List place:

Exam System Theory

of FHWS bachelor's degree courses electrical engineering and robotics

WS 2021

Prof. Dr. R. Hirn

Duration:	**90 minutes**
Tools:	**only legitimate calculators and the distributed formulary**
Max points:	**90 pt.** (15 + 14 + 16 + 14 + 14 + 17)
Tasks:	**6** (on 7 pages)

Last Name, First Name:	Solution
Matriculation No.:	

Hints:

- Write your name on each sheet!
- Do not remove any staples!
- Cheating is rated 5.0, i.e. "failed"!

Grade:	
First examiner:	
Second examiner:	

I wish you success!

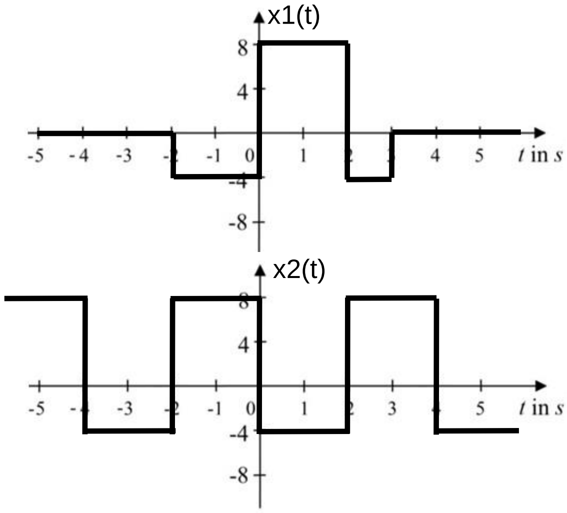
Task 1 — Points: 15

The signal $x_1(t)$ is time-limited, $x_2(t)$ is not time-limited but periodically.

Calculate the energy E of the signal $x_1(t)$ and the (average) power P of the signal $x_2(t)$.

$E_{x_1} = 4^2 \cdot 3 + 8^2 \cdot 2 = 176$

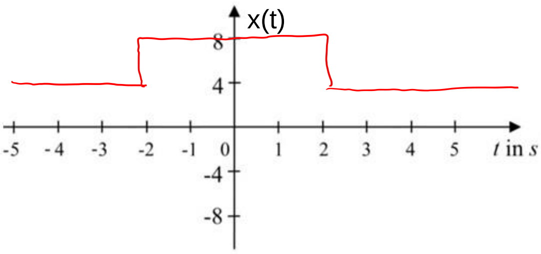
$P_{x_2} = \frac{1}{2}(8^2 + 4^2) = 40$



4

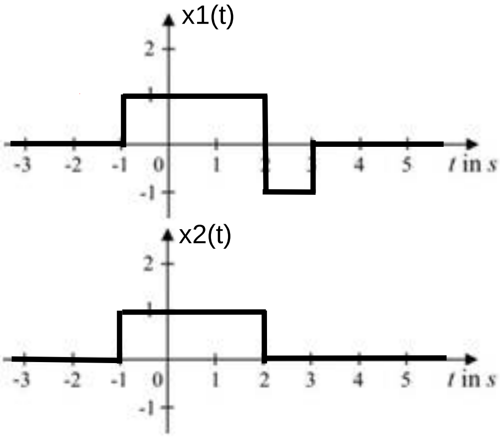
Sketch the time signal x(t):

$x(t) = 4 \cdot rect\left(\frac{t}{4}\right) + 4$

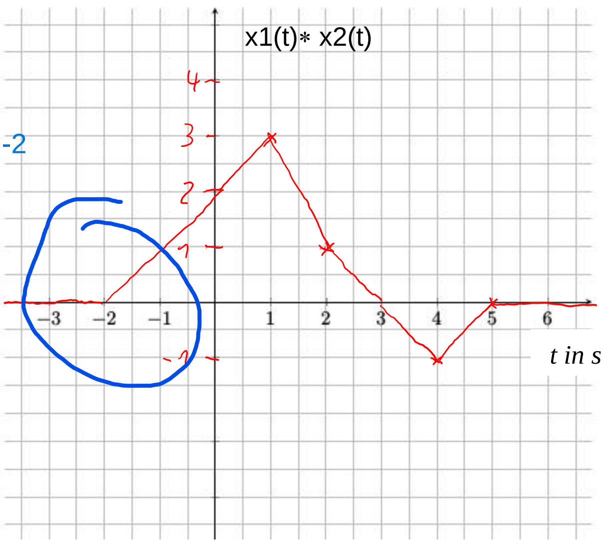


3

Sketch the convolution product of the two signals $x_1(t)$ and $x_2(t)$ (incl. correct axis labeling!).



