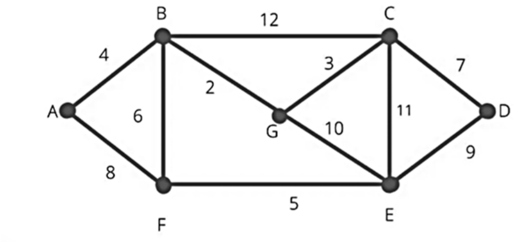
			Marks	BL	PO	CO
	iii.	Find minimal spanning tree for the given graph using Prim's algorithm	5	3	1, 2, 12	2



			Marks	BL	PO	CO
Q.5		Attempt any two:				
	i.	State and prove Lagrange's theorem.	5	3	1, 2, 12	2
	ii.	Prove that union of two subgroups is a subgroup if and only if one is contained in the other.	5	3	1, 2, 12	2
	iii.	Show that the set of fourth roots of unity {1, -1, i, -i} forms an abelian group with respect to multiplication.	5	3	1, 2, 12	2
Q.6		Attempt any two:				
	i.	Solve the recurrence relation: $9a_r - 6a_{r-1} + a_{r-2} = 0$ with $a_0 = 0, a_1 = 1$	5	4	1, 2, 12	4
	ii.	Solve the recurrence relation using generating function: $y_{h+2} + y_h = 5.2^h$	5	4	1, 2, 12	4
	iii.	Apply the generating function technique to solve the initial value problem: $y_{h+1} - 2y_h = 0$ with $y_0 = 1$	5	3	1, 2, 12	2

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem Examination Dec 2024

CA5BS04 Mathematics of Computer Applications

Programme: MCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

			Marks	BL	PO	CO	PSO
Q.1	i.	The value of n_{P_n} is: (a) $(n-1)!$ (b) $(n+1)!$ (c) $n!$ (d) None of these	1	2	1	1	
	ii.	The pigeon hole principle states that if n pigeons are kept into m pigeon holes with $n > m$ then- (a) Each pigeon hole will have atleast one pigeon (b) Atleast one pigeon hole will contain more than one pigeon (c) All pigeon holes will have the same number of pigeons (d) Each pigeon hole will be empty	1	2	1	1	
	iii.	The maximum number of edges in a simple graph with n vertices is- (a) $n(n-1)$ (b) $\frac{n(n+1)}{2}$ (c) $\frac{n(n-1)}{2}$ (d) None of these	1	2	1	1	
	iv.	If sum of degree of all vertices in a connected graph G is 16 then the number of edges in G is ______. (a) 6 (b) 7 (c) 8 (d) 32	1	2	1	1	

v. Which of the following statement is true for all trees? **1** 2 1 1
(a) They contain atleast one cycle
(b) They are always rooted
(c) They have equal number of vertices and edges
(d) Any two vertices are connected by exactly one path

vi. A spanning tree of a graph G is- **1** 2 1 1
(a) Any subgraph of G that includes all vertices and is a tree
(b) Any subgraph of G that is a cycle
(c) A tree that includes only the leaf nodes of G
(d) A graph that includes more edges than G

vii. If (G, .) is a group then $\forall\, a, b \in G, (ab)^{-1} = b^{-1}a^{-1}$ is called ____ law. **1** 2 1 1
(a) Commutative (b) Reversal
(c) Absorption (d) None of these

viii. The order of the element *'i'* of the multiplicative group G={1, -1, i, -i } is- **1** 2 1 1
(a) 1 (b) 2 (c) 3 (d) 4

ix. The order of the recurrence relation- **1** 2 1 1
$y_{h+2} + 5y_{h+1} - 3y_h = 3h + 4$ is
(a) 3 (b) 2
(b) (c) 1 (d) None of these

x. If the roots of second order linear recurrence relation are 2 and 3 respectively then the homogeneous solution is given by- **1** 2 1 1
(a) $C_1 2^r + C_2 3^r$ (b) $C_1 2^{-r} + C_2 3^{-r}$
(c) $(C_1 + C_2 r)2^r$ (d) None of these

Q.2 Attempt any two:

