1 Test 01

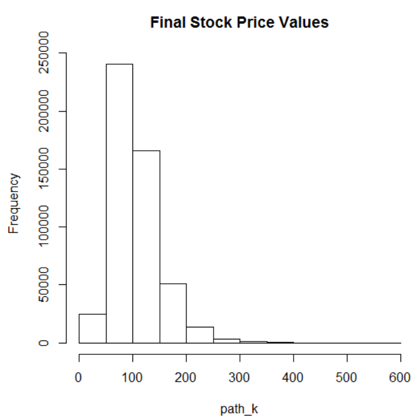
1.1 Lognormal Distribution Tests

The stock price is meant to be lognormally distributed. The following code plots and calculates theoretical vs numerical values for the final price S_T (given$S_0 = 100$,$r_f = .05$,$\sigma = .4$,$dt = 10^{-3}$and $T = 1$).

```
test01 <- function(){
  S_0=100;
  r_f=.05;
  vol=0.4;
  dt=.001;
  T=1;

  N<-500000
  path_k=array();

  for ( k in seq(N)){
    path = generate_path(S_0,r_f,vol,dt,T);
    path_k[k]=(path$values[length(path$values)])
  }

  hist(path_k,main="Final Stock Price Values")

  print(paste("mean S_T=",mean(path_k)))
  print(paste("var(log(S_T))=",var(log(path_k))))


  print(paste("theoretical mean S_T=",S_0*exp(r_f*T)));
  print(paste("theoretical var(log(S_T))=",vol*vol*T));
}
```

After running 500K runs (no antithetic technique employed), the values obtained are:

```
> source('validation.r');test01()
[1] "mean S_T= 105.111415714709"
[1] "var(log(S_T))= 0.160568654338744"
[1] "theoretical mean S_T= 105.127109637602"
[1] "theoretical var(log(S_T))= 0.16"
>
```



1.2 Visual Inspection

The stock-prices were all positive.

2 Test 02

2.1 Comparison of MonteCarlo Prices/Deltas with Theoretical values

The monte-carlo technique simply calculated the average of payoff $max(S_T - K, 0)$for call and $max(K - S_T, 0)$for put. The results are as follows:

```
[1] "Monte-Carlo call-option price = 18.0304317267674"
[1] "Monte-Carlo put-option price = 13.1529411879391"
[1] "theoretical BS CallOption price = 18.0229514502167"
[1] "theoretical BS CallOption delta = 0.627409464153284"
[1] "theoretical BS PutOption price = 13.1458939002881"
[1] "theoretical BS PutOption delta = 0.372590535846716"
```

.

2.2 Effect of number of intervals on path-independent options

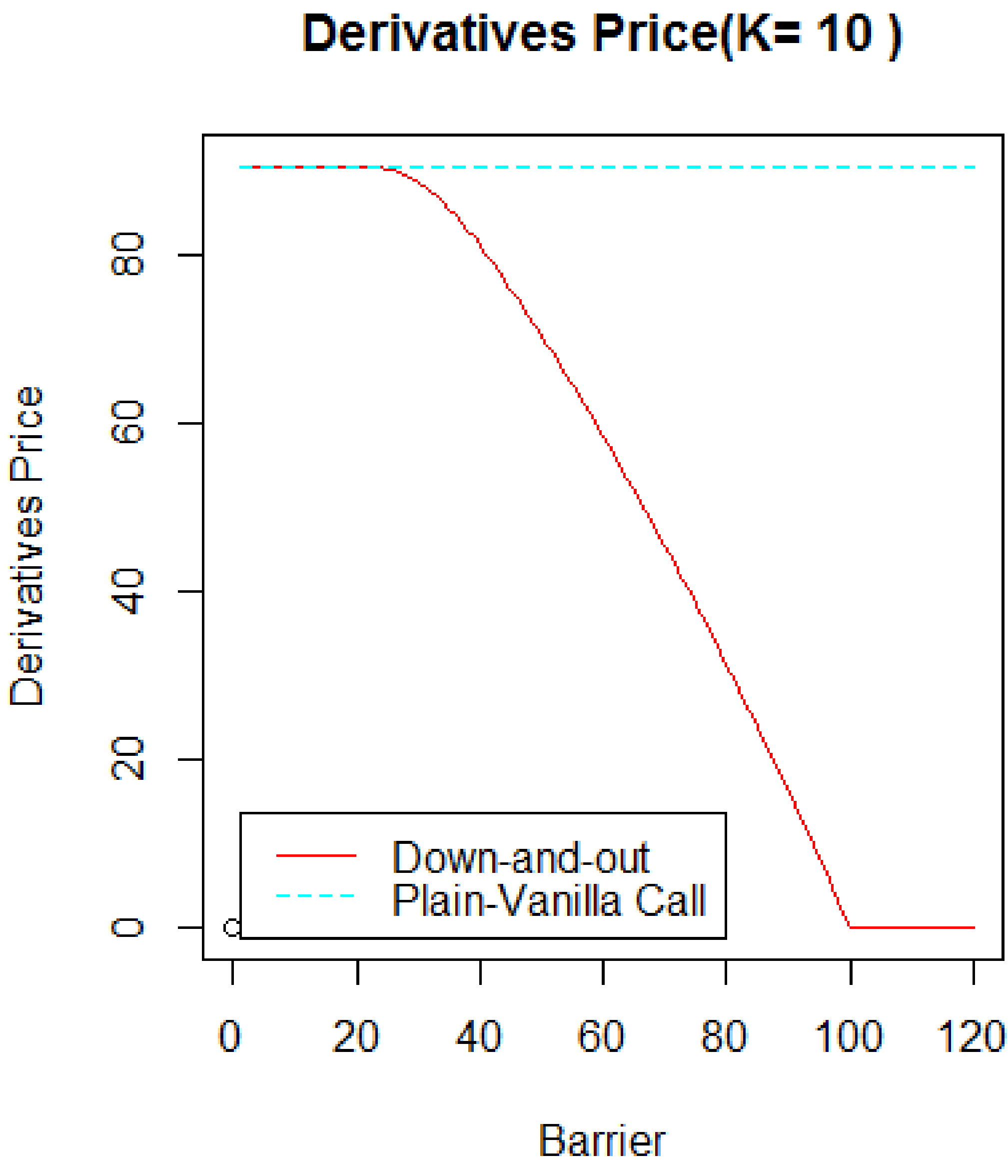
Widening the interval of simulatons *dt* slows the convergence of the Monte-carlo simulations. However, the converged values are unaffected by the choice of *dt*. With dt=.01 (instead of .001) we get following results (100K simulations):

[1] "sd(callpayoff_k)= 31.8117213164173"
[1] "Monte-Carlo call-option price = 17.4896755410705"
[1] "Monte-Carlo put-option price = 12.8769669719843"
[1] "theoretical BS CallOption price = 18.0229514502167"
[1] "theoretical BS PutOption price = 13.1458939002881"

3 Test 03

The implementation does not use the monte-carlo approach for valuing barriers. Since up-and-out option was not implemented as part of this exercise, the tests for up-and-out option have been postponed.

Notice that the price of a down-and-out barrier option does converge to that of the plain-vanilla option for low barriers. Following chart demonstrates this:



4 Test 04

The hedge-error variance increases with decreasing the number of steps.

dt	stdev
0.01	17.7548
0.02	17.7564
0.05	17.84027
0.1	17.92091