1)}) = 0.988$$

$$y^{(2)} = \frac{1}{52}(104 - 7x^{(2)} - 13z^{(1)}) = 1.412$$

$$z^{(2)} = \frac{1}{29}(71 - 3x^{(2)} - 8y^{(2)}) = 1.956$$ (+2)

Third Iteration :-

$$x^{(3)} = \frac{1}{83}(95 - 11y^{(2)} + 4z^{(2)}) = 1.051$$

$$y^{(3)} = \frac{1}{52}(104 - 7x^{(3)} - 13z^{(2)}) = 1.344$$

$$z^{(3)} = \frac{1}{29}(71 - 3x^{(3)} - 8y^{(3)}) = 1.969$$ (+1)

Fourth Iteration :-

$$x^{(4)} = \frac{1}{83}(95 - 11y^{(3)} + 4z^{(3)}) = 1.051$$

$$y^{(4)} = \frac{1}{52}(104 - 7x^{(4)} - 13z^{(3)}) = 1.34$$

$$z^{(4)} = \frac{1}{29}(71 - 3x^{(4)} - 8y^{(4)}) = 1.969$$

∴ the required Sol^n is $x = 1.051$, $y = 1.34$, $z = 1.969$ (+1)

Q.3(i) By Lagrange's Interpolation Formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} f(x_n)$$ (+2)

Here $x_0 = 1$ $x_1 = 2$ $x_2 = 4$ $x_3 = 8$ $x_4 = 10$

$f(x_0) = 0$ $f(x_1) = 1$ $f(x_2) = 5$ $f(x_3) = 21$ $f(x_4) = 27$

$$y = f(x) = x^4 - 15x^3 + \frac{211}{3}x^2 + 120x + 67 \quad (+1)$$

$$\frac{dy}{dx} = 4x^3 - 45x^2 + \frac{422x}{3} + 120$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 218.66 \quad (+1)$$

$$\frac{d^2y}{dx^2} = 12x^2 - 90x + \frac{422}{3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=4} = -27.33 \quad (+1)$$

<u>Alternative Method</u> <u>(Divided Difference Table)</u>

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	0				
		1			
2	1		1/3		
		2		0	
4	5		1/3		−1/144
		4		−1/16	
8	21		−1/6		
		3			
10	27				

(+2)

By Divided Diff Formula

$$y = f(x) = y_0 + (x-1)\Delta y_0 + (x-1)(x-2)\Delta^2 y_0$$
$$+ (x-1)(x-2)(x-4)\Delta^3 y_0 + (x-1)(x-2)(x-4)(x-8)\Delta^4 y_0$$

$$f(x) = x^4 - 15x^3 + \frac{211}{3}x^2 + 120x + 67 \quad (+1)$$

$$\frac{dy}{dx} = 4x^3 - 45x^2 + \frac{422}{3}x + 120$$

$$\left(\frac{dy}{dx}\right)_{x=4} = 218.66 \quad (+1)$$

$$\left(\frac{d^2y}{dx^2}\right) = 12x^2 - 90x + \frac{422}{3}$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=4} = -27.33 \quad (+1)$$

Q. 3(ii)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
4	48	52	15		
5	100	97	21	1	0
7	294	202	27	1	0
10	900	310	33	1	
11	1210	409			
13	2028				

(+3)

$$f(x) = f(x_0) + (x-x_0)\Delta f(x_0) + (x-x_0)(x-x_1)\Delta^2 f(x_0) \quad (+1)$$

$$+ - - - -$$

At $x = 8$

$$f(8) = 48 + 208 + 180 + 12$$

$$= 448 \quad (+1)$$

Q.3 (iii) Given $f(x) = x^3 - 2x^2 + x - 1$

Let $f(x) = Ax^{[3]} + Bx^{[2]} + Cx^{[1]} + D$

$\Rightarrow x^3 - 2x^2 + x - 1 = Ax(x-1)(x-2) + Bx(x-1) + Cx + D$ (+1)

Put $x = 0$ $\quad D = -1$

Put $x = 1$ $\Rightarrow C = 0$ (+1)

$x = 2 \Rightarrow B = 1$ (+1)

On eq^n coeff $\Rightarrow A = 1$ (+1)

$\therefore f(x) = x^{[3]} + x^{[2]} - 1$

$\Delta f(x) = 3x^{[2]} + 2x^{[1]}$

$\Delta^2 f(x) = 6x^{[1]} + 2$

$\Delta^3 f(x) = 6$

$\Delta^4 f(x) = 0$ (+1)

