Exercises

Time-Discrete Signals

1. The impulse response $h(n)$ is given. First, let $a = 2$.

$$h(n) = \delta(n+1) + \sum_{k=0}^{\infty} a^k \delta(n-k)$$

   a) Draw the impulse response $h(n)$ of the filter within the range $-2 \leq n \leq 3$ stating all characteristic values.

   In the following, let a be an arbitrary real number.

   b) For which a is $h(n)$ stable?

   c) For which a is the filter $h(n)$ anti-causal?

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2. The time-discrete signal $x(n)$ is given:

$$x(n) = \delta(n) + 2 \cdot \delta(n-1) + \delta(n-2)$$

   And the impulse response is

$$h_1(n) = \delta(n) + \delta(n-1)$$

   a) Calculate the result $y(n)$ of the time-discrete convolution:

$$y(n) = x(n) * h_1(n)$$

   In the following, we consider the transmission system $h_2(n)$.

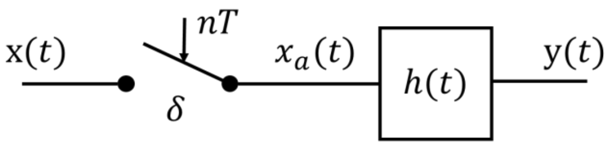
$$h_2(n) = \sum_{k=0}^{\infty} (0{,}5)^k \delta(n-k)$$

$$\text{mit} \qquad T = 1$$

   b) Draw the impulse response $h_2(n)$ within the range $-1 \leq n \leq 3$ stating all characteristic values.

   c) Determine the transfer function $H_2(f)$ of the system by making use of the formula sheet.

Now we consider the given sampling system.



$$x(t) = \mathrm{si}^2(3\pi t) \qquad H(f) = \mathrm{a} \cdot \mathrm{rect}\left(\frac{f}{6}\right)$$

$$X(f) = \frac{1}{3}\Lambda\left(\frac{f}{3}\right) \qquad = \int_{-\infty}^{\infty} h(t) \cdot e^{-j2\pi f t} dt$$

d) Specify the maximum sampling time T so that the sampling process does not generate an alias.

e) Determine the parameter a of the filter $H(f)$, so that for the under d) determined T the condition holds: $y(t) = x(t)$.

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3. The sampling time for the time-discrete signals and filters throughout the complete exercise is $T = 1$.

a) Determine $y(n)$:

$$y(n) = x(n) * h_1(n)$$
$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2)$$
$$h_1(n) = -\delta(n) + \delta(n-1)$$

b) Determine the transfer function $H_1(f)$ in relation to the given impulse response $h_1(n)$ and und calculate the real- and the imaginary-part of $H_1(f)$.

The impulse response $h_2(n)$ is given below:

$$h_2(n) = b\delta(n+1) + \delta(n) + \sum_{k=1}^{\infty} a^k \delta(n-k)$$

c) For which pair of values a and b is $h_2(n)$ causal? (justification required)

d) For which pair of values a and b is $h_2(n)$ stable? (justification required)

In the following we set $b = 0$ and $a = 0{,}5$.

e) Draw the resulting impulse response $h_2(n)$ within the range $-2 < n < 3$.

f) Determine the resulting transfer function $H_2(f)$.

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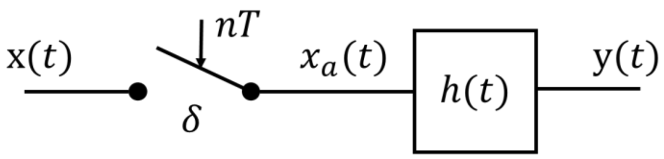
4. In the following, we consider the given sampling system.



$$x(t) = \mathrm{si}^2(2\pi t) \qquad T = \frac{1}{4}$$

$$X(f) = \frac{1}{2}\Lambda\left(\frac{f}{2}\right)$$

a) Draw the spectrum $X_a(f)$ of the sampled signal $x_a(t)$ stating all characteristic values within $-6 \leq f \leq 6$.

b) Name the function $h(t)$, for which the following condition holds: $x(t) = y(t)$.

In the following, we consider the time-discrete signal $s(n)$. The used sampling time is $T = 1$.

$$s(n) = a\delta(n+1) + \sum_{k=0}^{\infty} b^k \delta(n-k)$$

c) Determine $s(0)$.

d) For which pair of values a and b is $s(n)$ causal?

Now we set $a = 0$ and $b = \frac{3}{10}$.

e) Determine the Fourier-transform $S(f)$ of $s(n)$ by making use of the formula sheet.

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5. The Fourier transform of a discrete-time filter is given:

$$H(f) = 2\cos(3\pi f) + 2$$

a) By sampling in the time-domain a periodic spectrum is generated. Name two possible sampling times T_1 and T_2 , which might have generated the spectrum $H(f)$.

   Both sampling times shall be unequal to $T_0 = \frac{3}{2}$.

b) Determine the magnitude and phase of $H(f)$.

c) Is the filter $H(f)$ of linear-phase? (justification required)

Now the sampling time is $T_0 = \frac{3}{2}$.

d) Is $H(f)$ a low-, high-, bandpass- or a bandstopp-filter? (justification required)

e) Determine the impulse response $h(n)$.

f) Is $h(n)$ causal? (justification required)

6. A periodic time-continuous signal is given:

$$s(t) = 2\cos(3\pi t)$$

Via ideal sampling with the sampling time $T = 0{,}5$ we get the time-discrete signal $s_a(n)$.

a) How long is the shortest possible period of the signal $s(t)$?

b) Is $s(t)$ a power or an energy signal (justification required)

c) Draw the spectrum $S_a(f)$ of $s_a(n)$ within the range $-4 < f < 4$. (sketch stating all characteristic values)

d) Does the sampling generate alias? (justification required)

Via convolution of an ideal low-pass filter with the transfer function

$H(f) = \mathrm{rect}\left(\frac{f}{4}\right)$ we get (again) a time-continuous signal $g(t)$.

e) Determine the impulse response $h(t)$ of the ideal low-pass filter.

f) Determine

$$g(t) = h(t) * \left(s(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)\right) = h(t) * s_a(t)$$