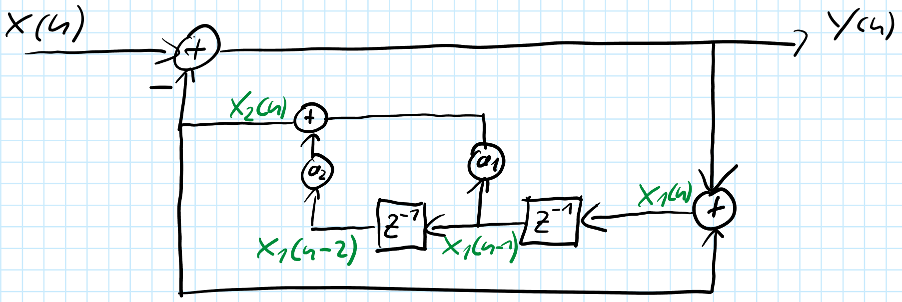
8. 22.05.2023

Montag, 22. Mai 2023 15:54

Z - Transform

Predictor of 2^{nd} order in "closed-loop" config



a) Determine the z-transform of the transfer function

- Determine intermediate signals after all adder

$x_1(n) = y(n) + x_2(n)$ $\quad$ ○—● (1) $X_1(z) = Y(z) + X_2(z)$

$x_2(n) = a_2 \cdot x_1(n-2) + a_1 \cdot x_1(n-1)$ $\quad$ ○—● (2) $X_2(z) = a_2 \cdot z^{-2} \cdot X(z) + a_1 z^{-1} \cdot X(z)$

$y(n) = x(n) - x_2(n)$ $\quad$ ○—● (3) $Y(z) = X(z) - X_2(z)$

[Z-Transform: $a \cdot x(n-n_0)$ ○—● $a \cdot z^{-n_0} \cdot X(z)$]

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = X(z) - X_2(z)$$

(2) in (3) $Y(z) = X(z) - (a_2 z^{-2} + a_1 z^{-1}) X(z)$

$$Y(z) = X(z)\left[1 - a_2 z^{-2} - a_1 z^{-1}\right]$$

$$H(z) = 1 - a_2 z^{-2} - a_1 \cdot z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 - a_2 z^{-2} - a_1 z^{-1}$$

b) Determine the difference equation $y(n) = \ldots$

$$Y(z) = X(z) \cdot (1 - a_2 z^{-2} - a_1 \cdot z^{-1})$$

$$= X(z) - a_2 z^{-2} X(z) - a_1 \cdot z^{-1} X(z)$$

$$\bullet\!\!-\!\!\circ$$

$$y(n) = x(n) - a_1 x(n-1) - a_2 x(n-2)$$

assume: $H(z) = \frac{Y(z)}{X(z)} = \frac{1 - a_1 z^{-1} - a_2 z^{-2}}{1 - b_1 z^{-1}}$

$$Y(z)\left(1 - b_1 z^{-1}\right) = X(z)\left(1 - a_1 z^{-1} - a_2 z^{-2}\right)$$

$$\bullet\!\!-\!\!\circ$$

$$y(n) - b_1 y(n-1) = x(n) - a_1 x(n-1) - a_2 x(n-2)$$

$$y(n) = x(n) - a_1 x(n-1) - a_2 x(n-2) + b_1 y(n-1)$$

c) Now set $a_1 = 1$ and $a_2 = -\frac{1}{2}$

Draw the pole-zero-diagram (Pol-Nullstellen-Diagramm)

$$H(z) = \left(1 - a_1 z^{-1} - a_2 z^{-2}\right)\frac{z^2}{z^2}$$

$$= \frac{z^2 - a_1 z^1 - a_2}{z^2} \overset{\substack{a_1 = 1 \\ a_2 = -\frac{1}{2}}}{=} \frac{z^2 - z^1 + \frac{1}{2}}{z^2}$$

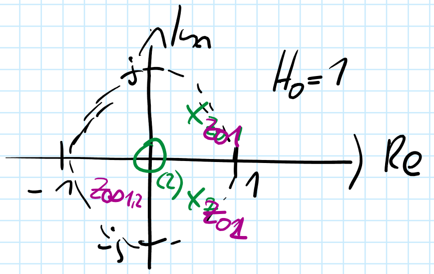
Pole: $z_{\infty\,1,2} = 0$

Null: $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{+1 \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot \frac{1}{2}}}{2 \cdot 1}$$

$$x_{1,2} = \frac{1}{2} \pm \frac{1}{2} j$$

$$1 \cdot \frac{z^2 - z + \frac{1}{2}}{z^2} = 1 \cdot \frac{\left(z - \frac{1}{2} - \frac{1}{2}j\right)\left(z - \frac{1}{2} + \frac{1}{2}j\right)}{z \cdot z}$$



$H_0 = 1$

Pole O ; Null X

amplification factor: $H_0 = 1$

Example for H_0

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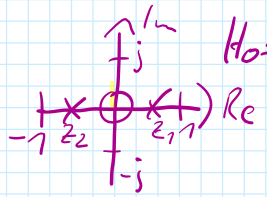
$$H(z) = \frac{3z^2 - 2}{2z}$$

Poles: 0

Nulls: $z_{1,2} = \pm\sqrt{\frac{2}{3}}$

$$H(z) = \frac{\left(z-\sqrt{\frac{2}{3}}\right)\left(z+\sqrt{\frac{2}{3}}\right)}{z} \cdot H_0 \qquad H_0 = \frac{3}{2}$$



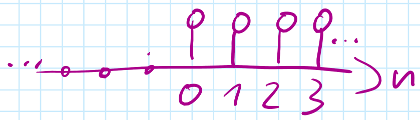
$H_0 = \frac{3}{2}$

d) Is the system stable?

