Total No. of Questions: 6 *Total No. of Printed Pages:3*

Enrollment No......................................

Faculty of Engineering

End Sem (Even) Examination May-2018

EN3BS02 Engineering Mathematics II

Programme: B.Tech. Branch/Specialisation: All

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. The Laplace transform of $\frac{\sin t}{t}$ is **1**

(a) $\cot^{-1}\frac{1}{s}$ (b) $\tan^{-1}s$ (c) $\tan^{-1}\frac{1}{s}$ (d) $\sin^{-1}s$

ii. The inverse Laplace transform of $\frac{e^{-3s}}{s^3}$ is **1**

(a) $(t-3)H(t-3)$ (b) $(t-3)^2H(t-3)$

(c) $(t-3)^2$ (d) $(t+3)H(t-3)$

iii. In the Fourier series expansion of $f(x)=|x|$ in $(-\pi,\pi)$, the value of b_n is equal to **1**

(a) π (b) 2π (c) 0 (d) $\frac{\pi}{2}$.

iv. If $f(x)=x\cos x$ in $(-\pi,\pi)$,then the value of a_n is equal to **1**

(a) 0 (b) $\frac{\pi}{2}$ (c) $-\pi$ (d) 2π

v. The partial differential equation for the relation $z=xy+f(x^2+y^2)$ **1**

(a) $qy-px=x^2-y^2$ (b) $qx-py=x^2+y^2$

(c) $qx-py=x^2-y^2$ (d) $qy-px=x^2+y^2$

vi. The solution of $xp+yq=z$ is **1**

(a) $f(x,y)=0$ (b) $f\left(\frac{x}{y},\frac{y}{z}\right)=0$

(c) $f(xy,yz)=0$ (d) $f(x^2,y^2)=0$

P.T.O.

vii. The value of p for which the vector field $\bar{v} = (2x+y)i + (3x-2z)j + (x+pz)k$ is solenoidal **1**

(a) 0 (b) 2 (c) -2 (d) 1

viii. The vector defined by $\vec{v} = e^x \sin y\, i + e^x \cos y\, j$ is **1**

(a) Rotational (b) Irrotational

(c) Solenoidal (d) Rotation in part of space

ix. In tossing a fair die, the probability of getting an odd number or a number less than 4 is **1**

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

x. The probability of having at least one tail in 4 throws with a coin is **1**

(a) $\frac{15}{16}$ (b) $\frac{1}{16}$ (c) $\frac{1}{4}$ (d) 1

Q.2 i. Evaluate $L\left[\frac{\cos 2t - \cos 3t}{t}\right]$ **4**

ii. Solve the differential equation by using Laplace transform **6**

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t\,, \quad x(0) = 0\ \&\ x'(0) = 1$$

OR iii. Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s}{\left(s^2+4\right)^2}\right]$ **6**

Q.3 i. Express $f(x) = x$ as a half-range sine series in $0 < x < 2$ **4**

ii. Find the Fourier series of **6**

$$f(x) = \begin{cases} \pi + x & if \quad -\pi < x < 0 \\ \pi - x & if \quad 0 < x < \pi \end{cases}, \qquad f(x+2\pi) = f(x)$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$

OR iii. Find the Fourier transform of $f(x) = e^{-a|x|}$, $a > 0$ **6**

Q.4 i. Solve the partial differential equation $\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$ **4**

ii. Solve the partial differential equation **6**

$$\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x+2y) + e^{2x+y}$$

OR iii. Use the method of separation of variables , to solve the partial differential equation **6**

$$4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u, \qquad u = 3e^{-x} - e^{-5x} \ \ at\ t = 0$$

Q.5 i Show that $\bar{v} = yz\,i + xz\,j + xy\,k$ is irrotational and find a scalar function u such that $\bar{v} = gard\, u$ **4**

ii Use the stroke's theorem to evaluate **6**

$\int_C \left[(x+2y)\,dx + (x-z)\,dy + (y-z)\,dz\right]$ where C is the boundary of the triangle with vertices (2,0,0) ,(0,3,0) and (0,0,6) oriented in the anticlockwise direction.

