13. 20.06.2023

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Orthogonality:

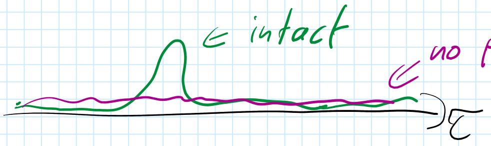
Are the signals similar to each other?

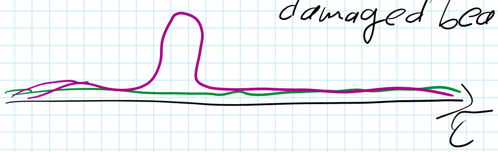
2 Prototype signal:

$s_1(t)$: intact bearing of motor

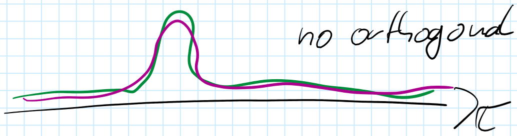
$s_2(t)$: defect bearing

- $\varphi^E_{s_1 g}(\tau)$
- $\varphi^E_{s_2 g}(\tau)$





just if s_1 and s_2 are orthogonal



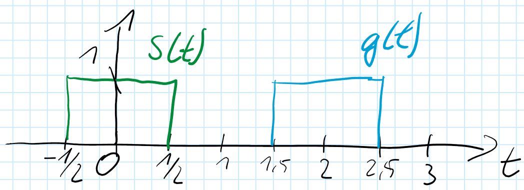
Time-Domain: $\varphi^E_{sg}(0) = 0$

$$\boxed{\int_{-\infty}^{\infty} s(t) \cdot g(t)\, dt = 0}$$

Frequency-Domain: $\varphi^E_{sg}(0) = \boxed{\int_{-\infty}^{\infty} S^*(f) \cdot G(f)\, df = 0}$

Test of orthogonality:
- test of overlap
- use symmetry of the signals

Example 1: $s(t) = rect(t) \qquad g(t) = rect(t-2)$

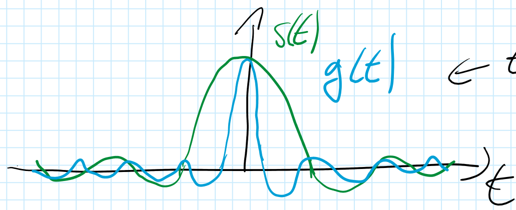


$$\varphi^E_{sg}(0) = \int_{-\infty}^{\infty} s(t) \cdot g(t)\, dt = 0$$

$\Rightarrow$ orthogonal because both signals don't overlap in time-domain.

Example 2: $s(t) = si(\pi t)$ $g(t) = si(\pi t) \cdot \cos(2\pi 3t)$

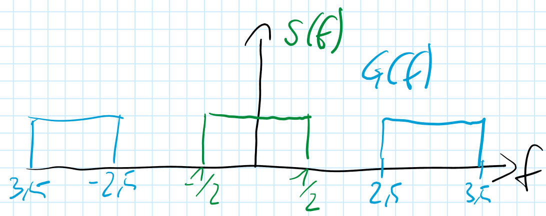
$\left[\cos(2\pi F t) \circ\!\!-\!\!\bullet \frac{1}{2}[\delta(f+F) + \delta(f-F)] \right]$



$\leftarrow$ too complicated

$\hookrightarrow$ Frequency Domain

$S(f) = rect(f)$ $G(f) = rect(f) * \frac{1}{2}[\delta(f+3) + \delta(f-3)]$



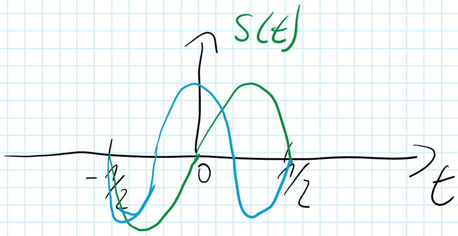
$\varphi_{sg}^{E}(\tau=0) \overset{!}{=} 0 = \int_{-\infty}^{\infty} s(t) \cdot g(t)\, dt$

$= \int_{-\infty}^{\infty} S^*(f) \cdot G(f)\, df$

$\Rightarrow$ At least one of the spectra is always 0

$\Rightarrow$ orthogonal because no overlap in frequency domain

Example 3: $s(t) = rect(t) \cdot \sin(2\pi t)$ $g(t) = rect(t) \cdot \cos(2\pi t)$



- overlap in time-domain
- overlap in frequency-domain

$\int_{-\infty}^{\infty} s(t) \cdot g(t)\, dt$

odd signal/function

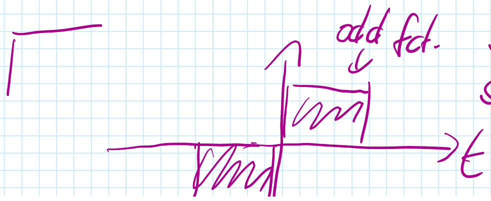
even signal/function

odd function: $s(t) = -s(-t)$

even function: $g(t) = g(-t)$

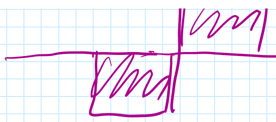
$\underbrace{\text{"odd"} \cdot \text{"even"}} = \text{"odd"}$

$\Rightarrow \int_{-\infty}^{\infty} \text{"odd fct" } dt = 0$



sum over surface cancel out

$\Rightarrow s(t)$ & $g(t)$ are orthogonal



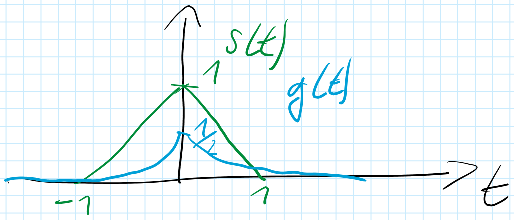
out

"$\int_{odd} = 0$" just for symmterical integration limits!

$\sin(2\pi F t) \circ\!\!-\!\!\bullet \frac{j}{2}[\delta(f+F) - \delta(f-F)]$

$\cos(2\pi F t) \circ\!\!-\!\!\bullet \frac{1}{2}[\delta(f+F) + \delta(f-F)]$

Example 4: $s(t) = \Delta(t)$ $\qquad g(t) = \frac{1}{2} \cdot e^{-|t|}$



$$\varphi_{sg}^{E}(0) = \int_{-\infty}^{\infty} s(t) \cdot g(t)\, dt$$

"even" · "even" = "even"

$s(t) \geq 0 \qquad g(t) > 0$

both signals are completely positiv and thus the integral over their surfaces cannot cancel out ⇒ not orthognal

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Time-Discrete Correlation-Functions

$$\int \ldots dt \to \sum$$

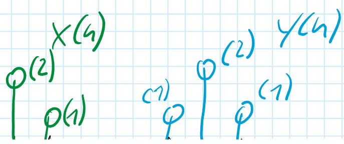
$$\varphi_{xy}^{E}(m) = x(-m) * y(m) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n-m)$$

orthogonality of Time-Discrete Functions:

$$\varphi_{xy}^{E}(0) = 0 = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n+0) \qquad \text{Time-Domain}$$

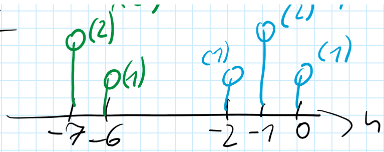
$$= \int_{-\infty}^{\infty} X(f)^* \cdot Y(f)\, df \qquad \text{Frequency-Domain}$$

Example 1:



orthogonal

Example 1:

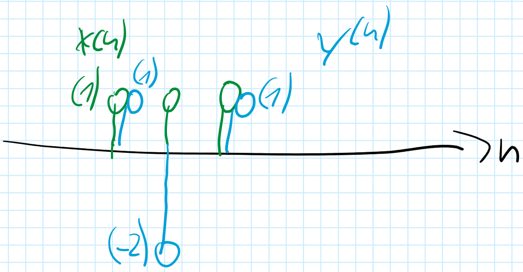


orthogonal

$r^E_{xy}(0) = 0$ because $x(n)$ and $y(n)$ don't overlap in time domain

Example 2:

$$\left.\begin{aligned} x(n) &= \delta(n) + \delta(n-1) + \delta(n-2) \\ y(n) &= \delta(n) - 2\delta(n-1) + \delta(n-2) \end{aligned}\right\} \text{orthogonal?}$$



$$r^E_{xy}(0) = \sum_{n=-\infty}^{\infty} x(n) \cdot y(n)$$

$$= 1 \cdot 1 + 1 \cdot (-2) + 1 \cdot 1 = 0$$

$\Rightarrow$ orthogonal

$$\left.\begin{aligned} x(n) &: \text{even} \\ y(n) &: \text{odd} \end{aligned}\right\} \int_{-\infty}^{\infty} \text{"odd"} = 0$$

$$s_1(t) \cdot \cos(2\pi f_1 t)$$
$$s_2(t) \cdot \cos(2\pi f_2 t)$$