

[4]

- ii. Find the Karl Pearson's coefficient of correlation for the following data **5**

x	2	4	4	7	5
y	8	8	5	6	2

- iii. Find the line of regression of y on x for the following data

x	5	2	1	4	3
y	5	8	4	2	10

5

Total No. of Questions: 6

Total No. of Printed Pages: 4

Enrollment No.....



Faculty of Engineering

End Sem (Odd) Examination Dec-2019

CA5BS01 Mathematical Foundation of Computer

Science

Programme: MCA

Branch/Specialisation: Computer Application

Maximum Marks: 60

Duration: 3 Hrs.

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

- Q.1 i. If $A = \{1,2,3,4,5\}$ and $B = \{3,4,5,6\}$ then $A - B$ is **1**
(a) {3,4,5} (b) {1,2} (c) {6} (d) None of these
- ii. If for any two sets A and B such that $A \cap B = \emptyset$ then A and B are **1** called
(a) Equal sets (b) Universal sets
(c) Disjoint sets (d) None of these
- iii. If $A = \{1,2,3\}$ and $B = \{a, b, c, d\}$ and if the relation **1**
 $R = \{(1, a), (1, b), (2, a)\}$, then domain of R is the set
(a) {1,2} (b) {1,2,3} (c) {a, b} (d) None of these
- iv. The range of the $f: R \rightarrow R$ defined by $f(x) = \sin x$, is **1**
(a) $[0,1]$ (b) $[-1,1]$ (c) $[-1,0]$ (d) $[-\pi, \pi]$
- v. A matrix M is called idempotent if **1**
(a) $M \cdot M = M$ (b) $M^T \cdot M = M$
(c) $M \cdot M = 0$ (d) $M \cdot M = I$
- vi. Every square matrix satisfies its **1**
(a) Characteristic polynomial (b) Characteristic equation
(c) Characteristic vectors (d) Characteristic roots
- vii. Calculating the difference between the largest and smallest figure **1** produces which figure?
(a) Variance (b) Range
(c) Quartile Deviation (d) None of these

P.T.O.

[2]

Q.2

Attempt any two:

- i. If A, B, C are three sets, prove that
(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

ii. A class has 100 students. The following is the description showing the number of students studying one or more of the following subjects in this class: Mathematics 32; Physics 20; Chemistry 45; Mathematics and Physics 7; Mathematics and Chemistry 15; Physics and Chemistry 30; and none of the three subjects 30. Find how many students study all the three subjects?

iii. If A, B, C are three sets then prove that $A \times (B - C) = (A \times B) - (A \times C)$. 5

03

Attempt any two:

- i. Prove that the relation R on the set $N \times N$ defined by 5
 $(a, b)R(c, d) \Leftrightarrow a + d = b + c$, for all $(a, b), (c, d) \in N \times N$ is an
equivalence relation.

ii. If $f: R \rightarrow R$ defined by $f(x) = x^2$ for all $x \in R$ and $g: R \rightarrow R$ defined 5
by $g(x) = \sin x$ for all $x \in R$ then show that $(f \circ g)x \neq (g \circ f)x$.

iii. Prove that the relation "x divides y" or $y = kx$, for some positive 5
integer k on the set of positive integer N is a partial order relation.

[3]

- Q.4 Attempt any two:

 - Define rank of a matrix and find the rank of matrix 5
 - $$A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$$
 - Test for consistency and solve 5

$$x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6.$$
 - Verify Cayley – Hamilton theorem for the matrix A and hence compute A^{-1} , 5
where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$

Q.2

Attempt any two:

- i. The mean height of 27 boys in a section A of a class is 150cms and the mean height of 33 boys in a section B of same class is 140cms. Find the overall mean height of 60 boys. 5

ii. An incomplete frequency distribution is given below 5

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	?	65	?	25	18

Find the missing frequencies when the total frequency is 229 and the median is 46.

iii. Calculate the standard deviation for the following data by step deviation method 5

03

06

- Attempt any two:

i. Following data are given: 5

	X	Y
Arithmetic mean	65	67
Standard deviation	2.5	305

The correlation coefficient between X and Y is 0.8. Find the two regression equations.

