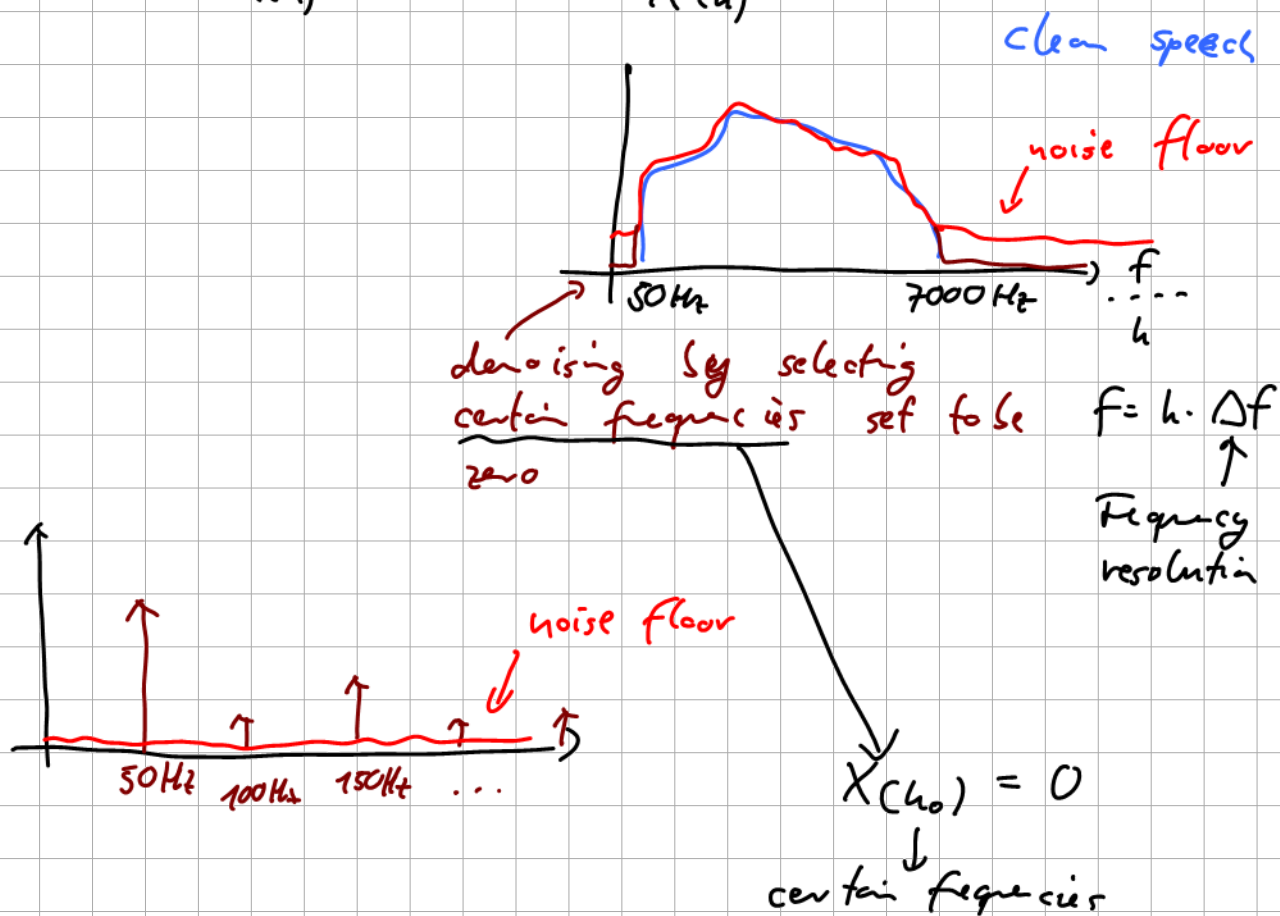


Removing noise by transforms?

time domain signal \longleftrightarrow frequency domain

$x(n)$ \longleftrightarrow $X(k)$



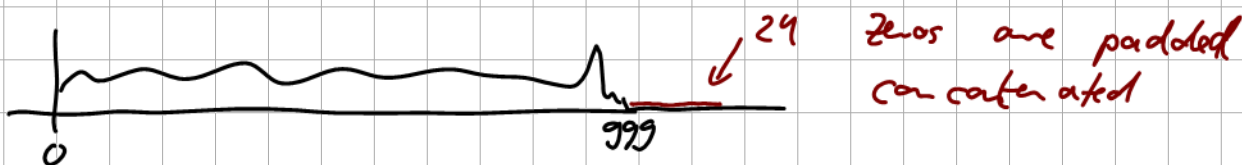
$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi \frac{nk}{N}}$$

zero padding is used if $K > N$

Blocklength

$N = 1000$ \Rightarrow $K = 1024 = 2^{10}$ \Rightarrow which results in faster evaluation (especially for small devices)

