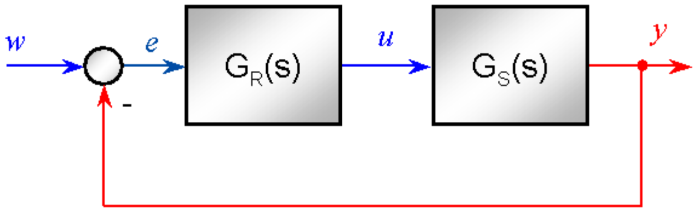


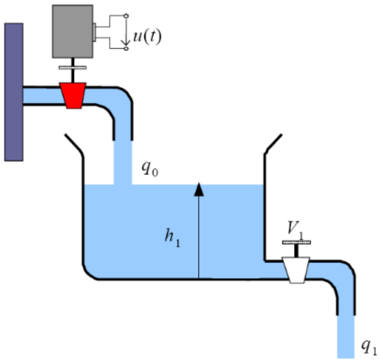

# Control Systems I

## Exercise Problems

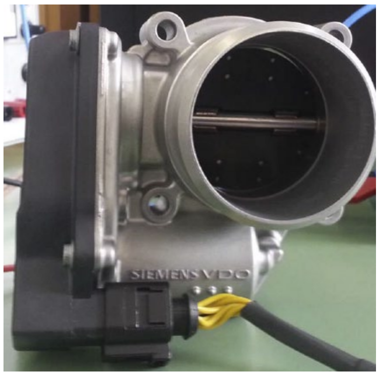
# Problem 1: *Basics*

1.1 Draw block diagrams of standard control loops for the following systems. You sketch must include control systems terms like controller, actuator, plant, sensor, set point, plant output, control deviation, control signal, disturbances. Please annotate each signal and block with its physical names like tank, valve, level, position etc.

a. Water level control: The inlet valve of the tanks system can be adjusted by applying the voltage $u$ to the input of the valve. The water level $h_1$ can be measured by using a suitable level sensor. The valve $V_1$ can be operated manually to influence the flow rate $q1$.




b. Position control of the throttle plate: An electronic throttle is used in the air system of a combustion engine. For proper operation of the engine, the position of the throttle plate have be controlled accurately. The reference value of the plate position is derived from the driver's gas pedal. A potentiometer measures the position of the throttle plate, which is driven by a dc motor. A transistor bridge actuates the motor depending on the control signal generated by the controller.




1.2 Explain the following controller components (control actions) briefly and give their equations in time-domain:

a. P controller
b. I controller
c. D controller

1.3 Describe the working principle of a digitally implemented controller with the help of a diagram explaining all the components need for the implementation.

# Problem 2: *Basics*

2.1 Explain the following terms briefly:
system; model; inputs; outputs; static system; dynamic system

2.2 Draw the following signals as functions of time. Please scale and label axes of your sketch. The signals are defined only for $t \geq 0$.

a. $y(t) = \varepsilon(t-1) + 3\varepsilon(t-2) - 5\varepsilon(t-4) + \varepsilon(t-5)$

b. $x(t) = 2\sin(\pi t - \frac{3\pi}{4})$

c. $v(t) = 2\sin\left(\pi t - \frac{3\pi}{4}\right) \cdot \epsilon\left(t - \frac{3}{4}\right)$

d. $u(t) = 1 - e^{-\frac{t}{2}}$

e. $z(t) = 4e^{-0.5t}\sin(\pi t)$

2.3 Transform the signals from problem 2.2 to s-domain using the table and properties of the Laplace transform.

# Problem 3: *Transfer Functions*

Given are the following models of different systems:

a. $\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} - 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 5y(t) = 5u(t)$.

b. $0.25\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = 2u(t)$

c. $0.2\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 0.8\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = 0.4\frac{\mathrm{d}u(t)}{\mathrm{d}t} + 1.4u(t)$

d. $2\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + \frac{\mathrm{d}y(t)}{\mathrm{d}t} = u(t)$

3.1 Calculate the transfer function and the steady-state gain of each system.

3.2 Plot a pole-zero map for each system and make a statement about its stability.

3.3 Now, calculate and plot impulse response of each system and justify stability statement with the help of the impulse response.

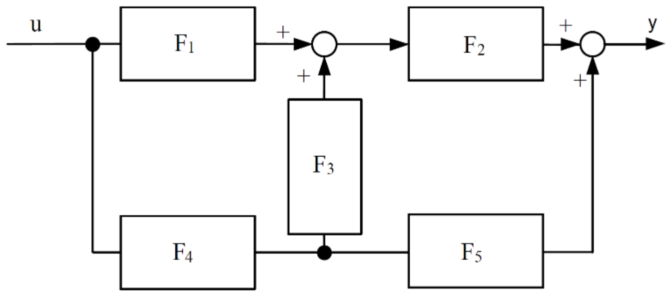
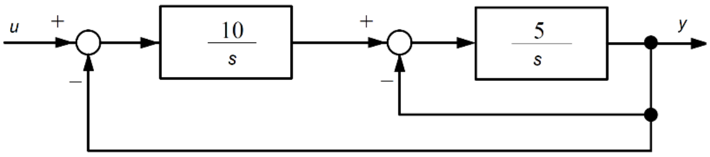
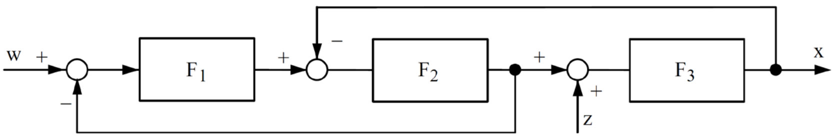
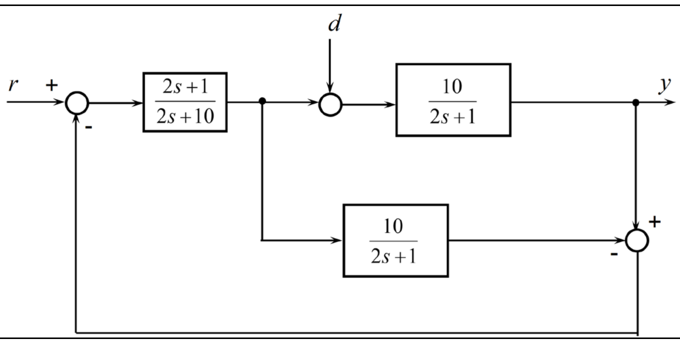
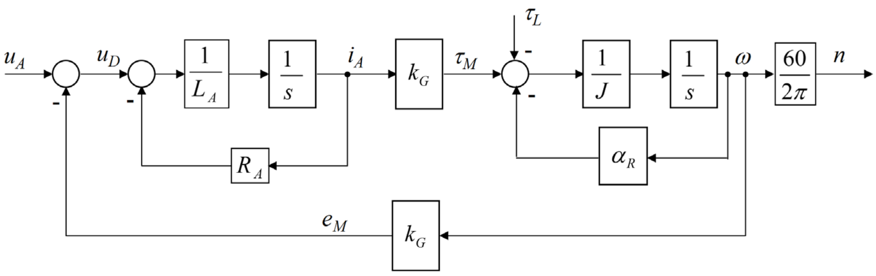
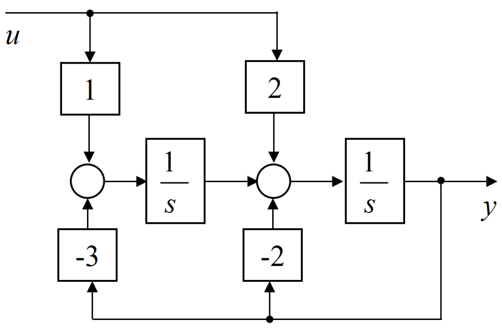
3.4 Sketch a block diagram for each system with input *u* and output *y*. You are not allowed to use a derivative block in your diagram.

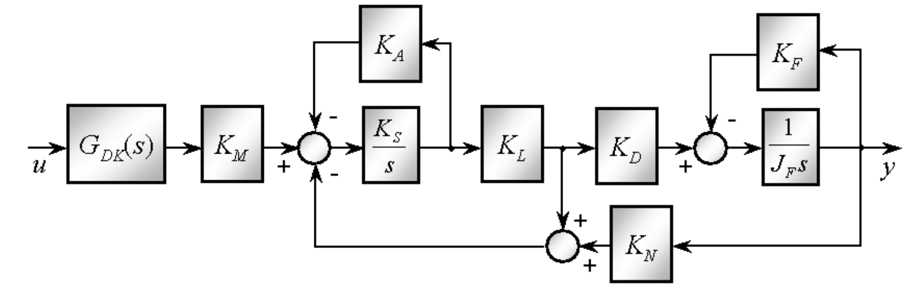
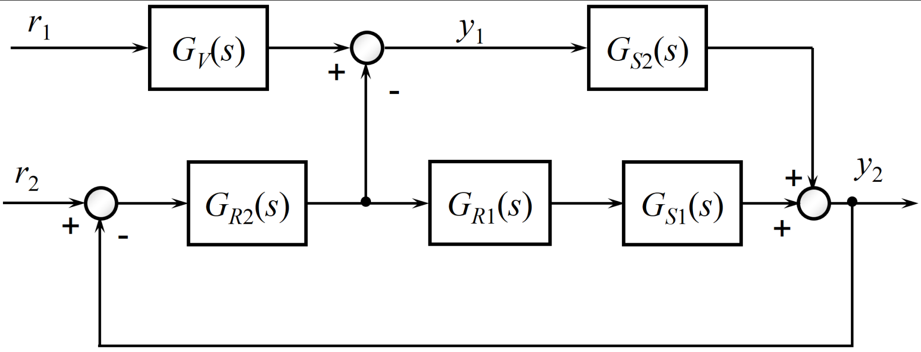
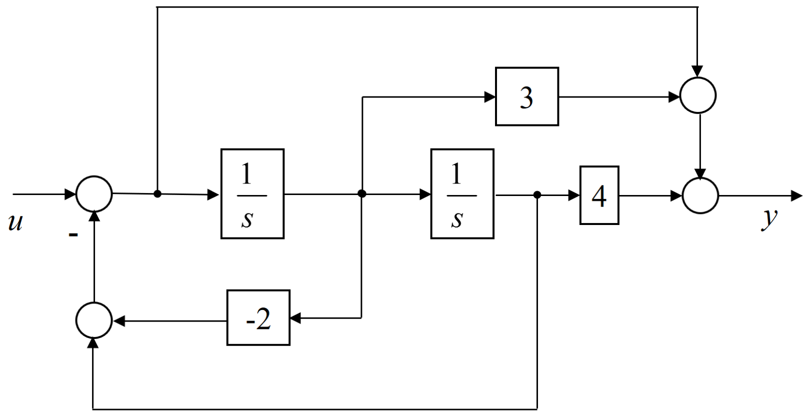
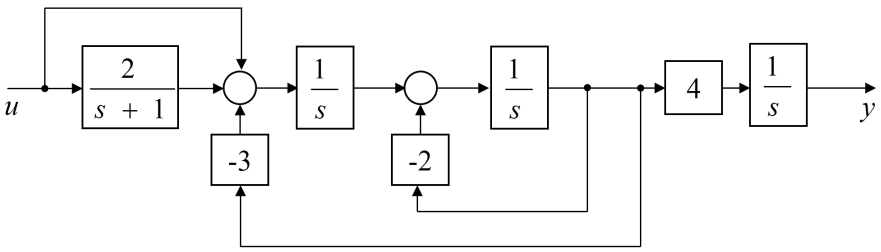
**Note:** You may use the following Matlab commands in order to verify your results

| Description | Command |
|---|---|
| Generate a tf object | `G=tf(num,den)` |
| Calculate poles | `pole(G)` |
| Calculate zeros | `zero(G)` |
| Steady-state gain | `dcgain(G)` |
| Stability check | `isstable(G)` |
| Pole-zero map | `pzmap(G)` |
| Impulse response | `impulse(G)` |
| Step response | `step(G)` |

