Image Acquisition

Optics & Calibration

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2D and 3D Geometry

- Euclidean Space
- Homogeneous Coordinates
- Line, Plane, Curve

Geometric Projections

- Central projection
- Intrinsics & Extrinsics
- Projections of Lines & Planes
- Camera Calibration I

Optics: The Lens

- Characteristic Values
- Thin Lense
- Imaging Errors
- Camera Calibration II

The Lens

Motivation

Standard Lens

Zoom Lens

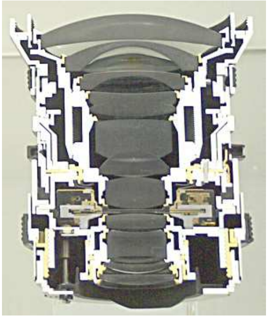
lens with variable focal length

Source: www.wikipedia.org

The Lens

Lens system with aperture

Every real camera has a lens with a collecting lens system and an aperture. The lens system influences the direction of propagation of light rays via diffraction, refraction and reflection. With the help of the aperture, the amount of light through the lens is specified. The three most important quantities are the focal length f, the aperture D and the principal distance b. These quantities influence e.g. the sharpness of the image capture or determine e.g. the reproduction scale. In the following, different characteristic values of a lens and their dependencies will be discussed in more detail.



Cross-section of a Zeiss Distagon

lens: lens system with diaphragm.

Source: Optical Museum Oberkochen

The Lens

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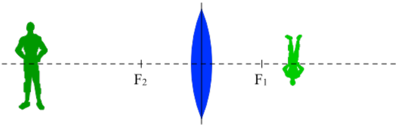
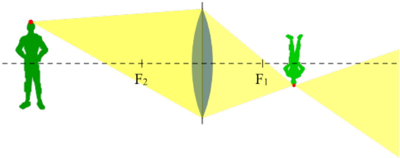
Image formation in lens imaging

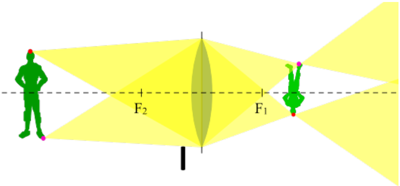
Collective lens without aperture



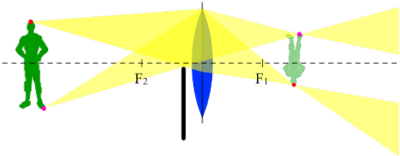
Head point image



Foot point image



Collective lens with aperture

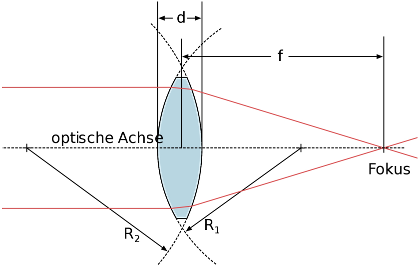


The Lens

Characteristics - focal length

The lens system of a lens usually consists of several convex (collecting) and concave (diverging) lenses. The optical properties of a lens are described by the focal length f. The focal length corresponds to the distance of the focal point F from the principal plane of a lens and is positive $f > 0$ for converging lenses. The focal length can be calculated from the refractive index of the lens material n, the refractive index of the surrounding medium n_0 (for air $n_0 = 1$), the radii of curvature R_1 and R_2, and the thickness d of the lens.

$$\frac{1}{f} = \frac{n - n_0}{n_0}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \underbrace{\frac{(n - n_0)^2 d}{n R_1 R_2}}_{\approx 0 \text{ for } d \ll R_1; R_2}$$



The Lens

Characteristics - Index of refraction

The refractive index n of the lens characterizes the refraction and reflection behavior of light waves when they hit the lens surface. It is given by the ratio of the propagation velocities of a light wave in vacuum c_0 and in the lens material c.

$$n = \frac{c_0}{c} > 1 \quad .$$

When light waves pass through a lens, the propagation speed c is reduced. The propagation speed of a wave depends on its wavelength λ and frequency ν:

$$c = \lambda\nu = \frac{c_0}{n} \quad .$$

The Lens

Characteristics - Refraction, focal length & color

Since the frequency never changes when it is transferred to another substance, the wavelength changes in the same way as the propagation speed:

$$\lambda = \frac{\lambda_0}{n} \quad (\lambda_0 = \text{Wellenlänge im Vakuum}) \quad .$$

The wavelength and the velocities of light in two different materials behave inversely to the refractive indices:

$$\frac{\lambda_1}{\lambda_2} = \frac{c_1}{c_2} = \frac{n_2}{n_1} \quad .$$

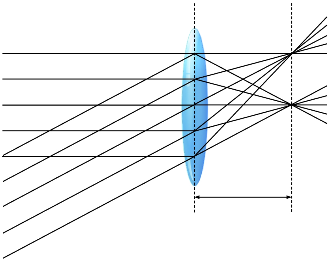
The refractive index n depends on the wavelength λ and therefore the focal length f is also dependent on the wavelength and therefore the color of the light:

$$f \propto \frac{\lambda}{\lambda_0 - \lambda} \quad , \quad \lambda < \lambda_0 \quad .$$

The Lens

Convex lense - Thin lense

The simplest way to focus light rays is via an aperture and a convex converging lens. If the thickness *d* of the lens is neglected, the model of the *thin lens* is obtained. It is assumed that the image plane is parallel to the lens plane and both are perpendicular to the optical axis, which passes through the center of projection of the lens. If only the refraction of light is considered and lens aberrations and diffraction phenomena are neglected, then all rays of a parallel light bundle meet in the focal plane and the radiation optics fundamental law is obtained.



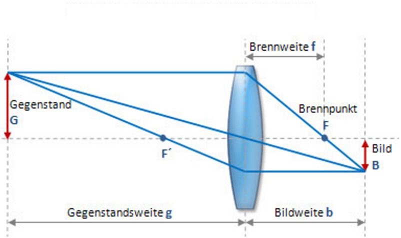
The Lens

Thin lens - Radiation optics fundamental law

This imaging equation for thin lenses ($d \rightarrow 0$) is obtained via the imaging scale

$$\beta = \frac{B}{G} = \frac{b}{g} = \frac{b-f}{f} \rightarrow \frac{1}{f} = \frac{1}{g} + \frac{1}{b}$$

from the ray theorem and the relations between object size G, image size B, object distance g and the principal distance b. Under the assumptions made, the principal distance corresponds to the purely geometrically defined camera constant $b = c$ (not the focal length!).



$$g = b\frac{G}{B} = \frac{b}{\beta} \quad \text{(working distance)}$$

$$\partial g = \frac{-f^2}{(b-f)^2}\partial b\,.$$

The Lens

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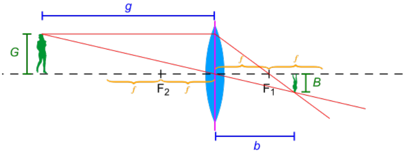
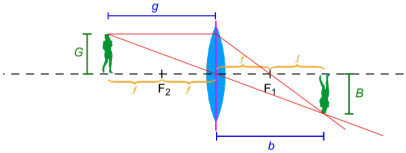
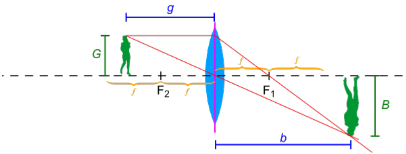
Thin lens - Aspect ratio



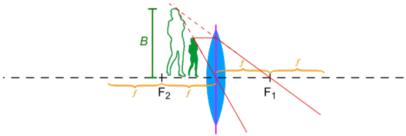
$g > 2f$ and $f < b < 2f$: $\rightarrow$ $B < G$



$g = 2f$ and $b = 2f$: $\rightarrow$ $B = G$



$f < g < 2f$ and $b > 2f$: $\rightarrow$ $B > G$



$0 < g < f$ and $b < 0$: $\rightarrow$ $B > G$
and $|b| > g$ (virtual image)

The Lens

Characteristics - Aspect ratio

The image scale β is defined as the ratio between image size B of an imaged object and the real object size G

$$\beta = \frac{B}{G} = \frac{b}{g} = \frac{b-f}{f}$$

- The smaller object distance g, the larger β.
- The longer focal length f, the larger β.

Every lens has a close-up limit. Below this minimum distance g_{min} to the object, it is no longer possible to focus on the object. A lens therefore has a maximum aspect ratio.

