Course „Control Systems 2“

Solution to Exercise Sheet 2

Task 1: Solution

a)

- **Physical modelling:**

  Assumption: Gravity is not considered

  - total force on m_1:

$$F_1 = -k_1(z_1 - z_{10}) - b(\dot{z}_1 - \dot{z}_2) \tag{1}$$

  - total force on m_2:

$$F_2 = F - k_2(z_2 - z_{20}) + b(\dot{z}_1 - \dot{z}_2) \tag{2}$$

  - Newton’s law for m_1 and m_2:

$$F_1 = m_1\ddot{z}_1$$

$$\text{with (1)} \rightarrow -k_1(z_1 - z_{10}) - b(\dot{z}_1 - \dot{z}_2) = m_1\ddot{z}_1 \tag{3}$$

$$F_2 = m_2\ddot{z}_2$$

$$\text{with (2)} \rightarrow F - k_2(z_2 - z_{20}) + b(\dot{z}_1 - \dot{z}_2) = m_2\ddot{z}_2 \tag{4}$$

- **Introduce state variables…**

$$x_1 = z_1$$
$$x_2 = \dot{z}_1$$
$$x_3 = z_2$$
$$x_4 = \dot{z}_2$$

- **… and input(s)**

$$u = F$$

- **State differential equations**

$$\dot{x}_1 = x_2$$
$$\text{with (3)} \rightarrow \quad \dot{x}_2 = -\frac{k_1}{m_1}x_1 - \frac{b}{m_1}x_2 + \frac{b}{m_1}x_4 + \frac{k_1}{m_1}z_{10}$$
$$\dot{x}_3 = x_4$$
$$\text{With (4)} \rightarrow \quad \dot{x}_4 = \frac{b}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b}{m_2}x_4 + \frac{1}{m_2}u + \frac{k_2}{m_2}z_{20}$$

- **Output equation(s)**

$$y = \dot{z}_2 = x_4$$

- **Summary: State equations**

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{k_1}{m_1}x_1 - \frac{b}{m_1}x_2 + \frac{b}{m_1}x_4 + \frac{k_1}{m_1}z_{10}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{b}{m_2}x_2 - \frac{k_2}{m_2}x_3 - \frac{b}{m_2}x_4 + \frac{1}{m_2}u + \frac{k_2}{m_2}z_{20}$$

$$y = x_4$$

b)

- The state equations are time-invariant, since the time variable t does not explicitly occur on the right hand side of the state equations.
- The state equations are not linear, because of the constant summands $\frac{k_1}{m_1}z_{10}$ and $\frac{k_2}{m_2}z_{20}$ on the right hand side of the state differential equations.