List-No.:

Exam System Theory

Bachelor Robotics at THWS

WS 2022/23

Prof. Dr. R. Hirn

Duration: **90 minutes**

Tools: **only legimitate calculators and the distributed formulary**

Max. points: **90 pts.** (12 + 16 + 15 + 17 + 17 + 13)

Tasks: **6** (on 7 pages)

Last Name, First Name:	
Matriculation-No.:	

Hints:

- Write your name on each sheet!
- Do not remove any staples!
- Cheating is rated 5.0, i.e. "failed"!

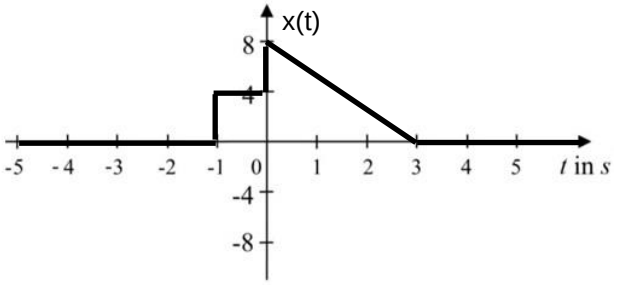
Grade:	
First examiner:	
Second examiner:	

I wish you success!

Task 1

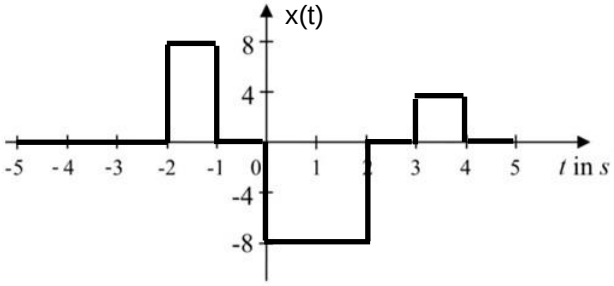
Points: 12

Write x(t) as a sum of elementary functions.



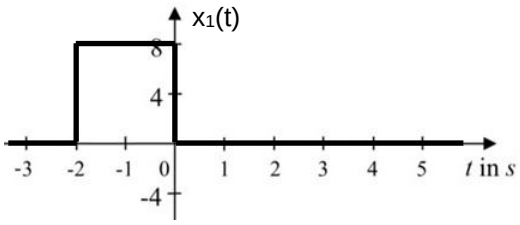
3

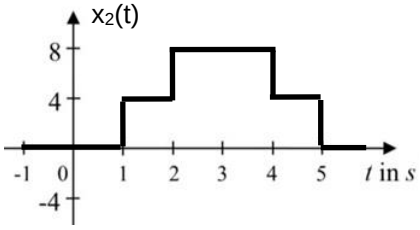
Determine the energy E_x of the signal x(t)

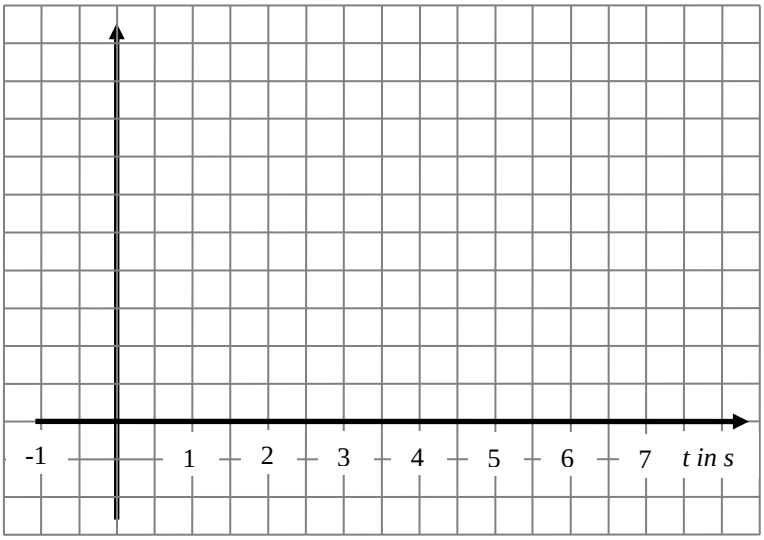


2

Sketch the convolution product of the two signals $x_1(t)$ and $x_2(t)$ (incl. correct axis labeling!).







