List-No.:

Exam System Theory

Bachelor Robotics at THWS

WS 2022/23

Prof. Dr. R. Hirn

Duration:	**90 minutes**	
Tools:	**only legimitate calculators and the distributed formulary**	
Max. points:	**90 pts.**	(12 + 16 + 15 + 17 + 17 + 13)
Tasks:	**6**	(on 7 pages)

Last Name, First Name:	Lösung
Matriculation-No.:	

Hints:

- Write your name on each sheet!
- Do not remove any staples!
- Cheating is rated 5.0, i.e. "failed"!

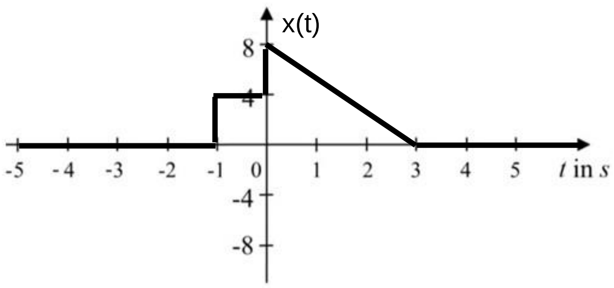
Grade:	
First examiner:	
Second examiner:	

I wish you success!

Task 1

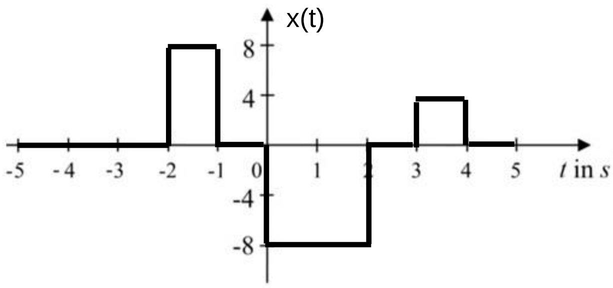
Points: 12

Write x(t) as a sum of elementary functions. (3)



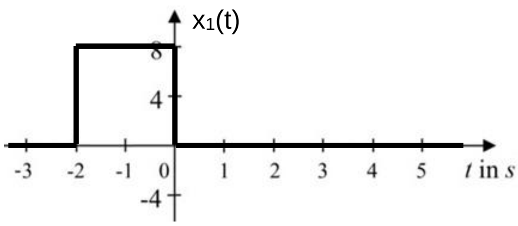
$x(t) = 4 \cdot \varepsilon(t+1) + 4 \cdot \varepsilon(t)$
$- \frac{8}{3} \cdot \rho(t) + \frac{8}{3} \rho(t-3)$

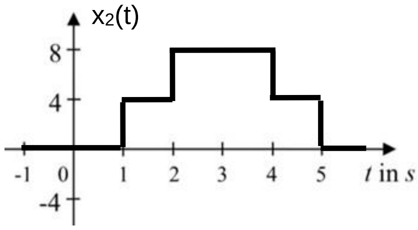
Determine the energy E_x of the signal x(t) (2)

