Termination using common Lyapunov function

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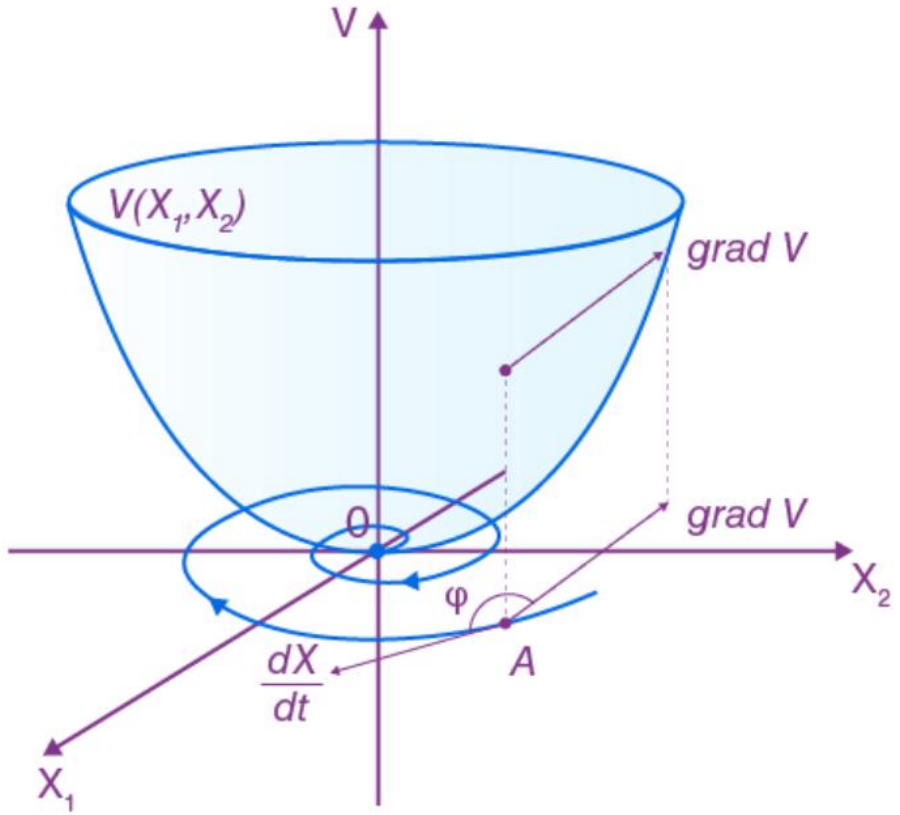
Lyapunov function

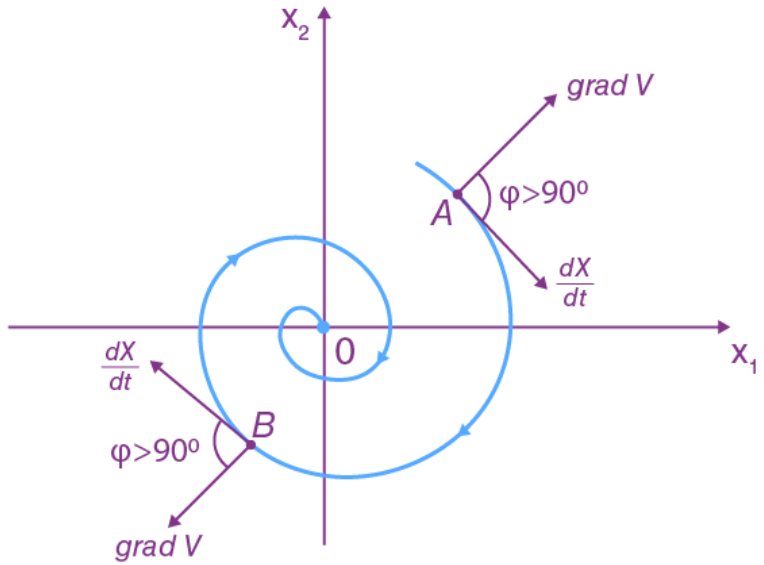
- A Lyapunov function helps us figure out if something will eventually stop or if it will go on forever by looking at the path it's following, like a car on a bumpy track.
- It's a tool that helps scientists and engineers understand how things behave in different situations.



