Probability and Statistics with R

Assignment 2

Submission Nov 16-2022 (Wednesday)

Problem 2 : Simulation Study to Understand Sampling Distribution

Part A Suppose $X_1, X_2, \cdots, X_n \overset{iid}{\sim} Gamma(\alpha, \sigma)$, with pdf as

$$f(x|\alpha,\sigma) = \frac{1}{\sigma^\alpha \Gamma(\alpha)} e^{-x/\sigma} x^{\alpha-1}, \quad 0 < x < \infty,$$

The mean and variance are $E(X) = \alpha\sigma$ and $Var(X) = \alpha\sigma^2$. Note that `shape =` α and `scale =` σ.

1. Write a `function` in `R` which will compute the MLE of $\theta = \log(\alpha)$ using `optim` function in `R`. You can name it `MyMLE`

```
MyMLE=function(n,shape,scale)
{
  n<<-n
  Negloglike=function(data,theta)
  {

    l=0
    for(i in 1:n)
    {
      l=l+log(dgamma(data[i], theta[1],scale =theta[2]))

    }
    return(-l)
  }

  theta=c(0.1,0.1)


  sim=rgamma(n,shape,scale)
  data=sim
  log(optim(par=theta,Negloglike,data=sim)$par[1])
}
```

2. Choose `n=20`, and `alpha=1.5` and `sigma=2.2`

   (i) Simulate $\{X_1, X_2, \cdots, X_n\}$ from `rgamma(n=20,shape=1.5,scale=2.2)`

```
rgamma(20,1.5,scale=2.2)
```

```
##  [1] 2.4272142 4.4802651 1.1637958 3.8201077 4.8896967 1.3788514 5.0654424
##  [8] 1.3458113 1.6865686 3.0225837 0.0856416 0.7889872 1.1823270 1.5184429
## [15] 1.7508496 1.1986160 7.1802282 2.6054386 7.9402429 7.0765110
```

(ii) Apply the 'MyMLE' to estimate θ and append the value in a vector

```
x=MyMLE(20,1.5,2.2)
x
```

```
## [1] 0.3551162
```

(iii) Repeat the step (i) and (ii) 1000 times

```
for (i in 1:1000){
  x=append(x,MyMLE(20,1.5,2.2))
}
x
```

```
##    [1]  0.355116166  0.355009460  0.630335225  0.953244241  0.931753465
##    [6]  0.617724643  0.148927155  0.705341601  0.907831377  0.312994967
##   [11]  1.065158981  0.461470040  1.300374327  0.409010162  0.859718658
##   [16] -0.070237218  0.501098980  0.328262183  0.945352819  1.015789766
##   [21]  0.306488514  0.464737051  0.366231911  0.351942017  0.815940838
##   [26]  0.474497022  0.554379668  0.118856631  0.038625865  0.298501017
##   [31]  0.416653366  0.776501694  0.557031939  0.140669187  1.001496404
##   [36]  0.654162813  0.296110314  0.158040505  0.751228505  0.363391862
##   [41]  0.643739959  0.147305627  0.506725687  0.567265720  0.577250835
##   [46]  0.447841731  0.588753932 -0.067032030  0.901476372  0.722303122
##   [51]  0.576136575  0.523658024  0.254122534  0.965113958  1.110346729
##   [56]  0.015932142  0.324338573  0.296343604  0.031961577  0.190551725
##   [61]  0.283083541  0.645953875  0.499483049  0.041369838  0.399752578
##   [66] -0.086704569  0.863066714  0.626224131  0.258688011  0.501621243
##   [71]  0.116364536  0.383848448 -0.194352720  0.700133351  0.390037176
##   [76]  0.388999322  0.526241195  0.749882610  0.678132760  0.665494762
##   [81]  0.523576378 -0.069083014  0.568918515  0.309689615  0.432154778
##   [86]  1.026543195 -0.297255978  0.372857779  0.879705061  0.475770552
##   [91]  1.385723480 -0.215450419  0.829661408  0.272461291  0.330461920
##   [96]  0.843136426  0.294926378  0.310636729  0.243720934  0.273379672
##  [101]  0.211572044  0.749485425  0.404881432 -0.229027233  0.331140808
##  [106]  0.670724897  0.422147692  0.179981126  0.710986711  0.090482901
##  [111]  0.389546389  0.059689931  0.209005986  1.023003853  0.574122362
##  [116]  0.597938198  0.619541632  0.392857635  0.265095702  0.761124267
##  [121]  0.642517878  0.347539252  0.506874502  0.085636930  0.959207566
##  [126]  0.312565578  0.022536888  0.667220634  0.748787367  0.753927320
##  [131]  0.054733681  0.193590671  0.674038055  0.753716488  0.722353742
##  [136]  0.722065288  0.292942485  0.814612135  1.012279730  0.489581762
##  [141]  0.639067215  0.809807620  0.125262279  0.612599316  0.560664705
##  [146]  0.960199934  1.348783824  0.651647542  0.624405307  0.380546165
```

```
##  [151]  0.306528257  0.829246154  0.207892612  0.634915429  0.303791143
##  [156]  0.344288438  0.444866535  0.947708207  0.423220635  0.321333898
##  [161]  1.169207516  0.523093174  0.506168784  0.299784449  0.108548053
##  [166]  0.469470227  0.284533172  0.174625592  0.011415695  0.533893274
##  [171]  0.657929999  0.983764808 -0.031553271  0.172419103  0.612585950
##  [176]  0.079575504  0.391373724  0.087853115  0.449907846  0.101935515
##  [181]  0.414007532  0.319575513  0.502197395  0.159852015  0.366105824
##  [186]  0.636105148  0.370764067  0.358198037  1.383941074  0.805851350
##  [191]  0.751432485  0.143037742  0.918745612  1.159208200  0.351401700
##  [196]  0.191776412  0.090209949  0.404922835  0.434277274  0.945767636
##  [201]  0.218363303  0.414717063  0.635196637  0.085276040  0.314323403
##  [206]  0.547944494  1.010727202  0.759122890  0.527482200  0.376864004
##  [211]  0.767483892  0.529367483  0.371582023  0.100895160 -0.091516091
##  [216]  0.358522713  0.158513275  0.485812351  1.189595626 -0.021291678
##  [221]  0.291144800  1.054798015  0.678246820  0.448161600 -0.152692475
##  [226]  1.091608647  0.916698726  0.362897130  0.266273728  0.730872042
##  [231]  0.913484900  0.188318370  0.654747169  0.700066696  0.374575443
##  [236] -0.167877109  0.576663987  0.337787040  0.128387480  0.327429145
##  [241]  0.450457403  0.523219549  0.437604152  0.231003685  0.469347775
##  [246]  0.497247960  0.396622643  0.511325802  0.343886701 -0.235204097
##  [251]  0.213094134  0.675296586  0.373369236  1.019507787  0.576331086
##  [256]  0.659428894  0.311890468  0.343357393  0.509765025  0.380181379
##  [261]  1.214583617  0.360046672  0.711694843  0.323799309  0.043940081
##  [266]  0.076449276  0.866622264  0.413428786  0.368650262  0.225304900
##  [271]  0.760404350 -0.033007181  0.184609320  0.798228296  0.216172559
##  [276]  0.597395308  0.637551245  0.301030230  0.115391582  0.677180840
##  [281]  0.272576667  0.714348127  0.140340585  0.353607711  0.488041357
##  [286]  0.455454798  0.376988381  0.422038928  0.608232609  0.526488468
##  [291]  0.477645194  0.689585649  0.405366252  0.285169853  0.037043949
##  [296]  0.558408395  0.126499163 -0.019380923  0.938224807  0.108526962
##  [301]  0.581391738  0.908791648  0.994906186  0.391419178  0.115767085
##  [306]  0.018151280  1.571526426  0.270907485  0.645092156  1.347640256
##  [311]  0.716176480 -0.015991005  0.586793207  0.746035893  0.990217053
##  [316]  0.893991266  0.914250039  0.759324354  0.631514115  0.057797978
##  [321]  0.453201553  0.684801036  0.704102549  0.546307839  0.262519695
##  [326]  0.209895752  0.708483119  0.307690197  0.911283417  1.178778188
##  [331]  0.348885019  0.022895179  0.238150432  0.408100045  0.695057916
##  [336]  0.724633440  0.657134270  0.335640071  0.270888676  0.514696630
##  [341]  0.149474245  0.505702761  0.135312197  0.775479664  0.708717232
##  [346] -0.045548398  0.266451350  0.286849945  0.100563058  0.545180182
##  [351]  0.263136271  0.574668062  0.283623346  0.327706768  1.114852658
##  [356]  0.384078249  0.676400988  0.511463578  0.313377321  0.066392596
##  [361]  0.728998212  0.680440564  0.658222498  0.283305396  0.600432448
##  [366]  0.645119574  0.076178419  0.848047072  1.000478250  0.580209339
##  [371]  0.083427552  0.653881546  0.329421851  0.231372187  0.050752836
##  [376]  0.534055857  1.044448902  0.925606562  0.466053398  0.266067504
##  [381]  0.571836724  0.622836477  0.505257639  0.247478178  0.174515689
##  [386]  0.812365529  0.369222177  0.401749428  1.053743017  0.333609557
##  [391]  0.919487805  0.193971599  0.262004978  0.438318550  0.852040856
##  [396]  0.319551287  1.453392891  0.692142739  0.065553722 -0.120106057
##  [401]  0.244988954  0.194277940  0.596606859  0.759057464  0.318527397
##  [406]  0.622118626  0.789577852  0.225535879  0.453334725  0.862699780
##  [411]  0.377369401  0.456575783  0.471160861  0.504799441  0.291795047
##  [416]  0.646808633  0.297624678  0.318562950  1.217492464  0.951572040
```

