thws Technical University of Applied Sciences Würzburg-Schweinfurt

# Statistics and Sensor Data Fusion

– Winter Term 2023/2024 –

Worksheet 6

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**Exercise 1.** The bivariate random vector $\boldsymbol{X} = (X_1, X_2)^T$ should be characterized by the joint probability density function (pdf)

$$f_{\boldsymbol{X}}(x_1, x_2) = \begin{cases} e^{-(x_1+x_2)}, & x_1 \geq 0,\, x_2 \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show that $f_{\boldsymbol{X}}(x_1, x_2)$ is actually a pdf.

(b) Compute the probability $P\big(\boldsymbol{X} \in [\frac{1}{2}, 1] \times [0, \frac{1}{2}]\big)$.

(c) Compute the probability that the value of the second component $X_2$ is larger than the value of the first component $X_1$, i.e. $P(X_2 > X_1)$.

(d) Compute the probability that the absolute value of the difference between the two components is smaller than 1, i.e. $P(|X_2 - X_1| < 1)$.

**Exercise 2.** The bivariate random vector $\boldsymbol{X} = (X_1, X_2)^T$ should be **normally distributed** with the joint pdf

$$f_{\boldsymbol{X}}(x_1, x_2) = c \cdot \exp\left(-\frac{2}{7}x_1^2 + \frac{2}{7}x_1 + \frac{6}{7}x_2 + \frac{2}{7}x_1 x_2 - \frac{4}{7}x_2^2 - \frac{4}{7}\right)$$

(a) Determine the mean vector $\boldsymbol{\mu} \in \mathbb{R}^2$, the covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{2\times 2}$ and the constant $c \in \mathbb{R}$.

(b) Determine the marginal densities $f_{X_1}(x_1)$ and $f_{X_2}(x_2)$.

(c) Are the components $X_1$ and $X_2$ statistically independent?