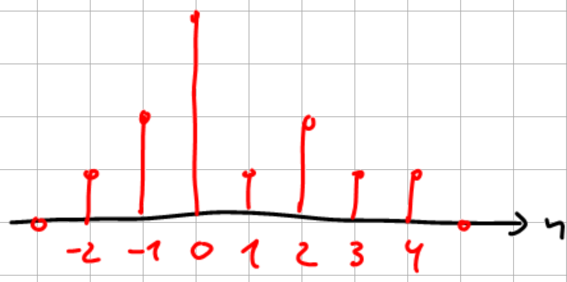
Fourier Transform

$$X(f) = \int_{-\infty}^{\infty} x(t) \cdot e^{j2\pi f t} \, dt$$

time continuous

Z - Transform

$$X(f) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{j2\pi f T}, \quad T = \frac{1}{r}$$



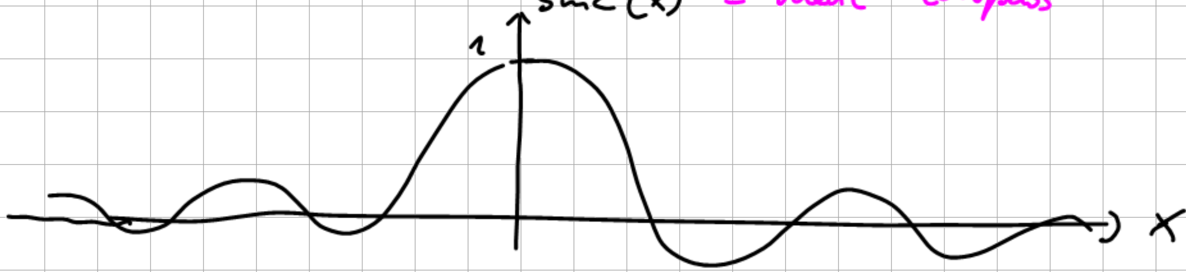
time discrete

$$\text{signum} \mathrel{\hat{=}} \text{sign} = \begin{cases} +1 & \text{for } x \geq 0 \\ -1 & \text{else} \end{cases}$$

$\sin(\ldots)$

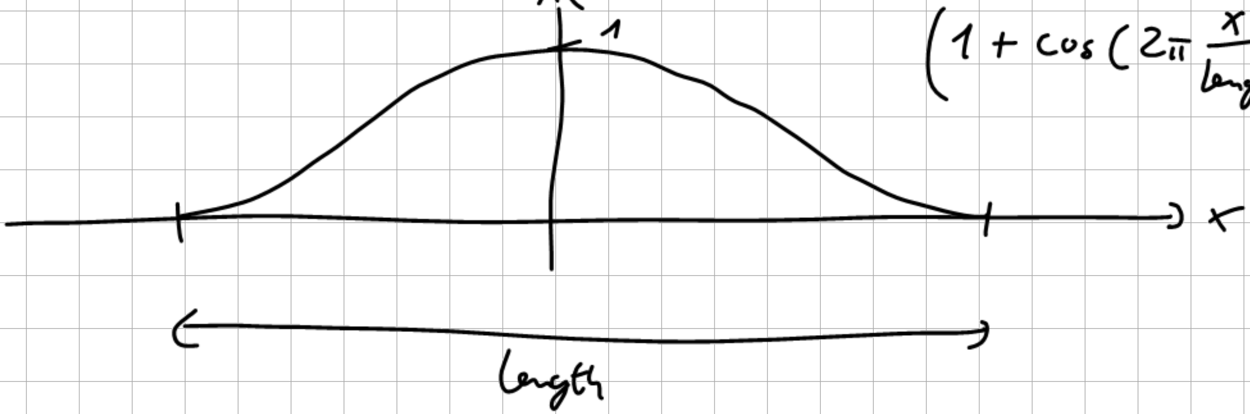
$$\text{sinc}(x) = \begin{cases} \frac{\sin(\pi x)}{\pi x}, & \text{else} \\ 1, & \text{for } x = 0 \end{cases}$$

