Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2022

EN3BS11 Engineering Mathematics -I

Programme: B.Tech.　　　　Branch/Specialisation: All

Duration: 3 Hrs.　　　　**Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If Eigen Values of a matrix is 1, -1,0 then the given matrix is- **1**

(a) Singular　　(b) Non-singular
(c) Can't predict　　(d) None of these

ii. The rank of Null Matrix is- **1**

(a) 0
(b) 1
(c) -1
(d) Without order can't be determined

iii. Euler's theorem is applicable for which of the following function **1**

(a) Continuous　　(b) Homogeneous
(c) Nonhomogeneous　　(d) None of these

iv. The derivative of function of several variable with respect to one of the variables keeping all other variable to be constant is called as- **1**

(a) Ordinary derivative　　(b) Partial derivative
(c) Both (a) and (b)　　(d) None of these

v. The value of $\Gamma 1/2$ is- **1**

(a) $\sqrt{\pi}$　　(b) π　　(c) 1　　(d) None of these

vi. The gamma function is defined as the definite integral $\Gamma n =$ ____. **1**

(a) $\int_0^\infty e^{-t}t^{n-1}$dt　　(b) $\int_0^1 e^{-t}t^{n-1}$dt
(c) $\int_0^\infty e^{-t}t^{n+1}$dt　　(d) None of these

vii. The order and degree of the differential equation $\left(1+\frac{dy}{dx}\right)^{3/2}=\frac{d^2y}{dx^2}$ is: **1**

(a) 1, 1　　(b) 2, 2　　(c) 3, 3　　(d) None of these

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viii. Solution of the differential equation $\frac{d^5y}{dx^5} - \frac{d^3y}{dx^3} = 0$ is: **1**

(a) $y = (c_1 + c_2x + c_3x^2) + c_4e^x + c_5e^{-x}$
(b) $y = (c_1x + c_2x + c_3x^2) + c_4e^x + c_5e^{-x}$
(c) $y = (c_1 + c_2 + c_3) + c_4e^x + c_5e^{-x}$
(d) None of these

ix. The equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ are: **1**

(a) Cauchy Reimann Equations (b) Cauchy Theorem statements
(c) Both (a) and (b) `` (d) None of these

x. A real valued function $u = u(x, y)$ is called harmonic function, if: **1**

(a) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = 0$ (b) $\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} = 0$
(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (d) None of these

Q2. i. Find the rank of matrix by reducing it into normal form- **3**

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$$

ii. Investigate for what values of λ and μ, the simultaneous equations $x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ Have (a) no solution (b) a unique solution (c) an infinite number of solutions. **7**

OR iii. Find all the Eigen values and Eigen Vectors of the matrix **7**

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q.3 Attempt any two:

i. If $u = f(r)$ where $r^2 = x^2 + y^2$ then show that- **5**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

ii. Expand $log_e x$ in powers of $(x - 1)$ and hence evaluate $\log 1.1$ correct to 4 decimal places **5**

iii. Discuss Maxima or Minima of Function, **5**

$$U = sinx + siny + sin(x + y)$$

Q.4 i. Using Definite Integral, find Sum of below given series: **3**

$$\lim_{n\to\infty}\left(\frac{1}{n+`1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}\right)$$

ii. Change the order of integration and hence evaluate the same$\int_0^1 \int_{x^2}^{2-x} xydxdy$? **7**

OR iii. State and prove relation between $\beta(m, n)$ and $\Gamma(n)$ functions? **7**

Q5 Attempt any two:

i. Solve the given differential equations $\frac{dy}{dx} = \frac{3xy+y^2}{3x^2}$ **5**

ii. Solve the given pair of simultaneous differential equations: **5**

$$\frac{dx}{dt} + y = sint \ \& \ \frac{dy}{dt} + x = cost$$

iii. Solve the given differential equation $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$ **5**

Q.6 Attempt any two:

i. If $f(z)$ is analytic function with constant modulus. Show that $f(z)$ is constant. **5**

ii. Prove that $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate function v. **5**

iii. Using Cauchy's integral formula, prove that $\int \frac{e^{2z}}{(z+1)^4}dz = \frac{8\pi e^{-2}}{3}i$ over the curve $|z| = 3$ **5**

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Q.1
(i) (a) Singular
(ii) (a) 0
(iii) (b) Homogeneous
(iv) (b) Partial derivative
(v) (a) $\sqrt{\pi}$
(vi) (a) $\int_0^\infty e^{-t} t^{n-1} dt$
(vii) (b) 2, 2
(viii) (a) $y = (c_1 + c_2 x + c_3 x^2) + c_4 e^{x} + c_5 e^{-x}$
(ix) (a) Cauchy Reimann Equations
(x) (c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Q.2
(i) $A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$

$R_{21}(-4),\ R_{31}(-2)$

$\sim \begin{bmatrix} 1 & -1 & 2 & -1 \\ 0 & 6 & -9 & 6 \\ 0 & 4 & -6 & 2 \end{bmatrix}$

$C_{21}(1),\ C_{31}(-2),\ C_{41}(1)$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & -9 & 6 \\ 0 & 4 & -6 & 2 \end{bmatrix}$ [1]

$C_2(1/6)$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -9 & 6 \\ 0 & 2/3 & -6 & 2 \end{bmatrix}$

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$R_{32}\left(-\frac{2}{3}\right)$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -9 & 6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$C_{32}(9)$, $C_4(1/2)$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$C_3 \sim C_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$C_{32}(-3)$, $R_3(-1)$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$= [I_3 \mid 0]$$

$\therefore \quad \rho(A) = 3$

(ii) $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ 4 \end{bmatrix}$$

Its augmented matrix is

$$[A \mid B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & 4 \end{array}\right]$$

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$R_{21}(-1)$, $R_{31}(-1)$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_{32}(-1)$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$ [4]

Case I : If $\lambda \neq 3$ & $\mu = 10$ or $\neq 10$

$\rho[A:B] = \rho[A] = 3$

System posses unique solution [5]

Case II : If $\lambda = 3$ & $\mu = 10$

$\rho[A:B] = \rho[A] < 3$

System posses infinite no. of solutions [6]

Case III : If $\lambda = 3$ & $\mu \neq 10$

$\rho[A:B] \neq \rho[A]$

System posses no solution. [7]

(iii) The characteristic equation of matrix A is

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$

$\Rightarrow \lambda = 2, 2, 8$ [2]

Eigen vector of A corresponding to $\lambda = 8$ is

$(A - 8I)X = 0$

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$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ [4]

$R_{21}(-1)$, $R_{31}(1)$

$$\sim \begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_{32}(-1)$

$$\sim \begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \quad -2x_1 - 2x_2 + 2x_3 = 0$

$-3x_2 - 3x_3 = 0 \Rightarrow x_2 = -x_3$

take $x_3 = 1 \Rightarrow x_2 = -1$ & $x_1 = 2$

$\therefore \quad X_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ is eigen vector corresponding to $\lambda = 8$ [5]

Eigen vector of A corresponding to $\lambda = 2$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_{21}(\frac{1}{2})$, $R_{31}(-\frac{1}{2})$

$$\sim \begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ [6]

$\therefore \quad 4x_1 - 2x_2 + 2x_3 = 0.$

take $x_1 = -1,\ x_2 = 0 \Rightarrow x_3 = 2$

~~f~~ if we take $x_1 = 1,\ x_2 = 2$ then $x_3 = 0$

$$\therefore X_2 = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix},\quad X_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$ [1]

are two linearly independent solutions of A corresponding to $\lambda = 2$.

3(i) As $r^2 = x^2 + y^2$

$$\therefore \frac{\partial r}{\partial x} = \frac{x}{r} \quad \& \quad \frac{\partial r}{\partial y} = \frac{y}{r}.$$ [1]

Now $u = f(r)$

$$\Rightarrow \frac{\partial u}{\partial x} = f'(r)\frac{\partial r}{\partial x} = f'(r)\frac{x}{r}$$ [2]

$$\therefore \frac{\partial^2 u}{\partial x^2} = f''(r)\frac{\partial r}{\partial x}\frac{x}{r} + f'(r)\left[\frac{r \cdot 1 - x\frac{\partial r}{\partial x}}{r^2}\right]$$

$$= f''(r)\frac{x^2}{r^2} + f'(r)\left[\frac{r - \frac{x^2}{r}}{r^2}\right]$$

$$= f''(r)\frac{x^2}{r^2} + f'(r)\left[\frac{r^2 - x^2}{r^3}\right]$$ [3]

