ii. Define cyclic group and prove that every cyclic group is an abelian group. **5** 4 3 3

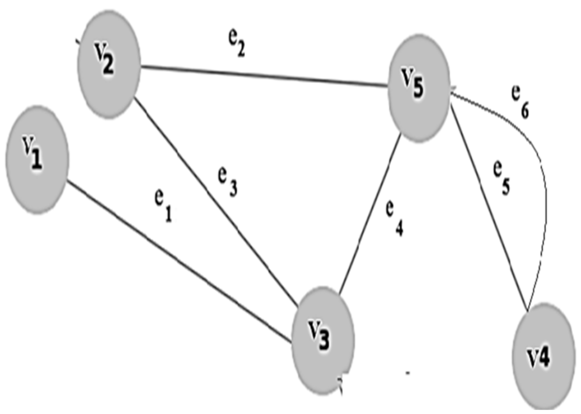
iii. Show that $\left(\{1,2,3,4,5,6,\},\times_7\right)$ is a cyclic group and 3 and 5 are the generators. **5** 3 2 2

Q.5 Attempt any two:

i. Define Complete graph, complete bipartite, planar, chromatic number for a graph, with examples. **5** 1 3 1

ii. Prove that in any graph, the number of vertices of odd degree is always even. **5** 3 1 2

iii. Write the incidence matrix of the following graph and mention various observations on it. **5** 4 5 3



Q.6 Attempt any two:

i. Solve the following recurrence relation- **5** 3 1 2

$$a_r - 5a_{r-1} + 6a_{r-2} = 5^r$$

ii. Prove that- **5** 3 1 2

$${}^{n}c_{r} = {}^{n-1}c_{r-1} + {}^{n-1}c_{r}.$$

iii. Determine the generating function of the numeric function a_r, **5** 3 1 2

$$a_r = \begin{cases} 2^r, & \text{if } r \text{ is even} \\ -2^r, & \text{if } r \text{ is odd} \end{cases}$$

******

Total No. of Questions: 6 *Total No. of Printed Pages:4*

Enrollment No......................................

Faculty of Engineering/Science
End Sem Examination Dec 2024
CS3BS04 / EC3BS02 / EE3BS02 / IT3BS01 / BC3BS05 Discrete Mathematics

Programme: B.Tech./B.Sc. Branch/Specialisation: CSE All/ EC/EE/IT/Computer Science

Duration: 3 Hrs. **Maximum Marks: 60**

Note: All questions are compulsory. Internal choices, if any, are indicated. Answers of Q.1 (MCQs) should be written in full instead of only a, b, c or d. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Marks BL PO CO PSO

Q.1 i. If A, B and C are the subsets of Universal set U then- **1** 2 3 1

$A-(B\cap C)=$______.

(a) $(A-B)-(B-C)$

(b) $(A-B)\cup(A-C)$

(c) $(A-B)\cap(A-C)$

(d) None of these

ii. If 18 pigeons are assigned to 5 pigeon holes then one of the pigeonholes most contain at least numbers of pigeons. **1** 3 2 2

(a) 4 (b) 3

(c) 5 (d) None of these

iii. If $(P,\leq)$ is a POSET then the relation $\leq$ satisfies- **1** 1 1 1

(a) Reflexive, symmetric, transitive properties.

(b) Reflexive, anti-symmetric, transitive properties

(c) Reflexive, asymmetric, transitive properties

(d) None of these

iv. The value of complete disjunctive normal form in 'n' variables is identically equals to- **1** 1 1 1

(a) One (b) Zero

(c) n (d) None of these

v. Which one of the following Algebraic structure is a abelian group- **1** 2 1 1

(a) $(\{-2,-1,0,1,2\},+)$

(b) $(I,-)$

(c) $(I,+)$

(d) None of these

vi. The generators of the cyclic group- **1** 2 3 1

$$\{a,a^2,a^3,a^4=e\}$$

(a) Only a (b) Only a^3

(c) Both (a) and (b) (d) None of these

vii. The following adjacency matrix **1** 2 3 1

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

represents the-

(a) Complete graph K_4

(b) Complement of Null Graph having four vertices

(c) Both K_4 and Complement of Null Graph having four vertices

(d) None of these

viii. The number of regions in the planar graph with 15 edges , 10 vertices is- **1** 3 2 2

(a) 5 (b) 7

(c) 3 (d) None of these

ix. How many numbers can be formed taking only 3 out of 5 digits 3, 4, 5, 6, 7? **1** 3 2 2

(a) ${}^5P_3$ (b) ${}^5C_3$

(c) ${}^5P_3 \text{ x } {}^5C_3$ (d) None of these

x. The generating function for $5.2^r, r \geq 0$ is- **1** 1 2 1

(a) $\dfrac{1}{1-5z}$ (b) $\dfrac{1}{1-2z}$

(c) $\dfrac{1}{1-z}$ (d) None of these

Q.2 Attempt any two:

i. Show that the relation- **5** 2 3 1

$R = \{(a,b) : a,b \in I$ and (a-b) is divisible by 3$\}$

is an equivalence relation.

ii. If A, B and C are three sets, prove that- **5** 2 3 1

$$(A-B)-C = A-(B \cup C).$$

iii. In a group of athletic team in a college, 21 are in basketball team, 26 are in hockey and 29 are in football team. If 14 play basketball and hockey; 12 play basketball and football; 15 play hockey and football; 8 paly all the three games. Find the number of player in all by using principle of inclusion & exclusion. **5** 3 2 2

Q.3 Attempt any two:

i. Define maximal element, minimal element, Least upper bound, Greatest lower bound in reference of POSET, with example of each. **5** 1 2 1

ii. State and prove De Morgan's law for Boolean Algebra$(B,\cup,\cap,',\phi,U)$, where U is the universal set. **5** 3 3 2

iii Express Boolean expression $F(x,y,z)$ into complete Disjunctive normal form. **5** 2 2 1

$$F(x,y,z) = (x+y'z)(y+z')$$

Q.4 Attempt any two:

i. If (G,o) be a group then prove that- **5** 4 3 3

(a) If $a,b,c \in G$ then $aob = aoc \Rightarrow b = c$

(b) $(aob)^{-1} = b^{-1}oa^{-1} \forall a,b \in G$

Faculty of Engg / Science

End Sem - Dec 2024

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Discrete Mathematics

Branch - CSE / EC / EE / IT / Comp Science

(10)

Qu. 1.
i. b) $(A-B) \cup (A-C)$
ii. a) ~~5~~ 4
iii. b). Reflexive, anti-symmetric, transitive properties
iv. a) one
v. c) $(I, +)$
vi. c) both (a) and (b)
vii. c) Both K_4 and complement of null graph having four vertices.
viii. b). 7
ix. a) ${}^5P_3$
x. d) None of these.

Qu. 2 (i) $R = \{(a,b) : a, b \in I$ & $(a-b)$ is divisible by $3\}$ is an equivalence relation if following properties are satisfied. +1

Reflexive property aRa

Let $a \in I$, $a-a=0$ is also divisible by 3.

i.e. $(a,a) \in R \quad \forall a \in I$.

Hence R is reflexive relation +1

Symmetric property $aRb \Rightarrow bRa$

If $a-b$ is divisible by 3, then $-(a-b)$ is also divisible by 3. [$(b-a)$ is divisible by 3]

i.e $(a,b) \in R \Rightarrow (b,a) \in R \quad \forall a,b \in I$

Hence R is symmetric relation +1

Transitive property $aRb, bRc \Rightarrow aRc$

If $a-b$ and $b-c$ are both divisible by 3, then $(a-b) + (b-c)$ is also divisible by 3.

$(a-c)$ is also divisible by 3.

i.e. $(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R \quad \forall a,b,c \in I$

Hence R is Transitive relation +1

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Now R is reflexive, Symmetric & transitive relation & hence R is an equivalence relation +1

(ii) · I $x \in (A-B)-C \Rightarrow x \in (A-B)$ and $x \notin C$

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow x \in A$ and $x \notin (B$ or $C)$

$\Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in A-(B \cup C)$ — (1)

$\therefore (A-B)-C \subseteq A-(B \cup C)$ +2

II

$x \in A-(B \cup C) \Rightarrow x \in A$ and $x \notin (B \cup C)$

$\Rightarrow x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $x \notin C$

$\Rightarrow x \in (A-B)$ and $x \notin C$

$\Rightarrow x \in (A-B)-C$

$\therefore A-(B \cup C) \subseteq (A-B)-C$ — (2) +2

By (1) & (2) $A-(B \cup C) = (A-B)-C$ +1

(iii). Let B, H, F denote the set of players of Basketball team, Hockey team & Football team respectively.

Given. $|B| = 21$, $|H| = 26$, $|F| = 29$

$|B \cap H| = 14$, $|B \cap F| = 12$, $|H \cap F| = 15$, $|B \cap H \cap F| = 8$. +1

We have to find $|B \cup H \cup F| = ?$

$\because |B \cup H \cup F| = |B| + |H| + |F| - |B \cap H| - |B \cap F|$

$- |H \cap F| + |B \cap H \cap F|$ +2

$= 21 + 26 + 29 - 14 - 12 - 15 + 8$ +1

$= 43$

There are 43 number of players in the group of athletic team. +1

(P)

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Qu. (3) (i) Maximal element | Marks

Let $(P, \leq)$ be a partially ordered set. An element m in P is said to be a maximal element if $m \leq x \Rightarrow m = x$ for $x \in P$ | +1

Minimal element

Let $(P, \leq)$ be a partially ordered set. An element n in P is said to be a minimal element if $x \leq n \Rightarrow n = x$ for $x \in P$ | +1

least upper bound

Let $(P, \leq)$ be a partially ordered set and $A \subseteq P$, If $u \in P$ is such an element that

$a \leq u \quad \forall\ a \in A$

Then u is called upper bound of A

If $u \leq v$ for all ~~all~~ upper bounds v of A then u is called the least upper bound or supremum of A. ~~and~~ | +1

Greatest lower bound

Let $(P, \leq)$ be a partially ordered set and $A \subseteq P$. If $l \in P$ is such an element that

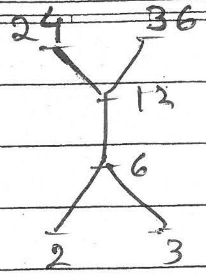
$l \leq a \quad \forall\ a \in A$

Then l is called lower bound of A.

If $m \leq l$ for all lower bounds of A then l is called the greatest lower bound or infimum of A. | +1

Eg. let $P = \{2, 3, 6, 12, 24, 36\}$ be a partially ordered set with the relation of divisibility. | +1

Marks



Then according to Hasse diagram
maximal element = 24, 36
minimal element = 2, 3
For $A = \{12, 24\}$
least upper bound = 24
Greatest lower bound = 12

(ii). For any two elements X and Y of Boolean algebra B, De Morgan's Laws are-

$(X \cup Y)' = X' \cap Y'$ and
$(X \cap Y)' = X' \cup Y'$ $\forall\ X, Y \in B$ +1

I To prove $(X \cup Y)' = X' \cap Y'$

It is sufficient to show $(X \cup Y) \cup (X' \cap Y') = U$
$(X \cup Y) \cap (X' \cap Y') = \phi$

$$(X \cup Y) \cup (X' \cap Y') = [X \cup Y \cup X'] \cap [X \cup Y \cup Y']$$
$$= [X \cup X' \cup Y] \cap [X \cup Y \cup Y']$$
$$= [U \cup Y] \cap [X \cup U]$$ +1
$$= U \cap U = U$$

$$(X \cup Y) \cap (X' \cap Y') = [X \cap X' \cap Y'] \cup [Y \cap X' \cap Y']$$
$$= [X \cap X' \cap Y'] \cup [Y \cap Y' \cap X']$$
$$= [\phi \cap Y'] \cup [\phi \cap X']$$
$$= \phi \cup \phi = \phi$$ +1

Hence $(X \cup Y)' = X' \cap Y'$

II To prove $(X \cap Y)' = X' \cup Y'$
It is sufficient to show $(X \cap Y) \cup (X' \cup Y') = U$
$(X \cap Y) \cap (X' \cup Y') = \phi$

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1. $(X\cap Y)\cup(X'\cup Y') = [X\cup X'\cup Y']\cap[Y\cup X'\cup Y']$
$= [X\cup X'\cup Y']\cap[Y\cup Y'\cup X']$
$= [U\cup Y']\cap[U\cup X']$ +1
$= U\cap U = U$

1. $(X\cap Y)\cap(X'\cup Y') = [X\cap Y\cap X']\cup[X\cap Y\cap Y']$
$= [X\cap X'\cap Y]\cup[X\cap Y\cap Y']$
$= [\phi\cap Y]\cup[X\cap\phi]$
$= \phi\cup\phi = \phi$ +1

Hence $(X\cap Y)' = X'\cup Y'$

(iii) $f(x,y,z) = (x+y'z).(y+z')$ +2
$= x.y + xz' + y'zy + y'zz'$
$= x\cdot y + xz' + 0 + 0$ $(aa'=0)$ $(a+0=a)$
$= x\cdot y\cdot 1 + x\cdot 1\cdot z'$ $(a\cdot 1=a)$
$= x\cdot y(z+z') + x(y+y')z'$ $(a+a'=1)$ +2
$= xyz + xyz' + xyz' + xy'z'$
$= xyz + xyz' + xy'z'$ $(a+a=a)$ +1

Qu. (4) (i) Given $(G,\circ)$ be a group.
Let $a\in G$ and e be the identity element. +1
$a\in G \Rightarrow a^{-1}\in G$ such that $a^{-1}\circ a\circ e = a\circ a^{-1}$
Now $a\circ b = a\circ c \Rightarrow a^{-1}\circ(a\circ b) = a^{-1}\circ(a\circ c)$
$\Rightarrow (a^{-1}\circ a)\circ b = (a^{-1}\circ a)\circ c$ (Associative law)
$\Rightarrow e\circ b = e\circ c$ $(a^{-1}\circ a = e)$
$\Rightarrow b = c$ +1.5

(ii). For $(a\circ b)^{-1} = b^{-1}\circ a^{-1}$
Let a^{-1} & b^{-1} are inverses of $a, b\in G$.
such that $a\circ a^{-1} = e = a^{-1}\circ a$
$b\circ b^{-1} = e = b^{-1}\circ e$ +1
where e is identity element

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As $(a_\circ b)_\circ(b^{-1}{}_\circ a^{-1}) = [a_\circ b_\circ b^{-1}]_\circ a^{-1}$
$= [a_\circ e]_\circ a^{-1}$
$= a_\circ a^{-1} = e$

$(b^{-1}{}_\circ a^{-1})_\circ(a_\circ b) = [b^{-1}{}_\circ a^{-1}{}_\circ a]_\circ b$
$= [b^{-1}{}_\circ e]_\circ b$
$= b^{-1}{}_\circ b. = e$ +1.5

we have $(a_\circ b)^{-1} = b^{-1}{}_\circ a^{-1}$ by the definition of inverse.

(ii). Cyclic group - A group $(G,\circ)$ is called cyclic if for $a \in G$, every element $x \in G$ is of the form a^n, where n is some integer. Symbolically $G = \{a^n \mid n \in I\}$

The element a is then called a generator of G. +2

let $G = \{a\}$ be a cyclic group, generated by a. let $x, y \in G$ arbitrarily then there exists integers r & s such that +1

$x = a^r$ & $y = a^s$.

$xy = a^r . a^s$
$= a^{r+s}$ (by law of indices)
$= a^{s+r}$
$= a^s a^r$
$= yx \implies \boxed{xy = yx}$ +2

Hence it is proved that cyclic group G is an abelian group.

