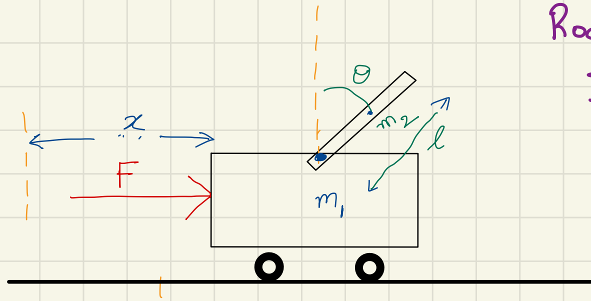
Rod with mass 'm'

$\bar{I} = I$



Lagrangian

Equations of Motion of above system:-

Translational K.E of cart — Translational K.E of Pole

$$\mathcal{L} = \underbrace{\frac{1}{2} m_1 \dot{x}^2}_{\text{Translational K.E of cart}} + \underbrace{\frac{1}{2} m_2 \left(\dot{x}^2 + \left(\frac{l}{2}\cos\theta\,\dot{\theta}\right)^2 + 2\frac{l}{2}\cos\theta\,\dot{\theta}\,\dot{x}\right) + \frac{1}{2} m_2 \left(\frac{l}{2}\sin\theta\,\dot{\theta}\right)^2}_{\text{Translational K.E of Pole}}$$

rotational K.E pole — P.E of Pole

$$+ \underbrace{\frac{1}{2} I \dot{\theta}^2}_{\text{rotational K.E pole}} - \underbrace{m_2 g \frac{l}{2}\cos\theta}_{\text{P.E of Pole}}$$

$$= \frac{1}{2}(m_1 + m_2)\dot{x}^2 + \frac{1}{2} m_2 \frac{l^2}{4}\dot{\theta}^2 + \frac{1}{2} m_2 l \cos\theta\,\dot{\theta}\,\dot{x} + \frac{1}{2} I \dot{\theta}^2 - m_2 g \frac{l}{2}\cos\theta$$

$$\frac{\partial\left(\frac{\partial L}{\partial \dot{x}}\right)}{\partial t} - \frac{\partial L}{\partial x} = F$$

$$F = (m_1 + m_2)\ddot{x} + \frac{1}{2} m l \cos\theta\,\ddot{\theta} - \frac{1}{2} m_2 l \sin\theta\,\dot{\theta}^2$$

$$\frac{\partial\left(\frac{\partial L}{\partial \dot{\theta}}\right)}{\partial t} - \frac{\partial L}{\partial \theta} = 0$$

$$d\left(\frac{1}{2} m_2 l \cos\theta\,\dot{x}\right)/dt$$

$$0 = m_2 \frac{l^2}{4}\ddot{\theta} + \frac{1}{2} m_2 l \cos\theta\,\ddot{x} - \frac{1}{2} m_2 l \sin\theta\,\dot{\theta}\,\dot{x} + I\ddot{\theta} + m_2 g \frac{l}{2}\sin\theta$$

Linearizing at $\theta = 0$, $\cos\theta \simeq 1$, $\sin\theta \approx \theta$, $\sin\theta\,\dot\theta \approx 0$

$$(m_1+m_2)\ddot{x} + \frac{1}{2}m_2 l \ddot\theta = F$$

$$\frac{1}{4}m_2 l^2 \ddot\theta + \frac{1}{2}m_2 l \ddot{x} + I\ddot\theta + \frac{1}{2}m_2 g l\theta = 0$$

$$\ddot{x} = \frac{m_2^2 l^2 g\theta + F l^2 m_2 + 4FI}{m_1 \cdot m_2 l^2 + 4m_1 I + 4m_2 I}$$

$$\ddot{th} = \frac{-2 l m_2 (F + g m_1 \theta + m_2 g\theta)}{m_1 m_2 l^2 + 4m_1 I + 4m_2 I}$$

System Equations are

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot\theta \\ \ddot\theta \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-m_2^2 l^2 g}{m_1 m_2 l^2 + 4(m_1+m_2)I} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{-2 l m_2 (m_1 g + m_2 g)}{m_1 m_2 l^2 + 4(m_1+m_2)I} & 0 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot\theta \end{bmatrix}}$$

$$+ \overset{B}{\begin{bmatrix} 0 \\ \frac{4I + m_2 l^2}{m_1 m_2 l^2 + 4(m_1+m_2)I} \\ 0 \\ \frac{-2 m_2 l}{m_1 m_2 l^2 + 4(m_1+m_2)I} \end{bmatrix}} \overset{u}{F}$$