DC Motor - Part 2

Actuators - IRO6

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DC Motor - Part 2

1. Dynamic equation system of the direct current machine
2. Discretization of the equations of state
3. Speed control with field weakening

Dynamic equation system of the direct current machine

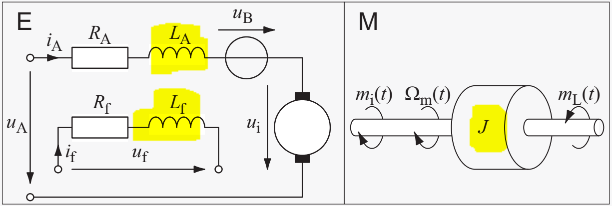
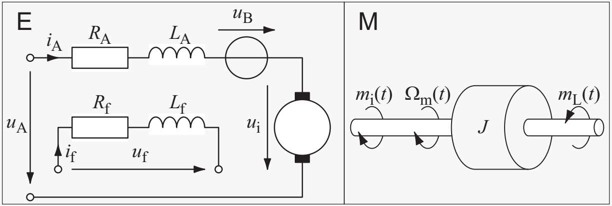


Figure Equivalent circuit diagram of DC motor

Dynamic equation system of the direct current machine



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Three energy storages:

Dynamic equation system of the direct current machine

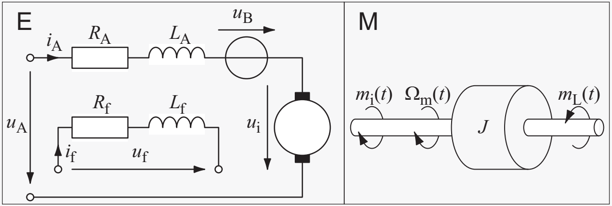


Figure Equivalent circuit diagram of DC motor

Three energy storages:

$$u_A(t) = R_A \cdot i_A(t) + L_A \frac{di_A}{dt} + u_i \qquad (3.1)$$

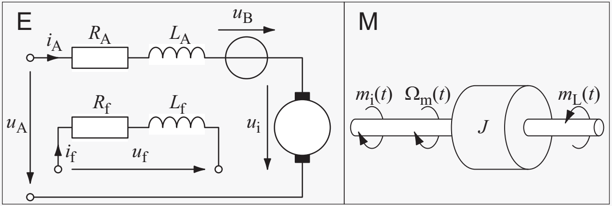
$$u_f(t) = R_f \cdot i_f(t) + \frac{d}{dt}(L_f i_f) \qquad (3.2)$$

(3.3)

Dynamic equation system of the direct current machine

X = steady state operation

x(t) = transient values over time



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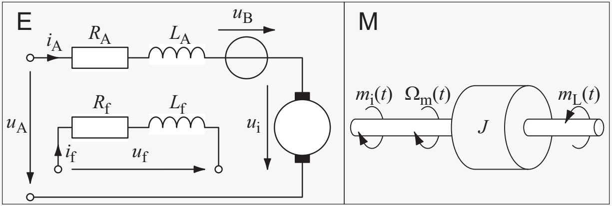
Dynamic equation system of the direct current machine



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$$m_i(t) = m_L(t) + M_{Fric} + J \cdot \frac{d\Omega_m}{dt} \qquad (3.3)$$

$$u_i(t) = c\Phi_f \cdot \Omega_m \qquad (3.4)$$

$$\Phi_f = f(i_f(t)) \qquad (3.5)$$

$$m_i(t) = c\Phi_f \cdot i_A(t) \qquad (3.6)$$

Dynamic equation system of the direct current machine I

Solving for the differentials:

$$\frac{L_A}{R_A}\frac{\mathrm{d}i_A}{\mathrm{d}t} = -i_A(t) + \frac{u_A(t)}{R_A} - \frac{c\Phi_f(t)}{R_A}\cdot\Omega_m(t) \qquad \Big| : I_{AN} \qquad (3.7a)$$

$$\frac{N_f}{R_f}\frac{\mathrm{d}\Phi_f}{\mathrm{d}t} = -i_f(t) + \frac{1}{R_f}u_f(t) \qquad \Big| : I_{fN} \qquad (3.7b)$$

$$J\frac{\mathrm{d}\Omega_m}{\mathrm{d}t} = c\Phi_f(t)\cdot i_A(t) - m_L(t) - M_{Fric} \qquad \Big| : M_N \qquad (3.7c)$$

$$\frac{1}{R_A}\left(u_A = R_A \cdot i_A + L_A \frac{di_A}{dt} + \overbrace{c\Phi_f \, \Omega_m}^{u_i} \right)$$

