Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering
End Sem Examination May-2023
RA3CO31 Automatic Control Systems

Programme: B.Tech. Branch/Specialisation: RA

Duration: 3 Hrs. Maximum Marks: 60

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. What is a traffic light system? **1**
(a) Closed loop system (b) Open loop system
(c) Both (a) and (b) (d) None of these

ii. Which of the following is an open loop control system? **1**
(a) Ward Leonard control (b) Metadyne
(c) Stroboscope (d) Field controlled D.C. motor

iii. Control system are normally designed to be- **1**
(a) Overdamped (b) Under damped
(c) Un damped (d) Critically damped

iv. What will be the nature of time response if the roots of the characteristic equation are located on the s-plane imaginary axis? **1**
(a) Oscillation (b) Damped oscillations
(c) No oscillations (d) Under damped oscillations

v. Which plots in frequency domain represent the two separate plots of magnitude and phase against frequency in logarithmic value? **1**
(a) Polar plots (b) Bode plots
(c) Nyquist plots (d) All of these

vi. How is the sinusoidal transfer function obtained from the system transfer function in frequency domain? **1**
(a) Replacement of 'jω' by 's' (b) Replacement of 's' by 'ω'
(c) Replacement of 's' by 'jω' (d) Replacement of 'ω' by 's'

vii. The order of the auxiliary polynomial is always- **1**
(a) Even (b) Odd
(c) May be even or odd (d) None of these

viii. Routh Hurwitz criterion gives- **1**
(a) Number of roots in the right half of the s-plane
(b) Value of the roots
(c) Number of roots in the left half of the s-plane
(d) Number of roots in the top half of the s-plane

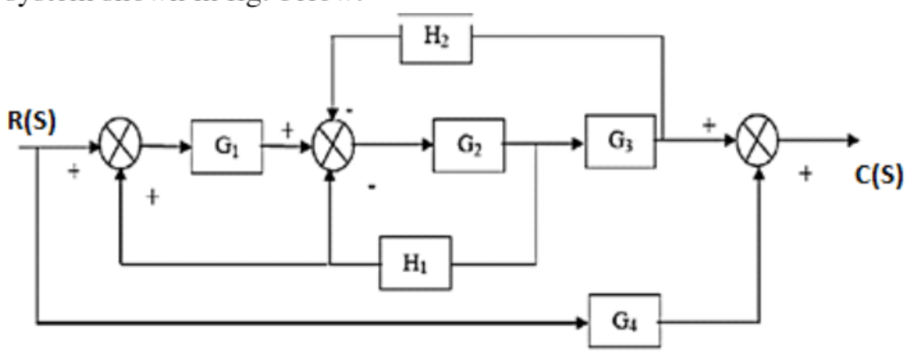
ix. The angles of asymptotes of the root loci of the equation $s^3+5s^2+(K+2)s+K=0$ are- **1**
(a) 0° and 270° (b) 0° and 180°
(c) 90° and 270° (d) 90° and 180°

x. If a feedback control system has its open loop transfer function $G(s)H(s) = K/(s-2)(s^2+3s+5)$ has the root locus plot which intersects the imaginary axis at s =0, then the value of K at this point will be- **1**
(a) -5 (b) 10 (c) 5 (d) -10

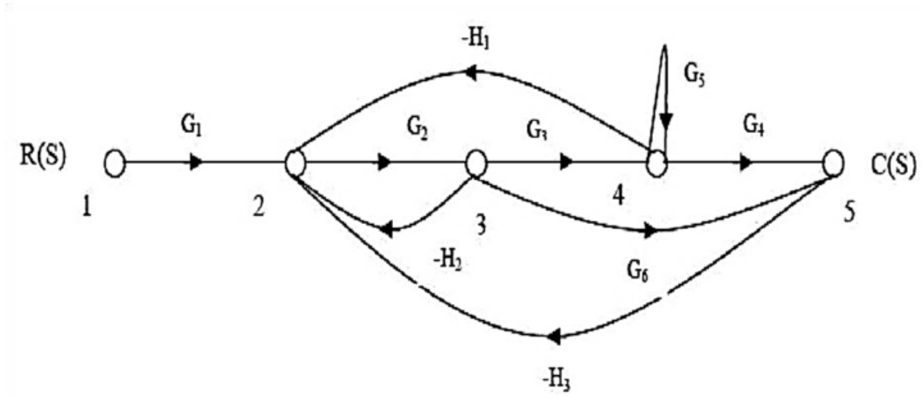
Q.2 i. Write masons gain formula. **2**

ii. Distinguish between open loop and closed loop control system. **3**

iii. Block diagram reduction technique finds the transfer function for the system shown in fig. below. **5**



OR iv. Find the overall gain C(S) / R(S) for the signal flow graph shown in fig. below. **5**



Q.3 i. Distinguish between type and order of a system. **2**

ii. Derive the expressions and draw the response of first order system for unit Step input. **8**

OR iii. With a neat block diagram and Derivation explain how PI, PD and PID compensation will improve the time response of the system. **8**

