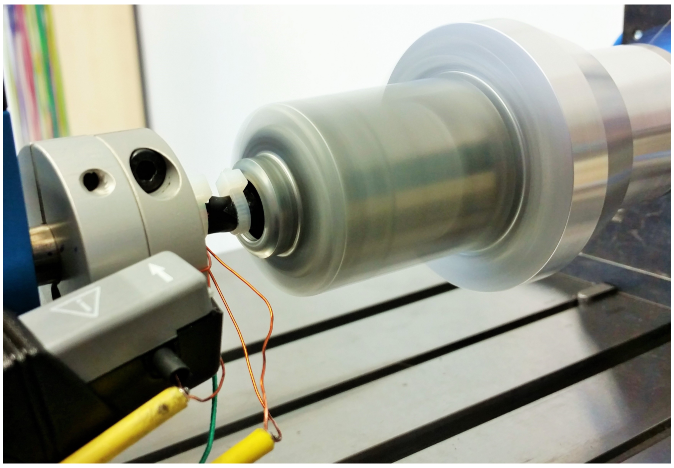

Actuators IRO6

Lecture for the degree
Robotics

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based on documents from **Prof. Dr.-Ing. Joachim Kempkes**

SoSe 2024

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Aktoren BRO6
Lecture script THWS
Stand SoSe 2024

Education is what is left when everything learned is forgotten!
Georg Kerschensteiner, reform pedagogue and high school teacher for mathematics and physics at the Gustav-Adolf-Gymnasium in Schweinfurt from 1890-1893

This is a script, not a book!

Errors/inconsistencies or suggestions for improvement are welcome to mercedes.herranz@thws.de.

Introduction

Actuators are important components of robot systems. By analogy with the human body, they are **like the muscles that make movements and apply forces**.

Figure 1 shows the position of the actuator in the overall system. Like the sensor, it is located at the interface between the information and the energy flow. Human operation or an automatic command serves as starting point in the system. The control variables for the actuator are then determined in the information processing subsystem. Thanks to the energy supply, the actuator becomes active in the mechanical system („it acts“). The response of the mechanical system and the environment is observed with sensors. These measured variables are in turn used in the information observation subsystem to determine the control variables. This closed structure forms the control loop.

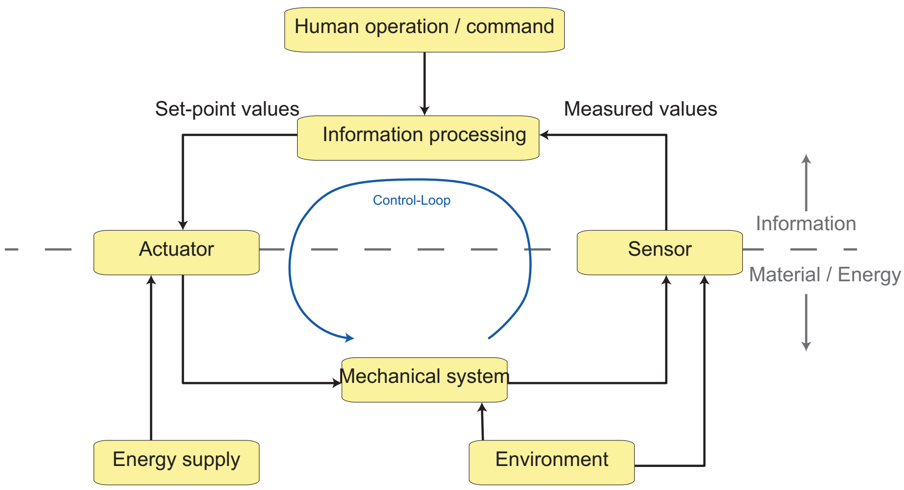


Figure 1: Actuator as a component in robotics or mechatronic systems (inspired by [5])

The actuator as a subsystem consists of 2 components (Figure 2). The control unit generates the input for the actual energy converter with the supply energy and the set-point value of the control variable. The energy converter transforms then the input energy into the form of energy required for the mechanical system (usually translational or rotational movement i.e. kinetic energy). In the first chapter of this course, the basic relationships in the mechanical system are discussed, in order to determine the actor´s keydata from the desired roboter movement.

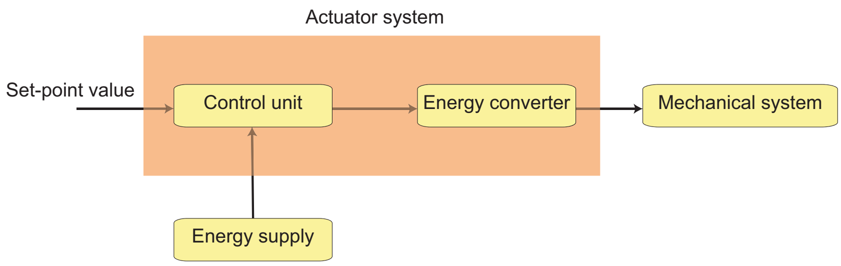


Figure 2: Components of the actuator subsystem

Hydraulic, pneumatic and electromechanical/electric drives are the three common actuator technologies. Electric drives are predominantly used in robot applications. The reasons for this are described in the second chapter of this course together with the basics of electrical drive technology. The different electric motor concepts are presented and the chapter closes with a discussion of the concepts that are best suited as robot drives. In the third, fourth and fifth chapters, these three concepts (permanent magnet DC motor, synchronous motor and stepper motor) are presented and their main equations are described. Also the control of each of them is discussed briefly. Since the synchronous motor works with three-phase current, the basics of three-phase systems can be found in a bonus chapter in the appendix. In the sixth chapter basics of converters for electric drives are presented. At the end of the course an example summarizes the most important contents.