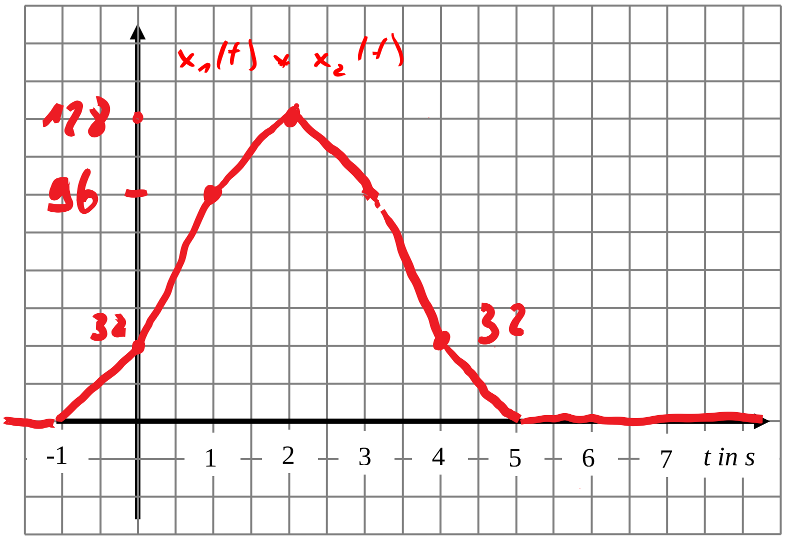


$E_x = \int_{-\infty}^{+\infty} |x(t)|^2 dt = 4^2 \cdot 1 + 8^2 \cdot 3$
$= 208$

Sketch the convolution product of the two signals $x_1(t)$ and $x_2(t)$ (incl. correct axis labeling!). (4)







Indicate with "Yes" or "No" whether the following system with input u(t) and output y(t), has the following properties (no further explainations are required). (3)

$$y(t) = \left(u(t+2)\right)^2 + u(t)$$

Linearity: no

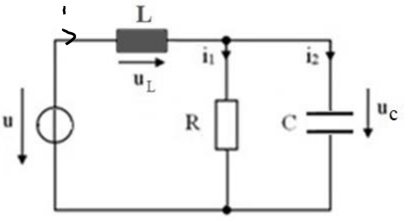
Stability: yes

Causality: no

Task 2 **Points: 16**

Required steps of modelling: 1) Determination of input variable u, state variables x_i and output variable y. 2) Determination of the coordinate systems. 3) Establishing the balance equations. 4) Isolating the derivatives of all state variables. 5) Drawing a block diagram.

The voltage source u(t) is ideal, the output shall be the voltage u_C(t)! 9
Creat a simulationsmodel following the required steps of modelling 1 to 5.



(1) $u = u(t)$ $y = u_C(t)$ $x_1 = i(t)$ $x_2 = u_C(t)$

(2) ✓

(3) $u_L = L \cdot \frac{di}{dt}$ $u_C = R \cdot i_1$ $i_2 = C \cdot \frac{du_C}{dt}$ $i_1 + i_2 = i$ $u_L + u_C = u$ (3)

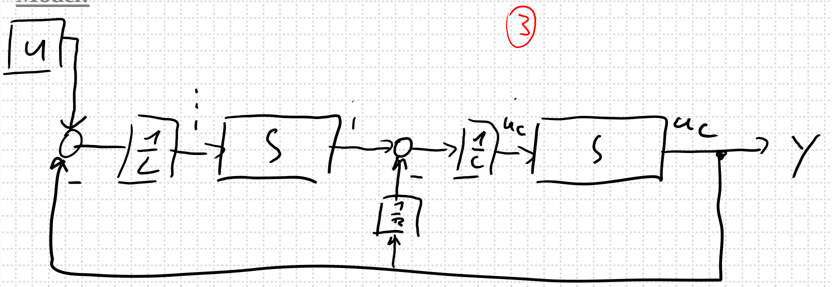
(4) $\dot{x} = f(x,u,t)$

$\dot{x}_1 = \dot{i} = \frac{u_L}{L} = \frac{1}{L}(u - u_C)$ ✓

$\dot{x}_2 = \dot{u}_C = \frac{i_2}{C} = \frac{1}{C}(i - i_1)$

$= \frac{1}{C}\left(i - \frac{u_C}{R}\right)$ ✓ (3)

Model: (3)



Determine the Laplace transform X(s) of the signal: $x(t) = (t-2) \cdot e^{-(t-2)} \cdot \varepsilon(t-2)$ 3