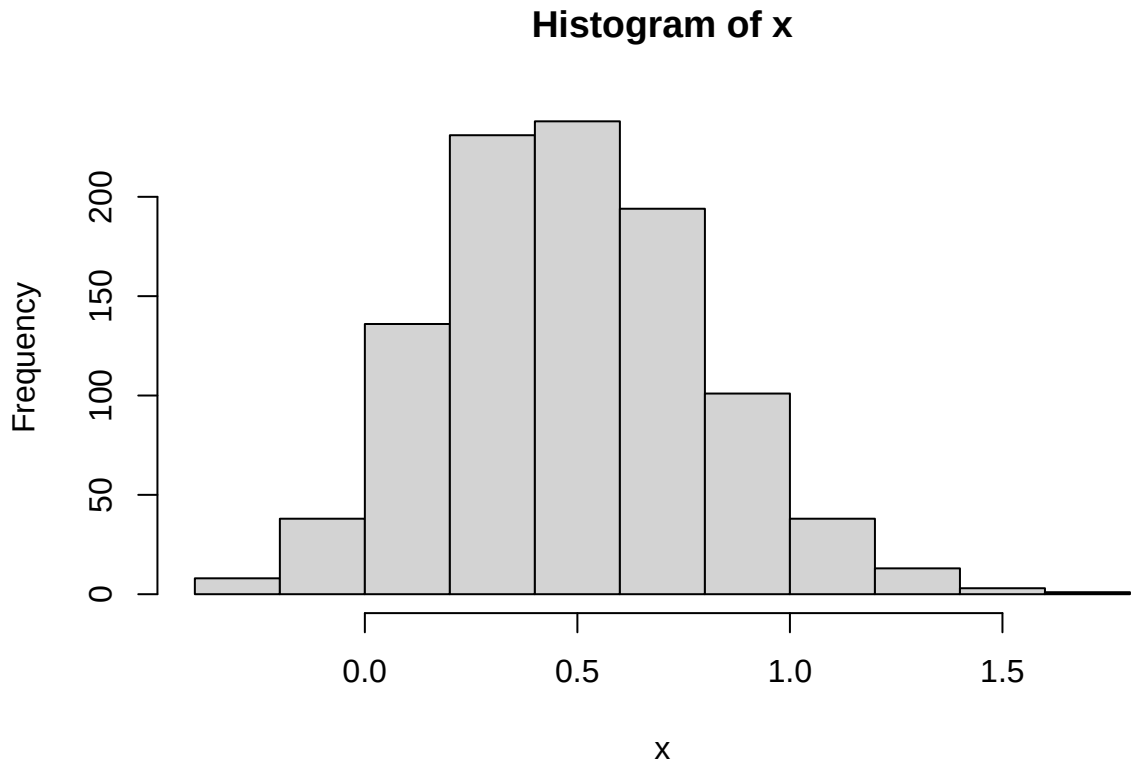
```
##  [421]  0.516494292  0.566534265  0.367704681  1.433733527  0.194722017
##  [426]  0.010365301  0.607917743  0.265530758  0.563846361  0.959015318
##  [431]  0.357165612  0.531842953  0.894130307  0.187103581  0.238909132
##  [436]  0.524077586  0.575720147  1.141597400  0.463178355  0.723129565
##  [441]  0.371915350  0.823503409  0.418172770  0.650388822  0.271455697
##  [446]  0.884398879  0.236714453  0.659077054  0.907100196  0.555614205
##  [451]  0.151053734  0.591133566  0.660169639  0.306645798  0.281871218
##  [456]  0.666442106  0.223871577  0.572324887  0.211912354  0.559829032
##  [461]  0.562372056  0.476578149  1.116316977  0.124837175  0.098630665
##  [466]  0.503973717  0.178743121  0.545000898  0.123463983  0.272942395
##  [471]  0.536209181  0.620115462  0.162105223  0.502820989  0.547178354
##  [476]  0.420415613  0.198497674  0.278377731  1.760709687  0.903361174
##  [481]  1.223072178  0.602370500  0.712886975  0.292039172  0.373941355
##  [486]  0.330607327  0.511182029  0.527705375  0.709192969  0.035582982
##  [491]  0.146717406  0.167680509  0.243753655  0.380461559  0.770378910
##  [496]  0.132310117  0.885951369  1.005443406  0.707823725  0.526496318
##  [501]  0.657291247  0.551918236  0.659056156  0.916503102  0.383689389
##  [506]  0.398648291  0.165030647  0.518765984  0.482991852  0.984137615
##  [511]  0.536055792  1.031683666  0.817064792  0.479702508  1.049274417
##  [516]  0.439603241  0.187173949  0.312883683  0.395636694  0.448361522
##  [521]  0.288388643  1.048487156  0.665490410  0.751135674  0.698120433
##  [526]  0.536089674  0.424034965  0.531485384  0.331061961  0.335941621
##  [531]  0.759733191  0.723908396  0.678979193 -0.019940540  0.403642188
##  [536]  0.916370283  0.524274514  0.153744643  0.147335045  0.290366888
##  [541]  0.910504728  0.316234998  0.661414987  0.107578493  0.789621464
##  [546]  0.570930250  0.813690259  0.318834339  0.349005894  0.525717438
##  [551]  0.349569950  0.276152070  0.688633076  0.406647704  0.165906379
##  [556]  0.829710950  0.593492389  0.279073269  0.532786631  0.765299081
##  [561]  0.579735745 -0.042673921  0.781625372  0.951864518  0.142444745
##  [566]  0.327533844  0.595380875  0.214876669  0.385697413  0.355793071
##  [571]  0.868403650  0.370120374  0.379354814  0.340178369  0.265020575
##  [576]  0.881662517  0.641216980  0.563790039  0.581646710  0.921970566
##  [581]  0.504171820  0.452811806  0.352820350  0.408758438 -0.025181728
##  [586]  0.038317465  0.435900860  0.429913130  0.516245080  0.812468755
##  [591]  0.212409879  0.514348137  0.463539742  0.842745062  0.202709591
##  [596]  0.382401713  0.044710485  0.569217480  0.106236794  0.582301190
##  [601]  0.598307968  0.265004106  0.188480584  0.021560999  1.025568448
##  [606]  0.333558023  0.744581641  0.705103006  0.047173441  0.654530173
##  [611]  0.881132020  0.391761314 -0.031712647  0.602768593  0.474640870
##  [616]  0.652269261  0.465071025  0.158464000  0.378463521  0.588307806
##  [621]  0.074317156  0.201244997  0.389140837  0.235172874  0.056639911
##  [626]  0.448135995 -0.280900209  0.763598469  0.634883837  0.402750490
##  [631]  0.244848002 -0.321079615  0.650549588  0.546845672  0.443223383
##  [636]  0.672547276  0.184453480  1.087095099  0.789583902  0.803814194
##  [641]  0.806817596  0.230300360  0.481635894  0.319194926  0.640821398
##  [646]  0.907946144 -0.050051714  0.224557005  0.799394606  0.430834413
##  [651] -0.157803296  0.278685705  0.768126986  0.178082935  0.200123908
##  [656]  0.223082784  0.678529563  0.703276104  0.537347586  0.196494432
##  [661]  0.209564894  0.242083589  0.766388402  0.514877621  0.520652686
##  [666]  0.561203627  0.404590962  0.732889842  0.761376424  0.460752137
##  [671]  0.720525418  0.949277988  1.208569471  0.215817754  0.406344097
##  [676]  0.440196083  0.689627844  0.848640200  0.709615743  0.699579763
##  [681]  0.602903132  0.640915147  0.611107063  0.792216410  0.143819462
##  [686]  0.699757651  0.509783538  0.623633367  0.428900751  0.071550990
```

```
##  [691]  0.523455321  0.846190573  1.137864631  0.470616713  0.274145584
##  [696]  0.543526741  0.475533569  0.744170356  0.702688304  0.090974728
##  [701]  0.222843970  0.368988082  0.414830628  0.420949605  0.382772695
##  [706]  0.696472436  0.720335208  0.315953962  0.495028524  0.325005524
##  [711]  0.598819597  0.461942590  0.729951154  0.194175902  0.566528948
##  [716]  0.143914993  0.824981603  0.321719677  1.170674105  0.183391389
##  [721]  0.865472155  0.587237688  0.673262054 -0.051640482  0.386813862
##  [726]  0.544118574  0.438767805  0.864705380  0.128865163  0.172928788
##  [731]  0.498475900  0.955864993  0.196968759  0.900578336  0.251376134
##  [736]  0.720820122  0.795243618  0.138156423  0.577524480  0.303613955
##  [741]  0.786770048  0.655005595  0.418710754  0.876734666 -0.068074871
##  [746]  0.317543345  0.186973111 -0.185741357  0.065262155  0.897415309
##  [751]  0.353014384  0.979781728  0.102444380  0.849111942  0.338873329
##  [756]  0.368883053  0.422157158  0.597484918  0.224443155  1.027274930
##  [761]  0.108834180  0.562133890  0.119966418  0.668970001  0.431119626
##  [766] -0.084139372  0.070123674  0.514767782 -0.061501832 -0.094272775
##  [771]  0.279122985  0.347231364  0.837679405  0.606684563  0.318547689
##  [776]  0.404285791  0.912470663  0.477804809  0.353889309  0.490701672
##  [781]  0.370380208  0.388891120  0.593535409  0.804623426  0.254644548
##  [786]  0.106404087  0.602286631  0.744322707  0.687171097  0.689181657
##  [791]  0.801420978  0.039213515  0.650015216  0.327490262  0.319335485
##  [796]  1.238820874  0.372339037  0.522232194  1.145543318  0.343325868
##  [801]  0.338590855  0.222201537  0.184546890 -0.028759915  0.595963730
##  [806]  0.746609434  0.346753206 -0.028333770 -0.250184119  0.367966106
##  [811]  0.615217892  0.807385439  0.003250944  0.848104878  0.408734752
##  [816]  0.652906097  0.088343219 -0.011278281  0.439240933  0.449006178
##  [821]  0.849514358  0.297529583  0.749585999  0.402574434 -0.068149410
##  [826]  0.634596745  0.833567962  0.649084301  0.191581292  0.782875628
##  [831]  0.653285305  0.646225884  0.406366397  0.225274360  0.379142277
##  [836]  0.434752585  0.166648675  0.966774564  1.090687129  0.534611751
##  [841]  0.574479519  0.762477009  0.873587079  1.032859584  0.804825146
##  [846]  0.618030839  0.523533272  0.870486333  1.255046676  0.300732853
##  [851]  0.949113932  0.727806000  0.045181553  0.381607987  0.189415814
##  [856]  0.351812932  0.436290537  0.658596068  0.543020639  0.238065753
##  [861]  0.340759856  0.833392904  0.684853721  0.448806993  0.609189413
##  [866]  0.310551381  0.712452205  0.244298736  0.148608892  0.166250987
##  [871]  0.358463335  0.501512429  0.693486464  0.528957290 -0.096200685
##  [876]  0.190195346  0.862230364  0.706515565  0.804357725  1.009721493
##  [881] -0.093353522  0.580680061  0.242113628  0.491388382 -0.017768672
##  [886]  0.056162014  0.497123615  0.518043740  0.758533771  0.125627574
##  [891]  0.554039852  0.313341702  0.634969915  0.640000130  0.155329081
##  [896]  0.585527489  0.083091269  0.784479157  0.051427874  0.417254738
##  [901]  0.543347531  0.688214354  0.993891267  1.188757126  0.763757570
##  [906]  0.680731335  0.686511841  0.692810685  0.279362715 -0.034456296
##  [911]  0.505241555  0.276955312 -0.069690946  0.450884978  0.306734375
##  [916]  0.638928734  0.574092973  0.308557446  0.923722644 -0.006242167
##  [921]  0.201563607  0.157389027  0.808677546 -0.209995214  0.383755238
##  [926]  0.564704649  0.429967904  0.321847785  0.007817618  0.698650254
##  [931]  0.501388805  0.176522022  1.374480071  0.512841710  0.408682847
##  [936]  0.304064613  0.087004492  0.622506893  0.679300777  0.622066546
##  [941]  0.951392707  0.467465676  0.788784797  0.323390161  0.578838387
##  [946]  0.836226784  0.578095938  0.594901109  0.384198005  0.739394331
##  [951]  0.523651798  0.814982778  0.796450275  0.174820835  0.393008278
##  [956]  0.543269055  0.146752369  0.154952463  0.554827512  0.786461090
```

```
##  [961]  0.486106413  0.312247494  0.599213756 -0.065667314  0.537048025
##  [966]  0.536424481  0.244851082  0.142876552  0.300500331  0.917289884
##  [971]  0.495875629  0.357565711  0.848129019  0.195188748  0.410533372
##  [976]  0.446422864  0.929002465  0.531199471  0.707900189  0.592806343
##  [981]  0.076024336  0.804005347  0.300305871  0.693740803  0.408611695
##  [986]  0.661027167  1.035462091  0.118082730  1.003602477  0.467905110
##  [991]  1.312333163  0.625949558  0.569109804  0.092568746  0.546399826
##  [996]  0.767541165  0.699566237  0.635723597  1.019720037  0.630605306
## [1001]  0.595906841
```

