1)

a) $z \to e^{j2\pi fT}$ $\quad T=1 \Rightarrow z \to e^{j2\pi f}$

$$H(z) = \frac{z-a}{z-0,5} \to H(f) = \frac{e^{j2\pi f}-a}{e^{j2\pi f}-0,5}$$

b) $H(f=0) = 0$

$$H(0) = \frac{e^{j0}-a}{e^{j0}-0,5} = \frac{1-a}{1-0,5} = 0 \rightsquigarrow 1-a=0$$

$$\underline{\underline{a=1}}$$

c) $H(f=\frac{1}{2T}) = 0$

$T=1 \Rightarrow H(f=\frac{1}{2}) = 0$

$$H(f=\tfrac{1}{2}) = \frac{e^{j2\pi\frac{1}{2}}-a}{e^{j2\pi\frac{1}{2}}-0,5} = 0$$

$$\Rightarrow e^{j\pi} - a = 0$$

$$-1-a=0 \Rightarrow \underline{\underline{a=-1}}$$

d) $$H(z) = \frac{Y(z)}{X(z)} = \frac{z-a}{z-0,5} \cdot \frac{z^{-1}}{z^{-1}} = \frac{1-a\cdot z^{-1}}{1-0,5\cdot z^{-1}}$$

$$Y(z)\cdot(1-0,5\,z^{-1}) = X(z)\cdot(1-a\cdot z^{-1})$$

$$y(n) - 0,5\,y(n-1) = x(n) - a\cdot x(n-1)$$

$$y(n) = x(n) - a\cdot x(n-1) + 0,5\cdot y(n-1)$$

2)

a) $y(n) = x(n) + b\cdot x(n-1) - a\cdot y(n-1)$

$$Y(z) = X(z) + b\cdot X(z)\cdot z^{-1} - a\cdot Y(z)\cdot z^{-1}$$

$$Y(z)\cdot(1+a\cdot z^{-1}) = X(z)\cdot(1+b\cdot z^{-1})$$

$$Y(z) \quad \underline{1+b\cdot z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + b \cdot z^{-1}}{1 + a \cdot z^{-1}}$$

b) für $a = 0$ und b beliebig

for $a = 0$ and b arbitrary

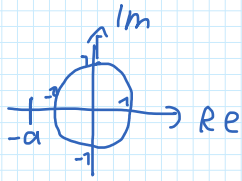
c) Instabil : Pol außerhalb Einheitskreis

instable : Pole outside unit circle

$$z_{Pole}: \quad 1 + a \cdot z_{pole}^{-1} = 0$$

$$z_{pole} + a = 0$$

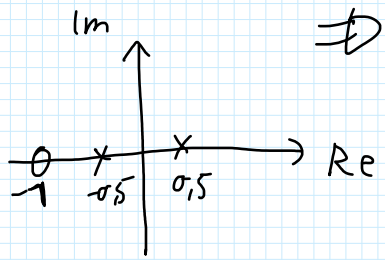
$$z_{pole} = -a$$



$\Rightarrow$ b beliebig & $a > 1$ bsp. $a = 2$

b arbitrary & $a > 1$ for example $a = 2$

3) a) $$H(z) = \frac{2 \cdot z + 2}{z^2 - \frac{1}{4}} = \frac{2 \cdot (z+1)}{(z - \frac{1}{2}) \cdot (z + \frac{1}{2})}$$



$\Rightarrow$ $H_0 = 2$

x : Pole
0 : Null

b) Ja, da alle Polstellen im Einheitskreis

yes, because all poles within unit circle

c) $$x(n) = \delta(n) - \frac{1}{2}\delta(n-1)$$

$$X(z) = 1 - \frac{1}{2} \cdot z^{-1}$$

d) $$Y(z) = X(z) \cdot H(z) = 1 - \frac{1}{2} \cdot z^{-1} \quad \frac{2(z+1)}{(\quad)(\quad)}$$

$Y(z) = \dots \qquad (z-\frac{1}{2})(z+\frac{1}{2})$

$$Y(z) = z^{-1}\left(z-\frac{1}{2}\right)\cdot\frac{2(z+1)}{(z-\frac{1}{2})(z+\frac{1}{2})}$$

$$= z^{-1}\,\frac{2(z+1)}{z+\frac{1}{2}} = z^{-2}\,\frac{2(z+1)}{1+\frac{1}{2}z^{-1}}$$

$$Y(z) = \frac{2z^{-1}+2z^{-2}}{1+\frac{1}{2}z^{-1}}$$

e)

$$Y(z) = 2(z^{-1}+z^{-2})\cdot\frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\downarrow \qquad * \qquad \downarrow$$

$$y(n) = 2[\delta(n-1)+\delta(n-2)] * \left(-\frac{1}{2}\right)^n\cdot\varepsilon(n)$$

Tab. 2 σ

$$y(n) = 2\cdot\left(-\frac{1}{2}\right)^{n-1}\cdot\varepsilon(n-1) + 2\cdot\left(-\frac{1}{2}\right)^{n-2}\cdot\varepsilon(n-2)$$