INDIAN STATISTICAL INSTITUTE

<u>Mid Semester Examination</u>

M.Tech CS

Computational Complexity

Date: 11 September 2025 | Maximum Marks: 50 | Duration: 2 hours

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General comment. Answer as much as you can, but the maximum you can score is 50.

(Q1) Give a parsimonious reduction of SAT to 3SAT. [10]

(Q2) (40 Marks) Recollect the definition of **ZPP**, **RP**, **coRP** and **BPP**. Prove that

$$\mathbf{ZPP} = \mathbf{RP} \cap \mathbf{coRP}.$$

[10]

(Q3) (40 Marks) Prove that **IP** $\subseteq$ **PSPACE**. [10]

(Q4) Construct (with a proof) a complete problem for the class coNP. [10]

(Q5) Suppose $L_1, L_2 \in NP$. Then is $L_1 \cup L_2 \in NP$? What about $L_1 \cap L_2$? [5]

(Q6) Show that $P \neq EXP$. [5]

(Q7) A language L is called unary if $L \subseteq 1^*$. Show that if a unary language is NP-complete, then $P = NP$. [10]

$y, (\bar{x} \vee z)$

$\cancel{y} = yx \vee \bar{y}z$

$\wedge \bar{z}\ z$

$L(x) = \cancel{M_1(x)} \wedge M_2(x)$

$\vee \overline{L(x)}\ M_1(x)$

INDIAN STATISTICAL INSTITUTE

End Semester Examination

M.Tech CS

Computational Complexity

Date: 24 November 2025 Maximum Marks: 50 Duration: 3 hours

General comment. Answer as much as you can, but the maximum you can score is 50.

Part A ($5 \times 5 = 25$ marks)

1. (a) Define the classes **P**, **NP**, and **coNP**.

   (b) Give an example of a problem in NP that is *not known* to be in P or NP-complete.

   (c) State whether the following inclusions are known to be true or false (justify briefly):

   i. $P \subseteq NP$

   ii. $NP \subseteq PSPACE$

   iii. $P = NP$

2. Explain what it means for a problem A to be *polynomial-time reducible* to a problem B. Give an example of one NP-complete problem and briefly describe how another problem can be reduced to it.

3. Explain in your own words how the **Time Hierarchy Theorem** is proved using diagonalization. What is the main idea behind the construction?

4. What does it mean for a proof technique to *relativize*? Give an example showing that relativization cannot resolve the P vs NP question.

5. (a) Define the *deterministic decision tree complexity* of a Boolean function.

   (b) What is the deterministic decision tree complexity of the OR function on n variables? Justify briefly.

Part B (5 × 12 = 60 marks)

1. Show that **3SAT** is polynomial-time reducible to **CLIQUE**. Clearly describe the reduction and argue correctness. (You may assume 3SAT is NP-complete.)

2. State the **Time Hierarchy Theorem** formally. Explain why it implies that $P \subsetneq EXP$. What would happen if the theorem did not hold? Discuss briefly.

3. (a) State (without proof) the **PCP theorem**.

   (b) Explain in simple terms what it means for NP to have a PCP verifier.

   (c) Mention one important implication of the PCP theorem in the context of *hardness of approximation*.

4. Suppose there exists an oracle A such that $P^A = NP^A$, and another oracle B such that $P^B \neq NP^B$.

   (a) What does this tell us about the limitations of relativizing proof techniques?

   (b) Which of the following techniques are **non-relativizing**: diagonalization, arithmetization, or probabilistic checking?

   (c) Why was the discovery of the PCP theorem considered a breakthrough in non-relativizing techniques?

5. (a) Define the **deterministic communication complexity** of a function $f(x, y)$.

   (b) For the **Equality** function

$$EQ(x, y) = \begin{cases} 1 & \text{if } x = y, \\ 0 & \text{otherwise,} \end{cases}$$

   where $x, y \in \{0, 1\}^n$, prove that its deterministic communication complexity is at least $n + 1$ bits. (Hint: Use the fooling set argument.)