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Signal correlation (Chapter 5)

signals with finite and infinite energy

Parseval theorem:

Energy of a signal can be calculated in time- and frequency domain:

$$E_s = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df$$

electrical power: $P = U \cdot I = \frac{U^2}{R} = I^2 \cdot R$

energy: $E = \int P dt = \int \frac{1}{R} U^2 dt$

$R = 1\Omega$

$E_s < \infty$ „energy signal"

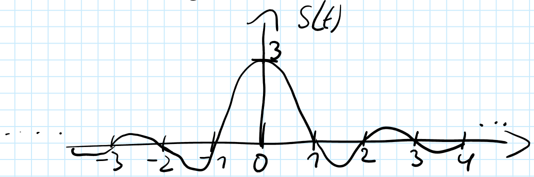
- $s(t) = rect(t)$
- piano
- discharging processes

$E_s \to \infty$ „power signal"

- $s(t) = \sin(2\pi f t)$
- PWM
- power plug 230V

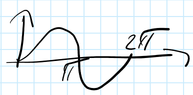
Example: Determine E_s of $s(t) = 3 \cdot si(\pi t)$

$$E_s = \int_{-\infty}^{\infty} |3 \cdot si(\pi t)|^2 dt$$

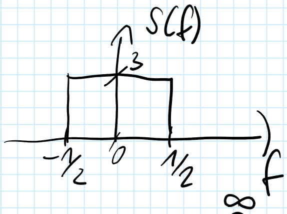


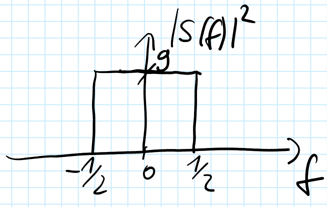
$si(\pi t) = \frac{\sin(\pi t)}{\pi t}$

$\sin(\pi t) \overset{!}{=} 0$



$\to$ frequency domain $3 \cdot si(\pi t) \circ\!\!-\!\!\bullet\ 3 \cdot rect(t)$





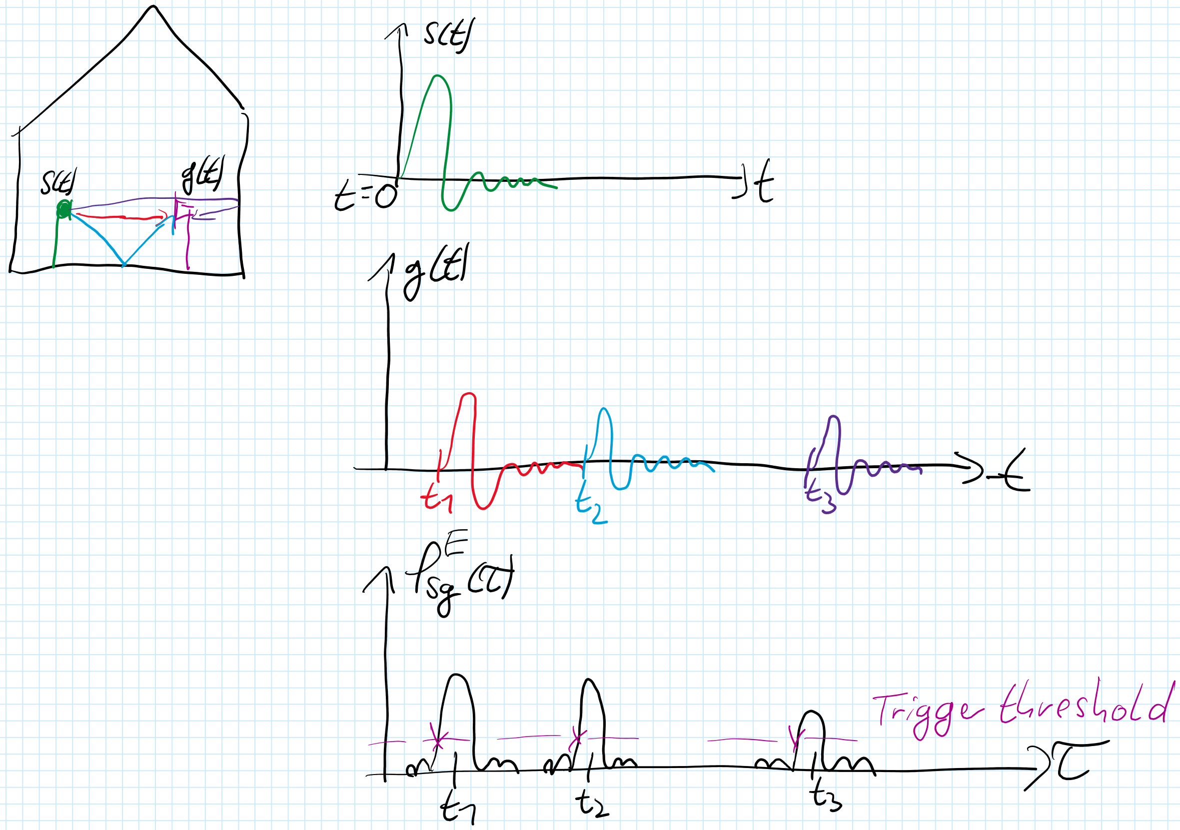
Parseval: $E_s = \underbrace{\int_{-\infty}^{\infty} |S(f)|^2 df}_{\text{Surface / area under the signal}} = 9$

