Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering / Science

End Sem (Odd) Examination Dec-2019

CA3CO04 Mathematics-I

Programme: BCA-MCA (Integrated) / BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If p is any statement, then $\{p \vee (\sim p)\}$ is **1**

(a) Contradiction (b) Tautology

(c) Not definite (d) None of these

ii. The symbol for connective word "And" is denoted by **1**

(a) $\vee$ (b) $\Leftrightarrow$ (c) $\Rightarrow$ (d) $\wedge$

iii. If a set A has 3 elements and the set B has 2 elements, then the product set $A \times B$ has _____ elements. **1**

(a) 6 (b) 2^6 (c) 2 (d) None of these

iv. If $A = \{x: x^2 + 4 = 0\}$, where x is a real number, then A is **1**

(a) Singleton set (b) Null set

(c) Universal set (d) None of these

v. Every identity relation is _______ relation but its converse is not true. **1**

(a) Transitive (b) Symmetric (c) Reflexive (d) None of these

vi. If $x, y \in R$ and $f = \{(x, y): y = 3x - 2\}; then\ f^{-1}$ is equal to **1**

(a) $\frac{(y+2)}{3}$ (b) $\frac{(y+3)}{2}$ (c) $(y + 2)$ (d) None of these

vii. $\lim_{x \to 0} (1 + x)^{1/x}$ is equal to **1**

(a) $\frac{1}{e}$ (b) e (c) $2e$ (d) None of these

viii. If $y = \sqrt{x}$ then $\frac{dy}{dx}$ is equal to **1**

(a) $2\sqrt{x}$ (b) $\frac{1}{\sqrt{x}}$ (c) $\frac{2}{\sqrt{x}}$ (d) None of these

P.T.O.

ix. The nullity of a square matrix of order n and rank r is **1**

(a) $n-r$ (b) $r-n$ (c) $n+r$ (d) None of these

x. The product of the eigen values of a square matrix A is equal to ____________ **1**

(a) Product of diagonal elements
(b) Sum of diagonal elements
(c) Determinant of a matrix
(d) None of these

Q.2 Attempt any two:

i. If $p\ and\ q$ are any two statements, then by using truth table prove that **5**

(a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$ (b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

ii. If $p \equiv It\ is\ a\ rainy\ season$, $q \equiv The\ mango\ is\ delicious$, construct the following statements: **5**

(a) $p \Rightarrow q$ (b) Converse of $p \Rightarrow q$
(c) Inverse of $p \Rightarrow q$ (d) Contrapositive of $p \Rightarrow q$
(e) Negation of $p \Rightarrow q$

iii. Define argument, valid argument and show that the argument $p, p \rightarrow q \vdash q$ is valid. **5**

Q.3 Attempt any two:

i. A class has 175 students. The following is the description showing the number of students studying one or more of the following subjects in this class: **5**

Mathematics 100; Physics 70; chemistry 46; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; and none of the three subjects 22. Find how many students study

(a) All the three subjects?
(b) Exactly one subject out of three?

ii. If A, B and C are any three sets then prove that **5**

(a) $A-(B \cap C)=(A-B) \cup (A-C)$
(b) $A \cap (B-C)=(A \cap B)-(A \cap C)$

iii. Define Power set, Intersection of two sets, complement of two sets, equal sets, Finite set with an example. **5**

Q.4 Attempt any two:

i. Define Equivalence Relation. Show that the relation $R=\{(a,b): a-b = even\ integer\ and\ a,b \in I\}$ in the set I of integers is an equivalence relation. **5**

ii. Define one-one mapping, onto mapping and many one mapping. Show that the mapping $f: R_+ \rightarrow R$ defined by $f(x)=\log x, x \in R_+$ is one-one onto where R_+ is the set of positive real numbers and R is the set of real numbers. **5**

iii. Prove that **5**

$(\sec A - cosec\ A)(1+\tan A+\cot A)=\tan A \sec A - \cot A\ cosec\ A$.

Q.5 Attempt any two:

i. Define limit of a function and evaluate $\lim_{x \to 2} \frac{x^2-4}{\sqrt{(3x-2)}-\sqrt{(x+2)}}$. **5**

ii. Differentiate the function $y=\frac{e^x+e^{-x}}{e^x-e^{-x}}$ with respect to x. **5**

iii. Define continuity of a function and show that the function $f(x)=3x^2+2x-1$ is continuous at $x=2$. **5**

Q.6 Attempt any two:

i. Find the rank and nullity of the matrix **5**

$$\begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

ii. Determine the eigen values and eigen vectors of the matrix **5**

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

iii. Test for consistency and solve **5**

$5x+3y+7z=4, 3x+26y+2z=9, 7x+2y+10z=5.$

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Medi-Caps University

Faculty of Science.

