$$y(n) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2)$$
$$+ a_0\, y(n) + a_1\, y(n-1) + a_2\, y(n-2)$$

$$\Rightarrow \quad y(n)\,(1 - a_0) = b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2)$$
$$+ a_1\, y(n-1) + a_2\, y(n-2)$$

$$y(n) = \Big[b_0\, x(n) + b_1\, x(n-1) + b_2\, x(n-2)$$
$$+ a_1\, y(n-1) + a_2\, y(n-2) \Big] \cdot \frac{1}{1-a_0}$$

$$y(n) = a \cdot y(n-1) + (1-a)\, x(n) \qquad a < 1$$
$$a \approx 1$$

typisch $a = 0.9$

$$y(n) = 0.9\, y(n-1) + 0.1\, x(n)$$

$$y(n) = a\, y(n-1) + (1-a)\, x(n)$$
$$\Big\downarrow$$
$$Y(z) = a\, z^{-1}\, Y(z) + (1-a)\, X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1-a}{1 - a z^{-1}} \qquad \boxed{z \,\hat{=}\, e^{j 2\pi \frac{f}{r}} \quad \text{assuming stability!}}$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{1-a}{1 - a\, e^{-j 2\pi \frac{f}{r}}}$$

$$H_{max} = H(f=0) = \frac{1-a}{1 - a\, e^{0}} = 1$$

$$|H(f_c)| = \frac{1}{\sqrt{2}} \cdot |H_{max}| = \frac{1}{\sqrt{2}}$$

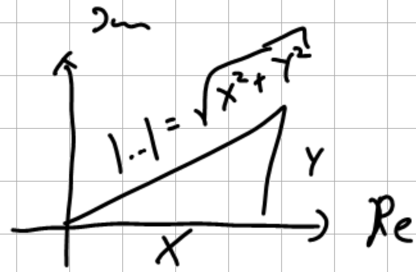
$$\left|\frac{1-a}{1-ae^{-j2\pi f/r}}\right| = \frac{1}{\sqrt{2}}$$

$$\frac{|1-a|}{|1-ae^{-j2\pi f/r}|} = \frac{1}{\sqrt{2}}$$

→ always positive

$$3-\sqrt{8} < a < 1$$

$$\frac{1-a}{\left|1 - a\cos\left(2\pi \frac{f}{r}\right) + ja\sin\left(2\pi f/r\right)\right|} = \frac{1}{\sqrt{2}}$$



$$\frac{(1-a)\sqrt{2}}{\sqrt{\left(1-a\cos\left(2\pi \frac{f}{r}\right)\right)^2 + a^2\sin^2\left(2\pi f/r\right)}} = 1$$

$$(1-a)\sqrt{2} = \sqrt{1-2a\cos\left(2\pi\frac{f}{r}\right) + \underbrace{a^2\cos^2\left(2\pi\frac{f}{r}\right) + a^2\sin^2\left(2\pi\frac{f}{r}\right)}_{a^2}}$$

$$(1-a)\sqrt{2} = \sqrt{1-2a\cos\left(2\pi\frac{f}{r}\right) + a^2}$$

$$(1-a)^2\cdot 2 = 1-2a\cos\left(2\pi\frac{f}{r}\right) + a^2$$

$$2-4a+2a^2 = 1-2a\cos\left(2\pi\frac{f}{r}\right) + a^2$$

$$0 = -1 + a\left(-2\cos\left(2\pi\frac{f}{r}\right) + 4\right) - a^2$$

$$2a\cos\left(2\pi\frac{f}{r}\right) = -1 + 4a - a^2$$

$$\cos\left(2\pi\frac{f}{r}\right) = \frac{-1+4a-a^2}{2a}$$

$$f = \frac{r}{2\pi}\arccos\left(\frac{-1+4a-a^2}{2a}\right)$$

Damping Parameter 2 Cutoff frequency ✓

$$a^2 + a\left(2\cos\left(2\pi\frac{f}{r}\right) - 4\right) + 1 = 0$$

$$a_{1,2} = -\frac{2\cos(2\pi\frac{f}{r})-4}{2} \pm \sqrt{\left(\frac{2\cos(2\pi\frac{f}{r})-4}{2}\right)^2 - 1}$$

