INDIAN STATISTICAL INSTITUTE

Semester Examination: 2024-2025

M. Tech (CS)

Computational Geometry

Date : 05.05.2025 Maximum Marks : 100 Duration : 3 hours

(Although the paper carries a total marks of 120, the maximum marks you can score is 100)

Question 1: Describe an $O(n \log h)$ time algorithm to compute the convex hull of a set P of n points assuming that the size h of the convex hull is known apriori. Analyze the running time of your algorithm. [6+6]

Question 2: (a) Given a simple polygon P with n vertices. Describe an $O(n)$ time algorithm to compute a diagonal of P from a split vertex v. (A vertex v is said to be a split vertex if the interior angle at v is greater than π and x-coordinate of its neighbors are greater than x-coordinate of v.)

(b) Given a x-monotone polygon P, describe an $O(n)$ time algorithm to obtain the list of vertices of P sorted with respect to x-coordinate.

(c) Write a linear time algorithm to find the intersection of two convex polygons (linear with respect to size of the sum of the vertices of two polygons). Analyze the running time of your algorithm. [3+3+3+3]

Question 3: Given a set P of n points in 2D, write an $O(n \log n)$ time algorithm to compute MST of P. Analyze running time of your algorithm. [6+6]

Question 4: Build a 2D orthogonal range tree T for the 8 points $P = \{(1,1),(2,4),(3,9),(4,16),(5,25),(6,36),(7,49),(8,64)\}$. For each internal node u of T, let $P(u)$ denotes the canonical subset of points associated with the leaves descended from u and $|P(u)|$ be the number of nodes rooted at u. Build the y-range for each internal node u in x-range tree where $|P(u)| \geq 4$. [7+5]

Question 5: Given a set P of n points in R^5. Give an algorithm that computes a pair of closest points in P in $O(n \log n)$ time. [12]

Question 6: Given an arrangement $\mathcal{A}$ of n lines, write an $o(n^3)$ time algorithm to find a vertical line segment of the smallest length that stabs k lines of $\mathcal{A}$. [12]

Question 7: (a) Let (X, R) be a given a range space. Write the following three definitions for a given a range $Q \in R$ when X is restricted to a set P of n points.

(i) the measure of Q with respect to P, (ii) ϵ-sample and (iii) ϵ-net.

(b) Show that there exists an ϵ-net of size $\frac{1}{\epsilon}$ where P is a set of n points in $1D$ and R is the collection of all possible intervals in 1D.

(c) Write an $O(n \log n)$ time algorithm to find an ϵ-net for question number (b). Analyze the running time of your algorithm. [1+1+1+3+3+3]

Question 8: (a) Write an $O(n \log n)$ time algorithm to count the number of inversions in a sequence of n numbers. Analyze running time of your algorithm.

(b) Given a set $P = \{p_1, \cdots, p_n\}$ of n points in 2D, where $p_i = (a_i, b_i)$. For $1 \leq i < j \leq n$, define

$$s_{i,j} = \frac{b_j - b_i}{a_j - a_i}.$$

Write an $O(n \log n)$ time algorithm to count the number of $s_{i,j}$ that lie in the interval $[0, \infty]$. Analyze the running time of your algorithm. [3+3+3+3]

Question 9: (a) Build a segment tree for the intervals shown in Figure 1. [6]

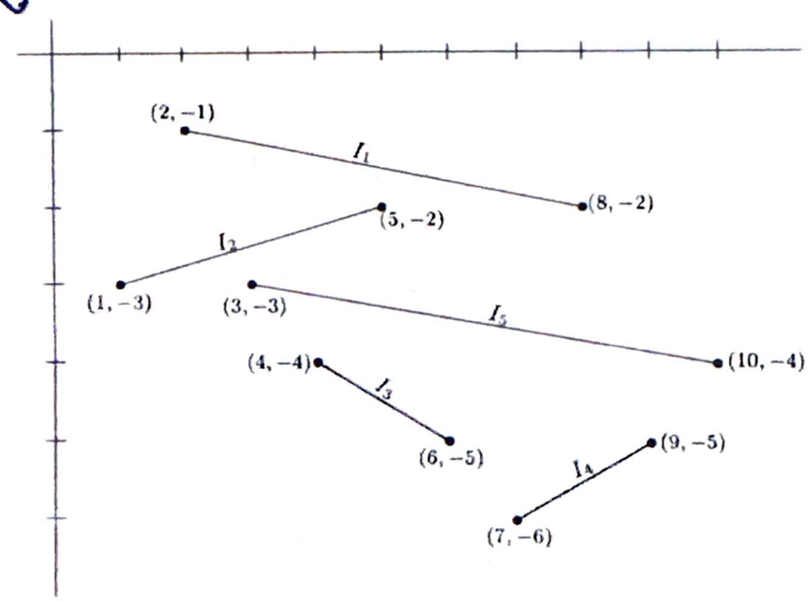

Figure 1: $I_1, I_2, I_3, I_4, and I_5$ are the intervals. For each i, the coordinates of the end-points of each interval are mentioned inside the parenthesis.

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(b) Show all the steps to build the upper horizon tree corresponding to the given cut $C = \{c_1, c_2, c_3, c_4, c_5\}$ for the five lines as shown in Figure 2. [6]

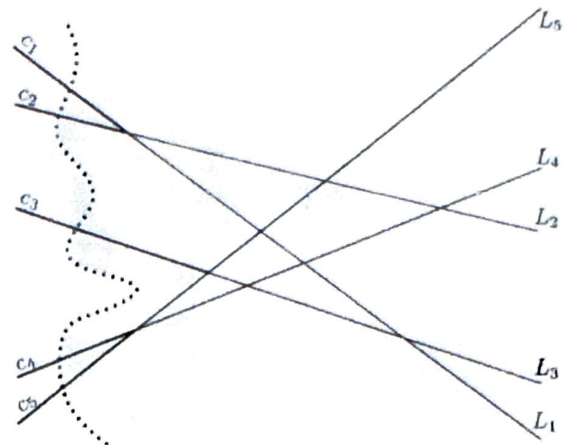

Figure 2: L_1, L_2, L_3, L_4, L_5 are the five lines.

Question 10: (a) Given a set T of n disjoint triangular obstacles in the plane. Given a vertex $v \in T$, write an $O(n \log n)$ time algorithm that finds all the vertices of T that are visible from v.

(b) Prove that the dual of the triangulated simple polygon is a tree. [6+6]