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Variables relative to nominal values with the abbreviations (additional index n):

Armature time constant

$$u_{An}(t) = \frac{u_A(t)}{U_{AN}} \qquad i_{An}(t) = \frac{i_A(t)}{I_{AN}} \qquad T_A = \frac{L_A}{R_A} \qquad r_A = \frac{R_A}{U_{AN}/I_{AN}} << 1 \quad (U_{RA} = U_A - U_i)$$

$$u_{fn}(t) = \frac{u_f(t)}{U_{fN}} \qquad i_{fn}(t) = \frac{i_f(t)}{I_{fN}} \qquad T_{fN} = \frac{L_{fN}}{R_f} \qquad r_f = \frac{R_f}{U_{fN}/I_{fN}} \rightarrow 1$$

$$\Phi_{fn}(t) = \frac{\Phi_f(t)}{\Phi_{fN}} \qquad N_f\Phi_{fN} = L_{fN}I_{fN}$$

$$\Omega_{mn}(t) = \frac{\Omega_m(t)}{\Omega_{m0N}} \qquad m_{Ln}(t) = \frac{m_L(t) + M_{Fric}}{M_N} \qquad T_J = \frac{\Omega_{m0N}\cdot J}{M_N}$$

$$U_{AN} = c\Phi_{fN}\,\Omega_{m0N} \qquad M_N = c\Phi_{fN}\cdot I_{AN}$$

No-load speed at nominal voltage

Dynamic equation system of the direct current machine II

The system of differential equations can thus be formulated in relative quantities:

$$T_{\mathrm{A}} \frac{\mathrm{d}i_{\mathrm{An}}}{\mathrm{d}t} = -i_{\mathrm{An}}(t) + \frac{u_{\mathrm{An}}(t)}{r_{\mathrm{A}}} - \frac{c\Phi_{\mathrm{fn}}(t)}{r_{\mathrm{A}}} \cdot \Omega_{\mathrm{mn}}(t) \tag{3.8a}$$

$$T_{\mathrm{f}} \frac{\mathrm{d}\Phi_{\mathrm{fn}}}{\mathrm{d}t} = -i_{\mathrm{fn}}(t) + \frac{u_{\mathrm{fn}}(t)}{r_{\mathrm{f}}} \tag{3.8b}$$

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$$T_{\mathrm{J}} \frac{\mathrm{d}\Omega_{\mathrm{mn}}}{\mathrm{d}t} = \Phi_{\mathrm{fn}}(t) \cdot i_{\mathrm{An}}(t) - m_{\mathrm{Ln}}(t) \tag{3.8c}$$

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$$u_{\mathrm{in}}(t) = \Phi_{\mathrm{fn}}(t) \cdot \Omega_{\mathrm{mn}}(t) \tag{3.8d}$$

$$\Phi_{\mathrm{fn}} = f(i_{\mathrm{fn}}(t)) \tag{3.8e}$$

$$m_{\mathrm{in}}(t) = \Phi_{\mathrm{fn}}(t) \cdot i_{\mathrm{An}}(t) \tag{3.8f}$$

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with Eq. 3.8a - Eq. 3.8f **physical block model**
(= model exclusively with integrators) in Figure 3.2:

