Enrollment No....................................

Faculty of Engineering / Science

End Sem (Odd) Examination Dec-2022

EN3BS11 / BC3BS01 Engineering Mathematics -I

Programme: B.Tech./ B.Sc(CS) Branch/Specialisation: All/ Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The nullity of a square matrix of order n and rank r is given by- **1**

(a) r − n (b) r (c) n − r (d) None of these

ii. A system of homogeneous linear equation AX= 0 is always- **1**

(a) Consistent (b) Inconsistent

(c) Both (a) and (b) (d) None of these

iii. A function is said to be continuous at x = a, if- **1**

(a) $\lim_{x\to a} f(x) = f(a)$ (b) $\lim_{x\to 0} f(x) \neq f(a)$

(c) $\lim_{x\to \infty} f(x) = f(a)$ (d) None of these

iv. The value of $\lim_{x\to 0}\frac{e^x-1}{x}$ is- **1**

(a) 1 (b) 0 (c) -1 (d) ∞

v. The value of integral doesn’t contains arbitrary constant then it is called- **1**

(a) Simple Integration (b) Double integration

(c) Triple Integration (d) Definite Integral

vi. $\lceil 5$ is equal to- **1**

(a) 24 (b) 12 (c) 36 (d) 48

vii. The power of highest derivative after removing the radicals and fractions is called: **1**

(a) Order of differential equation

(b) Degree of differential equation

(c) Both (a) and (b)

(d) None of these

viii. Integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$ is- **1**

(a) $e^{\int Pdx}$ (b) $e^{-\int Pdx}$ (c) $e^{\int Pdy}$ (d) $e^{-\int Pdy}$

ix. A complex variable z is written as- **1**

(a) $z = x + y$ (b) $z = x + iy$

(c) $z = x - y$ (d) None of these

x. Polar form of complex number is- **1**

(a) $z = r(cos\theta + i\ sin\theta)$

(b) $z = r(sin\theta + i\ \cot\theta)$

(c) $z = re^{i\theta}$

(d) Both (a) and (c)

Q.2 Attempt any two:

i. Test for consistency & solve the equations- **5**

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

ii. Find the rank of following matrix by reducing it into Normal form- **5**

$$A= \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

iii. Find eigen values and corresponding eigen vectors of the following matrix- **5**

$$A= \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Q.3 i. State Rolle's theorem. **2**

ii. If $z(x + y) = x^2 + y^2$, show that- **8**

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

OR iii. If u $= \log_e \frac{x^4+y^4}{x+y}$. By using Euler's theorem show that- **8**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3.$$

Q.4 i. Describe definite integral as limit of a sum. **3**

ii. Change the order of integration $\int_0^a \int_y^a \frac{x}{x^2+y^2} dy\, dx$ and hence evaluate it. **7**

OR iii. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\ dzdxdy.$ **7**

Q.5 i. Define order and degree of a differential equation with example. **4**

ii. Solve $\frac{dy}{dx} = \frac{x(2\ logx+1)}{siny+y\ cosy}$. **6**

OR iii. Solve $x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$. **6**

Q.6 Attempt any two:

i. Test the analytic behaviour of log z. **5**

ii. Find the analytic function $f(z) = u + iv$, **5**

$$if\ v = e^x(x\ siny + y\ cosy)$$

iii. Find $\int_c \frac{Z^2+5Z+6}{Z-2} dz$, whose **5**

(a) C is $|Z| = 1$

(b) C is $|Z| = 3$

(c) C is $|Z| = 2$

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Page No.: 1
Date:
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No. | Marks

EN3BS11 / BC3BS01
Engineering Mathematics - I

Q.1 (MCQs)

(i) (c) $n-r$

(ii) a consistent

(iii) d $\cancel{\lim_{x\to a} f(x) = f(a)}$ None of these

(iv) a 1

(v) d definite integral

(vi) a 24

(vii) b degree of differential equ^n

(viii) a $e^{\int P\,dx}$

(ix) b $z = x+iy$

(x) d both a and c.

