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RL = 20
MN = ~~8902689001~~ 01
FN = (65 + 78 + 85 + 82 + 65 + 78) % 70 = 453 % 70 = 33.
LN = (67 + 72 + 65 + 75 + 82 + 65 + 66 + 79 + 82 + 84 + 89) % 70
= 826 % 70 = 56.

FLN = 1279 % 70 = 19

RLM = 20 + 01 = 21
MFN = 01 + 33 = 34
MLN = 01 + 56 = 57

Q1. max. $Z = 34x_1 + 57x_2$

Subject to $x_1 + 20x_2 \leq 1000$
$x_1 + x_2 \leq 800$.
$x_1 + x_2 \leq 400$.
$x_1, x_2 \geq 0$.

Convert to canonical form by adding slack variables.

$Z - 34x_1 - 57x_2 + 0s_1 + 0s_2 + 0s_3 = 0$.
$x_1 + 20x_2 + s_1 = 1000$.
$x_1 + x_2 + s_2 = 800$
$x_1 + x_2 + s_3 = 400$.
$x_1, x_2 \geq 0$.
~~max. Z = 34x1 + 57x2~~

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For any iteration. Roll : 001610501020

$$\max.\ Z = C_B X_B + C_N X_N$$

s.t.

$$[B,\ N] \cdot \begin{bmatrix} X_B \\ X_N \end{bmatrix} = b \quad \text{and} \quad \begin{bmatrix} x_B \\ x_N \end{bmatrix} \geq 0.$$

$\because C_N = 0.$

$$\therefore\ Z = C_B X_B + C_N X_N = C_B X_B = C_B B^{-1} b.$$

$$B x_B + N x_N = b$$

$$B x_B = b - N x_N$$

$$x_B = B^{-1} b - B^{-1} N x_N$$

$$x_B = B^{-1} b.$$

$$\boxed{\begin{aligned} x_B &= B^{-1} b \\ Z &= C_B B^{-1} b. \end{aligned}}$$

Iteration 0

$$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$$

$$\therefore \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$$

$$C_B = [0\ \ 0\ \ 0] \qquad \therefore\ Z = [0\ \ 0\ \ 0] \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = 0$$

~~Iteration 1~~

~~$x_B =$~~

$$c_1 - z_1 = c_1 - y P_1$$

$$= \cancel{6}\ 34 - y \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

y is the dual.

$$y = C_B B^{-1} = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$c_1 - z_1 = \overset{34}{\cancel{6}} - [0\ 0\ 0] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 34$$

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Initialization   Iteration 0.

$c = [34 \quad 57]$ $\quad [A, I] = \left[\begin{array}{cc|ccc} 1 & 20 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right]$

$b = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad x_s = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}$

$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix}, \quad B = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = B^{-1}$

$x_B = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \end{bmatrix} = B^{-1}b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$

$C_B = [0 \quad 0 \quad 0]$ so $z = [0 \quad 0 \quad 0] \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = 0$.

Optimality test : $C_B B^{-1} A - c = [0\ 0\ 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \ 57]$

$= [-34 \quad -57]$

Now ~~all~~ there is atleast one $-$ve value.

$-c_2 = -57 < -c_1 = -34 \quad \therefore x_2$ is the entering variable.

Iteration 1

x_2 is entering variable.

To determine leaving variable.

$B^{-1}A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$

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We check only the column for x_2.

$a_{12} = 20, \quad a_{22} = 1, \quad a_{32} = 1.$

$$\frac{b_1}{a_{12}} = \frac{1000}{20} = 50., \quad \frac{b_2}{a_{22}} = \frac{800}{1}$$

$$\frac{b_3}{a_{32}} = \frac{400}{1} = 400.$$

$\frac{b_1}{a_{12}}$ is min so row 1, x_3 is the leaving variable

Entering : x_2 [2nd col]
Leaving : x_3 [1st row]
(1,2) is the pivot)

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix}$$

Obtain new B^{-1}.

$$\eta = \begin{bmatrix} \frac{1}{a_{12}} \\ -\frac{a_{22}}{a_{12}} \\ -\frac{a_{32}}{a_{12}} \end{bmatrix} = \begin{bmatrix} \frac{1}{20} \\ -\frac{1}{20} \\ -\frac{1}{20} \end{bmatrix}$$

$$E = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$B^{-1}_{new} = E B^{-1}_{old} = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix} = \begin{bmatrix} 50 \\ 750 \\ 350 \end{bmatrix}$$

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$$C_B B^{-1} A - C = [57 \ 0 \ 0] \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \ \ 57]$$

$$= \cancel{[57/20 \ 0 \ 0] \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} - [34 \ 57]}$$

$$= \cancel{[57/20 \ \ 57] - [34 \ \ 57]}$$

$$= \cancel{[-31.15 \ \ 0]} = [-31.15 \ \ -]$$

$c_1 < 0$ $\therefore$ still not optimal.

Iteration 2

x_1 is the entering variable.

To determine leaving variable.

$$A' = B^{-1} A = \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/20 & - \\ 19/20 & - \\ 19/20 & - \end{bmatrix}$$

We check only the column for x_1

$a_{11} = 1/20$ $\quad a_{12} = 19/20$ $\quad a_{13} = 21/20$.

$$\frac{b_1}{a'_{11}} = \frac{1000}{1/20} \qquad \frac{b_2}{a'_{12}} = \frac{800}{19/20} \qquad \frac{b_3}{a'_{13}} = \frac{400}{19/20}$$

row 3 is min.

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$\therefore x_5$ is the leaving varriable.

