

$$1) \quad h(n) = [1 \quad 2 \quad 1]$$

$$H(z) = 1 + 2z^{-1} + z^{-2} \quad z = e^{j2\pi fT}$$

$$T = \frac{1}{r} = 1$$

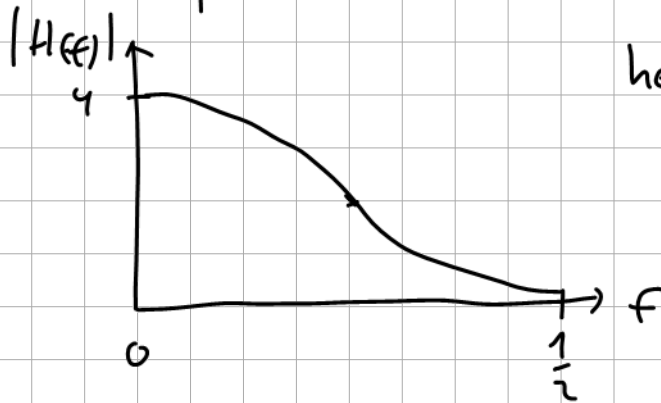
$$\begin{aligned} H(f) &= 1 + 2e^{-j2\pi f} + e^{-j4\pi f} \\ &= e^{-j2\pi f} (e^{j2\pi f} + 2 + e^{-j2\pi f}) \end{aligned}$$

$$|H(f)| = |e^{j2\pi f} + 2 + e^{-j2\pi f}|$$

$$= |\cos(2\pi f) + j\sin(2\pi f) + 2 + \cos(2\pi f) - j\sin(2\pi f)|$$

$$= |2 + 2\cos(2\pi f)| \quad 0 \leq f \leq \frac{r}{2} \quad \text{Sampling-theorem}$$

$$\text{here : } 0 \leq f \leq \frac{1}{2}$$



$\Rightarrow h(n)$ is a lowpass.

2) Converting a Lowpass to a Highpass:

$$h_{HP}(n) = h_{LP}(n) \cdot (-1)^n$$

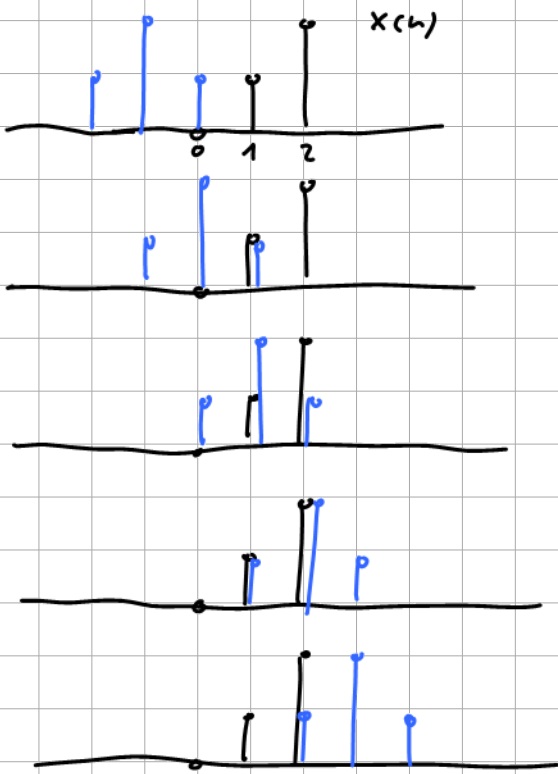
$$h_{LP}(n) = 1 \quad 2 \quad 1$$

$$h_{HP}(n) = 1 \quad -2 \quad 1$$

$$3) \quad x(n) = (0 \quad 1 \quad 2)$$

$$h(n) = (1 \quad 2 \quad 1)$$

graphical method:



$$\Rightarrow y(0) = 1 \cdot 0 = 0$$

$$\Rightarrow y(1) = 1 \cdot 1 + 2 \cdot 0 = 1$$

$$\Rightarrow y(2) = 1 \cdot 2 + 2 \cdot 1 = 4$$

$$\Rightarrow y(3) = 1 \cdot 1 + 2 \cdot 2 = 5$$

$$\Rightarrow y(4) = 1 \cdot 2 = 2$$

$$y(n) = 0 \quad \text{everywhere else!}$$

$$4) \quad h_{LP}(n) = \frac{\sin\left(2\pi f_c \frac{n}{r}\right)}{2\pi f_c \frac{n}{r}} \cdot \frac{1}{2} \left(1 + \cos\left(2\pi \frac{n}{N}\right)\right)$$

$$\text{with: } -2 \leq n \leq 2$$

$$\frac{f_c}{r} = \frac{1}{4}$$

$$h_{LP}(n) = \frac{\sin\left(\frac{\pi}{2} n\right)}{\frac{\pi}{2} n} \cdot \frac{1}{2} \left(1 + \cos\left(2\pi \frac{n}{5}\right)\right)$$

