iii. Find the Karl Pearson's coefficient of correlation between x and y for the following data: **5**

x	6	2	4	9	1	3	5	8
y	13	8	12	15	9	10	11	16

Q.6 Attempt any two:

i. Find the student's t-statistic for the following variable values in a sample: **5**

63, 63, 64, 65, 66, 69, 69, 70, 70, 71

Taking mean of the universe to be 65.

ii. A dice is rolled 60 times with the following results **5**

number turned up	1	2 or 3	4 or 5	6
frequency	6	18	24	12

Test the hypothesis that the dice is unbiased $\left(\chi^2_{0.1,\ 3} = 6.25\right)$

iii. In a test given two groups of students drawn from two normal populations, the marks obtained were as follows: **5**

Group A	18	20	36	50	49	36	34	49	41
Group B	29	28	26	35	30	44	46		

Examine at 5% level, whether the two populations have the same variance (Given that $F_{0.05}$= 4.15 for 8 and 6 degrees of freedom respectively).

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering
End Sem Examination Dec-2023
EN3BS15 Engineering Mathematics -III

Programme: B.Tech. Branch/Specialisation: AU/CE/FT/ME/RA

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Notations and symbols have their usual meaning.

Q.1 i. Let h be the interval of differencing, then Shifting operator is defined by? **1**

(a) $Ef(x) = f(x+h)$ (b) $Ef(x) = f(x)$

(c) $Ef(x) = f(x) - f(x-h)$ (d) $Ef(x) = f(x-h)$

ii. If $y = f(x)$ where the values of x are not equi-spaced then which of the following formula is best suited and should be applied to interpolate the corresponding value of y ? **1**

(a) Newton Backward Difference

(b) Newton forward Difference formula

(c) Newton Divided difference formula

(d)Central interpolation formula

iii. To solve ordinary differential equation $\frac{dy}{dx} = f(x, y)$, in which of the following methods the value of y is first predicted and then corrected? **1**

(a) Euler's Modified method (b) Taylor's series method

(c) Runge-Kutta method (d) Milne's method

iv. Which of the following formulae of numerical integration is also known as Parabolic formula? **1**

(a) Simpson's one-third formula

(b) Trapezoidal formula

(c) Weddle's formula

(d) None of these

v. For Poisson's distribution, which of the following relation is correct? **1**
(a) Mean > variance (b) Mean < variance
(c) Mean = variance (d) None of these

vi. The standard normal curve in a Normal distribution is symmetric about: **1**
(a) x-axis (b) y-axis
(c) Both (a) and (b) (d) None of these

vii. If the values of two regression coefficients b_{xy} *and* b_{yx} are 0.4 and 0.9 respectively, then the correlation coefficient between x *and* y is: **1**
(a) 0.4 (b) 0.9 (c) 0.6 (d) 0.36

viii. A coefficient of correlation r_{xy} is computed to be 0.95 means that: **1**
(a) The relationship between two variables x *and* y is perfect
(b) The relationship between two variables x *and* y is of low degree
(c) There is no correlation between two variables x *and* y
(d) The relationship between two variables x *and* y is of high degree

ix. Two samples of sizes 30 and 40 are independently drawn from two normal populations, where the unknown variances are assumed to be equal. The number of degrees of freedom for the equal-variances t-test statistic is: **1**
(a) 30 (b) 40 (c) 68 (d) 70

x. In Chi-square test, one of the assumptions are that, expected frequencies should **not** be: **1**
(a) Less than 5 (b) Greater than 5
(c) Greater than or equal to 5 (d) None of these

Q.2 Attempt any two:

i. Estimate the sale for year 1925 by applying Newton's Backward interpolation formula using the following data. **5**

Year	1891	1901	1911	1921	1931
Sale in thousands	46	66	81	93	101

ii. Apply Lagrange's interpolation formula to find the polynomial f(x) using following data and then find the value of f(6). **5**

x	1	2	7	8
f(x)	4	5	5	4

iii. If Δ is the forward difference operator, ∇ is the backward difference operator and E is the shifting operator provided that h is the interval of differencing, prove that: **5**
(a) $(1+\Delta)(1-\nabla) \equiv 1$
(b) $e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$

