INDIAN STATISTICAL INSTITUTE

Mid-Semestral Examination: 2025 – 26

Quantitative Finance: MStat II & Computational Finance: MTech (CS)

Date: 12 September 2025 Maximum Marks: 40 Duration: 2 Hours

Calculators are allowed, but no sharing between students

1. Critically explain the following:
   a) Mutual Fund Principle
   b) Reflection Principle (Hint: First passage time). [4 + 6 = 10]

2. Recall the iterative methods for solving a linear system of equations $Ax = b$, or minimizing loss functions of the form $\sum_{i=1}^{n} L(y_i - \beta_0 - \beta_1 x_i)$ in regression.

(a) Describe the iterative algorithm for Jacobi method. Show that if this algorithm converges, it will converge to the correct solution. Describe a sufficient condition for its convergence, and justify your answer.

(b) Describe a coordinate descent algorithm for a least absolute deviation regression with one predictor x: $\min_{\beta_0,\beta_1} |y_i - \beta_0 - \beta_1 x_i|$. [7 + 7 = 14]

3. (a) Using standard notation, prove the Put – Call parity of European option for the multi-period market. Is the same relation true for American options? – Prove or refute logically.

(b) For the European Compound option possibilities – Put on Call, Put on Put, Call on Call and Call on Put – what are the possible parity relations?

(c) What are the parity relations for double Barrier Knock-out options (with knock-out barriers both below and above)? [(4+2) + 4 + 6 = 16]

INDIAN STATISTICAL INSTITUTE

Semestral Examination: 2025 – 26

Quantitative Finance & Computational Finance

MStat (2nd Year) MTech (CS)

Date: 28 November 2025 Maximum Marks: 100 Duration: 3 Hours

Calculators are allowed, but no sharing between students

Answer mathematically, explaining all notation

1. (a) Define the Lookback and Chooser option contracts. For each of them, state the payoff function carefully, explaining all notation.

   (b) What is a Straddle portfolio arrangement? Explain and illustrate the payoff with a graph. [(5 + 5) + 7 = 17]

2. Consider the two period model $\{t = 0, 1, 2\}$. Stock price starts from S_0 = Rs. 100 and can go up or down by Rs. 10 in each period with probability ½. Assume zero rate of interest.

   (a) Compute the price of the following options

   (i) Up and out Barrier ($H = 115$) European type Call option with Strike $K = 105$.

   (ii) Down and out ($L = 85$) European type Put option with Strike $K = 95$.

   (b) How will the above prices change if we consider American type?

   [6 + 5 + 6 = 17]

3. Here $\{B_t\}$ is the standard Brownian Motion.

   (a) Prove directly from the definition of Ito integrals that
$$\int_0^t s dB_s = tB_t - \int_0^t B_s ds.$$
   (b) Check whether $X_t = t^2 B_t - 2\int_0^t sB_s ds$ is a Martingale. [9 + 8 = 17]

4. For the stochastic 1st order differential equation of the form
$$\left[a_0 \frac{\partial f}{\partial t}(t,\omega) + a_1 f(t,\omega)\right] dt = dB_t,$$
   find the general solution f_0 over R. Show that this solution is stationary with mean 0 and covariance function $\gamma_{f_0}(t) = \frac{\sigma^2}{2a_0a_1} e^{-a_0/a_1 |t|}, -\infty < t < \infty$.

   [9 + 8 = 17]

5. Consider a machine that can be in one of two states: good or bad. The Machine produces an item at the end of each period. The item is good or bad depending on the machine being in a good or bad state at the beginning of that period. Once the machine goes to the bad state it remains in that state until replaced. If the machine is in good state at the beginning of any period, then with probability 0.2 it will go to the bad state at the beginning of the next period. Once an item is produced, it may be inspected (at cost 1) or not inspected. If an inspected item is found to be bad, the machine is replaced with a machine in good state at cost 3. The overall cost incurred for producing a bad item is 2. Write a dynamic programming algorithm for the optimal inspection policy assuming the machine is initially in good state and a horizon of 8 periods. What is the optimal inspection rule?

[12 + 4 = 16]

6. Consider an elastic net regression with p variables and no intercept:

$$\frac{1}{2n}\sum_{i=1}^{n}(y_i - \beta^\top x_i)^2 + \lambda\,[\alpha||\beta||_2^2 + (1-\alpha)||\beta||_1].$$

Here $\alpha \in [0,1]$. Write down the univariate update formula for a coordinate descent algorithm. Assume the predictors are standardized as in the case of Lasso, i.e. $\sum_{i=1}^{n} x_{ij}^2 = n$, for every predictor $j = 1,\ldots,p$. [16]

~~ekhon~~ p - variate $\rightarrow$

~~at ite~~ $\cancel{(p-1)}$ initially $\beta_1^{(0)}, \beta_2^{(0)}, \ldots, \beta_p^{(0)}$.

iteration 1 $\rightarrow$ keep $(p-1)$ fixed, update β_1 say.

then, optimise ~~update~~ $\beta_2^{(1)} \longrightarrow \beta_1^{(1)}, \beta_3^{(0)}, \beta_4, \ldots, \beta_p^{(0)}$.

This way at each iteration, we update p variates,

$x_i y_j - x_i^2 \beta_j$.