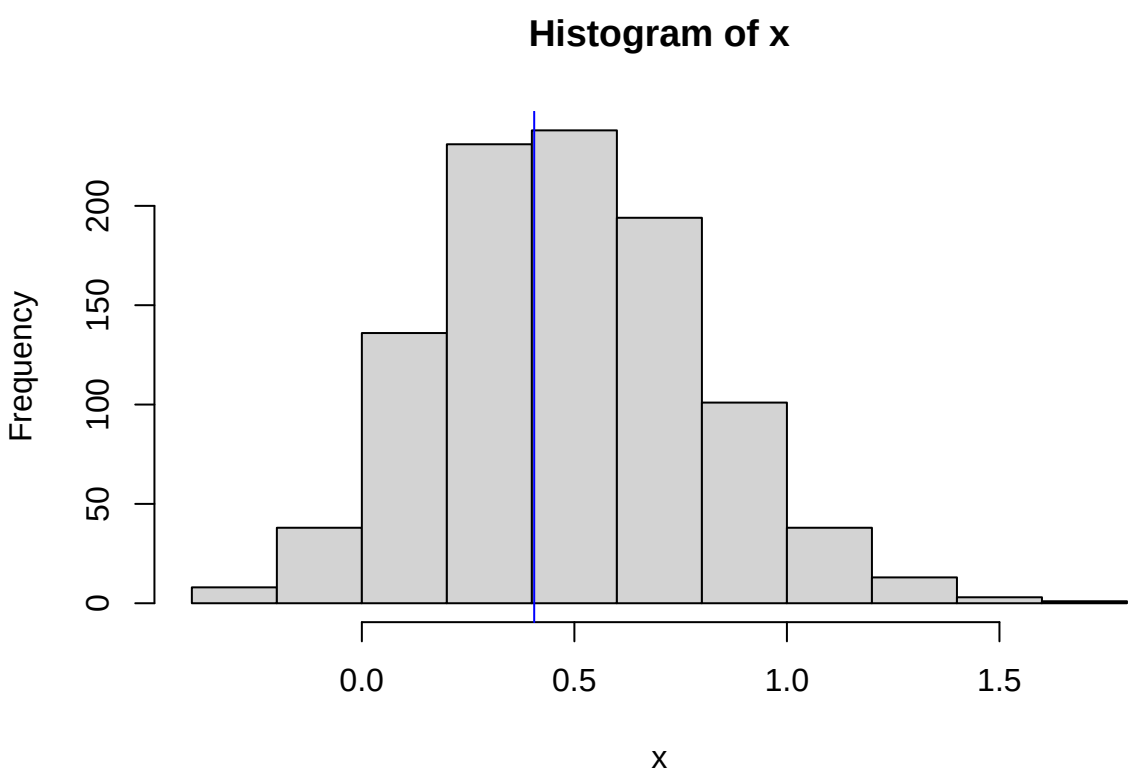
(iv) Draw histogram of the estimated MLEs of θ.

```
hist(x)
```



(v) Draw a vertical line using `abline` function at the true value of θ.

```
hist(x)
abline(v=log(1.5),col="blue")
```



(vi) Use 'quantile' function on estimated θ's to find the 2.5 and 97.5-percentile points.

```
y=quantile(x, probs = c(.025, .975))
y[2]-y[1]
```

```
##    97.5%
## 1.204897
```

3. Choose `n=40`, and `alpha=1.5` and repeat the (2). ##

   (i) Simulate $\{X_1, X_2, \cdots, X_n\}$ from `rgamma(n=20,shape=1.5,scale=2.2)`

```
rgamma(40,1.5,scale=2.2)
```

```
##  [1]  3.58252033  3.37217804  2.31656468  3.52765527  1.24986313  0.81002987
##  [7]  3.70375751  2.92911944  3.64379322  0.65795532  5.99017245  5.47409897
## [13]  0.87384483  3.01174152  1.32551800  6.61429728  6.50869026  2.51001704
## [19]  1.54318932  4.87081794  0.04660409  6.24564701  4.44547803  0.25034399
## [25] 10.14168066  5.21927213  1.21133727  2.18212436  1.28668463  6.13005227
## [31]  2.09820778  0.99523975  3.41165400  3.24172877  7.61927708  0.23732507
## [37]  5.45809553  1.02869012  3.60174121  7.39627494
```

(ii) Apply the 'MyMLE' to estimate θ and append the value in a vector

```
x=MyMLE(40,1.5,2.2)
x
```

```
## [1] 0.6092761
```

(iii) Repeat the step (i) and (ii) 1000 times

```
for (i in 1:1000){

  x=append(x,MyMLE(40,1.5,2.2))
}
x
```

```
##    [1]  0.609276138  0.400487229  0.433807729  0.317610826  0.467225904
##    [6]  0.823714947  0.568526480  0.533803267  0.371560676  0.246332006
##   [11]  0.449832316  0.689494882  0.215976794  0.374986432  0.768827988
##   [16]  0.643783227  0.384132156  0.558535614  0.757494561  0.699005995
##   [21]  0.641886640  0.304511619  0.527672259  0.481054218  0.345828419
##   [26]  0.153000650  0.091169806  0.253114692  0.644522997  0.219806957
##   [31]  0.687317418  0.241642009  0.511526175  0.448237427  0.346463393
##   [36]  0.173487005  0.337762714  0.405135527  0.662929720  0.424280995
##   [41]  0.596889443  0.325892117  0.842843598  0.561128727  0.273900356
##   [46]  0.614481937  0.689247059  0.635992273  0.304869362  0.657072502
##   [51]  0.499231788  0.190416147  0.452269114  0.455013810  0.524631883
##   [56]  0.268754795  0.763584284  0.246137490  0.659880780  0.527252937
##   [61]  0.423656588  0.784864157  0.578189335  0.610947219  0.337535965
##   [66]  0.750291444  0.469414601  0.123447678  0.530791560  0.168686275
##   [71]  0.676664610  0.245660238  0.317187378  0.534803031  0.564589626
##   [76]  0.363922732  0.283459309  0.681205224  0.381467953  0.510275875
##   [81]  0.531182476  0.172826571  0.678645921  0.381754962  0.941922957
##   [86]  0.641142888  0.477420366  0.487672822  0.644474557  0.288623077
##   [91]  0.558716764  0.897346423  0.536123797  0.554501382  0.315664196
##   [96]  0.538044795  0.823451113  0.159403149  0.446295224  0.299274656
##  [101]  0.253389514  0.567810401  0.263951921  0.567110499  0.192423118
##  [106]  0.662679188  0.496044055  0.572366106  0.671022632  0.439107082
##  [111]  0.285824458  0.664708372  0.438131407  0.676683322  0.775532098
##  [116]  0.573216253  0.384784824  0.261960360  0.521318208  0.578665495
##  [121]  0.682890353  0.040748350  0.104676898  0.549790304  0.415843168
##  [126]  0.170830278  0.138706712  0.235231839  0.677286288  0.449509062
##  [131]  0.312468179  0.325877716  0.408317852  0.293951410  0.498806537
##  [136]  0.544862256  0.277605571  0.673093576  0.702591008  0.360094817
##  [141]  0.614829306  0.525365171  0.718836951  0.727639439  0.312120756
##  [146]  0.861609411  0.692360556  0.370440444  0.222990950  0.717538361
##  [151]  0.633458785  0.002285646  0.343057662  0.416597029  0.525847827
##  [156]  0.252565283  0.535711634  0.539853984  0.439867581  0.965612810
##  [161]  0.206730657  0.636716213  0.572675932  0.378143170  0.627001821
##  [166]  0.623214350  1.108286489  0.359181169  0.326662883  0.236730254
##  [171]  0.412580338  0.151038392  0.226552290  0.634651632  0.893949032
##  [176]  0.494069500  0.752722525  0.644029779  0.916865648  0.622234360
##  [181]  0.636076188  0.131420577  0.229059160  0.570235028  0.206415160
##  [186]  0.827976182  0.567574331  0.267691488  0.365533249  0.189045457
##  [191]  0.612981640  0.364799206  0.580081037  0.210323540  0.381412302
```

