Total No. of Questions: 6 Total No. of Printed Pages:3

**Enrollment No......................................**

Faculty of Engineering / Science

End Sem Examination May-2024

EN3BS12 / BC3BS03 / SC3BS02

Engineering Mathematics -II

Programme: B.Tech./B.Sc. Branch/Specialisation: All

**Duration: 3 Hrs.** **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. The Laplace transform of $e^{-3t}t^3$ is given by- **1**

(a) $\frac{3!}{s^4}$ (b) $\frac{3!}{(s+3)^4}$ (c) $\frac{3!}{(s-3)^4}$ (d) None of these

ii. $L^{-1}\left\{\frac{s}{s^2-25}\right\}=$ ______. **1**

(a) $\cos 5t$ (b) $\sinh 5t$ (c) $\cosh 5t$ (d) None of these

iii. In the Fourier series expansion of function $f(x)=x\sin x$ in the range $-l\le x\le l$, the value of all sin terms coefficients i.e. $b_n$ will be- **1**

(a) 1 (b) 0 (c) $\frac{(-1)^n}{n}$ (d) None of these

iv. If $\overline{f}(s)$ is Fourier transform of $f(x)$ then as per scaling property Fourier transform of $f(ax)=$ ______. **1**

(a) $\frac{1}{a}\overline{f}\left(\frac{s}{a}\right)$ (b) $\overline{f}\left(\frac{s}{a}\right)$ (c) $a\overline{f}\left(\frac{s}{a}\right)$ (d) None of these

v. The complete integral of $f(p,q)=0$ is- **1**

(a) $z=ax+\phi(a)+c$ (b) $z=ax+\phi(a)y+c$

(c) $z=\varphi(b)+by+c$ (d) None of these

vi. The solution of $(D^2-2DD'+D'^2)z=0$ is ______, where $D\equiv\frac{\partial}{\partial x}$ and $D'\equiv\frac{\partial}{\partial y}$. **1**

(a) $z=f_1(y)+f_2(x)$ (b) $z=xf_1(y)+yf_2(x)$

(c) $z=f_1(y+x)+xf_2(y+x)$ (d) None of these

P.T.O.

vii. Normal vector $\vec{n}$ to the surface $\phi(x,y,z)=c$ is given by- **1**

(a) $curl\ \phi$ (b) $div\ \phi$ (c) $grad\ \phi$ (d) None of these

viii. If $\vec{F}$ is any continuous differentiable vector point function and $S$ is a surface bounded by $C$, then $\int_C \vec{F}.d\vec{r}=$______. **1**

(a) $\iint_S curl\ \vec{F}.\hat{n}$ ds (b) $\iint_S curl\ \vec{F}\text{x}\hat{n}$ ds

(c) $\iint_S div\ \vec{F}.\hat{n}$ ds (d) None of these

ix. Which one of the following methods not surely converges, to find root of equation *f(x)* = 0. **1**

(a) Newton Raphson method (b) Regula Falsi method

(c) Both (a) and (b) (d) None of these

x. The aim of elimination steps in Gauss elimination method is to reduce the coefficient matrix to ______. **1**

(a) Diagonal matrix (b) Lower triangular matrix

(c) Upper triangular matrix (d) None of these

Q.2 Attempt any two:

i. Find the Laplace transform of- **5**

$$\int_0^t \frac{\sin t}{t} dt$$

ii. Apply Convolution theorem to evaluate- **5**

$$L^{-1}\left\{\frac{1}{s(s^2-16)}\right\}$$

iii. Solve the following differential equation by Laplace transform- **5**

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=5e^t;\ \text{Given } y(0)=2, y'(0)=1$$

Q.3 i. Find the Fourier transform of- **3**

$$f(x)=\begin{cases}1, & for\ |x|\le \text{a}\\ 0, & for |x|>a\end{cases}$$

ii. Expand in Fourier series to represent the function, if **7**

$$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Hence deduce that,

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+---=\frac{\pi^2}{8}$$



OR iii. Obtain the half range Fourier sine series of $f(x)=x.\cos(x),\ x\in(0,\pi)$. **7**

