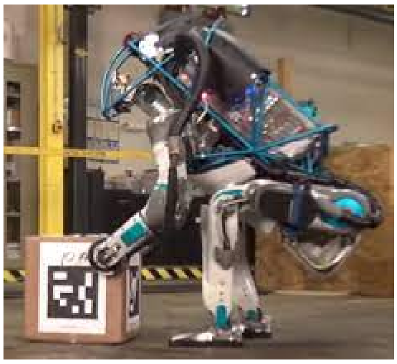
Exercise Image Processing

Prof. Dr.-Ing. Volker Willert

Sheet 2



In many applications of robotics and industrial image processing, the position and orientation of a camera relative to a known object should be determined from an image. For example, 3D coordinates of a workpiece may be known and the position of the camera is to be determined on the basis of projections of these points in the camera image. Other examples can be found in robot navigation. For example, the Boston Dynamics robot dog Spot recognizes its relative position to the charging station, or the humanoid robot Atlas knows how its hands are oriented to a box if the geometry of the box is known.

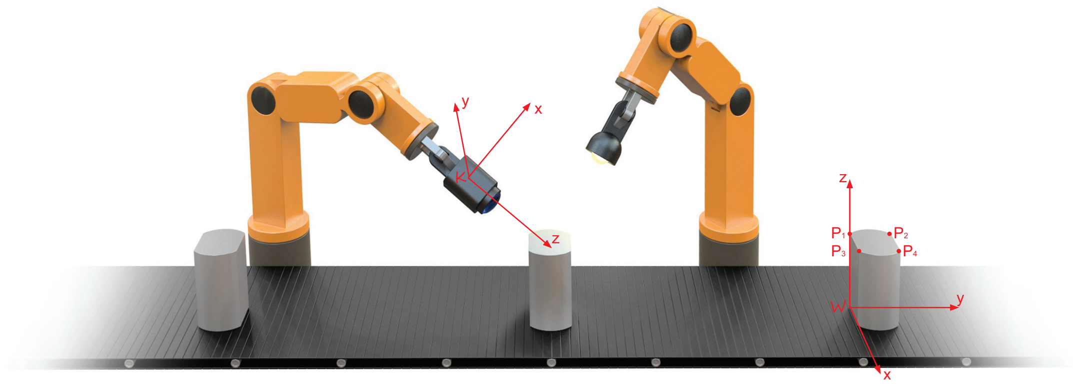


Figure 1: Where is the camera located in relation to the workpiece?

In this exercise we deal with the determination of the position of a geometrically calibrated camera in relation to an object, of which we know the coordinates of four points on a surface on the object. As an example we use the same scenario as in exercise sheet 1. The coordinates of the four vertices of the section with respect to the workpiece coordinate system are in millimeters: $\mathbf{p}_1 = [0, 0, 50]^T$, $\mathbf{p}_2 = [0, 20, 50]^T$, $\mathbf{p}_3 = [20, 0, 50]^T$ and $\mathbf{p}_4 = [20, 20, 50]^T$. The camera is calibrated and we have written an image processing algorithm that can detect the workpiece in the image and determine the projections of the vertices $\mathbf{x}'_1$, $\mathbf{x}'_2$, $\mathbf{x}'_3$ and $\mathbf{x}'_4$ with pixel accuracy.

Task 2.1: Determining the Pose of a Calibrated Camera

Calibration of the camera revealed the following intrinsic parameters:

$$\begin{aligned} \text{principal point}: &\quad o_x = 600\,[\mathrm{px}]\,, \quad o_y = 800\,[\mathrm{px}]\,, \\ \text{scale factors}: &\quad s_x = 400\,[\mathrm{px/mm}]\,, \quad s_y = 400\,[\mathrm{px/mm}]\,, \\ \text{camera constant}: &\quad c = 5[\mathrm{mm}]\,. \end{aligned}$$

The image processing algorithm measures the following coordinates with pixel accuracy:

$$\begin{aligned} \mathbf{x}'_1 &= [489, 689]^\top \\ \mathbf{x}'_2 &= [732, 405]^\top \\ \mathbf{x}'_3 &= [803, 1003]^\top \\ \mathbf{x}'_4 &= [1105, 777]^\top \end{aligned}$$

The measurement error is therefore in the range of $\Delta x, \Delta y = [0; 0.5]$ pixels.

Since all points lie on a plane, we can determine the pose of the camera by determining the homography between the bolt cross-sectional area and the image plane. We proceed as follows:

a) Define the coordinate system of the bolt such that the X and Y components do not change and the Z components all add up to zero.

b) Compute the inverse calibration matrix $\mathbf{K}^{-1}$ to determine the normalized image coordinates $\mathbf{x}_1$ to $\mathbf{x}_4$ of points p_1 to p_4.

c) Use the projection equation for points $\mathbf{X}_i$ on a plane $\mathbf{x}_i = \mathbf{H}\mathbf{X}_i$ to set up a homogeneous linear system of equations using the Direct Linear Transformation (DLT). What is the minimum number of points you need to uniquely determine the homography $\mathbf{H}$?

d) Reconstruct the pose $(\mathbf{R}, \mathbf{T})$ of the camera from the homography $\mathbf{H}$. Check if the rotation can be read directly? Are the conditions for a rotation matrix satisfied?

Task 2.2: Decomposition of a Projection Matrix

You have bought a new camera and in the data sheet the intrinsic parameters are not completely specified. Therefore, you perform a camera calibration and thereby determine the projection matrix:

$$\mathbf{P} = \begin{bmatrix} 2 & -\sqrt{3}/2 & 3/2 & 2+\sqrt{3} \\ 0 & \sqrt{3}/2 & 5/2 & 2+\sqrt{3} \\ 0 & -1/2 & \sqrt{3}/2 & 1 \end{bmatrix} = \mathbf{K}\left[\ \mathbf{R}|\mathbf{T}\ \right] = \left[\ \mathbf{KR}|\mathbf{KT}\ \right]$$

You must now decompose this into the individual calibration matrices of the intrinsics $\mathbf{K}$ and extrinsics $\mathbf{R}, \mathbf{T}$. To do this, perform the RQ decomposition (not QR decomposition!).

a) Let's assume a matrix equation $\mathbf{M} = \mathbf{RQ}$. What kind of decomposition results if we transpose the equation?

b) Using a permutation matrix $\mathbf{\Gamma}$ which, multiplied from the left, reverses the order of the rows and, multiplied from the right, reverses the order of the columns of a (3x3) matrix, try to transform the matrix equation from a) in such a way that a QR decomposition is possible.

$$\mathbf{\Gamma} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \text{ mit } \mathbf{\Gamma} = \mathbf{\Gamma}^T = \mathbf{\Gamma}^{-1}.$$

b) Test your solution with `Matlab`.

Task 2.3: Additional task: Determination of the world coordinates of the projection center via the projection matrix

In this task, the projection center in world coordinates corresponding to the null space of the projection matrix $\mathbf{\Pi}$ should be computed, which is formed by the following matrices $\mathbf{K}$, $\mathbf{R}$ and the vector $\mathbf{T}$, without decomposing the projection matrix into the individual matrices.

a) Use the singular value decomposition of $\mathbf{\Pi} = \mathbf{U}\mathbf{S}\mathbf{V}^\star$.

b) Check your solution using the single matrices $\mathbf{K}$, $\mathbf{R}$ and the vector $\mathbf{T}$ and the formula from the script.

The matrices are given as follows:

$$\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \sqrt{2} & 2 \\ -\frac{\sqrt{2}}{2} & 1 & \frac{\sqrt{2}}{2} & 1 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 1 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$