Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2019

EN3BS07 Introductory Topics in Statistics, Probability and Calculus

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If 'a' is the actual value and 'e' is its estimated value, the absolute error is **1**

(a) a-e (b) |a-e| (c) a/e (d) (a-e)/e

ii. Which of the following represents data? **1**

(a) A single value in a set (b) Only two values in a set

(c) A group of values in a set (d) At least two values in a set

iii. Which of the following is a one-dimensional diagram? **1**

(a) Bar diagram (b) Pie-chart

(c) Cylinder (d) A graph

iv. Define quartile deviation for ungrouped data. **1**

(a) Q_3-Q_1 (b) $(Q_3-Q_1)/2$ (c) Q_3+Q_1 (d) $(Q_3+Q_1)/2$

v. Given P(A)= 1/3, P(B)=3/4, P(AUB)=11/12 then find P(B/A) **1**

(a) 1/6 (b) 4/9 (c) 1/2 (d) 1/5

vi. Find E(2x+3) from the following probability distribution **1**

X	-3	-2	-1	0	1	2	3
P(X)	0.05	0.1	0.3	0	0.3	0.15	0.1

(a) 3 (b) 3.5 (c) 4.5 (d) 5

vii. If X is a Poisson random variable with mean 3, then $P\{|x-3|<1\}$ **1**

(a) 0.2240 (b) 0.3345 (c) 0.3456 (d) 0.7832

viii. The mean value of the Uniform distribution with pdf $f(x)=1/(\beta-\alpha)$ **1**

(a) $(\alpha\beta)/2$ (b) $2/(\alpha\beta)$ (c) $(\alpha-\beta)/2$ (d) $(\alpha+\beta)/2$

ix. Evaluate $\int_1^a \int_1^b \frac{1}{xy} dxdy$ **1**

(a) log(ab) (b) loga.logb (c) log(a/b) (d) loga + logb

P.T.O.

x. The formula for volume using triple integral. **1**

(a) $\iiint_E dxdydz$ (b) $\iiint_E xyz\ dxdydz$

(c) $\iiint_E (x+y+z)\ dxdydz$ (d) $\iiint_E x^y y^z z^x dxdydz$

Q.2 Attempt any two:

i. Discuss various methods of collecting the data. **5**

ii. Write about the applications of Statistics in various branches **5**

iii. What do you mean by Population and sample? Explain. **5**

Q.3 i. Find the standard deviation to the following data **2**

10, 12, 14, 15, 17, 18, 18, 24

ii. Calculate the standard deviation of the following data **8**

Class interval	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	3	61	132	153	140	51	2

OR iii. Calculate the Mean Deviation about mean of the following data. **8**

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	6	5	8	15	7	6	3

Q.4 i. The odds that a book will be favourably reviewed by three independent critics are 5 to 2, 4 to 3 and 3 to 4 respectively. What is the probability that, of three reviews, a majority will be favourable? **3**

ii. Of all the graduate students in the university, 70% are women and 30% are men. Suppose that 20% and 25% of the female and male population smoke cigarettes, what is the probability that a randomly selected graduate student is **7**

(a) A woman who smokes

(b) A man who smokes

(c) A smoker

OR iii. The contents of urns I, II and III are as follows: **7**

1 white, 2 black and 3 red balls

2 white, 1 black and 1 red balls

4 white, 5 black and 3 red balls.

One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urn III?

Q.5 i. The following probability function is given by **4**

X	-3	-2	-1	0	1	2	3
P(x)	0.05	0.10	2k	0	0.3	k	0.10

Compute

(a) K (b) E(x) (c) E(4x+5) (d) V(2x+3)

ii. Out of 800 families with 4 children each, how many families would be expected to have (assume equal probably for boys and girls) **6**

(a) Two boys and two girls (b) At least one boy

(c) At most two girls

OR iii. In a test on 2000 electric bulbs, it was found that the life of a particular type was normally distributed with an average life of 2040 hours and a standard deviation of 60 hours. Estimate the no. of bulbs likely to burn for **6**

(a) More than 2150 hours

(b) Less than 1950 hours

(c) More than 1920 hours but less than 2160 hours

Given that :

$P(0 \le Z \le 2) = 0.4772$

$P(0 \le Z < 1.83) = 0.4664$

$P(0 \le Z \le 1.5) = 0.4332$

Q.6 i. Evaluate $\int_0^1 \int_0^x (x+y)\, dx\, dy$. **2**

ii. Find the area bounded by the lines x=0, y=0 and x+y=1 **3**

iii. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$. **5**

OR iv. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\ dxdydz$. **5**

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Solution of EM3BS07

End Sem (odd) Exam. Dec-2019.

