

## Worksheet 5

### Exercise 1

$$\begin{aligned}P(X > 345) &= P(X = 346) + P(X = 347) = \\&= \binom{347}{346} \cdot 0.98^{346} \cdot 0.02 + \binom{347}{347} \cdot 0.98^{347} = \\&= 347 \cdot 0.98^{346} \cdot 0.02 + 0.98^{347} = \\&\approx 0.0064 + 0.0005 \approx \underline{\underline{0.007}}\end{aligned}$$

### Exercise 2

$X \hat{=}$  "number of correct pallets  
in spot check"

$A \hat{=}$  "no complete inventory"

$$\Rightarrow P(A) = P(X = 100) = \binom{100}{100} \cdot p^{100}$$

$$\text{with } p = 1 - \frac{2}{20000} = 0.9999$$

$$\Rightarrow P(A) \approx 0.99$$

$$\Rightarrow P(\bar{A}) = P(X < 100) \approx \underline{\underline{0.01}}$$

### Exercise 3

$$\begin{aligned}
 P(X \geq 9) &= P(X=9) + P(X=10) = \\
 &= \binom{10}{9} \cdot 0.8^9 \cdot 0.2 + \binom{10}{10} \cdot 0.8^{10} \\
 &\approx \underline{\underline{0.376}}
 \end{aligned}$$

#### Exercise 4

$$P(X=6) = 1 - \left( \frac{1}{3} + \frac{1}{4} + \frac{1}{6} \right) = \frac{1}{4}$$

Cumulative Distribution Function:

$$F(x) = P(X \leq x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{1}{3}, & \text{if } 1 \leq x < 2 \\ \frac{7}{12}, & \text{if } 2 \leq x < 4 \\ \frac{9}{12}, & \text{if } 4 \leq x < 6 \\ 1, & \text{if } x \geq 6 \end{cases}$$

$$\begin{aligned}
 P(X < 4) &= P(X=1) + P(X=2) \\
 &= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}
 \end{aligned}$$

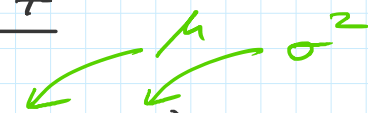
$$\begin{aligned}
 P(X \geq 2) &= 1 - P(X < 2) \\
 &= 1 - P(X=1) \\
 &= 1 - \frac{1}{3} = \frac{2}{3}
 \end{aligned}$$



$$= 1 - (1 - e^{-1}) = e^{-1} \approx 0.5134$$

### Exercise 7

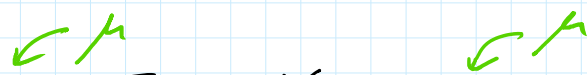
$$X \sim \mathcal{N}(30, 4)$$



$$\begin{aligned}
 (a) \quad P(24 \leq X \leq 36) &= F(36) - F(24) = \\
 &= F_0\left(\frac{36-30}{2}\right) - F_0\left(\frac{24-30}{2}\right) \\
 &= F_0(3) - F_0(-3) \\
 &= F_0(3) - (1 - F_0(3)) \\
 &= 2F_0(3) - 1 \approx 0.9973
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad P(X \geq 35) &= 1 - P(X \leq 35) = \\
 &= 1 - F(35) = 1 - F_0\left(\frac{35-30}{2}\right) = \\
 &= 1 - F_0\left(\frac{5}{2}\right) \approx 0.00621
 \end{aligned}$$

$$(c) \quad P(30 - \delta \leq X \leq 30 + \delta) \stackrel{!}{=} 0.98$$



$$\Leftrightarrow F(30 + \delta) - F(30 - \delta) = 0.98$$

$$\Leftrightarrow F_0\left(\frac{\delta}{2}\right) - F_0\left(-\frac{\delta}{2}\right) = 0.98$$

$$\Leftrightarrow 2F_0\left(\frac{\delta}{2}\right) - 1 = 0.98$$

$$\Leftrightarrow F_0\left(\frac{\delta}{2}\right) = 0.99$$

$$\Leftrightarrow \frac{\delta}{2} = 2.33 \quad \Leftrightarrow \delta = 4.66$$

Time Slot:  $[25.34, 34.66]$