	ii.	Derive the formula for Mean and Variance of Poisson distribution.	**7**
OR	iii.	A coin is tossed 4 times , what is the probability to getting (a) Two heads, (b) At least two heads.	**7**

Q.6		Attempt any two:	
	i.	Solve $y_{h+2} - 7y_{h+1} + 10y_h = 0$ with $y_0 = 0, y_1 = 3$ by the method of generating function.	**5**
	ii.	Solve the recurrence relation $a_r + 6a_{r-1} + 9a_{r-2} = 3$ given that $a_0 = 0, a_1 = 1$.	**5**
	iii.	Solve $y_{h+1} - y_h = h$ with $y_0 = 1$ by using the method of generating function	**5**

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Enrollment No.....................................

Faculty of Engineering

End Sem (Odd) Examination Dec-2017

CA5BS03 Mathematics of Computer Applications

Programme: MCA Branch/Specialisation: Computer Application

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d.

Q.1	i.	The Set $A = \{x : x \in N \text{ and } x < 5\}$ is in which of the following form: (a) Tabular Form (b) Set Builder Form (c) Both a and b (d) None	**1**
	ii.	If $A = \{1,2,3,4,5\}$ *and* $B = \{1,3,5,7,9\}$ then $F = \{(1,1),(1,3),(3,5),(3,7),(5,9)\}$ is a (a) Relation (b) Function (c) Both (a) and (b) (d) None of these	**1**
	iii.	The square matrix A is said to be idempotent if (a) $A^T = -A$ (b) $A^2 = A$ (c) $A^2 = I$ (d) None of these	**1**
	iv.	If the eigen values of a matrix A are 1,3,5 then determinant of the matrix is (a) 9 (b) 1 (c) 15 (d) None of these	**1**
	v.	The degree of pendant vertex is (a) 0 (b) 1 (c) 2 (d) None of these	**1**
	vi.	A tree T with n vertices has (a) (n-1) edges (b) n edges (c) (n+1) edges (d) None of these	**1**
	vii.	Normal Distribution is (a) Discrete Distribution (b) Continuous Distribution (c) Both a and b (d) None of these	**1**
	viii.	If the probability $p = \frac{2}{3}$ and $n = 18$ then mean of the binomial distribution is (a) 9 (b) 2 (c) 12 (d) None of these	**1**

ix. The value of $\Delta =$ **1**

(a) $E-1$ (b) $E+1$ (c) $(E-1)^2$ (d) None of these

x. Order of the differential equation $y_{h+3}-6y_{h+2}-3y_{h+1}=0$ is **1**

(a) 3 (b) 1 (c) 2 (d) None of these

Q.2 i. Let $A=\{1,2,3,4\}, B=\{a,b,c,d\}$ and $c=\{x,y,z\}$ consider the function $f:A\to B$ and $g:B\to C$ defined by $f=\{(1,a),(2,c),(3,b),(4,a)\}$ and $g=\{(a,x),(b,x),(c,y),(d,y)\}$ find the composition function $(g\circ f)$. **2**

ii. Given $A=\{1,2,3,4\}$ and $B=\{x,y,z\}$. Let R be the relation from A to B defined as $R=\{(1,x),(2,z),(3,x),(3,y),(3,z)\}$ **3**

(a) Determine the matrix of the relation ,

(b) Determine the domain and range of R .

iii. Show that the relation $R=\{(a,b): a-b = \text{an even integer and } a,b\in I\}$ in the set I of integers is an equivalence relation. **5**

OR iv. Out of 130 students, 60 are wearing hats, 51 are wearing scarves and 30 are wearing both hats and scarves. Of the 54 students who are wearing sweaters, 26 are wearing hats, 21 are wearing scarves and 12 are wearing both hats and scarves. Every one wearing neither a hat nor scarf is wearing gloves. **5**

(a) How many students are wearing gloves?

(b) How many students not wearing a sweater are wearing hats but not scarves?

