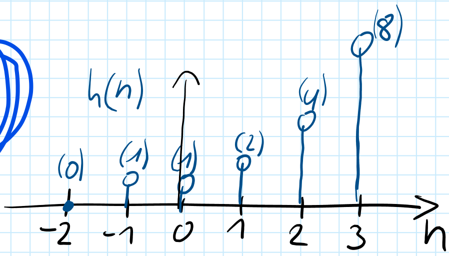
1) a)



$$h(n) = \delta(n+1) + \sum_{k=0}^{\infty} 2^k \delta(n-k)$$

$$\delta(n-k) \begin{cases} 1 & \text{für } n=k \\ 0 & \text{sonst} \end{cases}$$

$$\delta(n+1) \begin{cases} 1 & \text{für } n=-1 \\ 0 & \text{sons} \end{cases}$$

b) stabil wenn: $\sum_{n=-\infty}^{\infty} |h(n)| < \infty \Rightarrow |a| < 1$

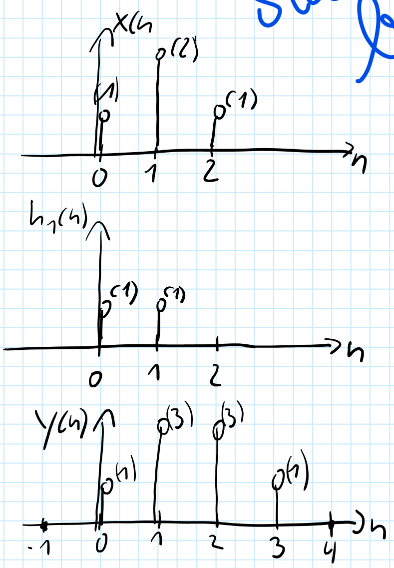
stable if: Amp. of signal $< \infty$

c) antikausal für $a=0$, weil $h(n) = \delta(n+1)$ antikausal

anticausal for, because anticausal

starts before $x=0$

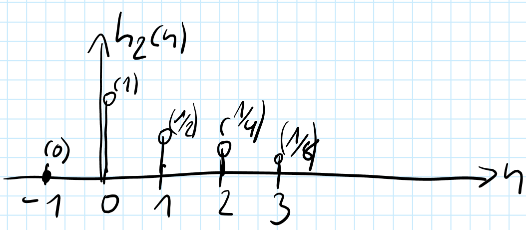
2) a)



Normal convolution

$$y(n) = \delta(n) + 3\delta(n-1) + 3\delta(n-2) + \delta(n-3)$$

b)



$$\sum_{k=0}^{\infty} (0{,}5)^k \delta(n-k)$$

$$= a^n \cdot \varepsilon(n)$$

c) $H_2(f) = \sum_{n=-\infty}^{\infty} h_2(n) \cdot e^{-j2\pi n f T}$

$$= \sum_{n=0}^{\infty} 0{,}5^n \cdot e^{-j2\pi n f} = \sum_{n=0}^{\infty} \Big(\overbrace{0{,}5 \cdot e^{-j2\pi f}}^{a}\Big)^n \quad c = 1$$

$$\boxed{\sum_{n=0}^{\infty} c \cdot a^n = \frac{c}{1-a}, \text{ für } |a| < 1}$$

$$= \frac{1}{1 - 0{,}5 e^{-j2\pi f}}$$

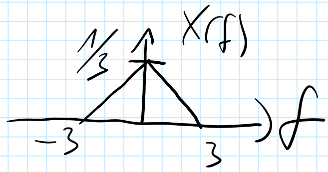
a/d

$$1-0{,}5e^{-j2\pi f}$$

alt.

$$h_2(n)=\sum_{k=0}^{\infty}0{,}5^k\,\delta(n-k)=0{,}5^n\cdot\varepsilon(n)\;\circ\!\!-\!\!\bullet\;\frac{1}{1-0{,}5e^{-j2\pi f}}=H_2(f)\qquad(\text{Tab. }18)$$

d)



$$\Rightarrow T=1/6$$

da $\frac{1}{T}\geq 2\cdot 3$ Abtasttheorem

sample theor.

e)

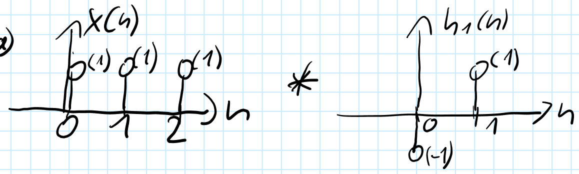
$$H(f)=a\cdot\mathrm{rect}(f/6)$$

$$X_a(f)=\frac{1}{T}\sum_{k=-\infty}^{\infty}X\left(f-\frac{k}{T}\right)$$

$$a\cdot\frac{1}{T}\overset{!}{=}1$$

$$\Rightarrow \underline{\underline{a=T=1/6}}$$

3) a)

