Enrollment No......................................

Faculty of Science

End Sem (Even) Examination May-2019

BC3CO07 Mathematics-II

Programme: B.Sc. (CS) Branch/Specialisation: Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1 i. If for the function $u = f(x, y)$ the value of $rt - s^2 > 0$ with $r > 0$ at (a, b) then the function have___________ at a point (a, b) is , **1**

(a) Maxima (b) Minima

(c) Neither (a) nor (b) (d) Doubtful case

ii. The $\lim\limits_{\substack{x\to 1\\ y\to 2}}\left(\dfrac{x^2 y}{x^2 - y^2 + 5}\right)$ is equal to **1**

(a) 1 (b) $\frac{1}{5}$ (c) $\frac{4}{3}$ (d) Does not exist.

iii. If equation of curve contains only even power of x and only odd powers of y, then the given curve is symmetrical about **1**

(a) x axis (b) y axis

(c) Both (a) and (b) (d) Can't say

iv. The value of $\int_0^1 \int_0^x dy dx =$ **1**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) None of these

v. The equation $M(x, y)dx + N(x, y)dy = 0$ is an exact differential equation if : **1**

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$

vi. The solution of the differential equation $p = \sin(y - xp)$, where $p = \frac{dy}{dx}$, is **1**

(a) $\sin^{-1} x = y$ (b) $y = cx + \sin^{-1} x$

(c) $(\sin^{-1} x) = c + (\sin^{-1} y)$ (d) $y = cx + \sin^{-1} c$

vii. The particular integral of $(D^2 + 1)y = \cos 2x$, where $D \equiv \frac{d}{dx}$ is equal to **1**

(a) $\frac{1}{3}\cos 2x$ (b) $\frac{1}{3}\sin 2x$ (c) $-\frac{1}{3}\cos 2x$ (d) $-\frac{1}{3}\sin 2x$

viii. If for the equation $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, 1+P+Q=0 then, one part of complimentary function **1**

(a) x (b) $\frac{1}{x}$ (c) e^x (d) e^{-x}

ix. If $L\{f(t)\} = \overline{f}(s)$ then $L\left\{\frac{f(t)}{t}\right\} =$ ______? provided $\lim_{t\to 0}\frac{f(t)}{t}$ exist. **1**

(a) $\frac{d}{ds}(\overline{f}(s))$ (b) $-\frac{d}{ds}(\overline{f}(s))$

(c) $\int_s^{\infty} \overline{f}(s)ds$ (d) $\int_0^{s} \overline{f}(s)ds$

x. $L^{-1}\left(\frac{s}{s^2+36}\right) =$ **1**

(a) $\frac{\sin 6t}{6}$ (b) $\cos 6t$ (c) $\sinh 6t$ (d) $\cosh 6t$

Q.2 i. Define limit of function of two variables $f(x, y)$ at a point (a,b). **2**

ii. Test continuity of $f(x, y) = \begin{cases} \frac{x^3y^3}{x^3+y^3}, & x \neq 0, y \neq 0 \\ 0 & x = 0, y = 0 \end{cases}$ at origin. **3**

iii. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that **5**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}.$$

OR iv. Discuss the maximum or minimum values or saddle point of the function $u = x^3 - 4xy. + 2y^2$ a. **5**

Q.3 Solve any two:

i. Change of order of integration for $\int_1^2\int_0^x \frac{dxdy}{x^2+y^2}$ and hence evaluate it. **5**

ii. Find the area enclosed between the parabolas $y^2 = 16x$ and $x^2 = 16y$. **5**

iii. Calculate the volume of solid bounded by the surface $z = 0, x^2 + y^2 = 1, x + y + z = 3$. **5**

Q.4 i. Solve $y = (x-a)p - p^2$, where $p = \frac{dy}{dx}$, **3**

ii. Solve $\frac{dy}{dx} - 2y\tan x = y^2 \tan^2 x$ **7**

OR iii. Solve $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$. **7**

Q.5 Solve any two:

i. Solve the differential equation $\frac{d^2y}{dx^2} - 4y = e^x + 4\cos^2 x$. **5**

ii. Solve $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$, if $y = e^x$ is one integral. **5**

iii. Solve $\frac{d^2y}{dx^2} + 4y = \sec 2x$ by the method of variation of parameter. **5**