Entering : x_4 (Col 1)

Leaving : x_5 (row 3)

$\therefore$ Pivot element (3,1)

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_4 \end{bmatrix}$$

$$\eta = \begin{bmatrix} -\frac{a'_{11}}{a'_{31}} \\ -\frac{a'_{21}}{a_{31}} \\ \frac{1}{a'_{31}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{19} \\ -1 \\ \frac{20}{19} \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & -\frac{1}{19} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{20}{19} \end{bmatrix}$$

$$B^{-1}_{New} = E B^{-1}_{old} = \begin{bmatrix} 1 & 0 & -\frac{1}{19} \\ 0 & 1 & -1 \\ 0 & 0 & \frac{20}{19} \end{bmatrix} \begin{bmatrix} \frac{1}{20} & 0 & 0 \\ -\frac{1}{20} & 1 & 0 \\ -\frac{1}{20} & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix}$$

$$c_B = [57 \quad 0 \quad 34]$$

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$$c_B B^{-1} A = [57 \;\; 0 \;\; 34] \begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix} \cancel{\begin{bmatrix} 1/20 \\ 19/20 \\ 19/20 \end{bmatrix}}$$

$$= \cancel{593/19} \; [23/19 \;\; 0 \;\; 623/19] \geq 0.$$

$$\cancel{c_B B^{-1} A - c_1 = 593/19 - 34 > 0}$$

~~coeffn of x_4~~

non basic variables are x_3, x_5.

$\therefore$ optimality reached.

$$x_B = \begin{bmatrix} x_2 \\ x_4 \\ x_4 \end{bmatrix} = B^{-1} b =$$

$$\begin{bmatrix} 1/19 & 0 & -1/19 \\ 0 & 1 & -1 \\ -1/19 & 0 & 20/19 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 400 \end{bmatrix}$$

$$= \begin{bmatrix} 600/19 \\ 400 \\ 7000/19 \end{bmatrix}$$

$$\therefore x_2 = \frac{600}{19} \qquad x_4 = \frac{7000}{19}$$

$$\boxed{\begin{aligned} x_4 &= 368.4211 \\ x_2 &= 31.5789 \\ z &= 34x_4 + 57x_2 = 14326.3158 \end{aligned}}$$

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Q2. A company knows that the demand of one of its most important products 1, 2, 3, 4 over the next 4 months. The company must plan the production of 10 units. If any production appears in a period, there is a setup cost of ₹5700k. In addition there is a production cost of ₹5700k for each produced unit. If a unit is put in the inventory there is an inventory cost of ₹1650k per unit. 5 units at most can be produced in a month and at most 4 units can be put in the inventory. How should the company plan their production to satisfy the demand and minimize production & inventory costs. There are no units in inventory at the beginning of month 1. Solve using dp after formulation.

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Setup cost = ₹5700k
Production cost per unit = ₹5700k
Inventory cost per unit (I_c) = ₹1650k.
Cost of making 'p' products
~~$c(p) = 5700$~~ All units are in multiple of 10^3.

$c(p) = 5700 + 5700p$.
$c(0) = 0$ ∵ no setup cost is needed.

no. of items in inventory at the beginning of month m, is i_m.

$i_m = i_{m-1} + P_{m-1} - d_{m-1}$

where P_{m-1} is the production in month m-1
and d_{m-1} is " demand in month m-1.

To formulate. Let
$f_m(x)$ denote the minimum cost at month m, when x items are available in the inventory

~~$f_m(p) = \min_p [c(p) + I_c(p$~~

$$f_m(x) = \min_p \left[c(p) + I_c(x+p-d_m) + f_{m+1}(x+p-D_m) \right]$$

$p \leq 5, \quad x \leq 4, \quad x+p-d_m \leq 4.,$
$x, p, d_m \geq 0.$

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Month 4 $\quad d_4 = 4.$

	p=0	p=1	p=2	p=3	p=4	f_4
x=0					28500	28500
x=1				24450		24450
x=2			20400			20400.
x=3		16350				16350.
x=4	660					

$\because$ month 4 is last $x+p = d_4 = 4.$

$c(4) = 5700 + 5700 \times 4 = ₹28500.$

$$f_4(0) = \min_p \left[c(p) + I_c(0+p-4) + 0 \right]$$

$$= \min_p \left[c(p) \right]$$

$$= ₹28500$$

$f_4(1) = c(3) = 22800.$

$f_4(2) = c(2) = 20400$

$f_4(3) = c(1) = 16350$

$f_4(4) = c(0) = 6600.$

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month 4 $\quad x+p=4 \quad \because$ last month.

	p=0	p=1	p=2	p=3	p=4	f_4
x=0					28500	28500
x=1				22800		22800
x=2			17100			17100
x=3		11400				11400
x=4	0					0 ← minimum.

$f_4(0) = c(4) = 28500.$

$f_4(1) = c(3) = 22800$

$f_4(2) = c(2) = 17100$

$f_4(3) = c(1) = 11400$

$f_4(4) = c(0) = 0$

$c(5) = 6 \times 5700$
$= 34200$

month 3

	p=0	p=1	p=2	p=3	p=4	p=5	f_3
x=0				51300	52950	54600	51300
x=1			45600	47250	48900	50550	45600.
x=2		39900	41550	43200	44850	40800	39900.
x=3	28500	35850	37500	39150	35100		28500.
x=4.	24450	31800	33450	35100.			24450.

$d_3 = 3$

$x+p-d \leq 4$. for next stage inventory.