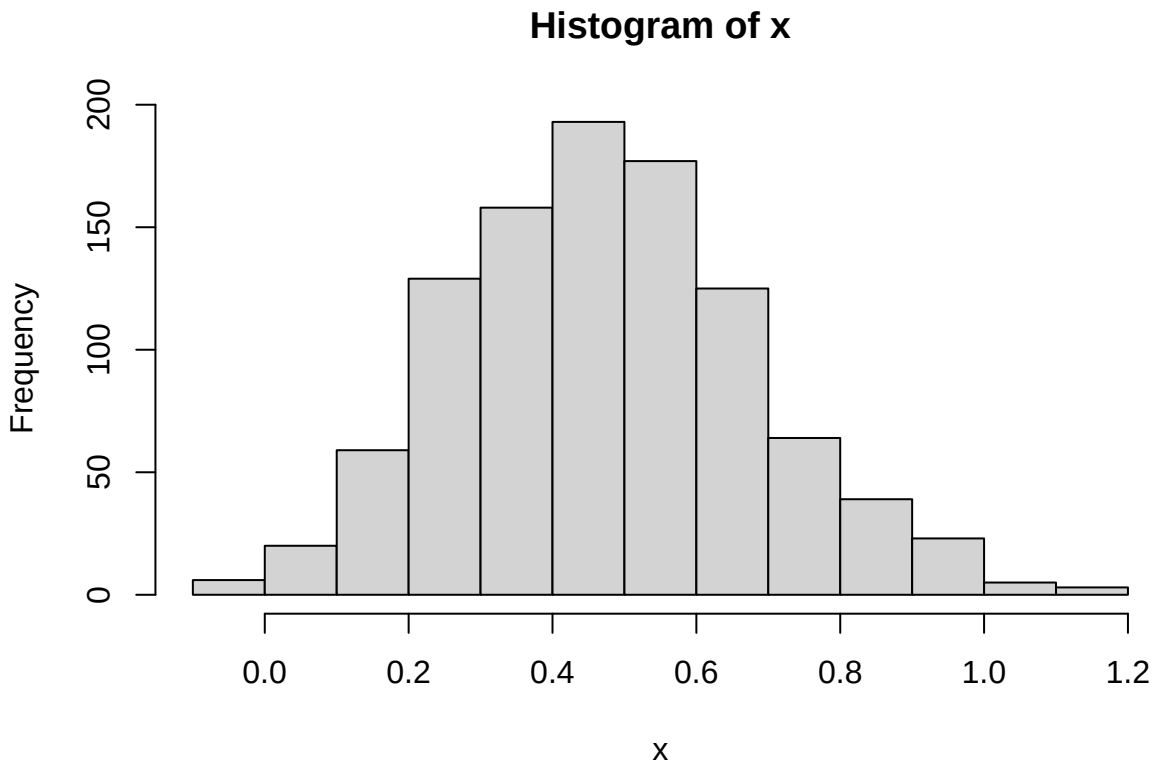
```
##  [196]  0.277994754  0.450770895  0.351256646  0.297424948  0.570972235
##  [201]  0.906257911  0.342453397  0.398291363  0.363195099  0.741977633
##  [206]  0.361417283  0.419754404  0.481288070  0.189158074  0.490920898
##  [211]  0.330775594  0.381364042  0.598263890  0.530773059  0.332745021
##  [216]  0.383371845  0.537865704  0.334709544  0.358432602  0.542419994
##  [221]  0.392259692  0.643942925  0.530288978  0.387664666  0.275827972
##  [226]  0.246153516  0.403933169  0.434831406  0.644553604  0.497752603
##  [231]  1.050045410  0.660663745  0.530918866  0.173539629  0.307572230
##  [236]  0.406196189  0.429490756  0.157699560  0.353377358  0.559109086
##  [241]  0.239231413  0.516988027  0.549431952  0.622977963  0.301318529
##  [246]  0.981268934  0.532177112  0.578386238  0.578195164  0.498445679
##  [251]  0.638349333  0.648458336  0.276906689  0.350178857  0.205889940
##  [256]  0.309361690  0.305169107  0.544967444  0.573203500  0.277046433
##  [261]  0.383978464  0.622441035  0.872020335  0.423376474  0.125057989
##  [266]  0.376856342  0.350044011 -0.012020385  0.353261285  0.429302107
##  [271]  0.410677962  0.465062269 -0.005125017  0.414596908  0.716135563
##  [276]  0.604529263  0.280989736  0.270473244  0.748307016  0.739434184
##  [281]  0.442036237  0.264823232  0.212469296  0.362815362  0.620679780
##  [286]  0.276926568  0.516750660  0.162373307  0.636109545  0.467084054
##  [291]  0.392599645  0.437500406  0.437237987  0.468657372  0.477331864
##  [296]  0.484152437  0.372573990  0.101895576  0.005156123  0.317969554
##  [301] -0.012837276  0.623473552  0.505794327  0.503472265  0.384366276
##  [306]  0.386016179  0.638667493  0.603366151  0.363014038  0.391505381
##  [311]  0.353099451  0.548073939  0.443063534  0.564967138  0.428069771
##  [316]  0.755669512  0.819184574  0.375456042  0.200975479  0.189246491
##  [321]  0.555833668  0.049873574  0.166851899  0.246052981  0.769777595
##  [326]  0.641903777  0.259452317  0.285031164  0.759936586  0.256684822
##  [331]  0.476118864  0.217511002  0.567055490  0.076939447  0.327348666
##  [336]  0.454744578  0.658553756  0.271608120  0.775736031  0.897569523
##  [341]  0.841853634  0.933452512  0.239113439  0.373130514  0.509682144
##  [346]  0.909369172  0.309787083  0.659645890  0.272068842  0.087882135
##  [351]  0.998935217  0.446309658  0.207769908  0.485030690  0.380164351
##  [356]  0.542204540  0.539210371  0.844281399  0.712115861  0.562521176
##  [361]  0.418593614  0.473468849  0.578529815  0.571751880  0.230867685
##  [366]  0.618263517  0.878479857  0.667494990  0.435870281  0.876314074
##  [371]  0.311983459  0.487824375  0.545556078  0.400290141  0.580514265
##  [376]  0.546021275  0.289009170  0.355285397  0.729441458  0.333959044
##  [381]  0.671337722  0.591776285  0.391874570  0.421286040  0.507314078
##  [386]  0.764212451  0.401669345  0.299917898  0.404623236  0.237104435
##  [391]  0.175467081  0.227225716  0.647435697  0.230471691  1.013893856
##  [396]  0.472285222  0.384717793  0.509085826  0.456956296  0.502599187
##  [401]  0.432425514  0.336593345  0.774976380  0.666057883  0.214704212
##  [406]  0.456313955  0.081344679  0.528447078  0.451230373  0.580545605
##  [411]  0.622885352  0.184101385  0.525982296  0.419216113  0.308486179
##  [416]  0.122897263  0.404500317  0.536586779  0.569643542  0.825179919
##  [421]  0.253968008  0.475419276  0.755660555  0.863414318  0.594583702
##  [426]  0.260146504  0.450595339  0.120007431  0.441896585  0.726563146
##  [431]  0.482915276  0.424357808  0.443748277  0.463432996  0.618209825
##  [436]  0.070153315  0.239375466  0.496734013  0.513547543  0.344457572
##  [441]  0.354307676  0.653270003  0.983187214  0.604655798  0.841914672
##  [446]  0.337886572  0.437372588  0.543926249  0.503136086  0.339548138
##  [451]  0.699091693  0.298270287  0.422233411  0.728945050  0.455891140
##  [456]  0.211130080  0.264355695  0.606002885  0.442431110  0.354189625
##  [461]  0.276726249  0.290726571  0.956906673  0.522328699  0.640472262
```

```
##  [466]  0.212263544  0.588118904  0.677771484  0.359423750  0.419175515
##  [471]  0.307180365  0.426597260  0.437324707  0.535983724  0.498236163
##  [476]  0.296775917  0.165288572  0.611021141  0.249979502  0.679791105
##  [481]  0.588451882  0.564722917  0.389430545  0.479365198  0.823422318
##  [486]  0.501749699  1.070523943  0.316497404  0.318688483  0.788983337
##  [491]  0.276259193  0.697392801  0.501044119  0.377936729  0.215885038
##  [496]  0.732700959  0.340052521  0.646394080  0.313671173  0.414556024
##  [501]  0.493620985  0.340969157  0.245242224  0.341639952  0.493599978
##  [506]  0.670696001  0.302970020  0.710530711  0.194457905  0.252783016
##  [511]  0.303414667  0.884086833  0.681896180  0.295957449  0.245818287
##  [516]  0.383387083  0.149958572  0.136846980  0.437073710  0.968042300
##  [521]  0.977062299  0.447819473  0.498678733  0.475735227  0.396428920
##  [526]  0.553656055  0.388768328  0.290172563  0.437058622  0.275904650
##  [531]  0.739976298  0.522439462  0.737931329  0.421416727  0.062326701
##  [536]  0.906460094  0.537026595  0.400120005  0.364928422  0.403901545
##  [541]  0.973501748  0.502105819  0.655879916  0.389748446  0.444464326
##  [546]  0.495880837  0.964386871  0.531675158  0.559312221  0.514782791
##  [551]  0.410781991  0.763378234  0.354240372  0.688971648  0.831775898
##  [556]  0.278947148  0.832100611  0.630533346  0.411149512  0.805211572
##  [561]  0.778998707  0.448293594  0.814855992  0.404002216  0.560780876
##  [566]  0.770273061  0.136338096  0.443204633  0.924171828  0.580179007
##  [571]  0.624727917  0.514832297  0.528861634  0.418200057  0.410859034
##  [576]  0.709726594  0.366255037  0.569790886  0.534077505  0.487883536
##  [581]  0.191360830  0.812946590  0.223932591  0.789944881  0.251272068
##  [586]  0.385605462  0.248723308  0.443714862  0.434128494  0.636454029
##  [591]  0.278228695  0.313349876  0.312270109  0.621337605  0.622128366
##  [596]  0.446989120  0.186712917  0.599619460  0.584508999  0.385439330
##  [601]  0.711375321  0.586969305  0.445614093  0.283321817  0.457452831
##  [606]  0.254504473  0.417441369  0.265537775  0.597033342  0.420198613
##  [611]  0.301053441  0.557906477  0.478341029  0.694900697  0.746022273
##  [616]  0.698311930  0.416110790  0.357986785  0.072055517  0.678322543
##  [621]  0.504995318  0.878741446  0.537620725  0.437298469  0.403467817
##  [626]  0.314693987  0.642485007  0.005542282  0.608863769  0.650802444
##  [631]  0.404057903  0.537006397  0.392807441  0.916478107  0.843086640
##  [636]  0.642456994  0.144270045  0.437363987  0.468041677  0.346408370
##  [641]  0.770938815  0.566023217  0.617703794  0.180523935  1.170454858
##  [646]  0.587170433  0.642661532  0.717824344  0.551336240  0.446927893
##  [651]  0.149032965  0.561039362  0.271812891  0.538913323  0.458938420
##  [656]  0.305346257  0.559454456  0.401852404  0.332636601  0.160097403
##  [661]  0.081285754  0.330513878  0.575885357  0.557783558  0.358620386
##  [666]  0.476901828  0.573450837  0.432363654  0.407166668  0.417778774
##  [671]  0.318826131  0.208368447  0.579433461  0.365742508  0.267201422
##  [676]  0.414875324  0.556726253  0.627350521  0.600332407  0.222751505
##  [681]  0.168602840  0.235910452  0.444080850  0.500490745  0.763048688
##  [686]  0.595179082  1.039766887  0.491968776  0.389357978  0.275589320
##  [691]  0.301888305  0.164640934  0.026972145  0.557032005  0.524381621
##  [696]  0.542729428  0.516233523  0.320300964  0.235640768  0.643795793
##  [701]  0.284002769  0.533806241  0.257939154  0.445757237  0.414829862
##  [706]  0.342156471  0.337500986  0.462388128  0.481621181  0.368013332
##  [711]  0.436294597  0.307185709  0.608836001  0.577539821  0.744409982
##  [716]  0.178093890  0.778411877  0.587148862  0.441767697  0.246849668
##  [721]  0.430857576  0.438621527  0.442291653  0.397602628  0.554691153
##  [726]  0.297266770  0.760066656  0.871687309  0.184772811  0.675585724
##  [731]  0.214542655  0.576679393  0.140563962 -0.010629922  0.558294557
```

```
##  [736]  0.552885941  0.356012106  0.287751854  0.413080413  0.702816125
##  [741]  0.753909232  0.204983639  0.376002940  0.853049334  0.649928645
##  [746]  0.449989014  0.343064087  0.431285258  0.726284874  0.617809075
##  [751]  0.585289180  0.286446835  0.844626153  0.602394310  0.565943848
##  [756]  0.475983923  0.715988571  0.410002440  0.384751630  0.508331797
##  [761]  0.176355038  0.517528831  0.284281577  0.521728978  0.593621230
##  [766]  0.591955911  0.440068439  0.012635224  0.465163180  0.976374346
##  [771]  0.618701637  0.433785350  0.344003561  0.265003161  0.465119823
##  [776]  0.512261461  0.355822195  0.750494168  0.639361304  0.450432485
##  [781]  0.457391196  0.461841510  0.726371162  0.426348905  0.507277534
##  [786]  0.479688035  0.645441641  0.619143893  0.679956976  0.167814969
##  [791]  0.829125797  0.617609601  0.596028044  0.264375198  0.615133782
##  [796]  0.526856818  0.508986618  0.413140744  0.471873069  0.429943341
##  [801]  0.291643452  0.198343183  0.338422067  0.610392036  0.074520474
##  [806]  0.332738169  0.458406772  0.535760276  0.365099375  0.549250911
##  [811]  0.620397566  0.342372313  0.368256509  0.078994974  0.989380764
##  [816]  0.301950672  0.660194221  0.526285500  0.142051664  0.280981070
##  [821]  0.530193672  0.485636967  0.532545492  0.395665942  0.555718113
##  [826]  0.471585812  0.424595998  0.286770857  0.032289338  0.200331224
##  [831]  0.197294550 -0.043322015  0.871780427  0.834890820  0.340974028
##  [836]  0.854991950  0.749639077  0.479606836  0.149342215  0.748904917
##  [841]  0.698145289  0.549688637  0.728206341  0.526073859  0.745026867
##  [846]  0.671864226  0.268718875  0.526781047  0.827303483  0.519768999
##  [851]  0.613554205  0.193505478  0.291256040  0.601049374  0.771535199
##  [856]  0.546315712  0.540169506  0.505719796  0.034452434  0.298453367
##  [861]  0.696023385  0.463624271  0.328247652  0.718295221  0.835072599
##  [866]  0.583093655  0.490332861  0.365582481  0.194293439  0.508763321
##  [871]  0.832131623  0.253987147  0.583158800  0.350219980  0.292376203
##  [876]  0.244959038  0.393959359  0.416121870  0.147163772  0.738771192
##  [881]  0.584256851  0.433756086  0.294412377  0.809055082  0.910892160
##  [886]  0.358700439  0.429208965  0.348203268  0.538164496  0.492494417
##  [891]  0.435454618  0.459644680  0.119166037  0.553521148  0.908966926
##  [896]  0.647865573  0.231694055  0.360388536  0.555353424  0.448384803
##  [901]  0.582746477  0.446031034  0.418931526 -0.010745438  0.999761118
##  [906]  0.711954686  0.335557278  0.188577644  0.537422979  0.544255832
##  [911]  0.497893820  0.211206320  0.467580598  0.365060946  0.452136766
##  [916]  0.466042802  0.451226158  0.596966025  0.305189114  0.396654470
##  [921]  0.440807369  0.162896237  0.248944561  0.427484462  1.137512633
##  [926]  0.288198918  0.273585396  0.724637950  0.412026717  0.372394020
##  [931]  0.630778917  0.416435102  1.006413260  0.380391362  0.432427243
##  [936]  0.623546093  0.308029610  0.589774476  0.228205902  0.769935094
##  [941]  0.748074605  0.233664500  0.624563619  0.518567245  0.398635378
##  [946]  0.701157289  0.566055927  0.627189890  0.694626101  0.621328323
##  [951]  0.538351637  0.523206147  0.431458018  0.601708560  0.791738219
##  [956]  0.297499180  0.620161634  0.848864040  0.648464317  0.410680783
##  [961]  0.247866645  0.234410573  0.096513419  0.375283706  0.526727411
##  [966]  0.421876549  0.236636884  0.326639148  0.539684949  0.326087194
##  [971]  0.556550807  0.736370665  0.619217875  0.682279498  0.407434804
##  [976]  0.341080162  0.414304521  0.349710186  0.567108379  0.435865095
##  [981]  0.210380089  0.432786822  0.100321141  0.355189324  0.976783777
##  [986]  0.339241903  0.415298940  0.287182058  0.173006777  0.634663203
##  [991]  0.440323756  0.517186792  0.151198456  0.253147605  0.366206311
##  [996]  0.666236584  0.887135664  0.153944975  0.282284773  0.103185081
## [1001]  0.309205302
```

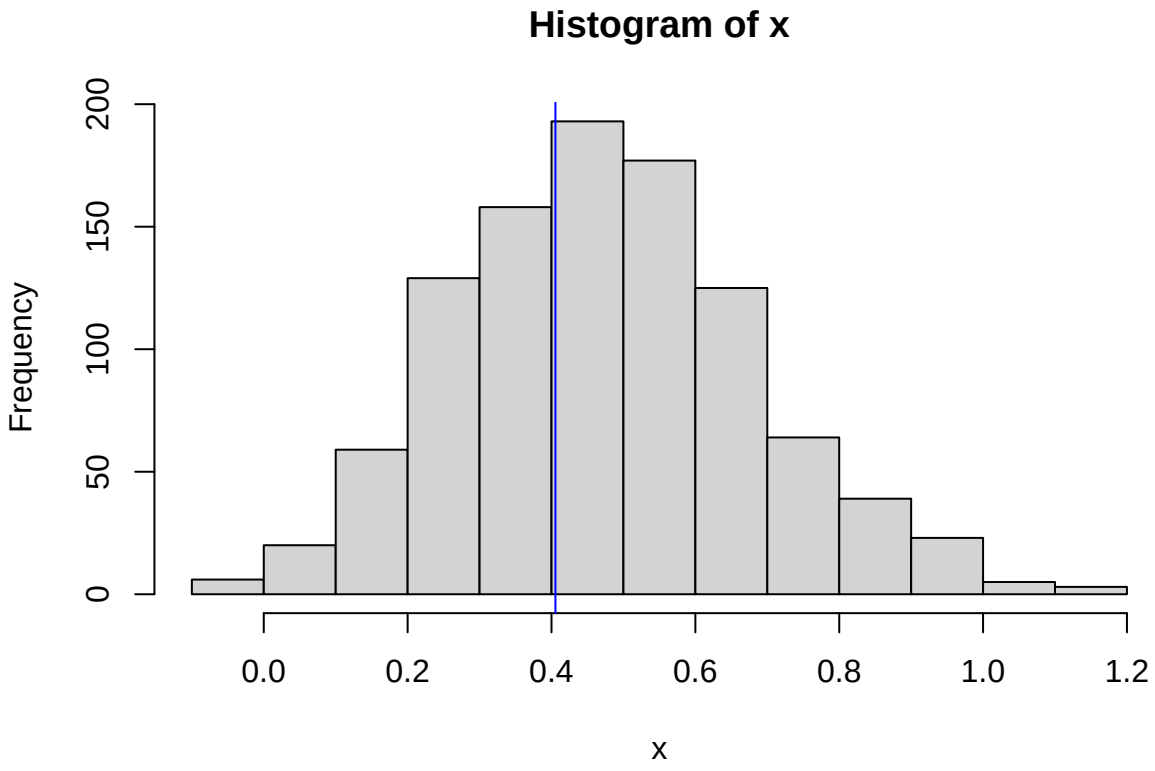
(iv) Draw histogram of the estimated MLEs of θ.

```
hist(x)
```



(v) Draw a vertical line using `abline` function at the true value of θ.

```
hist(x)
abline(v=log(1.5),col="blue")
```



(vi) Use 'quantile' function on estimated θ's to find the 2.5 and 97.5-percentile points.

```
y=quantile(x, probs = c(.025, .975))
y[2]-y[1]
```

```
##     97.5%
## 0.8199647
```

4. Choose `n=100`, and `alpha=1.5` and repeat the (2).

   (i) Simulate $\{X_1, X_2, \cdots, X_n\}$ from `rgamma(n=20,shape=1.5,scale=2.2)`

```
rgamma(100,1.5,scale=2.2)
```

```
##   [1]  3.4565311  2.9217362  2.5675841  2.4423461  3.7818831  2.2593046
##   [7]  0.9470851  7.3188463  3.8570015  0.8592851  1.1405510  2.2143958
##  [13]  2.4779075  5.0016295  2.2385964  3.5104273 11.4560129  1.1938904
##  [19]  1.7071494  4.1826279  2.2646934  1.9786545  2.6294327  4.5291233
##  [25]  3.7834283  2.3929370  3.7972137  5.2468806  2.1147860 10.7339374
##  [31]  3.4406680  2.9496224  3.9817473  1.5617403  0.9363336  1.5310814
##  [37]  2.9889658  0.8839924  7.3073343  3.9815402  4.3839715  9.5274910
##  [43]  0.8038668  0.6918969  4.9893904  0.5276709  1.9755873  2.8260465
```

```
##  [49]  2.2493122  1.7860933  1.6426932  1.2598931  0.1191012  0.1825402
##  [55]  3.5449298  2.2162032  2.0943701  0.7817875  3.1154280  0.6264143
##  [61]  4.2016147  0.1467697 10.3732635  4.9399551  1.8046835  2.2993162
##  [67]  3.2709538  2.9586236  1.4677147  2.2581693  0.6114394  0.9872268
##  [73]  7.9315279  1.1514288  2.0401495  1.6049329  2.0003077  0.6124063
##  [79]  2.7913473  0.8054410  1.1639889  3.2054308  1.2694738  4.7297298
##  [85]  3.8338789  3.2351725  1.6297678  0.2595032  1.3803276  3.2238675
##  [91]  1.5877201  2.3279498  1.6521238  0.6734307  0.4315581  1.3648676
##  [97]  5.2774393  1.5522489  7.8130378  2.9985635
```

(ii) Apply the 'MyMLE' to estimate θ and append the value in a vector

```
x=MyMLE(100,1.5,2.2)
x
```

```
## [1] 0.325064
```