Similarly

$$\frac{\partial^2 u}{\partial y^2} = f''(r)\frac{y^2}{r^2} + f'(r)\left(\frac{r^2 - y^2}{r^3}\right)$$ [4]

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r)\left(\frac{x^2+y^2}{r^2}\right) + f'(r)\left[\frac{2r^2 - [x^2+y^2]}{r^3}\right]$$

$$= f''(r) + f'(r)\frac{1}{r}.$$ [5]

H.P //

(ii) Let $f(x) = \log_e x = f(1 + x - 1)$

By

$f(x) = f(1 + (x-1)) = f(1) + (x-1) f'(1) + \frac{(x-1)^2}{2!} f''(1) + \cdots$ [1]

$f'(x) = \frac{1}{x}$ $\quad f'(1) = 1$ [2]

$f''(x) = -\frac{1}{x^2}$ $\quad f''(1) = -1$

$f'''(x) = \frac{2}{x^3}$ $\quad f'''(1) = 2$

$f^{iv}(x) = \frac{-6}{x^4}$ $\quad f^{iv}(1) = -6$ [3]

and $f(1) = 0$.

$\therefore \log_e x = (x-1) \cdot 1 + \frac{(x-1)^2}{2!}(-1) + \frac{(x-1)^3}{3!}(2)$

$+ \frac{(x-1)^4}{4!}(-6) + \cdots$ [4]

put $x = 1.1$ we get

$\log_e 1.1 = (.1) - \frac{(.1)^2}{2} + \frac{(.1)^3}{3} - \frac{(.1)^4}{4} + \cdots$

$= 0.0953$ [5]

(iii) $\frac{\partial u}{\partial x} = \cos x + \cos(x+y) = 0$

$\frac{\partial u}{\partial y} = \cos y + \cos(x+y) = 0$

$\Rightarrow \cos x = \cos y \Rightarrow x = y$. [1]

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$\Rightarrow \cos x + \cos 2x = 0$

$\Rightarrow \cos x + 2\cos^2 x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

or $\cos x = -1 = \cos\pi$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}$ or $x = 2n\pi \pm \pi$ [2]

$r = -\sin x - \sin(x+y)$

$s = -\sin(x+y)$

$t = -\sin y - \sin(x+y)$ [3]

at $x = y = \pi/3$

$r = -\sqrt{3}$

$s = -\frac{\sqrt{3}}{2}$

$t = -\sqrt{3}$

$rt - s^2 = \frac{9}{4} = +ve$

& r is $-ve$ so there exist [4]
maximal at $\left(2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{\pi}{3}\right)$

at $x = y = \pi$

$r = 0$, $s = 0$, $t = 0$

$\therefore rt - s^2 = 0$

So there exist neither maxima nor minima
at $(2n\pi \pm \pi, 2n\pi \pm \pi)$ [5]

Q 4

(i) r^{th} term of the series is $\frac{1}{n+r} = \frac{1}{n} \cdot \frac{1}{1+\frac{r}{n}}$

$$\therefore \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{1+\frac{r}{n}} \cdot \frac{1}{n}$$ [1]

lower limit of integration $= \lim_{n\to\infty} \frac{r}{n}; (r=1) = 0$

upper " " " $= \lim_{n\to\infty} \frac{r}{n}; (r=n) = 1$ [2]

