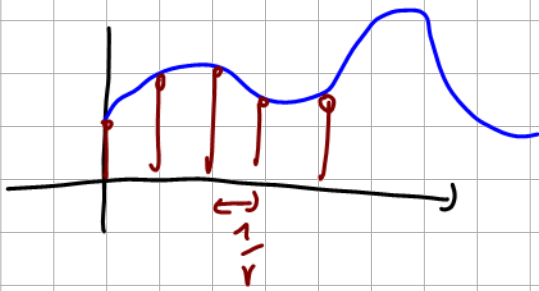


C2, 25, T1

$$x(t) = [\cos(7t)]^2$$

$$r = 10 \text{ kHz}$$

$$x(n) = ? \quad 0 \leq n \leq 5$$



sampling: $x(t) \rightarrow x(n)$ with

$$t = \frac{n}{r}$$

$$x(n=0) = [\cos(7 \cdot \frac{0}{r})]^2 = 1^2 = 1$$

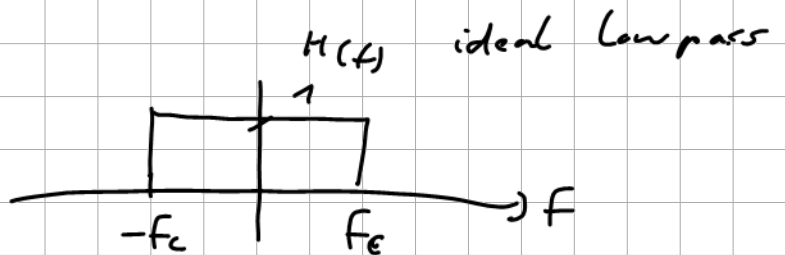
$$x(n=1) = [\cos(7 \cdot \frac{1}{10000})]^2 = 0.9999$$

cos in radian!

C2 25 T2

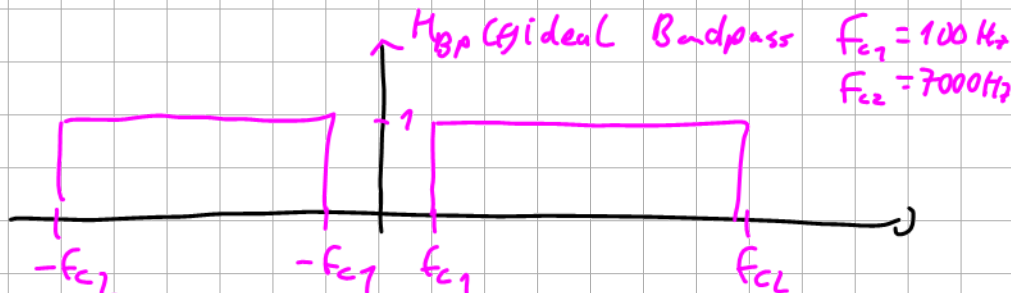
$$H(f) = \text{rect}\left(\frac{f}{2f_c}\right)$$

?



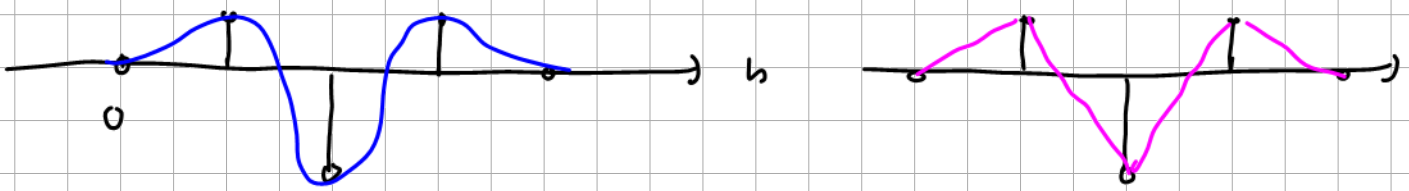
$$h(t) = 2f_c \cdot \text{sinc}(2f_c \cdot t)$$

$$= 2 \cdot f_c \cdot \text{si}(2 \cdot f_c \cdot \pi \cdot t) = 2 \cdot f_c \cdot \frac{\sin(2f_c \pi t)}{2f_c \pi t}$$



$$x(n) = [0; 1; -2; 1; 0]$$

linear interpolation



C2 J5 T4

has $z[n]$ a DC component?

