Normalizing Constant

Let's say you have a function like e^{-x^2}. This is not a valid Probability Density Function (PDF) because it doesn't satisfy the following property:

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

because:

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

Now, how do we get a function that has the same parameters as our original function, like mean and variance, and the densities of the input values, relative to each other, is unchanged? Well, wouldn't dividing $f(x)$ by $\int_{-\infty}^{\infty} f(x)dx$ do it? That is:

$$g(x) = \frac{f(x)}{\int_{-\infty}^{\infty} f(x)dx}$$

In our case:

$$g(x) = \frac{e^{-x^2}}{\sqrt{\pi}}$$

And, obviously:

$$\int_{-\infty}^{\infty} \frac{e^{-x^2}}{\sqrt{\pi}} dx = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-x^2} dx = \frac{1}{\sqrt{\pi}} \cdot \sqrt{\pi} = 1$$

Thus, scaling $f(x)$ with the reciprocal of $\int_{-\infty}^{\infty} f(x)dx$ does the job. Look at the graphs of the function with and without normalization for our example: