7. 15.05.2023

Montag, 15. Mai 2023 15:45

Cut off-frequencies

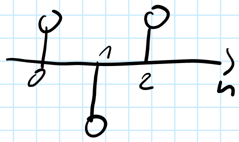
$$\sqrt{3}\,\sin(2\pi f T) = \frac{\sqrt{3}}{\sqrt{2}}$$

$$f_{c1} = \frac{1}{2\pi T}\underbrace{\arcsin\left(\frac{1}{\sqrt{2}}\right)}_{=\pi/4}$$

$$f_{c1} = \frac{\pi}{2\pi\, 4T} = \frac{1}{8T}$$

$$f_{c2} = f_1 - f_{c1}$$

$$h_2(n) = \frac{1}{2}\delta(n) - \delta(n-1) + \frac{1}{2}\delta(n-2)$$



$$H_2(f) = \frac{1}{2}e^{-j2\pi f\cdot 0T} - e^{-j2\pi fT} + \frac{1}{2}e^{-j2\pi f 2T}$$

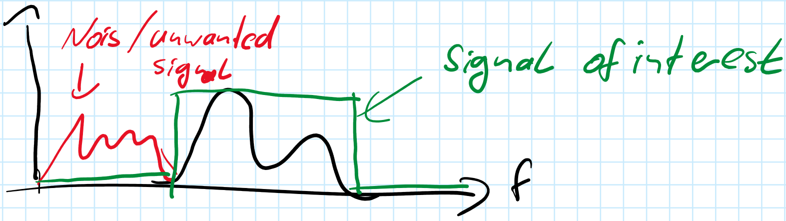
$$= e^{-j2\pi fT}\left(\frac{1}{2}e^{j2\pi fT} - 1 + \frac{1}{2}e^{-j2\pi fT}\right)$$

$$|H_2(f)| = \left|\cancel{\tfrac{1}{2}}\cdot\cancel{2}\cdot\cos(2\pi fT) - 1\right|$$

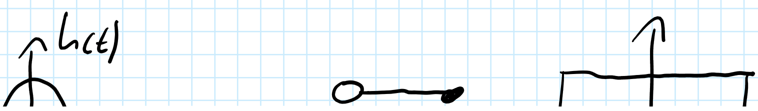
Bandpass

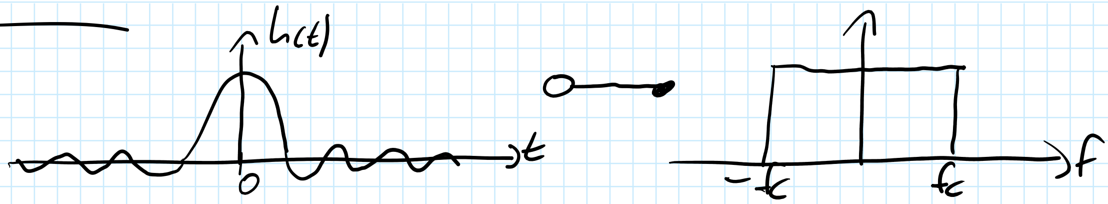
FIR Filter Design (finite impulse response)

Example



We know:





$H(f) = \mathrm{rect}\left(\frac{f}{2f_c}\right)$

$si(\pi t) ○\!\!-\!\!\bullet\ \mathrm{rect}(f)$

similarity theo. $\quad x(bt) ○\!\!-\!\!\bullet\ \frac{1}{|b|} \cdot X\left(\frac{f}{b}\right)$

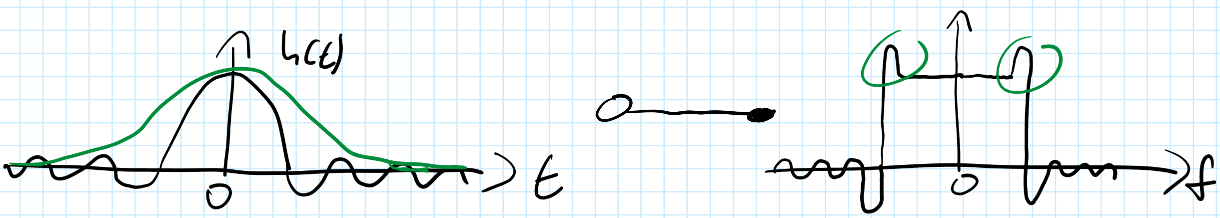
here: $\quad b = 2 \cdot f_c$

__Problems:__

- steep slopes (infinity high)
  - → infinite circuit effort
- impulse response is infinite
- not causal

__Solution:__

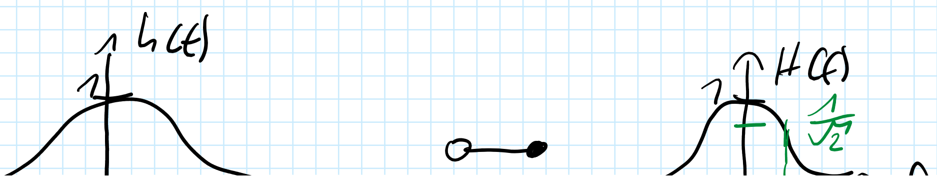
limit the response in time
typically with a window function
(eg. Hann, Hamming, Kaiser, ...)