This script provides the basic material for the course. In addition to the theoretical explanations, each chapter has three contents:

- Comprehension questions: the student can use this to check whether the basic concepts of the chapter have been understood.
- Examples: these arithmetic examples are solved together in class
- Tasks: these are used for self-study and are not solved completely in class. The final results are published in e-learning for checking.

This course was read for the first time in SS23. Course content and documents have been developed further for SS24. Comments or suggestions for improvement of the documents and the course in general are therefore most welcome.

1 Mechanical System

A **robot** must fulfill a specific mechanical task. This is done thanks to the actuator system. **Mechanical transmission elements** are often used between the mechanical load and the drive, for example to convert a linear movement into a rotary movement or to switch between two different speeds.

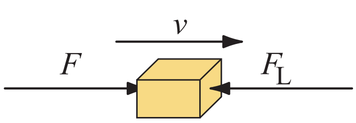
How can the required drive data be calculated from the "robot task"? How can mechanical transmission elements (e.g. gears) be taken into account? The answers to these two questions are outlined in this chapter, starting with the mechanical balance equation and the concept of transformed mass or inertia. The chapter concludes with a discussion of optimal gear ratios.

This chapter can only provide an overview of the most important concepts. Reference is made here to the literature (for example [6]) for further information. As a good practical guide, please refer to the formula collection from the manufacturer of micro drives Maxon [2].

1.1 Mechanical balance equations

1.1.1 Translation

A linear movement (translation) of a body with mass m can be described by an equation of motion as shown in Figure 1.1.



$$\sum F = 0: \qquad F - F_{\mathrm{L}} - m \cdot a = 0$$

Figure 1.1: Equation of motion for translational movement

In addition to the acceleration a, the following variables are relevant:

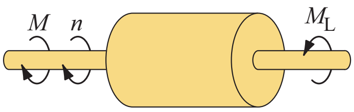
$$\begin{array}{llll}
s & \text{Displacement} & \text{in} & [\mathrm{m}] \\
v = \frac{\mathrm{d}s}{\mathrm{d}t} & \text{Speed} & \text{in} & \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \\
a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} & \text{Acceleration} & \text{in} & \left[\frac{\mathrm{m}}{\mathrm{s}^2}\right] \\
j = \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{\mathrm{d}^2 v}{\mathrm{d}t^2} = \frac{\mathrm{d}^3 s}{\mathrm{d}t^3} & \text{Jerk} & \text{in} & \left[\frac{\mathrm{m}}{\mathrm{s}^3}\right]
\end{array}$$

The jerk j as the third derivative of the location or as a change in acceleration a represents an important quantity for the mechanical stress. A „smooth acceleration“ avoids „blows“ in the process.

State variables for motion processes are the displacement s and the speed v, whereas the **cause** for the motion is the acceleration force $m \cdot a = F - F_L$. The jerk j, on the other hand, is a quantity that must be recorded and limited in some applications. (for example for rail vehicle drives in local transport or as a limited torque response time for servo drives in robotics)

1.1.2 Rotation

Analogous to the considerations of the linear movement in Figure 1.1, a rotating movement can be described in Figure 1.2.



$$\sum M = 0: \qquad M - M_L - J_{mech} \cdot \frac{\mathrm{d}\Omega_m}{\mathrm{d}t} = 0$$

Figure 1.2: Equation of motion for translational movement

Here too, the relevant variables can be defined analogous to linear movement:

$$\begin{array}{llll}
\varphi & \text{angle} & \text{in} & [\mathrm{rad}] \\
\Omega_m = \frac{\mathrm{d}\varphi}{\mathrm{d}t} & \text{angular velocity} & \text{in} & \left[\frac{\mathrm{rad}}{\mathrm{s}}\right] \\
\alpha_m = \frac{\mathrm{d}\Omega_m}{\mathrm{d}t} = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} & \text{angular acceleration} & \text{in} & \left[\frac{\mathrm{rad}}{\mathrm{s}^2}\right] \\
\sigma_m = \frac{\mathrm{d}\alpha_m}{\mathrm{d}t} = \frac{\mathrm{d}^2\Omega_m}{\mathrm{d}t^2} = \frac{\mathrm{d}^3\varphi}{\mathrm{d}t^3} & \text{angular jerk} & \text{in} & \left[\frac{\mathrm{rad}}{\mathrm{s}^3}\right]
\end{array}$$

1.1.3 Moment of inertia

The mass m during linear movement corresponds to the **moment of inertia** during rotation J_{mech} . The moment of inertia depends on the axis of rotation and the geometry of the rotating body. In drive technology, cylindrical bodies that rotate around their axis of symmetry are predominantly considered. The mass moment of inertia for a solid cylinder with diameter d_e and length l can be given as follows:

$$J_{mech} \left(= \int_V r^2 \rho_{mech}(r)\,\mathrm{d}V \right) = \frac{\pi}{2}\,\rho_{mech}\,l\,\left(\frac{d_e}{2}\right)^4 = \frac{\pi}{32}\,\rho_{mech}\,l\,d_e^4 = \frac{1}{8} m\,d_e^2 \tag{1.1}$$

Accordingly, the moment of inertia of a hollow cylinder with the outer diameter d_{e}, the inner diameter d_{i} and the length l can then be specified directly:

$$J_{\mathrm{mech}} = \frac{\pi}{32}\,\rho_{\mathrm{mech}}\,l\,\left(d_{\mathrm{e}}^4 - d_{\mathrm{i}}^4\right) = \frac{1}{8}m\,\left(d_{\mathrm{e}}^2 + d_{\mathrm{i}}^2\right) \tag{1.2}$$

