Bildanalyse

Contours & Structure

Technical University
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Recap: Image Gradient

- Gradient Vector
- Absolute Value & Direction

Description of Structure

- Structure Tensor
- Orientation & Coherence
- Corner Points

Finding Local Contours

Edges

The goal of derivative filters is to find the

- edges, corners and local extreme values

in the image space. This is mainly done by an evaluation of the

- gradient vector

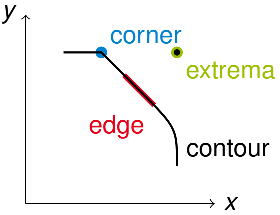
Edges are defined by the

- direction

and the

- absolute value

of the gradient vector.



Finding Local Contours

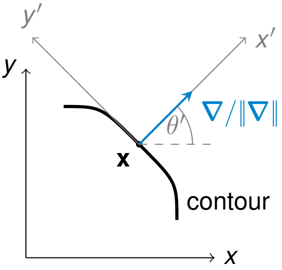
Gradient Vector

The partial derivatives of the image function along both dimensions at an image location **x** define the gradient vector ∇

$$\nabla(\mathbf{x}) = \left[\frac{\partial G(\mathbf{x})}{\partial x}, \frac{\partial G(\mathbf{x})}{\partial y}\right]^\top ,$$

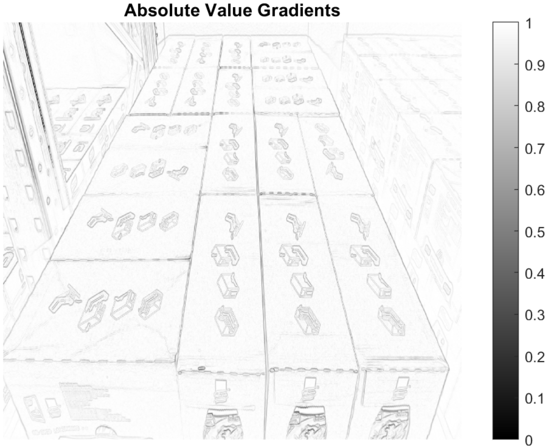
with the amount of gradient vector

$$\|\nabla\| = \sqrt{\nabla^\top \nabla} = \sqrt{\left(\frac{\partial G(\mathbf{x})}{\partial x}\right)^2 + \left(\frac{\partial G(\mathbf{x})}{\partial y}\right)^2} .$$



Finding Local Contours

Example Amount of Gradient



Finding Local Contours

Gradient Vector

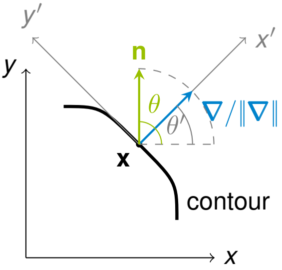
The direction derivative along the direction θ

$$\boldsymbol{\nabla}^\top \mathbf{n} = \cos(\theta)\frac{\partial G(\mathbf{x})}{\partial x} + \sin(\theta)\frac{\partial G(\mathbf{x})}{\partial y}$$

indicates the strength of the change along this direction and corresponds to the scalar product between direction vector $\mathbf{n}$ and gradient $\boldsymbol{\nabla}$

$$\langle \boldsymbol{\nabla}, \mathbf{n} \rangle = \|\boldsymbol{\nabla}\| \cos(\theta - \theta'),$$

whereas $\mathbf{n} = [\cos(\theta), \sin(\theta)]^\top$, with $\|\mathbf{n}\| = 1$.



Finding Local Contours

Gradient Vector

In the simplest case, the angle θ_0 of the steepest descent can be calculated by maximizing the directional derivative by direction:

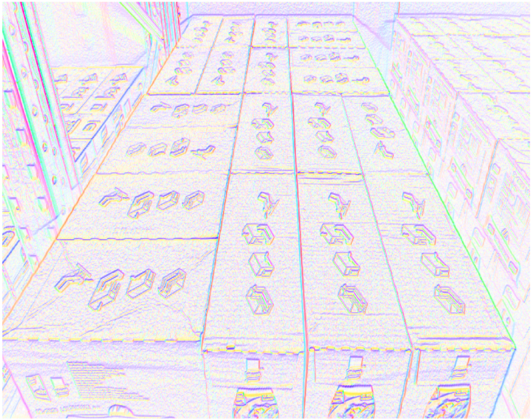
$$\frac{\partial}{\partial\theta}\left(\cos(\theta)\frac{\partial G(\mathbf{x})}{\partial x} + \sin(\theta)\frac{\partial G(\mathbf{x})}{\partial y}\right) \overset{!}{=} 0\,.$$

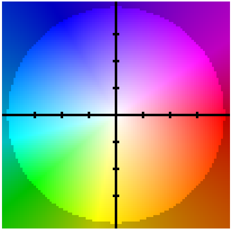
$$\tan(\theta_0) = \frac{\partial G(\mathbf{x})}{\partial y}\left(\frac{\partial G(\mathbf{x})}{\partial x}\right)^{-1} \quad \leftrightarrow \quad \theta_0 = \tan^{-1}\left(\frac{\partial G(\mathbf{x})}{\partial y}\left(\frac{\partial G(\mathbf{x})}{\partial x}\right)^{-1}\right)\,.$$

This corresponds to the direction of the gradient vector.

Finding Local Contours

Example Direction





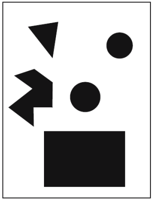
Local Structures

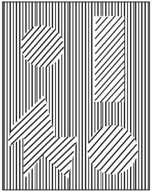
Structure Tensor - Motivation

An essential feature for the description of small image regions is the so-called structure. It is defined by quantities like

- the orientation and
- the coherence.

Unlike direction, which is defined in the range from zero to 360°, orientation describes a preferred direction without distinguishing between positive and negative derivatives. One would like to know whether a local structure has a certain preferred direction for the individual gradients and how strong this preferred direction is pronounced.





Structure Tensor - Definition

The structure tensor $\mathbf{J}(\mathbf{x})$ is given for a certain neighborhood $\mathcal{N}(\mathbf{x})$ with a certain weight $W(\mathbf{x}-\mathbf{x}')$ around the point $\mathbf{x}$ and defined as follows:

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} J_{11}(\mathbf{x}) & J_{12}(\mathbf{x}) \\ J_{12}(\mathbf{x}) & J_{22}(\mathbf{x}) \end{bmatrix} = \int\limits_{\mathbf{x}'\in\mathcal{N}(\mathbf{x})} W(\mathbf{x}-\mathbf{x}')\left(\nabla(\mathbf{x}')\nabla(\mathbf{x}')^\top\right) d\mathbf{x}' = \mathbf{W} * \left(\nabla\nabla^\top\right).$$

This corresponds in 2D to a symmetric 2×2 matrix, where the individual elements are weighted averages of the derivative combinations $(\partial G(\mathbf{x})/\partial x)^2$, $(\partial G(\mathbf{x})/\partial y)^2$ and $(\partial G(\mathbf{x})/\partial x)(\partial G(\mathbf{x})/\partial y)$.

The square of the projection $(\nabla(\mathbf{x})^\top \mathbf{n})^2$ between gradient vector and a preferred direction $\mathbf{n} = [\cos(\theta), \sin(\theta)]^\top$ describes the deviation to a certain orientation. The orientation which has the smallest average deviation to all gradient vectors in the neighborhood $\mathcal{N}(\mathbf{x})$ is defined as orientation of the local structure.

