Task 27:

Consider the plant described by the following LTI state equations (see Task 20 on Exercise Sheet 8, Task 22 on Exercise Sheet 9 and Task 25 on Exercise Sheet 11):

$$\dot{\underline{x}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x}$$

In the following, an observer-based feedback controller is designed which ensures the robust asymptotic compensation of constant disturbances.

a) First, the state feedback determined in Task 25b) which ensures the closed-loop eigenvalues $\lambda_{C,1} = \lambda_{C,2} = -5$ is extended by a feedforward part to the total control law $u = m_u w + \underline{k}^T(\underline{m}_x w - \underline{x})$. Determine the parameters m_u and $\underline{m}_x$ such that in steady state the relations $y = w$ and $\underline{m}_x w - \underline{x} = \underline{0}$ hold (assuming nominal operation without disturbances or model uncertainties).

b) Add a disturbance model for constant input disturbances to the plant description and state the corresponding extended state equations.

c) Design a Luenberger disturbance observer for the extended system model derived in subtask b) such that estimation errors decay according to the observer eigenvalues $\lambda_{o,1} = \lambda_{o,2} = \lambda_{o,3} = -1$. State the equations of the resulting observer algorithm.

d) The observer designed in subtask c) shall now be applied in order to

- realize the state feedback controller with y as only measurement signal;
- implement a robust asymptotic compensation of all constant disturbances.

Draw the overall block diagram and name all components of the final control algorithm.

e) Assume that the plant's input is limited to $-5 \leq u \leq 5$. Extend the block diagram derived in subtask d) such that windup is prevented in case the control algorithm requests higher absolute values of the control signal.