Cross - Correlation for $E_s < \infty$

$$\varphi_{sg}^E(\tau) = \int_{-\infty}^{\infty} s(t) \cdot g(t+\tau) dt = s(-\tau) * g(\tau)$$

$$\rho_{sg}^{E}(\tau) = \int_{-\infty}^{\infty} s(t) \cdot g(t+\tau)\,dt = s(-\tau) * g(\tau)$$

- measure for similarity of two signals $s(t)$ and $g(t)$
- find $g(t)$ in $s(t)$
- how far do we have to shift $g(t)$, that the similarity to $s(t)$ is maximum?



$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

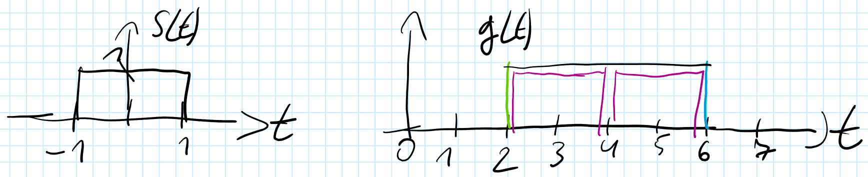
$\rho_{sg}^{E}(\tau)$ is able to automatically determine the time-delay and attenuation between $x(t)$ and $y(t)$.

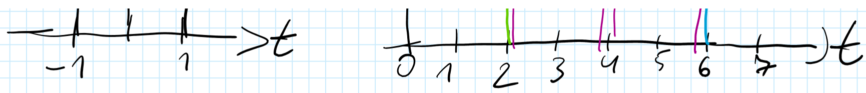
Example: $s(t) = \mathrm{rect}\left(\frac{t}{2}\right)$

$g(t) = \mathrm{rect}\left(\frac{t-4}{4}\right)$

$$s(t) \rightarrow \boxed{h(t)} \rightarrow g(t)$$

Which delay is caused by $h(t)$?





$$\frac{t-4}{4} = -\frac{1}{2} \Rightarrow t = 2 \text{ lower bound}$$

$$\frac{t-4}{4} = \frac{1}{2} \Rightarrow t = 6 \text{ upper bound}$$

$$\varphi_{sg}^{E}(\tau) = s(-\tau) * g(\tau) \quad \text{due to symmetry of rect}$$

$$= s(\tau) * g(\tau)$$

$$= \mathrm{rect}\left(\frac{\tau}{2}\right) * \mathrm{rect}\left(\frac{\tau}{2}\right) * [\delta(\tau-3) + \delta(\tau-5)]$$

