Control Systems Important Formulas and Topics

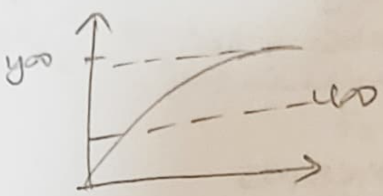
(A)

① Types of signals
Plotting them on graph

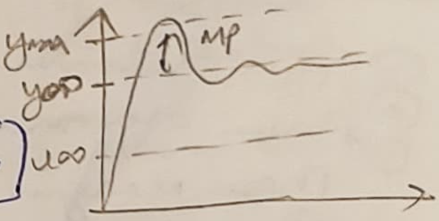
② Laplace Transformation and Anti-Laplace Questions
(Step/Impulse Response Questions)

③ PT1 General Form

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{Ts+1} \qquad \boxed{K = y_\infty / u_\infty}$$



④ PT2 General Form



$$G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\left(\frac{s}{\omega_0}\right)^2 + 2D\left(\frac{s}{\omega_0}\right) + 1} \qquad \boxed{K = y_\infty / u_\infty}$$

$$\boxed{M_p = \frac{y_{max} - y_\infty}{y_\infty}}$$ overshoot factor

⑤ Interconnection

Series = $G_1(s) \cdot G_2(s)$
Parallel = $G_1(s) + G_2(s)$
Feedback = $\frac{\text{~~Forward~~}}{1 - (\text{loop TF})} = \frac{G_1(s)}{1 + G_1(s) G_2(s)}$

⑥ Frequency Response
$A(\omega)$ Amplitude response
$\phi(\omega)$ Phase response

⑦ Bode Plotting (NO CLUE HOW TO DO – SORT OUT ASAP)
Corner Frequency calculation

⑧ CH3

$$G_{MR}(s) = \frac{G_c G_p}{1 + G_c G_p}$$ aim to make it 1

$$G_{MD}(s) = \frac{1}{1 + G_c G_p}$$ aim to make it 0

(9) STABILITY DIAGRAMS Page 35 and 36 Have a look

(10) NYQUIST CRITERION Page 70

* Theorems

No encirclement	CW	CCW
↓	↓	↓
Stable if no poles in RHP	Unstable	Stable if no of poles in RHP = Σ encirclements

(11) STABILITY MARGINS

(a) Gain crossover frequency $A(\omega_{gc}) = 1$

(b) Phase crossover frequency $\phi(\omega_{pc}) = -180^\circ$

(c) Phase Margin $\phi_M = \phi(\omega_{gc}) + 180^\circ$

(d) Gain Margin = $A_m = \frac{1}{A(\omega_{pc})}$

(12) CH4

PID controller

$$G_C(s) = K_C\left(1 + \frac{1}{T_I s} + T_D s\right)$$

(+) Modifiable → PIDT1

(−) Not implementable

(−) Derivative plays role of a high pass filter leading to noise and distortion.

↓

use low pass to suppress the noise

PIDT1 controller

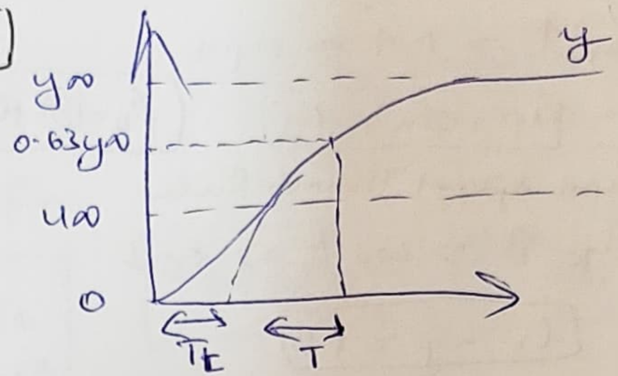
$$G_C(s) = K_C\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + T_1 s}\right)$$

{Speed Controller Example}

(13) EMPIRICAL METHODS   EXAM RELEVANT ???   (B)

ZIEGLER NICHOLAS $\boxed{\text{TUNING}}$

$$G_S(s) = \frac{k\, e^{-sT}}{Ts+1}$$



PID → $K_C = \frac{1.2T}{K_S T_t}$

→ $T_I = 2T_t$

→ $T_D = 0.5T_t$

$K_S = \frac{y_\infty}{u_\infty}$

T_t - rise time

T - @ $0.63y_\infty$

$\boxed{\text{MARGINAL STABILITY}}$

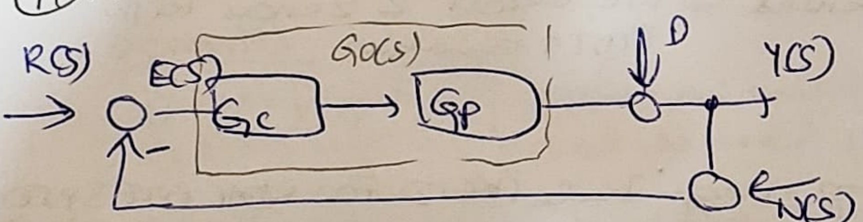
(1) Control plant using loop controller
(2) Increase controller gain K to critical value K_{CR} so closed loop is marginally stable
(3) Note values of K_{CR}, T_{CR}, Time period and oscillation
(4) Calculate controller parameters