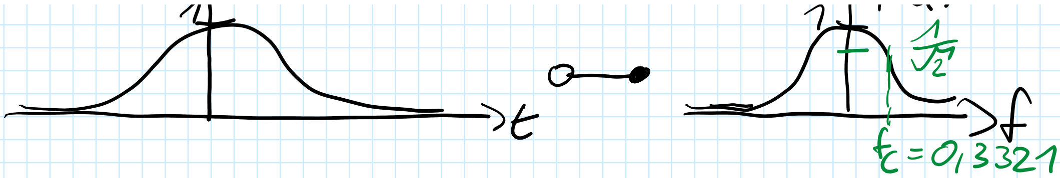


time-limited impulse response has an infinite spectrum.

__Example Exercise:__

$h(t) = e^{-\pi t^2} ○\!\!-\!\!\bullet\ H(f) = e^{-\pi f^2}$





a) Determine b $(b>0)$ so that $f_c = 1\,kHz$

similarity theorem: $x(bt) \circ\!\!-\!\!\bullet \frac{1}{|b|} X\left(\frac{f}{b}\right)$

$$x(t) = e^{-\pi t^2}$$

$$x(bt) = e^{-\pi (bt)^2} \circ\!\!-\!\!\bullet \frac{1}{|b|} e^{-\pi \left(\frac{f}{b}\right)^2}$$

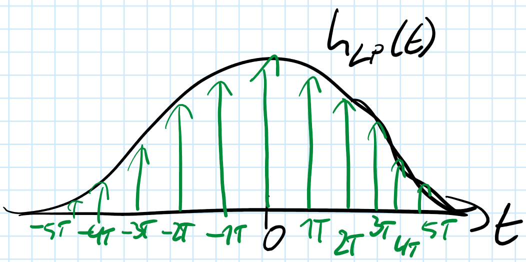
we know: $h(f_c) = e^{-\pi \cdot 0{,}3321^2} = \frac{1}{\sqrt{2}}$

$$e^{-\pi \left(\frac{1kHz}{b}\right)^2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \frac{1kHz}{b} = 0{,}3321 \Rightarrow b = 3010\,Hz$$

$$\Rightarrow h_{LP}(t) = \underset{\uparrow \text{neglect „Hz"}}{3010} \cdot e^{-\pi (3010Hz \cdot t)^2} \qquad LP \mathrel{\hat{=}} \text{Low pass}$$

$$H_{LP}(f) = 1 \cdot e^{-\pi \left(\frac{f}{3010Hz}\right)^2}$$

b) Determine $h(n)$ for $T = \frac{1}{16kHz}$ and $|n| \leq 5$



$n=0$: $h(0) = 3010$

$n=1$, $n=-1$: $h(1) = 3010 \cdot e^{-\pi \left(3010Hz \cdot \frac{1}{16kHz}\right)^2}$

$= 2693 = h(-1)$

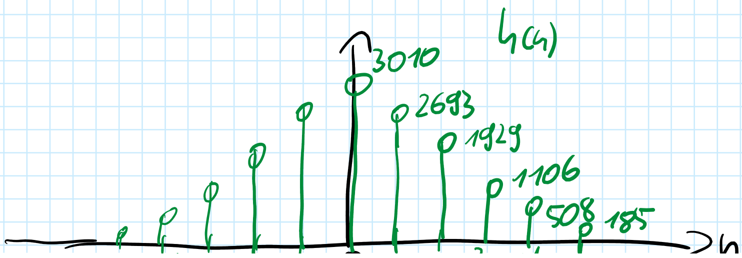
$n=2$, $n=-2$: $h(2) = h(-2) = 1929$

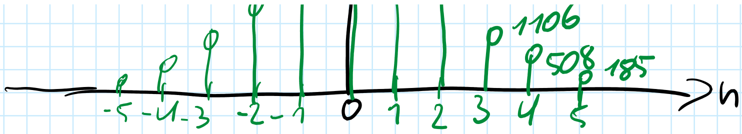
$n=3$, $n=-3$: $h(3) = h(-3) = 1106$

$n=4$, $n=-4$: $h(4) = h(-4) = 508$

$n=5$, $n=-5$: $h(5) = h(-5) = 185$

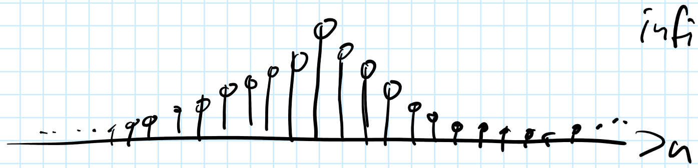
c) Sketch of $h(n)$ for $|n| \leq 5$



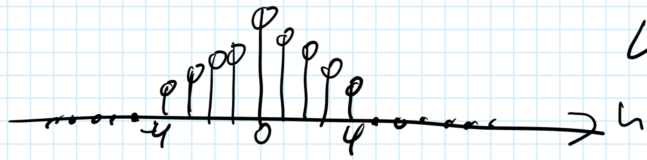


d) Determine the maximum deivation/error between the infinite impulse and the response limit to $|n| \leq 4$

