Q.5 i. If H and K are two finite subgroups of G. Then show that **4**

$$o(HK) = \frac{o(H)o(K)}{o(H \cap K)}$$

ii. Define Cyclic group. If $(G,*)$ is finite group of order n generated by a then a^m is also generator of $(G,*)$ iff m and n are relative primes. **6**

OR iii. Give definition of Abelian group. Prove that the set $G = \{1,3,7,9\}$ is an Abelian group under multiplication modulo 10. **6**

Q.6 Attempt any two:

i. Determine the numeric function for the corresponding generating function:

(a) $G(x) = \dfrac{9}{1-5x}$ **2**

(b) $G(x) = (1+x)^m + (1-x)^m$ **3**

ii. Solve the recurrence relation $a_r - 2a_{r-1} - 3a_{r-2} = 0, r \geq 2$ by the generating function method with initial conditions $a_0 = 3, a_1 = 1$ **5**

iii. Solve the recurrence relation: $a_r - 6a_{r-1} + 8a_{r-2} = r.4^r$ with $a_0 = 8, a_1 = 22$ **5**

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2018

CA5BS04 Mathematics of Computer Application

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If *(Kn+1)* pigeons are kept in n pigeon holes where *K* is a positive integer, what is the average no. of pigeons per pigeon hole? **1**

(a) *K+1/n* (b) *K-1/n* (c) *K/n* (d) *K+n*

ii. How many permutations of the letters of the word APPLE are there: **1**

(a) 110 (b) 120 (c) 240 (d) 60

iii. In a simple graph, the number of edges is equal to twice the sum of the degrees of the vertices. **1**

(a) True (b) False (c) Can't say (d) None of these

iv. Which of the following is true? **1**

(a) A graph may contain no edges and many vertices
(b) A graph may contain many edges and no vertices
(c) A graph may contain no edges and no vertices
(d) None of these

v. Dijkstra's Algorithm will work for both negative and positive weights? **1**

(a) True (b) False (c) Can't say (d) None of these

vi. A graph G is called a ……..... if it is a connected acyclic graph? **1**

(a) Cyclic graph (b) Regular graph
(c) Tree (d) Not a graph

P.T.O.

vii. The inverse of - i in the multiplicative group, {1, - 1, i, - i} is **1**

(a) 1 (b) -1 (c) I (d) -i

viii. Let G denoted the set of all n x n non-singular matrices with rational numbers as entries. Then under multiplication G is a/an **1**

(a) Subgroup

(b) Finite abelian group

(c) Infinite, non-abelian group

(d) Infinite, abelian

ix. A recurrence relation is defined as $u_{n+1} = 0.2u_n + 90$, $u_6 = 50$. What is the value of u_8? **1**

(a) 110 (b) 100 (c) 112 (d) 118

x. The recurrence relation for the sequence 9, 27, 81, … is: **1**

(a) $t_0 = 9$, $t_{n+1} = 9t_n$ (b) $t_0 = 9$, $t_{n+1} = t_{n+9}$

(c) $t_0 = 9$, $t_{n+1} = 3t_n$ (d) $t_0 = 9$, $t_{n+1} = t_n + 18$

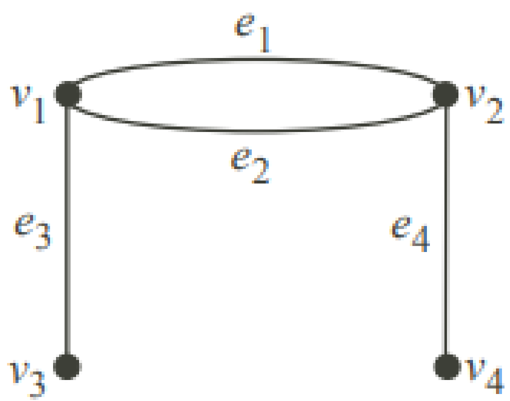
Q.2 i. Write short note on Combinatorics. **4**

ii. Using Pigeonhole principle, show that if any 14 numbers from 1 to 25 are chosen, then one of them is multiple of another. **6**

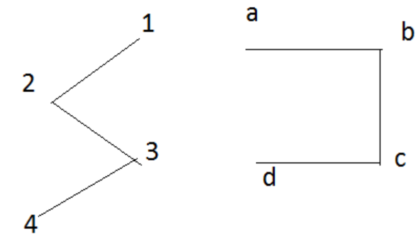
OR iii. Using Principle of Mathematical Induction Show that $n^5 - n$ is divisible by 5. **6**

