Q.6 i. Find the value of integral- **3**

$$\int_0^1 \int_0^1 \int_0^1 dx\, dydz$$

ii. Change the order of integration and hence evaluate the same- **7**

$$\int_0^1 \int_0^x e^{y/x} dxdy$$

OR iii. Evaluate the integral- **7**

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz\, dxdydz$$

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Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No.....

Faculty of Engineering
End Sem Examination Dec 2024
EN3BS07
Introductory Topics in Statistics, Probability & Calculus

Programme: B.Tech. Branch/Specialisation: CSBS

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

			Marks	BL	CO	PO	PSO
Q.1	i.	In statistics, a population consists of: (a) All people living in a country (b) All people living in the area (c) All objects whose characteristics are being studied (d) None of these	**1**	1	1	1	
	ii.	In the plural sense, statistics means: (a) Numerical data (b) Methods (c) Population data (d) Sample data	**1**	1	1	1	
	iii.	The measure of central tendency which is calculated by considering as most frequent occurring value as central value as classified as: (a) Mean (b) Mode (c) Median (d) None of these	**1**	2	1	2	
	iv.	The word ogive is also used for: (a) Frequency polygon (b) Cumulative frequency polygon (c) Frequency curve (d) Histogram	**1**	1	1	1	
	v.	E denotes Expectation, If E(x) = 2 and E(y) = 4, then E(2x+3y) is- (a) 16 (b)12 (c) 10 (d) None of these	**1**	3	2	2	

vi. Second central moment is also known as- **1** 1 1 1

(a) Mean (b) Mode

(c) Variance (d) None of these

vii. For Binomial distribution mean is 6, number of trials is 10 then probability of success is- **1** 3 2 2

(a) 3/8 (b) 3/5

(c) 3/4 (d) None of these

viii. For Poisson distribution, which of the following is true: **1** 1 1 1

(a) mean = variance (b) mean < variance

(c) mean >variance (d) None of these

ix. The derivative of $y = e^{-3x}$ is- **1** 2 1 1

(a) $-3e^{3x}$ (b) e^{3x}

(c) e^{x} (d) None of these

x. $\int 4x^2 \, dx$ is- **1** 2 1 1

(a) $x^4 + c$ (b) $\frac{4x^3}{3} + c$

(c) $x^{-2} + c$ (d) None of these

Q.2 i. Write any two definitions of statistics? **2** 1 1 1

ii. Define secondary data and its collection techniques. **3** 1 1 1

iii. Explain the procedure of the statistical investigation. **5** 1 1 1

OR iv. Write any five advantages of statistics. **5** 1 1 1

Q.3 i. Define range and coefficient of variation. **4** 1 1 1

ii. Calculate arithmetic mean from the following table- **6** 3 2 2

Class	20-30	30-40	40-50	50-60	60-70
Frequency	8	26	30	20	16

OR iii. An Incomplete distribution of the data is given as **6** 4 3 2

Class	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	12	30	?	65	?	25	18

Find the missing terms when the total of frequencies is 229 and median is 46.

Q.4 i. Write short note on- **4**

(a) Sample space

(b) Combinatorial probability

(c) Random experiment

(d) Addition theorem on probability

ii. Define mathematical expectation. Find the expected value of the number of points that will be obtained in a single throw with an ordinary die. **6**

OR iii. In a bolt factory, Machines A, B, C manufacture 50%, 30% and 20% of the total bolts. It is also known that 2% of the item manufactured by A are defective while 2% and 3% are defective manufactured by B and C respectively. a bolt is drawn at random and is found to be defective. What is the probability that it was manufactured by machines A, B and C? **6**

Q.5 i. Write four assumptions of Chi-Square distribution. **4**

ii. Fit a poisson distribution to the following data *given* $e^{-0.974} = 0.37756$. **6**

X	0	1	2	3	4
Y	46	38	22	9	1

OR iii. The random sample gives the following results- **6**

Sample	Size	Sample mean	Sum of square of deviation from the mean
1	10	15	90
2	12	14	108

Test whether sample come from the same normal population. (Given that $F_{0.05}$ (11,9) = 3.10, $F_{0.05}$ (9,11) = 2.90).

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Q-1 MCQ

(i) (c) All objectes whose characterstics are being studied. +1

(ii) (a) Numerical data +1

(iii) (b) Mode +1

(iv) (b) Cumulative frequency polygon +1

(v) (a) 16 +1

(vi) (c) Variance +1

(vii) (b) 3/5 +1

(viii) (a) Mean = Variance +1

(ix) (d) None of these +1

(x) (b) $\frac{4x^3}{3} + C$ +1

Q-2

(i) (a) Statistics is the method of decision making in the face of uncertainty on the basis of numerical data and calculated risks. +1

(b) Statistics is the science of estimates and Probabilities. +1

Q-2 (ii)

Secondary Data – Data which are not originally collected but are obtained from published or unpublished sources are known as secondary data. +1

Collection Techniques –

Secondary data comes from two main sources. namely published or unpublished.

(a) Published sources – +1

(i) Government publications
(ii) International publications
(iii) Journal or magzines or Newspaper
(iv) Publications of Research institutes

(b) Unpublished sources – +1

(i) Data which are not published and also available in files and office records of goverment and private organizations

Q-2 (iii)

Statistical Investigation –

Statistical investigation means statistical survey. Scientific and systematic collection of data and their analysis with the help of various statistical methods is known as statistical investigation. +1

procedure of statistical investigation –

(1) planning of investigation – To consider various points before the actual work of statistical investigation is termed as planning of statistical investigation. It is essential for the succesful completion and getting the best result at the least cost and time +2

(2) Collection of data

(3) Editing of data +1

(4) Presentation of data – classification, Tabulation Diagrams and graphs.

(5) Analysis of data +1

(6) Interpretation of data and drawing conclusion or prepration of Report.

Q-2 (iv)

(1) Statistics and planning – Statistics is indispensible to planning. In the modern age almost all organisations in the goverment or managements of busines are resorting to planning for effiect working and for formulating policy decisions. +1

(2) Statistics and Economics – Statistical methods are very much useful to understand economic concepts such as mandatory policy and public finance. Some of the important statistical techniques used in economic analysis are– Time series, Index Numbers, Estimation Theory etc. +1

(3) Statistics and Medicine – In medical field, Statistical methods are extensively used, If we look at the medical journal one can understand to what extent the Statistical techniques play a key ~~cote~~ role. +1

(4) Statistics and Agriculture – Experimentation and inference based on these experiments are the key features of general scientific methodology. Agricultural Scientist conduct experiments and make inference to decide whether the particular variety of crop gives a better yield than others. +1

(5) Statistical and Information Technology – IT is the application of computer and tele– +1

Communication equipments to store, retrieve, transmit and manipulate data. Now a days several industries are involved in IT and massive amount of data are stored every day.

Q-3 (i)

Range – The difference between highest and lowest value of the data is called the range of distribution. +2

Range = Highest value – Lowest value

Coefficient of Variation – Coefficient of variation is a relative measure of dispersion. Standard deviation cannot be used for comparing the variability of two or more series, thus coefficient of variation is used for this purpose. +2

Coefficient of variation is denoted by C.V. and is defined as.

$$C.V. = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100$$

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

Q-3 (ii)

class	freq. (f)	M.V. x	fx
20-30	8	25	200
30-40	26	35	910
40-50	30	45	1350
50-60	20	55	1100
60-70	16	65	1040
Total	$\Sigma f = 100$		$\Sigma fx = 4600$

+4

+1

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{4600}{100}$$

+1

$$\boxed{\text{Mean} = 46}$$

Q-3 (iii)

class	frequency	cum. freq.
10-20	12	12
20-30	30	42
30-40	x	42+x
40-50	65	107+x
50-60	y	107+x+y
60-70	25	132+x+y
70-80	18	150+x+y
	$\Sigma f = N = 229$	