4

Indicate with "Yes" or "No" whether the following system with input u(t) and output y(t), has the following properties (no further explainations are required).

$$y(t) = \big(u(t+2)\big)^2 + u(t)$$

Linearity:

Stability:

Causality: → if $u(t-2)$ → it is causal

3

Task 2

↗ content of the memory elements

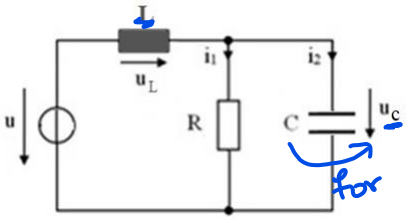
Points: 16

Required steps of modelling: 1) Determination of input variable u, state variables x_i and output variable y. 2) Determination of the coordinate systems. 3) Establishing the balance equations. 4) Isolating the derivatives of all state variables. 5) Drawing a block diagram.

The voltage source u(t) is ideal, the output shall be the voltage $u_C(t)$!
Creat a simulationsmodel following the required steps of modelling 1 to 5.

9

↗ always i



for

Model:

Determine the Laplace transform X(s) of the signal: $x(t) = (t-2) \cdot e^{-(t-2)} \cdot \varepsilon(t-2)$

3

common shift of all
so consider t, then make the shift

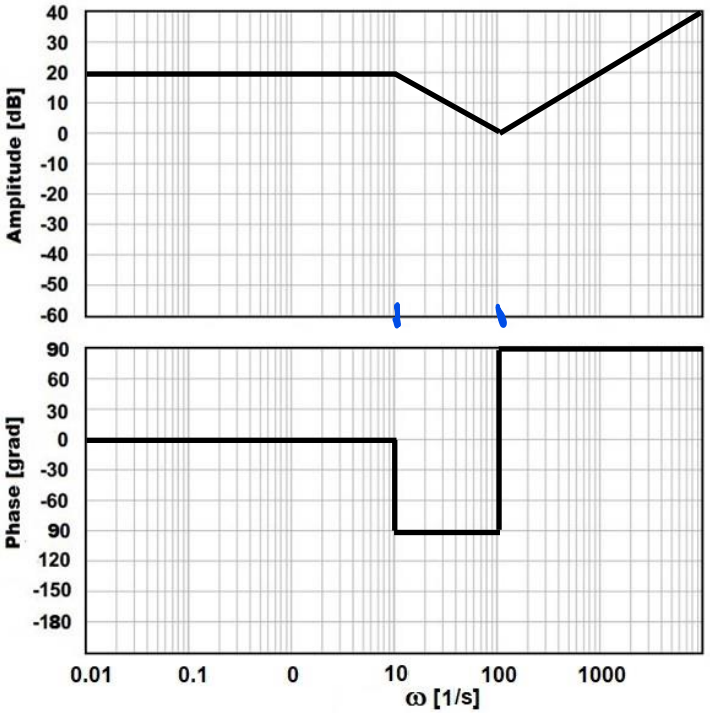
Solve this inital value problem by the tool Laplace tansform.

4

$$\ddot{x}(t) + x(t) = \delta(t) \qquad mit \qquad x(\mathbf{0}_-) = 3 \quad \text{und} \quad \dot{x}(\mathbf{0}_-) = 1$$

Task 3 **Points: 15**

A **non-causal** system has the shown Bode plot (only its asympotes are shown).
How many poles and zeros does this system have?
Find the transfer function G(s) which describes this system.



9

(+20dB) 2 zeroes = 100
falling = 10
1 poles → meaning poles
(-20dB)

zero - pulls up
poles - pulls down

if 1 zero, const. after 100
(non causal)

no. of poles ≠ no. of zeroes
$20\log(x) = A$

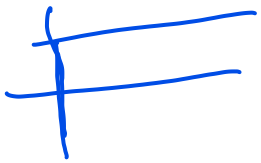
$G(s) = \frac{\text{zeroes}}{\text{poles}}$

Give the transfer function of an all-pass filter with a pole at s = -2 and a pole at s = -3 and sketch its amplitude response.

2

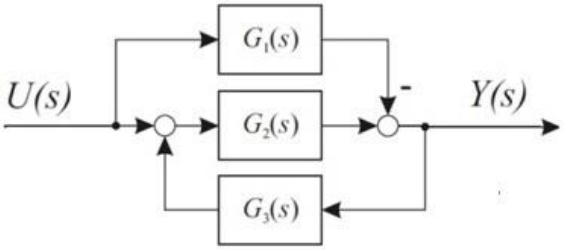
$G(s) = \frac{s-2}{s+2} \quad \frac{s-3}{s+3}$

eg.



Determine the transfer function $G(s) = \frac{Y(s)}{U(s)}$ of this system.

