1)

$$y(n) = a\,y(n-1) + (1-a)\,x(n)$$

z-Trafo

$$Y(z) = a z^{-1} Y(z) + (1-a)\,X(z)$$

$$Y(z)\,(1 - a z^{-1}) = (1-a)\,X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-a}{1-az^{-1}} = \frac{(1-a)\,z}{z-a}$$

$\Rightarrow$ pole at $z = a$

$\Rightarrow$ stable for $|a| < 1$

$\Rightarrow$ instable for all other a.

5) The mean value of a signal corresponds to the DC part of a signal.

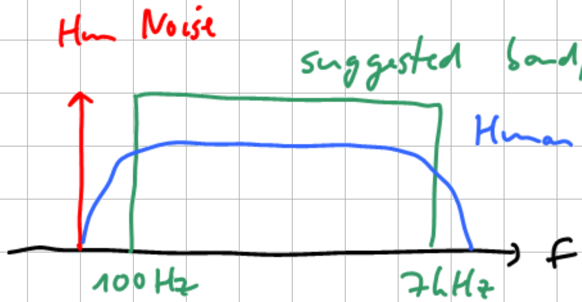
This DC part corresponds to the frequency $f = 0\,Hz$.

After a bandpass, no DC-component remains.

$\Rightarrow$ The signal has a zero mean after the bandpass.

Wideband human speech (good quality) passes this bandpass nearly unchanged $\Rightarrow$ human speech can pass this bandpass nearly without any problem.

Hum noise with $f = 50\,Hz$ lies below the lowest frequency of the bandpass ($\hat{=}$ 100 Hz). Therefore hum noise is strongly suppressed.



2) $$H(z) = \frac{1-a}{1-az^{-1}}$$

$$H(f) = \frac{1-a}{1-a\,e^{-j2\pi f/r}}$$

$$|H(f_c)| = \frac{|1-a|}{|1-a\,e^{-j2\pi f_c/r}|} = \frac{1}{\sqrt{2}} \cdot \underbrace{H(f=0\,Hz)}_{=1} = \frac{1}{\sqrt{2}}$$

$|1-a| = 1-a$ for stable filters

$$\Rightarrow \frac{1-a}{|1-a\cos(2\pi \frac{f_c}{r}) + j \cdot a \cdot \sin(2\pi \frac{f_c}{r})|} = \frac{1}{\sqrt{2}}$$

$$\frac{1-a}{\sqrt{(1-a\cos(2\pi \frac{f_c}{r}))^2 + a^2 \sin^2(2\pi \frac{f_c}{r})}} = \frac{1}{\sqrt{2}}$$

$$\frac{1-a}{\sqrt{1-2a\cos(2\pi \frac{f_c}{r}) + a^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1-2a+a^2}{1-2a\cos(2\pi \frac{f_c}{r}) + a^2} = \frac{1}{2}$$

$$2-4a+2a^2 = 1-2a\cos(2\pi \frac{f_c}{r}) + a^2$$

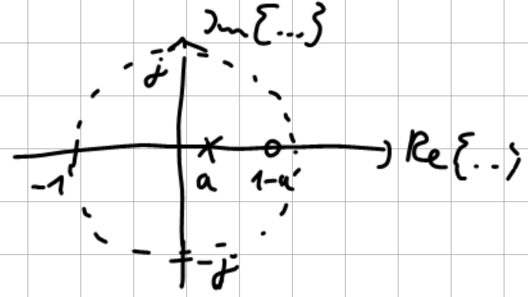
$$\frac{1-4a+a^2}{-2a} = \cos(2\pi \frac{f_c}{r})$$

$$\arccos\left(\frac{1-4a+a^2}{-2a}\right) = 2\pi \frac{f_c}{r}$$

$$\Rightarrow f_c = \frac{r}{2\pi} \arccos\left(\frac{1-4a+a^2}{-2a}\right)$$

3) $$H_{LP}(z) = \frac{1-a}{1-az^{-1}}$$

$$z_0 = 1-a$$
$$z_x = a$$



Switching poles and zeros converts a lowpass to a highpass:

$$H_{HP}(z) = \frac{1}{H_{LP}(z)} = \frac{1-az^{-1}}{1-a} = \frac{Y(z)}{X(z)}$$

$$Y(z)\cdot(1-a) = (1-az^{-1})\,X(z)$$

$$y(n)\cdot(1-a) = x(n) - a\,x(n-1)$$

$$y(n) = \frac{1}{1-a}\left(x(n) - a\,x(n-1)\right)$$

other solutions are possible.

4) $h(n_0) = h(0) \cdot e^{-5}$

$\Rightarrow (1-a) \cdot a^{n_0} = (1-a)\, e^{-5}$

$\Rightarrow a^{n_0} = e^{-5}$

$\Rightarrow n_0 = \log_a e^{-5}$

$= \dfrac{\log_e e^{-5}}{\log_e a}$

$= \dfrac{-5}{\log_e a}$