Q.3 i. Find the Eigen values of the matrix $A=\begin{bmatrix}1&0&0\\0&3&-1\\0&-1&3\end{bmatrix}$ **2**

ii. Test the consistency of system of equations: **3**

$x-2y+z-w=-1,\ 3x-2z+3w=-4,\ 5x-4y+w=-3$

iii. Find the inverse of the matrix $A=\begin{bmatrix}1&2&-1\\0&1&-1\\3&-1&1\end{bmatrix}$ **5**

OR iv Reduce the matrix A to its normal form where **5**

$$A=\begin{bmatrix}0&1&-3&-1\\1&0&1&1\\3&1&0&2\\1&1&-2&0\end{bmatrix}$$

Q.4 Attempt any two:

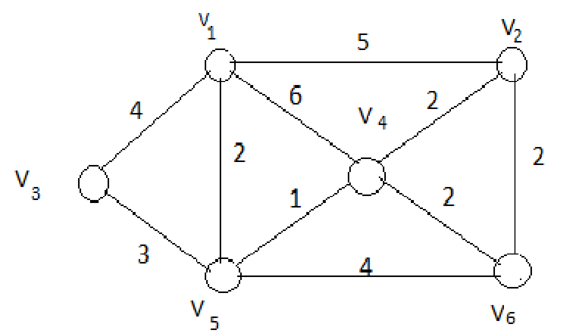
i. Define the following with example: **5**

(a) Isolated vertex (b) Regular graph (c) Euler graph

(d) Circuit (e) Tree

ii. Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$ **5**

iii. Using Prim's algorithm find the minimal spanning tree of the weighted graph **5**



Q.5 i. If $f(x)=Ce^{-x}, 0\le x\le\infty$, has a probability distribution function, then find the value of C. **3**

MCA - III sem (Aug - Dec. 2017)
'solution set'

1.1 (i) b) (ii) a) (iii) b) (iv) c) (v) b)
(vi) a) (vii) b) (viii) c) (ix) a) (x) c)

1.2 $gof = \{(1,x), (2,y), (3,x), (4,x)\}$ +2

1.2 $A = \{1,2,3,4\}$, $B = \{x,y,z\}$
and $R = \{(1,x), (2,z), (3,x), (3,y), (3,z)\}$

(i) Matrix of relⁿ R

	x	y	z
1	1	0	0
2	0	0	1
3	1	1	1
4	0	0	0

+1.5

Domain of R = {values of A} = {1, 2, 3, 4} +1.5

Range of R = {values of B} = {x, y, z}

Q.2

(iii) We know that R will be an equivalence relation if it is - reflexive, symmetric and transitive

a) R is reflexive: Let $a \in I$, then $a - a = 0$, which is an even integer. +1

$\therefore (a,a) \in R \ \forall a \in I$ +0.5

Hence R is reflexive relation.

b) R is symmetric: Let $a, b \in I$ +1

If $(a-b)$ is an even integer then $(b-a)$ is an even integer.

$\therefore (a,b) \in R \Rightarrow (b,a) \in R$ is true +0.5

Hence R is symmetric relation.

c) R is transitive: Let $a, b, c \in I$ +1

If $(a-b)$ and $(b-c)$ are even integers, then

$a - c = (a-b) + (b-c) =$ even integers

$(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$ is true +0.5

Hence R is transitive relation

Since in the set I, the relation R is reflexive, symmetric and transitive and therefore R is an equivalence relation +0.5

Q.2 (iv) Let X = total No. of std.

H = No. of std. wearing hats.

S = — " — scarves

P = — " — sweaters

G = — " — gloves.

$|X| = 130$, $|H| = 60$, $|S| = 51$, $|H \cap S| = 30$

$\therefore$ $|H \cup S| = |H| + |S| - |H \cap S| = 60 + 51 - 30 = 81$ +1

total no. of stds. wearing hat or scarves or both = 81.

Again $|P| = 54$, $|P \cap H| = 26$, $|P \cap S| = 21$, $|P \cap H \cap S| = 12$,

$\therefore$ $|H \cup S \cup P| = |H| + |S| + |P| - [|H \cap S| - |H \cap P| - |S \cap P| + |H \cap S \cap P|$ +1

$= 100$

Thus total no. of std. wearing one or more of hat, scarve and sweater = 100

(i) Total No. of std. wearing gloves +1

$= |G| = |X| - |H \cup S| = 49$

(ii) Let H_1 denote the set of stds. wearing hats only (neither wearing sweater nor wearing scarves). Then

$H_1 = H - P - S$

$|H_1| = |H - P - S| = |H| - |H \cap P| - |H \cap S| + |H \cap P \cap S|$ +1

$= 16$ +0.5

Q.3

(i) Eigen values are

$$|A - \lambda I| = 0$$

$$-\lambda^3 + 7\lambda^2 - 14\lambda - 1 = 0 \text{ or } (1-\lambda)(\lambda^2 - 6\lambda + 8) = 0$$ +1

$\lambda = 1, 2$ and 4. +1

Q.3 (ii)

$$[A:B] = \left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 3 & 0 & -2 & 3 & -4 \\ 5 & -4 & 0 & 1 & -3 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 5R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 0 & 6 & -5 & 6 & -1 \\ 0 & 6 & -5 & 6 & 2 \end{array}\right]$$ +1

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & -1 & -1 \\ 0 & 6 & -5 & 0 & -1 \\ 0 & 0 & 0 & 6 & 3 \end{array}\right]$$ +1

System is inconsisten and have no sol.

