Worksheet 5

Exercise 1

$$P(X > 345) = P(X = 346) + P(X = 347) =$$

$$= \binom{347}{346} \cdot 0.98^{346} \cdot 0.02 + \binom{347}{347} \cdot 0.98^{347} =$$

$$= 347 \cdot 0.98^{346} \cdot 0.02 + 0.98^{347} =$$

$$\approx 0.0064 + 0.005 \approx \underline{0.007}$$

Exercise 2

$X \hat{=}$ "number of correct pallets in spot check"

$A \hat{=}$ "no complete inventory"

$$\Rightarrow P(A) = P(X = 100) = \binom{100}{100} \cdot p^{100}$$

$$\text{with } p = 1 - \frac{2}{20000} = 0.9999$$

$$\Rightarrow P(A) \approx 0.99$$

$$\Rightarrow P(\bar{A}) = P(X < 100) \approx \underline{0.01}$$

Exercise 3

$$P(X \geq 9) = P(X = 9) + P(X = 10) =$$

$$= \binom{10}{9} \cdot 0.8^9 \cdot 0.2 + \binom{10}{10} \cdot 0.8^{10}$$

$$\approx \underline{0.376}$$

Exercise 4

$$P(X = 6) = 1 - \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{6}\right) = \frac{1}{4}$$

Cumulative Distribution Function:

$$F(x) = P(X \leq x) = \begin{cases} 0, & \text{if } x < 1 \\ \frac{1}{3}, & \text{if } 1 \leq x < 2 \\ \frac{7}{12}, & \text{if } 2 \leq x < 4 \\ \frac{9}{12}, & \text{if } 4 \leq x < 6 \\ 1, & \text{if } x \geq 6 \end{cases}$$

$$P(X < 4) = P(X = 1) + P(X = 2)$$

$$= \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - P(X = 1)$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(2 \leq X < 6) = P(X=2) + P(X=4)$$
$$= \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

$$\mu = \sum_{i=1}^{4} x_i \cdot P(X = x_i)$$
$$= 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{4} + 4 \cdot \frac{1}{6} + 6 \cdot \frac{1}{4} = 3$$

$$\sigma^2 = \sum_{i=1}^{4} x_i^2 \cdot P(X = x_i) - \mu^2$$
$$= 1 \cdot \frac{1}{3} + 4 \cdot \frac{1}{4} + 16 \cdot \frac{1}{6} + 36 \cdot \frac{1}{4} - 9 = 4$$

Exercise 5

$X \hat{=}$ "number of milkrun-trains per hour"

$X \sim Ps(4)$, λ, $P(X=k) = \frac{4^k}{k!} \cdot e^{-4}$

$\Rightarrow P(X=3) = \frac{4^3}{3!} \cdot e^{-4} \approx \underline{0.1954}$

Exercise 6

$X \sim Exp(\frac{1}{3})$, λ

(a) $\mu = \frac{1}{\lambda} = 3$ (years)

(b) $P(X \geq 7 \mid X \geq 2) =$ memoryless $P(X \geq 2) =$

$= 1 - P(X \leq 2) = 1 - F(2) =$

$1 - (1 - e^{-\frac{2}{3}})$ $e^{-\frac{2}{3}}$

$= 1 - (1 - e^{-2}) = e^{-2} \approx 0.5134$

Exercise 7

$X \sim N(30, 4)$ (μ, σ^2)

(a) $P(24 \leq X \leq 36) = F(36) - F(24) =$

$= F_0\left(\frac{36-30}{2}\right) - F_0\left(\frac{24-30}{2}\right)$

$= F_0(3) - F_0(-3)$

$= F_0(3) - (1 - F_0(3))$

$= 2F_0(3) - 1 \approx 0.9973$

(b) $P(X \geq 35) = 1 - P(X \leq 35) =$

$= 1 - F(35) = 1 - F_0\left(\frac{35-30}{2}\right) =$

$= 1 - F_0\left(\frac{5}{2}\right) \approx 0.00621$

(c) $P(30 - \delta \leq X \leq 30 + \delta) \overset{!}{=} 0.98$ (μ, μ)

$\Leftrightarrow F(30+\delta) - F(30-\delta) = 0.98$

$\Leftrightarrow F_0\left(\frac{\delta}{2}\right) - F_0\left(\frac{\delta}{2}\right) = 0.98$

$\Leftrightarrow 2F_0\left(\frac{\delta}{2}\right) - 1 = 0.98$

$$\Leftrightarrow \quad F_0\left(\frac{\delta}{2}\right) = 0.99$$

$$\Leftrightarrow \quad \frac{\delta}{2} = 2.33 \quad \Leftrightarrow \quad \delta = 4.66$$

Time Slot: $[25.34, 34.66]$