Date 20/1/2020
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part of D.M.

Why do we need G.T in C.S.

→ Graph theory is a wide topic that one study in D.M | D.S (BFS & DFS) | How to store a graph in a computer in a $\overline{m}$ | discussed in algo (minimum spanning tree | prim's | Kruskal, Dijkstra etc: deals with shortest path).

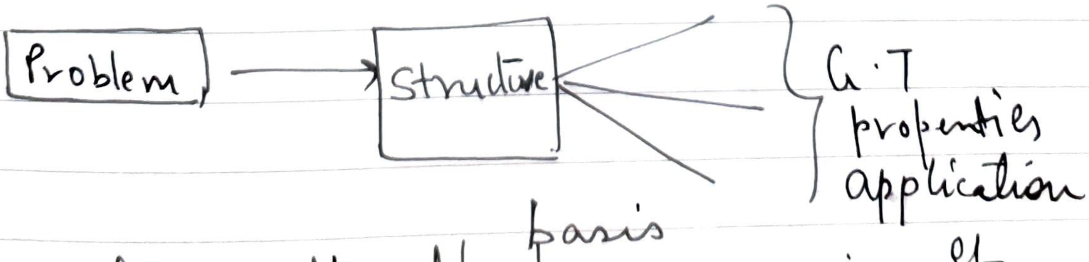
→ Used in Research as Tools ~~like~~ | models | structure | graph.

Based on app. Oriented: D.S, DBMS, Compiler, C.N

→ Problem is reduced in graph form, then we apply graph theory properties to solve it | prove it.

→ G.T has Top Priority as Pictorial rep^n whether coloring | connectivity & so on.

→ App on social n/w sites | e-commerce websites etc.



→ Mathematically basis we require it.

→ Asked in Interview.

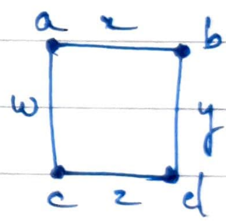
GRAPH THEORY :

diagram of points & lines connected to pts.

→ Pictorial representation of certain objects
→ May be these objects have pairs & are connected by links.
→ Objects are represented by points K/a as vertices(V) & the link connecting it is K/a edges(E)

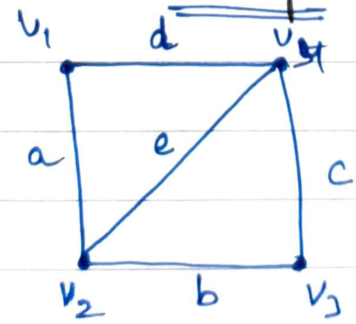
The definition is :
Graph is a pair of sets $G(V, E)$ where
structure

V is the set of vertices or node and
E is the set of edges.
$V\ (a, b, c, d)$
$E\ (x, y, z) \mid (ab, bd, cd, ac)$



NOTE :
Each edge has one/two vertices associated with it called its endpts.

Eg.



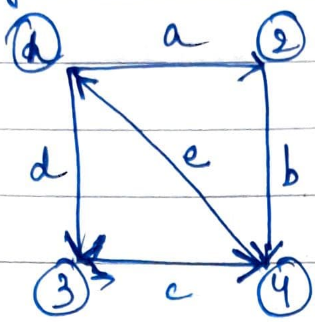
$V = \{v_1, v_2, v_3, v_4\}$; $E = \{a, b, c, d, e\}$

$\therefore G = (V, E)$.

* Two Major Kinds of Graph exists

Directed graphs

- $E(v_i, v_j)$
edge have direction



$a = (1,2)$ if directed edge can be represented by a pair of vertices.
as edge has start & end pt

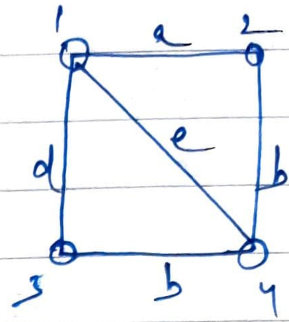
Start & end pt can be determined by direction

→ ORDERED PAIR of Vertices as graph has direction

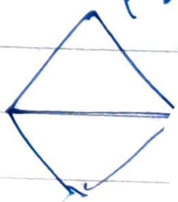
$a = (2,1)$ is not same.
So only saying abt edge is not enough.

Undirected graph.

$E(v_i, v_j)$



a $(1,2)$
$(2,1)$
Edge has no direction
Some relationship doesnt require direction always
Eg. Map | Simple when required



→ UNORDERED PAIR of Vertices.

Eg: In civil engg.
Open channel we have flow of water has to be shown.

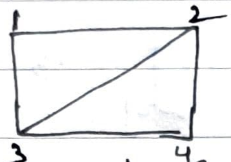
NOTE: → Undirected graph Examples will be taken throughout.

Adjacent Vertex:

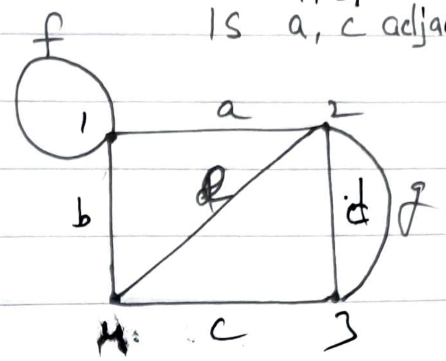
If two vertex are joined by the same edge they are called Adjacent vertex.

Is a, b / a, c adjacent

or



Is (1,4) / (3,2) adjacent ?!



Adjacent Edge:

If two ~~verti~~ edges are incident on same vertex they are called Adjacent edge.

Self Loop:

Edge having same vertex (v_1, v_2) as its end vertices they are known as Self loop.

→ end pts are same

Parallel Edge:

When we have more than one edge associated with a given pair of vertices such edges are called Parallel edges.

Parallel & adjacent different.

Two vertex with Two edge is an example of Parallel edge.

eg. d, g are not adjacent although it has common pt but they are having two common pt not one.

Scenario's

→ Google Map.

Two cities but have two/more paths to reach.

Multiple edges have be taken.

	Self loop	Parallel edges
Graph	✓	✓
Multi-graph	✗	✓
Pseudo graph	✓	✗
Simple graph	✗	✗

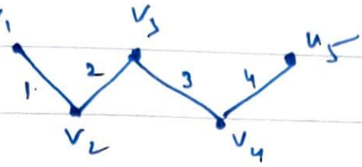
→ undirected graph applied to directed also

90% of time we will be talking about Simple graph when no Parallel edge or Self loop is not applied.

To decide category of graph.

Case Simple

Finite graph: A graph with no. of vertices as well as the finite no. of edges is called a finite graph.

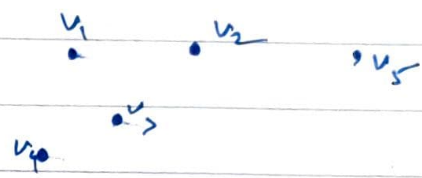


→ finite no. of vertices
" " " edges.

Null graph: A graph where vertex set is non-empty but edge set is empty is called Null graph

Eg. $V = \{v_1, v_2, v_3 \ldots v_n\}$

$E = \phi$



→ Graph with no edge.

e.g

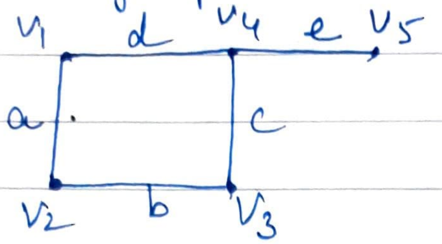
Trivial Graph: A graph where vertex set contains only one vertex and edgeset is empty is called Trivial graph.

→ <u>Smallest graph</u> possible.

Q. Draw a graph with '5' vertices & '7' edges.

Q Draw a simple with 5 vertices & 7 edge.

SIMPLE GRAPH: It is graph in which each edge connects two different vertices and where no two edges connects the same pair of vertices. OR A graph that has neither self nor parallel edge is called Simple graph.



Multi Graph: Graph may have multiple edges connecting with same vertices.
It can have some parallel edges.

Pseudo Graph: Graph that includes ~~have~~ ~~has~~ LOOPS and possibly multiple edges connecting same pair of vertices.

Case of Simple graph

Infinite Graph. A graph infinite no. of vertices as well as infinite no. of edges is called Infinite Graph.

