Exercises

Z-Transformation

1. Consider the following z-transformed signal $H(z)$:

$$H(z) = \frac{z - a}{z - 0.5}$$

In the remaining part of the exercise we set $T = 1$.

a) Determine the corresponding Fourier-transformed signal $H(f)$.

b) For which a is the DC-component filtered?
Note: In this case we have $H(f = 0) = 0$.

c) For which a is the Nyquist-frequency filtered?
Note: In this case we have $H\left(f = \frac{1}{2T}\right) = 0$.

d) Determine a difference equation which is associated with the z-transform $H(z) = \frac{Y(z)}{X(z)}$.

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2. The given difference equation is considered:

$$y(n) = x(n) + b \cdot x(n-1) - a \cdot y(n-1)$$

a) Determine the z-transformed signal of the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

b) Give a pair of values a, b, for which $H(z)$ is a FIR-filter.

c) Give a pair of values a, b, for which $H(z)$ is unstable.

3. A time-discrete signal $x(n)$ is transmitted over a system with the transfer function $H(z)$. The output signal is $y(n) = h(n) * x(n)$.

$$H(z) = \frac{2z + 2}{z^2 - \frac{1}{4}}$$

a) Draw the pole-zero diagram of $H(z)$ stating all characteristic values. Specify also the amplification factor H_0.

b) Is the system stable? (justification required)

In the following we set $x(n) = \delta(n) - \frac{1}{2}\delta(n-1)$.

c) Determine the z-transformed signal $X(z)$ of the input signal.

d) Determine the z-transformed signal $Y(z)$ of the output signal.

e) Determine the output signal $y(n)$ in time-domain. The result shall not contain any convolution operators.