why starting at -2



5

The signals x(t) and y(t) are both odd.
Underline the correct statement in each case:

The new signal

$x(t) + y(t)$ is a) even b) odd c) neither even nor odd

$x(t) - y(t)$ is a) even b) odd c) neither even nor odd

$x(t) + 2y(t)$ is a) even b) odd c) neither even nor odd

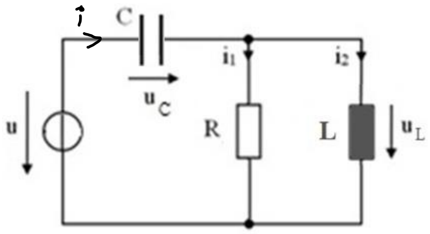
3

Task 2

Points: 14

Required steps of modeling: 1) Definition of the input variables u, the output variables y and the state variables x. 2) Establishing the coordinate systems. 3) Establishing the balance equations. 4) Isolating the first derivative of all state variables. 5) Sketching the model.

The voltage source u(t) is ideal, the output should be the voltage $u_L(t)$! Create a simulation model according to the steps 1 to 5 of above. 10



(1) $u = u(t)$ $y = u_L(t)$ $x_1 = u_C(t)$ $x_2 = i_2(t)$

(2) ✓

(3) $i = C \cdot \frac{du_C}{dt}$ $u_L = R \cdot i_1$ $u_L = L \cdot \frac{di_2}{dt}$ $i_1 + i_2 = i$

$u_C + u_L = u$

4

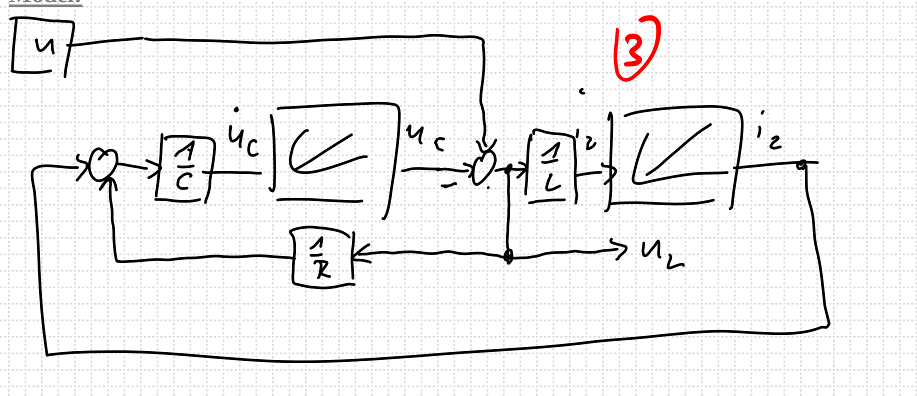
(4) $\dot{x} = f(x,u,p)$:

$\dot{x}_1 = \dot{u}_C = \frac{i}{C} = \frac{1}{C} \cdot (i_1 + i_2) = \frac{1}{C} \cdot \left(\frac{u_L}{R} + i_2 \right) = \frac{1}{C} \left(\frac{u - u_C}{R} + i_2 \right)$ ✓

$\dot{x}_2 = \dot{i}_2 = \frac{u_L}{L} = \frac{1}{L} (u - u_C)$ ✓

3

Model:



Step four of modeling an LTI system led to the following equations: (u is input, y is output, x_i are the state variables). 4

$$\dot{x}_1 = x_1 + 4x_2 + 2u$$
$$\dot{x}_2 = 2x_1 - 5x_3$$
$$\dot{x}_3 = 5x_2 - 5x_3 - u$$
$$y = x_1 + 2x_2$$

method?

1

Which of the following differential equations can in principle be used to describe the input/output behavior of this LTI system?
Mark the correct boxes (several answers are possible, no calculation is required).

