1)

$$y_j = f(x_j) = |x_j|$$

$$= \begin{cases} x_j , & \text{for } x_j \geq 0 \\ -x_j , & \text{else} \end{cases}$$

$$\frac{dy_j}{dx_j} = \begin{cases} 1 , & \text{for } x_j \geq 0 \\ -1 , & \text{else} \end{cases}$$

2) $L = \frac{1}{2}(y - o)^2$

$\frac{dL}{dy} = (y - o) = 7 - (-2) = 9$

3 a) $y_j = b_j + \sum_{i=0}^{I-1} x_i \, w_{j,i} = f(x)$

$$f(a x_1 + b x_2) = b_j + \sum_{i=0}^{I-1} (a x_{1i} + b x_{2i}) \, w_{ji}$$
$$= b_j + a \sum_{i=0}^{I-1} x_{1i} w_{ji} + b \sum_{i=0}^{I-1} x_{2i} \, w_{ji}$$

$$a \, f(x_1) + b \, f(x_2) = a \left(b_j + \sum_{i=0}^{I-1} x_{1i} \, w_{ji} \right) + b \cdot \left(b_j + \sum_{i=0}^{I-1} x_{2i} \, w_{ji} \right)$$
$$= (a + b) \cdot b_j + a \sum_{i=0}^{I-1} x_{1i} w_{ji} + b \sum_{i=0}^{I-1} x_{2i} w_{ji}$$

linear only if $b_j = 0$ $\Rightarrow$ non linearity is proven

3 b) $y_j = \begin{cases} x_j & , \text{ if } x_j \geq 0 \\ \alpha \cdot x_j & , \text{ else} \end{cases}$

counter example: $a = b = 1$

$x_1 = 3$

$x_2 = -1$

$$f(a \cdot x_1 + b \cdot x_2) = f(2) = 2$$

$$a f(x_1) + b f(x_2) = 1 \cdot f(3) + 1 \cdot f(-1)$$

$$= 1 \cdot 3 + 1 \cdot 0 = 3 \quad \lightning$$

non linearity is proven

3 c) $f(x) = \dfrac{1}{1+e^{-x}}$

counter example: $a = 1 \quad b = 1$

$x_1 = 1 \quad x_2 = 3$

$$f(ax_1 + bx_2) = f(1+3) = f(4) = \frac{1}{1+e^{-4}} = 0{,}982$$

$$af(x_1) + b\,f(x_2) = \frac{1}{1+e^{-1}} + \frac{1}{1+e^{-3}} = 1{,}68$$

non linearity is proven : $0{,}982 \neq 1{,}68$

4) For softmax layer, the output is normalized to a sum of one, therefore the missing third output can be evaluated by:

$$y_2 = 1 - y_0 - y_1 = 1 - 0.3 - 0.6 = 0.1$$

5) $y_j = x_j^{0.3}$

$x_0 = 1 \qquad a = 1 \qquad x_1 = 1 \qquad b = 1$

$f(ax_0 + bx_1) = f(2) = 2^{0.3} = 1.23$

$af(x_0) + bf(x_1) = 1^{0.3} + 1^{0.3} = 2$

$\Rightarrow$ non linearity is prove.

$$\frac{dy_j}{dx_j} = 0.3 \cdot x_j^{-0.7}$$