Indian Statistical Institute

Reinforcement Learning — M.Tech. CS

Mid-Semester Examination: 2025–26 Semester I

Full Marks: 80 Time: 2 Hours

Instructions: Answer all 8 questions. Each carries 10 marks. Justify your answers briefly and clearly.

1. (a) In a finite MDP, is it true that if there exists a deterministic optimal policy, then there is only one such deterministic optimal policy? Prove or give a counterexample.
   (b) If the deterministic condition is removed, what can be the number of optimal policies in a finite MDP?

2. Consider the 5×5 gridworld shown in Figure 1. The agent starts at the bottom middle cell and must reach the top middle cell. Moving to any non-terminal, non-bomb cell gives a reward of -1. Stepping on the bomb cell (the central square) ends the episode with a reward of -50. The agent learns using SARSA.

   Which path will be learnt under (a) SARSA with ε-greedy exploration, and (b) greedy SARSA ($\varepsilon = 0$)?
   Give a brief explanation for each case.

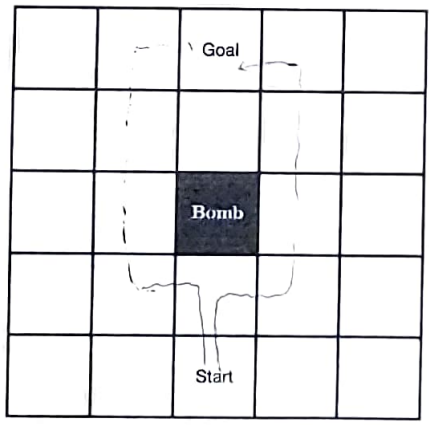


Figure 1: Gridworld with bomb in the center, start at bottom middle, goal at top middle.

3. (a) Why is the optimal state-value function v^* by itself usually not enough to find the optimal policy?

   (b) Why is the optimal action-value function q^* always enough? Give one situation where v^* alone can be enough.

4. Justify why policy iteration is guaranteed to converge to an optimal policy for a finite MDP.

5. A behavior policy b chooses each action uniformly at random. A target policy π is deterministic and greedy. Consider the episode sampled under b:

$$A \xrightarrow{r=2} B \xrightarrow{r=1} C \quad \text{(terminal)},$$

   with $\pi(A) = B,\ \pi(B) = C$. Assume each state has 2 possible actions.

   (a) Compute the importance sampling ratio ρ for this episode.

   (b) If the return is $G = 3$, what is the off-policy MC estimate of $v^\pi(A)$? How does this compare to the on-policy estimate?

6. The table below shows the runs scored by five famous Indian batters across six matches. The top scorer in each match is shown in bold, and the averages across all matches are given in the last row.

Match	Sachin	Dravid	Sehwag	Kohli	Rohit
1	**54**	33	0	40	39
2	2	2	7	**88**	34
3	7	42	37	**98**	81
4	**82**	8	1	46	27
5	53	**91**	50	6	35
6	**92**	34	78	41	6
Average	48.3	35.0	28.8	53.2	37.0

   The boss of *Cricinfo* asks his analyst: "Tell me the batting average of the best player." The analyst decides to do the following: for each match, he picks the highest individual score among all five players, then takes the average of these six maxima and reports it as the batting average of the best player.

   What has the analyst done, and why does this not correctly measure the batting average of the best player? Connect your explanation to the idea of maximization bias (as seen in multi-armed bandits).

7. State whether the following statements are **True or False**, and justify each briefly (1–2 lines). Each part carries 2 marks.

   (a) With function approximation, a single update can change the estimated values of multiple states.

   (b) Function approximation is always superior to tabular methods because it generalizes across states.

   (c) Using function approximation can introduce approximation error even if the algorithm is correct.

   (d) Semi-gradient TD(0) with linear function approximation may converge to a solution that is not equal to the true value function.

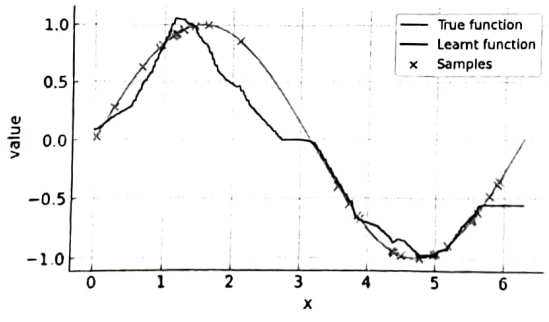
   (e) In the tabular case, TD(0) and Monte Carlo evaluation both converge to the true value function, given sufficient experience and a diminishing step-size schedule. With a fixed constant step-size, TD(0) only converges in expectation and continues to fluctuate around the true value.

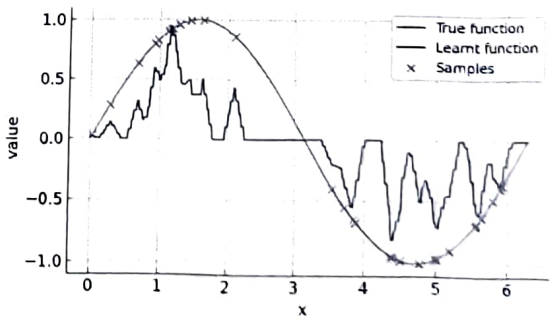
8. (a) Consider two tile-coding representations of $[0, 2\pi]$ with the same total number of binary features:

   - Case 1: 16 tilings $\times$ 10 tiles each.
   - Case 2: 4 tilings $\times$ 40 tiles each.

   Explain how the feature vector for a given point x differs in these two cases.

