Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering / Science

End Sem Examination Dec-2023

CA3CO17 Mathematics -I

Programme: BCA / BCA-MCA (Integrated) Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. If A = {a, b, c, d, e, f} and B = {a, b, p, q, 1, 2} then value of $A \cap B$ is **1**

(a) {1, 2} (b){a} (c) {p, q} (d) None of these

ii. If the element of set A is 4 and element of set B is 9 then elements of $A \times B$ will be- **1**

(a) 13 (b) 36 (c) 5 (d) None of these

iii. If $f(x) = \frac{x^2+1}{2x-1}$ then value of $\frac{f(3)}{f(1)}$ is- **1**

(a) 1 (b) 2 (c) 5 (d) None of these

iv. If $\log(3x + 10) - \log(3x + 5) = \log 2$ then value of x is- **1**

(a) 1 (b) 2 (c) 0 (d) None of these

v. The value of $\lim_{x\to 2} \frac{x^2+8}{(x+2)}$ is- **1**

(a) 1 (b) 3 (c) 5 (d) None of these

vi. If $f(x)$ and $g(x)$ are two continuous functions, then $f(x) - g(x)$ will be- **1**

(a) Discontinuous (b) Continuous

(c) Does not exists (d) None of these

vii. The value of $\int(3x^2 + 1)dx$ is- **1**

(a) x^2 (b) $x^3 + x + c$

(c) $x^{-3} + x$ (d) None of these

viii. The value of $\int_0^2 4x^3\, dx$ is- **1**

(a) 14 (b) 16 (c) 5 (d) None of these

P.T.O.

ix. If A = $\begin{vmatrix} 1 & 3 \\ 4 & 0 \end{vmatrix}$ the value of $|A|$ is- **1**

(a) 5 (b) 7 (c) -12 (d) None of these

x. If A = $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 9 & 2 \end{vmatrix}$ then value of cofactor a_{21} is- **1**

(a) 25 (b) 23 (c) -23 (d) None of these

Q.2 i. Explain Venn diagram with examples. **3**

ii. If U be the universal set and A, B are two finite subsets of U defined as follows $U = \{1, 2, 3, 7, 11, 14, 15\}$ $A = \{7, 11, 14, 15\}$ and $B = \{1, 2, 7\}$ then show that, **7**

(a) $(A \cup B)' = A' \cap B'$ (b) $(A \cap B)' = A' \cup B'$

OR iii. Explain with examples. **7**

(a) Union of two sets (b) Intersection of two sets
(c) Universal set (d) Compliment of set
(e) Difference of two sets (f) Certain product of sets
(g) Power of the set

Q.3 i. Define even and odd functions with suitable examples. **2**

ii. If two functions $f(x) and\ g(x)$ is defined as **8**

$$f(x) = \frac{2}{2-x} \quad and \quad g(x) = \frac{x-2}{x}$$

Then find-

(a) $f(5) + g(3)$ (b) $fog(x)\ and\ gof(x)$

OR iii. Define relation and types of relations. Check whether following relations on the set A is Reflexive symmetric and transitive? **8**

If A = {1, 2, 3, 4} and relations on the set A defined as follows-

R_1= { (1,1) (2,2), (3,3), (4,4), (1,2), (2,3)}

R_2 = { (1,2) (2,1), (1,3), (3,4), (1,1), (2,3)}

R_3 = { (1,2) (2,3), (1,3)}

Q.4 i. Solve : $\lim_{x \to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$ **3**

ii. Discuss the continuity of the function f(x) at x = 2 **7**

f(x) = $\begin{cases} 2-x & , \ x < 2 \\ 2+x & , \ x \geq 2 \end{cases}$

OR iii. Find the following derivatives- **7**

(a) If $y = (1 + x^2)\ cosx$ then find $\frac{dy}{dx}$

(b) If $y = \frac{e^x + e^{-x}}{e^x + e^{-x}}$ then find $\frac{dy}{dx}$

Q.5 i. Find the value of integral: $\int x.e^x dx$ **4**

ii. Find the value of integral: $\int \frac{sinx}{1+cos^2 x}\, dx$ **6**

OR iii. Show that : $\int_0^1 \frac{x^2}{1+x^6} dx$ **6**

Q.6 Attempt any two:

i. Define the properties of determinants with examples. **5**

ii. Show that: $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$ **5**

iii. Solve the following system of the equations by Cramer's rule- **5**

$$x + 3y + 4z = 8$$
$$2x + y + 2z = 5$$
$$5x + y + z = 7$$

******

23 TJ

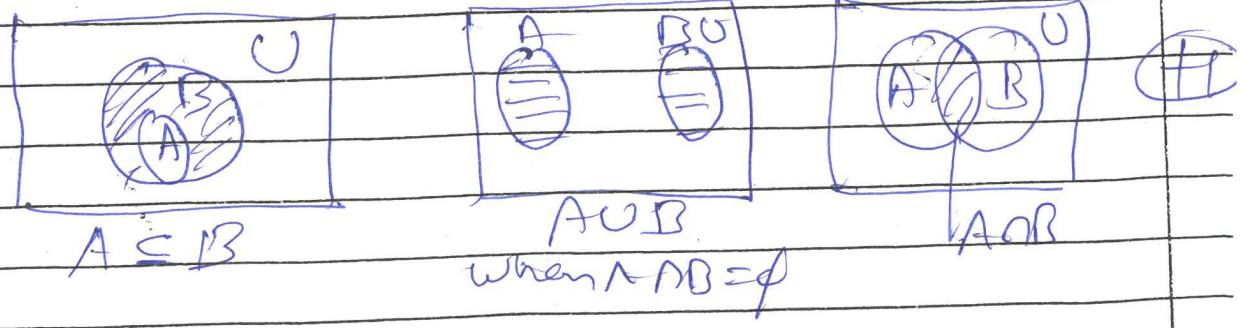
Medicaps University
End Sem Exam Dec 2023
BCA / BCA+MCA
CA3C017 Mathematics-I

Date :
P. No. :

Q1 MCQ (10)

i) d
ii) b 36 / d
iii) a 1
iv) c 0
v) b 3
vi) b Continuous
vii) b x^3+x+c
viii) b 16
ix) c -12
x) b 23

Q2 Venn diagram: The diagram drawn to represent sets are called Venn diagrams or Euler Venn diagrams. Here set U represented by rectangle and the subset A of set U is represented by the interior of a circle. If a set A is a subset of a set B then the circle representing A is drawn inside the circle representing B. If A and B are not equal but have common elements then to represent A and B by two intersecting circles. (12)



(1)

ii) Given

$U = \{1, 2, 3, 7, 11, 14, 15\}$

$A = \{7, 11, 14, 15\}$

$B = \{1, 2, 7\}$ +1

a) $A \cup B = \{1, 2, 7, 11, 14, 15\}$

$(A \cup B)' = \{3\}$ +1

$A' = \{1, 2, 3\}$, $B' = \{3, 11, 14, 15\}$

$A' \cap B' = \{3\}$ +1

$(A \cup B)' = A' \cap B'$

b) $A \cap B = \{7\}$ +1

$(A \cap B)' = \{1, 2, 3, 11, 14, 15\}$ +1

$A' \cup B' = \{1, 2, 3, 11, 14, 15\}$ +1

$\Rightarrow (A \cap B)' = A' \cup B'$ +1

OR

iii) Union Let U be a Universal set and A and B are two subsets of U then +1

$A \cup B = \{x : x \in A$ or $x \in B$ or $x \in A$ and B both$\}$

Intersection Let U be a Universal set A and B are two subsets of U then +1

$A \cap B = \{x : x \in A$ and $x \in B\}$

Eg $A = \{1, 2, 3\}$ $B = \{3, 5, 7\}$

$A \cap B = \{3\}$

c) Universal Set : Universal Set is a Sample or big set and rest all sets are subset of that set it is denoted by U

for eg $U = \{1, 2, 3, 4, 5, 6, 7\}$ is a Universal set and

$A = \{1, 2, 3, 4\}$, $B = \{3, 5, 6, 7\}$

$C = \{1, 5, 7, 8\}$ are subsets of U

d) Complement of a set : If U is a Universal set and A is a subset of set U then Complement of A is a set of elements of U which does not belongs to set A it is denoted by A^c or A'

ie

$$A' \text{ or } A^c = \{x : x \in U \text{ but } x \notin A\}$$

e) Difference of two sets : If U is a Universal set and A and B are two subsets of U then A-B is different of set such that elements of A and not elements of set B

$$A - B = \{x : x \in A \text{ but } x \notin B\}$$

Similarly

$$B - A = \{x : x \in B \text{ but } x \notin A\}$$

f) Certain Product of sets : Let A and B are two sets then Product of set is given by $A \times B$ and it has elements as order pairs.