$x+p \geq d.$

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$f_3(0) = \min\{h(0,p)\}$ 001610501020

$h(x,p) = c(p) + I_c(x+p-d) + f_4(x+p-d)$

$h(0,3) = c(3) + 0 + f_4(0)$

$= 22800 + 0 + 28500$

$= 51300.$

$h(0,4) = c(4) + I_c + f_4(1)$

$= 52950.$

$h(0,5) = c(5) + 2 \times I_c + f_4(2)$

$= 54600$

$h(1,2) = c(2) + 0 \times I + f_4(0)$

$= 17100 + 0 + 28500 = 45600.$

$h(1,3) = c(3) + I_c + f_4(1)$

$= 47250.$

$h(1,4) = c(4) + 2I_c + f_4(2)$

$= 48900.$

$h(1,5) = c(5) + 3I_c + f_4(3) = 50550$

$h(2,1) = c(1) + 0I_c + f_4(0) = 39900.$

$h(2,2) = c(2) + I_c + f_4(1) = 41550.$

$h(2,3) = c(3) + 2I_c + f_4(2) = 43200$

$h(2,4) = c(4) + 3I_c + f_4(3) = 44850$

$h(2,5) = c(5) + 4I_c + f_4(4) = 40800.$

$h(3,0) = c(0) + 0I_c + f_4(0) = 28500.$

$h(3,1) = c(1) + I_c + f_4(1) = 35850.$

$h(3,2) = c(2) + 2I_c + f_4(2) = 37500.$

$h(3,3) = c(3) + 3I_c + f_4(3) = 39150$

$h(3,4) = c(4) + 4I_c + f_4(4) = 35100.$

$h(4,0) = c(0) + I_c + f_4(1) = 24450.$

$h(4,1) = c(1) + 2I_c + f_4(2) = 31800.$

$h(4,2) = c(2) + 3I_c + f_4(3) = 33450$

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$h(4,3) = c(4) + 4I_c + f_4(4) = 35100.$

month 2

	p=0	p=1	p=2	p=3	p=4	p=5	f_2
x=0			68400.	70050	71700	67650	67650.
x=1		62700	64350	66000	61950	65250	61950.
x=2	51300	60300	56250	56250	59550		51300
x=3	47250	54600	50550	53850			47250.
x=4	43200	44850	48150				43200.

$d_2 = 2 \qquad x+p \geq 2$

$x+p-2 \leq 4.$

$h(0,2) = c(2) + 0 \cdot I_c + f_3(0) = 68400.$

$h(0,3) = c(3) + I_c + f_3(1) = 70050$

$h(0,4) = c(4) + 2I_c + f_3(2) = 71700$

$h(0,5) = c(5) + 3I_c + f_3(3) = 67650.$

$h(1,1) = c(1) + 0 \cdot I_c + f_3(0) = 62700.$

$h(1,2) = c(2) + I_c + f_3(1) = 64350$

$h(1,3) = c(3) + 2I_c + f_3(2) = 66000$

$h(1,4) = c(4) + 3I_c + f_3(3) = 61950$

$h(1,5) = c(5) + 4I_c + f_3(4) = 65250$

$h(2,0) = c(0) + 0 \cdot I_c + f_3(0) = 51300$

$h(2,1) = c(1) + I_c + f_3(1) = 58650$

$h(2,2) = c(2) + 2I_c + f_3(2) = 60300$

$h(2,3) = c(3) + 3I_c + f_3(3) = 56250$

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$h(2,4) = c(4) + 4 \times I_c + f_3(4)$ Roll : 001610501020

$= 59550$

$h(3,0) = c(0) + I_c + f_3(1) = 47250$

$h(3,1) = c(1) + 2I_c + f_3(2) = 54600$

$h(3,2) = c(2) + 3I_c + f_3(3) = 50550$

$h(3,3) = c(3) + 4I_c + f_3(4) = 53850$

$h(4,0) = c(0) + 2I_c + f_3(2) = 43200$

$h(4,1) = c(1) + 3I_c + f_3(3) = 44850$

$h(4,2) = c(2) + 4I_c + f_3(4) = 48150$

month 1

$d_1 = 1$. $x = 0$, $x + p - 1 \leq 4$

$x + p \geq 1$.

	p=0	p=1	p=2	p=3	p=4	p=5	f_1
x=0	-	79050	80700	77400	80700	84000	~~80700~~ 77400

$h(0,1) = c(1) + 0 \cdot I_c + f_2(0) = 79050$

$h(0,2) = c(2) + I_c + f_2(1) = 80700$

$h(0,3) = c(3) + 2I_c + f_2(2) = 77400$

$h(0,4) = c(4) + 3I_c + f_2(3) = 80700$

$h(0,5) = c(5) + 4I_c + f_2(4) = 84000$

$\therefore$ $\boxed{\text{min cost} = ₹80700\text{k}}$

Path followed : month 1 : p = 3

month 2 : p = 0

month 3 : p = 3

month 4 : p = 4

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~~Q3. Minimize $f(x_1, x_2) =$~~

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Q3 min. $f(x_1, x_2) = (x_1-1)^2 + (x_2+56)^2 + x_1x_2$

s.t. $x_1^2 + x_2^2 \leq 9$

$56x_1 + x_2 \geq 6.$

$x_1, x_2 \geq 0$

1. Determine a point x that satifies KKT conditions.

For KKT conditions.

min. $f(x)$

s.t. $g_i(x) \leq 0$

$x_1^2 + x_2^2 - 9 \leq 0$

$-56x_1 - x_2 + 6 \leq 0$

$$\nabla f(\bar{x}) + \sum_{i=1}^{m} \lambda_i \nabla g_i(\bar{x}) = 0.$$

$\bar{\lambda}_i \, g_i(\bar{x}) = 0 \quad \forall i$

$g_i(\bar{x}) \leq 0 \quad \forall i$ λ_i are KKT multipliers.

$\bar{\lambda}_i \geq 0$

if $\bar{x}$ is a point of minimum.

