1) DFT: $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{nk}{K}}$

$$X(K-k) = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{n}{K}(K-k)}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{n}{K} K + j 2\pi \frac{n}{K} k}$$

$$= \sum_{n=0}^{N-1} x(n)\, \underbrace{e^{-j 2\pi n}}_{=1 \text{ for } n = 0, 1, 2, \dots} e^{j 2\pi \frac{nk}{K}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{j 2\pi \frac{nk}{K}}$$

$$X^*(K-k) = \left( \sum_{n=0}^{N-1} x(n)\, e^{j 2\pi \frac{nk}{K}} \right)^*$$

$$= \sum_{n=0}^{N-1} x^*(n) \left( e^{j 2\pi \frac{nk}{K}} \right)^*$$

↳ $x(n)$ is assumed to be real valued

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{nk}{K}} = x(n)$$

2) $x(n) = \hat{x} \sin(2\pi f \cdot nT)$ with $T = \frac{1}{r}$

$$= 1 \cdot \sin\left(2\pi \frac{440\ Hz}{1000\ Hz} \cdot n\right)$$

$x(0) = 0$

$x(1) = \sin\left(2\pi \frac{440}{1000}\right) = 0{,}368$

$x(2) = \sin\left(2\pi \frac{440}{1000} \cdot 2\right) = -0{,}685$

$$X(k) = \sum_{n=0}^{2} x(n) \cdot e^{-j2\pi \frac{nk}{3}}$$

$$X(0) = 0 \cdot \underbrace{e^{-j2\pi \frac{0 \cdot 0}{3}}}_{=1} + 0{,}368 \cdot \underbrace{e^{-j2\pi \frac{1 \cdot 0}{3}}}_{=1} - 0{,}685 \cdot \underbrace{e^{-j2\pi \frac{2 \cdot 0}{3}}}_{=1}$$

$$= 0 + 0{,}368 - 0{,}685 = -0{,}316$$

$$X(1) = 0 \cdot e^{-j2\pi \frac{0 \cdot 1}{3}} + 0{,}368\, e^{-j2\pi \frac{1 \cdot 1}{3}} - 0{,}685 \cdot e^{-j2\pi \frac{2 \cdot 1}{3}}$$

$$= 0{,}158 - j\, 0{,}912$$

$$X(2) = X^*(1) = 0{,}158 + j\, 0{,}912$$

Parseval: $E_x = \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{K} \sum_{k=0}^{K-1} |X(k)|^2$

$$\sum_{n=0}^{N-1} |x(n)|^2 = 0^2 + 0{,}368^2 + 0{,}685^2 = 0{,}604$$

$$\frac{1}{K} \sum_{k=0}^{K-1} |X(k)|^2 = \frac{1}{3}\left[(-0{,}316)^2 + (0{,}158^2 + 0{,}912^2) \cdot 2\right]$$

$$= 0{,}604$$

3)- $\Delta f = \frac{1}{K \cdot T} = \frac{r}{K} = \frac{48000}{1024} Hz = 46{,}9 \ Hz$

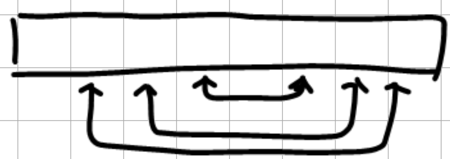
- yes, zero padding is used because of: $N < K$
  (without zero padding it must be $N = K$)

- $k=0$ : $f = k \cdot \Delta f = 0 \ Hz$

  $k=1$ : $f = k \cdot \Delta f = 46{,}9 \ Hz$ ← for $k=1$, the frequency 50 Hz is analyzed

  $k=2$ : $f = k \cdot \Delta f = 93{,}8 \ Hz$

- assuming $K=8$ :



3 coefficients can be dropped $\hat{=} \frac{K}{2} - 1$

$\Rightarrow K = 1024 \Rightarrow \frac{K}{2} - 1 = 511$ coefficients can be dropped

4) Advantages

- very fast
- linear frequency resolution is simple to analyze

Disadvantages

- periodicity in time domain is assumed
  → window functions are necessary
- spectrum is analyzed only for a set of (discrete) frequencies, depending on $\Delta f$
- logarithmic frequency resolution, e.g. for Bode plots or psychoacoustic measurements, are not straightforward to evaluate

5) $$X(f) = \sum_{n=-\infty}^{\infty} x(n) \; e^{-j2\pi f \frac{n}{r}}$$ Fourier Transformation of discrete signals

$$X(k) = \sum_{n=0}^{N-1} x(n) \; e^{-j2\pi \frac{nk}{N}}$$

Condition 1: $x(n) \neq 0$ if and only if $0 \leq n < N$

$\Rightarrow$ $x(n)$ must be resticted to the range $0 \leq n < N$

Condition 2: $f \cdot \frac{n}{r} = \frac{n \cdot k}{N}$

$\Rightarrow$ $f = k \cdot \frac{r}{N}$

Both outputs can only be analyzed at frequencies which are integer multiples of $\frac{r}{N} = \Delta f$