## Black holes and information theory

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During the past three decades investigators have unveiled a number of deep connections between physical information and black holes whose consequences for ordinary systems go beyond what has been deduced purely from the axioms of information theory. After a self-contained introduction to black hole thermodynamics, we review from its vantage point topics such as the information conundrum that emerges from the ability of incipient black holes to radiate, the various entropy bounds for non-black hole systems (holographic bound, universal entropy bound, etc) which are most easily derived from black hole thermodynamics, Bousso's covariant entropy bound, the holographic principle of particle physics, and the subject of channel capacity of quantum communication channels.

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#### I. INTRODUCTION

Black holes entered the stage of natural science—as an astrophysical paradigm—when British scientist and cleric John Michell [1] and French mathematician, astronomer and peer P. S. de Laplace [2] independently remarked that a star with a sufficiently large ratio of mass to radius cannot be observed by its own light because for such configuration the escape velocity exceeds the speed of the light corpuscles. General relativity, the modern gravity theory, sharpened the definition of 'black hole': no longer a mass from which light cannot issue to large distance, but a region of space rendered causally irrelevant to all its environment by gravity in the sense that no signal, by light, massive particles or whatever, can convey information about its nature and state to regions outside it. Information is the key concept here, as emphasised by Wheeler [3].

Nothing just said, however, prepared investigators for the surprising connections uncovered during the last three decades between the world of gravity and black holes, and the realms of thermodynamics and information. Though figuratively speaking a black hole is a tear in the fabric of spacetime, albeit one that weighs and moves along very much like any particle, physics demands that such an entity be endowed with thermodynamic attributes—entropy, temperature, etc [4, 5]—as well as quantum properties like the ability, first uncovered theoretically by Hawking [6], to radiate spontaneously in flouting defiance of the popular definition of 'black hole' as an entity incapable of shining on its own.

Hawking's radiance engenders a deep problem that has troubled researchers for over two decades. The matter which collapses to form a black hole can be imagined to be in a quantum pure state. After its formation the black hole radiates spontaneously, as just mentioned, and according to calculations, this radiation is in a thermal state, that is a mixed quantum state. When this radi-

ation has sapped all mass from the black hole so that it effectively evaporates, we are left with just a mixed quantum state of radiation. That is, the black hole has catalysed conversion of a pure state into a mixed state, in contradiction to the principle of unitary quantum evolution. Mixed means entropic, and so one can view what has happened as a loss of information. This is the gist of the information paradox, which cannot be said to have been settled to everybody's satisfaction, but which has stimulated thought in gravity theory both in the gravitation and particle physics camps.

Black hole entropy has been found to enter into the second law alongside its more common sibling, matter and radiation entropy. From the corresponding generalised second law (GSL)[4, 7], which has meanwhile received strong support from a variety of *qedanken* experiments, it can be inferred that it is physically impossible to pack arbitrarily large entropy into a region with given boundary area, or into a given mass with definite extension. These conclusions (holographic and universal entropy bounds) are quite at variance with expectations from extrapolation of daily experience (RAM memories are getting smaller as they shoot up in capacity), as well as with well understood consequences of quantum field theory, one of the pillars of contemporary theoretical physics. It has even been claimed that we stand at the threshold of a conceptual revolution in physics, one that will relax the tensions alluded to. A popular introduction to the matters just mentioned may be found in Ref. [8].

With one exception, this review focuses on those aspects of physical information which are not usually discussed by the standard methods of (quantum) information theory. Thus after the introduction of black hole physics (Sec. II) and thermodynamics (Sec. III) and discussion of the information paradox (Sec. IV), we review in Sec. V the holographic entropy and information bound from the point of view of the GSL, and in Sec. VI the intimately related holographic principle which acts as a bridge between information theory, particle physics and cosmology. There follows in Sec. VII an account of the universal entropy and information bound, and its origin. We continue with Sec. VIII, an essay on the uses of the GSL to set bounds on quantum channel capacity. This is

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a standard topic in information theory which is enriched by the methods here described. Sec. IX summarises the findings.

#### II. THE BLACK HOLE IN A NUTSHELL

One striking thing about the black hole phenomenon, in contrast to other astrophysically related paradigms, is that it is scale invariant. A planet is a planet only if its mass is between a fraction of Earth's and a few times Jupiter's, and a star shines as such only if its mass is between seven hundredths and about a hundred solar masses. Thus, even though Newton's gravity law has no preferred scale (the only constant involved is Newton's G), planets or stars exist in different mass ranges because of scales inherent in the matter making them up. By contrast, classically a black hole can have any mass, and its characteristics are the same for all masses. This comes about because the laws of relativistic gravity, Einstein's 1915 field equations, just as Newton's older formulation, do not involve a preferred scale, and because a black hole is conceptually distinct from any matter (which does have scales) out of which it originated. Quantum effects modify the above claims slightly: a black hole's mass cannot be below a Planck mass ( $\sim 2 \times 10^{-5}$  g) because if it where, the hole would then be smaller than its own Compton length, and would thus not exhibit the black hole hallmark, the event horizon.

The horizon is the boundary in spacetime between the region inaccessible to distant observers, and the outside world. It can be crossed only inward by particles and light. When viewed at a fixed time (more exactly on a spacelike surface), it has the topology of a two-sphere, namely it is a closed simply connected 2-D surface. When a black hole is nearly stationary it is useful to talk about a typical scale for it, e.g., its radius  $r_g$  if it is truly spherical. The mentioned scale invariance of black hole physics requires  $r_g$  to be proportional to the black hole's mass m, and its average density  $\langle \rho \rangle$  to be proportional to  $m^{-2}$ . For example, in general relativity a spherical electrically neutral black hole is described by the Schwarzschild solution [9], a particularly simple solution of Einstein's equations, which tells us that

$$r_g = \frac{2Gm}{c^2} = 1.49 \times 10^{-13} \left(\frac{m}{10^{15} \text{g}}\right) \text{cm}$$
 (1)

$$\langle \rho \rangle = \frac{3c^6}{32\pi G^3 m^2} = 7.33 \times 10^{52} \left(\frac{10^{15} \text{g}}{m}\right)^2 \frac{\text{g}}{\text{cm}^3}$$
 (2)

The arbitrary mass  $10^{15}$  g that we have introduced to make the orders of magnitude transparent is of the order of that of a modest mountain. It is clear why black holes are popularly regarded as smallish and hugely dense. But this is not always true: in our own Milky Way's core lurks a black hole 20 million kilometers across with an average density about that of water. Of course, black holes originating from stellar collapse, and those suspected to

have survived the rigours of the early universe, are much smaller and denser.