Q.4 Attempt any two:

i. Solve $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$. **5**

ii. Solve the following- **5**

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}-2\frac{\partial^2 z}{\partial y^2}=e^{x+2y}+y^2$$

iii. Solve by the method of separation of variables- **5**

$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0,\ \text{where } u(x,0)=4e^{-x}$$

Q.5 Attempt any two:

i. Prove that- **5**

$$div\ grad\ r^m=m(m+1)r^{m-2},\ \text{where } r=|\vec{r}|.$$

ii. Find the work done when a force $\vec{F}=(x^2-y^2+x)\hat{i}-(2xy)\hat{j}$ moves a particle in the XY-plane form $(0,0)$ to $(1.1)$ alone the parabola $y^2=x$. **5**

iii. Use Gauss's Divergence theorem to evaluate- **5**

$\iint_S (xdydz+ydzdx+zdxdy)$, where $S$ is the surface of sphere $x^2+y^2+z^2=a^2$.

Q.6 i. Find the relative and percentage error of $\frac{2}{3}$ approximated to 0.667. **4**

ii. Solve the equation $3x=\cos x+1$ using Newton Raphson method, correct upto 3 places of decimals. **6**

OR iii. Solve the following system of equation- **6**

$$10x+2y+z=9$$
$$-2x+3y+10z=22$$
$$x+10y-z=-22$$

Using Gauss Siedel Iterative method up to second iteration taking initial values as zero.

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Solution of Engineering Mathematics-II

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Q.1

i. (b) $\frac{3!}{(s+3)^4}$ [1]

ii. (c) $\cosh 5t$ [1]

iii. (b) 0 [1]

iv. (a) $\frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$ [1]

v. (b) $z = ax + \phi(a)y + c$ [1]

vi. (c) $z = f_1(y+x) + x f_2(y+x)$ [1]

vii. (c) grad $\phi$ [1]

viii. (a) $\iint_S \text{curl}\,\vec{F}\cdot\hat{n}\,ds$ [1]

ix. (a) Newton Raphson Method [1]

x. (c) Upper triangular matrix [1]

Date :
P. No. :

Q.2 [i] Evaluate $L\left[\int_0^t \frac{\sin t}{t} dt\right]$

As, $L[\sin t] = \frac{1}{s^2+1}$ [1]

$\therefore L\left[\frac{\sin t}{t}\right] = \int_s^\infty \frac{1}{s^2+1} ds$

$= (\tan^{-1} s)_s^\infty$ [2]

$= \frac{\pi}{2} - \tan^{-1} s = \cot^{-1} s$ [3]

As, $L\left[\int_0^t F(t) dt\right] = \frac{L[F(t)]}{s} = \frac{f(s)}{s}$ [4]

$\therefore L\left[\int_0^t \frac{\sin t}{t} dt\right] = \frac{L[\sin t/t]}{s} = \frac{\cot^{-1} s}{s}$. Ans [5]

Q.2 [ii]

Let $f(s) = \frac{1}{s}$ and $g(s) = \frac{1}{s^2-16}$

$\therefore F(t) = L^{-1}(f(s)) = 1$

& $G(t) = L^{-1}(g(s)) = \frac{1}{4}\sinh 4t$ [1]

$L^{-1}[g(s) f(s)] = \int_0^t G(u) F(t-u) du$ [2]

$= \int_0^t \frac{1}{4}\sinh 4u \cdot 1 \, du$ [3]

$= \frac{1}{4}\left(\frac{\cosh 4u}{4}\right)_0^t$ [4]

$$= \frac{1}{16}\left[\cosh 4t - 1\right]$$ [5]

Ans.

Q. 2 iii. Solve $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 5e^t$ given $y(0) = 2$, $y'(0) = 1$.

Sol. Taking Laplace transform on both the sides

$$L[y'' + 5y' + 6y] = L[5e^t]$$ [1]

$$s^2 L(y) - s y(0) - y'(0) + 5[s L(y) - y(0)] + 6 L(y) = 5 \cdot \frac{1}{s-1}$$

$$\Rightarrow [s^2 + 5s + 6] L(y) - 2s - 1 - 10 = \frac{5}{s-1}$$

$$\Rightarrow (s+2)(s+3) L(y) = 2s + \frac{5}{s-1} + 11$$ [2]

$$\Rightarrow L(y) = \frac{2s+11}{(s+2)(s+3)} + \frac{5}{(s-1)(s+2)(s+3)}$$

$$\Rightarrow L(y) = \frac{2(s+2)}{(s+2)(s+3)} + \frac{7}{(s+2)(s+3)} + \frac{5}{(s-1)(s+2)(s+3)}$$ [3]

$$\Rightarrow L(y) = \frac{2}{s+3} + 7\left[\frac{1}{s+2} - \frac{1}{s+3}\right] + \frac{5}{12}\left[\frac{1}{s-1}\right] - \frac{5}{3}\left[\frac{1}{s+2}\right] + \frac{5}{4}\left[\frac{1}{s+3}\right]$$ [4]

$$\therefore y = L^{-1}\left[-\frac{15}{4} \cdot \frac{1}{s+3}\right] + L^{-1}\left[\frac{16}{3} \cdot \frac{1}{s+2}\right] + L^{-1}\left[\frac{5}{12} \cdot \frac{1}{s-1}\right]$$

4

Date :
P. No. :

$$\Rightarrow y = \frac{-15}{4} e^{-3t} + \frac{16}{3} e^{-2t} + \frac{5}{12} e^{t}$$ Ans. [5]

GO TO NEXT PAGE

Q3 (i) find fourier transform of $f(x) = \begin{cases} 1, & \text{for } |x| \le a \\ 0, & \text{for } |x| > a \end{cases}$

Sol$^n$: The fourier transform of a function $f(x)$ is given by

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \int_{-\infty}^{-a} f(x)\, e^{isx}\, dx + \int_{-a}^{a} f(x)\, e^{isx}\, dx + \int_{a}^{\infty} f(x)\, e^{isx}\, dx \right] \quad [1]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ 0 + \int_{-a}^{a} 1 .\, e^{isx}\, dx + 0 \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{e^{isx}}{is} \right]_{-a}^{a} = \frac{1}{\sqrt{2\pi}} \frac{1}{is} \left[ e^{ias} - e^{-ias} \right] \quad [2]$$

$$= \frac{1}{\sqrt{2\pi}} \frac{2}{s} \frac{e^{ias} - e^{-ias}}{2i} = \frac{1}{\sqrt{2\pi}} \frac{2 \sin sa}{s}$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sin sa}{s} \quad [3]$$

Q3 (ii) given $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

Let $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$$= \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + \dots$$
$$+ b_1 \sin x + b_2 \sin 2x + \dots$$
$$\dots + b_n \sin nx$$

1 [1]

6

Date :
P. No. :

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\,dx + \int_0^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi(x)_{-\pi}^{0} + \left(\frac{x^2}{2}\right)_0^{\pi}\right] = \frac{1}{\pi}\left(-\pi^2+\frac{\pi^2}{2}\right) = \frac{-\pi}{2}$$

$$\boxed{a_0 = \frac{-\pi}{2}}$$

[2]

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\cos nx\,dx + \int_0^{\pi} x\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + \left(\frac{x\sin nx}{n} + \frac{\cos nx}{n^2}\right)_0^{\pi}\right]$$ [3]

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\right] = \frac{1}{\pi n^2}(\cos n\pi - 1)$$

$$\boxed{a_n = \frac{1}{\pi n^2}\left((-1)^n - 1\right)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$ [4]