OR iii Use Gauss Divergence theorem to evaluate **6**

$\iint_S xz^2 dydz + \left(x^2y - z^3\right)dzdx + \left(2xy + y^2z\right)dxdy$ where S is the surface of hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$.

Q.6 i. If $P(x) = \begin{cases} xe^{\frac{-x^2}{2}} & ; \quad x \geq 0 \\ 0 & ; otherwise \end{cases}$ **4**

(a) Show that P(x) is a p.d.f.

(b) Find its distribution function.

ii. A manufacturer knows that the condensers he makes contain on an average 1% of defectives. He packs them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensers? **6**

OR iii. In a male population of 1000, the mean height is 68.16 inches and standard deviation is 3.2 inches. How many men may be more than 6 feet (72 inches)? **6**

$[\phi(1.15) = 0.8749, \phi(1.2) = 0.8849, \phi(1.25) = 0.8944]$ where the argument is the standard normal variable.

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MEDICAPS UNIVERSITY, INDORE B.TECH EVEN SEM

END SEM EXAMINATION MAY-2018

EN3BS02 ENGINEERING MATHEMATICS-II

Que.1(i) (c) $\tan^{-1}\left(\frac{1}{s}\right)$ +1

(ii) (b) $(t-3)^2 H(t-3)$. +1

(iii) (c) 0 +1

(iv) (a) 0 +1

(v) (c) $qx - py = x^2 - y^2$ +1

(vi) (b) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$ +1

(vii) (c) -2 +1

(viii) (b) and (c) +1

(ix) (c) 2/3 +1

(x) (a) 15/16 +1

+1

Que. 2(i) $L\left\{\frac{\cos 2t - \cos 3t}{t}\right\}$

WKT by Division property $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty f(s)\,ds$

$\therefore L\left\{\frac{\cos 2t - \cos 3t}{t}\right\} = \int_s^\infty L\{\cos 2t - \cos 3t\}\,dt$ 01

$= \int_s^\infty \left(\frac{s}{(s^2+2^2)} - \frac{s}{(s^2+3^2)}\right) ds$ +01

$= \left[\frac{1}{2}\log(s^2+2^2) - \frac{1}{2}\log(s^2+3^2)\right]_s^\infty$

$= \frac{1}{2}\left[\log\frac{(s^2+2^2)}{(s^2+3^2)}\right]_s^\infty = \frac{1}{2}\left[0 - \log\frac{(s^2+2^2)}{(s^2+3^2)}\right]$ +01

$= \frac{1}{2}\log\left(\frac{s^2+3^2}{s^2+2^2}\right) - \log\left(\frac{s^2+3^2}{s^2+2^2}\right)^{1/2}$ +01

Answer.

Que.2(ii) $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$; $x(0)=0$, $x'(0)=1$

We can write $x''(t) + 2x'(t) + 5x(t) = e^{-t}\sin t$

taking laplace transform on both sides we get

$[s^2\bar{x} - s\,x(0) - x'(0)] + 2[s\bar{x} - x(0)] + 5\bar{x} = \frac{1}{(s+1)^2+1}$ 1

using $x(0)=0$, $x'(0)=1$, we get

$(s^2+2s+5)\,\bar{x} - 1 = 1/(s^2+2s+2)$

or $\bar{x} = \frac{1}{(s^2+2s+2)} \quad \frac{1}{(s^2+2s+5)} + \frac{1}{(s^2+2s+5)}$ +1

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or $L\{x(t)\} = \frac{1}{3}\left[\frac{1}{s^2+2s+2} - \frac{1}{s^2+2s+5}\right] + \frac{1}{s^2+2s+5}$

$= \frac{1}{3}\left[\frac{1}{s^2+2s+2} + \frac{2}{s^2+2s+5}\right]$ +1

$L\{x(t)\} = \frac{1}{3}\left[\frac{1}{(s+1)^2+1} + \frac{2}{(s+1)^2+2^2}\right]$

taking I.L.T. on both sides

$x(t) = \frac{1}{3}\left[L^{-1}\left\{\frac{1}{(s+1)^2+1^2}\right\} + \frac{1}{3}L^{-1}\left\{\frac{2}{(s+1)^2+2^2}\right\}\right.$ +2

