

$$1) \text{ DFT: } X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nk}{N}}$$

$$X(k-l) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{n}{N} (k-l)}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{n}{N} k + j2\pi \frac{n}{N} l}$$

$$= \sum_{n=0}^{N-1} x(n) \underbrace{e^{-j2\pi n}}_{=1 \text{ for } n=0, 1, 2, \dots} e^{j2\pi \frac{nl}{N}}$$

$$= \sum_{n=0}^{N-1} x(n) e^{j2\pi \frac{nl}{N}}$$

$$X^*(k-l) = \left(\sum_{n=0}^{N-1} x(n) e^{j2\pi \frac{nl}{N}} \right)^*$$

$$= \sum_{n=0}^{N-1} x^*(n) \left(e^{j2\pi \frac{nl}{N}} \right)^*$$

$\hookrightarrow x(n)$ is assumed to be real valued

$$= \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nl}{N}} = x(n)$$

$$2) \quad x(n) = \hat{x} \sin(2\pi f \cdot nT) \quad \text{with } T = \frac{1}{f}$$

$$= 1 \cdot \sin\left(2\pi \frac{440 \text{ Hz}}{1000 \text{ Hz}} \cdot n\right)$$

$$x(0) = 0$$

$$x(1) = \sin\left(2\pi \frac{440}{1000}\right) = 0,368$$

$$x(2) = \sin\left(2\pi \frac{440}{1000} \cdot 2\right) = -0,685$$

$$X(k) = \sum_{n=0}^2 x(n) \cdot e^{-j2\pi \frac{nk}{3}}$$

$$X(0) = 0 \cdot \underbrace{e^{-j2\pi \frac{0 \cdot 0}{3}}}_{=1} + 0,368 \cdot \underbrace{e^{-j2\pi \frac{1 \cdot 0}{3}}}_{=1} - 0,685 \cdot \underbrace{e^{-j2\pi \frac{2 \cdot 0}{3}}}_{=1}$$

$$= 0 + 0,368 - 0,685 = -0,316$$

$$X(1) = 0 \cdot e^{-j2\pi \frac{0 \cdot 1}{3}} + 0,368 e^{-j2\pi \frac{1 \cdot 1}{3}} - 0,685 \cdot e^{-j2\pi \frac{2 \cdot 1}{3}}$$

$$= 0,158 - j 0,912$$

$$X(2) = X^*(1) = 0,158 + j 0,912$$

Parseval: $E_x = \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{K} \sum_{k=0}^{K-1} |X(k)|^2$

$$\sum_{n=0}^{N-1} |x(n)|^2 = 0^2 + 0,368^2 + 0,685^2 = 0,604$$

$$\frac{1}{K} \sum_{k=0}^{K-1} |X(k)|^2 = \frac{1}{3} \left[(-0,316)^2 + (0,158^2 + 0,912^2) \cdot 2 \right]$$

$$= 0,604$$

$$3) - \Delta f = \frac{1}{K \cdot T} = \frac{r}{K} = \frac{48000}{1024} \text{ Hz} = 46,9 \text{ Hz}$$


- yes, zero padding is used because of: $N < K$
(without zero padding it must be $N = K$)

- $l=0 : f = l \cdot \Delta f = 0 \text{ Hz}$

$l=1 : f = l \cdot \Delta f = 46,9 \text{ Hz}$ ← for $l=1$, the frequency 50 Hz is analyzed

$l=2 : f = l \cdot \Delta f = 93,8 \text{ Hz}$

- assuming $K=8$:



3 coefficients can be dropped $\hat{=} \frac{K}{2} - 1$

$\Rightarrow K = 1024 \Rightarrow \frac{K}{2} - 1 = 511$ coefficients can be dropped

4) Advantages

- very fast
- linear frequency resolution is simple to analyze

Disadvantages

- periodicity in time domain is assumed
→ window functions are necessary
- spectrum is analyzed only for a set of (discrete) frequencies, depending on Δf
- logarithmic frequency resolution, e.g. for Bode plots or psychoacoustic measurements, are not straightforward to evaluate











