3 DC Motor

DC motors are mostly built in very large numbers as small motors. They are often used where a simple direct current source (battery, accumulators) is available. As universal motors, they are used on the AC mains in household appliances and power tools.

Just a few decades ago, the DC machine was practically the only electrical machine that could be easily controlled. Despite the progressing development in the field of three-phase power converters, DC drives with powers sometimes in the MW range are still used today.

3.1 Basic structure

The fixed part or stator of the DC machine consists of the yoke and the exciter or field poles, which hold the excitation or field winding. In the case of permanent-magnetically excited DC motors, the excitation poles are mostly realized with radially magnetized permanent magnets (see Fig. 3.1).

The yoke itself can be made of cast or rolled steel and, in addition to the magnetic return of the magnetic circuit, also serves as a mechanical supporting structure. The yoke and the exciter poles only have to be laminated due to the eddy current losses if the DC machine is operated with AC (universal motor) or if the current in the excitation winding is frequently adjusted. For industrial machines, the yoke and the exciter poles are also laminated for manufacturing reasons.

The rotor, which is often also called the armature in DC machines, is always laminated and accommodates the armature winding. The coils of the armature winding are placed in slots. The current is supplied to the armature coils via brushes made of graphite, which are mechanically connected to the housing via brush holders containing springs pressed against the commutator with a pressure of 2 to $2.5\,\mathrm{N/cm^2}$.

The commutator consists of copper segments, which are isolated from one another by thin mica sheets and mechanically connected to the rotor shaft by a pressed construction. The individual copper segments of the commutator are electrically

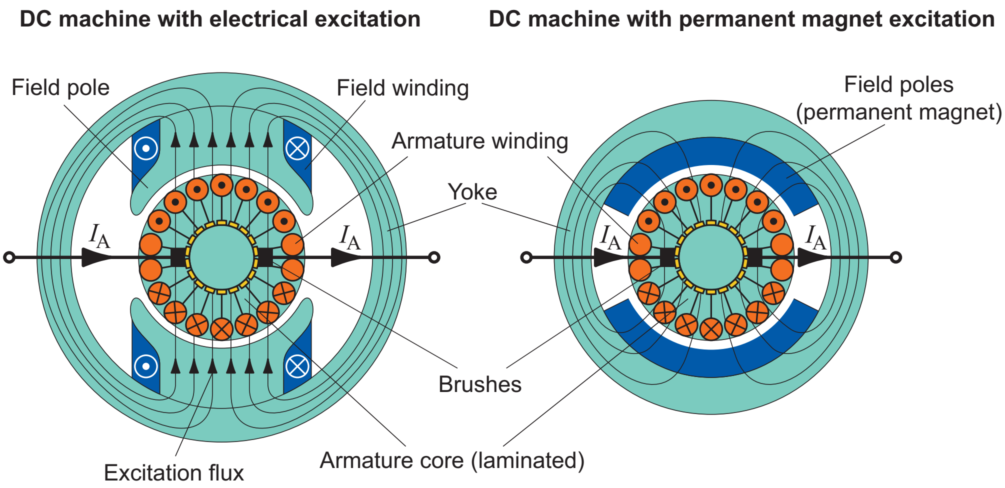

Figure 3.1: Electrically (left) and permanent magnet excited (right) DC motor

connected to the coil ends of the armature winding.

The commutator ensures that the currents in the individual rotor conductors always have the required direction, even when the armature is rotating.

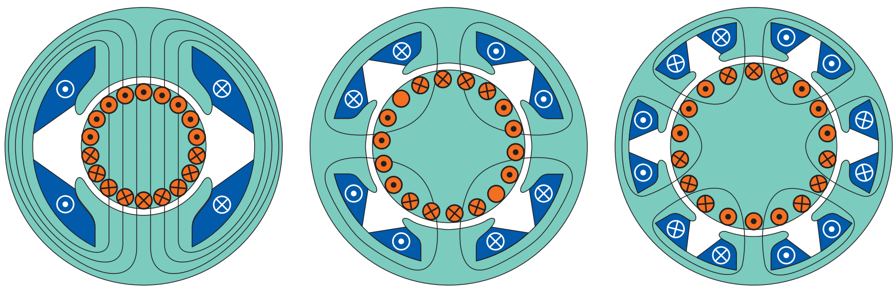

Figure 3.2: Structure of DC motors with number of poles $2p = 2, 4, 6$

In motor operation, the commutator acts like a mechanical inverter; in generator mode, however, like a rectifier. DC machines are often designed with four poles (2 pole pairs, number of pole pairs $p = 2$; Fig. 3.2) or six poles (3 pole pairs, number of pole pairs $p = 3$). This means that the structure of the DC machine is repeated $2p$ times around the circumference, with the +brushes and the -brushes having to be connected in parallel.

Since the overall magnetic flux of the machine is divided into $2p$ partial fluxes, the yoke cross-sections can be reduced. At the same time, the winding-ends of the field and armature windings also become smaller.

3.2 Armature construction

The winding construction in the armature must be designed so that the current direction under the excitation poles remains constant regardless of the rotor position: The conductors between the poles (shown dashed in Fig. 3.3) in the so-called neutral zone lie in the field-free space and therefore do not contribute to the torque.

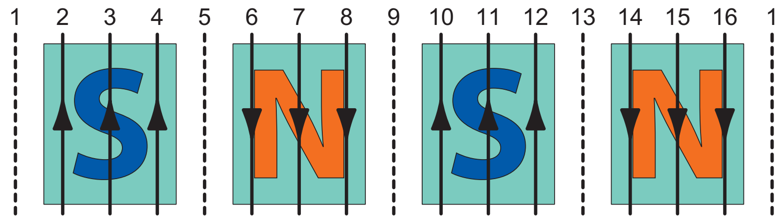

Figure 3.3: Current directions on a 4-pole machine with 16 slots

This means that the time these conductors are in the neutral zone can be used for commutation.

Image 3.4 shows an example of an armature winding.

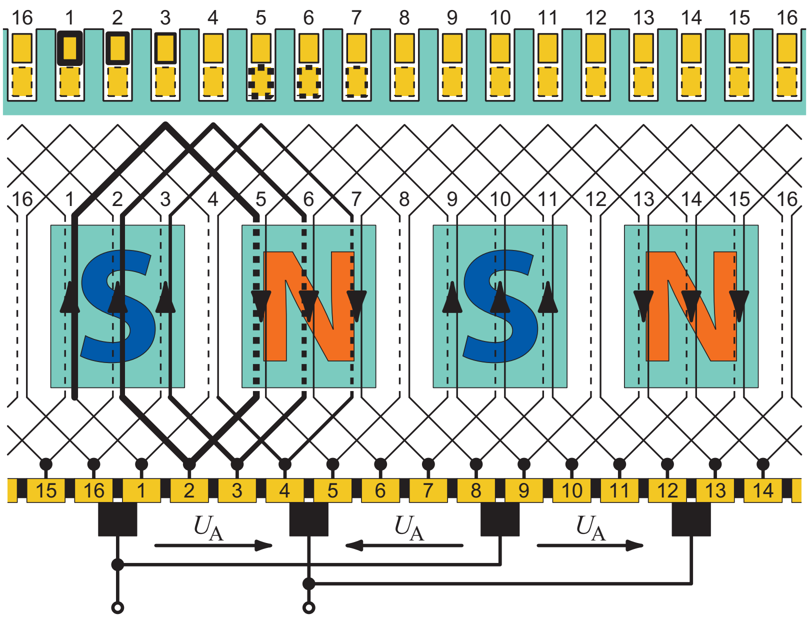

Figure 3.4: Winding diagram of an exemplary armature winding

Since the current in the neutral zone of the direct current machine is reversed electromechanically, there are disadvantages compared to AC machines:

- Brushes are subject to wear ⇒ Maintenance may be necessary;
- Maximum power is limited by speed and current-dependent brush fire (electric arc);
- Commutator requires extra overall length (higher moment of inertia);
- Additional losses due to brush friction and brush voltage drop ($\approx 0.6 ... 2\,\mathrm{V}$)

Picture 3.5 shows two different designs of direct current machines: an electrically excited direct current machine as used for industrial applications (left picture) and a small drive for a model railway. The dimensions in both applications are of course very different. The industrial drive has an output of approx. $20\,\mathrm{kW}$ and an axle height (= half the overall height) of $132\,\mathrm{mm}$ and a weight of approx. $160\,\mathrm{kg}$, whereas in the picture on the right the also electrically excited DC motor is built into the bogie of an H0 model locomotive with a gauge of $13.5\,\mathrm{mm}$.

