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Operations Research (Paper III)
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Practical 1

Aim: Use graphical method to solve the following LPP:

$$\text{Max } Z = 3x + 5y$$
w.r.t.
$$x + 2y \leq 2000,$$
$$x + y \leq 1500,$$
$$y \leq 600,$$
$$x, y \geq 0$$

Source Code:

```
require(lpSolve)

C ← c(3, 5)

A ← matrix(c(1, 2,
             1, 1,
             0, 1), nrow = 3, byrow = T)

B ← c(2000, 1500, 600)

constraint_direction ← c("⩽", "⩽", "⩽")

plot.new()
plot.window(xlim = c(0, 2000), ylim = c(0, 2000))
axis(1)
axis(2)
title(main = "LPP using graphical method", xlab = "X-axis", ylab = "Y-
axis")
box()

segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "red")
segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, col = "blue")

z ← lp(direction = "max",
```

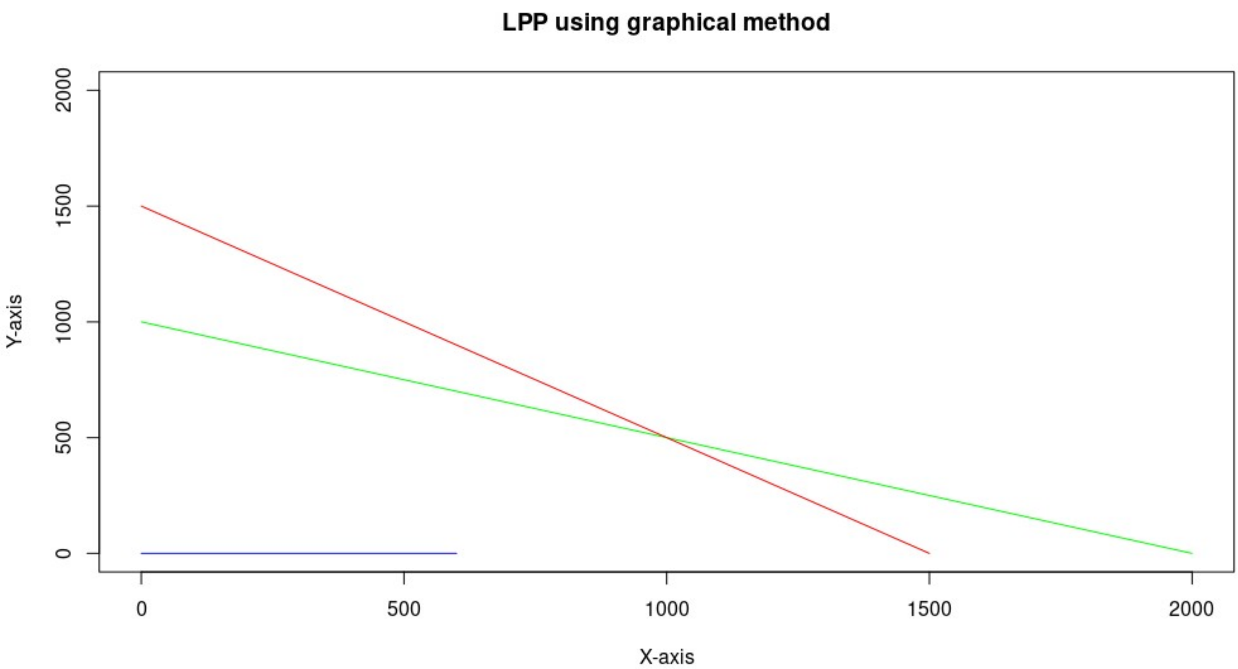
```
        objective.in = C,
        const.mat = A,
        const.dir = constraint_direction,
        const.rhs = B,
        all.int = T
        )

print(z$status)

best_sol <- z$solution
names(best_sol) <- c("x1", "x2")
print(paste("Total cost: ", z$objval, sep = ""))
```

Output:

```
> # Use graphical method to solve the following LPP:
> # Max z = 3x + 5y
> # w.r.t
> # x + 2y ≤ 2000,
> # x + y ≤ 1500,
> # y ≤ 600,
> # x, y ≥ 0
>
> require(lpSolve)
>
> C <- c(3, 5)
>
> A <- matrix(c(1, 2,
+               1, 1,
+               0, 1), nrow = 3, byrow = T)
>
> B <- c(2000, 1500, 600)
>
> constraint_direction <- c("≤", "≤", "≤")
>
> plot.new()
> plot.window(xlim = c(0, 2000), ylim = c(0, 2000))
```

```
> axis(1)
> axis(2)
> title(main = "LPP using graphical method", xlab = "X-axis", ylab = "Y-axis")
> box()
>
> segments(x0 = 2000, y0 = 0, x1 = 0, y1 = 1000, col = "green")
> segments(x0 = 1500, y0 = 0, x1 = 0, y1 = 1500, col = "red")
> segments(x0 = 0, y0 = 0, x1 = 600, y1 = 0, col = "blue")
>
> z <- lp(direction = "max",
+         objective.in = C,
+         const.mat = A,
+         const.dir = constraint_direction,
+         const.rhs = B,
+         all.int = T
+         )
>
> print(z$status)
[1] 0
>
> best_sol <- z$solution
> names(best_sol) <- c("x1", "x2")
```

```
> best_sol <- z$solution
> names(best_sol) <- c("x1", "x2")
> print(paste("Total cost: ", z$objval, sep = ""))
[1] "Total cost: 5500"
>
```



Practical 2

Aim: Use simplex method to solve the following LPP:

$$\text{Max } Z = 3x + 2y$$

w.r.t.

$$x + y \leq 4,$$
$$x - y \leq 2,$$
$$x, y \geq 0$$

Source Code:

In[1]:

```
from scipy.optimize import linprog

obj = [-3, -2]
lhs_ineq = [[1, 1], [1, -1]]

rhs_ineq = [4, 2]

bound = [(0, float("inf")), (0, float("inf"))]
```

In[2]:

```
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
        bounds = bound, method = "revised simplex")

z
```

In[3]:

```
print(z.fun)
print(z.success)
print(z.x)
```

Output:

In [1]:
```
from scipy.optimize import linprog

obj = [-3, -2]
lhs_ineq = [[1, 1],
            [1, -1]]

rhs_ineq = [4,
            2]

bound = [(0, float("inf")),
         (0, float("inf"))]
```

In [2]:
```
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
            bounds = bound, method = "revised simplex")

z
```

Out[2]:
```
     con: array([], dtype=float64)
     fun: -11.0
 message: 'Optimization terminated successfully.'
     nit: 2
   slack: array([0., 0.])
  status: 0
 success: True
       x: array([3., 1.])
```

In [3]:
```
print(z.fun)
print(z.success)
print(z.x)
```

```
-11.0
True
[3. 1.]
```

Practical 3

Aim: Use simplex method to solve the following LPP:

$$\text{Min } Z = x_1 - 3x_2 + 2x_3$$
w.r.t
$$3x_1 - x_2 + 3x_3 \leq 7,$$
$$-2x_1 + 4x_2 \leq 12,$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10,$$
$$x_1, x_2, x_3 \geq 0$$

Source Code:

In [1]:

```
from scipy.optimize import linprog

obj = [1, -3, 2]

lhs_ineq = [[3, -1, 3],
            [-2, 4, 0],
            [-4, 3, 8]]

rhs_ineq = [7,
            12,
            10]

bound = [(0, float("inf")),
         (0, float("inf")),
         (0, float("inf"))]
```

In [2]:

```
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
            bounds = bound, method = "revised simplex")

z
```

Output:

In [1]:

```
from scipy.optimize import linprog

obj = [1, -3, 2]

lhs_ineq = [[3, -1, 3],
            [-2, 4, 0],
            [-4, 3, 8]]

rhs_ineq = [7,
            12,
            10]

bound = [(0, float("inf")),
         (0, float("inf")),
         (0, float("inf"))]
```

In [2]:

```
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
            bounds = bound, method = "revised simplex")

z
```