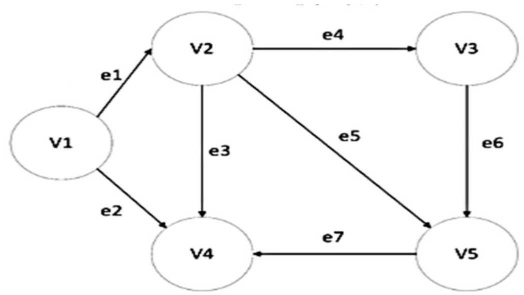
i. Prove that $7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25; $n \in N$ using mathematical induction. **5** 3 1, 2, 12 2

ii. How many different numbers can be formed by the digits 2,3,4,5,6,7 between 2000 and 3000? How many of these numbers are divisible by 5? **5** 3 1, 2, 12 2

iii. Show that $\sqrt{5}$ is an irrational number using contradiction method. **5** 3 1, 2, 12 2

Q.3 i. Define Hamiltonian graph with example. **2** 2 1 1

ii. Find the incidence matrix of the following directed graph. **3** 3 1, 2, 12 2

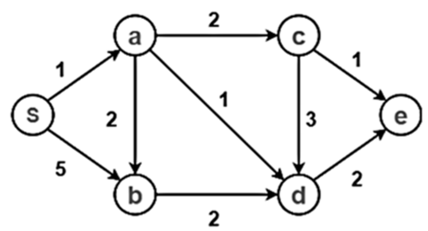


iii. Prove that the sum of the degrees of all vertices in a graph is equal to twice the number of edges. **5** 3 1, 2, 12 2

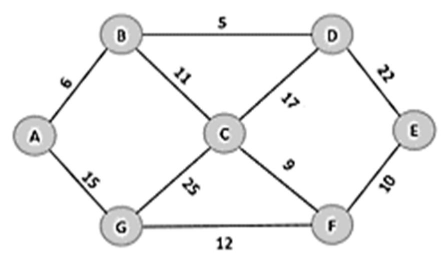
OR iv Prove that in any graph, the number of vertices of odd degree is always even. **5** 3 1, 2, 12 2

Q.4 Attempt any two:

i. Find the shortest path from the vertex 's' to 'e' for the given graph using Dijkstra's algorithm. **5** 3 1, 2, 12 2



ii. Find minimal spanning tree for the given graph using Kruskal's algorithm. **5** 3 1, 2, 12 2



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Programme: MCA/BCA-MCA(Int.) Branch: CA.

Q1.

i) (c) $n!$ 1
ii) (b) Atleast one pigeon hole will contain more than one pigeon. 1
iii) (c) $\frac{n(n-1)}{2}$ 1
iv) (c) 8 1
v) (d) Any two vertices are connected by exactly one path. 1
vi) (a) Any subgraph of G that includes all vertices and is tree. 1
vii) (b) Reversal 1
viii) (d) 4 1
ix) (b) 2 1
x) (a) $C_1 2^n + C_2 3^n$ 1

Q.2
(i) Let $P(n) = 7^{2n} + 2^{3n-3} \cdot 3^{n-1}$

$n=1$ $P(1) = 7^2 + 2^0 \cdot 3^0 + 49+1 = 50 = 25 \times 2$

$\Rightarrow$ $P(1)$ is divisible by 25. 1

Let $P(m)$ is divisible by 25.

$\Rightarrow$ $7^{2m} + 2^{3m-3} \cdot 3^{m-1} = 25k, \; k \in Z^+$ 2

Now $P(m+1) = 7^{2(m+1)} + 2^{3(m+1)-3} \cdot 3^{(m+1)-1}$

$= 7^{2m} \cdot 7^2 + 2^{3m-3} \cdot 2^3 \cdot 3^{m-1} \cdot 3$

$= 49 \cdot 7^{2m} + 24 \; 2^{3m-3} \cdot 3^{m-1}$

$= (50-1) 7^{2m} + (25-1) 2^{3m-3} \cdot 3^{m-1}$

$= 25(2.7^{2m} + 2^{3m-3}.3^{m} - 1) - (7^{2m} + 2^{3m-3}3^{m} - 1)$
$= 25(2.7^{2m} + 2^{3m-3}.3^{m} - 1) - 25k$ (1)

$\Rightarrow P(m+1)$ is divisible by 25

$\therefore$ By mathematical induction 25 divides $P(n)$ $\forall n \in \mathbb{N}$. (1)

Q2 ii.

