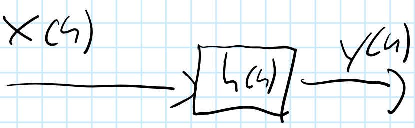
14. 26.06.2023

Montag, 26. Juni 2023 16:07

Time-Discrete Correlation Functions and LSI-Systems

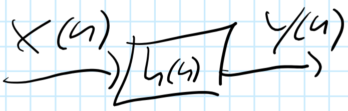
Linear-Shift-Invariant



$$y(n) = h(n) * x(n)$$

$$\begin{aligned}
\varphi^E_{xy}(m) &= x(-m) * y(m) \\
&= x(-m) * h(m) * x(m) \\
&= x(-m) * x(m) * h(m) \\
&= \varphi^E_{xx}(m) * h(m)
\end{aligned}$$

Example:



$x(n)$ & $y(n)$ are measured.

Wich attenuation and delay has our system?

System model: $h(n) = a \cdot \delta(n - n_0)$

Theory:

$$\begin{aligned}
\varphi^E_{xy}(m) &= \varphi^E_{xx}(m) * h(m) \\
&= \varphi^E_{xx}(m) * a \cdot \delta(m - n_0)
\end{aligned}$$

maximum at 0

maximum at 0

$$= a \cdot \underbrace{\varphi^E_{xx}(m-n_0)}_{\text{maximum at } n_0}$$

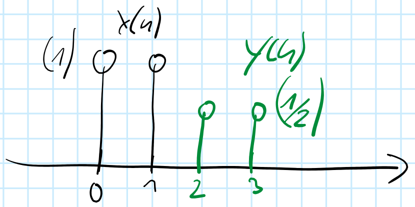
At position n_0: $\quad \varphi^E_{xy}(n_0) = a \cdot \varphi^E_{xx}(0)$

$$= a \cdot E_x$$

$$a = \frac{\varphi^E_{xy}(n_0)}{E_x} \triangleq \text{attenuation}$$

Example: $\quad x(n) = \delta(n) + \delta(n-1)$

$$y(n) = \tfrac{1}{2}\,\delta(n-2) + \tfrac{1}{2}\,\delta(n-3)$$

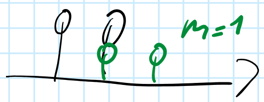


attenuation $a = 0{,}5$

delay $n_0 = 2$

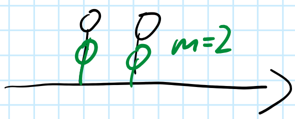
$m=0$: $\quad \varphi^E_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+0) = 0 \quad$ (orthogonal)

$m=1$:



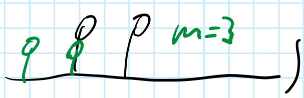
$\varphi^E_{xy}(1) = 1 \cdot \tfrac{1}{2} = \tfrac{1}{2} \quad \Big| = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+1)$

$m=2$:



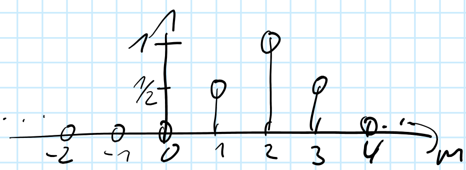
$\varphi^E_{xy}(2) = 1 \cdot \tfrac{1}{2} + 1 \cdot \tfrac{1}{2} = 1$

$m=3$:



$\varphi^E_{xy}(3) = 1 \cdot \tfrac{1}{2} = \tfrac{1}{2}$

$\varphi^E_{xy}(m) = 0 \quad$ for $m > 3$ and $m < 1$

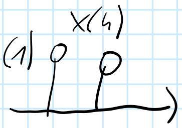


delay =? maximum of $\varphi^E_{xy}(m)$

$n_0 = 2$

attenuation = ?

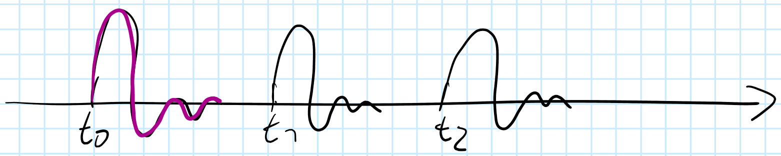
$$a = \frac{\varphi^E_{xy}(n_0)}{E_x} = \frac{1}{2}$$

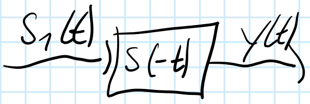


$$\Rightarrow E_x = \sum x^2(n) = 1^2 + 1^2 = 2$$

If you look for a reference signal / waveform inside a measurement, you can do a crosscorrelation!

$$\underbrace{s(t)}_{\text{reference signal}} * \left[\delta(t-t_0) + \delta(t-t_1) + \delta(t-t_2)\right] = \underbrace{s_1(t)}_{\text{meas. signal}}$$