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(-\pi)\sin nx\,dx + \int_0^{\pi} x\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi \cos nx}{n}\right)_{-\pi}^{0} + \left(\frac{-x\cos nx}{n} + \frac{\sin nx}{n^2}\right)_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left(1-\cos n\pi\right) - \frac{\pi}{n}\cos n\pi\right] = \frac{1}{n}\left(1-2\cos n\pi\right) \quad [5]$$

$$\boxed{b_n = \frac{1}{n}\left[1-2(-1)^n\right]}$$

by Putting $a_0, a_n, b_n$

$$\therefore f(x) = \frac{-\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \dots\right) \quad [6]$$

$$+ 3\sin x - \frac{\sin 2x}{2} + \frac{3\sin 3x}{3} - \frac{\sin 4x}{4} \quad \text{--- (2)}$$

putting $x=0$ in (2), we get

$$f(0) = \frac{-\pi}{4} - \frac{2}{\pi}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots \infty\right) \quad \text{--- (3)}$$

Now $f(x)$ is discontinuous at $x=0$

But $f(0-0) = -\pi$ and $f(0+0) = 0$

$$\therefore f(0) = \frac{1}{2}\left[f(0-0) + f(0+0)\right] = -\pi/2$$

from (3) and (4)

$$\frac{-\pi}{2} = \frac{-\pi}{4} - \frac{2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\Rightarrow \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \quad [7]$$

proved.

Date :
P. No. :

Q3 OR (iii) Half range fourier sine series of $f(x) = x.\cos x$, $x \in (0, \pi)$

Solⁿ:- Here $f(x) = x\cos x$ is an odd function over the interval $(-\pi, \pi)$ then $a_0 = 0$ and $a_n = 0$. Therefore it reduces to sine fourier series in half range interval $(0, \pi)$

$$\therefore \quad f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$\text{where } b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx \qquad [1]$$

$$= \frac{2}{\pi} \int_0^{\pi} x \cos x \sin nx \, dx$$

$$\left[\because 2 \sin A \cos B = \sin(A+B) + \sin(A-B)\right]$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} \{\sin(n+1)x + \sin(n-1)x\} dx + \int_0^{\pi} \{x \sin(n+1)x + x \sin(n-1)x\} dx\right] \qquad [2]$$

$$= \frac{1}{\pi}\left[x\left\{-\frac{\cos(n+1)x}{(n+1)}\right\} - 1\left\{-\frac{\sin(n+1)x}{(n+1)^2}\right\} + x\left\{-\frac{\cos(n-1)x}{(n-1)}\right\} - 1\left\{-\frac{\sin(n-1)x}{(n-1)^2}\right\}\right]_0^{\pi}, \text{ for } n \neq 1 \qquad [3]$$

$$= \frac{1}{\pi}\left[\pi\left\{-\frac{\cos(n+1)\pi}{n+1}\right\} + 0 + \pi\left\{-\frac{\cos(n-1)\pi}{(n-1)}\right\} + 0\right] \qquad [4]$$

for $n \neq 1$

$$\frac{(-1)^n}{(n+1)} + \frac{(-1)^n}{(n-1)} = \frac{2n(-1)^n}{n^2-1} \text{ if } n \neq 1$$

$$\& \; n = 2, 3, 4, \ldots$$

$$\left[ \because \cos(n+1)\pi = (-1)^{n+1} = -(-1)^n \right] \qquad [5]$$

Since $b_n$ doesnot exist for $n = 1$, so in order to find $b_1$ seperately

$$b_1 = \frac{2}{\pi} \int_0^{\pi} x \cos x \sin x \, dx$$

$$= \frac{1}{\pi} \int_0^{\pi} x \sin 2x \, dx = -\frac{1}{2}$$

$$\Rightarrow \boxed{b_1 = -\frac{1}{2}} \qquad [6]$$

Hence fourier series may be written as.

