Enrollment No......................................

Faculty of Science

End Sem (Odd) Examination Dec-2017

CA3CO04 Mathematics-I

Programme: BCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1	i.	Disjunction operator is represented by	**1**
		(a) $\wedge$ (b) $\vee$	
		(c) $\Rightarrow$ (d) $\Leftrightarrow$	
	ii.	If p = He is poor and q = He is laborious then the statement "It is false that he is poor or laborious" in the language of logic is	**1**
		(a) $p\Lambda(\sim q)$ (b) $\sim p\Lambda(\sim q)$	
		(c) $\sim(p\Lambda q)$ (d) $\sim(p \vee q)$	
	iii.	If A,B *and* C are any three nonempty sets then $A \cap (B \cup C) =$	**1**
		(a) $(A \cup B) \cap (A \cup C)$ (b) $(A \cup B) \cup (A \cup C)$	
		(c) $(A \cap B) \cup (A \cap C)$ (d) $A \cap (B \cap C)$	
	iv.	If A is any set then $A \oplus A =$	**1**
		(a) A (b) $\emptyset$	
		(c) 0 (d) None of these	
	v.	If $A = \{4,5,9\}$ and $R = \{(4,5),(4,9),(5,9)\}$ then relation R is	**1**
		(a) Reflexive (b) Symmetric	
		(c) Transitive (d) Anti $-$ symmetric	
	vi.	If $f: x \rightarrow \|x\|$ be a mapping, then the *f-image* of {-2,-1,0,1,2}is	**1**
		(a) $\{0\}$ (b) $\{0,1,2\}$	
		(c) $\{1,2\}$ (d) None of these	
	vii.	$\lim_{x\rightarrow 3}\frac{x^2-9}{x-3}$ is equals to	**1**
		(a) 0 (b) 3	
		(c) 6 (d) None of these	

viii. $If\ \ y = x^n\ +\ a^n\ \ then\ \frac{dy}{dx}$ is **1**

(a) $\frac{x^{n+1}}{n+1} + c$ (b) nx^{n-1}

(c) $\frac{x^{n-1}}{n-1} + c$ (d) 1

ix. Let A be a matrix such that there exists a square sub matrix of order r which is non-singular and every sub matrix of order $r+1$ or higher is singular, then the rank of A is **1**

(a) $= r + 1$ (b) $< r$

(c) $= r$ (d) $> r$

x. A system $AX = B$ having no solution if the rank of matrix A and augmented matrix $[A : B]$ is **1**

(a) 0 (b) Equal

(c) 1 (d) Unequal

Q.2 i. Check whether the statement $p \wedge (\sim p)$ is a contradiction or tautology. **2**

ii. State whether the argument given below is valid or not valid .If it is valid, identify the tautology. **3**

I will become famous or I will be writer.

I will not be a writer.

$\therefore I\ will\ become\ famous$

iii. If p *and* q are any statements then show that **5**

(a) $\sim (\,p \wedge q\,) \equiv (\sim p) \vee (\sim q)$ (b) $\sim (\,p \vee q\,) \equiv (\sim p) \wedge (\sim q)$

OR iv. If p, q and r are any three statements then show that **5**

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Q.3 i. If $A = \{1,2,3\}, B = \{3,4,5\}$ then find $A - B\ and\ B - A$. Also draw Venn-diagram. **2**

ii. If U= {1, 2,3,4,5,6} $A = \{1,2,3,4\}, B = \{2,3,4,5,6\}$ then find $(A \cup B)'and\ (A \cap B)'$. **3**

iii. If A, B, C are three sets then prove that **5**

$A\times(B \cap C) = (A \times B) \cap (A \times C)$.

