# Question Set

Hitesh Kumar - 2019201039

November 24, 2020

**Assignment Quesstion**
Q. For every Boolean function $f$

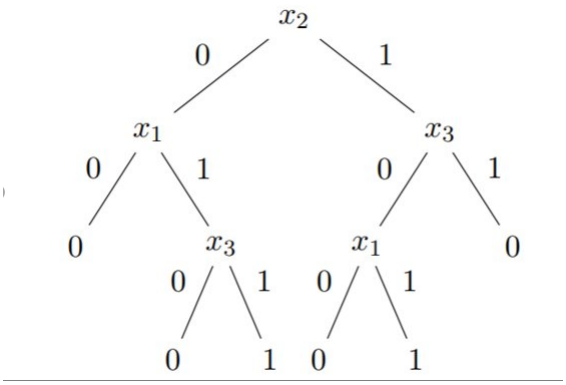
$$deg(f) \leq D(f)$$

The degree deg($f$) of $f$ is the degree of the unique multi-linear polynomial representation of $f$.
D($f$) is the decision tree complexity.

Ans: Fix a decision tree for $f$. For each leaf $\ell$ of the decision tree, define $P_l$ the polynomial which equals one on inputs that reach $\ell$ and 0 otherwise. That is, to form $P_l$ we multiply the terms $x_i$ for each internal node i for which we went "right" on the way to $\ell$ and $(1 - x_i)$ for each internal node i for which we went "left" on the way to $\ell$.

Now define P(x) := $\ell$ $T_l$ $P_l$(x), where $T_l$ is the label of the leaf $\ell$. This multi-linear polynomial P represents $f$ and has degree $max_\ell$ deg($P_l$(x)) = D($f$).

**LQ Question:**
Question: This decision tree correspond to which function?



**options:**

1. $x_1$ or $x_2$ and $x_3$
2. $x_1$ and $x_2$ and $x_3$
3. $x_1$ and $x_2$ or $x_3$
4. $x_1$ or $x_2$ or $x_3$

Ans: 3