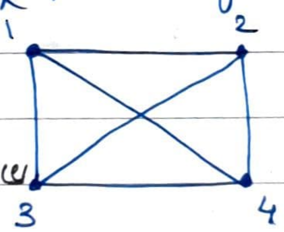
Ask?
Is vertices depend on edge or edge depend on vertices.

* For an existence a graph atleast <u>one vertex</u> is required.

* Without existence of edges, vertex existence is possible. but without existence of vertex, existence of edge not possible. $\boxed{v \propto e}$

Complete graph: A graph is said to be complete if there is an edge b/w every pair of vertex (K_n)

K_4 ↓ means 4 vertices

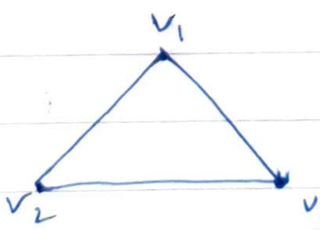


Largest graph

With no self loop & ||el edge

Q. Draw K_3

→ Means Complete graph, having 3 vertices & has a structure of triangle.



Q If a complete graph has 'n' vertices. Determine no. of edges.

Total no. of vertex

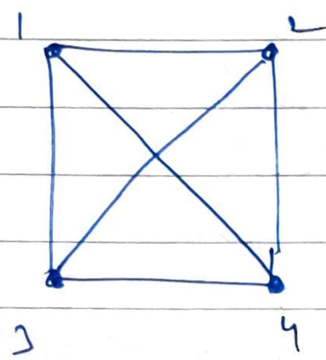
Every vertex has (n-1) no. of edges.

Total no. of edges = $n \times (n-1) / 2$

As counting vertex twice (1,2) | (2,1)

Conclusion: If a graph has 'n' no. of vertex then $\frac{n(n-1)}{2}$ edges will exist.

Eg.



$n = 4$.

$\therefore \frac{4(3)}{2} = 6$ edges.

Here no. of vertex = 4 i.e finite & no. of edges = 6 i.e also finite.

In the domain of simple graph, where no self loop & ||lel edge exists.

Regular Graph: Degree of each vertex is Same.

$v_1 = 2$
$v_2 = 2$
$v_3 = 2$

Types of graph.

1. Simple graph
2. Multi graph
3. Psuedo graph
4. Complete graph
5. Null graph
6. Regular graph.
7. Finite graph
8. Infinite graph
9. Trivial graph
10. Graph.

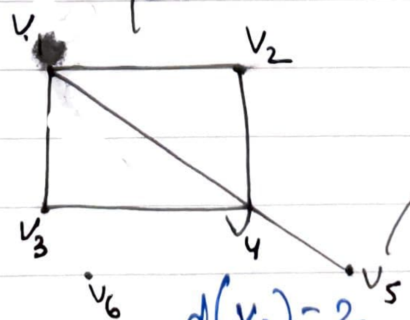
DEGREE OF VERTEX (v_i) → Undirected graph.

The no. of edges incident on vertex v_i with self-loop counted twice is called "degree of vertex". denoted by $d(v_i)$

Means how many edges is connected to an vertex.

Degree is always calculated of vertex not edge.

Eg.

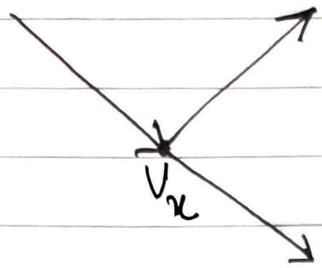


Pendant vertex

$d(v_1) = 3$
$d(v_2) = 2$
$d(v_3) = 2$
$d(v_4) = 4$
$d(v_5) = 1.$
$d(v_6) = 0$

Case : — Directed graph.

Q



No. of :

→ In degree (Edge coming towards vertex)

→ Out degree → Edge going away from vertex.

In : $d^+(V_x) = 1$

Out : $d^-(V_x) = 2$

NOTE :-

Eg: V_5 is <u>Pendant Vertex</u>.

A vertex of <u>degree 1</u> is K/a as Pendant vertex.