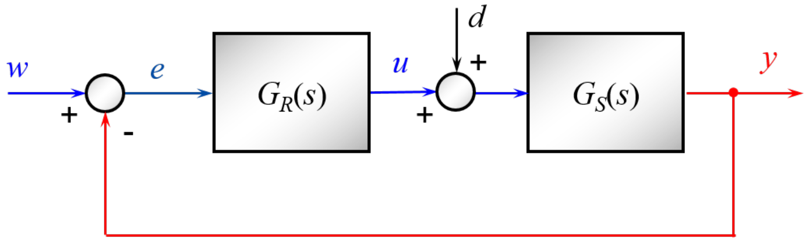
# **Problem 4:** *Block Diagrams*

Calculate the required transfer functions of each of the following block diagrams:

| | Block diagram | Transfer functions to be calculated |
|---|---|---|
| a |  | $G(s)=\dfrac{Y(s)}{U(s)}$ |
| b |  | $G(s)=\dfrac{Y(s)}{U(s)}$ |
| c |  | $G_{XW}(s)=\dfrac{X(s)}{W(s)}$ <br> $G_{XZ}(s)=\dfrac{X(s)}{Z(s)}$ |
| d |  | $G_{YR}(s)=\dfrac{Y(s)}{R(s)}$ <br> $G_{YD}(s)=\dfrac{Y(s)}{D(s)}$ |
| e |  | $G_i(s)=\dfrac{I_A(s)}{U_D(s)}$ <br> $G_{NU}(s)=\dfrac{N(s)}{U_A(s)}$ <br> $G_{NL}(s)=\dfrac{N(s)}{\tau_L(s)}$ |
| f |  | $G(s)=\dfrac{Y(s)}{U(s)}$ |

| | | |
|---|---|---|
| g |  | $G(s)=\frac{Y(s)}{U(s)}$ |
| h |  | $G_{11}(s)=\frac{Y_1(s)}{R_1(s)}$ $G_{12}(s)=\frac{Y_1(s)}{R_2(s)}$ $G_{21}(s)=\frac{Y_2(s)}{R_1(s)}$ $G_{22}(s)=\frac{Y_2(s)}{R_2(s)}$ |
| i |  | $G(s)=\frac{Y(s)}{U(s)}$ |
| j |  | $G(s)=\frac{Y(s)}{U(s)}$ |

# Problem 5: *Steady-State Behaviour*




5.1 Determine the following transfer functions:

a) $G_0(s)$: The open loop transfer function

b) $G_{YW}(s) = \dfrac{Y(s)}{W(s)}$  c) $G_{EW}(s) = \dfrac{E(s)}{W(s)}$

d) $G_{UW}(s) = \dfrac{U(s)}{W(s)}$  e) $G_{YD}(s) = \dfrac{Y(s)}{D(s)}$

5.2 Calculate the transfer functions of problem 5.1 by considering the following expressions for the plant and for the controller.

$$G_S(s) = \frac{B(s)}{A(s)} \quad \text{and} \quad G_R(s) = K_R$$

5.3 Now consider that there is no disturbance and the set point is changed like a step function:

$$d(t) = 0 \quad \text{and} \quad w(t) = \varepsilon(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Calculate the expression for control deviation $E(s)$ in s-domain.

5.4 Calculate steady-state control error $e_\infty$.

**Note:** $e_\infty = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$

5.5 Consider the following polynomials for the plant model:

$$B(s) = 4 \qquad A(s) = s^2 + 3s + 2$$

Controller gian $K_R$ can be calculated from the requirement about steady-state control error $e_\infty$. Calculate $K_R$ for the following cases:

a) $e_\infty = 0.2$,  b) $e_\infty = 0.03$,  c) $e_\infty = 0$

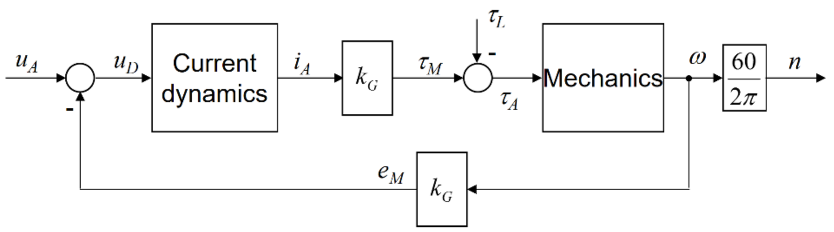
5.6 Now consider te disturbance rejection case, where

$$w(t) = 0 \quad \text{and} \quad d(t) = \varepsilon(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Calculate the expression for steady-state error $e_\infty$. Determine $e_\infty$ for the values of $K_R$ calculated in problem 5.5.

# Problem 6

A simplified model of a permanently excited dc motor is drawn Figure 6.1.




| | |
|---|---|
| $u_A$ | Armature voltage [V] |
| $i_A$ | Armature current [A] |
| $\tau_M$ | Motor torque [Nm] |
| $\tau_L$ | Load torque [Nm] |
| $e_M$ | Induced emf [V] |
| $n$ | Speed [rpm] |
| $\omega$ | Angular velocity [rad/s] |

Figure 6.1: Simplified block diagram of a dc motor

The following differential equation describes the dynamics of the mechanical part of the motor..

$$J\frac{d\omega}{dt}+\alpha\omega(t)=\tau_M(t)-\tau_L(t).$$

Dynamics of the armature current are described by the following differential equation:

$$L\frac{di_A}{dt}+Ri_A(t)=u_D(t).$$

The values for model parameters are given in the following table:

| Parameter | Description | Value |
|---|---|---|
| $L$ | Armature inductance | 1 mH |
| $R$ | Armatur resistance | 1 Ω |
| $J$ | Moment of inertia | 0.01 rad/(Nms$^2$) |
| $k_G$ | Generator/motor contant | 0.1 Nm/A |
| $\alpha$ | Coefficient of friction | 0.01 Nms/rad |

Table 6.1: Parameters of the machine

6.1 Derive the following transfer functions in s-domain.

a) Transfer function for current dynamics $G_i(s)=\dfrac{I_A(s)}{U_D(s)}$

b) Transfer function for the mechanics $G_M(s)=\dfrac{\omega(s)}{\tau_A(s)}$

c) $G_u(s)=\dfrac{N(s)}{U_A(s)}$

d) $G_L(s)=\dfrac{N(s)}{\tau_L(s)}$

6.2 Is the transfer function $G_u(s)$ stable? Please justify your answer.

6.3 What is the steady-state value of the speed $n$ at no load, if the input voltage is kept constant at 30V?

6.4 How much is the steady-state drop in rpm if a constant load of 0.3 Nm is applied to the motor?

# **Problem 7:** *Frequency Response*

Consider the following transfer function of a system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10}{1+2s}$$

7.1 Please determine the expression for the output signal $y(t)$, if the following signal is applied as input and all the transients have been decayed:

I. $u(t) = 3\text{ V}\sin(\omega t)$; with $\omega = 0.5$ [rad/s]

II. $u(t) = 3\text{ V}\sin(\omega t)$; with $\omega = 2$ [rad/s]

III. $u(t) = 5\text{V} + 3\text{ V}\sin(\omega t)$; with $\omega$=1000 [rad/s]

7.2 At which value of the frequency $\omega$ of the sinusoidal input are the amplitudes of the input and output signals same?

# **Problem 8:** *Lead, Lag Compensators*

The following transfer function can be configured/used as a lead or lag compensator:

$$G(s) = \frac{1+\dfrac{s}{\omega_Z}}{1+\dfrac{s}{\omega_P}}$$

Depending on the relative values of the corner frequencies $\omega_Z$ and $\omega_P$, it can modify the phase of a loop in positive (lead) or negative (lag) direction for a certain frequency band

8.1 Please sketch Bode diagrams for the following transfer functions:

$$G_1(s) = \frac{1+\dfrac{s}{1}}{1+\dfrac{s}{10}} \qquad G_2(s) = \frac{1+\dfrac{s}{10}}{1+\dfrac{s}{1}}$$

8.2 Now, considering the Bode plots from 8.1, make a general statement about frequencies $\omega_Z$ and $\omega_P$ for the following cases:

a. $G(s)$ is a lead compensator

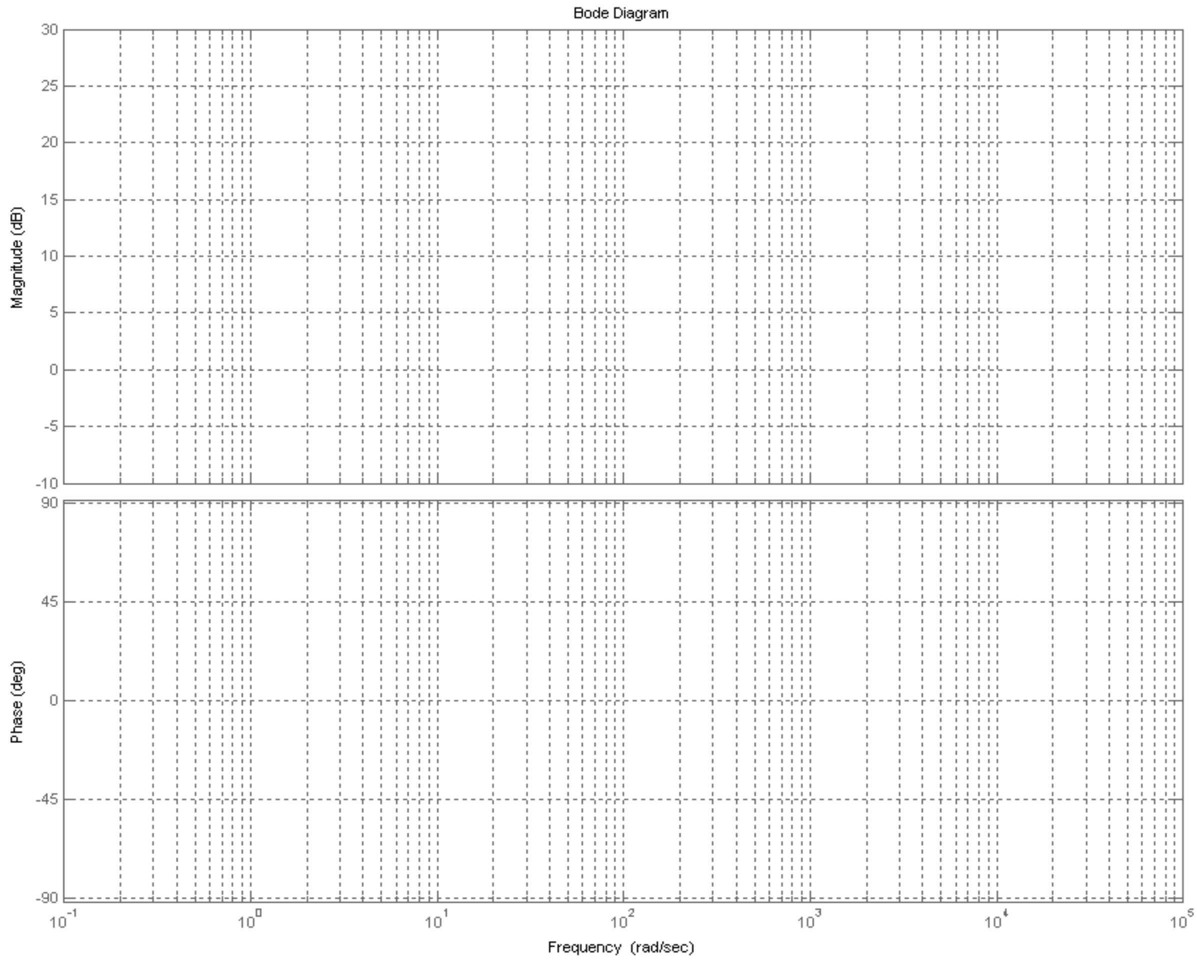
b. $G(s)$ is a lag compensator

8.3 What could be the use of these compensators? Give a brief description.

# Problem 9: Bode Plot

Please sketch the Bode plot of the following transfer function.

$$G(s) = \frac{(1+0.5s)(1+0.02s)}{0.5s(1+0.00125s)}$$




# Problem 10

Dynamic behaviour of a system is described by the following transfer function:

$$G_0(s) = \frac{10}{s(s+1)(0.1s+1)}$$

A sinusoidal signal 10V sin($\omega t$) with a variable frequency $\omega$ is applied on the input of the system. The output of the system is also a sinusoid with the same frequency.