$$f(x) = b_1 \sin x + \sum_{n=2}^{\infty} b_n \sin nx$$

$$= -\frac{1}{2} \sin x + \sum_{n=2}^{\infty} \frac{2n(-1)^n}{n^2-1} \sin nx. \qquad [7]$$

10

Date :
P. No. :

GO TO NEXT PAGE

Q4 (i)

Sol Here

$P = x^2(y-z)$, $Q = y^2(z-x)$ and
$R = z^2(x-y)$

$\therefore$ A.E are $\dfrac{dx}{x^2(y-z)} = \dfrac{dy}{y^2(z-x)} = \dfrac{dz}{z^2(x-y)}$

(i) Using multipliers as $\frac{1}{x^2}, \frac{1}{y^2}, \frac{1}{z^2}$, we get

each fraction $= \dfrac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{(y-z)+(z-x)+(x-y)}$ [1]

$= \dfrac{\frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz}{0}$

$\Rightarrow \frac{1}{x^2}dx + \frac{1}{y^2}dy + \frac{1}{z^2}dz = 0$

On Integrating,

$-\frac{1}{x} - \frac{1}{y} - \frac{1}{z} = c_1$ [2.5]

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = a$ $[\because a = -c_1]$

ii) Again, using multipliers as $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

we get.

each fraction $= \dfrac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{x(y-z)+y(z-x)+z(x-y)}$ [3.5]

$$\Rightarrow \frac{1/x\,dx + 1/y\,dy + 1/z\,dz}{0}$$

$$\Rightarrow \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

On integrating

$$\Rightarrow \log x + \log y + \log z = \log b$$

$$\Rightarrow \log(xyz) = \log b$$

$$\Rightarrow xyz = b$$ [4]

Hence general solution is $f(a,b) = 0$

$$\therefore f\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}, xyz\right) = 0$$ [5]

ii) Given:

$$(D^2 - DD' - 2D'^2)z = e^{x+2y} + y^2$$

For C.F : its A.E is

$$m^2 - m - 2 = 0$$

$$\Rightarrow (m-2)(m+1) = 0$$

$$\Rightarrow m = -1, 2$$ [1.5]

$\therefore$

$$C.F = f_1(y-x) + f_2(y+2x)$$ [2]

Now, $P.I = \frac{1}{D^2 - DD' - 2D'^2} e^{x+2y} + \frac{1}{D^2 - DD' - 2D'^2} y^2$

$$PI = I_1 + I_2$$

Date : 13
P. No. :

put $D = 1$ and $D' = 2$

$$I_1 = \frac{e^{x+2y}}{1^2 - 2 - 2\times4}$$

$$I_1 = \frac{e^{x+2y}}{-9}$$

$$I_2 = \frac{1}{D^2 - DD' - 2D'^2}\, y^2 \qquad [3]$$

$$= \frac{1}{D^2} \frac{1}{\left(1 - \frac{D'}{D} - \frac{2D'^2}{D^2}\right)} y^2$$

$$= \frac{1}{D^2}\left(1 - \frac{D'}{D} - \frac{2D'^2}{D^2}\right)^{-1} y^2$$

$$\left[\because (1-x)^{-1} = 1 + x + x^2 + x^3 \ldots\right.$$

$$= \frac{1}{D^2}\left(1 + \frac{D'}{D} + \frac{2D'^2}{D^2} + \frac{D'^2}{D^2} + \frac{4D'^4}{D^4} + \frac{4D'^3}{D^3} + \cdots\right) y^2$$

$$= \frac{1}{D^2} y^2 + \frac{D'}{D^3} y^2 + \frac{2D'^2}{D^4} y^2 + \frac{D'^2}{D^4} y^2 \qquad [4]$$

$$= \frac{x^2y^2}{2} + \frac{1}{D^3} 2y + \frac{1}{D^4} \cdot 4 + \frac{1}{D^4} \cdot 2$$

$$= \frac{x^2y^2}{2} + \frac{x^3 2y}{\cancel{6}\,3} + \frac{\cancel{4}x^4}{\cancel{24}\,6} + \frac{2x^4}{\cancel{24}\,12}$$