Miniature camera
24×36 mm

Panorama lens
f=17mm, FOV=93°

Standard lens
f=35mm, FOV=54°

Telephoto lens
f=200mm, FOV=10°

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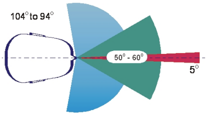
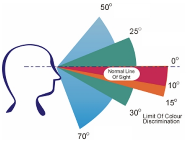
Characteristics - Field of view

As already shown in the section on geometric projections for the pinhole camera model, in practice the image plane is is spatially limited to a maximum image size B_{max}. This results in a restricted field of view *field of view* (FOV). If the distance to the subject is greater than the focal length $g > f$, the field of view or angle of view is given by

$$\theta = 2\arctan(\frac{B_{max}}{2f}) .$$

- For a flat image plane, this angle is always smaller than $\theta \leq 180°$.
- The smaller the focal length f, the larger the field of view θ.





The Lens

Characteristics - Example

If you want to obtain a constant image size B of an object for different object distances g sharply displayed, then the focal length f and the principal distance b must be adjusted accordingly, which also changes the angle of view θ.

f = 16mm small, g small, θ large

f = 85mm large, g large, θ small

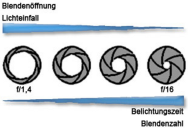
Characteristics - F-number & Exposure time

The f-number κ denotes the ratio of the focal length f to the diameter D of the entrance pupil of the lens. The entrance pupil is the aperture of the lens that limits the incident ray bundles.

$$\kappa = \frac{f}{D}$$

The smaller κ,

- the more light incidence available,
- the shorter exposure time required,
- the greater frame rate possible,
- the less motion blur occurs,
- the smaller the depth of field (see section Depth of Field).



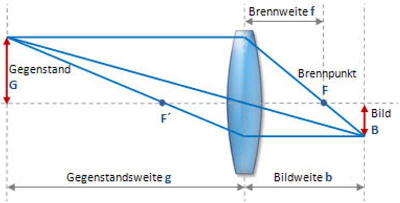
The lens

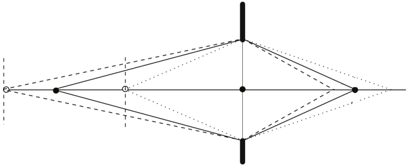
Thin lens - depth of sharpness & depth of field

If one neglects effects such as lens aberrations and diffraction phenomena, then there is for each object distance g a certain principal distance b in which a point-like reunion of all light rays occurs.

$$b = \frac{gf}{g - f}\,, \quad \partial b = \frac{-f^2}{(g - f)^2}\partial g\,. \qquad (1)$$

In this optimally adjusted image plane, an optimally sharp image of the object is created in the object plane. If the distance of the object is reduced, the image width must be increased.

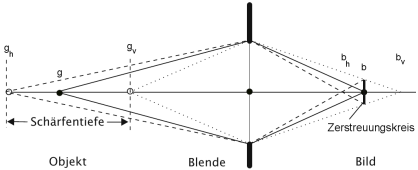




Thin lens - depth of sharpness & depth of field

All objects before g_v or behind g_h of the set object distance g are blurred on the image plane with distance b with a circle of confusion of diameter ϵ. This means that all objects in the range of depth of field $\Delta g = g_h - g_v$ will be imaged on the image plane with a circle of confusion smaller than ϵ. One calls ϵ the depth of sharpness, where for a digital camera a depth of sharpness of less than one pixel must be aimed at in order to achieve a sharp image as a function of the pixel size.

$$\epsilon \leq 1\text{pixel} \quad \text{(digital camera)}$$



Thin lens - depth of sharpness & depth of field

The aperture D and the depth of sharpness ϵ are related via the principal distances b, b_v and b_h as follows:

$$\frac{D}{b_v} = \frac{\epsilon}{b_v - b} \quad , \quad \frac{D}{b_h} = \frac{\epsilon}{b - b_h} \quad .$$

Substituting the relation (1) for b, b_v and b_h, we obtain with the f-number κ and a desired depth of sharpness ϵ, the depth of field range around a distance g at which an object is optimally sharp $\epsilon \to 0$:

$$g_v = \frac{gf^2}{f^2 + \kappa\epsilon(g - f)} \quad , \quad g_h = \frac{gf^2}{f^2 - \kappa\epsilon(g - f)} \quad .$$