Q.4 i. What are the main advantages of Bode plot **3**

ii. Sketch the Bode plot for the following transfer function and determine the phase margin and gain margin. **7**

$$G(S) = \frac{20}{S(1+3S)(1+4S)}$$

OR iii. Sketch the polar plot for the following transfer function and find gain cross over frequency, phase cross over frequency, gain margin and phase margin. **7**

$$G(S) = \frac{400}{S(S+2)(S+10)}$$

Q.5 i. Explain gain margin and phase margin. **4**

ii. Construct Routh array and determine the stability of the system whose characteristicequation is- **6**

$$S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16S+16 = 0$$

OR iii. Draw the Nyquist plot for the system whose open loop transfer function is- **6**

$$G(S) = \frac{K}{S(S+2)(S+10)}$$

Q.6 Attempt any two:

i. Explain the method of calculating breakaway point. **5**

ii. Describe the following terms: **5**
(a) Asymptotes (b) Centroid

iii. What is root locus? Explain with suitable example. **5**

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Solution: RA3CO31 Automatic control systems (faculty: Prabhat Pandey)

Q.1	i)	B. Open loop system
	ii)	D. Field controlled D.C. motor
	iii)	B. Under damped
	iv)	C. No oscillations
	v)	B. Bode plots
	vi)	C. Replacement of 's' by 'jω'
	vii)	A. Even
	viii)	A. Number of roots in the right half of the s-plane
	ix)	C. 90° and 270°
	x)	B. 10

Q2. i. Write masons gain formula

Answer: The relation between an input variable and an output variable of a signal flow graph is given by Mason's Gain Formula.

For determination of the overall system, the gain is given by:

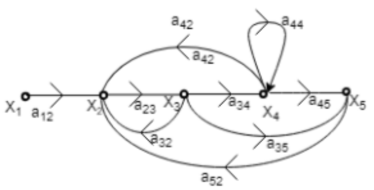
$$T = \frac{\sum_{K=1}^{n} P_K \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Where,

P_k = forward path gain of the K^{th} forward path.

Δ = 1 - [Sum of the loop gain of all individual loops] + [Sum of gain products of all possible of two non-touching loops] + [Sum of gain products of all possible three non-touching loops] + ...

Δ_k = The value of Δ for the path of the graph is the part of the graph that is not touching the K^{th} forward path.



Forward Path

From the above SFG, there are two forward paths with their path gain as -

$$P_1 = a_{12}\, a_{23} a_{34} a_{45}$$

$$p_2 = a_{12} a_{23} a_{35}$$

$L_1 = a_{23}a_{32}$

$L_2 = a_{23}a_{34}a_{42}$

$L_3 = a_{44}$

$L_4 = a_{23}a_{34}a_{45}a_{52}$

$L_5 = a_{23}a_{35}a_{52}$

Non-Touching Loops

There are two possible combinations of the non-touching loop with loop gain product as -

$$L_1.L_2 = a_{23}a_{32}a_{44}$$

$$L_5L_3 = a_{23}a_{52}a_{35}a_{44}$$

In above SFG, there are no combinations of three non-touching loops, 4 non-touching loops and so on.

Where,

$\Delta = 1 - [\ L_1 + L_2 + L_3 + L_4 + L_5\ ] + [\ L_1 \,.\, L_3 + L_5 \,.\, L_3\ ]$

$\Delta = 1 - [\ a_{23}a_{32} + a_{23}a_{34}a_{42} + a_{44} + a_{23}a_{34}a_{45}a_{52} + a_{23}a_{35}a_{52}\ ] + [a_{23}a_{32}a_{44} + a_{23}a_{35}a_{52}a_{44}\ ]$

$P_1 = a_{12}\, a_{23}\, a_{34}\, a_{45}$

$P_2 = a_{12}\, a_{23}\, a_{35}$

$\Delta_1 = 1$

$\Delta_2 = 1 - a_{44}$

$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

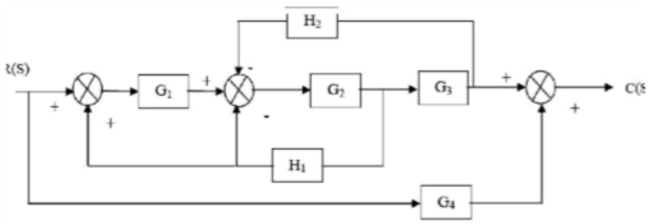
$$T = \frac{a_{12}a_{23}a_{34}a_{45}-1 + a_{12}a_{23}a_{35}(1-a_{44})}{[1-a_{23}a_{32}+a_{23}a_{34}a_{42}+a_{44}a_{23}a_{34}a_{45}a_{52}+a_{23}a_{35}a_{52}+a_{23}a_{32}a_{44}+a_{23}a_{35}a_{52}a_{44}]}$$

Q2. ii. difference between Open-Loop Control System and Closed-Loop Control System

Answered: The following table highlights all the major differences between open loop control system and closed loop control system –