End sem (odd) Examination Dec-19.

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Scheme / Solution

Q.1 (i) (b) Tautology +1

(ii) (d) $\wedge$ +1

(iii) (a) 6 +1

(iv) (b) Null set +1

(v) (c) Reflexive +1

(vi) (a) $(y+2)/3$ +1

(vii) (b) e +1

(viii) (d) None of these. +1

(ix) (a) $n-r$ +1

(x) (c) Determinant of a matrix +1

Q.2 (i) (a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$p \vee q$	$\sim(p\vee q)$	$\sim p$	$\sim q$	$(\sim p)\wedge(\sim q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

+2.5

(b) $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	$p \wedge q$	$\sim(p\wedge q)$	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

+2.5

Q.2 (ii) Let $p \equiv$ It is a rainy season.
$q \equiv$ The mango is delicious.

(a) If it is a rainy season, then the mango is delicious +1

(b) If the mango is delicious, then it is a rainy season +1

(c) If it is not a rainy season, then the mango is not delicious. +1

(d) If the mango is not delicious, then it is not rainy season. +1

(e) It is rainy season and mango is not delicious. +1

Q.2 (iii)

Argument:- An argument is a process which yield a conclusion from a given set of propositions, called premises. +1

ie $p_1, p_2, \dots p_n \vdash q$

Valid Argument:- An argument $p_1, p_2, \dots p_n \vdash q$ is called valid if q is true whenever all its premises $p_1, p_2, \dots, p_n$ are true. +1

In order to prove validity we shall show that

$[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

+3

Q.3 (i) Let X denote the set of all Students in the class. Let M, P and C denote the set of students studying Mathematics, Physics and Chemistry respt. Then we have

$|X| = 175$, $|M| = 100$, $|P| = 70$, $|C| = 46$, +1
$|M \cap P| = 30$, $|M \cap C| = 28$, $|P \cap C| = 23$,
$|(M \cup P \cup C)'| = 22$

$\therefore |(M \cup P \cup C)| = |X| - |(M \cup P \cup C)'|$
$= \underline{153}$

(a) By the principle of inclusion and exclusion we have.

$|M \cap P \cap C| = |M \cup P \cup C| - |M| - |P| - |C| + |M \cap P|$
$+ |M \cap C| + |P \cap C|$ +1

$= 153 - 100 - 70 - 46 + 30 + 28 + 23$

$|M \cap P \cap C| = \underline{\underline{18}}$.

$\therefore$ no. of Students who study all the 3 subjects $= \underline{\underline{18}}$.

(b) Let M_1, P_1, C_1 be the set of students who study only Mathematics, Physics, Chemistry respt.

$\therefore$ No. of students who study only Mathematics
$= |M_1| = |M - P - C|$

$= |M| - |M \cap P| - |M \cap C| + |M \cap P \cap C|$ +1

$= 100 - 30 - 28 + 18 = \underline{60}$

$\therefore$ No. of students who study only Physics

$|P_1| = |P - M - C| = 70 - 30 - 23 + 18$ +1

$= \underline{35}$

$\therefore$ No. of students who study only Chemistry

$|C_1| = |C - M - P|$

$= 46 - 28 - 23 + 18 = \underline{13}$

$\therefore$ the no. of students who study exactly one subject out of three +1

$= |M_1| + |P_1| + |C_1|$

$= \underline{108}$ Ans

Q.3 (i) (1) Let $x \in A - (B \cap C)$

then

$x \in A - (B \cap C) \Rightarrow x \in A$ and $x \notin (B \cap C)$

$\Rightarrow x \in A$ and $(x \notin B$ or $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow x \in (A - B)$ or $x \in (A - C)$ +1

$\Rightarrow x \in (A - B) \cup (A - C)$

$A - (B \cap C) \subset (A - B) \cup (A - C)$ — (1)

Now, let $x \in (A - B) \cup (A - C)$, then

$x \in (A - B) \cup (A - C) \Rightarrow x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow x \in A$ and $(x \notin B$ or $x \notin C)$

