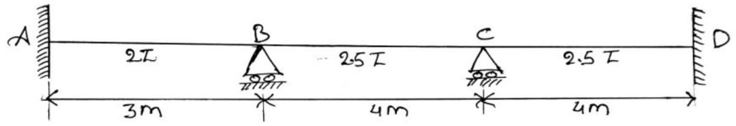
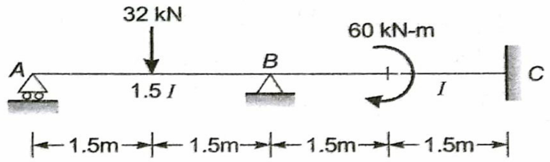
ii. By MDM determine the support moment if support B settles by 5mm. Drwa final bending moment diagram.Take EI=45000 KN-M **7**



OR iii. By SDM Analysis the beam loaded as shown in figure below using slpoe deflection method. Draw the BMD **7**



Q.6 Attempt any two:

i. A three hinged arch has a span of 30 meters and a rise of 10 m. The arch carries a uniformly distributed load of 60 KN per meter on the left half of its span. It also carries two concentrated load of 60 KN and 100 KN at 5 m and 10 m from the right end. Determine the horizontal thrust, at each support. **5**

ii. A cable carrying a load of 10KN per meter run of horizontal span is stretched between supports 100 meter apart. The supports are at the same level and the central dip is 8 meters. Find the greatest and the least tension in the cable. **5**

iii. A two hinged parabolic arch of span 25 m and rise 5 m carries a uniformly distributed load of 40 KN/m over the left half of the span and a concentrated load of 100 KN at the crown. Find the horizontal thrust at each support. **5**

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No....................................

Faculty of Engineering

End Sem (Even) Examination May-2018

CE3CO06 Structural Analysis -I

Programme: B.Tech. Branch/Specialisation: CE

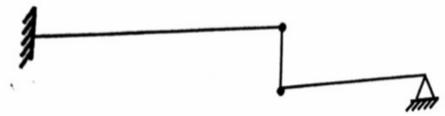
Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. A pin jointed plane frame is unstable if **1**

(a) (m +r)<2j (b) m + r = 2j (c) (m + r)>2j (d) None of these

ii. The beam shown in the figure is **1**



(a) Determinate stable (b) Stable

(c) Indeterminate (d) Unstable

iii. When the length of a simply supported beam, carrying a point load at mid span is doubled the deflection below the load becomes **1**

(a) Two times (b) Four Times

(c) Eight Times (d) None of these

iv. Bending Moment at any section X of the cantilever beam **1**

(a) $M = P^2X$ (b) $M = -PX$ (c) $M = PX^2$ (d) $M = -P^2X^2$

v. The ordinates of influence line diagram for bending moment always have the dimensions of **1**

(a) Force (b) Length

(c) Force x Length (d) Force / Length

vi. Influence line for horizontal thrust in a two hinged parabolic arch is **1**

(a) Parabolic (b) Cubic (c) Triangular (d) Rectangular

vii. Slope deflection method can be used for analysing **1**

(a) Statically determinate structure

(b) Statically indeterminate structure

(c) Unstable structure

(d) None of these

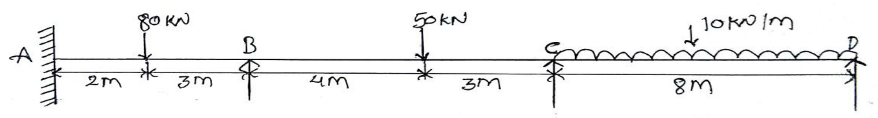
viii. Moment distribution method is nothing but a **1**
(a) Lateral translation method (b) Displacement method
(c) Rotational method (d) None of these

ix. In a solid arch, shear force acts **1**
(a) Vertically upward (b) Along the axis of the arch
(c) Perpendicular to the axis of arch (d) Tangentially to the arch

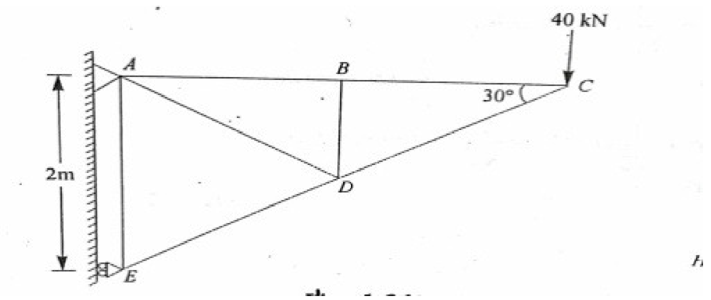
x. The horizontal component of reaction of three hinged arch is called **1**
(a) Horizontal thrust (b) Vertical thrust
(c) Inplane thrust (d) None of these

Q.2 i. Define kinematic indeterminacy. **2**

ii. Analyse beam by three moment equation. **8**

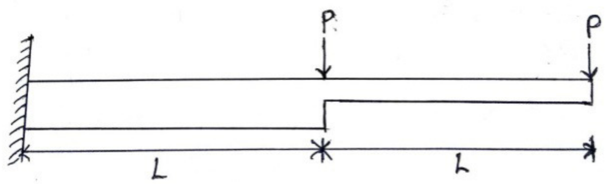


OR iii. Find the vertical and horizontal deflection of the joint C of the loaded truss shown in figure. The cross sectional areas of member CD and DE are each 2500 mm^2 and those of the other member are each 1250 mm^2. Take E = 200 KN/mm^2 **8**