However in order to apply KKT we have make sure the function is convex

$x_1^2 + x_2^2 - 9 \leq 0$ is convex as it is a circle

$-56x_1 - x_2 + 6 \leq 0$ is linear so convex.

$$\nabla f = [2(x_1-1) + x_2 \qquad 2(x_2+56) + x_1]$$

$$H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

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Find eigen values of H.

$$H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)^2 - 1 = 0$

$(2-\lambda)^2 = 1$

$(2-\lambda) = \pm 1$

$\lambda = 2 \pm 1$

$\lambda = 3, 1.$

$\therefore$ +ve definite.

$\therefore f(x)$ is convex and KKT may be applied. Suppose x'_0 is a point where KKT holds.

~~$\nabla f(x_0) + \lambda_1 g_1(x_0)$~~

$\nabla f(x') + \lambda_1 \nabla g_1(x'_0) + \lambda_2 \nabla g_2(x')$.

$= [2(x_1'-1) + x_2' \quad 2(x_2'+56) + x_1']$

$+ \lambda_1 [2x_1' \quad 2x_2'] + \lambda_2 [-56 \quad -1]$

$= 0.$

$= [2x_1' - 2 + x_2 + 2\lambda_1 x_1' - 56\lambda_2$

$2x_2' + 112 + 2x_1' + 2\lambda_1 x_2' - \lambda_2]$

$= 0.$

$(2+2\lambda_1)x_1' + x_2' - 2 - 56\lambda_2 = 0$

$2x_1' + (2+2\lambda_1)x_2' + 112 - \lambda_2 = 0.$

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$\lambda_1 g_1(x') = 0$ $\qquad$ $\lambda_1 \geq 0, \lambda_2 \geq 0$

$\lambda_1 (x_1'^2 + x_2'^2 - 9) = 0.$ $\qquad$ $x_1'^2 + x_2'^2 - 9 \leq 0$

$\lambda_2 (-56x_1' - x_2' + 6) = 0$ $\qquad$ $-56x_1' - x_2' + 6 \leq 0$

Case I: $\lambda_1 = 0, \lambda_2 = 0$

$$\boxed{\begin{aligned} &(2+2\lambda_1)x_1' + x_2' - 2 - 56\lambda_2 = 0 \\ &2x_1' + (2+2\lambda_1)x_2' + 112 - \lambda_2 = 0. \end{aligned}}$$

put $\lambda_1 = \lambda_2 = 0.$

$$\begin{aligned} 2x_1' + x_2' &= 2. \\ 2x_1' + 2x_2' &= -112 \\ \hline -x_2' &= 114 \end{aligned}$$

$$\boxed{\begin{aligned} x_2' &= -114. \\ x_1' &= 58 \end{aligned}}$$

This does not satisfy $x_1'^2 + x_2'^2 - 9 \leq 0$

$\therefore$ Discarded.

Case II: $\lambda_1 = 0., \quad 56x_1' + x_2' - 6 = 0.$

$$2x_1' + x_2' - 2 - 56\lambda_2 = 0.$$

$$2x_1' + x_2' + 112 - \lambda_2 = 0$$

Solving we get

$x_1' = \frac{2972}{1595}$ & $x_2' = \frac{-176002}{1595}$ $\quad \lambda_2 = -2.07$

Now, $x_1'^2 + x_2'^2 > 0$

$\therefore$ discarded.

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case III: $\lambda_2 = 0$, $x_1'^2 + x_2'^2 - 9 = 0$.

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$$(2+2\lambda_1)x_1' + x_2' - 2 = 0.$$

$$2x_1' + (2+2\lambda_1)x_2' + 112 = 0.$$

$$2x_1'\lambda_1 + 2x_1' + x_2' - 2 = 0.$$

$$2x_1' + 2\lambda_1 x_2' + 2x_2' + 112 = 0.$$

$$x_1'^2 + x_2'^2 - 9 = 0.$$

On solving this we get complex roots.

case IV: $x_1'^2 + x_2'^2 - 9 = 0.$

$$-56x_1' - x_2' + 6 = 0.$$

$$2x_1' + 2\lambda_1 x_1' + x_2' - 2 - 56\lambda_2 = 0$$

$$2x_1' + 2x_2' + 2\lambda_1 x_2' + 112 - \lambda_2 = 0$$

Solving we get

$x_1' = 0.16.$	$x_1' = 0.05$
$x_2' = -2.99.$	$x_2' = 2.99$
$\lambda_1 = -30.1$	$\lambda_1 = -100.34.$
$\lambda_2 = -0.25$	$\lambda_2 = -0.17$

$\because \lambda_i < 0$ $\therefore$ These are also reject.

Thus we get no such points.

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VDA

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Q2. $x' = (2,0)$

Putting $(2,0)$ in the constraints,

$x_1^2 + x_2^2 - 9 \le 0$

$-56x_1 - x_2 + 6 \le 0$

$4 - 9 \le 0$

$-56 \times 2 + 6 \le 0$

~~$\therefore$ Both the constraints are active.~~

~~Both constraints are satisfied.~~

~~Say $(2,0)$ is a KKT point.~~

None of the constraints are active as none of them are 0.

3. $\nabla f = [2(x_1 - 1) + x_2 \quad 2(x_2 + 56) + x_1]$.

H is +ve definite as shown before.

$\nabla f = 0$.

$2x_1 - 2 + x_2 = 0$

$x_2 + 56 + 2x_2 = 0$.

$2x_1 + x_2 = 2$

$x_1 + 2x_2 = -56$

$4x_1 + 2x_2 = 4$

$x_1 + 2x_2 = -56$

$3x_1 = 60$

$x_1 = 20$

$x_2 = -38$

$(x_1, x_2) = (20, -38)$ is a global minima

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Q4.

Factory.	W1	W2	W3	W4	Factory cap.
F1	20	56.	34	19	90
F2	1	19	57	20	1500.
F3	33	21	56.	1	1368.
Warehouse Requirement	20	1120	33	2850.	