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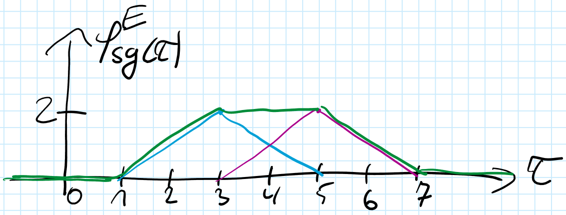
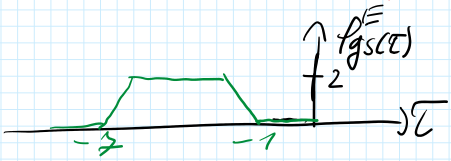
$$= 2 \cdot \Delta\left(\frac{\tau}{2}\right) * [\delta(\tau-3) + \delta(\tau-5)]$$



Note: $\varphi_{gs}^{E}(\tau) = \varphi_{sg}(-\tau)$ due to symmetry



$\varphi_{sg}^{E}(\tau)$ has its maximum within $3 \leq \tau \leq 5$

When we shift $s(t)$ by $t_0 = 3 \dots 5$ towards the right-hand side, then both signals have maximum similarity.

Autocorrelation

$$\varphi_{ss}^{E}(\tau) = \int_{-\infty}^{\infty} s(t) \cdot s(t+\tau)\, dt = s(-\tau) * s(\tau)$$

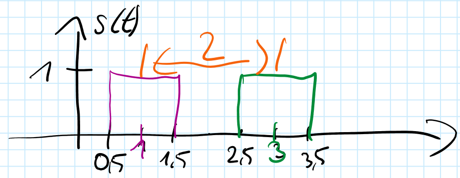
How similar is a signal $s(t)$ to itself when we shift it by τ?

Example:

$$s(t) = \mathrm{rect}(t) * [\delta(t-1) + \delta(t-3)]$$

Example:

$$s(t) = \text{rect}(t) * [\delta(t-1) + \delta(t-3)]$$



$$\rho_{ss}^E(\tau) = s(-\tau) * s(\tau)$$

$$= \text{rect}(-\tau) * [\delta(-\tau-1) + \delta(-\tau-3)] * \text{rect}(\tau) * [\delta(\tau-1) + \delta(\tau-3)]$$

Symmetry of rect $\quad -\tau-1=0 \quad\quad -\tau-3=0$

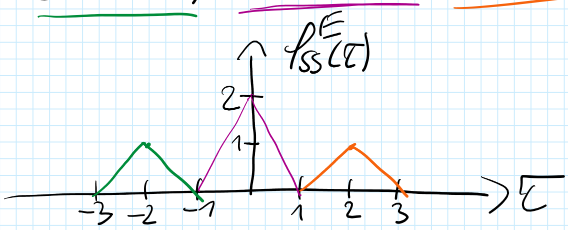
$\quad\quad\quad\quad\quad\quad\quad \tau=-1 \quad\quad\quad \tau=-3$

$$= \text{rect}(\tau) * [\delta(\tau+1) + \delta(\tau+3)] * \text{rect}(\tau) * [\delta(\tau-1) + \delta(\tau-3)]$$

$$= \text{rect}(\tau) * \text{rect}(\tau) * [\delta(\tau+1) * (\delta(\tau-1) + \delta(\tau-3)) + \delta(\tau+3) * (\delta(\tau-1) + \delta(\tau-3))]$$

$$= \Delta(\tau) * [\delta(\tau) + \delta(\tau-2) + \delta(\tau+2) + \delta(\tau)]$$

$$= \Delta(\tau) * [\delta(\tau+2) + 2 \cdot \delta(\tau) + \delta(\tau-2)]$$



- Shift by ± 2 maximum similarity to itself

  $\Rightarrow$ Period $= 2$

- Energy of the Signal: $E_s = \rho_{ss}^E(0) = 2$