DC Motor - Part 2

Actuators - IRO6

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Dynamic equation system of the direct current machine

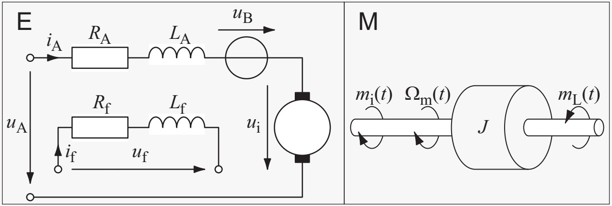

Figure Equivalent circuit diagram of DC motor

Three energy storages:

$$u_A(t) = R_A \cdot i_A(t) + L_A \frac{di_A}{dt} + u_i \qquad (3.1)$$

$$u_f(t) = R_f \cdot i_f(t) + \frac{d}{dt}(L_f i_f) \qquad (3.2)$$

$$m_i(t) = m_L(t) + M_{Fric} + J \cdot \frac{d\Omega_m}{dt} \qquad (3.3)$$

$$u_i(t) = c\Phi_f \cdot \Omega_m \qquad (3.4)$$

$$\Phi_f = f(i_f(t)) \qquad (3.5)$$

$$m_i(t) = c\Phi_f \cdot i_A(t) \qquad (3.6)$$

Dynamic equation system of the direct current machine I

Solving for the differentials:

$$\frac{L_\mathrm{A}}{R_\mathrm{A}}\frac{\mathrm{d}i_\mathrm{A}}{\mathrm{d}t} = -i_\mathrm{A}(t) + \frac{u_\mathrm{A}(t)}{R_\mathrm{A}} - \frac{c\Phi_\mathrm{f}(t)}{R_\mathrm{A}}\cdot\Omega_\mathrm{m}(t) \quad \Big| : I_\mathrm{AN} \qquad (3.7\mathrm{a})$$

$$\frac{N_\mathrm{f}}{R_\mathrm{f}}\frac{\mathrm{d}\Phi_\mathrm{f}}{\mathrm{d}t} = -i_\mathrm{f}(t) + \frac{1}{R_\mathrm{f}}u_\mathrm{f}(t) \quad \Big| : I_\mathrm{fN} \qquad (3.7\mathrm{b})$$

$$J\frac{\mathrm{d}\Omega_\mathrm{m}}{\mathrm{d}t} = c\Phi_\mathrm{f}(t)\cdot i_\mathrm{A}(t) - m_\mathrm{L}(t) - M_\mathrm{Fric} \quad \Big| : M_\mathrm{N} \qquad (3.7\mathrm{c})$$

Variables relative to nominal values with the abbreviations (additional index n):

$$u_\mathrm{An}(t) = \frac{u_\mathrm{A}(t)}{U_\mathrm{AN}} \qquad i_\mathrm{An}(t) = \frac{i_\mathrm{A}(t)}{I_\mathrm{AN}} \qquad T_\mathrm{A} = \frac{L_\mathrm{A}}{R_\mathrm{A}} \qquad r_\mathrm{A} = \frac{R_\mathrm{A}}{U_\mathrm{AN}/I_\mathrm{AN}}$$

$$u_\mathrm{fn}(t) = \frac{u_\mathrm{f}(t)}{U_\mathrm{fN}} \qquad i_\mathrm{fn}(t) = \frac{i_\mathrm{f}(t)}{I_\mathrm{fN}} \qquad T_\mathrm{fN} = \frac{L_\mathrm{fN}}{R_\mathrm{f}} \qquad r_\mathrm{f} = \frac{R_\mathrm{f}}{U_\mathrm{fN}/I_\mathrm{fN}}$$

$$\Phi_\mathrm{fn}(t) = \frac{\Phi_\mathrm{f}(t)}{\Phi_\mathrm{fN}} \qquad N_\mathrm{f}\Phi_\mathrm{fN} = L_\mathrm{fN}I_\mathrm{fN}$$

$$\Omega_\mathrm{mn}(t) = \frac{\Omega_\mathrm{m}(t)}{\Omega_\mathrm{m0N}} \qquad m_\mathrm{Ln}(t) = \frac{m_\mathrm{L}(t) + M_\mathrm{Fric}}{M_\mathrm{N}} \qquad T_\mathrm{J} = \frac{\Omega_\mathrm{m0N}\cdot J}{M_\mathrm{N}}$$

$$U_\mathrm{AN} = c\Phi_\mathrm{fN}\cdot\Omega_\mathrm{m0N} \qquad M_\mathrm{N} = c\Phi_\mathrm{fN}\cdot I_\mathrm{AN}$$