Introductory topics in Statistics, Probability and Calculus.

MCQ

Q1. (i)	(b)	$\|a-e\|$	1
(ii)	(c)	A group of values is a set	1
(iii)	(a)	Bar diagram.	1
(iv)	(b)	$(Q_3-Q_1)/2$	1
(v)	(c)	1/2	1
(vi)	(b)	3.5	1
(vii)	(a)	0.2246	1
(viii)	(d)	$(\alpha+\beta)/2$	1
(ix)	(b)	$\log a \cdot \log b$	1
(x)	(a)	$\iiint_E dx\,dy\,dz$	1

Q2. (i) Various methods of collecting data.

The statistical data may be already available or may have to be collected by an investigator or an agency. Data is termed primary when the reference is to data collected for the first time by the investigator and it is termed secondary when the data are taken from records or data already available. +1

There are two principal methods of data collection - through a census or through a sample survey. Census implies complete enumeration of each and every element of source.

When only some selected elements of the source are taken and measurements or observations of these selected elements are recorded is said to be sample data. +2

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Methods of collection of primary data

primary data is data collected for the first time through census or sample

There are several ways of collecting such data.

(i) direct personal interview or observation.
(ii) Indirect personal interview & observation
(iii) mailed questionnaire. +2
(iv) Schedules through enumerators.

(ii) Application of Statistics in various branches.

* It applies to almost all sciences : pure & applied, physical, natural, biological, medical, agricultural and Engineering.
* It also finds application in social and management Sciences, in commerce, business and industry. +2
* Social Sciences, economics leans most heavily on statistical methods for analysis of data relating to micro as well as to macro economics, from demand analysis to national income analysis.
* The impact of mathematics and statistics has led to the development like econometrics and Economic Statistics. There is wide scope of Statistics in handling and analysing data relating to Socio-economic, demographic and political processes. +2
* Statistical concepts and methods of quality control and reliability prove invaluable in industrial Engg. +1
* Applications of statistical methods to education and phychology have led to development of new dicipline "Psychometry".

(iii) Population :- A set of observations relating to a phenomenon under statistical investigation is known as population. It may be finite or infinite. Observations on population may relate to any source, animate or inanimate. human beings, families, commodities. so on. +2

The set of data actually collected through a process of observation or experimentation from selected items of the source is called a sample. or it is a part of population. it is always finite +3

A sample is taken in order to gather information about the true state and characteristic of the population. A sample has thus to be truly representative of the pop^{n}.

Q3-

(i) Standard deviation.

Two formulas we have.

$$\sigma^2 = \frac{1}{n}\Sigma(x_i - \bar{x})^2 \Rightarrow \sigma = \sqrt{\frac{1}{n}\Sigma(x_i-\bar{x})^2}$$

or $$\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$ +1/2

x	x^2
10	100
12	144
14	196
15	225
17	289
18	324
18	324
24	576

$$\sigma^2 = \frac{2178}{8} - \left(\frac{128}{8}\right)^2$$ +1/2

$$\sigma^2 = 272.25 - (16.)^2$$

$$\sigma = \sqrt{16.25}$$ +1

$$\sigma = 4.0311$$

(ii) When frequency is given we have two type of formula

$$\sigma^2 = h^2 \left[\frac{1}{N} \sum f d_i^2 - \left(\frac{\sum f d_i}{N} \right)^2 \right]$$

Where $d = \frac{x - A}{h}$ h = class interval

A = assumed mean. +1

or

$$\sigma^2 = h^2 \left[\frac{1}{N} \sum_{i=1}^{n} f_i (d_i - \bar{d})^2 \right]$$