Q.2 Attempt any two:-

Sol (i)

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

The given equ^n can be written in matrix form

$$AX = B$$

where
$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$ +1

Augmented matrix is given by

$$[A : B]$$

No. | Marks

$$[A \vdots B] = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 2 & 1 & -3 & 0 \\ 1 & 1 & 1 & 2 \end{array}\right]$$ +1

$R_3 \leftrightarrow R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & 0 \\ 1 & -1 & 1 & 4 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -4 \\ 0 & -2 & 0 & 2 \end{array}\right]$$

$R_3 \to R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -4 \\ 0 & 0 & 10 & 10 \end{array}\right]$$ +1

$R_3 \to R_3/10$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -1 & -5 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Clearly, $r[A \vdots B] = 3$ and $r[A] = 3$

i.e if $r[A \vdots B] = r[A] = 3 = 3$ (no. of unknown variables) +1
$\therefore$ the system is consistent and it has a unique solⁿ.

No. Marks

$x + y + z = 2$

$-y - 5z = -4$

$\boxed{z = 1}$

$\boxed{y = -1}$ and $\boxed{x = 2}$ +1

Thus, the unique solⁿ is

$x = 2,\ y = -1,\ z = 1$

Q.2 (ii)

$$A = \begin{bmatrix} 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & -3 & 1 & 2 \\ 2 & 1 & -3 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$$

$C_2 \to C_2 - C_1,$

$C_3 \to C_3 - C_1,$

$C_4 \to C_4 - 2C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & -2 & -4 \\ 0 & -1 & -5 & -10 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -5 & -10 \\ 0 & -6 & -2 & -4 \end{bmatrix}$$ +1

$R_2 \to -R_2$

$R_3 \to \frac{-R_3}{2}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 3 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$ +1

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & -14 & -28 \end{bmatrix}$$

$R_3 \to \frac{-R_3}{14}$

No. | Marks

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 10 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$C_4 \to C_4 - 2C_3$ +1

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$C_3 \to C_3 - 5C_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim [I_3 \vdots 0]$$

$$A \sim [I_3 \vdots 0]$$ +1

This is the required normal form +1
rank of the matrix A is $\underline{\underline{3}}$

Q 2 (iii) $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

Eigen values :-
The characteristic eqn of A is given by

$$|A - \lambda I| = 0.$$

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = 0$$ +1

On solving we get.

No. | Marks

$\lambda^3 - 7\lambda^2 + 11\lambda - 5 = 0$

$\Rightarrow \lambda = 1, 1, 5$ (eigen values) +1

Eigen vectors:-

Let $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the eigen vector corresponding to eigen value λ and is given by

$$[A - \lambda I]X = 0.$$

$$\Rightarrow \begin{bmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ +1

(1) case when $\lambda_1 = 5$

$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

on solving,

$$\frac{x}{2+2} = \frac{-y}{-3-1} = \frac{z}{6-2} = K.$$

$$\Rightarrow \frac{x}{4} = \frac{y}{4} = \frac{z}{4} = K$$

$$\Rightarrow x = y = z = K.$$ +1

$$\therefore X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K \\ K \\ K \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

No. | Marks

Case II when $\lambda_2 = 1$ (repeated root)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\because$ rank of coefficient matrix is 1

$\therefore$ $x + 2y + z = 0$.

Let $z = k_1$, $y = k_2$

$\Rightarrow$ $x = -k_1 - 2k_2$

$$\therefore X_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k_1 - 2k_2 \\ k_2 \\ k_1 \end{bmatrix}$$ +1

[or]

$$X_2 = \begin{bmatrix} -k_1 \\ 0 \\ k_1 \end{bmatrix} + \begin{bmatrix} -k_2 \\ k_2 \\ 0 \end{bmatrix}$$

Q.3 (i) Rolle's Theorem

If $f(x)$ be a real valued fun^n of x such that

(i) $f(a) = f(b)$ +1

(ii) $f(x)$ is continuous in $[a,b]$, $a \le x \le b$

(iii) $f(x)$ is differentiable in (a,b), $a < x < b$

then,

there exist at least one real value of $c \in (a,b)$ such that

$$f'(c) = 0$$ +1

No. | Marks

Q.3 (ii) Given $z(x+y) = (x^2+y^2)$

$\Rightarrow$

$$z = \frac{x^2+y^2}{x+y}$$

differentiate partially w.r.t. x, we get

$$\frac{\partial z}{\partial x} = \frac{(x+y)\frac{\partial}{\partial x}(x^2+y^2) - (x^2+y^2)\frac{\partial}{\partial x}(x+y)}{(x+y)^2} \quad +1$$