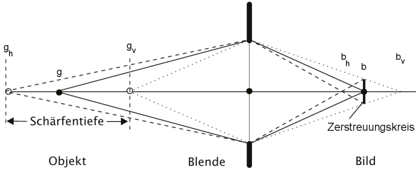
The lens

Thin lens - depth of sharpness & depth of field

If g_v and g_h of the workspace are given, then the object distance g is obtained, at which the depth of field ϵ is optimally small:

$$\frac{1}{g_h} + \frac{1}{g_v} = \frac{2}{g} \;\rightarrow\; g = \frac{2 g_h g_v}{g_h + g_v} \,.$$

If the depth of field range for a given ϵ is to be between $g_v(g)$ and $g_h \rightarrow \infty$, then g is given by:



$$g = \lim_{g_h \rightarrow \infty} \frac{2 g_h g_v}{g_h + g_v} = 2 g_v \;\rightarrow\; g = f\left(\frac{f}{\kappa\epsilon} + 1\right) \approx \frac{f^2}{\kappa\epsilon} \,.$$

The lens

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Thin lens - depth of sharpness & depth of field

If the set object width g and the actual object width g_i are known, then the resulting depth of sharpness can be calculated for $g_i \geq g$:

$$\epsilon_i = \frac{(1 - \frac{g}{g_i})f^2}{\kappa(g - f)} \quad .$$

And for objects being far away (teleradiography) $g \gg f$, $b \approx f$, the following approximation holds:

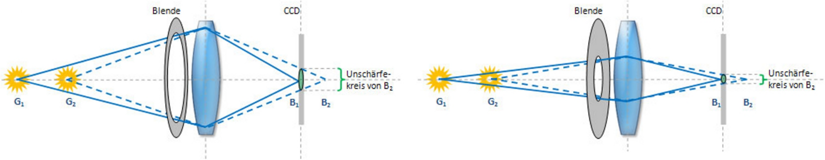
$$\frac{1}{g_h} - \frac{1}{g_v} = -\frac{2\kappa\epsilon(g - f)}{gf^2} \approx -\frac{2\kappa\epsilon}{f^2} \rightarrow g_h - g_v \approx \frac{2g^2\kappa\epsilon}{f^2}, \quad \text{because } g \approx g_h \approx g_v \,.$$

The Lens

Depth of Field - Aperture & Focal length

From this follows for the depth of field:

- The depth of field is inversely proportional to the square of the focal length. Thus smaller focal lengths result in a greater depth of field despite an image size proportional to f.
- A smaller aperture, and thus a larger f-number, increases the range of depth of field.
- A small aperture, however, reduces the amount of light, so the exposure time must be increased.



The Lens

Depth of Field - Examples

(Source 1: http://www.flickr.com/photos/foreby/2454926037/)

(Source 2: http://findimelda.com/483eweb/project02/photographysite/images/)

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Abberations - Overview

Imaging errors or aberrations are deviations from the ideal optical image, that cause an image to be blurred or distorted. There are a number of reasons for such deviations, which, among other things, influence the optical resolving power and contrast transmission:

- Diffraction blur
- Spherical aberration
- Astigmatism
- Chromatic aberration
- Distortions
- Field curvature
- Brightness distribution (natural light fall-off, vignetting)
- environmental influences (temperature, pollution, aging) see also vision-doctor.de

The Lens

Abberations - Diffraction blur

Due to the wave character of the light and its diffraction at the circular aperture of the lens, there is no ideal image point in the image plane of the camera, but a so-called diffraction slice with a theoretical total diameter:

$$u_{th} = 2.44\kappa\lambda$$

Practically, the diffraction blur corresponds approximately to the f-number

$$u \approx \kappa = \frac{f}{D}$$

- The smaller the aperture D, the greater the diffraction blur and the less light gets through the aperture.

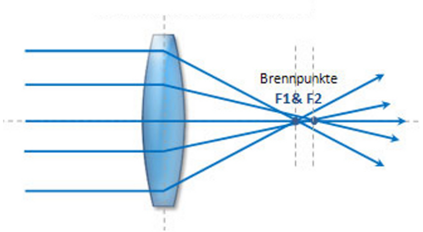
The Lens

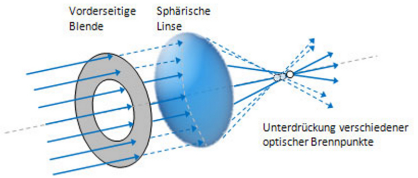
Abberations - Spherical aberration

With spherical abberation, spherically ground lenses no longer reproduce parallel light rays exactly at the focal point.