$\therefore$ The limit of summation of the given series is

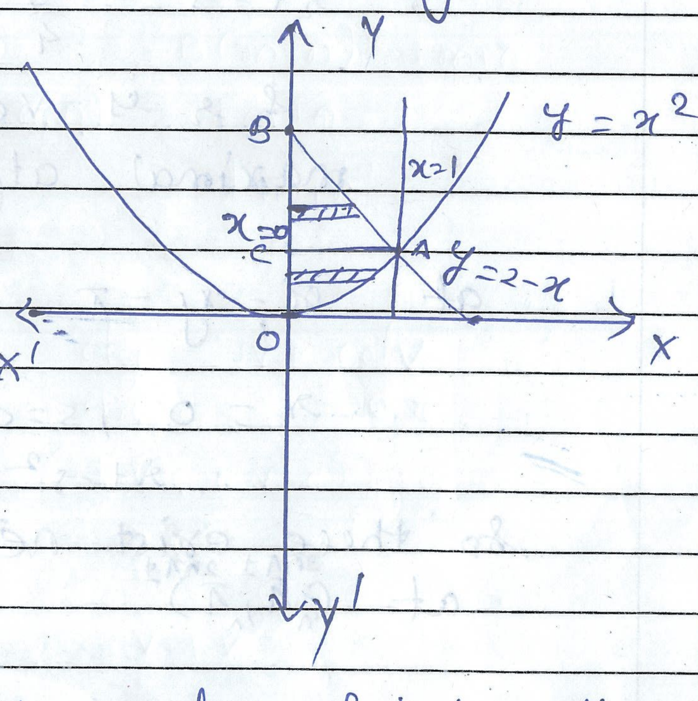
$$\int_0^1 \frac{1}{1+x}\,dx = \left[\log(1+x)\right]_0^1$$

$$= \log 2 - \log 1$$

$$= \log 2.$$ [3]

(ii) y varies from x^2 to $2-x$

$y = 2-x$



[2]

On changing the order of integration we get

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$$\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy = \int_0^1\int_0^{\sqrt{y}} xy\,dy\,dx$$

$$+\int_1^2\int_0^{2-y} xy\,dy\,dx \quad [3]$$

$$= \int_0^1 y\left(\frac{x^2}{2}\right)_0^{\sqrt{y}} dy + \int_1^2 y\left(\frac{x^2}{2}\right)_0^{2-y} dy$$

$$= \int_0^1 y\cdot\frac{y}{2} + \int_1^2 y\,\frac{(2-y)^2}{2}\,dy \quad [4]$$

$$= \left(\frac{y^3}{3\cdot 2}\right)_0^1 + \frac{1}{2}\int_1^2 (4y+y^3-4y^2)\,dy$$

$$= \frac{1}{6} + \frac{1}{2}\left[2y^2+\frac{y^4}{4}-\frac{4y^3}{3}\right]_1^2$$

$$= \frac{1}{6}+\frac{1}{2}\left[8+4-\frac{32}{3}-2-\frac{1}{4}+\frac{4}{3}\right] \quad [5]$$

$$= \frac{1}{6}+\frac{1}{2}\left[\frac{39}{4}-\frac{28}{3}\right]$$

$$= \frac{1}{6}+\frac{1}{2}\left[\frac{117-112}{12}\right] \quad [6]$$

$$= \frac{1}{6}+\frac{5}{24}$$

$$= \frac{9}{24} \quad [7]$$

(iii) As we know that

$$\Gamma n = \int_0^\infty e^{-zx}\,x^{n-1}\,z^n\,dx \quad [1]$$

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$$\therefore \Gamma n\, e^{-z} z^{m-1} = \int_0^\infty e^{-z(1+x)} x^{n-1} z^{m+n-1}\, dx \quad [3]$$

Integeate both sides with respect to z between the limits 0 to ∞ we get

$$\Gamma n \int_0^\infty e^{-z} z^{m-1} dz = \int_0^\infty x^{n-1}\left[\int_0^\infty e^{-z(1+x)} z^{m+n-1} dz\right] dx \quad [4]$$

$$\text{put } z(1+x) = y \Rightarrow dz = \frac{dy}{1+x} \quad [5]$$

$$\Gamma n \int_0^\infty e^{-z} z^{m-1} dz = \int_0^\infty x^{n-1}\left[\int_0^\infty e^{-y} \frac{y^{m+n-1}}{(1+x)^{m+n-1+1}} dy\right] dx \quad [6]$$

$$\Gamma n\, \Gamma m = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx \int_0^\infty e^{-y} y^{m+n-1} dy$$

$$\Gamma n\, \Gamma m = B(m,n)\, \Gamma m+n$$

$$\Rightarrow B(m,n) = \frac{\Gamma m\, \Gamma n}{\Gamma m+n} \quad [7]$$

Q5
(i)

$$\text{put } y = vx \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{3x\, vx + v^2x^2}{3x^2} \quad [1]$$

$$v + x\frac{dv}{dx} = \frac{3v + v^2}{3}$$

$$x\frac{dv}{dx} = \frac{3v + v^2}{3} - v \quad [2]$$

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$$x\frac{dv}{dx} = \frac{3v+v^2-3v}{3}$$

$$\frac{dv}{v^2} = \frac{dx}{3x}$$ [3]

