Applied Combinatorial Optimization

Exercise sheet 2

Dr. Bogdan Savchynskyy <bogdan.savchynskyy@iwr.uni-heidelberg.de>
Sebastian Stricker <sebastian.stricker@iwr.uni-heidelberg.de>

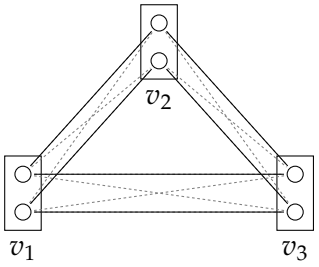
———— ⋆ ————

Exercise 2.1 Implement the representation of a labeling problem as an integer linear program (ILP) with Sherali-Adams and Fortet's linearization methods. Optimize the ILPs with an off-the-shelf solver and verify the solution.

Note: The Python-LP library (pulp[1]) allows to build and solve LP/ILPs conveniently.

Exercise 2.2 Construct both LP relaxations for a given input labeling problem programmatically. Optimize the respective LPs with an off-the-shelf solver and verify the solution. Implement the naïve rounding scheme to obtain an integer labeling. How do the LP optima, the rounded solutions and the ILP optimum compare in general?

Exercise 2.3 Optimize the local polytope relaxation of the following labeling problem. What is special about the solution of the problem? Round the solution to obtain an integer labeling and optimize the same model with an ILP solver. What do you observe? Explain why.



$$\theta_1 = [0.5, 0.5],$$
$$\theta_2 = [0.0, 0.0],$$
$$\theta_3 = [0.2, 0.2]$$

$$\forall \{u, v\} \in \mathcal{E} : \theta_{uv} = \begin{cases} 1 & \text{if } x_u = x_v, \\ 0 & \text{otherwise.} \end{cases}$$

Exercise 2.4 We provide multiple random acyclic and cyclic labeling problems (*model_2_4.py*). Compute the LP (local polytope) and ILP optima. What do you observe? Explain why.

acyclic problem	LP (local polytope)	LP(Fortet)	ILP
model 1	...	...	...
...	...	...	...

cyclic problem	LP (local polytope)	LP(Fortet)	ILP
model 1	...	...	...
...	...	...	...

[1] https://pypi.org/project/PuLP/

Exercise 2.5 One problem in computer vision is the reconstruction of depth information from two stereo image pairs. The basic idea is to rectifiy the stereo image pair and compute the disparity for each pixel. The task of this exercise is to reason about the practicability of using generic LP/ILP solvers for solving these (relatively large-scale) problems.

We provide a labeling problem formulation of the *tsukuba* scene (*model_2_5.py*) which is part of the Middlebury stereo benchmark. Additionally, we provide downsampled variants.

filename	image dimensions	#vars	#contrs	LP (time/value)	ILP (time/value)
`tsu_down_01.uai`	384x288	...	...	...	...
`tsu_down_02.uai`	192x144	...	...	...	...
`tsu_down_04.uai`	96x72	...	...	...	...
`tsu_down_08.uai`	48x36	...	...	...	...
`tsu_down_16.uai`	24x18	...	...	...	...
`tsu_down_32.uai`	12x9	...	...	...	...

How big (in terms of variables/constraints) are the resulting LP/ILP problems? How much time does the inference need? Can you solve all instances in reasonable time? How practical is it to solve these kind of vision problems with general LP/ILP-solvers? Give the above answers for the Sherali-Adams as well as for the Fortet linearization.

Exercise 2.6 Selecting a good set of segmentations out of a pool of possible candidates (over-segmentation) is a combinatorial optimization problem that can be formulated as max-weight independent set problem and has a natural ILP representation. We provide an archive containing 500 binary segmentation masks (*segments.zip*). Write a Python program that reads all the segmentation candidates and outputs a set of non-overlapping segmentations such that the total covered image area (number of pixels) is maximized by solving the ILP formulation of the problem. Visualize your results!