Black holes cannot shrink; this much seems clear from the fact that they can devour matter, radiation, etc. but cannot give any of these up. Penrose and Floyd [10], Christodoulou [11] and Hawking [12] independently showed that the said inference is correct in classical physics if precisely stated: in almost any transformation of a black hole, its horizon area will increase, and it cannot decrease under any circumstance. The 'classical' qualifier is critical; Hawking himself was soon to demonstrate the limitations of this "area theorem" once quantum processes intervene. But the theorem, though classical in scope, has turned out to be crucial to developments in black hole physics, not to mention to astrophysical applications.

Not only are black holes devoid of specific scales; they also lack the wide variety of individual traits that characterise stars and planets. A star's observable aspect, including its spectrum—the stellar fingerprint—depends very much on its chemical make up. Stars rich in the elements heavier than helium have more complicated spectra than do stars poor in them. There is a gamut of stellar chemical compositions and an equally wide range of spectral types. By contrast, in general relativity and similar gravity theories, all the black hole solutions describing stationary charged and rotating black holes form a single three-parameter family, the Kerr-Newman (KN) black hole solution [9].

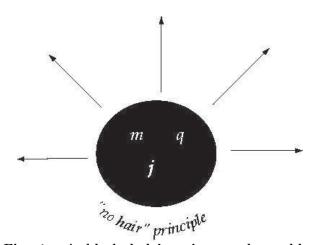


Fig. 1: A black hole's primary observables—gravitation and electromagnetic fields, Hawking emission (arrows)—as well a secondary ones, e.g. shape of the horizon, are entirely determined by its mass m, electric charge q and angular momentum j (this last is here responsible for the oblateness of the horizon.

Consequently, all of a black hole's observable traits are widely thought to depend only on its mass, angular momentum j and electric charge q (Fig. 1). Only these three and similar parameters (like the magnetic monopole, as yet unobserved in nature) are observables of a black hole.

Wheeler [3], who first emphasised the parsimony of a black hole's description, coined the maxim "black holes have no hair":

In standard gravity theory the most general stationary black hole exterior is described by the KN solution with m, q and j as its only parameters.

It might seem paradoxical to characterise the horizon as blocking all access to information about the black hole's interior while simultaneously maintaining that, say, electric charge of a black hole is observable. Actually there is no contradiction; the charge of the hole is the charge of the object that collapsed to make it. It could always be determined by Gauss theorem and, by charge conservation, always has the same value. Mass and angular momentum are similar in this respect; both can be determined from particular features of the hole's exterior spacetime geometry.

#### III. BLACK HOLE THERMODYNAMICS

How can a black hole—a blemish in spacetime—be endowed with thermodynamics? Thermodynamics is a successful description of a system provided it makes sense to describe the latter by merely a few parameters: energy, volume, magnetisation, etc., at least at sufficiently large scale. Otherwise a more complicated statistical mechanic or kinetic approach is indicated. We have seen that a black hole is fully described, as far as an outside observer is concerned, by just three parameters: m, q and j. No need to describe the matter that went to form the black hole in all gory detail. Hence thermodynamics seems an appropriate paradigm for black holes.

What are its variables? Black hole mass m, in the role of energy, is a typical thermodynamic parameter. Charged and rotating thermodynamic systems are, likewise, known, so q and j can be admitted. But to have a complete set of observable thermodynamic parameters for a black hole one still requires entropy. And it is plain that black hole entropy cannot be identified with the entropy of matter that went down the black hole, for it together with the matter becomes unobservable in the course of collapse. The fact that horizon area A tends to increase, and is definitely precluded from decreasing, suggests it represents the requisite black hole entropy. Various *qedanken* experiments together with Wheeler's remark [13] that the Planck length  $\ell_P \equiv (G\hbar/c^3)^{1/2}$  (the Compton length corresponding to Planck's mass) should play a crucial role here, motivated me to assert that black hole entropy,  $S_{BH}$ , is proportional to  $A/\ell_P^2$  [4, 5]. Is this reasonable?

The fact that the requisite  $S_{BH}$  must be exclusively a function of A (also when q and j do not vanish) is most clear from a latter argument by Gour and Mayo [14, 15]. For a KN black hole the area is easily calculated from the metric [9], namely

$$A = 4\pi[(M + \sqrt{M^2 - Q^2 - a^2})^2 + a^2],$$
 (3)

where  $M \equiv Gmc^{-2}$ ,  $Q \equiv \sqrt{Gqc^{-2}}$  and  $a \equiv jm^{-1}c^{-1}$  are three length scales which completely specify the black hole (M is just half our previous  $r_q$  in the Schwarzschild case q = j = 0). The black hole exists only when  $Q^2$  +  $a^2 \leq M^2$ . We infer from Eq. (3) that

$$d(mc^2) = \Theta dA + \Phi dQ + \Omega dj \tag{4}$$

with

$$\Theta \equiv c^4 (2GA)^{-1} (r_q - M) \tag{5}$$

$$\Phi \equiv q r_a (r_a^2 + a^2)^{-1} \tag{6}$$

$$\Phi \equiv q r_g (r_g^2 + a^2)^{-1}$$

$$\Omega \equiv j M^{-1} (r_g^2 + a^2)^{-1}.$$
(6)
(7)

Since  $mc^2$  is the energy, Eq. (4) has the aspect of the first law of thermodynamics  $TdS = dE - \Phi dQ - \Omega dj$ as applicable to a mechanical system whose electric potential and rotational angular frequency are  $\Phi$  and  $\Omega$ , respectively. Indeed, study of the motion of charged test particles about a KN black hole shows the above defined  $\Phi$  to be the electric potential at the hole's horizon, while  $\Omega$  is the uniform angular frequency with which infalling particles are entrained by the horizon—surely a good definition of rotational frequency of the black hole. Thus, if a black hole is to have a thermodynamics (first law at least), we must identify  $\Theta dA \leftrightarrow T_{BH} dS_{BH}$  with  $T_{BH}$  the hole's temperature. It follows that  $S_{BH} = f(A)$ ; black hole entropy depends on m,q and j only through the combination A. We then recognise that  $T_{BH} = \Theta/f'(A)$ . Why f(A) must be linear will be explained shortly.

Entropy lost into black holes cannot be kept track of, and so one should not, in ordinary circumstances, discuss entropy inside black holes. Thus the ordinary second law must be given a generalised form. Incorporating additivity of all entropy in analogy with other thermodynamics we get the generalised second law (GSL) [4, 7]:

> The sum of black hole entropies together with the ordinary entropy outside black holes cannot decrease.

Fig. 2 furnishes an example. The GSL reduces to the ordinary second law when black holes are absent, and to the area theorem if matter and radiation are absent (the last provided f'(A) > 0).

Which entropy exactly is covered by the stipulation "ordinary entropy" in the GSL's statement? After all, the entropy we associate with some matter depends on the 'resolution' of our description. If this last is rather coarse and ignores atoms, then we refer to the chemist's thermodynamic entropy. But if we include atomic and subatomic degrees of freedom, then there may be further contributions to the entropy at sufficiently high temperatures. It is easy to see that the "ordinary entropy" must be taken to mean the entropy calculated from statistical

mechanics applied to all degrees of freedom in matter and radiation, no matter how recondite. The reason is that the GSL is fundamentally a gravitational law, and gravitation is aware (via the equivalence principle) of energy residing in all degrees of freedom, no matter how deep they may lie. For example, string degrees of freedom should be taken into account if strings are taken as the fundamental entities.

