**Task 15**

a) The transformation is regular if the transformation matrix

$$\underline{T} = \begin{bmatrix} 1 & 0 & 1 \\ \alpha & \beta & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

is regular, i.e. if $\det(\underline{T}) \neq 0$. Since it is easy to show that $\det(\underline{T}) = 2\beta$, the transformation is regular for arbitrary values of $\alpha$ and for all values of the parameter $\beta$ except for $\beta = 0$.

b) By applying the transformation formulas we obtain the equivalent system description

$$\dot{\underline{\tilde{x}}} = \begin{bmatrix} 0 & 1 & \frac{1}{2} \\ -3 & 3 & 2 \\ 2 & 0 & 0 \end{bmatrix} \underline{\tilde{x}} + \begin{bmatrix} 3 \\ 7 \\ 4 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 2 & \frac{1}{2} \end{bmatrix} \underline{\tilde{x}} + 3u$$

**Task 16**

The system descriptions are equivalent if there exists a regular state transformation

$$\underline{\tilde{x}} = \underline{T}\underline{x}$$

which transforms the first system into the second and vice versa. The corresponding transformation matrix must be regular and it must satisfy the following equations

$$\underline{\tilde{A}} = \underline{T}\underline{A}\underline{T}^{-1} \quad \Rightarrow \quad \underline{\tilde{A}}\underline{T} = \underline{T}\underline{A}$$

$$\underline{\tilde{b}} = \underline{T}\underline{b}$$

$$\underline{\tilde{c}}^T = \underline{c}^T\underline{T}^{-1} \quad \Rightarrow \quad \underline{\tilde{c}}^T\underline{T} = \underline{c}^T$$

Substituting the given system matrices/vectors we obtain

$$\Rightarrow \quad \begin{bmatrix} 4 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -0.5 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \end{bmatrix}$$

In total we get a linear system of eight equations for the four unknown entries of the transformation matrix $T$. Solving this system of equations using e.g. the Gauss algorithm we find the solution

$$\underline{T} = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$$

Since $\det(\underline{T}) = 2 \neq 0$ this transformation matrix is regular. Thus, the two system descriptions are equivalent.