Q.6 Solve any two:

i. Find : **5**

(a) $L\left\{\frac{\sin t}{t}\right\}$ (b) $L^{-1}\left(\tan^{-1} s\right)$

ii. Use convolution theorem for Laplace transform to find $L^{-1}\left(\frac{1}{s(s^2+16)}\right)$. **5**

iii. Solve $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 2y = 0, y(0) = 0, y'(0) = 1$ using Laplace transform. **5**

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Faculty of Science
EndSem Exam May 2019
BC3CO07 Mathematics II

Programme: BSc. Speciali.: CS

Q.1.
(i) (b) Minima 1
(ii) (a) 1. 1
(iii) (b) about Y axis. 1
(iv) (b) $\frac{1}{2}$ 1
(v) (b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ 1
 1
(vi) (d) $y = cx + \sin^{-1}C$ 1
(vii) (c) $-\frac{1}{3}\cos 2x$ 1
(viii) (c) e^{x} 1
(ix) (c) $\int_{s}^{\infty} \bar{f}(s)\,ds$ 1
(x) (b) $\cos 6t$ 1

Q.2. (i) Let $f:(x,y) \to f(x,y)$ be the function defined on some domain $D \subseteq R^2$. The the real number L is said to be the limit of the function f as (x,y) approaches to (x_0, y_0) if for every $\epsilon > 0$, $\exists\ \delta > 0$ depending upon ϵ and (x_0, y_0) such that for every (x,y) in D.

$$0 < \left[\sqrt{(x-x_0)^2 + (y-y_0)^2}\right] < \delta$$

$\Rightarrow$ $|f(x,y) - L| < \epsilon$ [in terms of circular nbdhood] 2

or

$0 < |x - x_0| < \delta$

$0 < |y - y_0| < \delta \Rightarrow |f(x,y) - L| < \epsilon$ [in terms of squar nbdhood]

Symbollically we write

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

(2)

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(ii) $$f(x,y) = \begin{cases} \dfrac{x^3y^3}{x^3+y^3}, & x \neq 0, y \neq 0 \\ 0, & x=0, y=0. \end{cases}$$

Sol. As defined $f(0,0) = 0$

Let (x,y) approaches $(0,0)$ along the line $y = x$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{x^3x^3}{x^3+x^3} = \lim_{x\to 0} \frac{\cancel{x^6}}{2\cancel{x^3}}$$

$$= \frac{1}{2}\lim_{x\to 0} x^3 = 0.$$

Again let $(x,y) \to (0,0)$ along the cubic parabola $x = y^3$, then

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{y\to 0} \frac{y^9y^3}{y^9+y^3} = \frac{y^{12}}{\cancel{y^3}(y^6+1)} = \frac{y^9}{y^6+1}$$

$$= 0.$$ (2)

Since the limits obtained by two different approaches are same so

$\lim_{(x,y)\to(0,0)} f(x,y)$ exist. i.e. $(0,0)$ (1)

Consequently the given f^n is conti. at $(0,0)$

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(iii) $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then P.T.

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cdot \cos 2u}{4\cos^3 u}$$

Sol $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right) \Rightarrow \sin u = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right) = U$

so, $U = \frac{x\left[1+y/x\right]}{\sqrt{x}\left[1+\sqrt{(y/x)}\right]} = x^{1/2}\frac{\left[1+y/x\right]}{\left[1+\left(\sqrt{y/x}\right)\right]} = x^{1/2} f\left(y/x\right)$ 1

U is a homogeneous function of x & y of degree $\frac{1}{2}$. Hence By Euler's theorem.

$$x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial y} = \frac{1}{2}U$$

$$x\frac{\partial}{\partial x}\sin u + y\frac{\partial}{\partial y}\sin u = \frac{1}{2}\sin u$$

$$x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u \quad 1$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\cdot\frac{\sin u}{\cos u} \quad —(1)$$

Diff. (1) partially w.r.t. x.

$$x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} = \frac{1}{2}\sec^2 u \cdot \frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = \left(\frac{1}{2}\sec^2 u - 1\right)\frac{\partial u}{\partial x} \quad —(2)$$

1

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Diff (1) w.r.t. y

$$x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} = \frac{1}{2}\sec^2 u \cdot \frac{\partial u}{\partial y}$$

$$x\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = \left(\frac{1}{2}\sec^2 u - 1\right)\frac{\partial u}{\partial xy} \quad \text{— (3)}$$

1

Multiply (2) & (3) by x & y respectively and adding.