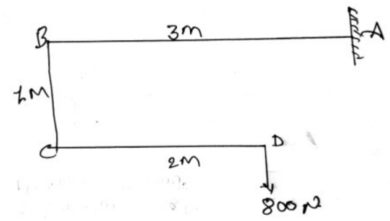


Q.3 i. Define principle of virtual work. **2**

ii. Determine the deflection and rotation at the free end of the cantilever. Use unit load method. **8**

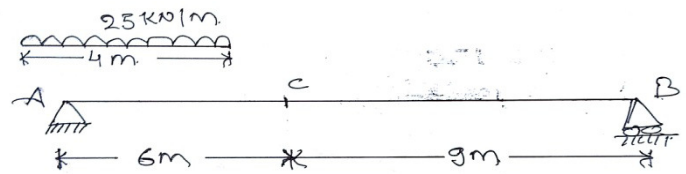


OR iii. A mild steel bar 100mm diameter is bent as shown in fig. It is fixed horizontally at A and a load of 800 N hangs at D. Draw the bending moment diagram for the parts AB, BC, and CD indicating the maximum values. Find the maximum bending stress. Find also the deflection at D. Take E = 2 x 10^5 N/mm^2 **8**



Q.4 i. Write down application of ILD. **3**

ii. A simply supported beam has a span of 15m. A uniformly distributed load of 25KN/m and 4m long crosses the girder from left to right. Drawn the influence line diagram for shear force and bending moment at a section 6m from the left support. Also calculate maximum shear force and bending moment at this section **7**



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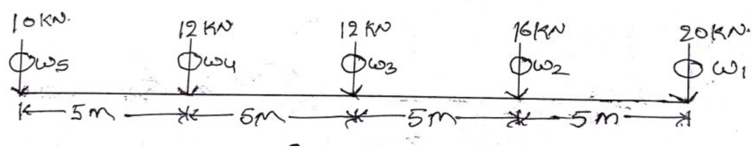
OR iii. Five point load of 10KN, 12KN,12KN, 16KN, and 20KN, spaced at 5m centre to centre rolls over a simply supported girder of 80m. The loads move left to right with 20KN load leading, then calculate position and magnitude maximum bending moment which may occur anywhere on the girder **7**



Q.5 i. Define Stiffness, Distribution Factor, Carry over Factor. **3**

Marking Scheme
CE3CO06 Structural Analysis -I

Question: - 1 objective (10 Marks)

(i) A pin jointed plane frame is unstable if
Ans. $(m + r) < 2j$

(ii) The beam shown in the figure is
Ans. Determinate stable

(iii) When the length of a simply supported beam, carrying a point load at mid span is doubled the deflection below the load becomes
Ans. Eight Times

(iv) Bending Moment at any section X of the cantilever beam
Ans. $M = -PX$

(v) The ordinates of influence line diagram for bending moment always have the dimensions of
Ans. Length

(vi) Influence line for horizontal thrust in a two hinged parabolic arch is
Ans. Cubic

(vii) Slope deflection method can be used for analysing
Ans. Statically indeterminate structure

(viii) Moment distribution method is nothing but a
Ans. Displacement method

(ix) In a solid arch, shear force acts
Ans. Perpendicular to the axis of arch

(x) The horizontal component of reaction of three hinged arch is called
Ans. Horizontal thrust

Question 2 (2, 8, or 8 Marks)

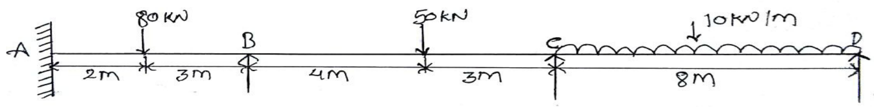
(i) Define kinematic indeterminacy.

Ans.

KINEMATIC INDETERMINACY

"The degree of kinematic Indeterminacy may be defined as the total number of degrees of freedom (independent displacement co-ordinates) at various joints in a skeletal structure".

(ii) Analyse beam by three moment equation.



$10 M_A + 5 M_B - 640 = 0$
$M_A + 0.5 M_B = 64$ — mark 0.5

0.5 mark: $M_A + 4.8 M_B + 1.4 M_C = 306$

0.5 mark: $M_B + 4.28 M_C = 317.6$

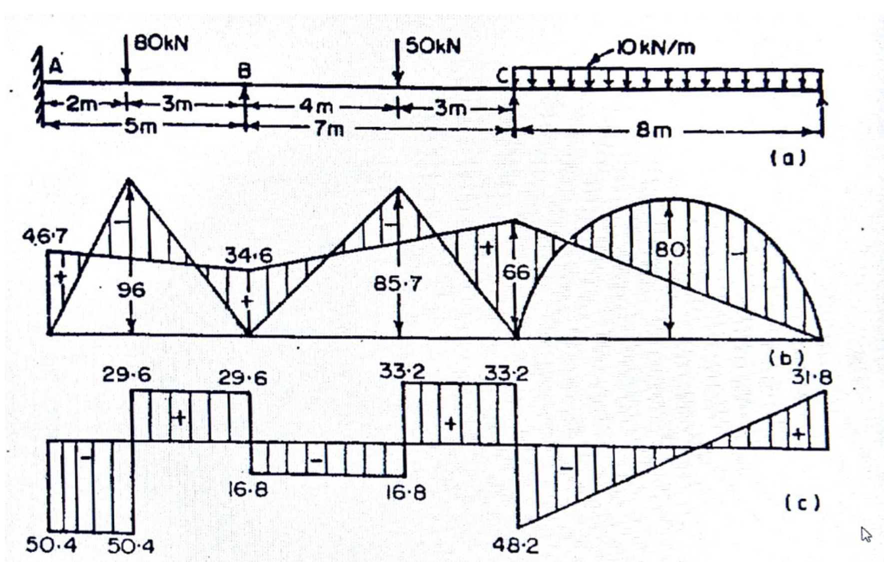
0.5 mark: $M_B + 0.325 M_C = 56.2$

1 mark:
$M_B = 34.6$ KN-m
$M_A = 46.7$ KN-m

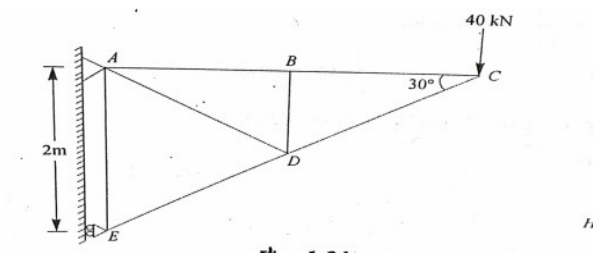
Reaction

0.5 mark each.
$R_D = 31.8$ KN
$R_C = 81.4$ KN
$R_A = 50.4$ KN
$R_B = 46.4$ KN.

B.M. S.F diagram have 1.5 marks each.



(iii) Find the vertical and horizontal deflection of the joint C of the loaded truss shown in figure. The cross sectional areas of member CD and DE are each 2500 mm^2 and those of the other member are each 1250 mm^2. Take E = 200 KN/mm^2



Ans. H_e, V_a, H_a, P_{ab}, P_{bc}, P_{cd}, P_{de}, P_{ae}, P_{ad}, P_{bd} = 0.5 mark each

Vertical deflection, Horizontal deflection = 0.5 mark each

Table = 1 mark

Diagram with force = 1 mark

We have,

$AC = 2\cos 30° = 2\sqrt{3}$ metre

Taking moments about A, $H_e \times 2 = 40 \times 2\sqrt{3}$

$H_e = 40\sqrt{3}\ kN \rightarrow$

$V_a = 40\ kN \uparrow$

$H_a = 40\sqrt{3}\ kN \leftarrow$

Joint C. Resolving vertically,

$P_{cd} \sin 30° = 40$

$\therefore \quad P_{cd} = 80$ kN (*compressive*)

Resolving horizontally,

$P_{cd} = 80 \cos 30° = 40\sqrt{3}\ kN$ (*tensile*)

Joint B. $P_{bd} = 0$; $P_{ba} = 40\sqrt{3}\ kN$ (*tensile*)

Joint D. $P_{da} = 0$; $P_{de} = 80\ kN$ (*compressive*)

Joint E. Resolving vertically,

$P_{ea} = 80 \sin 30°$

$= 40\ kN$ (*tensile*)

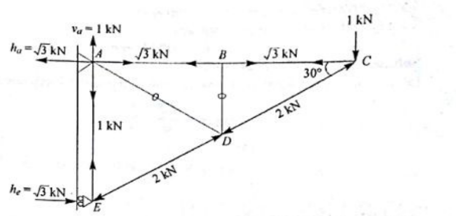


Fig. 1.86

To find the vertical deflection at C

Now remove the given load system and apply a vertical load of 1 kN at C. The forces in the members of the frame for this case will be $\frac{1}{40}$ of the forces in the previous case. See Fig. 1.86.

Vertical deflection of $C = y = \sum \frac{PKl}{AE}$

Member	$P\ (kN)$	$K\ (kN)$	$l\ (mm)$	$A\ (mm^2)$	$\frac{PKl}{A}$
AB	$-40\sqrt{3}$	$-\sqrt{3}$	$1000\sqrt{3}$	1250	166.3
BC	$-40\sqrt{3}$	$-\sqrt{3}$	$1000\sqrt{3}$	1250	166.3
CD	80	2	2000	2500	128
DE	80	2	2000	2500	128
AE	-40	-1	2000	1250	64
AD	0	0	2000	1250	0
BD	0	0	1000	1250	0
			Total		652.6

$\therefore$ Vertical deflection of C

$$= y = \sum \frac{PKl}{AE} = \frac{652.6}{200}$$

$= 3.263$ *mm*

To find the horizontal deflection of C.

In this case apply a unit load *i.e.*, 1 kN horizontally at C. The forces in the members for this loading are shown in Fig. 1.87.