$\Rightarrow x \in A$ and $x \notin (B \cap C)$ +1

$\Rightarrow x \in A - (B \cap C)$

$\therefore (A - B) \cup (A - C) \subset$ ~~xe~~ $A - (B \cap C)$ — (2)

from 1 & 2

$A-(B\cap C) = (A-B)\cup(A-C)$ +0.5

(ii) Let $x \in A-(B\cup C)$, then

$x \in A-(B\cup C) \Rightarrow x\in A$ and $x \notin (B\cup C)$

$\Rightarrow x \in A$ and ($x\notin B$ and $x \notin C$)

$\Rightarrow$ ($x\in A$ and $x\notin B$) and ($x\in A$ and $x\notin C$)

$\Rightarrow x\in(A-B)$ and $x\in(A-C)$

$\Rightarrow x\in(A-B)\cap(A-C)$ +1

$\therefore A-(B\cup C)\subseteq(A-B)\cap(A-C)$ —1

Let $x\in(A-B)\cap(A-C)$, then

$x\in(A-B)\cap(A-C)\Rightarrow x\in(A-B)$ and $x\in(A-C)$

$\Rightarrow$ ($x\in A$ and $x\notin B$) and ($x\in A$ and $x\notin C$)

$\Rightarrow x\in A$ and ($x\notin B$ and $x\notin C$)

$\Rightarrow x\in A$ and $x\notin(B\cup C)$

$\Rightarrow x\in A-(B\cup C)$ +1

$\therefore (A-B)\cap(A-C) \subseteq A-(B\cup C)$ —2

from 1 & 2

$A-(B\cup C) = (A-B)\cap(A-C)$ +.5

Q.3 (iii) Powerset:- Let A be a set. The set of all subsets of A is called the power set of A.

$P(A) = \{\text{set of all subsets of } A\}$ +1

eg If $A = \{a, b\}$, then

$P(A) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

Intersection of two sets: - Let A & B be two sets. The intersection of two sets A and B is the set of all those elements which are common to A & B both. +1

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

eg- If $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$.

$$A \cap B = \{2\}.$$

complement of sets: - Let A be any set and U be an universal set. The set U - A is called complement set of A and it is denoted by A' or A^c.

ie

$$A' = \{x : x \in U \text{ and } x \notin A\}$$ +1

If $U = \{1, 2, 3, \ldots, 10\}$, $A = \{2, 4, 6, 8, 10\}$

then $A^c = \{1, 3, 5, 7, 9\}$

Equal Sets: - Two sets A and B are equal if

(i) Every element of A is also an element of B

and (ii) Every element of B is also an element of A

ie $\boxed{A = B \Leftrightarrow A \subset B \text{ and } B \subset A}$ +1

If $A = \{1, 2, 3\}$, $B = \{1, 2, 3\}$

then $A = B$.

Finite set:- The set which contains finite no. of elements is called a finite set.

+1

eg- $A = \{a, e, i, o, u\}$.

Q. 4 (i) If R is a relⁿ in the set A, then R is called equivalence relⁿ if

(a) R is reflexive, $(a,a) \in R$; $\forall a \in A$. +1

(b) R is symmetric,
$(a,b) \in R \Rightarrow (b,a) \in R$, where $a, b \in A$

(c) R is transitive +1
$(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow (a,c) \in R$, where $a, b, c \in A$

$R = \{(a,b) : a-b = \text{even integer and } a, b \in I\}$.

(i) R is reflexive
Let $a \in I$, then $a - a = 0$, which is an even integer +1
$\therefore (a,a) \in R$, $\forall a \in I$
Hence, R is reflexive.

(ii) R is symmetric
Let $a, b \in I$ If $(a-b)$ is an even integer then $(b-a)$ is also an even integer
$\therefore (a,b) \in R \Rightarrow (b,a) \in R$ is true. +1
Hence, R is symmetric relⁿ.

(iii) R is transitive Let $a, b, c \in I$.
If $(a-b)$ and $(b-c)$ are even integers then, $(a-c) = (a-b) + (b-c) = \text{even integer}$ +1
$\therefore (a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ is true
Hence, R is transitive.

∵ the relⁿ R is reflexive, transitive and symmetric

∴ R is an equivalence relⁿ.

Q.4 (ii) One One Mapping

A mapping $f: A \to B$ is called one-one if different elements in A have different f-images in B i.e. +1

if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

Onto mapping

If a mapping $f: A \to B$ is such that the range of f is equal to the set B, i.e. $f(A) = B$, then f is called onto. +1

Many One mapping:-

A mapping $f: A \to B$ is called many one mapping if two or more than two different elements in A have the same f-images in B i.e. +1

$$x_1 \neq x_2 \Rightarrow f(x_1) = f(x_2).$$

A mapping $f: R_+ \to R$ defined by

$$f(x) = \log x, \; x \in R_+$$

One-One

Let $x_1, x_2 \in R_+$, then $\log x_1$ & $\log x_2$ exist.

$f(x_1) = f(x_2) \Rightarrow \log x_1 = \log x_2$ +1

$\Rightarrow x_1 = x_2$

∴ f is one-one

Onto

Let $y \in R$ be any arbitrary real no.

so that $y = f(x) \Rightarrow y = \log x$

$\Rightarrow x = e^y$

$\because$ we know that every value of y is, e^y is always (+) ve. Hence,

$e^y \in R_+$

and $f(e^y) = \log e^y$ +1

$= y$

Hence $\forall y \in R$ its pre-image $e^y \in R_+$

$\therefore f$ is onto.

Q. 4 (iii) on taking L.H.S :

$(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A)$

$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$ +1

$= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right)\left(\frac{\sin A \cos A + \sin^2 A + \cos^2 A}{\sin A \cos A}\right)$

$= \frac{(\sin A - \cos A)(\cos A \sin A + 1)}{\sin^2 A \cos^2 A}$ $[\because \sin^2\theta + \cos^2\theta = 1]$ +1

$= \frac{\sin A - \cos A + \sin^2 A \cos A - \sin A \cos^2 A}{\sin^2 A \cos^2 A}$ +1

$= \frac{-\cos A[1 - \sin^2 A] - \sin A \cdot [-1 + \cos^2 A]}{\sin^2 A \cos^2 A}$

+1

$= \frac{-\cos A \cdot \cos^2 A}{\sin^2 A \cos^2 A} + \frac{\sin^2 A \cdot \sin A}{\sin^2 A \cos^2 A}$

$= \tan A \sec A - \cot A \operatorname{cosec} A$ +1

$=$ R.H.S

Q.5 (i) Limit of a funt^n :- A funt^n $f(x)$ is said to tend to a limit l as x tends to a if the numerical difference bet^n $f(x)$ and l can be made as small as we please provided the numerical difference bet^n. x & a is taken sufficiently small. +1

ie $\lim_{x \to a} f(x) = l$.

$$\lim_{x \to 2} \frac{x^2-4}{\sqrt{3x-2}-\sqrt{x+2}}$$

$$= \lim_{x \to 2} \frac{(x^2-4)\left[\sqrt{3x-2}+\sqrt{x+2}\right]}{(\sqrt{3x-2}-\sqrt{x+2})(\sqrt{3x-2}+\sqrt{x+2})} \quad +1$$

$$= \lim_{x \to 2} \frac{(x^2-4)\left[\sqrt{3x-2}+\sqrt{x+2}\right]}{(3x-2)-(x+2)} \quad +1$$

$$= \lim_{x \to 2} \frac{(x-2)(x+2)\left[\sqrt{3x-2}+\sqrt{x+2}\right]}{2x-4}$$

$$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)\left[\sqrt{3x-2}+\sqrt{x+2}\right]}{2\cancel{(x-2)}} \quad +1$$

$$= \frac{4}{2}\left[\sqrt{4}+\sqrt{4}\right]$$

$$= 2\left[2+2\right] \quad +1$$

$$= 8 \text{ Ans}$$

Q. 5 (ii) $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$.

differentiate w.r.t. x, we get

$$\frac{dy}{dx} = \frac{(e^x - e^{-x})\frac{d}{dx}(e^x + e^{-x}) - (e^x + e^{-x})\frac{d}{dx}(e^x - e^{-x})}{(e^x - e^{-x})^2}$$ +1

$$\frac{dy}{dx} = \frac{(e^x - e^{-x})\cdot(e^x - e^{-x}) - (e^x + e^{-x})(e^x + e^{-x})}{(e^x - e^{-x})^2}$$ +1

$$\frac{dy}{dx} = \frac{(e^{2x} - e^0 - e^0 + e^{-2x}) - (e^{2x} + e^0 + e^0 + e^{-2x})}{(e^x - e^{-x})^2}$$