Dynamic equation system of the direct current machine II

The system of differential equations can thus be formulated in relative quantities:

$$T_\mathrm{A}\frac{\mathrm{d}i_\mathrm{An}}{\mathrm{d}t} = -i_\mathrm{An}(t) + \frac{u_\mathrm{An}(t)}{r_\mathrm{A}} - \frac{c\Phi_\mathrm{fn}(t)}{r_\mathrm{A}}\cdot\Omega_\mathrm{mn}(t) \tag{3.8a}$$

$$T_\mathrm{f}\frac{\mathrm{d}\Phi_\mathrm{fn}}{\mathrm{d}t} = -i_\mathrm{fn}(t) + \frac{u_\mathrm{fn}(t)}{r_\mathrm{f}} \tag{3.8b}$$

$$T_\mathrm{J}\frac{\mathrm{d}\Omega_\mathrm{mn}}{\mathrm{d}t} = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) - m_\mathrm{Ln}(t) \tag{3.8c}$$

$$u_\mathrm{in}(t) = \Phi_\mathrm{fn}(t)\cdot\Omega_\mathrm{mn}(t) \tag{3.8d}$$

$$\Phi_\mathrm{fn} = f(i_\mathrm{fn}(t)) \tag{3.8e}$$

$$m_\mathrm{in}(t) = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) \tag{3.8f}$$

with Eq. 3.8a - Eq. 3.8f **physical block model**
(= model exclusively with integrators) in Figure 3.2:

Physical block model of the DC motor

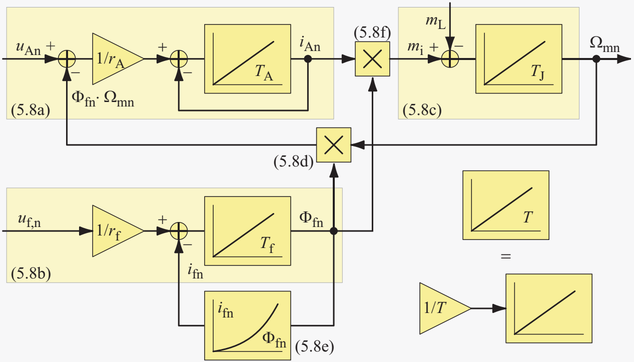

Figure Standard physical block model of the DC machine

- Consideration in the Laplace domain not more useful?

⇒ Saturation!

⇒ Multiplier!

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Discretization of the equations of state

- To describe dynamic processes:

⇒ Laplace if linear

⇒ Simulation if nonlinear

- numerical methods (solvers): e.g. Runge-Kutta → ODE45 in Simulink
- here: Recursion equations according to Euler-Cauchy (ODE1, backward Euler): Quotient of **differentials** → Quotient of **differences**

$$\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} = \lim_{T\to 0} \frac{y(t_k+T)-y(t_k)}{T}$$

Notation:

$$\left.\begin{array}{ll} y_k & := y(t_k) \\ y_{k+1} & := y(t_k+T) \end{array}\right\} \Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} \approx \frac{y_{k+1}-y_k}{T}$$

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Discretization of armature voltage equation

Differential quotient $\frac{\mathrm{d}i_\mathrm{A}}{\mathrm{d}t} \approx$ Difference quotient $\frac{i_{\mathrm{A},k+1} - i_{\mathrm{A},k}}{T}$:

$$T_\mathrm{A} \frac{\mathrm{d}i_\mathrm{An}}{\mathrm{d}t} = -i_\mathrm{An}(t) - \frac{u_\mathrm{An}(t)}{r_\mathrm{A}} - \frac{c\Phi_\mathrm{fn}(t)}{r_\mathrm{A}} \cdot \Omega_\mathrm{mn}(t) \qquad ((3.8a))$$

$$\frac{T_\mathrm{A}}{T}(i_{\mathrm{A},k+1} - i_{\mathrm{A},k}) = -i_{\mathrm{A},k} - \frac{u_{\mathrm{A},k}}{r_\mathrm{A}} - \frac{c\Phi_{\mathrm{f},k}}{r_\mathrm{A}} \cdot \Omega_{\mathrm{m},k}$$

⇒ Recursion equation $i_{\mathrm{A},k+1} = i_{\mathrm{A},k} + \ldots$ for numerical integration:

$$i_{\mathrm{A},k+1} = \left(1 - \frac{T}{T_\mathrm{A}}\right) i_{\mathrm{A},k} - \frac{T}{T_\mathrm{A} r_\mathrm{A}} (u_{\mathrm{A},k} - c\Phi_{\mathrm{f},k} \cdot \Omega_{\mathrm{m},k}) \qquad (3.9a)$$

- right side: values from the last (k-th) calculation step
- left side: new (($k + 1$)-th) value

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Discretization of field and motion equation