Q.3 Attempt any two:

i. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's 1/3 formula by dividing the interval into six equal parts. **5**

ii. Find the first two approximations using Picard's method for the solution of $\frac{dy}{dx} = x + y$; given that $y(0) = 1$ **5**

iii. Solve the differential equation $\frac{dy}{dx} = xy$, with initial condition $y(1) = 2$ by Runge-Kutta method of fourth order, at $x = 1.1$ with $h = 0.1$ **5**

Q.4 Attempt any two:

i. Write any five properties of a normal curve. **5**

ii. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective using Poisson's distribution. **5**

iii. Fit a Binomial distribution to the following data for finding the expected or theoretical frequencies: **5**

x	0	1	2	3	4
f	8	32	34	24	5

Q.5 Attempt any two:

i. Fit a straight line to the following data: **5**

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

ii. Prove that the coefficient of correlation is the geometric mean of the coefficient of regression. **5**

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MCQ

Q.1 (i) (a) $E f(x) = f(x+h)$ +1

(ii) (c) Newton divided difference formula. +1

(iii) (d) Milne's method +1

(iv) (a) Simpson's one-third formula +1

(v) (c) Mean = Variance +1

(vi) (b) y-axis +1

(vii) (c) 0·6 +1

(viii) (d) the relationship between two variables x and y is of high degree. +1

(ix) (c) 68 +1

(x) (a) less than 5. +1

Q. 2 (1)

Year	sale in thousand	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
1891	46				
1901	66	20	-5		
1911	81	15		2	-3
1921	93	12	-3	-1	
1931	101	8	-4		

+2

Newton backward interpolation formula is given by.

$$y = f(x) = y_n + \frac{u}{\lfloor 1} \nabla y_n + \frac{u(u+1)}{\lfloor 2} \nabla^2 y_n + \ldots$$ +1

$$u = \frac{x - x_n}{h} = \frac{1925 - 1931}{10}$$

$$\boxed{u = -0.6}$$ +1

Then, $y_{1925} = f(1925) =$

$$101 + \frac{(-0.6)}{\lfloor 1} \times 8 + \frac{(-0.6)(-0.6+1)}{\lfloor 2} \times -4 +$$

$$\frac{(-0.6)(-0.6+1)(-0.6+2)}{\lfloor 3} \times -1 +$$

$$\frac{(-0.6)(-0.6+1)(-0.6+2)(-0.6+3)}{\lfloor 4} \times -3$$

$$= 101 - 4.8 + 0.48 + 0.056 + 0.1008$$ +1

$y_{1925} = f(1925) = 96.84$ thousands

Ans.

Q.2 (ii)

x	1	2	7	8
$f(x)$	4	5	5	4

Here,

$x_0 = 1,\ x_1 = 2,\ x_2 = 7,\ x_3 = 8$ +1

$y_0 = 4,\ y_1 = 5,\ y_2 = 5,\ y_3 = 4$

Lagrange's formula is given by

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \cdots +$$

+1

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3 \quad \text{--- (1)}$$

$$f(x) = \frac{(x-2)(x-7)(x-8)}{(1-2)(1-7)(1-8)} \times 4 +$$

$$\frac{(x-1)(x-7)(x-8)}{(2-1)(2-7)(2-8)} \times 5 + \frac{(x-1)(x-2)(x-8)}{(7-1)(7-2)(7-8)} \times 5 +$$ +1

$$\frac{(x-1)(x-2)(x-7)}{(8-1)(8-2)(8-7)} \times 4$$

On solving we get.

$$f(x) = \frac{-x^2}{6} + \frac{3}{2}x + \frac{8}{3} \quad \text{--- (2)}$$ +1

Put $x = 6$ in eqn (2)

$f(6) = 5.66$ Ans +1

Q. 2 (iii) (a) By forward difference operator.

$$\Delta f(x) = f(x+h) - f(x)$$
$$\Delta f(x) = E f(x) - f(x)$$

$$\Rightarrow \quad \Delta = E - 1$$
$$\Rightarrow \boxed{E = 1 + \Delta} \quad \text{--- (1)}$$

+1

By Backward difference operator.

