# Lecture 28: Review

## Reading: All chapters in ISLR.

**STATS 202: Data mining and analysis**

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October 29, 2018

# Announcements

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- Where to submit your course evaluations?

  http://axess.stanford.edu

  Under the Student tab.

# Unsupervised learning

- In unsupervised learning, all the variables are on equal standing, no such thing as an input and response.

- **Two sets of methods:**
  1. PCA: find the main directions of variation in the data
  2. Clustering: find meaningful groups of samples
     - Hierarchical clustering (single, complete, or average linkage).
     - $K$-means clustering.

# PCA

1. Find the linear combination of variables

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- ▶ What is the proportion of variance explained? A scree plot?
- ▶ What is the effect of rescaling variables?

# $K$-means clustering

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- The algorithm starts from some assignment, and is guaranteed to decrease this average distance.

- This find a local minimum, not necessarily a global minimum, so we typically repeat the algorithm from many different random starting points.

# Hierarchical clustering



Average Linkage    Complete Linkage    Single Linkage

- Agglomerative algorithm produces a *dendrogram*.

- At each step we join the two clusters that are "closest":

    - **Complete:** distance between clusters is maximal distance between any pair of points.

    - **Single:** distance between clusters is minimal distance.

    - **Average:** distance between clusters is the average distance.

- Height of a branching point = distance between clusters joined.
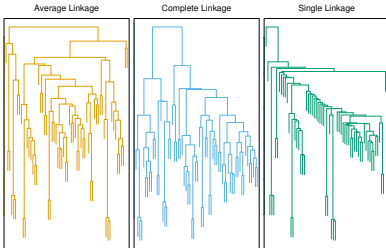
# Supervised learning

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Two classes of problem:

- Regression: $y_i$ is numerical

$$\mathsf{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

- Classification: $y_i$ is categorical

$$0 - 1 \text{ loss} = \sum_{i=1}^{n} \mathbf{1}(y_i \neq \hat{y}_i).$$

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We want to minimize the error on a very large test set which is sampled from the same process as the training data. This is called the *test error*.

# Bias-variance decomposition

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The expected test MSE of $\hat{f}$ has the following decomposition for any fixed $x$:

$$E([\hat{f}(x) - f(x)]^2) = \underbrace{E([\hat{f}(x) - E\hat{f}(x)]^2)}_{\mathsf{Var}(\hat{f}(x)) > 0} + \underbrace{[E(\hat{f}(x) - f(x))]^2}_{\text{Square bias of } \hat{f}(x). > 0} + \mathsf{Var}(\epsilon)$$

**Variance**: Increases with the flexibility of the model
**Bias**: Decreases as the flexibility of the model increases

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- No approach is superior to all others.

- What are the main differences? How do the bias and variance of the test error estimates compare? Which methods depend on the random seed?

# The Bootstrap

- **Main idea:** If we have enough data, the empirical distribution is similar to the actual distribution of the data.

- Resampling with replacement allows us to obtain pseudo-independent datasets.

- They can be used to:

  1. Approximate the standard error of a parameter (say, $\beta$ in linear regression), which is just the standard deviation of the estimate when we repeat the procedure with many independent training sets.

  2. **Bagging**: By averaging the *predictions* $\hat{y}$ made with many independent data sets, we eliminate the variance of the predictor.

# Regression methods

- Nearest neighbors regression

- Multiple linear regression

- Stepwise selection methods

- Ridge regression and the Lasso

- Principal Components Regression

- Partial Least Squares

- Non-linear methods:

    - Polynomial regression

    - Cubic splines

    - Smoothing splines

    - Local regression

    - GAMs: Combining the above methods with multiple predictors

- Decision trees, Bagging, Random Forests, and Boosting

# Classification methods

- Nearest neighbors classification

- Logistic regression

- LDA and QDA

- Stepwise selection methods

- Decision trees, Bagging, Random Forests, and Boosting

- Support vector classifier and support vector machines

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5. How does rescaling or transforming the variables affect the method?

6. In what situations does this method work well? What are its limitations?