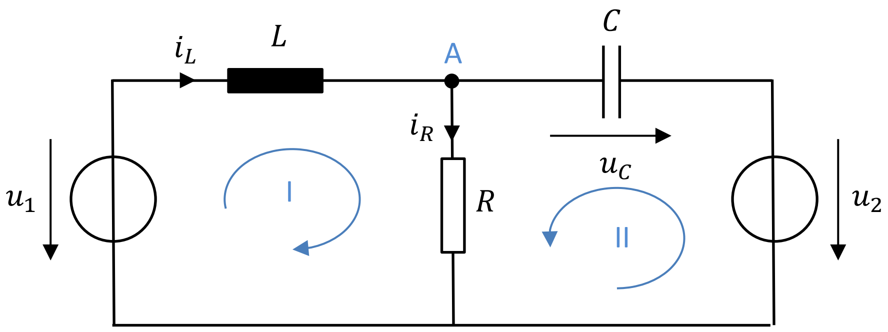
Task 1: Solution

- **Physical modelling:**



  - mesh rules:

$$\text{(I)} \qquad u_1 = L\frac{di_L}{dt} + Ri_R \tag{1}$$
$$\text{(II)} \qquad u_2 = -u_C + Ri_R \tag{2}$$

  - junction rule at point A:

$$i_L - i_R - C\frac{du_C}{dt} = 0 \tag{3}$$

  - using (3) we can eliminate i_R (note that i_R must not be present on the right-hand side of the state equations, since it is neither state nor input):

$$(3) \rightarrow i_R = i_L - C\frac{du_C}{dt}$$

  - substitute this expression in (1) and (2):

$$(1) \rightarrow u_1 = L\frac{di_L}{dt} + Ri_L - RC\frac{du_C}{dt} \tag{4}$$
$$(2) \rightarrow u_2 = -u_C + Ri_L - RC\frac{du_C}{dt} \tag{5}$$

- **Introduce state variables and solve for time-derivatives of the states:**

  - substitute $i_L = x_1$ and $u_C = x_2$ in (4) and (5):

$$(4) \rightarrow u_1 = L\dot{x}_1 + Rx_1 - RC\dot{x}_2 \tag{6}$$
$$(5) \rightarrow u_2 = -x_2 + Rx_1 - RC\dot{x}_2 \tag{7}$$

- solve for time derivatives $\dot{x}_1$ and $\dot{x}_2$:

$$\text{(7)} \rightarrow \boxed{\dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{RC}x_2 - \frac{1}{RC}u_2} \quad (8)$$

$$\text{(8) in (6)} \rightarrow \boxed{\dot{x}_1 = -\frac{1}{L}x_2 + \frac{1}{L}u_1 - \frac{1}{L}u_2} \quad (9)$$

- **Express output in terms of states (x_1 & x_2) and inputs (u_1 & u_2):**

$$\text{(2)} \rightarrow i_R = \frac{1}{R}u_C + \frac{1}{R}u_2$$

  - using $i_R = y$ and $u_C = x_2$:

$$\rightarrow \boxed{y = \frac{1}{R}x_2 + \frac{1}{R}u_2} \quad (10)$$

- **Sort equations and summarize:**
  - eqs. (9), (8) and (10) are the required state space description of the system:

$$\boxed{\begin{aligned} \dot{x}_1 &= -\frac{1}{L}x_2 + \frac{1}{L}u_1 - \frac{1}{L}u_2 \\ \dot{x}_2 &= \frac{1}{C}x_1 - \frac{1}{RC}x_2 - \frac{1}{RC}u_2 \\ y &= \frac{1}{R}x_2 + \frac{1}{R}u_2 \end{aligned}}$$