A Lyapunov function with one stable equilibrium

Lyapunov function

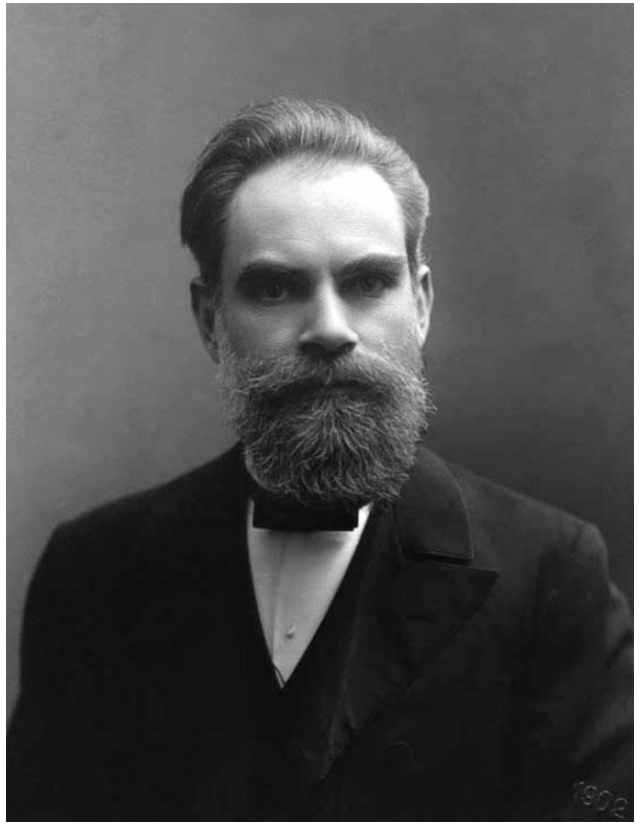
- Suppose V(X) be a continuously differentiable function in the origin's neighbourhood U. If the following requirements are satisfied, the function V(X) is known as the Lyapunov function for an autonomous system X' = f(x).



- V(X) >0 for all X ∈ U\ {0}
- (dV/dt) ≤ 0 for all X ∈ U
- V(0) = 0

Aleksandr Lyapunov

- Aleksandr Mikhailovich Lyapunov (1857–1918) was a Russian mathematician renowned for his pioneering work in stability theory and dynamical systems.
- Lyapunov's groundbreaking contributions include the development of Lyapunov functions, mathematical tools used to analyze the stability of dynamic systems.
- His work significantly impacted control theory and engineering, providing a framework to assess whether systems would remain stable or evolve unpredictably.
- His contributions in stability analysis remain integral to the study of complex systems across various disciplines.



Aleksandr Lyapunov in 1908

"What does 'termination' mean in the context of a Lyapunov function?"

- In the context of a Lyapunov function, "termination" refers to the behavior of a system reaching a stable and unchanging state, typically at an equilibrium point, as indicated by the Lyapunov function and its derivatives.

- A Lyapunov function is a mathematical tool used to assess the stability of an equilibrium point in a dynamical system. It helps determine whether, over time, the system's trajectories will converge to and remain within a certain vicinity of the equilibrium point, indicating stability.

- It does not refer to the termination of the Lyapunov function itself.

" How to Achieve this Equilibrium/ Stability?”

- If a derivative dV/dt along a phase trajectory is always negative, the trajectory will tend to the origin, indicating that the system is stable. Whenever the derivative dV/dt is positive, the system is unstable because the trajectory goes away from the origin.

- **Stability Theorem in the Lyapunov Sense**
- If a Lyapunov function V(X) exists in the neighbourhood U of an autonomous system’s zero solution X = 0, the system’s equilibrium point X = 0 is Lyapunov stable.

Reference/Resources

- https://www.researchgate.net/figure/Left-a-Lyapunov-function-with-one-stable-equilibrium-Right-a-Lyapunov-function-with_fig1_347796752
- https://en.wikipedia.org/wiki/Aleksandr_Lyapunov
- https://en.wikipedia.org/wiki/Lyapunov_function
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- https://www.youtube.com/watch?v=uXAx_641FPM
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THANK YOU!