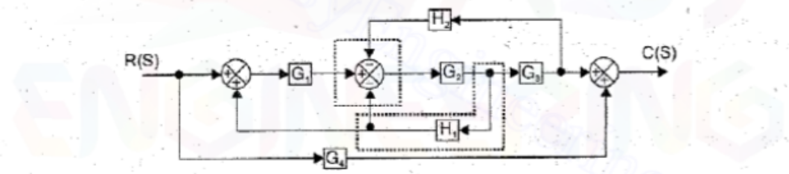
Basis of Difference	Open Loop Control System	Closed Loop Control System
Definition	A control system in which there is no feedback path is provided is called an *open loop control system.*	The control system in which there is a feedback path present is called a *closed loop control system.*
Also called	Open loop control system is also called non-feedback control system.	Closed loop control system is also called a feedback control system.
Control action	In open loop control system, the control action is independent of the output of the overall system.	In closed loop control system, the control action is dependent on the output of the system.
Design complexity	The design and construction of an open loop control system is quite simple.	Closed loop control system has comparatively complex design and construction.
Main Components	The major components of an open loop control system are – controller and plant.	The main components of a closed loop control system are – Controller, plant or process, feedback element and error detector (comparator).
Response	Open loop control system has fast response because there is no measurement and feedback of output.	The response of the closed loop control system is slow due to presence of feedback.
Reliability	The reliability of open loop control system is less.	The closed loop control system is more reliable.
Accuracy	The accuracy of open loop control system depends upon the system calibration and therefore, may be less.	Closed loop control system is comparatively accurate because the feedback maintains its accuracy.
Stability (in terms of output)	The stability of open loop control system is more, i.e., the output of the open loop system remains constant.	Closed loop control system is comparatively less stable.
Optimization	The open loop control system is not optimized.	Closed loop control system is optimized to produce the desired output.
Maintenance	Open loop control system requires less maintenance.	Comparatively more maintenance is needed in closed loop control system.
Implementation	Open loop control system is easy to implement.	The implementation of a closed loop control system is relatively difficult.
Cost	Open loop control system is less expensive.	The cost of the closed loop control system is relatively high.
Noise	Open loop control system has more internal noise.	In closed loop system, the internal noise in the system is less.
Examples	Common practical examples of open loop control systems are – automatic traffic light system, automatic washing machine, immersion heater, etc.	Examples of closed loop control systems include: ACs, fridge, toaster, rocket launching system, radar tracking system, etc.

Q2.iii. Block diagram reduction technique finds the transfer function for the system shown in fig

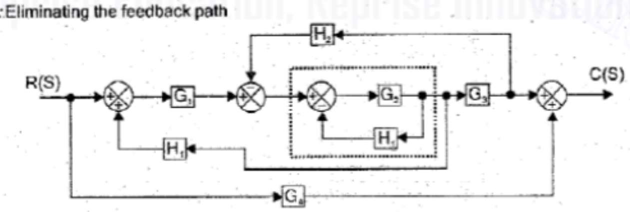


SOLUTION

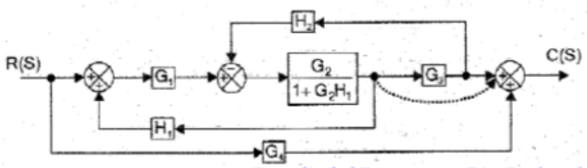
Step 1: Splitting the summing point and rearranging the branch points



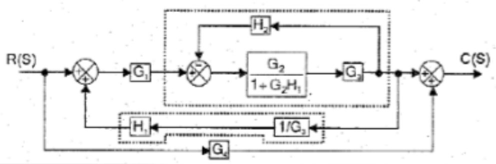
Step 2: Eliminating the feedback path



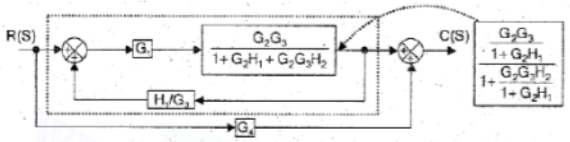
Step 3: Shifting the branch point after the block.



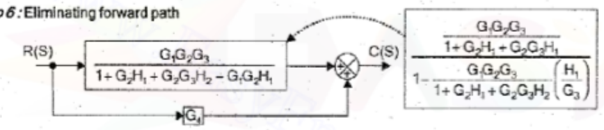
Step 4: Combining the blocks in cascade and eliminating feedback path



Step 5: Combining the blocks in cascade and eliminating feedback path

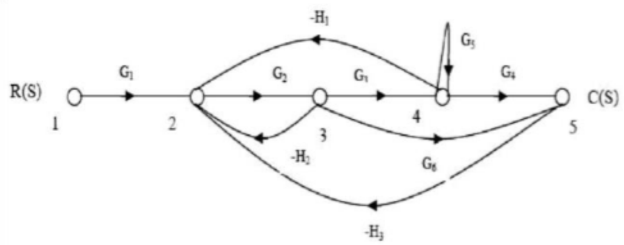


Step 6: Eliminating forward path



$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2-G_1G_2H_1} + G_4$$

OR IV .Find the overall gain C(S) / R(S) for the signal flow graph shown in fig.



SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore K=2$. Let the forward path gains be P_1 and P_2.

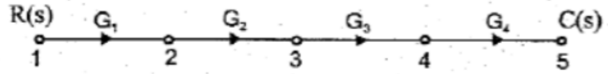


Fig 2 : Forward path-1

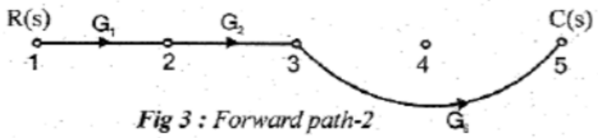


Fig 3 : Forward path-2

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

Individual Loop Gain

There are five individual loops. Let the individual loop gains be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

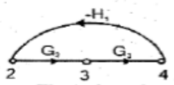


Fig 4 : loop-1

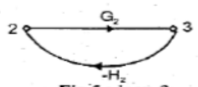


Fig 5 : loop-2

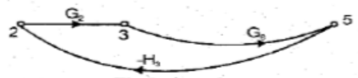


Fig 6 : loop-3

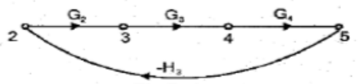


Fig 7 : loop-4

Fig 8 : loop-5

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.
Let the gain products of two non-touching loops be P_{12} and P_{22}.

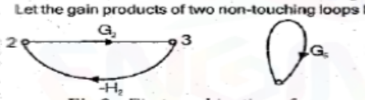


Fig 9 : First combination of two non-touching loops

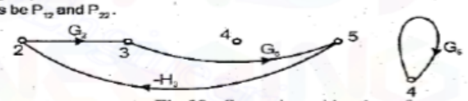


Fig 10 : Second combination of two non-touching loops

Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41}+P_{51})+(P_{12}+P_{22})$$
$$= 1-(-G_2G_3H_1-H_2G_2-G_2G_3G_4H_3+G_5-G_2G_6H_3)$$
$$+(-G_2H_2G_5-G_2G_5G_6H_3)$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

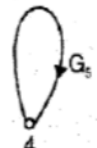


Fig 11

$\therefore \Delta_2 = 1 - G_5$

Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$T = \frac{1}{\Delta}\sum_K P_K\Delta_K$ (Number of forward path is 2 and so K = 2)

$$= \frac{1}{\Delta}[P_1\Delta_1+P_2\Delta_2] = \frac{1}{\Delta}[G_1G_2G_3G_4\times 1+G_1G_2G_6(1-G_5)]$$

$$= \frac{G_1G_2G_3G_4+G_1G_2G_6-G_1G_2G_5G_6}{1+G_2G_3H_1+H_2G_2+G_2G_3G_4H_3-G_5+G_2G_6H_3-G_2H_2G_5-G_2G_5G_6H_3}$$

Q3. 1 Distinguish between type and order of a system.

Answer: The Type of the system is the number of poles present at the origin. The Order of the system is the total number of poles.

- Type number is specified for loop transfer function but Order can be specified for any transfer function.
- The Type number is given by number of poles of loop transfer function lying at origin of S- Planebut the order is given by the number of poles of transfer function.

ii. Derive the expressions and draw the response of first order system for unit Step input.

Answer:

The type of system having '1' as the maximum power of 's' in the denominator of the transfer function of the control system is known as the **first-order system**. Thus, we can say, that the order of the system is specified by the highest power of s.

Basically the order of the system shows information regarding the closed-loop poles of the system. Thus for the first-order system, we can say that there will be one clo Time Response of First Order System

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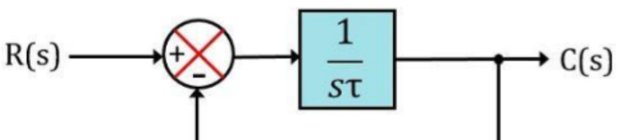
Basically the order of the system shows information regarding the closed-loop poles of the system. Thus for the first-order system, we can say that there will be one closed-loop pole.

Here we can see the block diagram of unity negative feedback first order control system:



Here in this article, we are about to discuss the time response of the first-order control system. Now the question arises-

What is time response?

The time response of the system is defined as the output of the system obtained by providing specific input to the system, where both input and output must be the function of time.

This means that the time response of the system provides an idea about the variation in output when a certain input is provided with respect to time.

Basically the time response of the system is composed of steady-state response and transient response and is given as:

$$c(t) = c_{ss}(t) + c_t(t)$$

The transient response represents the fluctuation in the output of the system on applying input before achieving the final value

While steady-state response represents the finally achieved output of the system.

Sometimes the finally achieved value shows variation from that of the desired value. Thus the difference of desired value and achieved value shows the steady-state error of the system, represented by e_{ss}.

The closed-loop transfer function of the system is given as:

$$\frac{G(s)}{1 + G(s)H(s)}$$

For unity negative feedback system, the characteristic equation gives the poles of the system

$$1 + G(s)H(s) = 0$$

So, in general form, the first order equation will be:

$$1 + \tau s = 0$$

Thus the closed-loop pole will be:

$$s = -\frac{1}{\tau}$$

Time Response of First-Order System

We know that to determine the response of the system, some input must be provided to it. So, here we will consider different inputs and will see the response of each input on the first-order control system.

For unit step signal as input

Since the open-loop gain of the first-order system is given as:

$$G(s) = \frac{1}{s\tau}$$

We know that for a unit step input

$r(t) = 1, t > 0$

Thus

$$r(t) = 1$$

Taking the Laplace transform of r(t). So,

$$R(s) = \frac{1}{s}$$

We know that the closed-loop transfer function is given as:

$$T(s) = \frac{C(s)}{R(s)}$$

: C(s) is the controlled output and R(s) is the reference input

Since $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

On substituting the open-loop gain, the above equation: $\frac{C(s)}{R(s)} = \frac{\frac{1}{s\tau}}{1 + \frac{1}{s\tau}}$

On simplifying,

$$\frac{C(s)}{R(s)} = \frac{1}{s\tau + 1}$$

Therefore,

$$C(s) = R(s)\frac{1}{s\tau+1}$$

As we have already determined, R(s), for unit step unit. Hence substituting the value of R(s), we will have

$$C(s) = \frac{1}{s}\frac{1}{s\tau+1}$$

On solving the partial fraction of the above equation,

$$C(s) = \frac{A}{s} + \frac{B}{s\tau+1}$$

$$\frac{1}{s}\frac{1}{(s\tau+1)} = \frac{A}{s} + \frac{B}{s\tau+1}$$

$$\frac{1}{s(s\tau + 1)} = \frac{A(s\tau + 1) + Bs}{s\,(s\tau + 1)}$$

On simplifying,

$$1 = A$$

Further substituting A = 1 in the general equation

$$1 = s\tau + 1 + Bs$$
$$1 = 1 + s\,(\tau + B)$$

Now, equating the coefficient of s, we will have

$$0 = \tau + B$$

Therefore,

$$B = -\,\tau$$

On substituting the values of A and B, we will have

$$C(s) = \frac{1}{s} + \frac{(-\tau)}{s\tau+1}$$

Further,

$$C(s) = \frac{1}{s} - \frac{(\tau)}{\tau(s+\frac{1}{\tau})}$$
$$C(s) = \frac{1}{s} - \frac{1}{(s+\frac{1}{\tau})}$$

Now, taking inverse Laplace transform of the above equation,

$$c(t) = L^{-1}[\frac{1}{s} - \frac{1}{(s+\frac{1}{\tau})}]$$

Since,

$$L^{-1}[\frac{1}{s}] = 1$$

While

$$L^{-1}[\frac{1}{(s+a)}] = e^{-at}\,u(t)$$

So,

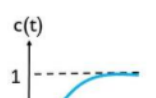
$$L^{-1}[\frac{1}{(s+\frac{1}{\tau})}] = e^{-1/\tau \,.\, t}\,u(t)$$

Thus,

$$c(t) = [1 - e^{-1/\tau \,.\, t}]\,u(t)$$
$$c(t) = u(t) - e^{-1/\tau \,.\, t}\,u(t)$$

Here u(t) represents the steady-state response while the other term is the transient response of the first-order control system.

The response of the unit step signal is given as:



OR iii. With a neat block diagram and Derivation explain how PI, PD and PID compensation will improve the time response of the system.

Print page 2.55 to 2.66

Q4 What are the main advantages of Bode plot? The main advantages are:

- Multiplication of magnitude can be into addition.
- A simple method for sketching an approximate log curve is available.
- It is based on asymptotic approximation. Such approximation is sufficient if rough information on the frequency response characteristic is needed.
- The phase angle curves can be easily drawn if a template for the phase angle curve of $1+j\Delta$ is available.

Q4. i

Solution

For the following transfer function draw bode plot and obtain gain cross-over frequency.

$$G(s)=\frac{20}{s\,(1+3s)\,(1+4s)}$$

SOLUTION

The sinusoidal transfer function of $G(j\omega)$ is obtained by replacing s by $j\omega$ in the given transfer function.

$$G(j\omega)=\frac{20}{j\omega\,(1+j3\omega)\,(1+j4\omega)}$$

MAGNITUDE PLOT

The corner frequencies are, $\omega_{c1}=\frac{1}{4}=0.25$ rad / sec, $\omega_{c2}=\frac{1}{3}=0.333$ rad / sec.

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their frequencies. Also the table shows the slope contributed by each term and change in slope at the corner frequency.

TABLE-1

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{20}{j\omega}$	–	-20	
$\frac{1}{1+j4\omega}$	$\omega_{c1}=\frac{1}{4}=0.25$	-20	$-20-20=-40$
$\frac{1}{1+j3\omega}$	$\omega_{c2}=\frac{1}{3}=0.33$	-20	$-40-20=-60$

Choose a frequency ω_l such that $\omega_l<\omega_{c1}$ and choose a high frequency ω_h such that $\omega_h>\omega_{c2}$.

Let, $\omega_l=0.15$ rad/sec and $\omega_h=1$ rad/sec.

Let, $A=|G(j\omega)|$ in db.

