Task 3

State equations of network in task 1 (see Exercise Sheet 1) in matrix form:

$$\dot{\underline{x}} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \underline{x} + \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} \\ 0 & -\frac{1}{RC} \end{bmatrix} \underline{u}$$

$$y = \begin{bmatrix} 0 & \frac{1}{R} \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & \frac{1}{R} \end{bmatrix} \underline{u}$$

Task 4

Calculate the results of the following matrix products:

a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 13 \\ 22 & 29 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = 20$

c) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 6 & 9 \\ 4 & 8 & 12 \end{bmatrix}$

Task 5

State for each of the following sets of vectors if the are linearly dependent (or not):

a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$: linearly independent

b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$: linearly independent

c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$: linearly dependent

d) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$: linearly dependent

e) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$: linearly dependent

f) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$: linearly independent

Task 6

Determine the rank of the following matrices:

a) $\operatorname{rk}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right) = 2$

b) $\operatorname{rk}\left(\begin{bmatrix}1 & 0\\3 & 4\end{bmatrix}\right) = 2$

c) $\operatorname{rk}\left(\begin{bmatrix}1 & 2 & 2\\3 & 4 & 5\end{bmatrix}\right) = 2$

d) $\operatorname{rk}\left(\begin{bmatrix}0 & 0 & 0\\3 & 4 & 5\end{bmatrix}\right) = 1$

e) $\operatorname{rk}\left(\begin{bmatrix}1 & 2 & 3\\3 & 9 & 6\\0 & -2 & 2\end{bmatrix}\right) = 2$

Task 7

Calculate the following determinants:

a) $\det\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = -2$

b) $\det\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 2 & 3\end{bmatrix} = 2$

c) $\det\begin{bmatrix}1 & 2 & 3\\3 & 9 & 6\\0 & -2 & 2\end{bmatrix} = 0$, see result e) of Task 6

Task 8

Solve the following homogeneous linear systems of equations. State the set of all solutions if more than one solution exists.

a) $\begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\1 & 2 & 3\end{bmatrix}\underline{x} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ → $\underline{x} = \begin{bmatrix}0\\0\\0\end{bmatrix}$, see result b) of Task 7

b) $\begin{bmatrix}1 & 2 & 3\\3 & 9 & 6\\0 & -2 & 2\end{bmatrix}\underline{x} = \begin{bmatrix}0\\0\\0\end{bmatrix}$ → $X = \left\{\underline{x} \in \mathbb{R}^3 \mid \underline{x} = c\begin{bmatrix}5\\-1\\-1\end{bmatrix}, c \in \mathbb{R}\right\}$

Task 9

Calculate the inverse of the following matrices (if it exists):

a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 9 & 6 \\ 0 & -2 & 2 \end{bmatrix}$ → inverse not existing, see result e) of Task 6

b) $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0.5 \\ 0 & -0.5 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & -1.5 & 0.5 \\ 0.5 & 1.5 & -0.5 \\ -0.5 & -0.5 & 0.5 \end{bmatrix}$

Task 10

Determine the eigenvalues of the following matrices:

a) $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ → $\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3$

b) $\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}$ → $\lambda_1 = 4, \lambda_2 = -8$

Task 11

Determine to each eigenvalue of the matrix $\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}$ a corresponding eigenvector.

$\lambda_1 = 4$ → $\underline{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda_2 = -8$ → $\underline{v}_2 = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$

Task 12

Calculate the matrix exponential function $\underline{e}^{\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}}$.

$$\underline{e}^{\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}} = \frac{1}{12}\begin{bmatrix} 7e^4 + 5e^{-8} & 5(e^4 - e^{-8}) \\ 7(e^4 - e^{-8}) & 5e^4 + 7e^{-8} \end{bmatrix}$$