+2

Here, $150 + x + y = 229$

$x + y = 79$ +1

$y = 79 - x$

Median is 46. Thus median class is 40-50

$l_1 = 40,\ l_2 = 50,\ f = 65,\ P.C.f. = 42 + x$

$$\text{Median} = l_1 + \left(\frac{\frac{N}{2} - P.C.f}{f}\right)(l_2 - l_1)$$

$$46 = 40 + \left[\frac{\frac{229}{2} - (42+x)}{65}\right](50 - 40)$$ +2

$$46 - 40 = \frac{114.5 - 42 - x}{65} \times 10$$

$6 \times 65 = (72.5 - x)\ 10$

$390 = 725 - 10x$

$10x = 335$

$x = 33.5 \approx 34$

$\boxed{x = 34}$ +1

$y = 79 - 34$

$\boxed{y = 45}$

Q-4 (i)

(a) Sample Space – The set of all possible outcomes in a random experiment is called Sample space. It is denoted by S. +1

Ex – When a coin is tossed

$S = \{H, T\}$.

(b) Combinatorial Probability – Combinatorial Probability is a branch of mathematics that calculates and deals with counting and calculating the Probability of events that involve combinations of objects or outcomes +1

(c) Random Experiment – A random experiment may be defined as an exprimant which when repeated under essential identical Conditions does not give unique results but may result in any one of the several possible outcomes. +1

(d) Addition Theorem – Let A and B be two events in a sample space. Then for addition theorm +1

(1) When events are mutually exclusive

$P(A \cup B) = P(A) + P(B)$

(2) When events are not mutually exclusive

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Q-4 (ii)

Mathematical Expectation – The mean of a probability distribution is known as mathematical expectation. It is denoted by $E(x)$.

If x is a discrete random variable +2

$$E(x) = \Sigma x \cdot p(x)$$

and for continuous random variable

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

When a single die is thrown all possible out comes are 1, 2, 3, 4, 5, 6

x :	1	2	3	4	5	6	+1
P :	1/6	1/6	1/6	1/6	1/6	1/6	

$$E(x) = \Sigma x \cdot p(x)$$

$$= \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(3 \cdot \frac{1}{6}\right) + \left(4 \cdot \frac{1}{6}\right) + \left(5 \cdot \frac{1}{6}\right) + \left(6 \cdot \frac{1}{6}\right) \quad +2$$

$$= \frac{1}{6}(1+2+3+4+5+6) \quad +1$$

$$= \frac{21}{6} = \boxed{\frac{7}{2}} = 3.5$$

Q-4 - (iii)

$$P(A) = \frac{50}{140}$$

$$P(B) = \frac{30}{140}$$ +1

$$P(C) = \frac{20}{140}$$

The Prob. of defective item, given that it is produced by machine A is $P\left(\frac{D}{A}\right) = \frac{2}{140}$ +1

by machine B is $P\left(\frac{D}{B}\right) = \frac{2}{140}$

by machine C is $P\left(\frac{D}{C}\right) = \frac{3}{140}$

$$P\left(\frac{A}{D}\right) = \frac{P(A)\,P(D/A)}{P(A)\cdot P(D/A) + P(B)\,P(D/B) + P(C)\cdot P(D/C)}$$ +2

$$= \frac{\frac{50}{140} \times \frac{2}{140}}{\left(\frac{50}{140} \times \frac{2}{140}\right) + \left(\frac{30}{140} \times \frac{2}{140}\right) + \left(\frac{20}{140} \times \frac{3}{140}\right)}$$

$$= \frac{140}{140+60+66} = \frac{140}{220} = \frac{5}{11}$$

$$P\left(\frac{B}{D}\right) = \frac{P(B)\cdot P(D/B)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)\cdot P(D/C)}$$ +1