[X] $\dddot{y} + 2\ddot{y} + 4y = 5u$
[X] $\ddot{y} - 3\dot{y} + 4y = 5u$
[X] $\ddot{y} + 4y = 2\dot{u} + 5u$
[ ] $\ddot{y} - 2t\ddot{y} - 3\dot{y} = 5u$
[ ] $\ddot{y} + 3\dot{y} + \sqrt{y} = 5u$

Task 3 — Points: 16

A system has the transfer function: **9**

$$G(s) = \frac{10 \cdot (s+1)^2}{s^2 (s+10)^2}$$

Construct the individual asymptotes and then the total asymptotes of the Bode plot.
(Please use different colors.)

$$G(s) = \frac{10 \cdot (s+1)^2}{100 \cdot s^2 \left(\frac{s}{10}+1\right)^2}$$

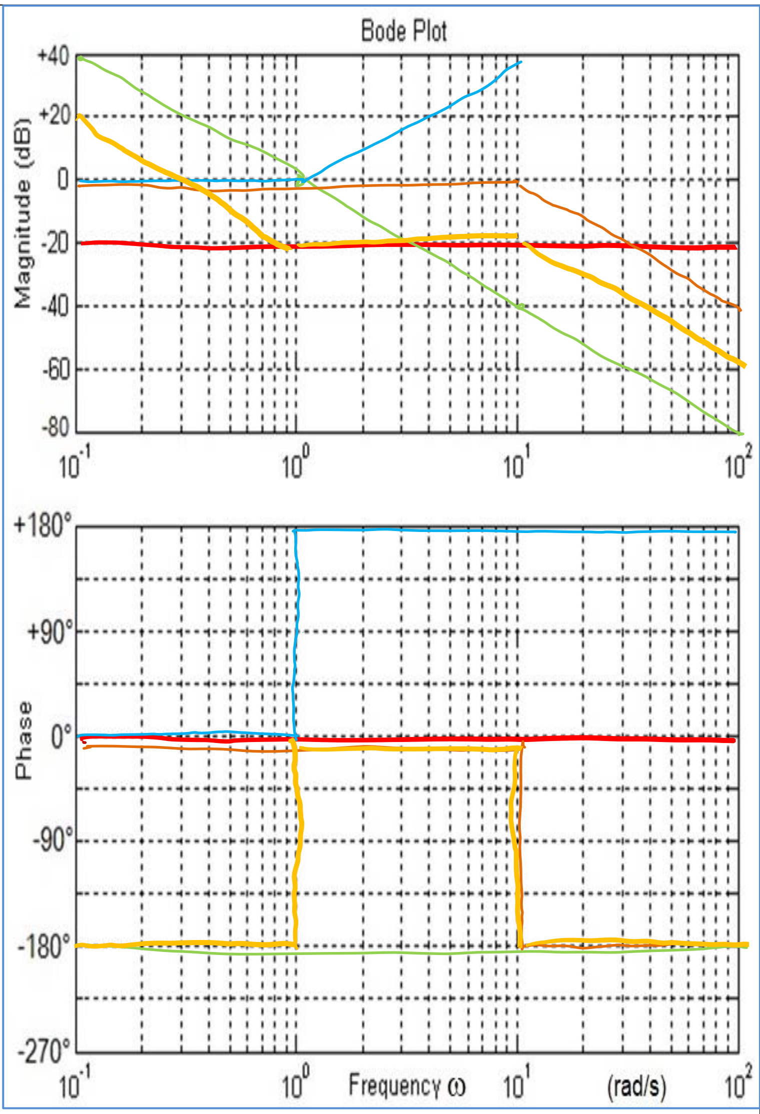
$P: \left(\frac{10}{100}\right): \; 20 \lg 0.1 = -20\,dB$

$Pol: \left(\frac{1}{s^2}\right): \; \omega_a = 0$

$NST: (s+1)^2: \; \omega_b = 1$

$Pol: \left(\frac{1}{\frac{s}{10}+1}\right)^2: \; \omega_a = 10$

Σ:



Calculate the solution y(t) of the following differential equation with initial condition using the Laplace transformation. **5**

$\dot{y}(t) + y(t) = e^{-t} \cdot \varepsilon(t)$ $\quad mit \quad$ $y(0\_) = 4$

$\mathcal{L}: \; sY(s) - y(0\_) + Y(s) = \frac{1}{s+1}$ (1)

laplace table given?