Effect: Blurred image, sharp core image is overlaid by a blurred one.

Remedy: Switching apertures in front

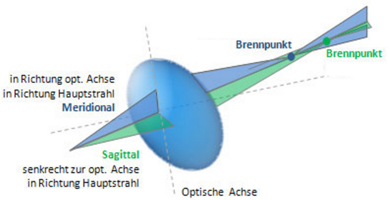




The Lens

Abberations - Astigmatism (Pointlessness)

If a diverging light beam does not strike the lens surface perpendicularly, but at an angle, and thus runs asymmetrically to the optical axis, astigmatism occurs on spherical lens surfaces. This is caused by different local radii of curvature of the lens. This results in different focal points and focal lengths for different partial ray paths.



Effect: The camera image no longer appears sharply focused to the viewer, since a scene point is no longer depicted as an image point but is blurred (pointless).

The Lens

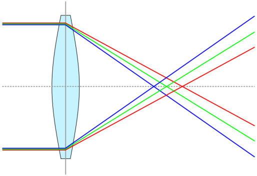
Abberations - Chromatic aberration

As already discussed beforehand, the refractive index n depends on the wavelength λ and therefore also the focal length f is dependent on the wavelength and therefore the color of the light:

$$f \propto \frac{\lambda}{\lambda_0 - \lambda} \quad , \quad \lambda < \lambda_0 \quad .$$

380 nm - 780 nm wave length

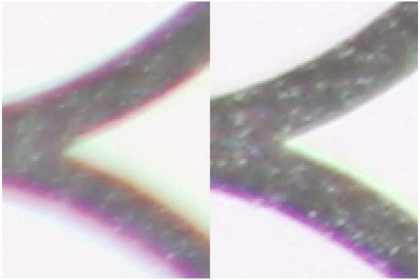
As a result, the optical resolving power also depends on the wavelength. Short-wave light is refracted more strongly than long-wave light, Therefore, the spectral components of the white light are focused differently and thus imaged at different locations on the image plane. This is also called chromatic aberration.



The Lens

Abberations - Example

Image

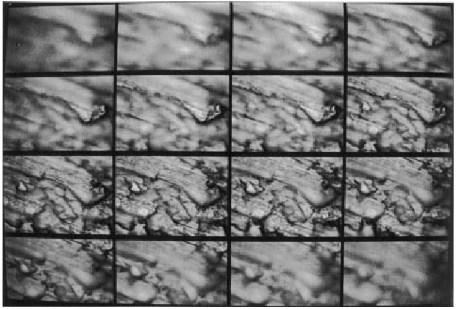


ROI (left lower corner), two different lenses

(Source: www.sump.org/blog/photography/)

Depth of Field

Application - 3D Reconstruction



Focus series with different focal planes

Reconstruced depth

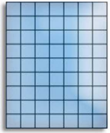
The lens

Abberations - Distortions

Distortion is a non-linear optical Abberation that locally changes the aspect ratio. It is caused e.g. by the deviation of the entrance and exit angle of a beam in a real optical system, the deviation of the mechanically realized principal distance to the optical principal distance or centering errors of the lenses. There are both radial and tangential distortions. Since the tangential distortion is usually one order of magnitude smaller than radial distortion, it can be neglected in many applications.

$$\mathbf{X}_{\mathcal{K}} \xrightarrow{\mathbf{K}_c\boldsymbol{\Pi}_0} \mathbf{x}_u \xrightarrow{\mathbf{K}_s} \mathbf{x}'_u \overset{?}{\leftrightarrows} \mathbf{x}'_d \quad .$$

$\mathbf{x}'_u$



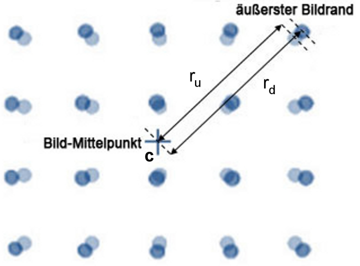
$\mathbf{x}'_d$
Pincushion

$\mathbf{x}'_d$
Barrel

The lens

Camera calibration II - radial distortions

Especially with wide-angle lenses, strong radial distortion can occur. It manifests itself in a barrel- or pincushion-shaped distortion of straight lines in the image. The parameters that characterize optical distortion also belong to the intrinsics of the camera.



