Total No. of Questions: 6 *Total No. of Printed Pages:2*

Enrollment No......................................

Faculty of Pharmacy
End Sem Examination Dec-2023
PY3RC02 Remedial Mathematics

Programme: B. Pharm. Branch/Specialisation: Pharmacy

Duration: 3 Hrs. **Maximum Marks: 35**

Note: All questions are compulsory. Internal choices, if any, are indicated. Assume suitable data if necessary. Notations and symbols have their usual meaning.

Q.1	i.	According to the property of Logarithm $\log a + \log b =$________.	**1**
	ii.	The Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is called ________.	**1**
	iii.	What is the Differentiation of $\cos x$ with respect to x?	**1**
	iv.	Integral of $\sin x + 1 =$ ____.	**1**
	v.	The Order of a Differential equation $3\frac{d^2y}{dx^2} + \frac{dy}{dx} = 5$ is ______.	**1**
Q.2	i.	If $f(x) = x^3 - 5\sin^3 x + 6x$, then prove that $f(x)$ is an odd function.	**6**
OR	ii.	Resolve the following into partial fractions $\frac{x+2}{(x-1)(x-2)^2}$.	**6**
Q.3	i.	If $A = \begin{bmatrix} 1 & -5 \\ 0 & 8 \\ 7 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ -6 & 7 \end{bmatrix}$ then find: (a) $(A+B)'$ (b) $A' - B'$	**6**
OR	ii.	Find the determinant of following matrics: (a) $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 5 \\ 4 & 3 \end{bmatrix}$	**6**
Q.4	i.	Find $\frac{dy}{dx}$ if $y = x^2.e^x + 2x$ - 4.	**6**
OR	ii.	Find the Derivative of $y = 3x^3 + 2sinx - \frac{logx}{cosx}$ with respect to x.	**6**

Q.5	i.	Solve: $\int_1^2 (x + 3x^2 - 2)\, dx$.	**6**
OR	ii.	Find the Integral of $I = x.e^x + sinx$.	**6**
Q.6	i.	Solve: $\frac{dy}{dx} + \frac{y}{x} = x^3$.	**6**
OR	ii.	Find Laplace transform of $L\{cosat + t^3 - e^{3t}\}$.	**6**

******

Faculty of Pharmacy
End Sem Examination Dec. 2023 (1)
6|I PY3RC02 Remedial Mathematics

Q.1 (i) $\log a + \log b = \log(a.b)$

(ii) The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is calld

Diagonal Matrix

(iii) differentiation of $\cos x = -\sin x$

(iv) $\int(\sin x + 1)dx = -\cos x + x + c$

(v) The order of $3\frac{d^2y}{dx^2} + \frac{dy}{dx} = 5$ is

'2'

Q.2 (i) $f(x) = x^3 - 5\sin^3 x + 6x$

Prove that $f(x)$ is an odd function

$f(x) = x^3 - 5\sin^3 x + 6x$ +½

Put $x = -x$ +½

$= (-x)^3 - 5\sin^3(-x) + 6(-x)$ +1

$= (-1)^3 x^3 - (-1)5\sin^3 x - 6x$ +1

$= -x^3 + 5\sin^3 x - 6x$ +½

$$f(x) = -\left[x^3 - 5\sin^3 x + 6x\right]$$ +1

$$f(x) = -f(x)$$ +1

Hence $f(x)$ is an odd function +1/2

Q.2 (OR) (ii) $\dfrac{x+2}{(x-1)(x-2)^2}$

Using Partial fraction

$$\frac{x+2}{(x-1)(x-2)^2} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$ +1