$Y(s) \cdot [s+1] = \frac{1}{s+1} + 4$

$\Rightarrow Y(s) = \frac{1}{(s+1)^2} + \frac{4}{s+1} \; \bullet\!\!\overset{\mathcal{L}}{—}\!\!\circ \; y(t) = [t \cdot e^{-t} + 4 \cdot e^{-t}] \cdot \varepsilon(t)$

(2) (2)

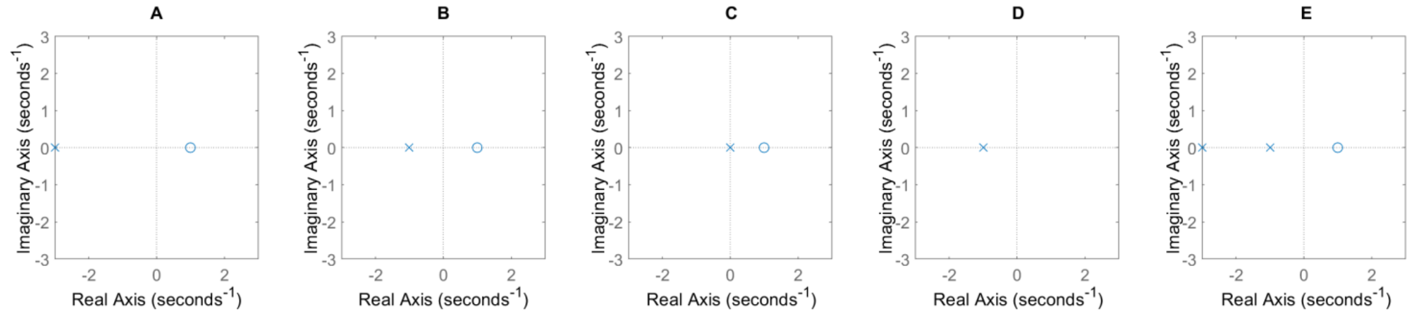
Determine the Laplace transform of the following signal x(t) (ε(t) is the unit-step function). **2**

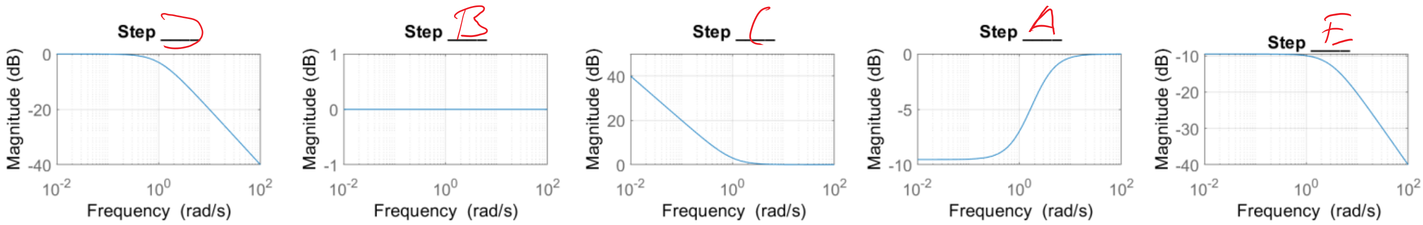
$x(t) = 4t \cdot \varepsilon(4t)$ $\qquad X(s) = \frac{4}{s^2}$ $= \frac{1}{4} \cdot \frac{1}{\left(\frac{s}{4}\right)^2} = \frac{16}{4} \cdot \frac{1}{s^2}$

Task 4

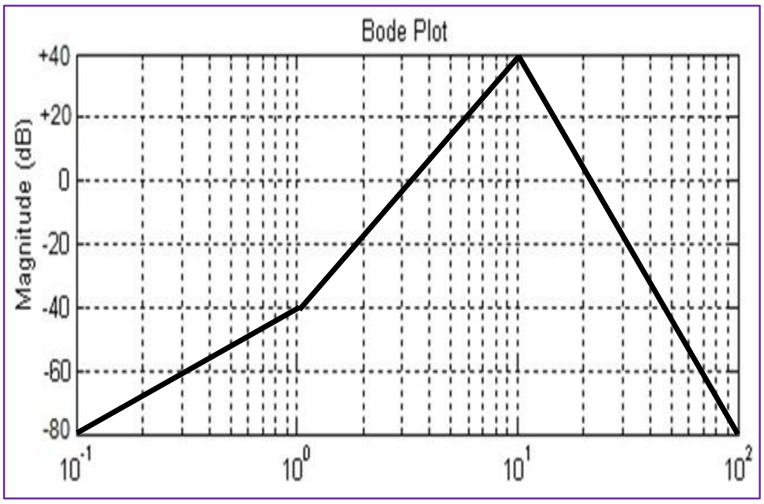
Points: 14

Above you see PN diagrams of five systems A to E, below five amplitude responses.
Find the pairs and fill in the correct letters below. 4