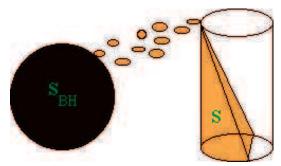


Fig. 2: Illustrating the GSL. The black hole entropy  $S_{BH}$  must increase at least by an amount equal to the entropy of the glassfull of tea poured down the black hole.

The GSL makes a linear f(A) seem most reasonable since addition of areas of black holes is a meaningful procedure (even in general relativity), and the 'area theorem' guarantees that the sum of areas (but not the sums of other combinations of black hole parameters) will increase. However, one might be leavy of the linear f(A)because it implies that doubling m, q and j of a KN black hole quadruples its  $S_{BH}$ , so that black hole entropy for one black hole is not extensive as for material systems. However, we now show that the seemingly more palatable alternative  $f(A) \propto \sqrt{A}$  is excluded together with all laws of the form  $f(A) \propto A^{\gamma}$  with  $\gamma \neq 1$ .

According to the area theorem, when a capsule containing some matter is dropped into a black hole, the latter's horizon area must increase. But is the consequent growth in  $S_{BH}$  sufficient to compensate for the loss of the capsule's entropy,  $S_{cap}$ , as the GSL demands ? One can try pushing the GSL against a wall by inserting the capsule (with rest mass  $\mu$  and radius b) in a most gentle manner so as to minimise the area increase  $\Delta A$ . Purely mechanical arguments [5, 16] show that for a generic KN black hole sufficiently big to accept the capsule and much more massive than it, there is a lower bound  $\Delta A \geq 8\pi G\mu bc^{-2}$  strikingly independent of the black hole parameters. It immediately follows that  $\Delta S_{BH} \geq f'(A) 8\pi G \mu b c^{-2} \geq S_{cap}$ . The second inequality comes from the GSL since in the infall the capsule's entropy is lost. Were we to choose  $f(A) \propto A^{\gamma}$  with  $\gamma$ constant, we would obviously be faced with a violation of the GSL for large A if  $\gamma < 1$ , or for small A if  $\gamma > 1$ .

We can conclude that f(A) cannot be an exact power law, except for the trivial one with  $\gamma = 1$ . Thus we adopt

$$S_{BH} = \eta A/\ell_P^2 \tag{8}$$

with  $\eta$  a constant;  $\ell_P$  is introduced for dimensional reasons. Our earlier result  $T_{BH} = \Theta/f'(A)$  thus gives (except where explicitly stated otherwise, temperature is in units of energy)

$$T_{BH} = (c\hbar/2\eta A)\sqrt{M^2 - Q^2 - a^2}.$$
 (9)

The smaller the hole, the hotter it is (see Eq. (11)). Black holes which have about as much angular momentum (or charge) as permitted are especially cool. Finding  $\eta$  is obviously the next logical step.

Attempts to understand the simple formula for black hole entropy from more fundamental points of view are legion. Bombelli, Kaul, Lee and Sorkin [17], and later and independently Srednicki [18] gave reasons to believe that black hole entropy is related to entanglement entropy arising from the tracing out of those degrees of freedom that are localised beyond the horizon. Early attempts of Thorne and Zurek [19] and independently 't Hooft [20] sought to identify black hole entropy with the entropy of the thermal radiative "atmosphere" of the black hole (see Sec. IV). We may also mention here some of the many attempts to relate it to the degrees of freedom of strings associated with the black hole [21, 22], or to the quantum gravity degrees of freedom of the horizon, be it in the context of conformal field theory [23, 24], of loop quantum gravity theory [25], or of more heuristic schemes [16, 26, 27].

#### HAWKING RADIATION AND THE IV. INFORMATION PARADOX

In the everyday world hot objects radiate. In 1974 Hawking demonstrated theoretically that a black hole formed by collapse does likewise [6]. In essence his quantum field theoretical calculation performed on a prescribed classical gravitational background shows that if a quantum field is in the vacuum state in the presence of an object which begins to collapse to a black hole, then as the object's radius nears the horizon's, the state of the field in the hole's exterior approaches that of thermally distributed radiation with a temperature of the form (9) with  $\eta = 1/4$ . This temperature is the same whatever the field, e.g. scalar, electromagnetic, neutrino, etc. Thus Hawking's result calibrated the black hole entropy and temperature formulae (8) and (9):

$$S_{BH} = 2.65 \times 10^{40} (m/10^{15} \text{g})^2 h_1$$
 (10)  
 $T_{BH} = 1.23 \times 10^{11} (10^{15} \text{g/m}) h_2^0 \text{K}$  (11)

$$T_{BH} = 1.23 \times 10^{11} (10^{15} \text{g/m}) h_2^{\ 0} \text{K}$$
 (11)

where  $h_1(Q/M, a/M)$  and  $h_2(Q/M, a/M)$  are two known dimensionless functions of order unity, both exactly equal to unity for Q = a = 0.

For comparison, the sun (mass  $2 \times 10^{33}$  g) has an entropy of order  $10^{58}$  and central temperature  $1.6 \times 10^{70}$  K. On the astronomical scale black holes are thus very entropic and cool. It is consistent with the GSL that a solar mass black hole have an entropy larger than that

of a solar mass star which might have been its predecessor. But why should the holes's entropy be the larger by many orders of magnitude? Boltzmann's principle that a system's entropy is the logarithm of the number of microscopic configurations compatible with that system's macroscopic properties, together with the "no hair" principle, suggests that black hole entropy is large because a black hole's aspect cannot tell us precisely which type of system gave rise to it. This extra lack of "composition information" over and above that about specific microscopic configurations may be what makes black hole entropy large. A black hole stands for a large amount of missing information.

Hawking noticed a conundrum when black hole radiation is considered in light of the unitarity principle of quantum theory [28]. One can imagine a black hole formed from matter in a pure state, e.g. a gravitating sphere of superfluid at  $T = 0^{0}$ K. The unitarity principle would thus require that the system always remain in a pure state. The fact that a black hole with large entropy forms is not in itself the real problem. Although nonzero entropy is a property of a mixed state, it is an everyday sight sanctioned by the second law that entropy can just appear when there was none before. This is understood as reflecting classical coarse graining or tracing out of some quantum degrees of freedom in a fundamentally pure state, both transpiring for operational reasons. The conundrum arises only in the aftermath of the black hole (and, incidentally, is unrelated to the oft made observation that according to some observers the black hole horizon never quite forms).