Physical block model of the DC motor

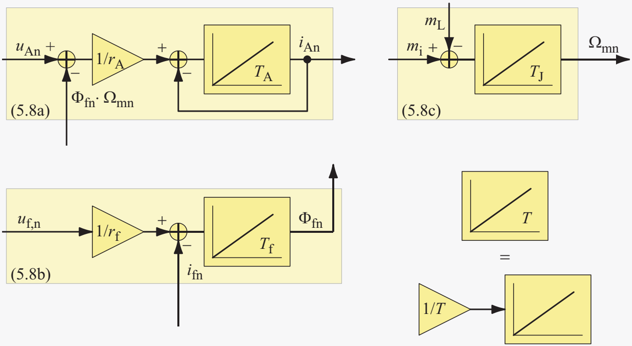

Figure Standard physical block model of the DC machine

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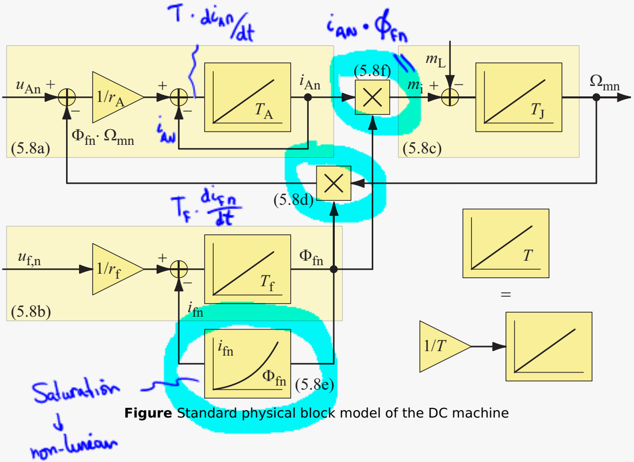

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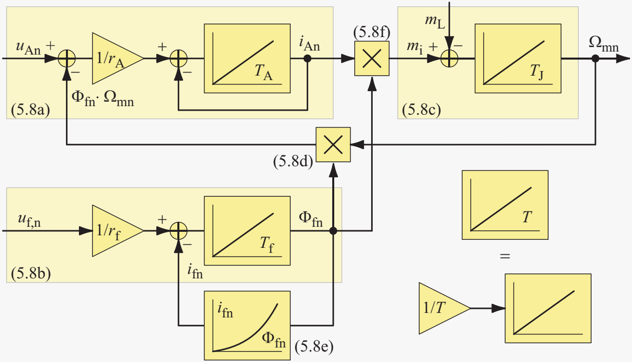

Figure Standard physical block model of the DC machine

- Consideration in the Laplace domain not more useful?

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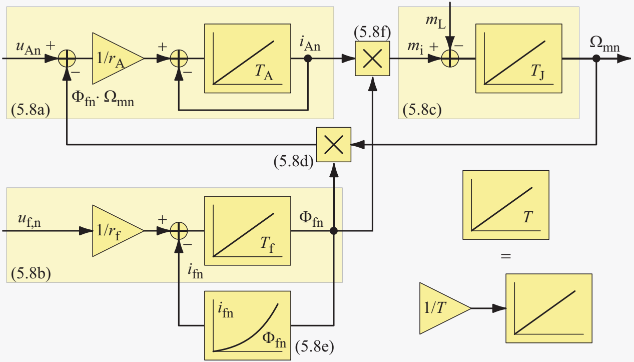

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⇒ Saturation!

Physical block model of the DC motor

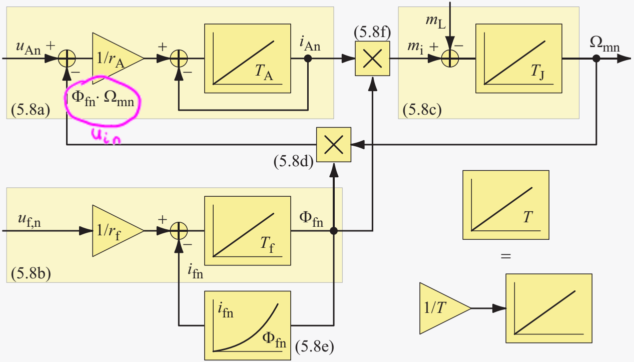

Figure Standard physical block model of the DC machine

- Consideration in the Laplace domain not more useful?

⇒ Saturation!

⇒ Multiplier!

