Task 28

Solution:

a) In task 25b) the following continuous-time Luenberger observer was found:

$$\dot{\underline{\hat{x}}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \underline{\hat{x}} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 8.5 \\ 6 \end{bmatrix} (y - \hat{y})$$
$$\hat{y} = [0 \quad 1] \underline{\hat{x}}$$

Approximate discrete-time state equations with respect to the sampling time T can be derived using the formula

$$\underline{\hat{x}}[k+1] = \left(T\underline{A} + \underline{I}\right)\underline{\hat{x}}[k] + T\underline{b}u[k] + T\underline{l}(y[k] - \hat{y}[k])$$
$$\hat{y}[k] = \underline{c}^T \underline{\hat{x}}[k]$$

Here:

$$\underline{\hat{x}}[k+1] = \begin{bmatrix} 1+T & -4T \\ 2T & 1-3T \end{bmatrix} \underline{\hat{x}}[k] + \begin{bmatrix} T \\ T \end{bmatrix} u[k] + \begin{bmatrix} 8.5T \\ 6T \end{bmatrix} (y[k] - \hat{y}[k])$$
$$\hat{y}[k] = [0 \quad 1] \underline{\hat{x}}[k]$$

Block diagram → see next page

b) If the sample time T is chosen according to the conservative design rule

$$|\lambda_i| \lesssim \frac{1}{10T}$$

where λ_i denotes all dominant open- and closed-loop eigenvalues, then the performance of the control-loop with digital state-space controller will almost certainly be very similar to the designed continuous-time dynamics.

Here:

- Open-loop eigenvalues = eigenvalues λ_1 and λ_2 of the plant:

$$\det\left(\lambda\underline{I} - \underline{A}\right) = \det\left(\begin{bmatrix} \lambda - 1 & 4 \\ -2 & \lambda + 3 \end{bmatrix}\right) = \lambda^2 + 2\lambda + 5 \;\Rightarrow\; \lambda_{1/2} = -1 \pm 2j \;\Rightarrow\; |\lambda_{1/2}| = \sqrt{5}$$

- Close-loop eigenvalues =
eigenvalues of closed-loop with state feedback and observer eigenvalues

$\Rightarrow$ $\lambda_{C,1} = \lambda_{C,2} = -5$ (see Task 22b) on Exercise Sheet 9)
$\lambda_{o,1} = \lambda_{o,2} = -4$ (see Task 25a) on Exercise Sheet 11)

→ Choose T such that $5\frac{1}{\text{sec}} \lesssim \frac{1}{10T} \;\Rightarrow\; T \lesssim \frac{1}{50}\text{sec} \;\Rightarrow\; T = 0.02\text{sec}$

a) Block diagram: