Exam:

State Space → Transfer Function

Formula for Controllability, Observability

Won't lose points if you don't indicate vectors and matrices

General solution would mostly be given

Can write answers in English

Should know formulae to solve exercise sheets

Theory questions will be there too

1) $\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} \rightarrow \det \begin{bmatrix} -1-\lambda & 5 \\ 7 & -3-\lambda \end{bmatrix} \overset{!}{=} 0$

$(1+\lambda)(\lambda+3) - 35 \overset{!}{=} 0 \Rightarrow \lambda^2 + 4\lambda - 32 \overset{!}{=} 0, \quad \lambda_{1,2} = -8, 4$

$(\lambda - (-8))(\lambda - 4)$

Almost made mistake here

$\lambda_{1,2}$ is not $8, -4$

$(\lambda - \lambda_1)(\lambda - \lambda_2) \overset{!}{=} 0$

2) Eigenvector $\underline{v_1}$ to λ_1

$\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} \underline{v_1} = -8\, \underline{v_1}$

$-v_{1,1} + 5v_{1,2} = -8v_{1,1} \Rightarrow 7v_{1,1} + 5v_{1,2} = 0$

$7v_{1,1} - 3v_{1,2} = -8v_{1,2} \Rightarrow 7v_{1,1} + 5v_{1,2} = 0$

$\underline{v_1} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$

Eigenvector $\underline{v_2}$ to λ_2

$\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} \underline{v_2} = 4\, \underline{v_2} \Rightarrow -v_{2,1} + 5v_{2,2} = 4v_2 \Rightarrow -5v_{2,1} + 5v_{2,2} = 0$

$\Rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\underline{V} = \begin{bmatrix} \underline{v}_1 & \underline{v}_2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -7 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} = \underline{V} \cdot \begin{bmatrix} -8 & 0 \\ 0 & 4 \end{bmatrix} \underline{V}^{-1}$$

$$e^{\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix} t} = \underline{e}^{\begin{bmatrix} 5 & 1 \\ -7 & 1 \end{bmatrix} \cdot \begin{bmatrix} -8t & 0 \\ 0 & 4t \end{bmatrix} \cdot \frac{1}{12} \begin{bmatrix} 1 & -1 \\ 7 & 5 \end{bmatrix}}$$

☆ How to find inverse of matrix

$$= \begin{bmatrix} 5 & 1 \\ -7 & 1 \end{bmatrix} \begin{bmatrix} e^{-8t} & 0 \\ 0 & e^{4t} \end{bmatrix} \cdot \frac{1}{12} \begin{bmatrix} 1 & -1 \\ 7 & 5 \end{bmatrix}$$

☆ Eigenvalues of triangle matrix are main diagonal

○ Matrix: $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ or $\begin{bmatrix} a & 0 \\ b & c \end{bmatrix}$

$$= \frac{1}{12} \begin{bmatrix} 5 & 1 \\ -7 & 1 \end{bmatrix} \begin{bmatrix} e^{-8t} & -e^{-8t} \\ 7e^{4t} & 5e^{4t} \end{bmatrix}$$

☆ Multiply right to left

$$= \frac{1}{12} \begin{bmatrix} 5e^{-8t} + 7e^{4t} & -5e^{-8t} + 5e^{4t} \\ -7e^{-8t} + 7e^{4t} & 7e^{-8t} + 5e^{4t} \end{bmatrix}$$