The amplitude response of a bandpass is given (only the asymptotes are drawn). 4



The associated transfer function is:

$$G(s) = c \cdot \frac{s^a \cdot (s+b)^m}{(s+10)^n}$$

Find the values a, b, m and n.

a = 2

b = 1

m = 2

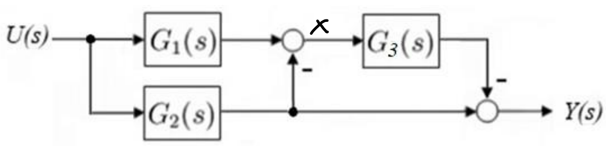
n = 10

Which undamped natural frequency, damping constant and proportional gain does the following PT2 element have: 2

$$G(s) = \frac{8}{4s^2 + 2s + 4}$$

$G(s) = \frac{2}{s^2 + 0.5s + 1}$ => $\boxed{\omega_0 = 1 \quad K = 2 \quad D = \frac{1}{4}}$

Calculate the total transfer function $G(s) = \frac{Y(s)}{U(s)}$ as function of the partial functions $G_i(s)$. 4



$Y = G_2 \cdot U - G_3 \cdot X = G_2 \cdot U - G_3 (G_1 \cdot U - G_2 \cdot U)$

$= U \left[G_2 - G_1 G_3 + G_2 G_3 \right]$

$\Rightarrow \quad G(s) = \frac{Y(s)}{U(s)} = G_2(s) - G_1(s)\, G_3(s) + G_2(s)\, G_3(s)$

Task 5

Points: 14

A time-discrete system is described by the following difference equation: (4)

$$4y[k] = 2y[k-1] - y[k-2] + 2u[k-1]$$

$4y[k] - 2y[k-1] + y[k-2] = 2u[k-1]$

Calculate the transfer function G(z) of the time-discrete system. $\circ\!\!-\!\!\bullet\ \mathcal{Z}$

z transform table?

$Y(z)\cdot[4 - 2z^{-1} + z^{-2}] = U(z)\cdot[2z^{-1}]$

$\Rightarrow G(z) = \frac{Y(z)}{U(z)} = \frac{2z^{-1}}{4 - 2z^{-1} + z^{-2}} \cdot \frac{z^2}{z^2} = \frac{2z}{4z^2 - 2z + 1}$

A time-discrete system has the following transfer function: $G(z) = \frac{z}{z-1} \cdot z^{-8}$ (2)

Calculate the impulse response g[k] in the time domain.

$G(z) \overset{\mathcal{Z}^{-1}}{\bullet\!\!-\!\!\circ}\ g[k] = \varepsilon[k-8]$

A time-discrete system is described by the following difference equation $y[k] = 0.5y[k-1] + 0.5u[k]$ and driven by a unit-jump on the input side (u[k] = ε[k]). (4)

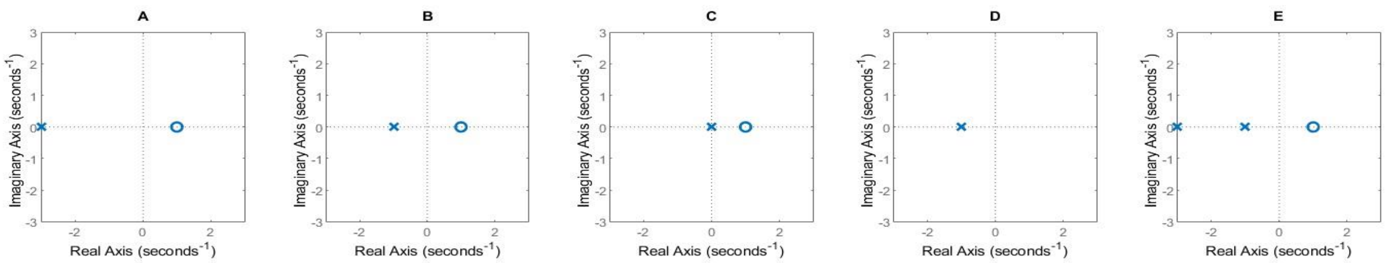
Add the missing values of the resulting output signal y[k] to the table (without major calculation):

