18 June, 2019

Homework

Problem 5

5.1

$\dfrac{\text{open loop TF}}{1 - \text{loop TF}} \leftarrow$ closed (only feedback part)

a) $G_{YR}(s) = \dfrac{Y(s)}{R(s)}$

b) $G_{YR}(s) = \dfrac{G_C(s) \quad G_P(s)}{1 + G_P(s) G_C(s)}$ ($1 + [..]$ because the sign in the adder is $\ominus$ve)

c) $G_{ER}(s) = \dfrac{E(s)}{R(s)}$; $= \dfrac{1}{1 + G_C(s) G_P(s)}$

d) $G_{UR}(s) = \dfrac{U(s)}{R(s)}$; $G_{UR}(s) = \dfrac{G_C(s)}{1 + G_C(s) G_P(s)}$

e) $G_{YD}(s) = \dfrac{Y(s)}{D(s)}$; $G_{YD}(s) = \dfrac{G_P(s)}{1 + G_C(s) G_P(s)}$

5.2

$G_P(s) = \dfrac{B(s)}{A(s)}$ & $G_C(s) = K_C$

b) $G_{YR} = \dfrac{G_C \quad G_P}{1 + G_P G_C} = \dfrac{\frac{B}{A} \cdot K_C}{1 + \frac{B}{A} K_C}$

$\dfrac{B K_C}{A}$; $\dfrac{A + B K_C}{A}$; $\dfrac{B K_C}{\cancel{A}} \cdot \dfrac{\cancel{A}}{A + B K_C} = \dfrac{B K_C}{A + B K_C}$

c) $G_{ER} = \dfrac{1}{1 + G_C(s) G_P(s)} = \dfrac{1}{1 + K_C \cdot \frac{B}{A}}$; $\dfrac{A}{A + B K_C} = \dfrac{A(s)}{A(s) + B(s) K_C(s)}$

d) $G_{UR} = \dfrac{G_C(s)}{1 + G_C(s) G_P(s)}$; $K_C \cdot \dfrac{A(s)}{A(s) + B(s) K_C(s)} =$

$= \dfrac{K_C \cdot A(s)}{A(s) + B(s) \cdot K_C}$

e) $[..] \Rightarrow \dfrac{B}{A} \cdot \dfrac{A}{A + B K_C} \Rightarrow \dfrac{B \cancel{A}}{\cancel{A}(A + B K_C)} = \dfrac{B}{A + B K_C}$

5.3

$$G_{ER}(s) = \frac{E(s)}{R(s)}$$

$E(s) = G_{ER}(s) \cdot R(s)$

$R(s) \Rightarrow$ given as a ~~trans~~ step function, which converted to (s)-domain given as $1/s$

$R(s) = 1/s$.

$$E(s) = \frac{1}{1 + G_c \cdot G_p} \cdot \frac{1}{s} = \frac{A(s)}{A(s) + K_c\, B(s)} \cdot \frac{1}{s}$$

5.4

$e_\infty = ?$

$$e_\infty = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\, E(s)$$

$$e_\infty = sE(0) = \frac{A(0)}{A(0) + K_c \cdot B(0)} \cdot \frac{1}{s} \cdot s = \frac{A(0)}{A(0) + K_c B(0)}.$$

5.5

$B(s) = 4 \qquad A(s) = s^2 + 3s + 2$

a) $e_\infty = 0{,}2$; $\quad e_\infty = \dfrac{A(0)}{A(0) + K_c B(0)}$

$$0{,}2 = \frac{s^2 + 3s + 2}{s^2 + 3s + 2 + K_c \cdot 4} = \frac{0 + 0 + 2}{0 + 0 + 2 + 4K_c} = \frac{2}{2 + 4K_c} = 0{,}2$$

$$0{,}2\,(2 + K_c\, 4) = 2 \Rightarrow 2 + K_c\, 4 = \frac{2}{0{,}2} \Rightarrow \boxed{K_c = 2}$$

b) $$0{,}03 = \frac{2}{2 + 4K_c} \Rightarrow 2 + 4K_c = \frac{2}{0{,}03} \Rightarrow 4K_c = 64{,}67$$

$$K_c = 16{,}15$$

c) $$0 = \frac{2}{2 + 4K_c} = 0 = \frac{1}{2 + K_c \cdot 4} \; ; \quad \frac{1}{\infty} = 0.$$

therefore $K_c = \infty$

Problem 6

6.1 a) $G_i(s) = \dfrac{I_A(s)}{U_D(s)} \Rightarrow$

$$i_D(t) = L \cdot \frac{di_A}{dt} + R i_A(t) + u_D(t)$$

$$L \cdot s \cdot I_A(s) - 0 + R \cdot I_A(s) = U_D(s)$$

$$L s I_A + R I_A = U_D(s)$$

$$\frac{I_A(s)}{U_D(s)} = \frac{1}{L \cdot s + R}$$

$$G_i(s) = \frac{1}{L \cdot s + R} \Rightarrow \frac{1}{0{,}001 s + 1}$$

b) $G_M(s) = \dfrac{w(s)}{\tau_A(s)}$

$$w(t) \Rightarrow J \cdot \frac{dw}{dt} + \alpha w(t) = \tau_M(t) - \tau_L(t)$$

Based on the block diagram, $\tau_A = \tau_M - \tau_L \Rightarrow \tau_M = \tau_A + \tau_L$

$$J \cdot \frac{dw}{dt} + \alpha w(t) = \tau_A + \cancel{\tau_L} - \cancel{\tau_L}$$

$$J \cdot s \cdot w(s) - 0 + \alpha w(s) = \tau_A(s)$$

$$w(s)\,(J \cdot s + \alpha) = \tau_A(s)$$

$$G_M = \frac{w(s)}{\tau_A(s)} = \frac{1}{Js + \alpha} \Rightarrow \frac{w(s)}{\tau_A(s)} = \frac{1}{0{,}01 s + 0{,}01}$$

c) $G_u(s) = \dfrac{N(s)}{U_A(s)}$;

$$\frac{N(s)}{U_A(s)} = \frac{\overset{G_1}{(\text{current dyn.})} \cdot K_G \cdot \overset{G_2}{(\text{Mech})} \cdot \frac{60}{2\pi}}{1 + (\text{curr. dyn.})(K_G)(\text{Mech})(K_G)}$$

$G_1 = G_I$; $G_2 = G_{IM}$

$G_I = \dfrac{1}{Ls+R}$ $\quad G_{IM} = \dfrac{1}{Js+\alpha}$

$$U_A(s) = \frac{\frac{1}{Ls+R} \cdot K_G \cdot \frac{1}{Js+\alpha} \cdot \frac{60}{2\pi}}{1 + \frac{1}{Ls+R} \cdot K_G \cdot \frac{1}{Js+R} \cdot K_G} \quad \begin{matrix} 1 \\ 3 \\ 2 \end{matrix}$$

1 $\dfrac{60 K_G}{2\pi (Ls+R)(Js+\alpha)}$

2 $1 + \dfrac{K_G^2}{(Ls+R)(Js+R)} = \dfrac{(Js+R)(Ls+R) K_G^2}{(Ls+R)(Js+R)}$

3 $\dfrac{\overset{30}{\cancel{60}} \cancel{K_G}}{\cancel{2}\pi (Ls+R)(\cancel{Js+\alpha})} \cdot \dfrac{\cancel{(Ls+R)}\cancel{(Js+R)}}{(Js+R)(Ls+R) K_G^2} = \dfrac{30}{\pi K_G (Js+R)(Ls+R)}$

$$\Rightarrow \frac{30}{\pi \cdot 0{,}1 (0{,}01s+1)(0{,}001s+1)} = \frac{30}{\pi} \cdot \frac{1}{0{,}1\left(\frac{s^2}{10.000} + \frac{1s}{100} + \frac{s}{1000} + 1\right)}$$

$$= \frac{30}{\pi} \cdot \frac{1}{\frac{s^2}{100000} + \frac{s}{1000} + \frac{s}{10000} + 0{,}1} = \frac{30}{\pi} \cdot \frac{1}{\frac{s^2 + 100s + 10s + 10000}{100000}} =$$

$$= \frac{30}{\pi} \cdot \frac{100000}{s^2 + 132s + 10000}$$

There's a calculation error somewhere.