4



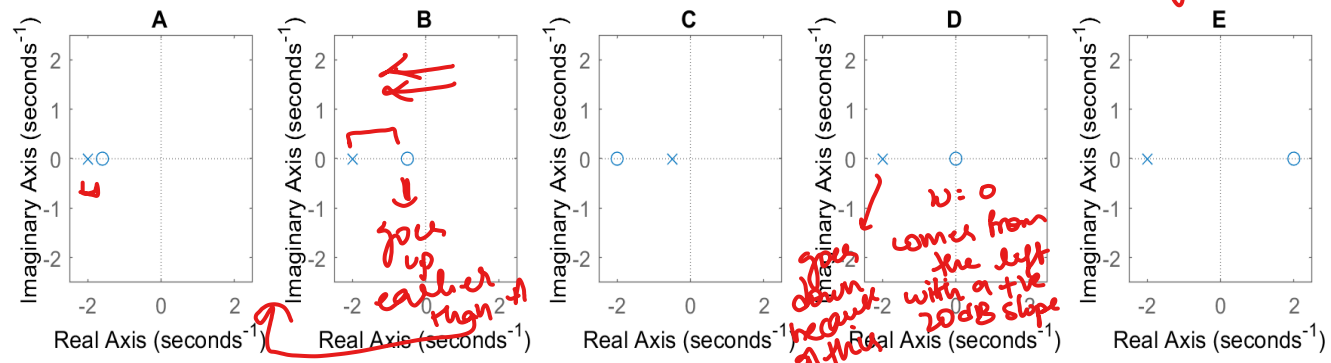
Task 4 **Points: 17**

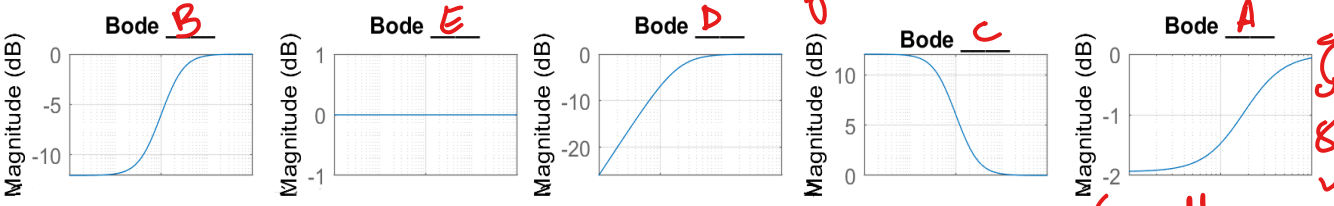
left to right

x poles o zeros

Above you see PZ-diagrams of five systems A to E, below five amplitude responses. Find the pairs and fill in the correct letters below. 4

symmetric





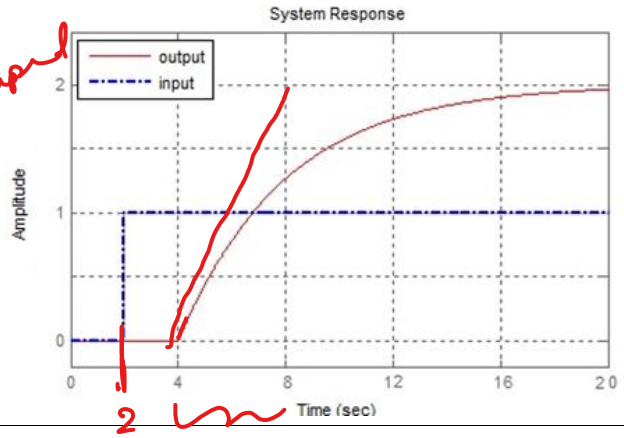
12 dB

An unknown system was tested with a unit-jump. Give a transfer function, which describes this system. (Pay attention also on the dead time!). 4

PT1 'S' shaped

$G(s) = \frac{2\,e^{-2.5s}}{4s+1}$

$P = \frac{A}{s}$



Determine the Fourier transform of the following signal: $x(t) = \frac{3}{t^2}$ 4

Hint: it applies: $\frac{1}{t^2} = -\frac{d}{dt}\left(\frac{1}{t}\right)$

A system with the frequency response $G(j\omega) = \frac{Y(j\omega)}{U(j\omega)} = \frac{j\omega}{j\omega+1}$ is stimulated with the following signal $u(t) = 5\sin(5t)$. Calculate and determine the expected steady-state output signal y(t). 5

