Course „Control Systems 2"   Exercise Sheet 3

Task 3

Write the state equations resulting from Task 1 (see Exercise Sheet 1) in matrix form.

Task 4

Calculate the results of the following matrix products:

a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

c) $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Task 5

State for each of the following sets of vectors if the are linearly dependent (or not):

a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

e) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

f) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

Task 6

Determine the rank of the following matrices:

a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 0 \\ 3 & 4 & 5 \end{bmatrix}$

e) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 9 & 6 \\ 0 & -2 & 2 \end{bmatrix}$

Task 7

Calculate the following determinants:

a) $\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

b) $\det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

c) $\det \begin{bmatrix} 1 & 2 & 3 \\ 3 & 9 & 6 \\ 0 & -2 & 2 \end{bmatrix}$

Task 8

Solve the following homogeneous linear systems of equations. State the set of all solutions if more than one solution exists.

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 9 & 6 \\ 0 & -2 & 2 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Task 9

Calculate the inverse of the following matrices (if it exists):

a) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 9 & 6 \\ 0 & -2 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

Task 10

Determine the eigenvalues of the following matrices:

a) $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}$

Task 11

Determine to each eigenvalue of the matrix $\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}$ a corresponding eigenvector.

Task 12

Calculate the matrix exponential function $e^{\begin{bmatrix} -1 & 5 \\ 7 & -3 \end{bmatrix}}$.