Right answer: $\boxed{\dfrac{30}{\pi} \cdot \dfrac{10000}{s^2 + 1001s + 2000}}$

d) $G_{1L}(s) = \dfrac{N(s)}{T_L(s)}$

$$G_L(s) = \frac{G_{12} \cdot \frac{60}{2\pi}}{1 + G_{12} \cdot K_{G_1} \cdot G_{11} \cdot K_{G_1}} =$$

$\frac{1}{10000}s^2 + 1{,}0100s + \frac{1}{100}$

$$\Rightarrow \frac{30}{(Js+\alpha)\pi} \quad ; \quad 1 + \frac{1}{(Js+\alpha)} \cdot \frac{1}{(Ls+R)} \cdot K_{G_1}^2 =$$

$$= \frac{(Js+\alpha)(Ls+R) + K_{G_1}^2}{(Js+\alpha)(Ls+R)}$$

$$\frac{30}{\pi} \cdot \frac{1}{\cancel{(Js+\alpha)}} \cdot \frac{\cancel{(Js+\alpha)}(Ls+R)}{(Js+\alpha)(Ls+R) + K_{G_1}^2} = \frac{30(Ls+R)}{\pi\left(JLs^2 + RJs + \alpha Ls + Rs + K_{G_1}^2\right)}$$

$$\frac{30}{\pi} \cdot \frac{(0{,}001s+1)}{10^{-5}s^2 + 0{,}01s + 10^{-5}s + s + 0{,}01} = \frac{30}{\pi} \frac{0{,}001s+1}{10^{-5}s^2 + 1{,}0100s + 0{,}01}$$

$$= \frac{30}{\pi} \frac{0{,}001s+1}{\frac{s^2 + 10100{,}1s + 100}{10000}} \Rightarrow \frac{30}{\pi} \cdot \frac{10s + 10000}{s^2 + 10100{,}1s + 100}$$

Algebraic Mistake somewhere.

Correct answer: $G_L(s) = \dfrac{100s + 100000}{s^2 + 1001s + 2000} \cdot \dfrac{30}{\pi}$

6.2

$$G_u(s) = \frac{10000}{s^2 + 1001s + 2000} \cdot \frac{30}{\pi}$$

$$\lim_{s \to \infty} G_u(s) = \frac{10000}{\infty} = 0 \rightsquigarrow \underline{\text{stable}}$$

6.3

$N(s) = ?$ @ $U_A = 30V$

$$G_u(s) = \frac{N(s)}{U_A(s)} \Rightarrow N(s) = G_u(s) \cdot \overset{30V}{U_A(s)}$$

$$N(s) = \frac{30}{\pi} \cdot \frac{10000}{s^2 + 1001s + 2000} \cdot 30V$$

$$n_\infty = \lim_{s \to 0} N(s) = \frac{30}{\pi} \cdot 30V \cdot \frac{10000}{0 + 0 + 2000} = 1432{,}4 \text{ rpm}$$

6.4

$N(s) = ?$ @ $\tau_L = 0{,}3Nm$

$$G_L(s) = \frac{N(s)}{\tau_L(s)} \Rightarrow N(s) = G_L(s) \cdot \overset{0{,}3Nm}{\tau_L(s)}$$

$$n_\infty = \lim_{s \to 0} N(s) = \frac{30}{\pi} \cdot 0{,}3Nm \cdot \frac{10.000}{2000} = 14{,}324 \text{ rpm}$$

13 July 2019

Homework

Control Systems 1.

Problem 12.1

$$G(s) = \frac{k}{1.5} \cdot \frac{1{,}5s+1}{s} \cdot \frac{e^{-0{,}5s}}{s+2}$$

$$G(j\omega) = \frac{k}{1.5} \cdot \frac{j1{,}5\omega+1}{j\omega} \cdot \frac{e^{-0{,}5j\omega}}{j\omega+2}$$

$$A_{dB} = |G(j\omega)| = \frac{K}{1{,}5} \cdot \frac{\sqrt{(1{,}5\omega)^2+1^2}}{\sqrt{\omega^2}} \cdot \frac{e^{\sqrt{(-0{,}5\omega)^2}}}{\sqrt{\omega^2+2^2}} =$$

$$\rightarrow e^{j\theta} = \cos(\theta) + j\sin\theta$$

$$= \frac{k}{1{,}5} \cdot \frac{\sqrt{2{,}25\omega^2+1}}{\omega} \cdot \frac{1}{\sqrt{\omega^2+4}} \qquad \Rightarrow \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = \underline{\underline{1}}$$

$$\phi = Arg(G(j\omega)) = \tan^{-1}\left(\frac{0}{k}\right) - \tan^{-1}\left(\frac{0}{1{,}5}\right) + \tan^{-1}\left(\frac{1{,}5\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{0}\right) +$$

$$(0{,}5\omega)\,rad = \left(0{,}5\omega \cdot \frac{180}{\pi}\right)^\circ$$

$$+ \left(-0{,}5\omega \cdot \frac{180}{\pi}\right)^\circ - \tan^{-1}\left(\frac{\omega}{2}\right)$$

Problem 12.2

$A(\omega_{gc}) = 1 \quad @ \quad \omega_{gc} = 2\ ^1/_s$

$$A(2) = 1\ ;\ 1 = \frac{2k}{3} \cdot \frac{\sqrt{2{,}25 \cdot 2^2+1}}{2} \cdot \frac{1}{\sqrt{2^2+4}} = \frac{k}{1{,}5} \cdot \frac{3{,}16}{2} \cdot \frac{1}{\sqrt{4{,}4}}$$

$$1 = \frac{\sqrt{10} \cdot k}{1{,}5\sqrt{8} \cdot 2} \Rightarrow \boxed{K = 2{,}68}$$

Problem 12.3

$$\phi(2) = \cancel{\tan^{-1}\left(\frac{0}{2{,}68}\right)}^{0} - \cancel{\tan^{-1}\left(\frac{0}{1{,}5}\right)}^{0} + \tan^{-1}\left(\frac{1{,}5 \cdot 2}{1}\right) - \cancel{\tan^{-1}\left(\frac{2}{0}\right)}^{\infty = 90^\circ} + \left(-0{,}5 \cdot 2 \cdot \frac{180}{\pi}\right) - \tan^{-1}\left(\frac{2}{2}\right) =$$

$$= 71{,}56^\circ - 90^\circ - 57{,}3^\circ - 45^\circ = -120{,}74^\circ$$

Problem 12.4

ω	0,1	1	10	100	1000
A_{dB}	19,1	3,17	-11,6	-31,4	-51,4
ϕ	-87,2	-88	-368	-2950	

Sketch provided in the worksheet.

Problem 13.1

$K_R = 5 \qquad T_D = 0{,}5s \qquad T_1 = 2s \qquad T_V = 0{,}1s$

[skipped]

Problem 14.1

$$G_c(s) = \frac{s+1}{s} \qquad G_p(s) = \frac{3}{s^2+3s+2}$$

$$G_0(s) = \frac{s+1}{s} \cdot \frac{3}{s^2+3s+2} = \frac{\cancel{(s+1)}}{s} \cdot \frac{3}{\cancel{(s+1)}(s+2)} = \frac{3}{s(s+2)}$$

Problem 14.2 & 14.3

$$G_0(j\omega) = \frac{3}{j\omega(j\omega+2)} \; ; \qquad A_{dB} = |G(j\omega)| = \frac{\sqrt{3^2}}{\sqrt{(j\omega)^2}} \cdot \frac{\sqrt{1^2}}{\sqrt{(j\omega)^2+2^2}} =$$

$$= \frac{3}{\omega} \cdot \frac{1}{\sqrt{\omega^2+4}}$$

$$\phi = Arg(G(j\omega)) = \underbrace{\tan^{-1}\left(\frac{0}{3}\right)}_{0} + \underbrace{\tan^{-1}\left(\frac{0}{1}\right)}_{0} - \tan^{-1}\left(\frac{\omega}{0}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) =$$

$$= -\underbrace{\cancel{\tan^{-1}(\omega)}}_{90} - \tan^{-1}\left(\frac{\omega}{2}\right) \Rightarrow -90° - \tan^{-1}\left(\frac{\omega}{2}\right)$$

14.4

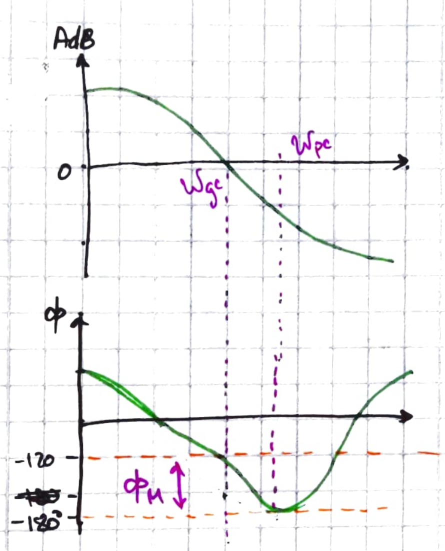
$A(\omega_{gc}) = 1$; $(1)^2 = \left(\frac{3}{\omega_{gc}} \cdot \frac{1}{\sqrt{\omega_{gc}^2+4}}\right)^2 \Rightarrow 1 = \frac{9}{\omega_{gc}^2} \cdot \frac{1}{\omega_{gc}^2+4}$.