Find initial basic feasible solution using Least Cost Method. For a cell the convention used is $\boxed{x \,|\, y}$

y = cost
x = allocation.

The problem is an unbalanced transportation problem where demand > supply.

Here ~~deman~~ total demand = 4023.

total supply = 2958

Excess demand = 1065.

∴ We add a dummy row F_d with transportation cost 0.

cell is $\boxed{x \,|\, y}$

x is allocation
y is cost

	W1	W2	W3	W4	supply.
F1	20	56	34	19	90
F2	1	19.	57	20	1500.
F3	33	21	56	1	1368
F_d	0	1065 | 0	0	0	1065
Demand	20	55 ~~1120~~	33	2850	4023.

Smallest transportation cost is 0. We can select any one cell say $F_d W_2$. We allocate. min (1065, 1120) and F_d row is removed.

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Step-2 Roll: 001610501020

	W1	W2	W3	W4	Supply
F1	20	56	34	19	90
F2	$\boxed{20}$ 1	19	57	20	1~~500~~ 1480
F3	33	21	56	$\boxed{1368}$ 1	1368
Fd	0	$\boxed{1065}$ 0	0	0	0
Demand	20	55	33	2850	

min cost is 1. select F3W4. Allocate ~~min (1500, 20) = 20~~. min (1368, 2850) = 1368

Step-3

	W1	W2	W3	W4	Supply
F1	20	56	34	19	90.
F2	$\boxed{20}$ 1	19	57	20	1500
~~F3~~	~~33.~~	~~21~~	~~56~~	$\boxed{1368}$ 1	0
~~Fd~~	~~0~~	$\boxed{1065}$ ~~0~~	~~0~~	~~0~~	~~0~~
Demand	20	55	33	1482	

~~Allocate~~ min. cost = 1. Select F2W1.
Allocate min (20, 1500) = 20.

Step-4

	W1	W2	W3	W4	Supply
F1	20	56	34	$\boxed{90}$ 19	90
F2	$\boxed{20}$ 1	19	57	20	1480
~~F3~~	~~33~~	~~21~~	~~56~~	$\boxed{1368}$ 1	0
~~Fd~~	~~0~~	$\boxed{1065}$ 0	0	0	0
Demand	~~20~~ 0	55	33	1482.	

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Step-4

min cost = 19. Select F1W4.

allocate min (90, 1482) = 90.

Step-5

	W1	W2	W3	W4	Supply
F1	20	56	34	$\boxed{90}$ 19	0
F2	$\boxed{20}$	$\boxed{55}$ 19	57	20	1480
F3	33	21	56	$\boxed{1368}$ 1	0.
Fd	0	$\boxed{1065}$ 0	0	0	0
Demand	0	55	33	1392	

min. cost = 19. Select F2W2. Allocate min (55, 1480) = 55.

Step-6

	W1	W2	W3	W4	Supply
F1	20	56	34	$\boxed{90}$ 19	0
F2	$\boxed{20}$ 1	$\boxed{55}$ 19	57	$\boxed{1392}$ 20	1425
F3	33	21	56	$\boxed{1368}$ 1	0
Fd	0	$\boxed{1065}$ 0	0	0	0
Demand	0.	0	33	1392	

min. cost = 20. Select F2W4.

Allocate min. (1425, 1392) = 1392.

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Step - 7

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	0
F2	[20] 1	[55] 19	57	[1130] 20	33
F3	33	21	56	[1368] 1	0
Fd	0	[1065] 0	0	0	0
Demand	0	0	33	0	

min. cost = 57. select F2W2. Allocate 33.

Step - 8

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	0
F2	[20] 1	[55] 19	[33] 57	[1130] 20	0
F3	33	21	56	[1368] 1	0
Fd	0	[1065] 0	0	0	0
Demand	0	0	0	0	0

∴ Initial feasible solution

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	90
F2	[20] 1	[55] 19	[33] 57	[1130] 20	1500
F3	33	21	56	[1368] 1	1368
Fd	0	[1065] 0	0	0	1065
Demand	20	1120	33	2850	

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Initial ~~feasible~~ cost $= 90\times19 + 20\times1 + 55\times19 + 33\times57 + 1392\times20 + 1368\times1 + 1065\times0$

$= 33864$.

number of allocated cells $= 7$.

Number of rows $= 4 = n$

Number of columns $= 4 = m$.

$\because$ No. of allocated cells $= n+m-1$

$\therefore$ Solution is non-degenerate.

(a) Solve using MODI method.

Assign u_i for rows and v_j for columns.

$c_{ij} = u_i + v_j$

	v_1	v_2	v_3	v_4		
	W1	W2	W3	W4	Supply	
F1	20	56	34	$\boxed{90}$ 19	90	u_1
F2	$\boxed{20}$ 1	$\boxed{55}$ 19	$\boxed{33}$ 57	$\boxed{1392}$ 20	1500	u_2
F3	33	21	56	$\boxed{1368}$ 1	1368	u_3
Fd.	0	$\boxed{1065}$ 0	0	0	1065	u_4
Demand	20	1120	33	2850		

$c_{14} = u_1 + v_4 = 19$

$c_{21} = u_2 + v_1 = 1$

$c_{22} = u_2 + v_2 = 19$

$c_{23} = u_2 + v_3 = 57$

$c_{24} = u_2 + v_4 = 20$.

$c_{34} = u_3 + v_4 = 1$

$c_{42} = u_4 + v_2 = 0$.