Date :

P. No. :

$$= \frac{x^2y^2}{2} + \frac{x^3y}{3} + \frac{x^4}{6} + \frac{x^4}{12}$$

$$= \frac{x^2y^2}{2} + \frac{x^3y}{3} + \frac{x^4}{4}$$

$$PI = I_1 + I_2$$

$$= \frac{e^{x+2y}}{-9} + \frac{x^2y^2}{2} + \frac{x^3y}{3} + \frac{x^4}{4}$$

Hence, complete solution is

$$z = CF + PI$$

$$z = f_1(y-x) + f_2(y+2x) + \left(\frac{e^{x+2y}}{-9}\right) + \frac{x^2y^2}{2} + \frac{x^3y}{3} + \frac{x^4}{4}$$ [5]

P.T.O

iii)

Sol Given

$$\frac{3\partial u}{\partial x} + \frac{2\partial u}{\partial y} = 0 \quad \text{--- (1)}$$

Let

$$u = X(x) \cdot Y(y) \quad \text{--- (2)} \qquad [1]$$

Where X is a function of x and Y is a function of y only.

Putting the value of u in equ (1) we get

$$3\frac{\partial (X \cdot Y)}{\partial x} + 2\frac{\partial (X \cdot Y)}{\partial y} = 0$$

$$3Y\frac{dX}{dx} + 2X\frac{dY}{dy} = 0 \qquad [2]$$

On separating the variables, we get

$$\frac{3}{X}\frac{dX}{dx} = -\frac{2}{Y}\frac{dY}{dy} = c$$

then

$$\frac{3\,dX}{X\,dx} = c \qquad [3]$$

$$\Rightarrow \frac{3\,dX}{dx} = cX$$

$$\Rightarrow 3DX - cX = 0$$

$(3D - c)X = 0$

A.E is

$$3m - c = 0$$
$$m = c/3$$
$$\therefore X = a\,e^{cx/3}$$

Similarly

$$\frac{2}{Y}\frac{dY}{dy} = c$$

$$\frac{2\,dY}{dy} = cY$$

$$2DY - cY = 0 \qquad [4]$$

$$(2D - c)Y = 0$$

A.E is

$$2m - c = 0$$
$$m = c/2$$

$$Y = b\,e^{cy/2}$$

putting the value of $X$ and $Y$ in (2) we have

$$u = a e^{cx/3} b e^{cy/2}$$

$$u = ab\,e^{(cx/3 + cy/2)} \quad \text{--- } 3$$

On putting $y = 0$ and $u = 4e^{-x}$

We get.

$$4e^{-x} = ab\, e^{cx/3}$$

$$\& \; ab = 4 \,, \quad c/3 = -1$$

put the values of ab and c in (3) we have

$$u = 4 e^{-x - 3/2\, y}$$ [5]

Ans.

P.T.O

Date :
P. No. :

GO TO NEXT PAGE

Q.5. (i) Prove that

$$\text{div grad } r^m = m(m+1)\, r^{m-2}, \text{ where } r = |\vec{r}|$$

Sol$^n$

$$\text{grad } r^m = \hat{i}\frac{\partial}{\partial x} r^m + \hat{j}\frac{\partial}{\partial y} r^m + \hat{k}\frac{\partial}{\partial z} r^m$$

Or $\nabla r^m = \hat{i}\frac{\partial}{\partial x} r^m + \hat{j}\frac{\partial}{\partial y} r^m + \hat{k}\frac{\partial}{\partial z} r^m$

$$= \hat{i}\, m r^{m-1}\frac{\partial r}{\partial x} + \hat{j}\, m r^{m-1}\frac{\partial r}{\partial y} + \hat{k}\, m r^{m-1}\frac{\partial r}{\partial z} \quad [1]$$

$$= m r^{m-1}\left\{\hat{i}\frac{x}{r} + \hat{j}\frac{y}{r} + \hat{k}\frac{z}{r}\right\}$$

$$\left[\because \frac{\partial r}{\partial x} = \frac{x}{r}, \frac{\partial r}{\partial z} = \frac{z}{r}, \frac{\partial r}{\partial y} = \frac{y}{r}\right]$$

$$= m r^{m-2}(x\hat{i} + y\hat{j} + z\hat{k}) \quad [2]$$

$$\nabla \cdot \nabla r^m = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right] \cdot \left[m r^{m-2}(x\hat{i} + y\hat{j} + z\hat{k})\right]$$

$$= \frac{\partial}{\partial x}(m r^{m-2} x) + \frac{\partial}{\partial y}(m r^{m-2} y) + \frac{\partial}{\partial z}(m r^{m-2} z) \quad [3]$$

But $\frac{\partial}{\partial x}(m r^{m-2} x) = m r^{m-2} + m x (m-2) r^{m-3}\frac{\partial r}{\partial x}$

$$= m r^{m-2} + m(m-2) r^{m-3} x \frac{x}{r}$$

$$= m r^{m-2} + m(m-2) r^{m-4} x^2$$

Similarly we can find other two expansions

$$\frac{\partial}{\partial y}\left(m r^{m-2} y\right) = m r^{m-2} + m(m-2) r^{m-4} y^2$$

$$\frac{\partial}{\partial z}\left(m r^{m-2} z\right) = m r^{m-2} + m(m-2) r^{m-4} z^2$$

$$\therefore \nabla \cdot \nabla r^m = 3 m r^{m-2} + m(m-2) r^{m-4} (x^2+y^2+z^2)$$ [4]

$$= 3 m r^{m-2} + m(m-2) r^{m-2}$$

$\{\because x^2+y^2+z^2 = r^2$

$$= r^{m-2}\left(3m + m^2 - 2m\right)$$

$$= r^{m-2}\left(m^2 + m\right)$$

$$= m(m+1) r^{m-2}$$

Hence $\text{div grad } r^m = m(m+1) r^{m-2}$ [5]

Proved

Q. 5 (ii)

Sol$^n$ Given $F = (x^2 - y^2 + x)\hat{i} - (2xy)\hat{j}$

Work done =

$$\int_C F \cdot d\vec{r} = \int_C \{(x^2 - y^2 + x)\hat{i} - (2xy)\hat{j}\} \cdot (dx\hat{i} + dy\hat{j}) \quad [1]$$

$\left\{ \begin{array}{l} \text{Integral is Along the Parabola } y^2 = x \\ \text{varies from } x = 0 \text{ to } x = 1 \\ \qquad y = 0 \text{ to } y = 1 \end{array} \right\}$

$$= \int_C (x^2 - y^2 + x)\,dx - \int_C 2xy\,dy \quad [2]$$

$$= \int_0^1 (x^2 - x + x)\,dx - \int_0^1 2y^2 y\,dy \quad [3]$$

$[\because y^2 = x]$

$$= \int_0^1 x^2\,dx - \int_0^1 2y^3\,dy$$

$$= \left[\frac{x^3}{3}\right]_0^1 - 2\left[\frac{y^4}{4}\right]_0^1 \quad [4]$$

$$= \frac{1}{3} - \frac{1}{2} = -1/6$$

$\therefore$ W.D $= 1/6$ [5]

20

(OR)

Sol$^n$ .5 (iii) Here we have

$$\iint_S [x\,dy\,dz + y\,dx\,dz + z\,dx\,dy]$$

$$= \iiint_V \left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right) dx\,dy\,dz \quad [1]$$

$$= \iiint_V \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) dx\,dy\,dz \quad [2]$$

$$\left[\because x^2+y^2+z^2=a^2\right]$$

$$= \iiint_V (1+1+1)\,dx\,dy\,dz = 3 \quad [3]$$

(volume of the sphere

$$= 3\left(\frac{4}{3}\pi a^3\right) \quad [4]$$

$$= 4\pi a^3 \quad [5]$$

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Date :
P. No. :

