Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2017

BC3CO03 Mathematics-I

Programme: B.Sc.(CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. How many real roots are there for the equation $2x^7 - x^5 + 4x^3 - 5 = 0$ **1**

(a) 2 (b) 3 (c) 5 (d) None of these

ii. A polynomial equation in x of nth degree always have **1**

(a) n real root (b) n roots (c) n-1 roots (d) n Complex roots

iii. If λ is an eigen value of Matrix A then eigen value of A^{-1} is **1**

(a) λ (b) $\frac{1}{\lambda}$ (c) λ^3 (d) None of these

iv. If the rank of augmented matrix [A:B]=Rank[A]= n the number of unknowns than the system of equation AX=B has **1**

(a) No solution (b) Unique solution

(c) Infinite solutions (d) None of these

v. The function $f(x) = \dfrac{x^3 - 8}{(x-2)(x+2)}$ is discontinuous for **1**

(a) $x = -2$ and $x = 2$ (b) $x = 3$

(c) $x = -2$ (d) None of these

vi. If $y = cos(ax+b)$ then the n^{th} derivative of y is equal to **1**

(a) $a^n \cos\left(ax+b+n\dfrac{\pi}{2}\right)$ (b) $a^n \sin\left(ax+b+n\dfrac{\pi}{2}\right)$

(c) $a^n \sin(ax+b)$ (d) None of these

vii. The radius of curvature is ρ , then curvature is **1**

(a) ρ (b) $\frac{1}{\rho}$ (c) ρ^2 (d) ρ^3

viii. The equation of tangent at point (3,1) on ellipse is **1**

(a) $4x+3y=15$ (b) $4x-3y=15$

(c) $3x-4y=15$ (d) $x-4y=15$

ix. The value of $\Gamma\left(\frac{1}{2}\right)$ is given by **1**

(a) $\sqrt{\pi}$ (b) $\frac{3}{4}\sqrt{\pi}$ (c) 1 (d) $\frac{1}{2}\sqrt{\pi}$

x. $\lim_{n\to\infty}\left\{\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}\right\}$ is equals to **1**

(a) $\log_e 2$ (b) $\log_e 4$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Q.2 i. Find the equation whose roots are the cubes of the roots of $x^3-3x^2+1=0$. **4**

ii. Solve the equation $2x^4-15x^3+35x^2-30x+8=0$, given that the roots are in geometric Progression. **6**

OR iii. Solve by Cardan's Method $x^3-6x^2+6x-5=0$. **6**

Q.3 i. Find the Eigen values of $A=\begin{bmatrix}2&1&1\\1&2&1\\0&0&1\end{bmatrix}$. **4**

ii. Discuss the consistency of following system of equation, If found consistent then Solve it. **6**

$2x+3y+4z=11, x+5y+7z=15, 3x+11y+13z=25$

OR iii. State and prove Cayley Hamilton Theorem. **6**

Q.4 i. Verify Lagrange mean value theorem for the function $f(x)=x^3-2x^2-x+3$ in $[0,1]$. **4**

ii. Discuss the continuity of the function $f(x)$ at $x=1$ and $x=2$ **6**

$$f(x)=\begin{cases}1+x, & -1<x\le 1\\ x, & 1<x<2\\ 4-x, & 2\le x<5\end{cases}$$

OR iii. Write statement of Roll's Theorem and Verify it for the function $f(x)=x^3$-$9x^2$+$26x$-24 in $[2,4]$. **6**

Q.5 i. Expand $y=e^x\cos x$, using Maclaurin's series. **4**

ii. If $y=e^{\tan^{-1}x}$ then using Leibnitz theorem, prove that **6**

$(1+x^2)y_{n+1}+(2nx-1)y_n+n(n-1)y_{n-1}=0$. .