10.1 At which value $\omega_{gc}$ of the frequency is the amplitude of the output signal 10V? Please calculate the phase of the output signal.

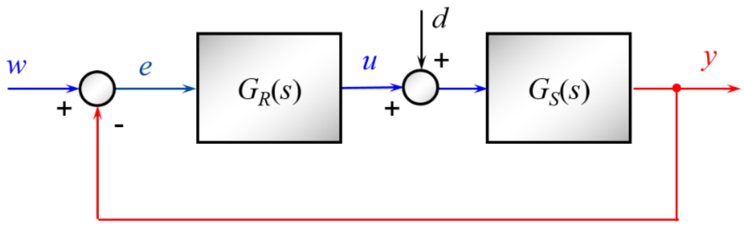
10.2 At which frequency value $\omega_{pc}$ is the output signal 180° out of phase of the input?

10.3 How large is the amplitude gain at very low frequencies (e.g. $\omega = 0$)?

Note: You may solve this problem with the help of Matlab.

# Problem 11

Please consider the following control loop:




Plant and controller have following transfer functions:

$$G_S(s) = \frac{2}{s+3} \qquad\qquad G_R(s) = \frac{b_1 s + b_0}{s + a_0}$$

11.1 Determine the closed-loop transfer function

$$G_W(s) = \frac{Y(s)}{W(s)}$$

11.2 Calculate the value of parameter $a_0$ so that the steady-state gain of the closed loop becomes 1.

Now assume $a_0$=0

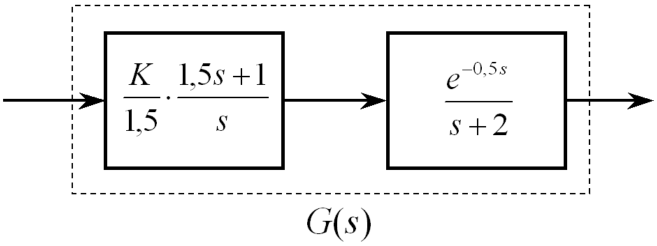
11.3 Write the denominator polynomial of $G_W(s)$ in a form so that the natural frequency $\omega_0$ and the damping ratio $D$ can be read directly. Give expressions for $\omega_0$ and $D$ as functions of the parameters $b_0$ and $b_1$.

11.4 Which values of $b_0$ and $b_1$ satisfy the following conditions?

$D = 0.707 \qquad \omega_0 = 2{,}828$ rad/s

# Problem 12

The dynamic behaviour of a system is described by the following block diagram.




*K* is an unknown parameter.

12.1 Calculate the mathematical expression for magnitude $A(\omega)$ and phase $\varphi(\omega)$ response of the system.

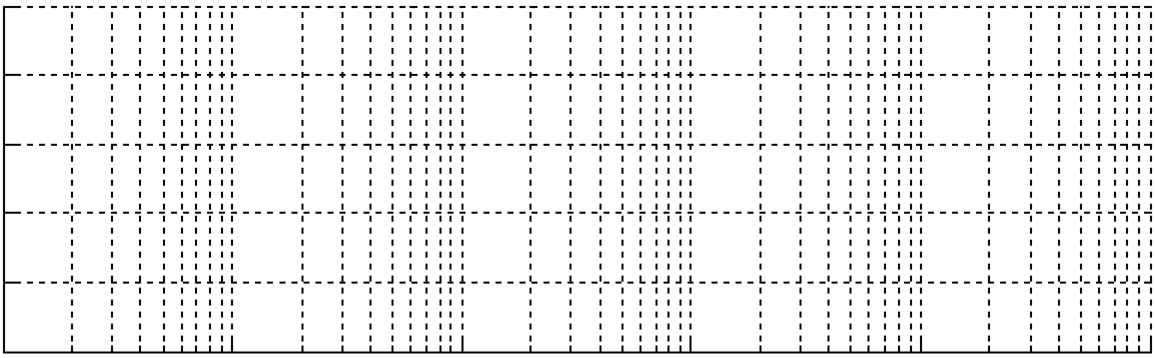
**Hint:** Be careful with the units while adding the phases of different factors.

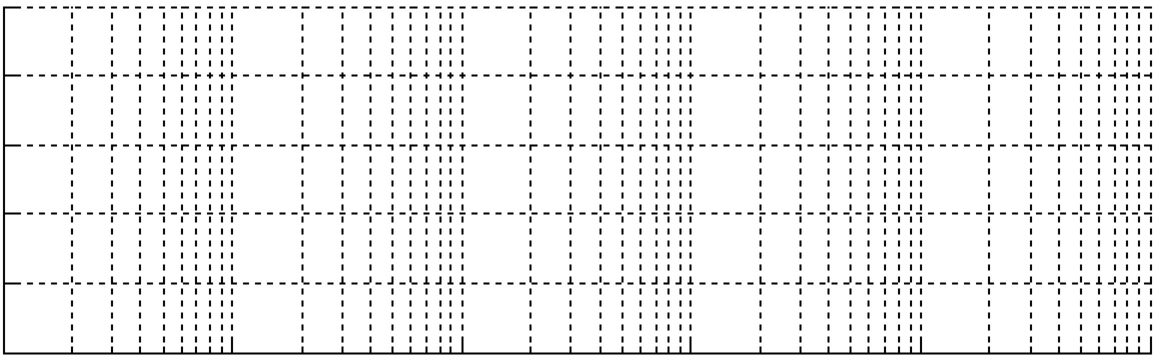
Now assume $\omega_{gc}$=2 rad/s

12.2 Calculate the value of the parameter *K* to satisfy te following conditions:
$A\,(\omega_{gc}) = 1$ with $\omega_{gc}$=2 rad/s

12.3 How much is the phase shift of the system at $\omega_{gc}$=2 rad/s?

12.4. Sketch the Bode plot of the system. Please scale and label your axes properly





# Problem 13 *PIDT1 controller*

Given are the following parameters of a PIDT1 controller:

$K_R = 5 \qquad T_I = 2s$

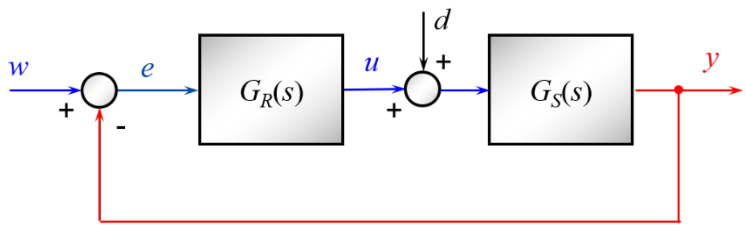
$T_D = 0.5s \qquad Tv = 0.1s$

13.1 Write the transfer function of the controller and draw its block diagram in Simulink notation.

13.2 Implement this controller as an analogue circuit diagram. Give proper values of all the resistors and capacitors used in this implementation.

# Problem 14 *Phase margin etc.*

Given is the following control loop:




$$G_R(s) = \frac{s+1}{s} \qquad G_S(s) = \frac{3}{s^2+3s+2}$$

14.1 Calculate the minimal realisation of the open loop transfer function $G_0(s) = G_R(s)\, G_S(s)$.
**Note:** A transfer function is in minimal realisation form if all of its cancelling pole/zero pairs are eliminated

14.2 Calculate the real and imaginary parts of the open-loop frequency response $G_0(j\omega)$.

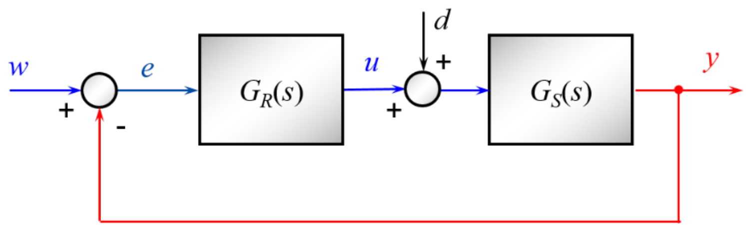
14.3 Calculate the expressions for its magnitude and phase response.

14.4 What is the value of the gain crossover frequency $\omega_{gc}$?

14.5 Calculate the phase margin $\phi_M$.

# Problem 14a *Stability margin*

Consider the following control loop:




$$G_R(s) = \frac{K}{s} \qquad G_S(s) = e^{-0,5s}$$

14a.1 Calculate the open loop transfer function $G_0(s) = G_R(s)\, G_S(s)$.

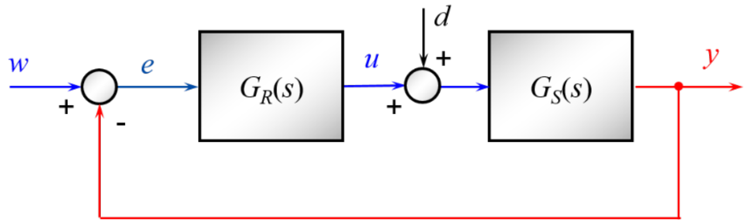
14a.2 Calculate the expressions for its magnitude and phase response.

14a.3 Up to which maximum value of the $K_{cr}$ of the parameter $K$ is the control loop stable? What is the oscillation frequency, when the closed loop becomes marginally stable?

14a.4 Now select $K=0.8K_{cr}$. Calculate the gain crossover frequency $\omega_{gc}$ and the phase margin $\phi_M$.

# **Problem 15** *The closed loop*

Consider the following control loop:




$$G_R(s) = K_R \qquad\qquad G_S(s) = \frac{2}{s(s+2)}$$

15.1 Calculate the tracking transfer function $Y(s)/W(s)$ of the closed loop.

15.2 Derive a relationship between steady-state control error $e_\infty$ and controller gain $K_R$ for the following cases:

a. the set-point $r$ is abruptly changed from 0 to 1 and $d$=0

b. the disturbance $d$ is abruptly changed from 0 to 1 and $r$=0?

