Worksheet 3

Exercise 1

(a) Smooth component:

$$G(t) = 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (t-5)\right) + 5 \cdot (20 + t)$$

$G = T + C$, T linear, C periodic

$$\Rightarrow T = T(t) = 100 + 5t$$

$$C = C(t) = 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (t-5)\right)$$

(b) Wavelength of C:

$$\frac{2\pi}{30} \cdot (t-5) \overset{!}{=} 0 \Rightarrow t = 5$$

$$\frac{2\pi}{30} \cdot (t-5) \overset{!}{=} 2\pi \Rightarrow t = 35$$

$\Rightarrow$ The wavelength of C is 30 months

(c) Forecast $T+C+S$ for the first quarter of 2023 $\longrightarrow$ months 61, 62, 63, 64:

January 2023 :

$$T + C + S = 100 + 5 \cdot \underline{61} + $$

$$+ 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (\underline{61} - 5)\right) - 40 \approx 398$$

February 2023 :

$$T + C + S = 100 + 5 \cdot \underline{62} +$$

$$+ 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (\underline{62} - 5)\right) - 50 \approx 400$$

March 2023 :

$$T + C + S = 100 + 5 \cdot \underline{63} +$$

$$+ 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (\underline{63} - 5)\right) - 30 \approx 431$$

April 2023 :

$$T + C + S = 100 + 5 \cdot \underline{64} +$$

$$+ 50 \cdot \cos\left(\frac{2\pi}{30} \cdot (\underline{64} - 5)\right) - 30 \approx 439$$

Exercise 2

t_i	x_i	x_i^*	$x_i - x_i^*$	$x_i - S_i$
1	10	/	/	9.5
2	12	/	/	10.5
3	8	11.5	-3.5	11.5
4	14	12.5	1.5	12.5
5	14	13.5	0.5	13.5
6	16	14.5	1.5	14.5
7	12	15.5	-3.5	15.5
8	18	16.5	1.5	16.5
9	18	17.5	0.5	17.5
10	20	18.5	1.5	18.5
11	16	/		19.5
12	22	/		20.5

Moving averages with a time window of length 4:

$$x_i^* = \frac{1}{4}\left(\frac{x_{i-2}}{2} + \frac{x_{i+2}}{2} + x_{i-1} + x_i + x_{i+1}\right)$$

Seasonal pattern for one year:

$$S = (S_1, S_2, S_3, S_4) = (0.5,\ 1.5,\ -3.5,\ 1.5)$$

(already standardized)

No component C present in the time series.

Exercise 3

Exponential smoothing for forecasting ($\alpha = 0.2$):

$$x_1^* = x_1, \quad x_{t+1}^* = \alpha x_t + (1-\alpha) x_t^*$$

$x_1^* = 10$

$x_2^* = 0.2 \cdot 10 + 0.8 \cdot 10 = 10$

$x_3^* = 0.2 \cdot 12 + 0.8 \cdot 10 = 10.4$

$x_4^* = 0.2 \cdot 8 + 0.8 \cdot 10.4 = 8.48$

$x_5^* = 0.2 \cdot 14 + 0.8 \cdot 8.48 \approx 9.58$

$x_6^* = 0.2 \cdot 14 + 0.8 \cdot 9.58 \approx 10.46$

$x_7^* = 0.2 \cdot 16 + 0.8 \cdot 10.46 \approx 11.57$

$x_8^* = 0.2 \cdot 12 + 0.8 \cdot 11.57 \approx 11.66$

$x_9^* = 0.2 \cdot 18 + 0.8 \cdot 11.66 \approx 12.93$

$x^* = 0.2 \cdot 18 + 0.8 \cdot 12.93 \approx 13.94$

$$x_{10}^{*} = 0.2 \cdot 18 + 0.8 \cdot 12.93 \approx 13.94$$

$$x_{11}^{*} = 0.2 \cdot 20 + 0.8 \cdot 13.94 \approx 15.15$$

$$x_{12}^{*} = 0.2 \cdot 16 + 0.8 \cdot 15.15 \approx 15.32$$

Forecast:

$$x_{13}^{*} = 0.2 \cdot 22 + 0.8 \cdot 15.32 \approx \underline{\underline{16.66}}$$