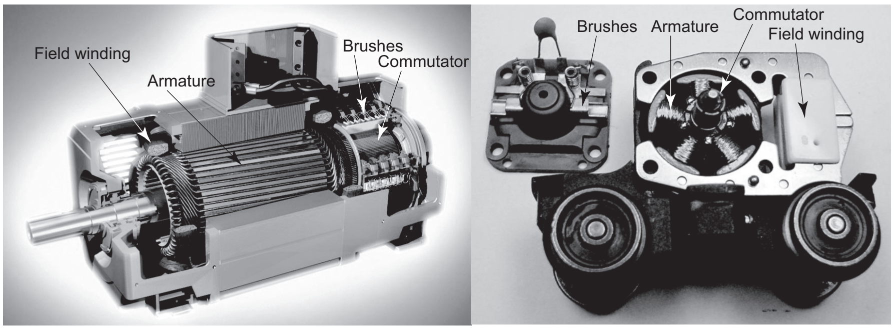

Figure 3.5: Example for an industrial and a micro drive (ABB/Märklin)

3.3 Induced voltage and Torque

In Fig. 3.6 the excitation field of a direct current machine with a slotless armature is plotted over two pole pitches.

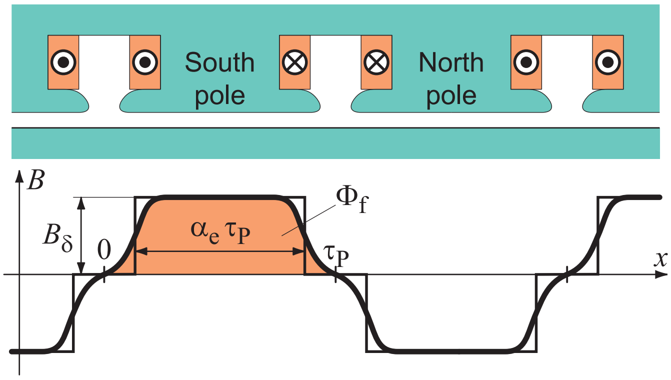

Figure 3.6: Excitation field and excitation flux

The excitation flux Φ_f then results as the integral of the air gap flux density B_δ over a pole pitch τ_P:

$$\Phi_f = l_{Fe} \cdot \int_0^{\tau_P} B(x)\,dx = l_{Fe} \cdot B_\delta \cdot \alpha_e \cdot \tau_P \qquad \text{mit } \tau_P = \frac{\pi\, d_{Re}}{2\,p} \tag{3.1}$$

α_e: ideal pole-covering

l_{Fe}: iron length

Due to the material properties of the iron core (B-H characteristic), the excitation flux depends on the excitation current:

$$\Phi_f = f(I_f) \tag{3.2}$$

In general, this relationship is non-linear and can be represented by a magnetization characteristic as shown in Figure 3.7.

The induced voltage in a single rotor bar of length l_{Fe}, moving under the pole with the speed v_A (armature circumferential speed) is:

$$u_{i,b} = l_{Fe} \cdot v_A \cdot B_\delta(x) \qquad \text{mit } v_A = \pi \cdot d_{Re} \cdot n$$

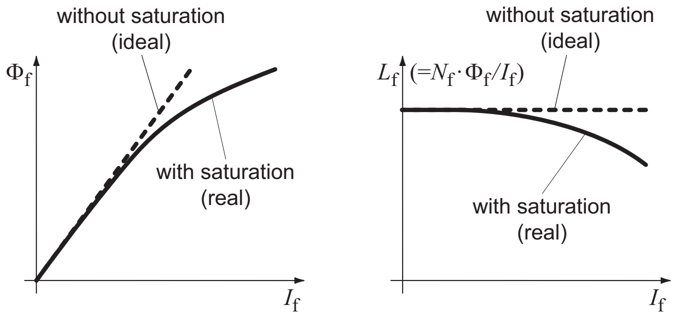

Figure 3.7: Excitation flux and inductance depending on the excitation current

The mean value for this time- or position-dependent voltage over half a period is then:

$$U_{i,b} = l_{Fe} \cdot v_A \cdot B_\delta \cdot \alpha_e \tag{3.3}$$

A coil connected to one of the commutator bars K has the number of turns N_{Coil}, so that the number of armature turns N_A relevant for the armature voltage is

$$N_A = \frac{K \cdot N_{Coil}}{2\,a}$$

The induced voltage, which can be measured directly at the brushes at no-load, is then:

$$U_i = 2 \cdot N_A \cdot U_{i,b}, \text{ (Factor 2 due to forward and return conductor)}$$

$$U_i = 2 \cdot N_A \cdot l_{Fe} \cdot \underbrace{\pi\, d_{Re} \cdot n}_{v_A} \cdot B_\delta \cdot \alpha_e \frac{2\,p}{2\,p}$$

$$U_i = \underbrace{\frac{\pi \cdot d_{Re}}{2\,p} \cdot l_{Fe} \cdot B_\delta \cdot \alpha_e}_{=\,\Phi_f} \cdot 2\,n \cdot N_A \cdot 2\,p = \underbrace{\frac{4\,p\,N_A}{2\,\pi}}_{=\,c} \cdot \Phi_f \cdot \underbrace{2\,\pi\,n}_{=\,\Omega_m}$$

$$\text{mit } c = \frac{4\,p\,N_A}{2\,\pi} = \frac{K}{a}\,\frac{p}{\pi}\,N_A$$

Resulting in the **1. Main Equation of the DC Machine**:

$$\boxed{U_i = c\,\Phi_f \cdot \Omega_m} \tag{3.4}$$

The torque of the DC machine can easily be determined from a power balance. The delivered mechanical power (including friction losses) must be equal to the electrical power:

$$\boxed{P_{i,mech} = P_{mech} + P_{Fric} = M_i \cdot \Omega_m = U_i \cdot I_A} \tag{3.5}$$

$$U_i \cdot I_A = c\,\Phi_f\,\Omega_m I_A \qquad \Rightarrow \qquad \boxed{M_i = c\,\Phi_f \cdot I_A} \tag{3.6}$$

The torque equation is also known as **2. Main Equation of the DC Machine**.

The winding resistance of the armature is the ohmic resistance between a plus and a minus brush. It can be determined using the copper losses:

$$P_{\mathrm{Cu}} = K\, N_{\mathrm{Coil}} \cdot \underbrace{\frac{2\, l_{\mathrm{av}}}{q_{\mathrm{C}}} \cdot \rho_{\mathrm{Cu}} \cdot \underbrace{\left(\frac{I_{\mathrm{A}}}{2\, a}\right)^2}_{\text{Current in one bar}}}_{\text{Losses in one bar}}$$

l_{av}: mean turn length

$l_{\mathrm{av}} \approx (l_{\mathrm{Fe}} + 0{,}9\tau_{\mathrm{P}})$ (approximation)

$$P_{\mathrm{Cu}} = K\, \underbrace{\frac{2\, a}{K}\, N_{\mathrm{A}}}_{N_{\mathrm{Sp}}} \cdot \frac{2\, l_{\mathrm{av}}}{q_{\mathrm{C}}} \cdot \rho_{\mathrm{Cu}} \cdot \left(\frac{I_{\mathrm{A}}}{2\, a}\right)^2 = \frac{N_{\mathrm{A}}}{2\, a}\, \frac{2\, l_{\mathrm{av}}}{q_{\mathrm{C}}}\, \rho_{\mathrm{Cu}} \cdot I_{\mathrm{A}}^2$$

$$\Rightarrow R_{\mathrm{A}} = \frac{N_{\mathrm{A}}}{2\, a}\, \frac{2\, l_{\mathrm{av}}}{q_{\mathrm{C}}}\, \rho_{\mathrm{Cu}} \tag{3.7}$$

This determination of the armature resistance can only be regarded as an estimate. The actual average winding length depends heavily on the design of the end winding.

For the complete voltage equation of the DC machine (**3rd Main Equation of the DC Machine**), the voltage drop at the brushes and, in the transient case, the corresponding inductances must also be taken into account:

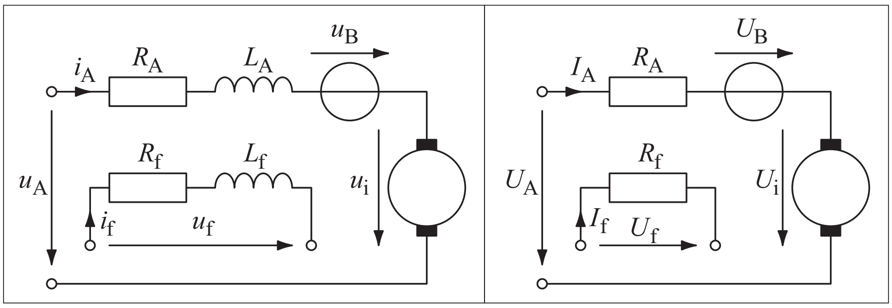

Figure 3.8: Transient (left) and steady-state (right) equivalent circuit of DC machine