1.1.4 Balance equation in relative values

It sometimes proves to be an advantage to work with relative values in dynamic processes. Torques are referred to the nominal torque and angular velocities to the nominal angular velocity:

$$\begin{aligned}
M - M_{\mathrm{L}} - J_{\mathrm{mech}}\,\frac{\mathrm{d}\Omega_{\mathrm{m}}}{\mathrm{d}t} &= 0 \qquad |:M_{\mathrm{N}} \\
m - m_{\mathrm{L}} - \frac{J_{\mathrm{mech}}}{M_{\mathrm{N}}}\,\frac{\mathrm{d}\Omega_{\mathrm{m}}}{\mathrm{d}t}\cdot\frac{\Omega_{\mathrm{m,N}}}{\Omega_{\mathrm{m,N}}} &= 0 \\
m - m_{\mathrm{L}} - T_{\mathrm{J}}\cdot\frac{\mathrm{d}\omega_{\mathrm{m}}}{\mathrm{d}t} &= 0 \\
\omega_{\mathrm{m}} &= \frac{\Omega_{\mathrm{m}}}{\Omega_{\mathrm{m,N}}} = \frac{n}{n_{\mathrm{N}}} \\
T_{\mathrm{J}} &= 2\pi\cdot\frac{J_{\mathrm{mech}}}{M_{\mathrm{N}}}n_{\mathrm{N}}
\end{aligned} \tag{1.3}$$

The ramp-up time T_{J} defined here is the time in which the unloaded drive can be accelerated from standstill to the nominal speed with nominal torque.

1.2 Transformed mass and moment of inertia

The application of this mechanical balance equation makes it possible to determine the forces and moments of the actors from the "robot task" when the load and the drive are directly coupled. If mechanical transmission elements (e.g. a gearbox) are used in the system, one difficulty arises: moving masses with different speeds and rotational speeds have to be taken into account in the balance equation. This can be overcome by the concept of transformed masses or monent of inertias. The law of conservation of energy is used to consider the mass of the load as an additional moment of inertia acting on the drive shaft. This related moment of inertia is determined here for three typical transmission elements:

- Case 1: Gear drive
- Case 2: Converter from rotational to linear movement by wheels or drums as used in vehicles, cranes, elevators or conveyor belts.
- Case 3: Spindle

Case 1: Gear transmission

Often different moments of inertia as shown in Figure 1.3 are connected to one another via a gear with the ratio i_{12}.

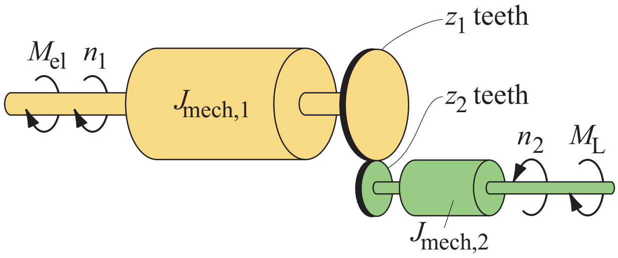


Figure 1.3: Basic kinematics of a gear transmission

In the case of a spur gear, the translation is determined by the number of teeth z_1 on the drive gear and the number of teeth z_2 on the driven gear.

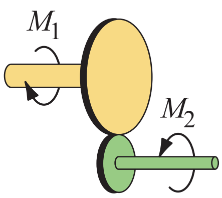
$$i_{12} = \frac{z_1}{z_2} \qquad \Rightarrow \qquad n_1 \cdot z_1 = n_2 \cdot z_2 \qquad \Rightarrow \qquad n_2 = \frac{z_1}{z_2} \cdot n_1 = i_{12} \cdot n_1 \tag{1.4}$$

First, the kinetic energy stored in the moment of inertia $J_{\text{mech},2}$ is considered:

$$\begin{aligned} W_{\text{kin}} &= \frac{1}{2} J_{\text{mech},2}\ \Omega_{\text{m},2}^2 = \frac{1}{2} J_{\text{mech},2} \cdot (i_{12} \cdot \Omega_{\text{m},1})^2 \\ &= \frac{1}{2} \left(i_{12}^2 \cdot J_{\text{mech},2}\right) \cdot \Omega_{\text{m},1}^2 \end{aligned}$$

$$W_{\text{kin}} = \frac{1}{2} \cdot J'_{\text{mech},2} \cdot \Omega_{\text{m},1}^2 \qquad \text{mit } J'_{\text{mech},2} = i_{12}^2 \cdot J_{\text{mech},2} \tag{1.5}$$

$J'_{\text{mech},2}$ is therefore the moment of inertia transformed (or effective) on shaft 1. In Figure 1.4 the same circumferential forces act on the two gears (neglecting friction losses!).



$$\frac{M_1}{z_1} = \frac{M_2}{z_2} \qquad \Rightarrow \qquad M_1 = i_{12} \cdot M_2$$

Figure 1.4: Equilibrium of forces in a gear transmission

The equation of motion on shaft 1 for the equivalent system shown in Figure 1.5a can be written as:

$$\Sigma M = 0: \quad M_{\text{el}} - i_{12} \cdot M_{\text{L}} - \left(J_{\text{mech},1} + J'_{\text{mech},2}\right) \cdot \frac{\mathrm{d}\Omega_{\text{m},1}}{\mathrm{d}t} = 0 \qquad \left(i_{12} = \frac{z_1}{z_2}\right) \tag{1.6}$$

In some applications it is more interesting to use the equation for shaft 1 (equivalent system in Figure 1.5b):

$$\Sigma M = 0: \quad M_{\text{el}}/i_{12} - M_{\text{L}} - \left(J_{\text{mech},1}/i_{12}^2 + J_{\text{mech},2}\right) \cdot \frac{\mathrm{d}\Omega_{\text{m},2}}{\mathrm{d}t} = 0$$

