BACHELOR OF COMP. SC. & ENGINEERING EXAMINATION, 2014

(3^{rd} year, 1st Semester)

COMPUTER GRAPHICS

Time: 3 hours | Full Marks: 100

Answer any FIVE questions.
(Parts of a question must be answered contiguously)

1. a) Develop the 2^{nd} order difference Mid – Point circle rasterization technique. Your starting point should be the equation of a circle; all other details must be developed/ derived and explained precisely. Finally, present the technique as a formal algorithm.

   b) Generate list of pixels necessary to display the complete circle centered at (5, 5) & having a radius of 8 using Bresenham's algorithm (integer version). Your generation procedure should be fully illustrated in tabular form giving numerical values of all parameters of the algorithm in all steps with short explanatory comments, where appropriate. **[(7+3)+10]**

2. a) Develop the Mid – Point ellipse rasterization technique. Your starting point should be the equation of an origin centered ellipse; all other details must be developed/ derived and explained precisely. Finally, present the technique as a formal algorithm.

   b) Generate a piecewise – linear approximation for complete ellipse with semi-major axis a = 4 and semi-minor axis b=1, inclined at 30° (CCW) to the horizontal and centered at (2, 2). The approximation should have 32 unique equispaced points on the ellipse. Explain all numerical details. **[(7+3)+10]**

3. a) A cube with sides of length S is placed such that one of its vertices is at the origin and three mutually perpendicular edges connected to this vertex are coincident with the positive coordinate – axes. Derive the transformation matrix needed to rotate this cube CCW by angle θ about the straight line passing through top right corner of the cube-face lying on the x – y plane and bottom left corner of the cube face parallel to but not coincident with the x – y plane.

   b) An unit cube is placed with three of its adjacent and mutually perpendicular faces on the positive x – y, y – z & z – x planes. This is to be translated by 5 units each, along positive x and y axes. Now, assume that this translated cube is to be finally projected onto the z = 0 plane from center of projection at (0, 0, 10) and do the following:
   i) Find position vectors for all vertices of the transformed cube after performing the translations and the necessary perspective transformation (**not actual projection**).
   ii) From the position vectors obtained in (i), compute actual vanishing point(s) geometrically and comment on the appropriateness or otherwise of the value(s) so obtained. **[10+(4+(5+1))]**

4. a) Develop the Liang – Barsky 2D line clipping technique. Your starting point should be the definition of a regular 2D window; all other details must be developed/ derived and explained precisely. Finally, present the technique as a formal algorithm.

b) The vertices of a convex 2D window are given by A(1, 0), B(2, 0), C(3, 1), D(3, 2), E(2, 3), F(1, 3) G(0, 2) and H(0, 1) in that order. Clip the line $P_1(-1, 1)$ to $P_2(3, 3)$ against this window using Cyrus – Beck algorithm. Full details of clipping should be presented in tabular form with numerical values of all important parameters shown in each step along with short explanatory comments, where appropriate. **[(9+3)+8]**

5. a) Develop the Sutherland – Hodgman polygon clipping technique. You can use any known method for clipping straight lines against convex window edge; all other details must be developed/ derived and explained precisely. Finally present the technique as a formal algorithm.

b) A regular square window, with sides = 120 units, has its lower left corner at (150, 10). Clip the polygon given by A(180, 70), B(220, -10), C(300, -10) and D(300, 430), in that order, against this window using Sutherland Hodgman algorithm. You may use any known method to clip a straight line against regular window clip-edge. Your clipping procedure should be fully illustrated in tabular form giving numerical details of all steps with short explanatory comments, where appropriate. **[(7+3)+10]**

6. a) Develop in details, the constraint(s) needed to join two cubic Bezier curves with C^2 continuity at the join. Explain the physical/ geometrical implications of the constraint(s) clearly.

b) Find the numerical values of t for which Bernstein basis functions $J_{3,1}(t)$ & $J_{3,2}(t)$ are maximized and also find the maximum values.

c) For a Bezier curve with 5 control points, show that

$$\sum_{i=0}^{4} J_{4,i}(t) = 1 \text{ for any } t, 0 \le t \le 1.$$

[(7+3)+5+5]

7. a) Derive in details the technique for generating a piece – wise cubic spline curve passing through n data points. Assume that slopes at the beginning & end of the curve are known and express your final result as a matrix equation in terms of parameter τ.

b) Consider position vectors $P_1[0\ 0]$, $P_2[1\ 1]$, $P_3[2\ -1]$ and $P_4[3\ 0]$ with tangent vectors at P_1 and P_4 both given by [1 1]. Determine the piece – wise cubic spline curve through them using the chord approximation for t_k's. Calculate intermediate points on the curve at τ = 1/3 & 2/3 for the segments. Numerical details for all relevant calculations must be shown. **[12+8]**

$1-4t=2t$ $6t=1$ $t=1/6$

8. a) The endpoints of two parallel lines are given in homogeneous coordinates. Show that after an arbitrary 2D transformation is performed on these two lines using a general 3×3 transformation matrix, the transformed pair of lines remain parallel to each other.

b) Prove by simple but precise arguments that any pair of intersecting straight lines remain intersecting even after undergoing arbitrary 2D transformation. Make no prior assumptions in support of your arguments.

c) Write short notes on any two.
   i) 3D Sutherland & Cohen line clipping.
   ii) Scan – line seed – fill technique.
   iii) Bit – Planes & Colour look – up tables.
   iv) Reconstruction of 3D object from its perspective projection(s).
   v) Splitting of Bezier curves.

[5+3+(6+6)]