Class I C.I	freq. f	middle Pt (x)	$d = \frac{x - 55}{10}$	fd	fd^2
20-30	03	25	-3	-9	27
30-40	61	35	-2	-122	244
40-50	132	45	-1	-132	132
50-60	153	55	0	0	0
60-70	140	65	1	140	140
70-80	51	75	2	102	204
80-90	02	85	3	6	18
	$N = 542$			$\sum fd = -15$	$\sum fd^2 = 765$

+5

$$\therefore \sigma^2 = 100 \times \left[\frac{765}{542} - \left(\frac{-15}{542} \right)^2 \right]$$

$$= 100 \times \left[\frac{765}{542} - (.028)^2 \right]$$

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$\sigma^2 = 100 \times 1.4107 = 141.07$ +2

$\therefore \sigma = \sqrt{141.07} = 11.88.$

(iii) Mean deviation about mean.

$$M.D(\bar{x}) = \frac{\sum f|x-\bar{x}|}{N} = \frac{\sum f_i|x_i-\bar{x}|}{N}$$

$\bar{x}$ = mean. $N = \sum f$

↳ $= \frac{\sum fx}{\sum f}$

class Interval	freq. f	midpt x	fx	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
0–10	6	5	30	28.4	170.4
10–20	5	15	75	18.4	92
20–30	8	25	200	8.4	67.2
30–40	15	35	525	1.6	24
40–50	7	45	315	11.6	81.2
50–60	6	55	330	21.6	129.6
60–70	3	65	195	31.6	94.8
	$N=50$		$\sum fx = 1670$		$\sum f\lvert x-\bar{x}\rvert = 659.2$

+6

$$\therefore M.D(\bar{x}) = \frac{\sum f_i|x-\bar{x}|}{\sum f}$$

$$= \frac{659.2}{50} = 13.184$$ +2

(here $\bar{x} = \frac{1670}{50} = 33.4$)

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Q4 (i) Let A, B, C are three reviewers.
given odds in favour so. $P(A) = \frac{5}{2+5} = \frac{5}{7}$

$P(B) = \frac{4}{3+4} = \frac{4}{7}$ and $P(C) = \frac{3}{3+4} = \frac{3}{7}$ +1

Now in the given situation four case arises

I A, B, C all review favourably Then probability is

$$P(A) \cdot P(B) \cdot P(C) = \frac{5}{7} \times \frac{4}{7} \times \frac{3}{7} = \frac{60}{343}$$

II A, B review favourably & C reviews unfavourably +1
Probability is $P(A) \cdot P(B) \cdot P(\bar{C}) = \frac{5}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{80}{343}$

III B, C, " & A " —
$P(B) \cdot P(C) \cdot P(\bar{A}) = \frac{4}{7} \times \frac{3}{7} \times \frac{2}{7} = \frac{24}{343}$

IV A, C " — & B " —

$P(A)\ P(C) \cdot P(\bar{B}) = 5/7 \times \frac{3}{7} \times 3/7 = \frac{45}{343}$

The probability that a majority will be favourable is +1

$$\frac{60}{343} + \frac{80}{343} + \frac{45}{343} + \frac{24}{343} = \frac{209}{343} = 0.86$$

(ii) Let F → Female
M → Male
S → Smoking
Then we are given

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$P(F) = 0.7 \quad P(M) = 0.3$ +1

$P(S/F) = 0.2 \quad P(S/M) = 0.25$

(a) A woman who smokes. i.e

$P(S \cap F) = P(S/F) \cdot P(F) = 0.2 \times 0.7 = 0.14$ +2

(b) $P(S \cap M) = 0.3 \times 0.25 = .075$ +2

(c) $P(S) = P(S \cap F) + P(S \cap M) = 0.14 + .075 = 0.215$ +2

(iii) A_1, A_2, A_3 be the event when the balls are chosen from urn I, II, III respectively.

we are given $P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$ +2

Let B be the event balls are chosen.

Also given two balls are drawn.

to be white & red. So,

$P(B/A_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1 \times 3}{6 \cdot 5} \times 2 \cdot 1 = \frac{1}{5}$ +1

$P(B/A_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{2 \times 1}{4 \times 3} \times 2 \times 1 = \frac{1}{3}$ +1

$P(B/A_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{4 \times 3}{12 \times 11} \times 2 \times 1 = \frac{2}{11}$ +1

using Bayes Theorem.

$$P(A_3/B) = \frac{P(A_3) \times P(B/A_3)}{\sum_{i=1}^{3} P(A_i) \cdot P(B/A_i)}$$

$$= \frac{\frac{1}{3} \times \frac{2}{11}}{\frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{2}{11}} \quad +2$$

$$= \frac{30}{118} = 0.2542$$

Q5 (i) (a) $\sum P(x) = 1$

$$.05 + .10 + 2k + 0 + 0.3 + k + 0.1 = 1$$

$$3k = 1 - 0.55$$

$$3k = 0.45 \quad +1$$

$$k = \frac{0.45}{3} = 0.15$$

(b) $E(x) = \sum x\, p(x)$

$$= -3 \times .05 + (-2) \times .10 + (-1) \times 2 \times 0.15 + 0 + 1 \times 0.3 + 2 \times (0.15) + 3 \times 0.10 \quad +1$$

$$= -0.15 - 0.2 - 0.3 + 0.3 + 0.3 + 0.3$$

$$= 0.25$$

(c) $E(4x+5)$ $+1$

$$= 4E(x) + 5.$$

$$= 4 \times 0.25 + 5 = 6$$

(d) $V(2x+3) = 4V(x) + V(3)$

$= 4 \times V(x) + 0$

Now

$V(x) = E(x^2) - [E(x)]^2$

& $E(x^2) = \sum x^2 P(x)$

$= 9 \times 0.05 + 4 \times 0.1 + 1 \times 0.3 + 1 \times 0.3 + 4 \times 0.15 + 9 \times 0.1$

$= 0.45 + 0.4 + 0.3 + 0.3 + 0.6 + 0.9$ +1

$= 2.95$

$\therefore V(x) = 2.95 - .0625 = 2.8875$

$\therefore V(2x+3) = 4 \times 2.8875 = 11.55$

(ii) Here $N = 800$ and $n = 4$.

prob of getting boy $(p) = 1/2$

getting girl $(q) = 1/2$

using binomial dist^n. (Prob. of getting x success out of n trials)

$P(x) = {}^nC_x \, p^x \, q^{n-x}$

(a) Two boys and two girls $(P(x=2))$

$= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} = \frac{3}{8} = 0.375$ +2

No of families 0.375×800

$= 300$.

(ii) At least one boy ($P x(\geq 1)$

$P(x \geq 1) = 1 - P(x < 1)$

$= 1 - P(x = 0)$

$= 1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$

$= 1 - \frac{1}{16} = \frac{15}{16} = 0.9375$ +2

no of families $= \frac{15}{16} \times 800^{100} = 750$

(iii) At most 2 girls $P(x \leq 2)$

$P(x=0) + P(x=1) + P(x=2)$

$= {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$

$= \frac{1}{16} + \frac{4^2 \times 1}{2 \quad 8/4} + \frac{4 \times 3}{2 \times 1} \left(\frac{1}{4_2}\right)\left(\frac{1}{4}\right)$

