Exercise Image Processing
Sample Solution

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Sheet 1

You are in your 3rd semester at a university of applied sciences and would like to earn some extra money while studying in a medium-sized company in the field of robotics. Your team leader has read in your application documents that you are currently listening to a lecture about image processing. That's why you're supposed to use an existing robot system for gripping workpieces that run along a conveyor belt with the help of a camera, so that the workpiece can be checked for defects in its shape. The camera is mounted directly on the robot arm above the gripper so that you can position the camera anywhere around the workpiece. The workpieces are all at predefined known positions with the same orientation on the conveyor belt and the conveyor belt is timed so that the workpieces always stop at the same position relative to the robot arm.

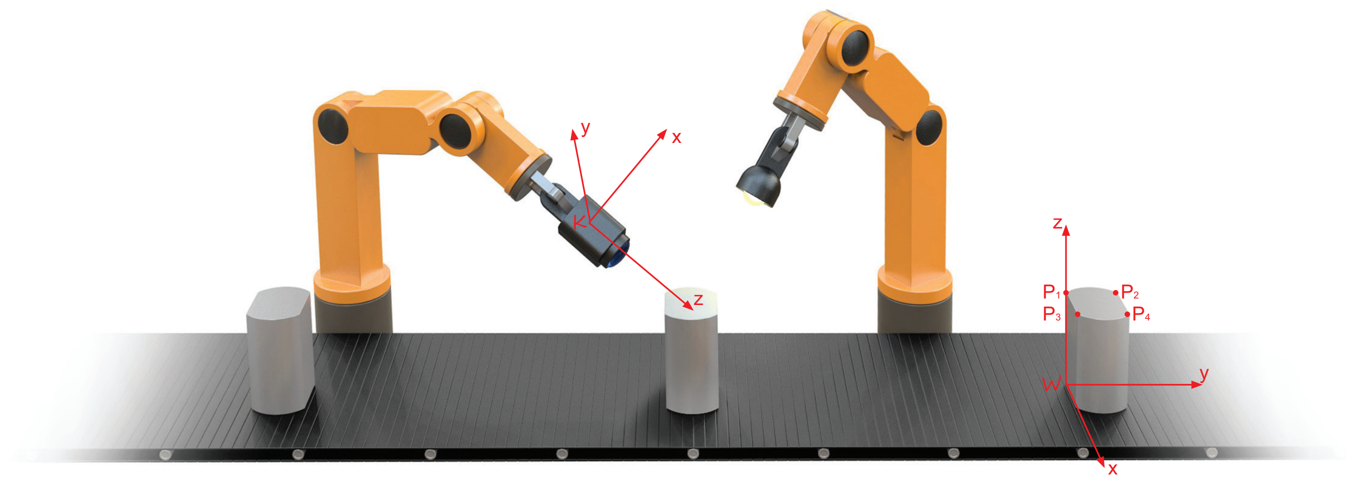

Figure 1: Robot system and conveyor belt with workpieces

The workpiece is a metallic cylindrical bolt with two parallel flattened sides, as shown in Figure 1. You will be assigned the following work package for the first week:

- Check the 2D geometry of the cut through the bolt for correct dimensions.

Task 1.1: Pose of the camera

The workshop manager has provided you with the following prior knowledge: The coordinate systems are defined as shown in Figure 1 and the coordinates of the four corner points of the section with respect to the workpiece coordinate system are as follows in millimeters: $\mathbf{p}_1 = [0, 0, 50]^T$, $\mathbf{p}_2 = [0, 20, 50]^T$, $\mathbf{p}_3 = [20, 0, 50]^T$ and $\mathbf{p}_4 = [20, 20, 50]^T$.

You start full of enthusiasm and buy from your budget a camera with a camera constant of 5 mm and an image plane of size 3 mm x 4 mm. After that you ask yourself:

a) What is the best way to align the camera so that I can check the bolt for correct contour?

Answer: The image plane must be parallel to the section of the bolt so that the image on the image plane corresponds to a scaled version of the top view of the bolt cut. The optical axis must lie on the symmetry axis of the bolt so that the image plane and the center of the circle of the bolt section both lie on the optical axis and the viewing area of the camera can be optimally utilized.

b) What is the minimum distance between the camera and the bolt so that the complete section of the bolt can be seen on the image plane? What can be derived from this for the camera pose?

