Exercise Image Processing
Sample Solution

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Sheet 3

In this exercise we will cover the chapters *lens*, *projective motions* and *illumination*. The questions are small-part and can be seen as examples of potential exam problems. Also use the formulary for the exam to complete the tasks.

Task 3.1: The lens

3.1a)

How does the focal length f change if the radii R_1 and R_2 of the lens remain constant, but a lens material with a larger refractive index n is chosen?

Answer: The focal length gets smaller.

3.1b)

For which configuration is the magnification β less than one?

- × The object width g is larger than twice the focal length f and the image width b is in the interval $]f; 2f[$.
- ☐ The image width b is larger than twice the focal length f and the object width g is in the interval $]f; 2f[$.
- ☐ The image width b is less than zero, the magnitude of the image width $|b|$ is greater than the object width g, and the object width lies in the interval $]0; f[$.
- ☐ The object width g and the image width b are both larger than twice the focal length f

3.1c)

Using a calculation, explain why the longer the focal length f, the larger the aspect ratio β for an image distance in the range $f < b < 2f$. What value can the aspect ratio not exceed in this range?

Answer: For the image scale is valid: $\beta = \frac{b-f}{f} = \frac{b}{f} - 1$. For the range $f < b < 2f$ is valid: $b = 2f - \epsilon$, where $0 < \epsilon < f$. This gives: $\beta = \frac{b}{f} - 1 = \frac{2f-\epsilon}{f} - 1 = 1 - \frac{\epsilon}{f}$. Thus, the longer the focal length is chosen, the larger the magnification becomes. However, the image scale $0 < \beta < 1$ is always smaller than one in this range $f < b < 2f$ of the image width.

3.1d)

How does the focal length f change when the wavelength λ of the light becomes shorter? Prove the relationship by an estimation.

Answer: The focal length becomes smaller. Reasoning: $f \propto \frac{\lambda}{\lambda_0 - \lambda} = \frac{1}{\lambda_0/\lambda - 1}$. If λ becomes smaller, then λ_0/λ becomes larger, then $f \propto \frac{1}{\lambda_0/\lambda - 1}$ becomes smaller.

3.1e)

Which statements regarding the aspect ratio β are correct?

- ☐ The smaller the object distance g, the smaller β.
- ☐ The longer the focal length f, the smaller is β if the image width is in the range $f < b < 2f$.
- × Every lens has a maximum aspect ratio because below a minimum distance g_{min} it is no longer possible to focus on an object.
- ☐ Every lens has a maximum aspect ratio, because above a maximum distance g_{max} it is no longer possible to focus on an object.

3.1f)

Calculate the working distance g for an image size of $b = 10mm$ and a focal length of $f = 5mm$. What does it follow for the image size B as a function of the object size G?

Answer: Working distance: $g = \frac{b}{\beta} = \frac{bf}{b-f} = \frac{10mm \cdot 5mm}{10mm - 5mm} = 10mm$. Because $\beta = \frac{b-f}{f} = 1$ the imge size gets $B = G$.

3.1g)

Calculate the depth of field range for a teleradiography for a working distance of $g = 10m$, an aperture of $D = 1cm$, a focal length of $2mm$ and a depth of sharpness of $\epsilon = 1px$. A pixel is square with side length of $s = 10\mu m$.

Answer: $g_h - g_v \approx \frac{2g^2\kappa\epsilon}{f^2} = \frac{2(10m)^2(2mm/1cm)(10\mu m)}{(2mm)^2} = 100m$.

3.1h)

Assume the model of a thin lens. With a constant focal length f, how must the object width g be changed, so that the object is in focus when the image width b increases by $\partial b > 0$, where: $b > f$?

Answer: • The object width depends on the image width as follows: $g(b) = \frac{bf}{b-f}$.

• The derivative of the object width with respect to the image width is given by: $\frac{\partial g}{\partial b} = -\frac{f^2}{(b-f)^2}$.

• Thus the change of the object width $\partial g < 0$ is negative and the object width g must be decreased by $\frac{f^2 \partial b}{(b-f)^2}$, so that the object is still sharply imaged when the image width is increased by $b + \partial b$.

3.1i)

How must the aperture be changed to increase the range of depth of field?

Answer: The aperture must be reduced to increase the depth of field range.

3.1j)

Which two effects have opposite effects on the depth of field when the aperture is varied?

Answer: A smaller aperture results in a larger f-number, which increases the range of depth of field. However, it also leads to greater diffraction blur, since the diffraction blur is approximately equal to the f-number.

3.1k)

What aberrations can a lens cause?

- ☐ chronic abberation
- ☐ bending blur
- ☐ spherical iteration
- × diffraction blur
- ☐ ternary distortions
- ☐ aspergerismn
- × chromatic abberation
- ☐ diffraction relation

Task 3.2: Illumination

3.2a)

What is the difference between radiometric and photometric quantities?

Answer: Measurement of the objectively emitted radiation and the radiation subjectively perceived by humans.

3.2b)

What kind of problem can occur on surfaces with directional reflection?

Answer: Occurs with reflective surfaces and creates specular highlights that shift with slight changes in the position of the object or camera.

3.2c)

Name two advantages of LED lighting?

Answer: Long life, mechanical robustness, ideal arrangement ability, choice of light color.

3.2d)

What type of illumination can be used to highlight edges of embossed or punched surfaces such as a coin in an image?

Answer: Dark field illumination

3.2e)

What is the difference between incident light and transmitted light?

Answer: Incident light: illumination and camera are both in front of the object. Transmitted light: Camera is in front of the object, illumination is behind the object.

Task 3.3: Projective movements

3.3a)

What assumption do you have to make if you want to calculate the ego-motion of a camera from the optical flow?

Answer: Projected points do not move (static scene)

3.3b)

Determine the calculation formula for the rotation speed ω_1 of a camera around the vertical axis X depending on the flux components u, v and the image point coordinates x, y. From which coordinate is the formula independent? If the camera rotates only around the vertical axis, then all translational motion components ν_1, ν_2, ν_3 are zero and all other rotational motion components ω_2, ω_3 as well. The flux component $u = 0$ does not exist.

Answer: $\omega_1 = \frac{v}{1-y^2}$. The formula does not depend on the x-component