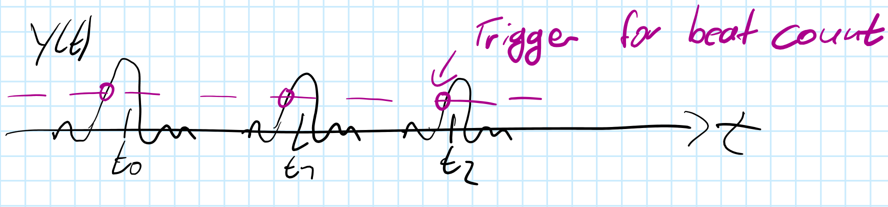




$$y(t) = s(-t) * s_1(t)$$

$$= s(-t) * s(t) * \left[\delta(t-t_0) + \delta(t-t_1) + \delta(t-t_2)\right]$$

$$= \underbrace{\underbrace{\varphi^E_{ss}(t)}_{\text{maxima at } t=0} * \left[\delta(t-t_0) + \delta(t-t_1) + \delta(t-t_2)\right]}_{\text{3 maxima at } t_0, t_1, t_2}$$



Exer. Sheet 1

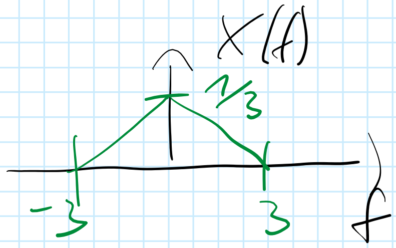
② $h_2(n) = \sum_{k=0}^{\infty} (0{,}5)^k \delta(n-k)$

$h_2(n) \circ\!\!-\!\!\bullet\; H_2(f)$

Tab 18:

$\sum_{k=0}^{\infty} a^k (n-k) = a^n \varepsilon(n) \circ\!\!-\!\!\bullet \dfrac{1}{1 - a \cdot e^{-j2\pi f T}} \quad |a| < 1$

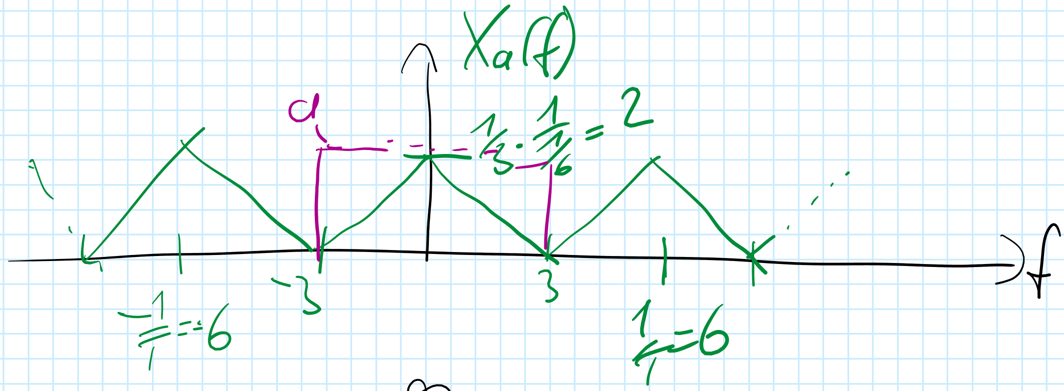
d)



$f_{max} = \pm 3 \qquad f_s \geq 2 \cdot f_{max}$

$\Rightarrow f_s = 6 \quad \Rightarrow T = \frac{1}{6}$

e)



$X_a(f) = X(f) * \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$ Tab. 14

$k = +\infty$

$$x(t) \stackrel{!}{=} y(t)$$

$$X_a(f) \cdot H(f) = X(f) = Y(f)$$

$$H(f) = a \cdot \mathrm{rect}\left(\frac{f}{6}\right)$$

$$2 \cdot a \stackrel{!}{=} \frac{1}{3}$$

$$\Rightarrow a = \frac{1}{6}$$

after Filtering: $2\Delta\left(\frac{f}{2}\right) \cdot \frac{1}{6} = \frac{1}{3}\Delta\left(\frac{f}{3}\right)$

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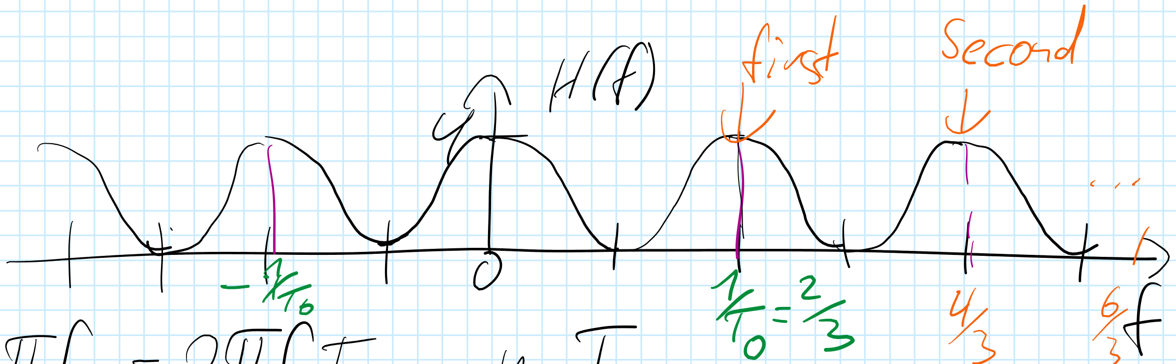
5 a)

$$H(f) = 2\cos(3\pi f) + 2$$



$$3\pi f = 2\pi f T_0 \qquad n \cdot T_0$$

$$\frac{3}{2} = T_0$$

next Periode $T_1 = \frac{3}{4}$

next after $T_2 = \frac{3}{6}$