DC Motor - Part 2

Discretization of the equations of state

- To describe dynamic processes:

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$$\left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} = \lim_{T\to 0} \frac{y(t_k+T)-y(t_k)}{T}$$

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Notation:

$$\left.\begin{array}{ll} y_k & := y(t_k) \\ y_{k+1} & := y(t_k+T) \end{array}\right\} \Rightarrow \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{y(t_k)} \approx \frac{y_{k+1} - y_k}{T}$$

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Discretization of armature voltage equation

Differential quotient $\frac{\mathrm{d}i_{\mathrm{A}}}{\mathrm{d}t} \approx$ Difference quotient $\frac{i_{\mathrm{A},k+1} - i_{\mathrm{A},k}}{T}$:

$$T_{\mathrm{A}} \frac{\mathrm{d}i_{\mathrm{An}}}{\mathrm{d}t} = -i_{\mathrm{An}}(t) - \frac{u_{\mathrm{An}}(t)}{r_{\mathrm{A}}} - \frac{c\Phi_{\mathrm{fn}}(t)}{r_{\mathrm{A}}} \cdot \Omega_{\mathrm{mn}}(t) \qquad ((3.8a))$$

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$$\frac{T_{\mathrm{A}}}{T}(i_{\mathrm{A},k+1} - i_{\mathrm{A},k}) = -i_{\mathrm{A},k} - \frac{u_{\mathrm{A},k}}{r_{\mathrm{A}}} - \frac{c\Phi_{\mathrm{f},k}}{r_{\mathrm{A}}} \cdot \Omega_{\mathrm{m},k}$$

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⇒ Recursion equation $i_{\mathrm{A},k+1} = i_{\mathrm{A},k} + \ldots$ for numerical integration:

$$i_{\mathrm{A},k+1} = \left(1 - \frac{T}{T_\mathrm{A}}\right) i_{\mathrm{A},k} - \frac{T}{T_\mathrm{A} r_\mathrm{A}}(u_{\mathrm{A},k} - c\Phi_{\mathrm{f},k} \cdot \Omega_{\mathrm{m},k}) \qquad (3.9a)$$

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- right side: values from the last (k-th) calculation step

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- right side: values from the last (k-th) calculation step
- left side: new (($k+1$)-th) value

Discretization of field and motion equation

Field excitation circuit:

$$T_\mathrm{f}\frac{\mathrm{d}\Phi_\mathrm{fn}}{\mathrm{d}t} = -i_\mathrm{fn}(t) + \frac{u_\mathrm{fn}(t)}{r_\mathrm{f}} \qquad ((3.8b))$$

$$\frac{T_\mathrm{f}}{T}\left(\Phi_{\mathrm{f},k+1} - \Phi_{\mathrm{f},k}\right) = -i_{\mathrm{f},k} + \frac{u_{\mathrm{f},k}}{r_\mathrm{f}}$$

$$\Rightarrow \qquad \Phi_{\mathrm{f},k+1} = \Phi_{\mathrm{f},k} + \frac{T}{T_\mathrm{f}}\left(\frac{u_{\mathrm{f},k}}{r_\mathrm{f}} - i_{\mathrm{f},k}\right) \qquad (3.9b)$$

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Equation of motion:

$$T_{\mathrm{J}} \frac{\mathrm{d}\Omega_{\mathrm{mn}}}{\mathrm{d}t} = \Phi_{\mathrm{fn}}(t) \cdot i_{\mathrm{An}}(t) - m_{\mathrm{Ln}}(t) \qquad ((3.8c))$$

$$\frac{T_{\mathrm{J}}}{T}\left(\Omega_{\mathrm{m},k+1} - \Omega_{\mathrm{m},k}\right) = \Phi_{\mathrm{f},k} \cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}$$

$$\Rightarrow \qquad \Omega_{\mathrm{m},k+1} = \Omega_{\mathrm{m},k} + \frac{T}{T_{\mathrm{J}}}\left(\Phi_{\mathrm{f},k} \cdot i_{\mathrm{A},k} - m_{\mathrm{L},k}\right) \qquad (3.9c)$$

Discrete. Equation system of the direct current machine

Simulation process:

1. Definition of constants:
   $T_\mathrm{A} = \ldots, \ T_\mathrm{f} = \ldots, \ T_\mathrm{J} = \ldots, \ T = \ldots (< T_\mathrm{A}/10), \ r_\mathrm{A} = \ldots, \ r_\mathrm{f} = \ldots (\approx 1)$
   $\tau_\mathrm{A} = T_\mathrm{A}/T, \ \tau_\mathrm{f} = T_\mathrm{f}/T, \ \tau_\mathrm{J} = T_\mathrm{J}/T$
2. Specify input variables:
   $u_{\mathrm{A},k} = \ldots, \ u_{\mathrm{f},k} = \ldots, \ m_{\mathrm{L},k} = \ldots, \ k = 0 \ldots k_{\mathrm{max}}$

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3. Initial values and default settings:
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0} \cdot i_{\mathrm{A},0}$

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4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c

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3. Initial values and default settings:
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0} \cdot i_{\mathrm{A},0}$
4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. Increment index k and check termination condition,
   otherwise back to 4.

