Fourier Transform $\quad X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$

z-Transform $\quad X(f) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi f \frac{n}{r}}$ $\quad t \hat{=} nT = \frac{n}{r}$

DFT(ransform) $\hat{=}$ FFT $\quad X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \cdot \frac{n \cdot k}{K}}$

Periodicity

$$e^{-j2\pi f \frac{n}{r}} \hat{=} e^{-j2\pi \frac{n \cdot k}{K}}$$

$$\frac{f}{r} \hat{=} \frac{k}{K}$$

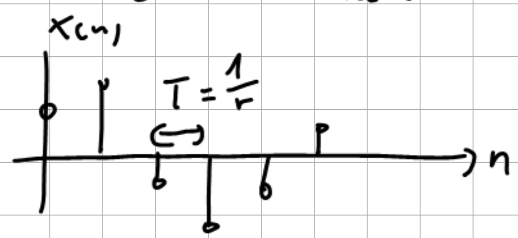
$$f \hat{=} \frac{k}{K} \cdot r = \Delta f \cdot k$$

const.

$$\Delta f = \frac{r}{K} = \text{const}$$

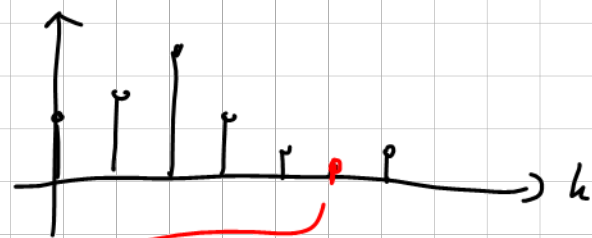
↳ Frequency resolution

time domain



DFT

IDFT



where is $t = 75\,\text{ms}$

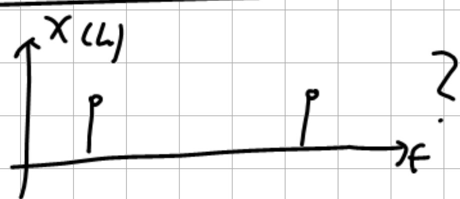
$\Rightarrow \quad t = nT = \frac{n}{r}$

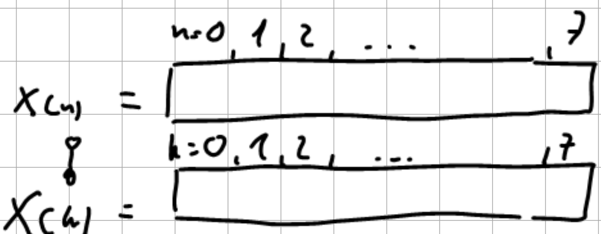
where is $f = 50\,\text{Hz}$

$\Rightarrow \quad f = k \cdot \Delta f = k \cdot \frac{r}{K}$

e.g. $\Delta f = 10\,\text{Hz}$

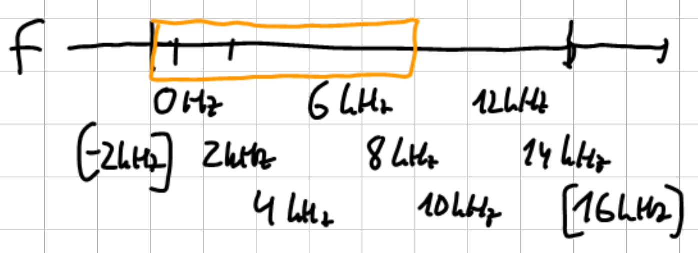
$x(t) = \sin(2\pi\ 100\,\text{Hz} \cdot t)$