Q.3 i. What is the difference between circuit and cycle? **2**

ii. (a) Find out the matrix representation of graph: **8**



(b) Show that the following graphs are isomorphic:



OR iii. Write short note (with example) on: **8**

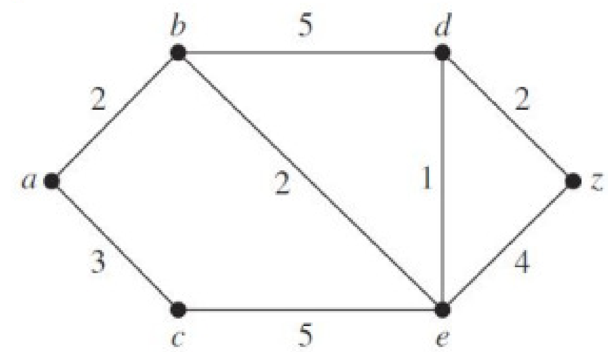
(a) Hamiltonian Graph (b) Digraphs

(c) Connected graph (d) Spanning sub graphs

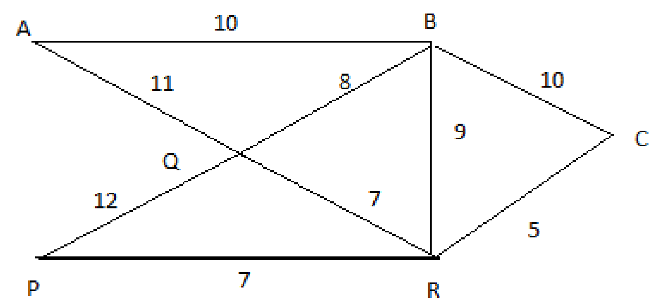
Q.4 i. Write short note on **3**

(a) Binary tree (b) Weighted graph

ii. Find the length and shortest path between a and z in the following weighted graph using Dijkstra's algorithm.: **7**



OR iii. Using the Kruskal's Algorithm, find a minimal spanning tree for the weighted graph given below: **7**



Q.1 (i) (a) {3} +1

(ii) {c} Transitive +1

(iii) {b} characteristic eqⁿ +1

(iv) (a) Both (I) & (II) are true +1

(v) (b) Isolated vertex +1

(vi) (b) Parallel edges +1

(vii) (a) $P(A \cap B) = P(A) \cdot P(B)$ +1

(viii) (b) np +1

(ix) (b) 2 +1

(x) (c) linear +1

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Q.2 (ii) Here $m = 14$, $n = 25$
and By Pigeonhole Principle
we have,

$$\left(\frac{n-1}{m}\right) + 1 \qquad +2$$

$$= \frac{25-1}{14} + 1 = 2 \qquad +2$$

OR $P(n) = n^5 - n$
for $n = 1$, $P(1) = 1^5 - 1 = 0$ and divisible by 5 +1
$\Rightarrow$ $P(n)$ is divisible by 5 for $n = 1$
let for some natural nos. $n = m$; $P(m)$ be divisible by 5, i.e. $m^5 - m$ be divisible by 5 +1

$$m^5 - m = 5K \quad \text{where } K \in N \qquad +1$$

~~$m^5 = 5K$~~

Now,

$$P(m+1) = (m+1)^5 - (m+1) \qquad +1$$

$$= (m+1)^3 (m+1)^2 - (m+1)$$

$$= (m^3 + 3m^2 + 3m + 1)(m^2 + 2m + 1) - (m+1)$$

$$= m^5 + 2m^4 + m^3 + 3m^4 + 6m^3 + 3m^2 + 3m^3 + 6m^2 + 3m + m^2 + 2m + 1 - (m+1) \qquad +1$$

$$= (m^5 - m) + 10m^3 + 5m^4 + 10m^2 + 5m$$

$$= (m^5 - m) + 5(m^4 + 2m^3 + 2m^2 + m)$$

so $P(m+1)$ is divisible by 5 if $P(m)$ is divisible by 5 +1

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Q.3 (i) Circuit : Traversing a graph Such that not an edge is repeated but vertex can be repeated and it is closed also $+\frac{1}{2}$

Eg. :- any circuit $+\frac{1}{2}$

(ii) Traversing a graph S.t. we do not repeat a vertex nor we repeat a edge but the starting and ending vertex must be same. $+\frac{1}{2}$

Eg. :- any cycle $+\frac{1}{2}$

(ii) (a) Matrix of given graph
There we draw only incidence matrix (b'coz of parallel edges) 1

$$\begin{array}{c|cccc} & e_1 & e_2 & e_3 & e_4 \\ \hline v_1 & 1 & 1 & 1 & 0 \\ v_2 & 1 & 1 & 0 & 1 \\ v_3 & 0 & 0 & 1 & 0 \\ v_4 & 0 & 0 & 0 & 1 \end{array}$$