$\text{sinc}(x) \mathrel{\hat{=}}$ ideal lowpass

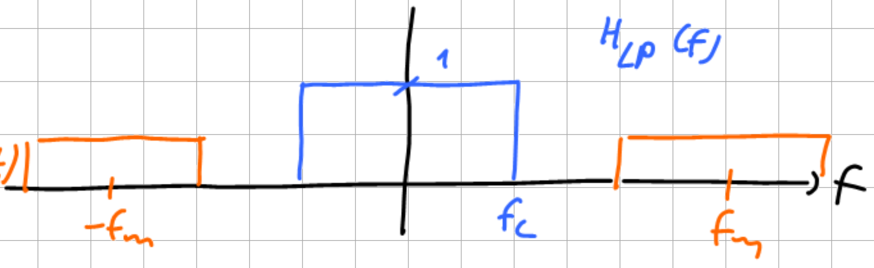


Hann-Window

$$\left(1 + \cos\left(2\pi \frac{x}{\text{length}}\right)\right) \cdot \frac{1}{2}$$

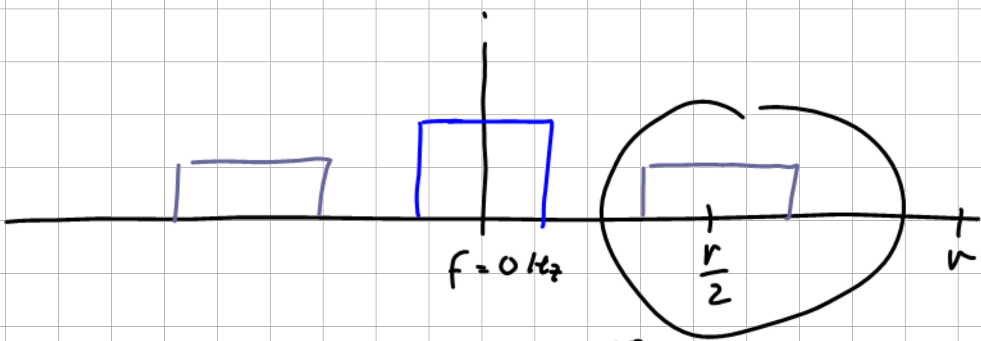


$h_{LP}(t) = \dots$



$h_{BP}(t) = h_{LP}(t) \cdot \cos(2\pi f_m \cdot t)$

$H_{BP}(f) = H_{LP}(f) * \frac{1}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right]$



$h_{BP}(n) = h_{LP} \cdot \cos\left(2\pi f_m \frac{n}{r}\right)$

$h_{HP}(n) = h_{LP} \cdot \cos\left(2\pi \frac{r}{2} \frac{n}{r}\right)$

$= h_{LP} \cdot \cos(\pi n)$

$= h_{LP} \cdot (-1)^n$

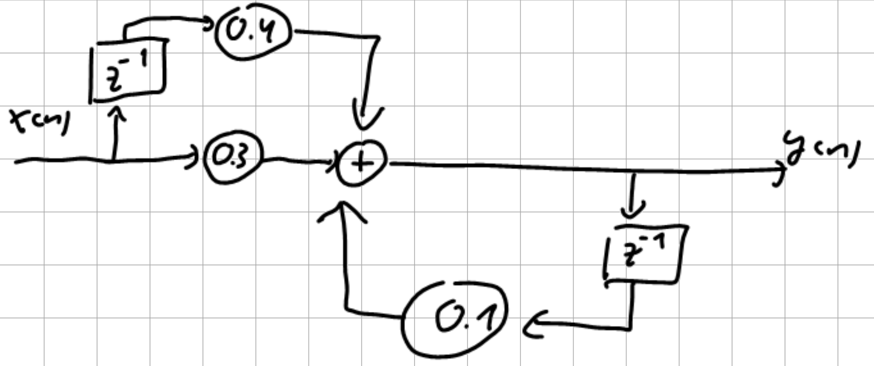
$$\begin{array}{cccc} & n=0 & n=1 & n=2 \\ h_{LP}(n) = & [1 & 2 & 1] \\ h_{HP}(n) = & [1 & -2 & 1] \end{array}$$