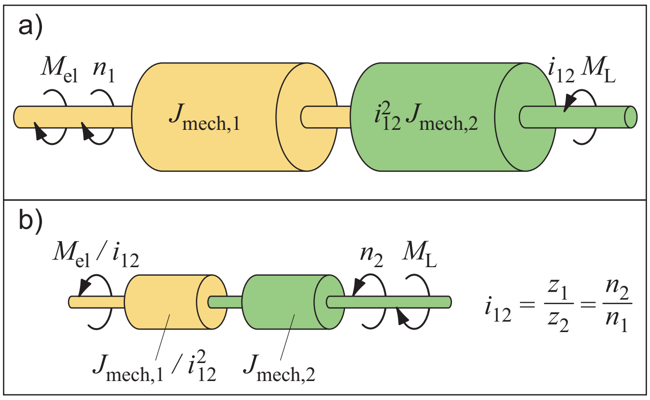
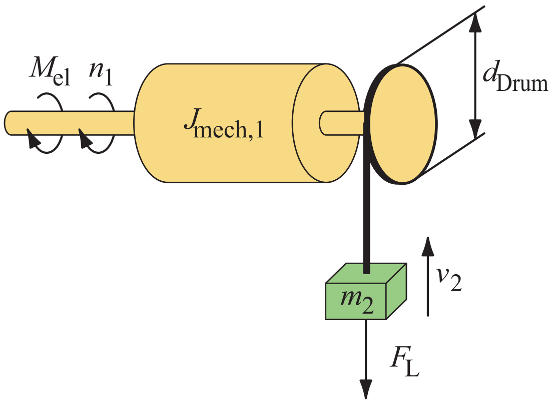


Figure 1.5: kinematic equivalent models

Case 2: Converter from rotational to linear movement by wheels or drums

In many applications the problem arises that a translational movement has to be converted into a rotational movement (e.g. vehicles, elevators, cranes, conveyor belts, ...). This case is examined here using the example of an elevator. Analogous equations arise for the other applications mentioned. Figure 1.6 shows the studied configuration, in which a mass m_2 is moved with an additional external force F_L and a speed v_2. The equation of motion should be given again in relation to the drive shaft (index 1).



$$v_2 = \Omega_{m,1} \frac{d_{Drum}}{2} = i_{12} \cdot \Omega_{m,1}$$

$$\Rightarrow \quad \Omega_{m,1} = \frac{v_2}{i_{12}} = \frac{v_2}{d_{Drum}/2}$$

Figure 1.6: Basic kinematics of an elevator

Here too, a gear ratio can be defined, but unlike the wheel gear, it is no longer dimensionless, as the diameter of the drum d_{Drum} is involved:

$$i_{12} = \frac{d_{Drum}}{2}.$$

In order to be able to convert the moving mass m_2 into an equivalent mass moment of inertia, the kinetic energy stored in m_2 is considered:

$$\begin{aligned} W_{kin} &= \frac{1}{2} m_2\, v_2^2 = \frac{1}{2} m_2 \left(i_{12} \cdot \Omega_{m,1}\right)^2 \\ &= \frac{1}{2} \left(i_{12}^2 \cdot m_2\right) \cdot \Omega_{m,1}^2 \\ &= \frac{1}{2} \cdot J'_{mech,2} \cdot \Omega_{m,1}^2 \qquad \text{mit } J'_{mech,2} = i_{12}^2 \cdot m_2 \end{aligned} \tag{1.7}$$

According to Figure 1.7, the tangential force acting on the drum results in a torque on shaft 1. This allows the equation of motion for shaft 1 to be written as:

$$\Sigma M = 0: \quad M_{\mathrm{el}} - i_{12} \cdot F_2 - \left(J_{\mathrm{mech},1} + J'_{\mathrm{mech},2}\right) \cdot \frac{\mathrm{d}\Omega_{\mathrm{m},1}}{\mathrm{d}t} = 0 \qquad \left(i_{12} = \frac{d_{\mathrm{Drum}}}{2}\right) \tag{1.8}$$

If the equation of motion should be established for the moving mass m_2, it can be done in exactly the same way as with the gear train.

$$\Sigma F = 0: \quad M_{\mathrm{el}}/i_{12} - F_2 - \left(J_{\mathrm{mech},1}/i_{12}^2 + m_2\right) \cdot \frac{\mathrm{d}v_2}{\mathrm{d}t} = 0 \qquad \left(i_{12} = \frac{d_{\mathrm{Drum}}}{2}\right) \tag{1.9}$$

Here, the moment of inertia $J_{\mathrm{mech},1}$ is converted into an equivalent mass $m'_1 = J_{\mathrm{mech},1}/i_{12}^2$, which apparently increases the moving mass m_2.

Case 3: Spindle

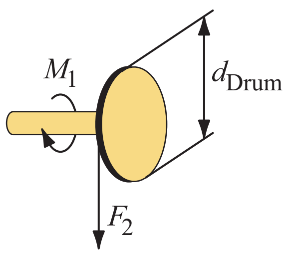
A ball spindle is a low friction mechanism to convert a rotational movement into a linear one. In Figure 1.8, a spindle is driven with the drive torque M_1 and the speed n_1, which with a screw-like movement moves the lead mass m_2 with the speed v_2 moves against a load F_{L}. When the spindle rotates through the angle $2\,\pi$, the lead mass moves further by the spindle pitch h_{Sp}.
As long as friction losses can be neglected, in the stationary state (no acceleration!) the mechanical work $M_1 \cdot 2\,\pi$ done on the shaft would be equal to the work done on the mass $F_2 \cdot h_{\mathrm{Sp}}$. The power would also be the same on both sides, so the following applies:

$$\begin{aligned} &\text{Work:} && M_1 \cdot 2\,\pi = F_2 \cdot h_{\mathrm{Sp}} && \Rightarrow && \frac{F_2}{M_1} = \frac{2\,\pi}{h_{\mathrm{Sp}}} \\ &\text{Power:} && M_1 \cdot \Omega_1 = F_2 \cdot v_2 && \Rightarrow && \frac{v_2}{\Omega_1} = \frac{M_1}{F_2} = \frac{h_{\mathrm{Sp}}}{2\,\pi} = i_{12} \end{aligned} \tag{1.10}$$

