Course „Control Systems 2“

Solution to Ex. Sheet 11

Task 25

We want to design a Luenberger observer for the LTI SISO system

$$\dot{\underline{x}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x}$$

<u>Solution:</u>

a) In Taks 20 on Ex. Sheet 8 we showed that the system is completely observable, such that the observer can be designed without restrictions (arbitrary observer eigenvalues are feasible).

The state equations of the corresponding Luenberger observer are given as

$$\dot{\hat{\underline{x}}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \hat{\underline{x}} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \underline{l}(y - \hat{y})$$
$$\hat{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{\underline{x}}$$

We want to determine the values of the vector $\underline{l}$ such that the observer eigenvalues $\lambda_{o,1} = \lambda_{o,2} = -4$ result.

Design equation:

$$\det\left(\lambda \underline{I} - \underline{A} + \underline{l}\underline{c}^T\right) = (\lambda + 4)^2$$

$$\Leftrightarrow \quad \det\left(\begin{bmatrix} \lambda - 1 & 4 + l_1 \\ -2 & \lambda + 3 + l_2 \end{bmatrix}\right) = \lambda^2 + 8\lambda + 16$$

$$\Leftrightarrow \quad \lambda^2 + (2 + l_2)\lambda + (2l_1 - l_2 + 5) = \lambda^2 + 8\lambda + 16$$

Equating the coefficients:

$$\Rightarrow \quad \begin{aligned} 2 + l_2 &= 8 \\ 2l_1 - l_2 + 5 &= 16 \end{aligned}$$

Solution of this linear system of two equations:

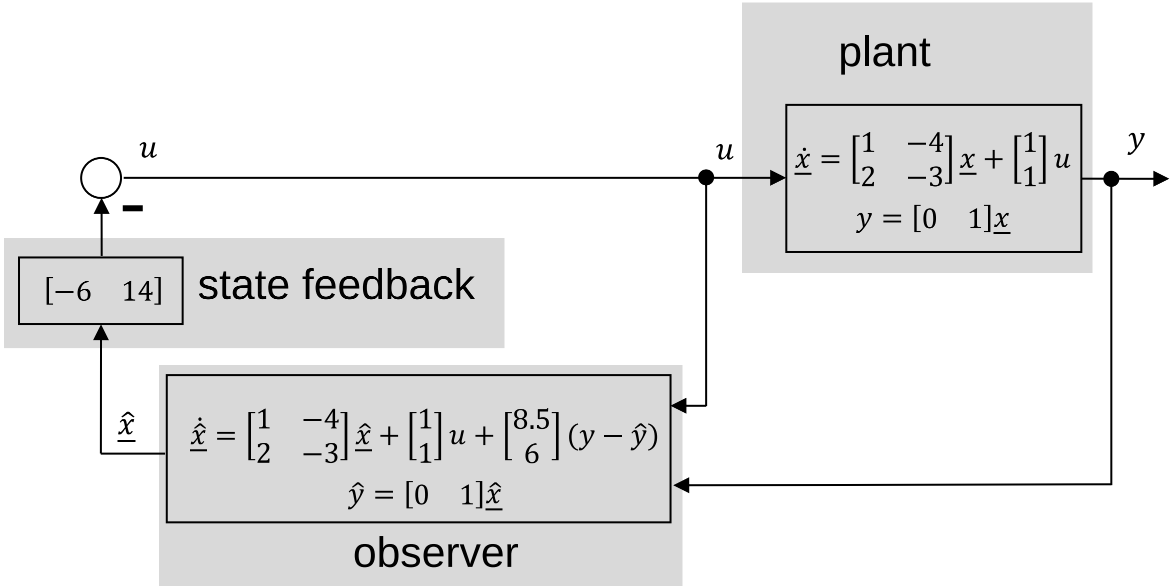
$$l_1 = 8.5 \text{ and } l_2 = 6$$

$$\Rightarrow \quad \underline{l} = \begin{bmatrix} 8.5 \\ 6 \end{bmatrix}$$

Thus, the final state equations of the requested Luenberger observer can be stated:

$$\dot{\hat{\underline{x}}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \hat{\underline{x}} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 8.5 \\ 6 \end{bmatrix} (y - \hat{y})$$
$$\hat{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{\underline{x}}$$

b) Block diagram:



c) If the plant contains a direct feedthrough part between input and output, then the control algorithm must be extended as follows:
We must generate the feedthrough-free auxiliary output variable

$$\tilde{y} = y - du$$

which can then be used as input to the observer.

<u>Here:</u>
Originally measured output: $\bar{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x} + 2u$
→ Auxiliary variable: $\tilde{y} = \bar{y} - du = \bar{y} - 2u$

→ Block diagram: