1) $h(n) = (1 \quad 2 \quad 1)$

$$H(z) = 1 + 2z^{-1} + z^{-2} \qquad z = e^{j2\pi f T} \qquad T = \frac{1}{r} = 1$$

$$H(f) = 1 + 2e^{-j2\pi f} + e^{-j4\pi f}$$

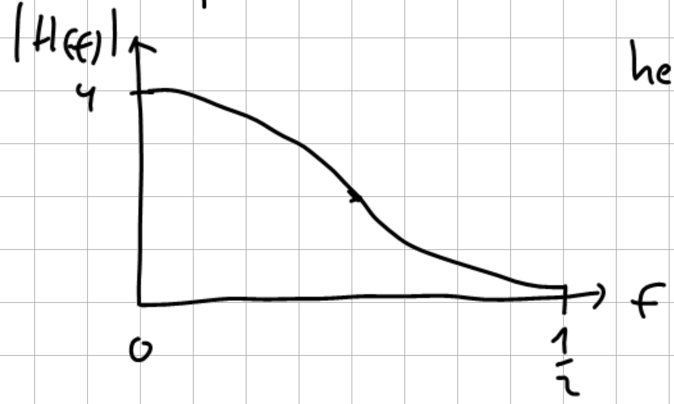
$$= e^{-j2\pi f}\left(e^{j2\pi f} + 2 + e^{-j2\pi f}\right)$$

$$|H(f)| = \left|e^{j2\pi f} + 2 + e^{-j2\pi f}\right|$$

$$= \left|\cos(2\pi f) + j\sin(2\pi f) + 2 + \cos(2\pi f) - j\sin(2\pi f)\right|$$

$$= \left|2 + 2\cos(2\pi f)\right| \qquad 0 \leq f \leq \frac{r}{2} \text{ Sampling-theorem}$$

here : $0 \leq f \leq \frac{1}{2}$



$\Rightarrow$ $h(n)$ is a lowpass .

2) Converting a lowpass to an highpass:

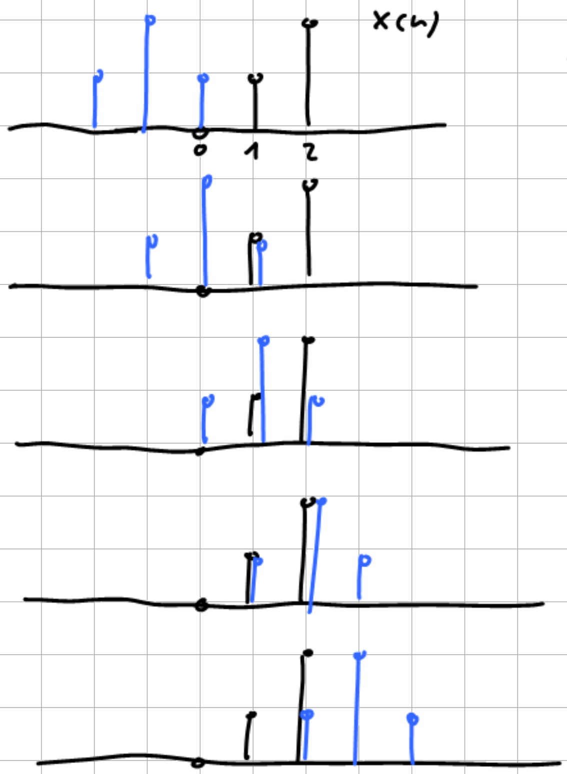
$$h_{HP}(n) = h_{LP}(n) \cdot (-1)^n$$

$$h_{LP}(n) = 1 \quad 2 \quad 1$$

$$h_{HP}(n) = 1 \quad -2 \quad 1$$

3) $x(n) = [0 \quad 1 \quad 2]$ $\qquad h(n) = [1 \quad 2 \quad 1]$

graphical method:



$\Rightarrow \quad y(0) = 1 \cdot 0 = 0$

$\Rightarrow \quad y(1) = 1 \cdot 1 + 2 \cdot 0 = 1$

$\Rightarrow \quad y(2) = 1 \cdot 2 + 2 \cdot 1 = 4$

$\Rightarrow \quad y(3) = 1 \cdot 1 + 2 \cdot 2 = 5$

$\Rightarrow \quad y(4) = 1 \cdot 2 = 2$

$y(n) = 0$ everywhere else !

4)
$$h_{LP}(n) = \frac{\sin\left(2\pi f_c \frac{n}{r}\right)}{2\pi f_c \frac{n}{r}} \cdot \frac{1}{2}\left(1 + \cos\left(2\pi \frac{n}{N}\right)\right)$$

with: $-2 \leq n \leq 2$

$$\frac{f_c}{r} = \frac{1}{4}$$

$$h_{LP}(n) = \frac{\sin\left(\frac{\pi}{2} n\right)}{\frac{\pi}{2} n} \frac{1}{2}\left(1 + \cos\left(2\pi \frac{n}{5}\right)\right)$$

$$h_{LP}(-2) = h_{LP}(2) = 0$$

$$h_{LP}(-1) = h_{LP}(1) = 0{,}417$$

$$h_{LP}(0) = 1$$

5)

$$y(n) = x(n) * h(n)$$

$$= \sum_{k=0}^{K-1} x(n-k) \cdot h(k) \qquad K = 501$$

$\Rightarrow$ K Multiplications

$K-1$ Additions

6) minimize $\sum_n \left(a \cdot y(n-T) - z(n)\right)^2 = f(a)$

first derivative: $\sum_n 2\left(a\, y(n-T) - z(n)\right) \cdot y(n-T) = \frac{df(a)}{da}$

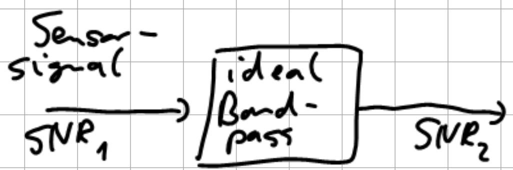
setting to 0: $\sum_n 2\left(a\, y(n-T) - z(n)\right) \cdot y(n-T) = 0$

$$\sum_n a\, y^2(n-T) - z(n)\, y(n-T) = 0$$

$$\sum_n a\, y^2(n-T) = \sum_n z(n)\, y(n-T)$$

$$a = \frac{\sum_n z(n)\, y(n-T)}{\sum_n y^2(n-T)}$$

7)



$$SNR_1 = 10 \log_{10} \frac{P_x}{P_{R_1}}$$

$$SNR_2 = 10 \log_{10} \frac{P_x}{P_{R_2}}$$

increase in SNR: $\quad SNR_2 - SNR_1 = 10 \left[\log_{10} \frac{P_x}{P_{R_2}} - \log_{10} \frac{P_x}{P_{R_1}} \right]$

$$= 10 \cdot \left[\log_{10} P_x - \log_{10} P_{R_2} - \log_{10} P_x + \log_{10} P_{R_1} \right]$$

$$= 10 \cdot \log_{10} \frac{P_{R1}}{P_{R_2}}$$

power of a signal is the integral over the power density function:

$$P_{R_1} = \int_0^{24000} |R(f)|^2 \, df = \int_0^{24000} c \cdot df$$

($|R(f)|^2$ is assumed to be constant)

$$= c \cdot 24000$$

$$P_{R_2} = \int_{100}^{7000} |R(f)|^2 \, df = 6900 \cdot c$$

SNR gain (increase) $= 10 \log_{10} \frac{24000c}{6900c} = 10 \log_{10} \frac{240}{69} = 5{,}4 \text{ dB}$

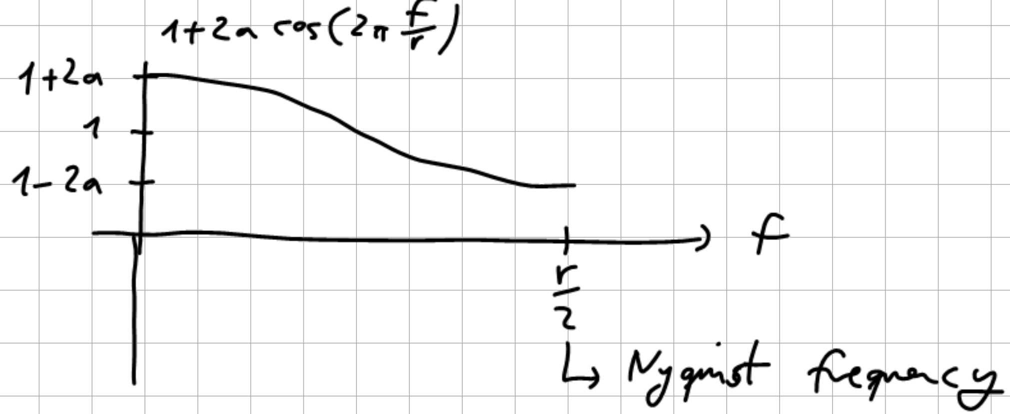
8) $h(n) = [a \quad 1 \quad a]$

$$H(f) = a + e^{-j2\pi \frac{f}{r}} + a e^{-j4\pi \frac{f}{r}}$$

$$= e^{-j2\pi \frac{f}{r}} \left(a e^{+j2\pi \frac{f}{r}} + 1 + a e^{-j2\pi \frac{f}{r}} \right)$$

$$|H(f)| = \left| a e^{j2\pi f/r} + 1 + a e^{-j2\pi \frac{f}{r}} \right|$$

$$= \left| 1 + 2a \cos(2\pi f/r) \right|$$



Condition: $\quad 1-2a > 0 \quad \Rightarrow \quad 1 > 2a \quad \Rightarrow \quad \underline{\underline{\frac{1}{2} > a}}$