OR iii. Find the radius of curvature of cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ **6**

Q.6 i. Show that $B(m,n)=B(m+1,n)+B(m,n+1)$ **4**

ii. Evaluate: **6**

$$\lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2}+\frac{2}{n^2}\sec^2\frac{4}{n^2}+\frac{3}{n^2}\sec^2\frac{9}{n^2}\text{..........}+\frac{1}{n}\sec^2 1\right]$$

OR iii. Evaluate the integral $\int_0^\infty e^{-a^2x^2}dx$ using Gamma function. **6**

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Faculty of Science — Answers/Solutions. (1)

End Sem Exam. Dec-2017

Q1 MCQ — BC3CO03 - Mathematics-I — BSc-CS

M. Marks - 60.

i. How many real roots are there for the equation $2x^7 - x^5 + 4x^3 - 5 = 0$ — 1

$\boxed{\text{Answer c. } 5}$

ii. A polynomial equation in x of nth degree always have — 1

Answer b. n ~~real~~ root n roots

iii. If λ is an eigen value of Matrix A then eigen value of A^{-1} is — 1

Answer $\boxed{\text{b. } \frac{1}{\lambda}}$

iv. If the rank of augmented matrix [A:B]=Rank[A]= n the number of unknowns than the system of equation AX=B has — 1

Answer $\boxed{\text{b) Unique solution}}$

v. The function $f(x) = \dfrac{x^3 - 8}{(x-2)(x+2)}$ is discontinuous for — 1

Answer $\boxed{\text{c) } x = -2}$

vi. If $y = cos(ax+b)$ then the n^{th} derivative of y is equal to . — 1

Answer $\boxed{\text{a) } a^n \cos\left(ax + b + n\frac{\pi}{2}\right)}$

vii. The radius of curvature is ρ , then curvature is — 1

Answer $\boxed{\text{b) } \frac{1}{\rho}}$

viii. The equation of tangent at point (3,1) on ellipse is — 1

Answer $\boxed{\text{a) } 4x + 3y = 15}$

ix. The value of $\Gamma\left(\frac{1}{2}\right)$ is given by — 1

Answer a) $\boxed{\sqrt{\pi}}$

x. $\lim_{n\to\infty}\left\{\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}\right\}$ is equals to — 1

Answer $\boxed{\text{a) } \log_e 2}$

Q2. (i) $x^3 - 3x^2 + 1 = 0$ — ① Let y be root of required eqn. and $y = x^3$ — ②. 1

① $\Rightarrow$ $[x^3+1]^3 = [3x^2]^3$

$x^9 + 3x^6 + 3x^3 + 1 = 27x^6$ 1

put $x^3 = y$

$y^3 - 24y^2 + 3y + 1 = 0$. 2

(ii) $2x^4 - 15x^3 + 35x^2 - 30x + 8 = 0$

Let roots are a/r^3, a/r, ar, ar^3

product of root is $a^4 = 4$ 2

Also the products of $a/r^3, ar^3, a/r, ar$ are each $a^2 = 2$

$\therefore$ The factors corresponding to $a/r^3, ar^3$, $a/r, ar$

are $(x^2 + px + 2)$ $(x^2 + qx + 2)$

Thus.

$(2x^4 - 15x^3 + 35x^2 - 30x + 8) = 2(x^2 + px + 2)(x^2 + qx + 2)$ 2

on comparing

$p = -9/2$ $q = -3$.

Thus given eqn is

$2(x^2 - \frac{9}{2}x + 2)(x^2 - 3x + 2) = 0$ 2.

hence, roots are $1, 2, 4, \frac{1}{2}$

(3)

or (iii) Cardon's method

$$x^3 - 6x^2 + 6x - 5 = 0$$

$y = x + \left(\frac{-6}{3}\right)$ $\quad y = x - 2$

put $\boxed{x = y+2}$ 1

$$(y+2)^3 - 6(y+2)^2 + 6(y+2) - 5 = 0$$

$$y^3 - 6y - 9 = 0 \quad \text{——— (1)}$$

$$y = u + v$$

$$y^3 = u^3 + v^3 + 3uv(u+v)$$

$$y^3 - 3uv(u+v) - (u^3+v^3) = 0$$ $2\frac{1}{2}$

$$y^3 - 3uv\,y - (u^3+v^3) \quad \text{——— (2)}$$

(1), (2) (comparing)