$$x+2 = A(x-2)^2 + B(x-1)(x-2) + C(x-1) \quad \text{—} (1)$$ +1/2

Put $x=2$ in (1) +1/2

$$4 = A(0) + B(2-1)(2-2) + C(2-1)$$ +1/2

$$\boxed{4 = C}$$ +1/2

Put $x=1$ in (1) +1/2

$$1+2 = A(1-2)^2 + B(1-1)(1-2) + C(1-1)$$

$$3 = A(-1)^2$$ +1/2

$$\boxed{A = 3}$$

Now Put $x=0$ in (1) $+\frac{1}{2}$

$$0+2 = A(0-2)^2 + B(0-1)(0-2) + C(0-1)$$

$$2 = 4A + 2B - C$$

$$2 = 4 \times 3 + 2 \times B - 4 \qquad +\frac{1}{2}$$

$$-6 = 2B$$

$$\boxed{B = -3} \qquad +\frac{1}{2}$$

Hence

$$\frac{x+2}{(x-1)(x-2)^2} = \frac{3}{(x-1)} + \frac{-3}{(x-2)} + \frac{4}{(x-2)^2} \qquad +\frac{1}{2}$$

Q. 3 (i) If $A = \begin{bmatrix} 1 & -5 \\ 0 & 8 \\ 7 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ -6 & 7 \end{bmatrix}$

$(A+B)' = ?$ $\qquad A' - B' = ?$

$$A+B = \begin{bmatrix} 1 & -5 \\ 0 & 8 \\ 7 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 3 & 1 \\ -6 & 7 \end{bmatrix} \qquad +\frac{1}{2}$$

$$A + B = \begin{bmatrix} 5 & 0 \\ 3 & 9 \\ 1 & 2 \end{bmatrix} \qquad +1$$

$$(A+B)' = \begin{bmatrix} 5 & 3 & 1 \\ 0 & 9 & 2 \end{bmatrix} \qquad +1$$

$$A' = \begin{bmatrix} 1 & 0 & 7 \\ -5 & 8 & -5 \end{bmatrix} \qquad +1$$

$$B' = \begin{bmatrix} 4 & 3 & -6 \\ 5 & 1 & 7 \end{bmatrix} \qquad +1$$

$$A' \overset{-}{\cancel{+}} B' = \begin{bmatrix} 1 & 0 & 7 \\ -5 & 8 & -5 \end{bmatrix} + \begin{bmatrix} 4 & 3 & -6 \\ 5 & 1 & 7 \end{bmatrix} \qquad +1/2$$

$$A' \overset{-}{\cancel{+}} B' = \cancel{\begin{bmatrix} 5 & 3 & 1 \\ 0 & 9 & 2 \end{bmatrix}} \begin{bmatrix} -3 & -3 & 13 \\ -10 & 7 & -12 \end{bmatrix} \qquad +1$$

Q. 3 (OR) (ii)

If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 4 \\ 1 & 3 \end{vmatrix}$ +1

$= 2 \times 3 - 4 \times 1$ +1

$= 6 - 4$

$= 2$ +1

If $B = \begin{bmatrix} -1 & 5 \\ 4 & 3 \end{bmatrix}$

$|B| = \begin{vmatrix} -1 & 5 \\ 4 & 3 \end{vmatrix}$ +1

$= -3 - 4 \times 5$ +1

$= -3 - 20$

$= -23$ +1

Q. 4 (i) If $y = x^2 \cdot e^x + 2x - 4$

$\frac{dy}{dx} = ?$

differentiating y w.r.to x +1

$\frac{d}{dx}[y] = \frac{d}{dx}\left((x^2 \cdot e^x) + 2x - 4\right)$ +1

$= \frac{d}{dx} x^2 . e^x + \frac{d}{dx} 2x - \frac{d}{dx} 4$ +1

$= x^2 \frac{d}{dx} e^x + e^x \frac{d}{dx} x^2 + 2 \frac{d}{dx} x$ +1

$- \frac{d}{dx}(4)$

$= x^2 e^x + e^x . 2x + 2 - 0$ +1

$= x^2 e^x + e^x . 2x + 2$ +1

Q.4 (OR) (ii) $y = 3x^3 + 2\sin x - \frac{\log x}{\cos x}$

differentiating y w.r.to x +1

$\frac{d}{dx}(y) = \frac{d}{dx}\left(3x^3 + 2\sin x - \frac{\log x}{\cos x} \right)$ +1

