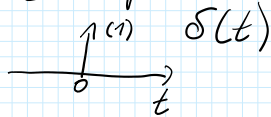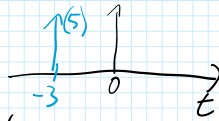# Dirac Impulse

$\uparrow^{(1)}$ $\delta(t)$
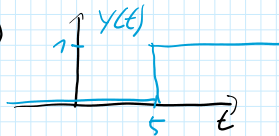
shift, weight:
$$h(t) = 5 \cdot \delta(t+3)$$

Neutral element of convolution

$$y(t) = x(t) * \delta(t) = x(t)$$

- Convolution with shifted Dirac

$$y(t) = x(t) * \delta(t-5) = x(t-5)$$

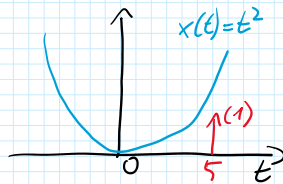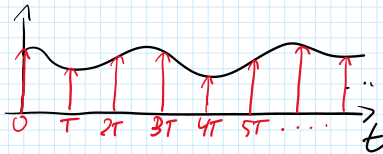example: $\varepsilon(t) * \delta(t-5)$

Multiplication with Dirac

$$x(t) = t^2$$
$$y(t) = x(t) \cdot \delta(t-5)$$
$$= x(5) \cdot \delta(t-5)$$
$$= 25 \cdot \delta(t-5)$$

outlook: sampling

"old" example

$$\varepsilon(t) = rect\left(t-\tfrac{1}{2}\right) + rect\left(t-\tfrac{3}{2}\right) + rect\left(t-\tfrac{5}{2}\right) + \cdots$$
$$= rect(t) * \delta\left(t-\tfrac{1}{2}\right) + rect(t) * \delta\left(t-\tfrac{3}{2}\right) + \cdots$$

Distributive Law of Convolution:

$$\varepsilon(t) = rect(t) * \left[\delta\left(t-\tfrac{1}{2}\right) + \delta\left(t-\tfrac{3}{2}\right) + \delta\left(t-\tfrac{5}{2}\right) + \cdots\right]$$
$$= rect(t) * \sum_{n=0}^{\infty} \delta\left(t - \tfrac{2n+1}{2}\right)$$

$$y(t) = h(t) * \varepsilon(t)$$
$$= \underline{rect(t) * rect(t)} * \sum_{n=0}^{\infty} \delta\left(t - \tfrac{2n+1}{2}\right)$$

Formular sheet table 11: $rect\left(\tfrac{t}{T}\right) * rect\left(\tfrac{t}{T}\right) = T \cdot \Delta\left(\tfrac{t}{T}\right)$
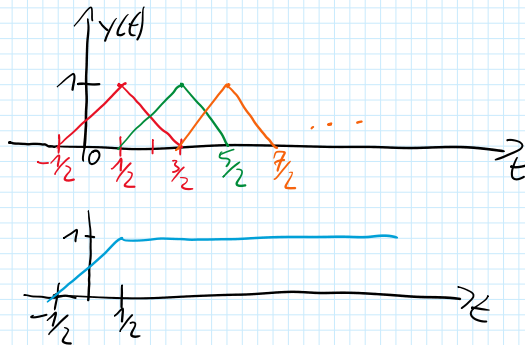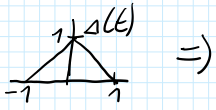
here: $T = 1$

$$y(t) = \Delta(t) * \sum_{n=0}^{\infty} \delta\left(t - \tfrac{2n+1}{2}\right)$$

$$y(t) = \Delta(t) \ast \sum_{n=0}^{\infty} \delta\left(t - \frac{2n+1}{2}\right)$$

Draw $y(t)$

$$= \Delta(t) \ast \left[\underline{\delta\left(t - \tfrac{1}{2}\right)} + \underline{\delta\left(t - \tfrac{3}{2}\right)} + \underline{\delta\left(t - \tfrac{5}{2}\right)} + \dots \right]$$

$$y(t) = h(t) \ast x(t) \qquad \text{with } h(t) = \text{rect}(t)$$
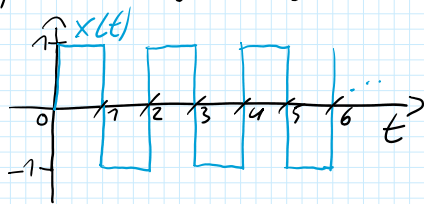
$$x(t) = \text{rect}(t) \ast \delta\left(t - \tfrac{1}{2}\right) - \text{rect}(t) \ast \delta\left(t - \tfrac{3}{2}\right) \dots$$

$$= \text{rect}(t) \ast \sum_{n=0}^{\infty} \delta\left(t - \frac{2n+1}{2}\right)(-1)^n$$

$$y(t) = \text{rect}(t) \ast \underline{\text{rect}(t)} \ast \sum_{n=0}^{\infty} \delta\left(t - \frac{2n+1}{2}\right)(-1)^n$$

formular sheet

$$= \Delta(t) \ast \sum_{n=0}^{\infty} \delta\left(t - \frac{2n+1}{2}\right)(-1)^n$$