Q.4 (i) $\int_a^b f(x)dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$ (+1)

$h = 1$

$\therefore \int_0^6 \frac{dx}{1+x^2} = \frac{1}{3}\left[(1+0.027) + 4(0.5+0.1+0.0385) + 2(0.2+.0588)\right]$ (+1)

$= 1.3662$

x	0	1	2	3	4	5	6
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	.0385	.027

(+2)

Q.4 (ii) Given $xy' = x - y$ $\quad y(2) = 2$

(OR)

$$y' = 1 - \frac{y}{x} = 0 \qquad y'(2) = 0 \quad (+1)$$

$$y'' = \frac{-y'}{x} + \frac{y}{x^2} ; \quad y''(2) = \frac{1}{2} \quad (+1)$$

$$y''' = \frac{-y''}{x} + \frac{2y'}{x^2} - \frac{2y}{x^3} \quad y'''(2) = \frac{-3}{4} \quad (+1)$$

$$y^{iv} = \frac{-y'''}{x} + \frac{3y''}{x^2} - \frac{6y'}{x^3} + \frac{6y}{x^4}$$

$$y^{iv}(x) = \frac{3}{2}$$

Taylor's Series

$$y = y_0 + (x-x_0)y_0' + \frac{(x-x_0)^2}{2!}y_0'' + \cdots \quad (+2)$$

$x = 2.1 \qquad x_0 = 2$

$$y = 2 + (x-0)\cdot 0 + \frac{1}{4}(x-2)^2 - \frac{1}{8}(x-2)^3 + \frac{1}{16}(x-2)^4 + \cdots$$

$$= 2 + .0025 - .000125 + .0000063$$

$$y = 2.00238 \quad (+1)$$

Q.4 (iii) (OR) $\frac{dy}{dx} = 1+y^2$ $x_0 = 0$, $y_0 = 0$, $h = 0.2$

$k_1 = hf(x_0, y_0)$; $K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$ $K_4 = hf(x_0+h, y_0+K_3)$

$K = \left(\frac{K_1 + 2K_2 + 2K_3 + K_4}{6}\right)$

$y = y_0 + K$ (+2)

y_1 at $x_1 = 0.2$; $K_1 = 0.2$; $K_2 = 0.202$

$y_1 = 0.20271$; $K_3 = .20204$; $K_4 = .20816$ (+2)

y_2 at $x_2 = 0.4$; $K_1 = .20822$; $K_2 = .21883$

$y_2 = .42279$; $K_3 = .21948$; $K_4 = .23565$ (+2)

Q.5 (i) $p = \frac{1}{3}$ $q = \frac{2}{3}$ $n = 6$

$p(x=r) = {}^nC_r \, p^r q^{n-r}$ (+1)

At least 3 $= p(3) + p(4) + p(5) + p(6)$

$= 233/729$ (+1)

$\therefore$ Total No. of Times we get at least three dice to show a five or six $= 729 \times \frac{233}{729}$

$= 233$ Times (+1)

Q.5 (ii)

x	0	1	2	3	4	
f	46	38	22	9	1	$\Sigma f = 116$
fx	0	38	44	27	4	$\Sigma fx = 113$

(+2)

$$m = \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{113}{116} = 0.974$$ (+1)

$$p(x=r) = \frac{e^{-m} \cdot m^r}{r!}$$ Here $\underline{N = 116}$ (+1)

theoretical frequencies at $x = r$ is

$$f(x=r) = N \times p(x=r) = N \times \frac{e^{-m} \cdot m^r}{r!}$$

For $r = 0$ $f(0) = 43.8$ (+1)
$r = 1$ $f(1) = 42.66$ (+1)
$r = 2$ $f(2) = 20.78$
$r = 3$ $f(3) = 6.74$
$r = 4$ $f(4) = 1.64$ (+1)

Q.5 (iii)
(OR)

$$\text{Mean} = \mu_1' = \int_{-\infty}^{x} x \cdot f(x)\, dx$$

where $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ $-\infty < x < \infty$

$$\therefore \mu_1' = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}\, dx$$

Let $\frac{x-\mu}{\sqrt{2}\sigma} = t \Rightarrow x = \mu + \sqrt{2}\sigma t$

$dx = \sqrt{2}\sigma\, dt$

$$\therefore \mu_1' = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\mu + \sqrt{2}\sigma t) e^{-t^2} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt + \sqrt{\frac{2}{\pi}}\, \sigma \int_{-\infty}^{\infty} t e^{-t^2} dt$$

