

- OR iii. If X_1, X_2, \dots, X_n are Poisson variates with parameter $\lambda = 2$, use central limit theorem to estimate $P(120 \leq S_n \leq 160)$ where $S_n = X_1 + X_2 + \dots + X_n$ and $n = 75$ [Given that $P(0 < Z < 2.45) = 0.4929$, $P(0 < Z < 0.82) = 0.2939$]

- Q.6**
- Attempt any two :
- i. Calculate Karl Pearson's coefficient of correlation from the advertisement cost and sales as per the data given below:
- Advertisement cost : 39 65 90 82 75 25 98 36 78
 Sales : 47 53 58 86 62 60 91 51 84
- ii. The correlation coefficient between the variable X and Y is $r = 0.60$. If $\sigma_x = 1.50$, $\sigma_y = 2.00$, $\bar{X} = 10$, $\bar{Y} = 20$, find the equation of the regression lines (a) Y on X (b) X on Y.
- iii. The following table gives the number of aircraft accidents that occurred during various days of the week, find whether the accidents are uniformly distributed over the week ($\chi^2_{0.05} = 12.59$).
- | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|
| Days : | Sun. | Mon. | Tue. | Wed. | Thu. | Fri. | Sat. |
| No. of accidents : | 14 | 15 | 8 | 20 | 11 | 9 | 14 |

6

Total No. of Questions: 6

Total No. of Printed Pages: 4

Enrollment No.....

Knowledge is Power

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

EC3BS03 / EI3BS03 Engineering Mathematics-III

Programme: B.Tech.

Branch/Specialisation: EC/EI

Maximum Marks: 60**Duration: 3 Hrs.**

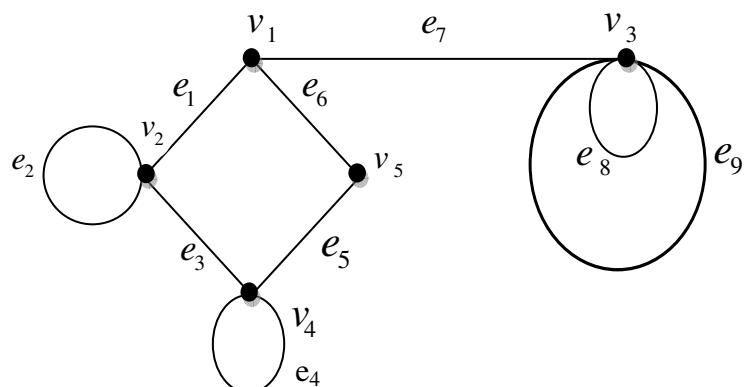
Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

- Q.1 i. Length of a walk W in a graph G is **1**
 (a) The number of vertices in walk W.
 (b) The number of edges in walk W.
 (c) Total number of edges in a graph G.
 (d) Total number of vertices in a graph G.
- ii. The degree of a vertex in any graph is **1**
 (a) The number of edges incident with vertex
 (b) Number of vertex in a graph
 (c) Number of vertices adjacent to that vertex
 (d) Number of edges in a graph
- iii. A terminal node in a binary tree is called **1**
 (a) Root (b) Leaf (c) Child (d) Branch
- iv. A tree has **1**
 (a) No circuit (b) 1 circuit (c) 2 circuit (d) Many circuits
- v. If $J_{n+1}(x) = \frac{2}{x} J_n(x) - J_0(x)$ where J_n is the Bessel function of first kind of order n. Then n is **1**
 (a) 0 (b) 2 (c) -1 (d) None of these
- vi. Let $P_n(x)$ be the Legendre polynomial .Then $P_n'(-x)$ is equal to **1**
 (a) $(-1)^{n+1} P_n'(x)$
 (b) $(-1)^n P_n'(x)$
 (c) $(-1)^n P_n(x)$
 (d) $P_n''(x)$

P.T.O.