15.3 Calculate the value of the controller gain $K_R$ for the following cases:

a. The closed loop has a repeated pole on the negative real axis.

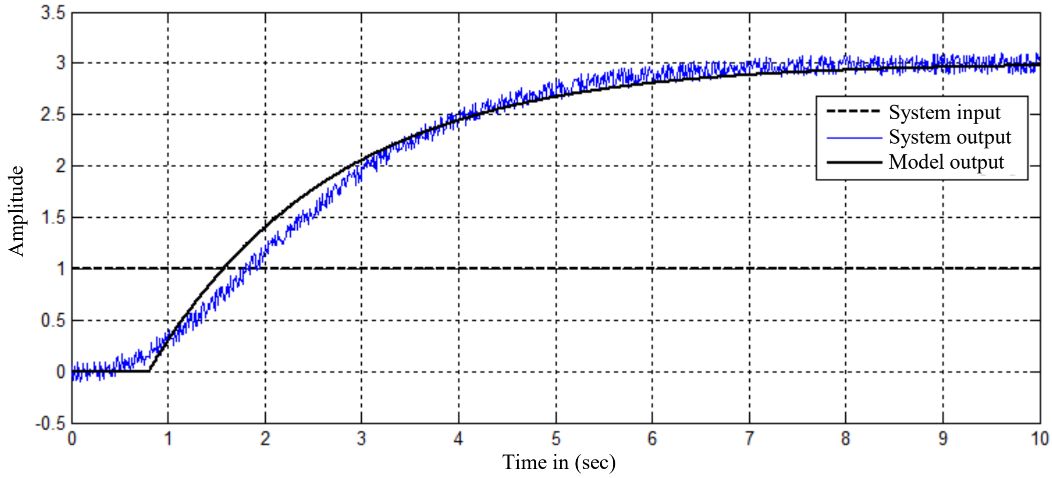
b. The closed loop has a damping ratio of 0.707

c. The natural frequency of the control loop is 1 rad/s.

d. The steady-state error due to step-like disturbance is 5%.

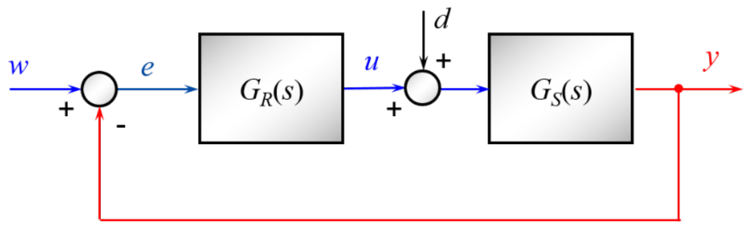
# **Problem 16** *Ziegler and Nichols' tuning rules*

In order to design a controller, the step response of a system is recorded. This response is approximated by a first order transfer function with transport delay. The measured response and the model output are drawn in the following plot. Please calculate the parameters of a PID controller for this plant using the first tuning method proposed by Ziegler and Nichols.



# Problem 17 *Controller design in frequency domain*

Given is the following control loop




Transfer functions for the plant and controller are also given

$$G_S(s) = \frac{5}{s^2 + 1{,}414s + 1}; \qquad G_R(s) = K_R\left(1 + \frac{1}{T_I s}\right)$$

The control loop should satisfy the following specifications for tracking of step-like set-point changes:

Rise time: $t_r = 3\text{s}$
Maximum overshoot: $M_P = 10\ \%$
Steady-state control error: $e_\infty = 0$

17.1 Calculate the open-loop frequency response properties using the following thumb rules:

| | |
|---|---|
| Rise time←→Gain crossover frequency | $t_r\ \omega_{gc} \approx 1{,}5$ |
| Maximum overshoot ←→Phase margin | $M_P + \phi_M \approx 70$ |
| Steady-state control error ←→$A(0)$ | $A(0) = 1/\ e_\infty - 1$ |

17.2 How the requirement ($e_\infty = 0 \rightarrow A(0) = \infty$) can be fulfilled?

17.3 Now determine the parameters $K_R$ and $T_I$ so, that all the requirements are fulfilled.

# Problem 17a *SISOTOOL (Frequency-domain method)*

Solve the Problem 17 using SISOTOOL (Control System Designer App) in Matlab. Please do the following steps:

17a.1 The controller transfer function $G_R(s)$ can also be written in the following format:

$$G_R(s) = K_R \frac{\left(s + \frac{1}{T_I}\right)}{s}.$$

The PI controller has a pole at $s$=0, and a zero at $s = -1/T_I$. Please set the initial values for $K_R$ and $T_I$ equal to 1 and generate tf objects Gr and Gs.

17a.2 Start SISOTOOL with the command `sisotool(Gp,Gc)`.

17a.3 In Bode Editor, drag/move the zero of the PI controller in horizontal direction so that the phase curve reaches the value $\phi_M$-180° at frequency $\omega_{gc}$.

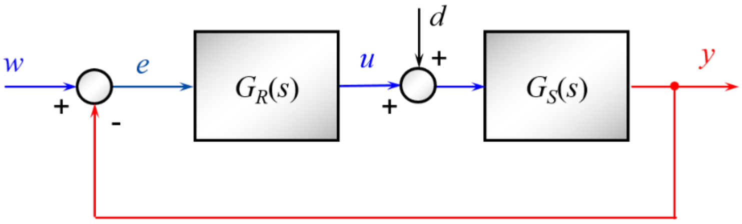
17a.4 Now drag/move the magnitude curve vertically so that it crosses the 0-dB line at $\omega_{gc}$.

17a.5 Read the values of the controller parameters from "Compensator Editor".

17a.6 Check if the specified requirements are fulfilled. For this purpose, you can calculate the closed-loop transfer function and draw its step response.

# **Problem 18** *Frequency-domain method and compensation of poles*

Given is the following control loop




With transfer functions for the plant and controller:

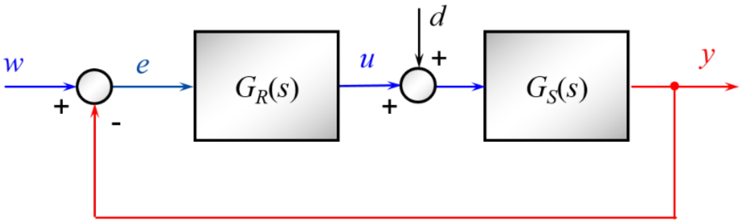
$$G_S(s) = \frac{20}{(s+3)(s+0.5)} \qquad G_R(s) = K_R\left(1 + \frac{1}{T_I s}\right)$$

Determine the values for parameters $K_R$ and $T_I$ so that the following conditions are satisfied:

- The slowest pole of the plant is compensated with a controller zero
- Phase margin of 60° is achieved

# **Problem 19** *Pole placement*

Consider the following control loop




$$G_R(s) = K_R\left(1 + \frac{1}{T_I s}\right) \qquad G_S(s) = \frac{4}{s+0.5}$$

19.1 Calculate the open-loop transfer function $G_0(s) = G_R(s)\, G_S(s)$.

19.2 Now determine the closed-loop transfer function $G_{YW}(s) = \frac{Y(s)}{W(s)}$.

19.3 Determine the parameters $K_C$ und $T_I$ so that the closed loop has the following poles: –2+j and –2-j.

19.4 Where is the zero of the closed-loop transfer function located?

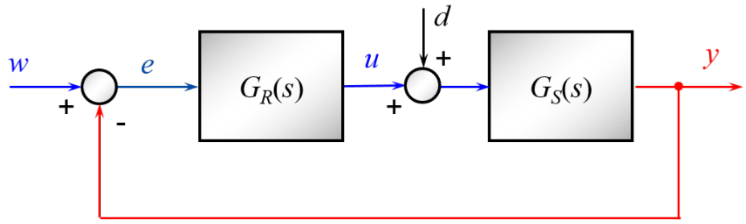
19.5 Draw the closed-loop step response in Matlab. Determine the maximum overshoot $M_P$. The closed-loop poles have a damping ratio of $D$=0.89. Please comment why the measured overshoot is much larger than the value expected from the damping ratio.

19.6 Now, calculate the closed-loop transfer function for disturbance rejection $G_{YD}(s) = \frac{Y(s)}{D(s)}$.

19.7 Determine poles and zeros of $G_{YD}(s)$.

# **Problem 20** *Pole placement*

Consider the following control loop




$$G_R(s) = K_R\left(1 + \frac{1}{T_I s} + T_d s\right) \qquad G_S(s) = \frac{2}{s^2 + 0{,}1s + 1}$$

20.1 Calculate the open-loop transfer function $G_0(s) = G_R(s)\, G_S(s)$.

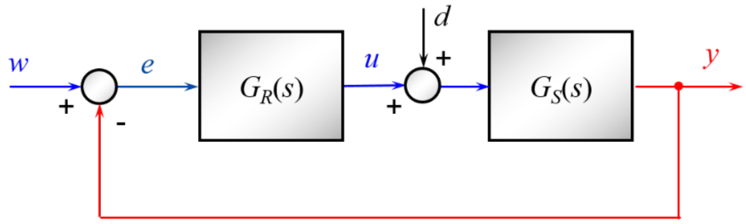
20.2 Now determine the closed-loop transfer function $G_{YW}(s)$.

20.3 Calculate the values of the parameters $K_R$, $T_I$ and $T_d$, so that the closed-loop has the following poles: $-1$, $-2$ and $-3$.

20.4 Where are the zeros of the closed loop located?

# **Problem 21** *Pole placement*

Consider the following control loop




$$G_S(s) = \frac{2}{s+2} \qquad G_R(s) = \frac{b_1 s + b_0}{s + a_0}$$

Controller parameters $b_1$, $b_0$ and $a_0$ are unknown.

21.1 Calculate the open-loop transfer function $G_0(s) = G_R(s)\, G_S(s)$.

21.2 Now determine the closed-loop transfer function $G_{YW}(s) = \dfrac{Y(s)}{W(s)}$.

21.3 Calculate the steady-state gain of the closed loop $K_G = \lim\limits_{s\to 0} G_{YW}(s)$.

21.4 Determine the values of the parameters $b_1$, $b_0$ and $a_0$ so that the following conditions are satisfied:

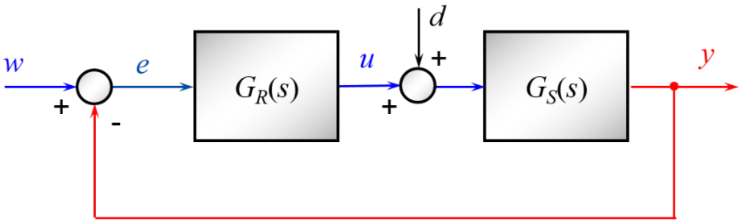
i) $K_G = 1$

ii) $G_{YW}(s)$ should have a conjugate complex pole pair at ($-4+j3$ and $-4-j3$).