OR iv. Out of 450 students in a school, 193 students read "Science" and 200 students read "Commerce", 80 students read neither. Find out how many read both. **5**

Q.4 i. Define floor and ceiling functions with example. **2**

ii. Let $A=\{1,2,3\}$, $B=\{a, b, c\}$ and $C=\{x, y, z\}$ and let $R_1 = \{(1, a), (2, c), (3, a), (3, c)\}$ and $R_2 = \{(b, x), (b, z), (c, y)\}$.Find the composition relation $R_1 \circ R_2$. Also find the matrices $M_{R_1}, M_{R_2}\ and\ M_{R_1 \circ R_2}$. **3**

iii. Show that the mapping $f: R_+ \to R$ defined by $f(x) = logx,\ x \in R_+$ is one -one, onto, where R_+ is the set of positive real numbers and R is the set of real numbers. **5**

OR iv. Show that the relation "less than or equal" ($\leq$) on the set of positive integers is a partial order relation. **5**

Q.5 i. Evaluate $\lim_{x \to 0} \frac{x}{\sqrt{1+x}\ -1.}$ **4**

ii. If $f(x) = \begin{cases} 2x + 3 & if\ x < 1 \\ 2 & if\ \ x = 1 \\ 7 - 2x & if\ x > 1. \end{cases}$ **6**

Check the continuity of $f(x)$ at $x=1$.

OR iii. If $y = loglog(logx)\ then\ find\ \ \frac{dy}{dx}$. **6**

Q.6 Attempt any two:

i. Investigate the values of λ and μ so that the equations **5**

2x + 3y + 5z = 9, 7x + 3y – 2z = 8, 2x + 3y +λz = μ

have (i) no solution (ii) a unique solution (iii) an infinite number of solution.

ii. Verify Cayley Hamilton theorem for the matrix **5**

$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and hence find A^{-1}.

iii. Find the Eigen values and Eigen vectors of the matrix **5**

$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

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Medi-Caps University

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Que.1 i (b) V | 1

(ii) (d) $\sim(p \vee q)$ | 1

(iii) (C) $(A \cap B) \cup (A \cap C)$ | 1

(iv) (b) ϕ | 1

(v) (c) Transitive | 1

(vi) (b) $\{0, 1, 2\}$ | 1

(vii) (C) 6 | 1

(viii) (b) nx^{n-1} | 1

(ix) (C) $= 2$ | 1

(x) (d) Unequal | 1

Que. 2(i) Let p be a statement. Truth table for $p \wedge (\sim p)$ is:

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

+1

Since all entries in the column of $p \wedge (\sim p)$ are F only, It is a contradiction. +1

2 (ii) Let p: I will become famous

q: I will be writer.

The given argument, in symbolic form,

$p \vee q$ (a premise)

$\sim q$ (a premise)

p (conclusion) +1

The given argument is. $(p \vee q) \wedge (\sim q) \vdash p$ will be valid if the statement $[(p \vee q) \wedge (\sim q)] \rightarrow p$ is a tautology. +0.5

Now we construct the truth table for the above statement.

p	q	$p \vee q$	$\sim q$	$(p \vee q) \wedge (\sim q)$	$[(p \vee q) \wedge (\sim q) \rightarrow p]$
T	T	T	F	F	T
T	F	T	T	T	T
F	T	T	F	F	T
F	F	F	T	F	T

Argument is valid +1.5

Q.2 (iii) Let p and q be two statements.

(a) Truth table for $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim(p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

It is clear from the above table that the statement $\sim(p \wedge q)$, $(\sim p) \vee (\sim q)$ both are logically equivalent since both column have same entries.

Thus $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$ +2.5

(b) Truth table for $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$(\sim p) \wedge (\sim q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

It is clear from the above table that the statement $\sim(p \vee q)$, $(\sim p) \vee (\sim q)$ both are logically equivalent, since both column have same entries.

Thus $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$ +2.5

R (iv)

p (1)	q (2)	r (3)	$(q \vee r)$ (4)	$p \wedge q$ (5)	$p \wedge r$ (6)	$p \wedge (q \vee r)$ (7)	$(p \wedge q) \vee (p \wedge r)$ (8)
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

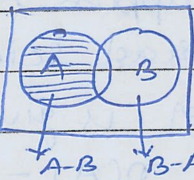
For (1,2,3) +1
For (4,5,6) +3
For (7,8) +1
Total +5

Since (7) and (8) columns are identitied. Hence given statement is tautology.

Que. 3(i) $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$

Then $A - B = \{1, 2\}$, $B - A = \{4, 5\}$ +1

$\because A - B = \{x : x \in A \text{ but } x \notin B\}$ +1



3(ii) $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4\}$ $B = \{2, 3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\} = U$ +1

$(A \cup B)' = U - (A \cup B) = \phi$ +1

$A \cap B = \{2, 3, 4\}$

$(A \cap B)' = U - (A \cap B) = \{1, 5, 6\}$ +1

3(iii) If A, B, C are three sets,

Let (x, y) be any arbitrary element of $A \times (B \cap C)$

$\therefore (x, y) \in A \times (B \cap C) \Rightarrow x \in A, y \in B \cap C$

$\Rightarrow x \in A, (y \in B \text{ and } y \in C)$

$\Rightarrow (x \in A, y \in B) \text{ and } (x \in A, y \in C)$

$\Rightarrow (x, y) \in (A \times B) \text{ and } (x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C) \longrightarrow (1)$ +2.5

Again let $(x_1, y_1) \in (A \times B) \cap (A \times C)$, then

$(x_1, y_1) \in (A \times B) \cap (A \times C) \Rightarrow (x_1, y_1) \in A \times B \text{ and } (x_1, y_1) \in A \times C$

$\Rightarrow (x_1 \in A, y_1 \in B) \text{ and } (x_1 \in A, y_1 \in C)$

$\Rightarrow x_1 \in A, (y_1 \in B \text{ and } y_1 \in C)$

$\Rightarrow x_1 \in A, y_1 \in (B \cap C)$

$\Rightarrow (x_1, y_1) \in A \times (B \cap C)$

$\therefore (x_1, y_1) \in (A \times B) \cap (A \times C) \subseteq A \times (B \cap C) \longrightarrow (2)$

from (1) and (2)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$. +2.5

Hence Proved.

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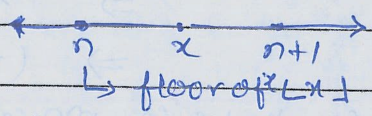
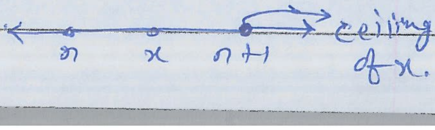
Que. 3(iv) Suppose A and B denote the set of students who read Science and Commerce, respectively.
It is given that
$n(A) = 193$, $n(B) = 200$, $n(U) = 450$
$n(A^c \cap B^c) = 80$ +1

Now we should find the number of those students who read science as well as commerce i.e. $n(A \cap B)$
Since $A^c \cap B^c = (A \cup B)^c$
$\therefore n(A \cup B)^c = 80.$ +1
But $n(A \cup B)^c = n(U) - n(A \cup B)$
$\Rightarrow 80 = 450 - n(A \cup B)$
$\therefore n(A \cup B) = 370$ +1
By principle of inclusion and exclusion
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ +1
$\Rightarrow 370 = 193 + 200 - n(A \cap B)$
or $n(A \cap B) = 23.$
Thus 23 students read both Science and Commerce. +1

Que-4 (i) Let x be any real number. The floor function, $f(x)$ defined for x is the greatest integer less than or equal to x. This function $f: R \to I$ is given by
$f(x) = \lfloor x \rfloor$ = greatest integer less than or equal to x.
Thus $\lfloor x \rfloor = n \Leftrightarrow n \leq x < n+1$
e.g. $\lfloor 4.25 \rfloor = 4$ +1



A ceiling function, $C(x)$ defined for x is the smallest (least) integer greater than or equal to x.
The notation $\lceil x \rceil$ is used for $C(x)$.
If x is a real number, and n is integer then
$\lceil x \rceil = n \Leftrightarrow n-1 < x \leq n.$ +1
e.g. ~~@~~ $\lceil 4.25 \rceil = 5$



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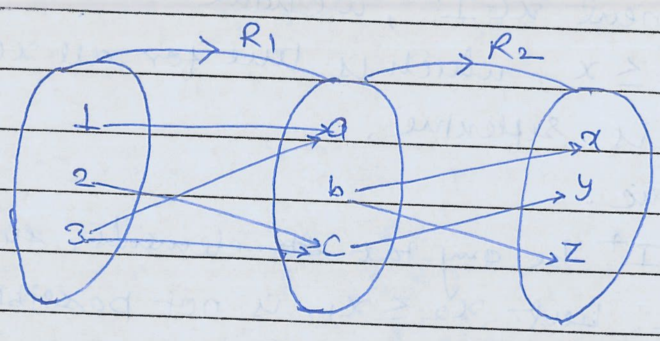
4(ii) $A=\{1,2,3\}$ $B=\{a,b,c\}$ $C=\{x,y,z\}$