Discrete. Equation system of the direct current machine

Simulation process:

1. Definition of constants:
   $T_A = \ldots,\ T_f = \ldots,\ T_J = \ldots,\ T = \ldots (< T_A/10),\ r_A = \ldots,\ r_f = \ldots (\approx 1)$
   $\tau_A = T_A/T,\ \tau_f = T_f/T,\ \tau_J = T_J/T$
2. Specify input variables:
   $u_{A,k} = \ldots,\ u_{f,k} = \ldots,\ m_{L,k} = \ldots,\ k = 0 \ldots k_{max}$
3. Initial values and default settings:
   $k = 0: \quad i_{A,0} = \ldots,\ \Phi_{f,0} = \ldots,\ \Omega_{m,0} = \ldots,\ i_{f,0} = f(\Phi_{f,0});\ m_{i,0} = \Phi_{f,0} \cdot i_{A,0}$
4. „Calculate"equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. Increment index k and check termination condition,
   otherwise back to 4.

Example: Excitation voltage is reduced linearly from $u_{fn} = 1 \rightarrow 0.5$ in $0.5\,s$.

Parameters: $T_A = 10\,ms,\ T_f = 200\,ms,\ T_J = 800\,ms,\ T = 2\,ms,\ r_A = 0.04,\ r_f = 1$

Result from Matlab simulation

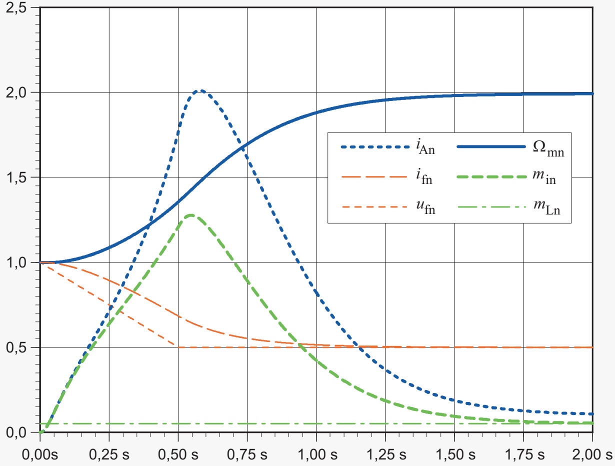

Figure Speed adjustment through field weakening

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Additional simulation

t

$0 \qquad\qquad\qquad 2$

$\Omega_{mON} \rightarrow 2\,\Omega_{mON} \rightarrow \frac{1}{2}\,\Omega_{mON}$

The motor should switch to half the nominal idle speed at 2 *s*?

1. What armature and excitation voltage are required for this?
2. What happens if both values are changed at the same time?
3. How can the armature current peak be reduced?

$u_{fn} = 1$

$u_{An} = 0.5$

↓ Change the armature voltage later than the excitation voltage.

DC Motor - Part 2

1. Dynamic equation system of the direct current machine
2. Discretization of the equations of state
3. Speed control with field weakening