   (b) Below are two plots of learning $\sin(x)$ on $[0, 2\pi]$ using 30 random samples. In both cases the total number of features is the same, but the tiling structure differs (as described above). Match each plot to Case 1 and Case 2, and explain briefly why.





Indian Statistical Institute

Reinforcement Learning — M.Tech. CS

End-Semester Examination: 2025–26 Semester I

Date: 20 November 2025

Full Marks: 100 Time: 3 Hours

Instructions: Questions 1-10 carry 7 marks each, and Questions 11-13 carry 10 marks each.

1. Suppose, in a deterministic MDP, two different deterministic optimal policies π_1 and π_2 exist. Let s be a state from which the two policies take different actions, i.e., $\pi_1(s) = a_1$ and $\pi_2(s) = a_2$. Then, prove or disprove: from state s both actions a_1 and a_2 must get the same immediate reward.

2. Why does Monte Carlo methods tend to have higher variance than temporal difference methods? Explain with an example.

3. Give an example of an episodic MDP where a Monte Carlo method updates action values for many state–action pairs after one episode, but TD(0) updates only one. Briefly explain why.

4. In the well-known cliff-walking MDP, ε-greedy SARSA avoids the path along the cliff even though it is optimal. Explain why. If ε is reduced gradually, will it cause any difference?

5. Consider the statement: A function approximation method may not be able to learn the exact value, or action-value functions, whereas a tabular method can. Show an example to explain this statement.

6. Prove or disprove the statement: Whenever a function approximation method fails to learn the exact value / action-value function, it will also fail to learn the optimal policy.

7. Explain, for which values of λ, the λ-return is equivalent to the one-step TD return and the Monte Carlo (MC) return.

8. Explain how planning (as in Dyna-Q) can accelerate learning compared to a purely model-free algorithm. Mention one example scenario where planning might not help much, or even may hurt.

9. Explain how experience replay improves both stability and sample efficiency (the ability to learn faster without many real samples) in DQN.

10. Why does an actor-critic algorithm need to learn both the policy and a value function, whereas REINFORCE needs to learn only the policy function?

11. As we see in Figure 1, Indiana Jones is standing at position 0 and there is a crown of high archaeological value at position 6. The tiles along positions 1-5 are made of glass. Some tiles (in this picture tiles 1,3 and 5) are fragile. Stepping on a fragile tile or jumping to some position greater than 6 means Dr. Jones falls into the gutter and has to start again from 0. Since this is a reinforcement learning assignment, we make him start over instead of killing him. He can see that the crown is at position 6, but he cannot understand whether a tile is fragile without actually stepping on it.

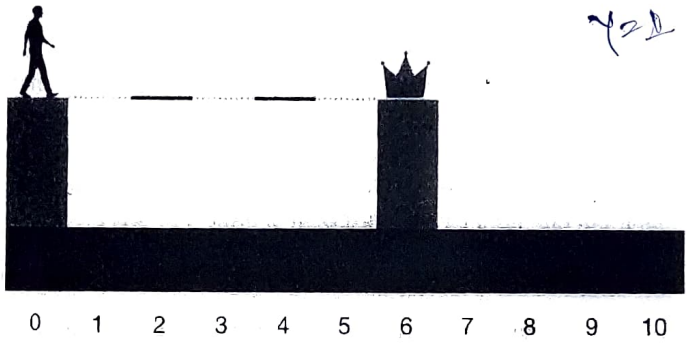


Figure 1: Indiana Jones and the Crown

The rules of the adventure are as follows. Dr. Jones has to jump from one tile to another, but he cannot just choose his length of jump. At each step, he has two choices, namely, to toss a coin or to roll a die. In case of the coin, he has to jump one position forward if head occurs, two positions otherwise. If he rolls the die, it will show a number $i \in \{1, 2, \ldots, 6\}$, and he has to jump i positions forward. In this process, if he manages to reach 6 (i.e., does not fall down before 6 and does not go overboard to position 7 or more), he gets the crown (reward 1). All other positions have reward 0. Assume that the coin and the die are unbiased. Answer the following questions based on this scenario.

(a) Compute the optimal value function $v_*(s)$ and optimal action-value function $q_*(s, a)$ for all non-terminal states s and all actions $a \in \{\text{toss, roll}\}$. Since you know the dynamics of the environment, you may use that for this question.

(b) From the optimal action-value function, determine an optimal policy. How many optimal policies are possible for this scenario?

(c) If Dr. Jones follows the optimal policy for one episode, what is the probability that he will get the crown?

$[(3 + 3) + (2 + 1) + 1 = 10]$

12. Assume that in an gridworld, each action generates a constant reward R for reaching all non-terminal states and R_T for reaching the terminal state. The goal of an agent is to learn the shortest path from any state to a terminal state. Which of the following setups will make sure that maximizing the cumulative return $G_t = \sum_{k=t}^{T-1} \gamma^{k-t} R_{k+1}$ aligns with the goal, i.e., learns the shortest path? Explain your answers briefly.

(a) $R = -1, R_T = 0, \gamma = 1$.

(b) $R = -1, R_T = -1, \gamma = 0.9$.

(c) $R = 0, R_T = 1, \gamma = 1$.

(d) $R = 0, R_T = 1, \gamma = 0.8$.

(e) $R = 1, R_T = 1, \gamma = 0.9$.

[2 × 5 = 10]

13. Briefly describe how the meta-RL algorithm RL^2 works using a diagram. Mark how the trials, episodes, states, rewards, hidden states and parameters of the network change.

[10]