$R_1=\{(1,a)\ (2,c),(3,a),(3,c)\}$

$R_2=\{(b,x),(b,z),(c,y)\}$



$R_1 \circ R_2 = \{(2,y),(3,y)\}$ +1.5

$$M_{R_1} = \begin{array}{c|c} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array} ; \quad M_{R_2} = \begin{array}{c|c} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \quad M_{R_1 \circ R_2} = \begin{array}{c|c} & \begin{matrix} x & y & z \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$ +1.5

1(iii) Let the mapping $f: R_+ \to R$ defined by

$f(x) = \log x \quad ; \quad x \in R_+$

Let x_1 and x_2 be positive real numbers then $\log x_1$ and $\log x_2$ exists.

$f(x_1) = f(x_2)$

$\Rightarrow \log x_1 = \log x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one. +2.5

Again let $y \in R$ be any arbitrary real number, so that

$y = f(x)$

$\Rightarrow y = \log x$

$\Rightarrow x = e^y$

We know that for every value of y (+ve or -ve) e^y is always positive. Hence $e^y \in R_+$ and $f(e^y) = \log(e^y) = y$

Hence for all $y \in R$, its pre image $e^y \in R^+$ $\therefore$ f is onto mapping +2.5

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Que. 4(iv) To prove the relation "$\le$" or R is a partial order relation, we prove the following three properties hold.

(1) Reflexive.
for any element $x \in I^+$, we have
$x \le x$, which is true for all $x \in I^+$
Hence, R is reflexive. +1.5

(2) Anti-symmetric.
Let $x_1, x_2 \in I^+$ be any two elements, such that
$x_1 \le x_2$ but $x_2 \le x_1$ is not possible
unless $x_1 = x_2$. +1.5
Hence, R is anti-symmetric.

(3) Transitive.
Let $x_1, x_2, x_3 \in I^+$ be any three elements.
such that $x_1 \le x_2$ and $x_2 \le x_3$.
It follows that $x_1 \le x_3$. +1.5
Hence R is transitive.

Since all the three properties of partial order relation +0.5
are satisfied, therefore the relation "$\le$" or R is partial order relation

Que 5(i) To evaluate $\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1} = \frac{0}{0}$ form; which is meaningless +1

$\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1} = \lim_{x \to 0} \frac{x}{\sqrt{1+x}-1} \times \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$ +1

$= \lim_{x \to 0} \frac{x(\sqrt{1+x}+1)}{1+x-1} = \lim_{x \to 0} \frac{x(\sqrt{1+x}+1)}{x}$

$= \lim_{x \to 0} \sqrt{1+x}+1$ +1

$= \sqrt{1+0}+1 = 1+1 = 2$ +1

i.e. $\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1} = 2$.

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5 (ii) Given

$$f(x) = \begin{cases} 2x+3 \; ; & x<1 \\ 2 \; ; & x=1 \\ 7-2x \; ; & x>1 \end{cases} \longrightarrow (A)$$

(1) Right hand limit of $f(x)$ at $x=1$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (7-2x) \quad (\because \text{By } (A))$$

$\therefore$ putting $x = 1+h$ and taking $h \to 0$ when $x \to 1$

$$f(1+0) = \lim_{h\to 0} f(1+h)$$

$$= \lim_{h\to 0} [7 - 2(1+h)]$$

$$= \lim_{h\to 0} (7-2-2h) = 5$$

i.e. $\lim_{x\to 1^+} = 5$ +2

(2) Left hand limit of $f(x)$ at $x=1$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (2x+3) \quad (\because \text{By } (A))$$

