Task 27

Solution:

a) Static feedforward design equation:

$$\begin{bmatrix} \underline{m}_x \\ m_u \end{bmatrix} = \begin{bmatrix} \underline{A} & \underline{b} \\ \underline{c}^T & d \end{bmatrix}^{-1} \begin{bmatrix} \underline{0} \\ 1 \end{bmatrix}$$

Here:

$$\begin{bmatrix} \underline{m}_x \\ m_u \end{bmatrix} = \begin{bmatrix} 1 & -4 & 1 \\ 2 & -3 & 1 \\ 0 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} X & X & -1 \\ X & X & 1 \\ X & X & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 5 \end{bmatrix}$$

→ $m_u = 5$ and $\underline{m}_x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

b) Disturbance model for constant disturbances:

$$\dot{x}_z = 0$$
$$z = x_z$$

Assuming an input disturbance and extending the plant state equation by the disturbance model yields

$$\underline{\dot{x}}_e = \begin{bmatrix} \underline{A} & \underline{b} \\ \underline{0}^T & 0 \end{bmatrix} \underline{x}_e + \begin{bmatrix} \underline{b} \\ 0 \end{bmatrix} u$$
$$y = [\underline{c}^T \quad 0] \underline{x}_e$$

where

$$\underline{x}_e = \begin{bmatrix} \underline{x} \\ x_z \end{bmatrix}$$

Here:

$$\underline{\dot{x}}_e = \underbrace{\begin{bmatrix} 1 & -4 & 1 \\ 2 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{\underline{A}_e} \underline{x}_e + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}_{\underline{b}_e} u$$

$$y = \underbrace{[0 \quad 1 \quad 0]}_{\underline{c}_e^T} \underline{x}_e$$

c) Design equation:

$$\det(\lambda \underline{I} - \underline{A}_e + \underline{l}_e \underline{c}_e^T) \stackrel{!}{=} \prod_i (\lambda - \lambda_{o,i})$$

<u>Here:</u>

$$\det\left(\begin{bmatrix} \lambda-1 & 4+l_{e,1} & -1 \\ -2 & \lambda+3+l_{e,2} & -1 \\ 0 & l_{e,3} & \lambda \end{bmatrix}\right) = (\lambda+1)^3$$

$$\Rightarrow\ \lambda^3 + (2+l_{e,2})\lambda^2 + (5-l_{e,2}+l_{e,3}+2l_{e,1})\lambda + l_{e,3} = \lambda^3+3\lambda^2+3\lambda+1$$

Equating the coefficients yields

$$\begin{aligned} &l_{e,3} = 1 \\ &2+l_{e,2} = 3 \ \Rightarrow\ l_{e,2} = 1 \\ &5-l_{e,2}+l_{e,3}+2l_{e,1} = 3 \ \Rightarrow\ 5-1+1+2l_{e,1} = 3 \ \Rightarrow\ l_{e,1} = -1 \end{aligned}$$

$$\rightarrow\ \underline{l}_e = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

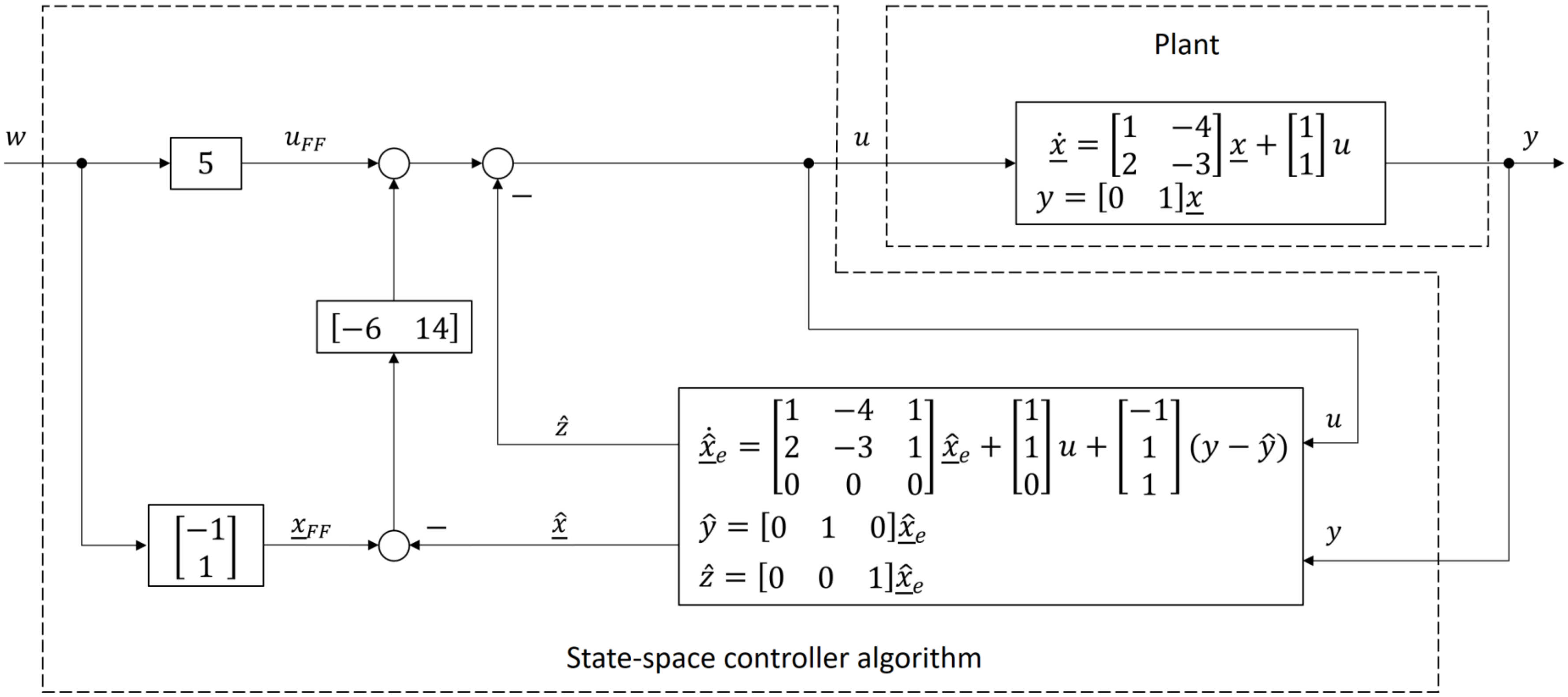
The final observer algorithm reads

$$\dot{\hat{\underline{x}}}_e = \begin{bmatrix} 1 & -4 & 1 \\ 2 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}\hat{\underline{x}}_e + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}u + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}(y-\hat{y})$$

$$\hat{y} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\hat{\underline{x}}_e$$

$$\hat{z} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}\hat{\underline{x}}_e$$

d) Block diagram:



e) Block diagram with anti-windup measures: