Problem 5: Computational Finance - Modelling Stock prices

Following piece of code download the prices of TCS since 2007

```
library(quantmod)
```

```
## Warning: package 'quantmod' was built under R version 4.2.2
```

```
## Loading required package: xts
```

```
## Warning: package 'xts' was built under R version 4.2.2
```

```
## Loading required package: zoo
```

```
## Warning: package 'zoo' was built under R version 4.2.2
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```
## Loading required package: TTR
```

```
## Warning: package 'TTR' was built under R version 4.2.2
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
getSymbols('TCS.NS')
```

```
## Warning: TCS.NS contains missing values. Some functions will not work if objects
## contain missing values in the middle of the series. Consider using na.omit(),
## na.approx(), na.fill(), etc to remove or replace them.
```

```
## [1] "TCS.NS"
```

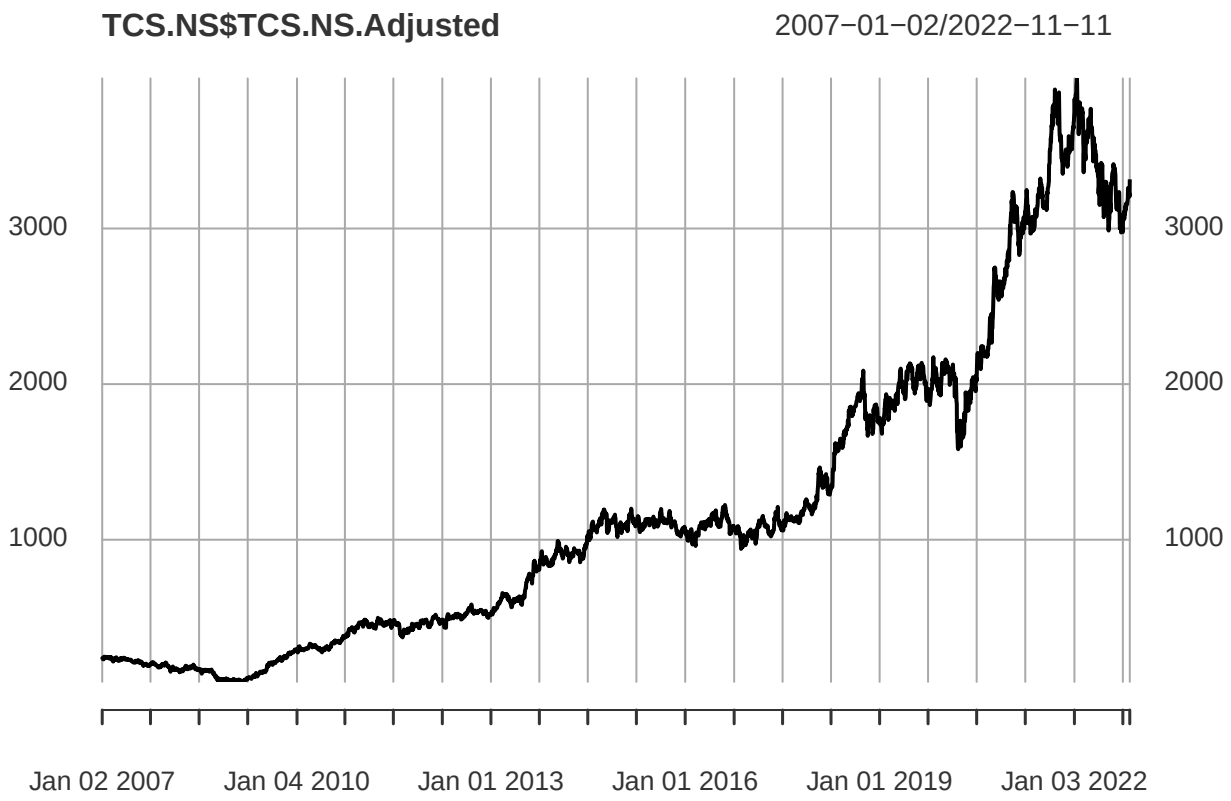
```
tail(TCS.NS)
```

```
##            TCS.NS.Open TCS.NS.High TCS.NS.Low TCS.NS.Close TCS.NS.Volume
## 2022-11-03     3228.05     3228.05    3195.00      3206.75       1422652
## 2022-11-04     3217.00     3220.05    3166.15      3217.40       1464013
## 2022-11-07     3229.00     3242.80    3195.10      3233.70       1474498
## 2022-11-09     3249.80     3249.80    3201.65      3216.05       1162267
## 2022-11-10     3170.00     3225.00    3170.00      3205.65       1573092
## 2022-11-11     3269.60     3341.60    3255.05      3315.95       3265394
##            TCS.NS.Adjusted
```

```
## 2022-11-03         3206.75
## 2022-11-04         3217.40
## 2022-11-07         3233.70
## 2022-11-09         3216.05
## 2022-11-10         3205.65
## 2022-11-11         3315.95
```

Plot the adjusted close prices of TCS

```
plot(TCS.NS$TCS.NS.Adjusted)
```



Download the data of market index Nifty50. The Nifty 50 index indicates how the over all market has done over the similar period.

```
getSymbols('^NSEI')
```

```
## Warning: ^NSEI contains missing values. Some functions will not work if objects
## contain missing values in the middle of the series. Consider using na.omit(),
## na.approx(), na.fill(), etc to remove or replace them.
```

```
## [1] "^NSEI"
```

```
tail(NSEI)
```

```
##            NSEI.Open NSEI.High NSEI.Low NSEI.Close NSEI.Volume NSEI.Adjusted
## 2022-11-03  17968.35   18106.3 17959.20   18052.70      213000      18052.70
## 2022-11-04  18053.40   18135.1 18017.15   18117.15      267900      18117.15
## 2022-11-07  18211.75   18255.5 18064.75   18202.80      314800      18202.80
## 2022-11-09  18288.25   18296.4 18117.50   18157.00      307200      18157.00
## 2022-11-10  18044.35   18103.1 17969.40   18028.20      256500      18028.20
## 2022-11-11  18272.35   18362.3 18259.35   18349.70      378500      18349.70
```

Plot the adjusted close value of Nifty50

```
plot(NSEI$NSEI.Adjusted)
```

Log-Return

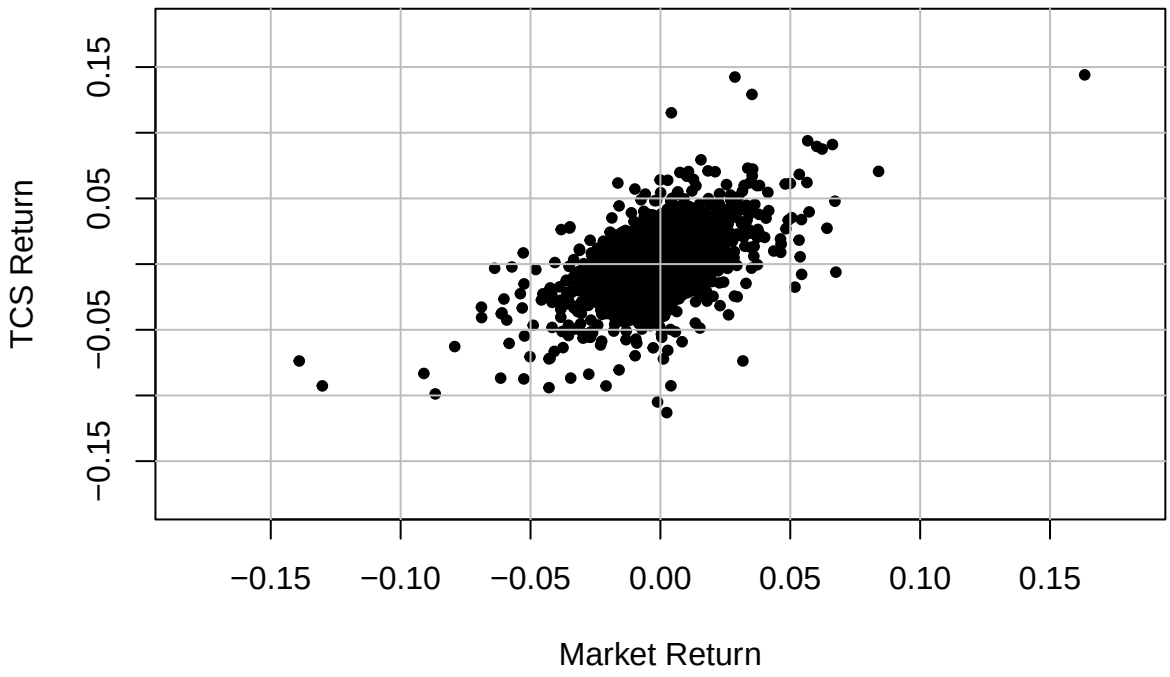
We calculate the daily log-return, where log-return is defined as

$$r_t = \log(P_t) - \log(P_{t-1}) = \Delta \log(P_t),$$

where P_t is the closing price of the stock on t^{th} day.

```
TCS_rt = diff(log(TCS.NS$TCS.NS.Adjusted))
Nifty_rt = diff(log(NSEI$NSEI.Adjusted))
retrn = cbind.xts(TCS_rt,Nifty_rt)
retrn = na.omit(data.frame(retrn))
```

```
plot(retrn$NSEI.Adjusted,retrn$TCS.NS.Adjusted
     ,pch=20
     ,xlab='Market Return'
     ,ylab='TCS Return'
     ,xlim=c(-0.18,0.18)
     ,ylim=c(-0.18,0.18))
grid(col='grey',lty=1)
```



- Consider the following model:

$$r_t^{TCS} = \alpha + \beta r_t^{Nifty} + \varepsilon,$$

where $\mathbb{E}(\varepsilon) = 0$ and $\mathbb{V}ar(\varepsilon) = \sigma^2$.

1. Estimate the parameters of the models $\theta = (\alpha, \beta, \sigma)$ using the method of moments type plug-in estimator discussed in the class.

Solution:

```
mu_y = mean(retrn$TCS.NS.Adjusted)
mu_x = mean(retrn$NSEI.Adjusted)
sigma_y = sd(retrn$TCS.NS.Adjusted)
```

```
sigma_x = sd(retrn$NSEI.Adjusted)
rho = cor(retrn$NSEI.Adjusted, retrn$TCS.NS.Adjusted)

alpha_0 = mu_y - rho * mu_x * (sigma_x/sigma_y)
beta_0 = rho *  (sigma_x/sigma_y)

retrn_mom = retrn
retrn_mom$mom_est_TCS.NS.Adjusted = alpha_0 + beta_0 * retrn_mom$NSEI.Adjusted

epsilon = retrn_mom$TCS.NS.Adjusted-retrn_mom$mom_est_TCS.NS.Adjusted
sigma_0 = sd(epsilon)

theta_0 = c(alpha_0, beta_0, sigma_0)

print( theta_0)
```

```
## [1] 0.0005848195 0.3904737999 0.0169171794
```

2. Estimate the parameters using the `lm` built-in function of `R`. Note that `lm` using the OLS method.

Solution:

```
lm_model = lm(TCS.NS.Adjusted~NSEI.Adjusted, data = retrn)

co = lm_model$coefficients

alpha_1 = matrix(co)[1,1]
beta_1 = matrix(co)[2,1]
sigma_1 = sd(lm_model$residuals)

theta_1 = c(alpha_1, beta_1, sigma_1)

print(theta_1)
```

```
## [1] 0.0004611203 0.7436971587 0.0161865311
```

3. Fill-up the following table

Parameters	Method of Moments	OLS
α		
β		
σ		

Solution:

Parameters	Method of Moments	OLS
α	0.0005848199	0.0004611208
β	0.3904739457	0.7436970826
σ	0.0169171740	0.0161865264

4. If the current value of Nifty is 18000 and it goes up to 18200. The current value of TCS is Rs. 3200/-. How much you can expect TCS price to go up?

Solution:

```
prediction_mom = function(Nifty_initial_value, Nifty_final_value, TCS_initial_value){
  beta = theta_0[2]
  alpha = theta_0[1]
  x = log(Nifty_final_value) - log(Nifty_initial_value)
  y = alpha + beta*x
  y1 = log(TCS_initial_value) + y
  y2 = exp(y1)
  return(y2)
}

prediction_ols = function(Nifty_initial_value, Nifty_final_value, TCS_initial_value){
  beta = theta_1[2]
  alpha = theta_1[1]
  x = log(Nifty_final_value) - log(Nifty_initial_value)
  y = alpha + beta*x
  y1 = log(TCS_initial_value) + y
  y2 = exp(y1)
  return(y2)
}
```

By the Method of Moments type method, we can say that we can expect TCS value to go up by

```
Nifty_initial_value = 18000
Nifty_final_value = 18200
TCS_initial_value = 3200

TCS_final_value = prediction_mom(Nifty_initial_value = Nifty_initial_value,
                                 Nifty_final_value = Nifty_final_value,
                                 TCS_initial_value = TCS_initial_value)

TCS_final_value - TCS_initial_value
```

```
## [1] 15.71684
```

By the OLS method, we can say that we can expect TCS value to go up by

```
TCS_final_value = prediction_ols(Nifty_initial_value = Nifty_initial_value,
                                 Nifty_final_value = Nifty_final_value,
                                 TCS_initial_value = TCS_initial_value)

TCS_final_value - TCS_initial_value
```

```
## [1] 27.89319
```

Thus we can expect TCS price to go up by some value around Rs. 15.72 and Rs. 27.89.