Enrollment No......................................

Faculty of Agriculture

End Sem (Odd) Examination Dec-2018

AG3EM01 Elementary Mathematics

Programme: B.Sc. (Ag.) Branch/Specialisation: Agriculture

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. Equation of X axis is **1**

(a) y=1 (b) y = 0 (c) x= 2 (d) x=a

ii. The distance between two points (11,12) and (14,15) is **1**

(a) $3\sqrt{2}$ (b) $5\sqrt{2}$ (c) $7\sqrt{2}$ (d) $9\sqrt{2}$

iii. The equation of a circle whose centre is origin and radius is 6 **1**

(a) $x^2 + y^2 = 6$ (b) $x^2 + y^2 = 36$

(c) $x^2 + y^2 = -6$ (d) None of these

iv. If the general equation of a circle is $x^2 + y^2 + 2gx + 2hy + c = 0$ then its centre is **1**

(a) (g, h) (b) (-g, -h) (c) (-g, h) (d) (g, -h)

v. $\lim_{x\to 0} \frac{\sin x}{x} =$ **1**

(a) 0 (b) 1 (c) 2 (d) Not exist

vi. $\frac{d}{dx}\sin x =$ **1**

(a) 1 (b) cosx (c) -cosx (d) cotx

vii. $\int e^x \, dx =$ **1**

(a) x +c (b) $e^x + c$ (c) e+c (d) None of these

viii. $\int a \, dx =$ **1**

(a) 0 +c (b) ax +c (c) 1+c (d) None of these

ix. In a determinant number of rows **1**

(a) Is less than number of columns

(b) Is equal to number of columns

(c) Is greater than number of columns

(d) All of these

x. Two matrices are said to be equal if **1**
(a) Their order is same
(b) Their order is same and corresponding elements are identical
(c) Their order is not same
(d) Their corresponding elements are identical

Q..2 Attempt any two:

i. Find the equation of a line which passes through (0,-4) and makes an angle of 60 degrees with the X axis. **5**

ii. Find the intercepts cut off by the line 2x-3y+18= 0 on the coordinate axes. **5**

iii. Find the point of intersection of lines **5**
$x\cos\alpha + y\sin\alpha = p$
$x\sin\alpha - y\cos\alpha + q = 0$

Q.3 Attempt any two:

i. Find the equation of the circle which passes through the points (0, 2), (3, 0) and (3, 2). **5**

ii. Find the equation of the tangent to the circle $x^2 + y^2 = 13$ at the points (3, -2). **5**

iii. Find the equation of the circle whose center is (1, -2) and which may touch the line x+y+5=0 **5**

Q.4 Attempt any two:

i. Find the value of $f(0), f(-1), f(\sin x), f(3)$ and $f\left(\frac{1}{3}\right)$ if $f(x) = x^2 + 2x - 3$ **5**

ii. Differentiate $x^n \cos x$ with respect to x. **5**

iii. Differentiate $\frac{2+\tan x}{2-\tan x}$ with respect to x. **5**

Q.5 Attempt any two:

i. Evaluate $\int xe^x dx$ **5**

ii. Prove that $\int_0^{\pi/2} \sin x\, dx = \int_0^{\pi/2} \cos x\, dx$ **5**

iii. Find the whole area of the circle $x^2 + y^2 = a^2$ **5**

Q.6 Attempt any two:

i. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$ then evaluate $A^2-4A-5I$ **5**

ii. Evaluate $\begin{vmatrix} a+b & c+d \\ -c+d & a-b \end{vmatrix}$ **5**

iii. Evaluate inverse of A if **5**
$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

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Q. 1. (i) (b) $y = 0$ (vi) (b) $\cos x$

(ii) (a) $3\sqrt{2}$ (vii) (b) $e^x + c$

(iii) (b) $x^2 + y^2 = 36$ (viii) (b) $9x + c$

(iv) (b) $(-g, -f)$ (ix) (b) is equal to the No. of col.

(v) (b) 1 (x) (b) Their order & corresponding elements are same.