PID → $K_C = 0.6K_{CR}$

→ $T_I = 0.5T_{CR}$

→ $T_D = 0.12T_{CR}$

(14) ANALYTICAL METHODS - CONTROLLER DESIGN



$$Y(S) = \frac{1}{1+G_o(s)} D(s) + \frac{G_o(s)}{1+G_o(s)} R(S) + \frac{G_o(s)}{1+G_o(s)} \cdot N(s)$$

Disturbance	Set Point	Disturbance
⇓	⇓	⇓
$G_{YD} = 0$	$G_{YR} = 1$	$G_{YN} = 0$
⇓	⇓	⇓
$G_o(s) \to \infty$	$G_o(s) \to \infty$	$G_o(s) \to 0$

Oscillation Behaviour Thumb Rule

$\phi_M \uparrow \Rightarrow D \uparrow \Rightarrow M_p \downarrow$

$\boxed{M_p + \phi_M \approx 70^\circ}$ $\boxed{\phi_M = 180 + \phi_M}$

increase in phase margin causes increase in damping ratio of closed loop and decrease in overshoot.

Response speed Thumb Rule

$\omega_{gc} \uparrow \Rightarrow \omega_0 \uparrow \Rightarrow t_r \downarrow$

$\boxed{t_r \cdot \omega_{gc} \approx 1.5}$

increase in gain crossover frequency causes increase in natural frequency of closed loop and decrease in rise time.

Steady State accuracy Thumb Rule

$|G_o(0)| = A(0) \uparrow \Rightarrow e_\infty \downarrow$

$\boxed{e_\infty = 1/[1 + A(0)]}$

$\Rightarrow \boxed{\frac{1}{e_\infty} - 1 = A(0)}$

open loop dc gain increase leads to a decrease in steady state error

(5) Direct cancellation of poles

(a) Responsible for slow response

→ cancel pole which is closest to zero

→ most likely a PI controller

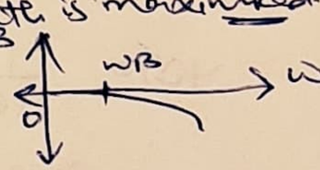
(b) Responsible for oscillation

→ we have 2 ~~zeroes~~ so we cancel 2 zeroes with PIDT1 equation

→ most likely

(6) optimisation of closed loop magnitude (VALID FOR REAL POLE SYSTEMS)

AIM calculation of parameters such that control bandwidth is maximized.



$$|G_{yR}(j\omega)| = \begin{cases} = 1 & \omega < 0 \\ \approx 1 & 0 \leq \omega \leq \omega_B \end{cases}$$

Current Control Example

$$\boxed{G_p(s) = \frac{K_p}{1 + a_1 s + \cdots a_n s^n}}$$

$$\boxed{T_I = a_1 + \frac{a_3 - a_1 a_2}{a_1^2 - a_2}}$$

$$\boxed{K_C = \frac{1}{2K_p} \cdot \left(\frac{a_1 (a_1^2 - a_2)}{a_1 a_2 - a_3} - 1 \right)}$$

Parameters from calculating $G_p(s)$

(17) SYMMETRICAL OPTIMUM {Speed Controller Example} ©

↳ Designing PI controller with an integrator already present

$$G_p(s) = \frac{K_p}{(T_1 s+1)\,T_2 s}$$

Point where $\phi_M = 45°$ (most stable)
ω_{gc} [GM of $1/T_1$ and $1/T_2$]

for given phase margin ϕ_M

$$\boxed{T_I = \left(\frac{1+\sin\phi_M}{\cos\phi_M}\right)^2 T_1}$$

$$\boxed{\omega_{gc} = \frac{1}{\sqrt{T_I \cdot T_1}}}$$

$$\boxed{K_C = \frac{1}{K_p}(\omega_{gc} T_2)}$$

<u>Situation</u>: plant with a large and small time constant given and task is to design a PI controller. Don't use POLE CANCELLATION, use symmetric optimum as the time constant ratio is very large.

