1)

$$y(n) = x(n) - a$$

$$\frac{1}{N}\sum_{n=0}^{N-1} y(n) = 0$$

$$\frac{1}{N}\sum_{n=0}^{N-1} (x(n) - a) = 0$$

$$\frac{1}{N}\sum_{n=0}^{N-1} x(n) - \frac{1}{N}\sum_{n=0}^{N-1} a = 0$$

$$\frac{1}{N}\sum_{n=0}^{N-1} x(n) - \frac{1}{N}\cdot N \cdot a = 0$$

$$\frac{1}{N}\sum_{n=0}^{N-1} x(n) = a$$

$$z(n) = y(n) \cdot b$$

$$\frac{1}{N}\sum_{n=0}^{N-1} z^2(n) = 1$$

$$\frac{1}{N}\sum_{n=0}^{N-1} (y(n) \cdot b)^2 = 1$$

$$\frac{1}{N}\sum_{n=0}^{N-1} y^2(n) \cdot b^2 = 1$$

$$\frac{b^2}{N}\sum_{n=0}^{N-1} y^2(n) = 1$$

$$b^2 = \frac{1}{\frac{1}{N}\sum_{n=0}^{N-1} y^2(n)}$$

$$b = \sqrt{\frac{N}{\sum_{n=0}^{N-1} y^2(n)}}$$

2) Symmetry of Pearson Correlation

$$\varphi(y,x) = \frac{\Sigma_n (y-\mu_y)\cdot(x-\mu_x)}{\sqrt{\Sigma_n (y-\mu_y)^2}\cdot\sqrt{\Sigma_n (x-\mu_x)^2}}$$

$$= \frac{\Sigma_n (x-\mu_x)\cdot(y-\mu_y)}{\sqrt{\Sigma_n (x-\mu_x)^2}\cdot\sqrt{\Sigma_n (y-\mu_y)^2}}$$

$$= \varphi(x,y) \qquad \text{q.e.d.}$$

3) Scale invariance of Pearson Correlation

$$\varphi(x, a\cdot y) = \frac{\Sigma_n (x-\mu_x)\cdot(a\cdot y - a\mu_y)}{\sqrt{\Sigma_n (x-\mu_x)^2}\cdot\sqrt{\Sigma_n (ay - a\mu_y)^2}}$$

$$= \frac{\Sigma_n (x-\mu_x)\cdot a\cdot(y-\mu_y)}{\sqrt{\Sigma_n (x-\mu_x)^2}\cdot\sqrt{a^2\cdot\Sigma_n (y-\mu_y)^2}}$$

$$= \frac{a \quad \Sigma_n (x-\mu_x)^2\cdot(y-\mu_y)}{|a|\sqrt{\Sigma_n (x-\mu_x)^2}\cdot\sqrt{\Sigma_n (y-\mu_y)^2}}$$

$$= \text{sign}(a)\cdot\varphi(x,y)$$

with $\text{sign}(a) = \begin{cases} 1, & \text{for } a > 0 \\ 0, & \text{for } a = 0 \\ -1, & \text{for } a < 0 \end{cases}$

$\Rightarrow$ For positive a, $a \in \mathbb{R}$, the Pearson Correlation is scale invariant.

4) poles: $z_p - 0.955 = 0 \Rightarrow z_p = 0.999$

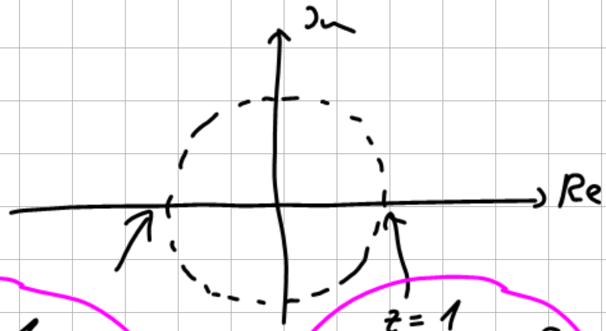
zeros: $z_z - 1 = 0 \Rightarrow z_z = 1$

a zero for $f = 0\,Hz$

- zero at $f = 0\,Hz$
- $f = 0\,Hz$ corresponds to DC part
- DC part is multiplied with zero

$\rightarrow$ $s(n)$ has no DC part

$\rightarrow$ $\underline{\underline{b = 0}}$



$z = -1$

$z \mathrel{\hat{=}} e^{j2\pi \frac{f}{r}}$

$z = -1 \Rightarrow f = \frac{r}{2}$

$z = 1$

$z \mathrel{\hat{=}} e^{j2\pi \frac{f}{r}}$

$z = 1 \Rightarrow f = 0\,Hz$