$3uv = 6$ $\quad \underline{u^3+v^3 = 9.}$

$uv = 2 \Rightarrow \underline{u^3v^3 = 8}$ $1\frac{1}{2}$

eqn having roots u^3, v^3 is

$$t^2 - (u^3+v^3)t + (u^3v^3) = 0$$

$$t^2 - 9t + 8 = 0$$

$$(t-1)(t-8) = 0$$

$t = 1, 8$ $\quad u^3 = 1 \quad v^3 = 8$

$u = 1, \omega, \omega^2$ $\quad v^3 - 8 = 0$

$v^3 - 2^3 = 0$

$(v-2)(v^2+2v+4) = 0$

$v = 2, -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$

$y = u + v$

$y = 3, \quad y = 2\omega \quad y = 2\omega^2$ 1

$x = 3+2 = 5, \quad 2+2\omega, \quad 2+2\omega^2$

Q3 (i) (4)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

$|A - \lambda I| = 0$. 1

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)\left[(2-\lambda)^2 - 1\right] = 0$

$(1-\lambda)(\lambda^2 - 4\lambda + 3) = 0$ 2

$(1-\lambda)(\lambda-3)(\lambda-1) = 0$ 1

$\lambda = 1, 1, 3$

ii) augmented matrix

$$\begin{bmatrix} 2 & 3 & 4 & 11 \\ 1 & 5 & \cancel{9}\ 7 & 15 \\ 3 & 11 & 13 & 25 \end{bmatrix}$$ 1

using only row transformation

$$\sim \begin{bmatrix} 1 & 5 & 7 & 15 \\ 0 & 1 & 10/7 & 19/7 \\ 0 & 0 & 4/7 & 16/7 \end{bmatrix}.$$ 2

Rank $[A] = R[A:B] = 3$. 1

Unique soln.

$$x + 5y + 7z = 15$$

$$y + \frac{10z}{7} = \frac{19}{7}$$

$$\frac{4z}{7} = \frac{16}{7} \Rightarrow z = 4.$$

By (2)

$$y = -3$$

$$x = 2 \qquad 2, -3, 4.$$ 2

(iii) Cayly Hamilton Th.

Statment
Every Square Matrix Satisfi its own.
Characteristic eqn.

If $|A - \lambda I| = (-1)^n (\lambda^n + a_1 \lambda^{n-1} + a_2 \lambda^{n-2} + \dots + a_n)$
be characteristic polynomial of matrix $(a_{ij})_{n \times n}$
Then the eqn

$$A^n + a_1 A^{n-1} + a_2 A^{n-2} + \dots + a_n I = 0$$ 2

P Since the elements of $(A - \lambda I)$ are at most the first degree in λ, the element of adjoint $(A - \lambda I)$ are at most $(n-1)$ degree in λ $\therefore$

$$\operatorname{adj}(A - \lambda I) = B_0 \lambda^{n-1} + B_1 \lambda^{n-2} + \dots + B_{n-1}$$

here $B_0, B_1, \dots B_{n-1}$ are $n \times n$ matrices
there elements being polynomial in λ'.

(6) 2

we know $(A-\lambda I)\,\mathrm{adj}(A-\lambda I) = |A-\lambda I|\, I$

$(A-\lambda I)(B_0\lambda^{n-1}+B_1\lambda^{n-2}+\dots+B_{n-1}) = (-1)^n(\lambda^n+a_1\lambda^{n-1}+\dots+a_n)I$

equating coefficient of like powers of λ:

$$\left.\begin{aligned} -IB_0 &= (-1)^n I \\ AB_0 - B_1 I &= (-1)^n a_1 I \\ AB_1 - B_2 I &= (-1)^n a_2 I \\ &\vdots \\ AB_{n-1} &= (-1)^n a_n I \end{aligned}\right\}$$

on multiplying the equation by A^n, $A^{n-1}, A^{n-2}, \dots I$ respecively and adding we obtain

$0 = (-1)^n\left[A^n + a_1A^{n-1}+\dots + a_n I\right]$ 2

HP

LMV Th.

Q4(i) ① $f(0) = 3$ 1

$f(1) = 2$ $f(1) = 1-2-1+3 = 1$

$f(0) \neq f(1)$

(ii) ② $f(x)$ is polynomial in x so cont in $[0,1]$ 1

③ $f'(x) = 3x^2-4x-1$ $3c^2-4c-1$ 1½

by LMV. T. $\exists\, c \in (0,1)$

st $f'(c) = 0$ $\frac{f(b)-f(a)}{b-a}$.