Given digits are 2, 3, 4, 5, 6, 7
$\therefore$ no. lie between 2000 and 3000
starts with '2' so, first place is fixed
Now the remaining 3 places ~~are~~ can
be filled by 6-digits i.e 2, 3, 4, 5, 6, 7. (1)

Number of different no. formed by 2, 3, 4, 5, 6, 7
b/w 2000 and 3000

$= {}^1C_1 \times {}^6C_1 \times {}^6C_1 \times {}^6C_1$

$= 1 \times 6 \times 6 \times 6$

$= \boxed{216}$ Ans. (2)

No. b/w 2000 to 3000 divisible by 5 and
formed by 6-digits 2, 3, 4, 5, 6, 7.

First place is fixed by 2 and last ~~digit~~
place is fixed by 5.

$= {}^1C_1 \times {}^6C_1 \times {}^6C_1 \times {}^1C_1$
$= 1 \times 6 \times 6 \times 1$
$= \boxed{36}$ Ans. (2)

Q2.iii)

We shall prove this result by contradictive method.

Let $\sqrt{5}$ is a rational number.

So, $\sqrt{5} = p/q$, $q \neq 0$, $p, q \in \mathbb{Z}$ and are co-primes

On squaring both the sides. 1

$5q^2 = p^2 \Rightarrow \cancel{5/p}\ 5 | p^2 = 5 | p$ — 1 1

$\Rightarrow \exists\ r \in \mathbb{Z}$ st.

$5r = p$

$\Rightarrow 25 \cdot r^2 = p^2$

$\Rightarrow 25 r^2 = 5 q^2$

$\Rightarrow 5 r^2 = q^2$

$\Rightarrow 5 | q^2 \Rightarrow 5 | q$ — 2 2

By 1 and 2

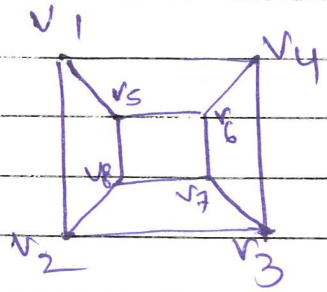
$\Rightarrow$ 5 divides p as well as q which is contradiction as p, q are co-primes

Hence our assumption is wrong.
So, $\sqrt{5}$ is an irrational number. 1

Q3.

i) Hamiltonian Graph:

A Graph $G(V, E)$ is called Hamiltonian graph if it contains a Hamiltonian circuit i.e. it must contain closed walk which transverses every vertix of G exactly once except the starting vertix. 1



is a hamiltonian graph.

$\because \exists$ closed walk.

$v_1 \to v_4 \to v_6 \to v_7 \to v_3 \to v_2 \to v_8 \to v_5 \to v_1$ 1

Q3.

ii)

$$\begin{array}{c|c} & \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

3

Q3

iii) <u>proof</u> : Let $G(V, E)$ be a graph
Let $|E| = e$

Then to prove $\sum_{v \in V} deg(v) = 2e$

We shall prove this by using Mathematical Induction.

If $|E| = e = 0$

$\Rightarrow deg\ v = 0 \quad \forall v \in V$

$\Rightarrow \sum_{v \in V} deg(v) = 0 = 2 \times 0 = 2e$

$\therefore$ case is true (1)

If $|E| = e = 1$

$\Rightarrow \exists\ v_1, v_2$ st. $deg(v_1) = deg(v_2) = 1$
or $deg(v_1) = 2$ (self loop)
& deg of remaining vertices is zero.

$\Rightarrow \sum_{v \in V} deg(v) = deg(v_1) + deg(v_2) + \dots$
$= 2 \times 1$
$= 2e$

$\therefore$ case is true. (1)

Now assume theorem is true for all graphs having $e-1$ edges.

Let G be a graph having e edges.
Now delete one edge say $e' = (a, b)$
from G

Thus, the new graph G'(say) obtained having e-1 edges where $G' = G - \{e'\}$

$\therefore$ In G' graph.

$$\Sigma \deg(v) = 2(e-1)$$ 1.5

Now, sum of edges in graph G
$= e-1+1$

So RHS $= 2(e-1+1) = 2e$.

LHS $= \Sigma \deg v = 2(e-1)+2$
$= 2e-2+2$
$= 2e$ 1.5

$\therefore$ LHS = RHS.

Hence the theorem.

Q.3
OR iv)

Let G(V,E) be a graph, where V is the set of vertices & E is the set edges in G.

Let $V_e \rightarrow$ set of vertices of even degree
$V_o \rightarrow$ set of vertices of odd degree.