$\downarrow$ (11/0)

odd funcⁿ

$$= \frac{\mu}{\sqrt{\pi}} \times 2 \int_{0}^{\infty} e^{-t^2} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \times 2 \times \frac{\sqrt{\pi}}{2}$$

$$\boxed{\mu_1' = \mu}$$ (+3)

Second Moment about origin

$$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$= \frac{1}{\sqrt{\pi}} \left[\int_{-\infty}^{\infty} \mu^2 e^{-t^2} dt + 2\sqrt{2}\sigma\mu \int_{-\infty}^{\infty} t e^{-t^2} dt + 2\sigma^2 \int_{-\infty}^{\infty} t^2 e^{-t^2} dt \right]$$

$$= \mu^2 + \sigma^2$$ (+3)

$$\text{Variance} = \mu_2' - (\mu_1')^2$$

$$= \mu^2 + \sigma^2 - \mu^2$$

$$\boxed{\text{Var} = \sigma^2}$$ (+1)

Q.6 (i) $f_{o11} = 230$ $f_{o13} = 151$

$f_{o12} = 148$ $f_{o14} = 471$

$N_1 = 378$ $N_3 = 381$

$N_2 = 622$ $N_4 = 619$

$N = 1000$

$$f_{e11} = \frac{N_1 \times N_2}{N} = 144$$

$$f_{e12} = \frac{N_1 \times N_4}{N} = 234$$

$$f_{e21} = \frac{N_3 \times N_2}{N} = 237$$

$$f_{e22} = \frac{N_4 \times N_2}{N} = 385$$ (+2)

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$ (+1)

$$= 51.36 + 31.61 + 31.21 + 19.21$$

$$= 133.29$$ (+1)

Degree of freedom $(\nu) = (m-1)(n-1)$

$= (2-1)(2-1)$

$= 1$

The Calculated Value of χ^2 at 5% Level of significance for 1 degree of freedom is 3.841

As Calculated Value $\nless$ Tabulated Value

$\therefore$ Null hypothesis is rejected

$\Rightarrow$ There is an association between Colours of son's eyes and Father's eyes. (+1)

Q.6 (ii) $n = 10 \quad \mu = 65 \quad \nu = n-1 = 9$

$$t = \frac{\bar{x} - \mu}{s}\sqrt{n}$$

where $s^2 = \dfrac{\Sigma(x-\bar{x})^2}{n-1}$ (+2)

Mean $\bar{x} = \Sigma x/n = 670/10 = 67$

x	$x - \bar{x}$	$(x-\bar{x})^2$
63	-4	16
63	-4	16
64	-3	9
65	-2	4
66	-1	1
69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
$\Sigma x = 670$		$\Sigma(x-\bar{x})^2 = 88$

(+2)

$s^2 = \frac{88}{9} = 3.13$ Inches

$t = \frac{(67-65)}{3.13}\sqrt{10}$

$= 2.024$ (+1)

As Calculated Value < Tabulated Value

∴ Null hypothesis is accepted.

⇒ Mean height of Universe is 65 inches

Q.6 (iii) Given $n_1 = 10$ $n_2 = 12$ $\nu = 9$ $\nu = 11$ $\bar{x} = 22$, $\bar{y} = 35$

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
20	-1.7	2.89	27	-8	64
16	-5.7	+32.49	33	-2	4
26	4.3	18.49	42	7	49
27	5.3	28.09	35	0	0
23	1.3	1.69	32	-3	9
22	0.3	0.09	34	-1	1
18	-3.7	+13.69	38	3	9
24	2.3	5.29	28	-7	49
25	3.3	10.89	41	6	36
16	-5.7	+32.49	43	8	64
			30	-5	25
			37	2	4
$\Sigma x = 217$		146.1	$\Sigma y = 420$		314

(+3)

$\bar{x} = \frac{\Sigma x}{n} = \frac{217}{10} = 21.7$ $\bar{y} = \frac{\Sigma y}{n} = 35$

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1} = \frac{146}{9} = 16.22$$

$$S_2^2 = \frac{\Sigma(Y-\bar{Y})^2}{n_2} = \frac{314}{11} = 28.5$$ (+1)

$$S_2^2 > S_1^2$$

$$F = \frac{S_2^2}{S_1^2} = 28.5 = 1.757$$ (+1)