Field excitation circuit:

$$T_\mathrm{f}\frac{\mathrm{d}\Phi_\mathrm{fn}}{\mathrm{d}t} = -i_\mathrm{fn}(t) + \frac{u_\mathrm{fn}(t)}{r_\mathrm{f}} \qquad ((3.8b))$$

$$\frac{T_\mathrm{f}}{T}\left(\Phi_{\mathrm{f},k+1} - \Phi_{\mathrm{f},k}\right) = -i_{\mathrm{f},k} + \frac{u_{\mathrm{f},k}}{r_\mathrm{f}}$$

$$\Rightarrow \qquad \Phi_{\mathrm{f},k+1} = \Phi_{\mathrm{f},k} + \frac{T}{T_\mathrm{f}}\left(\frac{u_{\mathrm{f},k}}{r_\mathrm{f}} - i_{\mathrm{f},k}\right) \qquad (3.9b)$$

Equation of motion:

$$T_\mathrm{J}\frac{\mathrm{d}\Omega_\mathrm{mn}}{\mathrm{d}t} = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) - m_\mathrm{Ln}(t) \qquad ((3.8c))$$

$$\frac{T_\mathrm{J}}{T}\left(\Omega_{\mathrm{m},k+1} - \Omega_{\mathrm{m},k}\right) = \Phi_{\mathrm{f},k}\cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}$$

$$\Rightarrow \qquad \Omega_{\mathrm{m},k+1} = \Omega_{\mathrm{m},k} + \frac{T}{T_\mathrm{J}}\left(\Phi_{\mathrm{f},k}\cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}\right) \qquad (3.9c)$$

Discrete. Equation system of the direct current machine

Simulation process:

1. Definition of constants:
   $T_\mathrm{A} = \ldots,\ T_\mathrm{f} = \ldots,\ T_\mathrm{J} = \ldots,\ T = \ldots (< T_\mathrm{A}/10),\ r_\mathrm{A} = \ldots,\ r_\mathrm{f} = \ldots (\approx 1)$
   $\tau_\mathrm{A} = T_\mathrm{A}/T,\ \tau_\mathrm{f} = T_\mathrm{f}/T,\ \tau_\mathrm{J} = T_\mathrm{J}/T$
2. Specify input variables:
   $u_{\mathrm{A},k} = \ldots,\ u_{\mathrm{f},k} = \ldots,\ m_{\mathrm{L},k} = \ldots,\ k = 0 \ldots k_\mathrm{max}$
3. Initial values and default settings:
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0} \cdot i_{\mathrm{A},0}$
4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. Increment index k and check termination condition,
   otherwise back to 4.

Example: Excitation voltage is reduced linearly from $u_\mathrm{fn} = 1 \rightarrow 0.5$ in $0.5\,s$.

Parameters: $T_\mathrm{A} = 10\,ms,\ T_\mathrm{f} = 200\,ms,\ T_\mathrm{J} = 800\,ms,\ T = 2\,ms,\ r_\mathrm{A} = 0.04,\ r_\mathrm{f} = 1$

Result from Matlab simulation

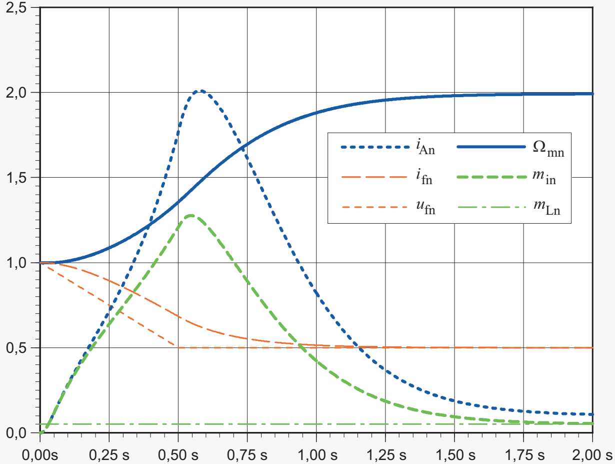

Figure Speed adjustment through field weakening

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Additional simulation

The motor should switch to half the nominal idle speed at 2 s?

1. What armature and excitation voltage are required for this?
2. What happens if both values are changed at the same time?
3. How can the armature current peak be reduced?

