Total No. of Questions: 6 Total No. of Printed Pages:3

**Enrollment No......................................**

Faculty of Engineering / Science

End Sem Examination Dec-2023

EN3BS12 / BC3BS03 Engineering Mathematics -II

Programme: B.Tech./ B.Sc. Branch/Specialisation: All

**Duration: 3 Hrs. Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1 i. Laplace transform of $sinat$ is **1**

(a) $\frac{s}{s^2+a^2}$ (b) $\frac{a}{s^2+a^2}$ (c) $\frac{a}{s^2-a^2}$ (d) None of these

ii. Laplace transform of $H(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$ is **1**

(a) $\frac{e^{as}}{s}$ (b) $e^{as}$ (c) $\frac{e^{-as}}{s}$ (d) None of these

iii. A function $f(x)$ is called an even function of $x$ if- **1**

(a) $f(-x) = f(x)$ (b) $f(-x) = -f(x)$

(c) $f(x) = 0$ (d) None of these

iv. For a periodic function $f(x)$ with period $T$, which of the following is true. **1**

(a) $f(x) = f(x+T)$ (b) $f(x) = f(x+\frac{T}{2})$

(c) $f(x) = f(T)$ (d) None of these

v. Equation $Pp + Qq = R$ is the standard form of (where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$)- **1**

(a) Charpit's equation

(b) Clairaut's equation

(c) Lagrange's linear equation

(d) None of these

vi. The particular integral of $(D^2 - 2DD' + D'^2)Z = e^{x+2y}$ is- **1**

(a) $e^{x+2y}$ (b) $\frac{1}{2}e^{x+2y}$ (c) $-\frac{1}{2}e^{x+2y}$ (d) None of these

vii. If vector $\vec{V}$ is solenoidal then **1**

(a) curl $\vec{V} = 0$

(b) div $\vec{V} = 0$

(c) Both curl $\vec{V} = 0$ and div $\vec{V} = 0$

(d) None of these

viii. $\frac{d}{dt}\left(\vec{a} \pm \vec{b}\right) =$ **1**

(a) $\frac{d\boldsymbol{a}}{dt} \pm \frac{d\boldsymbol{b}}{dt}$ (b) $\frac{d\boldsymbol{a}}{dt} + \frac{d\boldsymbol{b}}{dt}$ (c) $\frac{d\boldsymbol{a}}{dt} - \frac{d\boldsymbol{b}}{dt}$ (d) None of these

ix. If $f_n(x)$ is purely a polynomial in $x$, then $f_n(x) = 0$ is called- **1**

(a) An Algebraic equation

(b) Transcendental equation

(c) Both Algebraic and Transcendental equation

(d) None of these

x. What is the sufficient condition of convergence for the iterative method? **1**

(a) $|f'(x)| < 1$ (b) $|f'(x)| > 1$

(c) $|f(x)| < 1$ (d) None of these

Q.2 Attempt any two:

i. Find $L\left\{\frac{1-cos2t}{t}\right\}$ **5**

ii. Evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **5**

iii. Using Laplace transform solve $y'' - 2y' + 2y = 0$, given $y(0) = y'(0) = 1$ **5**

Q.3 i. Write Dirichlet's conditions for a Fourier series expansion. **3**

ii. Find a Fourier series to represent $x - x^2$ from $x = -\pi\ to\ x = \pi$. **7**

OR iii. Express $f(x) = x$ as a **7**

(a) Half range cosine series in $0 < x < 2$

(b) Half range sine series in $0 < x < 2$

Q.4 i. Form the partial differential equation from the relation $z = f(x + iy) + F(x - iy)$ where $f$ and $F$ are arbitrary functions. **3**

ii. Find the complete solution by Charpit's method $px + qy = pq$ **7**



OR iii. Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = ycosx$ **7**

Q.5 i. (a) Show that the vector field $\vec{F} = \nabla(x^3 + y^3 - 3xyz)$ is irrotational. **4**

(b) Find a unit vector normal to the surface $\emptyset = x^2 + y^2 - z$ at the point (1,2,5).

ii. Evaluate $\iint_S A.\hat{n}\, dS$ where A=18z i-12 j+3y k and $S$ is a part (i.e., surface) of the plane $2x + 3y + 6z = 12$ which is in the first octant. **6**