In the shown equivalent circuit diagram, it is assumed that the DC machine takes up electrical power and delivers mechanical power when rotating clockwise (when looking at the output shaft). The mesh equations for the armature and exciter circuit result in:

$$\text{steady-state:} \quad U_{\mathrm{A}} = U_{\mathrm{i}} + U_{\mathrm{B}} \cdot \mathrm{sign}(I_{\mathrm{A}}) + R_{\mathrm{A}}\, I_{\mathrm{A}} \tag{3.8a}$$

$$\text{transient:} \quad u_{\mathrm{A}}(t) = u_{\mathrm{i}}(t) + U_{\mathrm{B}} \cdot \mathrm{sign}(i_{\mathrm{A}}(t)) + R_{\mathrm{A}}\, i_{\mathrm{A}}(t) + L_{\mathrm{A}} \frac{\mathrm{d}i_{\mathrm{A}}}{\mathrm{d}t} \tag{3.8b}$$

$$\text{steady-state:} \quad U_{\mathrm{f}} = R_{\mathrm{f}}\, I_{\mathrm{f}} \tag{3.8c}$$

$$\text{transient:} \quad u_{\mathrm{f}}(t) = R_{\mathrm{f}}\, i_{\mathrm{f}}(t) + L_{\mathrm{f}} \frac{\mathrm{d}i_{\mathrm{f}}}{\mathrm{d}t} \tag{3.8d}$$

If the induced voltage has the same sign as the armature current ($P_{\mathrm{i,mech}} = I_\mathrm{A} \cdot U_\mathrm{i} > 0$), electrical power is consumed and mechanical power is delivered, i.e. **motor**operation is present. However, if the induced voltage and the armature current have different signs ($P_{\mathrm{i,mech}} = I_\mathrm{A} \cdot U_\mathrm{i} < 0$), electrical power is delivered and mechanical power is consumed, i.e. the DC machine is consequently operated as a **generator**.

The sign for the induced voltage results from the direction of rotation of the machine. ($U_\mathrm{i} > 0 \Rightarrow$ clockwise rotation of the machine, $U_\mathrm{i} < 0 \Rightarrow$ anti-clockwise rotation of the machine), whereas the sign of the armature current corresponds to the direction of the torque.

3.4 Operating behavior

Figure 3.9 shows the four different ways in which the armature and excitation windings of a DC machine can be connected. The type of connection influences the operating behavior of the DC machine.

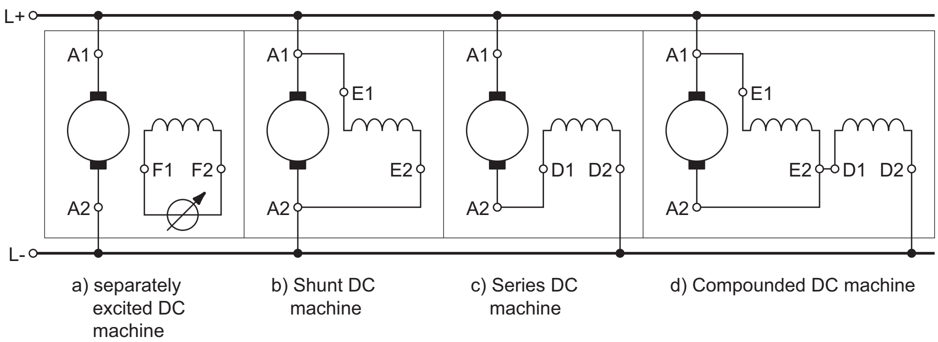

Figure 3.9: DC machine connections

With the **separately excited DC machine**, the excitation winding is fed by a separate voltage source, so that the excitation flux is independent of the armature and can be changed.

With the **shunt machine**, the field winding is parallel to the armature circuit. When operated on the mains, this leads to a constant and load-independent excitation flow.

With the **series machine**, the field winding is connected in series with the armature winding; the excitation flux is therefore load-dependent and theoretically equal to zero at no-load. However, this leads to a theoretically infinitely high no-load speed, which would inevitably lead to the destruction of the machine.

The **compounded machine** has an additional series winding compared to the shunt machine. With this additional winding, the negative effects of the armature reaction can be at least partially compensated.

3.4.1 Hazard warning

If the excitation winding is switched off or if the excitation current is too low, the speed of a direct current machine can assume impermissible high values. The machine can be destroyed by the centrifugal forces that occur! For this reason, the following **safety measures** regarding excitation must be observed with direct current machines:

- Always turn on the excitation current first and turn off the excitation current last!
- The excitation circuit must never be switched off during operation! Care must also be taken with the cables to ensure that the excitation winding cannot be accidentally interrupted.

For **robot systems** only direct current machines with **external excitation** are relevant due to the simple controllability. Therefore, only their operating behavior is described here.

3.4.2 Operating behavior of the separately excited DC machine

Starting from the 3rd main equation (Eq. 3.8a, brush transition voltage U_B neglected here), the 1st main equation (Eq. 3.4) can be used to substitute the induced voltage U_i by the angular velocity Ω_m and with the 2nd main equation (Eq. 3.6) the armature current can be replaced by the internal torque. With this, the dependency of the angular velocity Ω_m or the speed n with the load M_i can be described as:

$$U_A = U_i + R_A \cdot I_A + U_{Bü} \approx c\Phi_f \cdot \Omega_m + R_A \cdot \frac{M_i}{c\Phi_f}$$

$$\Omega_m = \frac{U_A}{c\Phi_f} - \frac{R_A}{(c\Phi_f)^2} M_i = \Omega_{m,0} - (\Omega_{m,0} - \Omega_{m,N}) \frac{M_i}{M_{i,N}} \tag{3.9a}$$

The result is a straight line equation $\Omega_m(M_i)$ with two terms:

- The first term being the quotient of armature voltage U_A and excitation flux $c\Phi_f$ is load-independent and can therefore be interpreted as **open-circuit / no-load angular velocity** $\Omega_{m,0}$. The no-load speed is therefore proportional to the armature voltage U_A and inversely proportional to the excitation flux $c\Phi_f$.
- The second term is proportional to the load torque M_i, i.e. if the load increases, the speed decreases accordingly. The reduction in speed at the same load (torque) increases if the armature resistance R_A increases, e.g. due to heating, or if the excitation flux $c\Phi_f$ is reduced to increase the no-load speed.

Instead of the excitation flux $c\Phi_f$, the field weakening factor f is often used as the inverse of the excitation flux $c\Phi_f$ relative to the nominal excitation flux $c\Phi_{fN}$:

$$\text{Field weakening factor } f := \frac{c\Phi_{fN}}{c\Phi_f} \tag{3.9b}$$

In Figure 3.10, the speed-torque characteristics result in a family of straight lines with the armature voltage U_A or the field weakening factor f as parameters. Two different areas can be distinguished:

- the **armature adjustment area or base speed area** (marked green in Figure 3.10), in which the speed can be adjusted via the armature voltage and

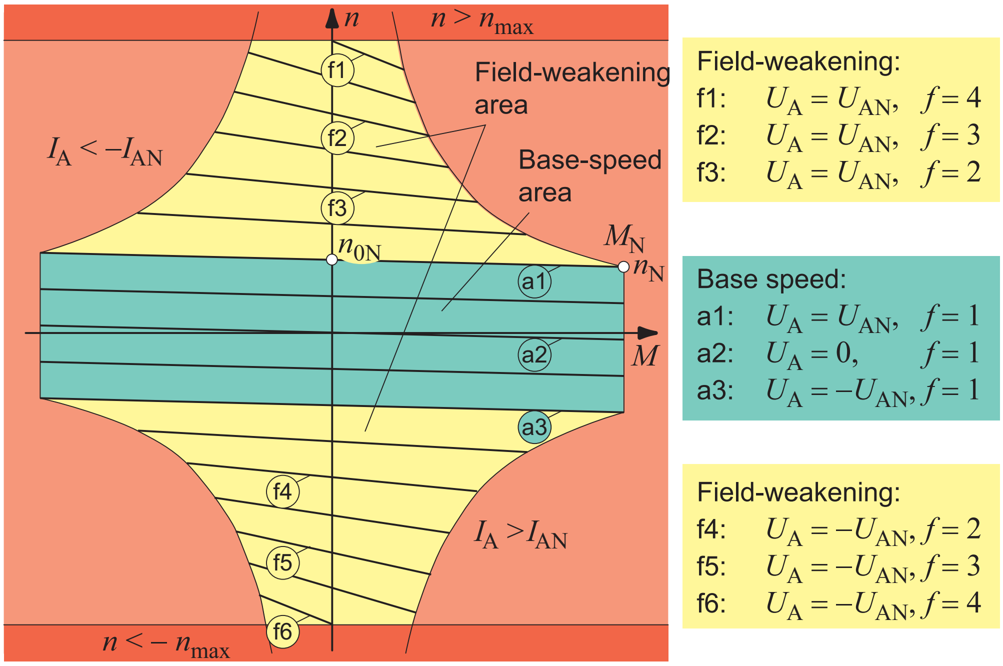

Figure 3.10: Operating diagram of the separately excited DC machine

- the **field weakening area** (marked yellow in Figure 3.10), in which the speed can be adjusted via the excitation current.

The operating limits for the machine are determined by the following factors:

- In continuous operation, the nominal current must not be exceeded due to the commutation and the permissible heating. This nominal current determines the maximum possible torque depending on the excitation flux.