no zero padding $\quad K = N = 8$

$r = 16\,\text{kHz} \Rightarrow \Delta f = 2\,\text{kHz}$



$f = k \cdot \Delta f$

Sampling theorem: $f_c = \frac{r}{2}$
$= 8\,kHz$

$X(1) = X^*(-1)$

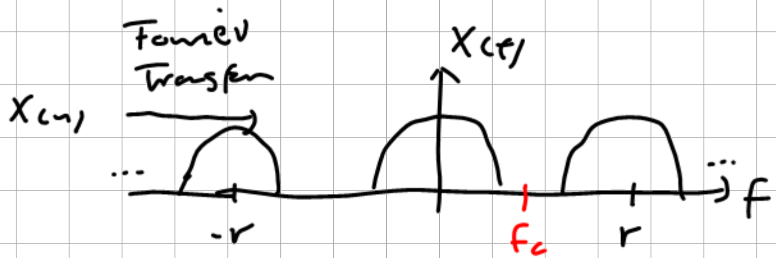
$X(2) = X^*(-2)$

Symmetry of Fourier Transform

$\Rightarrow$ for real $x(n)$

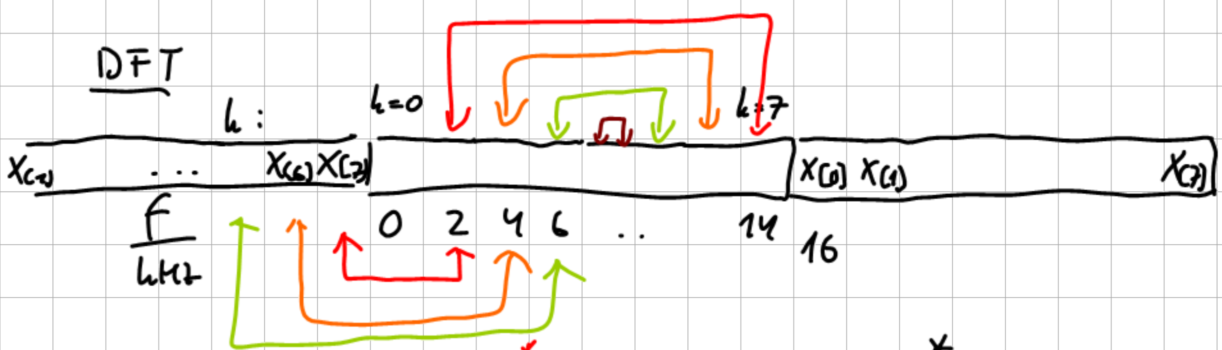
$\Rightarrow \quad X(k) = X^*(-k)$

complex conjugated



For sampled Signals $x(n)$ the spectrum is periodic.

DFT



$X[1] = X^*[-1] = X[7] \quad = X^*[K-k]$

$X[2] = X^*[-2] = X[6]$

$X[3] = X^*[-3] = X[5]$

$$X[0] = \sum_{n=0}^{N-1} x(n)\, e^{-j 2\pi \frac{nk}{K}}$$

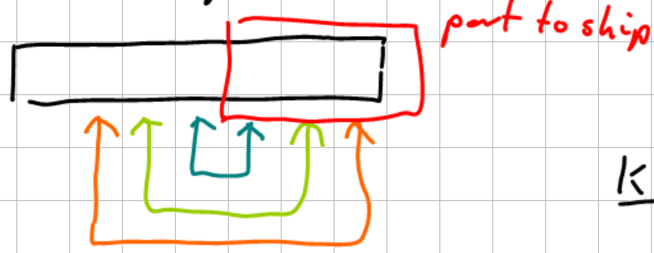
$$= \sum_{n=0}^{N-1} x(n) \cdot 1 = \sum_{n=0}^{N-1} x(n) \quad \text{always real valued}$$

$X[4] = X^*[-4] = X^*[4]$ also real valued

For $K \hat{=}$ even : keep $\frac{K}{2} + 1$ elements after DFT

e.g. $K = 8 \Rightarrow$ keep 5 elements

For $K \stackrel{?}{=}$ odd : e.g. $K=7$



$\Rightarrow K=7 \Rightarrow$ keep 4 Elements

$$\frac{K+1}{2}$$

for every K: $\left\lceil \frac{K+1}{2} \right\rceil$ ← rounded up

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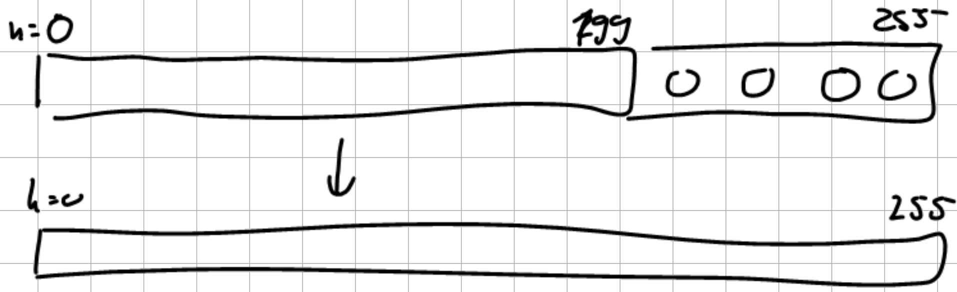
In literature: $$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi \frac{nk}{N}}$$