$\therefore$ putting $x = 1-h$ and taking limit $h\to 0$ when $x \to 1$

$$f(1-0) = \lim_{h\to 0} f(1-h)$$

$$= \lim_{h\to 0} (2(1-h)+3)$$

$$= \lim_{h\to 0} (2-2h+3) = 5$$

i.e. $\lim_{x\to 1^-} = 5$ +2

Value of function

Again when $x=1$, then $f(x) = 2$

Thus

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} \neq f(1)$$

Thus the given function is not continuous at $x=1$ +2

Que-5 (iii) If $y = \log\log(\log x) \rightarrow (1)$

on diff. equ-(1) w.r.t 'x' we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\log(\log(\log x))\right)$$ +1

$$= \frac{d}{dx}\log t \cdot \frac{dt}{dx}$$

put $\log(\log x) = t$

$$= \frac{1}{t} \cdot \frac{d}{dx}\left(\log(\log x)\right)$$ +1

$$= \frac{1}{\log(\log x)} \cdot \frac{d}{dx}\log u$$ +1

put $\log x = u$

$$= \frac{1}{\log(\log x)} \cdot \frac{1}{u} \cdot \frac{du}{dx}$$ +1

$$= \frac{1}{\log(\log x)} \cdot \frac{1}{(\log x)} \cdot \frac{d}{dx}(\log x)$$ +1

$$= \frac{1}{\log(\log x))} \cdot \frac{1}{(\log x)} \cdot \frac{1}{x}$$

i.e. $\frac{dy}{dx} = \frac{1}{x \log x \log\log(x)}$ Answer +1

Que-6 (i) The given system of equ. are.

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

The above system of equ. can be written in matrix form $AX = B$ as

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}; \quad B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ +0.5

Thus the augmented matrix $[A:B]$ is

$$[A:B] = \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 7 & 3 & -2 & : & 8 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$ +0.5

putting above values of A^2 and A^3 in equ. (2) we get | MARKS

$$A^3 - 6A^2 + 7A + 2I = 0$$ +1

To find A^{-1}; pre multiplying by A^{-1} in equ. (2) we get

$$A^2 - 6A + 7I + 2A^{-1} = 0$$

or $$A^{-1} = \frac{1}{2}(-A^2 + 6A - 7I)$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} -6 & 0 & 4 \\ -2 & 1 & 1 \\ 4 & 0 & -2 \end{bmatrix}$$. Answer. +1

OR
Que. 6(ii)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

characteristic equ. $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(2-\lambda)^2 - 1] = 0$$ +1

$$\lambda = 1, 1, 3. \text{ eigen values.}$$ +1

Case I let $X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vector corresponding to eigen value $\lambda = 3$. Then $(A - 3I)X_1 = 0$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus eigen vector $X_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ +1

Case II let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be the eigen vector corresponding to $\lambda = 1$

then $(A - I)X = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1 + x_2 + x_3 = 0$$

taking $x_1 = k_1$, $x_2 = k_2$ $\therefore x_3 = -k_1 - k_2$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ -k_1 - k_2 \end{bmatrix} = k_1\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$ +2

$\therefore X_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ $X_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ Answer

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On operating $R_2 \to R_2 - \frac{7}{2}R_1$, $R_3 \to R_3 - R_1$

$$[A:B] \sim \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 0 & -15/2 & -39/2 & : & -47/2 \\ 0 & 0 & \lambda-5 & : & \mu-9 \end{bmatrix}$$ +1

Case I No solution if $\rho(A:B) \neq \rho(A)$
when $\lambda - 5 = 0$ and $\mu - 9 \neq 0$
$\Rightarrow \lambda = 5$ and $\mu \neq 9$. +1

Case II. Unique solution if $\rho(A:B) = \rho(A) = 3$ (unknown)
when $\lambda - 5 \neq 0$ and μ have any value
i.e. $\lambda \neq 5$ and μ have any value. +1

Case III Infinite solution if $\rho(A:B) = \rho(A) = 2 < 3$
when $\lambda - 5 = 0$ and $\mu - 9 = 0$
$\lambda = 5$ and $\mu = 9$. +1

Que. 6(ii) $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

The characteristic equ. of matrix A is given by

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0$$ +0.5

$\Rightarrow \lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \to (1)$ +0.5

To verify Cayley Hamilton theorem, we show that

$A^3 - 6A^2 + 7A + 2I = 0 \to (2)$ +1

$$A^2 = A \cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$ +0.5

$$A^3 = A^2 \cdot A = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$ +0.5