03-Korrelationsfunktionen

Sonntag, 18. Juni 2023 18:29

①

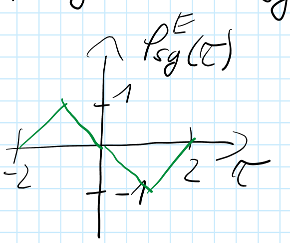
a) $\mathrm{rect}\left(\frac{t}{T}\right) * \mathrm{rect}\left(\frac{t}{T}\right) = T \cdot \Lambda\left(\frac{t}{T}\right)$

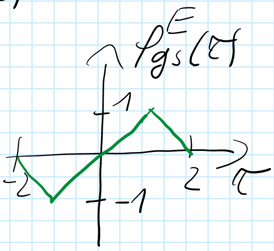
$T = 4 \quad x(t) = \frac{1}{\sqrt{4}} \cdot \mathrm{rect}\left(\frac{t}{4}\right) = \frac{1}{2}\,\mathrm{rect}\left(\frac{t}{4}\right)$

or: $x(t) = \frac{1}{2}\,\mathrm{rect}\left(\frac{t-10}{4}\right)$ — beliebig / arbitrary

b) $E_x = \varphi_{xx}^E(0) = 1$

c) $\varphi_{gs}^E(-\tau) = \varphi_{sg}^E(\tau)$





d) $\varphi_{sg}^E(0) = 0 \Rightarrow$ orthogonal

$\Rightarrow$ ja / yes

e) $\varphi_{sg}^E(\tau) = \Lambda(\tau+1) - \Lambda(\tau-1) \qquad \Lambda(t) \circ\!\!-\!\!\bullet\, si^2(\pi f)$

$\circ\!\!-\!\!\bullet$

$\Phi_{sg}^E(f) = si^2(\pi f) \cdot \left[e^{j2\pi f} - e^{-j2\pi f}\right]$

$= si^2(\pi f) \cdot \left[\cos(2\pi f) + j\sin(2\pi f) - \cos(2\pi f) + j\sin(2\pi f)\right]$

$= si^2(\pi f) \cdot 2j\sin(2\pi f)$

$\mathrm{Im}\{\Phi_{sg}^E(f)\} = 2 \cdot si^2(\pi f) \cdot \sin(2\pi f)$

$\mathrm{Re}\{\Phi_{sg}^E(f)\} = 0$

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②

a) Autokorrelation: $\varphi_{xx}^E(\tau) = \varphi_{xx}^E(-\tau)$

(2) a) Autokorrelation: $\varphi_{xx}^E(\tau) = \varphi_{xx}^E(-\tau)$

Kreuzkorrelation / cross-correlation: $\varphi_{xy}^E(\tau) = \varphi_{yx}^E(-\tau)$

b) $\Delta(t) \circ\!\!-\!\!\bullet\ si^2(\pi f)$

$\varphi_{ss}^E(\tau) = \frac{1}{3}\Delta\left(\frac{\tau}{3}\right) \circ\!\!-\!\!\bullet\ si^2(3\pi f)$

c) $E_s = \varphi_{ss}^E(0) = \frac{1}{3}$

d) „ iener Lee":

$\phi_{gg}^E(f) = \phi_{hh}^E(f) \cdot \phi_{ss}^E(f)$

e) $\phi_{sg}^E(f) = S^*(f) \cdot G(f) = S^*(f) \cdot S(f) \cdot H(f) = |S(f)|^2 \cdot H(f)$

$H(f) = j(\varepsilon(f) - \varepsilon(-f))$

$\phi_{sg}^E(f) = si^2(3\pi f) \cdot j(\varepsilon(f) - \varepsilon(-f))$

Orthogonal?

$$\int_{-\infty}^{\infty} \phi_{sg}^E(f)\, df \overset{!}{=} 0 = \int_{-\infty}^{\infty} \underbrace{\underset{\uparrow\ \text{gerade/even}}{|S(f)|^2} \cdot \underset{\nwarrow\ \text{ungerade/odd}}{H(f)}}\, df$$

symmetrisches Integral über ungerade Fkt. $= 0$

symmetrical integral over odd fct. $= 0$

$\Rightarrow$ orthogonal

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(3) a) $x(n)$ ist kausal, da $x(n) = 0$ für $n < 0$

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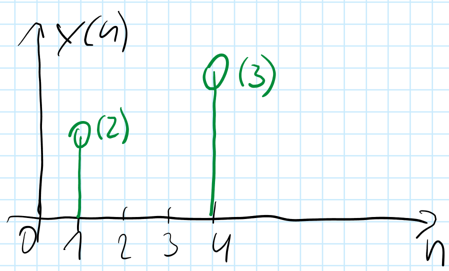
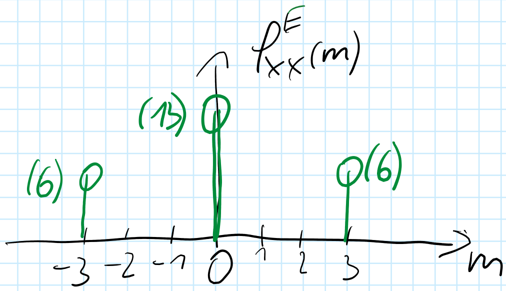
b)





$$\rho_{xx}^E(m) = 6\cdot\delta(m-3) + 13\,\delta(m) + 6\,\delta(m+3)$$

$$\rho_{xx}^E(-3) = 2\cdot 3 = 6 = \rho_{xx}^E(3)$$

$$\rho_{xx}^E(0) = 2\cdot 2 + 3\cdot 3 = 13$$

c) $$|X(f)|^2 = 6\cdot 2\cdot\cos(2\pi f\cdot 3\cdot 1) + 13$$

$$\underline{\underline{= 13 + 12\cos(6\pi f)}}$$

d) $$E_x = \rho_{xx}^E(0) = 13$$

e) $x(n)$ und $y(n)$ sind orthogonal, da sie sich im Zeitbereich nicht überlappen

$x(n)$ and $y(n)$ are orthogonal, because they don't overlap in time-domain.

f) $n_0 = 1 \Rightarrow$ Signale überlappen sich im Zeitbereich.

Signals overlap in time-domain

④ a) $\varphi^E_{sg}(\tau) = s(-\tau) * g(\tau)$

$= si(\pi\tau) * si(\pi\tau) * \delta(\tau - 42)$

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$= si(\pi\tau) * \delta(\tau-42) = si(\pi(\tau-42))$

b) $E_y = \sum_{n=-\infty}^{\infty} y^2(n) = 1 + 9 + 1 = \underline{\underline{11}}$

c) $\varphi^E_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n) = 1 \cdot 1 + 1 \cdot 3 + 1 \cdot 1 = 5$

d) $\varphi^E_{s1s1}(\tau)$: ja
- symmetrisch
- Maximum bei 0

yes
- symmetrical
- maximum at 0

$\varphi^E_{s2s2}(\tau)$: nein
- unsymmetrisch wegen t^3

no
- unsymmetrical due to t^3

$\varphi^E_{s3s3}(\tau)$: ja
- symmetrisch
- Maximum bei 0

yes
- symmetrical
- maximum at 0