# Exercise 3D Machine Vision

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Sheet 5

In this exercise we cover *triangulation*, *epipolar geometry* and the *discrete epipolar constraint*, as well as the *8-point-algorithm* and *rectification*. The questions are small-scale and can be seen as examples of potential exam questions.

## Task 5.1: Triangulation

### 5.1a)

Name a triangulation procedure and explain how it is implemented.

### 5.1b)

Which of the following triangulation procedures cannot be easily extended to more than two views?

- ☐ Geometric construction of minimum distances of rays
- ☐ Algebraic approach using projection equation
- ☐ Nonlinear approach via the reprojection error

### 5.1c)

Which triangulation method leads to an (over)determined linear system of equations? How can this system of equations be solved in a numerically stable way?

## Task 5.2: Epipolar Geometry

### 5.2a)

Construct the epipoles of a stereo camera system for any relative pose. Make a sketch for this purpose.

### 5.2b)

What points do all epipolar planes have in common? What kind of curve do these points describe?

## 5.2c)

What quantity of epipolar geometry corresponds to the null space of the essential matrix?

- ☐ epipolar line
- ☐ baseline
- ☐ epipolar plane
- ☐ epipole
- ☐ intersection of rays of two corresponding points
- ☐ translation vector between the optical centers
- ☐ rotation matrix of the relative pose between cameras

## 5.2d)

Which statements regarding the essential matrix are correct?

- ☐ the columns of the essential matrix correspond to the cross products between the translation vector and the columns of the rotation matrix
- ☐ the essential matrix equals the product of the skew symmetric matrix of the translation vector and the rotation matrix
- ☐ the essential matrix has full rank
- ☐ the essential matrix has two linearly independent row-vectors
- ☐ the eigenvalues of the essential matrix are non-zero
- ☐ the essential matrix has two identical eigenvalues

## Task 5.3: Epipolar Constraint & 8-Point-Algorithm

## 5.3a)

What is the minimum number of corresponding point pairs needed to compute the relative pose from the discrete epipolar constraint?

- ☐ 3
- ☐ 4
- ☐ 5
- ☐ 6
- ☐ 8

## 5.3b)

Explain why the eight-point algorithm requires a projection onto the essential space.

### 5.3c)

How many solutions result from the eight-point algorithm for which quantities?

### 5.3d)

Compute a first approximation of the essential matrix for the following relative pose: $\mathbf{R} = \begin{bmatrix} \cos(\pi/4) & 0 & sin(\pi/4) \\ 0 & 1 & 0 \\ -sin(\pi/4) & 0 & \cos(\pi/4) \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$.

### 5.3e)

The SVD of an essential matrix is given as follows:

$$\mathbf{E} = \mathbf{USV}^\top = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -\sqrt{2}/2 & 0 & -\sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix}.$$

From this, calculate all possible relative poses between the cameras.

## Task 5.4: Rectification

### 5.4a)

Why do you need rectification in a stereo system?

### 5.4b)

Where are the epipoles located after rectification?

### 5.4c)

The following normalized translation vector results from the eight-point algorithm: $\mathbf{T} = \frac{1}{\sqrt{6}}[1, 1, 2]^\top$. Construct the rotation matrix $\mathbf{R}_{rect}$ which generates parallel scan lines.