(18) POLE PLACEMENT METHOD (*)

<u>How to select closed loop poles?</u>

1) All poles located on LHP (Stability reasoning)
2) Natural frequency ω_0 is large enough (Speed of Response)
3) Damping ratio of poles large enough (Better Oscillation)
4) Natural frequency not too large (Avoid actuator Saturation)

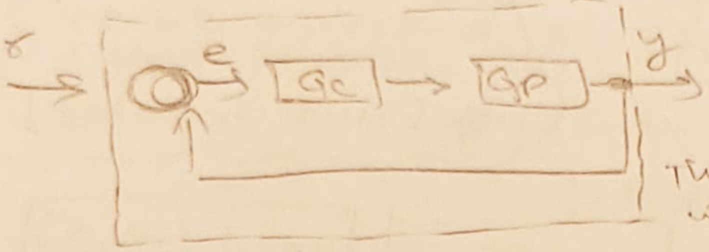
<u>Demerits</u>

1) No of poles depends on number of adjustable parameters
2) Poles are placed at desired position, zeroes are not considered.
3) Not applicable to plants with transport delay.

(20) DESIRED TRANSFER FUNCTION APPROACH

[COMPENSATION METHOD]

> express control specification in form of a transfer function. $K_W(s)$.



This loop should behave like $K_W(s)$ which we decide or may be given.

$$G_{YR}(s) = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)} = K_W(s)$$

$$\boxed{G_c(s) = \frac{K_W(s)}{1 - K_W(s)} \cdot \frac{1}{G_p(s)}}$$ } controller calculation when we know desired transfer fn.

$n \geq m$ $\qquad$ $\mu > v$

$$G_p(s) = \frac{B(s)}{A(s)} \qquad K_W(s) = \frac{\alpha(s)}{\beta(s)}$$

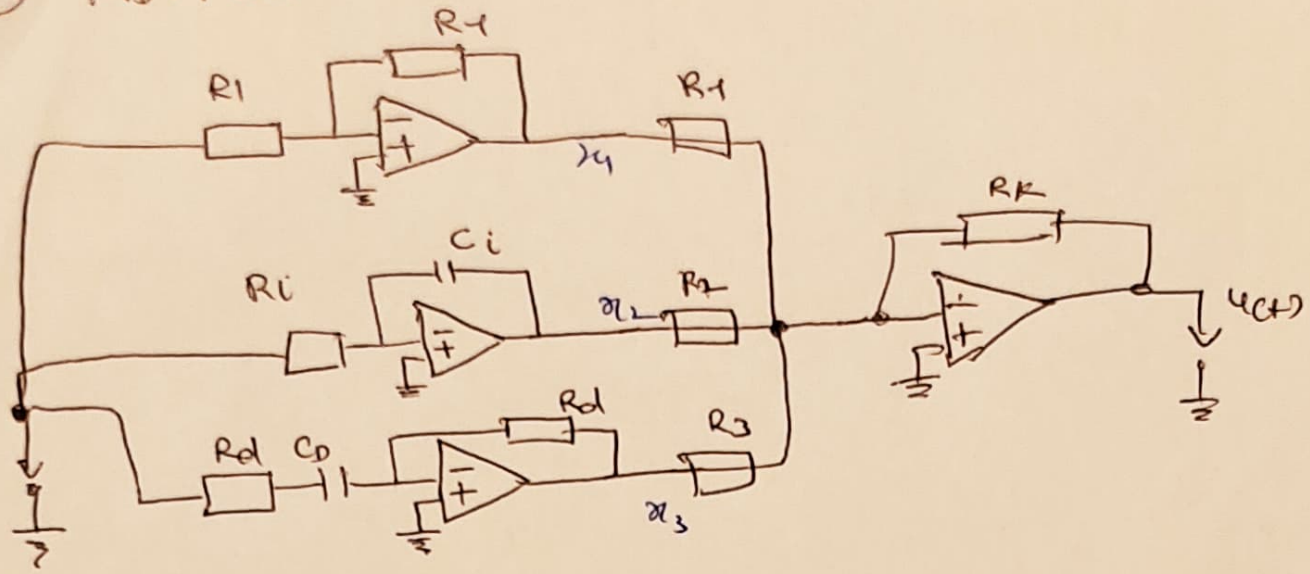
$$\boxed{G_c(s) = \frac{\alpha(s)}{\beta(s) - \alpha(s)} \cdot \frac{A(s)}{B(s)}}$$

Catalogue of requirements for $K_W(s)$ selection →

1) $K_W(s)$ must be stable [POLES IN LHP]
2) Speed of response acceptable [Application dependent]
3) Good Damping properties
4) Steady state Gain should be 1 ⇒ Steady State Error = 0
(*) 5) $\boxed{\mu - v \geq n - m}$
6) In order to avoid cancellation of plant zeroes from RHP with the controller zeroes, these zeroes are taken as zeroes of $K_W(s)$.

GOLDEN RULE ⇒ Never cancel zero/pole of plant with zero/pole of controller as it leads to instability.

(24) PIDTY ELECTRONIC IMPLEMENTATION (D)



$K_C = \frac{R_K}{R_1}$ $T_D = R_D C_D$ $T_I = R_I C_I$ $T_V = R_V C_D$

need resistors which are not too small because it leads to greater power loss so current stays in mA range
→ KΩ resistor range.