Cascade control with field weakening

- Subordinate current controls
- Setpoint $i_{f,Set}$ from Ω_m via characteristic curve

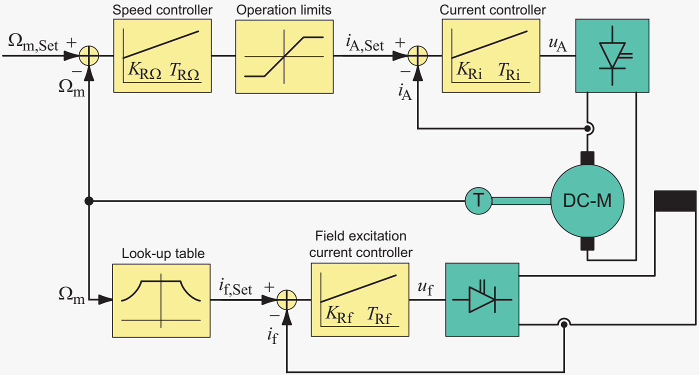

Figure Speed control with field weakening

Time-discrete PI controller

- All three controllers: PI controller (so stationary error → 0)
- Control deviation e as input variable → Control variable y as output variable

$$G_{\mathrm{PI}} = \frac{y(s)}{e(s)} = K_{\mathrm{R}} \cdot \frac{1 + sT_{\mathrm{R}}}{sT_{\mathrm{R}}}$$

$$sT_{\mathrm{R}} \cdot y(s) = K_{\mathrm{R}} \cdot e(s) + sT_{\mathrm{R}} K_{\mathrm{R}} \cdot e(s)$$

$$\Rightarrow T_{\mathrm{R}} \frac{\mathrm{d}y(t)}{\mathrm{d}t} = K_{\mathrm{R}} \cdot e(t) + T_{\mathrm{R}} K_{\mathrm{R}} \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

$$\text{discretize: } T_{\mathrm{R}} \frac{y_{k+1} - y_k}{T} = K_{\mathrm{R}} \cdot e_k + T_{\mathrm{R}} K_{\mathrm{R}} \frac{e_{k+1} - e_k}{T}$$

$$y_{k+1} = y_k + K_{\mathrm{R}}\, e_{k+1} - K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right) e_k$$

$$\Rightarrow y_{k+1} = y_k + q_0 e_{k+1} + q_1 e_k \qquad \text{mit } q_0 = K_{\mathrm{R}} \qquad (3.10)$$

$$q_1 = K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right)$$

Simulation process

1. Definition of constants
   $T_A = (10\,ms),\ T_f = (100\,ms),\ T_J = (800\,ms),\ T = (1\,ms),\ r_A = (0.04),\ r_f = (1)$
   $\tau_A = T_A/T,\ \tau_f = T_f/T,\ \tau_J = T_J/T$
   $K_{R\Omega} = (20),\ T_{R\Omega} = (100\,ms),\ K_{Ri} = (0.5),\ T_{Ri} = (10\,ms)$
   $K_{Rf} = (1),\ T_{Rf} = (50\,ms),\ i_{Amax} = (2),\ u_{Amax} = (1.2),\ u_{fmax} = (1)$
2. Specify input variables:
   $\Omega_{mSet,k} = 1 \ldots 2 \ldots,\ m_{L,k} = \ldots,\ k = 0 \ldots k_{max}$
3. Initial values and default settings:
   $k = 0: \quad i_{A,0} = \ldots,\ \Phi_{f,0} = \ldots,\ \Omega_{m,0} = \ldots,\ i_{f,0} = f(\Phi_{f,0});\ m_{i,0} = \Phi_{f,0} \cdot i_{A,0}$
   $i_{ASet,0} = \ldots,\ u_{A0} = \ldots,\ u_{f,0} = \ldots,\ e_{\Omega,0} = \ldots,\ e_{i,0} = \ldots,\ e_{f,0} = \ldots$
4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. „Calculate“control laws for 3(!) controllers and limit set variables
6. Increment index and check termination condition, otherwise return to 4.

Setpoint jump $0 \ldots 2n_N$

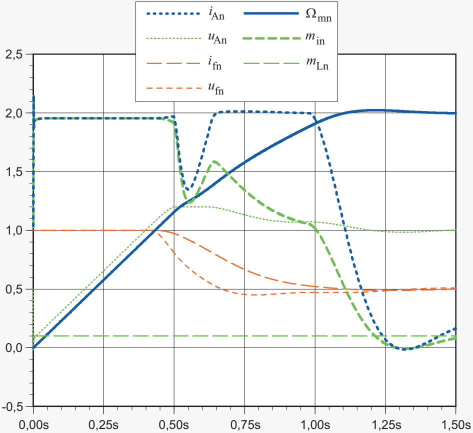

Figure Setpoint jump $0 \ldots 2n_N$

- till n_N: $u_{fn} = 1$
- $n > n_N$: $u_{fn} \downarrow$
- $u_f \to i_f$: PT1 behavior!
- $u_i(t)$ too large

⇒ Voltage reserve is not enough for $2I_{AN}$

- only shortly before n_{Set} is u_A slightly withdrawn
- i_{An} settles down to $2m_{Ln} = 0.2$.
- Nominal speed after 350 ms, setpoint reached after well 1 s
- without simulation: no chance

Additional simulations

1. In the previous simulation, how large does the armature voltage reserve have to be in order to operate permanently with double armature current?
2. The motor should now be switched off in a controlled way from nominal operation.