$$\nabla f(x) = f(x) - f(x-h)$$
$$\nabla f(x) = f(x) - E^{-1} f(x)$$

$$\Rightarrow \quad \nabla = 1 - E^{-1}$$

$$\boxed{E^{-1} = 1 - \nabla} \quad \text{--- (2)}$$

+1

On taking L.H.S

$$(1+\Delta)(1-\nabla) = (E)(E^{-1})$$
$$= 1 \quad \text{R.H.S.}$$

(b) $e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{E e^x}{\Delta^2 e^x}$

Sol^n $\left(\frac{\Delta^2}{E}\right) e^x = \frac{(E-1)^2}{E} e^x = \left(\frac{E^2 - 2E + 1}{E}\right) e^x$

$$= \left[E - 2 + E^{-1}\right] e^x$$

+1

$$= e^{x+h} - 2e^x + e^{x-h} \quad \text{--- (1)}$$

$$\frac{Ee^x}{\Delta^2 e^x} = \frac{e^{x+h}}{(E-1)^2 e^x} = \frac{e^{x+h}}{(E^2-2E+1)e^x}$$

$$= \frac{e^{x+h}}{e^{x+2h} - 2e^{x+h} + e^x}$$

$$\frac{Ee^x}{\Delta^2 e^x} = \frac{e^h}{e^{x+h} - 2e^x + e^{x-h}} \quad (2) \qquad +1$$

On taking R.H.S.

$$\left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = \left(\frac{e^{x+h} - 2e^x + e^{x-h}}{e^{x+h} - 2e^x + e^{x-h}}\right) \times e^x$$

$$= e^x = \text{L.H.S} \qquad +1$$

Hence proved.

Q.3 Attempt any two.

Soln 3 (i) We divide the range of integration in 6 equal parts taking

$$h = \frac{1-0}{6} = \frac{1}{6} \qquad +1$$

Given $f(x) = \frac{1}{1+x^2}$

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
$f(x) = \frac{1}{1+x^2}$	1	0.97297	0.9	0.8	0.69231	0.59016	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

+2

Simpson's $\frac{1}{3}$ rule is given by

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4(y_1+y_3+y_5+\cdots)\right]$$ +1

$$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{1/6}{3}\left[(1+0.5)+2(0.9+0.69231)+4(0.97297+0.8+0.59016)\right]$$

$$\int_0^1 \frac{1}{1+x^2}\,dx = \underline{\underline{0.78539}}$$ Ans +1

Sol^n 3 (ii) $\frac{dy}{dx} = x+y$, $y(0)=1$

given $f(x,y)=x+y$, $x_0=0$, $y_0=1$

Picards formula is given by

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,dx \quad \text{---}(1)$$ +1

I approximation

Put $n=1$ in eq^n (1)

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$y_1 = 1 + \int_0^x (x+1)\cdot dx$$

$$\boxed{y_1 = 1 + x + \frac{x^2}{2}} \quad (2)$$ +2

Second approximation

Put $n=2$ in equ (1)

$$y_2 = 1 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$$y_2 = 1 + \int_0^x \left[x + \left(1 + x + \frac{x^2}{2}\right)\right] dx$$

$$\boxed{y_2 = 1 + x + x^2 + \frac{x^3}{6}}$$ +2

Sol^n 3 (iii) $\frac{dy}{dx} = xy$, $y(1) = 2$

given $f(x,y) = xy$, $x_0 = 1$, $y_0 = 2$

Taking $h = 0.1$

$x_1 = x_0 + h = 1 + 0.1$ +1

$\boxed{x_1 = 1.1}$

R-K 4th order formula is given by

~~k~~

$K_1 = hf(x_0, y_0)$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$ +2

$K_4 = hf(x_0 + h, y_0 + K_3)$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$ +1

$K_1 = (0.1)(1 \times 2) = 0.2$

$K_2 = (0.1) f(1.05, 2.1) = 0.2205$

$K_3 = (0.1) f(1.05, 2.1103) = 0.2215$

$K_4 = (0.1) f(1.1, 2.2215) = 0.2443$

$K = 0.2583$

Now,

$y_1 = y_0 + K$

$y_1 = y(1.1) = 2 + 0.2583$

$y(1.1) = 2.2583$ +1

Ans

Q.4 Attempt any two! -

Sol^n 4(i)

(a) Normal curve is in bell-shaped. +1

(b) X-axis is asymptote to the curve. +1

(c) The curve is symmetrical about the line $x = \mu$ and x ranges from $-\infty$ $+\infty$ +1

(d) Mean, median and mode coincide at $x = \mu$ as the distribution is symmetrical. +1

(e) The mean deviation from the mean in normal distribution is equal to $\frac{4}{5}$ of its standard deviation. +1

Sol^n 4(ii) Given that $p = \frac{3}{100} = 0.03$ +1

$n = 100$.