Local Structures

Structure Tensor - Orientation

Thus, the orientation of the local environment $\hat{\mathbf{n}}(\mathbf{x})$ can be found via the following optimization problem:

$$\hat{\mathbf{n}}(\mathbf{x}) = \mathit{argmax}_{\mathbf{n}}\ \mathbf{n}^\top \mathbf{J}(\mathbf{x})\mathbf{n}\,.$$

The solution of this optimization problem is provided by the eigenvalue problem:

$$\mathbf{J}(\mathbf{x})\mathbf{e}(\mathbf{x}) = \lambda(\mathbf{x})\mathbf{e}(\mathbf{x})\,.$$

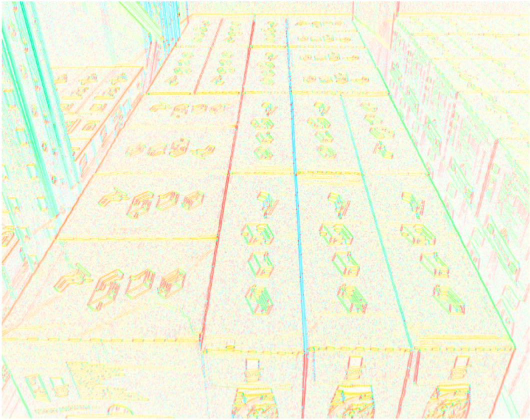
The eigenvector $\mathbf{e}(\mathbf{x})$ of the maximum eigenvalue $\lambda_{max}(\mathbf{x})$ corresponds to the Orientation of the local neighborhood $\hat{\mathbf{n}}(\mathbf{x})$. The solution is given by the following components of the structure tensor:

$$\hat{\mathbf{n}}(\mathbf{x}) = [J_{22} - J_{11}, 2J_{12}]^\top\,, \quad \text{wobei} \quad \tan(2\hat{\theta}) = 2J_{12}/(J_{22} - J_{11})\,.$$

Local Structures

Example Orientation

Orientation Structure Tensor



Local Structures

Structure Tensor - Coherence

The coherence is a measure for the isotropy of the gray value structure:

$$c(\mathbf{x}) = \frac{\sqrt{(J_{22} - J_{11})^2 + 4J_{12}^2}}{J_{11} + J_{22}} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2},$$

and varies between zero and one $c \in [0; 1]$. A structure with an ideal orientation has the value one ($\lambda_2 = 0, \lambda_1 > 0$) and a completely isotropic gray value structure has the value zero ($\lambda_1 = \lambda_2 > 0$).

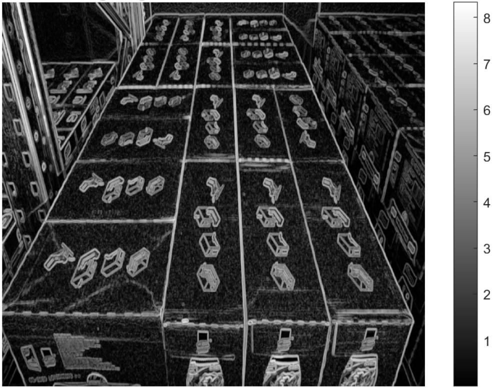
Thus, the eigenvalues characterize the structure:

- $\lambda_1 = \lambda_2 = 0$: The local environment is constant,
- $\lambda_1 > 0, \lambda_2 = 0$: Ideal orientation, all gradients are identically oriented.
- $\lambda_1 > 0, \lambda_2 > 0$: Gray values change in all directions.
- $\lambda_1 = \lambda_2 > 0$: Gray values change equally in all directions.