End Sem (odd) Examination Dec-2019 MARKS

CA5BS01 M.F.G.S. 3rd year 3rd sem

Programme - MCA.

$$Z^H = \{0\}, Q^S = \{1\}, Q^D = \{M\}, Q^I = \{X\}$$

Que. 1(i) (b) $\{1, 2\}$ $Q^S = \{0\} \cup \{1\} = \{0, 1\}$ $Q^D = \{M\}$ $Q^I = \{X\}$ +1

ii (c) Disjoint sets \Rightarrow two condition $\exists i \in \{1, 2\}$ $\forall j \in \{1, 2\}$ $i \neq j$ +1

iii (a) $\{1, 2\}$ +1

iv (b) $[-1, 1] = \{x \mid -1 \leq x \leq 1\} = \{0, 1, M\} \Rightarrow \{0, 1, M\}$ +1

(v) (a) M.M. = $M_0 A_0 I + M_1 A_1 I + \dots$ +1

vi (b) characteristic equation. +1

(vii) (b) Range $[0, M]$ $= \{x \mid 0 \leq x \leq M\}$ +1

viii (a) zero $Q^F = Q^S - Q^D = \{0\}$ +1

ix (b) geometric mean +1

(x) (b) Regression. $Z^H = Q^S - Q^D - Q^I = \{0, 1, M\}$ +1

\therefore $\text{Regression} \Rightarrow$ $\text{two points are } (0, 1)$ and $(M, 1)$

Que. 2(i) Let x be an arbitrary element of $A \cup (B \cap C)$. Then $\exists i \in \{1, 2, 3\}$

$x \in A \cup (B \cap C) \Leftrightarrow x \in A \text{ or } x \in (B \cap C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in A \text{ or } (x \in B \text{ and } x \in C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in (A \cup B) \text{ and } x \in (A \cup C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in (A \cup B) \cap (A \cup C)$ +1

Hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ +0.5

\therefore Ans $\text{points} \Rightarrow$ $(\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in A$ and $(\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in (B \cap C)$

(i) Let x be an arbitrary element of $A \cap (B \cup C)$. Then $\exists i \in \{1, 2, 3\}$

$x \in A \cap (B \cup C) \Leftrightarrow x \in A \text{ and } x \in (B \cup C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } x \in A \text{ and } (x \in B \text{ or } x \in C)$ +1

$\Leftrightarrow (\exists k \in \{1, 2, 3\}) \text{ s.t. } (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$ +1

$\Leftrightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$ +1

\therefore $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ +1

Hence $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ +0.5

Ques.2(iii) Let X denote the set of all students in the class MARKS

Let M , P and C denote the set of students studying Mathematics, Physics and Chemistry respectively, then we have

$$|X|=100, |M|=32, |P|=20, |C|=45$$

$$|MNP|=7 \quad |MNC|=15 \quad |PNC|=30 \quad |(M \cup P \cup C)'|=30 (+1)$$

By Principle of inclusion and exclusion

$$|MPPNC| = |MUPUC| = |M| - |P| - |C| + |MNP|$$

+ $\text{MnCl}_4^- + \text{C}_6\text{H}_5\text{CH}_2^+$ = Mn^{+3} in (+)

Hure

$$|\text{MUPUB}| = |\mathbf{x}| - |(\text{MUPUC})'|$$

$$|MUPUB| = 100 - 30 = 70$$

$$\therefore |MNPAC| = 70 - 32 - 20 - 45 + 7 + 15 + 30 = 25 (+2)$$

i.e. 25 students studying all the three subjects.

Q. 2 (iii) Let (x, y) be an arbitrary element of $(A \times (B - C))$, then

$$(x, y) \in A \times (B - C) \Rightarrow x \in A \text{ and } y \in (B - C)$$

$\Rightarrow x \in A$ and $(y \in B \text{ and } y \notin C)$

$\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \notin C)$

$$\Rightarrow (x, y) \in A \times B \text{ and } (x, y) \notin (A \times C) \\ \Rightarrow (x, y) \in (A \times B) - (A \times C) \quad (+2.5)$$

$$A \times (B - C) \subseteq (A \times B) - (A \times C) \xrightarrow{\text{eqn. (1)}}$$

Again let $(x, y) \in (A \times B) - (A \times C)$ be an arbitrary element

$$\text{they } (x, y) \in (A \times B) - (A \times C) \Rightarrow (x, y) \in A \times B \text{ and } (x, y) \notin A \times C$$

$\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \notin C)$

$\Rightarrow x \in A$ and $(y \in B \text{ and } y \notin C)$

$\Rightarrow x \in A$ and $y \in$

$$(D \cap A) \ni x \Rightarrow (x, y) \in A \times (B - C)$$

$$\text{from } \textcircled{1} \text{ and } \textcircled{2}, A \times (B - C) = (A \times B) - (A \times C) \quad (\text{+ 2.5})$$

Ques. 3(i) Suppose the relation R defined on $N \times N$ by $(a,b) R (c,d) \Leftrightarrow a+d = b+c \quad \forall (a,b), (c,d) \in N \times N$. MARKS

$$(a,b) R (c,d) \Leftrightarrow a+d = b+c \quad \forall (a,b), (c,d) \in N \times N \quad \text{by } \textcircled{1}$$

(i) Reflexive Relation.