$\omega_{gc}^2(\omega_{gc}^2+4) = 9$; $\cancel{\omega_{1,2} = 0}$ $\omega_{gc}^4 + 4\omega_{gc}^2 - 9 = 0$.

$\omega_{gc}^4 = x^2 \Rightarrow x^2 + 4x - 9 = 0 \Rightarrow x = 1{,}6$

$\omega_{gc} = \sqrt{x} = \sqrt{1{,}6} = 1{,}26$ 1/s.

14,5



1. Find $A_{dB}(\omega_{gc})$ (assuming $T_i = 1$)
2. Find $\phi(\omega_{gc})$
3. $180° + \phi(\omega_{gc}) = \phi_M$

$\phi(\omega_{gc}) = -90° - \tan^{-1}\left(\frac{\omega}{2}\right) = -90° - \tan^{-1}\left(\frac{1{,}26}{2}\right) = -122{,}2°$

$\phi_M = 180° + (-122{,}2°) = \underline{57{,}8°}$

Problem 14a

$G_c(s) = \frac{k}{s}$ $\quad G_p(s) = e^{-0,5s}$

14a.1

$G_o(s) = G_c(s) \cdot G_p(s) = \frac{k}{s} \cdot e^{-0,5s}$

$G_o(j\omega) = \frac{k}{j\omega} \cdot e^{-0,5j\omega}$

14a.2

$A_{dB}\left(|G(j\omega)|\right) = \frac{K}{\sqrt{\omega^2}} \cdot 1 = \frac{K}{\omega}$

$\phi(\omega_{gc}) = \tan^{-1}\left(\frac{\cancel{0}}{K}\right)^{0} - \tan^{-1}\left(\frac{\omega}{0}\right) \cdot \left(-\frac{1}{2}\omega \cdot \frac{180}{\pi}\right) =$

$= -90^\circ - \frac{1}{2}\omega \cdot \frac{180}{\pi}$.

14a.3 Essentially, at what ω & ϕ is the system at it's limits, before becoming unstable

$A(\omega_{cr}) = 1$; $1 = \frac{K_{cr}}{\omega_{cr}}$; $K_{cr} = \omega_{cr}$.

$\phi(\omega_{cr}) = -180^\circ$; $-90^\circ - \frac{1}{2}\omega_{cr} \cdot \frac{180}{\pi} = -180^\circ \Rightarrow \omega_{cr} = \pi$.

$\boxed{\omega_{cr} = K_{cr} = \pi = 3,14}$

14a.4

$K = 0,8 K_{cr}$; $K = 0,8 \cdot 3,14 = 2,512$

$A(\omega_{gc}) = 1$; $1 = \frac{K}{\omega_{gc}} \Rightarrow \omega_{gc} = 2,512$

$\phi_M = 180^\circ + \phi(\omega_{gc})$; $\phi(\omega_{gc}) = -90^\circ - \frac{1}{2}(2,512) \cdot \frac{180^\circ}{\pi} = \cancel{-161^\circ}$ -162°

$\phi_M = 180^\circ - 162^\circ = \underline{18^\circ}$.

Problem 15

$G_c(s) = K_c \qquad G_p(s) = \frac{2}{s(s+2)}$

15.1

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\overset{①}{G_c \cdot G_p}}{\underset{②}{1 + G_c \cdot G_p}} \quad ③$$

① $G_c \cdot G_p \Rightarrow \frac{2K_c}{s(s+2)}$

② $1 + G_c \cdot G_p \Rightarrow \frac{2K_c}{s(s+2)} + 1 = \frac{2K_c + s(s+2)}{s(s+2)}$

③ $\frac{2K_c}{\cancel{s(s+2)}} \cdot \frac{\cancel{s(s+2)}}{2K_c + s(s+2)} = \frac{2K_c}{2K_c + s(s+2)}$

15.2 a) $G_{ER} = ?$ $\quad G_{ER} = 1.$ (since there's nothing going on)

$\rightarrow G_{ER} = \frac{1}{1 + G_c \cdot G_p}$

b) $G_{YD} = ?$

the rest of the solution for both a & b is on the next page

In cases where we're looking for e_∞, we care about inputs & outputs of the system.

- For disturbance: disturbance affects the output, therefore we look for G_{YD}
- For error: error is created by the input, therefore we look for G_{ER}

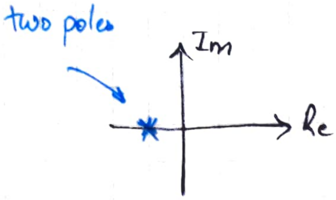
$$G_{YD} = \frac{G_p}{1 + G_c \cdot G_p} = \frac{2}{s(s+2)} \cdot \left(\frac{2K_c + s(s+2)}{s(s+2)}\right)^{-1} = \frac{2}{\cancel{s(s+2)}} \cdot \frac{\cancel{s(s+2)}}{2K_c + s(s+2)} =$$

$$= \frac{2}{2K_c + s(s+2)}$$

15.3 a)

$$G_0 = G_c \cdot G_p = \frac{2K_c}{s^2 + 2s}$$

We need to make denominator of G_{YR} in this form



$$(s+a)^2 = \underbrace{s^2 + 2as + a^2}$$

poles of function that has poles on the same spot.

$$G_{YR} = \frac{2K_c}{\underbrace{s^2 + 2s + 2K_c}}$$

Comparing the ~~too~~ two:

$2a = 2 \Rightarrow a = 1$

$2K_c = a^2 = 1 \Rightarrow K_c = \frac{1}{2}$

[continued]

15.2 a) $r = 0 \rightarrow 1 \qquad e_\infty = ?$

$$G_{ER} = \frac{1}{1 + G_c \cdot G_p} = \frac{s^2 + s}{s^2 + 2s + 2K_c}$$

$$G_{ER} = \frac{E(s)}{R(s)} \Rightarrow E(s) = G_{ER} \cdot R(s)$$

R(s) is a step function, since we know it goes from 0 to 1

$$e_\infty = \lim_{s \to 0} s\, E(s) = \lim_{s \to 0} s \cdot \boxed{\frac{1}{s}} \cdot G_{ER}$$

$$\Rightarrow \lim_{s \to 0} \frac{s^2 + s}{s^2 + 2s + 2K_c} = \frac{0}{2} = \underline{\underline{0}} \rightarrow e_\infty$$

$K_c > 0$ god knows why...

b) $d = 0 \rightarrow 1 \qquad e_\infty = ?$

$$G_{YD} = \frac{2}{2K_c + s(s+2)} \; ; \qquad G_{YD} = \frac{Y(s)}{D(s)} \Rightarrow Y(s) = G_{YD} \cdot D(s)$$

$$e_\infty = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot G_{YD} \Rightarrow \lim_{s \to \infty} \frac{2}{2K_c + s^2 + 2s} = \frac{1}{K_c} \rightarrow e_\infty$$

15.3

b) $s^2 + 2s + 2k_c = \left(\frac{s}{\omega_n}\right)^2 + \left(\frac{2\zeta}{\omega_n}\right)s + 1$

after matching the two equations, we get:

$K_c = 1.$

c) Damping ratios & natural frequency - keywords that tell us that we're dealing with a PT2 system.

