INDIAN STATISTICAL INSTITUTE

Mid-Semester Examination: 2024-2025

Image Processing - I

M.Tech.(Computer Science)

Date: 24.02.2025 **Full Marks: 40** **Time: 2 Hours**

Answer any **four** questions. All questions carry equal marks.

1. (a) Consider a 3-bit image of size 64×64 pixels. It has the intensity distribution as follows:

r_k	0	1	2	3	4	5	6	7
n_k	81	122	245	329	656	850	1023	790

where r_k denotes the k-th intensity level and n_k is the number of pixels that have intensity r_k.

(i) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(ii) Find out the intensity distribution of the histogram-equalized image of (i).

(b) Suppose a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization (on the histogram-equalized image) will produce exactly the same result as the first pass. [(5+2)+3=10]

2. (a) Assume continuous intensity values, and suppose that the histogram of an image can be approximated by the following probability density function (PDF):

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2} & \text{for } 0 \leq r \leq L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the transformation function that will map the input intensity values, r, into values, s, of a histogram-equalized image.

(ii) Find the transformation function that (when applied to the histogram-equalized intensities, s) will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3} & \text{for } 0 \leq z \leq L-1 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Any local histogram processing method requires that a histogram be computed at each neighborhood location. Propose a method for updating the histogram from one neighborhood to the next, rather than computing a new histogram each time.

[(3+4)+3=10]

3. (a) Consider the following two continuous functions $f(x)$ and $h(x)$:

$$f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(x) = \begin{cases} \frac{1}{2} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Compute the convolution function between $f(x)$ and $h(x)$.

(b) Prove that the differentiation of the output of a convolution, of a signal with a filter, can be achieved by convolving the signal with the derivative of that filter.

(c) Show that the orientation of the edge computed using Sobel filters is equal to its true orientation.

[3+3+4=10]

4. (a) State and prove the convolution theorem.

(b) Prove that the Fourier transform is linear.

(c) Prove that the origin of the Fourier transform of a two-variable function $f(x, y)$ can be moved to the center of its corresponding $N \times N$ frequency square by multiplying $f(x, y)$ by $(-1)^{x+y}$.

[(2+3)+2+3=10]

5. (a) Suppose an image is filtered with the Laplacian kernel. Prove that the sum of the pixel values in the filtered image is 0.

(b) Prove that the Laplacian is invariant to rotation. You may assume continuous quantities.

(c) Obtain the Fourier transform of $\Delta f(x, y)$, the Laplacian of a two-variable function $f(x, y)$. Assume that x and y are continuous variables.

[2+5+3=10]

6. (a) Define accumulator of the Hough transform, considering normal representation of a line.

(b) What is the advantage of the normal representation of a line over its slope-intercept form for linking edgels lying on lines?

(c) Write down the steps of Hough transform for linking edgels.

(d) Analyze the computational complexity of (c).

[2+2+4+2=10]

INDIAN STATISTICAL INSTITUTE

Second-Semestral Examination: 2024-2025

Image Processing - I

M.Tech.(Computer Science)

Date: 23.04.2025 **Full Marks: 100** **Time: 3 Hours**

Answer any **ten** questions. All questions carry equal marks.

1. (a) Assume that both object and background pixels of an image are normally distributed. Show that the minimum error threshold equation has two solutions.

   (b) What is the significance of the two solutions of the minimum error threshold equation?

   (c) Suppose the grey values of the object and the background pixels are distributed according to the following probability density function

$$p(x) = \frac{1}{2\sigma} \exp\left\{ \frac{|x-\mu|}{\sigma} \right\}$$

   with $\mu = 60$, $\sigma = 10$ for the objects, $\mu = 40$, $\sigma = 5$ for the background, and x being the grey value of the pixel. If the number of object pixels is two-third of the total number of pixels, determine the minimum error threshold.

[4+2+4=10]

2. Consider that $u(x)$ is the noise-free signal of interest and $g(x)$ is a filter for edge detection.

   (a) Prove that $g(x)$ has to maximize the following quantity to achieve good signal-to-noise ratio:

$$\mathcal{S} = \left\{ \int_{-\infty}^{+\infty} g(x)u(-x)dx \right\} \left\{ \int_{-\infty}^{+\infty} [g(x)]^2 dx \right\}^{-\frac{1}{2}}.$$

   (b) Using $\mathcal{S}$ of (a), show that $g(x)$ must be an odd function in order to find the edges of $u(x)$.

   (c) Prove that $g(x)$ is optimal with respect to good locality measure, if it maximizes the following quantity:

$$\mathcal{L} = \left\{ \int_{-\infty}^{+\infty} g''(x)u(-x)dx \right\} \left\{ \int_{-\infty}^{+\infty} [g'(x)]^2 dx \right\}^{-\frac{1}{2}}$$

   where $g'(x)$ and $g''(x)$ denote the first and second derivatives of $g(x)$, respectively.

[3+3+4=10]

3. (a) State and prove the correlation theorem.

   (b) Show that the Fourier transform of the autocorrelation function of a function $f(x)$ is the power spectrum of $f(x)$.

   (c) Obtain the Fourier transform of $\Delta f(x, y)$, the Laplacian of a two-variable function $f(x, y)$. Assume that x and y are continuous variables.

[(2+3)+2+3=10]

4. (a) Prove that the difference-of-Gaussians (DoG) function provides a close approximation to the scale-normalized Laplacian of Gaussian (LoG).

   (b) Consider the following Gaussian function:

$$G(x, y; \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{-(x^2 + y^2)}{2\sigma^2} \right\}.$$

   Prove that the zero crossings of the LoG occur at $x^2 + y^2 = 2\sigma^2$.

(c) Prove that in order for the LoG and DoG to have same zero crossings, the value of σ for the LoG, and σ_1, σ_2 for the DoG must be selected based on the following relation:

$$\sigma^2 = \frac{{\sigma_1}^2 {\sigma_2}^2}{{\sigma_1}^2 - {\sigma_2}^2} \ln\left[\frac{{\sigma_1}^2}{{\sigma_2}^2}\right] \quad \text{where } \sigma_1 > \sigma_2.$$

(d) Consider the following function that models a ramp edge:

$$u(x) = \frac{1}{1 + \exp(-sx)}$$

where the parameter s controls the slope of the edge. Show that the position of the edge is at $x = 0$.

[2+3+3+2=10]

5. (a) Prove that the histogram equalised version of an image conveys the maximum possible information the image may convey.

(b) Show that the information content of an image increases with the increase in the range of grey values of that image.

(c) Compute the mutual information between two co-registered images A and B, where

$$A = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 2 \end{pmatrix}; \qquad B = \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 2 & 2 & 1 \end{pmatrix}.$$

(d) What is bi-linear interpolation? [3+2+3+2=10]

6. (a) Consider the following 4×4 block of grey levels:

$$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 2 & 2 & 1 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & 2 & 2 & 2 \end{pmatrix}.$$

Construct the grey level co-occurrence matrices for angle $\theta = 0°$ and $\theta = 90°$, considering unit pixel distance.

(b) Consider the following 3×3 block of grey levels:

$$\begin{pmatrix} 6 & 1 & 3 \\ 5 & 4 & 7 \\ 9 & 2 & 8 \end{pmatrix}.$$

Compute the local binary pattern (LBP) and rotation invariant LBP for the central pixel.

[(3+3)+(2+2)=10]

7. (a) Consider a 4×4 block of grey levels $A = [a_{ij}]$, where $a_{ij} = \max\{i, j\}$ is the grey level of the (i, j)-th element of A. Encode the grey levels of A with the strings of 0's and 1's based on Huffman coding.

