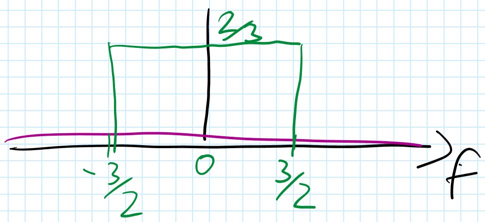
3. 03.04.2023

Montag, 3. April 2023 15:54

$$x(t) = 2 \cdot si(3\pi t) \circ\!\!-\!\!\bullet \frac{2}{3} \cdot rect\left(\frac{f}{3}\right) = X(f)$$

Task: Sketch the spectrum by real- and imaginary part:

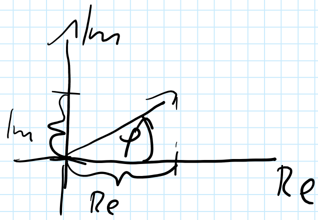


Sketch the spectrum by magnitude and phase:

Matgnitude: $|X(f)| = \sqrt{Re\{X(f)\}^2 + Im\{X(f)\}^2}$

$$\overset{here:}{=} \frac{2}{3} \cdot rect\left(\frac{f}{3}\right)$$

Phase: $\varphi(f) = \arctan \frac{Im\{X(f)\}}{Re\{X(f)\}}$



$$= 0$$

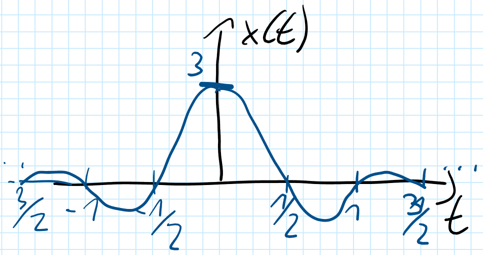
Parseval Theorem

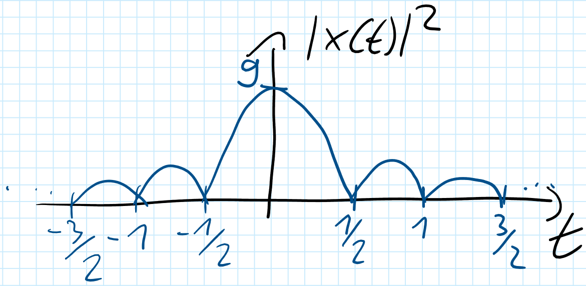
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df$$

The energy of a Signal can be calculated in time- and frequency domain.

Example: $x(t) = 3 \cdot si(2\pi t)$

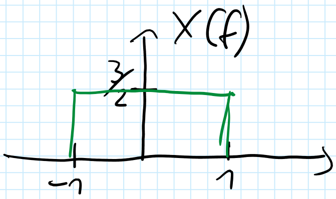
$$E_x = \int_{-\infty}^{\infty} |3 \cdot si(2\pi t)|^2 dt$$

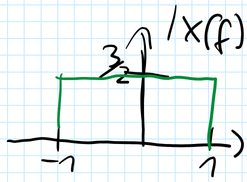


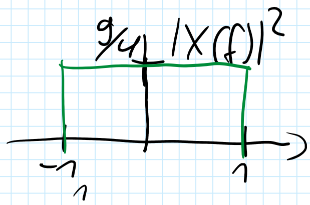


Integration to complicated

$$x(t) = 3 \cdot si(2\pi t) \circ\!\!-\!\!\bullet \frac{3}{2} rect\left(\frac{f}{2}\right) = X(f)$$







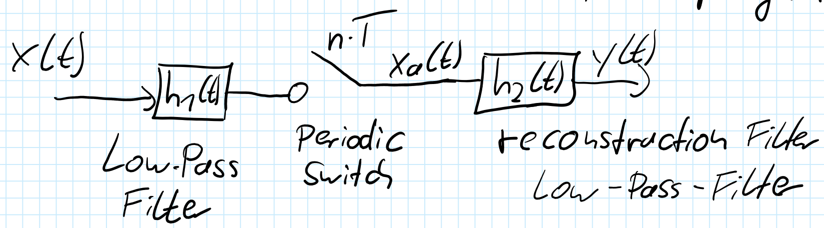
$$E_x = \int_{-\infty}^{\infty} |X(f)|^2 df = \int_{-\infty}^{\infty} \frac{9}{4} \cdot rect\left(\frac{f}{2}\right) df = \int_{-1}^{1} \frac{9}{4} df = \frac{9}{2}$$