V_6 is Isolated Vertex

Vertex having degree 0.

depends on edge connection

HAND SHAKING THEOREM / SUM OF DEGREE THEOREM. [Pg. 144, S.B. Singh]

$$\sum_{i=1}^{n} d(v_i) = 2|E| \quad 2E$$

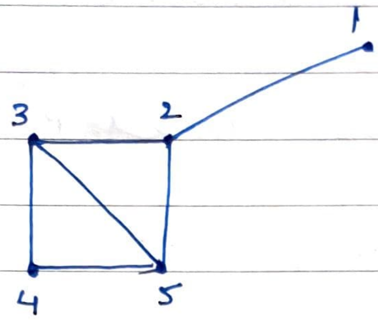
always even either case of even or odd.

$$\sum_{i=1}^{n} d(v_i) = \sum_{even} d(v_i) + \sum_{odd} (v_k).$$

Eg.

(1)

To prove Theorem 1.

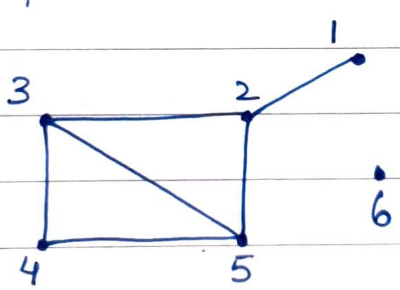


Total no. of degree = 1+3+3+2+3 = 12

Twice the no. of edge i.e 2(6).

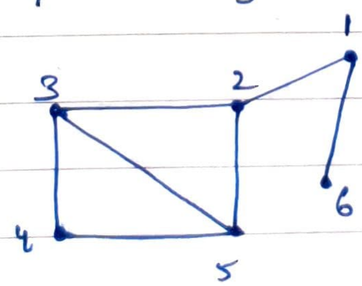
Hence proved.

(2)



No Affect on degree / Total sumation of degree.

(3)



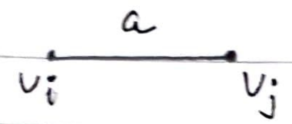
Addition of edge will affect degree of (1) & (6) and also on sumation of degree

NOTE: Hence degree is Calculated of vertex but highly dependent on edge

Creation of Edge

→ Contributed by two degree from v_i & v_j

Eg.



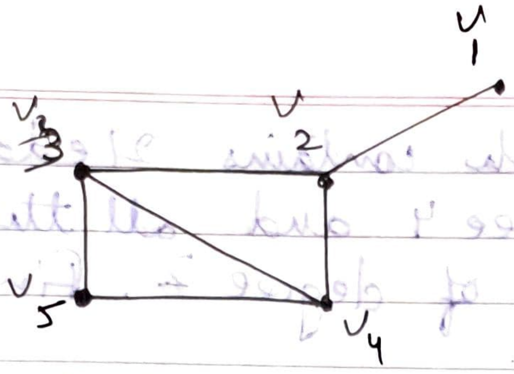
Edge created $= +2$ degree of vertices

Edge removed $= -2$ degree of vertices

Hence Proved (Theorem 1)

To Proof Theorem 2:

Eg.



To proof theorem 2.

(1) Find degree of vertex when it is even & odd.

$$\sum_{i=1}^{n} d(v_i) + \sum_{i=1}^{n} d(v_k)$$

result — even. even — odd. even.

For $d(v_k)$ =

$d(v_1) = 1$

$d(v_3) = 3$

$d(v_4) = 3$

$d(v_2) = 3$

for $d(v_i)$ =

$d(v_5) = 2$

no. of vertex with even degree results in even

$2+2=4$

$3+3=6$ even = even

$3+3+3 = 9$ odd vertices odd

NOTE:

No. of vertices with odd degree in a graph is always even.

Sumation of even degree results in even. $2+4+6=10$ $2+2+2+2=8$	odd time sum of odd result in odd. $3+3+3=9$ even time sum of odd no is even $3+3=6$

classmate

Date

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Q A graph contains 21 edges & 3 vertices of degree 4 and all the other vertices of degree 2. Find total no. of vertices.

$|E| = 21$.

