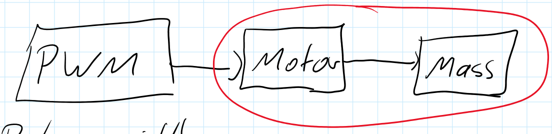
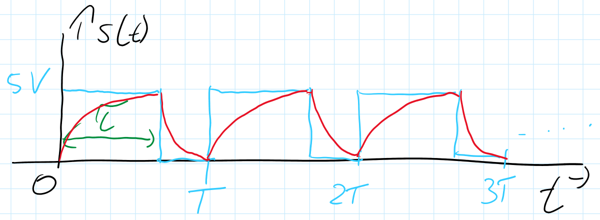


Pulse-width modulation

Low-Pass Filter

Mass reacts slow to acceleration



Period   τ: Pulse duration

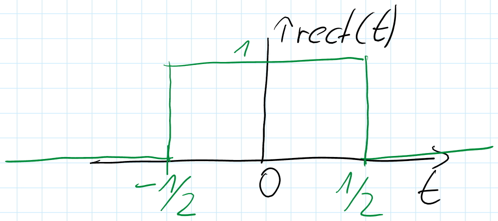
Duty Cycle: $DC = \frac{\tau}{T}$

How to descripe the PWM signal?

Elementary - Signals

rectangular signal

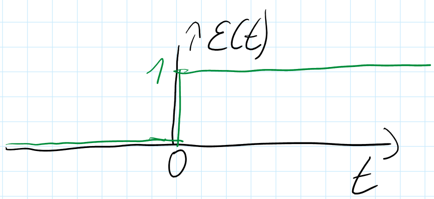
$$\text{rect}(t) = \begin{cases} 1, & \text{for } |t| \leq 0{,}5 \\ 0, & \text{else} \end{cases}$$



step function

$$\varepsilon(t) = \begin{cases} 1, & \text{for } t \geq 0 \\ 0, & \text{else} \end{cases}$$

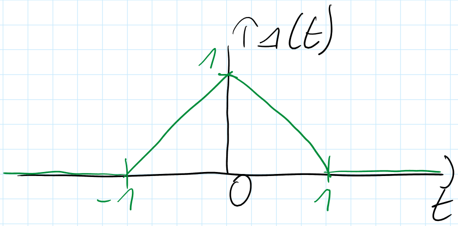
models switching process



triangular function

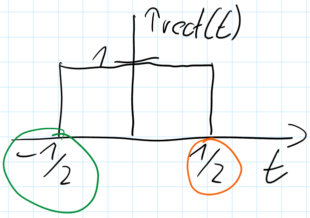
triangular function

$$\Delta(t) = \begin{cases} 1-|t| & , \text{for } |t| \leq 1 \\ 0 & , \text{else} \end{cases}$$



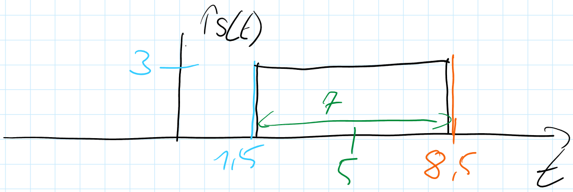
Amplitude, Shift, Expansion

$$s(t) = 3 \cdot \text{rect}\left(\frac{t-5}{7}\right)$$



$$\frac{t-5}{7} = -\frac{1}{2} \Rightarrow t-5 = -\frac{7}{2}$$
$$t = \frac{3}{2} = 1{,}5$$

$$\frac{t-5}{7} = \frac{1}{2} \Rightarrow t-5 = \frac{7}{2}$$
$$t = \frac{17}{2} = 8{,}5$$



Convolution

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

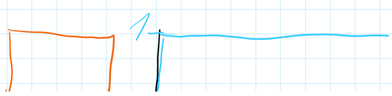
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$$