Let (a,b) be an arbitrary element of $N \times N$. Then by defn. $\textcircled{1}$

$$a+b = b+a \quad \text{which is always true} \quad \text{by commutativity of addition}$$

$$\therefore (a,b) R (a,b) \quad \text{by defn. } \textcircled{1}$$

Thus $(a,b) R (a,b) \quad \forall (a,b) \in N \times N$. (+) (+)

\Rightarrow Relation ' R ' is reflexive on $N \times N$. (+1)

(ii) Symmetry. Let $(a,b), (c,d) \in N \times N$. Then

$$(a,b) R (c,d) \Leftrightarrow a+d = b+c \quad \text{by defn. } \textcircled{1}$$

$$\Rightarrow c+b = d+a \quad \text{by commutativity of addition}$$

$$\Rightarrow (c,d) R (a,b) \quad \text{by defn. } \textcircled{1}$$

i.e. $(a,b) R (c,d) \Rightarrow (c,d) R (a,b)$ (+1)

\Rightarrow Relation ' R ' is symmetric. (+1)

(iii) Transitive. Let $(a,b), (c,d), (e,f) \in N \times N$. Then by $\textcircled{1}$

$$(a,b) R (c,d) \Rightarrow a+d = b+c \rightarrow (\star)$$

$$\text{and } (c,d) R (e,f) \Rightarrow c+f = d+e \rightarrow (\star\star)$$

$$\therefore (a,b) R (c,d) \Rightarrow (a+d) + (c+f) = (b+c) + (d+e) \quad \text{by } \textcircled{1} \text{ & } \textcircled{2}$$

$$\text{from } (\star) \text{ and } (\star\star) \Rightarrow a+f = b+e$$

$$\Rightarrow (a,b) R (e,f)$$

$\therefore R$ is transitive. (+1)

Thus, relation R is an equivalence relation on $N \times N$. (+1)

Ques. 3(ii) Given $f(x) = x^2$ and $g(x) = \sin x$

By defn. of composition of functions.

$$(g \circ f)(x) = g(f(x))$$

$$\therefore (g \circ f)(x) = g(x^2) = \sin x^2 \quad \text{(+2)}$$

$$\text{and } (f \circ g)(x) = f(g(x))$$

$$\therefore (f \circ g)(x) = f(\sin x) = \sin^2 x \quad \text{(+2)}$$

$$\therefore f \circ g(x) = (\sin x)^2 \quad \text{and } (g \circ f)(x) = \sin x^2$$

$$\text{clearly } (f \circ g)(x) \neq (g \circ f)(x) \quad \text{(+1)}$$

$$\text{Q.E.D.} \quad \text{Hence } (f \circ g)(x) \neq (g \circ f)(x) \quad \text{Q.E.D.} \quad \text{(+1)}$$

Que. 3(iii) The given relation x divides y or $y = kx$ for all $x, y \in N$. MARKS

for some positive integer k defined on the set of
positive integer N is given by

$$x \leq y \Leftrightarrow x | y \text{ or } y = kx \quad \forall x, y \in N.$$

(i) Reflexive. since x divides x i.e. $x | x \quad \forall x \in N$.

\therefore Relation $x \leq x$ or x divides x is reflexive. (++)

(ii) Anti-Symmetric. for $x, y \in N$.

Let $x \leq y$ i.e. $x | y$ or $y = k_1 x$ is 'g' possible.

and $y \leq x$ i.e. $y | x$ or $x = k_2 y$ (d,p) \Rightarrow $k_1 = k_2$

$$\Rightarrow y = k_1 \cdot (k_2 y) \text{ or } k_1 \cdot k_2 = 1 \Rightarrow k_1 = k_2 = 1.$$

Hence $x \leq y, y \leq x \Rightarrow x = y$. (++)

\therefore Relation $x | y$ is anti-symmetric.