$X(s) = \frac{1}{(s+1)^2} e^{-2s}$

Solve this inital value problem by the tool Laplace tansform. 4

$\ddot{x}(t) + x(t) = \delta(t)$ *mit* $x(0_-) = 3$ und $\dot{x}(0_-) = 1$

$\circ\!\!-\!\!\bullet$ X

$s^2 X(s) - s\,x(0_-) - \dot{x}(0_-) + X(s) = 1$

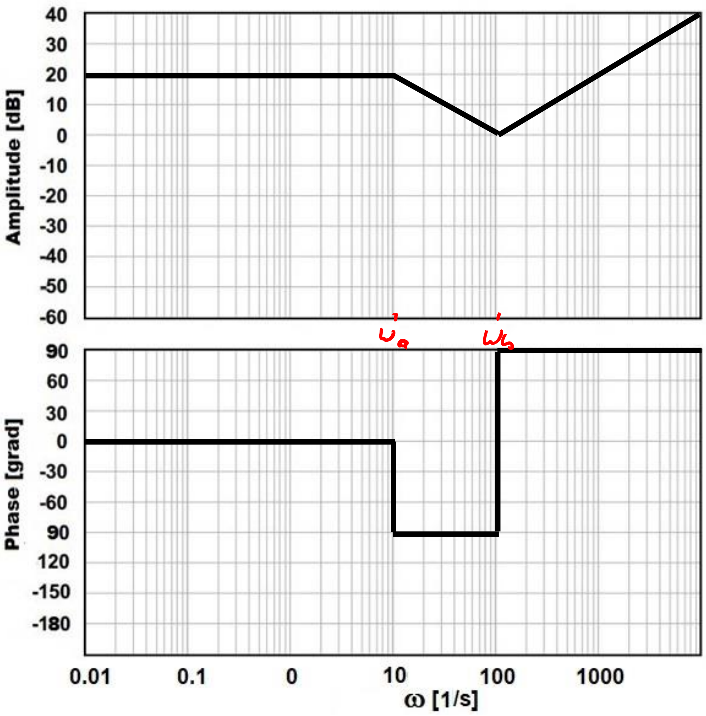
$X(s)\{s^2 + 1\} = 2 + 3s$

$X(s) = \frac{2}{s^2+1} + \frac{3s}{s^2+1}$ $\circ\!\!-\!\!\bullet$ $x(t) = \{2\sin(t) + 3\cos(t)\} \cdot \varepsilon(t)$

(3) (1)

Task 3 **Points: 15**

A **non-causal** system has the shown Bode plot (only its asympotes are shown).
How many poles and zeros does this system have?
Find the transfer function G(s) which describes this system. **9**



2 × NST , 1 × Pol

P: $K_0 = +20\,dB \Rightarrow 20 \lg K_0 = 20$
$\Rightarrow \lg K_0 = 1$
$\Rightarrow K_0 = 10^1 = 10$

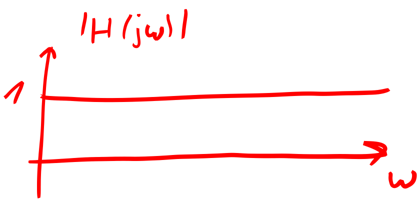
Pol: $\omega_a = 10$
$\Rightarrow G_a(s) = \left(\frac{s}{10} + 1\right)$

NST: $\omega_b = 100$
$\Rightarrow G_b(s) = \left(\frac{s}{100} + 1\right)^2$

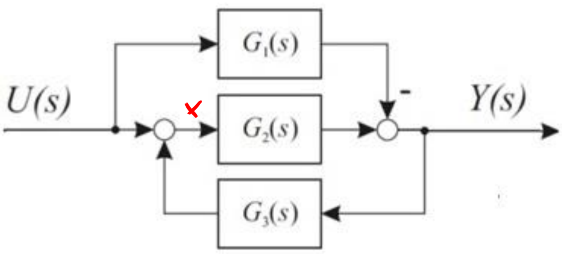
$$\Rightarrow G(s) = \frac{10 \cdot \left(\frac{s}{100} + 1\right)^2}{\frac{s}{10} + 1} = \frac{10 \cdot 10 \cdot 100^2 \left(\frac{s}{100} + 1\right)^2}{10 \cdot 100^2 \left(\frac{s}{10} + 1\right)} = \frac{(s+100)^2}{100\,(s+10)}$$