no, DC component has a frequency of $f=0$ Hz which cannot pass the Band-pass.

human voices understandable in $z[n]$?

old phone system / narrowband : 300 Hz ... 3400 Hz

new -" - / wideband : 50 Hz ... 7000 Hz

\Rightarrow the range from 100 Hz up to 7000 Hz:

human voice can pass

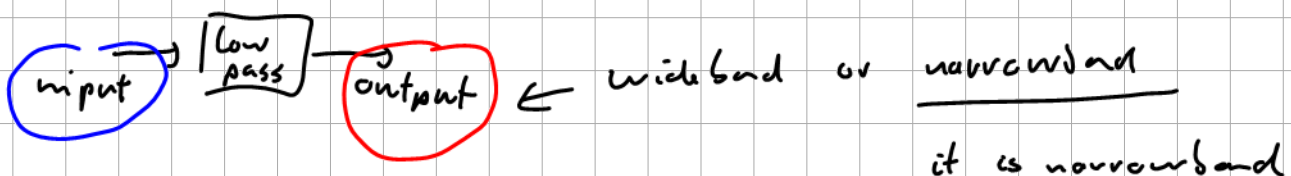
highest frequency in $y[n]$? 24 kHz

-" - in $z[n]$? 7 kHz

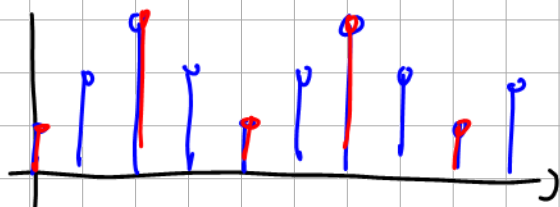
C2 J5 T3

$r = 32$ kHz

$f_c = 3400$ Hz



Sampling theorem : $f_c = 3400$ Hz \Rightarrow lowest allowed sampling rate = 6800 Hz



e.g. $c=2$

$$c=5 \Rightarrow 5 \cdot 6800 \text{ Hz} = 34000 \text{ Hz}$$

$$\boxed{c=4} \Rightarrow 4 \cdot 6800 \text{ Hz} = 27200 \text{ Hz} \quad \checkmark$$

$C2, 24, T3$

$$\begin{aligned} \text{Filter length} &= 47 = N \\ \text{delay} &= 23 \rightarrow \left\lfloor \frac{N}{2} \right\rfloor \leftarrow \text{rounded down} \end{aligned}$$

$C4, 2, T3$

Leaky ReLU



$$f(x) = \begin{cases} x, & x \geq 0 \\ \alpha x, & \text{else} \end{cases}$$

$$\alpha = 0.3$$

concrete example for nonlinearity

$$a = b = 1$$

$$x_1 = 1$$

$$x_2 = -1$$

$$f(a \cdot x_1 + b \cdot x_2) = a \cdot f(x_1) + b \cdot f(x_2)$$

$$f(1 \cdot 1 + 1 \cdot (-1)) = 1 \cdot f(1) + 1 \cdot f(-1)$$

$$f(0) = 1 \cdot 1 + 1 \cdot (-0.3 \cdot 1)$$

$$0 = 1 - 0.3 \quad \Sigma$$

Dense Layer: $y_j = \sum_{i=0}^{I-1} w_{ji} x_i + b_j = f(x_i)$

$$f(a \cdot x_1 + b \cdot x_2) = \sum_{i=0}^{I-1} w_{ji} (a \cdot x_{1i} + b \cdot x_{2i}) + b_j$$