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putting $u_2 = 0$. we get

$v_1 = 1$ $\qquad u_1 = c_{14} - v_4 = 19 - 20 = -1$

$v_2 = 19$ $\qquad u_2 = 0$

$v_3 = 57$ $\qquad u_3 = c_{34} - v_4 = 1 - 20 = -19$

$v_4 = 20$ $\qquad u_4 = c_{42} - v_2 = 0 - 19 = -19$

$$\boxed{\begin{array}{ll} v_1 = 1 & u_1 = -1 \\ v_2 = 19 & u_2 = 0 \\ v_3 = 57 & u_3 = -19 \\ v_4 = 20 & u_4 = -19 \end{array}}$$

find d_{ij}

$d_{11} = c_{11} - (u_1 + v_1) = 20$

$d_{12} = c_{12} - (u_1 + v_2) = 56 - (-1 + 19) = 38$

$d_{13} = c_{13} - (u_1 + v_3) = 34 - (-1 + 57) = -22$

~~$d_{31} = c_{31} - (u_3 + v_1) = 33 - (-1 + 57) =$~~

$d_{31} = c_{31} - (u_3 + v_1) = 33 - (-19 + 1) = 51$

$d_{32} = c_{32} - (u_3 + v_2) = 21 - (-19 + 19) = 21$

$d_{33} = c_{33} - (u_3 + v_3) = 56 - (-19 + 57) = 18$

$d_{41} = c_{41} - (u_4 + v_1) = 0 - (-19 + 1) = 18$

$d_{43} = c_{43} - (u_4 + v_3) = 0 - (-19 + 57) = -38$

$d_{44} = c_{44} - (u_4 + v_4) = 0 - (-19 + 20) = -1$

As all d_{ij} are not +ve $\therefore$ not optimal

choose d_{43} as it is most −ve.

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u_i		W_1	W_2	W_3	W_4	Supply
$u_1=-1$	F1	20	56	34	$\boxed{90}$ 19	90
$u_2=0$	F2	$\boxed{20}$ 1	$\boxed{55}$ (+) 19	$\boxed{33}$ (−) 57	$\boxed{[illegible]}$ 20	1500
$u_3=-19$	F3	33	21	56	$\boxed{1368}$ 1	1368
$u_4=-19$	Fd	0	$\boxed{1065}$ (−) 0	(+) 0	0	1065
v_j		$v_1=1$	$v_2=19$	$v_3=57$	$v_4=20$	
Demand		20	1120	33	2850	

minimum allocated value = 33. Subtract 33 if (−) else add 33.

	W1	W2	W3	W4	Supply
F1	20	56	34	$\boxed{90}$ 19	90
F2	$\boxed{20}$ 1	$\boxed{88}$ 19	57.	$\boxed{1392}$ 20.	1500
F3	33	21	56	$\boxed{1368}$ 1	1368
Fd	0.	$\boxed{1032}$ 0	$\boxed{33}$ 0	0	1065
Demand	20	1120	33	2850	

Again calculate u_i, v_j..

$c_{14} = u_1 + v_4 = 19$

$c_{21} = u_2 + v_1 = 1$

$c_{22} = u_2 + v_2 = 19$

$c_{24} = u_2 + v_4 = 20$

$c_{34} = u_3 + v_4 = 1$

$c_{42} = u_4 + v_2 = 0.$

$c_{43} = u_4 + v_3 = 0.$

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put $u_2 = 0$.

$v_1 = 1$ $\qquad$ $u_4 = -v_2 = -19$

$v_2 = 19$ $\qquad$ $u_3 = 1 - v_4 = -19$

$v_4 = 20$ $\qquad$ $u_1 = 19 - v_4 = -1$.

$v_3 = -u_4 = 19$ $\qquad$ $u_2 = 1 - v_1 = 0$.

$$\boxed{\begin{array}{ll} u_1 = -1 & v_1 = 1 \\ u_2 = 0 & v_2 = 19 \\ u_3 = -19 & v_3 = 19 \\ u_4 = -19 & v_4 = 20. \end{array}}$$

$d_{11} = c_{11} - (u_1 + v_1) = 20 - (-1+1) = 20.$

$d_{12} = c_{12} - (u_1 + v_2) = 56 - (-1+19) = 38.$

$d_{13} = c_{13} - (u_1 + v_3) = 34 - (-1+19) = 16.$

$d_{23} = c_{23} - (u_2 + v_3) = 57 - (0+19) = 38$

$d_{31} = c_{31} - (u_3 + v_1) = 33 - (-19+1) = 51$

$d_{32} = c_{32} - (u_3 + v_2) = 21 - (-19+19) = 21.$

$d_{33} = c_{33} - (u_3 + v_3) = 56 - (-19+19) = 56.$

$d_{41} = c_{41} - (u_4 + v_1) = 0 - (-19+1) = 18$

$d_{44} = c_{44} - (u_4 + v_4) = 0 - (-19+20) = -1.$

$d_{44} < 0$. So solution is not optimal.

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	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	90
F2	[20] 1	[88] (+) 19	57	[1392] (-) 20	1500
F3	33	21	56	[1368] 1	1368
Fd	0	[1032] (-) 0	0	(+) 0	1065
Demand	20	1120	33	2850	

min allocated value among all -ve position
= 1032.

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	90
F2	[20] 1	[1120] 19	57	[360] 20	1500
F3	33	21	56	[1368] 1	1368
Fd.	0.	0	[33] 0.	[1032] 0	1065
Demand	20	1120	33	2850.	

$c_{14} = u_1 + v_4 = 19$

$c_{21} = u_2 + v_1 = 1.$

$c_{22} = u_2 + v_2 = 19.$

$c_{24} = u_2 + v_3 = 20$

$c_{34} = u_3 + v_4 = 1.$

$c_{43} = u_4 + v_3 = 0$

$c_{44} = u_4 + v_4 = 0.$

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Roll : 001610501020

Put $v_4 = 0$.

$u_1 = 19$ $v_1 = 1 - u_2 = -19$

$u_2 = 20$ $v_2 = 19 - u_2 = -1$

$u_3 = 1$ $v_3 = -u_4 = 0$

$u_4 = 0$ $v_4 = 1 - u_3 = 0$.