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} =$$

$$\left(\frac{1}{2}\sec^2 u - 1\right)\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)$$

$$\left(\frac{1}{2}\sec^2 u - 1\right)\left(\frac{1}{2}\tan u\right)$$

$$\left(\frac{1}{2\cos^2 u} - \frac{1}{1}\right)\frac{1}{2}\left(\frac{\sin u}{\cos u}\right)$$

$$\left(\frac{1 - 2\cos^2 u}{2\cos^2 u}\right)\left(\frac{1}{2}\frac{\sin u}{\cos u}\right)$$

$$\cancel{= \frac{1}{4}\frac{\sin u}{\cos^3 u} - 2\cos^2 u}$$

$$- \left(\frac{\cos 2u}{2\cos^2 u}\right)\left(\frac{1}{2}\frac{\sin u}{\cos u}\right)$$

1

$$= - \frac{\sin u \cos 2u}{4\cos^3 u} \quad //$$

(OR) Date ___/___/______ (5)

(iv) Discuss maxima or minima values or saddle point of the f^n $u = x^3 - 4xy + 2y^2 a$.

Sol $u = x^3 - 4xy + 2ay^2$

$\frac{\partial u}{\partial x} = 3x^2 - 4y$; $\frac{\partial u}{\partial y} = -4x + 4ay$

For maxima or minima of u,

$\frac{\partial u}{\partial x} = 0$, $\frac{\partial u}{\partial y} = 0$

$3x^2 - 4y = 0$; $-4x + 4ay = 0$

$3x^2 = 4y$; $4y = \frac{4x}{a}$

$3x^2 = \frac{4x}{a} \Rightarrow 3ax^2 - 4x = 0$ (1)

$(3ax - 4)x = 0 \Rightarrow x = 0, \frac{4}{3a}$

$y = 0, \frac{4}{3a^2}$

Hence, the critical point $\left(\frac{4}{3a}, \frac{4}{3a^2}\right)$ (1)

$r = \frac{\partial^2 u}{\partial x^2} = 6x$

$s = \frac{\partial^2 u}{\partial x \partial y} = -4$

$t = \frac{\partial^2 u}{\partial y^2} = 4a$ (2)

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at point $\left(\frac{4}{3a}, \frac{4}{3a^2}\right)$

$rt - s^2 = 6x \cdot 4a - (-4)^2 = 24ax - 16$

$= 24 \cdot a\left(\frac{4}{3a}\right) - 16$

$r = 6\left(\frac{4}{3a}\right) = \frac{8}{a}$

$= 32 - 16 = 16$

Since $rt - s^2$ is +ve, but r is +ve or -ve depends on a.

1

Case I $rt - s^2 > 0$ and a is +ve then $r > 0$.
so function u is minimum at $\left(\frac{4}{3a}, \frac{4}{3a^2}\right)$

Case II $rt - s^2 > 0$ and a is -ve then $r < 0$.
So function u is maxi. at $\left(\frac{4}{3a}, \frac{4}{3a^2}\right)$

at point $(0, 0)$

$rt - s^2 = 6x \cdot 4a - (-4)^2 =$
$= 24ax - 16$ at $(0,0)$

$rt - s^2 = -16$

$r = 6(0) = 0.$

Case III $rt - s^2 = -ve$ i.e. u is neither maxi. nor minima at $(0, 0)$.