21.5 Put the values of the parameters into the controller transfer function $G_R(s)$. Which type of controller is it?

# Problem 22

Consider the following control loop




With

$$G_S(s) = \frac{5}{5s^3 + 11s^2 + 7s + 1}$$

22.1 Determine the controller so that the closed loop has the following transfer function:

$$\frac{Y(s)}{W(s)} = K_W(s) = \frac{1}{s^3 + 1{,}75s^2 + 2{,}15s + 1}$$

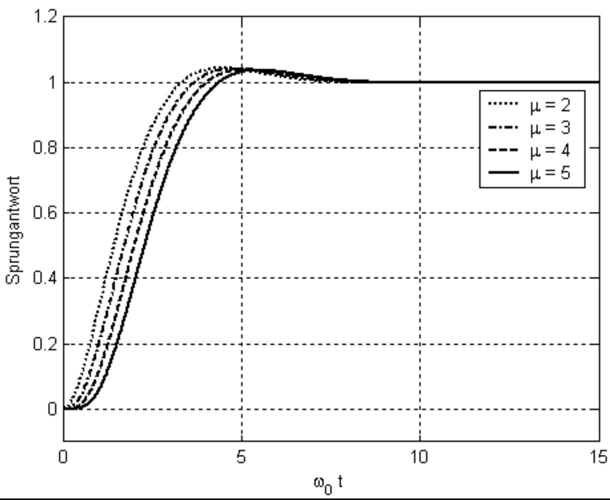
# Problem 23

The task is to design a compensation controller for the following plant:

$$G_S(s) = \frac{5}{s^2 + 0{,}2s + 1}$$

The denominator polynomial of the desired closed-loop transfer function should be given in Weber form with damping ratio of 0.7. The controller should be of minimum order. After a step-like change in the reference value, the actual system output should cross the desired value line after 0.5s.

Table 1: Transfer function and step response of $K_W(s)$ with denominator polynomial in Weber form ($D$=0.7)

| μ | $K_W(s)$ | Step response |
|---|---|---|
| 2 | $\dfrac{\omega_0^2}{s^2 + 1{,}4\omega_0 s + \omega_0^2}$ |  |
| 3 | $\dfrac{\omega_0^3}{(s^2 + 1{,}4\omega_0 s + \omega_0^2)(0{,}28s + \omega_0)}$ | |
| 4 | $\dfrac{\omega_0^4}{(s^2 + 1{,}4\omega_0 s + \omega_0^2)(0{,}28s + \omega_0)^2}$ | |
| 5 | $\dfrac{\omega_0^5}{(s^2 + 1{,}4\omega_0 s + \omega_0^2)(0{,}28s + \omega_0)^3}$ | |

# Problem 24

The input-output relationship of a system is described by the following differential equation:

$$2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t) = 5u(t)\,,$$

where $u(t)$ represents the input and $y(t)$ represents the output signal.

24.1 Write the transfer function of the system in s-domain.
24.2 Is this system stable? Justify your answer.
24.3 Sketch step response of the system.

In order to get a discrete-time model of the system, the differential operator is approximated by the forward difference operator given below:

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \approx \frac{y(k+1) - y(k)}{T_A}, \qquad (1)$$

where $y(k)$ and $y(k+1)$ are the sampled values of $y(t)$ and $T_A$ is the sample time.
24.4 Write the discrete-time difference equation of the system.
24.5 Determine its transfer function in z-domain.
24.6 Investigate the stability of the discrete-time system in following cases:
a. $T_A = 0.5$ s
b. $T_A = 5$ s.
24.7 What is the disadvantage of the approximation given by equation (1)?

# Problem 25

Given are the following transfer functions in z-domain:

a. $$G_1(z) = \frac{3}{z + 0.5}$$

b. $$G_2(z) = 0.5 + 0.3z^{-1} + 0.2z^{-2}$$

c. $$G_3(z) = \frac{1}{1 - z^{-1} + 0.25z^{-2}}$$

Convert these transfer functions into discrete-time difference equations, draw their block diagrams and sketch their step response for at least 5 samples.

# Problem 26

A plant is represented by the following transfer function in z-domain:

$$G_S(z) = \frac{z - 0{,}4}{z^2 - 1{,}6z + 0{,}89}$$

26.1 Determine its difference equation with input $u$ and output $y$.

26.2 Draw its block diagram.

26.3 Is this system stable? Justify your answer.

26.4 The system should be controlled using a digital PI controller. Write the transfer function of the controller with the following parameters:

Sample time: $T_A$

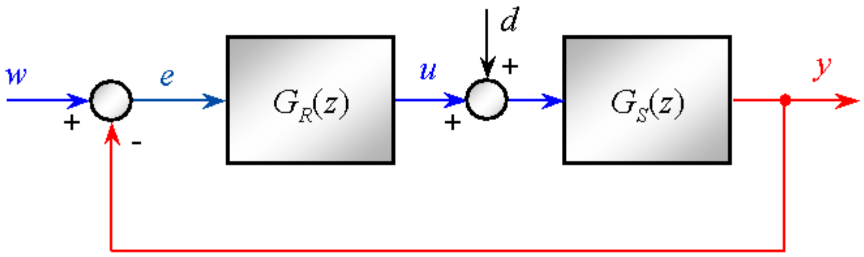
Controller gain $K_R$

Integration time constant $T_I$.

Integration method: Euler's forward rule.

# Problem 27

Consider the following digital control loop:




The plant model is given by

$$G_S(z) = \frac{0{,}2z^{-1}}{1 - 0{,}9z^{-1}}$$

The following transfer function was selected as controller:

$$G_R(z) = \frac{5 - 3z^{-1}}{1 - 0.7z^{-1}}$$

27.1 Determine the following transfer functions:

a) $G_{YW}(z) = \dfrac{Y(z)}{W(z)}$

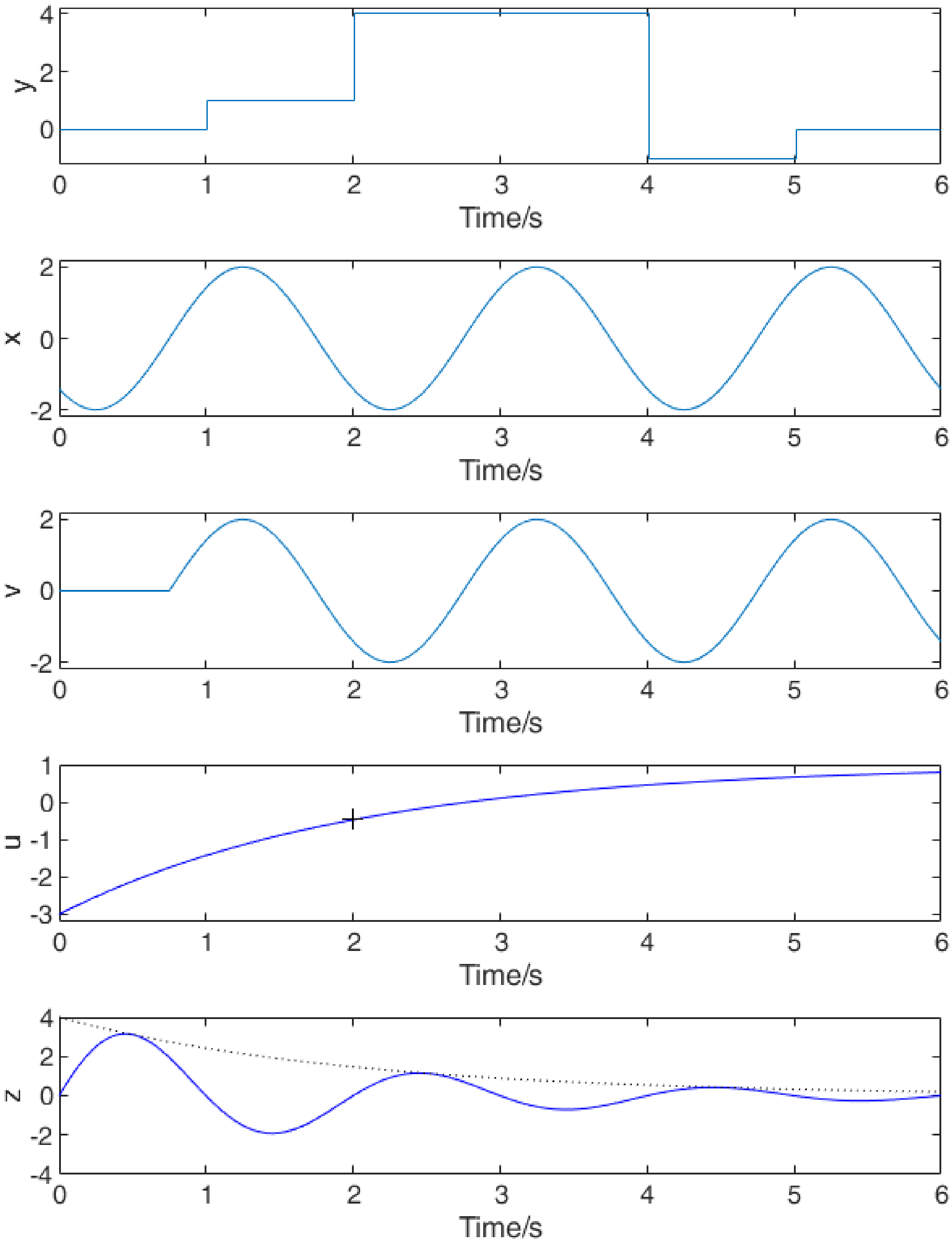
b) $G_{ED}(z) = \dfrac{E(z)}{D(z)}$

27.2 Calculate the steady-state value of the system output $y$ after a unit step in set-point $w$.

27.3 How large is the steady-state value of the control deviation e after a unit step of the disturbance $d$?

# Problem 2

2.2



2.3
a. $Y(s) = \frac{1}{s}\left(e^{-s} + 3e^{-2s} - 5e^{-4s} + e^{-5s}\right)$

b. $X(s) = -\frac{\sqrt{2}(s+\pi)}{s^2+\pi^2}$

c. $V(s) = \frac{\pi}{s^2+\pi^2} e^{-\frac{3}{4}s}$

d. $U(s) = \frac{1}{s} - \frac{4}{s+0.5}$

e. $Z(s) = \frac{4\,\pi}{(s+½)^2+\pi^2}$

# Problem 3

3.1

a. $G(s) = \frac{5}{s^2-2s+5}$ $K_0 = 1$

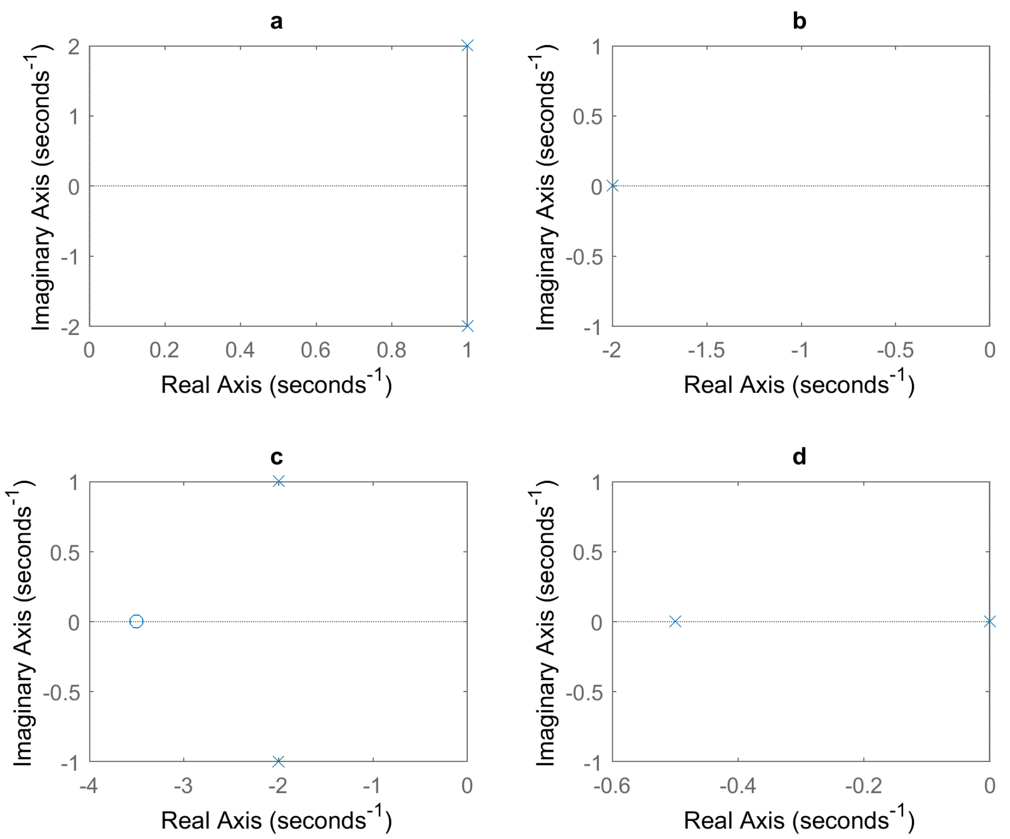
b. $G(s) = \frac{2}{0.25s^2+s+1}$ $K_0 = 2$