Hawking's radiation drains the hole's mass (also its angular momentum) on a finite time scale. Using the Stefan-Boltzmann radiation law  $P=(4\pi R^2)\sigma T^4$  and Eqs. (11) and (1), we estimate, c.f. Eq. (16) below, for each species of quanta radiated by a Schwarzschild black hole

$$\frac{dm}{dt} \approx -4.02 \times 10^{-6} \left(\frac{10^{15} \text{g}}{m}\right)^2 \text{g s}^{-1},$$
(12)

with an order of magnitude correction coming from the gravitational redshift, general relativistic geometrical factors, and particle statistics (Bose or Fermi). Obviously as the black hole looses mass, it radiates faster and the mass loss accelerates. Calculations give no hint that the evaporation can be arrested before m descends to Planck mass scale, by which time one is dealing with a pure quantum gravity phenomenon. Evaporation of the black hole to nothing, or at the very least to a Planck scale object, must thus take less than  $10^{20} (m/10^{15} \mathrm{g})^3 s$ . A black hole with a radius a little smaller than the proton's (see Eq. (1)) can thus have a lifetime briefer than our universe's age. The evaporation is thus slow but not an hypothetical phenomenon.

Hawking's original calculation [6] and many others since then showed that the radiation is thermal, both in its Planck-like spectrum and in the lack of correlations between different radiation modes. It thus seems that a

pure state can be converted into a mixed one through the catalysing influence of a black hole! Hawking [28] was led by this to assert that gravity violates the unitarity principle of quantum theory. This means the mixed character of the final state is dictated by physics, and is not the result of the way we choose to describe the system. Since the final state has a lot of entropy (of order of the intermediate black hole's entropy by the GSL), we are faced with a large intrinsic loss of information. To be sure. Hawking's inference has remained controversial: whereas general relativity investigators have tended to accept this conclusion, particle physicist have stood by the unitary principle and orthodox quantum theory. Many resolutions have been offered. In order to categorise them it is useful to draw an analogy between our problem and the following "experiment" attributed to S. Coleman.

A cold piece of coal is illuminated by a laser beam. The system is in a pure state: coal in its ground state and beam in a coherent state (analogous to the sphere of superfluid). Experience tells us the coal will heat up and radiate (black hole forming and radiating). The beam is interrupted (no matter is thrown into black hole after its formation). The coal cools while radiating thermally (Hawking radiation). The coal cools totally and returns to its ground state (black hole evaporates), which is, of course, pure. In both cases we are left with a mixed state—thermal radiation is as mixed as can be. Nobody doubts that unitary is respected in the coal-laser system. The information conundrum in that case is unravelled by the remark that subtle correlations between the early and late radiation take care to preserve the purity of the state once the correlation with the coal is broken by its reaching ground state. Any coarse inspection of the radiation, which would necessarily be local, would, by virtue of the implicit tracing out, reveal a mixed thermal state.

Many have argued by analogy that subtle correlations in the actual Hawking radiation preserve its overall pure state status after the black hole is gone. The fact that Hawking's and like calculations reveal no such correlation is thought to be due to their semiclassical character which ignores the quantum degrees of freedom of gravity. Certain model quantum gravity calculations have supported this point of view.

A different approach to resolving the paradox posits that a Planck scale remnant is always left after Hawking evaporation, and that it escapes the fate of total evaporation by means of quantum gravity modifications to Eq. (12). It is impossible, in the present state of the art of quantum gravity research, to check this possibility thoroughly. It does involve belief in objects of dimension  $10^{-33}$  cm whose information content corresponds at least to the entropy  $10^{40}$  characteristic of a black hole sufficiently light to evaporate to Planck scale in the lifetime of our universe (see Eq. (10) and conclusions stemming from (12)). But this is problematic [29]; as we shall see, such large information content in such small size conflicts with recent ideas.

A third way out of the information paradox which respects unitarity, is the supposition that a black hole always gives rise to another universe which may be reached, in principle, through the black hole. Certain black hole solutions of Einstein's equations, like Reissner and Nordström's one describing a nonrotating charged black hole [9], do show such a universe connected to the black hole's interior and lying to the future of the universe in which the black hole formed. It is not clear whether the later universe will indeed appear in the aftermath of realistic collapse. But if this point is granted, then the idea is that unitary evolution proceeds as always into the new universe so that no loss of information actually occurs from a universal point of view. The local observer in the universe where the black hole formed does retain the impression that information has been lost.

In summary, there may be resolutions to the information paradox. But the preservation of information is only manifest at an ideal level of scrutiny (being aware of all universes, analysing radiation emitted millions of years ago jointly with fresh one). For *practical purposes*, information does disappear in the presence of black holes.

#### V. THE HOLOGRAPHIC BOUND

The GSL immediately suggests the existence of information (or entropy) bounds for non-black hole objects. The first such derived in this way, the universal entropy bound [30], will be reviewed in the next section. Here we take up the holographic bound which is in many ways easier to comprehend.

L. Susskind [31] proposed the following gedanken experiment. Take a neutral nonrotating spherical object containing entropy S which fits entirely inside a spherical surface of area A, and allow it to collapse to a black hole, which by symmetry must be of the Schwarzschild type. Evidently the black hole's horizon area is smaller than A, but by the GSL  $S_{BH}$  must exceed S. It follows that

$$S \le \frac{\mathcal{A}}{4\ell_P^2}.\tag{13}$$

We have included the equality in order that a black hole itself partake of this, the holographic bound.

By the connection between entropy and information, bound (13) implies a generic "holographic" bound on the information inscribed in any isolated object, a bound which can be stated exclusively in terms of the area of some bounding surface (see Fig. 3). Like the holographic bound on entropy, this second bound is counterintuitive. Has it not been obvious for generations that, other things being equal, information capacity scales with volume of the information registering milieu? But if so, as the scale of the system goes up, the growth in volume must outstrip the growth of area bringing about a conflict with the assertion of the holographic bound. The resolution to the quandary is that before the crossover point is reached,

the information storage system has already collapsed to a black hole which, of course, cannot be used to store information useful to external observers [8].

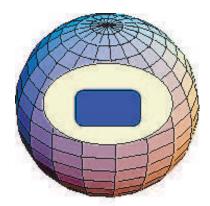


Fig. 3: Illustrating Susskind's version of the holographic bound. The information capacity in nits of an information storage of arbitrary construction (purple object) is bounded from above by a quarter of the area of an enclosing spherical (or other closed) surface measured in squared Planck lengths.

But how general is bound (13)? It might appear that the bound should not be used for an object which cannot spontaneously collapse, e.g. the Earth. But, at least in two kinds of circumstances, its applicability is justifiable even in this situation. If the system in question is weakly self-gravitating (as are most everyday, laboratory and astronomical systems), namely its mass  $\mu$  and radius R satisfy  $G\mu c^{-2} \ll R$ , Susskind's spherical collapse can be supplanted by the infall of the object into an already existing much larger and heavier black hole (see end of Sec. VII). Alternatively, for a strongly self-gravitating composite system  $(G\mu c^{-2} \sim R)$ , which is at the same time much larger and more massive than an elementary particle, e.g. a neutron star, a tiny preexisting black hole can be used to catalyse its eventual collapse, with the hole contributing little to the bookkeeping [32]. In both of the above cases the GSL can be used to recover Susskind's holographic bound (13).