Out[2]:

```
     con: array([], dtype=float64)
     fun: -11.0
 message: 'Optimization terminated successfully.'
     nit: 2
   slack: array([ 0.,  0., 11.])
  status: 0
 success: True
       x: array([4., 5., 0.])
```

Practical 4

Aim: Use simplex method to solve the following LPP:

$$\text{Max } Z = x + 2y$$
w.r.t.
$$2x + y \leq 20,$$
$$-4x + 5y \leq 10,$$
$$-x + 2y \geq -2,$$
$$-x + 5y = 15,$$
$$x, y \geq 0$$

Source code:

In [1]:
```
from scipy.optimize import linprog

obj = [-1, -2]

lhs_ineq = [[2, 1],
            [-4, 5],
            [1, -2]]

rhs_ineq = [20,
            10,
            2]

lhs_eq = [[-1, 5]]
rhs_eq = [15]

bound =  [(0, float("inf")),
         (0, float("inf"))]
```

In [2]:
```
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
            A_eq = lhs_eq, b_eq = rhs_eq,
            bounds = bound, method = "revised simplex")
z
```

Output:

```
In [1]:
from scipy.optimize import linprog

obj = [-1, -2]

lhs_ineq = [[2, 1],
            [-4, 5],
            [1, -2]]

rhs_ineq = [20,
            10,
            2]

lhs_eq = [[-1, 5]]
rhs_eq = [15]

bound =  [(0, float("inf")),
         (0, float("inf"))]
```

```
In [2]:
z = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
            A_eq = lhs_eq, b_eq = rhs_eq,
            bounds = bound, method = "revised simplex")

z
```

```
Out[2]:
     con: array([0.])
     fun: -16.818181818181817
 message: 'Optimization terminated successfully.'
     nit: 3
   slack: array([ 0.        , 18.18181818,  3.36363636])
  status: 0
 success: True
       x: array([7.72727273, 4.54545455])
```

Practical 5

Aim: Use Big M method to solve the following LPP:

$$\text{Min } Z = 4x_1 + x_2$$
w.r.t.
$$3x_1 + 4x_2 \geq 12,$$
$$x_1 + 5x_2 \geq 15,$$
$$x_1, x_2 \geq 0$$

Source code:

In [1]:
```
from scipy.optimize import linprog

obj = [4, 1]
lhs_ineq = [[-3, -4],
            [-1, -5]]

rhs_ineq = [-20,
            -15]

bound = [(0, float("inf")),
         (0, float("inf"))]
```

In [2]:
```
opt = linprog(c=obj,A_ub=lhs_ineq,b_ub=rhs_ineq,
              bounds=bound,method="interior-point")

opt
```

Output:

In [1]:
```
from scipy.optimize import linprog

obj = [4, 1]
lhs_ineq = [[-3, -4],
            [-1, -5]]

rhs_ineq = [-20,
            -15]

bound = [(0, float("inf")),
         (0, float("inf"))]
```

In [2]:
```
opt =linprog(c=obj,A_ub=lhs_ineq,b_ub=rhs_ineq,
               bounds=bound,method="interior-point")

opt
```

Out[2]:
```
     con: array([], dtype=float64)
     fun: 5.0000000002364455
 message: 'Optimization terminated successfully.'
     nit: 5
   slack: array([1.64270375e-10, 1.00000000e+01])
  status: 0
 success: True
       x: array([6.01160437e-11, 5.00000000e+00])
```

Practical 6

Aim: Use any method to solve the following resource allocation problem:

$$\text{Max } Z = 20x_1 + 12x_2 + 50x_3 + 25x_4 \text{ ....(profit)}$$
w.r.t.
$$x_1 + x_2 + x_3 + x_4 \leq 50 \text{ ....(manpower)}$$
$$3x_1 + 2x_2 + x_3 \leq 100 \text{ ....(material A)}$$
$$x_2 + 2x_3 \leq 90, \text{ ....(material B)}$$
$$x_1, x_2, x_3 \geq 0$$

Source code:

