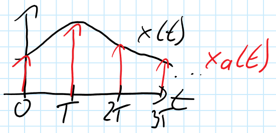
4. 17.04.2023

Montag, 17. April 2023 11:49

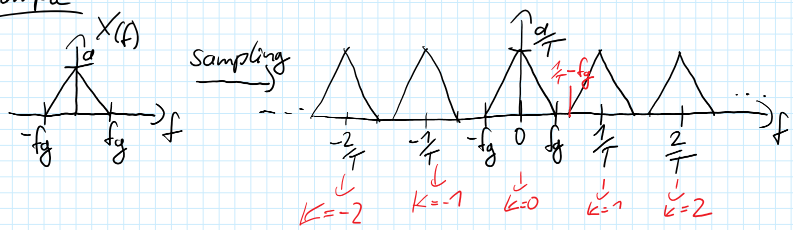
In Frequency-Domain

Time-Domain : $x(t)$ $\xrightarrow{\text{sampling}}$ $x_a(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT)$



$$X_a(f) = X(f) * \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

Example



That the spectra don't overlap:

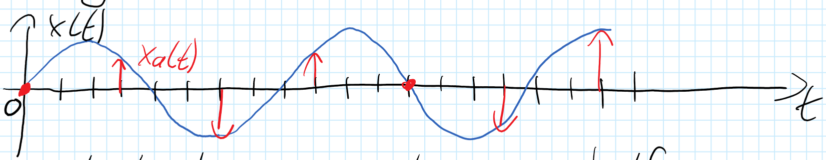
$$\frac{1}{T} - f_g \geq f_g \quad \Rightarrow 2 \cdot f_g \leq \frac{1}{T}$$

Sampling Theorem: $f_g \leq \frac{1}{2T}$ with $f_S = \frac{1}{T}$ ↖ Sample rate

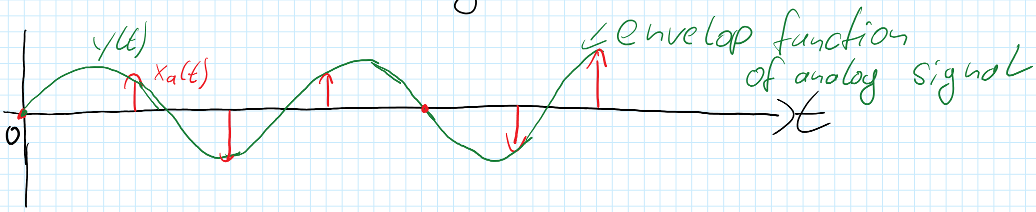
$$f_g \leq \frac{1}{2} f_S \quad | \; 2 f_g \leq f_S$$

„Nyquist Theorem"

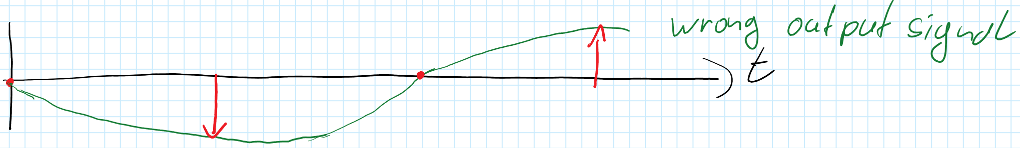
Sampling Theorem in Time-Domain representation



at least one sampling per half wave

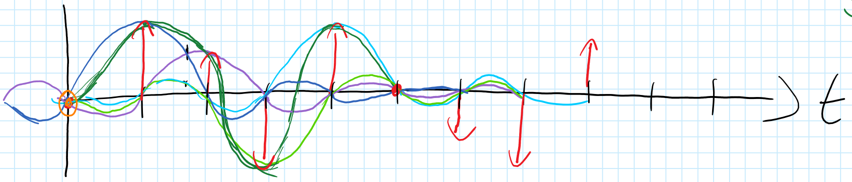


Violation of the sampling theorem



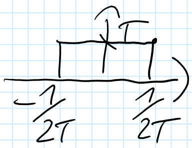
New Example

Reconstruction via si-function

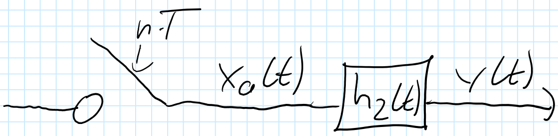


Sum of all shifted si-functions
→ signal envelope

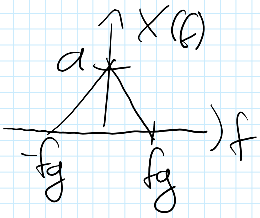
we know: $si(\pi \frac{t}{T})$ o—● $T \cdot rect(T \cdot f)$



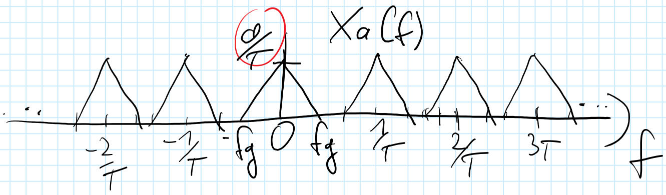
Reconstruction in frequency-domain via ideal interpolation