Horizontal deflection of $C = \sum \frac{PKl}{AE}$

$$= \frac{2(-40\sqrt{3})(-1)1000\sqrt{3}}{1250 \times 200} = 0.96\ mm.$$

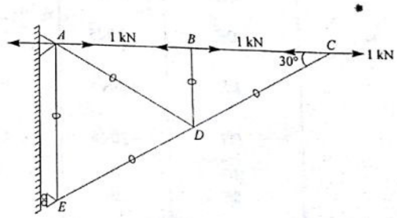


Fig. 1.87

Question: - 3 (2, 8, or 8 Marks)

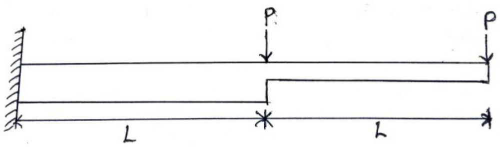
(i) Define principle of virtual work.

Ans: - The principle of virtual work state that in equilibrium the virtual work of the force applied to a system is zero.

or

In other word external work of system equal to internal work of system.

(ii) Determine the deflection and rotation at the free end of the cantilever. Use unit load method.



Ans.

Solution:

For given beam,

Deflection at free end is given by

2 Marks

Portion	CB	BA
Origin	C	B
Limit	O – L	O – L
M	$-Px$	$-P(L+x)+Px$
m_1	$-x$	$-(x+L)$
m_2	-1	-1
MOI	I	$2I$

$$\Delta = \sum \frac{Mm_1}{EI}dx \longrightarrow 1 \text{ mark}$$

$$\Delta = \int_0^L \frac{(-Px)(-x)}{EI}dx + \int_0^L \frac{[P(L+x)+Px](x+L)}{E(2I)} \cdot dx$$

$$= \int_0^L \frac{Px^2dx}{EI} + \int_0^L \frac{[P(x+L)^2 + Px(x+L)]\,dx}{2EI}$$

$$= \int_0^L \frac{Px^2dx}{EI} + \int_0^L \frac{P}{2EI}[x^2+L^2+2Lx+x^2+Lx]dx$$

MADE EASY Publications

1 marks

$$= \int_0^L \frac{Px^2dx}{EI} + \int_0^L \frac{P}{2EI}[2x^2+3Lx+L^2]dx$$

$$= \frac{P}{2EI}\left[\int_0^L 2x^2\,dx + \int_0^L (2x^2+3Lx+L^2)dx\right]$$

$$= \frac{P}{2EI}\left[2\left[\frac{x^3}{3}\right]_0^L + \left[2\cdot\frac{x^3}{3}+3\cdot\frac{Lx^2}{2}+L^2\cdot x\right]_0^L\right]$$

$$= \frac{P}{2EI}\left[2\cdot\frac{L^3}{3}+\frac{2}{3}L^3+\frac{3}{2}L^3+L^3\right]$$

$$\Delta = \frac{23PL^3}{12EI} \longrightarrow 1 \text{ mark}$$

Rotation at free end is given by,

$$\theta = \sum \frac{Mm_2}{EI}dx \longrightarrow 1 \text{ mark}$$

1 mark

$$= \int_0^L \frac{(-Px)(-1)dx}{EI} + \int_0^L \frac{[-P(L+x)+Px](-1)\,dx}{2EI}$$

$$= \int_0^L \frac{Px\,dx}{EI} + \int_0^L \frac{[P(L+x)+Px]}{2EI}dx$$

$$= \frac{P}{2EI}\left[\int_0^L 2x\,dx + \int_0^L (2x+L)dx\right]$$

$$= \frac{P}{2EI}\left[2\left[\frac{x^2}{2}\right]_0^L + 2\left[\frac{x^2}{2}\right]_0^L + L[x]_0^L\right]$$

$$\theta = \frac{P}{2EI}\left[2\cdot\frac{L^2}{2}+2\cdot\frac{L^2}{2}+L\cdot L\right] = \frac{3PL^2}{2EI} \longrightarrow 1\text{. mark}$$

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(iii) A mild steel bar 100mm diameter is bent as shown in fig. it is fixed horizontally at A and a load of 800 N hangs at D. Drwa the bending moment diagram for the parts AB, BC, and CD indicating the maximum values. Find the maximum bending stress. Find also the deflection at D. Take E = 2 x 10^5 N/mm^2

Ans: - B.M at D = 0

B.M at C (CD) = -1600 Nm **B.M.= 0.5 mark for each**

B.M. at C (CB) = 1600 Nm **Vertical deflection = 0.5 mark**

B.M. at B (AB) = 1600 Nm **Maximum bending stress = 2 Marks**

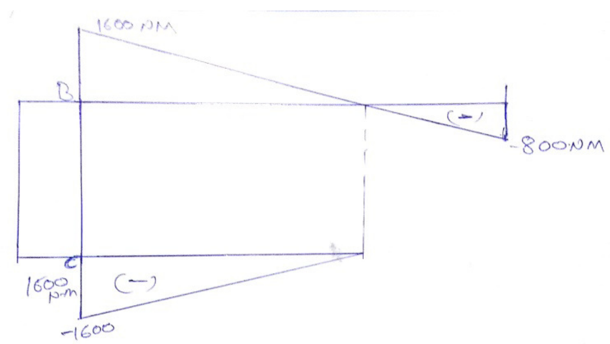
B.M. at A = -800 Nm

Maximum bending stress = (M / Z) = (1600 / 98174.7704) = 16.29 N/mm^2

Strain energy stored by bar =Summation of $\int M^2 ds/2EI$ ………………………. **1 Mark for formula**

= 29/6 x P^2/EI

Vertical deflection at D = 4.92 mm



2 Marks for B.M. Diagram

Question: - 4 (3, 7, or 7 Marks)

(i) Write down application of ILD

1 mark of each point

2.4 Application of ILD

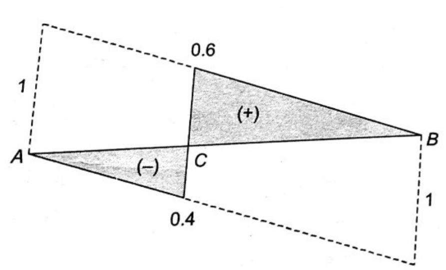
- ILD can be used to study the effect of a moving load on the structure.
- ILD can be used to find position of live load which will produce maximum value of a particular stress function.
- ILD can be used to calculate total value of a particular stress function due to a multiple load system for e.g.

  Net BM at *C* due to load system will be given by
  $M_C = P_1 y_D + P_2 y_E + w \times$ area of ILD below UDL.

Question 4 (ii)

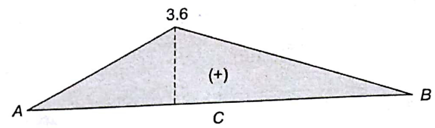
Solution:

ILD for SF at C:



1.5 marks

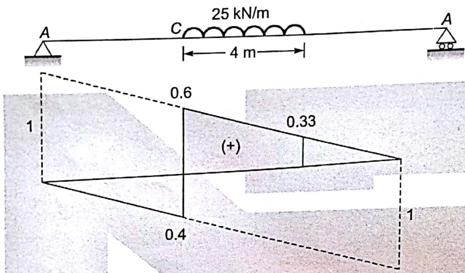
ILD for BM at C:



1.5 marks

$$\text{Ordinate of BM at section} = \frac{a(L-a)}{L} = \frac{6\times 9}{15} = 3.6 \text{ units}$$

(*i*) Maximum Positive Shear Force at Section

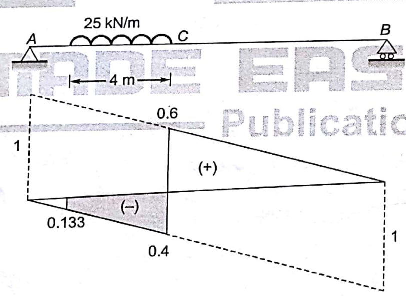


1.5 marks

$\therefore$ Maximum positive SF at C = Area of ILD below UDL $\times w$

$$= \frac{1}{2}\times(0.6+0.33)\times 4\times 25 = 46.67 \text{ kN}$$

(*ii*) Maximum Negative SF at *C*:



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1.5 marks

$\therefore$ Maximum negative SF at C = Area of ILD below UDL $\times w$

$$= \frac{1}{2}\times(0.4+0.133)\times 4\times 25 = 26.65 \text{ kN}$$

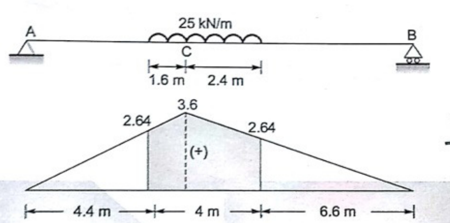
(*iii*) Maximum BM at *C*:

Maximum bending moment at C will occur when average loading just to the left of C is equals to average loading just to right of C.

$$\frac{a}{a'} = \frac{L-a}{L'-a'}$$

$$\Rightarrow \quad \frac{6}{a'} = \frac{9}{4-a'}$$

$$\therefore \quad a' = 1.6 \text{ m}$$



1.5 marks

$\therefore$ Maximum bending moment at C = Area of ILD below loading $\times w$

$$= \left[(2.64\times 4)+\left(\frac{1}{2}\times 4\times 0.96\right)\right]\times 25$$

$$= (10.56 + 1.92)\times 25 = 312 \text{ kN-m}$$

Maximum B.M. at C carries only 1 mark

Question 4 (iii)

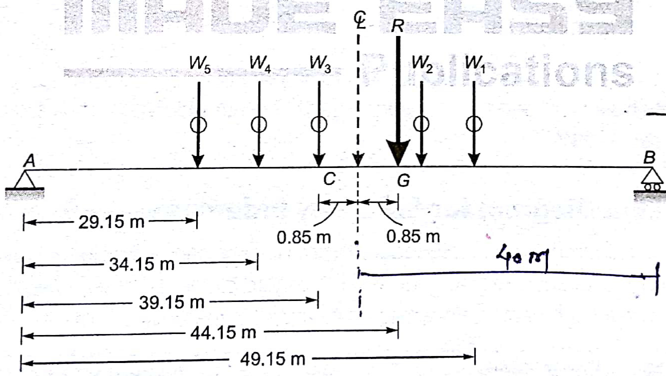
Solution:

Position of CG of load system,

$$\bar{X} = \frac{10\times0+12\times5+12\times10+16\times15+20\times20}{10+12+12+16+20} \quad \rightarrow 1 \text{ mark}$$

$$= 11.71 \text{ m} \quad \text{(From 10 kN load)}$$

Case-1: Let W_3 is load under consideration, hence maximum bending moment will occur below W_3 which is nearest to CG for maximum bending moment under W_3, load system should occupy such a position that the center of span is midway between CG of load system and load under consideration i.e. W_3.



— 1 mark

$$\Sigma M_A = 0$$

$$R_B \times 80 - 20 \times 49.15 - 16 \times 44.15 - 12 \times 39.15 - 12 \times 34.15 - 10 \times 29.15 = 0$$

$$R_B = 35.75 \text{ kN}$$

$$R_A = 70 - 35.75 = 34.25 \text{ kN} \quad \text{— 1 mark}$$

Absolute maximum bending moment = Bending moment under load W_3

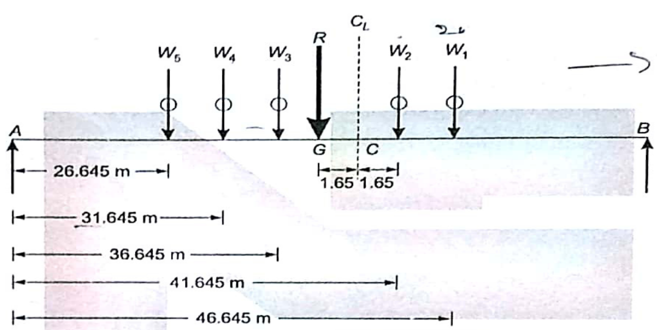
$= R_A \times 39.15 - 10 \times 10 - 12 \times 5$

$= 1180.80$ kN-m → 1 mark

Case-2: Let W_2 is load under consideration.

W_2 is heavier than W_3, however W_3 is nearest to C.G. Then maximum bending moment may occur below W_2.

Let's consider maximum BM occurs below W_2 when the load system occupy such a position that the center of span is midway between CG of load system and load under consideration i.e. W_2.



→ 1 mark

$$\Sigma M_A = 0 \quad -12\times31.645$$

$$R_A \times 80 - 20 \times 46.645 - 16 \times 41.645 - 12 \times 36.645 - 10 \times 26.645 = 0$$

$$R_B = 33.56 \text{ kN}$$

$$R_A = 36.44 \text{ kN} \quad \text{— 1 mark}$$

Absolute maximum bending moment = Bending moment below load W_2

$= R_A \times 41.645 - 10 \times 15 - 12 \times 10 - 12 \times 5$

$= 36.44 \times 41.645 - 150 - 120 - 60$

$= 1187.54$ kN-m → 1 mark

Hence absolute maximum bending moment is 1187.5 kN-m which occurs at a distance of 41.645 m from left support A under load W_2.

Question: - 5 (3, 7, or 7 Marks)

(i) Define Stiffness, Distribution Factor, Carry over Factor.

Stiffness :- Stiffness for a member at a joint is the moment (force) required to produce unit rotation (displacment) at the joint.
Stiffness depend upon the Condition and properties of cross-section.

1 marks

Distribution factor

Distribution factor for a member at a joint is the ratio of the stiffness (or relative stiffness) of the member to the total stiffness (or total relative stiffness)

1 mark

$$DF = \frac{\text{Stiffness of the member}}{\text{Total Stiffness of the Joint}}$$

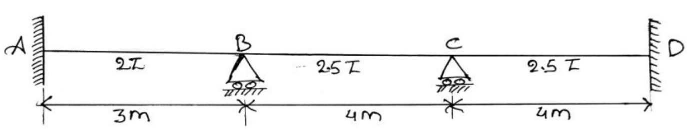
Carry over Factors

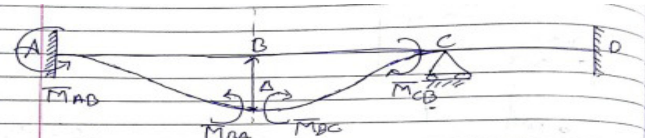
It is define as the ratio of moment at the fixed far end to the moment at the rotating near end.

$$CO = \frac{\text{Carry over moment at Far end}}{\text{Applied moment at near end}}$$

1 = mark.

(ii) By MDM determine the support moment if support B settles by 5mm. Drwa final bending moment diagram.Take EI=45000 KN-M





Fixed end moment

$\overline{M}_{AB} = \frac{-6EI\Delta}{l^2} = \frac{6E(2I)\,5\times10^{-3}}{3^2} = -300\,KN$

$\overline{M}_{BA} = \overline{M}_{AB} = -300$ KN-M

$\overline{M}_{BC} = \frac{6E(2.5I)\Delta}{4^2} = \frac{6E(2.5I)\,5\times10^{-3}}{4^2} = 211\,KN$

$\overline{M}_{CB} = \overline{M}_{BC} = 211$ KN-m.

