Total No. of Questions: 6 *Total No. of Printed Pages:2*

Enrollment No.....

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

EN2BS02 Mathematics-II

Programme: Diploma Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQ's) should be written in full instead of only a, b, c or d.

Q.1 i. $\lim_{x\to 0} \frac{\sin x}{x} = ?$ **1**

(a) 0 (b) 1 (c) ∞ (d) None of these

ii. $\frac{d}{dx}(x^n) = ?$ **1**

(a) nx^n (b) nx (c) nx^{n-1} (d) nx^{n+1}

iii. $\int e^x\, dx = ?$ **1**

(a) $e^x + c$ (b) $-e^x + c$ (c) $e^{-x} + c$ (d) $-e^x + c$

iv. $\int x\, dx = ?$ **1**

(a) $x^2 + c$ (b) $\frac{x^2}{2} + c$ (c) $-x^2 + c$ (d) $-\frac{x^2}{2} + c$

v. If m and n are the order and degree of the equation $\frac{d^3y}{dx^3} - 5(\frac{dy}{dx})^4 + 6y = x^2$ then: **1**

(a) m=4, n=3 (b) m=1, n=3 (c) m=3, n=4 (d) m=3, n=1

vi. The differential equation of the type $\frac{dy}{dx} + Py = Q$ is called: **1**

(a) Homogenous (b) Linear

(c) Non-homogenous (d) None of these

vii. Modulus of the vector $4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is: **1**

(a) 7 (b) $2\sqrt{9}$ (c) $\sqrt{29}$ (d) 3

viii. If $\vec{a}.\vec{b} = 0$ then angle between $\vec{a}$ and $\vec{b}$ will be: **1**

(a) 0^0 (b) 90^0 (c) 180^0 (d) 360^0

ix. The Mode of 4, 5, 9, 10, 11, 9, 7, 10, 9 is: **1**

(a) 9 (b) 10 (c) 11 (d) 4

x. Which of the following is a measure of dispersion? **1**

(a) Mean (b) Mode (c) Median (d) Standard Deviation

P.T.O

Q.2 Solve any two:

i. Find the value of the $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4}$. **5**

ii. Solve: $\frac{d}{dx}(x^2 \sin x)$ **5**

iii. If $y = \frac{e^x}{1+e^x}$ then find $\frac{dy}{dx}$. **5**

Q.3 Solve any two:

i. Evaluate : $\int_1^2 (3x^3 + 2x + 1)\, dx$ **5**

ii. Evaluate : $\int x\, e^x dx$ **5**

iii. Evaluate : $\int \frac{e^{tan^{-1}x}}{1+x^2}\, dx$ **5**

Q.4 Solve any two:

i. Solve by separation of variables: $\frac{dy}{dx} = (1 + x)(1 + y)$ **5**

ii. Solve the linear differential equation: $\frac{dy}{dx} + y = 1$ **5**

iii. Solve the homogeneous differential equation: $\frac{dy}{dx} = \frac{-y^2}{x(x-y)}$ **5**

Q.5 Solve any two:

i. Prove that $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ are mutually perpendicular. Also find the sum of $\vec{a}$ and $\vec{b}$ **5**

ii. Find the work done in moving a particle along a straight line from (3, 2, -1) to (2, 1, -4) in a force field given by $\vec{F} = 5\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ **5**

iii. Find a unit vector perpendicular to the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} + 4\hat{\jmath} - \hat{k}$. **5**

Q.6 Solve any two:

i. Find the median of the following data: 25,34,31,23,22,26,35,28,20,32 **5**

ii. Calculate the standard deviation for the set of numbers 3,4,9,11,13,6,8,10 **5**

iii. Find the mean of the following distribution. **5**

x	4	6	9	10	15
f(x)	5	10	10	7	8

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SOLUTION

Q1 (i) - (b) - 1

(ii) - (c) - nx^{n-1}

(iii) - (a) - $e^x + c$

(iv) - (b) - $\frac{x^2}{2} + c$

(v) - (d) - $m = 3, n = 1$

(vi) - (b) - Linear

(vii) - (c) - $\sqrt{29}$

(viii) - (b) - 90°

(ix) - (a) - 9

(x) - (d) - Standard deviation

Q2 (i) $\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x-3)}{(x-2)(x+2)}$ ——— (3)

$= \frac{(2-3)}{(2+2)} = \frac{-1}{4}$ ——— + (2)

Q2 (ii) Formula $\frac{d}{dx}(I \cdot II) = I \frac{d}{dx} II + II \frac{d}{dx} I$ ——— (2)

Applying the formula ——— + (1)

$\frac{d}{dx}(x^2 \sin x) = x^2 \cos x + 2x \sin x$ ——— + (2)

(iii) Formula $\frac{d}{dx}\left(\frac{I}{II}\right) = \frac{II \frac{d}{dx} I - I \frac{d}{dx} II}{(II)^2}$ —— (2)

Applying the formula —— + (1)