+1
+2

(b) Here we observe that
(i) Both the graph have 3 edges i.e.
(ii) Both the ___ ___ same no. of vertices (4) +1
(iii) deg (1) = deg(a) = 1
deg (2) = deg (b) = 2 +1
deg (3) = deg(c) = 2
deg (4) = deg(d) = 1 +1

Thus the vertices 1, 2, 3, 4 correspond to a, b, c, d resp. and edges (1,2), (2,3), (3,4) correspond to (a,b) (b,c), (c,d) resp. Hence two graph are isomorphic. +1

OR

(a) Hamiltonian graph:
A graph is called Hamiltonian graph if it contain a Hamitonian circuit +1
Eg. graph +1

(b) Diagraph: In a graph, a vertex is represented by a pt. and an edge by a line segment b/w v_i and v_j with an arrow directed from v_i to v_j +1
eg. : graph +1

(c) Connected Graph: A graph G is said to connected if there is at least one path b/w every pair of vertices in G. +1
eg. : +1

(d) Spanning Subgraph:
Let $G'(V', E')$ be a subgraph of a graph $G(V, E)$. If $V' = V$, then G' is said to be a spanning subgraph of G +1
eg. : +1

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Q.4. (i)(a) Binary Tree: A binary tree is defined as a tree in which there is exactly one vertex of degree two, and each of the remaining vertices is of degree one or three. +1

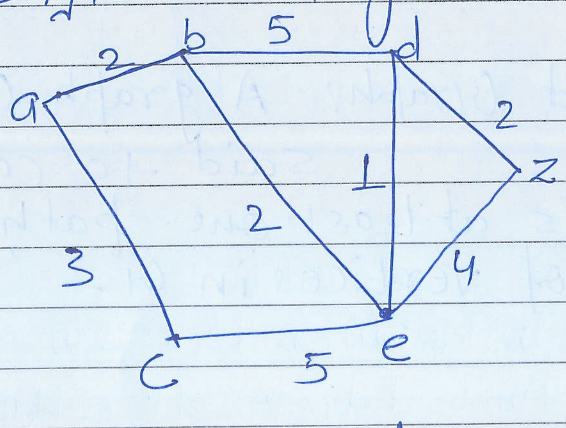
+1/2

eg:

(b) Weighted Graph: A weighted graph is a graph in which a non-negative real no. w(e) is assigned to its each edge e. +1
The no. w(e) is called the weight of the edge. +1/2

eg:

(ii) Dijkstra's Algorithm

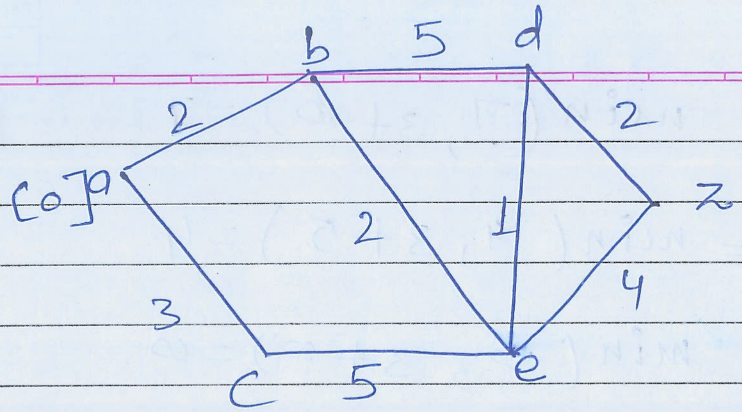


Assign the initial vertex a the permanent label 0 i.e.

$PL(a) = [0]$

& $TL(b) = TL(c) = TL(d) = TL(e)$ +1

$= TL(z) = \infty$



+1/2

Step 2

$TL(b) = \min(\infty, 0+2) = 2$

$TL(c) = \min(\infty, 0+3) = 3$

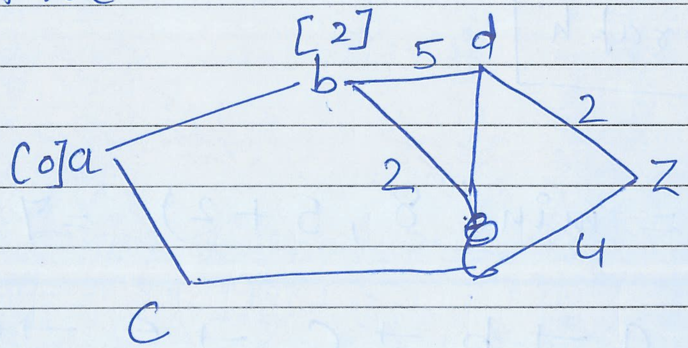
$TL(d) = \min(\infty, 0+\infty) = \infty$ +1

$TL(e) = \min(\infty, 0+\infty) = \infty$

$TL(z) = \infty$

step 3 - b has smallest new ~~TL~~) temp. label

$PL(b) = 2$



+1/2

$TL(c) = \min(3, 2+\infty) = 3$

$TL(d) = \min(\infty, 2+5) = 7$ +1

$TL(e) = \min(\infty, 2+2) = 4$

$TL(z) = \min(\infty, 2+\infty) = \infty$

step 4 - smallest is $c = 3$

$PL(c) = 3$

(graph)

$TL(d) = \min(7, 3+\infty) = 7$

$TL(e) = \min(4, 3+5) = 4$

$TL(z) = \min(\infty, 3+\infty) = \infty$ +1

Step 5: $PL(e) = 4$

$\boxed{\text{graph}}$ +1/2

$TL(d) = \min(7, 4+1) = 5$

$TL(z) = \min(\infty, 4+4) = 8$ +1/2

Step 6 : $PL(d) = 5$

$\boxed{\text{graph}}$ +1/2

$TL(z) = \min(8, 5+2) = 7$ +1/2

Path : $a \to b \to c \to e \to d \to z$

$= 7$

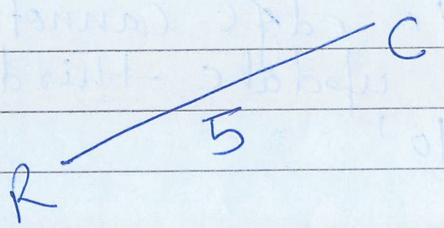
OR Kruskal's Algorithm

step 1 we arrange all the edges of the given graph in the increasing order of their weights

	Edges in increasing order	weights of edge	selection of the edge
1	R - C	5	yes
2	P - R	7	yes
3	A - R	7	yes
4	Q - B	8	yes
5	B - R	9	No
6	A - B	10	yes
7	B - C	10	No
8	A - Q	11	No
9	P - Q	12	No.

+2

Step 2 : consider the edge (R, C) having smallest weight of 5 and add it to the spanning tree by updating the third column of the first row represent the edge (R, C) to 'yes' +1



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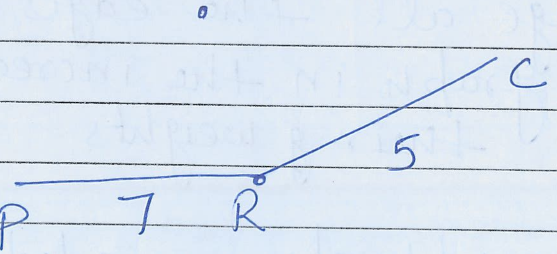
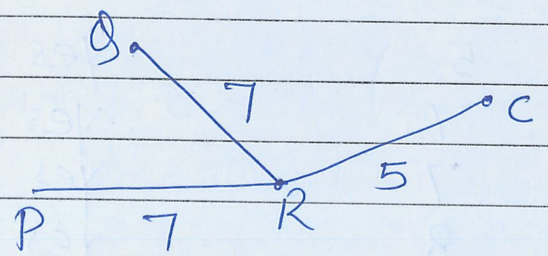
Step 3 : Consider the edge (P, R)



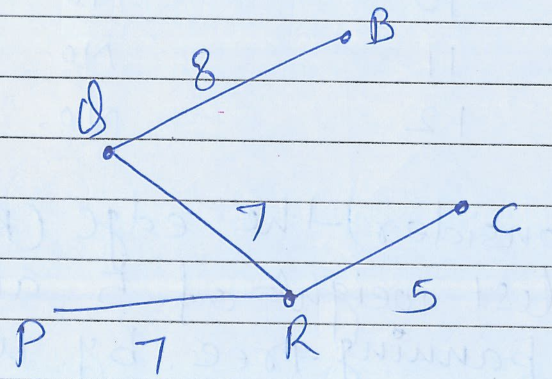
+1

Step 4 : Consider the edge (Q, R).