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Q3.(i) Change the order of integration.

$$\int_1^2 \int_0^x \frac{dx\,dy}{x^2+y^2}.$$

Sol. Let $I = \int_1^2 \int_0^x \frac{dx\,dy}{x^2+y^2}$ —— (1)

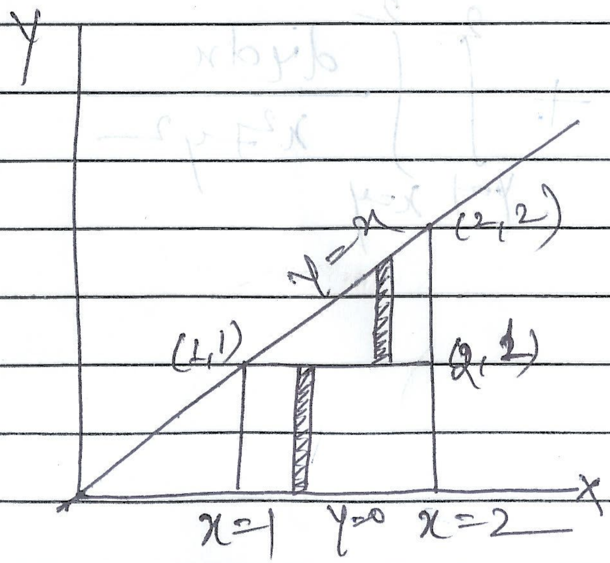
The region of integration is bounded by the following lines and curves:

(i) $x=1$ i.e. line parallel to Y axis.

(ii) $x=2$ i.e. line parallel to Y axis.

(iii) $y=0$ i.e. x axis — 1

(iv) $y=x$ i.e. st line passing through $(0,0)$.



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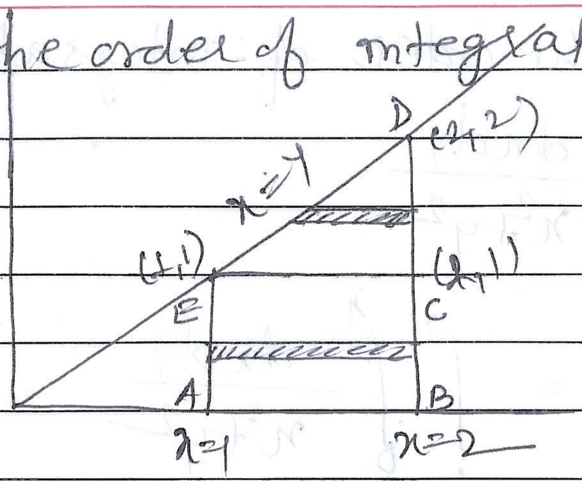
On changing the order of integration.



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So region of integration is ABCDEA

this ABCDEA = ABCE + ECDE

In ABCD
x varies from $x=1$ to $x=2$
y varies from $y=0$ to $y=1$.

In ECDE
x varies from $x=y$ to $x=2$ 1
y varies from $y=1$ to $y=2$

$$\therefore I = \int_{y=0}\int_{x=1}^{2} \frac{dy\,dx}{x^2+y^2} + \int_{y=1}^{2}\int_{x=y}^{2} \frac{dy\,dx}{x^2+y^2}$$ 1 // Ans

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Q5(ii) Given Eqⁿ of parabola are.

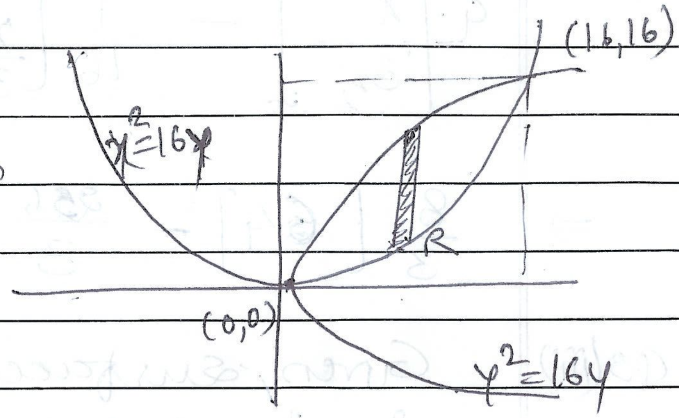
$y^2 = 16x$ — (1)

$x^2 = 16y$ — (2)