$\because \rho(A) = \rho(A;B) = 3 <$ no. of var. +1

ii) <u>Note</u>: In syllabus three methods for finding inverse :

(i) adjoint method

(ii) Gaussian elimination

(iii) Cayley Hamilton Theorem

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$$

By Gaussian elimination

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 3 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & -7 & 4 & -3 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 + 7R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -3 & 7 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & +1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -3 & -3 & 7 & 1 \end{array}\right]$$

$R_2 \to 3R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & -2 & 0 \\ 0 & 3 & 0 & 0 & -4 & -1 \\ 0 & 0 & -3 & -3 & 7 & 1 \end{array}\right]$$

$R_1 \to 3R_1 + R_3$ $\left[\begin{array}{ccc|ccc} 3 & 0 & 0 & 0 & 1 & 1 \\ 0 & 3 & 0 & 0 & -4 & -1 \\ 0 & 0 & -3 & -3 & 7 & 1 \end{array}\right]$

$R_1 \to \frac{1}{3}R_1 \,,\; R_2 \to \frac{1}{3}R_2 \,,\; R_3 \to \frac{-1}{3}R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/3 & 1/3 \\ 0 & 1 & 0 & 0 & -4/3 & -1/3 \\ 0 & 0 & 1 & 1 & -7/3 & -1/3 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1/3 & 1/3 \\ 0 & -4/3 & -1/3 \\ +1 & -7/3 & -1/3 \end{bmatrix}$$

+5

Q3 (iv)

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2 \,,\; R_4 \to R_4 - R_2$

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 1 & -3 & -1 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 + 3C_2$

$C_4 \to C_4 + C_2$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \phi & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 - C_1 \,,$

$C_4 \to C_4 - C_1$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_1 \leftrightarrow C_2$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{c|c} I_2 & 0 \\ \hline 0 & 0 \end{array}\right]$$

$$\rho(A) = 2$$

Q.4 (ii) Let G be a simple graph with n-vertices $v_1, v_2, \ldots v_n$ say. The vertex v_1 can be joined to the remaining $(n-1)$ vertices $v_2, v_3 \ldots v_n$ to obtain max: no. $n-1$ of edges, namely (v_1, v_2) (v_1, v_3) $\ldots$ (v_1, v_n).

The vertex v_2 can be joined to $(n-2)$ vertices $v_3, v_4, \ldots v_n$ to obtain a max. no. $n-2$ of edges, namely. (v_2, v_3) (v_2, v_4) $\ldots$ (v_2, v_n), Note that in this case we have not joined v_2 to v_1, since this edge (v_1, v_2) has already obtained. Proceeding in this manner, the vertex v_{n-1} will give us only one new edge (v_{n-1}, v_n)

Hence max. no. of edges in the graph G is

$$(n-1) + (n-2) + (n-3) + \cdots + 2 + 1 = \frac{n(n-1)}{2}$$

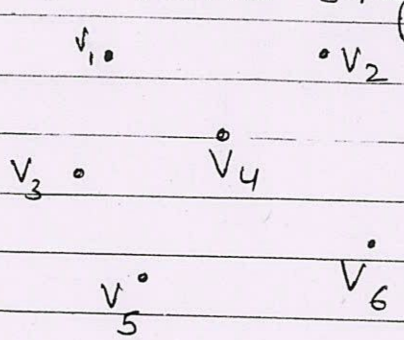
Q.4 iii) adjacency matrix

$$\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} - & 5 & 4 & 6 & 2 & - \\ 5 & - & - & 2 & - & 2 \\ 4 & - & - & - & 3 & - \\ 6 & 2 & - & - & 1 & 2 \\ 2 & - & 3 & 1 & - & 4 \\ - & 2 & - & 2 & 4 & - \end{bmatrix} \end{array}$$