(iii) Repeat the step (i) and (ii) 1000 times

```
for (i in 1:1000){

  x=append(x,MyMLE(100,1.5,2.2))
}
x
```

```
##    [1] 0.32506401 0.20072596 0.45914120 0.31513018 0.18590743 0.27725427
##    [7] 0.02767832 0.40333853 0.35815402 0.42222623 0.48447505 0.38105891
##   [13] 0.27053818 0.43292953 0.50088020 0.41128357 0.31755789 0.40929224
##   [19] 0.40366603 0.53718098 0.43538326 0.38928826 0.40015399 0.37074933
##   [25] 0.46220945 0.40188129 0.60656302 0.51377536 0.22360126 0.26817045
##   [31] 0.36039840 0.59643694 0.46129307 0.19156033 0.52467195 0.47925250
##   [37] 0.49211727 0.63453132 0.42833742 0.59190415 0.34862714 0.37627013
##   [43] 0.27634559 0.59194709 0.17365616 0.53046319 0.28833485 0.65359185
##   [49] 0.54168226 0.48127794 0.40456166 0.39329233 0.46508885 0.41852627
##   [55] 0.62553873 0.51686800 0.38097639 0.59737733 0.29555861 0.33495691
##   [61] 0.47259961 0.27808184 0.49264066 0.67533516 0.39566878 0.30169348
##   [67] 0.35911728 0.19641835 0.38537859 0.54184696 0.40467642 0.50490193
##   [73] 0.29542158 0.23570282 0.37855267 0.46665195 0.48013861 0.27855067
##   [79] 0.23163050 0.42900429 0.47396874 0.52562712 0.49421362 0.61998915
##   [85] 0.43450025 0.70195010 0.14407813 0.36225423 0.30369070 0.53609427
##   [91] 0.43074861 0.50278243 0.38054522 0.37207117 0.35029489 0.52834151
##   [97] 0.33813038 0.50148256 0.21318708 0.18562852 0.34655645 0.29670685
##  [103] 0.31405008 0.48031028 0.43860860 0.36406893 0.35959285 0.35737327
##  [109] 0.39816466 0.61446159 0.47655161 0.52123832 0.32729934 0.46631197
##  [115] 0.47264646 0.42869258 0.45002525 0.46553900 0.36630947 0.33156460
##  [121] 0.40209255 0.41801825 0.24280651 0.19351754 0.60513291 0.31501696
##  [127] 0.27536709 0.39091993 0.45098764 0.43889010 0.30184002 0.79183874
##  [133] 0.18906603 0.55967287 0.17738782 0.17211401 0.81684473 0.41816018
##  [139] 0.30879572 0.39888988 0.18439884 0.19701266 0.38106555 0.42598295
##  [145] 0.45002083 0.36360519 0.48416845 0.55271409 0.37587344 0.40621554
##  [151] 0.68058006 0.31646142 0.37612160 0.39997893 0.40201328 0.48291890
##  [157] 0.51896012 0.47240695 0.24661588 0.21495648 0.56449902 0.47996090
```

