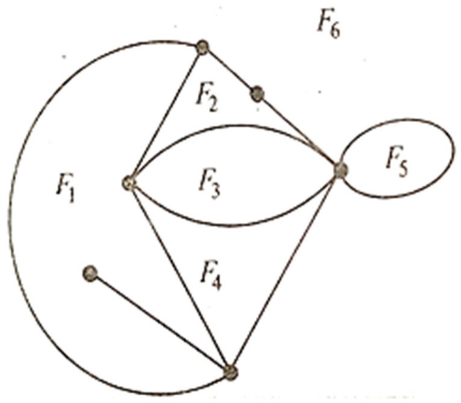
Q.5 Attempt any two:

i. Define the following with example: **5**
(a) Planar graph (b) Orthogonal space (c) Circuit subspace

ii. Draw the geometric dual of following planar graph and write observations we get from planar graph and its dual. **5**



iii. Prove that- The ring sum of two circuits in a graph is either a circuit or an edge disjoint union of circuits. **5**

Q.6 Attempt any two:

i. Let a and b be two nonadjacent vertices in a graph G. Let G' be a graph obtained by adding an edge between a and b. Let G'' be a simple graph obtained from G by fusing the vertices a and b together and replacing sets of parallel edges with single edges. Then prove that- **5**

$$P_n(\lambda) \text{ of } G = P_n(\lambda) \text{ of } G' + P_{n-1}(\lambda) \text{ of } G''.$$

ii. Define the following with example: **5**
(a) Complete bipartite graph.
(b) Covering of a graph with two observations.

iii. Prove that every tree with two or more vertices is 2-chromatic. **5**

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Enrollment No.....

Faculty of Science

End Sem (Odd) Examination Dec-2022

BC3EM01 Graph Theory

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If some closed walk in a graph contains all the edges of the graph, then the graph is called- **1**
(a) Euler graph (b) Regular graph
(c) Simple graph (d) None of these

ii. The number of vertices of odd degree in any graph is- **1**
(a) May be even may be odd (b) Always even
(c) Always odd (d) None of these

iii. What is the dimensions of circuit matrix? **1**
(a) Number of edges × number of edges
(b) Number of edges × number of vertices
(c) Number of vertices × number of vertices
(d) None of these

iv. The rank of incidence matrix of a connected graph with n-vertices is- **1**
(a) n (b) $n-1$ (c) n^2 (d) None of these

v. A graph with n vertices and has n-1 edges is called- **1**
(a) Complete graph (b) Tree
(c) Bipartite graph (d) None of these

vi. To apply Prim's algorithm, the given graph must be- **1**
(a) Weighted (b) Disconnected
(c) Directed (d) None of these

vii. Every cut set in a non-separable graph with more than two vertices contains _____. **1**
(a) At least two edges (b) At most two edges
(c) Exactly two edges (d) None of these

P.T.O.

viii. There will be total _____ sub graph of G which can be represented by unique linear combination of five basis vector. **1**
(a) 25 (b) 16 (c) 32 (d) None of these

ix. If G is a null graph, then chromatic number of G i.e. $\chi(G)$ =_____. **1**
(a) 2 (b) 1 (c) 3 (d) None of these

x. What is the number of perfect matching in a complete graph K_6? **1**
(a) 15 (b) 12 (c) 10 (d) None of these

Q.2 Attempt any two:

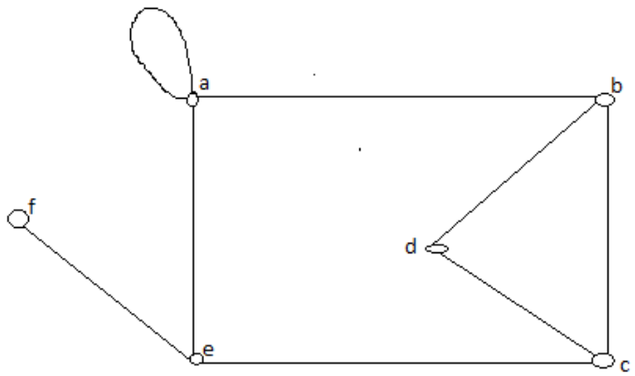
i. Define the following with example: **5**
(a) Path (b) Regular graph
(c) Hamiltonian graph (c) Isomorphic graph
(e) Spanning sub graph

ii. Prove that the sum of the degree of all vertices in a graph is twice the number of edges. **5**

iii. Prove that the maximum number of edges in a simple connected graph with n vertices is $\frac{n(n-1)}{2}$. **5**

Q.3 Attempt any two:

i. Write the adjacency matrix of the following graph. Also write any three observations you get from adjacency matrix representation of graph. **5**