OR iii. Find the value of $\int_C F.dr$, where $F = e^x sinyi + e^x cosyj$. The vertices of the rectangle C are (0,0), (1,0), (1,$\pi/2$), (0,$\pi/2$). **6**

Q.6 Attempt any two:

i. By applying Newton-Raphson method find a root of the equation: $x^3 - 3x - 4 = 0$ **5**

ii. Solve the following system of equations by Gauss elimination method: **5**

$$2x - y + 3z = 9, \quad x + y + z = 6, \quad x - y + z = 2$$

iii. Solve by Gauss-Seidal method: **5**

$5x + 2y + 3 = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$

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Solution

EN3BS12 / BC3BS03 Engg. Mathematics - II



Q1

(i) (b) $\frac{a}{s^2+a^2}$ 1

(ii) (c) $\frac{e^{-as}}{s}$ 1

(iii) (a) $f(-x) = f(x)$ 1

(iv) (a) $f(x) = f(x+T)$ 1

(v) (c) Langrange's linear equation 1

(vi) (a) The P.I. $= e^{x+2y}$ 1

(vii) (b) Div $\vec{V} = 0$ 1

(viii) (a) $\frac{d\vec{a}}{dt} \pm \frac{d\vec{b}}{dt}$ 1

(ix) (a) An algebric equation 1

(x) (a) $|f'(x)| < 1$ 1

Q. 2 (i) Find $L\left\{\frac{1-\cos 2t}{t}\right\}$

Let $F(t) = 1-\cos 2t$ +1

$\lim_{t \to 0} \frac{1-\cos 2t}{t} = \lim_{t \to 0} \frac{2\sin 2t}{1} = 0$

which exists.

Now

$$\mathcal{L}\{1-\cos 2t\} = \frac{1}{s} - \frac{s}{s^2+4} = f(s), \text{ say} \quad +1$$

$$\mathcal{L}\left\{\frac{1-\cos 2t}{t}\right\} = \int_s^\infty f(x)\,dx = \int_s^\infty \left[\frac{1}{x} - \frac{x}{x^2+4}\right] dx$$

$$= \left[\log|x| - \frac{1}{2}\log|x^2+4|\right]_s^\infty \quad +1$$

$$= \frac{1}{2}\left[\log \frac{x^2}{x^2+4}\right]_s^\infty \quad +1$$

$$= \frac{1}{2}\left[\lim_{x\to\infty} \log \frac{1}{1+4/x^2} - \log\frac{s^2}{s^2+4}\right] \quad +1$$

$$= \frac{1}{2}\log\frac{s^2+4}{s^2}$$

Q 2 (ii) Evaluate $\mathcal{L}^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$

Let $f(s) = \frac{s}{s^2+a^2}$ and $g(s) = \frac{s}{s^2+b^2}$

$\Rightarrow F(t) = L^{-1}\{f(s)\} = \cos at$

$h(t) = L^{-1}\{g(s)\} = \cos bt \quad +1$

$F(u) = \cos au$ and $h(t-u) = \cos(bt-bu)$

using convolution theorem.

$$L^{-1}\{f(s)\cdot g(s)\} = \int_{u=0}^{t} F(u)\,h(t-u)\,du \quad +1$$

[Since $2\cos A \cos B = \cancel{2}\cos(A+B) + \cos(A-B)$]

$$= \frac{1}{2}\int_{u=0}^{t} [\cos(au+bt-bu) + \cos(au-bt+bu)]\,du$$ +1

$$= \frac{1}{2}\left[\frac{\sin(au+bt-bu)}{(a-b)} + \frac{\sin(au-bt+bu)}{(a+b)}\right]_0^t$$

$$= \frac{1}{2}\left[\frac{(\sin at - \sin bt)}{(a-b)} + \frac{\sin at - \sin(-bt)}{(a+b)}\right]$$ +1

$$= \frac{1}{2}\left[\frac{2a\sin at - 2b\sin bt}{a^2-b^2}\right] = \frac{a\sin at - b\sin bt}{a^2-b^2}$$ +1