Integral is the area underneath the funtion

=> PDF : Power Density Function

Digital Sampling

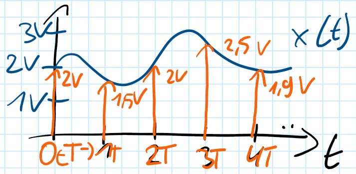
T : sampling time [s]
1/T : sampling rate [Hz]



AD → digital Signal DA →

Analog -> Digital   Digital-Analog

Simple case: ideal sampler



Time Domain:

$$x_a(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Frequency Domain:

$$X_a(f) = X(f) * \frac{1}{T} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$

mathematical mode:

$$x_a(t) = \underbrace{2V}_{\text{Amplitude}} \cdot \underbrace{\delta(t)}_{\text{time}} + 1{,}5V \cdot \delta(t-T) + 2V \cdot \delta(t-2T) \dots$$

digital signal

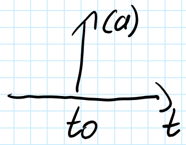
$$x_a(n) = [2; 1{,}5; 2; 1{,}9 \dots]$$

alternative representation in frequency domain

$$X_a(f) = X(f) * \frac{1}{T} \sum \delta\left(f - \frac{n}{T}\right)$$

alternative representation in frequency domain

$$X_a(f) = X(f) * \frac{1}{T}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T}\right)$$



$\circ\!\!-\!\!\bullet \quad a \cdot e^{-j2\pi f t_0}$

Formular Sheet

$$X_a(f) = 2 \cdot e^{-j2\pi f \cdot 0} + 1{,}5 \cdot e^{-j2\pi f \cdot T} + 2 \cdot e^{-j2\pi f \cdot 2T} + \dots$$

$$\Rightarrow X_a(f) = \sum_{n=-\infty}^{\infty} x_a(nT) \cdot e^{-j2\pi f \cdot nT}$$

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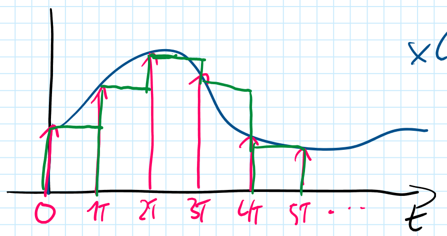
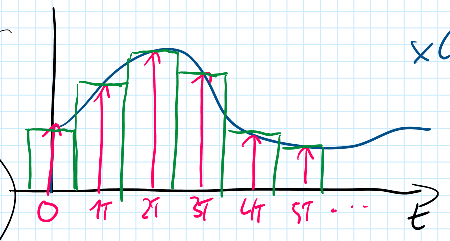
non-ideal sample: Sample & Hold



real sample & hold

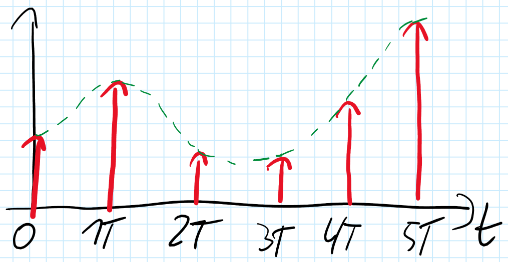
$$x_a(t) = \Big[\underbrace{x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)}_{\text{ideal sample}}\Big] * \mathrm{rect}\left(\frac{t}{T}\right) * \delta\left(t - \frac{T}{2}\right)$$



more simplified sample & hold

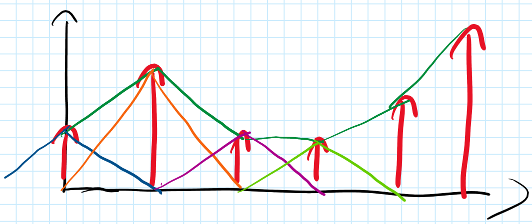
$$x_a(t) = \Big[x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)\Big] * \mathrm{rect}\left(\frac{t}{T}\right)$$

Reconstruction (Interpolation)



- we get a continous signal via the envelope function
- envelope functions are smooth
- low-Pass-Filter form envelope function

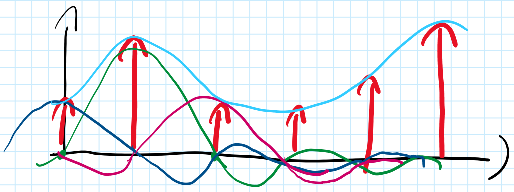
Linear Interpolation



Linear interpolated envelope:

$$y(t) = x_a(t) * \Delta\left(\frac{t}{T}\right)$$

Ideal Interpolation



$$y(t) = x_a(t) * si(\pi \frac{t}{T})$$

sum of all si-functions
leads to the envelope