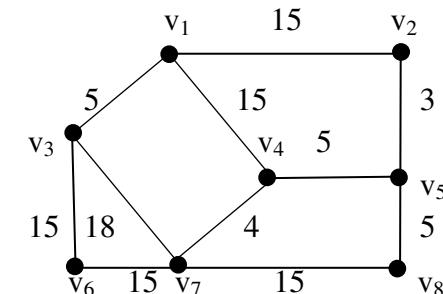
[2]

- vii. If X is a random variable which can take all values in an interval , then X is called **1**
 (a) A discrete random variable
 (b) A probability function
 (c) A continuous random variable
 (d) A probability density function.
- viii. A continuous random variable X has a probability density function $f(x)=k x^2 e^{-x}; x \geq 0$. Then the value of k is **1**
 (a) $1/3$ (b) $1/4$ (c) $1/6$ (d) $1/2$
- ix. The failure to reject a false null hypothesis is **1**
 (a) Type I error (b) Type II error
 (c) Type A error (d) Type B error
- x. When regression line passes through the origin, then: **1**
 (a) Intercept is zero (b) Regression coefficient is zero
 (c) Correlation is zero (d) Association is zero
- Q.2** i. Define walk, Path and circuit with example. **2**
 ii. For a simple complete graph of n vertices, prove that the number of edges is $\frac{1}{2}n(n-1)$. **3**
- iii. Prove that the number of vertices of odd degree in a graph is always even. **5**
- OR** iv. Define incidence and adjacency matrix of graph. Consider the multigraph below. Determine its adjacency matrix. **5**



[3]

- Q.3** i. Prove that there is one and only one path between every pair of vertices in a tree. **2**
- ii. Define minimum spanning tree. Determine the minimum spanning tree of the following weighted graph using Kruskal's Algorithm : **8**



- OR iii. Define flow augmenting path. Write the steps of Ford Fulkerson algorithm to determine the maximum flow for the network. **8**

- Q.4** i. Find the value of $J_{1/2}(x)$ and $J_{-1/2}(x)$ **3**
 ii. State and prove orthogonal properties of Legendre's polynomial **7**
- OR iii. Show that: **7**
 (a) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ (b) $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

- Q.5** i. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the probability distribution of (X, Y) and the marginal density functions of X and Y . **4**
- ii. The joint pdf of a two dimensional RV(X, Y) by **6**

$$f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1$$

Find:

- | | |
|---------------------|--------------------|
| (a) $P(X>1)$ | (b) $P(Y<1/2)$ |
| (c) $P(X<1 / Y <3)$ | (d) $P(X>1/Y<1/2)$ |

P.T.O.

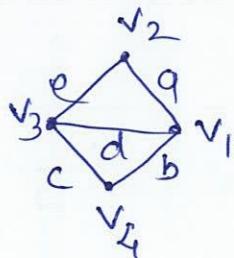
Ques

- (i) (b) The number of edges in walk w.
- (ii) (a) The number of edges incident with vertex.
- (iii) (b) leaf
- (iv) (a) No circuit
- (v) (d) None of these
- (vi) (a) $(-1)^{n+1} P_n(x)$
- (vii) (c) A continuous random variable
- (viii) (d) $1/2$
- (ix) (b) Type II error.
- (x) (a) intercept is zero.

Que 2
(i)

Walk: A walk is an alternating sequence of vertices and edges, beginning and ending with a vertex each edge is incident with the vertex preceding and following it, no edge appears more than once a vertex however may appear more than once.

Eg.



for eg. $v_1 \text{ } d \text{ } v_3 \text{ } c \text{ } v_4$ is a walk [1]
from v_1 to v_4 .

Path: An open walk in which no vertex appears more than once is called a path.

for eg. $v_1 \text{ } d \text{ } v_3 \text{ } c \text{ } v_4$ is a path from v_1 to v_4 . [1.5]

Circuit: A closed walk in which no vertex appears more than once except the ^{initial and} terminal vertex.

for eg. $v_1 \text{ } d \text{ } v_3 \text{ } e \text{ } v_2 \text{ } a \text{ } v_1$ is a circuit. [2]

(ii)

Let v_1, v_2, \dots, v_n be the n vertices of the graph. Since graph is complete, so every vertex is joined to every other vertex by means of an edge. So, the degree of every vertex ^{in G.} is $(n-1)$. Also sum of degree of all the vertices in a graph is twice the number of edges.

