Task 26

We want to design a static feedforward control unit for the system

$$\dot{\underline{x}} = \begin{bmatrix} 1.2 & 1.6 \\ 1.6 & -1.2 \end{bmatrix} \underline{x} + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \underline{x}$$

<u>Solution:</u>

a) We can realize any desired steady-state output, if the corresponding transfer function has no zero at $s = 0$.

<u>Here:</u> The transfer function from u to y is

$$F(s) = c^T (sI - A)^{-1} b + d = \frac{2(s+2)}{(s+2)(s-2)} = \frac{2}{s-2}$$

→ no zero at $s = 0$ → any arbitrary desired output is feasible in steady-state

b) Design equation:

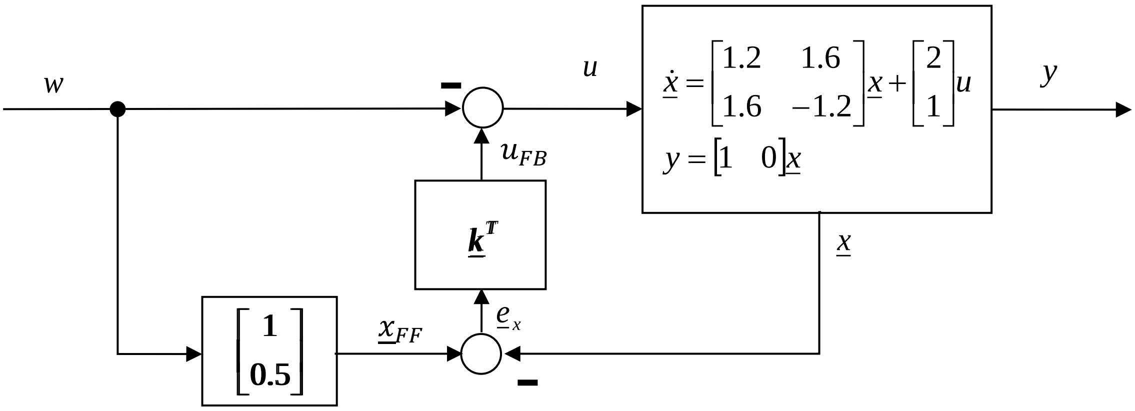
$$\begin{bmatrix} \underline{m}_x \\ m_u \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{b} \\ \underline{c}^T & d \end{bmatrix}^{-1} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

Here:

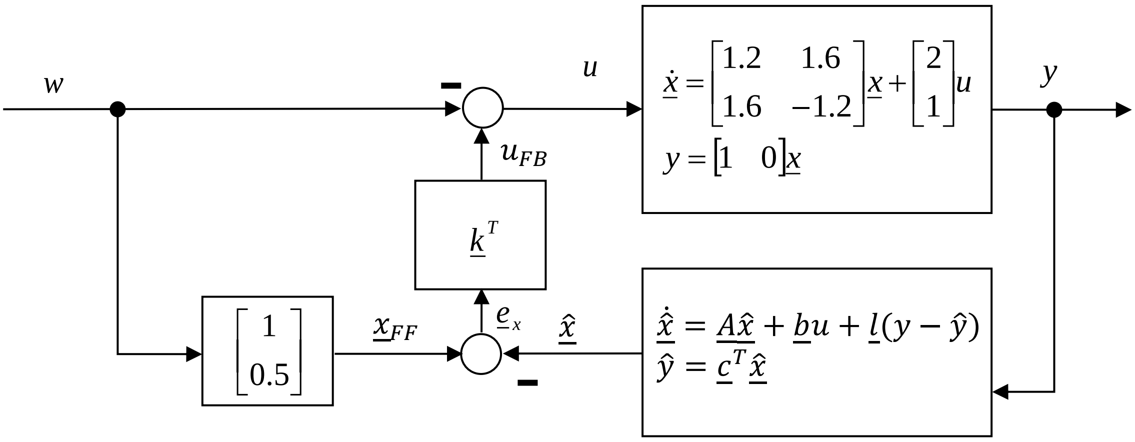
$$\begin{bmatrix} \underline{m}_x \\ m_u \end{bmatrix} = \begin{bmatrix} 1.2 & 1.6 & 2 \\ 1.6 & -1.2 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \mathrm{X} & \mathrm{X} & 1 \\ \mathrm{X} & \mathrm{X} & 0.5 \\ \mathrm{X} & \mathrm{X} & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ -1 \end{bmatrix}$$

→ $m_u = -1$ and $\underline{m}_x = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$

c) Block diagram:



d) Block diagram with observer:



Luenberger observer

At least if the mathematical model of the system used for observer design is correct, then the static feedforward control will still work (i.e. it still ensures that $y = w$ and that $\underline{e}_x = \underline{0}$ in steady state if no disturbances are acting on the system). This is the case, because in steady-state the estimated state $\hat{\underline{x}}$ will be equal to the true state $\underline{x}$. Consequently, in steady-state we have the same situation as in subtask b), i.e. $\underline{e}_x = \underline{0}$ and the feedforward control will generate the total input which is necessary to keep the system at the required operating point.