Exercises

Correlation Functions

1. The following auto-correlation function of $x(t)$ is given:

$$\varphi_{xx}^{E}(\tau) = \Delta\left(\frac{\tau}{4}\right)$$

a) Determine two possible signals $x(t)$, which have the given auto-correlation function.

b) Determine the energy of $x(t)$.

Now, the cross-correlation function between s(t) and g(t) is given:

$$\varphi_{sg}^{E}(\tau) = \Delta(\tau + 1) - \Delta(\tau - 1)$$

c) Sketch the cross-correlation function $\varphi_{gs}^{E}(\tau)$ stating all characteristic values.

d) Are the signals $s(t)$ and $g(t)$ orthogonal to each other?

e) Determine the Fourier-transform $\Phi_{sg}^{E}(\tau)$ of $\varphi_{sg}^{E}(\tau)$. State the real-part and the imaginary-part separately in the solution.

2. a) Which properties in symmetry have auto-correlation and cross-correlation functions?

The energy spectral density of s(t) is given:

$$\Phi_{ss}^{E}(f) = \mathrm{si}^2(3\pi f)$$

b) Determine the auto-correlation function of $s(t)$.

c) Determine the energy of $s(t)$.

$s(t)$ is now transmitted via an LTI system. The LTI system has the transfer function $H(f)$. The output signal is $g(t)$.

$$H(f) = \mathrm{j}(\varepsilon(f) - \varepsilon(-f))$$

d) Determine the energy spectral density $\Phi_{hh}^{E}(f)$ of $h(t)$.

e) Provide a formula to calculate $\Phi_{gg}^{E}(f)$ from $\Phi_{hh}^{E}(f)$ and $\Phi_{ss}^{E}(f)$.

f) Verify in frequency domain, whether $s(t)$ and $g(t)$ are orthogonal to each other.

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3. The two following signals are given:

$$x(n) = 2\delta(n-1) + 3\delta(n-4)$$
$$y(n) = 3\delta(n-2) + 2\delta(n-3)$$

a) Is $x(n)$ causal, anti-causal or non-causal? (justification required)

b) Determine the auto-correlation function $\varphi_{xx}^{E}(m)$.

c) Determine the energy spectral density $|X(f)|^2$. Note: The sampling time is T = 1.

d) Determine the energy of $x(n)$.

e) Are $x(n)$ and $y(n)$ orthogonal to each other? (justification required)

f) Determine the shift-factor n_0, so that $x(n-n_0)$ and $y(n)$ are not orthogonal.

4. a) Determine the cross-correlation function $\varphi_{sg}^{E}(\tau)$ for the given $s(t)$ and $g(t)$:

$$s(t) = si(\pi t)$$
$$g(t) = si(\pi(t-42))$$

The two following signals are given:

$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2)$$
$$y(n) = \delta(n) + 3\delta(n-1) + \delta(n-2)$$

b) Determine the energy of $y(n)$.

c) Determine the cross-correlation function $\varphi_{xy}^{E}(0)$.

d) Which of the following signals could be auto-correlation functions? (justification required)

$$\varphi_{s_1 s_1}^{E}(\tau) = 2 \cdot \Delta(\tau) \cdot (1-\tau^4)$$
$$\varphi_{s_2 s_2}^{E}(\tau) = 2 \cdot \mathrm{rect}(\tau) \cdot (1-\tau^3)$$
$$\varphi_{s_3 s_3}^{E}(\tau) = 2 \cdot \mathrm{rect}(\tau) \cdot (1-\tau^2)$$