By (2) squaring on both sides

$x^4 = (16)^2 y^2$

$x^4 = (16)^2 \, 16x$



$x^4 - (16)^3 x = 0$

$x(x^3 - 16^3) = 0$

$\Rightarrow x = 0, \quad x = 16$

Put in (1) $\quad y = 0, \quad y = 16$ 2

Required pnt of intersection are $(0,0)$ $(16,16)$

In the Region R, the limits are

limits of y : $y = \frac{x^2}{16}$ to $4\sqrt{x}$

Limits of x : $x = 0$ to $x = 16$

∴ Area b/w the parabolas A = Area of Region R

$$= \iint_R dx\,dy$$ 1

$$= \int_0^{16} \int_{x^2/16}^{4\sqrt{x}} dx\,dy = \int_0^{16} [y]_{x^2/16}^{4\sqrt{x}} dx$$

$$= \int_0^{16} \left[4\sqrt{x} - \frac{x^2}{16}\right] dx = \int_0^{16} \left(4 \cdot x^{1/2} - \frac{x^2}{16}\right) dx$$

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$$4 \cdot \left[\frac{x^{3/2}}{3/2}\right]_0^{16} - \frac{1}{16}\left[\frac{x^3}{3}\right]_0^{16} \Rightarrow \frac{8}{3}\left[(16)^{3/2}\right] - \frac{1}{48}\left[(16)^3\right]$$

$$\Rightarrow \frac{8}{3}[64] - \frac{256}{3} = \frac{512}{3} - \frac{256}{3} = \frac{256}{3} \text{ sq unit.}$$

2

10(iii) Given surfaces are

$x^2+y^2=1, \quad x+y+z=3, \quad z=0$

limits of z are from $z=0$ to $z=3-x-y$

" —— " y are from $y=-\sqrt{1-x^2}$ to $y=\sqrt{1-x^2}$

" —— " x are from $x=-1$ to 1

∴ Required volm.

$$V = \int_{x=-1}^{x=1}\int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}}\int_{z=0}^{z=3-x-y} 1\, dx\, dy\, dz$$

2

$$V = \int_{x=-1}^{x=1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} [z]_0^{3-x-y}\, dx\, dy$$

$$V = \int_{x=-1}^{x=1}\left[\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} [3-x-y]\, dy\right] dx$$

$$V = \int_{x=-1}^{x=1}\left[\left\{(3-x)y - \frac{y^2}{2}\right\}_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\right] dx$$

(11)

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$$V = \int_{-1}^{1} (3-x)\, 2\sqrt{1-x^2}\, dx$$

$$V = 2\left[\int_{-1}^{1} 3\sqrt{1-x^2}\, dx - \int_{-1}^{1} x\sqrt{1-x^2}\, dx\right]$$

$$= 2\left[6\int_{0}^{1}\sqrt{1-x^2}\, dx - 0\right] = 12\cdot\int_{0}^{1}\sqrt{1-x^2}\, dx \quad 2$$

$$1 \quad = 12\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_0^1$$

$$= 12\left[\left(0 + \frac{1}{2}\sin^{-1}1\right) - (0+0)\right]$$

$$1 \quad = 6\sin^{-1}(1) = \cancel{6}\frac{\pi}{\cancel{2}} = 3\pi$$

$$\boxed{V = 3\pi} \quad 1$$

1

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Q. 4(i) $y=(x-a)p-p^2 \qquad p=\frac{dy}{dx}$

Sol $y=(x-a)p-p^2$ — ①

The Given Eqn is in clairaut's form
Hence,

$$\frac{dy}{dx}=(x-a)\frac{dp}{dx}+p-2p\frac{dp}{dx}$$

$$\cancel{p}=(x-a)\frac{dp}{dx}+\cancel{p}-2p\frac{dp}{dx}$$

$$(x-a-2p)\frac{dp}{dx}=0 \qquad 1$$

$$\frac{dp}{dx}=0 \Rightarrow p=c$$

By Eqn ① $y=(x-a)c-c^2$ 1

$c^2-(x-a)c+y=0$ Quadractic in c.

$B^2-4AC=0$

$(x-a)^2-4ay=0$

$$\boxed{y=\frac{1}{4}(x-a)^2} \qquad 1$$

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Q4. (ii) $\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$

Sol. $\div$ by y^2

$$\frac{1}{y^2}\frac{dy}{dx} - \frac{2}{y}\tan x = \tan^2 x$$

$$y^{-2}\frac{dy}{dx} + y^{-1}(-2\tan x) = \tan^2 x \quad — (1)$$

Let $y^{-1} = v$

$$-y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

Put in (1)

$$-\frac{dv}{dx} + (-2\tan x)\cdot v = \tan^2 x$$

$$\frac{dv}{dx} + 2\tan x\cdot v = -\tan^2 x \quad (3)$$

$$IF = e^{\int 2\tan x\,dx} = e^{2\int \tan x\,dx}$$

$$IF = e^{2\log\sec x} = e^{\log\sec^2 x} = \sec^2 x$$

$$v\cdot IF = \int Q\cdot IF\,dx + C \quad (2)$$

$$v\cdot\sec^2 x = -\int \tan^2 x\cdot\sec^2 x\,dx$$

Let $\tan x = t$

$\sec^2 x\,dx = dt$

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$$v \sec^2 x = -\int t^2 dt$$

$$v \cdot \sec^2 x = -\frac{t^3}{3} + c$$

$$v \cdot \sec^2 x = -\frac{\tan^3 x}{3} + c$$

$$y^{-1} \sec^2 x = -\frac{\tan^3 x}{3} + c$$

$$\cancel{\sec^2 x = -\frac{\tan^3 x \cdot y}{3} + c}$$

2

$$\boxed{\frac{1}{y}\sec^2 x = \frac{1}{3}\tan^3 x + c_1} \quad \boxed{c_1 = -c}$$

3

(iii) $(1+e^{x/y})dx + e^{x/y}(1-x/y)dy = 0. \quad -①$

Sol Compare with $Mdx + Ndy = 0$

$M = 1+e^{x/y}$ and $N = e^{x/y}(1-x/y)$

2

$$\frac{\partial M}{\partial y} = e^{x/y}(-x/y^2) = -\frac{x}{y^2}e^{x/y}$$

$$\frac{\partial N}{\partial x} = e^{x/y}(0-1/y) + (1-x/y)e^{x/y}\left[\frac{y\frac{\partial}{\partial x}(x) - x\frac{\partial}{\partial x}(y)}{y^2}\right]$$

$$= e^{x/y}(-1/y) + (1-x/y)e^{x/y}(\cancel{y}/y^{\cancel{2}})$$

3

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$$\frac{\partial N}{\partial x} = \cancel{-\frac{1}{y} e^{x/y}} + \cancel{\frac{1}{y} e^{x/y}} - \frac{x}{y^2} e^{x/y}$$

$$\frac{\partial N}{\partial x} = -\frac{x}{y^2} e^{x/y}$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ Hence eq^n ① is exact

Thus the sol^n of eq^n ① is 2

$$\int_{y\text{-const}^t} M\,dx + \int (\text{terms of } N \text{ that do not contain } x)\,dy = C$$

$$\int_{y\text{-const}} (1 + e^{x/y})\,dx + \int 0\,dy = C$$

$$x + \frac{e^{x/y}}{1/y} = C$$

$$\boxed{x + y e^{x/y} = C}$$ // 2

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Q.5. (i) $\frac{d^2y}{dx^2} - 4y = e^x + 4\cos^2 x$

Sol $(D^2-4)y = e^x + 4\cos^2 x$

For C.F.

$m^2 - 4 = 0 \Rightarrow m = \sqrt{4} = \pm 2.$

C.F $=$ ~~[illegible]~~ $c_1 e^{2x} + c_2 e^{-2x}$ 2

For P.I $= \frac{1}{D^2-4}[e^x + 4\cos^2 x]$

$= \frac{1}{D^2-4} e^x + \frac{1}{D^2-4}(4\cos^2 x)$

$= \frac{1}{-3} e^x + \frac{1}{D^2-4} \cancel{4}^{2}\left(\frac{1+\cos 2x}{\cancel{2}}\right)$

$= -\frac{1}{3} e^x + \frac{1}{D^2-4}(2) + \frac{1}{D^2-4} 2\cos 2x.$

$= -\frac{1}{3} e^x + \frac{2}{-4} + \frac{2}{-2^2-4} \cos 2x$

$= -\frac{1}{3} e^x - \frac{1}{2} - \frac{1}{4} \cos 2x$ 2

The complete solution is

$y = c_1 e^{2x} + c_2 e^{-2x} - \left(\frac{1}{3} e^x + \frac{1}{2} + \frac{1}{4}\cos 2x\right)$ //

1

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Q5 (ii) $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$

given that $y = e^x$ is one integral.

