PROJECT

MODERN COMPLEXITY THEORY

DECISION TREES – III

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DISTRIBUTIONAL COMPLEXITY

DEFINITION 6 *The* **distributional complexity** $d(A, \mathcal{D})$ *of algorithm* A *given inputs distributed according to* $\mathcal{D}$ *is defined as:*

$$d(A, \mathcal{D}) = \sum_{x:input} \mathcal{D}(x) cost(A, x) = \mathbf{E}_{x \in \mathcal{D}}[cost(A, x)]$$

- Randomized complexity – Distribution on Decision trees.
- Distributed complexity – Distribution on inputs.
- Analogous to Average case complexity.

DISTRIBUTIONAL DECISION TREE COMPLEXITY

DEFINITION 7 *The* **distributional** *decision tree complexity,* $\Delta(f)$ *of function* f *is defined as:*

$$\Delta(f) = \max_{\mathcal{D}} \min_{A} d(A, \mathcal{D})$$

- A runs over set of decision trees which are deciders for f.
- distributional decision tree complexity measures the expected efficiency of the most efficient decision tree algorithm works given the worstcase distribution of inputs.
- Yao's lemma relates randomized decision tree complexity to this.

DISTRIBUTIONAL DECISION TREE COMPLEXITY

For all computational models in which both the set of inputs and the set of algorithms is finite, for any function f,

$$R(f) = \Delta(f).$$

- If we prove the lower bound for the Δ(f), it suffices the proof of lower bound of R(f).
- Yao's lemma is a version of Von neumann's minmax lemma.

SENSITIVITY

Another way of proving the lower bounds of decision tree complexity.

If $f : \{0, 1\}^n \rightarrow \{0, 1\}$ is a function and $x \in \{0, 1\}^n$, then the sensitivity of f on x, denoted $s_x(f)$, is the number of bit positions i such that $f(x) = f(x_i)$, where x_i is x with its i^{th} bit flipped. The sensitivity of f, denoted by s(f), is $\max_x\{sx(f)\}$.

SENSITIVITY

Examples:

- The Parity function has sensitivity n.
- The AND-of-OR function has sensitivity k when the input size is k^2.

BLOCK SENSITIVITY

The block sensitivity of f on x, denoted $bs_x(f)$, is the maximum number b such that there are disjoint blocks of bit positions $B_1, B_2, \ldots, B_b$ such that $f(x) = f(x^{Bi})$ where x^{Bi} is x with all its bits flipped in block B_i. The block sensitivity of f denoted bs(f) is $\max_x \{bs_x(f)\}$.

- 1) For any function, $s(f) \leq bs(f)$.
- 2) $C(f) \leq s(f)bs(f)$.

DEGREE METHOD

Definition 12.13. An n-variate polynomial $p(x_1, x_2, \ldots, x_n)$ over the reals represents $f : \{0, 1\}^n \rightarrow \{0, 1\}$ if $p(x) = f(x)$ for all $x \in \{0, 1\}^n$. The degree of f, denoted deg(f), is the degree of the multiline.

- The AND of n variables $x_1, x_2, \ldots, x_n$ is represented by the multilinear polynomial $\pi^n{}_{i=1} x_i$ and OR is represented by $1 - \pi^n{}_{i=1}(1 - x_i)$.
- The degree of AND and OR is n, and so is their decision tree complexity. In fact, $\deg(f) \leq D(f)$ for very function f.

THANK YOU