ie $A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

Date :
P. No. : 4

i) Power of the set ⇒ Let U be a Universal set and A be the subset of U then Power set of A is the set of all subsets of set A. (1)

A = {1, 2} then its Power set is

P[A] = {∅, {1}, {2}, {1, 2}} (1)

Q3

i) Even function: Let f: X → Y is a function such that y = f(x) then if f(-x) = f(x) then it is called even function. (1)

e.g. $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

Odd function: If f(-x) = -f(x) then it is odd function. (1)

e.g. $f(x) = x^3$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

---

ii) Given

$$f(x) = \frac{2}{2-x}, \quad g(x) = \frac{x-3}{x}$$

a) $f(5) = \frac{2}{2-5} = \frac{-2}{3}$, (1)

$g(3) = \frac{3-2}{3} = \frac{1}{3}$ (1)

$f(5) + g(3) = -\frac{2}{3} + \frac{1}{3} = -\frac{1}{3}$ (1)

5

Date :
P. No. :

b) $fog(x) = f\{g(x)\}$

$= f\left\{\frac{x-2}{x}\right\}$ (1)

$= \frac{2}{2-\left(\frac{x-2}{x}\right)}$

$= \frac{2x}{2x-x+2}$ (1)

$= \frac{2x}{x+2}$

$gof(x) = g\{f(x)\}$

$= g\left\{\frac{2}{2-x}\right\}$

$= \frac{\frac{2}{2-x}}{\frac{2}{2-x}}$ (1)

$= \frac{2-4+2x}{2}$

$= \frac{2x-2}{2} = \frac{2(x-1)}{2}$ (1)

$= x-1$ (1)

OR

ii) Relation: A relation R from a non empty set A to another non empty set B is a subset of $A \times B$ (1)

i.e. $R \subseteq A \times B$ or $R \subseteq \{(a,b) : a \in A, b \in B\}$

$(a,b) \in R \Leftrightarrow aRb$

Date :
P. No. :

Type

1) Reflexive: A relation R on a set A is said to be reflexive if $aRa, \forall a \in A$ (1)
i.e if $(a,a) \in R, \forall a \in A$

2) Symmetric: A relation R on a set A is said to be symmetric relation if (1)
$aRb \Rightarrow bRa \quad \forall a,b \in A$
i.e if $(a,b) \in R \Rightarrow (b,a) \in R \quad \forall a,b \in A$

3) transitive: A relation R on a set A is said to be a transitive if (1)
aRb and $bRc \Rightarrow aRc \quad \forall a,b,c \in A$
i.e $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$
$\forall a,b,c \in A$

Now $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1),(2,2),(3,3),(4,4),(1,2),(2,3)\}$
is reflexive only

$R_2 = \{(1,2),(2,1),(1,3),(3,4),(4,4),(2,3)\}$
not Reflexive, not symmetric, not transitive

$R_3 = \{(1,2),(2,3),(1,3)\}$ only transitive

Date :
P. No. :

Q4 i) $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$

Rationalizing

$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \times \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$ +1

$\lim_{x \to 0} \frac{(1+x) - (1-x)}{x(\sqrt{1+x} + \sqrt{1-x})}$

$\lim_{x \to 0} \frac{2x}{x(\sqrt{1+x} + \sqrt{1-x})}$ +1

$\lim_{x \to 0} \frac{2}{\sqrt{1+x} + \sqrt{1-x}}$

Putting $x = 0$ we get +1

$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = 1$

ii) Given

$$f(x) = \begin{cases} 2-x, & x < 2 \\ 2+x, & x \geq 2 \end{cases}$$

+1

At $x = 2$ $f(2) = 2+2 = 4$

RHL $\lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)$ +2

$= \lim_{h \to 0} 2 + 2 + h$

Putting $h \to 0$ $= 4$

LHL $\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h)$

$= \lim_{h \to 0} 2 - (2-h)$ +2

$= \lim_{h \to 0} h = 0$

$\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x) \neq f(2)$ (1/2)

Hence given function is not continuous (1/2)

OR (iii)

a) $y = (1+x^2)\cos x$

Differentiating with respect to x

$\frac{dy}{dx} = \frac{d}{dx}(1+x^2)\cos x$ (1/2)

$u \quad v$

$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (1+x^2)\frac{d}{dx}\cos x + \cos x \frac{d}{dx}(1+x^2)$ (1/2)