$d(v_i) = 4$. — Suppose 'n' no. of vertices.

$$\sum_{i=1}^{n} d(v_i) = 2|E|.$$

$$3(4) + (n-3)2 = 2(21$$

$$12 + 2n - 6 = 42.$$

Ans

Q. A simple graph G has 24 edges & degree of each vertex is 4. What is the no. of vertices.

let 'x' vertices be there in a simple graph.

$$4x = 2(24)$$

$$x = \frac{\cancel{2}(24)^{12}}{4\cancel{2}}$$

$$x = 12$$

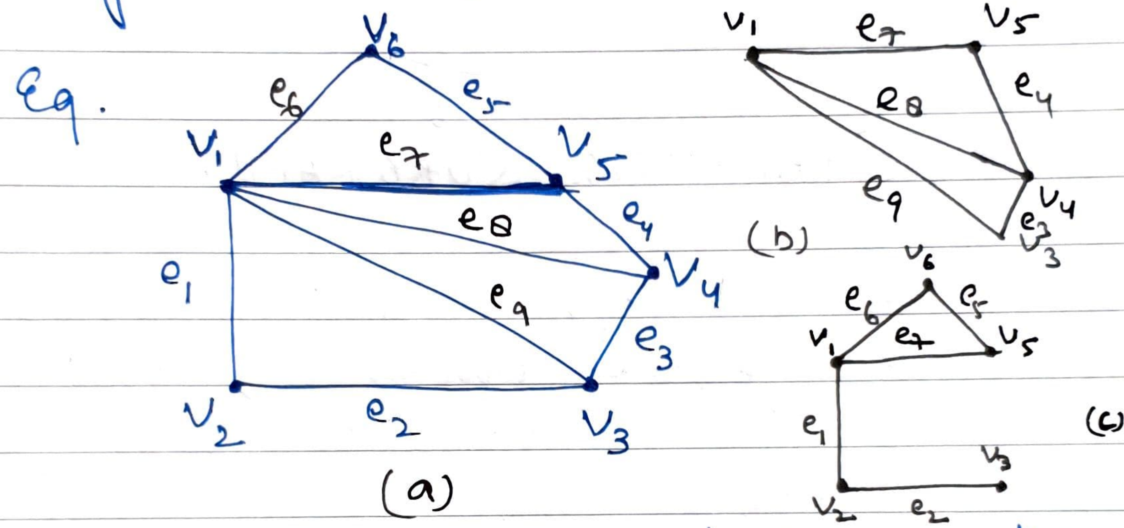
Ans

Similar to Subset of Set Theory

SUB GRAPHS (IN GRAPH)

Consider a graph $G = (V, E)$.
A graph ~~~~ $g = (v, e)$ is said to be a subgraph of G if e is a subset of E and v is a subset of V such that edges in 'e' are incident only with the vertices in v.

Eg.



A subgraph can be contained in another graph.

Following Observation is made:

* ~~A subgraph~~ A Graph G is a subgraph itself.

* A subgraph of a subgraph of G is a subgraph of G.

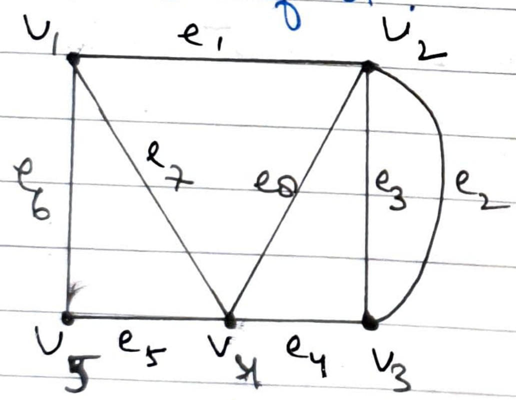
* A single vertex in a graph G is a subgraph of G.

* A single edge in G together with its end vertices is also a subgraph of G.

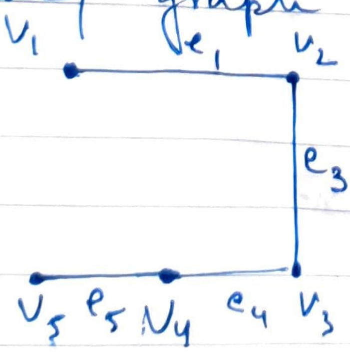
$G \subseteq G'$.

SPANNING SUBGRAPH.