Using the same comparison as in part b

$2K_c = 1$, thus $K_c = 0{,}5$

d) From 15.2b we know that $e_\infty = \frac{1}{K_c}$

When $e_\infty = 0{,}05$, $K_c = 20$

Problem 16

Taken from table on p.69

$G_c(s) = K_c\left(1 + \frac{1}{T_I s} + T_D \cdot s\right)$

$K_c = \frac{1{,}2}{K_\infty} \cdot \frac{T}{T_t}$ $\quad T_I = 2T_t \quad T_D = 0{,}5\,T_t$

$K_S \rightarrow K_\infty$

$T_I = 2T_t = 2 \cdot 0{,}8 = 1{,}6$

$T_D = 0{,}5\,T_t = \frac{1}{2} \cdot 0{,}8 = 0{,}4$

$T = 1{,}7$

$K_c = \frac{1{,}2}{K_\infty} \cdot \frac{T}{T_t} = \frac{1{,}2}{3} \cdot \frac{1{,}7}{0{,}8} = 0{,}8$

Problem 17

Solved all together

$\boxed{17.1}$ $\boxed{17.2}$ $\boxed{17.3}$

$$G_p(s) = \frac{5}{s^2 + 1{,}414s + 1} \qquad G_c(s) = K_c\left(1 + \frac{1}{T_I s}\right)$$

$t = 3s$ $\quad M_p = 10\%$ $\quad e_\infty = 0$ $\quad \to M_p = 10$ we forget that's % & take it as 10. It's just some rule that says so.

e_∞ is the error margin you get ~~from~~ between expected/desired y_∞ & the y_∞ I actually get.

$$t_r \cdot w_{gc} = 1{,}5 \Rightarrow w_{gc} = \frac{1{,}5}{3} = \frac{1}{2}.$$

$$M_p + \phi_M = 70 \Rightarrow \phi_M = 70 - 10 = 60.$$

$$G_o(s) = \frac{Y(s)}{R(s)} = \frac{5}{s^2 + 1{,}414s + 1} \cdot K_c + \frac{K_c}{T_I s} \Rightarrow$$ skipped solving it. Too tired.

$$G_o(s) = \frac{5k_c(T_I s + 1)}{T_I s(s^2 + 1{,}414s + 1)} \qquad (jw)^2 = -w^2$$

$$G_o(jw) = \frac{5k_c(T_I jw + 1)}{T_I jw(1 - w^2 + j1{,}414w)}$$

$$\sqrt{(\text{real } \#)^2 + (\text{Imag. } \#)^2}$$

$$A_{dB} = |G_o(jw)| = \frac{\sqrt{(5k_c)^2}\left(\sqrt{(T_I w)^2 + 1^2}\right)}{\sqrt{(T_I w)^2} \cdot \left(\sqrt{(1 - w^2)^2 + (1{,}414w)^2}\right)} =$$

$$= \frac{5k_c\sqrt{T_I^2 w^2 + 1}}{T_I w\left(\sqrt{(1 - w^2)^2 + 2w^2}\right)}$$

$\left.\begin{array}{l} T_I = 0{,}47s \\ w = w_{gc} = 0{,}5 \\ A(w) \overset{!}{=} 1 \end{array}\right\}$ plug into $A_{dB}(w)$ to find $K_c \Rightarrow K_c = 0{,}047$

$$\phi(w) = 0 + \tan^{-1}\left(\frac{T_I w}{1}\right) - \tan^{-1}\left(\frac{T_I w}{0}\right) - \tan^{-1}\left(\frac{1{,}414w^2}{1 - w^2}\right) =$$

$$= 0 + \tan^{-1}(T_I w) - 90° - \tan^{-1}\left(\frac{1{,}414w^2}{1 - w^2}\right)$$

$\left.\begin{array}{l} w = w_{gc} = \frac{1}{2} \\ \phi(w) = -180° + \phi_M = -120° \end{array}\right\}$ plug into $\phi(w)$ equation, solve for T_I

$T_I = 0{,}47s.$

Problem 18

based on no provided info, we assume to look for K_c & T_I for the open loop system

Given:

$\phi_M = 60°$

$$G_P(s) = \frac{20}{(s+3)(s+0,5)} \qquad G_C(s) = K_c\left(1 + \frac{1}{T_I s}\right)$$

$\phi_M = 180° + \phi(w_{gc})$

Compensating poles with controller zeroes

1. Find pole to be compensated:

$$G_P(s) = \frac{20}{(s+3)(s+0,5)} \Rightarrow 0 = (s+3)(s+0,5)$$

$$\therefore s = -3 \quad \text{or} \quad s = -0,5$$

Pole to be compensated : $\underline{\underline{-0,5}}$

(i.e. largest -ve number)

2. Rearrange numerator of $G_C(s)$ to match $(s+a)$

$$G_C(s) = K_c\left(1 + \frac{1}{T_I s}\right) = K_c\left(\frac{T_I s + 1}{T_I s}\right) = K_c\left(\frac{\cancel{T_I}\left(s + \frac{1}{T_I}\right)}{\cancel{T_I} s}\right) =$$

$$= K_c\left(\frac{s + \frac{1}{T_I}}{s}\right)$$

3. Solve for T_I using desired plant pole:

$$\left(s + \frac{1}{T_I}\right) = (s+0,5) \quad ; \quad \therefore \underline{\underline{T_I = 2}}$$

Now, we find K_c:

$$G_O = G_C \cdot G_P = \frac{20}{(s+3)(s+0,5)} \cdot \frac{K_c(s+0,5)}{s} =$$

plug in $T_I = 2$ into the new, rearranged equation of G_C

$$= \frac{20K_c}{s(s+3)} \quad ; \quad G_O(j\omega) = \frac{20k_c}{j\omega(j\omega+3)}$$

$$\phi(\omega) = 0 - 90° - \tan^{-1}\left(\frac{\omega}{3}\right) \Rightarrow \phi_M = 60 = -180° - 90° - \tan^{-1}\left(\frac{\omega_{gc}}{3}\right)$$

$$210° = \tan^{-1}\left(\frac{\omega_{gc}}{3}\right) \Rightarrow \omega_{gc} = 3\tan(210) = \sqrt{3}$$

$$A(\omega_{gc}) = 1$$

$$1 = \frac{20\,k_c}{\sqrt{3^2}\left(\sqrt{3^2} + 9\right)} \Rightarrow 36 = 20\,k_c \Rightarrow k_c = \frac{36}{20} = 1,8$$

Problem 19

$$G_P(s) = \frac{4}{s+0,5} \qquad G_C(s) = k_c\left(1 + \frac{1}{T_I s}\right)$$

19.1

$$G_o(s) = G_P(s) \cdot G_C(s) = \frac{4}{s+0,5} \cdot K_c\left(\frac{T_I s + 1}{T_I s}\right) =$$

$$= \frac{4}{(s+0,5)} \cdot \frac{K_c T_I s + K_c}{(T_I s)}$$

19.2

$$G_{YR}(s) = \frac{Y(s)}{R(s)} = \frac{G_C \cdot G_P}{1 + G_C \cdot G_P} =$$

$$= \frac{4(K_c T_I s + K_c)}{\cancel{(s+0,5)(T_I s)}} \cdot \frac{\cancel{(s+0,5)(T_I s)}}{4(K_c T_I s + K_c) + (s+0,5)(T_I s)} =$$

$$= \frac{4(K_c \cdot T_I s + K_c)}{4(K_c T_I s + K_c) + (s+0,5)(T_I s)} \Rightarrow$$

$$\Rightarrow \frac{4K_c(T_I s + 1)}{T_I s^2 + (0,5 T_I + 4K_c T_I)s + 4K_c}$$

19.3 $K_c, T_I = ?$ if closed loop has poles $-2 \pm j$

1. Reverse engineer the poles:

$$-2 \pm j \Rightarrow \frac{-4 \pm \sqrt{-4}}{2} \Rightarrow \sqrt{-4} \rightsquigarrow \sqrt{b^2 - 4ac} = \sqrt{16 - 4ac}$$

assume $a = 1$, $\underline{\underline{c = 5}}$, $b = 4$

So, we deduce:

$$s^2 + 4s + 5$$

2. Rearrange the $G_{YR}(s)$ equation so that the coefficient of s^2 is 1.

$$G_{YR}(s) = \frac{4k_c(T_I s + 1)}{T_I s^2 + 0{,}5 T_I s + 4k_c s T_I + 4k_c} \quad \Big| \begin{array}{l} \cdot \frac{1}{T_I} \\ \cdot \frac{1}{T_I} \end{array} \Rightarrow$$

$$\Rightarrow \frac{\frac{4k_c(T_I s + 1)}{T_I}}{s^2 + 0{,}5 s + 4k_c s + 4k_c \cdot \frac{1}{T_I}}$$