c. $G(s) = \frac{0.4s+1.4}{0.2s^2+0.8s+1}$ $K_0 = 1.4$

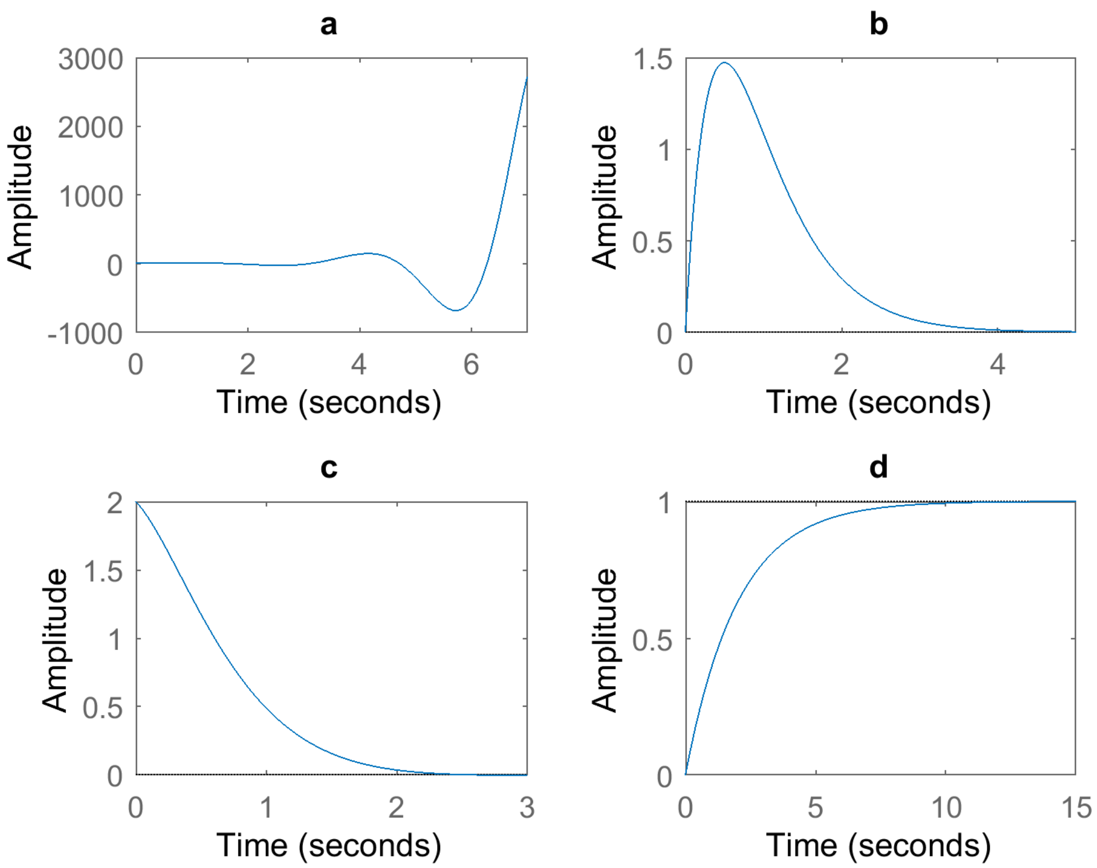
d. $G(s) = \frac{1}{2s^2+s}$ $K_0 = \infty$

3.2

a. Unstable, as both poles are in the RHP.
b. Stable, as both poles are in the LHP. (Double pole at -2)
c. Stable, as both poles are in the LHP.
d. Marginally stable, as one pole is on ω-axis and the other is in LHP.



3.3




a. $g(t) = 2.5e^{t}\sin(2t)$ $\lim_{t\to\infty} g(t) \to \infty$ => Unstable

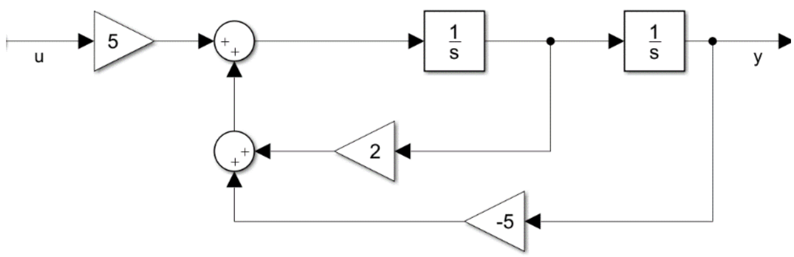
b. $g(t) = 8te^{-2t}$ $\lim_{t\to\infty} g(t) = 0$ => Stable

c. $g(t) = e^{-2t}(2\cos(t) + 3\sin(t))$ $\lim_{t\to\infty} g(t) = 0$ => Stable

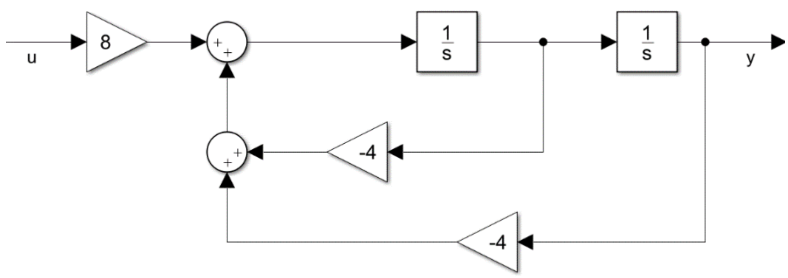
d. $g(t) = 1 - e^{-0.5t}$ $\lim_{t\to\infty} g(t) = 1$ => Marginally stable

3.4

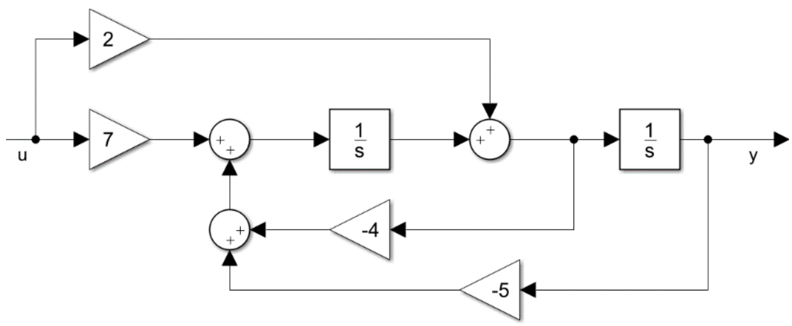
a.



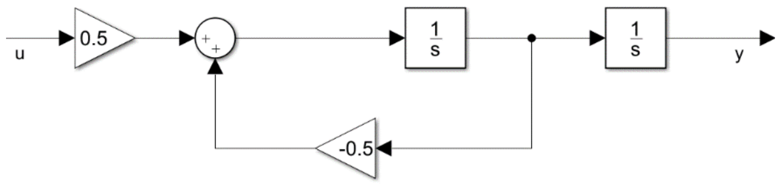

b.



c.




d.




# Problem 4

a. $G(s) = F_1F_2 + F_4F_3F_2 + F_4F_5$

b. $G(s) = \frac{50}{s^2+5s+50}$

c. $G_{XW}(s) = \frac{F_1F_2F_3}{F_1F_2 + F_2F_3 + 1}$ $\quad G_{XZ}(s) = \frac{(F_1F_2+1)F_3}{F_1F_2 + F_2F_3 + 1}$

d. $G_{YR}(s) = \frac{5}{s+5}$ $\quad G_{YD}(s) = \frac{10s}{(s+5)(2s+1)}$

e. $G_I(s) = \frac{1}{L_As+R_A}$ $\quad G_{NU}(s) = \frac{k_G}{JL_As^2+(JR_A + L\alpha)s+R_A\,\alpha + k_G^2} \cdot \frac{30}{\pi}$

…$G_{NL}(s) = -\frac{L_As+R_A}{JL_As^2+(JR_A + L\alpha)s+R_A\,\alpha + k_G^2} \cdot \frac{30}{\pi}$

f. $G(s) = \frac{2s+1}{s^2+2s+3}$

g. $G(s) = \frac{K_DK_LK_SK_MG_{DK}(s)}{J_Fs^2 + (K_F + K_SK_AJ_F + K_LK_SJ_F)s + K_LK_SK_F + K_LK_SK_DK_N + K_SK_AK_F}$

h. $G_{11}(s) = \frac{G_V(G_{R2}G_{R1}G_{S1} + 1)}{1+G_{R2}G_{R1}G_{S1} - G_{R2}G_{S2}}$ $\quad G_{12}(s) = \frac{-G_{R2}}{1+G_{R2}G_{R1}G_{S1} - G_{R2}G_{S2}}$

$G_{21}(s) = \frac{G_VG_{S2}}{1+G_{R2}G_{R1}G_{S1} - G_{R2}G_{S2}}$ $\quad G_{22}(s) = \frac{G_{R2}G_{R1}G_{S1} - G_{R2}G_{S2}}{1+G_{R2}G_{R1}G_{S1} - G_{R2}G_{S2}}$

i. $G(s) = \frac{S^2+3s+4}{S^2-2s+1}$

j. $G(s) = \frac{4(s+3)}{s(s+1)(s^2+2s+3)}$

# Problem 5

5.1 a) $G_0(s) = G_C(s)G_P(s)$

b) $G_{YR}(s) = \frac{G_C(s)G_P(s)}{1+G_C(s)G_P(s)}$ c) $G_{ER}(s) = \frac{1}{1+G_C(s)G_P(s)}$

d) $G_{UR}(s) = \frac{G_C(s)}{1+G_C(s)G_P(s)}$ e) $G_{YD}(s) = \frac{G_P(s)}{1+G_C(s)G_P(s)}$

5.2 a) $G_0(s) = \frac{K_C B(s)}{A(s)}$

b) $G_{YR}(s) = \frac{K_C B(s)}{A(s)+K_C B(s)}$ c) $G_{ER}(s) = \frac{A(s)}{A(s)+K_C B(s)}$

d) $G_{UR}(s) = \frac{K_C A(s)}{A(s)+K_C B(s)}$ e) $G_{YD}(s) = \frac{B(s)}{A(s)+K_C B(s)}$

5.3 $E(s) = \frac{A(s)}{A(s)+K_C B(s)} \cdot \frac{1}{s}$

5.4 $e_\infty = \frac{A(0)}{A(0)+K_C B(0)}$

5.5 a) $K_C = 2$ b) $K_C = 16.15$ c) $K_C = \infty$

5.6 $e_\infty = \frac{-B(0)}{A(0)+K_C B(0)}$; a) $e_\infty = -0.4$ b) $e_\infty = -0.06$ c) $e_\infty = 0$

# Problem 6

6.1 a) $G_i(s) = \frac{1}{0.001s+1}$ b) $G_M(s) = \frac{1}{0.01s+0.01}$

c) $G_u(s) = \frac{10000}{s^2+1001s+2000} \cdot \frac{30}{\pi}$ d) $G_u(s) = \frac{100s+100000}{s^2+1001s+2000} \cdot \frac{30}{\pi}$

6.2 $G_u(s)$ is stable as both of its poles (-998.998 and -2.002) are located in the left half plane.

6.3 $n_\infty = 1432.4\, rpm$

6.4 $\Delta n_\infty = 143.24\, rpm$

# Problem 7

7.1 I. $y(t)$=21.21 V $\sin(\omega t - 45°)$; mit $\omega = 0.5$ [rad/s]

II. $y(t)$=7.28 V $\sin(\omega t - 75.96°)$; mit $\omega = 2$ [rad/s]

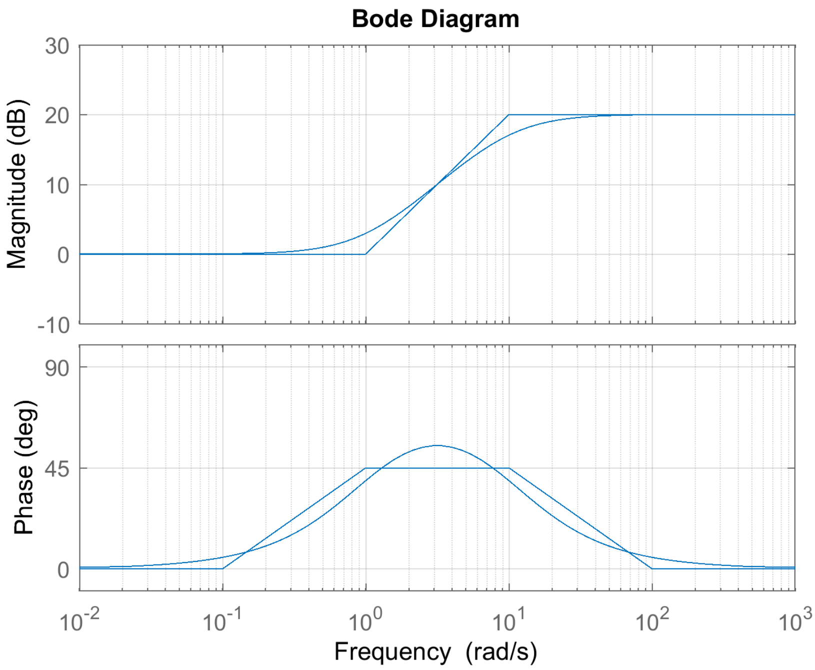
III. $y(t)$= 50V + 0.015 V $\sin(\omega t - 89.97°)$; mit $\omega = 1000$ [rad/s]

7.2 $\omega$ = 4.97 rad/s.

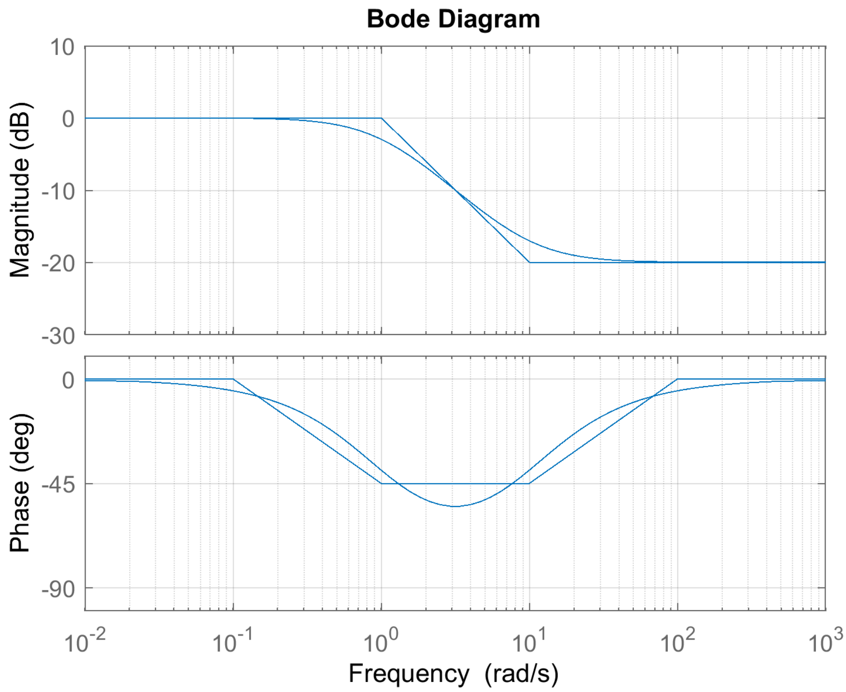
# Problem 8

8.1

$G_1(s)$




$G_2(s)$

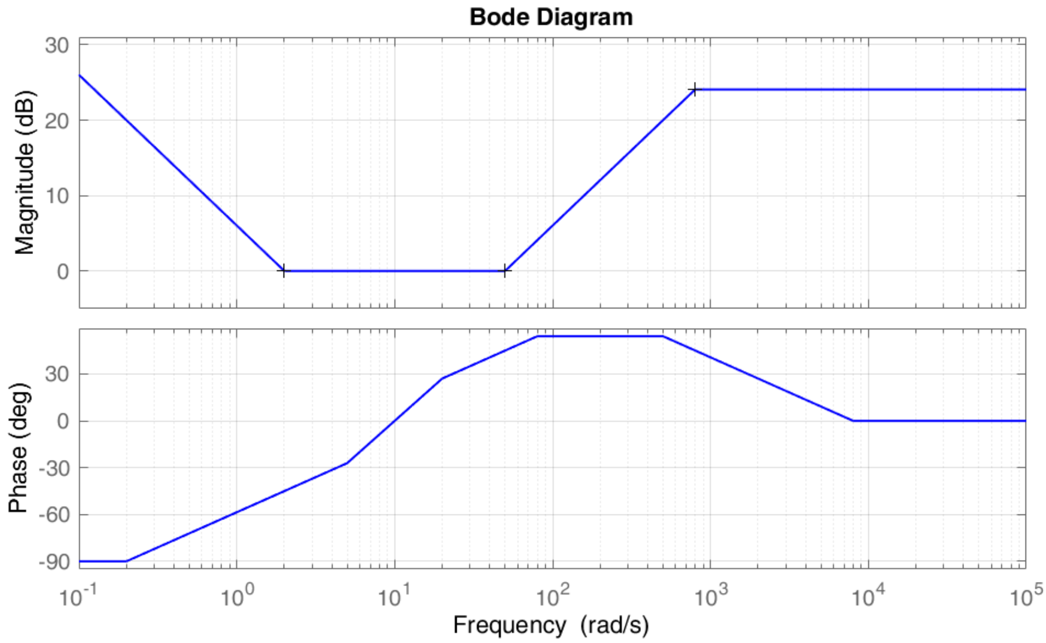



8.2 a. $G(s)$ is a lead compensator if $\omega_p > \omega_z$

b. $G(s)$ is a lag compensator if $\omega_z > \omega_p$

8.3 Lead and lag compensators can be used in the loop shaping. The lead compensator can influence the phase in positive direction in order to increase phase margin. Whereas, the lag compensator may be used to reduce the magnitude of the loop and hence increase the phase margin.

# Problem 9




# Problem 10

10.1 $\omega_{gc}$ = 3.0145 rad/s. $\varphi(\omega_{gc})$ = -178.42°.

10.2 $\omega_{pc}$ = 3.16 rad/s. 10.3 $A(0) = \infty$

# Problem 11

11.1 $G_W(s) = \dfrac{2(b_1 s + b_0)}{(s+3)(s+a_0) + 2(b_1 s + b_0)}$ 11.2 $a_0=0$

11.3 $P(s) = \dfrac{s^2}{\left(\sqrt{2b_0}\right)^2} + 2\dfrac{3+2b_1}{2\sqrt{2b_0}}\dfrac{s}{\sqrt{2b_0}} + 1$ 11.4 $b_0=4$; $b_1=0.5$

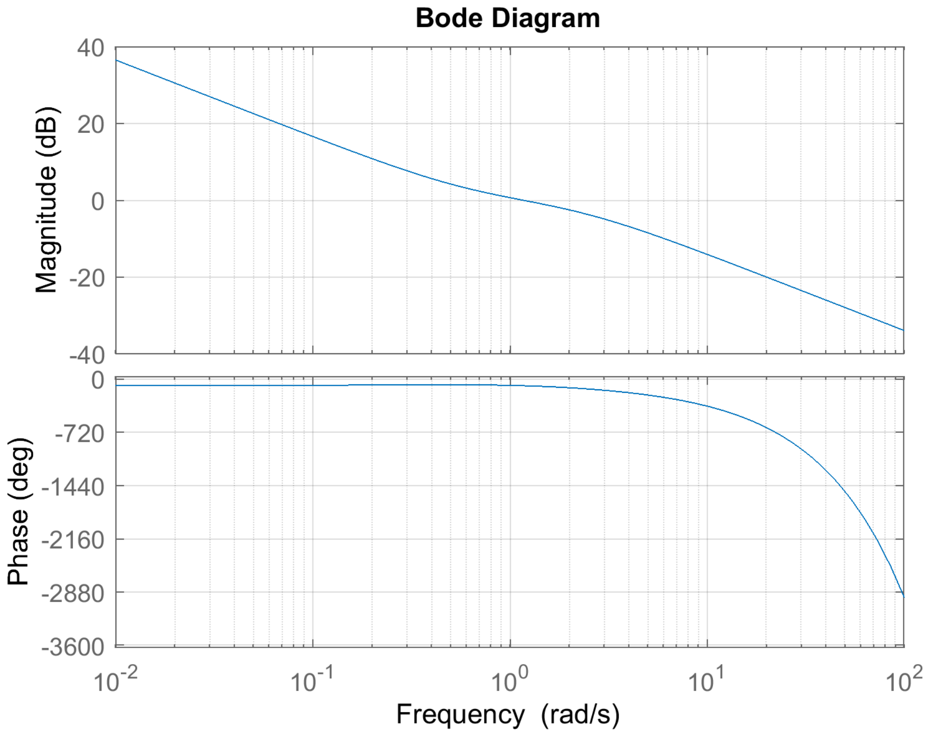
# Problem 12

12.1 $G(j\omega) = \dfrac{K(1+j1.5\omega)}{j1.5\omega(2+j\omega)}e^{-0.5j\omega}$ $A(\omega) = \dfrac{K\sqrt{1+2.25\omega^2}}{1.5\omega\sqrt{4+\omega^2}}$

$$\phi(\omega) = \arctan(1.5\omega) - 90° - \arctan(0.5\omega) - 0.5\omega\frac{180°}{\pi}$$

12.2 $K$=2.68 12.3 $\varphi(\omega_{gc})$ =-120.73°

12.4
```
s=tf('s')
bode(2*(1.5*s+1)*exp(-0.5*s)/1.5/s/(s+2))
grid on;
```




# Problem 13

See lecture notes.

# Problem 14

14.4 $\omega_{gc} = 1.27$ rad/s 14.5 $\phi_M = 57.6°$

# Problem 14a

14a.3 $K_{cr} = 3.14$; $\omega_{cr} = 3.14$ rad/s 14a.4 $\omega_{gc} = 2.51$ rad/s; $\phi_M = 18°$

# Problem 15

15.2 a. $e_\infty = 0$; for all $K_R > 0$ b. $e_\infty = -1/K_R$; for all $K_R > 0$

15.3 a. $K_R = 0.5$ b. $K_R = 1$ c. $K_R = 0.5$ d. $K_R = 20$

# Problem 16

$K_R = 0.95$; $T_I = 1.6$; $T_D = 0.4$;

# Problem 17/17a

$K_R$ =0.047; $T_I$ =0.47 s;

# Problem 18

$K_R$ = 0.299; $T_I$ =2 s;

# Problem 19

19.3 $K_R$ = 0.875; $T_I$= 0.7 s

# Problem 20

20.3 $K_R$=5; $T_I$ =1.67 s; $T_d$ = 0.59 s;

20.4 $s_{z1,2}$ = -0.85 ± j .55

# Problem 21

21.4 $b_1$=3; $b_0$ =12.5; $a_0$=0;

# Problem 22

$$G_R(s) = \frac{s^3 + 2.2s^2 + 1.4s + 0.2}{s^3 + 1.75s^2 + 2.15s}$$

# Problem 23

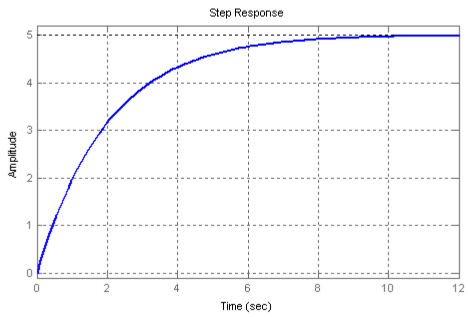
$$G_R(s) = \frac{8{,}64s^2 + 1{,}73s + 8{,}64}{s^2 + 9{,}2s}$$

# Problem 24

24.1 $G(s)=\dfrac{Y(s)}{U(s)}=\dfrac{5}{2s+1}$

24.2 Pole: $s_p$ = -0.5 lies in the left half-plane (LHP). The system is stable.

24.3




24.4 $\dfrac{2}{T_A}\cdot y(k+1)+\left(1-\dfrac{2}{T_A}\right)\cdot y(k)=5\cdot u(k)$

24.5 $G(z)=\dfrac{5T_A}{2z+T_A-2}$.

24.6 Pole: $z_p=1-\dfrac{T_A}{2}$

a. $T_A$=0.5 s ➔ $z_p$=0.75 ➔ $|z_p|<1$ ➔ stable

b. $T_A$=5 s ➔ $z_p$=-1.5 ➔ $|z_p|>1$ ➔ unstable

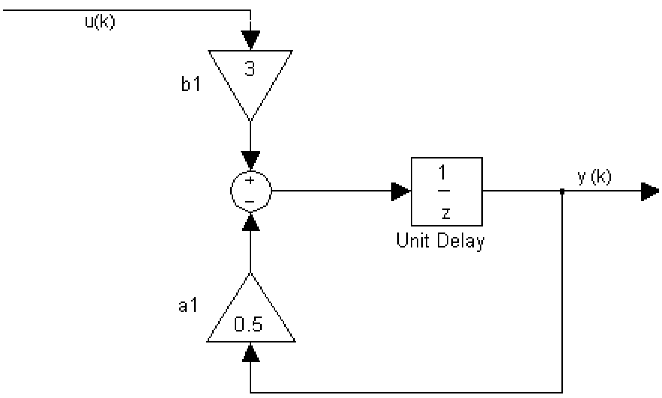
24.7 The stability of the sampled system also depends on sample time.

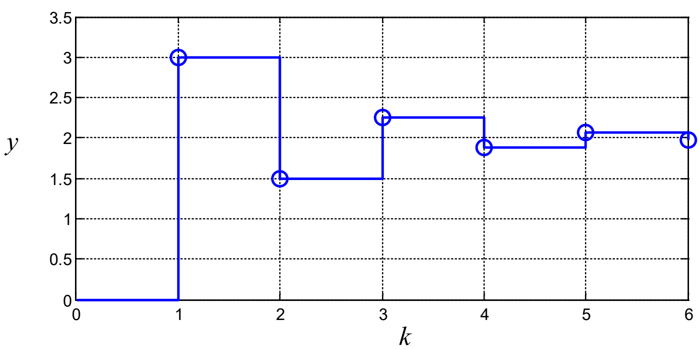
# Problem 25

a. $y(k+1)=3u(k)-0.5y(k)$

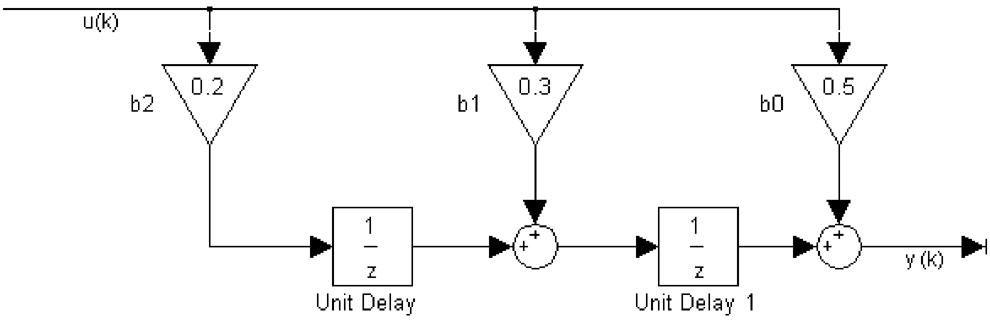
or

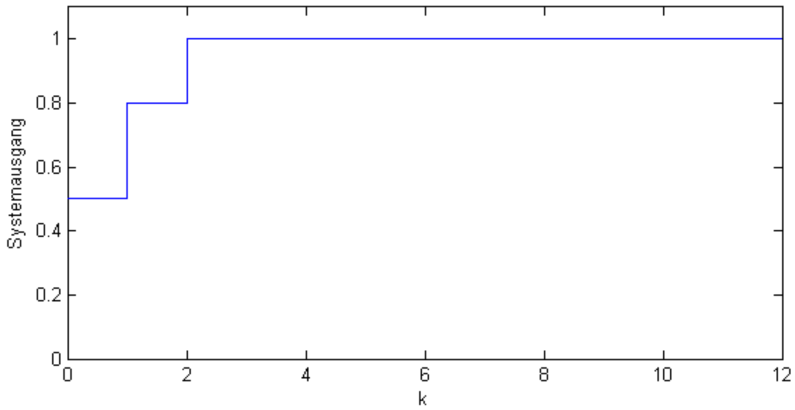
$y(k)=3u(k-1)-0.5y(k-1)$



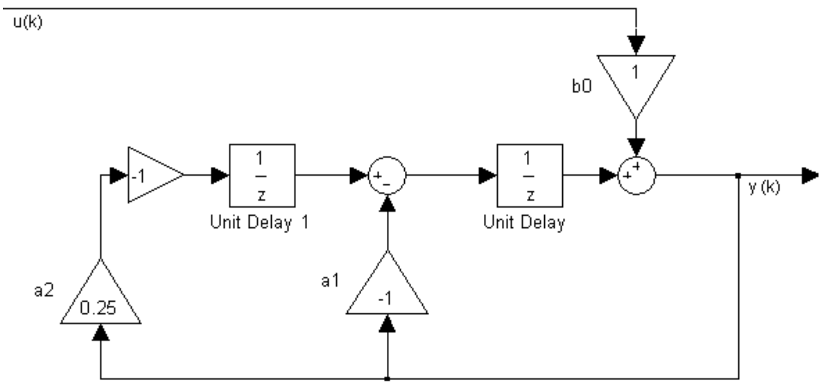



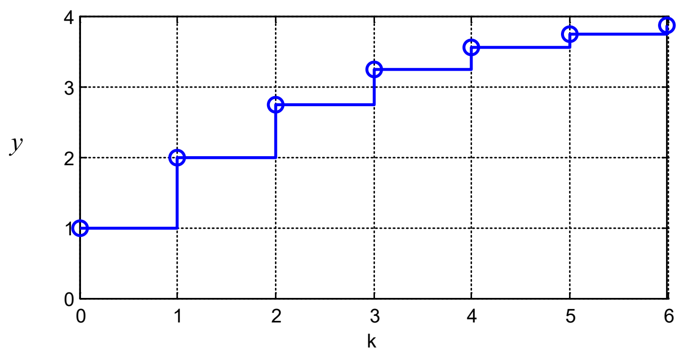
b. $y(k) = 0.5u(k) + 0.3u(k-1) + 0.2u(k-2)$



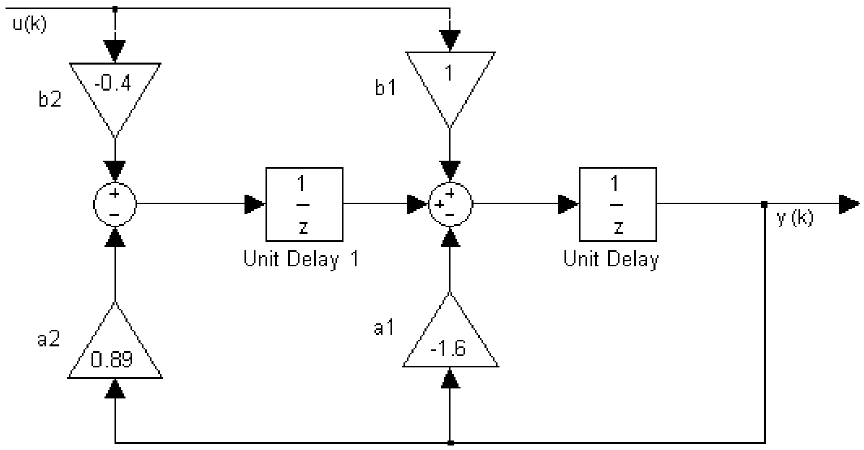

c. $y(k) - y(k-1) + 0.25y(k-2) = u(k)$



# Problem 26

26.1 $y(k) - 1{,}6\,y(k-1) + 0{,}89\,y(k-2) = u(k-1) - 0{,}4\,u(k-2)$




26.2

26.3 $|z_p| = |0{,}8 \pm j0{,}5| = 0{,}94 < 1$ → stable

26.4 $G_R(z) = K_R\left(1 + \dfrac{T_A}{T_I(z-1)}\right)$

# Problem 27

27.1 a) $G_{YW}(z) = \dfrac{z^{-1} - 0{,}6z^{-2}}{1 - 0{,}6z^{-1} + 0{,}03z^{-2}}$

b) $G_{ED}(z) = \dfrac{-0{,}2z^{-1} + 0{,}14z^{-2}}{1 - 0{,}6z^{-1} + 0{,}03z^{-2}}$

27.2 $y_\infty = 0{,}93$

27.3 $e_\infty = -0{,}14$