- A maximum speed due to the construction of the armature must not be exceeded.

Speed can be adjusted in three different ways:

1. Changing the terminal voltage in armature adjustment area, $|n| < n_{0N},\ f = 1$. The speed is proportional to the terminal voltage and decreases slightly under load. (Shunt behavior: low speed drop under load, speed drop proportional to the torque). If the terminal voltage is changed, the slope of the $n = f(M)$ characteristic and also the maximum torque that can be delivered remain constant. (Characteristics a1 to a3 in Figure 3.10 are all parallel to each other). The direction of rotation can be reversed by changing the polarity of the terminal voltage (characteristic a1 for $U_A = +U_{AN}$ and a3 for $U_A = -U_{AN}$ in Fig 3.10).

2. Increasing the field weakening factor in field weakening range, $|n| > n_{0N}, U_A = U_{AN}$. The no-load speed increases as the field weakening factor f increases; Due to the proportionality between flux and torque, however, the torque decreases with the same current (characteristic curves f3 to f1 or f4 to f6 in figure 3.10).Reversing the direction of rotation by changing the polarity of the excitation winding is not permitted, since the excitation flux becomes very small,

at least for a short time, and the speed can increase to impermissible high values.

3. Series resistor in the armature circuit. This method is hardly used any more because of the losses in the series resistor and the fact that power electronics are meanwhile cheaper.

With **permanent magnetic** excitation instead of electrical excitation, the excitation flux Φ_f is constant. **Field weakening operation is not possible** and the machine must be designed in such a way that the voltage limit in the armature is only reached at the maximum required speed.

3.5 Dynamic behavior and control

The stationary behavior of the direct current machine has already been fully discussed. However, if transient processes are to be examined or a control system for a direct current machine is to be designed, the state of the energy storage must also be taken into account.

3.5.1 System of equations

The DC machine has three energy storages: the armature inductance L_A, the field excitation inductance L_f and the mass moment of inertia J. For this reason, three differential equations must be established to describe the dynamic behavior:

$$u_\mathrm{A}(t) = R_\mathrm{A} \cdot i_\mathrm{A}(t) + L_\mathrm{A}\frac{\mathrm{d}i_\mathrm{A}}{\mathrm{d}t} + u_\mathrm{i} \tag{3.10}$$

$$u_\mathrm{f}(t) = R_\mathrm{f} \cdot i_\mathrm{f}(t) + \frac{\mathrm{d}}{\mathrm{d}t}\left(L_\mathrm{f} i_\mathrm{f}\right) \tag{3.11}$$

$$m_\mathrm{i}(t) = m_\mathrm{L}(t) + M_\mathrm{Fric} + J \cdot \frac{\mathrm{d}\Omega_\mathrm{m}}{\mathrm{d}t} \tag{3.12}$$

However, these three equations are linked to each other:

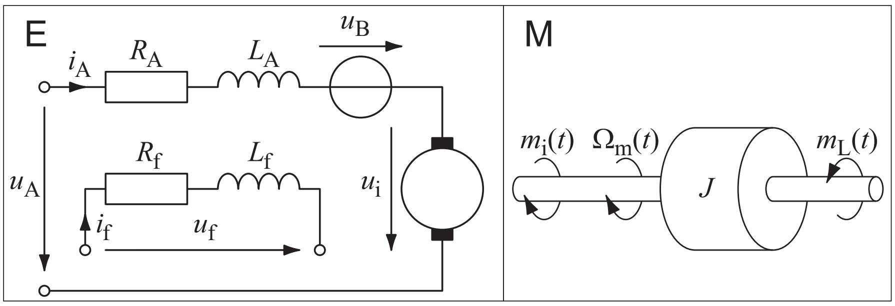

Figure 3.11: Equivalent circuit diagram of DC motor

$$u_\mathrm{i}(t) = c\Phi_\mathrm{f} \cdot \Omega_\mathrm{m} \tag{3.13}$$

$$\Phi_\mathrm{f} = f(i_\mathrm{f}(t)) \tag{3.14}$$

$$m_\mathrm{i}(t) = c\Phi_\mathrm{f} \cdot i_\mathrm{A}(t) \tag{3.15}$$

The three equations of state can be solved for the differentials:

$$\frac{L_\mathrm{A}}{R_\mathrm{A}}\frac{\mathrm{d}i_\mathrm{A}}{\mathrm{d}t} = -i_\mathrm{A}(t) + \frac{u_\mathrm{A}(t)}{R_\mathrm{A}} - \frac{c\Phi_\mathrm{f}(t)}{R_\mathrm{A}}\cdot\Omega_\mathrm{m}(t) \qquad \Big| : I_\mathrm{AN} \tag{3.16a}$$

$$\frac{N_\mathrm{f}}{R_\mathrm{f}}\frac{\mathrm{d}\Phi_\mathrm{f}}{\mathrm{d}t} = -i_\mathrm{f}(t) + \frac{1}{R_\mathrm{f}}u_\mathrm{f}(t) \qquad \Big| : I_\mathrm{fN} \tag{3.16b}$$

$$J\frac{\mathrm{d}\Omega_\mathrm{m}}{\mathrm{d}t} = c\Phi_\mathrm{f}(t)\cdot i_\mathrm{A}(t) - m_\mathrm{L}(t) - M_\mathrm{Fric} \qquad \Big| : M_\mathrm{N} \tag{3.16c}$$

It is helpful to use relative transient variables instead of absolute ones. They are related to their respective nominal values and marked with the additional index n. Hence the following abbreviations can be defined:

$$\begin{array}{llll}
u_\mathrm{An}(t) = \dfrac{u_\mathrm{An}(t)}{U_\mathrm{AN}} & i_\mathrm{An}(t) = \dfrac{i_\mathrm{An}(t)}{I_\mathrm{AN}} & T_\mathrm{A} = \dfrac{L_\mathrm{A}}{R_\mathrm{A}} & r_\mathrm{A} = \dfrac{R_\mathrm{A}}{U_\mathrm{AN}/I_\mathrm{AN}} \\
u_\mathrm{fn}(t) = \dfrac{u_\mathrm{f}(t)}{U_\mathrm{fN}} & i_\mathrm{fn}(t) = \dfrac{i_\mathrm{f}(t)}{I_\mathrm{fN}} & T_\mathrm{fN} = \dfrac{L_\mathrm{fN}}{R_\mathrm{f}} & r_\mathrm{f} = \dfrac{R_\mathrm{f}}{U_\mathrm{fN}/I_\mathrm{fN}} \\
\Phi_\mathrm{fn}(t) = \dfrac{\Phi_\mathrm{f}(t)}{\Phi_\mathrm{fN}} & N_\mathrm{f}\Phi_\mathrm{fN} = L_\mathrm{fN}I_\mathrm{fN} & & \\
\Omega_\mathrm{mn}(t) = \dfrac{\Omega_\mathrm{m}(t)}{\Omega_\mathrm{m0N}} & m_\mathrm{Ln}(t) = \dfrac{m_\mathrm{L}(t) + M_\mathrm{Fric}}{M_\mathrm{N}} & & T_\mathrm{J} = \dfrac{\Omega_\mathrm{m0N}\cdot J}{M_\mathrm{N}} \\
U_\mathrm{AN} = c\Phi_\mathrm{fN}\cdot\Omega_\mathrm{m0N} & M_\mathrm{N} = c\Phi_\mathrm{fN}\cdot I_\mathrm{AN} & &
\end{array}$$

The system of differential equations can thus be formulated in relative quantities:

$$T_\mathrm{A}\frac{\mathrm{d}i_\mathrm{An}}{\mathrm{d}t} = -i_\mathrm{An}(t) + \frac{u_\mathrm{An}(t)}{r_\mathrm{A}} - \frac{\Phi_\mathrm{fn}(t)}{r_\mathrm{A}}\cdot\Omega_\mathrm{mn}(t) \tag{3.17a}$$

$$T_\mathrm{f}\frac{\mathrm{d}\Phi_\mathrm{fn}}{\mathrm{d}t} = -i_\mathrm{fn}(t) + \frac{u_\mathrm{fn}(t)}{r_\mathrm{f}} \tag{3.17b}$$

$$T_\mathrm{J}\frac{\mathrm{d}\Omega_\mathrm{mn}}{\mathrm{d}t} = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) - m_\mathrm{Ln}(t) \tag{3.17c}$$

$$u_\mathrm{in}(t) = \Phi_\mathrm{fn}(t)\cdot\Omega_\mathrm{mn}(t) \tag{3.17d}$$

$$\Phi_\mathrm{fn} = f(i_\mathrm{fn}(t)) \tag{3.17e}$$

$$m_\mathrm{in}(t) = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) \tag{3.17f}$$

A physical standard model (= model exclusively with integrators) for the DC machine is shown in Figure 3.12. This model is not suitable to be considered in the Laplace domain, since the time-dependent variable excitation flux Φ_fn is on the one hand related to the time-dependent variable armature current i_An and, on the other hand, it is multiplied to the time-dependent variable angular velocity Ω_mn.

3.5.2 Discretization of the state equations

In general, the time behavior of the DC machine can only be reliably described via simulation due to the non-linearities (saturation, multiplication of time-dependent variables). The three differential equations for the DC machine then have to be solved numerically.

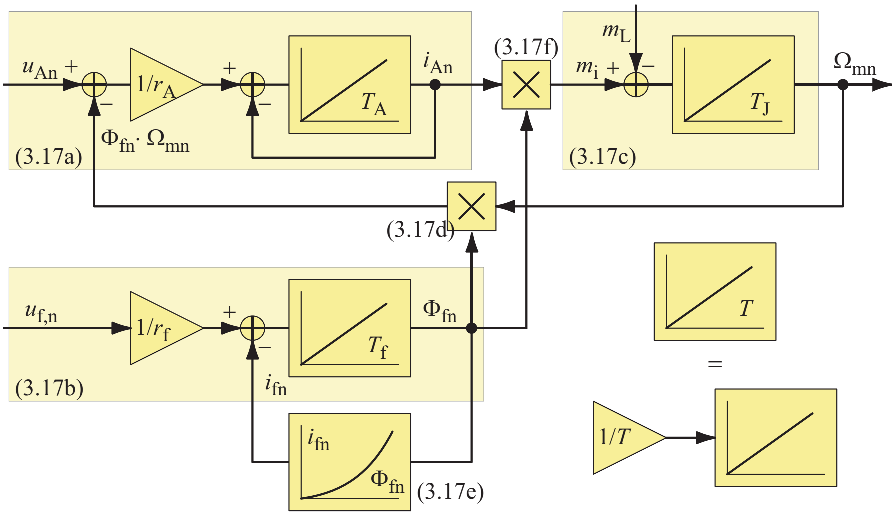

Figure 3.12: Standard physical model of the DC machine

The Runge-Kutta method, for example, is widely used for the numerical solution of differential equations. First of all, the state equations should be converted into recursion equations according to Euler-Cauchy. The principle is that the **Differential**quotients are converted into **Differences**quotients:

$$\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} = \lim_{T \to 0} \frac{y(t_k + T) - y(t_k)}{T}$$

$$\text{Notation:} \qquad \left.\begin{aligned} y_k &:= y(t_k) \\ y_{k+1} &:= y(t_k + T) \end{aligned}\right\} \Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} \approx \frac{y_{k+1} - y_k}{T}$$

As an example, the armature voltage equation Eq. 3.17a can be discretized, i.e. the differential quotient $\mathrm{d}i_{\mathrm{A}} / \mathrm{d}t$ on the left side is approximated by a difference quotient $(i_{\mathrm{A},k+1} - i_{\mathrm{A},k})/T$:

$$T_{\mathrm{A}} \frac{\mathrm{d}i_{\mathrm{An}}}{\mathrm{d}t} = -i_{\mathrm{An}}(t) + \frac{u_{\mathrm{An}}(t)}{r_{\mathrm{A}}} - \frac{\Phi_{\mathrm{fn}}(t)}{r_{\mathrm{A}}} \cdot \Omega_{\mathrm{mn}}(t) \qquad ((3.17a))$$

$$\frac{T_{\mathrm{A}}}{T}(i_{\mathrm{A},k+1} - i_{\mathrm{A},k}) = -i_{\mathrm{A},k} + \frac{u_{\mathrm{A},k}}{r_{\mathrm{A}}} - \frac{\Phi_{\mathrm{f},k}}{r_{\mathrm{A}}} \cdot \Omega_{\mathrm{m},k}$$

The last equation can be rearranged to isolate $i_{\mathrm{A},k+1}$ and one obtains a recursion equation for a numerical integration:

$$i_{\mathrm{A},k+1} = \left(1 - \frac{T}{T_{\mathrm{A}}}\right) i_{\mathrm{A},k} + \frac{T}{T_{\mathrm{A}} r_{\mathrm{A}}} \left(u_{\mathrm{A},k} - \Phi_{\mathrm{f},k} \cdot \Omega_{\mathrm{m},k}\right) \qquad (3.18a)$$

On the right side only values from the last (k-th) calculation step are used to determine the new ($(k+1)$-th) value. The other two differential equations can be discretized in the same way. From the voltage equation for the field excitation circuit follows:

$$T_\mathrm{f}\frac{\mathrm{d}\Phi_\mathrm{fn}}{\mathrm{d}t} = -i_\mathrm{fn}(t) + \frac{u_\mathrm{fn}(t)}{r_\mathrm{f}} \qquad ((3.17b))$$

$$\frac{T_\mathrm{f}}{T}\left(\Phi_{\mathrm{f},k+1} - \Phi_{\mathrm{f},k}\right) = -i_{\mathrm{f},k} + \frac{u_{\mathrm{f},k}}{r_\mathrm{f}}$$

$$\Rightarrow \qquad \Phi_{\mathrm{f},k+1} = \Phi_{\mathrm{f},k} + \frac{T}{T_\mathrm{f}}\left(\frac{u_{\mathrm{f},k}}{r_\mathrm{f}} - i_{\mathrm{f},k}\right) \qquad (3.18b)$$

The equation of motion can be converted into a recursion equation using the same principle:

$$T_\mathrm{J}\frac{\mathrm{d}\Omega_\mathrm{mn}}{\mathrm{d}t} = \Phi_\mathrm{fn}(t)\cdot i_\mathrm{An}(t) - m_\mathrm{Ln}(t) \qquad ((3.17c))$$

$$\frac{T_\mathrm{J}}{T}\left(\Omega_{\mathrm{m},k+1} - \Omega_{\mathrm{m},k}\right) = \Phi_{\mathrm{f},k}\cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}$$

$$\Rightarrow \qquad \Omega_{\mathrm{m},k+1} = \Omega_{\mathrm{m},k} + \frac{T}{T_\mathrm{J}}\left(\Phi_{\mathrm{f},k}\cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}\right) \qquad (3.18c)$$

This provides the necessary equations of state so that the direct current machine can be numerically simulated according to the following procedure:

1. Definition of constants:
   $T_\mathrm{A} = \ldots,\ T_\mathrm{f} = \ldots,\ T_\mathrm{J} = \ldots,\ T = \ldots(< T_\mathrm{A}/10),\ r_\mathrm{A} = \ldots,\ r_\mathrm{f} = \ldots(\approx 1)$
   $\tau_\mathrm{A} = T_\mathrm{A}/T,\ \tau_\mathrm{f} = T_\mathrm{f}/T,\ \tau_\mathrm{J} = T_\mathrm{J}/T$
2. Specify input variables:
   $u_{\mathrm{A},k} = \ldots,\ u_{\mathrm{f},k} = \ldots,\ m_{\mathrm{L},k} = \ldots,\ k = 0\ldots k_\mathrm{max}$
3. Initial values :
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0}\cdot i_{\mathrm{A},0}$
4. „Calculate“the state equations Eq. 3.18a, Eq. 3.18b and Eq. 3.18c
5. Increment run index k and check termination condition, otherwise back to 4.

As an example, a speed change from the idle (no-load) speed at nominal voltage to twice the nominal idle speed has been simulated. The excitation voltage has been reduced linearly from the nominal value ($u_\mathrm{fn} = 1$) to half the nominal value ($u_\mathrm{fn} = 0.5$) within $0.5\,\mathrm{s}$. The parameters have been set as follows:
$T_\mathrm{A} = 10\,\mathrm{ms},\ T_\mathrm{f} = 200\,\mathrm{ms},\ T_\mathrm{J} = 800\,\mathrm{ms},\ T = 2\,\mathrm{ms},\ r_\mathrm{A} = 0.04,\ r_\mathrm{f} = 1$

The Matlab script is listed on page 3-20. A linear relationship between field excitation current and flux was assumed (no saturation), resulting in the curves shown in Figure 3.13.
The excitation current follows the excitation voltage with a delay, causing the induced voltage to decrease. The induced voltage must be smaller than the armature voltage for an armature current to flow. If the excitation voltage is reduced too quickly, the armature current would increase too much; In the case examined here, the armature current reaches approximately twice the nominal value. The speed has reached its final value after approx. $1.5\,\mathrm{s}$.

The process shown can only be determined using a numerical simulation, as the two multiplication points could not be eliminated in the block diagram due to the

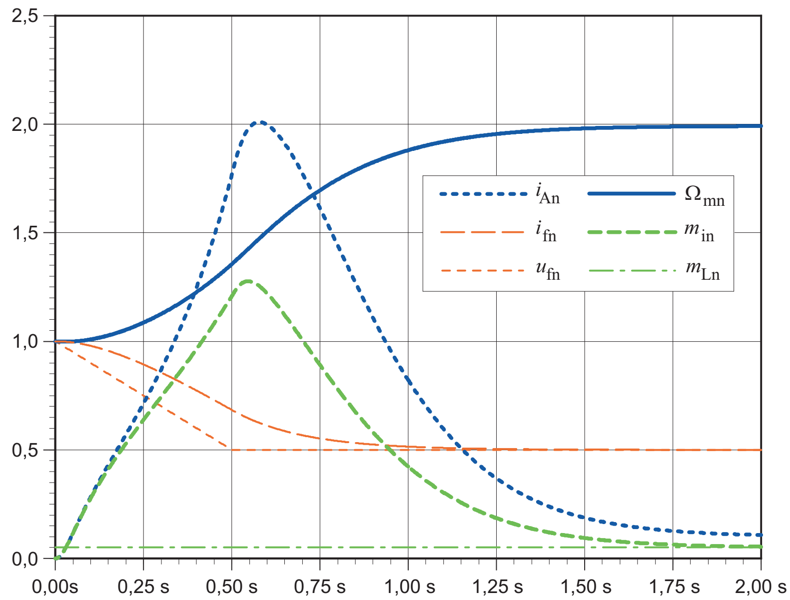

Figure 3.13: Speed regulation with field weakening

non-constant excitation flow. It also turns out that this type of speed adjustment does not occur particularly quickly, since the armature current does not even reach its nominal value on average during this process. A dynamically higher quality behavior can only be achieved with a closed-loop control.

3.5.3 Speed control with field weakening

The dynamic behavior of the DC machine can be significantly improved with closed-loop control. The simplest control structure is the so-called cascade control, in which the speed control loop is subordinated to a current control loop. Depending on the current speed, a setpoint for the excitation current is simultaneously generated via a look-up table, so that the DC machine can be operated in field weakening above the nominal speed:

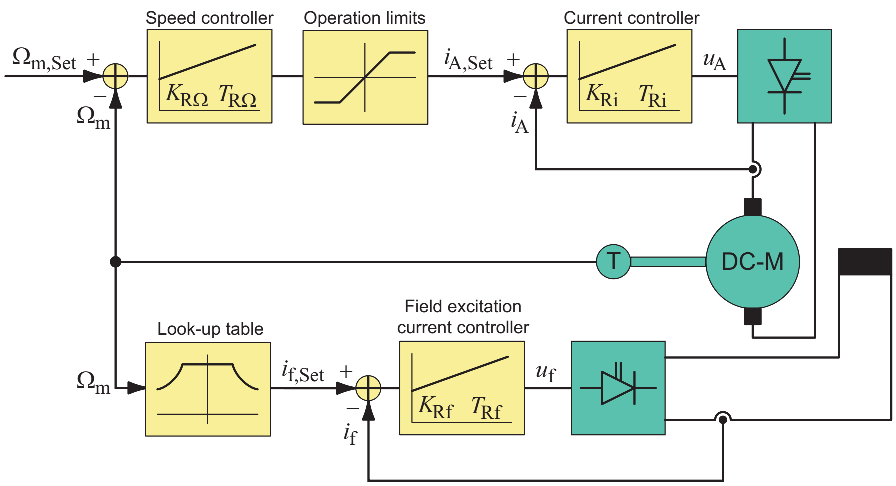

Figure 3.14: Cascade control of a DC motor including field weakening

All three controllers are designed as PI controllers in order to be able to regulate the stationary error to zero. The differential equation in the time domain can be specified from the transfer function of the PI controller with the control deviation e as the input variable and the output variable y as the reference variable:

$$G_{\mathrm{PI}} = \frac{y(s)}{e(s)} = K_{\mathrm{R}} \cdot \frac{1 + sT_{\mathrm{R}}}{sT_{\mathrm{R}}}$$

$$\begin{aligned} sT_{\mathrm{R}} \cdot y(s) &= K_{\mathrm{R}} \cdot e(s) + sT_{\mathrm{R}}\, K_{\mathrm{R}} \cdot e(s) \\ \Rightarrow T_{\mathrm{R}} \frac{\mathrm{d}y(t)}{\mathrm{d}t} &= K_{\mathrm{R}} \cdot e(t) + T_{\mathrm{R}}\, K_{\mathrm{R}} \frac{\mathrm{d}e(t)}{\mathrm{d}t} \end{aligned}$$

In order to be able to compare the behavior of a speed-controlled, externally excited DC machine with the previous simulation, the differential equation is also discretized:

$$T_{\mathrm{R}} \frac{y_{k+1} - y_k}{T} = K_{\mathrm{R}} \cdot e_k + T_{\mathrm{R}}\, K_{\mathrm{R}} \frac{e_{k+1} - e_k}{T}$$

$$\begin{aligned} y_{k+1} &= y_k + K_{\mathrm{R}}\, e_{k+1} - K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right) e_k \\ \Rightarrow y_{k+1} &= y_k + q_0 e_{k+1} + q_1 e_k \qquad \text{mit } q_0 = K_{\mathrm{R}} \\ & \qquad\qquad q_1 = K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right) \end{aligned} \tag{3.19}$$

This means that the process for simulating the DC machine can be extended with controller simulation:

1. Definition of constants
   The values given in brackets are assumed in the simulation. They are the same as in the previous section.
   $T_{\mathrm{A}} = (10\,\mathrm{ms}),\ T_{\mathrm{f}} = (100\,\mathrm{ms}),\ T_{\mathrm{J}} = (800\,\mathrm{ms}),\ T = (1\,\mathrm{ms}),\ r_{\mathrm{A}} = (0.04),\ r_{\mathrm{f}} = 1$
   $\tau_{\mathrm{A}} = T_{\mathrm{A}}/T,\ \tau_{\mathrm{f}} = T_{\mathrm{f}}/T,\ \tau_{\mathrm{J}} = T_{\mathrm{J}}/T$
   $K_{\mathrm{R}\Omega} = (20),\ T_{\mathrm{R}\Omega} = (100\,\mathrm{ms}),\ K_{\mathrm{Ri}} = (0.5),\ T_{\mathrm{Ri}} = (10\,\mathrm{ms})$
   $K_{\mathrm{Rf}} = (1),\ T_{\mathrm{Rf}} = (50\,\mathrm{ms}),\ i_{\mathrm{Amax}} = (2),\ u_{\mathrm{Amax}} = (1.2),\ u_{\mathrm{fmax}} = 1$

2. Specify input variables
   Only the curve for the speed setpoint and the load torque can be now specified.
   $\Omega_{\mathrm{mSet},k} = 1 \ldots 2 \ldots,\ m_{\mathrm{L},k} = \ldots,\ k = 0 \ldots k_{\max}$

3. Initial values:
   The integrator contents of the controllers and the deviations need to be preset.
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0} \cdot i_{\mathrm{A},0}$
   $i_{\mathrm{ASoll},0} = \ldots,\ u_{\mathrm{A}0} = \ldots,\ u_{\mathrm{f},0} = \ldots,\ e_{\Omega,0} = \ldots,\ e_{\mathrm{i},0} = \ldots,\ e_{\mathrm{f},0} = \ldots$

4. „Calculate“the state equations Eq. 3.18a, Eq. 3.18b and Eq. 3.18c

5. „Calculate“the control laws for the three(!) controllers according to Eq. 3.19 and apply the operational limit.

6. Increment running index and check termination condition, otherwise return to 4.

These procedure has been used to simulate a jump in the speed setpoint from the machine standstill to twice the nominal idle speed. (Matlab listing see page 3-21) The DC machine was already excited with the nominal field excitation current at standstill (initial values correspond to those!).

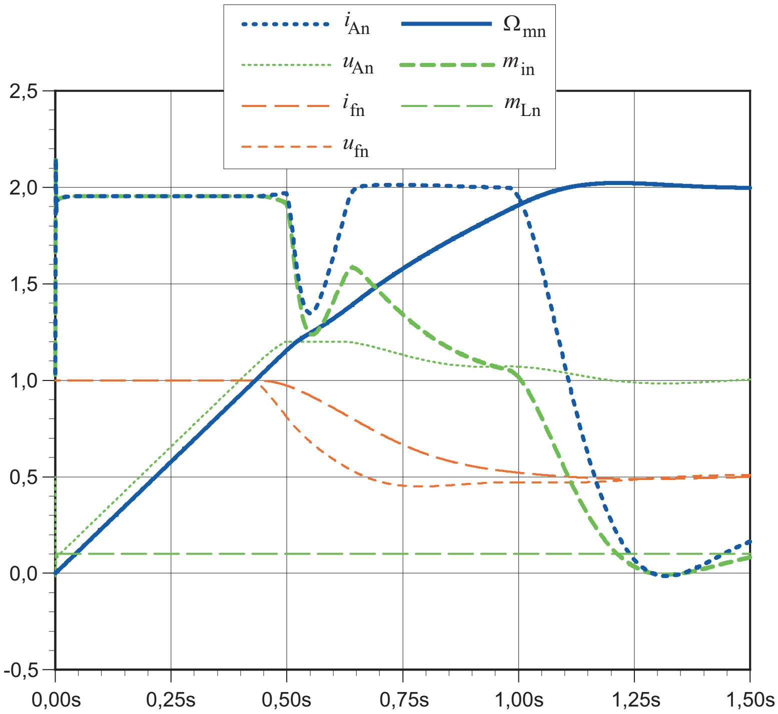

Figure 3.15: Setpoint jump $0 \ldots 2n_{0N}$

The nominal excitation is present up to the nominal speed. As soon as this speed has been reached, the excitation voltage is reduced by the excitation flux controller. However, due to the relatively large excitation time constant, the excitation current follows the excitation voltage with such a strong delay that the excitation voltage is even reduced to well below half the nominal value.

