Question Set

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Assignment Quesstion
Q. For every Boolean function f

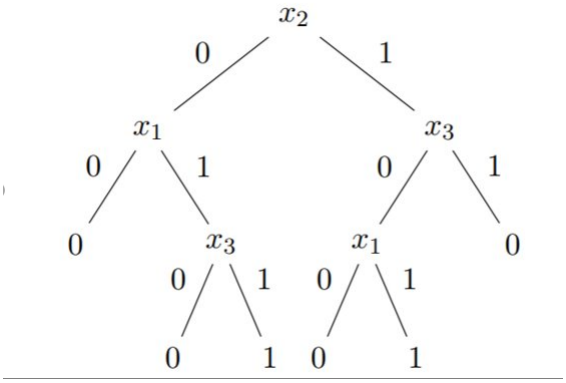
$$deg(f) \leq D(f)$$

The degree deg(f) of f is the degree of the unique multi-linear polynomial representation of f.
D(f) is the decision tree complexity.

Ans: Fix a decision tree for f. For each leaf ℓ of the decision tree, define P_l the polynomial which equals one on inputs that reach ℓ and 0 otherwise. That is, to form P_l we multiply the terms x_i for each internal node i for which we went "right" on the way to ℓ and $(1 - x_i)$ for each internal node i for which we went "left" on the way to ℓ.

Now define P(x) := ℓ T_l P_l(x), where T_l is the label of the leaf ℓ. This multi-linear polynomial P represents f and has degree max_ℓ deg(P_l(x)) = D(f).

LQ Question:



options:

1. x_1 or x_2 and x_3
2. x_1 and x_2 and x_3
3. x_1 and x_2 or x_3
4. x_1 or x_2 or x_3

Ans: 3