Exercise 3D Machine Vision

Sample solution

Prof. Dr.-Ing. Volker Willert

Technical University of Applied Sciences Würzburg-Schweinfurt

Exercise sheet 5

In this exercise we will cover *triangulation methods,* epipolar *geometry* and *discrete epipolar constraints,* as well as *the eight-point algorithm* and *rectification.* The questions are detailed and can be seen as examples of potential exam questions.

Task 5.1: Triangulation method

5.1a)

Name a triangulation method and explain how it is implemented.

Answer: Algebraic approach using projection equation: For each projected point xi, the projection equation for the unknown 3D point X can be set up with a known projection matrix ÿi , resulting in a homogeneous system of equations for each view: xiPiiX = 0. Two of these equations are linearly independent.
This results in four linear equations for the unknown point X for two views ÿ1 and ÿ2 and the projected coordinates x1 and x2 . If you solve this overdetermined system of linear equations, you have a good estimate for the triangulated 3D point.

5.1b)

Which of the following triangulation methods cannot easily be extended to more than two views?

× Geometric construction of minimum distances between rays

Algebraic approach using projection equation

Nonlinear approach via the reprojection error

5.1c)

Which triangulation method leads to an (over)determined linear system of equations? How can this system of equations be solved numerically in a stable manner?

Answer: Algebraic approach using projection equation: see above!

Task 5.2: Epipolar geometry

5.2a)

Construct the epipoles of a stereo camera system for any relative pose. Make a sketch.

Answer: See Epipolar Geometry slide set, page 15.

5.2b)

What points do all epipolar planes have in common? What kind of curve do these points describe?

Answer: All points that lie on the line connecting the two optical centers.

5.2c)

Which size of the epipolar geometry corresponds to the null space of the essential matrix?

- Epipolarline
- Base distance
- Epipolar plane
- × Epipole
- Intersection of the rays of two corresponding points
- Translation vector between the optical centers
- Relative pose rotation matrix

5.2d)

Which statements regarding the Essential Matrix are correct?

- × the columns of the essential matrix correspond to the cross products between the translation vector and the Columns of the rotation matrix
- × the product of the skew-symmetric matrix of the translation vector and the rotation matrix
- the Essential Matrix has full rank
- × the essential matrix has two linearly independent row vectors
- no eigenvalue of the essential matrix is zero
- × the essential matrix has two identical eigenvalues

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Task 5.3: Epipolar constraint & eight-point algorithm

5.3a)

What is the minimum number of corresponding point pairs required to calculate the relative pose from the discrete epipolar constraint?

- 3
- 4
- × 5
- 6
- 8th

5.3b)

Explain why the eight-point algorithm requires a projection onto essential space.

Answer: If the linear system of equations of the point correspondences of the discrete epipolar constraint is solved for the essential matrix, then this matrix does not yet fulfill all the properties of an essential matrix. By setting the eigenvalues of the essential matrix to the values [1, 1, 0], the properties are fulfilled. This process is called projection onto the essential space.

5.3c)

How many solutions does the eight-point algorithm produce for which sizes?

Answer: There are four solutions for the relative pose between two cameras.

5.3d)

Calculate a first approximation of the essential matrix for the following relative pose: $R = \begin{pmatrix} \cos(\pi/4) & 0 & \sin(\pi/4) \\ 0 & 1 & 0 \\ -\sin(\pi/4) & 0 & \cos(\pi/4) \end{pmatrix}$, $T = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$.

Answer: $E = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 0 & 2 & 0 \end{pmatrix}$

5.3e)

The SVD of an essential matrix is given as follows:

E = UPS = ÿ ÿ1 0 0 / 0 0 ÿ1 / ÿ 0 1 0 ÿÿÿÿ 2 0 0 / 0 2 0 / 0 0 0 ÿÿÿÿ ÿ ÿ 2/2 0 ÿ ÿ 2/2 / 0 1 0 / ÿ 2/2 0 ÿ ÿ 2/2 ÿ ÿ .

Calculate all possible relative poses between the cameras. Use the following formulas:

R = UR Z ± 2 ÿ V , T = URZ ± 2 ÿ SU ,

where:

R Z ± 2 ÿ = ÿ 0 ±1 0 / ÿ1 0 0 / ÿ 0 0 1 ÿ ÿ , T = ÿ 0 ÿTz Ty / Tz 0 ÿTx / ÿ ÿTy Tx 0 ÿ ÿ .

Answer: R1 = ÿ ÿ 2/2 0 ÿ ÿ 2/2 / 0 ÿ1 0 / ÿ ÿ ÿ 2/2 0 ÿ ÿ 2/2 ÿ ÿ, R2 = ÿ ÿ 2/2 0 ÿ 2/2 / 0 1 0 / ÿ ÿ ÿ ÿ 2/2 0 ÿ 2/2 ÿ ÿ, T1 = ÿ 0 0 0 / 0 0 1 / ÿ 0 ÿ1 0 ÿ ÿ, T2 = ÿ 0 0 0 / 0 0 ÿ1 / ÿ 0 1 0 ÿ ÿ

Task 5.4: Rectification

5.4a)

Why do you need rectification in a stereo system?

Answer: So that the correspondence search can be carried out along horizontal scan lines. An exact parallel alignment of the image planes is physically difficult, so a software-based readjustment, the so-called rectification, is required.

5.4b)

Where are the epipoles after rectification?

Answer: After rectification, the epipoles lie at infinity along the direction of the horizontal axes of the image planes.

5.4c)

1 The following normalized translation vector results from the eight-point algorithm: T/||T|| = ÿ 6 [1, 1, 2]. Construct Determine the rotation matrix Rrect, which generates parallel scan lines.

Answer: r1 = T/||T|| = 1/ÿ6 ÿ 1 / 1 / 2 ÿ ÿ,

r2 ÿ ÿ 0 / 0 / 1 ÿ × ÿ 1 / 1 / 2 ÿ = ÿ ÿ1 / 1 / 0 ÿ ÿ ÿ ÿ, 1 This means: r2 = ÿ 2 ÿ ÿ1 / 1 / 0 ÿ ÿ,

r3 = r1 × r2 = 1/ÿ6 ÿ 1 / 1 / 2 ÿ × 1/ÿ2 ÿ ÿ1 / 1 / 0 ÿ = 1/ÿ3 ÿ ÿ1 / ÿ1 / 1 ÿ