Marks

1. iii) $G = (\{1, 2, 3, 4, 5, 6\}, \times_7)$

If there exists an element $a \in G$ such that $o(G) = o(a) = 6$, then the group G will be a cyclic group and 'a' will be a generator of G. +1

For element 3

$1 = 3^6 = 3^5 \times_7 3 = 5 \times_7 3$ [The identity element]

$2 = 3^2 = 3 \times_7 3$

$3 = 3^1$

$4 = 3^4 = 3^3 \times_7 3 = 6 \times_7 3$

$5 = 3^5 = 3^4 \times_7 3 = 4 \times_7 3$

$6 = 3^3 = 3^2 \times_7 3 = 2 \times_7 3$. +1

Therefore G is a cyclic group and 3 is a generator of G.

$$G = \{3^6, 3^2, 3, 3^4, 3^5, 3^3\}$$

According to the theorem –

"If a is a generator of a cyclic group G, then a^{-1} is also a generator of G" +1

We can say that there is one more generator exists for G, that is 5

Because 5 is inverse element of 3 +1

$$5 \times_7 3 = \frac{15}{7} = 1$$

Hence it is clear that for given cyclic group 3 and 5 are the generators. +1

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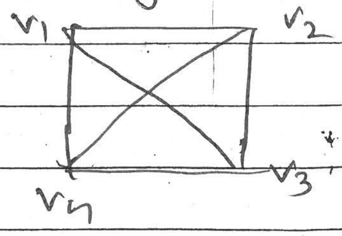
Que (5). (i). Complete graph-

Let (G = (V,E) be a graph. The graph G is called complete graph if every vertex in G is connected to every other vertex in G

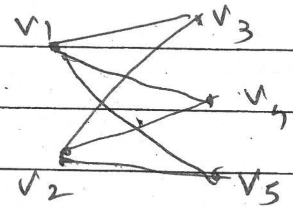


+ 1.25

Complete biparte graph- Let G (V,E) be a graph The graph G is called bipartite graph if its set V of vertices can be partitioned into two subsets H & S such that each edge of G connects a vertex of H to a vertex of S. If each vertex of H is connected to each vertex of S, then such a graph is called complete bipartite graph & denoted by $K_{m,n}$

$H = \{ v_1, v_2 \}$

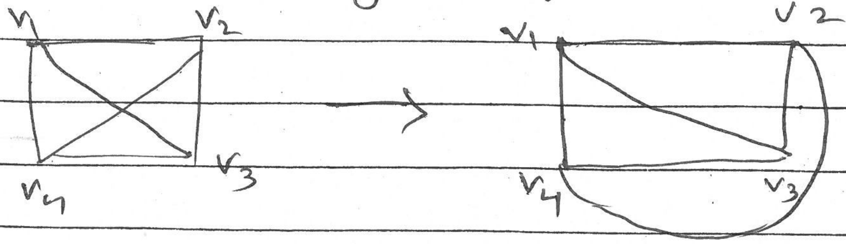
$S = \{ v_3, v_4, v_5 \}$



+ 1.25

Graph can be represent as $K_{2,3}$

Planar graph - A graph G is called planar if it can be drawn on a plane without intersecting edges.



This shows that the above graph is planar graph

+ 1.25

(5)

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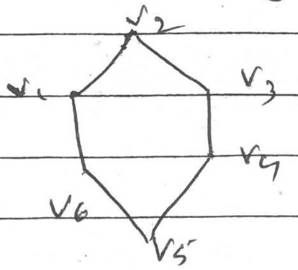
1. Chromatic number — The chromatic number of a graph G is the minimum number of colors required for proper coloring the vertices of G such that no two adjacent vertices have the same color. Chromatic number denoted by $\chi(G)$.



