# Faculty of Electrical Engineering

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## Course "Control Systems 2"

Solution to Ex. Sheet 14

#### Task 28

#### Solution:

a) In task 25b) the following continuous-time Luenberger observer was found:

$$\frac{\dot{\hat{x}}}{\hat{x}} = \begin{bmatrix} 1 & -4 \\ 2 & -3 \end{bmatrix} \hat{x} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 8.5 \\ 6 \end{bmatrix} (y - \hat{y})$$

$$\hat{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{x}$$

Approximate discrete-time state equations with respect to the sampling time T can be derived using the formula

$$\hat{\underline{x}}[k+1] = (T\underline{A} + \underline{I})\hat{\underline{x}}[k] + T\underline{b}u[k] + T\underline{l}(y[k] - \hat{y}[k])$$

$$\hat{y}[k] = \underline{c}^T\hat{\underline{x}}[k]$$

Here:

$$\hat{\underline{x}}[k+1] = \begin{bmatrix} 1+T & -4T \\ 2T & 1-3T \end{bmatrix} \hat{\underline{x}}[k] + \begin{bmatrix} T \\ T \end{bmatrix} u[k] + \begin{bmatrix} 8.5T \\ 6T \end{bmatrix} (y[k] - \hat{y}[k])$$

$$\hat{y}[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{\underline{x}}[k]$$

Block diagram → see next page

b) If the sample time T is chosen according to the conservative design rule

$$|\lambda_i| \lesssim \frac{1}{10T}$$

where  $\lambda_i$  denotes all dominant open- and closed-loop eigenvalues, then the performance of the control-loop with digital state-space controller will almost certainly be very similar to the designed continuous-time dynamics.

### Here:

• Open-loop eigenvalues = eigenvalues  $\lambda_1$  and  $\lambda_2$  of the plant:

$$\det(\lambda \underline{I} - \underline{A}) = \det\begin{pmatrix}\begin{bmatrix}\lambda - 1 & 4 \\ -2 & \lambda + 3\end{bmatrix}\end{pmatrix} = \lambda^2 + 2\lambda + 5 \Rightarrow \lambda_{1/2} = -1 \pm 2j \Rightarrow |\lambda_{1/2}| = \sqrt{5}$$

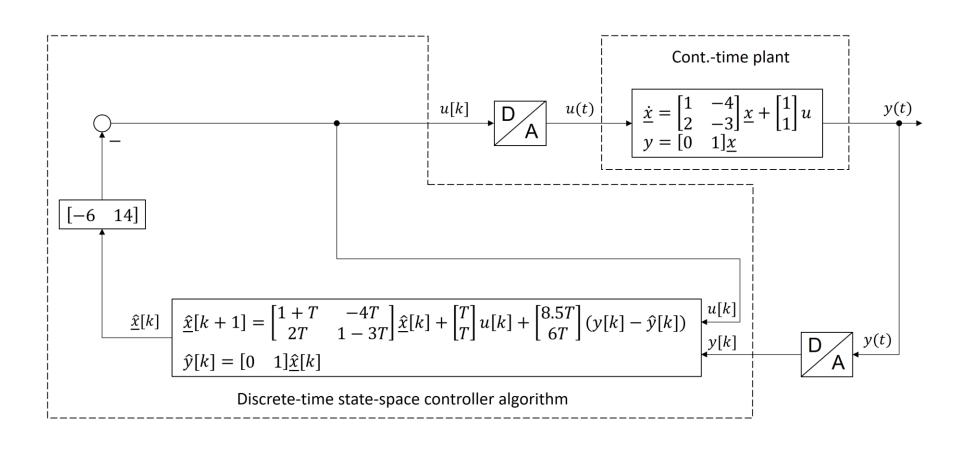
 Close-loop eigenvalues = eigenvalues of closed-loop with state feedback and observer eigenvalues

$$\Rightarrow$$
  $\lambda_{C,1} = \lambda_{C,2} = -5$  (see Task 22b) on Exercise Sheet 9)  $\lambda_{0,1} = \lambda_{0,2} = -4$  (see Task 25a) on Exercise Sheet 11)

→ Choose T such that  $5\frac{1}{\sec} \lesssim \frac{1}{10T} \Rightarrow T \lesssim \frac{1}{50} \sec \Rightarrow T = 0.02 \sec$ 

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a) Block diagram:



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