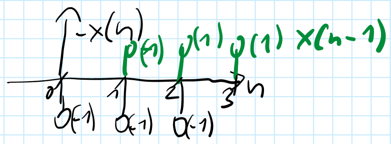


$$y(n)=x(n)*h_1(n)$$

$$=x(n)*[-\delta(n)+\delta(n-1)]$$

$$=-x(n)+x(n-1)$$

$$\underline{\underline{y(n)=-\delta(n)+\delta(n-3)}}$$



b)

$$h_1(n)\;\circ\!\!-\!\!\bullet\;H_1(f)$$

$$H_1(f)=\frac{1}{T}\sum_{n=-\infty}^{\infty}h_1(n)\,e^{-j2\pi nfT}\qquad|T=1$$

$$=\sum_{n=-\infty}^{\infty}h_1(n)\,e^{-j2\pi nf}$$

$$=\sum_{n=-\infty}^{\infty}[-\delta(n)+\delta(n-1)]\,e^{-j2\pi nf}$$

$$=-e^{-j2\pi f\cdot 0}+e^{-j2\pi f\cdot 1}=\underline{\underline{-1+e^{-j2\pi f}}}$$

alt.:

$$\left[\begin{array}{l}\delta(n)\;\circ\!\!-\!\!\bullet\;1\\ x(t-t_0)\;\circ\!\!-\!\!\bullet\;X(f)\cdot e^{-j2\pi f t_0}\end{array}\right]$$

$$H_1(f)=e^{-j2\pi f}-1=\cos(2\pi f)-1-j\sin(2\pi f)$$

$$H_1(f) = e^{-j2\pi f} \cdot 1 = \cos(2\pi f) - 1 \; - j\sin(2\pi f)$$

$$\mathrm{Re}\{H_1(f)\} = \cos(2\pi f) - 1$$

$$\mathrm{Im}\{H_1(f)\} = -\sin(2\pi f)$$

c) $b = 0$

a beliebig, da alle Impulse bei $n<0$ in diesem Fall verschwinden

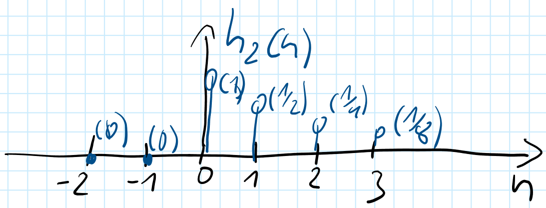
a arbitrary, because all impulses at $n<0$ disappear in this case

d) b: beliebig arbitrary

$|a|<1$ in diesem Fall konvergiert die geom. Reihe

in this case the geom. series converges

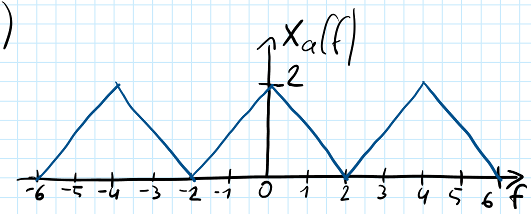
e)



f) $h_2(n) = \sum_{k=0}^{\infty} 0{,}5^k \cdot \delta(n-k) = 0{,}5^n \cdot \varepsilon(n)$ (Tab. 18)

$$0{,}5^n \varepsilon(n) \circ\!\!-\!\!\bullet \frac{1}{1 - 0{,}5\, e^{-j2\pi f}} = H_2(f)$$

4) a)



$$X_a(f) = X(f) * \tfrac{1}{4} \cdot \sum_{k=-\infty}^{\infty} \delta\left(f - \tfrac{k}{T}\right)$$

b) $H(f) = \frac{1}{4} \cdot \mathrm{rect}\left(\frac{f}{4}\right) \bullet\!\!-\!\!\circ \; \underline{si(4\pi t) = h(t)}$

b) $H(f) = \frac{1}{4} \cdot \text{rect}(f/4)$ •—○ $\underline{\underline{si(4\pi t) = h(t)}}$

c) $\underline{\underline{s(0)}} = b^0 \cdot \delta(0) \underline{\underline{= 1}}$

d) $a = 0$

$b =$ beliebig, arbitrary

weil bei $a=0$ kein Impuls von $n<0$

becaus with $a=0$ no impulse bevore $n<0$

e) $s(n) = \sum_{k=0}^{\infty} \left(\frac{3}{10}\right)^k \delta(n-k)$

$= \left(\frac{3}{10}\right)^k \cdot \varepsilon(n)$ ○—• $\frac{1}{1 - \frac{3}{10} \cdot e^{-j2\pi f}}$