Q: 2 (iii) Solve $y'' - 2y' + 2y = 0$, given $y(0) = y'(0) = 1$

Taking Laplace transform of both the sides

$$L[y'' - 2y' + 2y] = L(0)$$

$$[s^2\bar{y} - sy(0) - y'(0)] - 2[s\bar{y} - y(0)] + 2\bar{y} = 0$$ +1

$$\Rightarrow [s^2\bar{y} - s - 1] - 2[s\bar{y} - 1] + 2\bar{y} = 0$$

$$[s^2 - 2s + 2]\bar{y} = s - 1$$ +1

$$\bar{y} = \frac{s-1}{s^2-2s+2} = \frac{s-1}{(s-1)^2+1}$$ +1

$$\Rightarrow \quad y = L^{-1}\left\{\frac{s-1}{(s-1)^2+1}\right\}$$ +1

$$= e^t L^{-1}\left\{\frac{s}{s^2+1}\right\}$$ +1

$$= e^t \cos t$$

Q3 (i) Dirichlet's condition :-

(i) f(x) is periodic, single valued and finite. +1

(ii) f(x) has finite number of discontinuties in any on period. +1

(iii) f(x) has finite number of maxima and minima +1

~~(iii)~~

Q3 (ii) Fourier series $x-x^2$ from $x=-\pi$ to $x=\pi$

$$f(x) = x - x^2, \quad x \in (-\pi, \pi)$$

we know that Fourier series of f(x) over the interval $(-\pi, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$ +1

we have $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\,dx$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi}$$ +1

$$= \frac{1}{\pi}\left[\left(\frac{\pi^2}{2} - \frac{\pi^3}{3}\right) - \left(\frac{\pi^2}{2} + \frac{\pi^3}{3}\right)\right]$$

$$\boxed{a_0 = \frac{-2\pi^2}{3}}$$ +1

Now $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\cos nx\, dx$

$$= +\frac{1}{\pi}\left[\left\{(x-x^2)\frac{\sin nx}{n}\right\}_{-\pi}^{\pi} - \int_{-\pi}^{\pi}(1-2x)\frac{\sin nx}{n}\,dx\right]$$

+1

$$= \frac{1}{\pi}\left[(x-x^2)\frac{\sin nx}{n} - (1-2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(1-2\pi)\frac{\cos n\pi}{n^2} - (1+2\pi)\frac{\cos n\pi}{n^2}\right]$$

$$= \frac{1}{\pi}\left(-4\pi\frac{\cos n\pi}{n^2}\right)$$

$$\boxed{a_n = -4\frac{(-1)^n}{n^2}}$$ [$\sin n\pi = 0$ & $\cos n\pi = (-1)^n$] +1

Now $b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\sin nx\, dx$ +1

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{-\cos nx}{n}\right) - (1-2x)\left(\frac{-\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[(\pi^2-\pi)\frac{\cos n\pi}{n} - \frac{2\cos n\pi}{n^3} + (\pi-\pi^2)\frac{\cos n\pi}{n} + 2\frac{\cos n\pi}{n^3}\right].$$

$$= \frac{1}{\pi}\left[\frac{-2\pi\cos n\pi}{n}\right] = \frac{-2(-1)^n}{n}.$$

$$\boxed{b_n = \frac{-2(-1)^n}{n}}$$

+1

$$x - x^2 = -\frac{\pi^2}{3} + 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots\right] + 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right]$$

Q3(iii) $f(x) = x$

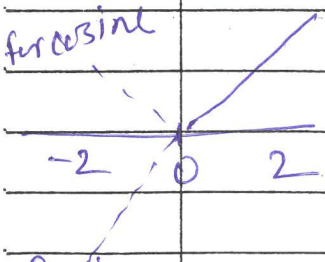
We know that half range sine series of $f(x)$ in $(0, l) = (0, 2)$.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) \qquad l = 2$$

+1

where $b_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}\,dx$

for cosine



for sin

$$= \frac{2}{2}\int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx.$$

$$= \left(\frac{x\cos\frac{n\pi x}{2}}{-\frac{n\pi}{2}}\right)_0^2 - \int_0^2 \frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(-\frac{n\pi}{2}\right)}dx$$