$$\therefore \text{no. of edges} = n(n-1)/2$$

[3]

[iii])

Let $G = (V, E)$ be the given graph. Let V_0, V_e be the set of vertices of odd & even degree respectively.

$$\therefore \sum_{v_i \in V} \deg v_i = \sum_{v_i \in V_0} \deg v_i + \sum_{v_i \in V_e} \deg v_i \quad [1]$$

As we know that sum of even numbers is always an even number.

$$\text{Let } \sum_{v_i \in V_e} \deg v_i = 2k \text{ say } k \in \mathbb{I} \quad [2]$$

Also, we know that $\sum_{v_i \in V} \deg v_i = 2e$. [3]

where, e is number of edges in G .

$$\therefore 2e = \sum_{v_i \in V_0} \deg v_i + 2k.$$

$$\Rightarrow \sum_{v_i \in V_0} \deg v_i = 2(e - k) \quad [4]$$

As, we know that sum of odd numbers is an even number, iff number of terms in the sum is even. So, number of vertices of odd degree in a graph is always even. [5]

[iv])

Incidence Matrix: Let G be a graph with n vertices, e edges and no self loops. Define an n by e matrix $A = [a_{ij}]$, whose n rows corresponds to the n vertices and e columns correspond to e edges, as follows;

$a_{ij} = 1$; if j th edge e_j is incident on i th vertex,

$= 0$; otherwise

[1.5]

Adjacency Matrix: The adjacency matrix of a graph G with n vertices and no parallel edges is an n by n symmetric binary matrix $X = [x_{ij}]$ defined as

$x_{ij} = 1$, if there is an edge between i^{th} and j^{th} vertices,

$= 0$, if there is no edge between them.

[3]

$$X = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 & v_5 \\ v_1 & 0 & 1 & 1 & 0 \\ v_2 & 1 & 0 & 0 & 1 \\ v_3 & 1 & 1 & 0 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 0 & 0 \\ v_5 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

[5]

Ques 3

(i) Let T be the tree. So T is connected graph, there must exist at least one path between every pair of vertices in T . Now, suppose that between two vertices a and b of T there are two distinct paths. The union of these two paths will contain a circuit and T can not be a tree so our assumption is wrong. Hence there exist exactly one path between every pair of vertices in a tree, T .

[1]

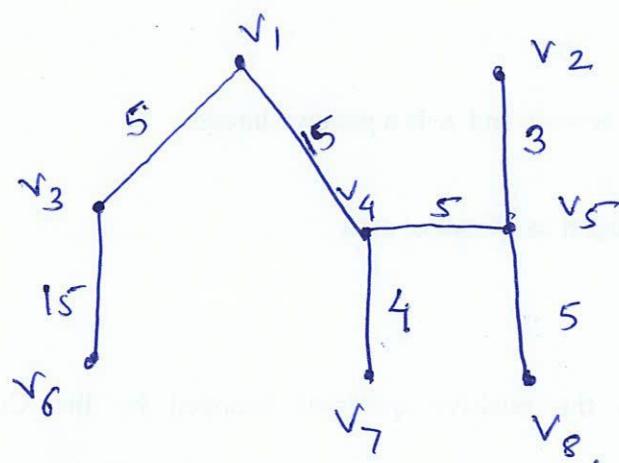
(ii)

Minimum Spanning Tree: A spanning tree with the smallest weight in a weighted graph is called Minimum Spanning Tree.

[1.5]

Edges in increasing order of their wts. Weights of edges Selection of the edge.

$v_2 - v_5$	3	Yes	
$v_4 - v_7$	4	Yes	
$v_1 - v_3$	5	Yes	
$v_4 - v_5$	5	Yes	
$v_5 - v_8$	5	Yes	
$v_1 - v_4$	15	Yes	[3]
$v_1 - v_2$	15	No	
$v_3 - v_6$	15	Yes	
$v_6 - v_7$	15	No	
$v_7 - v_8$	15	No	
$v_3 - v_7$	18	-	[4]



Weight of the minimal spanning tree is 52.