(iii) Transitive. Let $x, y, z \in N$, then (d,p) \Rightarrow (d,p)

$$x \leq y \Rightarrow y = k_1 x$$

$$y \leq z \Rightarrow z = k_2 y$$

$$\therefore z = k_2 \cdot k_1 \cdot x \text{ or } z = k_1 x \text{ where } k_1 \cdot k_2 = k$$

$$\therefore z = kx \text{ or } x \text{ divides } z. \quad (\text{i.e. } x | z)$$

Hence relation $x \leq y \Rightarrow x$ divides y or $y = kx$. (++)

is partial order relation, and thus (N, \leq) is a Poset.

(d,p) \Rightarrow (d,p)

Que. 4(i) Rank of a matrix. Let A be any matrix. A number r is called

the rank of a matrix A if it obeys the following two properties:

(i) there is at least one minor of A of order r which does not vanish.

(ii) every minor of A of order higher than r vanishes. (++)

Given

$$A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Operating $C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 + C_1$

$$\Rightarrow (x)(x^2) = (x^2)x + (x^2)x = (x^2)(x^2) \quad \therefore$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b^3-a^3) & (c^3-a^3) \end{bmatrix}$$

$$\Rightarrow (x)(x^2) + (x^2)(x^2) = (x^2)(x^2)$$

$$\therefore |A| = (b-a)(c-a)(a-b)(a+b+c) \rightarrow \text{eqn. ①} \quad (++)$$

242A11 Case I If $a=b=c$, then all minors of order 3 and 2 are zero MARKS
 $\therefore f(A)=1$ (++)

Case II. If $a=b \neq c$, then, by (1) (Introducing first row)

$$|A|_{3 \times 3} = 0 \text{ but } \begin{vmatrix} 1 & 1 \\ a & c \end{vmatrix} = c-a \neq 0$$

$$\therefore f(A)=2 \quad (++)$$

Case III. If $a \neq b \neq c$ but $a+b+c=0$

then by (1) $|A|=0$ but A has minors of order 2×2

$$\begin{vmatrix} 1 & 1 \\ a & b \end{vmatrix} = b-a \neq 0 \quad \therefore f(A)=2$$

Also if $a \neq b \neq c$ and $a+b+c \neq 0$

then by (1); $|A|_{3 \times 3} \neq 0$

$$\therefore f(A)=3 \quad (++)$$

Ques. If (i) we can write the above system of eqns. in matrix form

$$AX=B \text{ as } \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$\therefore [A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 1 & 2 & 3 & : & 4 \\ 1 & 4 & 9 & : & 6 \end{bmatrix} \quad (++)$$

Operating $R_3 \rightarrow R_3 - R_2$, $R_2 \rightarrow R_2 - R_1$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 1 & 2 & : & 1 \\ 0 & 0 & 6 & : & 2 \end{bmatrix} \quad (++)$$

$$R_3 \rightarrow R_3 \left(\frac{1}{2}\right) \quad (++)$$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 1 & 2 & : & 1 \\ 0 & 0 & 1 & : & 1 \end{bmatrix} \quad (++)$$

$$R_3 \rightarrow R_3 - R_2$$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & 1 & 2 & : & 1 \\ 0 & 0 & 1 & : & 1 \end{bmatrix} \quad \text{which is in upper triangular form.} \quad (++)$$

2023/24 MATHS Q. 2.5. If $f(A; B) = f(A) = 3$ then find x, y, z if $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ MARKS

L.H.S
Thus the given system of equations is consistent
and have a unique solution. (1+1)

Also by back substitution

$$x+y+z=3, \quad y+2z=1, \quad z=0 \quad \text{Add } (1) + (2)$$

$$\text{we get } z=0, \quad y=1, \quad x=2.$$

(L.H.S)

(1+1)

Q. 4 (i) Given

$$\text{Ans} \quad A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \quad | \quad 0 = 2+4+0 \quad \text{Add } (1) + (2) + (3) \quad \text{IIT 2008}$$

we know that $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0 \quad | \quad 0 = 2+4+0 \quad \text{Add } (1) + (2) + (3) \quad \text{IIT 2008}$$

(L.H.S)

on solving we get characteristic eqn,

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0 \rightarrow (1) \quad (\text{IIT 2008})$$