$$
\begin{aligned}
r_u &= r_d + \Delta r_d(r_d)\,, \\
&= r_d L(r_d)\,. \\
L(r_d) &= (1 + a_1 r_d + a_2 r_d^2 + a_3 r_d^3 + a_4 r_d^4 + \dots)\,, \\
r_d &= ||\mathbf{x}'_d - \mathbf{c}||\,.
\end{aligned}
$$

The lens

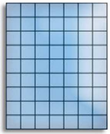
Camera calibration II - radial distortions

Depending on which model is chosen to describe the radial distortions, a different number of distortion parameters are needed $\{L'(r_d), \mathbf{c}\}$. By determining the parameters, one can use these models to determine the actual distortion $\mathbf{x}'_d = (x'_d, y'_d)^\top$ of an image and obtain a rectified image $\mathbf{x}'_u = (x'_u, y'_u)^\top$.

$$
\begin{aligned}
\text{general model:} \quad \mathbf{x}'_u &= \mathbf{c} + L'(r_d)(\mathbf{x}'_d - \mathbf{c})\,, \\
r_d &= ||\mathbf{x}'_d - \mathbf{c}||\,, \\
\text{sufficient model:} \quad \mathbf{x}'_u &= \mathbf{c} + (1 + a_2 r_d^2)(\mathbf{x}'_d - \mathbf{c})\,, \\
\text{(0.1 pixel accuracy)} \quad &= \mathbf{x}'_d + a_2 r_d^2(\mathbf{x}'_d - \mathbf{c})\,.
\end{aligned}
$$

The center of distortion $\mathbf{c}$ and the distortion correction factors $a_1, a_2, a_3, a_4, \ldots$ are to be determined.

$\mathbf{x}'_u$



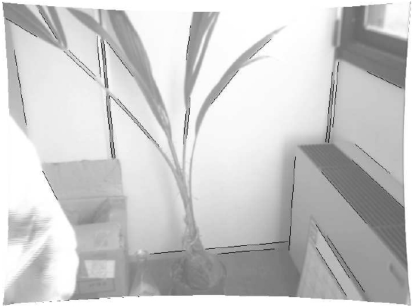
$\mathbf{x}'_d$

$\mathbf{x}'_d$

The lens

Camera calibration II - radial distortions

radially distorted image



rectified image

(Source: Devernay and Fougeras 1995)

The lens

Camera calibration II - radial distortions

Advantage: Capture a field of view of 180°

Camera with “Fish-eye lense”

rectified image detail

(Source: http://www.mobotix.com)

The lens

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Camera calibration II - Normalized Coordinates

Let us assume for the projection $\lambda\bar{\mathbf{x}}' = \mathbf{K}\Pi_0\bar{\mathbf{X}}_{\mathcal{K}}$ it requires a calibrated camera, where

$$\mathbf{K} = \begin{bmatrix} cs_x & cs_\theta & o_x \\ 0 & cs_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

and the model $\mathbf{r}$ of radial distortion

$$\mathbf{x}'_u = \mathbf{r}(\mathbf{x}'_d) \overset{z.B.}{=} \mathbf{x}'_d + a_2 r_d^2(\mathbf{x}'_d - \mathbf{c})$$

is known, then the following normalized image coordinates $\bar{\mathbf{x}}$ result:

$$\underbrace{\lambda}_{Z_{\mathcal{K}}}\underbrace{\mathbf{r}(\bar{\mathbf{x}}'_d)}_{\bar{\mathbf{x}}'_u} = \mathbf{K}\Pi_0\bar{\mathbf{X}}_{\mathcal{K}} \quad \rightarrow \quad \underbrace{\lambda}_{Z_{\mathcal{K}}}\underbrace{\mathbf{K}^{-1}\mathbf{r}(\bar{\mathbf{x}}'_d)}_{\bar{\mathbf{x}}} = \Pi_0\bar{\mathbf{X}}_{\mathcal{K}} \ .$$

Ideal normalized image coordinates result from a purely canonical projection. The better the camera calibration is performed, the closer one comes to this ideal.

Digital Cameras

Additional Animations

https://ciechanow.ski/cameras-and-lenses/