$\frac{1}{16} + \frac{1}{4} + \frac{3}{4} = \frac{11}{16} = 0.6875$

Number of families

$= \frac{11}{16} \times 800 = 550$ +2

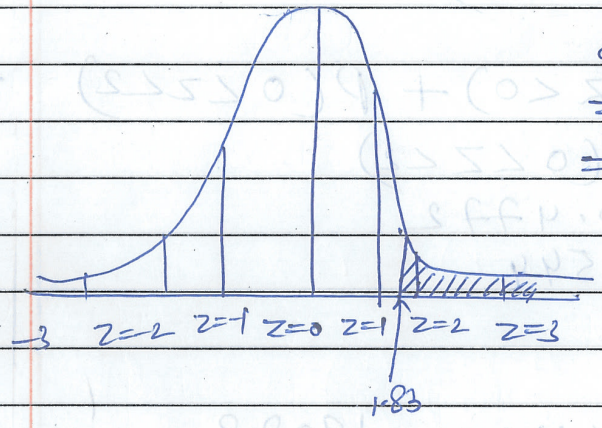
(iii) given $N = 2000$

$\mu = 2040 \quad \sigma = 60$

$z = \frac{x - 2040}{60}$ as $z = \frac{x-\mu}{\sigma}$

(a) $P(x > 2150) = P(z > 1.83)$

$z = \frac{2150 - 2040}{60} = 1.83$



$\therefore P(z > 1.83)$

$= 0.5 - P(0 < z < 1.83)$

$= 0.5 - 0.4664$

$= 0.0336$

+2

Estim no of bulb $= 0.0336 \times 2000 = 67.2$

≈ 67 bulbs.

(b) $P(x < 1950)$

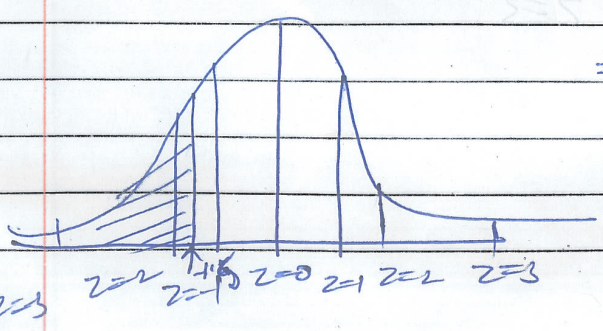
$= P(z < -1.5)$

$z = \frac{1950 - 2040}{60}$

$= -1.85 = -1.5$

$= 0.5 - P(0 < z < 1.5)$

$= 0.0668$



No. of bulbs 0.0668×2000

$= 133.6$

$\approx 134.$

+2

(c) More than 1920 but less than 2160

$P(1920 < x < 2160)$

$$z_1 = \frac{1920 - 2040}{60} = -2$$

$$z_2 = \frac{2160 - 2040}{60} = 2$$

$P(-2 < z < 2)$

$= P(-2 < z < 0) + P(0 < z < 2)$ +2

$= 2 \times P(0 < z < 2)$

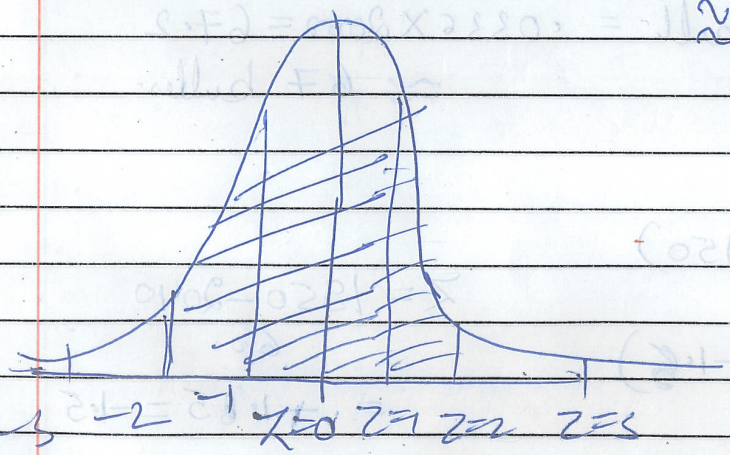
$= 2 \times 0.4772$

$= 0.9544$

Number of bulbs.

$= 0.9544 \times 2000 = 1908.8$

≈ 1909 bulb.



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Q6. (i) $\int_0^1 \int_0^x (x+y)\, dx\, dy$

$= \int_0^1 \left(xy + \frac{y^2}{2}\right)_0^x dx$ +1

$= \int_0^1 \left(x^2 + \frac{x^2}{2}\right) dx = \int_0^1 \frac{3}{2} x^2\, dx$

$= \frac{3}{2}\left(\frac{x^3}{3}\right)_0^1 = \frac{3}{2} \times \frac{1}{3} = \frac{1}{2}$ +1

(ii) area bounded by the lines $x=0$

$y=0$ and $x+y=1$.

using the concept of double integral area is

$$A = \iint_{\text{Region}} dx\, dy$$

given lines are. $x=0$, $y=0$ — (1)

