Convolution via shift & summation

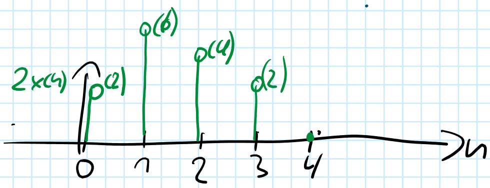
$$y(n) = x(n) * h(n) \qquad x(n) = \delta(n) + 3(n-1) + 2\delta(n-2) + \delta(n-3)$$
$$h(n) = 2\delta(n) + \delta(n-1)$$

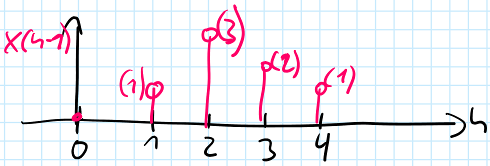
$$y(n) = x(n) * [2\delta(n) + \delta(n-1)]$$

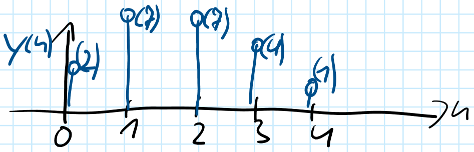
Distributive law

$$= x(n) * 2\delta(n) + x(n) * \delta(n-1)$$
$$= \underline{2 \cdot x(n)} + \underline{x(n-1)}$$

convolution with Delta-Impulse equals a shift in „time"



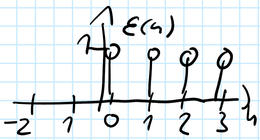




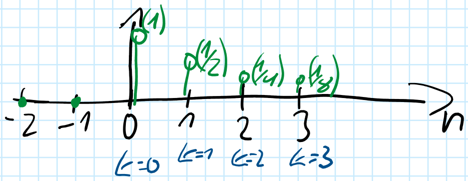
this method is much faster if $x(n)$ or $h(n)$ few values $\neq 0$

Example:

$$x(n) = \delta(n) + \tfrac{1}{2}\delta(n-1)$$
$$h(n) = \sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k \cdot \delta(n-k) = \left(\tfrac{1}{2}\right)^n \cdot \varepsilon(n) \leftarrow \text{time discrete unit step}$$



a) Draw $h(n)$ with specification of all charakteristic values.



b) Is $h(n)$ stable? According to BIBO (Bounded Input Bounded Output)

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left| \left(\tfrac{1}{2}\right)^n \cdot \varepsilon(n) \right|$$

$$\sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = 2$$

$\Rightarrow 2 < \infty \Rightarrow h(n)$ is stable

[FS: geometrical series $\sum_{n=0}^{\infty} c \cdot a^n = \frac{c}{1-a}$

c) Is $h(n)$ causal?

$h(n)$ is causal, because $h(n) = 0$ for $n < 0$

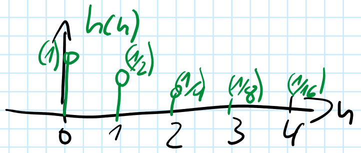
[anti causal: $h(n) = 0$ for $n > 0$

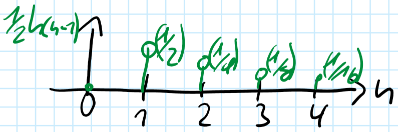
non causal: $h(n) \neq 0$ for $n > 0$ and $n < 0$

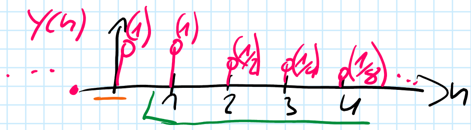
d) Determine $y(n) = x(n) * h(n) = h(n) * x(n)$

$$y(n) = \left[\sum_{k=0}^{\infty} \left(\tfrac{1}{2}\right)^k \cdot \delta(n-k)\right] * \left[\delta(n) + \tfrac{1}{2}\delta(n-1)\right]$$

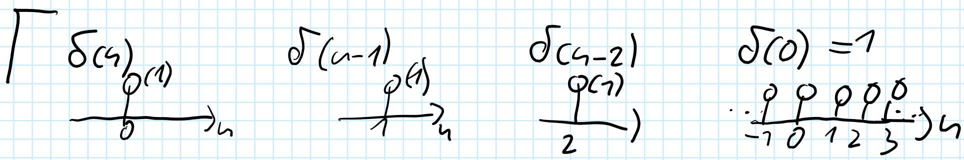
$$= h(n) + \tfrac{1}{2} h(n-1)$$







$$y(n) = \underline{\delta(n)} + \underline{\left(\tfrac{1}{2}\right)^{n-1} \cdot \varepsilon(n-1)} = \delta(n) + h(n-1)$$