$h \;\circ\!\!-\!\!\bullet\; H(f)$

z-Transform

IIR-Filter

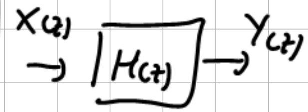
$y(n) = \underline{0.3}\, x(n) + 0.4\, x(n-1) + \underline{0.1\, y(n-1)}$ differential equation



$$y(n) = 0.3\,x(n) + 0.4\,x(n-1) + 0.1\,y(n-1)$$

$$Y(z) = 0.3\,X(z) + 0.4\,z^{-1}X(z) + 0.1\,z^{-1}Y(z)$$

$$\begin{aligned} y(n) &\to Y(z) \\ x(n) &\to X(z) \\ x(n-1) &\to X(z)\cdot z^{-1} \end{aligned}$$



$$X(z)\cdot H(z) = Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z)\,(1 - 0.1\,z^{-1}) = X(z)\cdot(0.3 + 0.4\,z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.3 + 0.4\,z^{-1}}{1 - 0.1\,z^{-1}}$$

$$H(z) = \frac{1-a}{1 - a\cdot z^{-1}}$$

$$H(f) = \frac{1-a}{1 - a\cdot e^{-j2\pi f/r}}$$

$$H_{max}(f=0) = \frac{1-a}{1-a\cdot 1} = \underline{\underline{1}}$$

$$|H(f_c)| = \frac{H_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$f_c = f(a, r)$$

$$\left| \frac{1-a}{1-a\,e^{-j2\pi f_c/r}} \right| = \frac{1}{\sqrt{2}}$$

$$\frac{1-a}{|1-a\cos(2\pi f_c/r) + ja\sin(2\pi f_c/r)|} = \frac{1}{\sqrt{2}}$$

$$\frac{1-a}{\sqrt{(1-a\cos(2\pi f_c/r))^2 + a^2\sin^2(2\pi f_c/r)}} = \frac{1}{\sqrt{2}}$$

$$\frac{(1-a)^2}{1-2a\cos(2\pi f_c/r) + \underbrace{a^2\cos^2(2\pi f_c/r) + a^2\sin^2(2\pi f_c/r)}_{a^2}} = \frac{1}{2}$$

$$\frac{1-2a+a^2}{1-2a\cos(2\pi f_c/r)+a^2} = \frac{1}{2}$$

$$2-4a+2a^2 = 1-2a\cos(2\pi f_c/r)+a^2$$

$$1-4a+a^2 = -2a\cos(2\pi f_c/r)$$

$$\frac{1-4a+a^2}{-2a} = \cos(2\pi f_c/r)$$

$$\arccos\left(\frac{1-4a+a^2}{-2a}\right) = 2\pi f_c/r$$

$\cos^{-1}$

$$f_c = r \cdot \frac{\arccos\left(\frac{1-4a+a^2}{-2a}\right)}{2\pi}$$

$$1-4a+2a\cos(2\pi f_c/r)+a^2 = 0$$

$$1-a\left(4-2\cos(2\pi f_c/r)\right)+a^2 = 0$$

$$ax^2+bx+c=0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$p_1x^2+p_2x+p_3=0$$

$p1 = 1$ $\qquad p2 = -4 + 2\cos(2\pi f_c/r)$ $\qquad p_3 = 1$

$$x1 = \frac{-p2 + \sqrt{p_2^2 - 4\,p1\,p3}}{2\,p1}$$

$x2 = \ldots$