$$= \frac{60}{220} = \frac{3}{11}$$

$$P(x) = \frac{e^{-m} m^x}{x!}$$

$$P(x) = \frac{e^{-0.974} (0.974)^x}{x!}$$

x	$P(x)$	Expected freq $= N \cdot P(x)$
0	$P(0) = \frac{e^{-0.974} \times (0.974)^0}{0!}$ $= 0.37756$	116×0.37756 $= 43.796$
1	$P(1) = \frac{e^{-0.974} (0.0974)}{1!}$ $= 0.03677$	116×0.03677 ~~$= 4.265$~~ $= 42.65$ +3
2.	$P(2) = \frac{e^{-0.974} (0.974)^2}{2!}$ $= 0.1790$	116×0.1790 $= 20.77$
3.	$P(3) = \frac{e^{-0.974} (0.974)^3}{3!}$ $= 0.05814$	116×0.5814 $= 6.744$
4.	$P(4) = \frac{e^{-0.974} (0.974)^4}{4!}$ $= 0.01415$	116×0.0145 $= 1.642$
		Total = 116

$$P\left(\frac{C}{D}\right) = \frac{P(C)\cdot P(D/C)}{P(A)\cdot P(D/A) + P(B)\cdot P(D/B) + P(C)\cdot P(D/C)}$$ +1

$$= \frac{60}{220} = \frac{3}{11}$$

Q-5 (i) The Sample Observations should be independent. +1

(i) (ii) Constraints on the cell frequencies, should be linear $\Sigma O_i = \Sigma E_i$. +1

(iii) N, the total frequency should be reasonably large, say, greater than 50. +1

(iv) No theoretical cell frequency should be less than 5. +1

Q-5 (ii)

x	y	xy
0	46	0
1	38	38
2	22	44
3	9	27
4	1	4
	$\Sigma y = 116$	$\Sigma xy = 113$

+2

+1

$$\text{Mean } (m) = \frac{\Sigma xy}{\Sigma y} = \frac{113}{116} = 0.974$$

Q-5 (iii)

$H_0: \mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

$n_1 = 10$, $n_2 = 12$, $\bar{x_1} = 15$, $\bar{x_2} = 14$ +1

$\Sigma (x_1 - \bar{x_1})^2 = 90$ $\Sigma (x_2 - \bar{x_2})^2 = 108$

$$S_1^2 = \frac{1}{n_1 - 1} \Sigma (x_1 - \bar{x_1})^2$$ +1

$$= \frac{90}{9} = 10$$

+1

$$S_2^2 = \frac{1}{n_2 - 1} \Sigma (x_2 - \bar{x_2})^2$$

$$= \frac{108}{11} = 9.82$$

Since $S_1^2 > S_2^2$ +2

$$F = \frac{S_1^2}{S_2^2}$$

$$= \frac{10}{9.82} = 1.018$$

+1

$F_{0.05}(9, 11) = 2.90$

$F_{cal} < F_{tab}$

Null hypothesis of equality of variance is accepted.

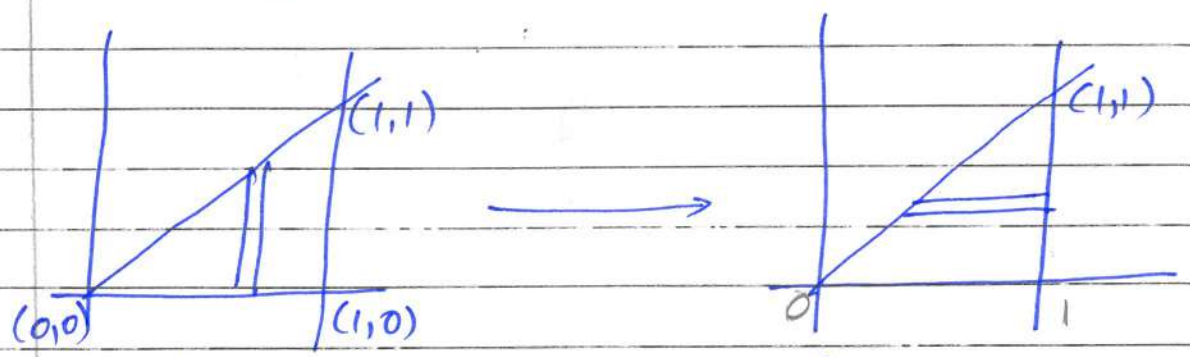
Q6 i) $\int_0^1 \int_0^1 \int_0^1 dx\,dy\,dz$.