In [1]:

```
from scipy.optimize import linprog

obj = [-20, -12, -40, -25]

lhs_ineq = [[1, 1, 1, 1],
        [3, 2, 1, 0],
        [0, 1, 2, 3]]

rhs_ineq = [50,
        100,
        90]
```

In [2]:

```
opt = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
method="revised simplex")

opt
```

Output:

In [1]:

```
from scipy.optimize import linprog

obj = [-20, -12, -40, -25]

lhs_ineq = [[1, 1, 1, 1],
            [3, 2, 1, 0],
            [0, 1, 2, 3]]

rhs_ineq = [50,
            100,
            90]
```

In [2]:

```
opt = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq, method="revised simplex")

opt
```

Out[2]:

```
    con: array([], dtype=float64)
    fun: -1900.0
message: 'Optimization terminated successfully.'
    nit: 2
  slack: array([ 0., 40.,  0.])
 status: 0
success: True
      x: array([ 5.,  0., 45.,  0.])
```

Practical 7

Aim: Use simplex method to solve the following LPP:

$$\begin{aligned}
&\text{Max } Z = 200x + 300y \\
&\text{w.r.t.} \\
&2x + 3y \geq 1200, \\
&x + y \leq 400, \\
&2x + 1.5y \geq 900, \\
&x, y \geq 0
\end{aligned}$$

Source code:

In [1]:

```
from scipy.optimize import linprog

obj = [-200, 300]

lhs_ineq = [[-2, -3],
        [1, 1],
        [-2, -3/2]]

rhs_ineq = [-1200, 400, -900]

bnd=[(0, float("inf")),
   (0, float("inf"))]
```

In [2]:

```
opt = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
          bounds = bnd,
          method = "revised simplex")

opt
```

Output:

In [1]:
```python
from scipy.optimize import linprog

obj = [-200, 300]

lhs_ineq = [[-2, -3],
         [1, 1],
         [-2, -3/2]]

rhs_ineq = [-1200, 400, -900]

bnd=[(0, float("inf")),
    (0, float("inf"))]
```

In [2]:
```python
opt = linprog(c = obj, A_ub = lhs_ineq, b_ub = rhs_ineq,
           bounds = bnd,
           method = "revised simplex")

opt
```

Out[2]:
```
     con: array([], dtype=float64)
     fun: 120000.0
 message: 'The problem appears infeasible, as the phase one auxiliary problem terminated successfully with a residual of 3.0e+02, greater than the tolerance 1e-12 required for the solution to be considered feasible. Consider increasing the tolerance to be greater than 3.0e+02. If this tolerance is unacceptably large, the problem is likely infeasible.'
     nit: 1
   slack: array([   0.,    0., -300.])
  status: 2
 success: False
       x: array([  0., 400.])
```

Practical 8

Aim: Use dual simplex method to solve the following LPP:

$$\text{Max } Z = 40x_1 + 50x_2$$
w.r.t.
$$2x_1 + 3x_2 \leq 3,$$
$$8x_1 + 4x_2 \leq 5,$$
$$x_1, x_2 \geq 0$$

Source code:

```
require(lpSolve)
f.obj <- c(40, 50)
f.con <- matrix(c(2, 3,
                  8, 4), nrow = 2, byrow = T)
f.dir <- c("<=", "<=")
f.rhs <- c(3, 5)

lp("max", f.obj, f.con, f.dir, f.rhs)$solution
lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$sens.coef.from
lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$sens.coef.to
lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$duals
lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$duals.to
```

Output:

```
> # Maximize the following:
> # z = 40x1 + 50x2
> # w.r.t
> # 2x1 + 3x2 ≤ 3
> # 8x1 + 4x2 ≤ 5
> # x1, x2 ≥ 0
>
> require(lpSolve)
Loading required package: lpSolve
> f.obj <- c(40, 50)
> f.con <- matrix(c(2, 3,
+                   8, 4), nrow = 2, byrow = T)
> f.dir <- c("≤", "≤")
> f.rhs <- c(3, 5)
>
> lp("max", f.obj, f.con, f.dir, f.rhs)$solution
[1] 0.1875 0.8750
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$sens.coef.from
[1] 33.33333 20.00000
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$sens.coef.to
[1] 100  60
```

```
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$duals
[1] 15.00  1.25  0.00  0.00
> lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens = T)$duals.to
[1] 3.75e+00 1.20e+01 1.00e+30 1.00e+30
>
```