If one considers the kinetic energy stored in the moving mass m_2, the equivalent mass moment of inertia J'_{m} related to shaft 1 can be derived:

$$\begin{aligned} &\frac{1}{2} m_2 v_2^2 = \frac{1}{2} J'_{\mathrm{m}} \Omega_1^2 \\ \Rightarrow \quad & J'_{\mathrm{m}} = m_2 \left(\frac{v_2}{\Omega_1}\right)^2 = m_2 \cdot i_{12}^2 = m_2 \left(\frac{h_{\mathrm{Sp}}}{2\,\pi}\right)^2 \end{aligned} \tag{1.11}$$

The moment of inertia of the spindle $J_{\mathrm{mech},1}$ can also be modeled with an equivalent mass m'_1, which moves with the speed v_2:



$$M_1 = F_2 \cdot \frac{d_{\mathrm{Drum}}}{2} = i_{12} \cdot F_2$$

Figure 1.7: Equilibrium of forces in an elevator mechanism

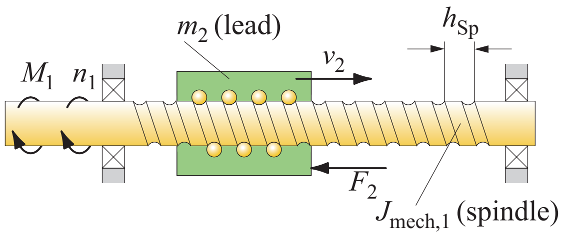


Figure 1.8: Linear axis with ball roller spindle

$$\frac{1}{2} m_1' v_2^2 = \frac{1}{2} J_{\text{mech},1} \Omega_1^2$$

$$\Rightarrow \qquad m_1' = J_{\text{mech},1} \left(\frac{\Omega_1}{v_2}\right)^2 = J_{\text{mech},1} \cdot i_{12}^{-2} = J_{\text{mech},1} \left(\frac{2\,\pi}{h_{\text{Sp}}}\right)^2 \tag{1.12}$$

1.3 Optimal gear ratio

The time required for a positioning process depends very much on the gear ratio selected. If a very high gear ratio is selected in order to be able to use the smallest possible motor with the same power, the speed (and also the angular acceleration!) on the output side will be too small for a quick positioning process.

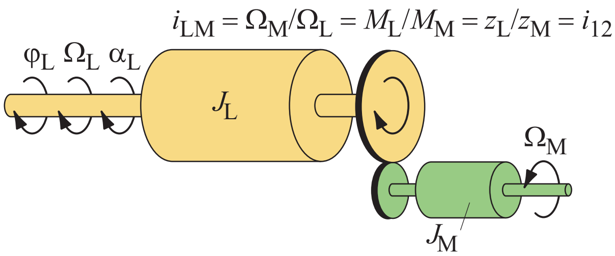


Figure 1.9: Drive with gear

However, if the gear ratio is chosen too small (in extreme cases, no gear with a direct drive), the transformed load moment of inertia at the motor shaft becomes too large and the available acceleration becomes smaller again. There must therefore be an optimum. In the following, a drive (index M) that is connected to the load (index L) via a gearbox will be considered.

Assuming that no load or friction torque occurs, the angular acceleration at the load can be represented as follows for a motor torque M_{M}:

$$\alpha_{\text{L}} = \frac{i_{12} \cdot M_{\text{M}}}{J_{\text{L}} + i_{12}^2 J_{\text{M}}} = \frac{M_{\text{M}}}{i_{12} \cdot J_{\text{M}} + J_{\text{L}}/i_{12}} \tag{1.13}$$

To determine the optimal gear ratio, a distinction must be made as to whether the maximum speed during acceleration is limited or not.

Case 1: (Time-)optimal gear ratio without speed limitation

If the load is to be positioned from position φ_1 to position φ_2 in the optimal time, the load is accelerated up to half of the distance with the maximum motor torque and then braked again with the maximum torque.

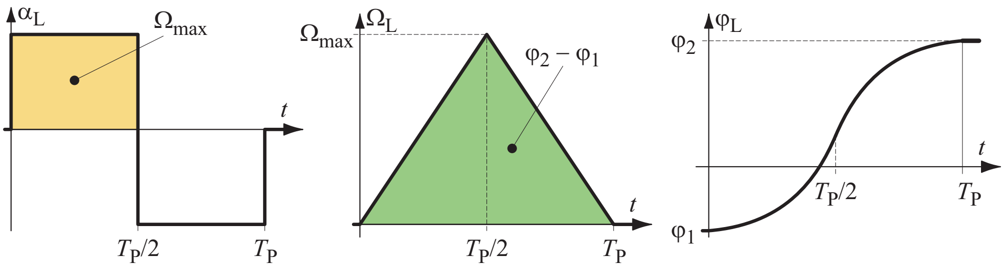


Figure 1.10: Time-optimal positioning process without limitation

The speed characteristic is therefore made up of straight line pieces as shown in Figure 1.10, while the position is made up of two parabolic pieces. The following applies:

$$\alpha_L = \frac{d\Omega_L}{dt} = \frac{\Omega_{max}}{T_P/2} \qquad \Rightarrow \qquad \Omega_{max} = \alpha_L \cdot \frac{T_P}{2}$$

$$\Delta\varphi_L = \varphi_2 - \varphi_1 = \frac{T_P \cdot \Omega_{max}}{2} = \frac{\alpha_L}{4} \cdot T_P^2$$

$$\Rightarrow \quad T_P = \sqrt{\frac{4\Delta\varphi_L}{\alpha_L}} = \sqrt{\frac{4\Delta\varphi_L}{M_M} \cdot \left(i_{12} \cdot J_M + \frac{J_L}{i_{12}}\right)} \tag{1.14}$$