$3c^2 - 4c - 1 = \frac{\cancel{4}-3}{1-0} = -\cancel{1}$ $\left\{ f'(c) = \frac{f(b)-f(a)}{b-a} \right\}$ 1

$3c^2 - 4c \cancel{+1} = 0$ $3(c-1)(c-1) = 0$

$\Rightarrow c = \frac{\cancel{4}}{3} \in (0,1)$ $c = \frac{1}{3}$

①, ②, ③ verified.

(ii) L.H.L at $x=1$

$= \lim_{x \to 1^-} f(x) = 2$

$RHL = \lim_{x \to 1^+} f(x) = 1$ 1

& $f(1) = 2$

$LHL \neq RHL$

at $x=1$ not continuous. 2

Now at $x=2$

$LH.L = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = 2.$ 1

$RH.L \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)$ 1

$= \lim_{h \to 0} 4-(2+h) = 2$

& $f(2) = 4-2 = 2$ 1

$f(2) = LHL = RHL \Rightarrow x=2$ continuous

OR Q4.

(iii) Rolle Th. :- $f(x)$ be a real valued function of x 8

s.t (i) $f(a) = f(b)$

(ii) $f(x)$ is a conti funct^n in $[a, b]$

(iii) $f(x)$ is diff in (a, b) 2

$\exists$ at least one real value of $c \in (a, b)$ s.t

$f'(c) = 0$

$f(x) = x^3 - 9x^2 + 26x - 24$ in $[2, 4]$

$f(2) = 0$

$f(4) = 0$

(i) $f(2) = f(4)$

(ii) Since $f(x)$ is a polynomial function in x. $\Rightarrow f(x)$ 1

is continuous in $[2, 4]$ 1

(iii) Since $f(x)$ is polynomial in x, it can be differentiate

s.t $f'(x) = 3x^2 - 18x + 26$

By Rolle's th. $\exists$ $c \in (2, 4)$ s.t. $f'(c) = 0$ $1\frac{1}{2}$

$3c^2 - 18c + 26 = 0 \Rightarrow c = 3 \pm \frac{1}{\sqrt{3}}$

$c = 3.5773, 2.4226$

Hence verified Rolle th

Q5. (i)

$y = e^x \cos x$

$y_0 = 1$ $(y_1)_0 = 1$ $(y_2)_0 = 0$

$(y_3)_0 = -2$ $(y_4)_0 = -4$ 3

$(y_5)_0 = -4$

Maclar series

$$y = y_0 + x(y_1)_0 + \frac{x^2}{\lfloor 2}(y_2)_0 + \frac{x^3}{\lfloor 3}(y_3)_0 + \frac{x^4}{\lfloor 4}(y_4)_0 + \cdots$$

$$\Rightarrow \quad y = 1 + x + \frac{x^2}{2} \times 0 + \frac{x^3}{\lfloor 3}(-2) + \frac{x^4}{\lfloor 4}(-4) + \frac{x^5}{\lfloor 5}(-4)$$

$$e^x \cos x = 1 + x - \frac{2x^3}{\lfloor 3} - \frac{2^2 x^4}{\lfloor 4} - \frac{2^2 x^5}{\lfloor 5} \cdots$$ 1

(ii)

$$y = e^{\tan^{-1}x}$$

diff wrt x, $y_1 = e^{\tan^{-1}x} \frac{1}{(1+x^2)}$ 1

$$(1+x^2) y_1 = y$$ 1

diff n times by leibnitz th.

$$(1+x^2) y_{n+1} + n y_n (2x) + \frac{n(n-1)}{2} y_{n-1} \cdot 2 = y_n$$ 1

$$\vdots$$

$$(1+x^2) y_{n+1} + (2nx - 1) y_n + n(n-1) y_{n-1} = 0$$ 2

use

Lebinitz rule

$$\frac{d^n}{dx^n}(u \cdot v) = {}^nC_0 u_n \cdot v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2$$ 1

$$+ \cdots {}^nC_2 u_{n-2} \cdot v_2 + \cdots + {}^nC_n u \cdot v_n.$$

here $u_n \rightarrow$ n^{th} differentiation.