Q2 1. sol (i) Let the eq^n of line passes through a point (x_1, y_1) be

$y - y_1 = m(x - x_1)$ — (1) (1)

eq^n (1) passes through $(0, -4)$ then,

$y - (-4) = m(x - 0)$ (1)

eq^n (1) makes angle of 60° with X axis

i.e. $m = \tan 60° = \sqrt{3}$. (1)

$\Rightarrow$ $y + 4 = \sqrt{3}x$ (1)

$\Rightarrow$ $\boxed{y = \sqrt{3}x - 4}$ // (1)

Q2 sol (ii) Given Eq^n of the line is $2x - 3y + 18 = 0$

$\Rightarrow$ $2x - 3y = -18$ (1)

$\Rightarrow$ Divide the above equation by -18

$\Rightarrow$ $\frac{\cancel{2}x}{-\cancel{18}_9} - \frac{\cancel{3}y}{-\cancel{18}_6} = \frac{\cancel{-18}}{\cancel{-18}}$

$\Rightarrow$ $\frac{x}{-9} + \frac{y}{6} = 1$

$\Rightarrow$ Intercept on X axis $a = -9$ (2)

Intercept on Y axis $b = 6$ (2)

Sol (iii) Given Eqn of lines are

$$x\cos\alpha + y\sin\alpha - p = 0$$

$$x\sin\alpha - y\cos\alpha + p = 0$$

Point of Intersection of two lines are given by

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1} \quad (1)$$

By the Given Equations

$a_1 = \cos\alpha \quad b_1 = \sin\alpha \quad c_1 = -p$

$a_2 = \sin\alpha \quad b_2 = -\cos\alpha \quad c_2 = q \quad (1)$

$$\frac{x}{(\sin\alpha)(q) - (-\cos\alpha)(-p)} = \frac{y}{(-p)(\sin\alpha) - (q)(\cos\alpha)} = \frac{1}{(\cos\alpha)(-\cos\alpha) - (\sin\alpha)(\sin\alpha)}$$

$$\frac{x}{q\sin\alpha - p\cos\alpha} = \frac{y}{-p\sin\alpha - q\cos\alpha} = \frac{1}{-(\cos^2\alpha + \sin^2\alpha)} \quad (1)$$

$$\frac{x}{q\sin\alpha - p\cos\alpha} = \frac{y}{-p\sin\alpha - q\cos\alpha} = -1 \quad (1)$$

$$\boxed{x = p\cos\alpha - q\sin\alpha} \quad \boxed{y = p\sin\alpha + q\cos\alpha} \quad (1)$$

Q.3 Sol (i) Let the Eqn of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \; - (i) \quad (1)$$

Eqn (i) passes through (0,2) then $4 + 4f + c = 0$ — (ii)

Eqn (i) passes through (3,0) then $9 + 6g + c = 0$ — (iii)

Eqn (i) passes through (3,2) then $13 + 6g + 4f + c = 0$ (iv) (2)

Add eqn (ii) & (iii) we get

$13 + 4f + 6g = -2c$

Put in (iv)

$-2c + c = 0 \Rightarrow c = 0$

from (ii) $f = -1$, from (iii) $g = -3/2$

Centre of circle $(-g,-f) = (1, 3/2)$

radius of circle $r = \sqrt{g^2+f^2-c} = \sqrt{1+9/4-0} = \frac{\sqrt{13}}{2}$

Required Eq^n of circle is

$$(x-1)^2 + (y-3/2)^2 = \left(\frac{\sqrt{13}}{2}\right)^2 \qquad (2)$$

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3 Sol (ii) Given Eq^n of circle is $x^2+y^2=13$

$$\Rightarrow x^2+y^2 = (\sqrt{13})^2 \qquad (1)$$

The point at which the tangent is drawn $(3,-2)$

$x_1 = 3,\ y_1 = -2$

Hence Eq^n of tangent is $xx_1 + yy_1 = a^2$ (2)

$$\Rightarrow x(3) + y(-2) = (\sqrt{13})^2$$

$$\Rightarrow \boxed{3x-2y=13} \ // \qquad (2)$$

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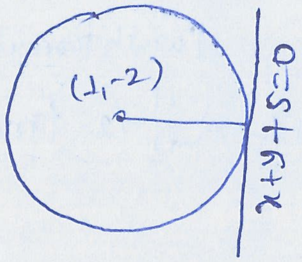
3 Sol (iii)