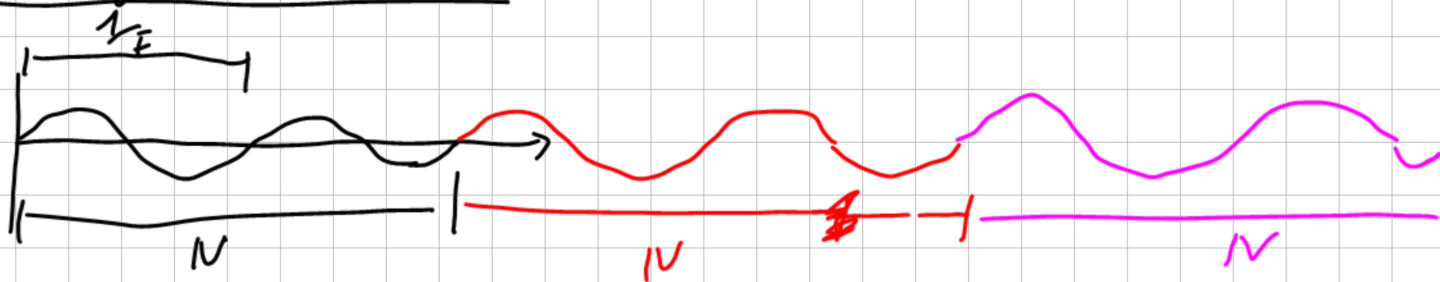
↑ Input length = Output length

$x[n]$ = Block of 200 Samples

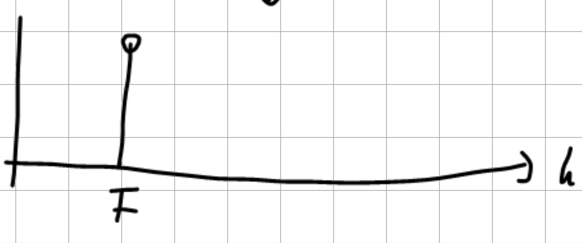
Fast Implementation für $N = 256 = 2^8$

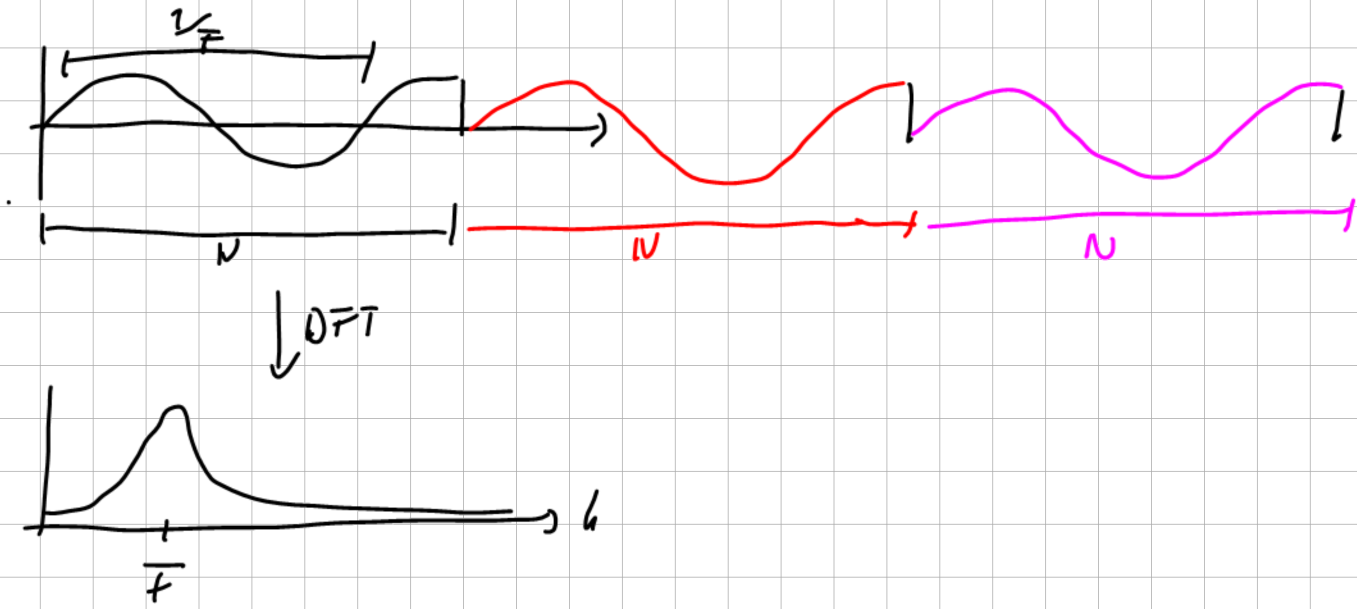


Periodicity in time domain

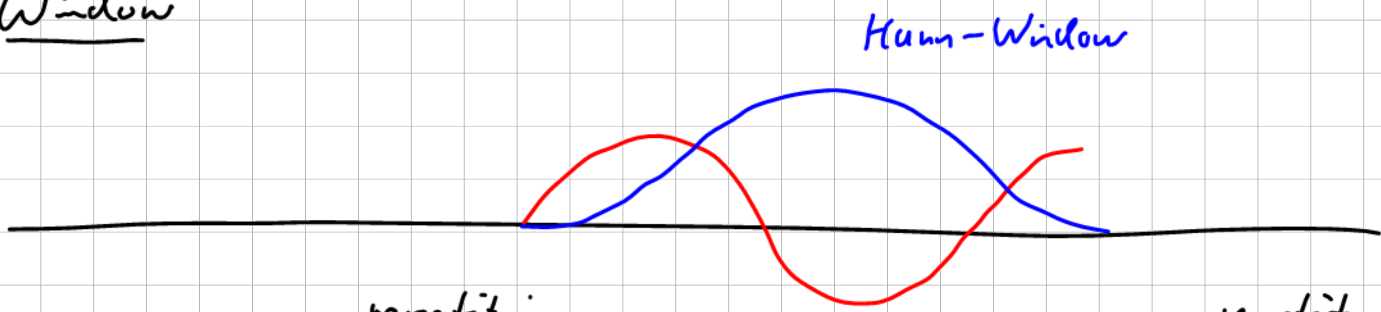


↓ DFT

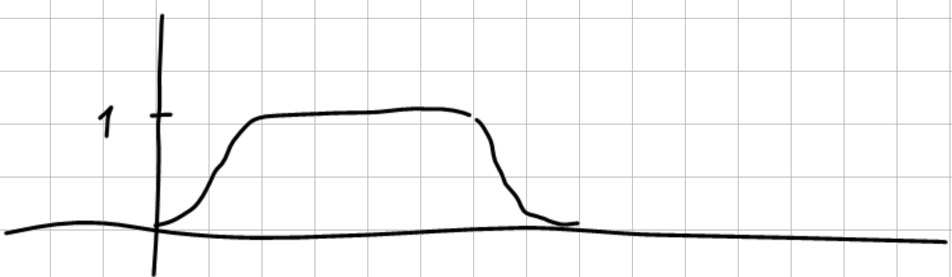




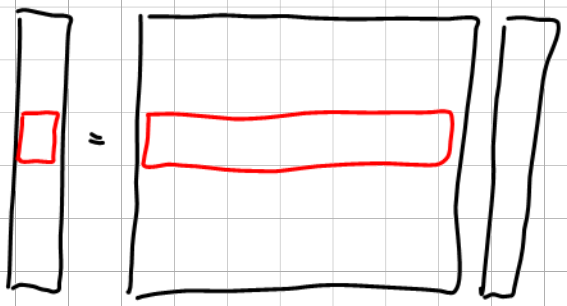
Window



„Other Windows"



Matrix Multiplication



$$X_{(k)} = T \cdot x_{(n)}$$

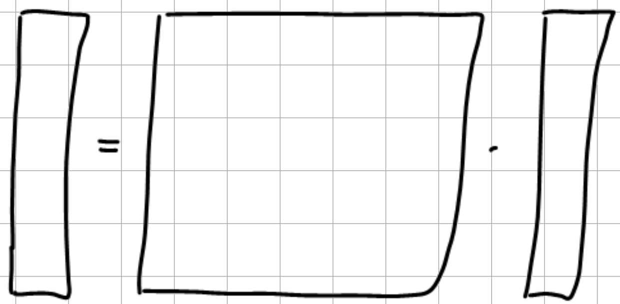
$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi \frac{nk}{K}}$$

Scalar product: elementwise Multiplication

Sum over it

$$x_w(n) = w(n) \cdot x(n)$$

elementwise multiplication



$x_w \qquad T_w \qquad X$