On integration we get

$$\frac{v^{-1}}{-1} = \frac{1}{3}\log x + \log c.$$ [4]

$$\frac{1}{y/x} + \frac{1}{3}\log x + \log c = 0$$

$$\frac{x}{y} + \frac{\log x}{3} + \log c = 0$$ [5]

(ii) $$Dx + y = \sin t$$
$$x + Dy = \cos t$$

$$D^2x + Dy = \cos t$$ [1]
$$x + Dy = \cos t$$

$$\therefore D^2x - x = 0$$ [2]

Its AE'q $m^2 - 1 = 0$
$$m = \pm 1$$

$$\therefore \boxed{x = c_1e^t + c_2e^{-t}}$$ [3]
$$Dx = c_1e^t - c_2e^{-t}$$

$$c_1e^t - c_2e^{-t} + y = \sin t$$
$$\boxed{y = \sin t - c_1e^t + c_2e^{-t}}$$ [5]

(iii) $$D^3y + 3D^2y + 2Dy = x^2$$

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Its A.E is

$m^3 + 3m^2 + 2m = 0$

$m[m^2 + 3m + 2] = 0$

$m(m+1)(m+2) = 0$

$m = 0, -1, -2$ [2]

Its C.F is

$C_1 e^{0x} + C_2 e^{-x} + C_3 e^{-2x}$ [3]

Its P.I is

$$\frac{1}{D^3 + 3D^2 + 2D} x^2$$

$$= \frac{1}{2D\left[1 + \frac{D^2}{2} + \frac{3}{2}D\right]} x^2$$ [4]

$$= \frac{1}{2D}\left[1 + \left(\frac{D^2}{2} + \frac{3D}{2}\right)\right]^{-1} x^2$$

$$= \frac{1}{2D}\left[\left(1 - \left(\frac{D^2}{2} + \frac{3D}{2}\right) + \frac{(-1)(-2)}{2!}\left(\frac{D^2 + 3D}{2}\right)^2 + \ldots\right] x^2\right]$$

$$= \frac{1}{2D}\left[x^2 - \left(1 + \frac{3}{2}(2x)\right) + \left(\frac{0 + 9(2) + 0}{4}\right)\right]$$

$$= \frac{1}{2D}\left[x^2 + \frac{7}{2} - 3x\right]$$

$$= \frac{1}{2}\left[\frac{x^3}{3} + \frac{7}{2}x - \frac{3x^2}{2}\right]$$

$$= \frac{x^3}{6} + \frac{7x}{4} - \frac{3x^2}{4}$$ [5]

$y = c_1 + c_2 e^{-x} + c_3 e^{2x} + \frac{x^3}{6} + \frac{7x}{4} - \frac{3x^2}{4}$

Q 6
(i) Let $f(z)$ be an analytic function with constant modulus.

Let $f(z) = u + iv$

By C-R Equation

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

also $|f(z)| = \text{constant}$

$\therefore \; |u+iv| = c \quad \text{(say)}$

$\sqrt{u^2+v^2} = c$ [1]

$\Rightarrow u^2+v^2 = c^2$.

Partially diff. wrt x we get

$$2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x} = 0 \Rightarrow u u_x + v v_x = 0 \quad (1)$$

Partially diff. wrt y we get

$$2u\frac{\partial u}{\partial y} + 2v\frac{\partial v}{\partial y} = 0 \Rightarrow u u_y + v v_y = 0$$

using C-R eq.

$$-u v_x + v u_x = 0 \quad (2)$$ [2]

Squaring (1) & (2) & adding we get

$$(u u_x + v v_x)^2 + (-u v_x + v u_x)^2 = 0$$

$$u^2 u_x^2 + v^2 v_x^2 + \cancel{2uv u_x v_x} + u^2 v_x^2 + v^2 u_x^2 - \cancel{2uv u_x v_x} = 0$$

$$u^2(u_x^2 + v_x^2) + v^2(u_x^2 + v_x^2) = 0$$

$$(u_x^2+v_x^2)(u^2+v^2)=0$$

$$(u_x^2+v_x^2)\,c^2=0$$

$$\Rightarrow u_x^2+v_x^2=0$$

$$\Rightarrow u_x=0,\ v_x=0$$ [3]

