INDIAN STATISTICAL INSTITUTE

End Semester Examination

M.Tech CS, 2025 – 2026

Computational Game Theory

Date: 22 November 2025 Maximum Marks: 50 Duration: 2 hours

General comment. Answer as much as you can, but the maximum you can score is 50.

(Q1) The following is a 2 person zero sum game given by the following pay-off matrix where the row player has 5 strategies $R_1.R_2, \ldots, R_5$ and column player has 5
10 strategies $C_1, C_2, \ldots C_5$.

$$\begin{array}{cc} & \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \\ R_5 \end{array} & \begin{bmatrix} 3 & 1 & 2 & 2 & 2 \\ 0 & 2 & 2 & 3 & 1 \\ 5 & 4 & 5 & 5 & 5 \\ 2 & 3 & 3 & 3 & 3 \\ 4 & 6 & 5 & 5 & 5 \end{bmatrix} \end{array}$$

(a) Use dominant strategies to reduce the game.

(b) Solve the game using graphical method.

[10]

(Q2) Given a 2 person zero sum game with pay-off matrix $A_{m\times n}$, let the optimal strategy for player 1 be completely mixed. Let y^* denote an optimal for player 2.

(a) Let v be the value of the game. Express v in terms of A and y^*.

(b) Give an algorithm to find y^*. Write the algorithm, its overview, proof of correctness, and running time.

[10+10]

(Q3) Given a 2 person zero sum game with pay-off matrix $A_{m\times n}$, let the value of the game $v = 0$. Suppose, every optimal strategy for player 1 is completely mixed. Then show that $m - 1 \leq rank(A) \leq n - 1$.

[10]

(Q4) Decide for which of the set(s) X listed below an analog of Brouwer's fixed point theorem is valid (i.e. every continuous function $f : X \mapsto X$ has a fixed point). If it is valid, derive it from Brouwer's fixed point theorem, and if not, describe a function f witnessing it.

(a) X is a circle in the plane (we mean the curve, not the disk bounded by it);

(b) X is a circular disk in the plane;

(c) X is a triangle in the plane with one interior point removed;

(d) X is a sphere in the 3-dimensional space (a surface);

[5+5+5+5 = 20]

(Q5) Does there exist a solution to the LCP given by the below M and q? If yes, write how to find the solution, otherwise show no solution can exist.

$$M = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, q = \begin{bmatrix} -5 \\ -6 \end{bmatrix}$$ [10]

(Q6) In a cooperative game (N, ν), let x, y, and z be three imputations such that $x \succ y$ and $y \succ z$. Then, either write a proof to show the following statement is true, or give a counterexample:

$$x \succ z$$

[10]

(Q7) Write an algorithm to find core in a proper simple game (N, ν). [10]

(Q8) Prove that any social choice function which is not a dictatorship (i.e., the choice is not made according to the preferences of a single voter), and has at least three alternatives in its range, can be strategically manipulated. [10]