+1

$$= \frac{-2}{n\pi}\left[(2\cos n\pi - 0) - \left(\frac{\sin\frac{n\pi x}{2}}{\left(\frac{n\pi}{2}\right)}\right)_0^2\right]$$

$$= \frac{-2}{n\pi}\left[2(-1)^n - 0 - 0\right]$$

$$= -4\frac{(-1)^n}{n\pi}$$

Hence series is

$$x = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2} \qquad +1/2$$

(b) Half range cossin series in $0<x<2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{2}. \qquad +1/2$$

where $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$$a_0 = \frac{2}{2}\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 \qquad +1$$

$$= 2$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{2}\int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx. \qquad +1$$

$$= \left[x\left(\frac{2}{n\pi}\sin\frac{n\pi x}{2}\right) - 1\left(\frac{-4}{n^2\pi^2}\cos\frac{n\pi x}{2}\right)\right]_0^2$$

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$$a_n = \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

+1

$$a_1 = \frac{-8}{\pi^2}, \quad a_3 = \frac{-8}{3^2\pi^2}, \quad a_2 = a_4 = a_6 = 0$$

Hence required Half range cosine series is

$$x = 1 - \frac{8}{\pi^2}\left[\frac{\cos(\pi x/2)}{1^2} + \frac{\cos(3\pi x/2)}{3^2} + \cdots\right]$$

Q: 4(i) $\quad z = f(x+iy) + F(x-iy)$

Differentiating two times w.r.t $x$ and $y$ we get

$$\frac{\partial z}{\partial x} = f'(x+iy) + F'(x-iy)$$

+1

$$\frac{\partial z}{\partial y} = f'(x+iy)\,i + F'(x-iy)(-i)$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x+iy) + F''(x-iy)$$

+1

$$\frac{\partial^2 z}{\partial y^2} = f''(x+iy)(i^2) + F''(x-iy)(-i)^2$$

$$= -f''(x+iy) + F''(x-iy)$$

$$= -\left[f''(x+iy) + F'(x-iy)\right] = -\frac{\partial^2 z}{\partial x^2}$$

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$$\Rightarrow \quad \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$ +1

Q4 (ii) $px + qy = pq$

Let $f(x, y, z, p, q) = px + qy - pq = 0$ —(1)

charpits auxiliary eq^n is given by.

$$\frac{dp}{\frac{\partial f}{\partial x} + p\frac{\partial f}{\partial z}} = \frac{dq}{\frac{\partial f}{\partial y} + q\frac{\partial f}{\partial z}} = \frac{dz}{-p\frac{\partial f}{\partial p} - q\frac{\partial f}{\partial q}}$$ 1

$$= \frac{dx}{-\frac{\partial f}{\partial p}} = \frac{dy}{-\frac{\partial f}{\partial q}} = \frac{dF}{0}$$

we get $\frac{\partial f}{\partial x} = p$, $\frac{\partial f}{\partial y} = q$

1

$\frac{\partial f}{\partial z} = 0$, $\frac{\partial f}{\partial p} = x - q$, $\frac{\partial f}{\partial q} = y - p$

$$\frac{dp}{p} = \frac{dq}{q} = \frac{dz}{-p(x-q) - q(y-p)} = \frac{dx}{q - x}$$

$$= \frac{dy}{p - y} = \frac{dF}{0}$$

Taking 1st and 2nd ratios we get

$$\frac{dp}{p} = \frac{dq}{q}$$

+1

$$\log p = \log q + \log a$$

$$\Rightarrow \quad p = aq \qquad -(2)$$

where a is any arbitary constant
substituting the above value p in (1)
we get.

$$q\,(ax+y) = aq^2$$

+2

$$q = \frac{ax+y}{a} \qquad -(3)$$

from (2) & (3)

$$p = ax+y \qquad -(4)$$

Now putting value of p & q in

$$dz = p\,dx + q\,dy$$

+1

$$dz = (ax+y)\,dx + \frac{(ax+y)}{a}\,dy$$

$$a\,dz = (ax+y)(a\,dx+dy)$$

$$a\,dz = (ax+y)\,d(ax+y)$$

$$az = \frac{(ax+y)^2}{2} + \frac{b}{2}$$

+1

$$2az = (ax+y)^2 + b$$

Q4 (iii) Solve

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$$

$$(D^2 + DD' - 6D'^2)z = y\cos x$$

A.E is $M^2 + M - 6 = 0$

$\Rightarrow (M+3)(M-2) = 0$

$\Rightarrow M = -3, 2$ (roots) +1

C.F. $= f_1(y-3x) + f_2(y+2x)$ +1

Now, P.I. $= \frac{1}{(D+3D')(D-2D')} y\cos x$

$= \frac{1}{(D+3D')}\left[\frac{1}{D-2D'} y\cos x\right]$ +1

[ Since $D-2D' = 0$ or $M-2=0 \Rightarrow M=2$

So put $y = c - Mx = c - 2x$ ]

$\therefore$ P.I. $= \frac{1}{D+3D'}\int (c-2x)\cos x\, dx$ +1

on integrating by parts, we get

$= \frac{1}{(D+3D')}\int (c-2x)\cos x\, dx.$

on intigrating by parts. we get +1

$= \frac{1}{D+3D'}\left[\int (c-2x)\sin x - \int (-2)\sin x\, dx\right]$

$$= \frac{1}{D+3D'} [y \sin x - 2\cos x]$$

[since $D+3D' = 2$ or $m = -3$ so put $c - mx = y$
$= c + 3x$]

$$= \int [(c+3x)\sin x - 2\cos x]\, dx$$

+1

$$= (c+3x)(-\cos x) + 3\sin x - 2\sin x$$

$$= -y\cos x + \sin x. \qquad [c = y - 3x]$$

Hence

$z = C.F. + P.I.$ +1

$$z = f_1(y-3x) + f_2(y+2x) - y\cos x + \sin x.$$

Q 5(i) (a). $\vec{F} = \nabla (x^3 + y^3 - 3xyz)$

$\vec{F}$ to be irrotational

Curl ~~div~~ $\vec{F} = 0$ +1

$$\vec{F} = \nabla (x^3 + y^3 - 3xyz)$$

$$= \sum i \frac{\partial}{\partial x} (x^3 + y^3 \cancel{+z^3} - 3xyz)$$

$$\vec{F} = i(3x^2 - 3yz) + \hat{j}(3y^2 - 3xz)$$ +1

$$\text{div } \vec{F} = \frac{\partial}{\partial x}(3x^2 - 3yz) + \frac{\partial}{\partial y}(3y^2 - 3zx)$$

$$= \cancel{6x + 6y}$$

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$$\text{curl}\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2-3yz & 3y^2-3zx & 0 \end{vmatrix}$$ +1

$$= \hat{i}\,(-3x+3x) + \hat{j}\,(-3y+3y) + \hat{k}\,(0)$$

$$= 0.$$

Since $\text{curl}\vec{F} = 0$ hence $\vec{F}$ is +1
irrotational vector.

5 (ii) (b) $\phi = x^2 + y^2 - z$

unit normal to the surface is given by.

$$\hat{n} = \frac{\text{grad}\,\phi}{|\text{grad}\,\phi|} \text{ at } (1,2,5)$$

$$\text{grad}\,\phi = \nabla\phi = \Sigma i \frac{\partial}{\partial x}(x^2+y^2-z)$$

$$= 2x\hat{i} + 2y\hat{j} - \hat{k}$$ +1

grad $\phi$ at $(1,2,5)$

$$= 2\hat{i} + 4\hat{j} - \hat{k}$$

$$|\text{grad}\,\phi| = \sqrt{(2^2) + 4^2 + 1^2}$$ +1

$$\hat{n} = \frac{2\hat{i} + 4\hat{j} - \hat{k}}{\sqrt{20+1}} = \frac{2\hat{i}+4\hat{j}-\hat{k}}{\sqrt{21}}$$

5 (ii) The eqn of the plane is

$$\phi = 2x + 3y + 6z - 12$$

Now $\text{grad}\,\phi = \nabla\phi = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$\hat{n} = \text{grad}\,\phi / |\text{grad}\,\phi|$$

$$= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{4 + 9 + 36}} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$ +1

$$\vec{F}\cdot\hat{n} = (18z\hat{i} - 12\hat{j} + 3y\hat{k})\cdot\frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$$ +1

$$= \frac{18}{7}(y + 2z - 2)$$

Let R be the projection of S on the xy-plane then

$$ds = \frac{dx\,dy}{|\hat{n}\cdot\hat{k}|} \text{ and } R : 2x + 3y = 12$$

$[\because z = 0]$ +1

Limits $y = 0$ to $(12 - 2x)/3$.