St. $V = V_e \cup V_o$ and $V_e \cap V_o = \phi$

Hence,

$$\sum_{v \in V} \deg v = \sum_{v \in V_e} \deg v + \sum_{v \in V_o} \deg v \quad \text{—} \ (1)$$ 1

$\because v \in V_e \therefore \sum_{v \in V_e} \deg v = 2K$ (even no.) 1

And also $\sum_{v \in V} \deg v = 2e$ where $|E| = e$ (1)

by (1)

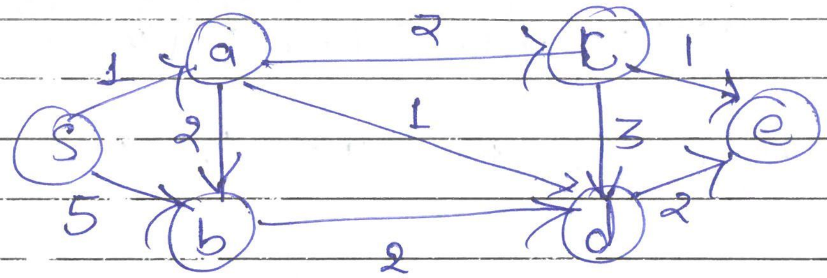
$$2e = 2k + \sum_{v \in V_0} \deg(v)$$

$$\sum_{v \in V_0} \deg v = 2e - 2k$$
$$= 2(e-k) \text{ is an even no.}$$ (2)

Hence the theorem

Q4.

i)



Let $G(V, E)$ be a graph

Here $V = \{a, b, c, d, e, s\}$

$$P_1 = \{s\} \quad T_1 = \{a, b, c, d, e\}$$

$l(b) = 5 \quad l(a) = 1 \quad l(c) = \infty \quad l(d) = \infty \quad l(e) = \infty$

$\therefore a \in T_1$ has min index $= 1$ (1)

Taking $P_2 = \{s, a\} \quad T_2 = \{b, c, d, e\}$

$$l(b) = \min\{5, 1+2\} = 3$$
$$l(c) = \min\{\infty, 1+2\} = 3$$
$$l(d) = \min\{\infty, 1+1\} = 2$$
$$l(e) = \min\{\infty, 1+\infty\} = \infty$$

$\therefore d \in T_2$ has min index $= 2$ (1.5)

Taking $P_3 = \{s, a, d\}$, $T_3 = \{b, c, e\}$

$l(b) = \min\{3, 2+\infty\} = 3$

$l(c) = \min\{3, 2+\infty\} = 3$

$l(e) = \min\{\infty, 2+2\} = 4$

$c \in T_3$ has min index $= 3$

Taking $P_4 = \{s, a, d, c\}$ $T_4 = \{b, e\}$

$l(b) = \min\{3, 3+\infty\} = 3$

$l(e) = \min\{4, 3+1\} = 4$

$b \in T_4$ has min index $= 3$. 1.5

Taking $P_5 = \{s, a, d, c, b\}$ $T_5 = \{e\}$

$l(e) = \min\{4, 3+\infty\} = 4$

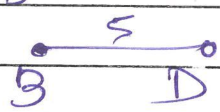
Shortest path $s \to a \to c \to e$. 1

Q4

ii)

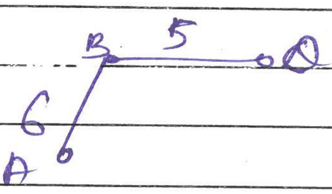
Edge	(B,D)	(A,B)	(C,F)	(F,E)	(B,C)	(G,F)	(A,G)	(C,D)	(D,E)	(C,G)
Cost	5	6	9	10	11	12	15	17	22	25

min cost $= 5$ from $B \to D$ 1



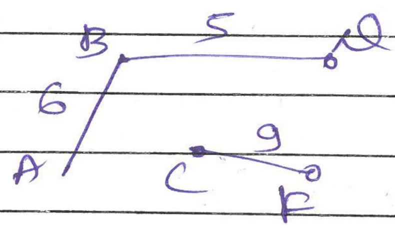
Next min cost

$B \to A = 6$



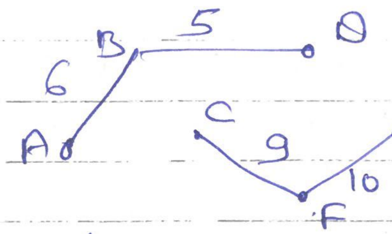
next least cost $C \to F = 9$

$6 + 5 + 9 = 20$

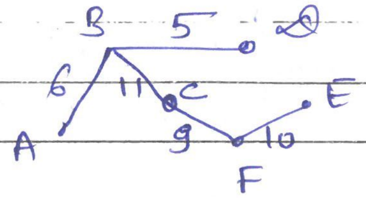


c

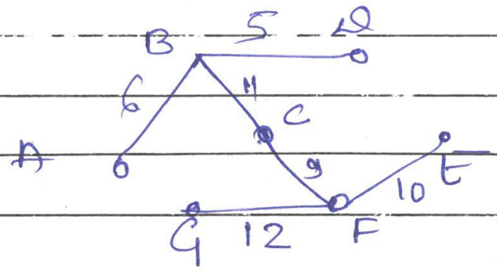
Next min cost F → E = 10
min cost = 5 + 6 + 9 + 10 = 30



next min cost B → C = 11
min cost = 5 + 6 + 9 + 10 + 11 = 41 1



next min cost G → F = 12
min cost = 5 + 6 + 9 + 10 + 11 + 12 = 53 1



The result obtained is the minimum spanning tree of the given graph with cost = 53. 1

~~Q4~~
~~(iii)~~

Q4
iii)

$$
\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ F \\ G
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\
\left[\begin{array}{ccccccc}
- & 4 & \infty & \infty & \infty & 8 & \infty \\
4 & - & 12 & \infty & \infty & 6 & 2 \\
\infty & 12 & - & 7 & 11 & \infty & 3 \\
\infty & \infty & 7 & - & 9 & \infty & \infty \\
\infty & \infty & 11 & 9 & - & 5 & 10 \\
8 & 6 & \infty & \infty & 5 & - & \infty \\
\infty & 2 & 3 & \infty & 10 & \infty & -
\end{array}\right]
\end{array}
$$

4

$A \rightarrow B \rightarrow G \rightarrow C \rightarrow D \rightarrow E \rightarrow F$ 1

$4 + 2 + 3 + 7 + 9 + 5$

$= 16 + 14$

$= \boxed{30}$

Q5
i) Lagrange Theorem: If H is a subgroup of the finite group G, then the order of H divides the order of group G. 1

~~proof~~ proof: Let H be a subgroup, $o(H) = m$ & let G be a group $o(G) = n, n \geq m$.

Consider the left coset decomposition of G relative to H.

Let $a \in G$ then aH is left coset of G.

To prove aH has m[$m = o(H)$] distinct elements

Suppose $h_1, h_2, \ldots h_m \in H$ (are m distinct element)
then, $aH = \{ah_1, ah_2, \ldots ah_m\}$

$\because h_i \neq h_j, i \neq j \quad h_i, h_j \in H$
~~$\Rightarrow ah_i \neq ah_j \quad i \neq j$~~

If $ah_i = ah_j \quad i \neq j$
$\Rightarrow h_i = h_j \quad i \neq j$ which is a contradiction.

Therefore, each left coset of H has m distinct members. 2

Now we can decompose G into disjoint left coset of H in G.
& no. of such cosets will be finite say (K) Since $o(G)$ is finite.

$$G = a_1H \cup a_2H \cup \cdots \cup a_kH \ , k \in \mathbb{Z}$$
$\&$ $|a_iH| = m \quad \forall i$

$\Rightarrow \ o(G) = mk \ , k \in \mathbb{Z}$
$n = mk$
$k = n/m$

$k = o(G)/o(H)$

$\Rightarrow o(H)$ divides $o(G)$ 2

Hence the Theorem.

Q5
ii) proof: Suppose $H_1 < G$, G is a group.
$H_2 < G$
[$\Rightarrow$] Let $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$
Then
$H_1 \cup H_2 = H_1$ or H_2
$\Rightarrow H_1 \cup H_2$ is a subgroup of G. 2

[$\Leftarrow$] Suppose $H_1 \cup H_2$ is a Subgroup of G.
To prove $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$

On a contrary assume $H_1 \nsubseteq H_2$ and $H_2 \nsubseteq H_1$

$\Rightarrow \exists \ a \in H_1$ and $a \notin H_2$ — 1
$\exists \ b \in H_2$ and $b \notin H_1$ — 2
by 1 & 2 1
$b, a \in H_1 \cup H_2$

$\because H_1 \cup H_2$ is a subgroup of G.