Let us calculate A at ω_l, ω_{c1}, ω_{c2} and ω_h.

$$\text{At } \omega=\omega_l,\ A=|G(j\omega)|=20\log\left|\frac{20}{0.15}\right|=42.5\text{ db}$$

$$\text{At } \omega=\omega_{c1},\ A=|G(j\omega)|=20\log\left|\frac{20}{0.25}\right|=38\text{ db}$$

$$\text{At } \omega=\omega_{c2},\ A=\left[\text{Slope from }\omega_{c1}\text{ to }\omega_{c2}\times\log\frac{\omega_{c2}}{\omega_{c1}}\right]+A_{(at\ \omega=\omega_{c1})}$$

$$=-40\times\log\frac{0.33}{0.25}+38=33\text{ db}$$

$$\text{At } \omega=\omega_h,\ A=\left[\text{Slope from }\omega_{c2}\text{ to }\omega_h\times\log\frac{\omega_h}{\omega_{c2}}\right]+A_{(at\ \omega=\omega_{c2})}$$

$$=-60\times\log\frac{1}{0.33}+33=4\text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10 db on y-axis. The frequencies are marked in decades from 0.01 to 10 rad/sec on logarithmic scales on x-axis. Fix the points a, b, c and d on the graph sheet. Join the points by a straight line and mark the slope in the respective region.

PHASE PLOT

The phase angle of $G(j\omega)$, $\phi=-90^0-\tan^{-1}3\omega-\tan^{-1}4\omega$

The phase angle of $G(j\omega)$ are calculated for various values of ω and listed in table-2.

TABLE-2

ω, rad/sec	$\tan^{-1} 3\omega$, deg	$\tan^{-1} 4\omega$, deg	$\phi = \angle G(j\omega)$, deg	Points in phase plot
0.15	24.22	30.96	$-145.18 \approx -146$	e
0.2	30.96	38.66	$-159.61 \approx -160$	f
0.25	36.86	45.0	$-171.86 \approx -172$	g
0.33	44.7	52.8	$-187.5 \approx -188$	h
0.6	60.14	67.38	$-218.32 \approx -218$	i
1	71.56	75.96	$-237.56 \approx -238$	j

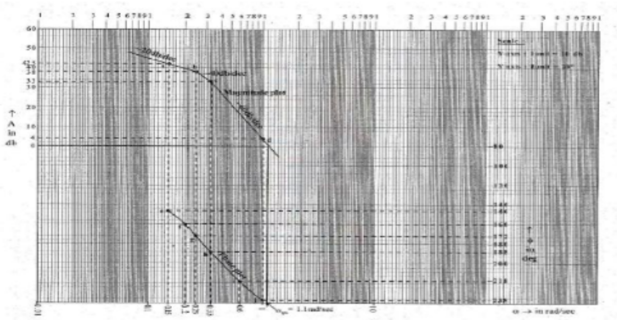


Fig 3.5.1 : Bode plot for transfer function, $G(j\omega) = \frac{20}{j\omega(1+j3\omega)(1+j4\omega)}$

Q5. i. Explain gain margin and phase margin.

Answer: Gain margin is a factor by which the system gain can be decreased to drive the system to the verse of instability.
Phase margin is the additional phase lag at the gain cross over frequency to bring the system to verge of instability.

ii Construct Routh array and determine the stability of the system whose characteristicequation is,

$S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16S+16 = 0$.

The Routh array formed is

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0
s^2				
s^1				
s^0				

As we see, s^3 row is completely zero. Hence the auxiliary polynomial is formed from s^4 row, i.e. $2s^4 + 12s^2 + 16 = 0$

Or $s^4 + 6s^2 + 8 = 0$

Differentiating the polynomial w.r.t s, we get

$4s^3 + 12s = 0$

The zeros in the s^3 row are now replaced by the coefficients of derivative of auxiliary polynomial. The Routh array now formed will be

s^6	1	8	20	16
s^5	2	12	16	
s^4	1	6	8	
s^3	4	12	0	
s^2	3	8		
s^1	1/3			
s^0	8			

In above array, there is no change of sign. Hence the system will be marginally or limitedly stable.

Also, if we solve and find the roots of auxiliary polynomial

$s^4 + 6s^2 + 8 = 0$

The roots are $s = \pm j\sqrt{2}$ and $s = \pm j2$

These two pair of roots are also among the roots of given characteristic equation.

Question :5 iii.

Draw the Nyquist plot for the system whose open loop transfer function is, $G(s)H(s) = \dfrac{K}{s\,(s+2)\,(s+10)}$.

Determine the range of K for which closed loop system is stable.

SOLUTION

Given that, $G(s)H(s) = \dfrac{K}{s\,(s+2)\,(s+10)} = \dfrac{K}{s \times 2\left(\dfrac{s}{2}+1\right) \times 10\left(\dfrac{s}{10}+1\right)} = \dfrac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$

The open loop transfer function has a pole at origin. Hence choose the Nyquist contour on s-plane enclosing the entire right half plane except the origin as shown in fig 4.13.1.

The Nyquist contour has four sections C_1, C_2, C_3 and C_4. The mapping of each section is performed separately and the overall Nyquist plot is obtained by combining the individual sections.

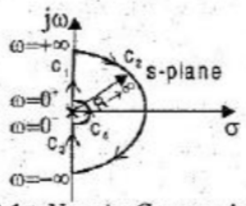


Fig 4.13.1 : *Nyquist Contour in s-plane*

MAPPING OF SECTION C_1

In section C_1, ω varies from 0 to +∞. The mapping of section C_1 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from 0 to ∞. This locus is the polar plot of $G(j\omega)H(j\omega)$.