Obviously the positioning time T_P depends on the gear ratio i. In order to determine at which ratio i_{opt} the positioning time becomes minimum, the derivative of the positioning time with respect to i_{12} must be found and equaled to zero:

$$\frac{\partial T_P}{\partial i_{12}} = \frac{\partial}{\partial i_{12}} \sqrt{\frac{4\Delta\varphi_L}{M_M} \cdot \left(i_{12} \cdot J_M + \frac{J_L}{i_{12}}\right)} = \sqrt{\frac{4\Delta\varphi_L}{M_M}} \cdot \frac{\partial}{\partial i_{12}} \sqrt{i_{12} \cdot J_M + \frac{J_L}{i_{12}}}$$

$$= \sqrt{\frac{4\Delta\varphi_L}{M_M}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{i_{12} \cdot J_M + \frac{J_L}{i_{12}}}} \cdot \left(J_M - \frac{J_L}{i_{12}^2}\right) \stackrel{!}{=} 0$$

$$\Rightarrow \quad J_M - \frac{J_L}{i_{opt}^2} = 0 \quad \Rightarrow \quad i_{opt} = \sqrt{\frac{J_L}{J_M}} \tag{1.15}$$

This means that the positioning time is minimal when the load moment of inertia transformed to the motor shaft is as large as the motor moment of inertia itself.

However, the gear ratio often cannot be optimized only with regard to the positioning time and other aspects must be considered (e.g. available drive power from the catalogs). To find the best compromise, the dependence of the positioning time with the gear ratio becomes important.

Inserting the result from Eq. 1.15 into Eq. 1.14, the minimum time can be found:

$$T_{P,opt} = \sqrt{\frac{4\Delta\varphi_L}{M_M} \cdot \left(\sqrt{\frac{J_L}{J_M}} \cdot J_M + \sqrt{\frac{J_M}{J_L}} \cdot J_L\right)} = \sqrt{\frac{4\Delta\varphi_L}{M_M} \cdot \left(\sqrt{J_L \cdot J_M} + \sqrt{J_M \cdot J_L}\right)}$$

$$= \sqrt{\frac{8\Delta\varphi_L}{M_M} \cdot \sqrt{J_L \cdot J_M}}$$

The ratio of the positioning time from Eq. 1.14 to the optimal one can be then interpreted very clearly graphically in Figure 1.11 :

$$\frac{T_{\mathrm{P}}}{T_{\mathrm{P,opt}}} = \frac{\sqrt{\frac{4\Delta\varphi_{\mathrm{L}}}{M_{\mathrm{M}}}\cdot\left(i_{12}\cdot J_{\mathrm{M}}+\frac{J_{\mathrm{L}}}{i_{12}}\right)}}{\sqrt{\frac{8\Delta\varphi_{\mathrm{L}}}{M_{\mathrm{M}}}\cdot\sqrt{J_{\mathrm{L}}\cdot J_{\mathrm{M}}}}} = \sqrt{\frac{1}{2}\cdot\left(\frac{i_{12}\cdot J_{\mathrm{M}}}{\sqrt{J_{\mathrm{L}}\cdot J_{\mathrm{M}}}}+\frac{J_{\mathrm{L}}}{i_{12}\cdot\sqrt{J_{\mathrm{L}}\cdot J_{\mathrm{M}}}}\right)}$$

$$= \sqrt{\frac{1}{2}\cdot\left(\frac{i_{12}}{\sqrt{J_{\mathrm{L}}/J_{\mathrm{M}}}}+\frac{\sqrt{J_{\mathrm{L}}/J_{\mathrm{M}}}}{i_{12}}\right)} = \sqrt{\frac{1}{2}\cdot\left(\frac{i_{12}}{i_{\mathrm{opt}}}+\frac{i_{\mathrm{opt}}}{i_{12}}\right)} \tag{1.16}$$

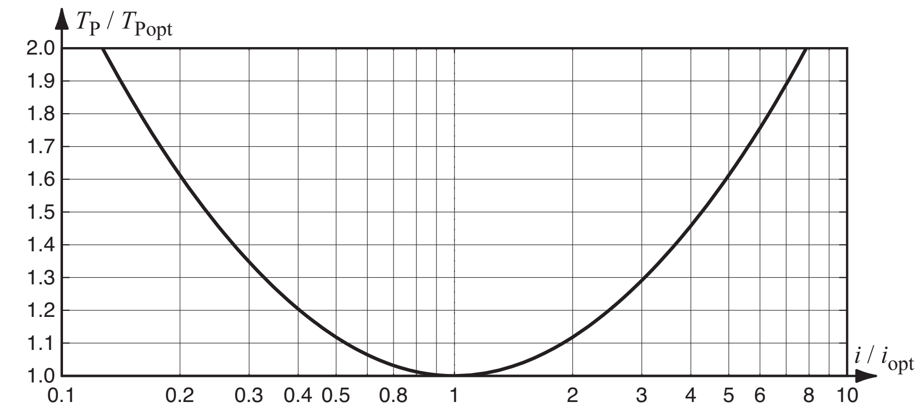


Figure 1.11: Dependence of the positioning time with the gear ratio

Case 2: (Time-)optimal gear ratio with limited load speed

The load speed is often limited for mechanical reasons. Excessive centrifugal forces can occur when a certain speed is exceeded, or intolerable mechanical vibrations can occur. If the load is to be positioned from a position φ_1 to a position φ_2 in the optimal time, a trapezoidal surface is now created by the speed curve.