Susskind's argument will also apply to a charged object provided it indeed collapses to a black hole in spite of the Coulomb repulsion. Due to the quirks of relativity it is less clear if collapse to a rotating (Kerr) black hole of a very compact rotating object necessarily involves a contraction of the bounding area, though this seems likely. Therefore, the holographic bound's range of applicability is broad. It is, however, inappropriate to apply bound (13) to a system which is not well isolated from its surroundings. Gravitational collapse is not controllable, and cannot be expect to affect the system while exempting surrounding objects from like fate.

All the above is not to say that the holographic bound, as just stated, is flawless. For instance, it fails if applied to our whole universe, particularly if the latter is infinite,

as suggested by contemporary cosmological data. In the standard cosmological model the universe contains entropy (principally of radiation) with some uniform density. A sufficiently large sphere can thus contain more entropy than allowed by (13) if  $\mathcal{A}$  is interpreted as its bounding area, because then  $\mathcal{A}$  scales only as the square of the radius of the said volume. And if the universe is fairly uniform, the above mentioned crossover point is not accompanied by collapse. Likewise, a spherical system already inside the black hole of it own making will eventually violate (13), because as it inexorably contracts (as it must by general relativity), its bounding area eventually shrinks to zero while, by the ordinary second law, the entropy it contains cannot decrease. (Note that we here take the view of the *interior* observer).

R. Bousso [33, 34] introduced a reinterpretation of formula (13), the covariant entropy bound, which makes it more broadly valid. We shall not go into technical details. Suffice it to say that A now refers to the area of any 2-D surface, closed or open, which satisfies mild technical restrictions. The S refers to all the entropy that "gets illuminated" by a hypothetical brief flash of light emitted perpendicularly from one side of the surface, with entropy beyond the point where light rays start crossing not counted. In cases where the original holographic bound applies, it can be derived from Bousso's [33]. As of this writing, Bousso's bound has navigated successfully a number of classical hurdles [34]. It is, however, known that quantum radiation, e.g. Hawking's, can cause the Bousso bound's failure [35]. A generalisation of the bound to extend its validity to this situation—but still short of quantum gravity—has been proposed by A. Strominger and D. Thompson [36].

#### VI. THE HOLOGRAPHIC PRINCIPLE

Much of the contemporary status of the holographic bound, in whatever formulation, is due to its intimate connection with G. 't Hooft's holographic principle [37] (in fact the adjective 'holographic's was applied by 't Hooft at the outset to the principle). Many workers regard the holographic principle as providing guidelines for the final theory of nature. The holographic principle asserts that physical processes in a universe of  $\mathcal{D}$  spacetime dimensions as described by some physical theory, e.g. string theory or a field theory, are reflected in processes taking place on the  $\mathcal{D}-1$  dimensional boundary of that universe (provided it has one) which are described by a different physical theory formulated in  $\mathcal{D}-1$  dimensions. There is an equivalence between theories of different sorts written in spacetimes of different dimensions [38, 39].

A concrete example is the equivalence of string theory operating in 5-D anti-deSitter spacetime and a conformal field theory operating in the 4-D flat spacetime which constitutes 5-D anti-deSitter spacetime's boundary. DeSitter spacetime is a solution of Einstein's equations, as augmented by a positive cosmological constant, representing a highly symmetric (and formally empty) universe. There is much astronomical evidence that our universe may be headed for a deSitter like phase. AntideSitter spacetime is obtained from deSitter's solution by switching the sign of the cosmological constant. A consequence of the above mentioned equivalence is the correspondence between properties of a black hole, a still mysterious entity here conceived in string theory terms. which resides in the anti deSitter universe and those of black body radiation (by now a trite subject) in the flat spacetime [39]. Correspondences of this sort have been used to simplify difficult calculations. And the equivalence between the laws is undeniably of philosophical import. Thus far it has not proved possible to set up a like holographic correspondence involving deSitter's uni-

The relation between holographic principle and bound is an informational one. If processes in the bulk spacetime can be understood by correspondence with processes on its boundary, then in some sense the measure of information about the bulk is not so large that it cannot be bounded in terms of the extent of the boundary, which is the natural measure for information therein. For systems in three-dimensional space, this suggests an information content that scales no faster than the area of the boundary of the space, as in the holographic bound. There is also contrast between bound and principle. The holographic bound is applicable also to part of the space and the corresponding boundary (provided, as mentioned above, that the system in question is truly isolated). This is in stark contrast to the full holographic principle which only asserts detailed equivalence of the physics in two different 'universes'.

Although in the public's mind the holographic principle is associated with string theory, the two actually stand in conflict, just as do the principle and quantum field theory. Fields are continuous, and they live in a continuum spacetime. As a result a field has an infinity of possible different (orthogonal) states in a typical volume (say one whose boundary is not unduly convoluted), and certainly in a whole universe. But it is already clear from the holographic bound that to the given volume can be associated only a finite entropy, *ergo* a finite number of states. Likewise, a string, quite different from a field in other respects, also has an infinite number of possible states (think of the number of distinct vibrations of a taut cord). Since the string can be confined to a given volume, this conclusion clashes with the holographic bound.

The clash is not confined to finite systems. The de-Sitter 'universe' has *infinite* volume, much of it hidden behind an event horizon very like that of a black hole in many respects, but encompassing the whole 'sky' of the observer. This horizon has a finite area (whose size is set by the value of the cosmological constant—a parameter of the physics). Since Gibbons and Hawking's early paper on thermodynamics of deSitter spacetime [40], entropy has been ascribed to deSitter's horizon according

to the usual black hole rule (8), and the GSL is known to apply [41]. It follows that the entropy of matter (or radiation) hidden behind the horizon is always finite, even though the universe is infinite. This obviously raises challenges for string theory as much as for field theory.

# VII. THE UNIVERSAL INFORMATION BOUND

The holographic information bound is simple; it is also extremely lax. For example, an object the size of a music compact disk would be allowed by the bound an information capacity of up to 10<sup>68</sup> bits. Present technology can only store 10<sup>10</sup> bits on it, and is expected to improve only by a few orders of magnitude. It is clear from this and other examples that the holographic bound, important though it be for matters of principle, is of no great practical use. Can an alternative do better while still being generally correct? Indeed, the hoary universal entropy bound [30] does much better than the holographic bound. The original argument for it involves fine points of general relativity; it has also been attacked on the grounds that it does not properly account for the phenomenon of quantum buoyancy [42]. Therefore, we provide here a much simplified approach to the said entropy bound [43].

