CHBE 553 – Mathematical operations – Assignment 6 (02/24/2019)

OBJECTIVE: To compute and plot $\Theta(\eta) = 1 - \int_0^{\eta} e^{-y^3}$, 0<η<2

Gauss quadrature algorithm can give exact solution of integrals of the form

$$\int_a^b w(x)f(x)dx$$

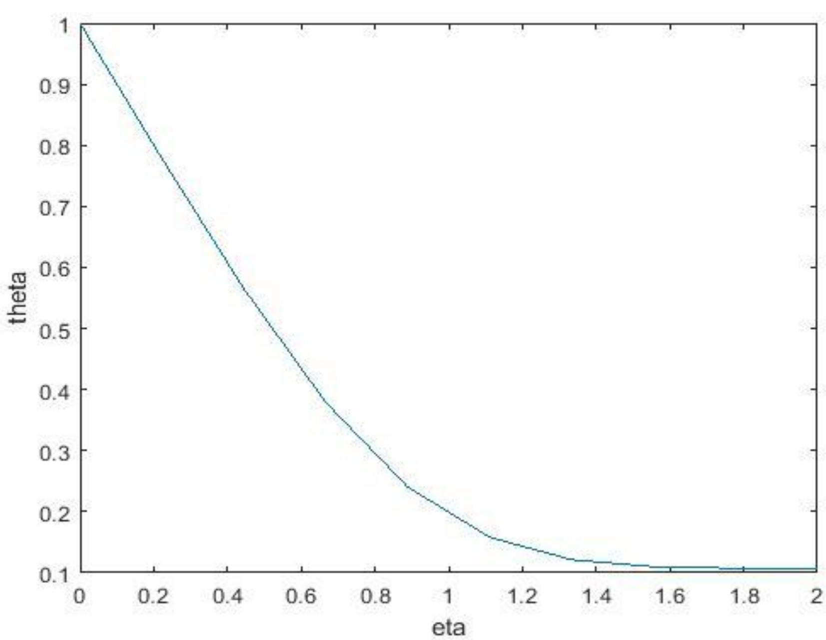
where w(x) is called the weighting function. The integral is calculated as follows:

$$\int_a^b w(x)f(x)dx = \sum_{i=1}^{n} w(i)f(xi) \qquad 1$$

where w(i) is called the weights and x(i) is called the abscissa. For most common functions, the weights and abscissa have been tabulated in "M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables, AMS-55, 1964 [1]". The nodes and the weights are calculated such that (1) yields the exact integral if f(x) is a polynomial of degree <= 2n-1.

For the given problem, the weighting function w(x) is 1 so it represents the legendre polynomials.

The weights and the abscissa for the legendre polynomial is taken from [1]. The code is programmed to stop when the difference between the iterations are of the order of 10^{-5}. There were 8 iterations to evaluate the integral from 0<η<2 to produce an integral of required accuracy. The value of $\Theta(\eta)$ ranges from 1 to 0.1070 for 0<η<2. The final plot of $\Theta(\eta)$ for 0<η<2 is shown below.



eta	theta
0	1
0.222222	0.778386
0.444444	0.56507
0.666667	0.378809
0.888889	0.240377
1.111111	0.158128
1.333333	0.121401
1.555556	0.109804
1.777778	0.107367
2	0.107046

MATLAB CODE:

```
%Assignment6
%limits
a = 0; %lower limit
b = linspace(0,2,10); %upper limit from 0<eta<2
c = length(b);
I_old = 0;
%for loop to evaluate the integral bewtween different limits
for j = 1: c
    %n is the number of nodes
    for n = 1:20
        %function call statement to leg(n), which possesses the values of
        %weights and abscissa for legendre function depending on the number
        %of nodes

        %yold = abscissa, w = weights
        [yold, w] = Leg(n);

        %to check if the limits (a,b)are between 1 and -1, since the
        %function is only valid between [-1,1]
        if a ~= -1 && b(j) ~= 1
            %to evaluate ynew, which is between [-1,1]
            ynew = ((b(j)-a)/2)*yold + (b(j)+a)/2;
        end
        % gauss quadrature integration
        Iini = 0;
        for i = 1:length(yold)
            %integral is evaluated using the weights and the function value
            %at ynew
            I = ((b(j)-a)/2)*(w(i)*exp(-ynew(i)^3));
            I = Iini + I;
            Iini = I;
        end
        %stoppage condition
        if (abs(I-I_old) < 10^-5)
            break;
        end
        I_old = I;

    end
    %final answer
    ans(j) = 1 - I;
end
plot(b,ans)
xlabel('eta');
ylabel('theta');

%function that contains the legendre polynomial weights and abscissa
function [ydata, wdata] = Leg(n)
Legendre = xlsread('Ass6Data.xlsx',n);
ydata = Legendre(:,1);
wdata = Legendre(:,2);
end
```