Statistics and Sensor Data Fusion

– Winter Term 2023/2024 –

Worksheet 7

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Exercise 1. The time X necessary to complete a multi-level production order in minutes is assumed to be a normally distributed random variable, i.e. $X \sim \mathcal{N}(\mu, \sigma^2)$. A recorded random sample provides the following values:

$$158, 135, 128, 140, 162, 128, 132, 136, 148, 140.$$

a) Without knowing both the mean μ and the variance σ^2, determine estimates for these parameters based on the corresponding unbiased estimators.

b) Now, for the true variance it should hold that $\sigma^2 = 9$. Use the corresponding estimate from a) to construct a symmetric confidence interval for the unknown mean μ at a confidence level of $1 - \alpha = 0.95$.

c) How many measurement values would be necessary to halve the length of the confidence interval from b) in the case that the confidence level remains unchanged?

d) The variance σ^2 is now assumed to be unknown. Recompute the confidence interval from b) under this condition and compare the result with the result from b).

e) The variance σ^2 is again considered to be unknown. Compute a one-sided (lower) confidence interval at a level of significance of $\alpha = 0.1$.

Exercise 2. Consider the observed realizations

$$\boldsymbol{x}_1 = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}, \quad \boldsymbol{x}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \quad \boldsymbol{x}_3 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad \boldsymbol{x}_4 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$$

of a three-dimensional random vector

$$\boldsymbol{X} = (X_1, X_2, X_3)^T$$

and determine unbiased estimates of the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$ of the underlying multivariate probability distribution.

Exercise 3. A parallel fusion network consists of four sensors S_1, S_2, S_3, S_4 which are characterized by the local error probabilities

$$p_{f_1} = 0.1, \quad p_{f_2} = 0.2, \quad p_{f_3} = 0.3, \quad p_{f_4} = 0.4$$

$$p_{m_1} = 0.3, \quad p_{m_2} = 0.3, \quad p_{m_3} = 0.2, \quad p_{m_4} = 0.1$$

According to the relative frequency of the presence of targets in the surveillance area, the prior probabilities of $\mathcal{H}_0$ and $\mathcal{H}_1$ are given by

$$\pi_0 = 0.8 \quad \text{and} \quad \pi_1 = 0.2$$

During operation, the fusion center receives the local decisions

$$u_1 = 1, \quad u_2 = 0, \quad u_3 = 0, \quad u_4 = 1$$

What would be the result $u_0 \in \{0, 1\}$ of the optimal fusion rule?