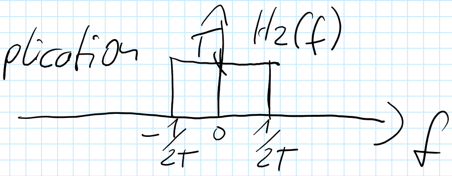
$h_2(t) = si(\pi \frac{t}{T})$

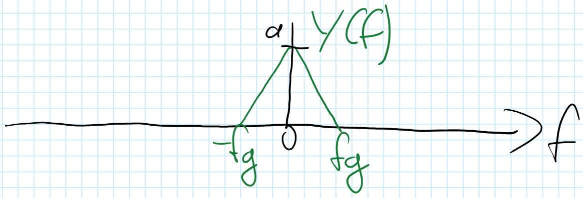


sampling



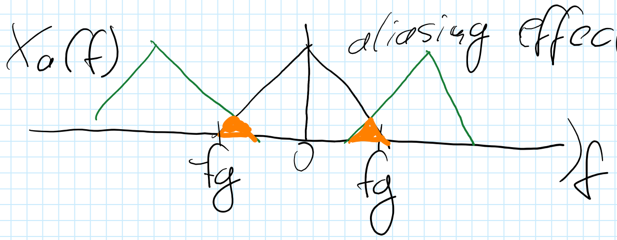
Multiplication





Sampling Theorem
$f_g \leq \frac{1}{2T}$

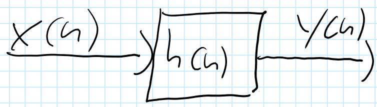
anti-example $f_g > \frac{1}{2T}$



Discrete Convolution

$$y(n) = x(n) * h(n) = \sum_{n=\infty}^{\infty} x(m) \cdot h(n-m)$$

Model



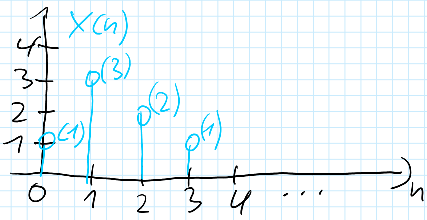
LTI

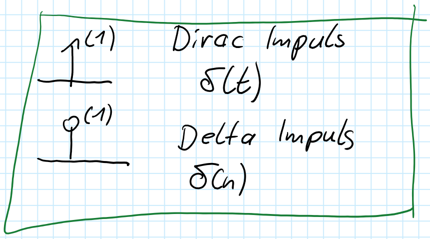
LSI → "linear-shift invariant"

Example

$$x(n) = \delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$$

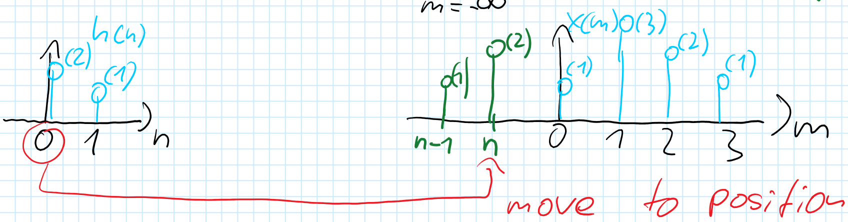
$$x = [1 \quad 3 \quad 2 \quad 1]$$





$$h(n) = 2\delta(n) + \delta(n-1)$$

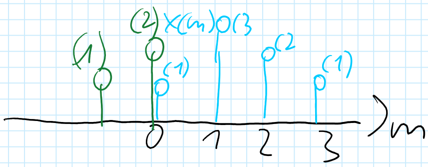
$$y(n) = x(n) * h(n) = \sum_{m=-\infty}^{\infty} x(m) \cdot h(n-m)$$ shift + flip (mirror)



move to position "n" during convolution

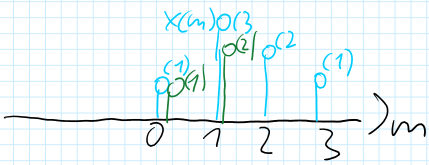
$n<0$: $y(n) = 0$

$n=0$:



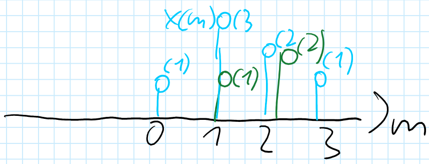
$$y(0) = 1\cdot 0 + 2\cdot 1 = 2$$

$n=1$:



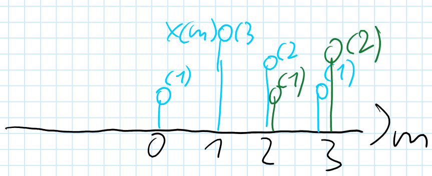
$$y(1) = 1\cdot 1 + 2\cdot 3 = 7$$

$n=2$



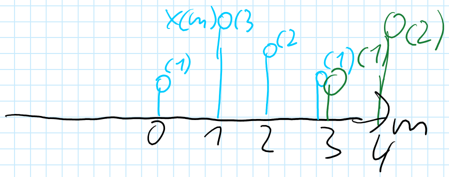
$$y(2) = 1\cdot 3 + 2\cdot 2 = 7$$

$n = 3$



$y(3) = 1 \cdot 2 + 2 \cdot 1 = 4$

$n = 4$



$y(4) = 1 \cdot 1 + 2 \cdot 0 = 1$

$n > 4 \qquad y(n) = 0$

$$y(n) = 2 \cdot \delta(n) + 7 \cdot \delta(n-1) + 7\delta(n-2) + 4 \cdot \delta(n-3) + 1 \cdot \delta(n-4)$$