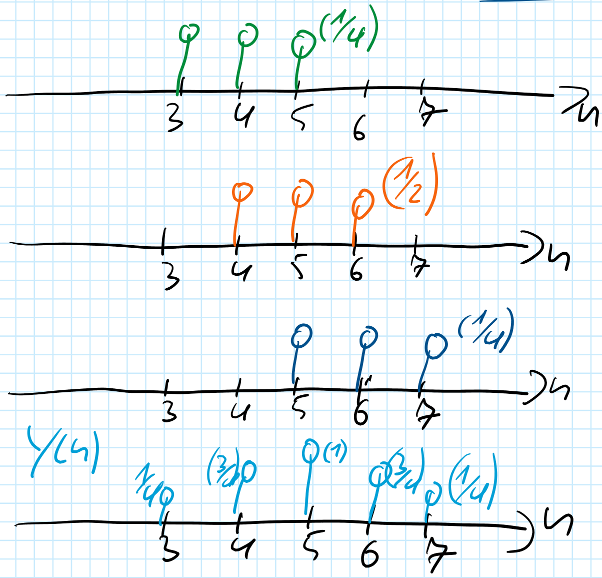
non causal Filter

$$x(n) = \delta(n-3) + \delta(n-4) + \delta(n-5)$$

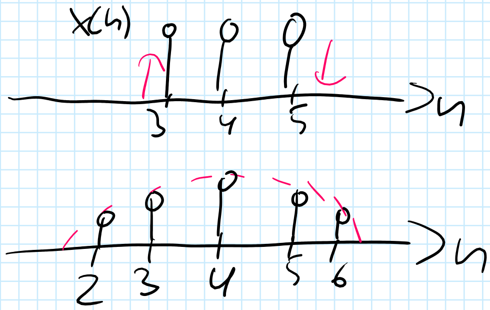
low pass filter, kausal: $h(n) = \frac{1}{4}\delta(n) + \frac{1}{2}\delta(n-1) + \frac{1}{4}\delta(n-2)$

$$y(n) = x(n) * h(n)$$

$$= x(n) * [\underline{\tfrac{1}{4}\delta(n)} + \underline{\tfrac{1}{2}\delta(n-1)} + \underline{\tfrac{1}{4}\delta(n-2)}]$$

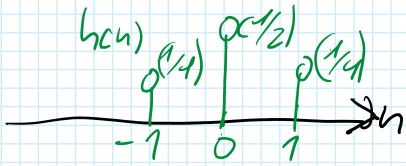


reflect:

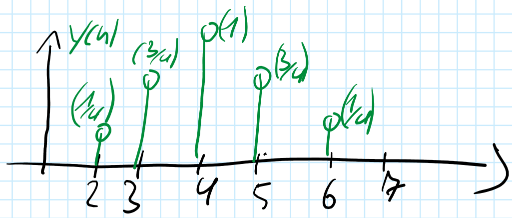


non causal

$$h(n) = \tfrac{1}{4}\delta(n+1) + \tfrac{1}{2}\delta(n) + \tfrac{1}{4}\delta(n-1)$$



y(n)



Fourier Transformation for time discrete signals

Exercise: $X(k) = \sum^{\infty} x(n) \cdot \cos\left(\frac{\pi}{N}(n+\tfrac{1}{2})k\right)$ Discrete cosinus

Exercise: $X(k) = \sum_{k=0}^{\infty} x(n) \cdot \underbrace{\cos\left(\frac{\pi}{N} (n+1/2) k\right)}_{\text{basis function } h_k(n)}$

Discrete Cosinus Transformation DCT.

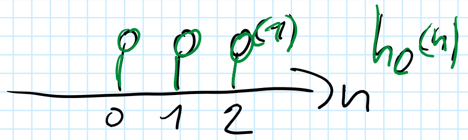
d) Derive 3 Filter for $0 \leq k < N$, $N=3$

$0 \leq n < N$

$k=0$: $n=0$: $\cos\left(\frac{\pi}{3}(0+1/2)\cdot 0\right) = 1$

$n=1$: $\cos\left(\frac{\pi}{3}\, \frac{3}{2} \cdot 0\right) = 1$

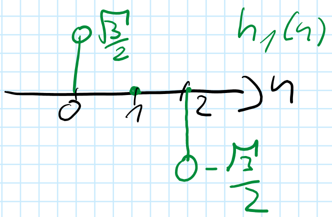
$n=2$: $\cos\left(\frac{\pi}{3}\, \frac{5}{2} \cdot 0\right) = 1$



$k=1$: $n=0$: $\cos\left(\frac{\pi}{3} \cdot \frac{1}{2} \cdot 1\right) = \frac{\sqrt{3}}{2}$

$n=1$: $\cos\left(\frac{\pi}{3} \cdot \frac{3}{2} \cdot 1\right) = 0$

$n=2$: $\cos\left(\frac{\pi}{3} \cdot \frac{5}{2} \cdot 1\right) = -\frac{\sqrt{3}}{2}$



$k=2$ $n=0$: $\cos\left(\frac{\pi}{3} \cdot \frac{1}{2} \cdot 2\right) = 1/2$

$n=1$: $\cos\left(\frac{\pi}{3} \cdot \frac{3}{2} \cdot 2\right) = -1$

$n=2$: $\cos\left(\frac{\pi}{3} \cdot \frac{5}{2} \cdot 2\right) = 1/2$