Give the transfer function of an all-pass filter with a pole at s = -2 and a pole at s = -3 and sketch its amplitude response. **2**

$$A(s) = \frac{(s-2)(s-3)}{(s+2)(s+3)}$$



Determine the transfer function $G(s) = \frac{Y(s)}{U(s)}$ of this system. **4**



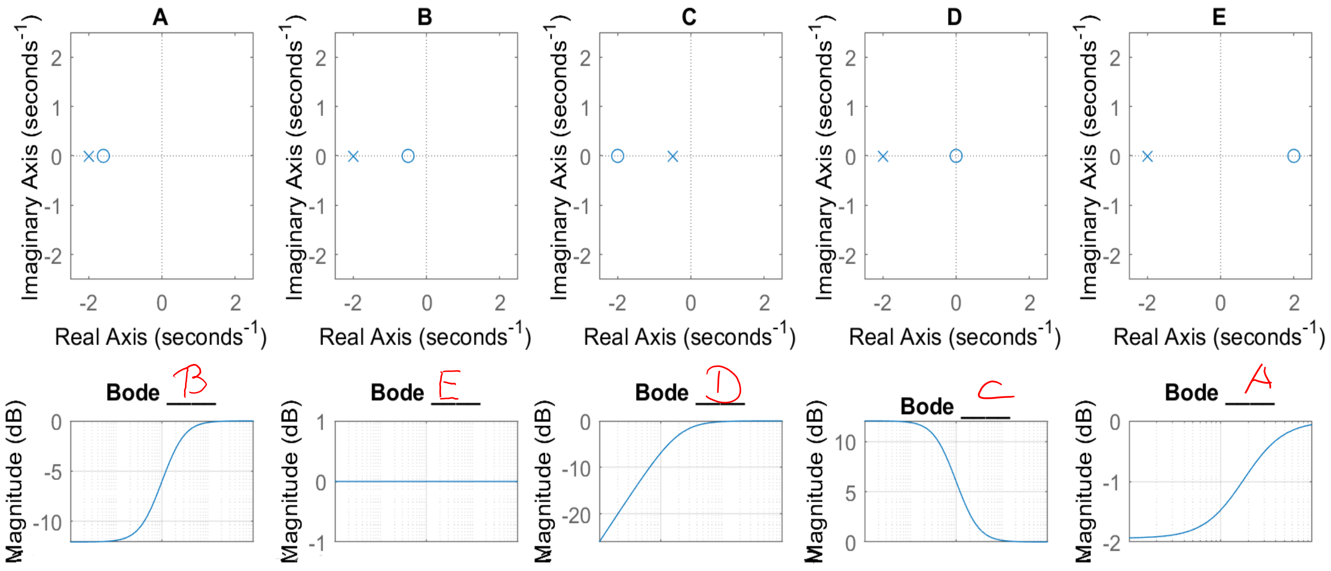
$y = -G_1 \cdot u + G_2 \cdot x = -G_1 u + G_2 (u + G_3 y)$

$y(1 - G_2 G_3) = u(-G_1 + G_2)$

$$\Rightarrow G(s) = \frac{Y(s)}{U(s)} = \frac{G_2(s) - G_1(s)}{1 - G_2(s)\, G_3(s)}$$

Task 4 **Points: 17**

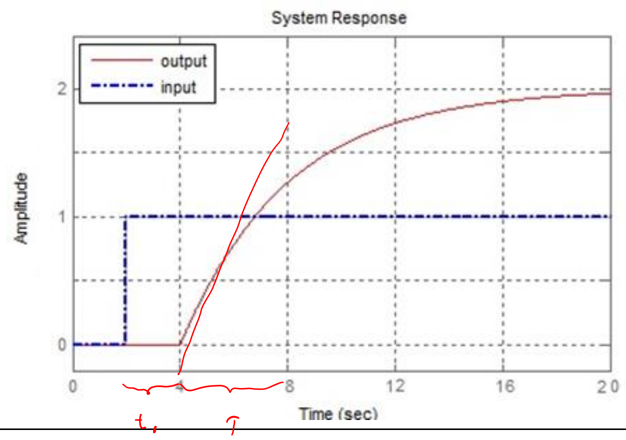
Above you see PZ-diagrams of five systems A to E, below five amplitude responses.
Find the pairs and fill in the correct letters below. 4



An unknown system was tested with a unit-jump.
Give a transfer function, which describes this system.
(Pay attention also on the dead time!). 4