Answer: To position the camera as close as possible to the surface, the longer side of the image plane must be aligned along the larger diameter of the cross section of the bolt. The radius of the circle is $r = \sqrt{10^2 + 10^2} = \sqrt{200}$. This gives the following possible minimum distances:

$$x_{\text{max}} = c\frac{X}{Z_{\text{min,x}}} \quad \rightarrow \quad Z_{\text{min,x}} = c\frac{X}{x_{\text{max}}} = 5\frac{10}{1,5} \approx 33.3$$
$$y_{\text{max}} = c\frac{Y}{Z_{\text{min,y}}} \quad \rightarrow \quad Z_{\text{min,y}} = c\frac{Y}{y_{\text{max}}} = 5\frac{\sqrt{200}}{2} \approx 35.4$$

From this follows a minimum distance of $Z_{\text{min}} = \max(Z_{\text{min,x}}, Z_{\text{min,y}}) = 35.4mm$, a rotation of $-180°$ around the X_W-axis and a subsequent rotation of $-90°$ around the Z_W-axis, so that the image plane is aligned correctly, and a translation of $T_K = [-10, -10, 50 + Z_{\text{min}}]^T$ (-10,-10 to place the optical axis at the workpiece center, 50 for the workpiece height) so that the cut is completely visible in the image. The total rotation results to:

$$\mathbf{R}_{KW} = \begin{pmatrix} \cos(-90°) & \sin(-90°) & 0 \\ -\sin(-90°) & \cos(-90°) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-/+180°) & \sin(-/+180°) \\ 0 & -\sin(-/+180°) & \cos(-/+180°) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Here the coordinate system of the camera is rotated counterclockwise.

c) What are the coordinates of the four corner points p_1, p_2, p_3 and p_4 of the cross section in camera coordinates, if you choose a distance of the camera to the cross section center of 40mm?

Answer: With translation $T_K = [-10, -10, 50 + 40]^T$ and the formula for coordinate transformation $\mathbf{X}_K = \mathbf{R}_{KW}\mathbf{X}_W + T_K$ we get the following camera coordinates: $\mathbf{p}_1 = [-10, -10, 40]^T$, $\mathbf{p}_2 = [10, -10, 40]^T$, $\mathbf{p}_3 = [-10, 10, 40]^T$ und $\mathbf{p}_4 = [10, 10, 40]^T$.

Task 1.2: Projection of the bolt cross section

After you have set up the camera properly, you want to determine the expected ideal projection for an error-free measured contour. You proceed according to the following flow chart:

a) What projection should result if the bolt is manufactured without defects and the image plane is aligned parallel to the bolt cross-section?

Answer: Two parallel straight line segments between (p_1, p_3) and (p_2, p_4) and two circular segments between (p_1, p_2) and (p_4, p_3).

b) Calculate the image coordinates $\mathbf{x}_1$ to $\mathbf{x}_4$ of the points p_1 to p_4.

Answer: With projection equations $x = c\frac{X}{Z}$ and $y = c\frac{Y}{Z}$ the image coordinates are as follows:

$$x_1 = c\frac{X_1}{Z_1} = 5\frac{-10}{40} = -1.25\,, \quad y_1 = c\frac{Y_1}{Z_1} = 5\frac{-10}{40} = -1.25$$
$$\mathbf{x}_1 = (-1.25, -1.25)\,, \quad \mathbf{x}_2 = (1.25, -1.25)\,, \quad \mathbf{x}_3 = (-1.25, 1.25)\,, \quad \mathbf{x}_4 = (1.25, 1.25)\,.$$

c) Set up the straight line equations for the point pairs (p_1, p_3) and (p_2, p_4) in homogeneous coordinates $\overline{\mathbf{x}}$ and determine the parameters $\mathbf{l}_{13}$ and $\mathbf{l}_{24}$. This corresponds to the expected position of the two flattened sides of the bolt.

Answer: Straight line representation in homogeneous coordinates:

$$\mathbf{l}_{13} = \hat{\overline{\mathbf{x}}}_1 \overline{\mathbf{x}}_3 = [-2.5, 0, -3.125]^\top$$
$$\mathbf{l}_{24} = \hat{\overline{\mathbf{x}}}_2 \overline{\mathbf{x}}_4 = [-2.5, 0, 3.125]^\top \qquad \text{(Symmetry!)}$$

d) Determine the 2nd order curve $\overline{\mathbf{x}}^T \mathbf{C} \overline{\mathbf{x}} = 0$ for the circular segments of the bolt. For the matrix holds:

$$\mathbf{C} = \begin{pmatrix} c_1 & 0 & c_2 \\ 0 & c_1 & c_3 \\ c_2 & c_3 & c_4 \end{pmatrix}.$$

How many points on the circle do you need to solve for the circle parameters? Solve the resulting definite linear homogeneous system of equations.

Answer: To set up the certain homogeneous system of equations for the circle parameters c_1-c_4 you need three points, e.g. p_1, p_2 and p_3. Since the system of equations is homogeneous, you can set one parameter arbitrarily, e.g. $c_1 = 1$ (alternatively you could also set the norm of the matrix $\|\mathbf{C}\| = 1$). For each point you get an equation:

$$[x_i^2 + y_i^2, 2x_i, 2y_i, 1][c_1, c_2, c_3, c_4]^\top = 0\,.$$

For $c_1 = 1$ the equation system reads:

$$\begin{aligned} p_1: &\quad 3.125 - 2.5c_2 - 2.5c_3 + c_4 = 0 \\ p_2: &\quad 3.125 + 2.5c_2 - 2.5c_3 + c_4 = 0 \\ p_3: &\quad 3.125 - 2.5c_2 + 2.5c_3 + c_4 = 0 \end{aligned}$$

This gives the parameters: $c_2 = c_3 = 0$ and $c_4 = -3.125 = -r^2$. This gives the matrix:

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3.125 \end{pmatrix}.$$

Thus $\overline{\mathbf{x}}^T \mathbf{C} \overline{\mathbf{x}} = 0$ corresponds to the circle equation: $x^2 + y^2 = r^2 = 3.125$.

Task 1.3: Checking the bolt cross section

You have written an algorithm that uses an edge image to extract (measure) the contours of the bolt cut and then fitted two straight lines and a circle into the corresponding measured contour sections.

a) You will get the following measured 2D lines of the bolt cut for the flattenings: $\hat{\mathbf{l}}_{13}^T = [-2.5, 0.5, -3.125]^T$, $\hat{\mathbf{l}}_{24}^T = [-2.5, 0, 3.235]^T$. Determine the deviation of the measured line segments with the calculated references by calculating the scalar product between the parameter vectors.

Answer: The scalar products lead to the following scalars:

$$\hat{\mathbf{l}}_{13}^\top \mathbf{l}_{13} = [-2.5, 0.5, -3.125][-2.5, 0, -3.125]^\top \approx 16.01$$
$$\hat{\mathbf{l}}_{24}^\top \mathbf{l}_{24} = [-2.5, 0, -3.235][-2.5, 0, 3.125]^\top \approx -3.86$$

These values are not very meaningful because the parameters of the straight lines have not been normalized.

b) Normalize the parameters of $\mathbf{l}_{13}$ and $\mathbf{l}_{24}$ to Hessian normal form: $\mathbf{l}^\top \overline{\mathbf{x}} = \mathbf{n}^\top \mathbf{x} - d = 0$. Now determine the differences in distance and orientation of the calculated and measured straight lines.

Answer: Normalizing the parameters $\mathbf{l}$ to the Hessian normal form $\tilde{\mathbf{l}} = [\mathbf{n}, -d]^\top$ yields for the first two parameters the normal vector $[\tilde{l}_1, \tilde{l}_2]^\top \stackrel{!}{=} \mathbf{n}$ and for the third parameter the negative distance $\tilde{l}_3 \stackrel{!}{=} -d$ to the origin:

$$\tilde{l}_1 = \pm \frac{l_1}{\sqrt{l_1^2 + l_2^2}}, \qquad \tilde{l}_2 = \pm \frac{l_1}{\sqrt{l_1^2 + l_2^2}}, \qquad \tilde{l}_3 = \pm \frac{l_3}{\sqrt{l_1^2 + l_2^2}},$$

where the sign must be chosen negative if $l_3 > 0$ so that the condition $\tilde{l}_3 \stackrel{!}{=} -d$ is satisfied. We obtain the following normalized straight line parameters:

$$\tilde{\mathbf{l}}_{13} = [-1, 0, -1.25]^\top, \qquad \tilde{\hat{\mathbf{l}}}_{13} = [-0.98, 0.19, -1.23]^\top,$$
$$\tilde{\mathbf{l}}_{24} = [1, 0, -1.25]^\top, \qquad \tilde{\hat{\mathbf{l}}}_{24} = [1, 0, -1.29]^\top.$$

The differences in orientation result in:

$$\Delta\alpha_{13} = \cos^{-1} \mathbf{n}_{13}^\top \hat{\mathbf{n}}_{13} = 11.4^\circ,$$
$$\Delta\alpha_{24} = \cos^{-1} \mathbf{n}_{24}^\top \hat{\mathbf{n}}_{24} = 0^\circ.$$

The differences in distance to the origin result in:

$$\Delta d_{13} = 1.25 - 1.23 = 0.02,$$
$$\Delta d_{24} = 1.29 - 1.25 = 0.04.$$