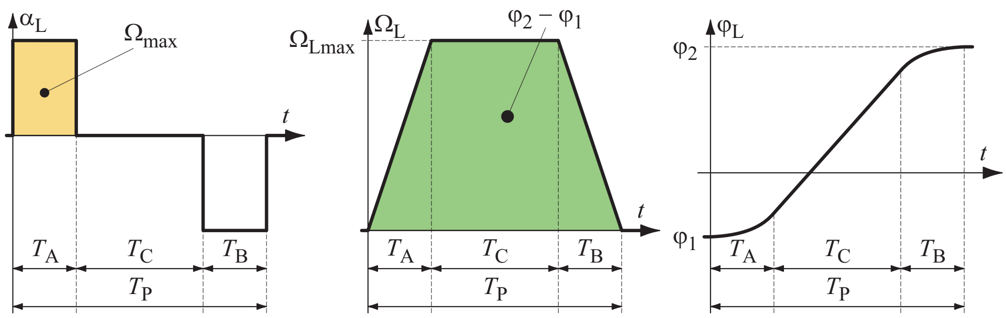


Figure 1.12: Time-optimal positioning process with limited load speed

The positioning time T_{P} consists of a start-up time T_{A} (accelerating), a deceleration time T_{B} (braking) and a time T_{C} (continuous), at which the speed remains constant.

According to Eq. 1.13, the following applies to the angular acceleration:

$$\alpha_\mathrm{L} = \frac{\mathrm{d}\Omega_\mathrm{L}}{\mathrm{d}t} = \frac{\Omega_\mathrm{Lmax}}{T_\mathrm{A}} = \frac{\Omega_\mathrm{Lmax}}{T_\mathrm{B}} = \frac{M_\mathrm{M}}{i_{12} \cdot J_\mathrm{M} + J_\mathrm{L}/i_{12}}$$
$$\Rightarrow \quad T_\mathrm{A} = T_\mathrm{B} = \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \left(i_{12} \cdot J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}} \right)$$

The trapezoidal area under the angular velocity in Figure 1.12 is equal to the displacement angle $\Delta\varphi = \varphi_2 - \varphi_1$:

$$\Delta\varphi = \varphi_2 - \varphi_1 = \left(\frac{T_\mathrm{A} + T_\mathrm{B}}{2} + T_\mathrm{C} \right) \cdot \Omega_\mathrm{Lmax} \quad \Rightarrow \quad T_\mathrm{C} = \frac{\Delta\varphi}{\Omega_\mathrm{Lmax}} - T_\mathrm{A}$$

This allows the positioning time T_P to be specified depending on the gear ratio:

$$T_\mathrm{P} = T_\mathrm{A} + T_\mathrm{C} + T_\mathrm{B} = 2T_\mathrm{A} + T_\mathrm{C} = 2T_\mathrm{A} + \frac{\Delta\varphi}{\Omega_\mathrm{Lmax}} - T_\mathrm{A}$$
$$= \frac{\Delta\varphi}{\Omega_\mathrm{Lmax}} + \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \left(i_{12} \cdot J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}} \right)$$

Here too, the positioning time must be derived according to the gear ratio, whereby the first term is omitted due to the lack of dependence on the gear ratio:

$$\frac{\partial T_\mathrm{P}}{\partial i_{12}} = \frac{\partial}{\partial i_{12}} \left[\frac{\Delta\varphi}{\Omega_\mathrm{Lmax}} + \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \left(i_{12} \cdot J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}} \right) \right]$$
$$= \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \frac{\partial}{\partial i_{12}} \left(i_{12} \cdot J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}} \right) = \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \left(J_\mathrm{M} - \frac{J_\mathrm{L}}{i_{12}^2} \right) \stackrel{!}{=} 0$$
$$\Rightarrow \quad i_\mathrm{opt} = \sqrt{\frac{J_\mathrm{L}}{J_\mathrm{M}}}$$

This means that the limitation of the load speed plays no role in selecting the optimal gear ratio.

Case 3: (Time) optimal gear ratio with limited motor speed

Since the size and therefore the costs of a drive motor essentially depend on the torque, one is generally inclined to use the whole available speed range of a motor as much as possible. In this case, however, there is a different value for the optimal gear ratio.

In terms of quality, the same curves result in this case as in the last section; the expression shown on the right can also be used for the positioning time. It only has to be taken into account that the maximum output speed Ω_Lmax now depends on the gear ratio:

$$T_\mathrm{P} = \frac{\Delta\varphi}{\Omega_\mathrm{Lmax}} + \frac{\Omega_\mathrm{Lmax}}{M_\mathrm{M}} \cdot \left(i_{12} \cdot J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}} \right) \qquad \text{here:} \quad \Omega_\mathrm{Lmax} = \frac{\Omega_\mathrm{Mmax}}{i_{12}}$$
$$= \frac{i_{12} \cdot \Delta\varphi}{\Omega_\mathrm{Mmax}} + \frac{\Omega_\mathrm{Mmax}}{M_\mathrm{M}} \cdot \left(J_\mathrm{M} + \frac{J_\mathrm{L}}{i_{12}^2} \right)$$

To determine the optimal gear ratio, the positioning time must be derived again according to the ratio i_{12} and the zero point of the derivation must be determined:

$$\frac{\partial T_\mathrm{P}}{\partial i_{12}} = \frac{\Delta\varphi}{\Omega_\mathrm{Mmax}} + \frac{\Omega_\mathrm{Mmax}}{M_\mathrm{M}} \cdot \left(0 - 2\frac{J_\mathrm{L}}{i_{12}^3}\right) \stackrel{!}{=} 0$$

$$\Rightarrow \quad i_\mathrm{opt} = \sqrt[3]{\frac{2\,\Omega_\mathrm{Mmax}^2 \cdot J_\mathrm{L}}{\Delta\varphi \cdot M_\mathrm{M}}} \tag{1.17}$$

Since the optimal gear ratio now also depends on the adjustment angle $\Delta\varphi$, this consideration is almost purely academic and practically meaningless for an application. It makes more sense to select the engine in such a way that the determination of the gear ratio is not restricted by a limited motor speed.

Case 4: (Energy-)Optimal Movement

The positioning time is often the most important (optimization) criterion in robotics but also in machine tooling construction (i.e. for the positioning of tools and workpieces) and also in process engineering (i.e. in conveyor technology).

But there are also applications where the minimum energy requirement is also in the focus. This is in robotics the case, for example, in battery-operated autonomous mobile robots. How considering the energy requirement can change the positioning process is shown here using the example of an "S-Bahn". For urban railway, an average stop distance of around $1000\,\mathrm{m}$ can be assumed. Depending on the traffic volume and the associated varying lengths of people changing times, a travel time of $70\,\mathrm{s}$ to $80\,\mathrm{s}$ is typical in order to be able to adhere to the sometimes very tight timetable.

With this energy-optimal movement, an attempt is made to let the vehicle roll with the drives switched off for as long as possible. It would be ideal to control the timetable in such a way that the energy fed back during braking can also be made available to a train departing from the same stop at the same time in order to minimize line losses.

The actual energy consumption for such a driving cycle has been shown in Figure 1.13. The vehicle is accelerated to a speed of $60\,\mathrm{km/h}$ to $80\,\mathrm{km/h}$. After reaching the so-called switch-off speed, the drives are switched off. The vehicle rolls and is initially only slowed down by the driving resistance. At the end it is braked electrically shortly before reaching the next platform to control the exact location and time.

If a travel time of $80\,\mathrm{s}$ is permitted, in this case only $9.5\,\mathrm{kWh}$ will be consumed per driving cycle. With a required travel time of $70\,\mathrm{s}$ during rush hour, already $17.4\,\mathrm{kWh}$ are necessary!

In the case of traction drives, the required driving power per drive wheel axle ultimately determines the size of the traction motor, whereby the force required for the vehicle when starting should be achieved with the best possible motor efficiency. The gear ratio can then be determined on this basis.

As a rule, however, one will try to choose the gear ratio as large as possible so that the drive power can be provided with the lowest possible drive torque. The size of an electric motor is essentially determined by the torque. A smaller motor also tends to produce smaller losses - even if the motor speed and thus the speed-dependent losses increase.

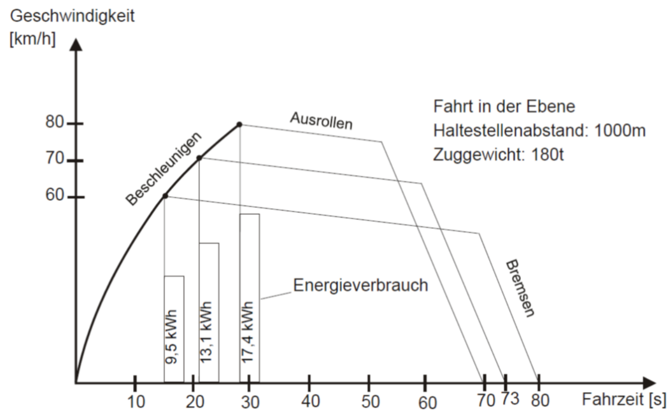


Figure 1.13: Speed-time diagram of an S-Bahn, source: Hamburger Hochbahn
Geschwindigkeit = Speed
Fahrzeit = Travel time
Beschleunigen = Accelerating
Ausrollen = Rolling
Bremsen = Braking
Fahrt in der Ebene = Plain terrain
Haltestellenabstand = Distance between stations
Zuggewicht = Train weight

1.4 Examples and tasks

Example 1-1: Operating point drive/load

Given is a system consisting of an electric motor and a mechanical load, e.g. a robot arm. The speed-torque equation for the motor is $M = M_0 \cdot (1 - 0,1\frac{n}{n_{max}})$ with $M_0 = 2\,\mathrm{Nm}$ and $n_{max} = 3000\,\mathrm{min}^{-1}$. The equation for the load torque acting on the motor shaft is $M_L = M_{0,L} \cdot (1 + 3\frac{n}{n_{max}})$ with $M_{0,L} = 0.75\,\mathrm{Nm}$.

a. What is the stationary operating point of the system?
b. Thanks to an inverter, M_0 for the motor can be varied as a function of the input voltage. The acceleration during startup should be as constant as possible. Draw qualitatively the operating points of the motor in the M-n characteristic.

Example 1-2: Drives for a passenger elevator

A passenger elevator with a maximum payload of $650\,\mathrm{kg}$ (max. total mass of the elevator car $1900\,\mathrm{kg}$, mass of the counterweight $1565\,\mathrm{kg}$) should cover the distance from the basement (2nd basement floor) to the 20th floor in $30\,\mathrm{s}$. Each floor has a height of $3.5\,\mathrm{m}$

a. Estimate the necessary drive power and driving speed!
b. What values do you estimate for the acceleration, the acceleration time and the distance covered in the acceleration phase?
c. By what factor would the torque be greater during the acceleration phase than during the time at constant speed?
d. What would be the gear ratio with a cable drum diameter of $320\,\mathrm{mm}$ and a motor speed of $1500\,\mathrm{min}^{-1}$?
e. Is this ratio time-optimal? How much can the travel time be shortened? Remember that it is very unlikely that an elevator can travel 22 floors without stopping! (The motor moment of inertia can be assumed to be $0.04\,\mathrm{kg\,m}^2$)

1.5 Comprehension questions

Q1-1: How can the necessary drive data for a robot application be derived?
Q1-2: What is the mechanical balance equation for rotational movements?
Q1-3: What stationary operating point occurs in a drive-load system?
Q1-4: What is the transformed moment of inertia?
Q1-5: Is there “the” optimal gear ratio for robotics? Which parameters play a role in the selection?

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