$= \frac{1}{3}\left[e^{-t}\sin t + 2\cdot\frac{1}{2}\cdot e^{-t}\sin 2t\right]$ (using shifting theorem

i.e. $x(t) = \frac{1}{3}e^{-t}(\sin t + \sin 2t)$ +1

Answer

Que 2 (iii) Convolution theorem to evaluate $L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$

sol we can write $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = L^{-1}\left\{\frac{s}{(s^2+2^2)^2}\right\}$

Let $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\cdot\frac{s}{(s^2+a^2)}\right\}$

let $f(s) = \frac{s}{(s^2+a^2)}$ and $g(s) = \frac{1}{(s^2+a^2)}$ 01

$\Rightarrow F(t) = L^{-1}\{f(s)\} = L^{-1}\left\{\frac{s}{(s^2+a^2)}\right\} = \cos at$

and $G(t) = L^{-1}\{g(s)\} = L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{a}\sin at$

By convolution theorem

$L^{-1}\{f(s)\cdot g(s)\} = \int_{u=0}^{t} f(u)\, g(t-u)\,du$ +1

$= \frac{1}{a}\int_{u=0}^{t} \cos au \cdot \sin(at-au)\,du$

$= \frac{1}{2a}\int_{u=0}^{t} 2\sin(at-au)\cdot\cos au\,du$

$(\because 2\sin A\cos B = \sin(A+B)+\sin(A-B))$ +2

$= \frac{1}{2a}\int_{u=0}^{t}\{\sin at + \sin(at-2au)\}du$

$= \frac{1}{2a}\left[\sin at\int_{u=0}^{t}du + \int_{u=0}^{t}\sin(at-2au)du\right]$

$$= \frac{1}{2a}\left[\sin at\,(u)_{u=0}^{t} - \left(\frac{\cos(at-2au)}{-2a}\right)_{u=0}^{t}\right]$$

or $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{1}{2a}\left[t\sin at + \frac{1}{2a}(\cos at - \cos at)\right]$

+1

i.e. $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\} = \frac{t}{2a}\sin at \longrightarrow (1)$

Finally to evaluate $L^{-1}\left\{\frac{s}{(s^2+2^2)^2}\right\} = L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$

put $a=2$ in (1) we get +1

$$L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\} = \frac{t}{4}\sin 2t.$$

Answer.

Que-3(i) $f(x) = x$ as a half range sine series in $0<x<2$.

We know that the half range sine series of $f(x)$ in $(0, l)$ is given by $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$

clearly here $l=2$

$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) \longrightarrow (1)$

where $b_n = \frac{2}{l}\int_0^{l} f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)dx$ +1

$$= \frac{2}{2}\int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx$$

$$= \left(\frac{x\cos\left(\frac{n\pi x}{2}\right)}{-\frac{n\pi}{2}}\right)_0^2 - \int_0^2 \frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(-\frac{n\pi}{2}\right)}dx$$ +1

$$= \frac{-2}{n\pi}\left[(2\cos n\pi - 0) - \left\{\frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{n\pi}{2}\right)}\right\}_0^2\right]$$

$$= \frac{-2}{n\pi}\left[2(-1)^n - 0 - 0\right]$$

i.e. $b_n = \frac{-4(-1)^n}{n\pi}$ +1

Hence eqn (1) becomes

$$f(x) = x = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\left(\frac{n\pi x}{2}\right)$$ +1

Answer

Que-3(ii) Given $f(x) = \begin{cases} \pi + x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi. \end{cases}$

and $f(x+2\pi) = f(x)$.

Here we observe that

$f(-x) = \pi - x$ in $(-\pi, 0) = f(x)$ in $(0, \pi)$

$f(-x) = \pi + x$ in $(0, \pi) = f(x)$ in $(-\pi, 0)$

Thus $f(x)$ is an even function of x in $(-\pi, \pi)$.