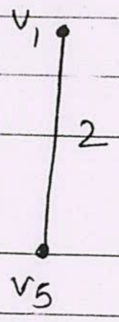
+1

8

a) initial stage

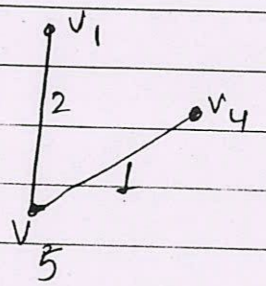


b) stage 1

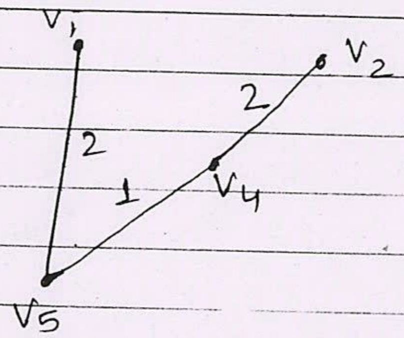


+1

stage 2

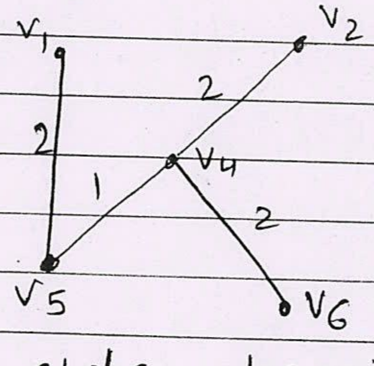


stage 3

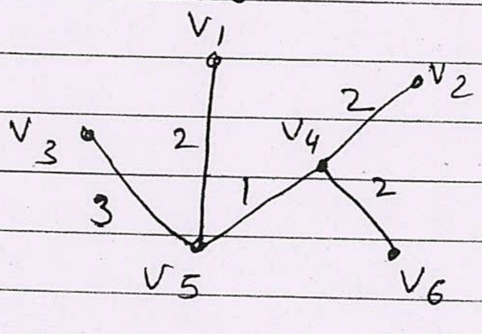


+1

stage 4



stage 5



+1

If steps describe with imges

+1

Q.5 (i) By probability distribution function we have

(i) $f(x) \geq 0 \ \forall x$ (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$ +1

$\because e^{-x}$ is +ve $\forall\ 0 \leq x < \infty$

$\therefore$ cond^n is satisfied if $c \geq 0$

again by cond^n (ii)

$$\int_0^{\infty} c e^{-x}\,dx = 1 \Rightarrow \left(-c e^{-x}\right)_0^{\infty} = 1$$ +1

$$\boxed{c = 1}$$ +1

Q.5 (ii) Mean of P.D

$$\mu_1' = \sum_{r=0}^{\infty} r\, p(r)$$

$$= \sum_{r=0}^{\infty} r . \frac{e^{-m} m^r}{r!}$$ +1

$$= \sum_{r=1}^{\infty} \frac{e^{-m} m^r}{(r-1)!}$$ ~~+0.5~~

$$= \frac{e^{-m} . m}{0!} + \frac{e^{-m} m^2}{1!} + \frac{e^{-m} m^3}{2!} + \cdots$$ +1

$$= e^{-m} m \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots\right]$$ +1

$$= m e^{-m} e^m$$

$$\mu_1' = m.$$

Variance of P.D. $\mu_2 = \mu_2' - \mu_1'^2$ +1

$$\mu_2' = \sum_{r=0}^{\infty} r^2 p(r)$$

$$= \sum_{r=0}^{\infty} \left(r(r-1) + r\right) p(r)$$ +1

$$= \sum_{r=0}^{\infty} r(r-1) p(r) + \sum_{r=0}^{\infty} r p(r)$$

$$= \sum_{r=0}^{\infty} r(r-1) \frac{e^{-m} m^r}{r!} + m$$ +1

$$= \sum_{r=2}^{\infty} \frac{e^{-m} m^r}{(r-2)!} + m$$ +1

$$= m^2 + m$$

Variance $= m$

1.5 (iii) Given $n=4$, let $p=$ prob. of getting a head in single throw of one coin
i.e. $p = 1/2$, $q = 1/2$ +1

By binomial dist. $P(r) = {}^{n}C_{r}\, q^{n-r} p^{r}$ +1

(i) $P(\text{two heads}) = P(r=2) = {}^{4}C_{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$ +1

$= 0.375$ +1

(ii) $P(\text{at least two heads})$

$P(r = 2 \text{ or } 3 \text{ or } 4$

$= P(r=2) + P(r=3) + P(r=4)$ +1

$= {}^{4}C_{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 + {}^{4}C_{3} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 + {}^{4}C_{4} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4$ +1