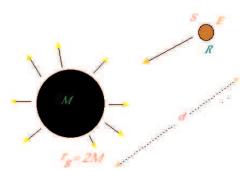


Fig. 4: Free infall of a macroscopic object into a Schwarzschild black hole can be used to derive a weak version of the universal entropy bound.

Consider the following gedanken experiment. Drop a composite system  $\mathcal{U}$  (not an elementary particle) of radius R, total energy E (its rest energy) and entropy  $\mathcal{S}$  into a Schwarzschild black hole of mass  $m \gg Ec^{-2}$  from a large distance  $d \gg M$  away; d is so chosen that the Hawking radiance carries away energy equal to E while  $\mathcal{U}$  is falling to the horizon where it is effectively assimilated by the black hole. This is depicted in Fig. 4. Upon completion of the process the black hole mass is back to m and its entropy has not changed. Were the emission reversible, the radiated entropy would be  $E/T_{BH}$  with  $T_{BH} \equiv c\hbar(8\pi M)^{-1}$  (see Eq. (9)). The curvature of spacetime makes the entropy emitted a factor  $\nu$  larger; typical values, depending on particle species, are  $\nu = 1.35$ –1.64 [44]. Thus the overall change in world entropy is

$$\Delta S = \nu E / T_{BH} - \mathcal{S}. \tag{14}$$

One can certainly choose  $M = Gmc^{-2}$  larger than R, say, by an order of magnitude so that the system will fall into the hole without being torn up:  $M = \zeta R$  with  $\zeta$  of the order a few. Thus by the GSL (the black hole is unchanged) we obtain

$$S < 8\pi\nu\zeta RE/c\hbar.$$
 (15)

This bound applies to an arbitrary composite system. This means we must require  $R \gg c\hbar/E$  ( $\mathcal{U}$  much larger than its own Compton length; even a nucleon qualifies). Additionally, in the derivation  $\mathcal{U}$  is not allowed to be strongly gravitating (which would entail  $GEc^{-4} \sim R$ ) because then m could not be large compared to E, as we have assumed, if we really insist that  $\zeta$  is of order a few. We thus have to assume in addition  $GEc^{-4} \ll R$ . Note, however, that the resulting bound on entropy, (15), is G independent; gravity was central in the derivation, but has been swept under the rug in the result.

Note also that it is impossible to infer (15) by using a plain heat reservoir in lieu of a black hole. A reservoir which has gained energy E upon  $\mathcal{U}$ 's assimilation, and has returned to its initial energy by radiating, does not necessarily return to its initial entropy, certainly not until  $\mathcal{U}$  equilibrates with the rest of the reservoir. But a (nonrotating uncharged) black hole whose mass has not changed overall, retains its original entropy because that depends only on its mass (any equilibration here is on a dynamical scale, and thus extremely rapid). In addition, for the black hole mass and radius are related in a simple way; this allowed us to replace  $T_{BH}$  in terms of R. By contrast, for a generic reservoir, size is not simply related to temperature. So black holes are crucial in obtaining (15) without delying into the thermodynamics of  $\mathcal{U}$ .

The above derivation cavalierly ignores the effect of Hawking radiation pressure. Could it blow  $\mathcal{U}$  outwards? We can get an idea of the effect by calculating the Hawking radiation flux  $\mathcal{F}$  via Stefan-Boltzmann's law as applicable to a sphere with temperature  $T_{BH}$  and radius 2M. At distance r from the hole

$$\mathcal{F}(r) = \frac{c^2 \bar{\Gamma} \mathcal{N} \hbar}{61,440(\pi M r)^2}.$$
 (16)

Here  $\mathcal{N}$ , a natural constant, is the effective number of massless species radiated (photons contribute 1 to  $\mathcal{N}$  and each neutrino species 7/16), and  $\bar{\Gamma}$  corrects for general relativistic effects, including the fact that the radiating area is actually a bit larger than  $4\pi(2M)^2$ :  $\bar{\Gamma}\sim 2$  [45]. This energy (and momentum) flux results in a radiation pressure force  $f_r(r)=\pi R^2 c^{-1}\mathcal{F}(r)$  on  $\mathcal{U}$ . More precisely, species which reflect well off  $\mathcal{U}$  are approximately twice as effective at exerting force as just stated, while those (neutrinos and gravitons) which go right through  $\mathcal{U}$  contribute very little.

Since the Newtonian gravitational force on  $\mathcal{U}$  is  $f_{\rm g}(r) = GmEc^{-2}r^{-2}$ ,

$$\frac{f_{\rm r}(r)}{f_{\rm g}(r)} = \frac{c\bar{\Gamma}\mathcal{N}_{\rm eff}\,\hbar R^2}{61,440\pi M^3 E}.\tag{17}$$

We write here an effective number of species,  $\mathcal{N}_{\rm eff}$ , because, as mentioned, some species just pass through  $\mathcal{U}$  without exerting force on it. In addition, only those radiation species actually represented in the radiation flowing out during  $\mathcal{U}$ 's infall have a chance to exert forces. An Hawking quantum, just as any quantum in thermal radiation, bears an energy of order  $T_{BH}$ , so the number of quanta radiated together with energy E is approximately  $8\pi ME/c\hbar$ . Our assumption that  $\mathcal{U}$  is composite  $(R\gg c\hbar/E)$  and our stipulation that  $M=\zeta R>R$  together make this number large compared to unity. Since a species can exert pressure only if it is represented by at least one quantum, one obviously has  $\mathcal{N}_{\rm eff}<8\pi ME/c\hbar$ . Therefore,

$$\frac{f_r(r)}{f_g(r)} < \frac{\bar{\Gamma}R^2}{7680M^2} = \frac{\bar{\Gamma}}{7680\zeta^2} \ll 1$$
 (18)

Radiation pressure is thus negligible.

We must still check our tacit assumption that  $d\gg M$ , which, by making most of the infall take place in the Newtonian regime, exempts us from having to deal with general relativistic corrections. We recall that d must be such that the infall time equals the time t for the hole to radiate energy E. Newtonially speaking, the time t for free fall of a test body, from d to 2M in the field of a mass  $m=c^2MG^{-1}$  is given implicitly by  $d\approx 2(c^2t^2M/\pi^2)^{1/3}$ , while Eq. (16) gives the estimate  $t\approx 7680\pi EM^2c^{-2}\hbar^{-1}\mathcal{N}^{-1}$  (we have taken  $\tilde{\Gamma}\approx 2$  and  $\mathcal{N}$  as the full species number). From these equations and  $M=\zeta R$  we get that

$$d \approx 780 (\zeta ER/\mathcal{N}c\hbar)^{2/3}M.$$
 (19)

Thus for  $\mathcal{N} < 10^2$  (conservative estimate of our world's massless particle content) and taking into account  $R \gg c\hbar/E$ , we have  $d \gg 36\zeta^{2/3}M$  for all weakly gravitating composite systems  $\mathcal{U}$ . For all these we have thus justified the entropy bound (15).