Sol Standard form of given Eqⁿ ⓪ is

$$\frac{d^2y}{dx^2} - \left(2 - \frac{1}{x}\right)\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = 0 \quad —(1)$$

$P = -2 + \frac{1}{x}$, $Q = 1 - \frac{1}{x}$, $R = 0$ (1)

Given $y = e^x$ is one integral.
Hence $u = e^x$ is a part of CF of eqⁿ (1)
Then v is given by

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left(-\cancel{2} + \frac{1}{x} + \frac{\cancel{2}}{\cancel{e^x}}\cancel{e^x}\right)\frac{dv}{dx} = 0$$

$$\frac{d^2v}{dx^2} + \frac{1}{x}\frac{dv}{dx} = 0$$

Let $\cancel{v} = \frac{dq}{dx}$ $\quad \frac{dv}{dx} = q$

$$\Rightarrow \frac{d^2v}{dx^2} = \frac{dq}{dx}$$

(2)

then

$$\frac{dq}{dx} + \frac{q}{x} = 0 \quad —— (2)$$

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$$\frac{dq}{dx} = -\frac{q}{x}$$

$$\Rightarrow \int \frac{dq}{q} = -\int \frac{1}{x} dx$$

$$\Rightarrow \log q = -\log x + \log C_1$$

$$\Rightarrow \log q\cdot x = \log(C_1/x) \Rightarrow qx = C_1$$

$$\Rightarrow \frac{dv}{dx} = \frac{C_1}{x}$$ 1

$$\Rightarrow \int dv = \int \frac{C_1}{x} dx$$

$$\Rightarrow v = C_1 \log x + C_2$$

$$\Rightarrow y = u\cdot v$$

$$\Rightarrow y = e^x(C_1 \log x + C_2) \;//$$ 1

Q5(iii) $$\frac{d^2y}{dx^2} + 4y = \sec 2x.$$

Sol. $(D^2 + 2^2)y = \sec 2x.$

$$m^2 + 2^2 = 0 \Rightarrow m = \pm 2i$$ 2

$$CF = C_1 \cos 2x + C_2 \sin 2x$$

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for P.I. $y_1 = \cos 2x$, $y_2 = \sin 2x$ $R(x) = \sec 2x$

$$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$$= 2(\cos^2 2x + \sin^2 2x) = 2$$

$$P.I. = -y_1 \int \frac{y_2 R(x)}{W} dx + y_2 \int \frac{y_1 R(x)}{W} dx$$ 2

$$= -\cos 2x \int \frac{\sin 2x \cdot \sec 2x}{2} dx + \sin 2x \int \frac{\cos 2x \cdot \sec 2x}{2} dx$$

$$= -\frac{\cos 2x}{2} \int \tan 2x \, dx + \frac{\sin 2x}{2} \int dx$$

$$= \frac{1}{2^2} \cos 2x \log \cos 2x + \frac{\sin 2x}{2} \cdot x.$$

$$= \frac{\cos 2x}{4} \log(\cos 2x) + \frac{x}{2} \sin 2x$$ 1

$$y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}(\cos 2x)(\log \cos 2x) + \frac{x}{2} \sin 2x$$

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Q.6 (i) $L\left\{\frac{\sin t}{t}\right\}$

Sol $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(s)\,ds$

$2\frac{1}{2}$

$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty L\{\sin t\}\,dt = \int_s^\infty \frac{1}{s^2+1}\,ds$

$L\left\{\frac{\sin t}{t}\right\} = \left(\tan^{-1}s\right)_s^\infty = \frac{\pi}{2} - \tan^{-1}s$

$= \cot^{-1}s$ //

(ii) $L^{-1}\{\tan^{-1}s\}$

Sol $f(s) = \tan^{-1}s$

Diff. w.r.t. s both sides

$f'(s) = \frac{1}{s^2+1}$

$(-1)\frac{d}{ds}f(s) = \frac{-1}{s^2+1}$

$L\{t\cdot F(t)\} = \frac{-1}{s^2+1}$

$tF(t) = -L^{-1}\left\{\frac{1}{s^2+1}\right\}$

$2\frac{1}{2}$

$F(t) = \frac{-\sin t}{t}$ //