$T=4$ $K=2$ $t_0=2$

$$G(s) = \frac{2}{4s+1} \cdot e^{-2s}$$



Determine the Fourier transform of the following signal: $x(t) = \frac{3}{t^2}$ 4

Hint: it applies: $\frac{1}{t^2} = -\frac{d}{dt}\left(\frac{1}{t}\right)$

$$X(j\omega) = -j\omega \cdot 3(-j\pi\, \mathrm{sgn}(\omega)) = -\pi\omega \cdot \mathrm{sig}(\omega)\, 3$$

A system with the frequency response $G(j\omega) = \frac{Y(j\omega)}{U(j\omega)} = \frac{j\omega}{j\omega+1}$ is stimulated with the following signal $u(t) = 5\sin(5t)$. 5
Calculate and determine the expected steady-state output signal y(t).

$$|G(j5)| = \left|\frac{j5}{j5+1}\right| = \frac{5}{\sqrt{25+1}} = \frac{5}{\sqrt{26}} = 0,98$$

$$\angle G(j5) = \angle j5 - \angle(j5+1) = \frac{\pi}{2} - \arctan\left(\frac{5}{1}\right) \approx 0,2$$

$$\Rightarrow \boxed{y(t) = \underbrace{0,98 \cdot 5}_{4,9} \sin(5t + 0,2)}$$

Task 5 — Points: 17

An unstable PT$_1$-System besitzt has the following transfer function: $G(s) = \frac{1}{s-1}$. (8)

Using the jump invariant transformation, calculate the corresponding z-transfer function $G_{HS}(z)$ for a sampling time T = 1s.

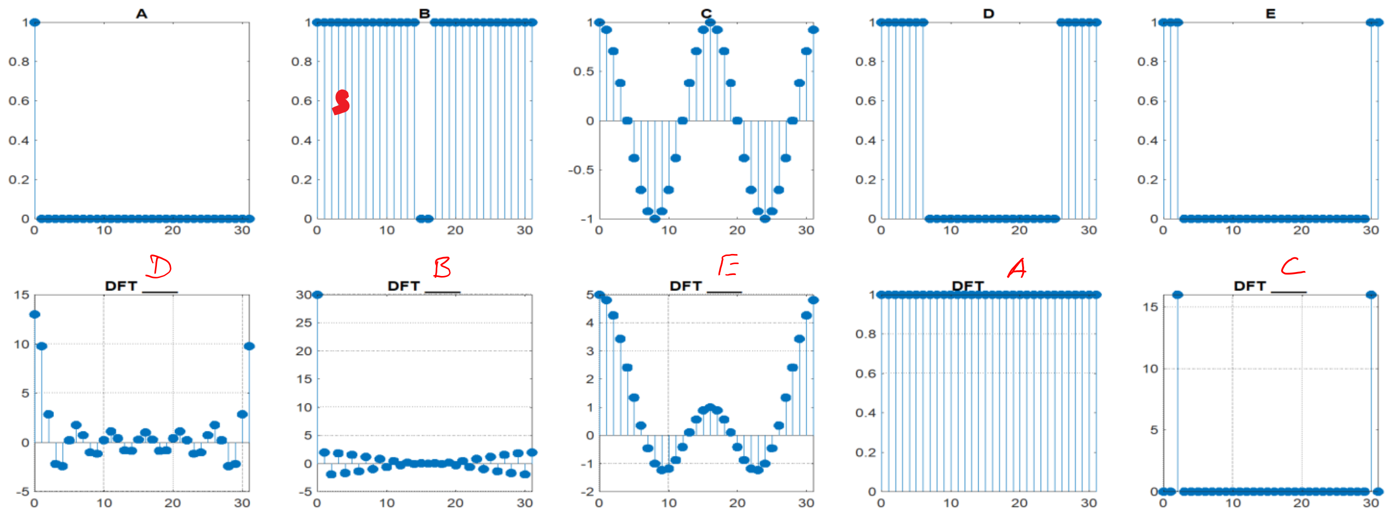
$$G_{HS}(z) = \frac{z-1}{z} \cdot \mathcal{Z}\left\{ \mathcal{L}^{-1}\left(\frac{1}{s(s-1)}\right)\Big|_{t=kT} \right\} = \frac{z-1}{z} \cdot \mathcal{Z}\left\{ -(1-e^{t}) \cdot \varepsilon(t) \Big|_{t=kT} \right\}$$