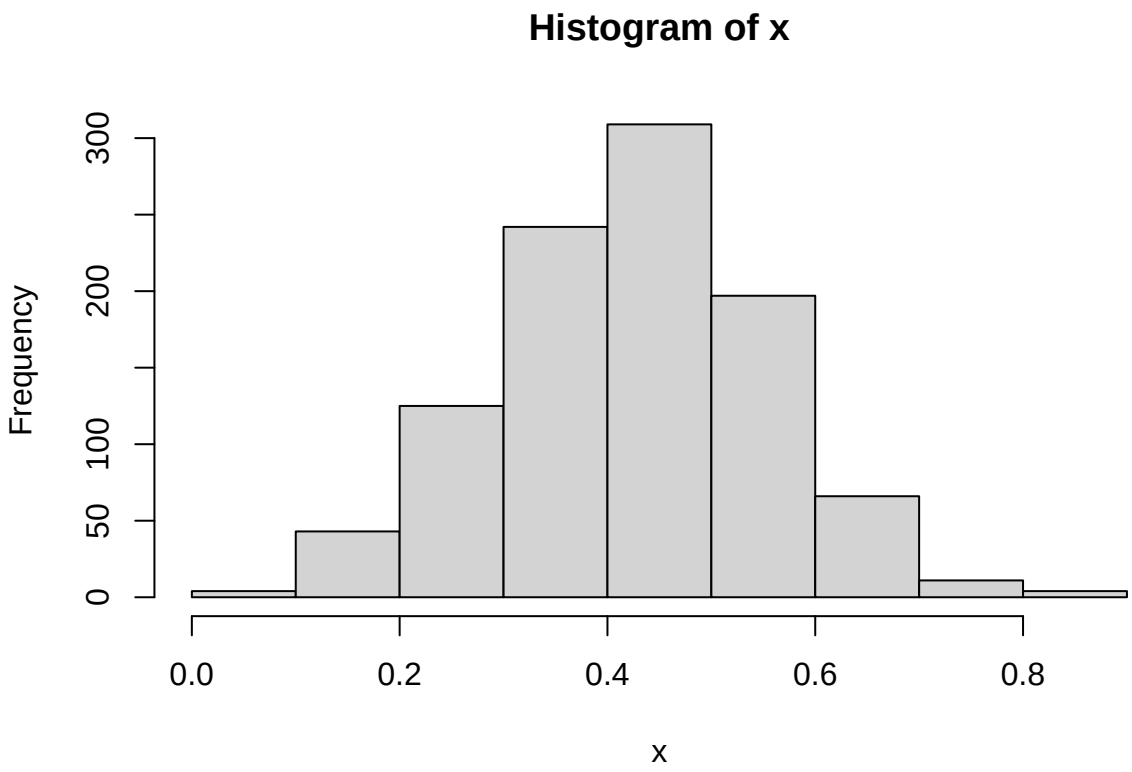
```
##  [163] 0.56847515 0.30115825 0.35571868 0.51390130 0.28041460 0.39052699
##  [169] 0.47205754 0.45961424 0.64228303 0.42796139 0.43294265 0.28774978
##  [175] 0.67799785 0.32987258 0.59092206 0.54350239 0.43734546 0.54973750
##  [181] 0.28043922 0.29453887 0.44409862 0.36075802 0.41446053 0.44854391
##  [187] 0.31832467 0.40466072 0.53762650 0.56552861 0.31743302 0.41440516
##  [193] 0.35377314 0.22529923 0.12375242 0.37580420 0.59540753 0.45401512
##  [199] 0.52264484 0.54212712 0.49104701 0.28872199 0.28793951 0.34513552
##  [205] 0.34764389 0.57607246 0.48496698 0.15669892 0.33670554 0.36029834
##  [211] 0.36630873 0.47653356 0.56298259 0.47042359 0.54636567 0.31436466
##  [217] 0.22480671 0.51189052 0.47814934 0.21617334 0.45818880 0.53276963
##  [223] 0.57906614 0.34691730 0.45648914 0.53667888 0.28245907 0.56390906
##  [229] 0.19095376 0.59009408 0.37499744 0.64847386 0.41856993 0.30221381
##  [235] 0.41297227 0.58120286 0.47007357 0.61415241 0.53383695 0.35763520
##  [241] 0.18391744 0.66556447 0.43611645 0.39117751 0.48354235 0.32394206
##  [247] 0.58654182 0.28056685 0.42669016 0.47733366 0.56708102 0.52742312
##  [253] 0.31293518 0.35433773 0.28477849 0.51717789 0.68389416 0.45745637
##  [259] 0.54902425 0.30060652 0.42759789 0.18551667 0.67136561 0.42168884
##  [265] 0.33661792 0.25582164 0.51961765 0.54210286 0.47555534 0.35702917
##  [271] 0.19459110 0.28612221 0.57931610 0.45891979 0.24994315 0.49872317
##  [277] 0.22135750 0.50972396 0.60155511 0.51083202 0.22109945 0.58136328
##  [283] 0.61898018 0.50996054 0.36526208 0.40197036 0.48486757 0.36480301
##  [289] 0.30466928 0.36536702 0.44614819 0.51896257 0.38902029 0.35055004
##  [295] 0.39433813 0.38418171 0.40611015 0.42079892 0.28229412 0.48633196
##  [301] 0.63498775 0.78350088 0.23088135 0.48815044 0.41557792 0.58619231
##  [307] 0.60987547 0.39201342 0.26875575 0.35511584 0.47519724 0.31296237
##  [313] 0.57873010 0.39374717 0.46366318 0.28761669 0.27712147 0.34346312
##  [319] 0.44509681 0.36028051 0.35751456 0.24460115 0.62280288 0.41157029
##  [325] 0.33766380 0.41457638 0.47067998 0.50171469 0.30254075 0.17254974
##  [331] 0.59189751 0.17592742 0.45363714 0.30054015 0.31352192 0.50421597
##  [337] 0.30914082 0.31543539 0.55093307 0.31311564 0.42716421 0.58636932
##  [343] 0.43190556 0.53825456 0.36222422 0.32591198 0.54587951 0.39881497
##  [349] 0.54170325 0.37899203 0.49902362 0.16484131 0.25530449 0.36892857
##  [355] 0.32245864 0.45505312 0.13014935 0.40186548 0.49544254 0.49460832
##  [361] 0.43942455 0.59401527 0.51400067 0.43202539 0.54488112 0.55477808
##  [367] 0.48053917 0.44377213 0.41543748 0.37697862 0.62376510 0.57492238
##  [373] 0.48819601 0.62217586 0.42762349 0.52901510 0.37731811 0.40020186
##  [379] 0.29974093 0.53624110 0.52684499 0.42253677 0.36563780 0.32064413
##  [385] 0.65378405 0.23839144 0.29356522 0.44365011 0.22134022 0.40907256
##  [391] 0.49799481 0.58574120 0.46610804 0.45286325 0.38455290 0.38989149
##  [397] 0.41785096 0.38620269 0.47803754 0.53123441 0.49096190 0.34822910
##  [403] 0.46160374 0.36691210 0.45037646 0.21565496 0.59508078 0.36962211
##  [409] 0.72619609 0.36840481 0.30299585 0.41279234 0.51899164 0.37270707
##  [415] 0.07460841 0.41085795 0.48822521 0.41641563 0.14763125 0.56450732
##  [421] 0.25120361 0.60040407 0.44660572 0.43005147 0.55569109 0.64341256
##  [427] 0.20545835 0.55861002 0.39169400 0.19019013 0.57367432 0.45455662
##  [433] 0.46274357 0.33192026 0.48616102 0.20252576 0.48382016 0.39507584
##  [439] 0.40454556 0.35825407 0.42448096 0.55532086 0.37262394 0.18118943
##  [445] 0.52847682 0.63481533 0.32294953 0.28830608 0.27018031 0.18029337
##  [451] 0.38905439 0.38096940 0.31048019 0.26783837 0.47503188 0.30197043
##  [457] 0.31897139 0.42410867 0.45372488 0.51199028 0.53337941 0.31650585
##  [463] 0.26047793 0.20578268 0.45595589 0.23344971 0.48785063 0.36969971
##  [469] 0.39204647 0.29950297 0.47713740 0.31756370 0.49144175 0.41728409
##  [475] 0.83169825 0.34925592 0.53722286 0.46477056 0.44500083 0.60709700
##  [481] 0.53379655 0.47627125 0.59655562 0.53761633 0.44904491 0.38669083
```

```
##  [487] 0.38964697 0.41774453 0.40280163 0.59454656 0.51520195 0.31434426
##  [493] 0.46136524 0.40888092 0.37750259 0.57010010 0.64178112 0.27302395
##  [499] 0.32741629 0.29062250 0.46821211 0.32343449 0.42141434 0.46651362
##  [505] 0.39055804 0.50282111 0.49216216 0.44665203 0.53765197 0.48761266
##  [511] 0.41641989 0.27060097 0.32631944 0.45404248 0.84040061 0.54095239
##  [517] 0.38605406 0.33170282 0.41474718 0.37103616 0.54138089 0.43711468
##  [523] 0.23425061 0.47064672 0.51129955 0.21825122 0.46049924 0.26616980
##  [529] 0.52917730 0.56154058 0.59598611 0.41570324 0.39584402 0.54285125
##  [535] 0.60071963 0.49322229 0.13225142 0.45604662 0.31434559 0.44762232
##  [541] 0.25901605 0.29613083 0.53325072 0.57339886 0.49589633 0.53257107
##  [547] 0.61917441 0.37045622 0.26696362 0.28771527 0.38257789 0.17742887
##  [553] 0.40190886 0.24719459 0.73504124 0.54323943 0.40061710 0.34939569
##  [559] 0.36609749 0.43081863 0.35158942 0.51593677 0.16539777 0.41805166
##  [565] 0.24743418 0.28702247 0.38988674 0.29898046 0.53117252 0.13207135
##  [571] 0.30789964 0.46153049 0.21313651 0.56977879 0.61282360 0.53161878
##  [577] 0.32124201 0.40876214 0.39535802 0.27446943 0.45069139 0.48596090
##  [583] 0.52522814 0.58102579 0.22712758 0.57915615 0.31466090 0.18583750
##  [589] 0.45047045 0.40942877 0.49752620 0.55359063 0.38475345 0.13199996
##  [595] 0.33519183 0.39537643 0.44899274 0.41518577 0.59797601 0.31355762
##  [601] 0.36553351 0.38690397 0.31732176 0.38315685 0.43038334 0.57813430
##  [607] 0.37395445 0.73634375 0.52304150 0.45646562 0.37774813 0.58721308
##  [613] 0.43056467 0.51231964 0.48353183 0.19877071 0.30485053 0.42586865
##  [619] 0.23829013 0.53067490 0.46141116 0.70272757 0.60905798 0.29446633
##  [625] 0.27013605 0.53619969 0.58250817 0.27560561 0.59813849 0.29694059
##  [631] 0.68753175 0.22954068 0.40050105 0.39817913 0.34465102 0.42858395
##  [637] 0.36988216 0.54225639 0.47715250 0.53938561 0.54062655 0.41546813
##  [643] 0.52750151 0.27409230 0.44350242 0.35959809 0.47337394 0.34224941
##  [649] 0.30405619 0.34004487 0.44266789 0.39673034 0.58537682 0.64590131
##  [655] 0.42211246 0.65477939 0.44476840 0.33861384 0.53141151 0.41242262
##  [661] 0.35966079 0.56968707 0.62526293 0.36205689 0.51554756 0.52562794
##  [667] 0.43891161 0.47924301 0.34745572 0.51482693 0.56207629 0.48671987
##  [673] 0.51608564 0.23390476 0.14650027 0.49242094 0.46507629 0.33880256
##  [679] 0.62404890 0.31481143 0.43006238 0.17752204 0.63477992 0.29231841
##  [685] 0.44696313 0.33756962 0.47029205 0.40090838 0.46051889 0.15771014
##  [691] 0.40423964 0.33895590 0.35574087 0.53782093 0.31128188 0.62280465
##  [697] 0.41886655 0.48751166 0.37633871 0.64719161 0.61637171 0.25488646
##  [703] 0.51689570 0.52558090 0.33467850 0.41212358 0.42074870 0.41230064
##  [709] 0.22797826 0.80712283 0.21135735 0.15574350 0.45971375 0.20723754
##  [715] 0.43475825 0.38206129 0.37926619 0.32477330 0.19397147 0.29190137
##  [721] 0.59037158 0.45835147 0.53160350 0.36953145 0.60518192 0.56639983
##  [727] 0.50473442 0.09686445 0.40289322 0.57687635 0.51154368 0.38867507
##  [733] 0.41507259 0.36027489 0.39018610 0.54683942 0.49821240 0.68992640
##  [739] 0.27760797 0.60160081 0.40706849 0.41544363 0.45328049 0.41860683
##  [745] 0.47357299 0.49496448 0.40836363 0.14206864 0.34867075 0.43588271
##  [751] 0.53790709 0.40138582 0.43524129 0.30967461 0.42241286 0.31881499
##  [757] 0.50052852 0.51756231 0.49483278 0.20476052 0.48854180 0.61922239
##  [763] 0.48733578 0.48784385 0.59062181 0.50504748 0.40925876 0.74549237
##  [769] 0.17205167 0.31687229 0.63297810 0.38796916 0.53365173 0.49050941
##  [775] 0.36837855 0.55218660 0.55952921 0.34968267 0.69373640 0.55560038
##  [781] 0.55381206 0.50546861 0.42718974 0.43890604 0.29573263 0.45263822
##  [787] 0.49171833 0.52831578 0.48881721 0.30425128 0.29199558 0.45775594
##  [793] 0.34846355 0.16047385 0.46864097 0.23133091 0.48658685 0.49032806
##  [799] 0.26369935 0.30895044 0.40710580 0.42181573 0.21873611 0.52236057
##  [805] 0.32076573 0.54184947 0.40627893 0.67944122 0.45517989 0.43849490
```

```
##  [811] 0.25296478 0.58697043 0.49230222 0.52171470 0.40992460 0.47669856
##  [817] 0.32772971 0.42936021 0.30458188 0.45009905 0.61045247 0.41698558
##  [823] 0.39579722 0.35283592 0.53965028 0.73355138 0.24906500 0.45683736
##  [829] 0.46831601 0.36185762 0.53226645 0.32620765 0.34927851 0.22893293
##  [835] 0.42406443 0.50084136 0.26839439 0.35304132 0.27194828 0.48236547
##  [841] 0.50825431 0.33253487 0.42996765 0.46897900 0.52369641 0.44768254
##  [847] 0.51602527 0.29361731 0.51354945 0.45538103 0.40796498 0.46342109
##  [853] 0.13908160 0.25873824 0.67187532 0.32404821 0.53249286 0.27201923
##  [859] 0.43744499 0.35501858 0.48641734 0.23179581 0.55656925 0.24442094
##  [865] 0.52761067 0.53496345 0.50944242 0.45582852 0.67782025 0.45719330
##  [871] 0.35504395 0.26612636 0.42210625 0.45956503 0.08968252 0.37508659
##  [877] 0.59925248 0.45862015 0.57829551 0.29888461 0.32046773 0.16237403
##  [883] 0.47338400 0.32359932 0.47196605 0.64285630 0.35762029 0.44093866
##  [889] 0.26231212 0.54541083 0.54488394 0.53366699 0.28818741 0.27458272
##  [895] 0.27076423 0.33313887 0.61315940 0.65798532 0.47123360 0.40078795
##  [901] 0.42734653 0.35926252 0.49386821 0.37294408 0.51084693 0.46972000
##  [907] 0.41425514 0.67714155 0.56837342 0.29668219 0.34689031 0.44610102
##  [913] 0.49127090 0.45824837 0.41655671 0.42419084 0.34528099 0.60917728
##  [919] 0.34379825 0.55267653 0.34652546 0.33873206 0.54389058 0.31541725
##  [925] 0.38433772 0.41048738 0.39687784 0.38700341 0.56709876 0.25330880
##  [931] 0.49970545 0.49916418 0.48839236 0.24672001 0.21907904 0.64818291
##  [937] 0.43790112 0.23083836 0.34353483 0.29034027 0.33759347 0.33973877
##  [943] 0.29128162 0.25393700 0.39362466 0.43594921 0.66968185 0.59505895
##  [949] 0.40677309 0.34193573 0.78478992 0.33867244 0.44489376 0.25429671
##  [955] 0.46757454 0.65919708 0.37968511 0.43619108 0.29310818 0.48037706
##  [961] 0.29170710 0.42335322 0.66583894 0.56299669 0.56332977 0.43030435
##  [967] 0.61142590 0.53621657 0.38503902 0.35872968 0.39008125 0.37417153
##  [973] 0.58499741 0.47620731 0.53507050 0.61442178 0.47646121 0.68545958
##  [979] 0.51206587 0.56132324 0.61774455 0.55773307 0.31518003 0.23604796
##  [985] 0.33079504 0.56518069 0.46748041 0.45716571 0.54743253 0.23357100
##  [991] 0.62572714 0.41469565 0.45905229 0.44453984 0.54589347 0.33140495
##  [997] 0.78976182 0.52602482 0.46785899 0.48711709 0.34201199
```

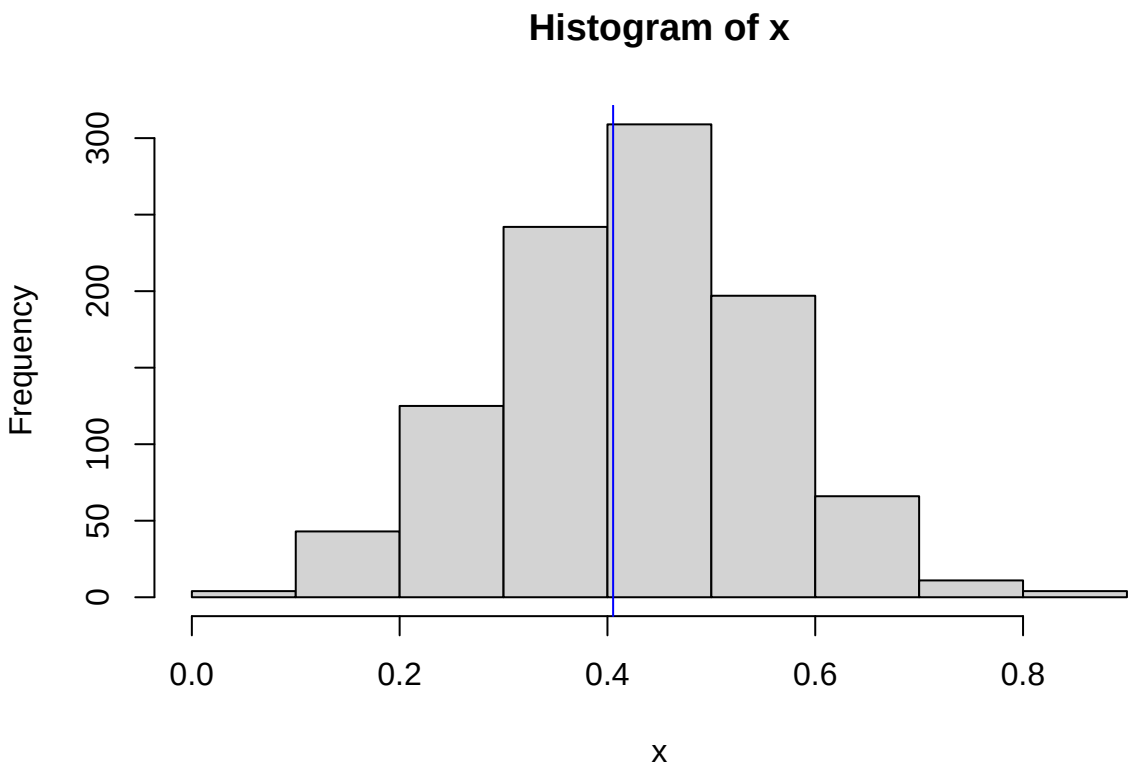
(iv) Draw histogram of the estimated MLEs of θ.

```
hist(x)
```



