Exemplary Exam Questions

Warning: The exemplary exam questions provided here are only intended to give an orientation with respect to scope and level of difficulty. There is **no** guarantee that similar questions will appear in the actual exam.

Problem 1

For a automatic assembly line, the total time necessary to complete the construction of a customized product in minutes was recorded during a working week on five consecutive days:

Day	1	2	3	4	5
Number of measurements	15	30	20	25	10
Arithmetic mean	46	36	58	38	42
Variance	6	4	5	6	7

Compute the overall arithmetic mean and the overall standard deviation of the measurements. Which kind of variation in the measurements has the main influence on the overall variance?

Problem 2

The table below presents bivariate data concerning multi-level production orders. Indicated are both the respective number of production levels and the average time for completion in minutes.

Number of production levels	2	3	4	5	6
Average time for completion	25	35	42	46	48

(a) Determine the corresponding coefficients of correlation of Spearman and Bravais-Pearson and give a short interpretation of the values of the two coefficients.

(b) Compute an estimate for the completion time of a single-level production order based on linear regression analysis.

Problem 3

The probability mass function (pmf) of a discrete random variable X is given by

x_i	0	1	2	3	4	5
p_i		$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$		$\frac{1}{8}$

where $p_i = P(X = x_i)$.

(a) Find the missing values of the pmf under the assumption that the mean of X is $\mu = 2$.

(b) Compute the probabilities $P(X \leq 2)$ and $P(1 \leq X < 4)$.

(c) Determine the variance σ^2 of the discrete random variable X.

Problem 4

The bivariate random vector $\boldsymbol{X} = (X_1, X_2)^T$ should be characterized by the joint probability density function (pdf)

$$f_{\boldsymbol{X}}(x_1, x_2) = \begin{cases} \frac{1}{2}x_1^3 x_2, & 0 \leq x_1 \leq 2, \ \ 0 \leq x_2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Compute the probability $P(\boldsymbol{X} \in [1,2] \times [\frac{1}{2}, \frac{3}{2}])$.

(b) Determine the mean vector $\boldsymbol{\mu} \in \mathbb{R}^2$ of $\boldsymbol{X}$.

(c) Determine the densities of the marginal distributions.

(d) Are the components X_1 and X_2 statistically independent?
Justify your answer!

(e) Indicate the conditional pdfs $f_{X_1|X_2}(x_1|x_2)$ and $f_{X_2|X_1}(x_2|x_1)$.

Problem 5

Consider a binary classification problem, where the two class-conditional densities of the bivariate feature vector $\boldsymbol{X} = (X_1, X_2)^T$ are given by

$$f_k(\boldsymbol{x}) = \frac{1}{2\pi|\boldsymbol{\Sigma}_k|^{1/2}} \cdot \exp\left(-\tfrac{1}{2}(\boldsymbol{x} - \boldsymbol{\mu}_k)^T \boldsymbol{\Sigma}_k^{-1}(\boldsymbol{x} - \boldsymbol{\mu}_k)\right), \quad k = 1, 2$$

where

$$\boldsymbol{\mu}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{\mu}_2 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma}_1 = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix}, \quad \boldsymbol{\Sigma}_2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}.$$

Now, the observed feature vector is $\boldsymbol{x} = (1, 2)^T$. Determine the decision of the Bayes optimal classifier by calculating the corresponding posterior probabilities $p(k|\boldsymbol{x})$ in the case of the prior probabilities

$$\pi_1 = 0.6, \quad \pi_2 = 0.4.$$

Problem 6

The contamination of an alloy by impurities (in %) is modeled by a normally distributed random variable. An analysis of 10 samples taken from the metallurgical process provides the following values:

$$5.8,\ 6.4,\ 6.7,\ 5.5,\ 6.8,\ 4.8,\ 6.1,\ 4.9,\ 5.7,\ 6.3.$$

(a) In the case that the true variance is known to be $\sigma^2 = \frac{3}{2}$, construct a symmetric confidence interval for the unknown mean μ at a level of significance $\alpha = 0.03$.

(b) How many measurements would be necessary to reduce the length of the confidence interval from (a) by at least one third in the case that the level of significance remains unchanged?

(c) The true variance σ^2 is assumed to be unknown. Recompute the symmetric confidence interval for the mean μ under this assumption at a level of significance $\alpha = 0.05$.