Date : 23
P. No. :

Q6. (i) Here $x = \frac{2}{3}$ and $x_a = 0.667$

$\therefore$ Relative Error $E_r = \frac{E_a}{x} = \frac{|x - x_a|}{x}$

$= \frac{|\frac{2}{3} - 0.667|}{\frac{2}{3}} = \frac{1}{2000} = 0.0005$ [2]

Percentage Error $= E_R \times 100$

$= 0.0005 \times 100 = 0.05\%$ [4]

(ii) Let $f(x) = 3x - \cos x - 1 = 0$, $f'(x) = 3 + \sin x$

$f(0) = 0 - 1 - 1 = -2$

$f(0.6) = 1.8 - \cos 0.6 - 1 = -0.0253$

$f(1) = 3 - \cos 1 - 1 = 1.4597$ [1]

As $|\phi'(x)| < 1$ for $x_0 = 0.6$ and also 0.6 is nearer to zero, than $f(1)$. so, we take first approximate root as 0.6

By Newton-Raphson Method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ [2]

$$x_1 = x - \frac{(3x - \cos x - 1)}{3 + \sin x}$$

$$= 0.6 - \frac{(3(0.6) - \cos 0.6 - 1)}{3 + \sin 0.6} = 0.6071$$ [3]

$$x_2 = 0.6071 - \frac{3(0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)}$$ [5]

$$= 0.6071$$

Since $x_2 = x_1$, $\therefore$ the real root of the given equation is 0.6071

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Since $x_2 = x_3 = 0.607$ correct to 3 decimal Places.

$\therefore$ Root of given equation is 0.607. [6]

(iii) Since in each Equation one of the Coefficient is Larger than the other, satisfying the condition for gauss-seidal. i.e,

$$|10| > |2|+|1|, \quad |10| > |-2|+|3|, \quad |10| > |1|+|-1|. \quad [1]$$

Now, we Write the given equations in the following form:

$$\left.\begin{aligned} x &= \frac{1}{10}(9-2y-z) \\ y &= \frac{1}{10}(-22-x+z) \\ z &= \frac{1}{10}(22+2x-3y) \end{aligned}\right\} \text{—— } (1)$$

$x^{(i+1)} = \frac{1}{10}(9 - 2y^{(i)} - z^{(i)})$; $y^{(i+1)} = \frac{1}{10}[-22 - x^{(i+1)} + z^{(i)}]$; $z^{(i+1)} = \frac{1}{10}[22 + 2x^{(i+1)} - 3y^{(i+1)}]$

We Start with $x^{(0)} = y^{(0)} = z^{(0)} = 0$ and using the most recent values of $x, y, z$, we get. [2]

Take $i=0$

first iteration;

$$x^{(1)} = \frac{9}{10} = 0.9$$

$$y^{(1)} = \frac{1}{10}(-22 - x^{(1)})$$

$$= -2.29$$

$$z^{(1)} = \frac{1}{10}\left(22 + 2x^{(1)} - 3y^{(1)}\right) = 3.067$$ [4]

Take $i = 1$

Second iteration:

$$x^{(2)} = \frac{1}{10}\left(9 - 2y^{(1)} - z^{(1)}\right) \simeq 1.0513$$

$$y^{(2)} = \frac{1}{10}\left(-22 - x^{(2)} + z^{(1)}\right) = -1.9984$$

$$z^{(2)} = \frac{1}{10}\left(22 + 2x^{(2)} - 3y^{(2)}\right) \simeq 3$$

$\therefore$ The Required solution is

$$x = 1.0513,\ y = -1.9984,\ z = 3$$ [6]

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P. No. :