$$\frac{\partial z}{\partial x} = \frac{x^2+2xy-y^2}{(x+y)^2} \quad +1$$

Similarly,

$$\frac{\partial z}{\partial y} = \frac{(x+y)\frac{\partial}{\partial y}(x^2+y^2) - (x^2+y^2)\frac{\partial}{\partial y}(x+y)}{(x+y)^2} \quad +1$$

$$\frac{\partial z}{\partial y} = \frac{y^2+2xy-x^2}{(x+y)^2} \quad +1$$

On taking L.H.S

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = \left[\frac{x^2+2xy-y^2}{(x+y)^2} - \frac{y^2+2xy-x^2}{(x+y)^2}\right] \quad +1$$

$$= \frac{4(x-y)^2}{(x+y)^2} \quad +1$$

On taking R.H.S

No. Marks

$$4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)=4\left[1-\left(\frac{x^2+2xy-y^2}{(x+y)^2}\right)-\left(\frac{y^2+2xy-x^2}{(x+y)^2}\right)\right] \quad +1$$

$$=\frac{4(x-y)^2}{(x+y)^2} \quad +1$$

Hence L.H.S = R.H.S

$$\Rightarrow \left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2=4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$$

Hence proved.

Q.3 (iii) $u=\log_e\left(\frac{x^4+y^4}{x+y}\right)$ (given)

$$\Rightarrow e^u=\left(\frac{x^4+y^4}{x+y}\right) \quad +1$$

Now, let $z=e^u$

$$\Rightarrow z=\frac{x^4+y^4}{x+y} \quad +1$$

t-test

$$z(xt,yt)=\frac{t^4x^4+t^4y^4}{tx+ty}=t^3z(x,y) \quad +2$$

Clearly, z is a homogeneous futn. of x and y of degree 3. +1

No. | Marks

Then, by Euler's theorem

$$x\frac{\partial z}{\partial x}+y\cdot\frac{\partial z}{\partial y}=3\cdot z$$ +2

$$\Rightarrow x\frac{\partial}{\partial x}(e^u)+y\cdot\frac{\partial}{\partial y}(e^u)=3\cdot e^u$$

$$\Rightarrow e^u\left[x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right]=3\cdot e^u$$

$$\Rightarrow \boxed{x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3}$$ +1

Hence proved.

Q.4 (i) ~~Defi~~

Definite integral as limit of a sum:-
Let f(x) be a continuous fut^n defined on
Close interval [a, b].
The definite integral $\int_a^b f(x)dx$ is the

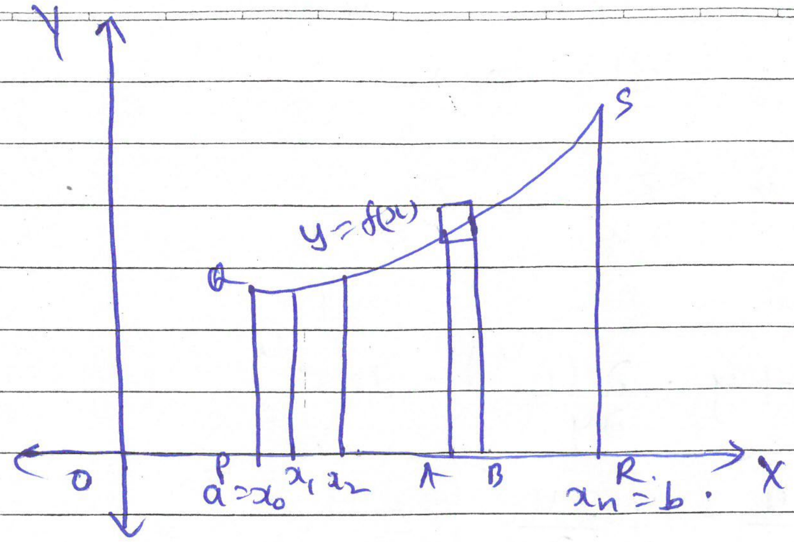
area bounded by the curve $y=f(x)$, +1
the ordinates $x=a$, $x=b$ and the x-axis
Suppose the closed interval [a, b] dividing
into n-equal subintervals of each
of length h such that