(1)

Centre of the circle is $(1,-2)$, Radius is to find to get the Eq^n. Therefore

Radius of circle = length of perpendicular from $(1,-2)$ on the line $x+y+5=0$

$$r = \left|\frac{(1)+(-2)+5}{\sqrt{1^2+1^2}}\right| = \left|\frac{4}{\sqrt{2}}\right| = 2\sqrt{2} \qquad (2)$$

Hence the centre radius form of circle is

$$(x-1)^2 + (y+2)^2 = (2\sqrt{2})^2$$

$$\Rightarrow x^2+1-2x+y^2+4+4y = 8$$

$$\Rightarrow \boxed{x^2+y^2-2x+4y-3=0} \ // \qquad (2)$$

Q 4 sol (i) Given function is $f(x) = x^2 + 2x - 3$

$\Rightarrow f(0) = 0^2 + 2(0) - 3 = -3$ (1)

$\Rightarrow f(-1) = (-1)^2 + 2(-1) - 3 = -4$ (1)

$\Rightarrow f(\sin x) = \sin^2 x + 2\sin x - 3$ (1)

$\Rightarrow f(3) = (3)^2 + 2(3) - 3 = 12$ (1)

$\Rightarrow f(1/3) = (1/3)^2 + 2\left(\frac{1}{3}\right) - 3 = \frac{-20}{9}$ (1)

Q 4 sol (ii) $y = x^n \cos x$ (1)

$\Rightarrow \frac{dy}{dx} = x^n \frac{d}{dx} \cos x + \cos x \frac{d}{dx} x^n$ [with Formula] (2)

$= x^n(-\sin x) + \cos x \cdot n x^{n-1}$ (1)

$= \boxed{-\sin x \cdot x^n + n x^{n-1} \cos x}$ // (1)

Q 4 sol (iii) $y = \frac{2 + \tan x}{2 - \tan x}$

[with formula]

$\Rightarrow \frac{dy}{dx} = \frac{(2 - \tan x)\frac{d}{dx}(2 + \tan x) - (2 + \tan x)\frac{d}{dx}(2 - \tan x)}{(2 - \tan x)^2}$ (2)

$\Rightarrow \frac{dy}{dx} = \frac{(2 - \tan x)(\sec^2 x) - (2 + \tan x)(-\sec^2 x)}{(2 - \tan x)^2}$

$= \frac{2\sec^2 x - \cancel{\tan x \sec^2 x} + 2\sec^2 x + \cancel{\tan x \sec^2 x}}{(2 - \tan x)^2}$ (2)

$= \boxed{\frac{4\sec^2 x}{(2 - \tan x)^2}}$ // (1)

Q5 sol (i) $\int x e^x \, dx$

$\Rightarrow \int f_1 f_2 \, dx = f_1 \int f_2 \, dx - \int \left[\frac{d}{dx} f_1 \int f_2 \, dx\right] dx$

$\Rightarrow \int x e^x \, dx = x \int e^x \, dx - \int \left[\frac{d}{dx} x \int e^x \, dx\right] dx$ (2)

$\Rightarrow \int x e^x \, dx = x e^x - \int [1 \cdot e^x] \, dx$ (1)

$= x e^x - \int e^x \, dx = x e^x - e^x$ (1)

$\Rightarrow \boxed{\int x e^x \, dx = e^x (x-1)}$ // (1)

Q5 sol (ii) $\int_0^{\pi/2} \sin x \, dx = \int_0^{\pi/2} \cos x \, dx$

$\Rightarrow \int_0^{\pi/2} \sin x \, dx = \left[-\cos x\right]_0^{\pi/2} = \left[\cos x\right]_{\pi/2}^{0}$ (1)