So the fourier series of $f(x)$ in $(-\pi, \pi)$ must reduce to Fourier cosine series in the half range interval $(0, \pi)$.

Thus fourier series in $(0, \pi)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)\,dx \longrightarrow (1) \qquad — 1$$

where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx = \frac{2}{\pi}\int_0^{\pi}(\pi - x)\,dx$

$$= \frac{2}{\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{\pi} = \frac{2}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \frac{2}{\pi}\cdot\frac{\pi^2}{2}$$

i.e. $\boxed{a_0 = \pi}$

and $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot\cos nx\,dx \qquad + 2$

$$= \frac{2}{\pi}\int_0^{\pi}(\pi - x)\cdot\cos nx\,dx$$

$$= \frac{2}{\pi}\left[(\pi - x)\cdot\left(\frac{\sin nx}{n}\right) - (0 - 1)\cdot\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{(\pi - x)\sin nx}{n} - \frac{\cos nx}{n^2}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(0 - \frac{\cos n\pi}{n^2}\right) - \left((\pi\cdot 0) - \frac{\cos 0}{n^2}\right)\right]$$

$$a_n = \frac{2}{\pi}\left[-\frac{\cos n\pi}{n^2} + \frac{1}{n^2}\right] = \frac{2}{n^2\pi}\left[1 - (-1)^n\right] \qquad + 2$$

putting these values in (1)

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{2}{n^2\pi}\left(1 - (-1)^n\right)\cos nx$$

$$= \frac{\pi}{2} + \frac{2}{\pi}\left[\frac{2}{1^2}\cos x + 0 + \frac{2\cdot\cos 3x}{3^2} + 0 + \frac{2\cdot\cos 5x}{5^2} + \cdots\right] \qquad + 1$$

$$f(x) = \frac{\pi}{2} + \frac{4}{\pi}\left[\frac{\cos x}{1^2} + \frac{\cos 3x}{3^2} + \frac{\cos 5x}{5^2} + \cdots\right] \longrightarrow (2)$$

putting $x = 0$ in (2) and noting that $f(0) = \pi$ (from given function)

$$\pi = \frac{\pi}{2} + \frac{4}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right] \Rightarrow \frac{\pi^2}{8} = \left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right]$$

Answer

Que.3(ii) $f(x) = e^{-a|x|}$; $a>0$

By definition of Fourier transform $F\{f(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cdot e^{isx}dx$

$$F\{f(x)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-a|x|}\cdot e^{isx}\,dx$$ 1

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{-a|x|}e^{isx}dx + \int_{0}^{\infty} e^{-a|x|}e^{isx}dx\right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{+ax}e^{isx}dx + \int_{0}^{\infty} e^{-ax}e^{isx}dx\right]$$ +2

$$\left\{\because\ |x| = \begin{cases} -x ; & x\le 0 \\ x ; & x\ge 0\end{cases}\right.$$

$$= \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{0} e^{(a+is)x}dx + \int_{0}^{\infty} e^{-(a-is)x}dx\right]$$

$$= \frac{1}{\sqrt{2\pi}}\left\{\left[\frac{e^{(a+is)x}}{(a+is)}\right]_{-\infty}^{0} + \left[\frac{e^{-(a-is)x}}{-(a-is)}\right]_{0}^{\infty}\right\}$$ +2

$$= \frac{1}{\sqrt{2\pi}}\left\{\left[\frac{1}{(a+is)} - 0\right] + \left[0 - \left(\frac{1}{-(a-is)}\right)\right]\right\}$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{1}{(a+is)} + \frac{1}{(a-is)}\right]$$ +1

i.e. $f(x) = \frac{1}{\sqrt{2\pi}}\ \frac{2a}{(a^2+s^2)}$ Answer

or $f(x) = \sqrt{\frac{2}{\pi}}\cdot\frac{a}{(a^2+s^2)}$ Answer

Que.4(i) Given p.d.e. is $(x^2-y^2-z^2)p + 2xyq = 2xz$.