$$= \frac{z-1}{z} \cdot \mathcal{Z}\{ -\varepsilon[k] + e^{k} \cdot \varepsilon[k] \}$$

$$= \frac{z-1}{z}\left(\frac{-z}{z-1} + \frac{z}{z-e} \right) = -1 + \frac{z-1}{z-e} = \frac{e \; + z - 1}{z-e}$$

$$= \boxed{\frac{e-1}{z-e} = \frac{1,7}{z-2,7}}$$

Above you see five time discrete signals x[k] with length N = 32 labeled as A to E, below five DFTs. Find the pairs and fill in the correct letters below. (4)



Given is the following z-transfer function G(z) of time-discrete system: $G(z) = \frac{Y(z)}{U(z)} = \frac{2z^2+4}{z^2-2}$ (5)

Is this system stable? Yes why? pole is RHP

Give the corresponding difference equation to calculate the output y[k].

(i.e. y[k] should be explicitly calculable, i.e. y[k] shall be be alone on the left side of your equation!)

$$Y(z) \cdot (z^2 - 2) = U(z) \cdot (2z^2 + 4) \quad \| : z^2$$

$$z^2 - 2 = 0$$

$$z^2 = 2$$

$$|z_{\infty 1,2}| = \pm\sqrt{2} \,| > 1$$

$$\Rightarrow \text{instabil}$$

$$Y(z) \cdot (1 - 2z^{-2}) = U(z) \cdot (2 + 4z^{-2})$$

$$\circ\!\!-\!\!\bullet \; \mathcal{Z}^{-1}$$

$$y[k] - 2y[k-2] = 2u[k] + 4u[k-2]$$

$$\Rightarrow \boxed{y[k] = 2y[k-2] + 2u[k] + 4u[k-2]}$$

Task 6

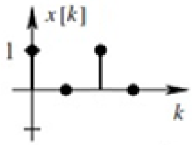
Points: 13

A time-discrete systems has the following z-transfer function $G(z) = \frac{z+1}{z-0.5}$.
Give the frequency response of this system. 2

$|z_\infty| = 0,5 < 1$
$\Rightarrow$ stabil

$$G(j\Omega) = \frac{e^{j\Omega}+1}{e^{j\Omega}-0,5}$$

Above you see a signal x[k], below the definition of the DFT.
Determine the second value of the corresponding DFT for the length N = 4.
Hint: requested is X[1], i.e. n = 1. 3



$$X[n] = DFT\{x[k]\} = \sum_{k=0}^{N-1} x[k] \cdot e^{-j\left(2\pi \frac{n}{N} k\right)}$$

$X[1] = 1 - 1 = 0$

What is the long form of the abbreviation FIR? Give an arbitrary transfer function that has FIR behavior. 2

FIR: Finite impulse response

$G_{FIR}(s) = \frac{z+1}{z^3}$

Complete the following sentence with at least one correct mathematical statement: "The spectrum of a non-periodic, analog and odd time signal is stets kontinuierlich, ungerade und imaginär" 2

The steps 1-4 of the modelling of an electric system resulted in the following equations: 4

1) Input: $u = u_E$, Output: $y = u_C$, State variables: $x_1 = u_C$ and $x_2 = i_L$

4) $\dot{x} = f(x, u, p)$: $\dot{x}_1 = \frac{du_C}{dt} = \frac{1}{RC}(u_E - u_C)$ $\dot{x}_2 = \frac{di_L}{dt} = \frac{u_E}{L}$

Give the state space representation (matrix-form) of this systems, i.e.: $\dot{x} = Ax + Bu$, $y = Cx + Du$

The state space vector is predefined and shall be: $x = \begin{bmatrix} u_C \\ i_L \end{bmatrix}$

$$\begin{bmatrix} \dot{u}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} \\ \frac{1}{L} \end{bmatrix} \cdot u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} u_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \cdot u$$