Q.	No.	Question	Marks	BL	PO	CO
Q.6		Attempt any two:				
	i.	If $f(x) = Kx^2, 0 < x < 1$, has probability density function, determine K and find: $P(\frac{1}{3} < x < \frac{1}{2})$ and find $'a'$ if $P(X > a) = 0.05$.	**5**	3	1, 2, 12	2
	ii.	In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for (a) More than 2150 hours (b) Less than 1950 hours. Given: $P(0 < z < 1.83) = 0.4664$, $P(0 < z < 1.5) = 0.4332$	**5**	3	1, 2, 12	2
	iii.	A random variable X has an exponential distribution with Probability density function given by $f(x) = \begin{cases} 2e^{-2x}, & if\ x > 0 \\ 0, & if\ x \le 0 \end{cases}$ Compute $P(X \nless 3)$ and mean.	**5**	3	1, 2, 12	2

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No....................................

Faculty of Science / Engineering
End Sem Examination Dec 2024
CA3CO20 Mathematics -III

Programme: BCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.	No.	Question	Marks	BL	PO	CO	PSO
Q.1	i.	Let x be an exact value and x_a be the approximate value then relative error is: (a) $\left\|\frac{x-x_a}{x_a}\right\|$ (b) $\|x-x_a\|$ (c) $\left\|\frac{x-x_a}{x}\right\|$ (d) None of these	**1**	2	1, 2, 12	1	
	ii.	The condition for iterative method is: (a) $\|f'(x)\|<1$ (b) $\|f'(x)\|>1$ (c) $\|f(x)\|<1$ (d) None of these	**1**	2	1, 2, 12	1	
	iii.	Let h be the finite difference, then backward difference operator ∇ is defined by (a) $\nabla f(x) = f(x+h) - f(x)$ (b) $\nabla f(x) = f(x-h) + f(x)$ (c) $\nabla f(x) = f(x) - f(x-h)$ (d) None of these	**1**	2	1, 2, 12	1	
	iv.	In factorial notation $x^{(n)}$, n is a positive integer and h is the constant difference, then $\Delta^{n+1}x^{(n)} =$ (a) $n!$ (b) 0 (c) $n!\,h^n$ (d) None of these	**1**	2	1, 2, 12	1	

v. Trapezoidal rule for evaluation of $\int_{x_0}^{x_0+nh} f(x)dx$ is given by- **1** 2 1, 2, 12 1

(a) $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \_\_\_\_\_\_\_ + y_{n-1})]$

(b) $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{3}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \_\_\_\_\_\_\_ + y_{n-1})]$

(c) $\int_{x_0}^{x_0+nh} f(x)dx = \frac{h}{2}[(y_0 + y_n) + 3(y_1 + y_2 + y_3 + \_\_\_\_\_\_\_ + y_{n-1})]$

(d) None of these

vi. Runge-Kutta method is used to solve any- **1** 2 1, 2, 12 1

(a) Ordinary differential equation
(b) Partial differential equation
(c) Integral equation
(d) None of these

vii. The mean for binomial distribution is given by- **1** 2 1, 2, 12 1

(a) np (b) npq
(c) n^2p^2 (d) None of these

(Where n =number of trails, p =probability of success, q = probability of failure)

viii. The probability of getting an even number in a single throw of unbiased dice is- **1** 2 1, 2, 12 1

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) None of these

ix. The normal curve is symmetrical about______. **1** 2 1, 2, 12 1

(a) Mean (b) Median
(c) Mode (d) All of these

x. Total area in an exponential distribution is ___. **1** 2 1, 2, 12 1

(a) 2 (b) 1 (c) 0 (d) None of these

Q.2 Attempt any two:

i. By using Newton-Raphson's method find the root of the equation $x^4 - x - 10 = 0$, which is nearer to 2, correct to three places of decimal. **5** 3 1, 2, 12 2

ii. Find a real root of the equation $x^3 - 9x + 1 = 0$ by the Regula Falsi method. **5** 3 1, 2, 12 2

iii. Solve the following by Gauss-Seidel iteration method. (only 3 iterations) **5** 3 1, 2, 12 2