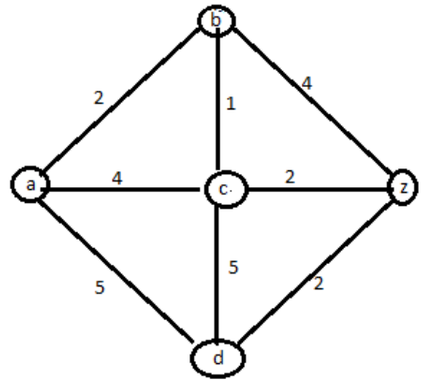
ii. Define the following with example: **5**
(a) Cut set matrix (c) Path matrix

iii. (a) Define fundamental circuit matrix with example. **5**
(b) Draw a graph for the following incidence matrix:

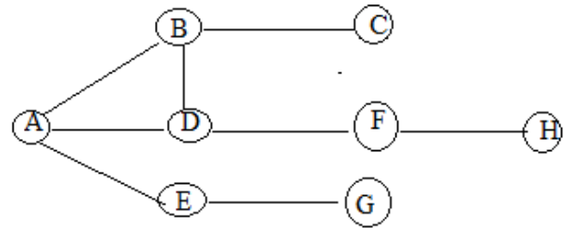
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Q.4 Attempt any two:

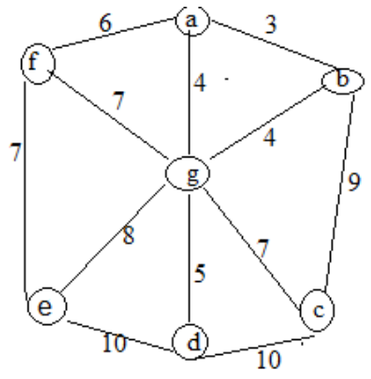
i. Using Dijkstra's algorithm, find the shortest path from a to z of the following weighted graph **5**



ii. Traverse the graph using Breadth First search algorithm starting from vertex A **5**



iii. Find minimal spanning tree of the following graph using Kruskal's algorithm **5**



P.T.O.

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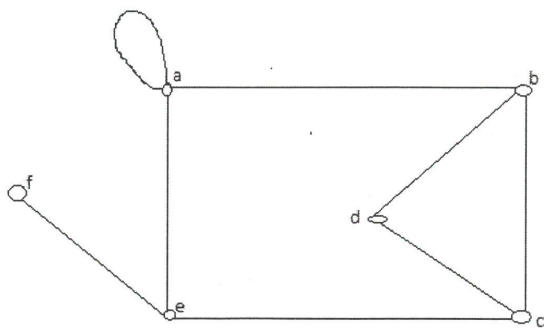

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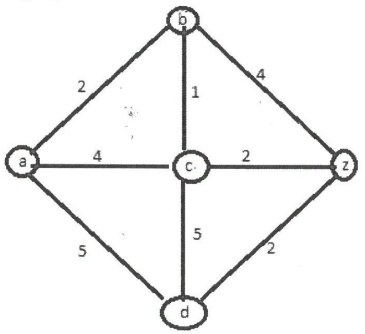

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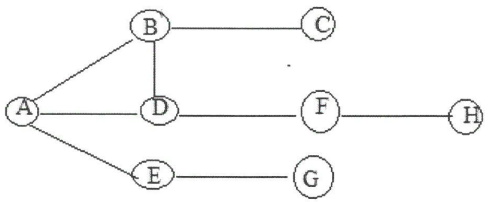

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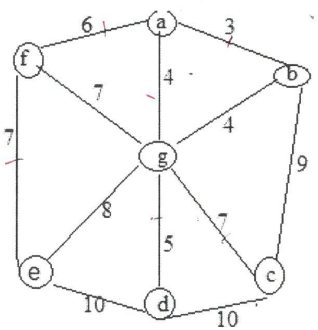

Two subspaces are said to be orthogonal to each other [4] if every vector in one is orthogonal to every vector in the other.

Q.5 Attempt any two:

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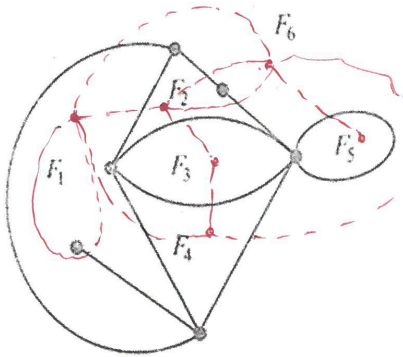
The set of all circuit vectors in W_6 forms a subspace W_r.



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In a graph G the set of edges is said to cover G if every vertex in G is incident on atleast one edge in.

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(a) Complete bipartite graph.

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A set of edges that covers G is said to be edge covering

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Total No. of Questions: 6 1 | 4 *Total No. of Printed Pages:4*

Enrollment No.....

Faculty of Science

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BC3EM01 Graph Theory

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