$= (1+x^2)(-\sin x) + \cos x \, 2x$ (1/2)

b) $y = \frac{e^x + e^{-x}}{e^x + e^{-x}}$ (1/2)

$\frac{dy}{dx} = \frac{d}{dx}\frac{e^x + e^{-x}}{e^x + e^{-x}}$

$= 0$

Date :
P. No. :

Q5 i) $I = \int \underset{I}{x} \underset{II}{e^{+x}} dx$

By Parts

$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}u \cdot \int v\,dx\right]dx$ (+1)

$= x\int e^{+x}dx - \int\left[\frac{d}{dx}x\int e^{+x}dx\right]dx$ (+1)

$= +xe^{+x} - \int(+e^{+x})\,dx$

$= +xe^{+x} + \int e^{+x}dx$

$= +xe^{+x} + e^{+x} + C$ (+1)

$\int xe^{+x}dx = e^{+x}(x+1) + C$ (+1)

ii) $I = \int \frac{\sin x}{1+\cos^2 x}\,dx$

by substitution method

$\cos x = t$ (+2)

$-\sin x\,dx = dt$

$\sin x\,dx = -dt$

$= \int \frac{-dt}{1+t^2}$ (+2)

$= -\tan^{-1}t + C$ $\left\{\int \frac{dx}{1+x^2} = \tan^{-1}x + C\right\}$ (+1)

$= -\tan^{-1}\cos x + C$ (+1)

iii) $I = \int_0^1 \frac{x^2}{1+x^6}\,dx$

$= \int_0^1 \frac{x^2}{1+(x^3)^2}\,dx$ (+1)

Date :
P. No. :

Put $x^3 = t$

$3x^2 dx = dt$ (+2)

$x^2 dx = \frac{dt}{3}$

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

$$I = \int_0^1 \frac{dt/3}{1+t^2}$$ (+1)

$$= \frac{1}{3}\left[\tan^{-1} t\right]_0^1$$

$$= \frac{1}{3}\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$ (+1)

$$= \frac{1}{3}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{12}$$ (+1)

Q6

7) Properties

1) The value of a determinant remains unaltered when rows are changed into corresponding columns and columns are changed into corresponding rows.

Example: According to above one

2) If any two rows or two columns of a determinant are interchanged then the sign of determinant is changed and numerical value unaltered.

Example: According to above one

$= (b-a)(b-a)[1.(c+a-b-a)]$

$= -(a-b)(c-a)(c-b)$

$= (a-b)(b-c)(c-a) = RHS$ (11)

(iii) Given equations

$x + 3y + 4z = 8$

$2x + y + 2z = 5$

$5x + y + z = 7$

from equation we obtained

$$|D| = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 5 & 1 & 1 \end{vmatrix}$$ (11)

$= 1(1-2) - 3(2-10) + 4(2-5)$

$= -1 + 24 - 12 = 11$

$$|D_x| = \begin{vmatrix} 8 & 3 & 4 \\ 5 & 1 & 2 \\ 7 & 1 & 1 \end{vmatrix}$$ (11)

$= 8(1-2) - 3(5-14) + 4(5-7)$

$= -8 + 27 - 8 = 11$

$$|D_y| = \begin{vmatrix} 1 & 8 & 4 \\ 2 & 5 & 2 \\ 5 & 7 & 1 \end{vmatrix}$$

Date :
P. No. :

3) If two rows or columns of a det are identical then the value of det is zero

Example: Accordingly any one

4) If the elements of any row or any column of a det be each multiplied by the same factor k then the value of the det is multiplied by k

Example: Accordingly any one +5

5) The determinant become zero on Putting $x = \alpha$ then we say that $(x-\alpha)$ is a factor of det

10) $$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$ +2

$$= \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & (b-a)(b+a) \\ 0 & (c-a) & (c-a)(c+a) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$ +2

on expansion we get

13 Date : P. No. :

$|D_y| = 1(5-14) - 8(2-10) + 4(14-25)$

$= -9 + 64 - 44 = 11$

$|D_8| = \begin{vmatrix} 1 & 3 & 8 \\ 2 & 1 & 5 \\ 5 & 1 & 7 \end{vmatrix}$

$= 1(7-5) - 3(14-25) + 8(2-5)$

$= 2 + 33 - 24 = +11$

$x = \frac{|D_x|}{|D|}, \quad y = \frac{|D_y|}{|D|} \quad z = \frac{|D_8|}{|D|}$

$x = 1 \qquad y = 1 \qquad z = +1$