Now, we compare it to $s^2 + 4s + 5$

- $0{,}5s + 4k_c s = 4s \Rightarrow 4k_c \cancel{s} = 3{,}5\cancel{s} \Rightarrow \boxed{k_c = \frac{3{,}5}{4} = 0{,}875}$
- $4k_c \cdot \frac{1}{T_I} = 5 \Rightarrow 4 \cdot 0{,}875 \cdot \frac{1}{T_I} = 5 \Rightarrow \boxed{T_I = 0{,}7}$

Zeroes of a f-on → numerator

polees of a f-ou → denominator

19.4

$4k_c(T_I s + 1) = 0$

$T_I s + 1 = 0 \Rightarrow s = -1{,}43$

19.5 Skipped, because it's a MATLAB problem

19.6

$$G_{YD}(s) = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_p G_c} \Rightarrow$$

$$1 + G_p G_c = 1 + \frac{4k_c\left(1 + \frac{1}{T_I s}\right)}{(s + 0{,}5)} = \frac{(s + 0{,}5) + 4k_c\left(1 + \frac{1}{T_I s}\right)}{(s + 0{,}5)}$$

19.7

$$\frac{4}{\cancel{s+0{,}5}} \cdot \frac{\cancel{(s+0{,}5)}}{s + 0{,}5 + 4k_c\left(1 + \frac{1}{T_I s}\right)} = \frac{4}{(s+0{,}5) + 4k_c\left(1 + \frac{1}{T_I s}\right)}$$

$K_c = 0{,}875$

$T_I = 0{,}7$

$$\frac{4}{(s+0{,}5) + 4 \cdot 0{,}875\left(1 + \frac{1}{0{,}7s}\right)} ; \quad \frac{4 \cdot T_I s}{4k_c T_I s + 4k_c + T_I s^2 + 0{,}5 T_I s}$$

after expanding the brackets

$4 \cdot T_I s = 0 \Rightarrow s = 0$

$4 \cdot 0{,}875 \cdot 0{,}7s + 4 \cdot 0{,}875 + 0{,}7s^2 + 0{,}5 \cdot 0{,}7 \cdot s = 0 \quad 2{,}45s + 3{,}5 + 0{,}7s^2 + 0{,}35s = 0$

$0{,}7s^2 + 2.8s + 3{,}5 = 0 \Rightarrow s = -2 \pm j$ not sure if this answer is right though. But the procedure to solving it is right.

Problem 20

20.1 $G_c(s) = K_c\left(1 + \frac{1}{T_I s} + T_d s\right)$ $\quad G_p(s) = \frac{2}{s^2 + 0{,}1s + 1}$

$$G_o(s) = G_p(s) \cdot G_c(s) \Rightarrow \frac{2}{s^2+0{,}1s+1} \cdot K_c\left(1 + \frac{1}{T_I s} + T_d s\right) \Rightarrow$$

$$\Rightarrow \left(K_c + \frac{K_c}{T_I s} + K_c \cdot T_d s\right) \frac{2}{s^2+0{,}1s+1} \Rightarrow$$

$$\Rightarrow \frac{2K_c + \frac{2K_c}{T_I s} + 2K_c \cdot T_d s}{s^2 + 0{,}1s + 1}$$

20.2 $1 + G_o(s) = \dfrac{s^2 + 0{,}1s + 1 + 2K_c + \frac{2K_c}{T_I s} + 2K_c \cdot T_d s}{s^2 + 0{,}1s + 1}$

$$G_{yR}(s) = \frac{G_o(s)}{1+G_o(s)} = \frac{2K_c + \frac{2K_c}{T_I s} + 2K_c T_d s}{s^2 + 0{,}1s + 1 + 2K_c + \frac{2K_c}{T_I s} + 2K_c \cdot T_d s} \cdot \left|\begin{matrix} \cdot s \\ \cdot s \end{matrix}\right.$$

$$= \frac{2K_c s + \frac{2K_c}{T_I} + 2K_c T_d \cdot s^2}{s^3 + 0{,}1s^2 + 2K_c T_d s^2 + s + 2K_c s + \frac{2K_c}{T_I}} =$$

$$= \frac{2K_c s + \frac{2K_c}{T_I} + 2K_c T_d \cdot s^2}{s^3 + s^2(0{,}1 + 2K_c T_d) + s(1 + 2K_c) + \frac{2K_c}{T_I}}.$$

20.3

poles: $-1, -2, -3$

$$s^3 + s^2(0{,}1 + 2K_c T_d) + s(1+2K_c) + \frac{2K_c}{T_I} = \underbrace{(s+1)(s+2)(s+3)}_{s^3 + 6s^2 + 11s + 6}$$

$s^2 \rightarrow 0{,}1 + 2K_c T_d = 6 \Rightarrow K_c T_d = 2{,}95 \Rightarrow T_d = 0{,}59$

$s \rightarrow 1 + 2K_c = 11 \Rightarrow K_c = 5$

$\frac{2K_c}{T_I} = 6 \Rightarrow 6 T_I = 2K_c \Rightarrow T_I = \cancel{3{,}3}\ \underline{\underline{1{,}67}}$

20.4

$$2k_c s + \frac{2k_c}{T_I} + 2k_c T_d \cdot s^2 = 0$$

$K_c = 5$

$T_I = 1,67$

$T_d = 0,59$

$$2\cdot5\cdot s + \frac{2\cdot5}{1,67} + 2\cdot5\cdot0,59\,s^2 = 0$$

$$10s + 6 + 5,9s^2 = 0 \Rightarrow s = -\frac{50}{59} \pm j\frac{3\sqrt{109}}{59}$$

Problem 21

$$G_p(s) = \frac{2}{s+2} \qquad G_c(s) = \frac{b_1 s + b_0}{s+a_0}$$

21.1 $$G_0(s) = G_p(s)\cdot G_c(s) = \frac{2}{s+2}\cdot\frac{b_1 s + b_0}{s+a_0} =$$

$$= \frac{2b_1 s + 2b_0}{s^2 + a_0 s + 2s + 2a_0}$$

21.2 $$G_{yR}(s) = \frac{Y(s)}{R(s)} = \frac{G_0(s)}{1+G_0(s)}$$

$$1 + G_0(s) = \frac{2b_1 s + 2b_0}{s^2+(a_0+2)s+2a_0} + 1 = \frac{2b_1 s + 2b_0 + s^2 + (a_0+2)s + 2a_0}{s^2+(a_0+2)s+2a_0}$$

$$\frac{2b_1 s + 2b_0}{\cancel{s^2 + a_0 s + 2s} + 2a_0} \cdot \frac{\cancel{s^2+(a_0+2)s+2a_0}}{2b_1 s + 2b_0 + s^2 + (a_0+2)s + 2a_0} = \frac{2b_1 s + 2b_0}{s^2 + s(2b_1 + a_0 + 2) + 2b_0 + 2a_0}$$

21.3 $$\lim_{s\to0} G_{yR}(s) = \frac{2b_1\cdot0 + 2b_0}{0 + 0 + 2b_0 + 2a_0} = \frac{2b_0}{2b_0+2a_0} = \frac{b_0}{b_0+a_0} = K_{yR}$$

21.4

Continued ~~on the~~ on the next page

21.4 $K_{YR} = 1$; poles of $G_{YR}(s) = -4 \pm j3$

$$-4 \pm j3 = \frac{\quad}{2}$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

I need: $\sqrt{b^2-4ac} = j6$ or -36.

$b = 8$ (because $-\frac{b}{2a} \stackrel{!}{=} -4$) $\to b = 8$

$a = 1$ (simplest & safest option)

$b^2 - 4ac = -36 \Rightarrow 64 - 4 \cdot 1 \cdot c = -36 \Rightarrow \underline{\underline{c = 25}}$

$$\frac{-8 \pm \sqrt{64-4 \cdot 1 \cdot 25}}{2 \cdot 1} = \frac{-8 \pm \sqrt{-36}}{2} = \underline{\underline{-4 \pm j3}}$$

$\Rightarrow s^2 + 8s + 25.$

$$\frac{b_0}{b_0 + a_0} = 1 \Rightarrow \boxed{a_0 = 0}$$

$s^2 + s(2b_1 + \cancel{a_0} + 2) + 2b_0 + \cancel{2a_0} \Rightarrow s^2 + s(2b_1 + 2) + 2b_0$

$8 \to 2b_1 + 2 = 8 \Rightarrow \boxed{b_1 = 3}$

$2b_0 = 25 \Rightarrow \boxed{b_0 = 12{,}5}$

21.5 $G_C(s) = \frac{b_1 s + b_0}{s + a_0} = \frac{3s + 12{,}5}{s} = \frac{3s}{s} + \frac{12{,}5}{s} =$

$= 3 + \frac{12{,}5}{s} \to K_C\left(1 + \frac{1}{T_I \cdot s}\right) \to$ PI controller.

