Task 20

We are looking for the input-output transfer function of the LTI SISO system

$$\dot{\underline{x}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x}$$

<u>Solution:</u>

$$F(s) = c^T (sI - A)^{-1} b + d = \frac{s+1}{s^2 + 2s + 5}$$

a) The system is completely controllable, since all eigenvalues are also poles of the input-output transfer function.

b) The system is also completely observable, since all eigenvalues are also poles of the input-output transfer function.

Task 21:

We are to analyzing the input-output behavior of the system

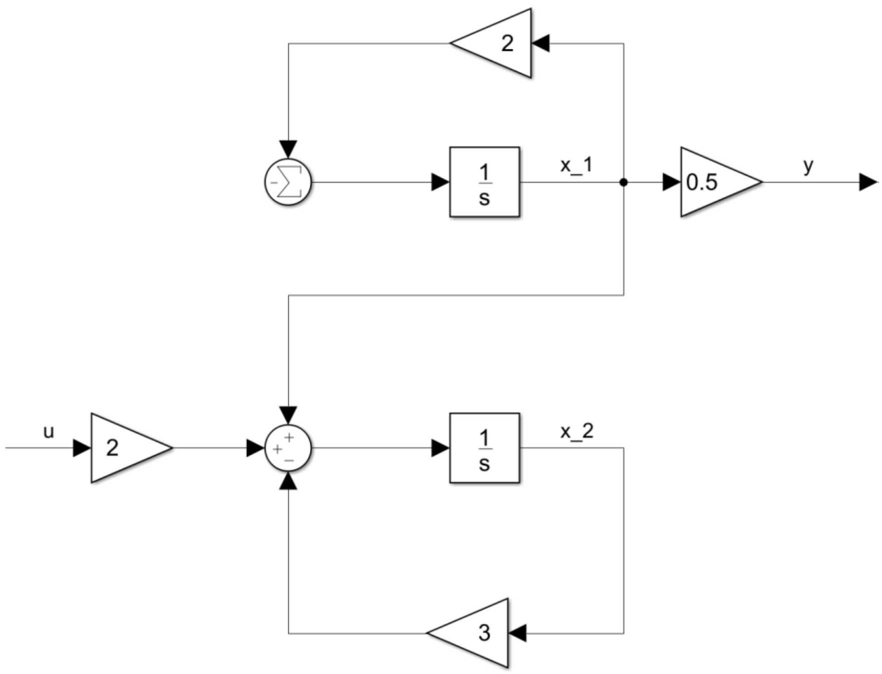
$$\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \underline{x}$$

Note that in Task 19 on Exercise Sheet 7 we have already seen that this system is neither completely controllable nor completely observable.

<u>Solution:</u>

a)
$$F(s) = c^T (sI - A)^{-1} b + d = 0$$

b) During Task 19 on Exercise Sheet 7 we have derived the following block diagram of the system above:



Obviously, the state x_1 (and in particular the corresponding eigenfunction for the eigenvalue $\lambda_1 = -2$) cannot be manipulated by the input. In the input-output transfer behavior, such a controllability defect means that this eigenvalue $\lambda_1 = -2$ is not visible as pole of the transfer function.
The block diagram also shows that the value of the second state x_2 has no impact on the output y, such that the state x_2 (and in particular the corresponding eigenfunction for the eigenvalue $\lambda_2 = -3$) cannot be observed at the output y. Such an observability defect means that the corresponding eigenvalue $\lambda_2 = -3$ is also not visible as pole of the transfer function. As the system has no direct feedthrough and the complete system dynamics is not visible in the input-output behavior, the transfer function from the input u to the output y is equal to zero.