To verify Cayley Hamilton theorem we show that

$$A^3 - 6A^2 + 9A - 4I = 0 \rightarrow (2) \quad \text{IIT 2008}$$

$$A^2 = \begin{bmatrix} 6 & -5 & -5 \\ -5 & 6 & -5 \\ -5 & -5 & 6 \end{bmatrix} \quad (\text{IIT 2008})$$

(L.H.S)

and

$$A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} \quad \text{putting values in eqn. (1)} \quad (\text{IIT 2008})$$

putting above values in eqn. (2) we get

$$A^3 - 6A^2 + 9A - 4I = 0$$

(IIT 2008)

To find A^{-1} ; pre multiplying by A^{-1} in eqn. (2)

$$A^2 - 6A + 9I - 4A^{-1} = 0 \quad \text{or} \quad A^{-1} = \frac{1}{4}(A^2 - 6A + 9I) \quad (\text{IIT 2008})$$

$$\therefore A^{-1} = \frac{1}{4} \begin{bmatrix} 6 & -5 & -5 \\ -5 & 6 & -5 \\ -5 & -5 & 6 \end{bmatrix} \quad (\text{IIT 2008})$$

(L.H.S)

values of A are

most important

Ques 5(i) Given $n_1 = 27$, $m_1 = 150 \text{ cm}$. $n_2 = 33$, $m_2 = 140 \text{ cm}$. MARKS (+1)

$$\therefore \text{mean height of 60 students} = \frac{m_1 \cdot n_1 + m_2 \cdot n_2}{n_1 + n_2} \quad \text{Ans} \quad \text{(+1)}$$

$$= \frac{(150 \times 27) + (140 \times 33)}{27 + 33} \quad \text{(+1)}$$

$$= \frac{8670}{60} = 144.5 \quad \text{(+1)}$$

Hence the mean height of 60 students is 144.5 cm. (+1)

5(ii) Let the missing frequency of the class 30-40 is "a" and that of 60-70 be "b" then

Cumulative frequency Table

Class	frequency	C.F.
10-20	12	12
20-30	$30 - 22 = 8$	42
30-40	a	$42 + a$
40-50	$50 - 22 = 28$	$107 + a$
50-60	b	$107 + a + b$
60-70	25	$132 + a + b$
70-80	18	$150 + a + b$

$$\text{Total} \quad N = 150 + a + b$$

According to given condition $N = 229$

$$\Rightarrow 229 = 150 + a + b$$

$$\Rightarrow a + b = 79 \rightarrow \text{eqn. ①}$$

Since median is 46 so median class is 40-50. Thus

$$Md = l_1 + \left\{ \frac{\left(\frac{N}{2} \right) - C}{f} \right\} \times i \quad \text{(+1)}$$

$$\therefore 46 = 40 + \left\{ \frac{\left(\frac{229}{2} \right) - (42+a)}{65} \right\} \times 10$$

$$\therefore a = 72.5 - 39 = 33.5 \quad \text{(+1)}$$

$$\text{and } b = 79 - 33.5 = 45.5 \quad \text{Ans}$$

$$\therefore a = 33.5 \text{ and } b = 45.5 \quad \text{(+1)}$$

Que-5(iii)

Standard deviation table

MARKS

(L+)	class interval	frequency	Mid value (x)	$U = \frac{x-10}{4}$	U^2	$f.U^2$	$f.U$
(1+)	0-4	4	2.	-2	4	16	-8
(1+)	4-8	8	6.	-1	1	8	-8
(1+)	8-12	2.	10.	0	0	0	0
(1+)	12-16	1	14.	1	1	1	1
	Total	15			25	-15	

(L+) Ans: Here $U = (x-a)/i$ where $a=10$, $i=4$ Let the assumed mean $a=10$, $i=\text{class interval}=4$

By step deviation method

$$\text{S.D. } S = i \sqrt{\frac{\sum fU^2 - (\sum fU)^2}{N}} \quad (1)$$

(Step Deviation Method)

$$\therefore S = 4 \sqrt{\frac{25 - (-15)^2}{15}} \quad (1)$$

$$= 4 \sqrt{1.666 - 1} = 4 \sqrt{0.666} \quad (1)$$

$$\therefore S = 4 \times (0.8160) = 3.26 \quad (1)$$

$$\therefore S = 3.26 \quad (1)$$

(2+)