$$y = A \cdot x$$

$$x = \underbrace{A \backslash}_{\text{stable inversion}} y$$

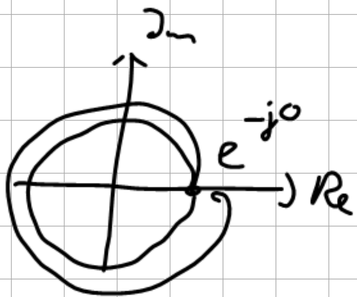
C2, J8, EP1

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \frac{nk}{K}}$$

$$X(k) = X^*(K-k) = \left(\sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \frac{n}{K}\cdot(K-k)}\right)^*$$

$$= \left(\sum_{n=0}^{N-1} x(n)\, \underbrace{e^{-j2\pi n \frac{K}{K}}}_{=1}\, e^{-j2\pi n \frac{-k}{K}}\right)^*$$

$$= \left(\sum_{n=0}^{N-1} x(n)\, e^{+j2\pi n \frac{k}{K}}\right)^*$$



$$= \sum_{n=0}^{N-1} \left(x(n)\, e^{+j2\pi \frac{nk}{K}}\right)^*$$

$$= \sum_{n=0}^{N-1} x(n) \left(e^{+j2\pi \frac{nk}{K}}\right)^*, \quad x(n) \in Re$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi \frac{nk}{K}}$$