Problem 22

$$G_P(s) = \frac{5}{5s^3+11s^2+7s+1}$$

22.1

$$\frac{Y(s)}{R(s)} = K_w = \frac{1}{s^3+1{,}75s^2+2{,}15s+1}$$

$$G_{yR}(s) = \frac{G_c \cdot G_p}{1+G_c G_p} = \frac{1}{s^3+1{,}75s^2+2{,}15s+1}$$

$$1+G_c G_p = 1+\frac{G_c}{5s^3+11s^2+7s+1}$$

$$\frac{G_c(s)\cdot 5}{5s^3+11s^2+7s+1} \cdot \frac{5s^3+11s^2+7s+1}{5s^3+11s^2+7s+1+G_c} = \frac{G_c \cdot 5}{5s^3+11s^2+7s+1+G_c}$$

$$\frac{G_c \cdot 5}{5s^3+11s^2+7s+1+G_c} = \frac{1}{s^3+1{,}75s^2+2{,}15s+1}$$

$$\boxed{G_c = \frac{K_w}{1-K_w}\cdot\frac{1}{G_P}} = \frac{1}{s^3+1{,}75s^2+2{,}15s+\cancel{1}-\cancel{1}} \cdot \frac{\overset{2{,}2}{\cancel{5}s^3+\cancel{11}s^2}+\overset{1{,}4}{\cancel{7}s}+\overset{0{,}2}{\cancel{1}}}{\cancel{5}\,1} =$$

$$\Rightarrow G_c = \frac{s^3+2{,}2s^2+1{,}4s+0{,}2}{s^3+1{,}75s^2+2{,}15s}$$

from the script. (Idk where exactly from)

K_w - Weber's transfer function.

Problem 23

$$G_p(s) = \frac{5}{s^2 + 0{,}2s + 1} \qquad G_c(s) = ?$$

$t = 0{,}5s \qquad \zeta = 0{,}7$

$$G_c(s) = \frac{K_w}{1 - K_w} \cdot \frac{1}{G_p}$$

$K_w = ?$ $\boxed{\mu - r \geq n - m}$ General equation when trying to find Kw

μ - order of closed loop den.
r - — " — numerator
n - order of open-loop den.
m - — " — numerator.

$\mu - r = n - m$ - when trying to find minimal order of Kw, this equation is used.

$$\mu = n - m + r = 2 - 0 + 0 = 2.$$

since no info is given about a feed-back, it is assumed that the system is an open-loop system.

$$\omega_0 = ? \qquad \omega_0 = \frac{t_{\text{from graph}}}{t_{\text{given}}} = \frac{3}{0{,}5} = 6\ \tfrac{1}{s}$$

(given) for $\mu = 2 \rightarrow$ $$K_w = \frac{\omega_0^2}{s^2 + 1{,}4\omega_0 s + \omega_0^2} = \frac{36}{s^2 + 8{,}4s + 36}$$

$$\boxed{G_c = \frac{36}{s^2 + 8{,}4s} \cdot \frac{s^2 + 0{,}2s + 1}{5}}$$

18 July, 2019

Homework

Control Systems 1.

Problem 7

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{1+2s}$$

7.1)

$$G(j\omega) = \frac{10}{1+2j\omega} \; ; \quad A_{dw}(\omega) = \frac{\sqrt{10^2}}{\sqrt{1^2+(2\omega)^2}} = \frac{10}{\sqrt{1+4\omega^2}}$$

$$\phi(\omega) = Arg(G(j\omega)) = \cancel{\tan^{-1}\left(\frac{0}{10}\right)}^{0} - \tan^{-1}\left(\frac{2\omega}{1}\right) = 0 - \underbrace{\tan^{-1}(2\omega)}_{-45^\circ}$$

1. $u(t) = 3V \sin(\omega t)$ @ $\omega = 0{,}5$ rad/s

$y(t) = 3V \cdot 7{,}07 \sin(\cancel{45} \cdot t \cdot 0{,}5 - 45^\circ)$

2. @ $\omega = 2$ rad/s ; $\phi(\omega) = -76^\circ$ $A = 2{,}42$

$y(t) = 3V \cdot 2{,}42 \cdot \sin(2t - 76^\circ)$

3. @ $u(t) = 5V + 3V \sin(\omega t)$ @ $\omega = 1000$ rad/s

$y(t) = 5V \cdot 10 + 3V \cdot \sin(1000t - 90^\circ) \cdot 3\times10^{-3}$

$A = 3\times10^{-3}$; $\phi = -90^\circ$

Problem 10

$$G_0(s) = \frac{10}{s(s+1)(0.1s+1)} \quad ; \quad u(t) = 10V \sin(wt)$$

10.1) @ $w_{gc} = ?$ output $= 10V$.

$$G_0(w_{gc}) = \frac{10}{w_{gc}(w_{gc}+1)(0{,}1\,w_{gc}+1)} = 10V.$$

$$w_{gc}(w_{gc}+1)(0{,}1\,w_{gc}+1) = 1V \Rightarrow (w_{gc}^2 + w_{gc})(0{,}1\,w_{gc}+1) \Rightarrow$$

$$\Rightarrow 0{,}1\,w_{gc}^3 + w_{gc}^2 + 0{,}1\,w_{gc}^2 + w_{gc} = 0$$

$$0{,}1\,w_{gc}^3 + 1{,}1\,w_{gc}^2 + w_{gc} + 0 = 0.$$

$$w_1 = -1{,}13 \,; \quad w_2 = -8{,}87 \,; \quad w_3 = 0.$$

$$G_0(jw) = \frac{10}{jw(jw+1)(0.1jw+1)}$$

$$A_{dB} = |G_0(jw)| = \frac{\sqrt{10^2}}{\sqrt{(w_{gc})^2}\left(\sqrt{w_{gc}^2+1}\right)\left(\sqrt{(0{,}1\,w_{gc})^2+1^2}\right)} =$$

$$= \frac{10}{w_{gc}\left(\sqrt{w_{gc}^2+1}\right)\left(\sqrt{0{,}01\,w_{gc}^2+1}\right)} = 1$$

$$\Rightarrow \left(w_{gc} \cdot \sqrt{w_{gc}^2+1}\,\sqrt{0{,}01\,w_{gc}^2+1}\right)^2 = (10)^2 \Rightarrow w_{gc}^2\left(w_{gc}^2+1\right)\left(0{,}01\,w_{gc}^2+1\right) = 100.$$

$$\left(w_{gc}^4 + w^2\right)\left(0{,}01\,w_{gc}^2+1\right) = 100 \Rightarrow 0{,}01\,w_{gc}^6 + w_{gc}^4 + w_{gc}^4 \cdot 0{,}01 + w_{gc}^2 - 100 = 0.$$

$$x^2 = w_{gc}^6 \Rightarrow 0{,}01x^3 + 1{,}1x^2 + x - 100 = 0. \Rightarrow$$

8 im. numbers

$$w_{gc}^6 = x^2 \Rightarrow w_{gc}^3 = x \Rightarrow w_{gc} = \sqrt[3]{4{,}23} = \boxed{w_{gc} = 3{,}01}$$

some small mistake in calculations

$$\phi(w_{gc}) = \left[\frac{10}{jw(jw+1)(0.1jw+1)}\right] \Rightarrow \tan^{-1}\left(\frac{0}{}\right) - \tan^{-1}\left(\frac{w}{0}\right) - \tan^{-1}(1w) - \tan^{-1}\left(\frac{0.1w}{1}\right) =$$

$$= -90 - \tan^{-1}(3{,}01) - \tan^{-1}\left(\frac{0{,}1 \cdot 3{,}01}{1}\right) = -178{,}37^\circ$$

10.2

$$-180^\circ = -90^\circ - \tan^{-1}(\omega_{pc}) - \tan^{-1}(0.1\omega_{pc}) \Rightarrow \tan^{-1}(\omega_{pc}) + \tan^{-1}(0.1\omega_{pc}) = -270^\circ$$

10.3

$$A = 10^{\frac{A_{dB}}{20}} \quad ; \quad A_{dB}(0) = \infty \quad ; \quad A = 10^{\frac{\infty}{20}} = \underline{\underline{\infty}}$$

Problem 11

$$G_p(s) = \frac{2}{s+3} \qquad G_c(s) = \frac{b_1 s + b_0}{s + a_0}$$

11.1

$$G_{Ry}(s) = \frac{y(s)}{R(s)} = \frac{G_c(s) G_p(s)}{1 + G_c(s) G_p(s)}$$

$$G_c \cdot G_p = \frac{2}{s+3} \cdot \frac{b_1 s + b_0}{s + a_0} = \frac{2(b_1 s + b_0)}{(s+3)(s+a_0)}$$

$$1 - G_c G_p = 1 + \frac{2(b_1 s + b_0)}{(s+3)(s+a_0)} \Rightarrow \frac{(s+3)(s+a_0) + 2(b_1 s + b_0)}{(s+3)(s+a_0)}$$

$$G_{Ry}(s) = \frac{2(b_1 s + b_0)}{\cancel{(s+3)(s+a_0)}} \cdot \frac{\cancel{(s+3)(s+a_0)}}{(s+3)(s+a_0) + 2(b_1 s + b_0)} = \frac{2b_1 s + b_0 \cdot 2}{s^2 + s(a_0 + 2b_1 + 3) + 3a_0 + 2b_0}$$

11.2

$$G_{Ry}(0) = 1 \Rightarrow \frac{2b_0}{3a_0 + 2b_0} \Rightarrow 3a_0 + 2b_0 = 2b \Rightarrow a_0 = 0.$$

11.3 @ $a_0 = 0$.

$$s^2 + s(a_0 + 2b_1 + 3) + 3a_0 + 2b_0 \Rightarrow \frac{k_c}{\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$

$$= s^2 + s(2b_1 + 3) + 2b_0 \quad \Big| \cdot \frac{1}{2b_0}$$

$$\frac{s^2}{2b_0} + \frac{s(2b_1+3)}{2b_0} + 1 \Rightarrow \frac{k_c}{\left(\frac{s}{\sqrt{2b_0}}\right)^2 + 2\zeta \quad \ldots}$$

11.4 $\zeta = 0{,}707$ $\omega_n = 2.$ $b_0, b_1 = ?$

$$\frac{s}{\omega_n} = \frac{s}{2} ;$$

$$\frac{s}{\sqrt{2b_0}} = \frac{s}{2} \Rightarrow \sqrt{2b_0} = 2 \Rightarrow \underline{\underline{b_0 = 2}}$$

$$\frac{2\zeta \cdot s}{\omega_n} = \frac{(1{,}5 + b_1)\cancel{2}}{\cancel{2}b_0} \cdot \cancel{s} = \frac{\cancel{2} \cdot 0{,}707\,\cancel{s}}{\cancel{2}}$$

$$\Rightarrow \frac{1{,}5 + b_1}{2} = 0{,}707 \Rightarrow \underline{\underline{b_1 = -0{,}086}}$$

Problem 12

$$G_C(s) = \frac{k}{1{,}5} \frac{1{,}5s+1}{s} \qquad G_P(s) = \frac{e^{-0{,}5s}}{s+2}$$

12.1 $A(\omega) = ?$ $\phi(\omega) = ?$

$$G_0(s) = G_{YR}(s) = G_C \cdot G_P = \frac{k(1{,}5s+1)}{1{,}5s} \cdot \frac{e^{-0{,}5s}}{(s+2)}$$

$$G_0(j\omega) = \frac{k(1{,}5j\omega+1)}{1{,}5j\omega} \cdot \frac{e^{-0{,}5j\omega}}{(j\omega+2)}$$

$$A_{dB} = |G_0(j\omega)| = \frac{\sqrt{k^2} \cdot (\sqrt{(1{,}5\omega)^2+1})}{(1{,}5\omega)^2} \cdot \frac{e^{\sqrt{(-0{,}5j\omega)^2}}}{\sqrt{\omega^2+4}} =$$

$$= \frac{k \cdot (\sqrt{2{,}25\omega^2+1})}{2{,}25\omega^2} \cdot \frac{e^{-0{,}5\omega}}{\sqrt{\omega^2+4}}$$

$$\phi(\omega) = \cancel{\tan^{-1}(0)}^{\,0} + \tan^{-1}(1.5\omega) - \cancel{\tan^{-1}(\infty)}^{\,90} - \tan^{-1}\left(\tfrac{1}{2}\omega\right) - 0{,}5\omega\cdot\frac{180}{\pi} \Rightarrow$$

$$\Rightarrow \tan^{-1}(1{,}5\omega) - \tan^{-1}\left(\tfrac{1}{2}\omega\right) - 0{,}5\omega\cdot\frac{180}{\pi} - 90^\circ$$

12.2

$$A(\omega) = A(\omega_{gc}) = \frac{K\cdot\sqrt{2{,}25\omega^2+1}}{2{,}25\omega^2}\cdot\frac{e^{-0{,}5\omega}}{\sqrt{\omega^2+4}} = 1.$$

$$@\ \omega_{gc} = 2\ \text{rad/s} \Rightarrow \frac{K\sqrt{2{,}25\cdot 4+1}}{2{,}25\cdot 4}\cdot\frac{e^{-0.5\cdot 2}}{\sqrt{4+4}} = 1.$$

$$\frac{K\cdot\sqrt{10}}{9}\cdot\frac{e^{-1}}{\sqrt{8}} = 1 \Rightarrow K = \frac{9\sqrt{8}}{e^{-1}\sqrt{10}} = 21.88$$

Right answer: $K = 2{,}68$

Due to some sort of a calculation mistake

12.3 $\phi(\omega_{gc}) = ?$ $\quad \omega_{gc} = ?$

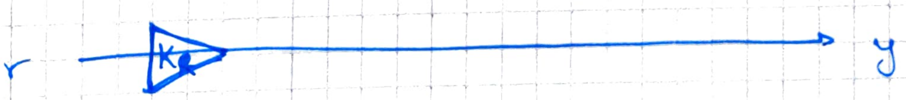
$$\phi = \tan^{-1}(1{,}5\cdot 2) - \tan^{-1}(0{,}5\cdot 2) - 0{,}5\cdot 2\cdot\frac{180}{\pi} \overset{-90^\circ}{=} \cancel{2{,}08}\quad -120{,}73^\circ$$

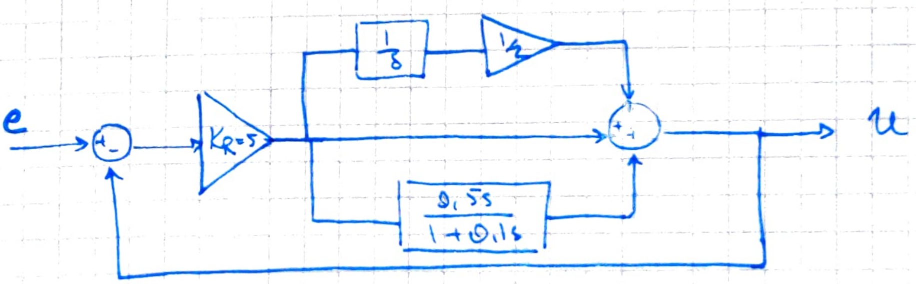
Problem 13:

$K_R = 5$ $\quad T_D = 0{,}5\,s$ $\quad T_I = 2s$ $\quad T_V = 0{,}1s$

$$G_c(s) = K_c\cdot\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1+T_V\cdot s}\right) = 5\left(1 + \frac{1}{2s} + \frac{0{,}5s}{1+0.1s}\right)$$

$$= K_c + \frac{K_c}{T_I s} + \frac{K_c T_D s}{1+T_V\cdot s}$$





Problem 14

14 $G_c(s) = \frac{s+1}{s}$ $\qquad G_p(s) = \frac{3}{s^2+3s+2}$

14.1 $G_o(s) = \frac{s+1}{s} \cdot \frac{3}{s^2+3s+2} = \frac{\cancel{(s+1)}}{s} \cdot \frac{3}{\cancel{(s+1)}(s+2)} = \frac{3}{s(s+2)}$

14.2 $G_o(j\omega) = \frac{3}{j\omega(j\omega+2)}$

14.3 $A_{dB} = |G_o(j\omega)| = \frac{\sqrt{3^2}}{\sqrt{\omega^2}\left(\sqrt{\omega^2+4}\right)} = \frac{3}{\omega\left(\sqrt{\omega^2+4}\right)}$