However, since the excitation cannot be reduced quickly enough, the converter for the armature circuit is not able to produce twice the nominal current even after the field weakening range has been reached due to the induced voltage being too high, despite a voltage reserve of 20% ($u_{An,max} = 1.2$).

As a result, the armature current drops significantly over a period of approx. $150\,\mathrm{ms}$, so that the full torque is not available for acceleration during this time. Shortly before the target speed is reached, the armature voltage is reduced slightly and the armature current settles to its stationary final value. ($i_{An} = m_{Ln} = 0.1$).

But the nominal speed is already reached after $350\,\mathrm{ms}$ and the setpoint speed after another approx. $650\,\mathrm{ms}$. Without control, the same machine only completed the run-up from nominal speed to twice the nominal speed after approx. $2\,\mathrm{s}$. However, it turns out that speed adjustment via excitation cannot be done nearly as dynamically as in the armature adjustment range.

These results can only be achieved via simulation with reasonable effort.

3.6 Tasks

Example 3-1: Data of a DC motor

For a DC motor with a power of $3\,\mathrm{kW}$ at $1500\,\mathrm{min}^{-1}$ at the shaft and a nominal voltage of $230\,\mathrm{V}$ for the armature and excitation winding, estimate the equivalent circuit diagram data for both the armature and the field winding. An overall efficiency of 84% can be assumed with a distribution of losses: armature winding 8%, field winding 5% and friction 3% of the input electrical power. The voltage drop in the brushes can be neglected.

Example 3-2: Nameplate and M/n characteristic

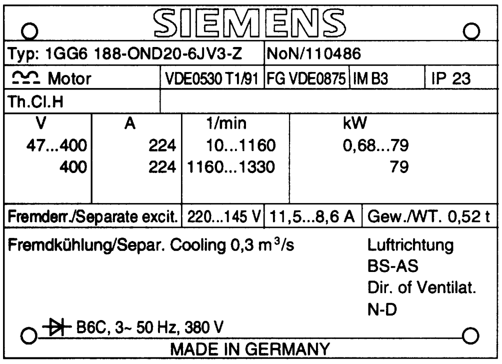

Figure 3.16: Nameplate of a DC motor (Siemens AG)

a. Use the rating plate in figure 3.16 to determine the rated values for armature voltage and current, excitation voltage and current, as well as power and speed.
b. Determine the armature resistance from the nominal data.
c. Determine from the data for $n = 1160\,\mathrm{min}^{-1}$ and $n = 1330\,\mathrm{min}^{-1}$ the excitation resistance and explain the difference. What are the no-load speeds at:
   c1. $U_\mathrm{A} = 400\,\mathrm{V}$ / $U_\mathrm{f} = 220\,\mathrm{V}$ and
   c2. $U_\mathrm{A} = 400\,\mathrm{V}$ / $U_\mathrm{f} = 145\,\mathrm{V}$?
d. Sketch the speed-torque characteristics for
   d1. $U_\mathrm{A} = 400\,\mathrm{V}$ / $U_\mathrm{f} = 220\,\mathrm{V}$ and
   d2. $U_\mathrm{A} = 400\,\mathrm{V}$ / $U_\mathrm{f} = 145\,\mathrm{V}$.

Example 3-3: Separately excited DC motor

Given is a machine with the nominal data $10\,\mathrm{kW}$ / $1300\,\mathrm{min}^{-1}$, armature: $440\,\mathrm{V}$ / $25\,\mathrm{A}$, excitation circuit $200\,\mathrm{V}$ / $2\,\mathrm{A}$. The brush voltage drop is estimated as $1\,\mathrm{V}$ per brush.

In a no-load test (externally driven machine with an open armature circuit, i.e. $I_\mathrm{A} = 0$), at $1000\,\mathrm{min}^{-1}$ the armature voltage U_A was measured as a function of the excitation current I_f (figure 3.17). Iron losses and armature reaction can be neglected. The friction torque can be assumed to be constant.

a. Provide the power flow diagram for the nominal point with the numerical values for all losses (including friction and brush losses). What is the efficiency at the nominal point? Sketch the equivalent circuit with all numerical values for the nominal point.

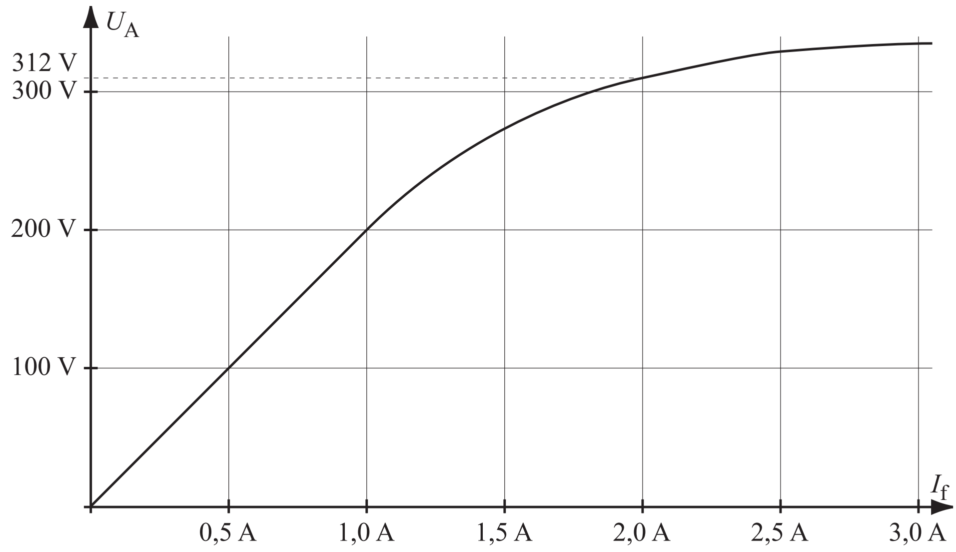

Figure 3.17: Saturation curve (measured values)

b. What is the rotational speed when the DC machine is completely unloaded starting from the rated point? (i.e. $M_{\text{Last}} = 0$, but $M_{\text{Fric}} > 0$)
c. Assuming that the calculated equivalent circuit data are valid for the machine at operating temperature. What speed would be reached during operation at rated voltages if the machine were loaded with the rated torque when cold?
Note: ϑ_{U} = 20°C; ϑ_{W} = 120°C; $\alpha_{\text{Cu},20}$ = $0.004\,\text{K}^{-1}$; $R(\vartheta) = R_{20}(1 + \alpha_{\text{Cu},20} \cdot \Delta\vartheta)$

Example 3-4: Regulated DC Motor
A DC motor with the following nominal data is given: Nominal data $3\,\text{kW} / 1500\,\text{min}^{-1}$, Armature: $220\,\text{V}$ / $14.5\,\text{A}$, Excitation circuit $220\,\text{V}$ / $1.2\,\text{A}$. The armature and excitation circuits of the speed-controlled motor are each fed via their own power converter, with the excitation current being regulated in inverse proportion to the speed from the rated speed. ($I_\text{f}/I_{\text{fN}} = n_\text{N}/n$ for $n > n_\text{N}$) With the exception of the Joule heat losses, all losses can be neglected.

a. Determine the armature and excitation resistance.
b. What is the efficiency and the nominal torque at the nominal point?
c. At rated speed, the motor is loaded with half the rated torque. What are the values for the armature voltage and the excitation voltage?
d. The speed setpoint is now increased to 150% of the nominal speed, whereby the motor is now only loaded with 25% of the nominal torque. What are the values for the armature voltage and the excitation voltage?
e. What is the maximum torque that the motor can deliver at this speed if the electrical ratings can not be exceeded?

Example 3-5: Simulation model of the DC machine in Simulink
The simulation results in Figure 3.13 and Figure 3.15 are to be verified with a Simulink model. The simulation models are to be recreated and further developed. Find the prepared files (syntax: `gm<n>.mdl`: Simulink model, `gm<n>_init.m`: initialization routine in which the parameters for the simulation are set). You can access the files in e-Learning. The parameters are already preset as previously.

a) Create a simulation model in Simulink (`gm1.mdl`) with relative magnitudes as shown in Figure 3.12. Build the model so that each of the three equations Eq. 3.18a, Eq. 3.18b and Eq. 3.18c is modeled in its own subsystem and give the inputs and outputs meaningful names!

b) Verify the results in Figure 3.13 with the model `gm2.mdl` (the initialization routine `[gm]=gm2_init.mdl` is called automatically when the simulation starts).

c) The DC motor should now be examined in controlled operation in the armature adjustment area. In the `gm3.mdl` model, the DC motor is combined as a subsystem and in the `gm4.mdl` model it is implemented together with the two PI controllers for speed and current control. The setpoint for the armature voltage is given directly to the input of the motor model. Compare the setpoint and actual value of the armature current during startup! Why are they different? Adjust the controller parameters and repeat the simulation!

d) In the model `gm5.mdl` the field current controller is also taken into account. Verify the results in Figure 3.15. Why is the speed control no longer able to set the setpoint in the field weakening range? What change needs to be made?