$= \left[\cos 0 - \cos \frac{\pi}{2}\right] = [1 - 0] = 1$ (1)

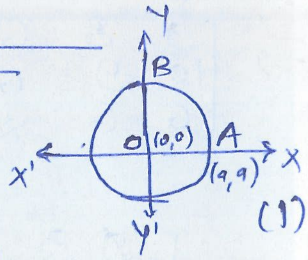
$\Rightarrow \int_0^{\pi/2} \cos x \, dx = \left[\sin x\right]_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0$ (1)

(1)

$= 1 - 0 = 1$

Hence, $\int_0^{\pi/2} \sin x \, dx = 1 = \int_0^{\pi/2} \cos x \, dx$ (1)

Q5 sol (iii) Given Eqn of circle is $x^2 + y^2 = a^2$



(1)

$\Rightarrow y^2 = a^2 - x^2 \Rightarrow y = \sqrt{a^2 - x^2}$

$\therefore$ Area of circle $= 4 \cdot$ Area of quadrant OAB (1)

$= 4 \cdot A_{OAB}$

$= 4 \int_0^a y \, dx$

$= 4 \int_0^a \sqrt{a^2 - x^2} \, dx$

$= 4 \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$ (2)

$$= 4\left[0 + \frac{a^2}{2}\sin^{-1}\left(\frac{a}{a}\right) - \left(0 + \frac{a^2}{2}\sin^{-1}\left(\frac{0}{a}\right)\right)\right]$$

$$= 4\left[\frac{a^2}{2}\sin^{-1}(1) - 0\right] = \frac{4}{2}a^2 \cdot \frac{\pi}{2}$$

$$= \pi a^2 \text{ square units.} \quad (1)$$

6 sol (i)

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

$$\Rightarrow A \cdot A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix} =$$

$$\Rightarrow A^2 = \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+0 \\ 2+2+4 & 4+1+4 & 4+2+0 \\ 2+4+0 & 4+2+0 & 4+4+0 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 6 \\ 10 & 9 & 6 \\ 6 & 6 & 8 \end{bmatrix} \quad (1)$$

$$\Rightarrow 4A = 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 0 \end{bmatrix} \quad (1)$$

$$\Rightarrow 5I = 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad (1)$$

$$A^2 - 4A - 5I$$

$$\Rightarrow \begin{bmatrix} 9 & 8 & 6 \\ 10 & 9 & 6 \\ 6 & 6 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \quad (1)$$

$$\Rightarrow \cancel{\begin{bmatrix} 9 & 8 & 6 \\ 10 & 9 & 6 \\ 6 & 6 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} 5 & 0 & -2 \\ 2 & 5 & -2 \\ -2 & -2 & 8 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -2 \\ 2 & 0 & -2 \\ -2 & -2 & 3 \end{bmatrix} \quad (1)$$

Q.6 sol (ii) $\begin{bmatrix} a+b & c+d \\ -c+d & a-b \end{bmatrix}$

$\Rightarrow (a+b)(a-b) - (c+d)(-c+d)$ (1)

$\Rightarrow a^2-b^2+(c+d)(c-d)$ [With Formula or Product] (2)

$\Rightarrow a^2-b^2+c^2-d^2$ (2)

Q6 sol (iii) $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{\text{Adj}\, A}{|A|}$

$|A| = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{vmatrix}$

$= 1(3-1) - 2(3-1) + 5(2+3)$ (1)

$= 2-4+25 = 23$

$C_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} = 2$ $C_{12} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = -3$ $C_{13} = \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 5$

$C_{21} = \begin{vmatrix} 2 & 5 \\ 1 & 1 \end{vmatrix} = 3$ $C_{22} = \begin{vmatrix} 1 & 5 \\ -1 & 1 \end{vmatrix} = 6$ $C_{23} = \begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} = -3$ (2)

$C_{31} = \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = -13$ $C_{32} = \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = 9$ $C_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

Cofactors matrix of A $= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}$ (1)

$\text{Adj}\, A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$ $A^{-1} = \dfrac{1}{23}\begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$ (1)