Find d_{ij}.

$d_{11} = c_{11} - (u_1 + v_1) = 20 - (19 - 19) = 20$.

$d_{12} = c_{12} - (u_1 + v_2) = 56 - (19 - 1) = 38$

$d_{13} = c_{13} - (u_1 + v_3) = 34 - (19 + 0) = 15$

$d_{23} = c_{23} - (u_2 + v_3) = 57 - (20 + 0) = 37$

$d_{31} = c_{31} - (u_3 + v_1) = 33 - (1 - 19) = 51$

$d_{32} = c_{32} - (u_3 + v_2) = 21 - (1 - 1) = 21$

$d_{33} = c_{33} - (u_3 + v_3) = 56 - (1 + 0) = 55$

$d_{41} = c_{41} - (u_4 + v_1) = 0 - (0 - 19) = 19$

$d_{42} = c_{42} - (u_4 + v_2) = 0 - (0 - 1) = 1$.

$\because$ all $d_{ij} \geq 0$ $\therefore$ The solution obtained is optimal.

min total cost $= 90 \times 19 + 20 \times 1 + 55 \times 19 + 33 \times 57 + 1392 \times 20 + 1368 \times 1 + 1065 \times 0$

$= 31578$.

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b) Solved using stepping stone method.

Initial feasible solution :

Only lowest -ve cost path is shown in diagram.

	W1	W2	W3	W4	Supply
F1	20	56.	34	[90] 19	90
F2	[20] 1	[55] 19 (+)	[33] (-) 57	[1392] 20.	1500
F3	33	21	56	[1368] 1	1368
Fd.	0	[1065] 0 (-)	(+) 0	0	1065
Demand	20	1120	33	2850.	

Now for every unoccupied cell we have to construct closed path &

Unoccupied cell	Path	cost
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20−19+20−1=20
F1W2	F1W2 → F1W4 → F2W4 → F2W2	56−19+20−19=38
F1W3	F1W3 → F1W4 → F2W4 → F2W3	34−19+20−57=−22
F3W1	F3W1 → F3W4 → F2W4 → F2W1	33−1+20−1=51
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21−1+20−19=21
F3W3	F3W3 → F3W4 → F2W4 → F2W3	56−1+20−57 =18
FdW1	FdW1 → FdW2 → F2W2 → F2W1	0−0+19−1=18
FdW3	FdW3 → FdW2 → F2W2 → F2W3	0−0+19−57=−38
FdW4	FdW4 → FdW2 → F2W2 → F2W4	0−0+19−20=−1

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Lowest -ve value is -38

minimum allocated value on -ve path is 33.

New allocation.

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	90.
F2	[20] 1	[88] (+) 19	57	[1392] (-) 20	1500
F3	33	21	56	[1368] 1	1368
Fd	0	[1032] (-) 0	[33] 0	(+) 0	1065
Demand	20	1120	33	2850	

Again find closed paths.

Unoccupied cell	Path.	cost.
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20-19+20-1 = 20
F1W2	F1W2 → F1W4 → F2W4 → F2W2	56-19+20-19 = 38
F1W3	F1W3 → F1W4 → F2W4 → F2W2 → FdW2 → FdW3 → F1W3	34-19+20-19+0-0 = 16
F2W3	F2W3 → F2W2 → FdW2 → FdW3	57-19+0-0 = 38
F3W1.	F3W1 → F3W4 → F2W4 → F2W1	33-1+20-1 = 51
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21-1+20-19 = 21.
F3W3	F3W3. → F3W4. → F2W4 → F2W2 → FdW2 → FdW3 → F3W3	56-1+20-19+0-0 = 56.
FdW1	FdW1 → FdW2 → F2W2 → F2W1	0-0+19-1 = 18
FdW4	FdW4 → FdW2 → F2W2 → F2W4	0-0+19-20 = -

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Lowest -ve value is -1 ∴ Not optimal.

min. allocated value of -ve path is 1032.

New allocation.

	W1	W2	W3	W4	Supply
F1	20	56	34	[90] 19	90.
F2	[20] 1	[1120] 19	57	[360] 20	1500
F3	33	21	56	[1368] 1	1368
Fd	0	0	[33] 0	[1032] 0	1065
Demand	20	1120	33	2850	

Again find closed paths.

Unoccupied cell.	Path	Cost.
F1W1	F1W1 → F1W4 → F2W4 → F2W1	20−19+20−1 = 20
~~F2~~ F1W2	F1W2 → F1W4 → F2W4 → F2W2	56−19+20−19 = 38.
F1W3	F1W3 → F1W4 → ~~F2~~ FdW4 → FdW3	34−19+0−0 = 15
F2W3	F2W3 → F2W4 → FdW4 → FdW3	57−20+0−0 = 37
F3W1	F3W1 → F3W4 → F2W4 → F2W1	33−1+20−1 = 51.
F3W2	F3W2 → F3W4 → F2W4 → F2W2	21−1+20−19 = 21.
F3W3.	F3W3 → F3W4 → FdW4 → FdW3	56−1+0−0 = 55
FdW1	FdW1 → FdW4 → F2W4 → F2W1	0−0+20−1 = 19.
FdW2	FdW2 → FdW4 → F2W4 → F2W2	0−0+20−19 = 1.

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Since there is no gain ∴ the allocation is optimal.