$= \int_0^1 dx \int_0^1 dy \int_0^1 dz$ +2

$= (x)_0^1 (y)_0^1 (z)_0^1 = 1 \times 1 \times 1 = 1$. +1

ii) Change the order of integration

$$\int_0^1 \int_{y=0}^{x} e^{y/x}\,dx\,dy.$$



+2

$$\int_{y=0}^{1} \int_{x=y}^{x=1} e^{y/x}\,dx\,dy = \int_0^1 \left(\frac{e^{y/x}}{-y/x^2}\right)_y^1 dy.$$ +

$$\int_0^1 \left[\frac{-x^2}{y}\left(e^{y/x}\right)\right]_y^1 dy = \int_0^1 \left(\frac{-1}{y} e^{y} + \frac{y x^2}{y} e\right) dy$$ +3

$$- \int_0^1 \frac{e^y}{y}\,dy + e \int_0^1 y\,dy$$

$$-\int_0^1 \frac{e^y}{y}\,dy + e\left(\frac{y^2}{2}\right)_0^1$$

$$= -\int_0^1 \frac{e^y}{y}\,dy + \frac{e}{2}$$ +2

iii)

$$\int_0^1\int_0^{1-x}\int_0^{1-x-y} xyz\,dx\,dy\,dz$$ (2)

$$\int_0^1\int_0^{1-x} xy\left(\frac{z^2}{2}\right)_0^{1-x-y} dx\,dy$$

$$\int_0^1\int_0^{1-x} \frac{xy}{2}(1-x-y)^2\,dx\,dy$$

$$\int_0^1\int_0^{1-x} \frac{xy}{2}\left(1+(x+y)^2-2(x+y)\right)dx\,dy$$

$$\int_0^1\int_0^{1-x} \frac{xy}{2}\left(1+x^2+2xy-2x-2y+y^2\right)dx\,dy$$

$$\int_0^1\int_0^{1-x} \frac{xy}{2}+\frac{x^3y}{2}+x^2y^2-2x^2y-xy^2\,dx\,dy$$

$$\int_0^1\left[\frac{x}{2}\left(\frac{y^2}{2}\right)_0^{1-x}+\frac{x^3}{2}\left(\frac{y^2}{2}\right)_0^{1-x}+x^2\left(\frac{y^3}{3}\right)_0^{1-x}-x^2\left(\frac{y^2}{2}\right)_0^{1-x}-x\left(\frac{y^3}{3}\right)_0^{1-x}\right]dx$$

Q-6

iii) $$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dx\,dy\,dz$$

$$= \int_0^1 \int_0^{1-x} xy \left[\frac{z^2}{2}\right]_0^{1-x-y} dx\,dy$$

$$= \int_0^1 \int_0^{1-x} \frac{xy}{2}(1-x-y)^2 \, dx\,dy \qquad (+3)$$

$$= \int_0^1 \int_0^{1-x} \frac{xy}{2}\left[1+(x+y)^2-2(x+y)\right] dx\,dy$$

$$= \int_0^1 \int_0^{1-x} \frac{xy}{2}\left[1+x^2+2xy+y^2-2x-2y\right] dx\,dy$$

$$= \int_0^1 \int_0^{1-x} \left[\frac{xy}{2}+\frac{x^3y}{2}+x^2y^2+\frac{xy^3}{2}-x^2y-xy^2\right] dx\,dy$$

$$= \int_0^1 \left\{\frac{x}{2}\left(\frac{y^2}{2}\right)_0^{1-x}+\frac{x^3}{2}\left(\frac{y^2}{2}\right)_0^{1-x}+x^2\left(\frac{y^3}{3}\right)_0^{1-x}\right.$$
$$\left.+\frac{x}{2}\left(\frac{y^4}{4}\right)_0^{1-x}-x^2\left(\frac{y^2}{2}\right)_0^{1-x}-x\left(\frac{y^3}{3}\right)_0^{1-x}\right\} dx$$

$$= \int_0^1 \left[\frac{x}{2}\frac{(1-x)^2}{2}+\frac{x^3}{2}\frac{(1-x)^2}{2}+x^2\frac{(1-x)^3}{3}\right.$$
$$\left.+\frac{x}{2}\frac{(1-x)^4}{4}-\frac{x^2}{2}(1-x)^2-\frac{x}{3}(1-x)^3\right] dx$$

$$= \int_0^1 \left[\frac{x(1-2x+x^2)}{4}+\frac{x^3(1-2x+x^2)}{4}\right.$$

$$+\frac{x^2}{3}(1-3x+3x^2-x^3)+\frac{x}{8}(1-4x+6x^2-4x^3+x^4)$$
$$-\frac{x^2}{2}(1-2x+x^2)\ -\frac{x}{3}(1-3x+3x^2-x^3)\Big]dx$$

$$=\int_0^1 \frac{1}{4}(x-2x^2+x^3)+\frac{1}{4}(x^3-2x^4+x^5)$$
$$+\frac{1}{3}(x^2-3x^3+3x^4-x^5)+\frac{1}{8}(x-4x^2+6x^3-4x^4+x^5)$$

(+2) $$-\frac{1}{2}(x^2-2x^3+x^4)-\frac{1}{3}(x-3x^2+3x^3-x^4)\Big]dx$$

$$=\frac{1}{4}\left[\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right]_0^1+\frac{1}{4}\left[\frac{x^4}{4}-\frac{2x^5}{5}+\frac{x^6}{6}\right]_0^1$$
$$+\frac{1}{3}\left[\frac{x^3}{3}-\frac{3x^4}{4}+\frac{3x^5}{5}-\frac{x^6}{6}\right]_0^1+$$
$$\frac{1}{8}\left[\frac{x^2}{2}-\frac{4x^3}{3}+\frac{6x^4}{4}-\frac{4x^5}{5}+\frac{x^6}{6}\right]_0^1$$
$$-\frac{1}{2}\left[\frac{x^3}{3}-\frac{2x^4}{4}+\frac{x^5}{5}\right]_0^1-\frac{1}{3}\left[\frac{x^2}{2}-\frac{3x^3}{3}+\frac{3x^4}{4}-\frac{x^5}{5}\right]_0^1$$

$$=\frac{1}{4}\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\right]+\frac{1}{4}\left[\frac{1}{4}-\frac{2}{5}+\frac{1}{6}\right]$$
$$+\frac{1}{3}\left[\frac{1}{3}-\frac{3}{4}+\frac{3}{5}-\frac{1}{6}\right]+\frac{1}{8}\left[\frac{1}{2}-\frac{4}{3}+\frac{3}{2}-\frac{4}{5}+\frac{1}{6}\right]$$
(+2)
$$-\frac{1}{2}\left[\frac{1}{3}-\frac{1}{2}+\frac{1}{5}\right]-\frac{1}{3}\left[\frac{1}{2}-1+\frac{3}{4}-\frac{1}{5}\right]$$

~~0 0 0 8 4 6 6~~

$$= 0.1041 + 0.0041 + 0.0055 +$$

$$0.00416 - 0.0166 - 0.0166$$

$$= 0.08466$$