$\therefore$ mean $= np = 100 \times 0.03$

$\underline{\underline{m = 3}}$ +1

By Poisson distribution

$$P(r) = e^{-m} \frac{m^r}{\lfloor r}, \quad r = 0, 1, 2, \ldots 100$$ +1

$\therefore$ P (exactly five bulbs are defective)

$= P(r = 5)$

$$= \frac{e^{-3}(3)^5}{\lfloor 5} = \frac{0.04979 \times 243}{120}$$ +1

$$P(r = 5) = \underline{\underline{0.1008}}$$ +1

Sol^n 4 (iii)

x	0	1	2	3	4
f	8	32	34	24	5

$n = 4$, $N = \Sigma f = 103$.

$\text{Mean} = m = \frac{\Sigma fx}{\Sigma f} = \frac{192}{103} = 1.864$ +1

$n = 4$ and mean $= np = 1.864$.

$p = \frac{1.864}{4} = 0.466$ +1

$q = 1 - p = 0.534$

Expected frequencies ~~is~~ given by

$f(r) = N \times {}^nC_r q^{n-r} p^r$ +1

$f(r) = 103 \times {}^4C_r (0.534)^{4-r} (0.466)^r$.

at $r = 0$ $f(0) = 103 \times {}^4C_0 (0.534)^4 (0.466)^0$
$= 8.37 \approx 9$

$r = 1$ $f(1) = 29.235 \approx 29$

$r = 2$ $f(2) = 38.268 \approx 38$ +2

$r = 3$ $f(3) = 22.26 \approx 22$.

$r = 4$ $f(4) = 4.857 \approx 5$

put all the value in normal equns.

$$0 = 9a + 0$$
$$57 = 0 + 60b$$

$\Rightarrow$

$$a = 0, \quad b = 0.95$$

$$Y = 0 + 0.95X$$

$$y - 12 = 0.95(x - 5)$$

$$\boxed{y = 7.25 + 0.95x}$$ +1

Sol^n 5 (ii) Let regression coefficient of x on y is denoted by b_{xy} and is given by

$$b_{xy} = r\frac{\sigma x}{\sigma y}$$ +2

Regression coefficient of y on x is denoted by b_{yx} and is given by

$$b_{yx} = r\frac{\sigma y}{\sigma x}$$ +2

Now,

$$b_{xy} \times b_{yx} = \frac{r\sigma x}{\sigma y} \times r\frac{\sigma y}{\sigma_x}$$

$$b_{xy} \times b_{yx} = r^2$$ +1

Q. 5. Attempt any two :-

~~Sol^n 5(i) Let $y = a + bx$, be the straight line to be fitted.~~

Sol^n. 5(i)

x	y	$X = x-5$	$Y = y-12$	XY	X^2
1	9	-4	-3	12	16
2	8	-3	-4	12	9
3	10	-2	-2	4	4
4	12	-1	0	0	1
5	11	0	-1	0	0
6	13	1	1	1	1
7	14	2	2	4	4
8	16	3	4	12	9
9	15	4	3	12	16
		$\Sigma X = 0$	$\Sigma Y = 0$	$\Sigma XY = 57$	$\Sigma X^2 = 60$

+2

Mean of x-series is 5
" " y-series is 12
Let $X = x-5$, $Y = y-12$

Let the straight line to be fitted be
$$Y = a + bX$$
The normal equ^ns are

$$\Sigma Y = na + b\Sigma X$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

+2

Hence $r = \pm \sqrt{b_{xy} \times b_{yx}}$

Hence proved.

Sol^n 5 (iii)

x	y	$u_i = x-5$	$v_i = y-12$	uv	u^2	v^2
6	13	1	1	1	1	1
2	8	-3	-4	12	9	16
4	12	-1	0	0	1	0
9	15	4	3	12	16	9
1	9	-4	-3	12	16	9
3	10	-2	-2	4	4	4
5	11	0	-1	0	0	1
8	16	3	4	12	9	16
		$\Sigma u_i = -2$	$\Sigma v_i = -2$	53	56	56

+3

Let $u = x-5$, $v = y-12$
where 5, 12 are assumed mean of x & y respt.

