

Exercises

Correlation Functions

1. The following auto-correlation function of $x(t)$ is given:

$$\varphi_{xx}^E(\tau) = \Delta\left(\frac{\tau}{4}\right)$$

- a) Determine two possible signals $x(t)$, which have the given auto-correlation function.
- b) Determine the energy of $x(t)$.

Now, the cross-correlation function between $s(t)$ and $g(t)$ is given:

$$\varphi_{sg}^E(\tau) = \Delta(\tau + 1) - \Delta(\tau - 1)$$

- c) Sketch the cross-correlation function $\varphi_{gs}^E(\tau)$ stating all characteristic values.
- d) Are the signals $s(t)$ and $g(t)$ orthogonal to each other?
- e) Determine the Fourier-transform $\Phi_{sg}^E(\omega)$ of $\varphi_{sg}^E(\tau)$. State the real-part and the imaginary-part separately in the solution.

2. a) Which properties in symmetry have auto-correlation and cross-correlation functions?

The energy spectral density of $s(t)$ is given:

$$\Phi_{ss}^E(f) = \text{si}^2(3\pi f)$$

- b) Determine the auto-correlation function of $s(t)$.
c) Determine the energy of $s(t)$.

$s(t)$ is now transmitted via an LTI system. The LTI system has the transfer function $H(f)$. The output signal is $g(t)$.

$$H(f) = j(\varepsilon(f) - \varepsilon(-f))$$

- d) Provide a formula to calculate $\Phi_{gg}^E(f)$ from $\Phi_{hh}^E(f)$ and $\Phi_{ss}^E(f)$.
e) Verify in frequency domain, whether $s(t)$ and $g(t)$ are orthogonal to each other.

3. The two following signals are given:

$$x(n) = 2\delta(n-1) + 3\delta(n-4)$$

$$y(n) = 3\delta(n-2) + 2\delta(n-3)$$

- a) Is $x(n)$ causal, anti-causal or non-causal? (justification required)
b) Determine the auto-correlation function $\varphi_{xx}^E(m)$.
c) Determine the energy spectral density $|X(f)|^2$. Note: The sampling time is $T = 1$.
d) Determine the energy of $x(n)$.
e) Are $x(n)$ and $y(n)$ orthogonal to each other? (justification required)
f) Determine the shift-factor n_0 , so that $x(n - n_0)$ and $y(n)$ are not orthogonal.

4. a) Determine the cross-correlation function $\varphi_{sg}^E(\tau)$ for the given $s(t)$ and $g(t)$:

$$s(t) = \sin(\pi t)$$

$$g(t) = \sin(\pi(t - 42))$$

The two following signals are given:

$$x(n) = \delta(n) + \delta(n - 1) + \delta(n - 2)$$

$$y(n) = \delta(n) + 3\delta(n - 1) + \delta(n - 2)$$

- b) Determine the energy of $y(n)$.
- c) Determine the cross-correlation function $\varphi_{xy}^E(0)$.
- d) Which of the following signals could be auto-correlation functions? (justification required)

$$\varphi_{s_1 s_1}^E(\tau) = 2 \cdot \Delta(\tau) \cdot (1 - \tau^4)$$

$$\varphi_{s_2 s_2}^E(\tau) = 2 \cdot \text{rect}(\tau) \cdot (1 - \tau^3)$$

$$\varphi_{s_3 s_3}^E(\tau) = 2 \cdot \text{rect}(\tau) \cdot (1 - \tau^2)$$