How big is the factor  $\nu\zeta$ ? It seems safe to assert that  $4\nu\zeta < 10^2$ . Indeed, the original argument [30] gave  $2\pi$  for the numerical coefficient in the bound; with this choice it is referred to as the *universal entropy bound*. A pleasant spinoff of the choice  $2\pi$  is that the bound then formally applies also to black holes if we identify E with  $mc^2$  and R with  $(A/4\pi)^{1/2}$  (with A defined by Eq. (3)). The Schwarzschild black hole is the only one to saturate the entropy bound [30].

In our derivation of bound (15) R evidently stands for the *largest* radius of the system  $\mathcal{U}$ . It has been claimed [46, 47] that R in the universal bound can actually be interpreted as a smaller dimension, if such is available. The derivation of this improved bound relies on a generalised form of the purely classical Bousso bound. The validity of this generalisation has, however, lately been cast in doubt by a counterexample of V. Husain [48].

This is also the point at which to notice that the condition of weak self-gravity,  $GEc^{-4} \ll R$  allows us to infer immediately that the holographic bound (13) is satisfied

with plenty of room to spare just as a consequence of the universal entropy bound. In other words, for weakly self–gravitating systems (which include most systems known), the universal entropy bound is much tighter than the holographic one.

Being a statement about entropy of a system, the universal bound can also be derived directly from statistical mechanics for simple (quantum) systems [49, 50]. None of these analytical or numerical arguments have the simplicity or broad applicability of the argument described above. But whatever its derivation, the universal bound automatically provides us with a bound on information. In words, the ceiling on the capacity in bits is of the order of the ratio of the largest dimension of the system to its formal Compton length. To use our earlier illustration, the bound on the information capacity of a compact disk is now about 10<sup>40</sup> bits, 28 orders tighter than the holographic information bound. But the universal bound is still many orders above any foreseeable capacity. Can one do better? One situation where this is possible is for systems which are extensive in the thermodynamic sense. For these Gour [51] has shown that  $S < (ER/\hbar c)^{3/4}$  up to a numerical coefficient dependent on the number of species. For composite systems  $(ER/\hbar c \gg 1)$  Gour's bound is tighter than the universal one, and while the former's scope is more limited, it is nonetheless useful. Other tight bounds for restricted situations are sure to exist.

#### VIII. BOUNDS ON INFORMATION FLOW

Information theory has perennially been very much a theory of communication channel capacity. Can one use black holes to obtain new insights into natural limitations on information flow rate? Indeed one can, though in contrast to the case of the entropy bounds discussed earlier, the results on information flow were actually known in some form from early work on quantum communication channels.

As before, we shall make use of the GSL. The first thing to notice is that the power in Hawking radiation of a Schwarzschild black hole can be written as, c.f. Eq. (16),

$$P_{BH} = \frac{c^2 \bar{\Gamma} \mathcal{N} \hbar}{15,360 \pi M^2} \tag{20}$$

According to out comments in connection with Eq. (14), the entropy outflow  $\dot{S}_r$  is  $\nu P_{BH}/T_{BH}$ . We can thus write

$$\dot{S}_r = \left(\frac{\pi \nu^2 \bar{\Gamma} \mathcal{N} P_{PH}}{240\hbar}\right)^{1/2} \tag{21}$$

(In Ref. [43] the numerical coefficient under the radical is a factor of 2 greater than here because there we consider photons as two separate helicity species.)

This is our key formula. Notably the entropy outflow versus power relation here is not that of a typical 3-D hot body, for which  $\dot{S}_r \propto P^{3/4}$ . In terms of its radiative properties, a black hole is more like a body thermally radiating in 1-D for which  $\dot{S}_r \propto P^{1/2}$  [43, 52]. This observation leads us right into the subject of communication channels which often are just 1-D conduits of radiation.

We define a communication channel  $\mathcal{C}$  very generally: a collection of modes for quantum particles, massless or massive, possibly of various species, bosonic or fermionic. The modes may span a narrow or a broad range of frequencies, as well as a range of directions, but all like modes differing only in the time at which they are emitted must be included provided they all move from source to receiver. Signals are conveyed by populating the modes with quanta in accordance with a probability distribution for the various quantum states (density matrices). We know that von Neumann's entropy is an upper bound on the information conveyed [53]. When no record of the actual choice of state of the fields in the channel is kept,  $\mathcal{C}$  carries only entropy, and we wish to set a bound on it.

Our restriction that the whole time sequence of modes with given quantum numbers be included allows us to speak of steady state channel power P and von Neumann entropy flux rate  $\dot{S}(P)$  for C. To get our result we imagine directing the channel upon a Schwarzschild black hole in such a way that the channel's energy all goes down the black hole. To prevent part of the incident radiation from being scattered out of the hole, we require that the channel only include suitable modes. For example, spherical-like modes (perhaps so shaped by a mirror system) are acceptable so long as the wavelengths spectrum has an upper cutoff  $\lambda_c$  substantially smaller than the hole's scale M. So are wavepacket modes with small spread of direction and transversal dimensions well under M, which means they also have a long wavelength cutoff  $\lambda_c$  considerably shorter than M. We pick the black hole scale  $M = \xi \lambda_c$  with  $\xi$  of order a few, so that virtually all the channel's energy goes into the hole (see Fig. 5). For any given C it is convenient to define a characteristic black hole power, c.f. Eq. (20),

$$P_c \equiv \frac{c^2 \bar{\Gamma} \mathcal{N} \hbar}{15,360 \pi \lambda_c^2} \approx 10^{-4} c^2 \hbar \lambda_c^{-2}.$$
 (22)

For example, for optical wavelengths,  $P_c \sim 1/30 \text{ erg s}^{-1}$ . We must emphasise that if the signal carriers are massive particles, power here must include the rest energy flux.

Now  $(P - P_{BH})c^{-2}$ , the net rate of gain of black hole mass, causes a gain in black hole entropy at a rate  $\dot{S}_{BH} = (P - P_{BH})/T_{BH}$  (both of these quantities might be negative). In addition, the emitted black hole power is accompanied by radiation entropy rate  $\dot{S}_{BH} = \nu P_{BH}/T_{BH}$ . The (obviously positive) sum of these two entropy contributions, with the substitution  $T_{BH} \to \hbar c/8\pi M$ , provides, by the GSL, a upper bound on  $\dot{S}(P)$ , from which we infer one on communication rate  $\dot{\mathcal{I}}$  (always expressed in bits s<sup>-1</sup>):

$$\dot{\mathcal{I}}(P) < \frac{8\pi\lambda_c}{\hbar c} \left[ \xi P + \frac{\nu - 1}{\xi} P_c \right] \log_2 e \tag{23}$$

The bound is smallest for  $\xi = [(\nu - 1)P_c/P]^{1/2}$ , at which value the two terms in bound (23) are equal. This optimisation makes sense only if  $\xi$  comes out of order a few at least, that is for  $P \ll P_c$ . In this case

$$\dot{\mathcal{I}}(P) < \left(\frac{\pi(\nu - 1)\bar{\Gamma}\mathcal{N}P}{60\hbar}\right)^{1/2}\log_2 e \tag{24}$$

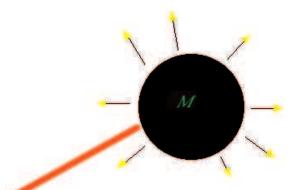


Fig. 5: A communication channel is directed onto a Schwarzschild black hole whose scale M is chosen large compared to the longest wavelength in the channel (which determines its width). Meanwhile the hole radiates  $a\ la$  Hawking.