$\phi(\omega) = \mathrm{Arg}\left(G_o(j\omega)\right) = \cancel{\tan^{-1}\left(\frac{0}{3}\right)}^{0} - \cancel{\tan^{-1}\left(\frac{\omega}{0}\right)}^{90°} - \tan^{-1}\left(\frac{\omega}{2}\right) =$

$= -90° - \tan^{-1}(0{,}5\omega)$

14.4 $A_{dB}(\omega) = A_{dB}(\omega_{gc}) = 1.$

$\frac{3}{\omega_{gc}\sqrt{\omega_{gc}^2+4}} = 1 \Rightarrow 9 = \omega_{gc}^2 \cdot (\omega_{gc}^2 + 4)$

~~$9 = \omega_{gc}^3 + 4gc\ 4\omega_{gc}^2 \Rightarrow \omega_{gc} = 1{,}3$~~

$\omega_{gc}^4 + 4\omega_{gc}^2 - 9 = 0 \Rightarrow \omega_{gc} = 1{,}6\ 1/s.$

14.5 $\cancel{\phi(\omega_{gc})}$ $\phi_M = 180 + \phi(\omega)$

$\phi_M = 180 + (-128{,}66°) \Rightarrow \phi_M = \underline{\underline{51{,}34°}}$

Problem 14a

$$G_c = \frac{k}{s} \qquad G_p = e^{-0,5s}$$

14a.1 $G_o(s) = G_c \cdot G_p = \frac{k}{s} \cdot e^{-0,5s}$

14a.2

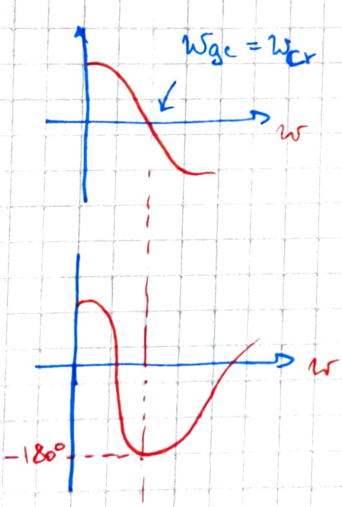
$$G_o(j\omega) = \frac{k}{j\omega} \cdot e^{-0,5j\omega}$$

$$A_{dB} = |G(j\omega)| = \frac{\sqrt{k^2}}{\sqrt{\omega^2}} \cdot e^{\sqrt{(-0,5j\omega)^2}} = \frac{k}{\omega} \cdot 1$$

$$\phi = Arg(G_o(j\omega)) = \cancel{\tan^{-1}\left(\frac{0}{k}\right)}^{0} - \cancel{\tan^{-1}\left(\frac{\omega}{0}\right)}^{90^\circ} - 0,5\omega \cdot \frac{180}{\pi}$$

$$= -90^\circ - 0,5\omega \cdot \frac{180}{\pi}$$

14a.3



$$\phi_M = 180^\circ + \phi(\omega)$$

$$\phi_M = 180^\circ + (-180^\circ) = 0.$$

$$A_{dB}(\omega_{gc}) = A_{dB}(\omega_{cr})$$

$$\phi(\omega_{cr}) = -180^\circ; \quad -180^\circ = -90^\circ - 0,5\,\omega_{cr} \cdot \frac{180}{\pi} \rightarrow$$

$$\rightarrow \omega_{cr} = \pi.$$

$$A_{dB}(\omega_{cr}) = 1 \Rightarrow \frac{K}{\omega} = 1 \Rightarrow K = \omega = \underline{\underline{\pi}}$$

14a.4 $K = 0,8\,K_{cr} = 0,8\pi$

$$A_{dB}(\omega_{gc}) = \frac{k}{\omega_{gc}} = 1 \Rightarrow \omega_{gc} = k = 0,8\pi = 2,51 \text{ rad/s}$$

$$\phi_M = 180^\circ + \phi(\omega_{gc}) = 180^\circ - 90^\circ - \frac{1}{2} \cdot \overset{2,51}{\cancel{\pi}} \cdot \frac{180}{\pi} = 18^\circ.$$

Problem 15

$$G_c(s) = K_c \qquad G_p(s) = \frac{2}{s(s+2)}$$

15.1

$$G_o = G_c \cdot G_p = \frac{2K_c}{s(s+2)} \quad ; \quad 1 + G_o = \frac{s(s+2) + 2k_c}{s(s+2)}$$

$$G_{yR}(s) = \frac{2k_c}{\cancel{s(s+2)}} \cdot \frac{\cancel{s(s+2)}}{s(s+2)+2k_c} = \frac{2k_c}{s^2+2s+2k_c}$$

$$G_o(j\omega) = \frac{2K_c}{-\omega^2 + 2j\omega + 2K_c}$$

$$A_{dB}(j\omega) = |G_o(j\omega)| = \frac{\sqrt{(2k_c)^2}}{\sqrt{(-\omega^2+2k_c)^2 + 4\omega^2}} =$$

$$= \frac{2k_c}{\sqrt{+\omega^4 - 4\omega^2 k_c + 4k_c^2 + 4\omega^2}} = \frac{2k_c}{\sqrt{\omega^4 + \omega^2(-4k_c+4) + 4k_c^2}}$$

$$\phi = \operatorname{Arg}(G_o(j\omega)) = \tan^{-1}\left(\frac{0}{k_c}\right) - \tan^{-1}\left(\frac{2\omega}{}\right.$$

15.2

a) $r = 0 \to 1 \qquad d = 0.$

$$G_{ER}(s) = \frac{E(s)}{R(s)} = \frac{1}{1 + G_c \cdot G_p} = \frac{s^2+2s}{s^2+2s+2K_c}$$

$$E(s) = G_{ER}(s) \cdot R(s) = G_{ER}(s) \cdot \frac{1}{s}$$

$$e_\infty = \lim_{s\to 0} s \cdot E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot G_{ER}(s) = \frac{0}{2k_c} = 0.$$

b) $d = 0 \to 1 \qquad r = 0$

$$G_{yD}(s) = \frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + G_c \cdot G_p} = \frac{2}{2k_c + s^2 + 2s}$$

$$Y(s) = G_{yD}(s) \cdot D(s) = G_{yD}(s) \cdot \frac{1}{s}$$

$$e_\infty = \lim_{s \to 0} s \cdot Y(s) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot Y(s) = \frac{2}{2k_c} = \frac{1}{k_c}$$

15.3

a) $s^2 + 2s + 2k_c = (s+a)^2 \Rightarrow \cancel{s^2} + 2s + 2k_c = \cancel{s^2} + 2as + a^2$

$\cancel{2}s = \cancel{2}as \Rightarrow a = 1$

$2k_c = a^2 \Rightarrow 2k_c = 1 \Rightarrow k_c = \frac{1}{2}.$

b) $\zeta = 0{,}707$; $\left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1.$

$s^2 + 2s + 2k_c = \left(\frac{s}{\omega_n}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1.$

- $s^2 = \frac{s^2}{\omega_n} \Rightarrow \omega_n = 1.$
- $\cancel{2}s = \cancel{2} \cdot 0.707 \frac{s}{\omega_0} \Rightarrow \omega_0 = 0{,}707$
- $2k_c = 1 \Rightarrow k_c = \frac{1}{2}$

c) $2k_c = 1 \Rightarrow k_c = \frac{1}{2}$

d) $\frac{1}{k_c} = 0{,}05 \Rightarrow k_c = (0{,}05)^{-1} = \underline{20}.$

Problem 16

$K_c = \frac{1{,}2}{K_s} \cdot \frac{T}{T_t}$; $T_I = 2T_t$ $\quad T_D = 0{,}5\,T_t$

$\cancel{K_c = \frac{1{,}2}{K_s}}$. $G_c(s) = K_c \cdot \left(1 + \frac{1}{T_I \cdot s} + T_D \cdot s\right)$

$K_c = \frac{1{,}2}{3} \cdot \frac{2{,}5 - 0{,}8}{0{,}8} = 0{,}85$

$T_I = 2 \cdot 0{,}8 = 1{,}6$

$T_D = 0{,}5 \cdot 0{,}8 = 0{,}4$

$T_t \rightarrow$ time it starts rising

$T \rightarrow$ time @ 63% $- T_t$

$G_c(s) = \cancel{K_c\left(1 + \frac{T}{T_T}\right.}$ $\underline{0{,}85 \cdot \left(1 + \frac{1}{1{,}6s} + 0{,}4s\right)}$