Only speed regulation via voltage regulation

→ Slow speed change

→ Current peaks

⇒ Control needed

Cascade control with field weakening

- Subordinate current controls

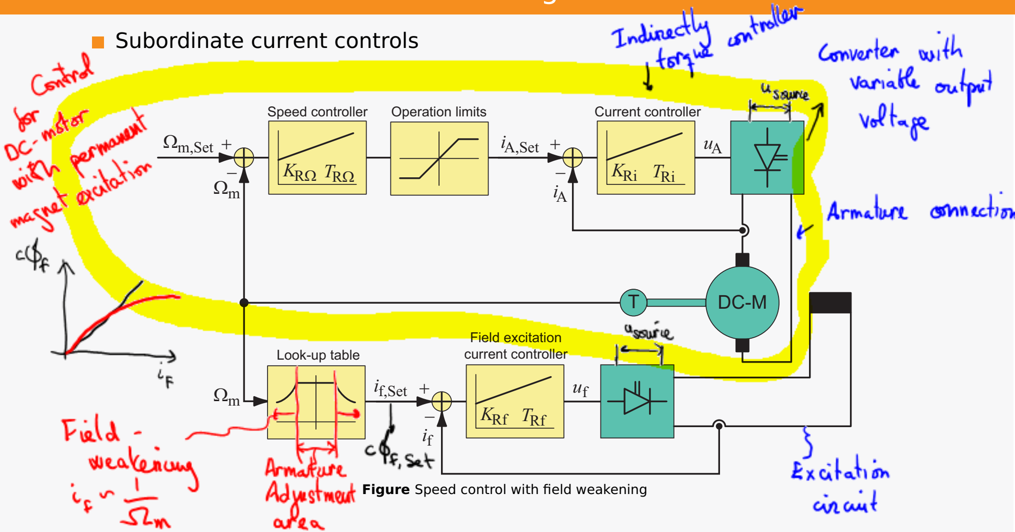

Figure Speed control with field weakening

Cascade control with field weakening

- Subordinate current controls
- Setpoint $i_{f,Set}$ from Ω_m via characteristic curve

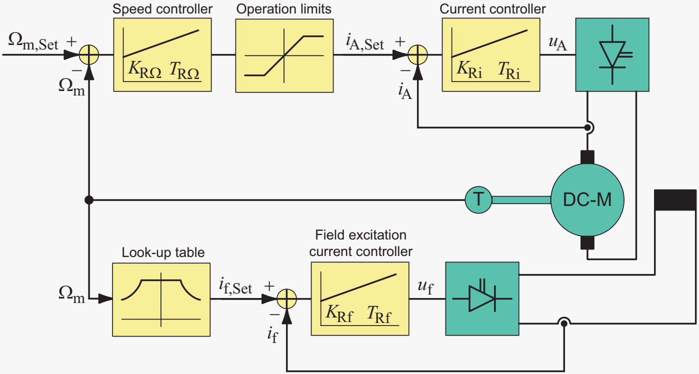

Figure Speed control with field weakening

Time-discrete PI controller

- All three controllers: PI controller (so stationary error → 0)
- Control deviation e as input variable → Control variable y as output variable

$$G_{\mathrm{PI}} = \frac{y(s)}{e(s)} = K_{\mathrm{R}} \cdot \frac{1 + sT_{\mathrm{R}}}{sT_{\mathrm{R}}}$$

change to the time domain

$$sT_{\mathrm{R}} \cdot y(s) = K_{\mathrm{R}} \cdot e(s) + sT_{\mathrm{R}} K_{\mathrm{R}} \cdot e(s)$$

$$\Rightarrow T_{\mathrm{R}} \frac{\mathrm{d}y(t)}{\mathrm{d}t} = K_{\mathrm{R}} \cdot e(t) + T_{\mathrm{R}} K_{\mathrm{R}} \frac{\mathrm{d}e(t)}{\mathrm{d}t}$$

Differential quotient
↓ Differenes quotient

$$\text{discretize: } T_{\mathrm{R}} \frac{y_{k+1} - y_k}{T} = K_{\mathrm{R}} \cdot e_k + T_{\mathrm{R}} K_{\mathrm{R}} \frac{e_{k+1} - e_k}{T}$$

$$y_{k+1} = y_k + K_{\mathrm{R}}\, e_{k+1} - K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right) e_k$$

$$\Rightarrow y_{k+1} = y_k + q_0 e_{k+1} + q_1 e_k \qquad \text{mit } q_0 = K_{\mathrm{R}} \tag{3.10}$$

$$q_1 = K_{\mathrm{R}} \left(1 - \frac{T}{T_{\mathrm{R}}}\right)$$

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Simulation process

1. Definition of constants
$T_\mathrm{A} = (10\,ms),\ T_\mathrm{f} = (100\,ms),\ T_\mathrm{J} = (800\,ms),\ T = (1\,ms),\ r_\mathrm{A} = (0.04),\ r_\mathrm{f} = (1)$
$\tau_\mathrm{A} = T_\mathrm{A}/T,\ \tau_\mathrm{f} = T_\mathrm{f}/T,\ \tau_\mathrm{J} = T_\mathrm{J}/T$
$K_{\mathrm{R}\Omega} = (20),\ T_{\mathrm{R}\Omega} = (100\,ms),\ K_\mathrm{Ri} = (0.5),\ T_\mathrm{Ri} = (10\,ms)$
$K_\mathrm{Rf} = (1),\ T_\mathrm{Rf} = (50\,ms),\ i_\mathrm{Amax} = (2),\ u_\mathrm{Amax} = (1.2),\ u_\mathrm{fmax} = (1)$
2. Specify input variables:
$\Omega_{\mathrm{mSet},k} = 1 \ldots 2 \ldots,\ m_{\mathrm{L},k} = \ldots,\ k = 0 \ldots k_\mathrm{max}$

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Simulation process

1. Definition of constants
   $T_{\mathrm{A}} = (10\,ms),\ T_{\mathrm{f}} = (100\,ms),\ T_{\mathrm{J}} = (800\,ms),\ T = (1\,ms),\ r_{\mathrm{A}} = (0.04),\ r_{\mathrm{f}} = (1)$
   $\tau_{\mathrm{A}} = T_{\mathrm{A}}/T,\ \tau_{\mathrm{f}} = T_{\mathrm{f}}/T,\ \tau_{\mathrm{J}} = T_{\mathrm{J}}/T$
   $K_{\mathrm{R}\Omega} = (20),\ T_{\mathrm{R}\Omega} = (100\,ms),\ K_{\mathrm{Ri}} = (0.5),\ T_{\mathrm{Ri}} = (10\,ms)$
   $K_{\mathrm{Rf}} = (1),\ T_{\mathrm{Rf}} = (50\,ms),\ i_{\mathrm{Amax}} = (2),\ u_{\mathrm{Amax}} = (1.2),\ u_{\mathrm{fmax}} = (1)$
2. Specify input variables:
   $\Omega_{\mathrm{mSet},k} = 1 \ldots 2 \ldots,\ m_{\mathrm{L},k} = \ldots,\ k = 0 \ldots k_{\mathrm{max}}$
3. Initial values and default settings:
   $k = 0: \quad i_{\mathrm{A},0} = \ldots,\ \Phi_{\mathrm{f},0} = \ldots,\ \Omega_{\mathrm{m},0} = \ldots,\ i_{\mathrm{f},0} = f(\Phi_{\mathrm{f},0});\ m_{\mathrm{i},0} = \Phi_{\mathrm{f},0} \cdot i_{\mathrm{A},0}$
   $i_{\mathrm{ASet},0} = \ldots,\ u_{\mathrm{A}0} = \ldots,\ u_{\mathrm{f},0} = \ldots,\ e_{\Omega,0} = \ldots,\ e_{\mathrm{i},0} = \ldots,\ e_{\mathrm{f},0} = \ldots$

Simulation process

1. Definition of constants
$T_A = (10\,ms),\ T_f = (100\,ms),\ T_J = (800\,ms),\ T = (1\,ms),\ r_A = (0.04),\ r_f = (1)$
$\tau_A = T_A/T,\ \tau_f = T_f/T,\ \tau_J = T_J/T$
$K_{R\Omega} = (20),\ T_{R\Omega} = (100\,ms),\ K_{Ri} = (0.5),\ T_{Ri} = (10\,ms)$
$K_{Rf} = (1),\ T_{Rf} = (50\,ms),\ i_{Amax} = (2),\ u_{Amax} = (1.2),\ u_{fmax} = (1)$
2. Specify input variables:
$\Omega_{mSet,k} = 1 \ldots 2 \ldots,\ m_{L,k} = \ldots,\ k = 0 \ldots k_{max}$
3. Initial values and default settings:
$k = 0: \quad i_{A,0} = \ldots,\ \Phi_{f,0} = \ldots,\ \Omega_{m,0} = \ldots,\ i_{f,0} = f(\Phi_{f,0});\ m_{i,0} = \Phi_{f,0} \cdot i_{A,0}$
$i_{ASet,0} = \ldots,\ u_{A0} = \ldots,\ u_{f,0} = \ldots,\ e_{\Omega,0} = \ldots,\ e_{i,0} = \ldots,\ e_{f,0} = \ldots$
4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c

