Institute for Theoretical Physics Heidelberg University
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Nonlinear dynamics Summer term 2024

Assignment 2

Handout 29.04.2024 – Return 06.05.2024 – Discussion 09/10.05.2024

Exercise 1 [8 points]: Overfishing

Consider the model for a fish population discussed in the lecture:

$$\dot{u} = \epsilon u \left(1 - \frac{u}{c}\right) - r,$$

where $u \geq 0$ is the fish population, $\epsilon > 0$ its reproduction rate, $c > 0$ the capacity of the habitat and $r \geq 0$ the rate of removal, i.e. the fishing rate.

1. Non-dimensionalize the equation. How many parameters are left? Use the non-dimensional fishing rate $\bar{r}$ as the control parameter in the following. *(2 points)*

2. Determine the fixed points and their stability. Plot the (rescaled) steady-state population size as a function of (rescaled) fishing rate. Which bifurcation type is it? *(2 points)*

3. Why is the model problematic (consider small populations and high fishing rate r)? *(1 point)*

4. Show that these problems are resolved when one replaces the fishing rate $r \to r\frac{u}{b+u}$. What is the interpretation? Determine again the fixed points and plot the steady-state population size as a function of r. Which bifurcation type is it now? *(3 points)*

Exercise 2 [7 points]: Mutual inhibition and coexistence of species

Consider the model for two populations discussed in the lecture:

$$\begin{aligned} \dot{x} &= \alpha x(1-x) - xy\,, \\ \dot{y} &= \alpha y(1-y) - xy\,, \end{aligned}$$

with $x, y \geq 0$ and $\alpha > 0$.

1. Draw the nullclines and calculate the fixed points. *(2 points)*

2. Calculate the stability of all fixed points. *(3 points)*

3. You will find a critical value α_c, where the fixed point describing coexistence (where x and y are both finite) changes stability. Plot the nullclines, fixed points and the phase space flow for both $\alpha < \alpha_c$ and $\alpha > \alpha_c$. *(2 points)*
   HINT: The nullclines and flows are simple enough to sketch them by hand, but you may also want to familiarize yourself plotting them using *python*.