graph required minimum 2 colors for proper coloring. +1.25
i.e. $\chi(G) = 2$

(ii) . Let $G = (V, E)$ be a graph, where V is the set of vertices & E is the set of edges.
Let $V_e = \{$ vertices of even degree $\}$
$V_o = \{$ vertices of odd degree $\}$

i.e. $V_e \cap V_o = \phi$ +1

$$\because \sum_{v \in V} \deg(v) = \sum_{v \in V_o} \deg(v) + \sum_{v \in V_e} \deg(v)$$

$$2e = \sum_{v \in V_o} \deg(v) + 2K$$ +2

Because sum of degrees of all vertices in a graph is equal to twice of number of edges

Now $\sum_{v \in V_o} \deg(v) = 2e - 2K =$ an even number +1

To make the sum an even number, the number of terms in the sum must be even. Hence it is proved that the number of vertices of odd degree is always even +1

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(iii). Incidence matrix

$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ v_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_3 & 1 & 0 & 1 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_5 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}$$

+2

Observations are —

1. Sum of each row is equal to the degree of the corresponding vertex
2. Each column has exactly two 1's.
3. There is no isolated vertex as there exist no zero row.
4. In case of parallel edges, corresponding columns are identical.
5. Graph is connected
6. ~~No self loop is there~~

+3

As per observations (1) v_1 is pendant vertex,
As per observation (4) e_5 & e_6 are parallel edges

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Qu. 6 (i). $a_r - 5a_{r-1} + 6a_{r-2} = 5^r$

Sol Total sol $a_r = a_r^{(h)} + a_r^{(p)}$ — ①

Homogeneous sol

$$\frac{m^r - 5m^{r-1} + 6m^{r-2} = 0}{m^{r-2}}$$

$$m^2 - 5m + 6 = 0$$

$$m = 2, 3 \qquad +1$$

$$a_r^{(h)} = c_1 2^r + c_2 3^r \quad — ② \qquad +1$$

Particular sol

Required trial sol $a_r^{(t)} = A \cdot 5^r$ +1

Substitute in given eq

$$A \cdot 5^r - 5A \cdot 5^{r-1} + 6A \cdot 5^{r-2} = 5^r$$

$$A\left[6 \cdot 5^{r-2}\right] = 5^r \Rightarrow A = \frac{25}{6} \qquad +1$$

$$a_r^{(p)} = A \cdot 5^r = \frac{25}{6} 5^r = \frac{5^{r+2}}{6} \quad — ③ \qquad +1$$

By ① $a_r = c_1 2^r + c_2 3^r + \frac{5^{r+2}}{6}$

(ii). ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

By RHS. ${}^{n-1}C_{r-1} + {}^{n-1}C_r$

$$= \frac{(n-1)!}{(r-1)!\,(n-1-r+1)!} + \frac{(n-1)!}{r!\,(n-1-r)!} \qquad +1$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r\,(r-1)!\,(n-1-r)!} \qquad +1$$

$$= \frac{(n-1)!}{(r-1)!\,(n-1-r)!}\left[\frac{1}{n-r} + \frac{1}{r}\right] \qquad +1$$

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$$= \frac{(n-1)!}{(r-1)!\,(n-1-r)!}\left[\frac{1}{n-r}+\frac{1}{r}\right]$$

$$= \frac{(n-1)!}{(r-1)!\,(n-r-1)!}\left[\frac{\cancel{r}+n-\cancel{r}}{(n-r)\,r}\right]$$ +1

$$= \frac{n\,(n-1)!}{r(r-1)!\;(n-r)\;(n-r-1)!} = \frac{n!}{r!\,(n-r)!}$$ +1

$$= {}^{n}C_{r}$$

iii . $a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$

Generating function $A(z) = a_0 z^0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots$ +2

$a_0 = 1,\ a_1 = -2,\ a_2 = 2^2,\ a_3 = -2^3,\ a_4 = 2^4 \cdots$ +1

$A(z) = 1 - 2z + 2^2 z^2 - 2^3 z^3 + 2^4 z^4 - 2^5 z^5 \cdots$ +1

$$= (1+2z)^{-1} = \frac{1}{1+2z}$$ +1