5 a) min. Periode : $\frac{1}{T_0} = \frac{2}{3}$

min. period :

nächste Periode: $\frac{1}{T_1} = \frac{4}{3} \Rightarrow T_1 = \frac{3}{4}$

next period: $\frac{1}{T_2} = \frac{6}{3} \Rightarrow T_2 = \frac{1}{2}$

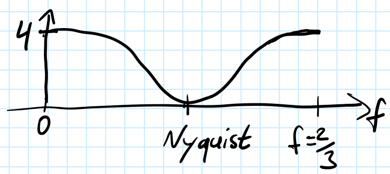
b) $|H(f)| = 2\cos(3\pi f) + 2$

$\varphi(f) = 0$

c) ja, weil $\varphi(f) = c \cdot f$ hier: $c=0$

yes, because $\varphi(f) = c \cdot f$ here: $c=0$

d) $H(f) = 2\cos(3\pi f) + 2$



Tiefpass, da Maximum bei $f=0$

und Minimum bei $f = \frac{1}{2T_0}$

Low-Pass, because maximum at $f=0$

and minimum at $f = \frac{1}{2T_0}$

e) $H(f) = 2\cos(3\pi f) + 2$

$\delta(n+n_0) + \delta(n-n_0)$ ○—• $2\cos(2\pi f n_0 T)$

$\Rightarrow 2 n_0 T_0 = 3 \Rightarrow n_0 = \frac{3}{2T_0} = \frac{3}{2} \cdot \frac{2}{3} = 1$

$\Rightarrow h(n) = 2\delta(n) + \delta(n-1) + \delta(n+1)$

f) nein, da $h(n) \neq 0$ für $n < 0$

$\Rightarrow h(n) = 2\delta(n) + \delta(n-1) + \delta(n+1)$

f) nein, da $h(n) \neq 0$ für $n < 0$

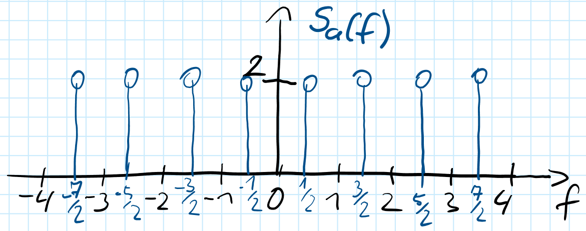
no, because $h(n) \neq 0$ for $n < 0$

⑥ a) $3\pi t = 2\pi \frac{3}{2} t \Rightarrow \underline{\underline{T_0 = \frac{2}{3}}}$

b) $\int_{-\infty}^{\infty} 2\cos(3\pi t)\,dt \to \infty$

$\Rightarrow$ Leistungssignal, power signal

c)



$s(t) \circ\!\!-\!\!\bullet\, S(f)$

$s(t) = 2\cos(3\pi t) \circ\!\!-\!\!\bullet\, \delta(f+\frac{3}{2}) + \delta(f-\frac{3}{2}) = S(f)$

$$S_a(f) = S(f) * \frac{1}{0{,}5} \sum_{k=-\infty}^{\infty} \delta(f + \frac{k}{0{,}5})$$

$$= S(f) * 2 \sum_{k=-\infty}^{\infty} \delta(f + 2k)$$

d) Ja, da die Signalfrequenz $(\frac{3}{2}) > \frac{1}{2}$ Abtastfrequenz (1)

$\Rightarrow$ Abtasttheorem $2f_g \leq \frac{1}{T}$ nicht erfüllt

Yes, because signal frequency $(\frac{3}{2}) > \frac{1}{2}$ sampling frequency (1)

$\Rightarrow$ sampling theorem $2f_c \leq \frac{1}{T}$ not fulfilled

e) $H(f) \bullet\!\!-\!\!\circ\, h(t)$

$\mathrm{rect}(\frac{f}{4}) \bullet\!\!-\!\!\circ\, 4\,\mathrm{si}(4\pi t)$

$\mathrm{rect}(f) \bullet\!\!-\!\!\circ\, \mathrm{si}(\pi t)$

$\frac{1}{b}\mathrm{rect}(\frac{f}{b}) \bullet\!\!-\!\!\circ\, \mathrm{si}(\pi b t)$

f) Signalausschnitt durch $H(f)$: $H(f) \cdot S_a(f)$

f) Signalausschnitt durch $H(f)$: $\quad H(f) \cdot S_a(f)$

signal cutout due to $H(f)$:

$$2\left[\delta(f+\tfrac{1}{2}) + \delta(f-\tfrac{1}{2})\right] + 2\left[\delta(f+\tfrac{3}{2}) + \delta(f-\tfrac{3}{2})\right]$$

$$\bullet\!\!-\!\!\circ \quad 4\cos(2\pi \tfrac{1}{2} t) + 4\cos(2\pi \tfrac{3}{2} t)$$

$$\Rightarrow g(t) = 4\left[\cos(\pi t) + \cos(3\pi t)\right]$$