Now

$$f'(z)=\frac{dw}{dz}=\frac{\partial w}{\partial x}$$

$$=\frac{\partial(u+iv)}{\partial x}$$

$$=0+i0$$

$$=0$$

$$\Rightarrow f(z)=\text{constant}.$$ [5]

i) $u=e^{-2xy}\sin(x^2-y^2)$

$$\frac{\partial u}{\partial x}=-2ye^{-2xy}\sin(x^2-y^2)+e^{-2xy}2x\cos(x^2-y^2)$$ [1]

$$\frac{\partial^2 u}{\partial x^2}=(-2y)(-2y)e^{-2xy}\sin(x^2-y^2)$$

$$+(-2y)e^{-2xy}2x\cos(x^2-y^2)$$

$$+2\Big[(-2y)e^{-2xy}x\cos(x^2-y^2)$$

$$+e^{-2xy}\left[\cos(x^2-y^2)+x\cdot 2x(-\sin(x^2-y^2))\right]\Big]$$

$$=4y^2e^{-2xy}\sin(x^2-y^2)-4xye^{-2xy}\cos(x^2-y^2)$$

$$-4xy\,e^{-2xy}\cos(x^2-y^2)+2e^{-2xy}\cos(x^2-y^2)$$

$$-4x^2e^{-2xy}\sin(x^2-y^2)$$ [2]

$$\frac{\partial u}{\partial y}=-2xe^{-2xy}\sin(x^2-y^2)+e^{-2xy}(-2y)\cos(x^2-y^2)$$

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$$\frac{\partial^2 u}{\partial y^2} = (-2x)(-2x)e^{-2xy}\sin(x^2-y^2) + (-2x)e^{-2xy}(-2y)\cos(x^2-y^2)$$

$$+ (-2x)e^{-2xy}(-2y)\cos(x^2-y^2)$$

$$-2e^{-2xy}\left[1\cos(x^2-y^2) + y(-\sin(x^2-y^2)\cdot(-2y))\right]$$

$$= 4x^2e^{-2xy}\sin(x^2-y^2) + 4xy\,e^{-2xy}\cos(x^2-y^2)$$

$$+ 4xy\,e^{-2xy}\cos(x^2-y^2) - 2e^{-2xy}\cos(x^2-y^2)$$

$$- 4y^2e^{-2xy}\sin(x^2-y^2)$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$\Rightarrow$ u is harmonic. [3]

$$\phi_1(x,y) = \frac{\partial u}{\partial x}$$

$$\therefore \phi_1(z,0) = 2z\cos z^2$$

$$\phi_2(z,0) = -2z\sin z^2$$

$$f(z) = \int[\phi_1(z,0) - i\phi_2(z,0)]dz + ic$$ [4]

$$= \int(2z\cos z^2 + i\,2z\sin z^2)\,dz + ic$$

put $z^2 = t \Rightarrow 2z\,dz = dt$

$$= \int(\cos t + i\sin t)\,dt + ic$$

$$= \int e^{+it}\,dt$$

$$= -ie^{+it}$$

$$= -ie^{+iz^2}$$

$$= -ie^{+i(x^2-y^2+2xiy)}$$

$$= -ie^{+i(x^2-y^2)-2xy}$$

$$= -ie^{-2xy}[\cos(x^2-y^2) + i\sin(x^2-y^2)]$$

$= e^{-2xy}\left[\sin(x^2-y^2) - i\cos(x^2-y^2)\right]$

$\therefore\ v = -e^{-2xy}\cos(x^2-y^2)$ [5]

iii) as (-1) lies inside curve

$\therefore \int_C \frac{e^{2z}}{(z+1)^4} = \frac{2\pi i}{3!} f'''(-1)$; where $f(z) = e^{2z}$ [7]

$\therefore\ f'''(z) = 8e^{2z}$ [3]

hence $\int_C \frac{e^{2z}}{(z+1)^4} = \frac{\pi i}{3}\, 8e^{-2}$

$= \frac{8\pi i e^{-2}}{3}$ [5]