

CHBE 553 –Assignment 5 – 02/14/2019

OBJECTIVE 1: Obtain derivatives of D with respect to t using direct forward difference method

Forward Euler difference method is given as:

$$f'_i = (f_{i+1} - f_i) / h$$

where, $h = x_{i+1} - x_i$

OBJECTIVE 2: Smaller step size 'h'

Central difference method was used to evaluate the first derivative of D using small step size 'h' = 10^{-6}

Central difference method is given as:

$$f'_i = (f_{i+h} - f_{i-h}) / 2h, \text{ where } h \text{ is very small}$$

OBJECTIVE 3: Direct differentiation of $D = at^b$

$$\frac{dD}{dt} = a * b * t^{(b-1)}$$

The derivative was calculated for each value of t and the derivative was plotted against 't'.

RESULT AND OBSERVATION

The MATLAB code was executed and the following plot was obtained:

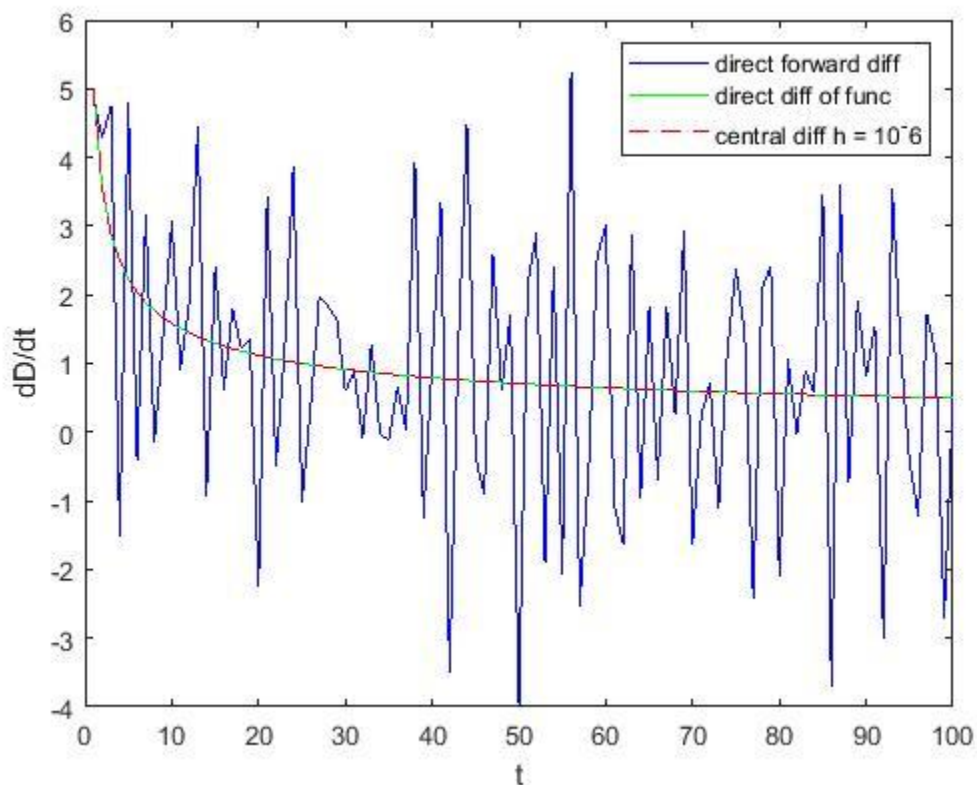


Fig 1 : dD/dt vs t

From fig 1, it can be seen that the derivative obtained from the direct Euler's method using larger step size $x_{i+1} - x_i$ is an unstable curve with fluctuations. It can also be observed that when step is lowered ($\sim 10^{-6}$), the function becomes stable and overlaps with the derivative obtained from the analytical differentiation of the function directly. This may be due to the increase in the global truncation error (e) when using larger step size. Global truncation error is the cumulative of local truncation error committed at each step and is proportional to 'h'. As the step size decreases, the global truncation error is lowered, but the round-off error increases, as it is proportional to h^{-1} . Hence there's a tradeoff between the truncation and round-off error. It can be concluded that, step size should be sufficiently small, to obtain a stable approximation of the derivative of the function.

MATLAB CODE:

```
%assignment5
%part1
%import data file
data = xlsread('datafile1.xlsx');
xdata = data(:,1);
ydata = data(:,2);

%forward difference to calculate dD/dt
for i = 1:(size(xdata)-1)
    dD_dt(i) = (ydata(i+1) - ydata(i))/(xdata(i+1)-xdata(i));
    if i+1 > size(xdata)
        break;
    end
end

plot(xdata,dD_dt,'b')
hold on

%analytical differentiation
%part2
%dD/dt = a*b*t^(b-1)
a = 10;
b = 0.5;
%dD/dt using direct differentiation
for i = 1:size(xdata)
    dD_dt2(i) = a*b*(xdata(i)^(b-1));
end

plot(xdata,dD_dt2,'g-')
hold on

%Central difference method using step size = 10^-6
%central difference to calculate dD/dt
h1 = 10^-6;
for i = 1:size(xdata)
    dD_dt3(i) = (y_eval(xdata(i)+h1) - y_eval(xdata(i)-h1))/(2*h1);
end
```

```
plot(xdata,dD_dt3,'r--')

xlabel('t');
ylabel('dD/dt');
legend({'direct forward diff','direct diff of func','central diff h = 10^-6'}, 'location','northeast');
hold off

function y_new = y_eval(x)
a = 10;
b = 0.5;
y_new = a*x^b;
end
```