Exam in “Digital Signal Processing and State Space Control“ (Part: State Space Control)			
10 Pages		**Examiners:**	Dr. B. Müller Dr. T. Rommel
Date:	**11/02/2023**		**WS 22/23**
Duration:	**90 Minutes (both parts)**	**Points / Grade:**	/
Name:		**Matriculation no.:**	

Task:	1	2	3	Σ
Max. Points:	13	13	15	41
Achieved Points:				

Authorized aids: - non-programmable calculator

$$\det(A-\lambda I)=0$$

Please note:

- The examination is to be done independently and without any help. Cheating and attempted cheating will always be sanctioned.
- Mobile phones, notebooks or programmable calculators are not permitted.
- Mobile phones must be switched off and put on a desk in the front row.
- Please write down your name and matriculation no. on the task sheets.
- Write your solutions directly on the respective task sheet.
- Only if your approach to a solution/answer is written down comprehensibly and transparently, it is marked and graded. If you give more than one approach to a solution, only the one that is highlighted is marked and graded.
- Please hand back all task sheets and write your name on them!

Please **do not use** a red pen.

Good luck!

$$M_0 = \begin{bmatrix} C^T \\ C^T A \end{bmatrix} \quad \det(M_0) \neq 0 \Rightarrow \text{observable.}$$

$$M_C = [B, AB, A^2B] \quad \text{rank} = 3 +$$

$$F(s) = C^T(sI-A)^{-1}B+D$$

$$Q_C = M_C = \begin{bmatrix} B & AB \end{bmatrix} \quad \det \neq 0$$

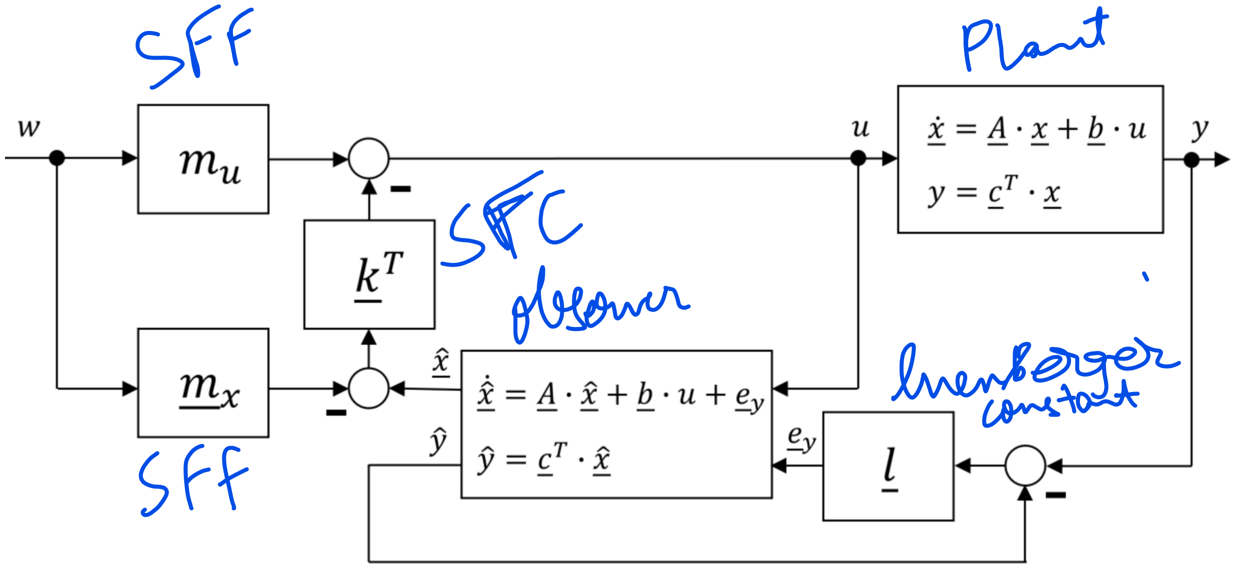
Task 1: Questions **13 Points**

a) Consider the block diagram below showing a state-space-controlled system. Mark the following subsystems in the block diagram:

- plant
- observer
- state feedback controller KT
- static feedforward D → Brainless

Note: Your solution must clearly and uniquely assign each component of the block diagram to one of the subsystems above. One subsystem may consist of more than one block.

mx



b) A system is described by the state equations

$$\dot{x}_1 = -2x_1 + 3.7x_2 + 1.2u$$
$$\dot{x}_2 = 0.3x_1 + 7.12x_2 - 0.25u$$
$$y = 12u + 2(x_1 + x_2)$$

What is the order of the system? 2 (no. of state var.)

Is the system linear or nonlinear? Explain briefly!

Linear ?

