1)

$$y_{(n)} = a\,y_{(n-1)} + x_{(n)} + 0.9\,x_{(n-1)}$$

$$Y_{(z)} = a\,z^{-1}\,Y_{(z)} + X_{(z)} + 0.9\,z^{-1}\,X_{(z)}$$

$$Y_{(z)}\,(1 - a z^{-1}) = X_{(z)}\,(1 + 0.9\,z^{-1})$$

$$H_{(z)} = \frac{Y_{(z)}}{X_{(z)}} = \frac{1 + 0.9\,z^{-1}}{1 - a z^{-1}}$$

$$z \leftarrow e^{j2\pi f T}$$

$$H_{(f)} = \frac{1 + 0.9\,e^{j2\pi f T}}{1 - a\,e^{j2\pi f T}}$$

$$H_{(z)} = \frac{1}{1 - a z^{-1}} + 0.9\,z^{-1}\,\frac{1}{1 - a z^{-1}}$$

$$h_{(n)} = a^{n}\,\varepsilon_{(n)} + 0.9\,a^{n-1}\,\varepsilon_{(n-1)})$$

n	$h_{(n)}$
-1	0
0	1
1	$a + 0.9$
2	$a^2 + 0.9\,a$
3	$a^3 + 0.9\,a^2$

$$H^{-1}_{(z)} = \frac{1 - a z^{-1}}{1 + 0.9 z^{-1}} = \frac{z - a}{z + 0.9}$$

pole at $z_p = -0.9$

$|z_p| < 1 \implies stable$

$$\frac{X_{(z)}}{Y_{(z)}} = H^{-1}_{(z)} = \frac{1 - a z^{-1}}{1 + 0.9 z^{-1}}$$

$$X_{(z)} (1 + 0.9 z^{-1}) = Y_{(z)} (1 - a z^{-1})$$

$$X_{(z)} + 0.9 z^{-1} X_{(z)} = Y_{(z)} - a z^{-1} Y_{(z)}$$

$$x_{(n)} + 0.9 \, x_{(n-1)} = y_{(n)} - a \, y_{(n-1)}$$

$$x_{(n)} = y_{(n)} - a \, y_{(n-1)} - 0.9 \, x_{(n-1)})$$