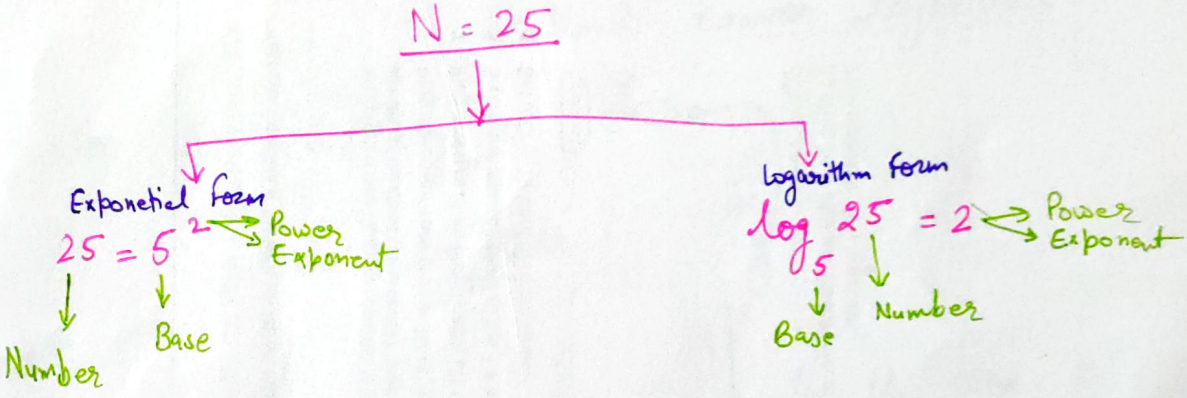
LOGARITHMS

Any positive real number N can be expressed in two ways

① Exponential representation

② Logarithm representation

For Example:



Definition: For each positive real number a, $a \neq 1$ the unique real number x is called the logarithm of N to the base a if and only if

$$a^x = N$$

i.e. $$\log_a N = x \iff a^x = N$$

where $N > 0$ & $a > 0, a \neq 1$

Note: ① 'log' being the abbreviation of the word "logarithm"

② we define the log of only positive real no. to a positive base other than 1

i.e. Negative no. & zero have no logarithms.

Examples

Q 1) Express the following in the form of logarithms.

(i) $512 = 8^3$

Sol^n $512 = 8^3$

The logarithmic form is $\log_3 512 = 8$

(ii) $10^{-3} = 0.001$

The logarithmic form is $\log_{10} 0.001 = -3$

(iii) $3^3 = 27$

the logarithmic form is $\log_3 27 = 3$

(iv) $(\sqrt{2})^2 = 2$

Logarithmic form is $\log_{\sqrt{2}} 2 = 2$

(v) $3^4 = 81$

logarithmic form is $\log_3 81 = 4$

(vi) $10^3 = 1000$

logarithmic form is $\log_{10} 1000 = 3$

Example 2) Find the value of each of following

1) $\log_3 27$

Sol^n Let $\log_3 27 = x$ $\quad (\because \log_a N = x \Rightarrow a^x = N)$

$\Rightarrow 3^x = 27$

$\Rightarrow 3^x = (3)^3$ $\quad (\because a^m = a^n \Rightarrow m = n)$

$\Rightarrow x = 3$

$\therefore \log_3 27 = 3$

2) $\log_2 \sqrt{32}$

Sol^n Let $\log_2 \sqrt{32} = x$

$\Rightarrow 2^x = \sqrt{32}$

$\Rightarrow 2^x = (32)^{\frac{1}{2}} = (2^5)^{\frac{1}{2}} = 2^{5/2}$

$\Rightarrow x = \frac{5}{2}$

$\therefore \log_2 \sqrt{32} = \frac{5}{2}$

Important Properties (from definition)

(i) $\log_a 1 = 0$ because $a^0 = 1$

(ii) $\log_a a = 1$ $(\because a^1 = a)$

(iii) $\log_a a^x = x$ $(\because a^x = a^x)$

Common logarithms : Logarithms to the base 10 are called common logarithms.

For Ex: (i) $\log_{10} 100$

(ii) $\log_{10} 0.01$

Natural logarithms : Logarithms to the base 'e' are called natural logarithms.

Where $e = 2.7182818284.....$ is the number

For Ex: (i) $\log_e x$

Note: (i) The function $f(x) = \log_e x$ is called logarithmic function. Denoted by $\ln$ or $\log x$.

(ii) In all theoretical problems, when base is not mentioned it taken as 'e'

In all numerical problems, when base is not mentioned it taken as '10'.

Laws Of Logarithms

1. $\log_a m + \log_a n = \log_a (mn)$
2. $\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$
3. $\log_a m^n = n \log_a m$

Change of Base

$$\log_a b = \frac{\log_c b}{\log_c a}$$, for any base c, $c > 0$, $c \neq 1$

Example: 1) Evaluate $\log_2 16\sqrt{8}$

Soln:

$$\log_2 16\sqrt{8} = \log_2 16 + \log_2 \sqrt{8} \quad (\because \log_a mn = \log_a m + \log_a n)$$

$$= \log_2 (2^4) + \log_2 8^{\frac{1}{2}}$$

$$= 4 \log_2 2 + \frac{1}{2} \log_2 8 \quad (\because \log_a m^n = n \log_a m)$$

$$= 4 + \frac{1}{2} \log_2 2^3 \quad (\because \log_a a = 1)$$

$$= 4 + \frac{1}{2} \cdot 3 \log_2 2$$

$$= 4 + \frac{3}{2} = \frac{11}{2}$$