i)

$$x = a(\theta + \sin\theta), \quad y = a(1 - \cos\theta).$$

given eqn of cycloid $\dot{x} = \frac{dx}{d\theta}$

$$\dot{x} = a(1 + \cos\theta) \quad \dot{y} = a \sin\theta \quad \dot{y} = \frac{dy}{d\theta}$$ 1

(10)

Two ways to solve

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad (1) \quad \text{OR.} \quad \rho = \left(\frac{(\dot{x}^2+\dot{y}^2)^{3/2}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}\right)$$

$$\ddot{y} = \frac{d^2y}{d\theta^2}, \quad \ddot{x} = \frac{d^2x}{d\theta^2}$$

OR

$$\frac{dy}{dx} = \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \tan\theta/2$$

$$\frac{d^2y}{dx^2} = \frac{1}{4a}\sec^4\theta/2 \text{ as. } \left\{\frac{d}{dx}(\tan\theta/2)\right\} = \left\{\frac{d}{d\theta}(\tan\theta/2)\frac{d\theta}{dx}\right\}.$$ 2

Using (1)
$$\rho = \frac{(1+\tan^2\theta/2)^{3/2}}{\frac{1}{4a}\sec^4\theta/2}$$ 1

$$\rho = 4a\cos\theta/2$$

5(i) Prove

$$B(m,n) = \frac{\Gamma m+1\,\Gamma n}{\Gamma m+1+n} + \frac{\Gamma m\,\Gamma n+1}{\Gamma m+1+n}$$ 2

R.H.S.

$$= \frac{m\Gamma m\,\Gamma n}{\Gamma m+n+1} + \frac{\Gamma m\,\Gamma n\; n}{\Gamma m+1+n}$$

$$\Gamma m+1+n = m+n\,\Gamma m+n$$

$$= \frac{\Gamma m\,\Gamma n}{\Gamma m+n}\,\frac{\cancel{(m+n)}}{\cancel{(m+n)}} = B(m,n) = \underline{LHS}$$ 2

(ii) Let
$$S = \lim_{n\to\infty}\left[\frac{1}{n^2}\sec^2\frac{1}{n^2} + \frac{2}{n^2}\sec^2\frac{4}{n^2} + \frac{3}{n^2}\sec^2\frac{9}{n^2} + \dots \frac{1}{n}\sec^2 1\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{r}{n^2}\sec^2\left(\frac{r}{n}\right)^2$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r}{n}\right)\sec^2\left(\frac{r}{n}\right)^2\cdot\frac{1}{n}.$$ 2

by summation of Series

$$\frac{r}{n} \to x$$
$$\frac{1}{n} \to dx$$
$$\Sigma \to \int$$
$$\text{L.L. } \lim_{n\to\infty} \left(\frac{r}{n}\right)_{r=1} = 0$$
$$\text{U.L. } \lim_{n\to\infty} \left(\frac{r}{n}\right)_{r=n} = 1$$ 2

$$S = \int_0^1 x \sec^2 x^2 \, dx$$
$$x^2 = t, \quad 2x\,dx = dt$$
$$S = \frac{1}{2}\int_0^1 \sec^2 t \, dt = \frac{1}{2}\tan 1.$$ 2

ii) Let $I = \int_0^\infty e^{-a^2x^2} \, dx.$

$$a^2x^2 = y$$
$$x = \frac{\sqrt{y}}{a}$$
$$dx = \frac{1}{2a\sqrt{y}} dy$$ $1\frac{1}{2}$

$$I = \int_0^\infty e^{-y} \frac{1}{2a\sqrt{y}} dy$$
$$= \frac{1}{2a}\int_0^\infty e^{-y} y^{-1/2} dy$$ 2
$$= \frac{1}{2a}\int_0^\infty e^{-y} y^{\frac{1}{2}-1} dy$$

using

$$\frac{1}{2a}\Gamma\frac{1}{2} \qquad \int_0^\infty e^{-x} x^{n-1} dx = \Gamma n \quad n>0$$ $2\frac{1}{2}$

$$= \frac{\sqrt{\pi}}{2a}.$$

END