On comparing with Lagrange's L.P.D.E. $Pp+Qq=R$;

here $P = (x^2-y^2-z^2)$ $Q = 2xy$ $R = 2xz$

Lagrange's auxiliary equations are

$$\frac{dx}{(x^2-y^2-z^2)} = \frac{dy}{2xy} = \frac{dz}{2xz} \longrightarrow (1)$$ 1

taking last two fractions we get

$$\frac{dy}{2xy} = \frac{dz}{2xz} \Rightarrow \frac{dy}{y} = \frac{dz}{z}$$; on Integrating we get

$$\log y = \log z + \log c_1 \quad \text{or} \quad \frac{y}{z} = c_1 \longrightarrow (2)$$ +1

Now taking multipliers x, y, z we get

$$\frac{x\,dx + y\,dy + z\,dz}{+x(y^2+z^2+x^2)} = \frac{dz}{2xz}$$

or $\dfrac{2(xdx+ydy+zdz)}{(x^2+y^2+z^2)} = \dfrac{dz}{z}$

on integrating, we get

$\log(x^2+y^2+z^2) = \log z + \log C_2$

or $C_2 = \dfrac{(x^2+y^2+z^2)}{z}$ +1

Thus the general sol. of given p.d.e. is

$f(C_1, C_2) = 0 \Rightarrow f\left(\dfrac{y}{z}, \dfrac{(x^2+y^2+z^2)}{z}\right) = 0$ +1

Answer

ue-4(ii) Given p.d.e. $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{x+2y} \longrightarrow (1)$

A.E. of (1) by putting $D = m$, $D' = 1$

$m^3 - 7m - 6 = 0 \Rightarrow (m^2+3m+2)(m-3) = 0$

$\Rightarrow (m+2)(m+1)(m-3) = 0$

$\Rightarrow m = -2,\ m = -1,\ m = 3.$

$C.F. = \phi_1(y+m_1x) + \phi_2(y+m_2x) + \phi_3(y+m_3x)$ 02

$\therefore CF = \phi_1(y+3x) + \phi_2(y-x) + \phi_3(y-2x) \longrightarrow (2)$

Now

$P.I. = \dfrac{1}{f(D,D')} f(x,y)$

$\therefore PI = \dfrac{1}{(D^3-7DD'^2-6D'^3)}\{\sin(x+2y) + e^{2x+y}\}$

$PI = \dfrac{1}{(D^3-7DD'^2-6D'^3)}\sin(x+2y) + \dfrac{1}{(D^3-7DD'^2-6D'^3)} e^{2x+y}$ +1.5

Now $\dfrac{1}{(D^3-7DD'^2-6D'^3)}\sin(x+2y) = \dfrac{1}{[1^3-7(1)(2)^2-6(2)^3]}\iiint \sin v\,(dv)^3$

where $v = x+2y$ and $f(1,2) \neq 0$

$\therefore \dfrac{1}{(D^3-7DD'^2-6D'^3)}\sin(x+2y) = \dfrac{-1}{75}\cos v = \dfrac{-1}{75}\cos(x+2y)$

Also $\dfrac{1}{(D^3-7DD'^2-6D'^3)} e^{2x+y} = \dfrac{e^{2x+y}}{(2^3-7\times 2\cdot 1^2-6\times(1)^3)} = \dfrac{e^{2x+y}}{-12}$ +1.5

$\left(\because \dfrac{1}{f(D,D')} e^{ax+by} = \dfrac{1}{f(a,b)} e^{ax+by};\ \text{when } f(a,b) \neq 0\right)$

$\therefore P.I = \dfrac{-1}{75}\cos(x+2y) - \dfrac{1}{12} e^{2x+y}$ +1

Hence Complete sol.

$z = CF + PI \Rightarrow z = \phi_1(y+3x) + \phi_2(y-x) + \phi_3(y-2x) - \frac{1}{75}\cos(x+2y)$ [illegible]

Que-4(iii) Given p.d.e. $4\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$. $\rightarrow$ (1)

Also Given $u = 3e^{-x} - e^{-5x}$ at $t = 0$.

