Worksheet 4

Exercise 1

(a) Events:

$A_1 \mathrel{\hat{=}}$ "4, 5 or 6 on the first toss"

$A_2 \mathrel{\hat{=}}$ "1, 2, 3 or 4 on the second toss"

$$
\begin{aligned}
P(A_1 \cap A_2) &= P(A_1) \cdot P(A_2 \mid A_1) \\
&= P(A_1) \cdot P(A_2) \\
&= \frac{3}{6} \cdot \frac{4}{6} = \frac{12}{36} = \frac{1}{3}
\end{aligned}
$$

(b) Events:

$A_1 \mathrel{\hat{=}}$ "6 on first toss"

$A_2 \mathrel{\hat{=}}$ "6 on second toss"

$$
\begin{aligned}
P(A_1 \cup A_2) &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\
&= P(A_1) + P(A_2) - P(A_1) \cdot P(A_2) \\
&= \frac{1}{6} + \frac{1}{6} - \frac{1}{6} \cdot \frac{1}{6} = \frac{11}{36}
\end{aligned}
$$

Exercise 2

Events:

$L_i \mathrel{\hat{=}}$ "Homer decides for Lake i"

$S \mathrel{\hat{=}}$ "Homer is successful at fishing"

(a) Are L_3 and S independent?

$P(S|L_3) = 0.8$

Law of total probability:

$$P(S) = P(S|L_1)\cdot P(L_1) + P(S|L_2)\cdot P(L_2) +$$
$$+ P(S|L_3)\cdot P(L_3)$$
$$= 0.4 \cdot \frac{1}{3} + 0.6 \cdot \frac{1}{3} + 0.8 \cdot \frac{1}{3}$$
$$= 0.6$$

It holds that

$$P(S|L_3) \neq P(S)$$

$\Rightarrow$ The events L_3 and S are <u>not</u> statistically independent

(b) (i):

$$P(L_1|S) = \frac{P(L_1 \cap S)}{P(S)} = \frac{P(S|L_1)\cdot P(L_1)}{P(S)}$$
$$= \frac{0.4 \cdot \frac{1}{3}}{0.6} = \frac{2}{9}$$

(ii):

$$P(L_2|S) = \frac{P(L_2 \cap S)}{P(S)} = \frac{P(S|L_2)\cdot P(L_2)}{P(S)}$$
$$= \frac{0.6 \cdot \frac{1}{3}}{0.6} = \frac{1}{3}$$

(iii):

$$P(L_3|S) = \frac{P(L_3 \cap S)}{P(S)} = \frac{P(S|L_3)\cdot P(L_3)}{P(S)}$$
$$= \frac{0.8 \cdot \frac{1}{3}}{0.6} = \frac{4}{9}$$

Exercise 3

Events:

$A_i \mathrel{\hat{=}}$ "component K_i is intact",

$i = 1, \ldots, 5$

$$\Rightarrow \quad P(A_i) = 1 - P(K_i)$$

$B_j \mathrel{\hat{=}}$ "path j is intact", $j = 1, 2, 3$

$$\Rightarrow \quad P(B_1) = P(A_1 \cap A_2)$$
$$= P(A_1) \cdot P(A_2)$$
$$= (1 - P(K_1)) \cdot (1 - P(K_2))$$

$$P(B_2) = P(A_3) = 1 - P(K_3)$$

$$P(B_3) = P(A_4 \cap A_5)$$
$$= P(A_4) \cdot P(A_5)$$
$$= (1 - P(K_4)) \cdot (1 - P(K_5))$$

$C_j \mathrel{\hat{=}}$ "path j fails", $j = 1, 2, 3$

$$\Rightarrow \quad P(C_j) = 1 - P(B_j)$$

$F \mathrel{\hat{=}}$ "network fails"

$$\Rightarrow \quad F(F) = P(C_1 \cap C_2 \cap C_3) =$$
$$= P(C_1) \cdot P(C_2) \cdot P(C_3) =$$
$$= (1 - P(B_1)) \cdot (1 - P(B_2)) \cdot (1 - P(B_3)) =$$
$$= (1 - (1 - P(K_1)) \cdot (1 - P(K_2))) \cdot$$

$$\cdot \left(1-(1-P(K_3))\right) \cdot$$

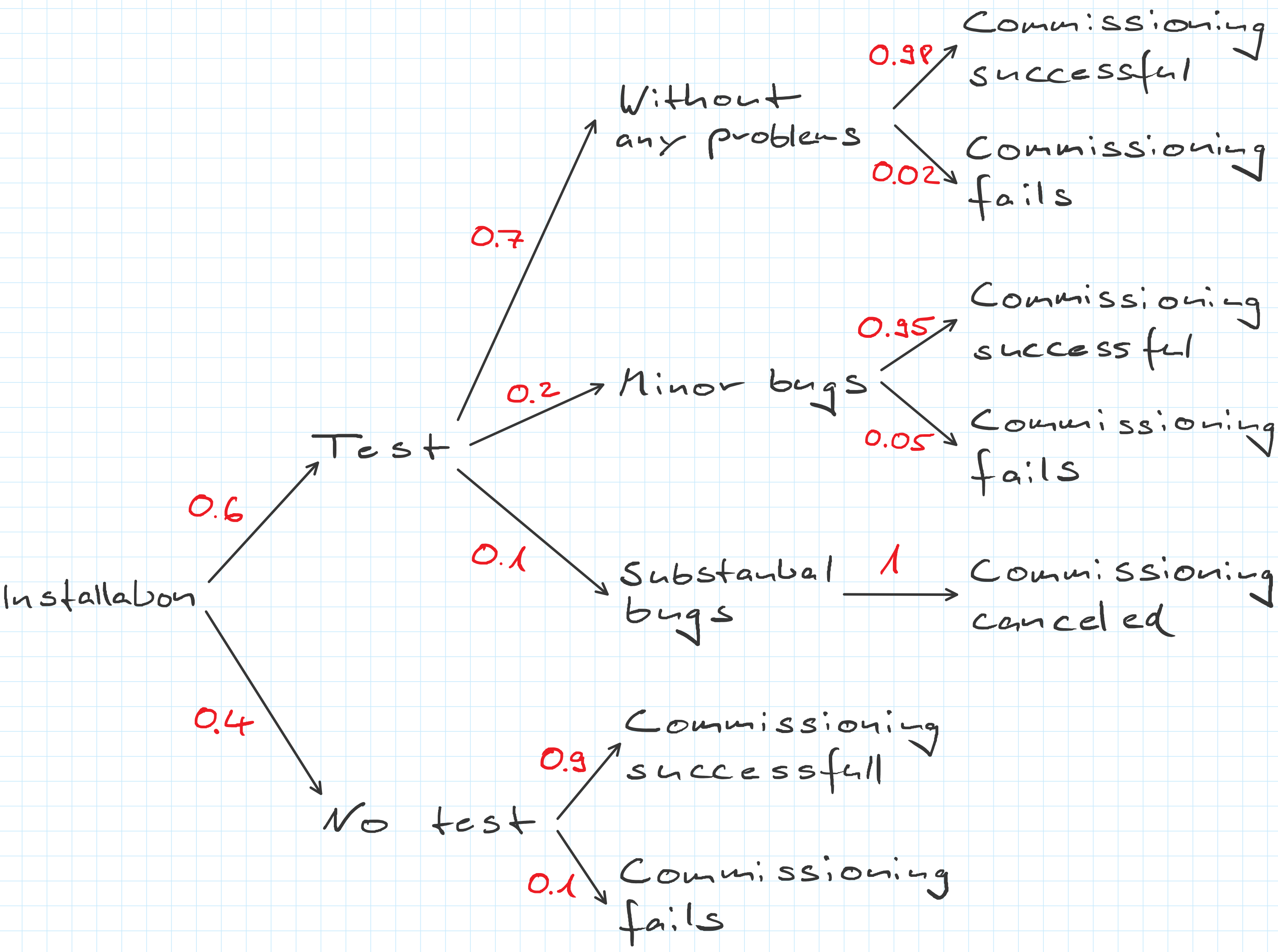
$$\cdot \left(1-(1-P(K_4))\cdot(1-P(K_5))\right) =$$

$$= (1-0.7\cdot 0.6)\cdot 0.5 \cdot (1-0.8\cdot 0.9) =$$

$$= 0.0812$$

$$\Rightarrow P(\overline{F}) = 1 - P(F) = \underline{\underline{0.9188}}$$

Exercise 4



(a) $P(\text{"Commissioning canceled"}) = 1 \cdot 0.1 \cdot 0.6 = \underline{\underline{0.06}}$

$$= \underline{\underline{0.06}}$$

(b) P("Commissioning successful") =

$$= 0.98 \cdot 0.7 \cdot 0.6 + 0.95 \cdot 0.2 \cdot 0.6 + 0.9 \cdot 0.4 = \underline{\underline{0.8856}}$$

(c) P("Commissioning fails") =

$$= 1 - 0.06 - 0.8856 = \underline{\underline{0.0544}}$$

(d) P("Commissioning successful" | "Test required") =

$$= 0.98 \cdot 0.7 + 0.95 \cdot 0.2 = \underline{\underline{0.876}}$$

(e) Events are <u>not</u> independent.