Lecture 27: Review

Reading: All chapters in ISLR.

STATS 202: Data mining and analysis

December 6, 2017

Announcements

Final exam:

- Tuesday, December 12, 8:30-11:30 am, in the following rooms:
  Last names A-L: Skilling Auditorium (here)
  Last names M-S: Gates B03
  Last names T-Z: Thornton 201.

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- Closed book, closed notes. No calculators or computers.
- The final covers everything we did in class except non-linear dimensionality reduction, missing data, and learning from relational data. In other words, everything in the book.
- Practice problems and practice final on the website.

Announcements

The Kaggle project is due this Thursday - upload Homework 8 by 9am.
There is no class Friday. Instructor and TAs will have their usual office hours. Instructor is out of town Friday – Monday.

Unsupervised learning

- In unsupervised learning, all the variables are on equal standing, no such thing as an input and response.
- **Two sets of methods:**
  1. PCA: find the main directions of variation in the data
  2. Clustering: find meaningful groups of samples
     - Hierarchical clustering (single, complete, or average linkage).
     - K-means clustering.

PCA

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$$\theta_{11}X_1 + \theta_{12}X_2 + \cdots + \theta_{1p}X_p$$

with $\sum_i \theta_{1i}^2 = 1$, which has the largest variance.

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- What is a biplot, how is it interpreted?
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- What is the effect of rescaling variables?

K-means clustering

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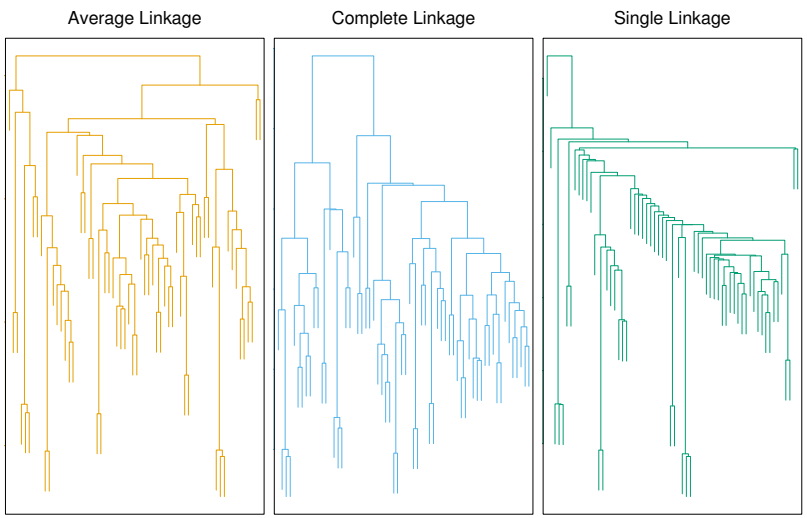
- This find a local minimum, not necessarily a global minimum, so we typically repeat the algorithm from many different random starting points.

Hierarchical clustering



- Agglomerative algorithm produces a *dendrogram*.
- At each step we join the two clusters that are "closest":
  - **Complete:** distance between clusters is maximal distance between any pair of points.
  - **Single:** distance between clusters is minimal distance.
  - **Average:** distance between clusters is the average distance.
- Height of a branching point = distance between clusters joined.

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Two classes of problem:

- Regression: y_i is numerical

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- Classification: y_i is categorical

$$0-1 \text{ loss} = \sum_{i=1}^{n}\mathbf{1}(y_i \neq \hat{y}_i).$$

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Both the MSE for regression, and the 0-1 loss for classification can be computed:

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We want to minimize the error on a very large test set which is sampled from the same process as the training data. This is called the *test error*.

Bias-variance decomposition

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The expected test MSE of $\hat{f}$ has the following decomposition for any fixed x:

$$E([Y - \hat{f}(x)]^2) = \underbrace{E([\hat{f}(x) - E\hat{f}(x)]^2]}_{\text{Var}(\hat{f}(x)) > 0} + \underbrace{[E(\hat{f}(x) - f(x))]^2}_{\text{Square bias of } \hat{f}(x). > 0} + \text{Var}(\epsilon)$$

Variance: Increases with the flexibility of the model
Bias: Decreases as the flexibility of the model increases

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- No approach is superior to all others.
- What are the main differences? How do the bias and variance of the test error estimates compare? Which methods depend on the random seed?

The Bootstrap

- **Main idea:** If we have enough data, the empirical distribution is similar to the actual distribution of the data.
- Resampling with replacement allows us to obtain pseudo-independent datasets.
- They can be used to:
  1. Approximate the standard error of a parameter (say, β in linear regression), which is just the standard deviation of the estimate when we repeat the procedure with many independent training sets.
  2. **Bagging**: By averaging the *predictions* $\hat{y}$ made with many independent data sets, we reduce the variance of the predictor.

Regression methods

- Nearest neighbors regression
- Multiple linear regression
- Stepwise selection methods
- Ridge regression and the Lasso
- Principal Components Regression
- Partial Least Squares
- Non-linear methods:
  - Polynomial regression
  - Cubic splines
  - Smoothing splines
  - Local regression
  - GAMs: Combining the above methods with multiple predictors
- Decision trees, Bagging, Random Forests, and Boosting

Classification methods

- Nearest neighbors classification
- Logistic regression
- LDA and QDA
- Stepwise selection methods
- Decision trees, Bagging, Random Forests, and Boosting
- Support vector classifier and support vector machines

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4. How is the method related to other methods, mathematically and in terms of bias, variance?
5. How does rescaling or transforming the variables affect the method?
6. In what situations does this method work well? What are its limitations?