Practical 9

Aim: Solve following transportation problem in which each cell represents unit costs:

	Customers				**Supply**
	1	**2**	**3**	**4**	
Suppliers	10	2	20	11	15
	12	7	9	20	25
	4	14	16	18	10
Demand	5	15	15	15	

Source code:

```
library(lpSolve)
cost <- matrix(c(10, 2, 20, 11,
                 12, 7, 9, 20,
                 4, 14, 16, 18), nrow = 3, byrow = T)

colnames(cost) <- c("Customer 1", "Customer 2", "Customer 3", "Customer
4")
rownames(cost) <- c("Supplier 1", "Supplier 2", "Supplier 3")

row.signs <- rep("<=", 3)
row.rhs <- c(15, 25, 10)

col.signs <- rep(">=", 4)
col.rhs <- c(5, 15, 15, 15)

total.cost <- lp.transport(cost, "min", row.signs, row.rhs, col.signs,
col.rhs)
total.cost$solution
print(total.cost)
```

Output:

```
> ##SOLVE FOLLOWING TRANSPORTATION PROBLEM IN WHICH CELL ENTRIES REPRESENT UNIT COSTS USING R PROGRAMMIN
G.
>
> #             "Customer 1", "Customer 2", "Customer 3", "Customer 4"  SUPPLY
> #SUPPLIER 1     10          2                20            11          15
> #SUPPLIER 2     12          7                9             20          25
> #SUPPLIER 3     4           14               16            18          10
> #DEMAND         5           15               15            15
>
> library(lpSolve)
> cost <- matrix(c(10, 2, 20, 11,
+                  12, 7, 9, 20,
+                  4, 14, 16, 18), nrow = 3, byrow = T)
>
> colnames(cost) <- c("Customer 1", "Customer 2", "Customer 3", "Customer 4")
> rownames(cost) <- c("Supplier 1", "Supplier 2", "Supplier 3")
>
> row.signs <- rep("≤", 3)
> row.rhs <- c(15, 25, 10)
>
> col.signs <- rep("≥", 4)
> col.rhs <- c(5, 15, 15, 15)
```

```
> total.cost <- lp.transport(cost, "min", row.signs, row.rhs, col.signs, col.rhs)
> total.cost$solution
     [,1] [,2] [,3] [,4]
[1,]    0    5    0   10
[2,]    0   10   15    0
[3,]    5    0    0    5
> print(total.cost)
Success: the objective function is 435
```

Practical 10

Aim: Solve following assignment problem represented in this matrix:

		Jobs		
		1	**2**	**3**
Workers	**1**	15	10	9
	2	9	15	10
	3	10	12	8

Source Code:

```
library(lpSolve)

cost <- matrix(c(15, 10, 9,
                 9, 15, 10,
                 10, 12, 8), nrow = 3, byrow = T)

cost
answer <- lp.assign(cost)
answer$solution
```

Output:

```
> #SOLVE FOLLOWING ASSIGNMENT PROBLEM REPRESENTED IN FOLLOWING MATRIX USING R PROGRAMMING
> # Assignment Problem
> #       JOB1    JOB2    JOB3
> #W1     15      10      9
> #W2     9       15      10
> #W3     10      12      8
>
> library(lpSolve)
>
> cost <- matrix(c(15, 10, 9,
+                  9, 15, 10,
+                  10, 12, 8), nrow = 3, byrow = T)
>
> cost
     [,1] [,2] [,3]
[1,]   15   10    9
[2,]    9   15   10
[3,]   10   12    8
> answer <- lp.assign(cost)
```

```
> answer <- lp.assign(cost)
> answer$solution
     [,1] [,2] [,3]
[1,]    0    1    0
[2,]    1    0    0
[3,]    0    0    1
>
```