$$h_{LP}(-2) = h_{LP}(2) = 0$$

$$h_{LP}(-1) = h_{LP}(1) = 0,417$$

$$h_{LP}(0) = 1$$

5)

$$y(n) = x(n) * h(n)$$
$$= \sum_{l=0}^{K-1} x(n-l) \cdot h(l) \quad K = 501$$

\Rightarrow K Multiplications

$K-1$ Additions

$$b) \text{ minimize } \sum_n (a \cdot y(n-T) - z(n))^2 = f(a)$$

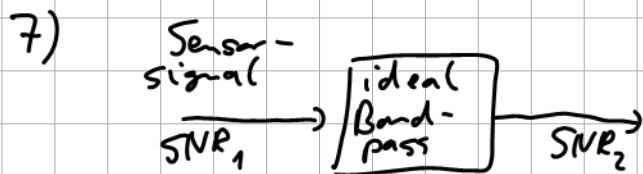
$$\text{first derivative: } \sum_n 2(a y(n-T) - z(n)) \cdot y(n-T) = \frac{df(a)}{da}$$

$$\text{setting to 0: } \sum_n 2(a y(n-T) - z(n)) \cdot y(n-T) = 0$$

$$\sum_n a y^2(n-T) - z(n) y(n-T) = 0$$

$$\sum_n a y^2(n-T) = \sum_n z(n) y(n-T)$$

$$a = \frac{\sum_n z(n) y(n-T)}{\sum_n y^2(n-T)}$$



$$SNR_1 = 10 \log_{10} \frac{P_x}{P_{R_1}}$$

$$SNR_2 = 10 \log_{10} \frac{P_x}{P_{R_2}}$$

increase in SNR: $SNR_2 - SNR_1 = 10 \left[\log_{10} \frac{P_x}{P_{R_2}} - \log_{10} \frac{P_x}{P_{R_1}} \right]$

$$= 10 \cdot \left[\log_{10} P_x - \log_{10} P_{R_2} - (\log_{10} P_x - \log_{10} P_{R_1}) \right]$$

$$= 10 \cdot \log_{10} \frac{P_{R_1}}{P_{R_2}}$$

power of a signal is the integral over the power density function:

$$P_{R_1} = \int_{0 \text{ Hz}}^{24000 \text{ Hz}} |R(f)|^2 df = \int_{0 \text{ Hz}}^{24000 \text{ Hz}} c \cdot df \quad (|R(f)|^2 \text{ is assumed to be constant})$$

$$= c \cdot 24000$$

$$P_{R_2} = \int_{100 \text{ Hz}}^{7000 \text{ Hz}} |R(f)|^2 df = 6900 \cdot c$$

$$SNR \text{ gain (increase)} = 10 \log_{10} \frac{24000c}{6900c} = 10 \log_{10} \frac{240}{69} = 5,4 \text{ dB}$$

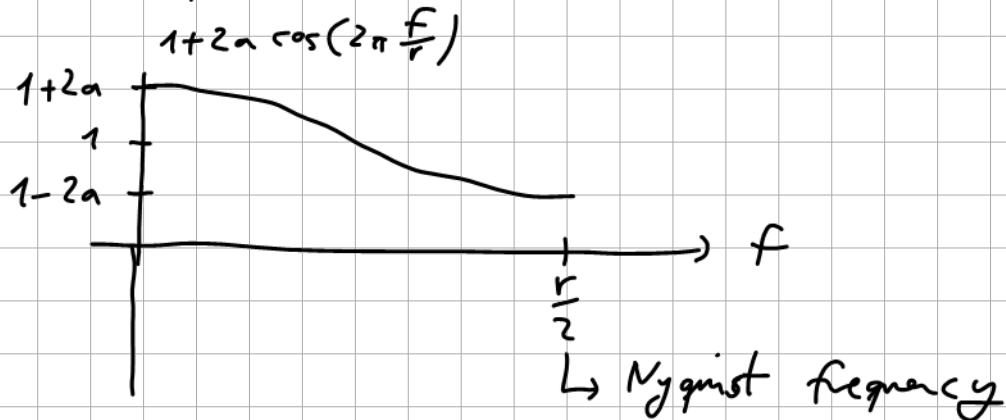
$$8) \quad h(n) = [a \quad 1 \quad a]$$

$$H(f) = a + e^{-j2\pi \frac{f}{r}} + a e^{-j4\pi \frac{f}{r}}$$

$$= e^{-j2\pi \frac{f}{r}} \left(a e^{+j2\pi \frac{f}{r}} + 1 + a e^{-j2\pi \frac{f}{r}} \right)$$

$$|H(f)| = \left| a e^{j2\pi f/r} + 1 + a e^{-j2\pi \frac{f}{r}} \right|$$

$$= \left| 1 + 2a \cos(2\pi f/r) \right|$$



Condition: $1 - 2a > 0 \Rightarrow 1 > 2a \Rightarrow \underline{\underline{\frac{1}{2} > a}}$