k	-2	-1	0	1	2	3
u[k]	0	0	1	1	1	1
y[k]	0	0	0.5	0.75	0.875	0.9375

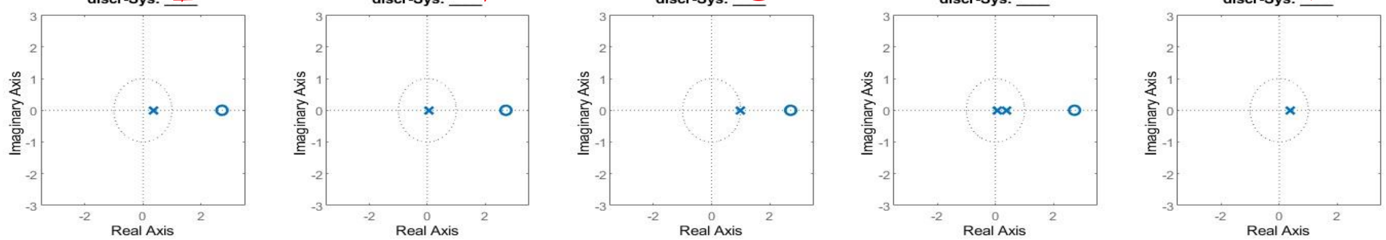
The exact (matched) transformation can be used to convert time-continuous to time-discrete systems. In this case, all the poles and zeros of the continuous-time system are simply converted into the new poles and zeros of the time-discrete system with the rule: $z = e^{sT}$ (4)

In the first row you can see the PN diagrams of five time-continuous systems A to E, in the second row the PN diagrams of five time-discrete systems (generated using the rule $z = e^{sT}$ with T = 1).

Find the pairs and fill in the correct letters below.



discr-Sys: B | discr-Sys: A | discr-Sys: C | discr-Sys: E | discr-Sys: D



Task 6 — Points: 17

A filter has the following transfer function (the parameter a is variable): $G(s) = \frac{as+4}{s+3}$ — 6

First write down the frequency response G(jω) of this filter.
Calculate the value of a at which the filter has for a frequency of ω = 4 a gain of exactly 0 dB.
(One value is enough, since two different values for a are even possible).

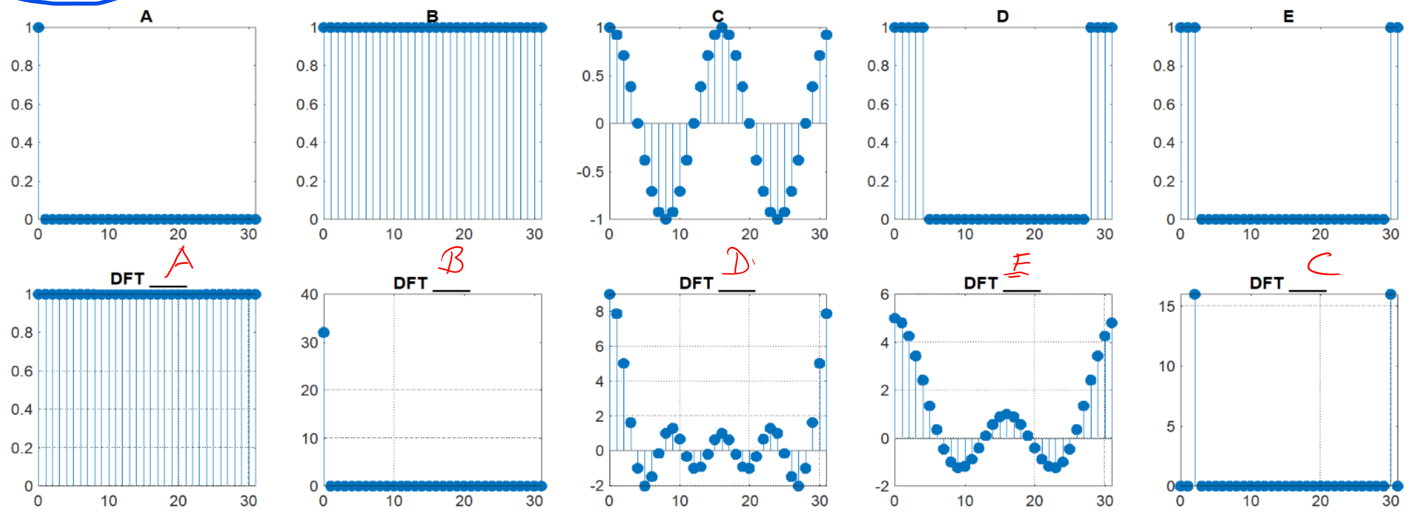
$G(j\omega) = \frac{aj\omega + 4}{j\omega + 3}$ 0

$0\,dB \stackrel{!}{=} 1$

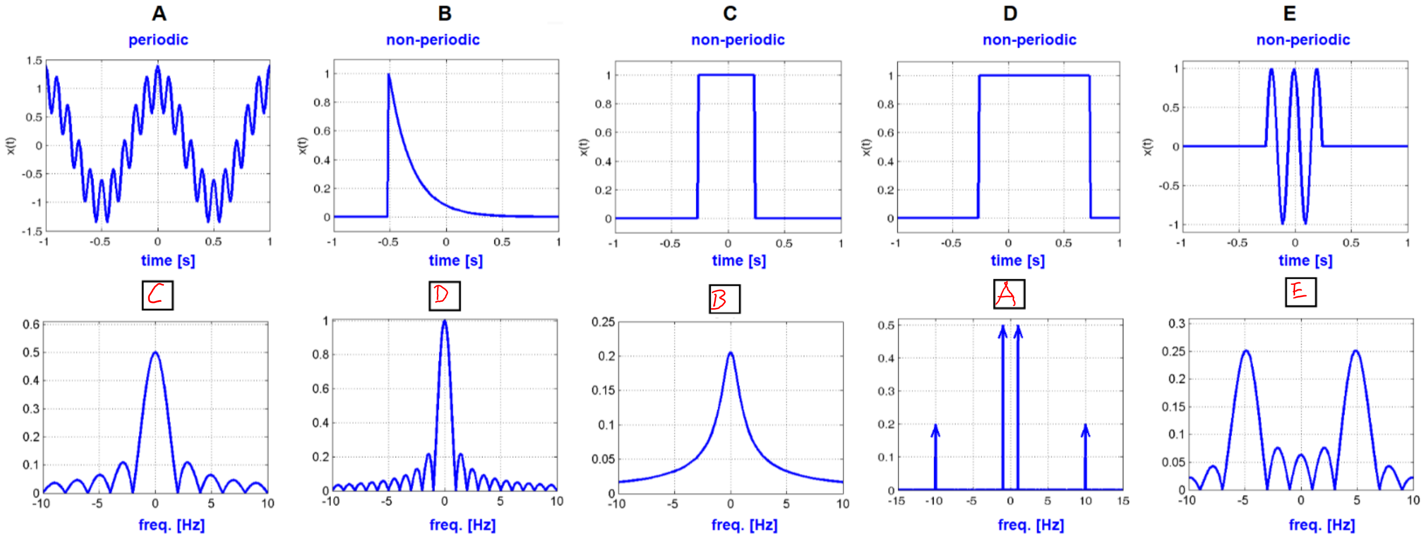
$|G(j4)| = \left|\frac{aj4 + 4}{j4+3}\right| \stackrel{!}{=} 1 \Rightarrow \frac{\sqrt{(a4)^2 + 4^2}}{\sqrt{3^2+4^2}} = 1 \Rightarrow a4 = \pm 3 \Rightarrow \boxed{a = \pm\frac{3}{4}}$

(2) (1) (1) (1)

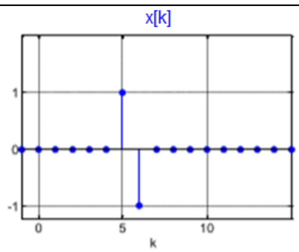
Above you see five time-discrete real signals x[k] of length N = 32, below you see five DFTs.
Find the pairs and fill in the correct letters below. — 4



Above you can see five time signals x(t), below five spectra X(jω) (only it's absolute values are shown).
Find the pairs and fill in the correct letters below. — 4



A time-discrete signal x[k] consisting of only two pulses is given.
Find the associated Discrete-Time Fourier transform X(jΩ). — 3



$x[k] = \delta[k-5] - \delta[k-6]$

$\updownarrow$ DTFT

$X(j\Omega) = e^{-j5\Omega} - e^{-j6\Omega}$