$x = 0$ to $6$

$$\hat{n}\cdot\hat{k} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})\cdot\hat{k} = \frac{6}{7}$$

$$\iint_S \vec{F}\cdot\hat{n}\,ds = \iint_R \vec{F}\cdot\hat{n}\,\frac{dx\,dy}{|\hat{n}\cdot\hat{k}|}$$ +1

$$= \iint_R (3y + 6z - 6)\,dx\,dy.$$

$$= \iint_R (3y + (12 - 2x - 3y) - 6)\,dx\,dy$$

$[2x + 3y + 6z = 12]$

5(ii)
- continue

$$= \int_{x=0}^{x=6} \int_{y=0}^{\frac{12-2x}{3}} (6-2x)\,dy\,dx.$$ +1

$$= \frac{4}{3} \int_0^6 (3-x)(6-x)\,dx$$

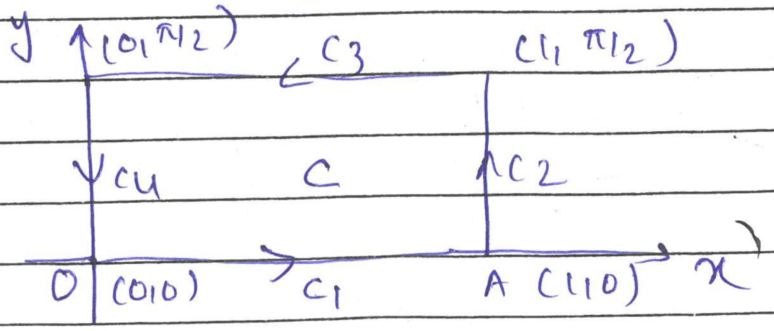
$$= \frac{4}{3} \left[ 18x - \frac{9}{2}x^2 + \frac{x^3}{3} \right]_0^6$$ +1

$$= 24$$

OR 5(iii) $\vec{F} = e^x \sin y\, i + e^x \cos y\, j$

$C$: $(0,0)$ $(1,0)$ $(1,\pi/2)$ $(0,\pi/2)$

To calculat $\int_C F\,dr$ line gntegral




+2

$$\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3} + \int_{C_4}$$

For $C_1$ : OA $y=0 \Rightarrow dy=0$ & limit $x \to 0$ to $1$
for $C_2$ : AB $x=1 \Rightarrow dx=0$ & $y=0$ to $\pi/2$.
for $C_3$ : BC $y=\pi/2 \Rightarrow dy=0$ & $x \to 1$ to $0$ +2
for $C_4$ : CO $x=0 \Rightarrow dx=0$ & $y \to \pi/2$ to $0$

Hence

$$\int_c \vec{F}\cdot d\vec{r} = \int_0^1 0 + \int_0^{\pi/2} e\cos y\, dy$$ +1

$$+ \int_{x=1}^{0} e^x \sin(\pi/2)\, dx$$

$$+ \int_{y=\pi/2}^{0} \cos y\, dy$$

$$= e(1-0) - (e-1) + (0-1) = 0$$ +1

[This may be solved using Green's Theorem the answer will remain same]

Q 6(i) $f(x) = x^3 - 3x - 4$ by Newton-Raphson

Let $f(x) = x^3 - 3x - 4$

$f(0) = -4 = -ive$

$f(1) = -6 = -ive$

$f(2) = -2 = -ive$ +1

$f(3) = 4 = +ive$

$f(2)$ and $f(3)$ are opposite signs
Hence root lies between 2 and 3.

$\therefore$ taking $x_0 = 2$

By Newton Raphson formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ +1

$$f'(x) = 3x^2 - 3.$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 3x_n - 4)}{(3x_n^2 - 3)}$$ +1

$$x_{n+1} = \frac{2x_n^3 + 4}{3(x_n^2 - 1)} \quad \text{--- (1)}$$

putting $n=0$ in (1)

$$x_1 = \frac{20}{9} = 2.222$$ +1

$$x_2 = \frac{25.9472}{11.8118} = 2.197$$

$$x_3 = 2.196$$

$$x_4 = 2.196$$ +1

$x_3 = x_4 = 2.196$ correct to 3 decimal places.

6 (ii) By Gauss elimination

$$2x - y + 3z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

The given matrix form

$$AX = B.$$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$ +1

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

Augmented matrix $[A \,\vdots\, B] =$ +1

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right] \quad -(1)$$

operating $R_2 - \frac{R_1}{2}$ and $R_3 \to R_3 - \frac{R_1}{2}$

$$[A \,\vdots\, B] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 9 \\ 0 & 3/2 & -1/2 & 3/2 \\ 0 & -1/2 & -1/2 & -5/2 \end{array}\right]$$ +1

$R_3 \to R_3 + \frac{R_2}{3}$ we get

$$[A \,\vdots\, B] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 9 \\ 0 & 3/2 & -1/2 & 3/2 \\ 0 & 0 & -2/3 & -2 \end{array}\right] \quad -(2)$$ +1

which is upper triangular matrix form

from (2) we get

$$2x - y + 3z = 9$$
$$(3/2)y - (1/2)z = 3/2$$
$$(-2/3)z = -2$$

+1

using back substitution, we get

$z = 3$, $y = 2$ and $x = 1$

5(iii) Solve by Gauss Seidel Method.

| | |
|---|---|
| $5x + 2y + \overset{z}{3} = 12$ | $5x + 2y = 12 - 3 = 9$ |
| $x + 4y + 2z = 15$ | $x + 4y + 2z = 15$ |
| $x + 2y + 5z = 20$ | $x + 2y + 5z = 20$ |

(Marks may be awarded to both).

Since in each equation one of the coefficient is larger than the other i.e. satisfying the condition for Gauss Seidel method we can write the given eqn in following form.

$$x = \frac{1}{5}[12 - 2y - z]$$

+1

$$y = \frac{1}{4}[15 - x - 2z]$$

$$z = \frac{1}{5}[20 - x - 2y]$$

Starting with $y = 0$ & $z = 0$ and using the most recent value we get

First iteration

$x^{(1)} = 2.4$
$y^{(1)} = 3.15$ +1
$z^{(1)} = 2.26$

Second iteration

$x^{(2)} = 0.688$
$y^{(2)} = 2.448$
$z^{(2)} = 2.8832$ +1

Third iteration

$x^{(3)} = 0.8442$
$y^{(3)} = 2.0974$
$z^{(3)} = 2.9922$

Fourth iteration

$x^{(4)} = 0.9626$ +1
$y^{(4)} = 2.0133$
$z^{(4)} = 3.0022$

Fifth iteration

$x^{(5)} = 0.9942$
$y^{(5)} = 2.0003$
$z^{(5)} = 3.0010.$ +1

Since fourth and fifth iteration are partially same hence solution of given system.

$x \simeq 1 \quad y \simeq 2 \quad z \simeq 3$

— x —

9 < (viii)

$$5x + 2y = 12 - 3 = 9$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

Equations are satisfying the condition for Gauss seidal method. i.e. in each eq^n one of coefficent is larger than the other.

$$x = \frac{1}{5}[9 - 2y]$$
$$y = \frac{1}{4}[15 - x - 2z]$$
$$z = \frac{1}{5}(20 - x - 2y)$$

First

$$x = \frac{1}{5}[9 - 2(0)] = 1.8$$

$$y = \frac{1}{4}[15 - 1.8 - 2(0)] = 3.3$$

$$z = \frac{1}{5}[12 - (1.8) - 2(3.3)] = \frac{3.6}{5}$$

$$= 0.72$$

second

$$x = \frac{1}{5}[9 - 2(3.3)] = 0.48$$

$$y = \frac{1}{4}[15 - 0.48 - 2(0.72)]$$

$$= \frac{13.08}{4} = 3.27$$

$$z = \frac{1}{5}[20 - 0.48 - 2(3.27)]$$

$$= 2.596$$

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$$x = \frac{1}{5}\left[9 - 2(3.271)\right] = \frac{2.458}{5}$$

$$= 0.4916$$

$$y = \frac{1}{4}\left[15 - (0.4916) - 2(2.596)\right]$$

$$= \frac{9.3164}{4} = 2.329]$$

$$z = \frac{1}{5}\left[20 - (0.4916) - 2(2.3291)\right]$$

$$= \frac{1}{5}(2014.8504) = 2.97008$$

— x —