$$[\underset{=a}{x_0}, x_1], [x_1, x_2], [x_2, x_3], \ldots [x_{n-1}, x_n=b]$$

where $x_n=x_0+nh=a+nh$

and $h=\frac{b-a}{n}$

No. | Marks

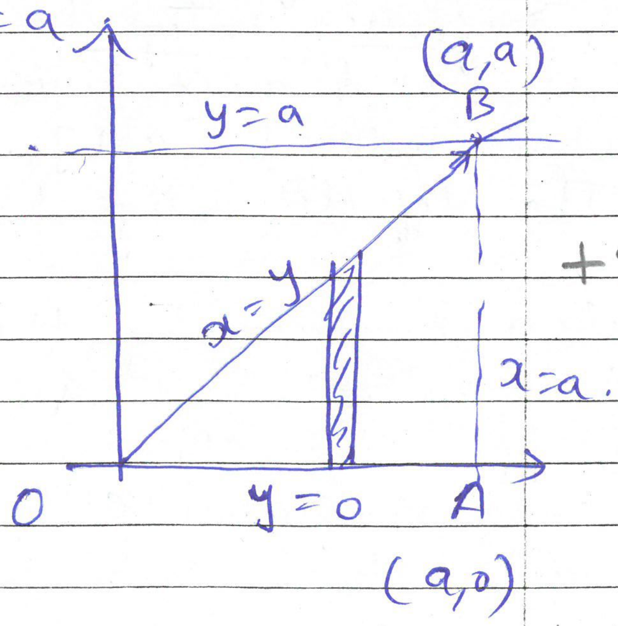


+1

$$\int_a^b f(x)\,dx = \lim_{h\to 0} h\left[f(a) + f(a+h) + \cdots + f\{a+(n-1)h\}\right]$$ +1

Q.4 (ii) The region of given integral is made of the following lines.

$y=0,\ y=a,\ x=y,\ x=a$



The new limits of integration becomes

y varies from

$y=0$ to x

+2

and x varies from

$x=0$ to a.

+2 (for limits)

No. | Marks

$$I=\int_{y=0}^{y=a}\int_{x=y}^{x=a}\frac{x}{x^2+y^2}\,dy\,dx$$ +1

$$=\int_{x=0}^{x=a}\int_{y=0}^{y=x}\frac{x}{x^2+y^2}\,dx\,dy$$

$$=\int_0^a x\left[\int_0^x\frac{1}{x^2+y^2}\,dy\right]dx$$

$$=\int_0^a x\cdot\frac{1}{x}\left[\tan^{-1}\frac{y}{x}\right]_0^x dx$$ +1

$$=\int_0^a\left[\tan^{-1}1-\tan^{-1}0\right]dx$$

$$=\frac{\pi}{4}\int_0^a dx$$

$$=\frac{\pi}{4}\left[x\right]_0^a$$

$$=\frac{a\pi}{4}$$ +1

Ans

Q.4 (iii)

Let $I=\int_0^1\int_{y^2}^1\left[\int_0^{1-x}x\,dz\right]dx\,dy$

$$=\int_0^1\int_{y^2}^1\left[xz\right]_0^{1-x}dx\,dy$$ +1

No. | Marks

$$= \int_0^1 \int_{y^2}^1 x(1-x)\,dx\,dy$$

$$= \int_0^1 \left[\int_{y^2}^1 (x - x^2)\,dx\right] dy \quad +1$$

$$= \int_0^1 \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{y^2}^1 dy \quad +1$$

$$= \int_0^1 \left[\left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{y^4}{2} - \frac{y^6}{3}\right)\right] dy$$

$$= \int_0^1 \left(\frac{1}{6} - \frac{y^4}{2} + \frac{y^6}{3}\right) dy \quad +1$$

$$= \left[\frac{1}{6}y - \frac{y^5}{10} + \frac{y^7}{21}\right]_0^1$$

$$= \frac{1}{6} - \frac{1}{10} + \frac{1}{21} \quad +1$$

$$= \frac{24}{210} = \frac{4}{35} \text{ Ans} \quad +1$$

Q.5 (i) The order of a differential equ^n is the order of the highest differential coefficient present in the equ^n. +1

eg- $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 6y = e^x$ is of order 2. +1

No. | Marks

The degree of a differential equ^n is the degree of the highest derivative after removing the radical sign and fraction. +1

eg - $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 6y = e^x$ is of degree 1. +1

Q.5 (ii) $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$

$\Rightarrow x(2\log x+1)\,dx - (\sin y + y\cos y)\,dy = 0$ — (1) +1

on compairing with

$$M\,dx + N\,dy = 0$$

$M = x(2\log x+1)$ $\quad$ $N = -(\sin y + y\cos y)$

$\frac{\partial M}{\partial y} = 0$ $\quad$ $\frac{\partial N}{\partial x} = 0$

$\because \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ +1

$\therefore$ equ (1) is an exact differential equ^n.