Final allocation

	W1	W2	W3	W4	Supply
F1	20	56	34	$\boxed{90}$ 19	90.
F2	$\boxed{20}$ 1	$\boxed{1120}$ 19	57.	$\boxed{360}$ 20	1500.
F3	33	21	56	$\boxed{1368}$ 1	1368
F_d	0	0	$\boxed{33}$ 0	$\boxed{1032}$ 0	1065
Demand	20	1120	33	2850	

Total. min. transportation cost =

$$90\times19 + 20\times1 + 1120\times19 + 360\times20$$
$$+1368\times1 + 33\times0 + 1032\times0$$
$$= 31578.$$

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Q5(a) Optimization is the most essential ingredients component in machine learning algorithms. In most machine learning algorithms some kind of loss is defined and then this loss function is optimized by adjusting the weights. This loss function is optimized using some optimization routine. The choice optimization algorithm can make a difference between getting a good accuracy in hours or days.

Examples:

1. Logistic Regression:

Here the loss function is optimised.

Output = 0 or 1.

Hypothesis $Z = WX + B$.

$h_\theta(x) = \text{sigmoid}(Z)$.

Now, we define the cost function.

$$\text{cost}(h_\theta(x), Y) = -\log(h_\theta(x)) \text{ if } y = 1$$
$$= -\log(1 - h_\theta(x)) \text{ if } y = 0.$$

This may be written as

$$C = \text{cost}(h_\theta(x), y) = -y \log(h_\theta(x)) - (1-y)\log(1 - h_\theta(x))$$

$\therefore$ The formulation is

$$\underset{\theta}{\text{minimize}}\left(\text{cost}(h_\theta(x), y)\right)$$

This may be done using gradient descent.

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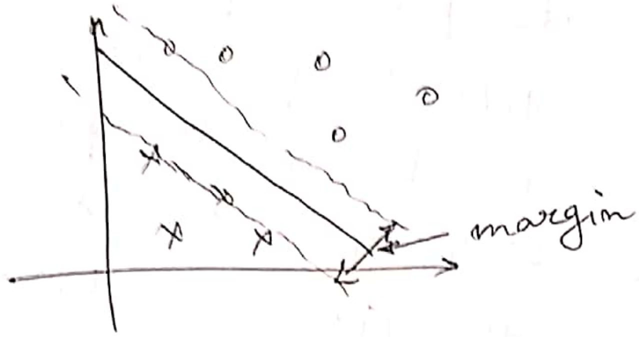
2. SVM.

In SVM we optimize the function so as to maximize the margin between the data points and the hyperplane. This loss function is known as hinge loss.



for $y_i = +1$, $\quad wx_i + b > 0$.

$y_i = -1$, $\quad wx_i + b < 0$.

Scaling we get

$y_i = 1 \quad wx_i + b > 1 \quad$ — ①

$y_i = -1 \quad wx_i + b < -1 \quad$ — ②

max. margin width is

$$M = (x^+ - x^-) \cdot n = (x^+ - x^-) \frac{w}{\|w\|} = \frac{2}{\|w\|}$$

$x^+ - x^- = 2$ from ① and ②

maximize $\frac{2}{\|w\|}$ such that

$wx_i + b > +1 \quad y_i = +1$

$wx_i + b < -1 \quad y_i = -1$

$$\boxed{\text{minimize } \frac{1}{2}\|w\|^2 \text{ such that } y_i(wx + b) \geq 1}$$

This can optimized using Quadratic Programming

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Q5 (b) when we solve a non-linear problem it is very important to know if the problem is convex or not. The convexity characteristics decide which solution method is suitable to use and what solution quality we can expect when applying this method. If the problem is convex then each local optimum will also be a global optimum. Since most methods are search methods that only guarantee to find a local optimum, the convexity properties decide if we with certainty or not can announce that the solution we have found is the global optimum.

if $f(x)$ is twice differentiable. then,

- $f(x)$ is an convex function if hessian matrix H is positive semi-definite for all $x \in X$.
- $f(x)$ is concave if H is $-$ve semi-definity $\forall x \in X$.

~~Say, for example $f(x)$ = $-x_1^2$~~

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$f(x_1, x_2) = -x_1^2 - 4x_2^2 + x_1x_2$ 001610501020

$\nabla = \begin{bmatrix} -2x_1 + x_2 & -8x_2 + x_1 \end{bmatrix}$

$H = \begin{bmatrix} -2 & 1 \\ 1 & -8 \end{bmatrix}$

Now find eigen value.

$|H - \lambda I| = 0$

or, $(-2-\lambda)(-8-\lambda) - 1 = 0.$

$\lambda^2 + 10\lambda + 15 = 0.$

$\therefore \lambda_1 = -5 + \sqrt{10} \quad \lambda_2 = -5 - \sqrt{10}.$

$\because$ All eigen values are < 0 $\therefore f(x_1, x_2)$ is not convex.

say $f(x_1, x_2) = x_1^2 + x_2^2$

$\nabla = \begin{bmatrix} 2x_1 & 2x_2 \end{bmatrix}$

$H = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$|H - \lambda I| = 0$

$\begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = 0.$

$\lambda = 2.$

$\therefore$ +ve definite H.

$\therefore f(x_1, x_2)$ is convex.

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Q5(c) In duality we ~~convert~~ consider each constraint as a variable and the RHS as its coefficient. Basically solving a dual problem is equivalent to solving its primal problem.

In LP models, the parameters are usually not exact. ~~In LP,~~ The parameters may change withing certain limits without changing the optimal solution. This is referred to as sensitivity analysis.

Relaxation is expanding the feasible region by making constraints "less restrictive".

- Removing a constraint
- Increasing RHS of $\leq$ constraint
- Decreasing RHS of $\geq$ constraint.

When we relax a binding constraint, the optimal objective function value will improve or stay the same. For non-~~bin~~ binding it remains same.

Restriction is contracting the feasible region by making constraints more restrictive.

- Adding a new constraint.
- Decreasing RHS $\leq$ constraint
- Increasing RHS $\geq$ constraint.

Objective functions worsens or remains same

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