A subgraph of 'G' is said to be a spanning subgraph if it contains all the vertices of G

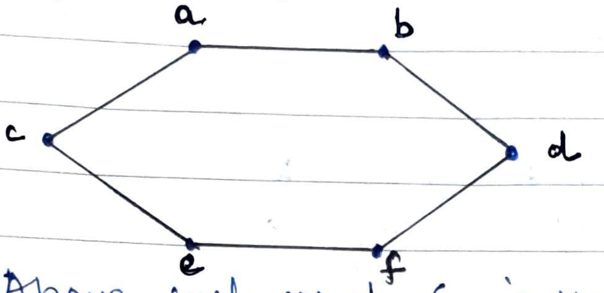


Spanning graph will be



Cycle Graph: A connected graph whose edges form a cycle of length 'n' is called a Cycle graph of order 'n'. Cycle graph are denoted by C_n.

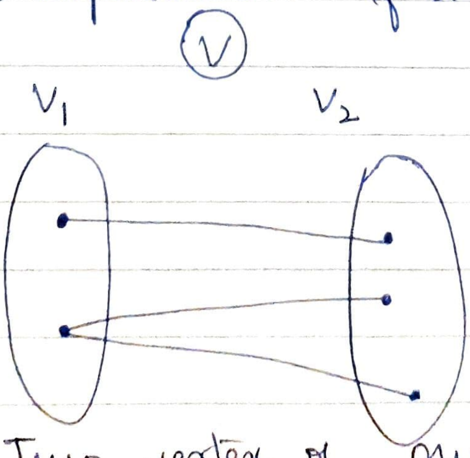
Graph of class C_6 is shown as:



Note: Above cycle graph C_6 is regular of degree 2

Bipartite graph: A graph $G(V, E)$ is called Bi-partite if its vertex set $V(G)$ can be partitioned into two non-empty disjoint subsets $V_1(G)$ and $V_2(G)$ in such a way that each edge $e \in E(G)$ has its one end point in $V_1(G)$ and other end point in $V_2(G)$, the partition $V = V_1 \cup V_2$ called Bipartition of G.

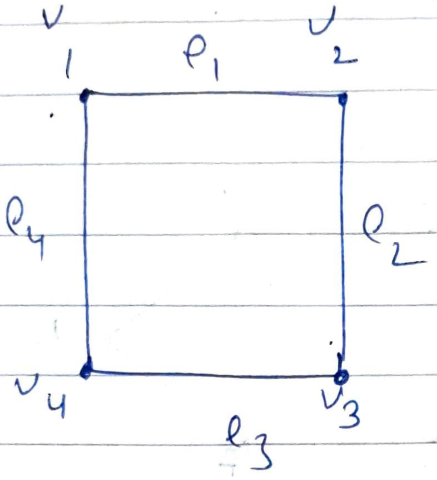
Eg.



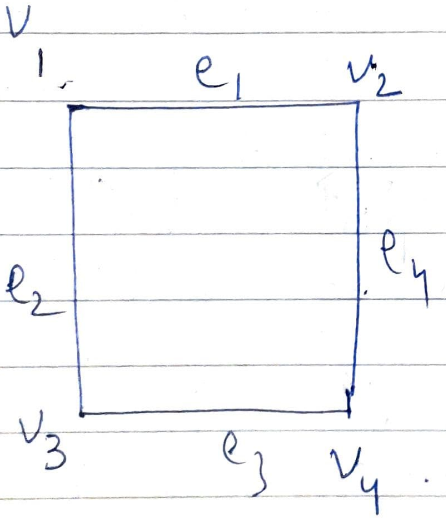
NOTE:- Two vertex of one subset should not be adjacent or should not have edge b/w it.

(ii) Check whether following graph is a Bipartite graph or not.

(a)



(b)

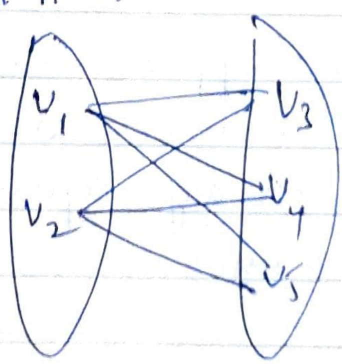


Complete Bipartite Graph :

* It has to be Bipartite graph ($\overset{V}{\wedge}_{V_1 V_2}$)
* Denoted by $K_{m,n}$. (eg. $K_{2,3}$)
* It ~~was~~ should has complete structure means every vertex of one subset has to connected to every vertex of another subset then it is a Complete Bipartite graph.

$|V_1| = m$ $|V_2| = n$

Eg.



then $K_{m,n}$.