$10x + y + z = 12,$
$2x + 10y + z = 13,$
$2x + 2y + 10z = 14$

Q.3 Attempt any two:

i. Using Newton's Forward Interpolation Formula, find the value of $f(1.6)$, if **5** 3 1, 2, 12 2

x:	1	1.4	1.8	2.2
y:	3.49	4.82	5.96	6.5

ii. Apply Lagrange's formula to find $f(5)$ given that $f(1) = 2,\ f(2) = 4,\ f(3) = 8,\ f(4) = 16,$ $f(7) = 128$ **5** 3 1, 2, 12 2

iii. By using Newton's divided difference formula, find the value of $f(15)$ from the following table: **5** 3 1, 2, 12 2

x:	4	5	7	10	11	13
y:	48	100	294	900	1210	2028

Q.4 Attempt any two:

i. Calculate $\int_2^{10} \frac{dx}{1+x} dx$ approximately by dividing the range into eight equal parts using Simpson's one-third rule. **5** 3 1, 2, 12 2

ii. Use Picard's method to solve: **5** 3 1, 2, 12 2

$\frac{dy}{dx} = 1 + xy, with\ x_0 = 2, y_0 = 0$

(Upto three approximation)

iii. Employ Taylor's series method to obtain approximate value of y at $x = 0.2$ for the differential equation $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0.$ **5** 3 1, 2, 12 2

Q.5 Attempt any two:

i. Define random variable and probability mass function. **5** 3 1, 2, 12 2

ii. Find variance for Poisson's distribution. **5** 3 1, 2, 12 2

iii. Six dice are thrown 729 times. How many times do you expect at least three dice to show a five or six. (Using Binomial distribution) **5** 3 1, 2, 12 2

Medicaps University
End Sem Exam Dec 2024
BCA Mathematics - III CA3CO20

Q.1 MCQ: +10

i) c $\left| \frac{x - x_a}{x} \right|$

ii) a $|f'(x)| < 1$

iii) e $\nabla f(x) = f(x) - f(x-h)$

iv) b 0

v) a $\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\dots+y_{n-1})\right]$

vi) a Ordinary Differential Equation

vii) a np

viii) b $\frac{1}{2}$

ix) d All of these

x) b 1

Q.2

i) Let

$f(x) = x^4 - x - 10 = 0$ —(1)

$f(0) = -10$ (-ve) +1

$f(1) = -10$ (-ve)

$f(2) = 4$ (+ve)

Clearly $f(1)$ and $f(2)$ are opposite signs

∴ root of equation (1) lies between 1 & 2

Taking initial approx. $x_0 = 1.5$ $|\phi'(x)| < 1$

By Newton Raphson Method

+½

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2 \dots$$

Since $f(x) = x^4 - x - 10$, $f'(x) = 4x^3 - 1$

$$x_{n+1} = x_n - \frac{x_n^4 - x_n - 10}{4x_n^3 - 1}$$

$$x_{n+1} = \frac{3x_n^4+10}{4x_n^3-1} \quad \text{--- (3)} \qquad (+1/2)$$

Putting $n=0$ in (3) we get

Ist Appx. $x_1 = \frac{3x_0^4+10}{4x_0^3-1} = \underline{\underline{2.015}}$ (+1)

IInd Apprx.

$x_2 = \frac{3x_1^4+10}{4x_1^3-1} = \underline{\underline{1.874}}$ (+1)

IIIrd Apprx

$x_3 = \frac{3x_2^4+10}{4x_2^3-1} = \underline{\underline{1.856}}$ (+1)

IVth Apprx. $x_4 = \frac{3x_3^4+1}{4x_3^3-1} = \underline{\underline{1.856}}$

Since $x_3 = x_4 = 1.856$ Correct to 3 decimal places.

iii) Let $f(x) = x^3 - 9x + 1$

$f(2) = -9 \quad f(3) = 1 \Rightarrow f(2)$ & $f(3)$ are of opposite signs (+1/2)

$\therefore$ root lies between 2 and 3

taking $x_0 = 2,\ x_1 = 3$ so that $f(x_0) = -9$

$f(x_1) = 1$

$\therefore$ By Regula falsi method, Ist Approx.

$$x_2 = x_0 - \frac{(x_1 - x_0)}{f(x_1) - f(x_0)} f(x_0) \qquad (+1/2)$$

$$x_2 = 2 + \frac{9}{10} = \underline{2.9416}$$

Now $f(x_2) = f(2.9) = -0.711$ (+1)