$$= \frac{(e^{2x} - 1 - 1 + e^{-2x}) - (e^{2x} + 1 + 1 + e^{-2x})}{(e^x - e^{-x})^2}$$ +1

$$= \frac{e^{2x} - 2 + e^{-2x} - e^{2x} - 2 - e^{-2x}}{(e^x - e^{-x})^2}$$ +1

$$\frac{dy}{dx} = \frac{-4}{(e^x - e^{-x})^2}$$ +1

Ans

Q. 5 (iii) Continuity of a fut^n: - fut^n $f(x)$ is continuous at $x = a$ if the R.H.L exists, L.H.L exist and both are equal to the value of the fut^n $f(x)$ at $x = a$. +1

i.e. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

$f(x) = 3x^2 + 2x - 1$.

(i) value of fut^n at $x = 2$

$f(2) = \underline{\underline{15}}$ +1

(ii) R.H.L

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (3x^2 + 2x - 1)$

$\lim_{h \to 0} f(2+h) = \lim_{h \to 0} [3(2+h)^2 + 2(2+h) - 1]$

$= \lim_{h \to 0} [3h^2 + 14h + 15]$ +1

$= \underline{\underline{15}}$

(iii) L.H.L

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x^2 + 2x - 1)$

$\lim_{h \to 0} f(2-h) = \lim_{h \to 0} [3(2-h)^2 + 2(2-h) - 1]$

$= \lim_{h \to 0} [3h^2 - 14h + 15]$ +1

$= \underline{\underline{15}}$

$\because f(2) = L.H.L = R.H.L$ +1

$\therefore$ given fut^n is continuous at $x = 2$

Q.6 (i) $A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

On applying the transformations

$R_1 \leftrightarrow R_3$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$A \sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$ +1

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$, $C_4 \to C_4 - 2C_1$

$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$ +1

On applying

$R_2 \leftrightarrow R_3$, $R_2 \to -R_2$, $R_3 \to \frac{R_3}{-2}$

$R_3 \to R_3 - 3R_2$

$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 10 \\ 0 & 0 & -14 & -28 \end{bmatrix}$ +1

on applying

$C_3 \to C_3 - 3C_2$, $C_4 \to C_4 - 10C_2$

$C_3 \to \frac{C_3}{-14}$, $C_4 \to \frac{C_4}{-28}$,

$C_4 \to C_4 - 2C_3$ +1

$A \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$, rank of $A = 3$ Ans

$= [I_3 \,|\, 0]$ +1

Q.6 (ii) The characterist eqn is given by

$$|A-\lambda I|=0 \quad +1$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0.$$

$$\lambda = 2, 2, 8 \quad +1$$

Eigenvectors $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\cancel{2x - y + z = 0}$ $\quad +0.5$

$\cancel{-2x + y - z = 0}$

$R_1 \to R_1 + 2R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x + y - z = 0.$$

$$\Rightarrow x = \frac{z-y}{-2} \Rightarrow \boxed{x = \frac{y-z}{2}}$$

Let $y = K_1$, $z = K_2$

$$x = \frac{K_1 - K_2}{2}$$

eigenvector $X = \begin{bmatrix} \frac{K_1 - K_2}{2} \\ K_1 \\ K_2 \end{bmatrix}$

$$= \begin{bmatrix} K_1/2 \\ K_1 \\ 0 \end{bmatrix} + \begin{bmatrix} -K_2/2 \\ 0 \\ K_2 \end{bmatrix}$$ +1

$$= K_1 \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + K_2 \begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$$

when $\lambda = 8$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

+0.5

$$-2x - 2y + 2z = 0$$
$$-2x - 5y - z = 0$$

$$\frac{x}{2} = \frac{y}{-1} = \frac{z}{1} = K$$

+1

eigenvector $X = K \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Q.6 (iii) The matrix form is $AX = B$.
where

$$A = \begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$ +1

$$[A:B] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_2 \to 5R_2 - 3R_1$, $R_3 \to 5R_3 - 7R_1$

$$[A:B] \sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$$ +1

$R_2 \to \frac{1}{11} R_2$

$$[A:B] \sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{array}\right]$$ +1

$R_3 \to R_3 + R_2$

$$[A:B] \sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\because \rho[A:B] = \rho(A) = 2 <$ no. of unknowns +1
$\therefore$ System is consistent & have infine sol^ns

$$x = \frac{7-16K}{11}, \quad y = \frac{3+K}{11}, \quad z = K$$ +1