$x(t) = \varepsilon(t)$ switch on at $t=0$

$h(t) = \text{rect}(t)$ short-time integrator / low pass

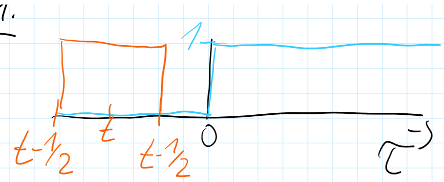
$$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \varepsilon(\tau) \cdot \text{rect}(t-\tau)\, d\tau$$

Case 1:



upper bound: $t - \tau = -\frac{1}{2}$ | lower bound: $t - \tau = \frac{1}{2}$

Case 1:

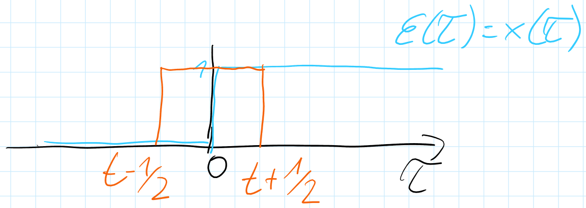


$$t-\tau = -\tfrac{1}{2} \qquad\qquad t-\tau = \tfrac{1}{2}$$
$$-\tau = -\tfrac{1}{2}-t \qquad\qquad -\tau = \tfrac{1}{2}-t$$
$$\tau = \tfrac{1}{2}+t \qquad\qquad \tau = t-\tfrac{1}{2}$$
$$\tau = t+\tfrac{1}{2}$$

$t+\frac{1}{2} < 0 \quad t < -\frac{1}{2}$: $y(t) = 0$ because at least one signal is equal to 0.

or: Both signals don't overlap

Case 2:

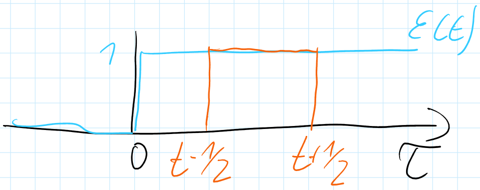


$$\left.\begin{aligned} t+\tfrac{1}{2} \geq 0 \quad &\Leftrightarrow \quad t \geq -\tfrac{1}{2} \\ t-\tfrac{1}{2} < 0 \quad &\Leftrightarrow \quad t < \tfrac{1}{2} \end{aligned}\right\} -\tfrac{1}{2} \leq t < \tfrac{1}{2}$$

$$y(t) = \int_{-\infty}^{\infty} \varepsilon(\tau) \cdot \mathrm{rect}(t-\tau)\, d\tau$$

$$= \int_{0}^{t+\frac{1}{2}} 1 \cdot 1 \, d\tau = [\tau]_0^{t+\frac{1}{2}} = t+\tfrac{1}{2} - 0 = t+\tfrac{1}{2}$$

Case 3:



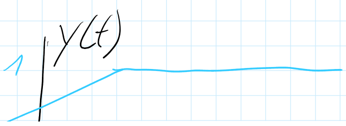
$$t-\tfrac{1}{2} \geq 0 \Rightarrow t \geq \tfrac{1}{2}$$

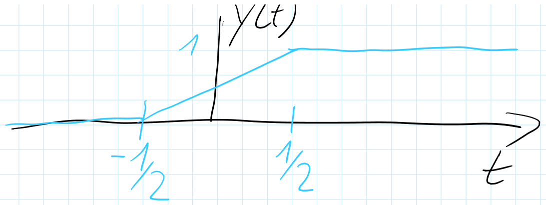
$$y(t) = \int_{-\infty}^{\infty} \varepsilon(\tau) \cdot \mathrm{rect}(t-\tau)\, d\tau$$

$$= \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} 1 \cdot 1 \, d\tau = [\tau]_{t-\frac{1}{2}}^{t+\frac{1}{2}}$$

$$= t+\tfrac{1}{2} - (t-\tfrac{1}{2}) = \underline{\underline{1}}$$

$$y(t) = \begin{cases} 0, & \text{for } t \leq -\tfrac{1}{2} \\ t+\tfrac{1}{2}, & \text{else} \\ 1, & \text{for } t \geq \tfrac{1}{2} \end{cases}$$





$\Delta(t)$, $\Lambda(t)$