STOCHASTIC SIMULATION USING THE DISCRETIZATION SCHEMES OF EULER AND MILSTEIN

For this script, I considered the following stochastic differential equation,

$$dX(t) = aX(t)dt + bX(t)dW(t),$$

here $X(0) = 100$, $a = 0.05$, and $b = 0.2$, as given by the text. This is a standard Log-Normal process, which can be solved using Ito's lemma, so the exact solution of the SDe is defined as follows.

$$X(t) = X(0)e^{(a-\frac{b^2}{2})t+bW(t)}$$

The Euler Scheme of discretization is given as

$$X_{n+1} = X_n + aX_n\Delta t + bX_n\Delta W_n, \tag{1}$$

whereas the Milstein scheme is a modification of the prior

$$X_{n+1} = X_n + aX_n\Delta t + bX_n\Delta W_n + \frac{1}{2}b^2X_n((\Delta W_n)^2 - \Delta t), \tag{2}$$

where Δt is the discretization step and ΔW_n is a normal random variable with 0 mean and Δt variance.

All content listed here is credited to the book "Stochastic Simulation and Applications in Finance with MATLAB Programs" by Huu Tue Huynh, Van Son Lai, Issouf Soumaré.