Institute for Theoretical Physics — Heidelberg University
PD Dr. Falko Ziebert
**Nonlinear dynamics** — Summer term 2024

# Assignment 1

Handout 22.04.2024 – Return 29.04.2024 – Discussion 02./03.05.2024

**Exercise 1 [5 points]: Landau equation**

In the lecture we discussed the Landau equation

$$\dot{A} = \epsilon A - gA^3.$$

It is one of the rare cases of nonlinear equations that can be integrated up analytically. Do so (as a function of the initial condition!) and plot all types of solutions in a diagram $A$ vs. $t$.
HINT: Separation of variables and partial fraction decomposition.

**Exercise 2 [4 points]: (Im-)Possibility of oscillations**

1. Find arguments why the equation

   $$\dot{x} = f(x)$$

   can not display oscillations on the line $-\infty < x < \infty$. *(2 points)*
   HINT: You might e.g. introduce a potential. Or you may assume a period $T$ and investigate $\int_t^{t+T} f(x)\dot{x}dt$ to find a contradiction.

2. You all know the harmonic oscillator equation

   $$m\ddot{x} = -kx\,.$$

   Why does this equation display oscillations (give a phase space-based argument[1])? Perform a linear stability analysis of the fixed point $x^* = 0$ and discuss the eigenvalues. *(2 points)*
   HINT TO[1]: Show that there are closed trajectories in phase space.

**Exercise 3 [6 points]: Fixed points and stability**

Use linear stability analysis to classify the fixed points $x^*$ of the following systems $\dot{x} = f(x)$. Plot the $\dot{x}$ vs. $x$ diagrams to verify your calculation.
HINT: If linear stability analysis fails, because $\frac{df}{dx}(x^*) = 0$, use a graphical argument to decide the stability.

1. $\dot{x} = x(1-x)$ *(2 points)*
2. $\dot{x} = x(1-x)(2-x)$ *(2 points)*
3. $\dot{x} = 1 - \exp(-x^2)$ *(2 points)*