Exercise Image Processing
Sample Solution

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Sheet 6

In this exercise we cover the chapters *Nonlinear Filters*, *Geometric Transformations*, and *Structure*. The questions are small-part and can be seen as examples of potential exam problems. Also use the formulary for the exam to work through the problems.

Task 6.1: Nonlinear Filters

6.1a)

Apply non-maximum suppression to the following 3x3 neighborhood without considering orientation:
Answer: Without considering the orientation, the 8 at the anchor point becomes a 0.

0	3	9
3	8	3
9	3	0

Draw the direction of the steepest descent. What value results from the non-maximum suppression according to Canny?

Answer: The direction of the steepest ascent runs along the diagonal of the image area. After Canny, the 8 remains at the anchor point.

6.1b)

What class does the anchor point of the following Canny classification into weak (1) and strong (2) edges receive when a hysteresis filter is applied?
Answer: The weak edge becomes a strong.

1	2	1
0	1	0
0	0	0

What result do you get if the anchor point was not classified as an edge?

Answer: If the anchor point was not classified as a weak or strong edge, then the classification does not change.

Task 6.2: Geometric Transformations & Interpolation

6.2a)

Which coordinate transformations can be realized with an affine 2D mapping?

- [x] Translation
- [ ] Point reflection
- [ ] Bilinear mapping
- [x] Shear
- [ ] Axis mirroring
- [ ] Destilation

6.2b)

given is the following geometric transformation:

$$\begin{aligned} x' &= 1 + 2x \\ y' &= 4 - 2y \end{aligned}$$

Is the inverse mapping existent? if yes, what does the inverse mapping look like?

Answer: The inverse mapping is existent, because it holds: $|\mathbf{J}(\mathbf{M}(\mathbf{x}))| = -4 \neq 0$. The inverse mapping reads as follows:

$$\begin{aligned} x &= 0.5(x' - 1) \\ y &= -0.5(y' - 4) \end{aligned}$$

6.2c)

The table shows different types of interpolation and their properties are given. Decide which properties belong to which interpolation types.

	Nearest-Neighbor Interpolation	Bilinear Interpolation	Bikubic Interpolation
light smoothing, few artifacts			×
false high frequencies are created, lots of artefacts	×		
high frequencies are dampened, strong smoothing		×	

6.2d)

Perform bilinear interpolation for the gray value at location $G(6.5, 8.5)$ when the gray values $G(6,8)$, $G(6,9)$, $G(7,8)$, $G(7,9)$ are given as follows:
What value would you get from a nearest neighbor interpolation?

	6	7
8	6	8
9	2	4

Answer: The coefficients are: $a_{00} = G(\mathbf{p}_1) = 6\,, a_{10} = G(\mathbf{p}_2) - G(\mathbf{p}_1) = 8 - 6 = 2\,, a_{01} = G(\mathbf{p}_3) - G(\mathbf{p}_1) = 2 - 6 = -4\,, a_{11} = G(\mathbf{p}_1) - G(\mathbf{p}_2) - G(\mathbf{p}_3) + G(\mathbf{p}_4) = 6 - 8 - 2 + 4 = 0$ The interpolated gray value is calculated as follows: $G(6.5, 8.5) = a_{00} + a_{10}x + a_{01}y + a_{11}xy = 6 + 2 \cdot 0.5 - 4 \cdot 0.5 = 5$.

Task 6.3: Gradient & Structure

6.3a)

Which scalar derived quantity of the Hessian matrix is invariant to rotations?

Answer: The trace of the Hessian matrix.

6.3b)

Show by a short calculation that the magnitude of the 2D gradient is invariant to rotations of the coordinate system.

Hint: A 2D rotation by the angle ϕ is given by the rotation matrix $\mathbf{R} = \begin{bmatrix} cos(\phi) & sin(\phi) \\ -sin(\phi) & cos(\phi) \end{bmatrix}$.

Answer:

$$|| \begin{pmatrix} cos(\phi) & sin(\phi) \\ -sin(\phi) & cos(\phi) \end{pmatrix} [\mathbf{I}_x, \mathbf{I}_y]^\top || = || [cos(\phi)\mathbf{I}_x + sin(\phi)\mathbf{I}_y, -sin(\phi)\mathbf{I}_x + cos(\phi)\mathbf{I}_y]^\top ||$$
$$= \left((cos(\phi)\mathbf{I}_x + sin(\phi)\mathbf{I}_y)^2 + (-sin(\phi)\mathbf{I}_x + cos(\phi)\mathbf{I}_y)^2\right)^{0.5}$$
$$= \left(cos^2(\phi)\mathbf{I}_x^2 + 2cos(\phi)\mathbf{I}_x sin(\phi)\mathbf{I}_y + sin^2(\phi)\mathbf{I}_y^2 + sin^2(\phi)\mathbf{I}_x^2 - 2cos(\phi)\mathbf{I}_x sin(\phi)\mathbf{I}_y + cos^2(\phi)\mathbf{I}_y^2\right)^{0.5}$$
$$= \left((cos^2(\phi) + sin^2(\phi))\mathbf{I}_x^2 + (sin^2(\phi) + cos^2(\phi))\mathbf{I}_y^2\right)^{0.5} = (\mathbf{I}_x^2 + \mathbf{I}_y^2)^{0.5}$$