1) to matrix con

$\dot{x} = f(x, u)$

$y = h(x, u)$

$\frac{\partial \dot{x}}{\partial x}$

$\frac{\partial y}{\partial y}$

c) <u>One</u> of the following three transfer functions

- $F_1(s) = \frac{(s+4)}{(s+2)(s+3)(s+5)}$ -5 is not eigenvalue
- $F_2(s) = \frac{(s+20)}{(s+2)(s+7)}$ $(s+3)$ was cancelled by zero on top
- $F_3(s) = \frac{1}{(s+2)(s+3)(s+4)}$ → -4 is not eigen.

describes the input-output behavior of the plant with the state equations

$$\underline{\dot{x}} = \begin{bmatrix} -7 & 0 & 0 \\ 4 & -3 & 0 \\ 5 & 6 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} u$$

eigenvalues $= -7, -3, -2$

$$y = \begin{bmatrix} 1 & 4 & 2 \end{bmatrix} \underline{x}$$

Specify which one is the correct transfer function by excluding the other two given possibilities. Explain (briefly)!

<u>Hint:</u> There is no calculation needed in order to answer this question!

$(s+7)(s+3)$

$(s+2)$

Task 2: Solution of the State Equations **13 Points**

A system is given by the following state equations:

$$\dot{x} = 3x + 3u$$
$$y = -3x$$

$A(1\times1)$ B

$\dot{x} = [3]\,x + [3]\,u$

$y = [-3]\,x + [0]\,u$

C D

a) What is the order of the system? 1

b) Is the system asymptotically stable? Why (not)? Not (eigenvalues in left)

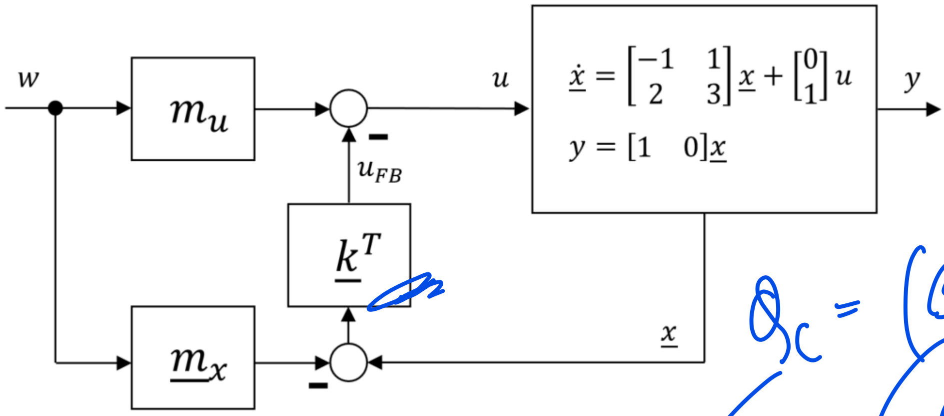
c) Determine the output signal $y(t)$ for $t \geq 0$ assuming the initial state $x_0 = x(0) = 1$ and the constant input $u(t) = -2$ for $t \geq 0$.

<u>Hint:</u> The general solution formula for an LTI SISO system in state space form is

$$\underline{x}(t) = \underline{e}^{\underline{A}t}\underline{x}_0 + \int_0^t \underline{e}^{\underline{A}(t-\tau)}\underline{b}u(\tau)d\tau$$

Task 3: State Space Controller Design 15 Points

Consider the state-space-controlled system



$$Q_C = \begin{pmatrix} \underline{b} & A\underline{b} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 3 \end{pmatrix} = 2$$

a) Show that the plant in this block diagram is completely controllable!

b) Calculate $\underline{k}^T$, m_u and $\underline{m}_x$ such that

- the closed-loop system has the eigenvalues $\lambda_{C,1} = -1$ and $\lambda_{C,2} = -2$;
- $y = w$ and $u_{FB} = 0$ holds in steady state for arbitrary constant inputs w. ?

Hint: $\begin{bmatrix} -1 & 1 & 0 \\ 2 & 3 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ -3 & 1 & -5 \end{bmatrix}$

$$\underline{b}\underline{k}^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} \lambda+1 & -1 \\ -2 & \lambda-3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}\right)$$

$$\Rightarrow \det\begin{bmatrix} \lambda+1 & -1 \\ k_1-2 & \lambda-3+k_2 \end{bmatrix} = (\lambda+1)(\lambda+2)$$

$$\Rightarrow (\lambda+1)(\lambda+k_2-3) + k_1 - 2 = \lambda^2 + 3\lambda + 2$$

$\Rightarrow \lambda^2 + k_2\lambda - 3\lambda$
$+ \lambda + k_2 - 3$
$+ k_1 - 2$

$\Rightarrow \lambda^2 + \lambda(k_2 - 2) + k_1 + k_2 - 5$
$= \lambda^2 + 3\lambda + 2$

$k_2 - 2 = 3 \Rightarrow k_2 = 5$

$k_1 + k_2 - 5 = 2 \Rightarrow k_1 = 2$

Control Law $\rightarrow$

$u = -k^T \underline{x}$
$= -\begin{bmatrix} 2 & 5 \end{bmatrix} \underline{x}$
$= -2x_1 - 5x_2$

$$Q_0 = \begin{bmatrix} c^T \\ c^T A \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_1 A_1 & c_1 A_2 \\ c_n A & c_2 A \end{bmatrix}$$

$$\frac{M_x}{M_u} = \begin{bmatrix} A & b \\ c^T & d \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \hline 1 \end{bmatrix}$$

$$\frac{M_x}{M_u} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ -3 & 1 & -5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ & -5 \end{bmatrix}$$