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Simulation process

1. Definition of constants
   $T_A = (10\,ms),\ T_f = (100\,ms),\ T_J = (800\,ms),\ T = (1\,ms),\ r_A = (0.04),\ r_f = (1)$
   $\tau_A = T_A/T,\ \tau_f = T_f/T,\ \tau_J = T_J/T$
   $K_{R\Omega} = (20),\ T_{R\Omega} = (100\,ms),\ K_{Ri} = (0.5),\ T_{Ri} = (10\,ms)$
   $K_{Rf} = (1),\ T_{Rf} = (50\,ms),\ i_{Amax} = (2),\ u_{Amax} = (1.2),\ u_{fmax} = (1)$
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   $k = 0: \quad i_{A,0} = \ldots,\ \Phi_{f,0} = \ldots,\ \Omega_{m,0} = \ldots,\ i_{f,0} = f(\Phi_{f,0});\ m_{i,0} = \Phi_{f,0} \cdot i_{A,0}$
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4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. „Calculate“control laws for 3(!) controllers and limit set variables

Simulation process

1. Definition of constants
   $T_A = (10\,ms),\ T_f = (100\,ms),\ T_J = (800\,ms),\ T = (1\,ms),\ r_A = (0.04),\ r_f = (1)$
   $\tau_A = T_A/T,\ \tau_f = T_f/T,\ \tau_J = T_J/T$
   $K_{R\Omega} = (20),\ T_{R\Omega} = (100\,ms),\ K_{Ri} = (0.5),\ T_{Ri} = (10\,ms)$
   $K_{Rf} = (1),\ T_{Rf} = (50\,ms),\ i_{Amax} = (2),\ u_{Amax} = (1.2),\ u_{fmax} = (1)$
2. Specify input variables:
   $\Omega_{mSet,k} = 1 \ldots 2 \ldots,\ m_{L,k} = \ldots,\ k = 0 \ldots k_{max}$
3. Initial values and default settings:
   $k = 0: \quad i_{A,0} = \ldots,\ \Phi_{f,0} = \ldots,\ \Omega_{m,0} = \ldots,\ i_{f,0} = f(\Phi_{f,0});\ m_{i,0} = \Phi_{f,0} \cdot i_{A,0}$
   $i_{ASet,0} = \ldots,\ u_{A0} = \ldots,\ u_{f,0} = \ldots,\ e_{\Omega,0} = \ldots,\ e_{i,0} = \ldots,\ e_{f,0} = \ldots$
4. „Calculate“equations of state Eq. 3.9a, Eq. 3.9b and Eq. 3.9c
5. „Calculate“control laws for 3(!) controllers and limit set variables
6. Increment index and check termination condition, otherwise return to 4.

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Setpoint jump $0 \ldots 2n_N$

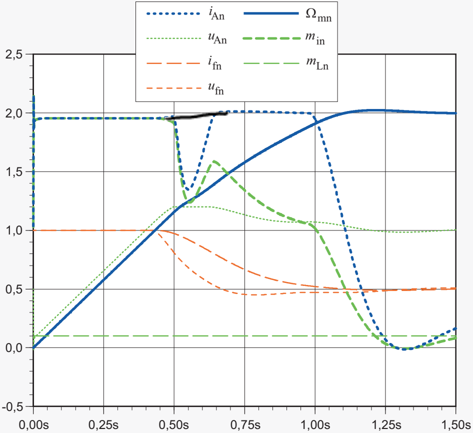

Figure Setpoint jump $0 \ldots 2n_N$

- till n_N: $u_{fn} = 1$
- $n > n_N$: $u_{fn} \downarrow$
- $u_f \rightarrow i_f$: PT1 behavior!
- $u_i(t)$ too large

⇒ Voltage reserve is not enough for $2I_{AN}$

- only shortly before n_{Set} is u_A slightly withdrawn
- i_{An} settles down to $2m_{Ln} = 0.2$.
- Nominal speed after 350 ms, setpoint reached after well 1 s
- without simulation: no chance

Additional simulations

1. In the previous simulation, how large does the armature voltage reserve have to be in order to operate permanently with double armature current?
2. The motor should now be switched off in a controlled way from nominal operation.

1.25