$$= -\cos(2\pi\frac{f}{r}) + 2 \pm \sqrt{\left(\cos(2\pi\frac{f}{r})-2\right)^2 - 1}$$

$$a_1 = -\cos(2\pi\frac{f}{r}) + 2 + \sqrt{\left(\cos(2\pi\frac{f}{r})-2\right)^2 - 1}$$

$$a_2 = -\cos(2\pi\frac{f}{r}) + 2 - \sqrt{\left(\cos(2\pi\frac{f}{r})-2\right)^2 - 1}$$

only one is in the valid range!

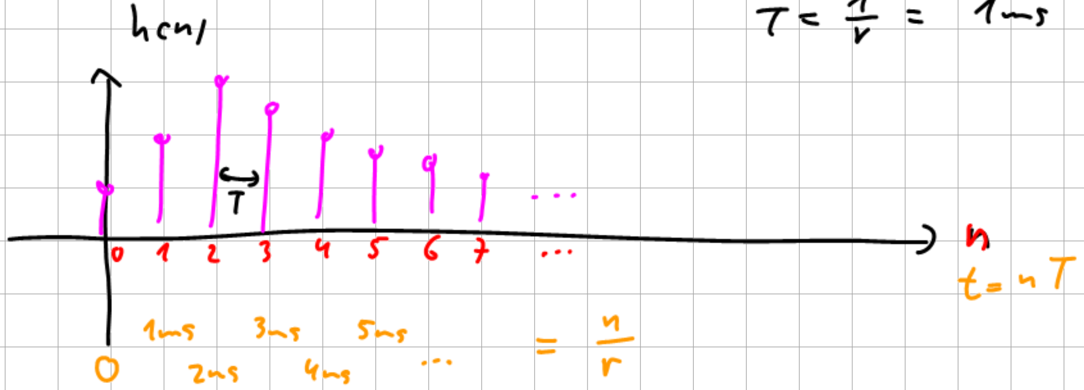
Latency

Latency In Samples = 17

$r = 1\,kHz$

T Abtastdauer

$T = \frac{1}{r} = 1\,ms$

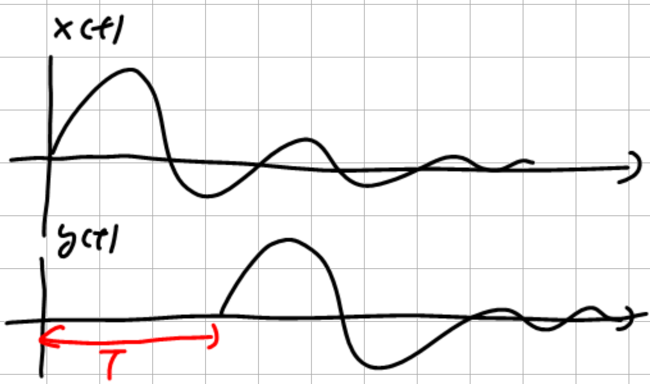


Cross Correlation

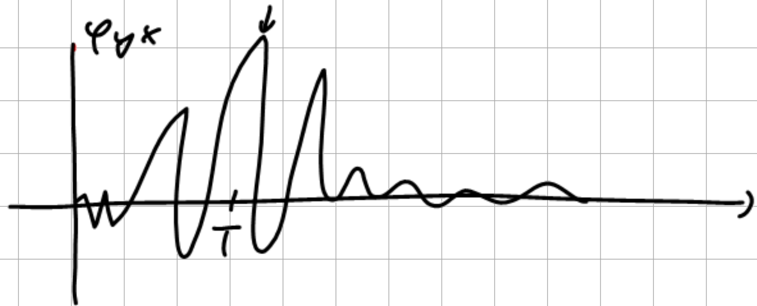
Latenz mit Kreuzkorrelation

$h(t) = \delta(t-T)$

$y(t) = x(t-T)$