For  $P > P_c/10$ , the optimal  $\xi$  is no longer in a safe range for total absorption. For all such channels we simply fix  $\xi = M/\lambda_c$  at some large value (we shall take  $\xi = 10$  for illustration), and use the full bound (23). If in addition  $P > P_c$ , then by Eqs. (20) and (22) the second term in Eq. (23) becomes negligible. Thus for  $P > P_c$ 

$$\dot{\mathcal{I}}(P) < \frac{8\pi\xi\lambda_c P}{\hbar c}\log_2 e. \tag{25}$$

This bound is formally independent of the black hole characteristic parameters  $\bar{\Gamma}$  and  $\nu$  and even of  $\mathcal{N}$ .

An immediate corollary of bound (25) is sometimes useful. Because of the cutoff  $\lambda_c$ , any information packet in the channel cannot be of shorter duration than  $\sim \lambda_c/c$ . Thus the total energy E of such packet must exceed  $P\lambda_c/c$ , so that

$$\dot{\mathcal{I}} < \frac{8\pi\xi E}{\hbar} \log_2 e. \tag{26}$$

This is Bremermann's bound [54] (usually stated with a smaller coefficient). It has been obtained in a variety of ways, including derivation from the universal entropy bound [50, 55, 56]. While independent of many details and good for first orientation, it is rather lax.

What to compare bounds (23)–(25) with? Let us restrict attention to channels employing massless particles for signalling. These include phonons in solids as well as photons in an optical fibre. If the channel has no short wavelength cutoff, then on dimensional grounds an information packet should have a maximum information

depending on its total energy E, its duration  $\tau$  and the cutoff  $\lambda_c$ . No other variables seem relevant. Let us write

$$\mathcal{I}_{\text{max}} = F\left(\frac{\lambda_c^2 E^2}{c^2 \hbar^2}, \frac{E\tau}{\hbar}\right) \tag{27}$$

where F is a positive function of the indicated dimensionless variables (no other independent ones can be formed from E,  $\tau$  and  $\lambda_c$ ). If the packet's duration is long in some sense, we can think of the flow of information as a steady state with flow rate  $\dot{\mathcal{I}} = \mathcal{I}/\tau$  and power  $P = E/\tau$ . The channel capacity  $\dot{\mathcal{I}}_{\text{max}}$  deriving from (27) should obviously depend only on P and  $\lambda_c$ , but not E and  $\tau$  separately. This is possible only if F is homogeneous of degree 1/2 with respect to both its arguments, x and y. But the most general such function can be written  $F(x,y) = \sqrt{y} \, f(x/y)$  with f(z) another positive function. Thus

$$\dot{\mathcal{I}}_{\text{max}} = (P/\hbar)^{1/2} f(\lambda_c^2 P/c^2 \hbar). \tag{28}$$

We are being cavalier in treating  $\tau$  in these proceedings as if it were dimensionless. However, it is clear that Eq. (28) is dimensionally correct. It should be remarked that f(z) should be a monotonically increasing function because the longer the cutoff  $\lambda_c$ , the more modes are employed in the information transport for one and the same P.

Let us consider the limit  $\lambda_c \to \infty$ . The capacity of such a cutoff-free channel is known [50, 57]. Regardless of the dispersion relation obeyed by the waves

$$\dot{\mathcal{I}}_{\text{max}} = (n\pi P/3\hbar)^{1/2} \log_2 e,$$
 (29)

where n stands for an effective number of information carrier species. Comparing with Eq. (28) we conclude that

$$f(\infty) = (n\pi/3)^{1/2} \log_2 e. \tag{30}$$

Evidently Eq. (29) should be applicable for any  $\lambda_c$  provided P is sufficiently large. By comparing with Eq. (22) it is clear that the case before us is the high power case of the black hole derived bound (23). Indeed, bound (25) is found to exceed the capacity (29) even for P somewhat below  $P_c$  and for n as large as  $\mathcal{N}$  ( $\mathcal{N}$  is evidently of the order of the maximum n allowed).

Passing now to the low P limit of capacity (28), we enter the  $P \ll P_c$  regime for which bound (24) is relevant. Comparing the two we conclude that

$$f(0) < [\pi(\nu - 1)\bar{\Gamma}\mathcal{N}/60]^{1/2} \log_2 e.$$
 (31)

This inequality together with Eq. (30) are generally consistent with the requirement of monotonic behaviour of

f(z). A problem could arise only if (31) is very close to saturation and  $\mathcal{N}$  is larger than 20 (because we may always chose n=1, for example). However, there is no reason to expect that bound (24) is anywhere close to equality, and the number of massless species in nature is, after all, rather modest.

### IX. SUMMARY

The GSL is the unifying thread of this review. We have seen why it is required for black hole containing systems. It differs from the ordinary second law in that black hole horizon area becomes a proxy for the entropy that has gone beyond the horizon. This black hole entropy complicates the usual question of consistency between unitary evolution and the irreversibility required by the second law, thus engendering the information paradox. We have seen that there exist several resolutions to this quandary in the spirit of unitary evolution, but that as a practical matter, information is lost in black holes.

The GSL provides a good way to obtain Susskind's form of the holographic bound on the information storage capacity of a spatially finite system. We have seen that this simple bound can fail in certain circumstances: it must then be replaced by Bousso's covariant form of the holographic principle. This very general bound is one way to get at the universal bound on the entropy of any finite weak self-gravity system, which bound is much tighter than the primitive holographic bound. We have also supplied a GSL based derivation of the universal entropy bound. The holographic bounds are intimately related to 't Hooft's holographic principle, the claim that the physics of a system is equivalent to a theory restricted to its spatial boundary, but this principle, thus far only partly established, is a stronger claim than those made by the various entropy bounds.

Finally, we have seen how to apply the GSL to the issue of quantum communication channel capacity. In contrast to the direct calculation of the entropy for a precisely specified channel, as customary in information theory, we have given an example of how to derive a bound on such entropy for a more vaguely specified channel by applying the GSL to its interaction with a black hole. Variations of this example should be illuminating.

#### Acknowledgments

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