+1

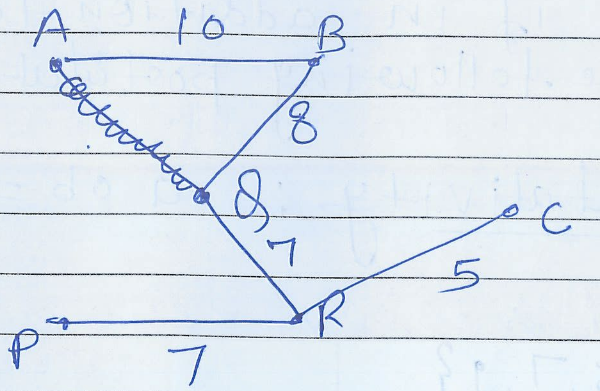
Step 5 : Consider the edge (Q, B)



+1

Step 6 : Consider the edge (B, R), As the inclusion of this edge to the spanning tree will create a circuit, so this edge cannot be ~~ind~~ taken. and update third column say 'No'

Step 7 ; Consider the edge (A, B)



Step 8 : Consider the edge (B, C)
it also forms a circuit so skip this edge.

Step 9 : ~~Consider the edge~~ The algorithm stops as the tree already contains n-1 no. of edge (n - no of vertex) +1

Q.5 (ii) Cyclic Group : A group (G, •) is called cyclic is for $a \in G$, every element $x \in G$ is of the form a^n, where n is some integer symbolically, $G = \{a^n | n \in I$ +2

The element a is then called a generator of G

Proof of theorem +3

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OR. (iii) Abelian Group: A group (G, o) is said to be abelian or if in addition to four axioms the following postulate is also satisfied

Commutativity: $a o b = b o a$ +2

$G\{1, 3, 7, 9\}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

+1

1) Closure +1

2) Associative +1

3) Identity +1

4) Inverse – $a = 1$ $a^{-1} = 1$, $a = 3$, $a^{-1} = 7$ +1
$a = 7$ $a^{-1} = 1$, $a = 9$, $a^{-1} = 9$

5) Commutativity +1

Q. 6 (i) (a) $G(x) = \frac{9}{1-5x}$

$= 9(1-5x)^{-1}$

$= 9[1 + 5x + 5^2x^2 + \cdots + 5^r x^r + \cdots)$

$= 9 + 3^2 5x + 3^2 5^2 x^2 + 3^2 5^3 x^3 + \cdots$

$+ 3^2 . 5^r x^r + \cdots$ +1

Hence the numeric function a_r corresponding to $A(z)$ is

$a_r = 3^2 . 5^r$ where $r = 0, 1, 2, 3 \cdots$ +1

(ii) $G(x) = (1+x)^m + (1-x)^m$

$= \left[1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}\right.$

$\left. x^3 + \cdots\right] + \left[1 - mx + \frac{m(m-1)}{2!}x^2\right.$

$\left. - \frac{m(m-1)(m-2)}{3!}x^3 + \cdots\right]$ +1

$= 2 + 2\frac{m(m-1)}{2!}x^2 + 2\frac{m(m-1)(m-2)(m-3)}{4!}x^4$ +1

$+ \cdots + 2$

$a_r = 2 . {}^mC_{2r} x^{2r}$ +1

(iii) $a_r - 6a_{r-1} + 8a_{r-2} = r.4^r$ — (1)

and $a_0 = 8$, $a_1 = 22$

$m^2 - 6m + 8 = 0$ — (2)

$m = 2, 4$

$a_r^{(h)} = C_1 2^r + C_2 4^r$ — (3) +1

and $a_r^{(P)} = (C_3 r + C_4 r^2) 4^r$ — (4)

Substituting in (1)

$4^r C_3 r + C_4 r^2 4^r - \frac{C_3 6}{4} r 4^r + \frac{C_3 6}{4} 4^r + 6C_4 \frac{(r-1)^2}{4} 4^r + \frac{8C_3}{16}(r-2) 4^r + \frac{8C_4}{16}(r-2)^2 4^r$

$= r.4^r$

and comparing Coeff., find C_3 & C_4 +1

then put in (4)

$a_r = = a_r^{(h)} + a_r^{(P)}$ — (5) +1

then ~~find~~ put $r=0$ & $r=1$ in (5)

find C_1 & C_2 +1

then complete the soln +1

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Q.5(i) let H and K be the finite subgroup of group G then HK is a subset of G +1

By Lagrange's theo. +1

$$m = \frac{o(K)}{O(D)}$$

distinct right cosets of D in K

$\therefore$ $HK = H(\bigcup_{i=1}^{m} Dk_i)$ +1

we shall show that the coset are pairwise distinct ~~0~~

$\therefore$ $O(HK) = m \times (o(H)$ +1