Sketch $y(t)$:
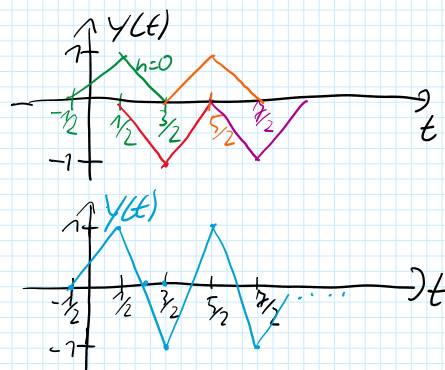
$n=0 : \Delta(t)\, \delta\left(t - \tfrac{1}{2}\right)(-1)^0$

$n=1 : \Delta(t)\, \delta\left(t - \tfrac{3}{2}\right)(-1)^1$

$n=2 : \Delta(t)\, \delta\left(t - \tfrac{5}{2}\right)(-1)^2$

$n=3 : \Delta(t)\, \delta\left(t - \tfrac{7}{2}\right)(-1)^3$

Similarity - Theorem of the Fourier - Transform
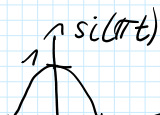
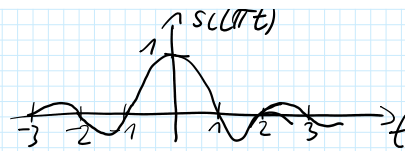example : Describe the spectrum of the function

$$x(t) = 2 \cdot si(3\pi t) \qquad \qquad si(\pi t) = sinc(t)$$

$$si(\pi t) = \frac{sin(\pi t)}{\pi t} \qquad \qquad si(0) = 1$$

Nulls: $\pi t = n \cdot t$

$\text{si}(\pi t)$

Nulls: $\pi t = n \cdot t$

sketch $x(t) = 2\,\text{si}(3\pi t)$ Nulls: $3\pi t = n \cdot t$

$$\Rightarrow t = \frac{n}{3}$$

$x(t)$

Time Domain   Frequency-Domain
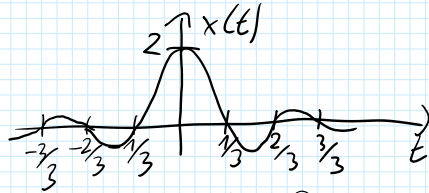
Formular Sheet: $\text{si}(\pi t) \;\circ\!\!-\!\!\bullet\; \text{rect}(f)$ Tab. 13

similarity: $x(bt) \;\circ\!\!-\!\!\bullet\; \frac{1}{|b|} \cdot X\!\left(\frac{f}{b}\right)$ Tab 12

$$b \neq 0$$

expansion $b < 1$

compression $b > 1$

$\text{si}(\pi t) = \text{sinc}(t)$?

$x(t) = 2 \cdot \text{si}(3\pi t) \;\circ\!\!-\!\!\bullet\; 2 \cdot \frac{1}{3} \cdot \text{rect}\!\left(\frac{f}{3}\right) = X(f)$

$2 \cdot \text{sinc}(3t) \;\circ\!\!-\!\!\bullet\; 2 \cdot \frac{1}{3} \cdot \text{rect}\!\left(\frac{f}{3}\right)$

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t}\,dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j2\pi f t}\,dt = \frac{1}{-j2\pi f} \cdot e^{-j2\pi f t} \Big|_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{1}{-j2\pi f}\left[e^{-j2\pi f \frac{1}{2}} - e^{j2\pi \frac{1}{2}} \right] = \frac{-2j\sin(2\pi f \frac{1}{2})}{-j\,2\pi f \frac{1}{2}} = \frac{\sin(\pi f)}{\pi f} = \text{si}(\pi f)$$

Eeuler $2j\sin x = e^{jx} - e^{-jx}$

$-2j\sin x = e^{-jx} - e^{jx}$

$$\Rightarrow \text{si}(bt) \;\circ\!\!-\!\!\bullet\; \frac{\pi}{|b|}\,\text{rect}\!\left(\frac{\pi f}{b}\right)$$

$$\text{rect}(bt) \;\circ\!\!-\!\!\bullet\; \frac{1}{|b|}\,\text{si}\!\left(\frac{\pi f}{b}\right)$$