- flow augmenting path : A flow augmenting path in a network with a given flow f_{ij} on each edge (i,j) is a path $P : s \rightarrow t$ such that
- (i) no forward edge is used to capacity; then $f_{ij} < c_{ij}$ for these.

iii) no backward edge has flow 0; thus $f_{ij} > 0$ for these. (4)

For Fulkerson Algorithm:

- 1) Start with a initial flow as 0 (5)
- 2) while there is an augmenting path from source to sink. Add this path flow to flow. (7)
- 3) Return flow. (8)

Ques
(i)

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{n+2k}}{k! \sqrt{n+k+1}} \quad (1)$$

$$\text{Put } n = \frac{x}{2}$$

$$J_{\frac{x}{2}}(x) = \left(\frac{x}{2}\right)^{\frac{x}{2}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k! 2^{2k} \sqrt{k+3/2}} \quad (0.5)$$

$$= \left(\frac{x}{2}\right)^{\frac{x}{2}} \left[\frac{1}{\sqrt{3/2}} + \frac{(-1)x^2}{4\sqrt{5/2}} + \frac{(-1)^2 x^4}{2! 2^4 \sqrt{7/2}} + \dots \right]$$

$$= \sqrt{\frac{x}{2}} \left[\frac{1}{\sqrt{1/2\pi}} - \frac{x^2}{4 \cdot \frac{3}{2} (\sqrt{2\pi})} + \frac{x^4}{2! \cdot 16 \cdot \frac{5}{2} \cdot \frac{3}{2} (\sqrt{2\pi})} + \dots \right]$$

$$= 2\sqrt{\frac{x}{2\pi}} \left[1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \right] \quad (1)$$

$$= \frac{2}{x} \sqrt{\frac{x}{2\pi}} \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \sin x \quad (1.05)$$

Put $n = -\frac{1}{2}$ in (1)

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{k! 2^{2k} \sqrt{k+\frac{1}{2}}} \quad .$$

$$\begin{aligned}
 &= \sqrt{\frac{2}{\pi}} \left[\frac{1}{\Gamma_2} + \frac{(-1) x^2}{2^2 \Gamma_3 \Gamma_2} + \frac{(-1)^2 x^4}{2^4 \cdot 2! \Gamma_5 \Gamma_2} + \dots \right] \\
 &= \sqrt{\frac{2}{\pi}} \left[\frac{1}{\sqrt{\pi}} - \frac{x^2}{2^2 \frac{1}{2} \sqrt{\pi}} + \frac{x^4}{2^4 \cdot 2 \cdot \frac{3}{2} \cdot \Gamma_2 \cdot \sqrt{\pi}} + \dots \right] \\
 &= \sqrt{\frac{2}{\pi x}} \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right] \\
 &= \sqrt{\frac{2}{\pi x}} \cos x.
 \end{aligned}$$

(ii)

Orthogonality of Legendre polynomials:

$$\int_{-1}^1 P_m(x) P_n(x) dx = \begin{cases} 0 & ; m \neq n \\ \frac{2}{2n+1} & ; m = n \end{cases}$$

Case I: when $m \neq n$

We know that $P_m(x)$ and $P_n(x)$ are soln. of eqs.

$$(1-x^2)u'' - 2xu' + m(m+1)u = 0 \quad (1)$$

$$\& (1-x^2)v'' - 2xv' + n(n+1)v = 0 \quad (2)$$

Multiplying (1) by v and (2) by u and subtracting.

$$(1-x^2)(u''v - v''u) - 2x(u'v - v'u) + [m(m+1) - n(n+1)]uv = 0$$

$$\Rightarrow \frac{d}{dx} [(1-x^2)(u'v - v'u)] + (m-n)(m+n+1)uv = 0$$

$$(n-m)(n+m+1)uv = \frac{d}{dx} [(1-x^2)(u'v - v'u)]$$

Integrate w.r.t x from -1 to 1 we get

$$(n-m)(n+m+1) \int_{-1}^1 uv dx = [(1-x^2)(u'v - v'u)] \Big|_{-1}^1$$

$$\Rightarrow \int_{-1}^1 P_m(x) P_n(x) dx = 0.$$

(3)

Case II: when $m=n$

from generating function we have.

$$(1-2xh+h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x) \quad (4)$$

Squaring both sides

$$\begin{aligned} (1-2xh+h^2)^{-1} &= \left(\sum_{n=0}^{\infty} h^n P_n(x) \right)^2 \\ &= \sum_{n=0}^{\infty} h^{2n} [P_n(x)]^2 + 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h^{m+n} P_m(x) P_n(x) \end{aligned} \quad (5)$$

Integrate w.r.t x b/w limits from -1 to 1 we get

$$\begin{aligned} \sum_{n=0}^{\infty} \int_{-1}^1 h^{2n} [P_n(x)]^2 dx + 2 \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \int_{-1}^1 h^{m+n} P_m(x) P_n(x) dx \\ = \int_{-1}^1 \frac{dx}{1-2xh+h^2} \end{aligned}$$

$$\Rightarrow \sum_{n=0}^{\infty} \int_{-1}^1 h^{2n} [P_n(x)]^2 dx = -\frac{1}{2h} [\log(1-2xh+h^2)]_{-1}^1 \quad (6)$$

$$= -\frac{1}{2h} [\log(1-h)^2 - \log(1+h)^2]$$

$$= \frac{1}{h} [\log(1+h) - \log(1-h)]$$

$$= \frac{1}{h} \left[h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} + \dots \right] + \left[h + \frac{h^2}{2} + \frac{h^3}{3} + \dots \right] \quad (6.3)$$

$$= \frac{2}{h} \left[h + \frac{h^3}{3} + \frac{h^5}{5} + \dots \right]$$

$$= 2 \left[1 + \frac{h^2}{3} + \frac{h^4}{5} + \dots + \frac{h^{2n}}{2n+1} + \dots \right]$$

Equating the coefficient of x^{2n} both sides

$$\int_{-1}^1 [P_n(x)]^2 dx = \frac{2}{2n+1}$$

(7)

$$\begin{aligned}
 (iii) (a) \frac{d}{dx} [x^n J_n(x)] &= \frac{d}{dx} \left[x^n \sum_{l=0}^{\infty} \frac{(-1)^l (x/2)^{n+2l}}{l! \Gamma(n+l+1)} \right] \\
 &= \frac{d}{dx} \left[\sum_{l=0}^{\infty} \frac{(-1)^l x^{2n+2l}}{2^{n+2l} l! \Gamma(n+l+1)} \right] \\
 &= \sum_{l=0}^{\infty} \frac{(-1)^l 2(n+l) x^{2n+2l-1}}{2^{n+2l} l! (n+l) \Gamma(n+l)} \\
 &= x^n \sum_{l=0}^{\infty} \frac{(-1)^l (x/2)^{n+2l-1}}{l! \Gamma((n-1)+l+1)} = x^n J_{n-1}(x)
 \end{aligned}$$

(2)

$$\begin{aligned}
 (b) \frac{d}{dx} [x^{-n} [J_n(x)]] &= \frac{d}{dx} \left[x^{-n} \sum_{l=0}^{\infty} \frac{(-1)^l (x/2)^{n+2l}}{l! \Gamma(n+l+1)} \right] \\
 &= \frac{d}{dx} \left[\sum_{l=0}^{\infty} \frac{(-1)^l x^{2l}}{2^{n+2l} l! \Gamma(n+l+1)} \right] \\
 &= \sum_{l=0}^{\infty} \frac{(-1)^l 2l x^{2l-1}}{2^{n+2l} l! (l-1)! \Gamma((n+1)+(l-1)+1)} \\
 &= -x^{-n} \sum_{l=0}^{\infty} \frac{(-1)^{l-1} (x/2)^{n+2l-1}}{(l-1)! \Gamma((n+1)+l-1+1)} \\
 &\quad \text{replace } l-1 \text{ by } k. \\
 &= -x^{-n} J_{n+1}(x)
 \end{aligned}$$

(4)

(6)

(8.)

Ques

(i) Let X denote the number of white balls taken from the box.

$\therefore X$ takes the value 0, 1, and 2.

Let Y denote the number of red balls taken from the box.

$\therefore Y$ takes the values 0, 1, 2 and 3.

$$P(X=0, Y=0) = \frac{4C_3}{9C_3} = \frac{1}{21}$$

$$P(X=0, Y=1) = \frac{3C_1 \times 4C_2}{9C_3} = \frac{3}{14}$$

$$\begin{aligned} P(X=0, Y=2) &= P(2 \text{ red and 1 black ball}) \\ &= \frac{3C_2 \times 4C_1}{9C_3} = \frac{1}{7} \end{aligned}$$

$$P(X=0, Y=3) = \frac{3C_3}{9C_3} = \frac{1}{84}$$

$$P(X=1, Y=0) = \frac{2C_1 \times 4C_2}{9C_3} = \frac{1}{7}$$

$$P(X=1, Y=1) = \frac{2C_1 \times 3C_1 \times 4C_1}{9C_2} = \frac{2}{7}$$

$$P(X=1, Y=2) = \frac{2C_1 \times 3C_2}{9C_3} = \frac{1}{14}$$

$$P(X=1, Y=3) = 0$$

$$P(X=2, Y=0) = \text{not } \frac{2C_2 \times 4C_1}{9C_3} = \frac{1}{21}$$

$$P(X=1, Y=1) = \frac{2C_2 \times 3C_1}{9C_3} = \frac{1}{28}$$

$$P(X=2, Y=2) = P(X=2, Y=3) = 0$$

(1)

(2)

(3)

(3)

x\y	0	1	2	3
0	y_{21}	$\frac{3}{14}$	y_7	y_{84}
1	y_7	$\frac{2}{7}$	$\frac{1}{14}$	0
2	y_{21}	y_{28}	0	0

(4)

(ii) the marginal density function of X

$$f(x) = \int_{-\infty}^{\infty} f(x,y) dy = \int_0^1 \left(xy^2 + \frac{x^2}{8} \right) dy$$

$$= \left(\frac{xy^3}{3} + \frac{x^2y}{8} \right) \Big|_0^1 = \frac{x}{3} + \frac{x^2}{8}; 0 < x < 2$$

$$f(y) = \int_0^2 \left(xy^2 + \frac{x^2}{8} \right) dx = \left(\frac{x^2y^2}{2} + \frac{x^3}{24} \right) \Big|_0^2 \\ = 2y^2 + \frac{1}{3} \quad ; 0 < y < 1.$$

$$P(X > 1) = \int_1^2 f(x) dx = \int_1^2 \left(\frac{x}{3} + \frac{x^2}{8} \right) dx = \frac{19}{24}$$

$$P(Y < \frac{1}{2}) = \int_0^{\frac{1}{2}} f(y) dy = \int_0^{\frac{1}{2}} \left(2y^2 + \frac{1}{3} \right) dy = \frac{1}{4}$$

$$P(X > 1, Y < \frac{1}{2}) = \int_0^{\frac{1}{2}} \int_1^2 \left(xy^2 + \frac{x^2}{8} \right) dx dy \\ = \int_0^{\frac{1}{2}} \left(\frac{x^2y^2}{2} + \frac{x^3}{24} \right) \Big|_1^2 dy \\ = \int_0^{\frac{1}{2}} \left(2y^2 + \frac{1}{3} - \frac{y^2}{2} - \frac{1}{24} \right) dy.$$

(2)

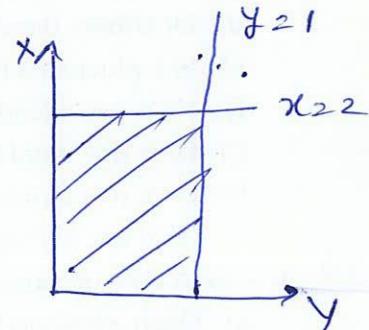
$$= \int_0^{y_2} \left(\frac{3y^2}{2} + \frac{7}{24} \right) dy.$$

