1 Order Book Liquidity

There are mainly two groups of liquidity providers in the market, namely the market makers and the limit order traders[1]. Harris and Hasbrouck provide evidence that placing limit orders at or better than the prevailing quotes is a better trading strategy than simply using market orders, even after accounting for non-execution risks[2].

The cost-to-trade measures gauge the ability of the limit order book to handle a sudden surge in the demand for liquidity by market orders.

The deeper the book is, the less it penalizes liquidity demanders[1].

Market participants monitor trading activity, and accordingly revise their expectations of future price and liquidity levels. For institutional investors trading in large volumes, the effect of order size on security price and market depth is of significant importance. If there is too much volume available at or close to the best price, an order with limit price further away from the best price may never get filled. Similarly, if the market impact is wider, with thin volumes at the best price (or close to it), there is then an increased probability of price-improving order being filled.[3].

If the probability that the information is long lived is high, then informed traders are more likely to place limit orders than market orders[4].

The open-to-close average price change for buys is only 0.34% . The behavior of prices from the open to the trade can be attributable to short-run liquidity costs, prior release of information, or positive-feedback trading by managers. The post-trade behavior of prices exhibits a sharp difference between buys and sells. The price continues to rise after purchases - the principal-weighted average return from the trade to the closing price is 0.12% whereas it tends to correct itself after sales - the reversal is 0.10%.The post-trade reversal for sells is consistent with the existence of short-run liquidity costs, whereas the post-purchase behavior of prices for buys is consistent with information effects or imperfectly elastic demand curves[5].

1.1 Order Book Filling

In the current model, it is assumed that the only two order precendence rules are based on price and time (a more favorable price - lowest bid and highest ask is selected for filling the order.

An order book may look like the following:

BidSize	BidPrice	AskSize	AskPrice
2000	10.5	1000	11.2
1000	10.4	2000	11.1
1000	10.3	2000	11.0
500	10.2	1000	10.9
500	10.1	500	10.8
1000	10.0	500	10.6

If somone buys a lot then, the market moves (i.e. stock-price increases

when you buy more) - since the the new prices are not known any more (See: http://money.stackexchange.com/questions/15156/how-do-exchanges-match-limit-orders) . The depth of the book is a a good measure of liquidity and liquidity ratio is a good proxy for the depth of book. An increase (decrease) in the liquidity ratio is associated with an increase (decrease) in market depth or liquidity[6].

The Amivest Ratio LR_i is defined as:

$$LR_i = \frac{\sum_y V_{id}}{\sum_y |R_{id}|} = \frac{\sum_{n=1}^{N_t} P_n \nu_n}{\sum_y |100(\frac{P_{id}}{P_{i(d-1)}} - 1)|}$$

Here V_{id} =Daily Volume of the stock
R_{id} =Return of the stock i on day d
P_n =The price of stock at the transaction n
N_t =Total number of transactions
ν_n =Number of traded stocks in the transaction n

Amivest ratio captures the notion that large amounts can be traded in a liquid stock without any significant changes in the stock price[7]. It provides the trading volume needed to change a stock's price by 1% (positive or negative).

Given a certain range of Amivest ratio, we know the percentage by which the stock price would move and thus can calculate our PNL.

1.2 Close to Maturity

When we're close to maturity, the by no-aribtrage (which assumes lack of liquidity constraints), the call-option price would be $S_T - K$ whenever option is in the money. For an in-the-money option, our hedged portfolio moves as$\Pi_T = \alpha B_T + \Delta \cdot S_T$. The value of the two should be the same and Δ should thus approach unity. If the option is out of money, then $S_T > K$ and the price of the option should be zero (no position in stock necessary as the out-of-the-money option is worthless around maturity when stock-price remains constant).

2 Current Project

2.1 Transaction Costs vs Liquidity

Setting high transaction costs would discourage trading which may provide liquidity. Let's say we have $N = 1e + 6$ number of shares of a company in the market. Let the Market-capitalization of the company be $X = 10e + 6$ (Share price=10). Now, buying $b = 200K$ of its shares ($b < N$ at price$\frac{X}{N} = 10$) would decrease the total number of shares to $N - b = 800K$. We own $\frac{b}{N} = 20\%$ of the company but $\frac{1-b}{N} = 80\%$ of the company is still in the market. It's value would be higher because bid-prices are lower than the ask price (I would sell at a higher value than what I bought at).

Trading a lot of underlying rather than the free-float. The more shares you trade, the less favorable conditions would be. The hedging function would also make this evident.

If one hedges such position (daily-rebalancing) one realizes the riskiness of the position. Calculating VaR shows this.

The current activities analyze effect of liquity when an underlying is traded. First, the behaviour of delta is explored with the hedging portfolio.

$$d\Pi = \alpha dB + \Delta dS$$

Equating above with Ito's Lemma gives us Black-Scholes equation. This has been valued empirically using a simulated approach.

The Hedging PNL is compared for plain call option and the barrier option in light of the trading of underlying. The trading of underlying itself affects the price itself - thus posing costs on top of the flat transaction costs.

2.2 Tasks

1. Calculate Amivest Ratio
2. Assume a certain Amivest ratio $A = \frac{\partial P}{\partial V}$
3. In the hedging portfolio, modify the PNL function to simulate the change of price in response to traded volume. The PNL change in response to bought shares dV can be approximated as $\Delta PNL = (P + A\partial V) \cdot (V + \partial V) - PV = (P + AV)\partial V$. In other words, the simulated PNL evolves as $\frac{\partial(PNL)}{\partial V} = P + AV$. With transaction costs, this becomes $\frac{\partial(PNL)}{\partial V} = P + AV - c$ where c =the rate of transaction costs (per unit of bought volume).
4. Observe Hedging PNL with transactions costs of a regular option
5. Observe Hedging PNL with transaction costs of a down-and-out option

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