$A = 5\,\frac{5}{\sqrt{26}} = \left|\frac{5j}{j5+1}\right|$

$A\sin(5t+\phi)$

Task 5 **Points: 17**

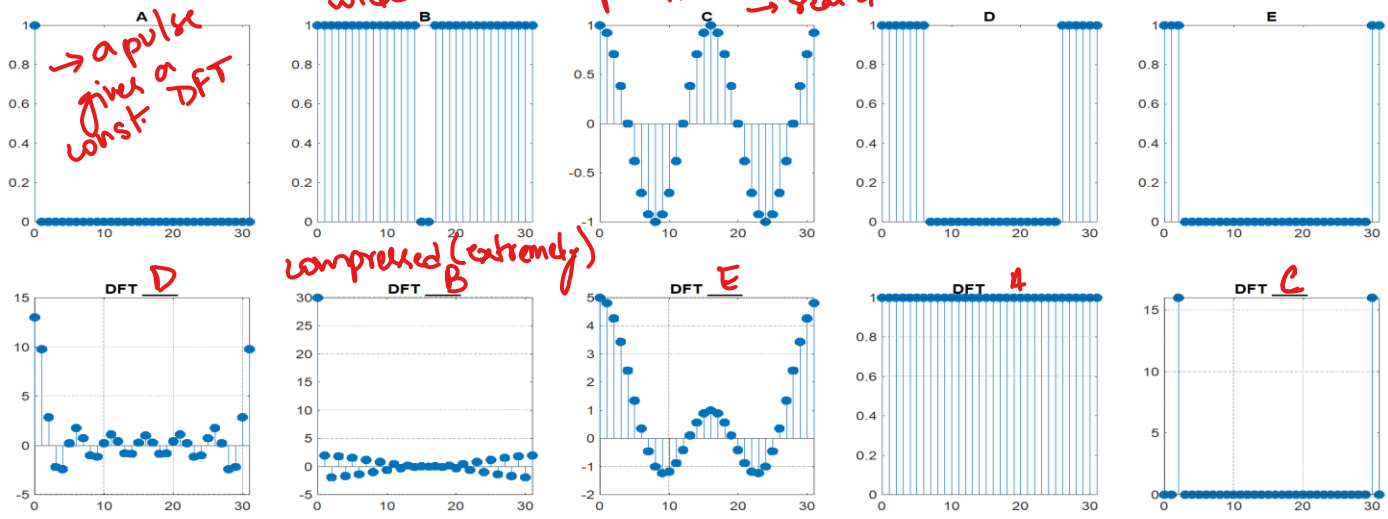
An unstable PT_1-System besitzt has the following transfer function: $G(s) = \frac{1}{s-1}$. 8

Using the jump invariant transformation, calculate the corresponding z-transfer function $G_{HS}(z)$ for a sampling time T = 1s.

Duality → wide in one, small in the other
↳ vice versa

BDE → even rect. signals
↳ $\frac{\sin(x)}{x}$ spectrum → not purely but looks like it

Above you see five time discrete signals x[k] with length N = 32 labeled as A to E, below five DFTs. 4
Find the pairs and fill in the correct letters below.



→ a pulse gives a const. DFT
wide
→ cos ⇒ FFT 2 direc pulse → real & even
compressed (extremely)
DFT D
DFT B
DFT E
DFT A
DFT C

Given is the following z-transfer function G(z) of time-discrete system: $G(z) = \frac{Y(z)}{U(z)} = \frac{2z^2+4}{z^2-2}$ 5

Is this system stable? NO

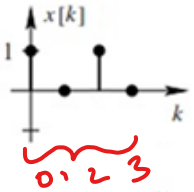
Give the corresponding difference equation to calculate the output y[k].
(i.e. y[k] should be explicitly calculable, i.e. y[k] shall be be alone on the left side of your equation!)

if on in the circle, stable
Here $\pm\sqrt{2} > 1$ instable.

Task 6 **Points: 13**

A time-discrete systems has the following z-transfer function $G(z) = \frac{z+1}{z-0.5}$.
Give the frequency response of this system. 2

Above you see a signal x[k], below the definition of the DFT.
Determine the second value of the corresponding DFT for the length N = 4.
Hint: requested is X[1], i.e. n = 1. 3



0, 1, 2, 3

$$X[n] = DFT\{x[k]\} = \sum_{k=0}^{N-1} x[k] \cdot e^{-j\left(2\pi \frac{n}{N} k\right)}$$

What is the long form of the abbreviation FIR? Give an arbitrary transfer function that has FIR behavior. 2

spectrum of discrete is periodic

Complete the following sentence with at least one correct mathematical statement: "The spectrum of a 2

non-periodic, analog and odd time signal is _____ _____"

non-periodic ↓ continuous — imaginary & odd

The steps 1-4 of the modelling of an electric system resulted in the following equations: 4

1) Input: $u = u_E$, Output: $y = u_C$, State variables: $x_1 = u_C$ and $x_2 = i_L$

4) $\dot{x} = f(x, u, p)$: $\dot{x}_1 = \frac{du_C}{dt} = \frac{1}{RC}(u_E - u_C)$ $\dot{x}_2 = \frac{di_L}{dt} = \frac{u_E}{L}$

Give the state space representation (matrix-form) of this systems, i.e.: $\dot{x} = Ax + Bu$, $y = Cx + Du$

The state space vector is predefined and shall be: $x = \begin{bmatrix} u_C \\ i_L \end{bmatrix}$