length of $x(n)$ length of $X(k)$

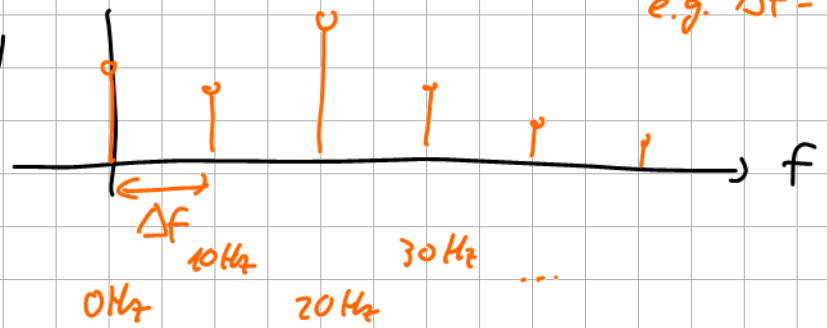


$$\Delta f = \frac{1}{K \cdot T} = \frac{f}{K}$$

the greater K , the finer the frequency resolution

$$x[n] \rightarrow X[k]$$

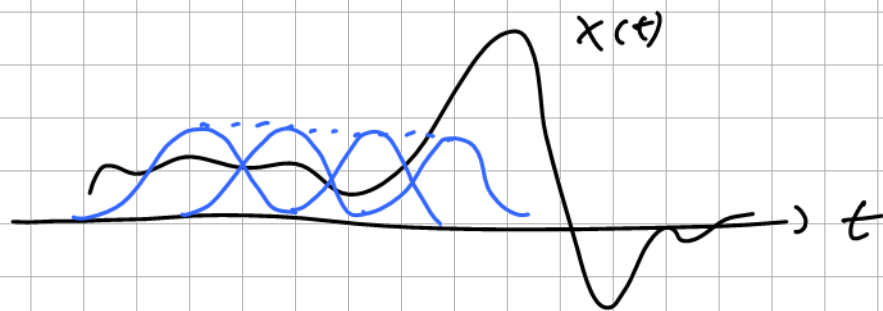
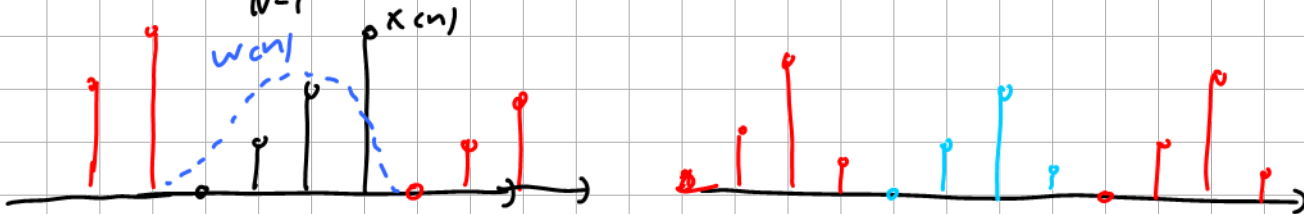
$$\text{e.g. } \Delta f = 10 \text{ Hz}$$



DFT assumes periodicity in time domain

$N=4$

$w[n]$



Symmetry

$$X[k] = X^*[K-k]$$

$$r = 16 \text{ kHz}$$

$$N=K=8$$

$$\left. \begin{array}{l} r = 16 \text{ kHz} \\ N=K=8 \end{array} \right\} \Delta f = \frac{r}{K} = 2 \text{ kHz}$$

$$x[n] = \text{[impulse response]}$$

$$X[k] = \text{[frequency response]}$$

$$k = 0 \quad 1 \quad 2 \quad 3 \quad \dots \quad 7$$

$$f = 0 \text{ kHz} \quad 2 \text{ kHz} \quad 4 \text{ kHz} \quad 6 \text{ kHz}$$

$$8 \text{ kHz} \quad 10 \text{ kHz} \quad 12 \text{ kHz} \quad 14 \text{ kHz}$$

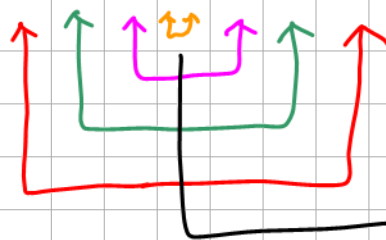
$$X[k] = X^*[8-k]$$

$$X[1] = X^*[7]$$

$$X[2] = X^*[6]$$

$$X[3] = X^*[5]$$

$$X[4] = X^*[4] \Rightarrow \text{real valued}$$



Nyquist frequency

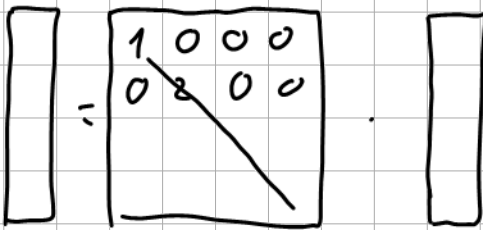
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nk}{N}}$$



$$x(n) = 2 \ 1 \ 2 \ 1$$

$$w(n) = 1 \ 2 \ 3 \ 2$$

$$x_w(n) = w(n) \cdot x(n)$$



$$\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{aligned} 1 \cdot 2 + 0 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 &= 2 \\ 0 \cdot 2 + 2 \cdot 1 + 0 \cdot 2 + 0 \cdot 1 &= 2 \\ 0 \cdot 2 + 0 \cdot 1 + 3 \cdot 2 + 0 \cdot 1 &= 6 \\ 0 \cdot 2 + 0 \cdot 1 + 0 \cdot 2 + 2 \cdot 1 &= 2 \end{aligned}$$

$$x_w = T_w \cdot x$$

$$X = T_{DFT} \cdot x_w$$

$$= T_{DFT} \cdot T_w \cdot x$$

$$= T \cdot x$$

↓

$$T_{DFT} \cdot T_w \stackrel{!}{=} \text{np.matmul}(T_{DFT}, T_w)$$