$= 3\frac{d}{dx} x^3 + 2\frac{d}{dx} \sin x - \frac{d}{dx} \frac{\log x}{\cos x}$ +1

$= 3 \times 3x^2 + 2\cos x - \left[\frac{\cos x \frac{d}{dx} \log x - \log x \frac{d}{dx} \cos x}{(\cos x)^2} \right]$ +1

$$= 9x^2 + 2\cos x - \left[\frac{\cos x\left(\frac{1}{x}\right) - \log x(-\sin x)}{(\cos x)^2}\right] \quad +1$$

$$= 9x^2 + 2\cos x - \frac{\frac{\cos x}{x} + \log x\,(\sin x)}{\cos^2 x} \quad +1$$

Q.5 (i) Solve $\int_1^2 (x + 3x^2 - 2)\,dx$

$$= \int_1^2 x\,dx + 3\int_1^2 x^2\,dx - 2\int_1^2 1\,dx \quad +1$$

$$= \left[\frac{x^2}{2}\right]_1^2 + 3\left[\frac{x^3}{3}\right]_1^2 - 2\left[x\right]_1^2 \quad +1$$

$$= \left(\frac{2^2}{2} - \frac{1^2}{2}\right) + 3\left(\frac{2^3}{3} - \frac{1^3}{3}\right) - 2\left(2 - 1\right) \quad +1$$

$$= \left(\frac{4}{2} - \frac{1}{2}\right) + 3\left(\frac{8}{3} - \frac{1}{3}\right) - 2(1) \quad +1$$

$$= 3/2 + 3\left(\frac{7}{3}\right) - 2 \quad +1$$

$$= \frac{3}{2} + \frac{21}{3} - 2 \quad +1$$

Q.5 (OR) (ii) $I = x.e^x + \sin x$

$$\int (x.e^x + \sin x)\,dx$$ +1

$$\int x.e^x\,dx + \int \sin x\,dx$$ +1

$$\left\{ \int u.v\,dx = u\int v\,dx - \int\left(\frac{d}{dx}u \int v\,dx\right)dx \right\}$$ +1

$$= x\int e^x dx - \int\left(\frac{d}{dx}x \int e^x dx\right)dx + (-\cos x)$$ +1

$$= xe^x - \int 1.e^x dx - \cos x$$ +1

$$= xe^x - e^x - \cos x$$

$$= (x-1)e^x - \cos x$$ +1

Q.6 (i) Solve $\frac{dy}{dx} + \frac{y}{x} = x^3$

linear differential Equation of the type

$$\frac{dy}{dx} + py = Q$$

$p = \frac{1}{x}$, $Q = x^3$ +1

I.F. $e^{\int p\,dx}$ +1

$= e^{\int \frac{1}{x} dx}$

$= e^{\log x}$

$= x$ +1

Solution

$y \cdot I.F. = \int Q \cdot I.F.\, dx + c$ +1

$y \cdot x = \int x^3 \cdot x \, dx + c$ +1

$= \int x^4 \, dx + c$

$y \cdot x = \frac{x^5}{5} + c$ +1

Q. 6 (OR) (ii) $L\{\cos at + t^3 - e^{3t}\}$

$\Rightarrow L\{\cos at\} + L\{t^3\} - L\{e^{3t}\}$ +1

$L\{\cos at\} = \frac{s}{s^2+a^2}$ +1

$L\{t^n\} = \frac{n!}{s^{n+1}}$ +1

$L\{e^{at}\} = \frac{1}{s-a}$ +1

Date :
P. No. : 10

$$\Rightarrow \frac{s}{s^2+a^2} + \frac{3!}{s^{3+1}} - \frac{1}{s-3}$$ +1

$$\Rightarrow \frac{s}{s^2+a^2} + \frac{6}{s^4} - \frac{1}{s-3}$$ +1

— X — X —