Worksheet 2

Exercise 1

x : number of employees

y : average duration of employment

Ranks :

$x_1 = 221 \Rightarrow R(x_1) = 8$ smallest company

$x_2 = 251 \Rightarrow R(x_2) = 7$

$x_3 = 346 \Rightarrow R(x_3) = 6$

$x_4 = 376 \Rightarrow R(x_4) = 5$

$x_5 = 401 \Rightarrow R(x_5) = 4$

$x_6 = 421 \Rightarrow R(x_6) = 3$

$x_7 = 471 \Rightarrow R(x_7) = 2$

$x_8 = 481 \Rightarrow R(x_8) = 1$ largest company

$y_1 = 9.7 \Rightarrow R(y_1) = 1$

$y_2 = 7.9 \Rightarrow R(y_2) = 3$

$y_3 = 8.6 \Rightarrow R(y_3) = 2$

$y_4 = 7.2 \Rightarrow R(y_4) = 5$

$y_5 = 7.3 \Rightarrow R(y_5) = 4$

$$y_5 = 7.3 \Rightarrow R(y_5) = 4$$

$$y_6 = 7.1 \Rightarrow R(y_6) = 6$$

$$y_7 = 7.0 \Rightarrow R(y_7) = 7$$

$$y_8 = 6.8 \Rightarrow R(y_8) = 8$$

Spearman's Rank Correlation Coefficient:

$$r_s = 1 - \frac{6 \sum_{i=1}^{n} \left(R(x_i) - R(y_i)\right)^2}{(n-1) \cdot n \cdot (n+1)}$$

$$= 1 - \frac{6 \cdot 164}{504} \approx -0.95$$

Coefficient of Bravais - Pearson:

$$r = \frac{\sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \cdot \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 371$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = 7.7$$

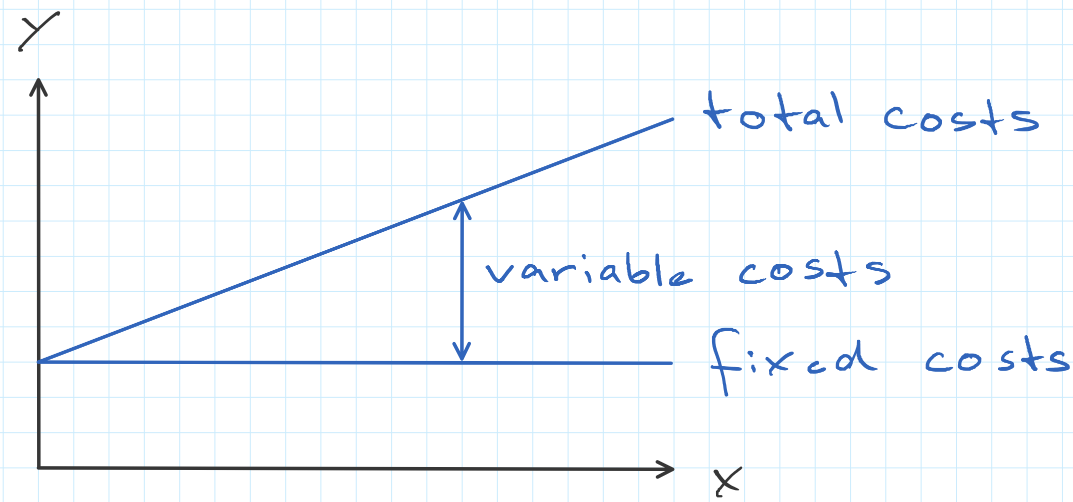
$$\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \approx 251.1$$

$$\sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2} \approx 2.63$$

$$\sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y}) = -560$$

$$\Rightarrow \quad r = \frac{-560}{251.1 \cdot 2.63} \approx -0.85$$

Exercise 2



Regression analysis:

$$y = ax + b$$

$$a = \frac{\sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$b = \bar{y} - a\bar{x}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = 12$$

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = 13$$

$$\sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{x}) = 5 \quad)$$

$$\left.\begin{aligned} \sum_{i=1}^{n} (x_i - \bar{x}) \cdot (y_i - \bar{x}) &= 5 \\ \sum_{i=1}^{n} (x_i - \bar{x})^2 &= 14 \end{aligned}\right\} \Rightarrow a = \frac{5}{14}$$

$$\Rightarrow b = 13 - \frac{5}{14} \cdot 12 \approx 8.71$$

Fixed costs: 8.71 Mio EUR

Variable costs: 357 T EUR

Coefficient of determination:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \hat{u}_i^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

$$\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (y_i - y(x_i))^2 = 0.213$$

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 = 2$$

$$\Rightarrow R^2 = 1 - \frac{0.213}{2} = 0.8935$$

Exercise 3

x_i	1	5	7	9
y_i	4	120	253	418

Assumed regression function :

$$y = b x^a$$

Consider applying the natural logarithm :

$$\underbrace{\ln(y)}_{\tilde{y}} = \ln(b x^a) = \underbrace{\ln(b)}_{c} + a \cdot \underbrace{\ln(x)}_{\tilde{x}}$$

$$\Rightarrow \quad \tilde{y} = a \cdot \tilde{x} + c$$

$\tilde{x}_i$	0	1.61	1.95	2.20
$\tilde{y}_i$	1.39	4.79	5.53	6.04

$$\bar{\tilde{x}} = \frac{1}{4} \sum_{i=1}^{4} \tilde{x}_i = 1.44$$

$$\bar{\tilde{y}} = \frac{1}{4} \sum_{i=1}^{1} \tilde{y}_i = 4.44$$

$$\sum_{i=1}^{4} (\tilde{x}_i - \bar{\tilde{x}}) \cdot (\tilde{y}_i - \bar{\tilde{y}}) = 6.2234$$

$$\sum_{i=1}^{4} (\tilde{x}_i - \bar{\tilde{x}})^2 = 2.9402$$

$$\Rightarrow \quad a = \frac{6.2234}{2.9402} \approx 2.12$$

$$c = \bar{\tilde{y}} - a\bar{\tilde{x}}$$

$$= 4.44 - 2.12 \cdot 1.44$$

$$\approx 1.39$$

$$\Rightarrow \quad b = e^c \approx 4$$

$$\Rightarrow \quad y = 4 \cdot x^a, \quad a = 2.12$$

Exercise 4

(a) Right

(b) Wrong

(c) Right

(d) Right