$x+y=1$ — (2)

points on (2) are (0,1) and (1,0) +1

first we draw the given lines

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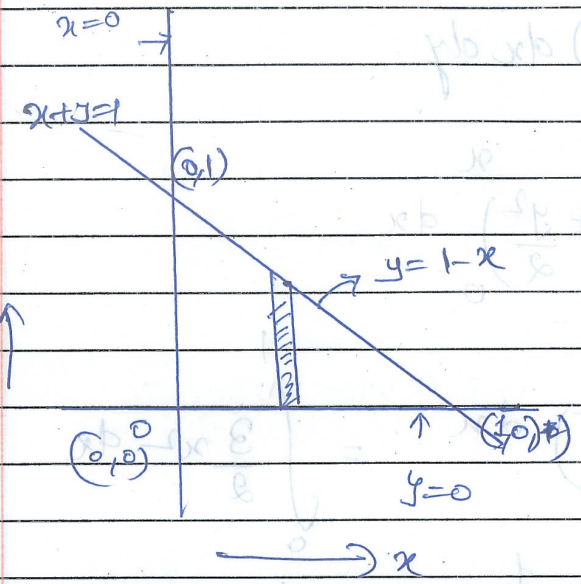

taking the strip ∥ to y-axis

limits for y, $y=0$ to $y=1-x$

and $x=0$ to $x=1$

$$\therefore \text{Area} = \int_0^1 \int_0^{1-x} dx\, dy$$

$$= \int_0^1 (1-x)\, dx$$

$$= \left(x - \frac{x^2}{2}\right)_0^1$$

$$= 1 - \frac{1}{2} = \frac{1}{2} \text{ sq. unit}$$

(iii) $$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz.$$

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$$= \int_0^a \int_0^x e^{x+y} \left\{ \int_0^{x+y} e^z dz \right\} dx\, dy$$

$$= \int_0^a \int_0^x e^{x+y} \left(e^z\right)_0^{x+y} dx\, dy, \quad \text{taking } x, y \text{ constants} \qquad +1$$

$$= \int_0^a \int_0^x e^{x+y} \left(e^{x+y} - e^0\right) dx\, dy$$

$e^0 = 1$

$$= \int_0^a \left\{ \int_0^x \left\{ e^{2(x+y)} - e^{x+y} \right\} dy \right\} dx \qquad +1$$

$$= \int_0^a \left(\frac{e^{2x} e^{2y}}{2} - e^x e^y \right)_0^x dx$$

$$= \int_0^a \left\{ \frac{e^{4x}}{2} - \frac{e^{2x}}{2} - \left(e^{2x} - e^x\right) \right\} dx$$

$$= \frac{1}{8}\left(e^{4a} - 1\right) - \frac{3}{4}\left(e^{2a} - 1\right) + \left(e^a - 1\right)$$

$$= \frac{1}{8}\left(e^{4a} - 6e^{2a} - 8e^a - 3\right) \qquad +3$$

(iv) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dx \, dy \, dz$

$= \int_0^1 \int_0^{\sqrt{1-x^2}} xy \left(\frac{z^2}{2}\right)_0^{\sqrt{1-x^2-y^2}} dx\,dy$ +1

$= \int_0^1 \int_0^{\sqrt{1-x^2}} xy \, \frac{(1-x^2-y^2)}{2} \, dx\,dy$

$= \int_0^1 \frac{x}{2} \left\{ \frac{xy^2}{2} - \frac{x^2y^2}{2} - \frac{y^4}{4} \right\}_0^{\sqrt{1-x^2}} dx$ +1

$= \frac{1}{8} \int_0^1 \left[(1-x^2)^2 2x - (1-x^2)^4 . x \right] dx$

$= \frac{1}{8} \int_0^1 (x - 2x^3 + x^5) \, dx$

$= \frac{1}{8} \left(\frac{x^2}{2} - \frac{2x^4}{4} + \frac{x^6}{6} \right)_0^1$

$= \frac{1}{8} \left(\frac{1}{2} - \frac{2}{4} + \frac{1}{6} \right) = \frac{1}{48}$ +3.

End of the Solution.