Exercise Image Processing
Sample Solution

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Sheet 5

In this exercise we cover the chapters *convolution*, *correlation*, and *linear and nonlinear filter design*. The questions are small-part and can be seen as examples of potential exam problems. Also use the formulary for the exam to work through the problems.

Task 5.1: Convolution and Correlation

5.1a)

What condition must a 2D convolution mask satisfy to be separable?

Answer: It must correspond to the multiplication of a column vector with a row vector. (This is also called dyadic product).

5.1b)

Specify how many arithmetic operations (specify additions/subtractions and multiplications/divisions separately) can be saved with a separable convolution compared to a non-separable convolution depending on the mask size ($K \times L$).

Answer: Computational complexity for a non-separable convolution: $K \cdot L$ mutliplies/divisions, $K \cdot L - 1$ additions/subtractions.
Computational cost for a separable convolution: $K+L$ mutliplications/divisions, $K+L-2$ additions/subtractionsTherefore, one saves $K \cdot L - K - L$ mutliplications/divisions and $K \cdot L - K - L + 1$ additions/subtractions.

5.1c)

What is the difference between convolution and correlation?

Answer: In convolution the convolution kernel $h_{-m',-n'}$ is mirrored and in correlation the convolution kernel $h_{m',n'}$ is not mirrored.

5.1d)

What condition must a convolution mask satisfy for the result to correspond to a correlation?

Answer: The convolution mask must be point symmetric: $h_{-m',-n'} = h_{m',n'}$.

5.1e)

Name two different ways of border handling when convolving an image?

Answer: extrapolate or fill with zeros

Task 5.2: Linear Filters

5.2a)

What properties should an ideal smoothing filter have?

Answer: An ideal smoothing filter should

- be shift-free,
- preserve the mean value,
- have a monotonically decreasing and
- have isotropic transfer function.

5.2b)

Name three different smoothing filters and one property common to all of these filter masks?

Answer: Box-filter, Binomial-filter, Gaussian-filter. All these filters are linear ans separable.

5.2c)

Specify the result of the box filter for the following image area:

6	4	0
9	1	7
3	5	2

Answer: The solution is: $37/9 \approx 4,11$.

5.2d)

How many summations and how many multiplications are saved for an r-times convolution with a 1×2 rectangular filter compared to a convolution with a 1D binomial filter of order r per pixel?

Answer: For an r-times convolution with a 1×2 rectangular filter one needs r additions and 1 division. For a 1D convolution with a 1D binomial filter of order r one needs r additions and $r + 1$ multiplications. So one saves r multiplications and no addition.

5.2e)

What properties should an ideal difference filter have?

Answer: An ideal derivative filter should

- be shift-free,
- suppress the mean value and
- have an isotropic transfer function.

5.2f)

What filter results when the following two filter masks are **convolved**:

$$\begin{array}{|c|c|c|} \hline -1 & 1 & 0 \\ \hline \end{array} \quad * \quad \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|c|c|} \hline -1 & 1 & 1 & -1 & 0 \\ \hline \end{array}$$

5.2g)

Which 2D filter mask results, for the following separable filter:

$$\begin{array}{|c|} \hline 1 \\ \hline -1 \\ \hline 1 \\ \hline \end{array} \quad \cdot \quad \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline -1 & -2 & -1 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

5.2h)

In a 2D regularized derivative, does it make a difference whether you first average along one direction and then differentiate along the orthogonal direction, or reverse the order? Justify your answer.

5.2i)

Various filters are given in the table. Assign the order of the derivatives involved.

	symmetric difference	Laplace-Filter	Sobel-Filter	LoG-Filter	DoG-Filter
1st order derivative	×		×		
2nd order derivative		×		×	×

Task 5.3: Nonlinear Filters

5.3a)

Which rank order filter applied to a binary image corresponds to which set operation.

	Minimum-Filter	Median-Filter	Maximum-Filter
Dilation			×
Erosion	×		

5.3b)

Explain the morphological operator: opening.

Answer: Opening first performs erosion followed by dilation. This eliminates elements in a binary image that are smaller than the filter mask and separates objects that touch at points smaller than the filter mask.

5.3c)

What feature can you extract in an image when you subtract the result of an erosion with a 3×3 mask from a dilation with a 3×3 mask?

Answer: The difference between the minimum and maximum value of the filter mask is calculated. This has a comparable effect to the calculation of the gradient strength.

5.3d)

Specify the result of the median filter for the following image area:

9	13	34	39	16
43	31	52	17	16
6	4	0	3	2
9	1	7	8	5
3	5	2	12	11

Answer: The result of the median filter is: 9.