(S+)

$$d = D + \frac{f_1 f_2}{f_1 + f_2} \quad (1)$$

$$d = D + \frac{0.21 \times 0.21}{0.21 + 0.21} = 11 \quad (1)$$

$$D = d - \frac{f_1 f_2}{f_1 + f_2} = 11 - \frac{0.21 \times 0.21}{0.21 + 0.21} = 10.97 \quad (1)$$

$$(P) D = d - \frac{f_1 f_2}{f_1 + f_2} = 11 - \frac{0.21 \times 0.21}{0.21 + 0.21} = 10.97 \quad (1)$$

(L+)

$$D = d - \frac{f_1 f_2}{f_1 + f_2} = 11 - \frac{0.21 \times 0.21}{0.21 + 0.21} = 10.97 \quad (1)$$

(L+)

$$D = d - \frac{f_1 f_2}{f_1 + f_2} = 11 - \frac{0.21 \times 0.21}{0.21 + 0.21} = 10.97 \quad (1)$$

(L+)

$$D = d - \frac{f_1 f_2}{f_1 + f_2} = 11 - \frac{0.21 \times 0.21}{0.21 + 0.21} = 10.97 \quad (1)$$

Que. 6(i) Given $\bar{x} = 65$ $\sigma_x = 2.5$

$$\bar{y} = 67 \quad \sigma_y = 3.05 \quad f = 0.8 \quad (+1)$$

MARKS

The regression line of y on x is

$$(1) (y - \bar{y}) = f \cdot \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \quad (+1)$$

$$\Rightarrow (y - 67) = (0.8) \times \frac{(3.05)}{(2.5)} (x - 65) \quad (+1)$$

$$(y - 67) = (0.8) \times (1.22) (x - 65) = (97.6)(x - 65)$$

$$(y - 67) = 97.6x - 6277$$

$$\therefore y = 97.6x - 6277 \quad (+1)$$

(Q-H) The regression line of x on y is

$$(x - \bar{x}) = f \cdot \frac{\sigma_x}{\sigma_y} (y - \bar{y}) \quad (+1)$$

$$\therefore (x - 65) = (0.8) \times \frac{2.5}{3.05} (y - 67) = 0.655(y - 67) \quad (+1)$$

$$(x - 65) = 0.00655y + 65 - 0.4393$$

$$\Rightarrow x = 0.00655y + 64.5606 \quad (+1)$$

Que. 6(ii) We know that Karl Pearson's coefficient of correlation is

$$r_{xy} = \frac{\sum xy - \frac{1}{n} (\sum x, \sum y)}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}} \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}} \quad (+1)$$

Table for calculation

x	y	x^2	y^2
2	8	16	64
4	8	32	64
4	5	20	25
7	6	49	36
5	2	25	4
$\sum x = 22$		$\sum y = 29$	$\sum y^2 = 193$
$\sum xy = 120$		$\sum x^2 = 110$	

$$\begin{aligned} \text{Q. 2. } \hat{x}_{xy} &= \frac{120 - \{(22 \times 29)/5\}}{\sqrt{110 - \frac{(22)^2}{5}}} \sqrt{193 - \frac{(29)^2}{5}} \\ &= \frac{-7.6}{\sqrt{13.2}} = -0.420 \end{aligned}$$

$$\therefore \hat{x}_{xy} = -0.420 \quad \text{Ans.} \quad (+)$$

Q. 6 (iii) Table for calculation. $\hat{x}(y-x) = (\bar{x}-x)(y-\bar{y})$

x	y	$x \cdot y$	x^2	y^2
2	8	16	04	64
1	5	5	01	25
4	2	8	16	04
3	10	30	09	100
$\sum x = 15$		$\sum y = 29$	$\sum xy = 83$	$\sum x^2 = 55$
				$\sum y^2 = 209$

The regression coefficient of y on x is

$$b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = (\bar{x}-x) \quad (+)$$

$$= \frac{5 \times 83 - 15 \times 29}{5 \times 55 - (15)^2} = \frac{8 - 20}{50} = -0.4$$

$$\therefore b_{yx} = -0.4 \quad (+)$$

$$\text{Also } \bar{x} = \frac{\sum x}{n} = \frac{15}{5} = 3 \text{ and } \bar{y} = \frac{\sum y}{n} = \frac{29}{5} = 5.8 \quad (+)$$

The regression line of y on x is

$$(y - \bar{y}) = b_{yx} (x - \bar{x})$$

$$\Rightarrow (y - 5.8) = (-0.4)(x - 3) = -0.4x + 1.2$$

$$\therefore y = -0.4x + 7 \quad (+)$$