Course „Control Systems 2“

Exercise Sheet 7

Task 18:

Consider the LTI SISO system

$$\dot{\underline{x}} = \begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x}$$

a) Is the system completely controllable? Why (not)?

b) Is the system completely observable? Why (not)?

Task 19:

Consider the system

$$\dot{\underline{x}} = \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \underline{x}$$

a) Draw the block diagram of the system. Is it possible (for this particular system) to make conclusions about controllability and/or observability by analyzing the structure of the block diagram? Explain!

b) Use the Kalman criteria to show that the system is neither controllable nor observable.

c) Apply the linear state transformation

$$\underline{\tilde{x}} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underline{x}$$

and show that the resulting equivalent system with state vector $\underline{\tilde{x}}$ has the same controllability and observability properties as the original system.

d) Show in general that the controllability and observability properties of an LTI SISO system will not change by applying any regular state transformation $\underline{\tilde{x}} = \underline{T x}$.