Modell $y(t) = x(t-T)$
passt solala



nahe zu perfektes
Signalmodell

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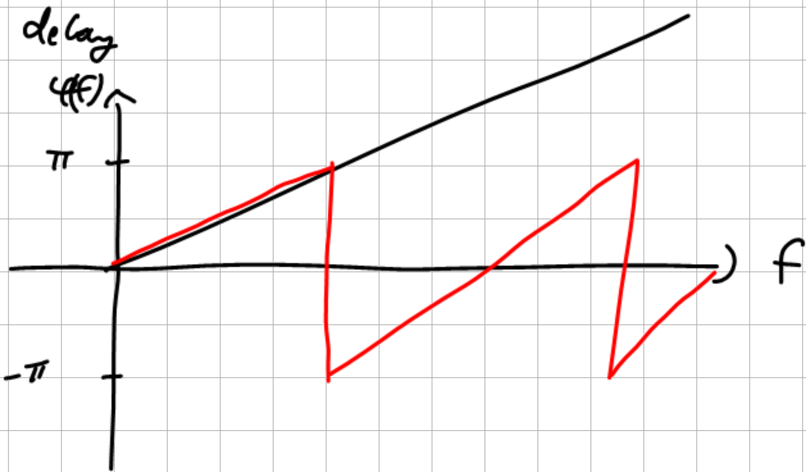
phase delay – unwrapping (ausrollende Phase)

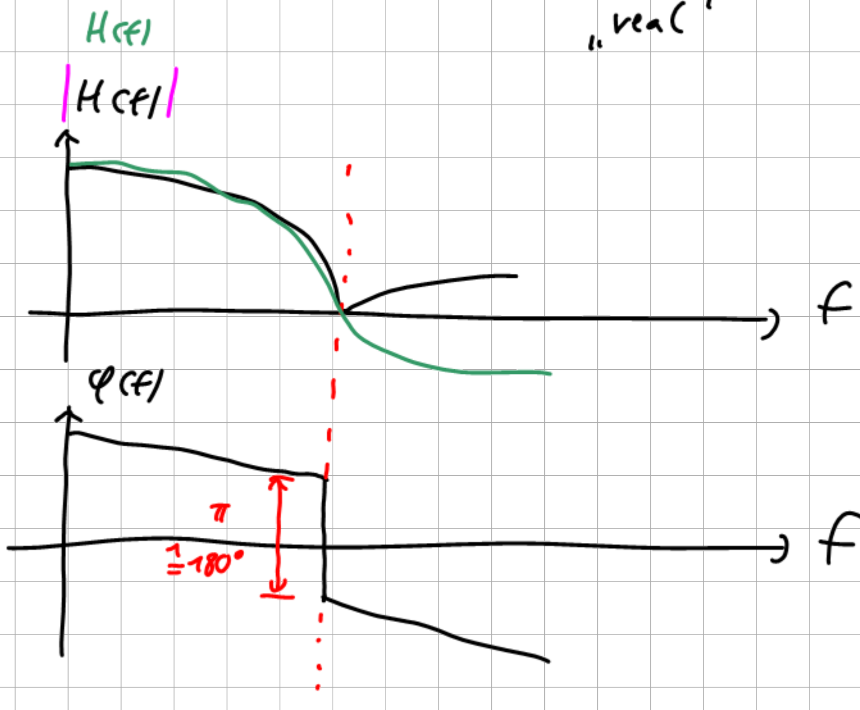
$$T = -\frac{\varphi(f)}{2\pi f} \qquad \text{Phase delay}$$

$$\varphi(f) = -2\pi T \cdot f$$

$$T = 3ns \quad \Rightarrow$$



Unwrapping ≙ rekonstruiere $\underbrace{\text{schwarze Phase}}_{\text{„real"}}$ aus $\underbrace{\text{rote Phase}}_{\text{„gemessen / angezeigt"}}$



Phasensprung von π
entspricht ein
Vorzeichenwechsel

$$e^{-j\pi} = -1$$