$n = 8$

Now,

$$r = \frac{n\Sigma uv - (\Sigma u)(\Sigma v)}{\sqrt{n\Sigma u^2 - (\Sigma u)^2} \times \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$ +1

$$= \frac{8(53) - (-2)(-2)}{\sqrt{8(56) - (-2)^2} \times \sqrt{8(56) - (-2)^2}}$$

$$= \frac{420}{444}$$

$r = 0.946$ Ans +1

Solⁿ Q.6 Attempt any two:-

Solⁿ 6(i)

x	$x-\bar{x}$ $(x-67)$	$(x-\bar{x})^2$
63	−4	16
63	−4	16
64	−3	9
65	−2	4
66	−1	1
69	2	4
69	2	4
70	3	9
70	3	9
71	4	16
$\Sigma x = 670$		88

+1

$\mu = 65$ (given)

$$\text{Mean } (\bar{x}) = \frac{\Sigma x}{n} = \frac{670}{10} = \underline{\underline{67}}$$ +1

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{88}{9}} = \underline{\underline{3.13}}$$ +1

t-test is given by

$$t = \frac{(\bar{x}-\mu)\sqrt{n}}{S}$$ +1

$$t = \frac{(67-65)}{3.13} \times \sqrt{10}$$

$$t = \underline{\underline{2.02}} \text{ Ans}$$ +1

Sol^n 6(ii)

number turned up	frequency (fo)	fe	(fo-fe)	$(fo-fe)^2$	$\frac{(fo-fe)^2}{fe}$
1	6	$\frac{1}{6}\times 60 = 10$	-4	16	1.6
2 or 3	18	$\frac{1}{3}\times 60 = 20$	-2	4	
4 or 5	24	$\frac{1}{3}\times 60 = 20$	4	16	
6	12	$\frac{1}{6}\times 60 = 10$	2	4	

+2

Step1 Null Hypothesis (H_0) - The dice is unbaised. +1

Step2 calculation of expected frequencies (fe)

$N = 60$

$fe(x) = N \times P(x)$

Step3:- Calculation of χ^2 - Statistic

$$\chi^2 = \Sigma\left\{\frac{(fo-fe)^2}{fe}\right\}$$ +1

$$\chi^2 = \frac{16}{10} + \frac{4}{20} + \frac{16}{20} + \frac{4}{10}$$

$$\chi^2 = 3.$$

Also, degree of freedom $= 4-1 = 3$.

Step4:- Calculated value < tabulated value.

$3 < 6.25$

Step5 : Calculated value is < tabulated value

So, Null hypothesis is accepted. +1

Sol^n 6(iii) Given that.

$$n_1 = 9 \text{ and } n_2 = 7$$

$$\bar{x} = \frac{\Sigma x}{n_1} = \frac{333}{9} = 37$$

$$\bar{y} = \frac{\Sigma y}{n_2} = \frac{238}{7} = 34$$

Step1 :- Null Hypothesis (H_0) – two populations have same variance.

$$\sigma_1^2 = \sigma_2^2 .$$ +1

Step2 Calculation of F-Statistics

Group A | Group B

x	$x-\bar{x}$	$(x-\bar{x})^2$	y	$y-\bar{y}$	$(y-\bar{y})^2$
18	−19	361	29	−5	25
20	−17	289	28	−6	36
36	−1	1	26	−8	64
50	13	169	35	1	1
49	12	144	30	−4	16
36	−1	1	44	10	100
34	−3	9	46	12	144
49	12	144			
41	4	16			
$\Sigma x = 333$		1134		$\Sigma y = 238$	386

+1

we have

$$S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{1134}{8} = 141.75$$

$$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1} = \frac{238}{6} = 64.33$$

Clearly, $S_1^2 > S_2^2$ +2

$$F = \frac{S_1^2}{S_2^2} = \underline{\underline{2.203}}$$

Step 3 tabulated value of F at 5% level of significance and for d.o.f. 8 and 6 is 4.15.

Step 4:- ∵ Calculated value < tabulated. +1

⇒ null hypothesis is accepted.

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Verified
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