$$= a \cdot \sum_{i=0}^{I-1} w_{ji} x_{1i} + b \cdot \sum_{i=0}^{I-1} w_{ji} x_{2i} + b_j$$

$$\begin{aligned} a \cdot f(x_1) + b \cdot f(x_2) &= a \cdot \left(\sum_{i=0}^{I-1} w_{ji} x_{1i} + b_j \right) + b \cdot \left(\sum_{i=0}^{I-1} w_{ji} x_{2i} + b_j \right) \\ &= a \sum_{i=0}^{I-1} w_{ji} x_{1i} + b \cdot \sum_{i=0}^{I-1} w_{ji} x_{2i} + a \cdot b_j + b \cdot b_j \end{aligned}$$

not linear

Fourier Transform
(discrete)

$$y(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{nk}{K}} = f(x(n))$$

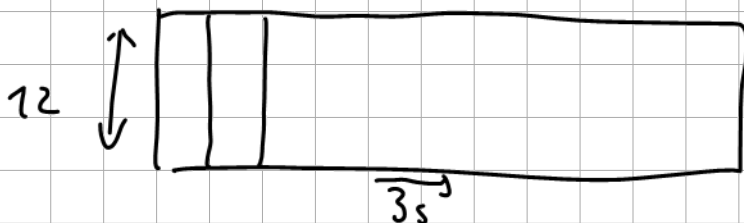
$$\begin{aligned} f(a \cdot x_1 + b \cdot x_2) &= \sum_{n=0}^{N-1} (a x_1(n) + b \cdot x_2(n)) e^{-j2\pi \frac{nk}{K}} \\ &= \sum_{n=0}^{N-1} a x_1(n) e^{-j2\pi \frac{nk}{K}} + b x_2(n) e^{-j2\pi \frac{nk}{K}} \end{aligned}$$

$$\begin{aligned} a \cdot f(x_1) + b \cdot f(x_2) &= a \cdot \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi \frac{nk}{K}} + b \cdot \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi \frac{nk}{K}} \\ &= \sum_{n=0}^{N-1} a x_1(n) e^{-j2\pi \frac{nk}{K}} + b \cdot x_2(n) e^{-j2\pi \frac{nk}{K}} \quad \checkmark \end{aligned}$$

Fourier Transform is linear.

C3, 25, 75

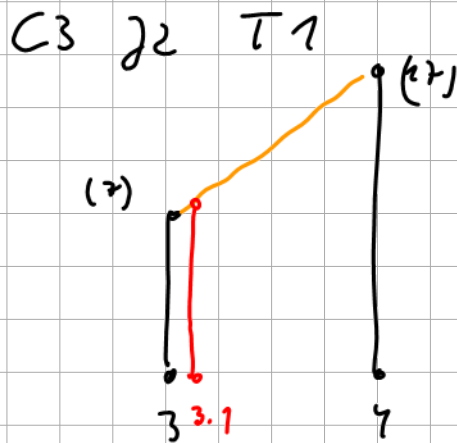
audio 3 seconds, hopsize 10ms, how many coefficients in the MFCC matrix?



each hopsize introduces a new column

$$\Rightarrow 300 \text{ cols} = \frac{3s}{10ms}$$

$$\Rightarrow 12 \cdot 300 = 3600 \text{ MFCC Feature}$$



$$f(x) = mx + n$$

$$7 = m \cdot 3 + n$$

$$17 = m \cdot 4 + n$$

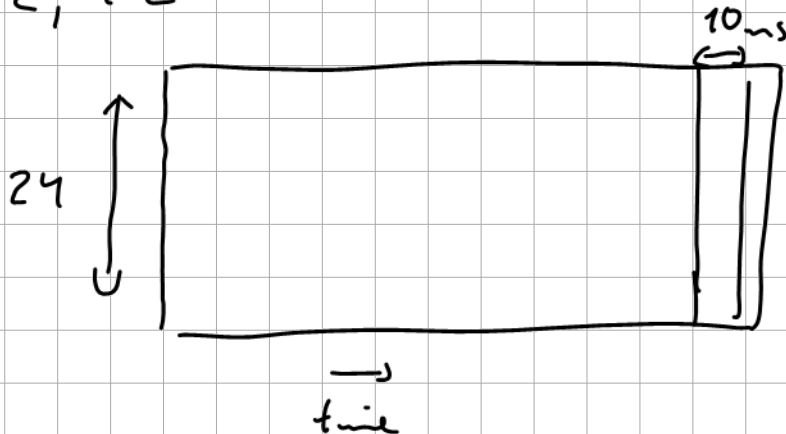
$$m = \frac{17 - 7}{4 - 3} = 10$$

$$7 = 10 \cdot 3 + n \Rightarrow n = -23$$

$$f(x) = 10 \cdot x - 23$$

$$F(3.1) = 10 \cdot 3.1 - 23 = 31 - 23 = 8$$

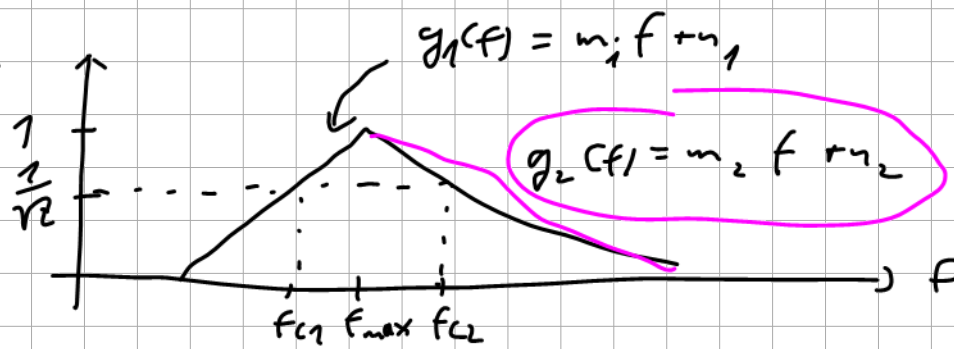
C3, J2, T2



4 Bytes / Bandpass
output

$$\begin{aligned} \text{data rate} &= 24 \text{ Bandpasses} \cdot 4 \text{ Bytes / Bandpass} \cdot 100 \frac{1}{s} \\ &= 24 \cdot 4 \cdot 100 \frac{\text{Bytes}}{s} \end{aligned}$$

C3, J3, T3



$$-3 \text{ dB} = 20 \log_{10} a \quad \Rightarrow \quad a = 10^{\frac{-3}{20}} = 0.707 = \frac{1}{\sqrt{2}}$$

$$\frac{F}{\text{kHz}} = 1.96 \cdot \frac{b + 0.53}{26.28 - b}$$

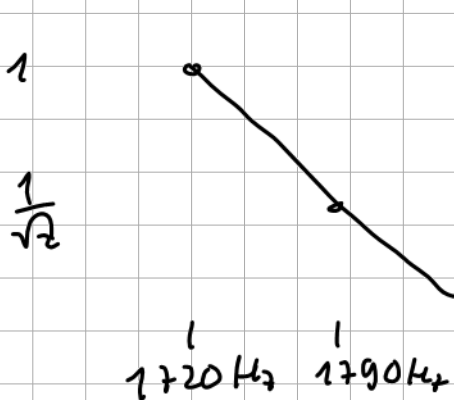
$$f_m = 12 \text{ kHz}$$

$$\rightarrow f_m = 1.72 \text{ kHz} = 1720 \text{ Hz}$$

$$f_{c1} = 11.75 \text{ kHz}$$

$$f_{c2} = 12.25 \text{ kHz}$$

$$\rightarrow f_{c2} = 1.79 \text{ kHz} = 1790 \text{ Hz}$$



$$m_2 = \frac{\frac{1}{\sqrt{2}} - 1}{1790 \text{ Hz} - 1720 \text{ Hz}} = -4.18 \cdot 10^{-3} \text{ s}$$

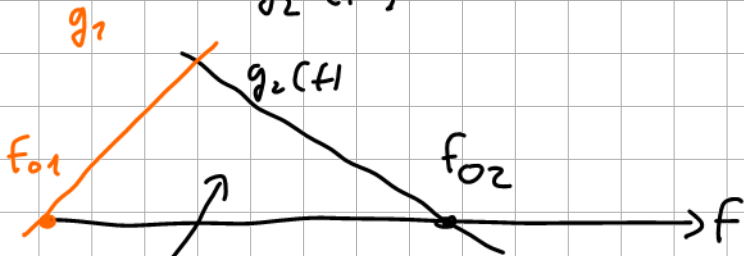
$$1 = m_2 \cdot 1720 \text{ Hz} + n_2$$

$$n_2 = 1 - m_2 \cdot 1720 \text{ Hz}$$

$$= 1 - (-4.18 \cdot 10^{-3} \text{ s}) \cdot 1720 \text{ Hz}$$

$$= 8.20$$

$$g_2(f) = -4.18 \cdot 10^{-3} \text{ s} \cdot f + 8.20$$



$$\text{Area} = \frac{1}{2} \cdot 1 \cdot (f_{02} - f_{01})$$

$$g_2(f) = 0 \quad (\Leftrightarrow) \quad -4.18 \cdot 10^{-3} \text{ s} \cdot f_{02} + 8.20 = 0$$

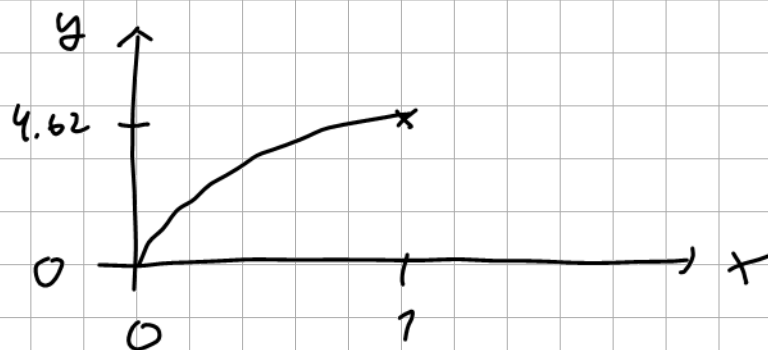
$$f_{02} = \frac{-8.20}{-4.18 \cdot 10^{-3} \text{ s}}$$

$$= 1961 \text{ Hz}$$

C3 J4 T1

$$y = \log_e(100x + 1)$$

$$0 \leq x < 1$$



$$\log_e (100 \cdot 1 + 1) = 4.62$$

C3 24 T3

$$f(x) = \dots$$

sensitivity: $\frac{df}{dx}$ first derivative

e.g. $f(x) = \log x$

$$\text{sens: } \frac{d \log x}{dx} = \frac{df(x)}{dx}$$

C2 28 T3

$$N = 1000$$

$$K = 1024$$

zero padding?

yes: $K > N$

no: $K = N$

$K < N$ has no practical application

$$\Delta f = \frac{1}{K \cdot T}$$

$$T = \frac{1}{r}$$

$$= \frac{r}{K} = \frac{48 \text{ kHz}}{1024} = 46.9 \text{ Hz}$$

$$h = 0 \quad 1 \quad 2 \quad 3 \quad \dots$$

$$X(h) = \begin{array}{c} \hline \uparrow \quad \uparrow \quad \uparrow \\ 0 \text{ Hz} \quad \uparrow 2 \Delta f \quad \uparrow \\ \Delta f \quad 3 \Delta f \end{array}$$

$$h = 1 \hat{=} f = h \cdot \Delta f = 46.9 \text{ Hz} \approx 50 \text{ Hz}$$

$$h = \frac{f}{\Delta f}$$

e.g.: $f = 432 \text{ kHz} \Rightarrow h = \frac{432 \text{ Hz}}{46.7 \text{ MHz}} = 9.21 \approx 9$