(v) Draw a vertical line using `abline` function at the true value of θ.

```
hist(x)
abline(v=log(1.5),col="blue")
```



(vi) Use 'quantile' function on estimated θ's to find the 2.5 and 97.5-percentile points.

```
y=quantile(x, probs = c(.025, .975))
y[2]-y[1]
```

```
##     97.5%
## 0.4994077
```

5. Check if the gap between 2.5 and 97.5-percentile points are shrinking as sample size `n` is increasing?

```
#Yes, It does.
```

Hint: Perhaps you should think of writing a single `function` where you will provide the values of `n`, `sim_size`, `alpha` and `sigma`; and it will return the desired output.

Problem 4: Modelling Insurance Claims

Consider the `Insurance` datasets in the `MASS` package. The data given in data frame `Insurance` consist of the numbers of policyholders of an insurance company who were exposed to risk, and the numbers of car insurance claims made by those policyholders in the third quarter of 1973.

This data frame contains the following columns:

`District` (factor): district of residence of policyholder (1 to 4): 4 is major cities.

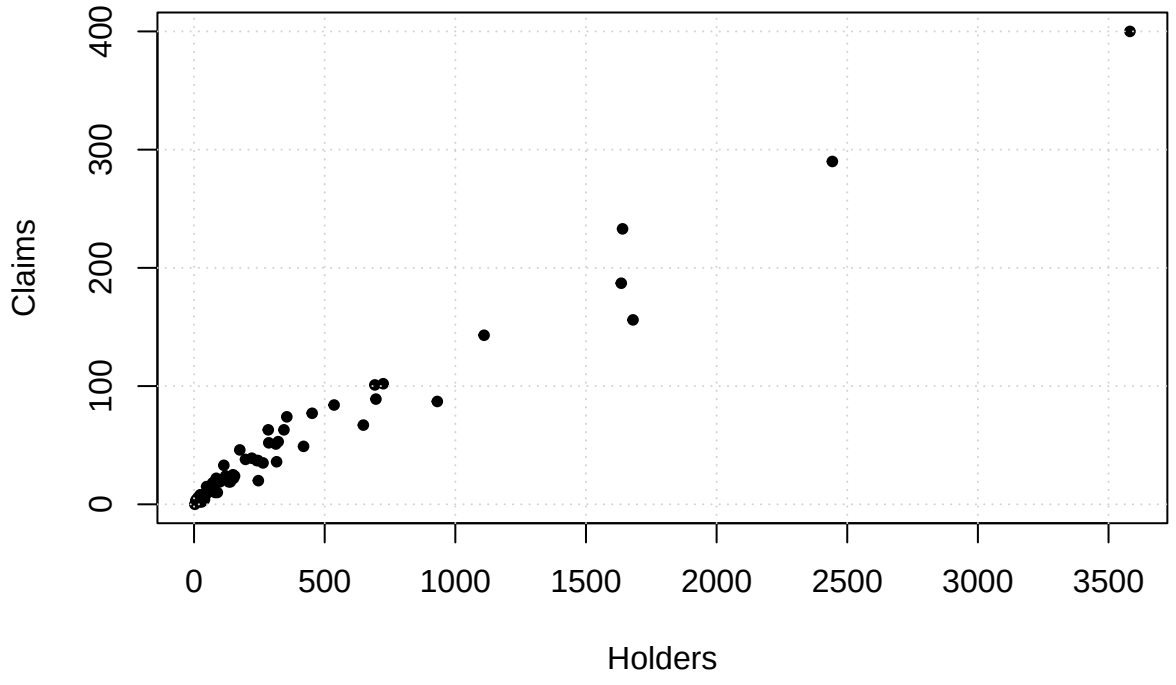
`Group` (an ordered factor): group of car with levels <1 litre, 1–1.5 litre, 1.5–2 litre, >2 litre.

`Age` (an ordered factor): the age of the insured in 4 groups labelled <25, 25–29, 30–35, >35.

`Holders` : numbers of policyholders.

`Claims` : numbers of claims

```
library(MASS)
plot(Insurance$Holders,Insurance$Claims
     ,xlab = 'Holders',ylab='Claims',pch=20)
grid()
```



Note: If you use built-in function like `lm` or any packages then no points will be awarded.

Part A: We want to predict the `Claims` as function of `Holders`. So we want to fit the following models:

$$\texttt{Claims}_i = \beta_0 + \beta_1\ \texttt{Holders}_i + \varepsilon_i, \quad i = 1, 2, \cdots, n$$

Assume : $\varepsilon_i \sim N(0, \sigma^2)$. Note that $\beta_0, \beta_1 \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$.

The above model can alse be re-expressed as,

$$\texttt{Claims}_i \sim N(\mu_i, \sigma^2), \quad where$$

$$\mu_i = \beta_0 + \beta_1 \, \texttt{Holders}_i + \varepsilon_i, \quad i = 1, 2, \cdots, n$$

(i) Clearly write down the negative-log-likelihood function in `R`. Then use `optim` function to estimate MLE of $\theta = (\beta_0, \beta_1, \sigma)$

```
library(SciViews)
library(MASS)
```

```
library(jmuOutlier)
Holders=Insurance$Holders
Claims=Insurance$Claims
data=data.frame(cbind(Claims,Holders))
data=data[-61,]
n=length(Holders)-1

y=data[,1]
x=data[,2]
```

```
Negloglike=function(data,theta)
{
  l=0
  for(i in 1:n)
  {
    l=l+log(dnorm(y[i], theta[1]+theta[2]*x[i],theta[3]))

  }
  return(-l)
}

theta=c(0.1,0.1,50)
fit=optim(theta,Negloglike,data=data)
##Estimated value of theta is:

c(fit$par[1],fit$par[2],fit$par[3])
```

```
## [1]  8.3084803  0.1125138 11.9133879
```

(ii) Calculate **Bayesian Information Criterion** (BIC) for the model.

```
BIC_A=ln(n)*(length(fit$par))+2*fit$value
#BIC value is:
BIC_A
```

```
## [1] 503.405
```

Part B: Now we want to fit the same model with change in distribution:

$$\texttt{Claims}_i = \beta_0 + \beta_1 \, \texttt{Holders}_i + \varepsilon_i, \quad i = 1, 2, \cdots, n$$

Assume : $\varepsilon_i \sim Laplace(0, \sigma^2)$. Note that $\beta_0, \beta_1 \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$.

(i) Clearly write down the negative-log-likelihood function in `R`. Then use `optim` function to estimate MLE of $\theta = (\beta_0, \beta_1, \sigma)$

```
Negloglike=function(data,theta)
{
  l=0
  for(i in 1:n)
  {
    l=l+log(dlaplace(y[i], theta[1]+theta[2]*x[i],theta[3]))

  }
  return(-l)
}

theta=c(0.1,0.1,50)
fit=optim(theta,Negloglike,data=data)
##Estimated value of theta is:

c(fit$par[1],fit$par[2],fit$par[3])
```

```
## [1]  5.2021496  0.1165771 11.6746589
```

(ii) Calculate **Bayesian Information Criterion** (BIC) for the model.

```
BIC_B=ln(n)*(length(fit$par))+2*fit$value
#BIC value is:
BIC_B
```

```
## [1] 491.7071
```

Part C: We want to fit the following models:

$$\texttt{Claims}_i \sim LogNormal(\mu_i, \sigma^2), where$$

$$\mu_i = \beta_0 + \beta_1 \log(\texttt{Holders}_i), \quad i = 1, 2, ..., n$$

Note that $\beta_0, \beta_1 \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$.

(i) Clearly write down the negative-log-likelihood function in `R`. Then use `optim` function to estimate MLE of $\theta = (\alpha, \beta, \sigma)$

```
Negloglike=function(data,theta)
{
  l=0
  for(i in 1:n)
  {
    l=l+log(dlnorm(y[i], theta[1]+theta[2]*log(x[i]),theta[3]))

  }
  return(-l)
}
```

```
theta=c(0.1,0.1,1)
fit=optim(theta,Negloglike,data=data)
##Estimated value of theta is:

c(fit$par[1],fit$par[2],fit$par[3])
```

```
## [1] -1.0243551  0.8479072  0.3293700
```

(ii) Calculate **Bayesian Information Criterion** (BIC) for the model.

```
BIC_C=ln(n)*(length(fit$par))+2*fit$value
#BIC value is:
BIC_C
```

```
## [1] 452.6034
```

Part D: We want to fit the following models:

$$\texttt{Claims}_i \sim Gamma(\alpha_i, \sigma), where$$

$$log(\alpha_i) = \beta_0 + \beta_1 \log(\texttt{Holders}_i), \quad i = 1, 2, ..., n$$

(i) Clearly write down the negative-log-likelihood function in `R`. Then use `optim` function to estimate MLE of $\theta = (\alpha, \beta, \sigma)$

```
e=2.718281828459045
Negloglike=function(data,theta)
{
  l=0
  for(i in 1:n)
  {
    l=l+log(dgamma(y[i], e^(theta[1]+theta[2]*log(x[i])),theta[3]))

  }
  return(-l)
}

theta=c(0.1,0.1,0.1)
fit=optim(theta,Negloglike,data=data)

##Estimated value of theta is:

c(fit$par[1],fit$par[2],fit$par[3])
```

```
## [1] -1.6430902  0.8371016  0.4858613
```

(ii) Calculate **Bayesian Information Criterion** (BIC) for the model.

```
BIC_D=ln(n)*(length(fit$par))+2*fit$value
#BIC value is:
BIC_D
```

```
## [1] 437.3382
```

(iii) Compare the BIC of all three models

```
c(BIC_A,BIC_B,BIC_C,BIC_D)
```

```
## [1] 503.4050 491.7071 452.6034 437.3382
```