Let the sol. of equ. (1) be $u(x,t) = X(x).T(t)$ $\rightarrow$ (2)

where X is a function of x and T is a function of 't' only.

Now diff. equ. (2) partially w.r.t x and t we get

$\frac{\partial u}{\partial x} = X'.T$ and $\frac{\partial u}{\partial t} = X.T'$ 01

using these in equ. (1) we get

$4.X.T' + X'T = 3XT$

$\Rightarrow \frac{4XT'}{XT} + \frac{X'T}{XT} = 3$ or $\frac{4T'}{T} + \frac{X'}{X} = 3$

or $\frac{4T'}{T} = 3 - \frac{X'}{X} = k$ (say) 01

$\therefore \frac{4T'}{T} = k \Rightarrow \frac{T'}{T} = \frac{k}{4}$ and $3 - \frac{X'}{X} = k \Rightarrow \frac{X'}{X} = -k+3$

on integrating we get

$\log T = \frac{kt}{4} + \log c_1$ and $\log X = (3-k)x + \log c_2$ 02

$\therefore \log T - \log c_1 = \frac{kt}{4}$ and $\log X - \log c_2 = (3-k)x$

$\log\left(\frac{T}{c_1}\right) = \frac{kt}{4}$ and $\log\left(\frac{X}{c_2}\right) = (3-k)x$

$\therefore \frac{T}{c_1} = e^{(kt/4)}$ and $\frac{X}{c_2} = e^{(3-k)x}$

$\therefore T = c_1.e^{(kt/4)}$. and $X = c_2 e^{(3-k)x}$

Hence the sol. (2) becomes

$u(x,t) = c_1.c_2 e^{(3-k)x}.e^{(kt/4)}$ $\rightarrow$ (3)

Now putting $t = 0$ in (3) we get

$u(x,0) = 3e^{-x} - e^{-5x} = c_1.c_2.e^{(3-k)x}$ 01

Hence the required solution in (2) becomes

$u(x,t) = 3e^{t-x} - e^{2t-5x}$ 01

Que. 5(i) Given $\vec{v} = yz\hat{i} + xz\hat{j} + xy\hat{k}$

Consider $\text{curl } \vec{v} = \nabla \times \vec{v}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix} \qquad 01$$

$$= \hat{i}\left(\frac{\partial}{\partial y}xy - \frac{\partial}{\partial z}xz\right) - \hat{j}\left(\frac{\partial}{\partial x}xy - \frac{\partial}{\partial z}yz\right) + k\left(\frac{\partial}{\partial x}xz - \frac{\partial}{\partial y}yz\right)$$

$$= \hat{i}(x - x) - \hat{j}(y - y) + \hat{k}(z - z)$$

i.e. $\text{curl } \vec{v} = 0 \Rightarrow \vec{v}$ is irrotational. 01

To find scalar potential

Let u be the scalar potential, then $\vec{v} = \text{grad}u = \nabla u$

Now $du = \nabla u . d\vec{r} = \vec{v} . d\vec{r}$

$$= (yz\hat{i} + xz\hat{j} + xy\hat{k})(\hat{i}dx + \hat{j}dy + \hat{k}dz) \qquad 01$$

$$du = yz\,dx + xz\,dy + xy\,dz$$

On integrating

$$u = xyz + xyz + xyz + C = 3xyz + C. \quad \text{Answer} \qquad 01$$

Que. 5(ii) By Stoke's theorem

$$\int_C \vec{F}.d\vec{r} = \iint_S \text{curl } \vec{F}.\hat{n}\,ds \quad \rightarrow (1)$$

Here $\vec{F} = (x+2y)\hat{i} + (x-z)\hat{j} + (y-z)\hat{k} \quad \rightarrow (2)$