$\frac{d}{dx}\left(\frac{e^x}{1+e^x}\right) = \frac{(1+e^x)\cdot e^x - e^x \cdot (e^x)}{(1+e^x)^2}$ —— + (2)

$= \frac{e^x + (e^x)^2 - (e^x)^2}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2}$

Q3 (i) Applying formulae —— (2)

$\int_1^2 (3x^3 + 2x + 1)\,dx = \left(\frac{3x^4}{4} + \frac{2x^2}{2} + x\right)_1^2$

Applying limit

$= \left[3 \times \frac{16}{4} + \frac{2 \times 4}{2} + 2\right] - \left[\frac{3}{4} + 1 + 1\right]$ —— + (2)

$= 18 - \frac{11}{4} = \frac{72 - 11}{4} = \frac{61}{4}$ —— + (1) Ans

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Q3 (ii) $\int x e^x\,dx$

Applying formula $\int I \cdot II\,dx = I\int II - \int\left\{\frac{d}{dx} I \int II\,dx\right\}$ —— (2)

$\int x e^x\,dx = x e^x - \int [1 \cdot e^x]\,dx$ —— + (2)

$= x e^x - e^x + c$ —— + (1)

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3 (iii) put $\tan^{-1} x = t$ —— (2)

so that $\frac{1}{1+x^2}\,dx = dt$

$\therefore \int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx = \int e^t\,dt = e^t + c = e^{\tan^{-1}x} + c$ + (3)

(i) Seperating the variables (2)

$$\frac{dy}{(1+y)} = (1+x)\,dx$$

Integrating (3)

$$\log(1+y) = x + \frac{x^2}{2} + C$$

Q4 (ii) Comparing with std. form of L.D.E

$$\frac{dy}{dx} + Py = Q \quad (1)$$

$$P = 1, \quad Q = 1$$

$$I.F = e^{\int P\,dx} = e^{\int 1\,dx} = e^x \quad (2)$$

Soln is

$$y \cdot e^x = e^x + C \quad (2)$$

$$y = 1 + Ce^{-x}$$

Q4 (iii) put $y = vx$

$$\& \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{-(vx)^2}{x(x - vx)} = \frac{-v^2}{1-v} \quad (2)$$

$$x\frac{dv}{dx} = \frac{-v}{1-v}$$

$$\frac{1-v}{v}dv = -\frac{1}{x}dx \quad (+2)$$

On integrating

$$\log v - v = -\log x + C$$

$$\log vx - v = C \Rightarrow vx = e^{v+C} \Rightarrow y = C_1 e^{y/x}$$

$$C_1 = e^C \quad (+1)$$

(i) $\vec{a} \cdot \vec{b} = -2 - 3 + 5 = 0$ (2)

$\therefore \vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a}$ & $\vec{b}$ are $\perp$ + (1)

$\vec{a} + \vec{b} = \hat{i} + 2\hat{j} + 6\hat{k}$ + (2)

Q5 (ii) P.V. of A $= 3\hat{i} + 2\hat{j} - \hat{k}$

P.V. of B $= 2\hat{i} + \hat{j} - 4\hat{k}$ (1)

Displacement $\vec{d}$ = P.V of B − P.V. of A

$= -\hat{i} - \hat{j} - 3\hat{k}$ + (1)

Total W.D $= \vec{F} \cdot \vec{d}$ + (1)

$= (5i - 3j + 2k)(-i - j - 3k)$

$= -5 + 3 - 6$

$= -11 + 3$ + (2)

$= -8$ units

Q5. (iii) $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} = -3i + 5\hat{j} + 11\hat{k}$ (2)

$|\vec{a} \times \vec{b}| = \sqrt{9 + 25 + 121} = \sqrt{155}$ + (1)

Let $\hat{n}$ be the unit vector $\perp$ to both $\vec{a}$ & $\vec{b}$ then,

$\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \dfrac{-3i + 5j + 11k}{\sqrt{155}}$ + (2)

Q6 (i) Arrange in ascending order (1)

20, 22, 23, 25, 26, 28, 31, 32, 34, 35

$n = 10$ (even):

Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{ term} + \left(\frac{n}{2} + 1\right)^{th} \text{ term}}{2}$ + (+2)

Median $= \frac{5^{th} + 6^{th} \text{ term}}{2} = \frac{26+28}{2} = 27$ (+2)

Q6 (ii)

S.N.	Number(x)	Deviation $= (x - M)$	$(x-M)^2$
1	3	-5	25
2	4	-4	16
3	9	1	01
4	11	3	09
5	13	5	25
6	6	-2	04
7	8	0	00
8	10	2	04
Total	$\Sigma x = 64$		$\Sigma (x-M)^2 = 84$

(3)

$N = 8$,

Mean $M = \frac{\Sigma x}{N} = \frac{64}{8} = 8$

S.D $(\sigma) = \sqrt{\frac{\Sigma (x-M)^2}{N}} = \sqrt{\frac{84}{8}} = 3.24$ Ans (+2)

Q6 (iii)

x	f(x)	x · f(x)
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
Total	40	360

(+3)

Mean $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{360}{40} = 90$ (+2)

Ans.