Hence, the sol^n is given

$\int_{y\,as\,const} M\,dx + \int (\text{terms of } N \text{ not containing } x)\,dy = c$. +1

$\int (2\underset{I}{x}\underset{II}{\log x} + x)\,dx - \int (\sin y + y\cos y)\,dy = c$ +1

~~$\left[2\left(\log x \cdot \frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx\right) + \frac{x^2}{2}\right] - \left[-\cos y + (y\right.$~~

No. | Marks

$$\left[2\left(\log x\cdot\frac{x^2}{2}-\int\frac{1}{x}\cdot\frac{x^2}{2}\,dx\right)+\frac{x^2}{2}\right]-$$

$$\left[-\cos y+\left(y\sin y-\int\sin y\,dy\right)\right]=c$$ +1

$$\left[2\left(\frac{x^2}{2}\log x-\frac{x^2}{4}\right)+\frac{x^2}{2}\right]-\left[-\cos y+y\sin y+\cos y\right]=c$$

$$\boxed{x^2\log x-y\sin y=c}$$ +1

Ans

Q.5 (iii) $x(x-1)\frac{dy}{dx}-(x-2)y=x^2(2x-1)$

$$\Rightarrow \frac{dy}{dx}-\frac{(x-2)}{x(x-1)}y=\frac{\cancel{x^2}(2x-1)}{\cancel{x}(x-1)}$$

$$\Rightarrow \frac{dy}{dx}-\frac{x-2}{x(x-1)}y=\frac{x(2x-1)}{x-1}$$ +1

On compairing ~~comp.~~ with $\frac{dy}{dx}+Py=Q$

$$P=-\left(\frac{x-2}{x(x-1)}\right),\quad Q=\frac{x(2x-1)}{x-1}$$ +1

$$I.F=e^{\int P\,dx}=e^{-\int\frac{x-2}{x(x-1)}dx}$$

$$=e^{\int\left(\frac{1}{x-1}-\frac{2}{x}\right)dx}=e^{[\log(x-1)-2\log x]}$$

$$I.F=e^{\log\left(\frac{x-1}{x^2}\right)}$$ +2

No. | Marks

$$I \cdot F = \frac{x-1}{x^2}$$

Now, sol^n is given by

$$y \times I \cdot F = \int (I \cdot F \times Q)\, dx + C$$ +1

$$y \times \left(\frac{x-1}{x^2}\right) = \int \left(\frac{\cancel{x-1}}{x^{\cancel{2}}}\right) \times \cancel{x} \frac{(2x-1)}{\cancel{(x-1)}}\, dx + C$$

$$\frac{y(x-1)}{x^2} = \int \left(\frac{2x-1}{x}\right) dx + C$$

$$y\left(\frac{x-1}{x^2}\right) = \int \left(2 - \frac{1}{x}\right) dx + C$$

$$y\frac{(x-1)}{x^2} = (2x - \log x) + C$$ +1

$= sol^n$

Q.6 (i) Let $f(z) = \log z$

$$u + iv = \log(x + iy)$$ +1

$$u + iv = \log(r\cos\theta + i r\sin\theta)$$

$$u + iv = \log(re^{i\theta})$$ +1

$$u + iv = \log r + i\theta$$ +1

equating real and imaginary parts, we get

$$u = \log r, \quad v = \theta$$ +1

No. Marks

Differentiate partially w.r.t. r and θ:

$$\frac{\partial u}{\partial r} = \frac{1}{r}, \quad \frac{\partial v}{\partial r} = 0$$ +1

$$\frac{\partial u}{\partial \theta} = 0 \text{ and } \frac{\partial v}{\partial \theta} = 1$$ +1

It is clear that:

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$$ +1

$\Rightarrow$ The fut^n log z is analytic except at origin.