$\Rightarrow ab \in H_1 \cup H_2$

$\Rightarrow ab \in H_1$ or $ab \in H_2$

Suppose $ab = c \in H_1 \Rightarrow b = a^{-1}c \in H_1$

which is a contradiction of $b \notin H_1$

or

Suppose $ab = c \in H_2 \Rightarrow a = b^{-1}c \in H_2$

which is a contradiction of $a \notin H_2$

Hence either $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$ 2

Q5

iii) let $G = \{1, -1, i, -i\}$ set of fourth root of unity. To prove $(G, \cdot)$ is an abelian group.

composition table.

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

1

1 Closure property:

$\because$ every entry in a composition table is belong to set G

$\Rightarrow$ statisfy closure property. 0.5

ii) Associative property : This property is purely based on the binary operation & multiplication satisfy associative property.

0.5

iii) Existence of identity :
from the composition table we can easily observe that.

1

$\forall a \in G \; \exists \; 1 \in G$ st $a.1 = a = 1.a$.

$\Rightarrow 1 \in G$ is an identity element.

iv) Existence of inverse :
from the composition table we can observe that.

$\forall a \in G \; \exists \; a^{-1} \in G$ st $aa^{-1} = 1 = a^{-1}a$.

i.e $1.1 = 1 = 1.1$

$-1 \, -1 = 1 = -1 \, -1$

$-i \; i = 1 = i(-i)$

1

v) Commutative law :
Since composition table is symmetric about diagonal

$\Rightarrow ab = ba \; \forall \; a, b \in G$

i.e $i(-i) = (-i)i = 1$ etc.

Hence $(G, \circ)$ is an abelian group.

1

Q 6
i) The characteristic equation is

$$9m^2 - 6m + 1 = 0$$
$$(3m-1)^2 = 0$$
$$m = 1/3, 1/3$$

The general solution of given recurrence formula is.

$$a_r = (C_1 + C_2 r)(1/3)^r \quad \text{---} (1)$$ +2

putting $r=0$ in (1)

$$a_0 = C_1$$
$$\Rightarrow \boxed{0 = C_1}$$ 1

and.

$r = 1$ in (1)

$$a_1 = (C_1 + C_2)\, 1/3$$
$$a_1 = (0 + C_2) 1/3 = C_2\, 1/3$$
$$1 = C_2(1/3)$$
$$\Rightarrow \boxed{C_2 = 3}$$ 1

put C_1 & C_2 in (1)

$$a_r = 3r(1/3)^r \text{ or } \frac{r}{3^{r-1}}$$ 1

Q6 ii)

$$y_{n+2} + y_n = 5.2^n$$

Characteristic equation. $m^2 + 1 = 0$

$\Rightarrow m = \pm i$

$= \cos \pi/2 \pm i \sin \frac{\pi}{2}$ (1)

Homogeneous solution

$A \cos \frac{\pi n}{2} + B \sin \frac{\pi n}{2}$ (1)

Particular solution. $= \frac{1}{1+E^2} 5.2^n$

$= \frac{1}{1+2^2} 5.2^n$ (2)

$= 2^n$

General solution

$y_n = A \cos \frac{\pi n}{2} + B \sin \frac{\pi n}{2} + 2^n$ (1)

Q6 iii)

Given $y_{n+1} - 2y_n = 0$ —(1)

Consider the generating function $Y(t)$

$Y(t) = \sum_{n=0}^{\infty} y_n t^n = y_0 + y_1 t + y_2 t^2 + \cdots$ —(2) (+1)

multiplying (1) by t^n & summing from $n=0$ to $n=\infty$

we get

$$\sum_{n=0}^{\infty} y_{n+1} t^n - 2 \sum_{n=0}^{\infty} y_n t^n = 0$$ (+1)

$$(y_1 + y_2 t + y_3 t^2 + \cdots) - 2Y(t) = 0 \quad [\text{by } ②]$$

$$\frac{1}{t}(y_1 t + y_2 t^2 + \cdots) - 2Y(t) = 0$$

$$\frac{Y(t) - y_0}{t} - 2Y(t) = 0$$

$$Y(t) - y_0 - 2t\,Y(t) = 0$$

$$(1-2t)\,Y(t) = y_0$$

$$Y(t) = \frac{y_0}{1-2t} = \frac{1}{1-2t} \quad [y_0 = 1]$$

$$Y(t) = (1-2t)^{-1}$$

+2

$$\sum_{n=0}^{\infty} y_n t^n = 1 + 2t + (2t)^2 + \cdots + (2t)^n + \cdots$$

Equating the coefficient of t^n

we get $y_n = 2^n$ which is a required sol^n +1