1 $H(z) = \frac{z-a}{z-0{,}5}$

a) $z \to e^{j2\pi fT}$; $T=1 \Rightarrow z \to e^{j2\pi f}$

$$H(f) = \frac{e^{j2\pi f} - a}{e^{j2\pi f} - 0{,}5}$$

b) $H(f) = 0$

$$H(0) = \frac{e^{j0} - a}{e^{j0} - 0{,}5} = \frac{1-a}{1-0{,}5} = 0$$

$\Rightarrow 1 - a = 0$

$\underline{\underline{a = 1}}$

c) $H(f = \frac{1}{2T}) = 0$

$T = 1 \Rightarrow H(f = \frac{1}{2}) = 0$

$$H(f = 1/2) = \frac{e^{j2\pi 1/2} - a}{e^{j2\pi 1/2} - 0{,}5}$$

$\Rightarrow e^{j\pi} - a = 0$

$-1 - a = 0 \Rightarrow \underline{\underline{a = -1}}$

d) $$H(z) = \frac{Y(z)}{X(z)} = \frac{z-a}{z-0{,}5} \cdot \frac{z^{-1}}{z^{-1}} = \frac{1 - a \cdot z^{-1}}{1 - 0{,}5 \cdot z^{-1}}$$

$$Y(z) \cdot (1 - 0{,}5 \cdot z^{-1}) = X(z) \cdot (1 - a z^{-1})$$

$$y(n) - 0{,}5\, y(n-1) = x(n) - a \cdot x(n-1)$$

$$\underline{y(n) = x(n) - a \cdot x(n-1) + 0{,}5 \cdot y(n-1)}$$

$$y(n) = x(n) - a \cdot x(n-1) + 0{,}5 \cdot y(n-1)$$

② $y(n) = x(n) + b \cdot x(n-1) - a \cdot y(n-1)$

a) $Y(z) = X(z) + b \cdot X(z) \cdot z^{-1} - a \cdot Y(z) \cdot z^{-1}$

$$Y(z)\left(1 + a \cdot z^{-1}\right) = X(z)\left(1 + b \cdot z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + b \cdot z^{-1}}{1 + a \cdot z^{-1}}$$

b) für $a = 0$ und b beliebig

for $a = 0$ and b arbitrary

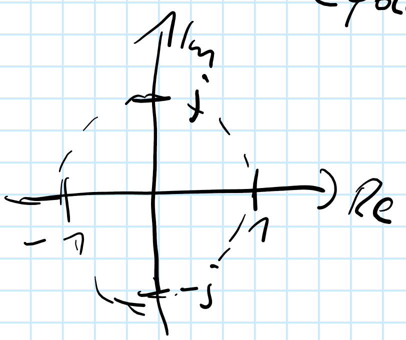
c) Instabil: Pol außerhalb Einheitskreis

instable: Pole outside unit circle

z_{pole}: $\quad 1 + a \cdot z_{pole}^{-1} = 0$

$$z_{pole} + a = 0$$

$$z_{pole} = -a$$



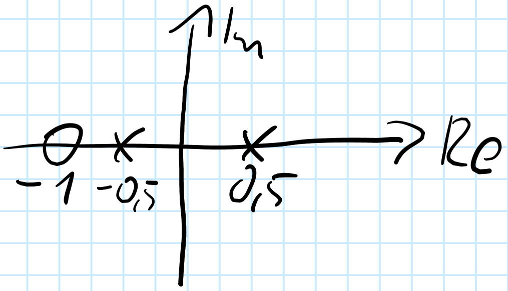
$\Rightarrow$ b beliebig & $a > 1$

b arbitrary & $a > 1$

b arbitrary & $a > 1$

3) $H(z) = \dfrac{2z+2}{z^2 - 1/4}$

a) $H(z) = \dfrac{2\cdot(z+1)}{(z-1/2)\cdot(z+1/2)} \Rightarrow H_0 = 2$



X : Pole

O : Null

b) Ja, da alle Polstellen im Einheitskreis

yes, because all poles within unit circle

$x(n) = \delta(n) - \frac{1}{2}\delta(n-1)$

c) $X(z) = 1 - \frac{1}{2}\cdot z^{-1}$

d) $Y(z) = X(z)\cdot H(z)$

$$= \left(1 - \frac{1}{2}\cdot z^{-1}\right)\cdot \frac{2(z+1)}{(z-\frac{1}{2})(z+\frac{1}{2})}$$

$$= z^{-1}\left(z - \frac{1}{2}\right)\cdot \frac{2(z+1)}{(z-\frac{1}{2})(z+\frac{1}{2})}$$

$$= z^{-1}\ \frac{2(z+1)}{} = z^{-2}\ 2(z+1)$$

$$= z^{-1}\,\frac{2(z+1)}{z+\frac{1}{2}} = z^{-2}\,\frac{2(z+1)}{1+\frac{1}{2}z^{-1}}$$

$$\underline{\underline{Y(z) = \frac{2z^{-1}+2z^{-2}}{1+\frac{1}{2}z^{-1}}}}$$

e) $$Y(z) = 2\left(z^{-1}+z^{-2}\right)\cdot\frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\bullet\!-\!\circ \qquad * \qquad \bullet\!-\!\circ$$

Tab. 20

$$y(n) = 2\left[\delta(n-1)+\delta(n-2)\right] * \left(-\tfrac{1}{2}\right)^{n}\cdot\varepsilon(n)$$

$$\underline{\underline{y(n) = 2\cdot\left(-\tfrac{1}{2}\right)^{n-1}\cdot\varepsilon(n-1) + 2\left(-\tfrac{1}{2}\right)^{n-2}}}\cdot\varepsilon(n-2)$$