Q. 6 (ii) $v = e^x(x\sin y + y\cos y)$

$$\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y) + e^x \sin y$$

$$\frac{\partial v}{\partial x} = \phi_2(x,y)$$ +1

$$\frac{\partial v}{\partial y} = e^x(x\cos y + \cos y - y\sin y)$$

$$= \phi_1(x,y)$$ +1

By Milne's Thomson method

$$f(z) = \int [\phi_1(z,0) + i\,\phi_2(z,0)]\,dz + C$$ +2

$$= \int [e^z(z+1) + i(0)]\,dz + C$$

$$= (z+1)e^z - \int e^z dz + C$$

No. | Marks

$$= (z+1)e^z - e^z + c$$

$$f(z) = z e^z + c \quad +1$$

Ans

OR

Given: $v = e^x(x\sin y + y\cos y)$ — (1)

$$\frac{\partial v}{\partial y} = e^x(x\cos y + y\sin y + \cos y)$$

By C-R eqn $\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}$

So, $\frac{\partial u}{\partial x} = e^x(x\cos y + y\sin y + \cos y)$ +1

on integration w.r.to x

$$u = x e^x \cos y + e^x y \sin y + \phi(y) \text{ — (2)}$$

then find $\partial u/\partial y$

$$\frac{\partial u}{\partial y} = -x e^x \sin y + e^x y \cos y + e^x \sin y + \phi'(y) \text{ — (3)} \quad +1$$

from (1)

$$\frac{\partial v}{\partial x} = e^x(x\sin y + y\cos y) + e^x \sin y \text{ — (4)}$$

and we have, By C-R eqn

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} \quad +1$$

Hence from (3) & (4)

$$-2e^x(y\cos y + \sin y) = \phi'(y)$$

on integration w.r.to y

$$\phi(y) = -2e^x(y\sin y - \cos y) + 2e^x\cos y$$

$$\phi(y) = -2e^x y \sin y + c \quad +1$$

from (2) $u = xe^x\cos y - e^x y \sin y + c$

$$f(z) = (xe^x\cos y - e^x y\sin y + c) + i\, e^x(x\sin y + y\cos y) \quad +1$$

$$= (x+iy)e^x(\cos y + i\sin y) + c$$

$$= z \cdot e^z + c$$

No. | Marks

Q.6 (iii) Here, we have

$$\int_C \frac{z^2+5z+6}{(z-2)} dz,$$

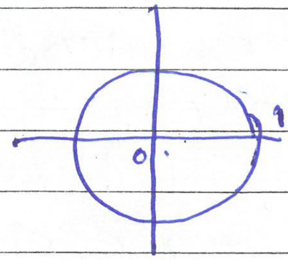
Poles of the integrand is $z=2$. +1

$f(z) = z^2+5z+6$.

By Cauchy integral formula

$$\int \frac{f(z)}{z-a} dz = 2\pi i \, f(a)$$ +1

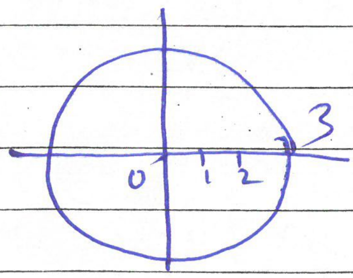
(1) C is $|z|=1$



The poles lies outside the given circle.

So $\int \frac{z^2+5z+6}{z-2} dz = 0$ +1

(2) C is $|z|=3$



The pole $z=2$ lies inside the given circle.

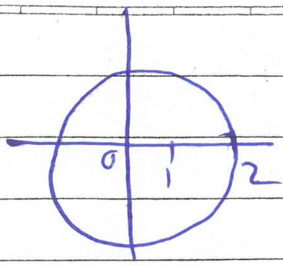
So $\int_C \frac{z^2+5z+6}{z-2} dz = 2\pi i \, f(2)$

$= 2\pi i \times 20$

$= \underline{40\pi i}$ +1

Marks

No. 3 C is $|z| = 2$



The pole $z = 2$ lies on the circle

$$\therefore \int \frac{z^2+5z+6}{z-2}\,dz = 2\pi i\, f(2)$$

$$= 2\pi i \times 20$$

$$= 40\pi i \quad \text{Ans}$$

+1