$= 0.6875$ +1

6 (i) we have

$$y_{n+2} - 7y_{n+1} + 10y_n = 0 \quad \text{--- (1)}$$

given $y_0 = 0$, $y_1 = 3$

consider the generating function

$$Y(t) = \sum_{n=0}^{\infty} y_n t^n = y_0 + y_1 t + y_2 t^2 + \cdots \quad \text{--- (2)} \qquad +1$$

Multiplying (1) by t^n and summing from $n=0$ to ∞

$$\sum_{n=0}^{\infty} y_{n+2} t^n - 7\sum_{n=0}^{\infty} y_{n+1} t^n + 10\sum_{n=1}^{\infty} y_n t^n = 0 \qquad +1$$

$$\left(y_2 + y_3 t + \cdots\right) - 7(y_1 + y_2 t + \cdots) + 10\,Y(t) = 0$$

$$\frac{Y(t) - y_0 - y_1 t}{t^2} - 7\,\frac{Y(t) - y_0}{t} + 10\,Y(t) = 0 \qquad +1$$

$$\frac{Y(t) - 3t}{t^2} - 7\,\frac{Y(t)}{t} + 10\,Y(t) = 0$$

$$Y(t) - 3t - 7t\,Y(t) + 10t^2\,Y(t) = 0$$

$$(1 - 7t + 10t^2)\,Y(t) = 3t$$

$$Y(t) = \frac{3t}{(1 - 7t + 10t^2)} = \frac{1}{1-5t} - \frac{1}{1-2t} \qquad +1$$

$$\sum_{n=0}^{\infty} y_n t^n = (1-5t)^{-1} - (1-2t)^{-1}$$

$$= \left(1 + 5t + (5t)^2 + \cdots + (5t)^n + \cdots\right) - \left(1 + 2t + (2t)^2 + \cdots\right)$$

equating coeff of t^n $\qquad +1$

$$y_n = 5^n - 2^n.$$

6 (ii) $a_r + 6a_{r-1} + 9a_{r-2} = 3$ — (1) given $a_0 = 0$, $a_1 = 1$

The characteristic eq^n

$$m^2 + 6m + 9 = 0$$ +1

$$(m+3)^2 = 0$$

$$m = -3, -3$$

$a_r^{(h)} = (C_1 + C_2 r)(-3)^r$ - Homogeneous sol^n +1

The particular sol^n (trial sol^n) corresponding to the term '3' is A_0

$a_r^{(P)} = A_0$ — (2) +1

substituting (2) in (1)

$$A_0 + 6A_0 + 9A_0 = 3$$

$$16A_0 = 3$$

$$A_0 = 3/16$$

$a_r^{(P)} = 3/16$ +1

Total sol^n of (1) is

$a_r = (C_1 + C_2 r)(-3)^r + 3/16$ +1

.6 (iii) $y_{n+1} - y_n = n$ — ① with $y_0 = 1$

Consider generating function

$$Y(t) = \sum_{n=0}^{\infty} y_n t^n = y_0 + y_1 t + y_2 t^2 + \cdots \quad \text{— ②}$$ +1

multiplying ① by t^n and summing from $n=0$ to $n=\infty$, we have

$$\sum_{n=0}^{\infty} y_{n+1} t^n - \sum_{n=0}^{\infty} y_n t^n = \sum_{n=0}^{\infty} n t^n$$ +1

$$(y_1 + y_2 t + y_3 t^2 + \cdots) - Y(t) = t + 2t^2 + 3t^3 + \cdots$$

$$\frac{Y(t) - y_0}{t} - Y(t) = t(1 + 2t + 3t^2 + \cdots)$$ +1

$$Y(t) - 1 - t\,Y(t) = t^2(1-t)^{-2} \quad \begin{bmatrix} \text{given} \\ y_0 = 1 \end{bmatrix}$$

$$(1-t)\,Y(t) = \cancel{1+}t^2(1-t)^{-2}.$$

$$Y(t) = \frac{1}{1-t} + \frac{t^2}{(1-t)^3}$$

$$= (1-t)^{-1} + t^2(1-t)^{-3}$$ +1

$$\sum_{n=0}^{\infty} y_n t^n = \sum_{n=0}^{\infty} t^n + t^2 \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} t^n$$

$$= \sum_{n=0}^{\infty} t^n + \sum_{n=0}^{\infty} \frac{(n+1)(n+2)}{2} t^{n+2}$$

equating coefficient of t^n on both sides, we have +1

$$y_n = 1 + \frac{1}{2}(n-1)n = 1 + \frac{1}{2}n(n-1)$$