Example Eigenvalues Structure Tensor

- Large eigenvalues mapped nonlinearly: $\ln(1 + \lambda_1)$

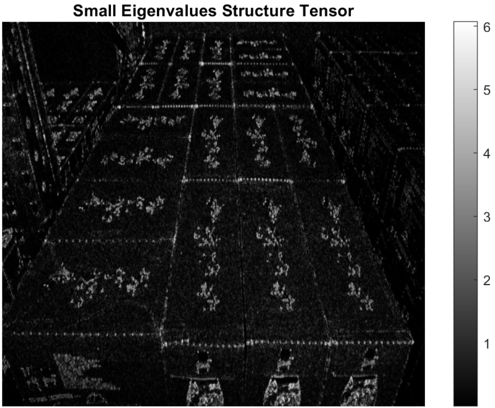
Large Eigenvalues Structure Tensor



Local Structures

Example Eigenvalues Structure Tensor

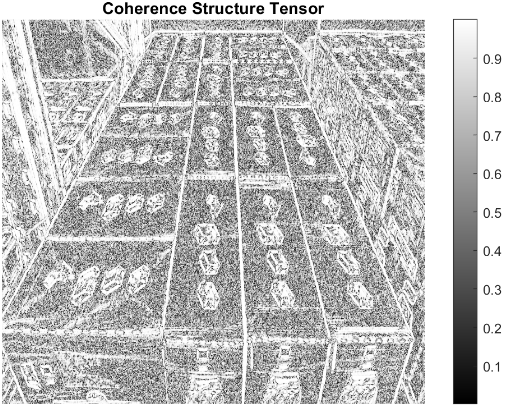
- Small eigenvalues mapped nonlinearly: $\ln(1 + \lambda_2)$



Local Structures

Example Coherence

- Coherence: $(\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2)$



Local Structures

Structure Tensor - Corner Detector

From the eigenvalues and the relation to the coherence, a very simple corner detector can be defined: If the smallest eigenvalue exceeds a certain predefined threshold τ, then a corner point is present.

Shi-Tomasi corner detector

- $\min(\lambda_1, \lambda_2) > \tau$: corner point exists.

Another variation of this type of corner point detector was proposed by Harris and Stephens in 1988:

Harris corner detector

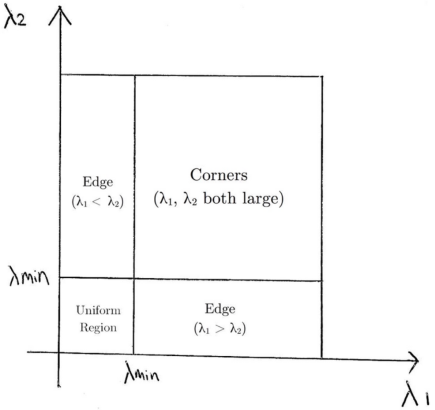
- $det(\mathbf{J}) + k \cdot trace^2(\mathbf{J}) = \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2 > \tau$: corner point exists.

With $k = [0.04; 0.06]$ the sensitivity to the gray level change can be adjusted along a preferred direction.

Local Structures

Structure Tensor - Shi-Tomasi Corner Detector

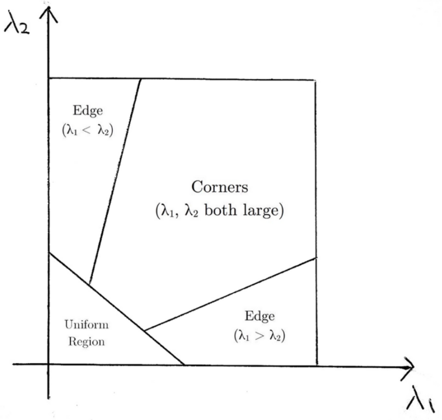
- $\min(\lambda_1, \lambda_2) > \tau$: Corner is detected.



Local Structures

Structure Tensor - Harris Corner Detector

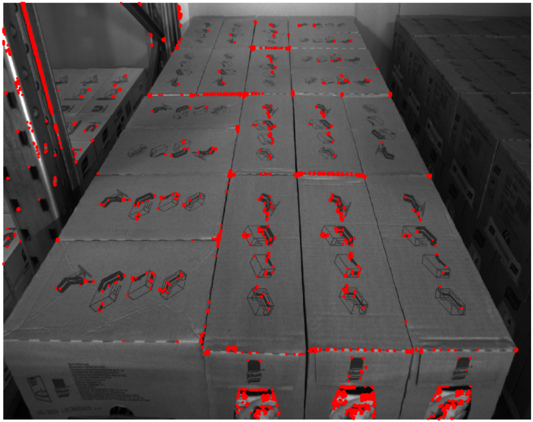
- $det(\mathbf{J}) + k \cdot trace^2(\mathbf{J}) = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2 > \tau$: Corner is detected.



Local Structures

Comparison - Harris vs. Shi-Tomasi

Harris Corners



Comparison - Harris vs. Shi-Tomasi

Shi-Tomasi Corners