Joint	member	Stiffness	T.S.	D.F
B	BA	2I/3	1.29 I	0.51
	BC	2.5I/4		0.49
C	CB	2.5I/4	1.25 I	0.58
	CD	2.5I/4		0.5

		0.51	0.49	0.5	0.5	
F.E.M	-300	-300	211	211	0	0
B.M.		45.39	43.61	-105.5	-105.5	
CoM	22.70		-52.75	21.80		-52.75
BM		+26.91	25.84	-10.9	-10.9	
CoM	-13.45		-5.45	12.92		-5.45
BM		+2.78	+2.67	-6.46	-6.46	
	1.39		-3.23	1.335		-3.23
		+1.65	+1.58	-0.667	-0.667	
	0.825		-3.23	1.335		-0.333
		0.17	0.163	-0.667	-0.395	
	0.085		-0.197	0.081		-0.197
		+0.099	0.098	-0.395	-0.04	
F.E.M.	-261.55	-223	+223	123.96	-123.96	-61.96

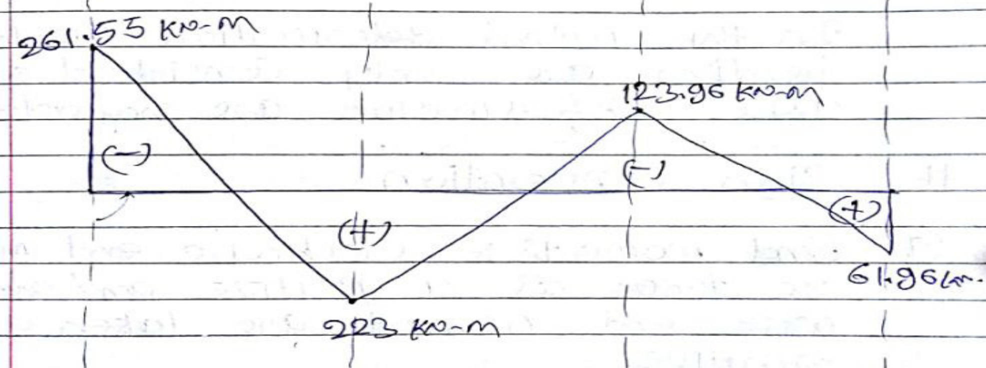
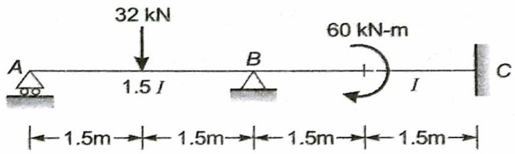
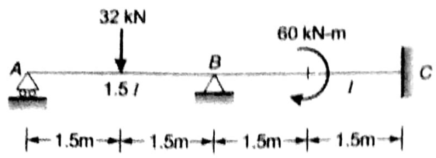


Diagram -> 1
Fixed end moment -> 2
Table D.F. -> 1
F.E.M. table -> 2
B.M. Diagram -> 1.

(iii) By SDM Analysis the beam loaded as shown in figure below using slpoe deflection method. Draw the BMD





Solution:

Fixed end moments:

0.25 mark each.

$$\overline{M}_{AB} = -\frac{PL}{8} = -\frac{32\times 3}{8} = -12 \text{ kN-m}$$

$$\overline{M}_{BA} = +\frac{PL}{8} = +12 \text{ kN-m}$$

$$\overline{M}_{BC} = +\frac{M_0}{4} = +\frac{60}{4} = +15 \text{ kN-m}$$

$$\overline{M}_{CB} = +\frac{M_0}{4} = +15 \text{ kN-m}$$

Slope deflection equations:

Member AB:

$$M_{AB} = \overline{M}_{AB} + \frac{2E(1.5I)}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{AB} = -12 + \frac{3EI}{3}(2\theta_A + \theta_B)$$

0.5 mark

$$M_{AB} = -12 + 2\,EI\theta_A + EI\theta_B \quad \text{...(i)}$$

and

$$M_{BA} = \overline{M}_{BA} + \frac{2E(1.5I)}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$M_{BA} = +12 + \frac{3EI}{3}(2\theta_B + \theta_A)$$

0.5 mark

$$M_{BA} = +12 + 2\,EI\theta_B + EI\theta_A \quad \text{...(ii)}$$

Member BC:

$$M_{BC} = \overline{M}_{BC} + \frac{2E(I)}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right) = +15 + \frac{2EI}{3}(2\theta_B - 0)$$

0.5 mark

$$M_{BC} = 15 + \frac{4EI\theta_B}{3} \quad \text{...(iii)}$$

$$M_{CB} = \overline{M}_{CB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right) = +15 + \frac{2EI}{3}(0 + \theta_B)$$

0.5 mark

$$M_{CB} = +15 + \frac{2}{3}EI\theta_B \quad \text{...(iv)}$$

MADE EASY Publications

Equilibrium equations:

There are two rotational unknowns θ_A and θ_B. Hence two joint equilibrium conditions are required.

$$M_A = 0 \qquad \text{(Hinged)}$$

$$\therefore \quad -12 + 2\,EI\theta_A + EI\theta_B = 0 \qquad \text{...(A)}$$

0.5 mark

Consider joint equilibrium at B,

$$M_{BA} + M_{BC} = 0$$

$$+12 + 2EI\theta_B + EI\theta_A + 15 + \frac{4}{3}EI\theta_B = 0$$

$$27 + 3.33\,EI\theta_B + EI\theta_A = 0$$

$$EI\theta_A + 3.33\,EI\theta_B = -27 \qquad \text{...(B)}$$

0.5 mark

On solving equation (A) and (B), we get

$$EI\theta_A = 11.823$$

$$EI\theta_B = -11.647$$

1 mark.

Final end moments

0.25 mark each.

$$M_{AB} = 0$$

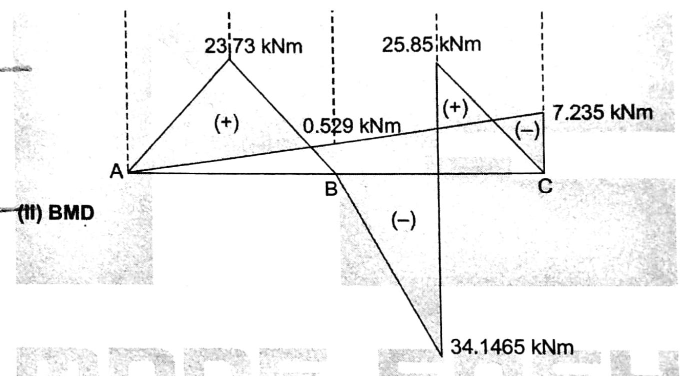
$$M_{BA} = 12 + 2EI\theta_B + EI\theta_A$$

$$= 12 + 2(-11.647) + 11.823 = 0.529 \text{ kNm}$$

$$M_{BC} = 15 + \frac{4}{3}EI\theta_B = 15 + \frac{4}{3}(-11.647) = -0.529 \text{ kNm}$$

$$M_{CB} = 15 + \frac{2}{3}EI\theta_B = 15 + \frac{2}{3}(-11.647) = 7.235 \text{ kNm}$$

1 mark.



Question: - 6 Attempt any two: (5, 5, or 5 Marks)

(i) A three hinged arch has a span of 30 meters and a rise of 10 m. The arch carries a uniformly distributed load of 60 KN per meter on the left half of its span. It also carries two concentrated load of 60 KN and 100 KN at 5 m and 10 m from the right end. Determine the horizontal thrust, at each support.

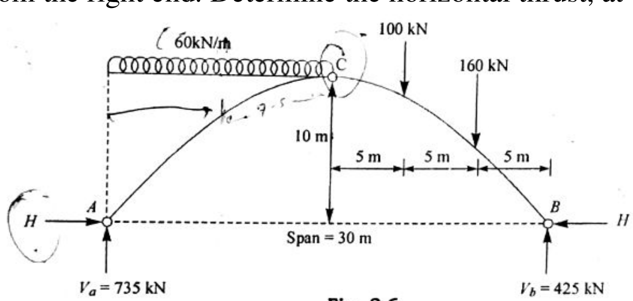


Fig. 2.6

marks

Diagram -> 2

Va = 718.33 KN -> 1

Vb = 341.67 KN -> 1

H = 402.45 KN -> 1

Solution. Let V_a and V_b be the vertical reactions at the supports A and B

Let H be the horizontal thrust.

Taking moments about the left end A, we have,

$$V_b \times 30 = 60 \times 15 \times \frac{15}{2} + 100 \times 20 + 160 \times 25$$

$$\therefore \quad V_b = 425\ kN; \qquad \therefore \quad V_a = \{(60 \times 15) + 100 + 160\} - 425 = 735\ kN$$

Taking moments about C of the forces on the left hand side of C, we have,

$$735 \times 15 = 60 \times 15 \times \frac{15}{2} + H \times 10; \qquad \therefore H = 427.5\ kN.$$

... moments about C of the forces on the right hand side of C, we have

(ii) A cable carrying a load of 10KN per meter run of horizontal span is stretched between supports 100 meter apart. The supports are at the same level and the central dip is 8 meters. Find the greatest and the least tension in the cable.

Solution. Each vertical reaction $= V = \frac{wl}{2} = \frac{10 \times 100}{2} = 500\,kN$ -> 1 mark Diagram -> 1

Horizontal reaction $= H = \frac{wl^2}{8h} = \frac{10 \times 100^2}{8 \times 8} kN = 1562.5\,kN$ -> 1 mark

Maximum tension $= T_{max} = \sqrt{V^2 + H^2} = \sqrt{500^2 + 1562.5^2} = 1640.55\,kN$ 1- mark

Minimum tension $= T_{min} = H = 1562.5\,kN$ 1- mark

(iii) A two hinged parabolic arch of span 25 m and rise 5 m carries a uniformly distributed load of 40 KN/m over the left half of the span and a concentrated load of 100 KN at the crown. Find the horizontal thrust at each support.

2 Marks

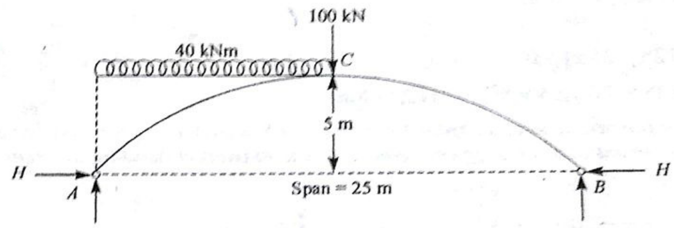


Fig. 2.84

Solution. Horizontal thrust due to uniformly distributed load on the left half of the span.

$$H_1 = \frac{wl^2}{16h} = \frac{40 \times 25^2}{16 \times 5} = 312.5\ kN$$ 1 - marks

Horizontal thrust due to point load at the crown,

$$H_2 = \frac{25}{128}\frac{Wl}{h} = \frac{25}{128} \times \frac{100 \times 25}{5} = 97.66\ kN$$ -> 1 mark

Total horizontal thrust,

$$H = H_1 + H_2 = 312.50 + 97.66 = 410.16\ kN$$ -> 1 Mark