Yes, because all poles are within the unit circle.

Example unit-step

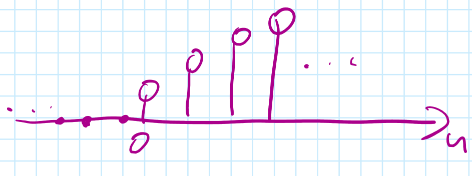
ma-ginal stable



stable: because the amplitude does not get bigger

not stable: because the amplitude never vanish

ex. $h(n)$ for Poles outside of UC

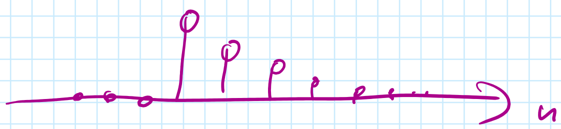


system with rising amplitude

$|h(n)| \to \infty$

$\Rightarrow$ not stable

ex. $h(n)$ for Poles inside of UC



system with damping amplitude

$|h(n)| \to 0$

$\Rightarrow$ stable

BIBO: $\sum^{\infty} |h(n)| < \infty$

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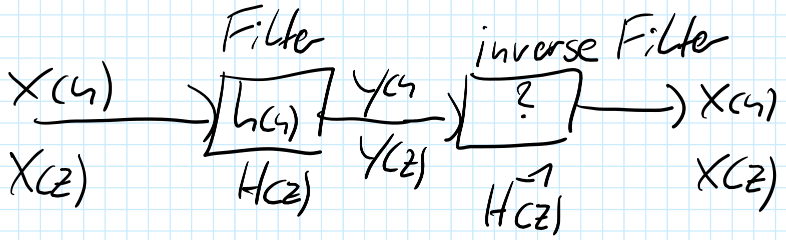
e)



$$Y(z) = X(z) \cdot H(z) \Rightarrow X(z) = \frac{1}{H(z)} \cdot Y(z) = H^{-1}(z) \cdot Y(z)$$

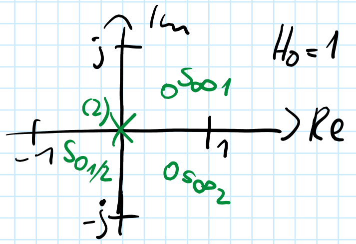
Determine the inverse filter $H^{-1}(z)$

$$H(z) = \frac{z^2 - z + 1/2}{z^2} \Rightarrow H^{-1}(z) = \frac{z^2}{z^2 - z + 1/2}$$

f) Pole-Zero-Diagramm of $H^{-1}(z)$

Poles become Zeros

Zeros become Poles



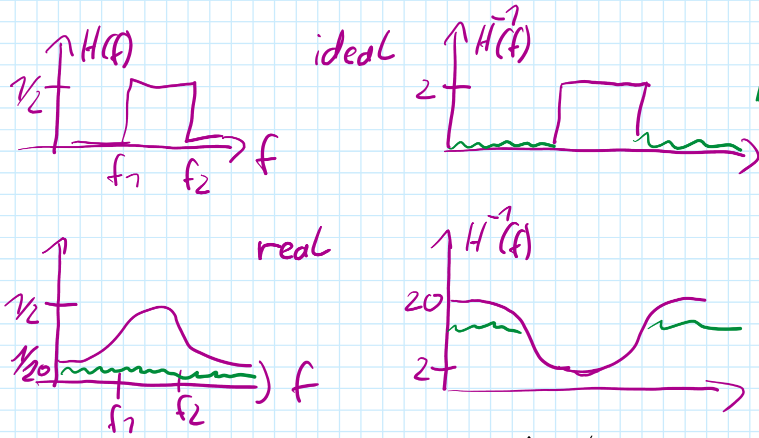
$H_0 = 1$

Zeros: X

Poles: O

$H^{-1}(z)$ is stable

Example of a non-invertable System



Division by zero

noise will be multiplide with 20

$\Rightarrow$ Amplification of noise

Solution: Apply window function

Relation: z-transform & Fourier-transform

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$$z \to e^{sT} \qquad s = \sigma + j\omega = \sigma + j2\pi f$$

$$z \to e^{j2\pi fT} \qquad T: \text{sample rate}$$

$$H(z) = \frac{z^2 - z + \frac{1}{2}}{z^2} = 1 - z^{-1} + \frac{1}{2} z^{-2}$$

$$H(f) = 1 - e^{-j2\pi fT} + \frac{1}{2} e^{-j4\pi fT}$$