(b) Calculate the average code-word length. [8+2=10]

8. Consider the following 6×6 block of grey levels:

$$\begin{pmatrix} 1 & 3 & 3 & 2 & 0 & 0 \\ 0 & 3 & 2 & 2 & 1 & 0 \\ 0 & 2 & 2 & 2 & 1 & 1 \\ 0 & 2 & 3 & 2 & 0 & 1 \\ 1 & 3 & 3 & 2 & 1 & 0 \\ 1 & 3 & 3 & 2 & 0 & 0 \end{pmatrix}.$$

Construct the run-length matrices for angle $\theta = 0°$ and $\theta = 45°$, and compute the run percentage for each case. [(4+4)+2=10]

9. (a) Let $\overline{m}_{ij}$ be the (i,j)-th central moment of an image f and θ represents the slope of the principal axis. Prove that

$$\tan^2\theta + \frac{\overline{m}_{20} - \overline{m}_{02}}{\overline{m}_{11}} \tan\theta - 1 = 0.$$

(b) Show that the above equation is equivalent to

$$(\overline{m}_{11}\tan\theta + \overline{m}_{20})^2 - (\overline{m}_{20} + \overline{m}_{02})(\overline{m}_{11}\tan\theta + \overline{m}_{20}) + (\overline{m}_{20}\overline{m}_{02} - \overline{m}_{11}^2) = 0.$$

(c) Prove that the principal axis is in the direction of the eigenvector corresponding to the larger eigenvalue of the following matrix:

$$\begin{pmatrix} \overline{m}_{20} & \overline{m}_{11} \\ \overline{m}_{11} & \overline{m}_{02} \end{pmatrix}.$$

[4+2+4=10]

10. (a) Assume that an image, with minimum grey value $I_{\min}$, has a bright object on a dark background. Show that Otsu thresholding method finds the optimal threshold for segmenting the image by maximizing the following objective function:

$$\mathcal{F}(t) = \frac{[\mu(t) - \mu\theta(t)]^2}{\theta(t)[1-\theta(t)]},$$

where $t > I_{\min}$ denotes the threshold, μ is the mean grey value of the image, $\theta(t)$ denotes the fraction of pixels having grey values between $I_{\min}$ and t, and $\mu(t) = \tilde{\mu}\theta(t)$, $\tilde{\mu}$ being the mean grey value of the pixels having grey values between $I_{\min}$ and t.

(b) How can Otsu method be extended to obtain two thresholds?

[7+3=10]

11. (a) Define hue, saturation and intensity of the HSI color model.

(b) Write down the expressions for converting colors from RGB model to HSI model.

(c) Assume that the RGB values are normalized to the range $[0,1]$. Using (b), compute the hue, saturation and intensity of a color point $P = (1,0,1)$ of RGB model.

(d) What color does P of (c) represent?

[3+3+3+1=10]

12. The locally adaptive unsharp masking is an image enhancement technique, where the local variance is used as the basis for making changes that depend on image characteristics in a neighborhood about each pixel in an image. Suppose a low-contrast image $f(x,y)$ is subjected to local enhancement using the locally adaptive unsharp masking. Let (x,y) denote the coordinates of any pixel in the given image $f(x,y)$ and S_{xy} be a neighborhood of specified size, centered on (x,y). The local variance $\sigma(x,y)$ at (x,y) is the variance of the grey values of the pixels inside the window S_{xy} and is a measure of intensity contrast in that neighborhood.
Write pseudocode for computing the local variance $\sigma(x,y)$, given the image $f(x,y)$, with the help of the following allowed operations:

- **DFT**($f(x,y)$): returns $F(u,v)$, the discrete Fourier transform of a 2D function $f(x,y)$
- **IDFT**($F(u,v)$): returns $f(x,y)$, the inverse discrete Fourier transform of a 2D function $F(u,v)$
- **CONV_FD**($F(u,v)$, $G(u,v)$): returns $[F(u,v) * G(u,v)]$, the convolution between two 2D functions $F(u,v)$ and $G(u,v)$ in frequency domain
- **HP_FD**($F(u,v)$, $G(u,v)$): returns $[F(u,v)G(u,v)]$, the Hadamard product between two matrices $F(u,v)$ and $G(u,v)$ in frequency domain
- **DIFF_FD**($F(u,v)$, $G(u,v)$): returns $[F(u,v) - G(u,v)]$, the difference between $F(u,v)$ and $G(u,v)$.

You are also allowed to define 2D functions of your choice in frequency domain only.

[10]