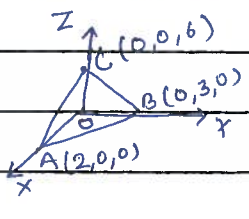
$\therefore$ $\text{curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (x+2y) & (x-z) & (y-z) \end{vmatrix} \qquad 01$

$$= \hat{i}\left\{\left(\frac{\partial}{\partial y}(y-z) - \frac{\partial}{\partial z}(x-z)\right)\right\} - \hat{j}\left\{\frac{\partial}{\partial x}(y-z) - \frac{\partial}{\partial z}(x+2y)\right\}$$

$$+ \hat{k}\left\{\frac{\partial}{\partial x}(x-z) - \frac{\partial}{\partial y}(x+2y)\right\}$$

$$= \hat{i}\{1 - (-1)\} - \hat{j}\{0 - 0\} + \hat{k}\{1 - 2\}$$

i.e. $\text{curl } \vec{F} = 2\hat{i} - 1\hat{k} \quad \rightarrow (3)$



Let C be the boundary of the triangle ΔABC and let S be the surface of ΔABC, then equation of triangular plane be

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1 \Rightarrow 3x + 2y + z = 6 \qquad +1$$

Let $\phi = 3x + 2y + z - 6$

$$\therefore \hat{n} = \frac{\text{grad}\phi}{|\text{grad}\phi|} = \frac{3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{9+4+1}} = \frac{1}{\sqrt{14}}(3\hat{i} + 2\hat{j} + \hat{k}) \qquad +1$$

Suppose projection of the surface S is xy-plane is R (i.e. $\triangle ABC$)

so that $ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$

$\therefore\ ds = \frac{dx\,dy}{\frac{1}{\sqrt{14}}\cdot(3\hat{i}+2\hat{j}+\hat{k})\cdot\hat{k}} = \sqrt{14}\,dx\,dy.$

Hence eqn. (1) becomes

$\int_C \vec{F}\cdot d\vec{r} = \iint_R \text{curl}\,\vec{F}\cdot\hat{n}\cdot ds = \iint_R (2\hat{i}-7\hat{k})\cdot\frac{(3\hat{i}+2\hat{j}+\hat{k})}{\sqrt{14}}\cdot\sqrt{14}\,dx\,dy$

$= \iint_R (6-7)\,dx\,dy = 5\iint_R dx\,dy.$ +1

$= 5\int_{x=0}^{2}\int_{y=0}^{(6-3x)/2} dx\,dy$ $\left\{\because z=0 \text{ in } \phi \quad 3x+2y=6 \Rightarrow y = 6-3x/2\right.$ +1

$= 5\int_{x=0}^{2}\left(\frac{6-3x}{2}\right)dx = \frac{5}{2}\cdot\left(6x-\frac{3x^2}{2}\right)_0^2$

$= \frac{5}{2}\cdot\left(6\cdot 2-\frac{3\times 2^2}{2}\right) = \frac{5}{2}\cdot\frac{(12-12)}{2} = 15$ Answer +1

Que. 5(iii) Here $\vec{F} = xz^2\,\hat{i} + (x^2y - z^3)\hat{j} + (2xy+y^2z)\hat{k}$

$\text{div}\vec{F} = \hat{i}\frac{\partial\vec{F}}{\partial x} + \hat{j}\frac{\partial\vec{F}}{\partial y} + \hat{k}\frac{\partial\vec{F}}{\partial z}$ +1

$\therefore\ \text{div}\vec{F} = x^2+y^2+z^2$ +1

By Gauss divergence theorem

$\iint_S \vec{F}\cdot\hat{n}\,ds = \iiint_V \text{div}\vec{F}\,dV$ +1

$= \iiint_V (x^2+y^2+z^2)\,dV$

Let $x = r\sin\theta\cos\phi;\ y = r\sin\theta\sin\phi \quad z = r\cos\theta$ +1

$= \iiint r^2(r^2\sin\theta\,dr\,d\theta\,d\phi)$

$= \int_0^{2\pi} d\phi\int_0^{\pi/2}\sin\theta\,d\theta\int_0^a r^4\,dr$ +1

$=$

$= [\phi]_0^{2\pi}\cdot(-\cos\theta)_0^{\pi/2}\cdot\left(\frac{r^5}{5}\right)_0^a$