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)}$$

Let $s = j\omega$.

$$\therefore G(j\omega)H(j\omega) = \frac{0.05K}{j\omega\,(1+j0.5\omega)\,(1+j0.1\omega)} = \frac{0.05K}{j\omega\,(1+j0.6\omega-0.05\omega^2)} = \frac{0.05K}{-0.6\omega^2 + j\omega\,(1-0.05\omega^2)}$$

When the locus of $G(j\omega)H(j\omega)$ crosses real axis the imaginary term will be zero and the corresponding frequency is the phase crossover frequency, ω_{pc}.

$$\therefore \text{At } \omega = \omega_{pc}, \quad \omega_{pc}(1-0.05\omega_{pc}^2) = 0 \quad \Rightarrow \quad 1-0.05\omega_{pc}^2 = 0 \quad \Rightarrow \quad \omega_{pc} = \sqrt{\frac{1}{0.05}} = 4.472 \text{ rad/sec}$$

$$\text{At } \omega = \omega_{pc} = 4.472 \text{ rad/sec}, \quad G(j\omega)H(j\omega) = \frac{0.05K}{-0.6\omega^2} = -\frac{0.05K}{0.6 \times (4.472)^2} = -0.00417K$$

The open loop system is type-1 and third order system. Also it is a minimum phase system with all poles. Hence the polar plot of $G(j\omega)H(j\omega)$ starts at −90° axis at infinity, crosses real axis at −0.00417K and ends at origin in second quadrant. The section C_1 and its mapping are shown in fig 4.13.2. and 4.13.3.

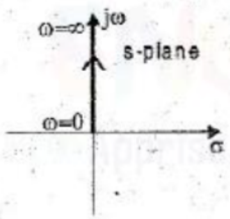


Fig 4.13.2 : Section C_1 in s-plane

⇒

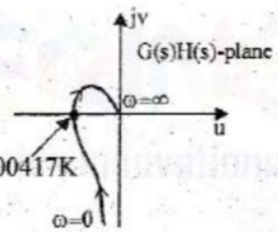


Fig 4.13.3 : *Mapping of section C_1 in*

G(s)H(s)-plane

MAPPING OF SECTION C_2

The mapping of section C_2 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R\to\infty}{Lt} R\,e^{j\theta}$ in G(s)H(s) and varying θ from $+\pi/2$ to $-\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to \infty$, the G(s)H(s) can be approximated as shown below, [i.e., $(1+sT) \approx sT$].

$$G(s)\,H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s \times 0.5s \times 0.1s} = \frac{K}{s^3}$$

Let, $s = \underset{R\to\infty}{Lt}\; Re^{j\theta}$.

$$\therefore\; G(s)H(s)\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{s^3}\Bigg|_{s=\underset{R\to\infty}{Lt} Re^{j\theta}} = \frac{K}{\underset{R\to\infty}{Lt}\,(Re^{j\theta})^3} = 0e^{-j3\theta}$$

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{-j3\frac{\pi}{2}}$ .....(1)

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = 0e^{+j3\frac{\pi}{2}}$ .....(2)

From the equations (1) and (2) we can say that section C_2 in s-plane (fig 4.13.4.) is mapped as circular arc of zero radius around origin in G(s)H(s)-plane with argument (phase) varying from $-3\pi/2$ to $+3\pi/2$ as shown in fig 4.13.5.

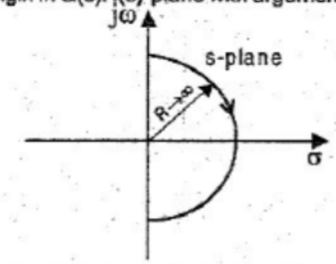


Fig 4.13.4 : Section C_2 in s-plane

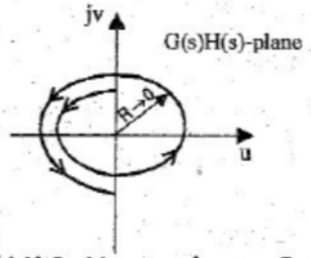


Fig 4.13.5 : *Mapping of section C_2 in G(s)H(s)-plane*

G(s)H(s)-plane

MAPPING OF SECTION C_3

In section C_3, ω varies from $-\infty$ to 0. The mapping of section C_3 is given by the locus of $G(j\omega)H(j\omega)$ as ω is varied from $-\infty$ to 0. This locus is the inverse polar plot of $G(j\omega)H(j\omega)$.

The inverse polar plot is given by the mirror image of polar plot with respect to real axis. The section C_3 in s-plane and its corresponding contour in G(s)H(s) plane are shown in fig 4.13.6 and fig 4.13.7.