$$= \left(\frac{y^3}{2} + \frac{7y}{24} \right)_0^{y_2} = \frac{5}{24}$$

$$\begin{aligned} P(X > 1 / Y < y_2) &= \frac{P(X > 1, Y < y_2)}{P(Y < y_2)} \\ &= \frac{\frac{5}{24}}{\frac{1}{4}} = \frac{5}{6}. \end{aligned} \quad (4)$$

$$P(X < 1 / Y < 3) = \frac{P(X < 1, Y < 3)}{P(Y < 3)}$$

$$\begin{aligned} P(X < 1, Y < 3) &= \int_0^1 \int_0^1 f(x, y) dx dy \\ &= \int_0^1 \int_0^1 (xy^2 + x^2) dx dy \\ &= \int_0^1 \left(\frac{xy^3}{3} + \frac{x^3}{8} \right)_0^1 dx \\ &= \int_0^1 \left(\frac{x}{3} + \frac{x^2}{8} \right) dx \\ &= \left(\frac{x^2}{6} + \frac{x^3}{24} \right)_0^1 = \frac{5}{24}. \end{aligned}$$



$$\begin{aligned} P(Y < 3) &= \int_0^1 f(y) dy \\ &= \int_0^1 (2y^2 + \frac{1}{3}) dy = \left(\frac{2y^3}{3} + \frac{y}{3} \right)_0^1 = 1 \end{aligned}$$

$$P(X < 1 / Y < 3) = \frac{y_3}{1} = \frac{1}{3},$$

(6)

(iii) Given that X_1, X_2, \dots, X_n are Poisson variates

\therefore By Poisson distribution

$$\text{mean} = \lambda = \text{variance} = 2$$

$$\therefore \sigma = \sqrt{2}$$

$$\sigma = \sqrt{2}$$

$$S_n \sim N(n\mu, \sigma\sqrt{n}) \quad (2)$$

$$S_n \sim N(150, \sqrt{150}) \quad \text{By central limit theorem}$$

$$\text{Standard Normal variate } z = \frac{X - \mu}{\sigma} = \frac{X - 150}{\sqrt{150}}$$

$$\begin{aligned} \therefore P[120 \leq S_n \leq 160] &= P\left[\frac{120-150}{\sqrt{150}} \leq z \leq \frac{160-150}{\sqrt{150}}\right] \\ &= P[-2.45 \leq z \leq 0.82] \\ &= P[0 \leq z \leq 2.45] + P[0 \leq z \leq 0.82] \\ &= 0.4929 + 0.2939 \\ &= 0.7868. \end{aligned} \quad (4)$$

Ques
(i)

x	y	x^2	y^2	xy
39	47	1521	2209	1833
65	53	4225	2809	3445
90	58	8100	3364	5220
82	86	6724	7396	7052
75	62	5625	3844	4650
25	60	625	3600	1500
98	91	9604	8281	8918
36	51	1296	2601	1836
78	84	6084	7056	6552
588	592	43804	41166	41006

$$\begin{aligned} r &= \frac{n \sum xy - \sum x \sum y}{\sqrt{(n \sum x^2 - (\sum x)^2)(n \sum y^2 - (\sum y)^2)}} \\ &= \frac{20958}{\sqrt{48492 \times 19976}} \\ &= \frac{20958}{31123.56329} = 0.673 \end{aligned} \quad (4) \quad (5)$$

$$(i) (a) y - \bar{y} = r \frac{\sigma_x}{\sigma_y} (x - \bar{x})$$

$$y - 20 = (\frac{-0.6}{1.5})(2)(x - 10)$$

$$y = -0.8x + 12$$

(Q. 5)

$$(b) x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 10 = (\frac{-0.6}{2})(1.5)(y - 20)$$

$$x = -0.45y + 1$$

(5)

(ii)

H_0 : The accidents are uniformly distributed over the week.

(1)

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2 / f_e$
14	13	1	1	0.0769
15	13	2	4	0.3076
8	13	-5	25	1.9230
20	13	7	49	3.7692
11	13	-2	4	0.3076
9	13	-4	16	1.2307
14	13	1	1	0.0769
91	91			7.6919

(3)

Calculated value of $\chi^2 <$ tabulated value of χ^2

So, accept the Null hypothesis that the accidents are uniformly distributed over the week.

(4)

(5)