$= 2\pi(-0+1)\frac{a^5}{5}$ +1

i.e. $= \frac{2\pi a^5}{5}$ Answer

Que. 6(i) Given $P(x) = \begin{cases} x \cdot e^{-x^2/2} & ; \quad x \geq 0 \\ 0 & ; \quad \text{otherwise.} \end{cases}$

(i) For $P(x)$ to be a p.d.f. $\int_{-\infty}^{\infty} P(x)\,dx = 1$

$$\therefore \int_{-\infty}^{\infty} P(x)\,dx = \int_{-\infty}^{\infty} x \cdot e^{-x^2/2}\,dx$$

let $\frac{x^2}{2} = t \Rightarrow x\,dx = dt$ +1

when $x \to 0 \Rightarrow t \to 0$

when $x \to \infty \Rightarrow t \to \infty$

$$\therefore \int_{-\infty}^{\infty} P(x)\,dx = \int_0^{\infty} e^{-t}\,dt$$

$$= \left(\frac{e^{-t}}{-1}\right)_0^{\infty} = 1 \qquad +1$$

$\therefore P(x)$ is a p.d.f.

(ii) Distribution Function $F(x) = P(X \leq x)$

$$= \int_{-\infty}^{x} P(x)\,dx = \int_{-\infty}^{0} P(x)\,dx + \int_0^{x} P(x)\,dx$$

$$= \int_{-\infty}^{x} x e^{-x^2/2}\,dx = \int_{-\infty}^{0} P(x)\,dx + \int_0^{x} x e^{-x^2/2}\,dx \qquad +1$$

~~Again putting $x^2/2 = t \Rightarrow x\,dx = dt$~~

$$= \int_0^{x} e^{-t}\,dt = \left[\frac{e^{-t}}{-1}\right]_0^{x} = -\left[e^{-x} - e^{-0}\right]$$

$$= -\left[e^{-x} - 1\right] = 1 - e^{-x} \quad \underline{\text{Answer}} \qquad +1$$

Que. 6(ii) Here $p = 1\% = 0.01$, $n = 100$

$\therefore m = np = 100 \times 0.01 = 1$ +1

$$P(r) = \frac{e^{-m} \cdot m^r}{\lfloor r} = \frac{e^{-1}\,1^r}{\lfloor r} = \frac{e^{-1}}{\lfloor r} \qquad +1$$

$\therefore P(4 \text{ or more faulty condensers}) = P(4) + P(5) + \cdots + P(100)$

$$= 1 - [P(0) + P(1) + P(2) + P(3)] \qquad +1$$

$$= 1 - \left[\frac{e^{-1}}{\lfloor 0} + \frac{e^{-1}}{\lfloor 1} + \frac{e^{-1}}{\lfloor 2} + \frac{e^{-1}}{\lfloor 3}\right]$$

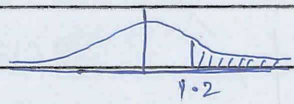
$$= 1 - e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6}\right] = 1 - \frac{8}{3e} \qquad +2$$

i.e. $P(4 \text{ or more faulty condensers}) = 1 - 0.981 = 0.019$ Answer +1

Que. 6(iii) Male population = 1000

Mean height = 68.16 inches

standard deviation = 3.2 inches



Men more than 72 inches = ?

$\phi(1.15) = 0.8749$, $\phi(1.2) = 0.8849$

$\phi(1.25) = 0.8944$ +1

$z = \dfrac{x-\bar{x}}{6}$ or $\dfrac{x-\mu}{6} = \dfrac{72-68.16}{3.2} = 1.2$ +2

$\phi(1.2) = 0.8849$

For more than 1.2 $= 1 - 0.8849 = 0.1151$ +2

Men more than 72 inches $= 1000 \times 0.1151 = 115.1$

$= 115$ (say). +1

—x—