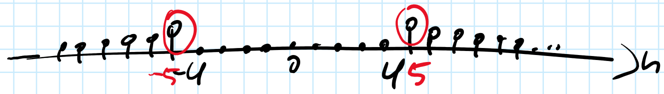
infinite



Limited to $|n| \leq 4$



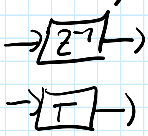
Error

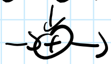


maximum error: 185 at $|n| = 5$

e) Sketch the block diagramm for a causal realisation of $h(n)$

with: multiplication, delay, addition





$h(n) = [a_0 \ \ a_1 \ \ a_2 \ \ldots a_8]$

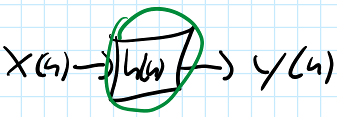
$a_0 = a_8 = 508$

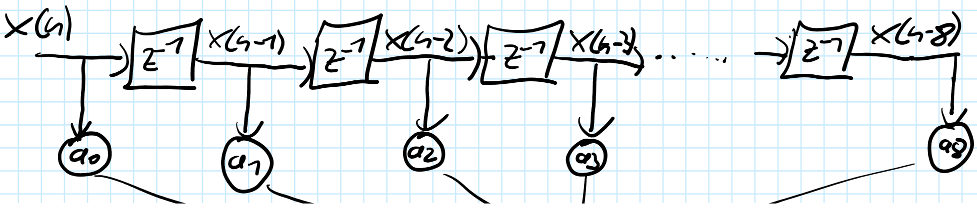
$a_1 = a_7 = 1106$

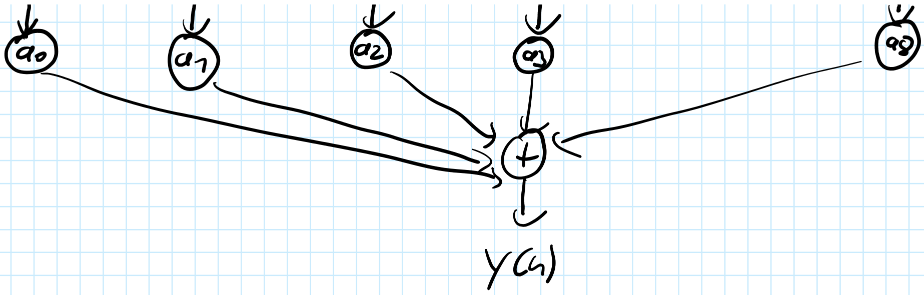
$a_2 = a_6 = 1929$

$a_3 = a_5 = 2693$

$a_4 = 3010$







time dicrete convolation:

$$y(n) = h(n) * x(n) = \sum_{m=-\infty}^{\infty} h(m)\, x(n-m)$$

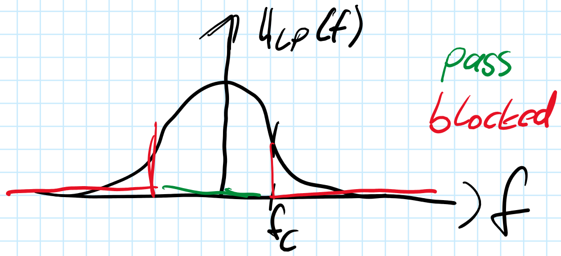
here m : $a_0 \dots a_8$

$$\Rightarrow y(n) = \sum_{m=0}^{8} h(m) \cdot x(n-m)$$

$$= h(0) \cdot x(n-0) + h(1) \cdot x(n-1) + h(2) \cdot x(n-2) + \dots + h(8) \cdot x(n-8)$$

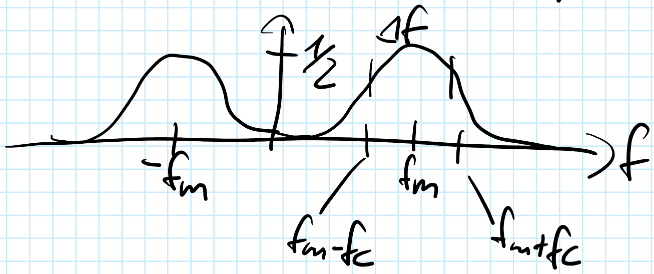
f) Bandpass filter with center frequency $f_m = 4kHz$

Derive $h_{BP}(t)$, $H_{BP}(f)$ and the bandwith Δf



$$H_{BP}(f) = H_{TP}(f) * \frac{1}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$$

$$h_{BP}(t) = h_{TP}(t) \cdot \cos(2\pi f_m T)$$



$\Delta f = 2 f_c = 2kHz$