C3 25 T 2

minimize: $\sum_r (m \cdot r + n - NG(r))^2$



$$\frac{d}{dm} = \sum_r 2(mr + n - NG(r)) \cdot r = 0$$

$$\frac{d}{dn} = \sum_r 2(mr + n - NG(r)) = 0$$

$$\sum_r m \cdot r^2 + \sum_r n \cdot r = \sum_r r \cdot NG(r)$$

$$\sum_r m \cdot r + \sum_r n = \sum_r NG(r)$$

$$\begin{pmatrix} \sum_r r^2 & \sum_r r \\ \sum_r r & \sum_r 1 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} \sum_r r \cdot NG(r) \\ \sum_r NG(r) \end{pmatrix}$$

$NG(r) =$	0.166	2.385	0.505	3.1	0.836
$r =$	0	1	2	3	4

restrict to the first 5 elements

$$\sum_r r^2 = 0 + 1 + 4 + 9 + 16 = 30$$

$$\sum_r r = 0 + 1 + 2 + 3 + 4 = 10$$

$$\sum_r 1 = 5$$

$$\begin{aligned} \sum_r r \cdot NG(r) &= 0 \cdot 0.166 + 1 \cdot 2.385 + 2 \cdot 0.505 + 3 \cdot 3.1 + 4 \cdot 0.836 \\ &= 16.04 \end{aligned}$$

$$\sum_r NG(r) = 0.166 + 2.385 + 0.505 + 3.1 + 0.836 \\ = 6.99$$

$$\begin{pmatrix} 30 & 10 \\ 10 & 5 \end{pmatrix} \begin{pmatrix} m \\ n \end{pmatrix} = \begin{pmatrix} 16.09 \\ 6.99 \end{pmatrix} - 3 \cdot 2 -$$

$$0 \cdot m + (-5) \cdot n = 16.09 - 3 \cdot 6.99$$

$$n = \frac{16.09 - 3 \cdot 6.99}{-5} = 0.786$$

$$30 \cdot m + 10 \cdot 0.786 = 16.09$$

$$m = \frac{16.09 - 0.786}{30} = 0.206$$

$$\begin{pmatrix} \sum_r i^2 & \sum_r r \\ \sum_r r & \sum_r 1 \end{pmatrix} \dots$$

$$r = \text{np.arange}(10)$$

$$\left[\begin{array}{l} \leftarrow r = \text{np.zeros}((20)) \\ \text{for } i \text{ in range}(r.\text{shape}[0]): \\ \quad r[i] = i \end{array} \right] \leftarrow$$

$$a = \text{np.sum}(r * r) \quad \text{or} \quad \text{np.sum}(r ** 2)$$

$$b = \text{np.sum}(r)$$

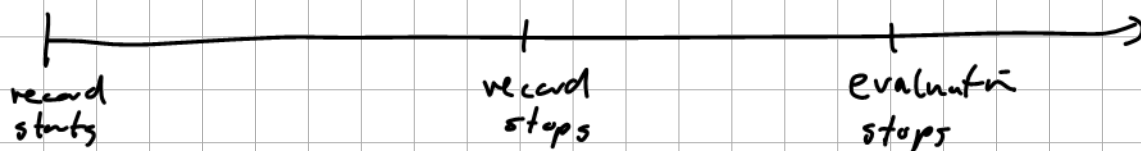
$$c = r.\text{shape}[0] \quad \text{or} \quad \text{np.sum}(r * 1)$$

$$A = \text{np.array}([[a, b], [b, c]])$$

C4 26 T2

and io 30 s

evaluation 28 s



$$\text{Latency} = 58 \text{ s}$$

$$\text{real time factor} = \frac{28}{30} < 1$$

\Rightarrow in principle capable of real time evaluation.