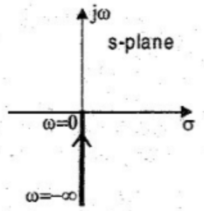


Fig 4.13.6 : Section C_3 in s-plane

$\Rightarrow$

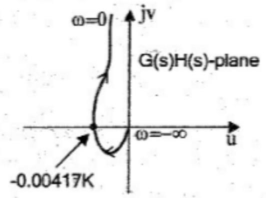


Fig 4.13.7 : *Mapping of section C_3 in G(s)H(s)-plane*

MAPPING OF SECTION C_4

The mapping of section C_4 from s-plane to G(s)H(s)-plane is obtained by letting $s = \underset{R \to 0}{\mathrm{Lt}} \; R\,e^{j\theta}$ in G(s)H(s) and varying θ from $-\pi/2$ to $+\pi/2$. Since $s \to R\,e^{j\theta}$ and $R \to 0$, the G(s) H(s) can be approximated as shown below, [i.e., $(1+sT) \approx 1$].

$$G(s)H(s) = \frac{0.05K}{s\,(1+0.5s)\,(1+0.1s)} \approx \frac{0.05K}{s \times 1 \times 1} = \frac{0.05K}{s}$$

Let $s = \underset{R \to 0}{\mathrm{Lt}} \; Re^{j\theta}$.

$$\therefore\; G(s)H(s)\Big|_{s = \underset{R\to 0}{\mathrm{Lt}} Re^{j\theta}} = \frac{0.05K}{s}\Big|_{s = \underset{R\to 0}{\mathrm{Lt}} Re^{j\theta}} = \frac{0.05K}{\underset{R\to 0}{\mathrm{Lt}}\,(Re^{j\theta})} = \infty e^{-j\theta}$$

When $\theta = -\frac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{+j\frac{\pi}{2}}$ .....(3)

When $\theta = \frac{\pi}{2}$, $\quad G(s)H(s) = \infty e^{-j\frac{\pi}{2}}$ .....(4)

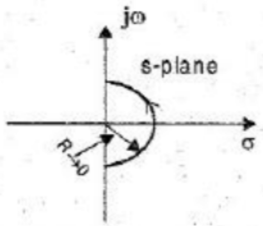


Fig 4.13.8 : Section C_4 in s-plane

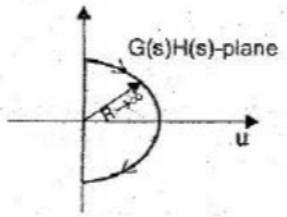


Fig 4.13.9 : *Mapping of section C_4 in G(s)H(s)-plane*

COMPLETE NYQUIST PLOT

The entire Nyquist plot in G(s)H(s)-plane can be obtained by combining the mappings of individual sections, as shown in fig 4.13.10.

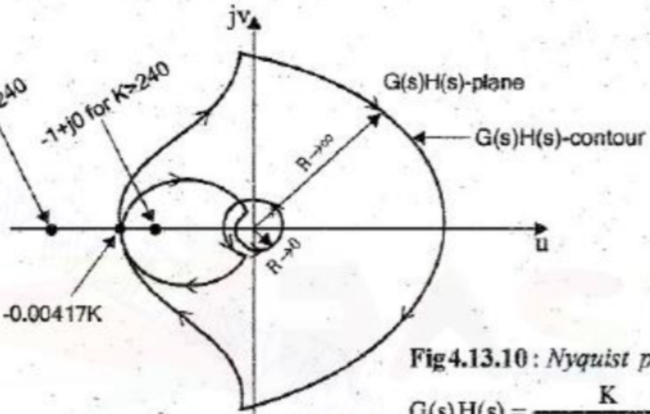


Fig 4.13.10 : *Nyquist plot of*

$$G(s)H(s) = \frac{K}{s(s+2)(s+10)}$$

STABILITY ANALYSIS

When, $-0.00417K = -1$, the contour passes through $(-1+j0)$ point and corresponding value of K is the limiting value of K for stability.

$$\therefore \text{Limiting value of } K = \frac{1}{0.00417} = 240$$

From the equations (3) and (4) we can say that section C_4 in s-plane (fig 4.13.8.) is mapped as a circular arc of infinite radius with argument (phase) varying from $+\pi/2$ to $-\pi/2$ as shown in fig 4.13.9.

When K < 240

When K is less than 240, the contour crosses real axis at a point between 0 and $-1+j0$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is not encircled. Also the open loop transfer function has no poles on the right half of s-plane. Therefore the closed loop system is stable.

When K > 240

When K is greater than 240, the contour crosses real axis at a point between $-1+j0$ and $-\infty$. On travelling through Nyquist plot along the indicated direction it is found that the point $-1+j0$ is encircled in clockwise direction two times. [Since there are two clockwise encirclement and no right half open loop poles, the closed loop system has two poles on right half of s-plane]. Therefore the closed loop system is unstable.

RESULT

The value of K for stability is $0 < K < 240$

Q6 . i. Explain the method of calculating breakaway point.
At break away point the root locus breaks from the real axis to enter into the complex plane. At break in point the root locus enters the real axis from the complex plane. To find the break away or break in points, form a equation for K from the characteristic equation and differentiate the equation of K with respect to s. Then find the roots of the equation dK/dS = 0. The roots of dK/dS = 0 are break away or break in points provided for this value of root the gain K should be positive and real.

Q6. ii
Describe the terms: (i) Asymptotes (ii).Centroid (iii). Breakaway point
Asymptotes are straight lines which are parallel to root locus going to infinity and meet the rootlocus at infinity.
The meeting point of asymptotes with real axis is called centroid.

The Point at which the root locus breaks from the real axis to enter into the complex planeis called as Break away point.