Institute for Theoretical Physics Heidelberg University
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Nonlinear dynamics Summer term 2024

Assignment 8

Handout 10.06.2024 – Return 17.06.2024 – Discussion 20./21.06.2024

Exercise 1 [6 points]: A global bifurcation

Consider the following dynamical system with $x, y \in \mathbb{R}$:

$$\begin{aligned} \dot{x} &= y\,, \\ \dot{y} &= -\epsilon y + x - x^5\,. \end{aligned} \tag{1}$$

1. First consider the case $\epsilon = 0$ (what is the meaning of ϵ?). Determine the fixed points and their stability. *(2 points)*
2. Determine the homoclinic path through the origin and encircling the other fixed points. Give an implicit formula for the homoclinic path and sketch it. *(2 points)*
3. Now consider varying ϵ. How does the stability of the fixed points change? Sketch the cases $\epsilon < 0$ and $\epsilon > 0$ and compare to $\epsilon = 0$. *(2 points)*

Exercise 2 [9 points]: Amplitude equation for the SH model without $\pm u$-symmetry

Consider the Swift-Hohenberg (SH) equation discussed in the lecture with an additional quadratic nonlinearity ($\alpha > 0$)

$$\partial_t u = \left[\epsilon - (q_0^2 + \partial_x^2)^2\right] u + \alpha u^2 - u^3\,.$$

Derive the amplitude equation.
Is the amplitude equation qualitatively different from the one in the absence of the u^2-term?
HINT: In order $\mathcal{O}(\epsilon)$, you will get a contribution from the quadratic nonlinearity that is not resonantly driving but makes u_1 different from u_0. You hence have to calculate a particular solution for u_1. In turn, u_1 will lead to new contributions in $\mathcal{O}\left(\epsilon^{3/2}\right)$, including new resonant terms contributing to the amplitude equation.