C4 J6 T3

not capable of real time application \Rightarrow whisper tales

use experience

much longer than 500ms

C4 J6 T1 Levenshtein distance

evaluated
by each
letter

each insertion: distance + 1
each missing: distance + 1
each replacing: distance + 1

2
7
2

correct sentence:

11 = Levenshtein distance

Today it is much too cold.
Two day it is cold.

Diagram illustrating Levenshtein distance between "Today it is much too cold." and "Two day it is cold." with arrows showing substitutions and deletions. Circled numbers 1, 5, and 2 indicate specific distance calculations at different points in the alignment.

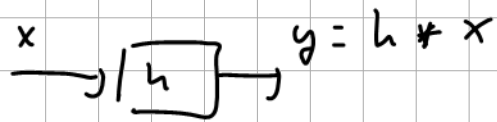
C4 J7 T2

$$p(1) = \frac{1}{n \cdot \ln(1.78 \cdot 10^5)}$$

$$p(1) = \frac{1}{1 \cdot \ln(1.78 \cdot 10^5)} = \dots$$

$$p(2) = \frac{1}{2 \cdot \ln(1.78 \cdot 10^5)} = \dots$$

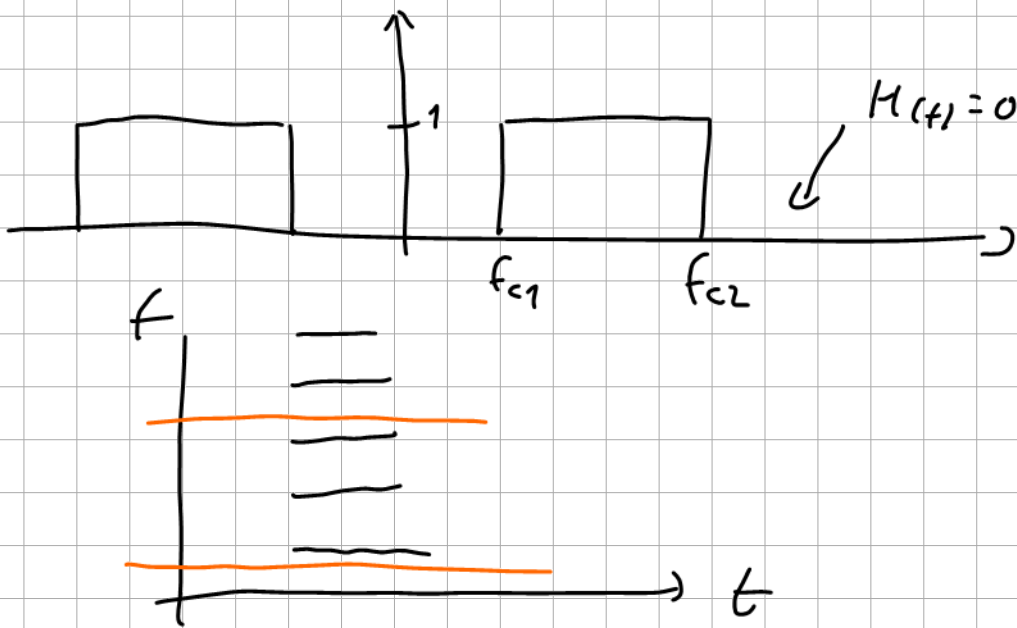
C4 25 T1



$$Y(f) = H(f) \cdot X(f)$$

$$X(f) = \frac{1}{H(f)} \cdot Y(f)$$

$H(f)$ ideal Bandpass



C4 24







