12. 19.06.2023

Montag, 19. Juni 2023 15:46

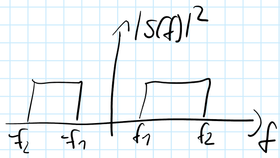
Correlation functions – Fourier Transform

CCF: $\varphi_{sg}^{E}(\tau) = s(-\tau) * g(\tau)$

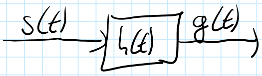
$$\Phi_{sg}^{E}(f) = S^{*}(f) \cdot G(f)$$

ACF: $\varphi_{ss}^{E}(\tau) = s(-\tau) * s(\tau)$

$$\Phi_{ss}^{E}(f) = S^{*}(f) \cdot S(f) = |S(f)|^2$$

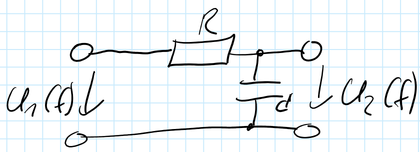


- if energy is finite: energy density
- for energy infinite: power density Spectrum

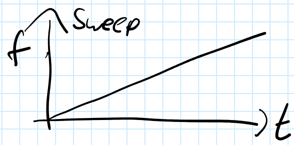


How much energy do we have at the output?

Example: · RC – Low-Pass (signal model of a cable)



· $s(t)$ has constant spectrum: $S(f) = 1$



· Dirac Impuls
· white noise
· sweep (frequency)

$$U_2(f) = U_1(f) \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = U_1(f) \cdot \frac{1}{1 + j\omega RC}$$

$$H(f) = \frac{U_2(f)}{U_1(f)} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\,2\pi f RC}$$

$$s(t) \rightarrow \boxed{h(t)} \rightarrow g(t) \quad \hat{=} \quad u_1(t) \rightarrow \boxed{h_1(t)} \rightarrow u_2(t)$$

$$G(f) = H(f) \cdot \underbrace{S(f)}_{=1}$$

$$G(f) = \frac{1}{1 + j2\pi f RC}$$

Determine the Energy Spectral Density of $G(f)$:

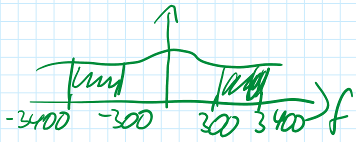
$$|G(f)|^2 = \frac{1}{\sqrt{1^2 + (2\pi f RC)^2}^{\,2}} = \frac{1}{1 + (2\pi f RC)^2}$$

How much energy do we have at the output?

Parseval: $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 \, df$

Energy within 300 Hz and 3400 Hz

$$E = \int_{300\,Hz}^{3400\,Hz} |G(f)|^2 df + \int_{-3400\,Hz}^{-300\,Hz} |G(f)|^2 df$$

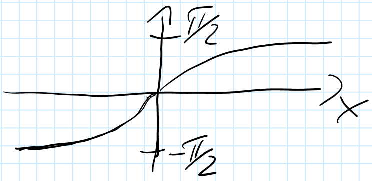


$$= 2 \cdot \int_{300\,Hz}^{3400\,Hz} |G(f)|^2 df$$

e.g. Bronstein

here: $E_g = \int_{-\infty}^{\infty} \frac{1}{1 + (2\pi f RC)^2} df = \left[\frac{2}{\sqrt{\Delta}} \cdot \arctan\left(\frac{2af}{\sqrt{\Delta}} \right) \right]_{-\infty}^{\infty}$

with $\Delta = 4a$; $a = (2\pi RC)^2$



$$\Rightarrow E_g = \frac{2}{\sqrt{\Delta}} \cdot \frac{\pi}{2} - \frac{2}{\sqrt{\Delta}} \left(-\frac{\pi}{2} \right) = 2 \cdot \frac{2\pi}{\sqrt{\Delta} \cdot 2}$$

$$= 2 \cdot \frac{\pi}{\sqrt{4 (2\pi RC)^2}} = \underline{\underline{\frac{1}{2RC}}}$$

Correlation Functions and LTI-Systems

$$s(t) \rightarrow \boxed{h(t)} \rightarrow g(t) = h(t) * s(t)$$

CCF: $\varphi_{sg}^E(\tau) = s(-\tau) * g(\tau)$
$$= s(-\tau) * h(\tau) * s(\tau)$$
$$= s(-\tau) * s(\tau) * h(\tau)$$
$$= \varphi_{ss}^E(\tau) * h(\tau)$$

$$\circ\!\!-\!\!\bullet$$

$$\phi_{sg}^E(f) = |S(f)|^2 \cdot H(f)$$

ACF: $\varphi_{gg}^E(\tau) = g(-\tau) * g(\tau)$
$$= h(-\tau) * s(-\tau) * h(\tau) * s(\tau)$$
$$= h(-\tau) * h(\tau) * s(-\tau) * s(\tau)$$
$$\varphi_{gg}^E(\tau) = \varphi_{hh}^E(\tau) * \varphi_{ss}^E(\tau)$$

Wiener-Khinchin-Theorem / Wiener-Lee

$$\circ\!\!-\!\!\bullet$$

$$|G(f)|^2 = |H(f)|^2 \cdot |S(f)|^2$$
$$\phi_{gg}^E(f) = \phi_{hh}^E(f) \cdot \phi_{ss}^E(f)$$

Symmetry of the Correlation Functions

ACF: $\varphi_{gg}^E(\tau) = \varphi_{gg}^E(-\tau)$

$$\phi_{gg}^E(f) = \phi_{gg}^E(-f)$$

CFF: $\varphi_{gs}^E(\tau) = \varphi_{sg}^E(-\tau)$

$$\phi_{gs}^E(f) = \phi_{ss}^E(f) \cdot H(-f)$$

Orthogonality:

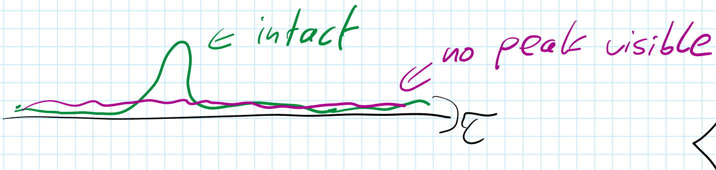
Are the signals similar to each other?

2 Prototype signal:

$s_1(t)$: intact bearing of motor

$s_2(t)$: defect bearing

- $\rho^E_{s_1 g}(\tau)$
- $\rho^E_{s_2 g}(\tau)$



just if s_1 and s_2 are orthogonal