Clearly $f(x_2) < 0 \quad f(x_1) > 0$

roots lie between $2.9\cancel{416}$ and 3

Taking $x_0 = 2.9$, $x_1 = 3$

$f(x_0) = -0.0711$ $f(x_1) = 1$ +1

$$x_3 = 2.9 - \frac{(3-2.9)}{1+0.711}(-0.711) = 2.9416$$

$f(x_3) = f(2.9416) = -0.0207$

$f(x_1) > 0$, $f(x_3) < 0$ ∴ roots lie between 2.9416 and 3

taking $x_0 = 2.9416$, $x_1 = 3$

$f(x_0) = -0.0207$ $f(x_1) = 1$ +1

$$x_4 = 2.9416 - \frac{0.0584}{1.0207}(-0.0207)$$

$$= \underline{\underline{2.9428}}$$

$f(x_4) = -0.0003$

∴ roots lie between 2.9428 and 3

taking $x_0 = 2.9428$, $x_1 = 3$ +1

$$x_5 = 2.9428 - \frac{0.0572}{1.0003}(-0.0003)$$

$x_5 = 2.9428$

Clearly $x_4 = x_5 = 2.9428$

Hence the root is 2.9428

iii) Given equation

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

Since each equation one of the coefficient is larger than the other satisfying the condition for Gauss Siedel method we write the given equations in the following form

$$x = \frac{1}{10}(12 - y - z)$$

$$\left.\begin{aligned} y &= \frac{1}{10}(13-2x-z) \\ z &= \frac{1}{10}(14-2x-2y) \end{aligned}\right\} \longleftarrow (1)$$

+1

We start with $y=0$, $z=0$ and using most recent values of x, y, z we get

+1

$$x^{(1)} = \frac{12}{10} = 1.2$$

$$y^{(1)} = \frac{1}{10}\left(13 - 2x^{(1)}\right) = 1.06$$

$$z^{(1)} = \frac{1}{10}\left(14 - 2x^{(1)} - 2y^{(1)}\right) = 0.948$$

IInd Iteration

$$x^{(2)} = \frac{1}{10}\left(12 - y^{(1)} - z^{(1)}\right) = 0.9992$$ +1

$$y^{(2)} = \frac{1}{10}\left(13 - 2x^{(2)} - z^{(1)}\right) = 1.0068$$

$$z^{(2)} = \frac{1}{10}\left(14 - 2x^{(2)} - 2y^{(2)}\right) = 1.00024$$

IIIrd iteration

$$x^{(3)} = \frac{1}{10}\left(12 - y^{(2)} - z^{(2)}\right) = 0.9992$$ +1

$$y^{(3)} = \frac{1}{10}\left(13 - 2x^{(3)} - z^{(2)}\right) = 1.0001$$

Cheque up to 3 iterations only

$$z^{(3)} = \frac{1}{10}\left(14 - 2x^{(3)} - 2y^{(3)}\right) = 1.0001$$

IV Iteration

$$x^{(4)} = \frac{1}{10}\left(12 - y^{(3)} - z^{(3)}\right) = 0.99998$$

$$y^{(4)} = \frac{1}{10}\left(13 - 2x^{(4)} - z^{(3)}\right) = 0.9999$$

$$z^{(4)} = \frac{1}{10}\left(14 - 2x^{(4)} - 2y^{(4)}\right) = 1.00002$$

V Iteration

$$x^{(5)} = \frac{1}{10}\left(12 - y^{(4)} - z^{(4)}\right) = 1.000$$

$$y^{(5)} = \frac{1}{10}\left(13 - 2x^{(5)} - z^{(4)}\right) = 0.9999$$

$$z^{(5)} = \frac{1}{10}\left(14 - 2x^{(5)} - 2y^{(5)}\right) = 1.00002$$

$\therefore$ The required solution is (+1)

$x = 1.000 \quad y = 0.9999 \quad z = 1.0002$

Q.3

i) Here given interval is equal (+2)

and $h = 0.4$

forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	3.49			
		1.33		
1.4	4.82		-0.19	
		1.14		-0.41
1.8	5.96		-0.6	
		0.54		
2.2	6.5			

We know that, Newton forward interpolation (+1)

$$y(x) = f(x) = y_0 + \frac{u}{\lfloor 1}\Delta y_0 + \frac{u(u-1)}{\lfloor 2}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{\lfloor 3}\Delta^3 y_0 + \ldots$$

$$u = \frac{x - x_0}{h} = \frac{1.6 - 1}{0.4} = 1.5$$ (+1)

$$y(1.6) = f(1.6) = 3.49 + 1.5 \times 1.33 + \frac{1.5 \times 0.5}{2} \times (-0.19) + \frac{1.5 \times 0.5 \times -0.5}{6} \times (-0.41)$$

$$= 3.49 + 1.995 - 0.0712 + 0.026 = 5.4898$$ (+1)

iii) The divided difference table is:

x	$y=f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
4	48				
		52			
5	100		15		
		97		1	
7	294				0
			21		
10	900	202		1	
			27		0
11	1210	310		1	
			33		
13	2028	409			

+2

$\therefore$ Newton's divided difference formula

$$f(x) = f(x_0) + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + \cdots$$

+1

$$f(15) = 48 + (15-4)(52) + (15-4)(15-5)(15)$$ +1

$$+ (15-4)(15-5)(15-7)(1) + 0$$

$$= 48 + 572 + 1650 + 880 = 3150$$ +1

ii) Here $x_0 = 1$ $x_1 = 2$ $x_2 = 3$ $x_3 = 4$ $x_4 = 7$

$y_0 = 2$ $y_1 = 4$ $y_2 = 8$ $y_3 = 16$ $y_4 = 128$ +1

Using lagranges Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0$$ +1

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \cdots$$

$$f(x) = \frac{(x-2)(x-3)(x-4)(x-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2$$ +1

$$+ \frac{(x-1)(x-3)(x-4)(x-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(x-1)(x-2)(x-4)(x-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8$$

$$+ \frac{(x-1)(x-2)(x-3)(x-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(x-1)(x-2)(x-3)(x-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

Putting $x = 5$

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(-1)(-2)(-3)(-6)} \times 2$$ +1

$$+ \frac{(5-1)(5-3)(5-4)(5-7)}{(1)(-1)(-2)(-5)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{2 \times 1 \times (-1) \times (-4)} \times 8$$

$$+ \frac{(5-1)(5-2)(5-3)(5-7)}{3 \times 2 \times 1 \times (-3)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{6 \times 5 \times 4 \times 3} \times 128$$

$$f(5) = \frac{-24}{36} + \frac{-64}{-10} + \frac{-128}{8} + \frac{-768}{-18} + \frac{3072}{360}$$ +1

$$= \underline{\underline{40.93}}$$

Q 4

i) Here we have

$n = 8 \quad h = \frac{b-a}{n} = \frac{10-2}{8} = 1$ +1/2

Since

$x_n = x_0 + nh, \ n = 0, 1, 2, 3 - - 8$

$x: \quad x_0 = 2 \quad x_1 = 3 \quad x_2 = 4 \quad x_3 = 5 \quad x_4 = 6$ +2

$x_5 = 7 \quad x_6 = 8 \quad x_7 = 9 \quad x_8 = 10$

$y = \frac{1}{1+x} \quad y_0 = \frac{1}{3} \quad y_1 = \frac{1}{4} \quad y_2 = \frac{1}{5} \quad y_3 = \frac{1}{6} \quad y_4 = \frac{1}{7}$

$y_5 = \frac{1}{8} \quad y_6 = \frac{1}{9} \quad y_7 = \frac{1}{10} \quad y_8 = \frac{1}{11}$

By Simpson's 1/3 rule +1/2

$$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}\left[(y_0 + y_8) + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6)\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{3} + \frac{1}{11}\right) + 4\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right) + 2\left(\frac{1}{5} + \frac{1}{7} + \frac{1}{9}\right)\right]$$ +1

$$= \frac{1}{3}\left(0.42 + 4 \times 0.64 + 2 \times 0.45\right] = 1.29$$ +1

ii) Here

$f(x, y) = 1 + xy, \ x_0 = 2, \ y_0 = 0$

First Approx. +1.5

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$= 0 + \int_2^x (1 + x \cdot 0)\,dx$$

$$= 0 + [x]_2^x = x - 2$$

$$= x - 2$$

Second Approx.

$$y_2 = y_0 + \int_2^x f(x, y_1)\,dx \qquad +1.5$$

$$= 0 + \int_2^x [1 + x(x-2)]\,dx$$

$$= \int_2^x 1 + x^2 - 2x \; dx$$

$$= \left[x - \cancel{2}\frac{x^2}{\cancel{2}} + \frac{x^3}{3}\right]_2^x$$

$$= \left(x - x^2 + \frac{x^3}{3}\right) - \left(2 - 4 + \frac{8}{3}\right)$$

$$= x - x^2 + \frac{x^3}{3} - \frac{2}{3}$$

Third Approx.

$$y_3 = y_0 + \int_2^x f(x, y_2)\,dx \qquad +2$$

$$= 0 + \int_2^x 1 + x\left(x - x^2 + \frac{x^3}{3} - \frac{2}{3}\right) dx$$

$$= \int_2^x \left(1 + x^2 - x^3 + \frac{x^4}{3} - \frac{2}{3}x\right) dx$$

$$= \left[x - \frac{1}{3}x^2 + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{15}\right]_2^x$$

$$= \left(x - \frac{1}{3}x^2 + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{15}\right) - \left(2 - \frac{4}{3} + \frac{8}{3} - 4 + \frac{32}{15}\right)$$

$$= x - \frac{x^2}{3} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{15} - \frac{22}{15}$$

iii) Given

$$\frac{dy}{dx} = 2y + 3e^{x}, \; x_0 = 0, \; y_0 = 0$$

Since $y' = 2y + 3e^{x} \Rightarrow y_0' = 3$

Differentiating (+2)

$$y'' = 2y' + 3e^{x} \Rightarrow y_0'' = 9$$

$$y''' = 2y'' + 3e^{x} \Rightarrow y_0''' = 21$$

$$y'''' = 2y''' + 3e^{x} \Rightarrow y_0'''' = 45$$

and so on

Using Taylor's series

(+1)

$$y(x) = y_0 + (x - x_0)\, y_0' + \frac{(x-x_0)^2}{2!} y_0'' + \frac{(x-x_0)^3}{3!} y'''(0) + \cdots$$

$$= 0 + x(3) + \frac{x^2}{2!}(9) + \frac{x^3}{3!}(21) + \frac{x^4}{4!}(45) + \cdots$$

(+1)

$$= 3x + \frac{9x^2}{2} + \frac{7x^3}{2} + \frac{15x^4}{8} + \cdots$$

When $x = 0.2$

$$y(0.2) = 3(0.2) + \frac{9}{2}(0.2)^2 + \frac{7}{2}(0.2)^3 + \frac{15}{8}(0.2)^4 + \cdots$$

(+1)

$$\boxed{y(0.2) = 0.8110}$$

Q.5

i) Random variable: Let S be the sample space associated with a given random experiment then a real valued function X which assigns to each outcome $x \in S$ to a unique real number $X(x)$ is called a random variable. (2.5)

It is of two types:

1 Discrete random variable: it takes discrete values

2 Continuous random variable: it takes all possible values between certain limit.

Probability Mass Function (+2.5)

If x is a discrete random variable then its Probability function $p(x)$ is a discrete Probability function it is also called Probability Mass function.

For $x_1, x_2, -- x_n$ n different values of a variable with Probability $p(x_1), p(x_2)$ $-- p(x_n)$ then

1) $p(x_i) \geq 0 \quad i = 1, 2, 3 -- n$

2) $\sum p(x) = 1 \quad i = 1, 2, 3 -- n$

$p(x)$ is p.m.f of the variable X

ii) Variance of Poisson Distribution

$$\sigma^2 = \sum_{r=0}^{\infty} r^2 p(r) - \mu^2 \quad (+\tfrac{1}{2})$$

$$= \sum r^2 \frac{\lambda^r \cdot e^{-\lambda}}{r!} - \lambda^2 \quad (+\tfrac{1}{2})$$

$$= e^{-\lambda} \sum \frac{r^2 \lambda^r}{r!} - \lambda^2 \quad (+\tfrac{1}{2})$$

$$= e^{-\lambda} \left[\frac{1^2 \lambda}{1!} + \frac{2^2 \lambda^2}{2!} + \frac{3^2 \lambda^3}{3!} + -- \right] - \lambda^2 \quad (+1.5)$$

$$= \lambda e^{-\lambda} \left\{ \left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + -- \right) + \left(\frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + -- \right) \right\} - \lambda^2$$

$$= \lambda e^{-\lambda} \{ e^{\lambda} + \lambda (1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots) \} - \lambda^2$$ (+1)

$$= \lambda e^{-\lambda} (e^{\lambda} + \lambda e^{\lambda}) - \lambda^2$$

$$= \lambda + \cancel{\lambda^2} - \cancel{\lambda^2}$$ (+1)

$$\boxed{\text{Variance } \sigma^2 = \lambda}$$

iii) p = Prob of throwing 5 or 6 with one die $= \frac{2}{6} = \frac{1}{3}$ (+1/2)

$q = 1 - p = 2/3$

By Binomial distribution

$$P(x=r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$ (+1/2)

$$P(x=r) = {}^{6}C_{r} \left(\frac{1}{3}\right)^{r} \left(\frac{2}{3}\right)^{6-r}$$ (+1/2)

P(at least 3 dice show 5 or 6)