3.7 Comprehension questions

Q3-1: What is meant by armature reaction? How does it affect performance?

Q3-2: Draw the speed-torque characteristics for a separately excited DC machine! What is the relationship between torque and armature current?

Q3-3: What are the options for adjusting the speed of separately excited DC machines and permanently excited DC machines?

Q3-4: Name the most important safety measures when operating DC machines!

Q3-5: Draw a four-pole DC machine! Also show the the individual current directions!

Q3-6: Is the armature of a DC machine laminated or solid? Why? Are the excitation poles laminated? Why not)?

Q3-7: Why does the armature reaction limit the field weakening area?

Q3-8: What limits the size of the field weakening factor?

Q3-9: Use the equivalent circuit diagram of the separately excited DC machine to provide a complete system of differential equations for the separately excited DC machine! Which energy storage devices are available in the DC machine and with which time constants can they be described?

Q3-10: Qualitatively sketch the speed curve $n(t)$ when the armature voltage jumps from $U_\mathrm{A} = 0$ to $U_\mathrm{A} = U_\mathrm{AN}$ with an idle DC machine ($n(t = 0) = 0$) for different total moments of inertia! When can the nominal speed be exceeded (only qualitatively)?

Q3-11: Qualitatively sketch the speed curve $n(t)$ with a load jump from $M_\mathrm{L} = 0$ to $M_\mathrm{L} = M_\mathrm{N}$ with an idle DC machine $(n(t = 0) = n_{0\mathrm{N}})$ for different total moments of inertia (qualitatively!)! When does the greatest drop in speed occur?

Q3-12: How would the maximum armature current change if in Figure 3.13 (starting point is no-load operation, separately excited DC machine at nominal voltage, excitation voltage is within $0.5\,\mathrm{s}$ reduced from the nominal value to half the nominal value) an additional moment of inertia would be coupled to the DC machine? Why? What measure can be used to achieve twice the nominal current as the maximum value again (qualitatively!)?

Q3-13: Sketch the structure of a speed control for a separately excited DC machine that is to be operated up to twice the nominal no-load speed!

Q3-14: How does increasing or reducing the moment of inertia affect the armature current of the speed-controlled DC motor in Figure 3.15? Why?

3.8 Matlab-Listings

3.8.1 Listing for case 1: Speed regulation with field weakening

```
function case_1()
%---------------------------------------------------------------
% 1. Constant definitions
%
T_A=0.01; % Armature time constant
T_f=0.2; % Excitation time constant
T_J=0.8; % Ramp-up time
T_abt=0.002; % Sampling time
tau_A=T_A/T_abt; tau_f=T_f/T_abt; tau_J=T_J/T_abt;
r_A=0.04; % Relative armature resistor (r_A = R_A/(U_AN/I_AN))
r_f=1; % Relative field excitation resistor (r_f = R_f/(U_fN/I_fN))
k_max=2/T_abt;
%---------------------------------------------------------------
% 2. Input values / equations
%
k_hoch=0.5/T_abt;
for k=1:k_hoch+1
u_A(k)=1;
m_L(k)=0.05;
u_f(k)=1-(k-1)/k_hoch*0.5;
end
for k=k_hoch+1:k_max+1
u_A(k)=1;
m_L(k)=0.05;
u_f(k)=0.5;
end
%---------------------------------------------------------------
% 3. Start values
%
i_A(1)=0;Phi_f(1)=1;Om(1)=1;
i_f(1)=1;m_i(1)=i_A(1)*Phi_f(1);
%---------------------------------------------------------------
% 4./5. Calculation
%
for k=1:k_max
i_A(k+1) =(1-1/tau_A)*i_A(k)+(u_A(k)-Om(k)*Phi_f(k))/tau_A/r_A;
Phi_f(k+1)=Phi_f(k)+(u_f(k)/r_f-i_f(k))/tau_f;
i_f(k+1) =Phi_f(k+1);
m_i(k+1) =Phi_f(k+1)*i_A(k+1);
```

```
Om(k+1) =Om(k)+(m_i(k)-m_L(k))/tau_J;
end
%-----------------------------------------------------------------------
% 6. Graphical output
%
zeit=linspace(0,2,k_max+1);
% plot(zeit,Om,zeit,u_f,zeit,i_f,zeit,i_A,zeit,m_i)
plot(zeit,Om,'b',zeit,u_f,'r:',zeit,i_f,'r--',zeit,i_A,'b:',zeit,m_i,'g--')
grid on
```

3.8.2 Listing for case 2: Speed control with field weakening

```
function case_2()
%-------------------------------------------------------------
% 1. Constant definitions
%
T_A=0.01; % Armature time constant
T_f=0.2; % Excitation time constant
T_J=0.8; % Ramp-up time
T_abt=0.002; % Sampling time
tau_A=T_A/T_abt; tau_f=T_f/T_abt; tau_J=T_J/T_abt;
r_A=0.04; % Relative armature resistor (r_A = R_A/(U_AN/I_AN))
r_f=1; % Relative field excitation resistor (r_f = R_f/(U_fN/I_fN))
k_max=1.5/T_abt;
K_R_om=20; T_R_om=0.1; % Parameter speed controller
q0_om =K_R_om; q1_om=K_R_om*(T_abt/T_R_om -1);
K_R_A=0.5; T_R_A=0.01; % Parameter armature current controller
q0_A =K_R_A; q1_A=K_R_A*(T_abt/T_R_A -1);
K_R_f=1; T_R_f=0.05; % Parameter field current controller
q0_f =K_R_f; q1_f=K_R_f*(T_abt/T_R_f -1);
%-------------------------------------------------------------
% 2. Input values / equations
%
k_hoch=0.5/T_abt;
for k=1:k_max+1
Om_soll(k)=2;
m_L(k)=0.1;
end
%-------------------------------------------------------------
% 3. Initial values
%
i_A(1)=0;Phi_f(1)=1;Om(1)=0;
i_f(1)=1;m_i(1)=i_A(1)*Phi_f(1);
i_ASoll(1)=0;u_A(1)=0;u_f(1)=1;i_fSoll(1)=1;
e_om(1)=0;e_A(1)=0;e_f(1)=0;
for k=1:k_max
%-------------------------------------------------------------
% 4. Calculate DC motor equations
%
i_A(k+1) =(1-1/tau_A)*i_A(k)+(u_A(k)-Om(k)*Phi_f(k))/tau_A/r_A;
Phi_f(k+1)=Phi_f(k)+(u_f(k)/r_f-i_f(k))/tau_f;
i_f(k+1) =Phi_f(k+1);
m_i(k+1) =Phi_f(k+1)*i_A(k+1);
```

```
Om(k+1) =Om(k)+(m_i(k)-m_L(k))/tau_J;
%-----------------------------------------------------------------
% 5. Calculate controller equations
%
e_om(k+1)=Om_soll(k+1)-Om(k+1);
i_ASoll(k+1)=i_ASoll(k)+q0_om*e_om(k+1)+q1_om*e_om(k);
i_ASoll(k+1)=local_begrenze(i_ASoll(k+1),2.0);
e_A(k+1)=i_ASoll(k+1)-i_A(k+1);
u_A(k+1)=u_A(k)+q0_A*e_A(k+1)+q1_A*e_A(k);
u_A(k+1)=local_begrenze(u_A(k+1),1.2);
if abs(Om(k+1)) > 1
i_fSoll(k+1)=1/abs(Om(k+1));
else
i_fSoll(k+1)=1;
end
e_f(k+1)=i_fSoll(k+1)-i_f(k+1);
u_f(k+1)=u_f(k)+q0_f*e_f(k+1)+q1_f*e_f(k);
u_f(k+1)=local_begrenze(u_f(k+1),1.0);
end
%-----------------------------------------------------------------
% 6. Graphical representation
%
zeit=linspace(0,1.5,k_max+1);
% plot(zeit,Om,zeit,u_f,zeit,i_f,zeit,i_A,zeit,m_i)
plot(zeit,i_A,'b:',zeit,Om,'b',zeit,u_A,'g:',zeit,m_i,'g--',...
zeit,i_f,'r--',zeit,u_f,'r-.');
grid on
%-----------------------------------------------------------------
% Limit als local function
%
function back = local_begrenze(wert,grenze)
if wert > grenze
back = grenze;
elseif wert < -grenze
back = -grenze;
else
back = wert;
end
return
```