$= P(3) + P(4) + P(5) + P(6)$ (+1/2)

or $= 1 - [P(x=0) + P(x=1) + P(x=2)]$

$= {}^6C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^3 + {}^6C_4\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_5\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5$ (+1)

$+ {}^6C_6\left(\frac{1}{3}\right)^6$

$= \frac{1}{3^3}\left[20 \times \frac{8}{27} + 15 \times \frac{4}{27} + 6 \times \frac{2}{27} + \frac{1}{27}\right]$

(+1)

$= \frac{233}{729}$

Now $N = 729$ therefore

Required number $= NP = 729 \times \frac{233}{729}$ (+1)

$= \underline{\underline{233}}$

Q6

1) Since f(x) is a p.d.f so

$\int_a^b f(x)\,dx = 1$ (+1)

$\Rightarrow \int_0^1 kx^2\,dx = 1$

$\Rightarrow k\left[\frac{x^3}{3}\right]_0^1 = 1$

$\Rightarrow \boxed{k = 3}$

$\Rightarrow$ f(x) is a p.d.f

Now

$$P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \int_{1/3}^{1/2} f(x)\,dx \qquad +2$$

$$= \int_{1/3}^{1/2} 3x^2\,dx$$

$$= \left[x^3\right]_{1/3}^{1/2} = \frac{1}{8} - \frac{1}{27}$$

$$= \frac{19}{216}$$

To find 'a'

Since $P(x > a) = 0.05$ $\quad +2$

$$\Rightarrow \int_0^{\infty} f(x)\,dx = 0.05$$

$$\Rightarrow \int_a^1 3x^2\,dx = 0.05$$

$$\Rightarrow \left[x^3\right]_a^1 = 0.05$$

$$\Rightarrow 1 - a^3 = 0.05$$

$$\Rightarrow a^3 = 0.95$$

$$\Rightarrow a = (0.95)^{1/3}$$

ii) let x be a random variate measuring burn of bulb

here mean $\mu = 2040$, $\sigma = 60$ hours

The S.N.V $z = \frac{x-\mu}{\sigma} = \frac{x-2040}{60}$ (+1)

When $x = 2150$ hours

$z = \frac{2150-2040}{60} = 1.883$ (+1/2)

$\therefore$ $P(\text{more } 2150) = P(x > 2150) = P(z > 1.883)$

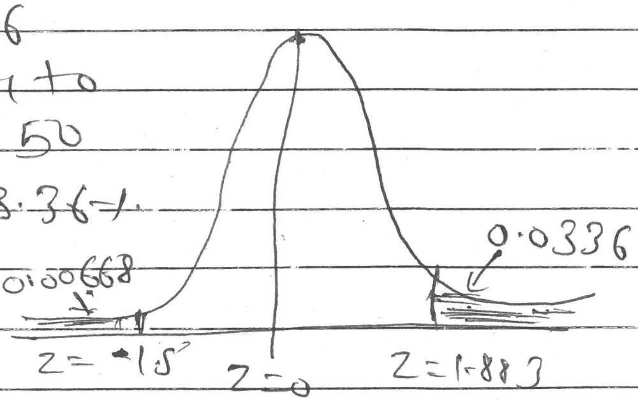
$= P(0 < z < \infty) - P(0 < z < 1.883)$ (+1)

$= \cancel{0.5 - 0.4884}\ 0.5 - 0.4664$

$= 0.0336$

$\therefore$ % of bulbs likely to burn more than 2150

$= 0.0336 \times 100 = 3.36\%$



When $x = 1950$

$z = \frac{1950-2040}{60} = -1.5$ (+1/2)

$P(\text{less } 1950) = P(x < 1950) = P(z < -1.5)$ (+1)

$= P(z > 1.5) = P(0 < z < \infty) - P(0 < x < 1.5)$

$= 0.5 - 0.4332$

$= 0.00668$ (+1)

$\therefore$ % of bulbs likely to burn less 1950

$= 0.00668 \times 100 = 6.68\%$

iii) $f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$

$f(x) = \int_0^\infty \lambda e^{-\lambda x}$

$\frac{1}{\lambda}$ Mean

$\lambda = 2$

$P(X \not< 3) = P(X \geq 3)$ +1

$= \int_0^\infty f(x)\,dx$:

+2

$= \int_3^\infty 2e^{-2x}\,dx$

$= 2\left(\frac{e^{-2x}}{-2}\right)_3^\infty$ +1

$= -[0 - e^{-6}] = e^{-6}$ +1

Mean $= \frac{1}{\lambda} = \frac{1}{2}$ +1